Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/35-1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [958]. This is test number [35].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (958)	0.00 (0)
Mathematica	98.33 (942)	1.67 (16)
Maple	75.99 (728)	24.01 (230)
Fricas	68.79 (659)	31.21 (299)
Giac	49.58 (475)	50.42 (483)
Maxima	32.57 (312)	67.43 (646)
Mupad	28.81 (276)	71.19 (682)
Sympy	27.66 (265)	72.34 (693)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

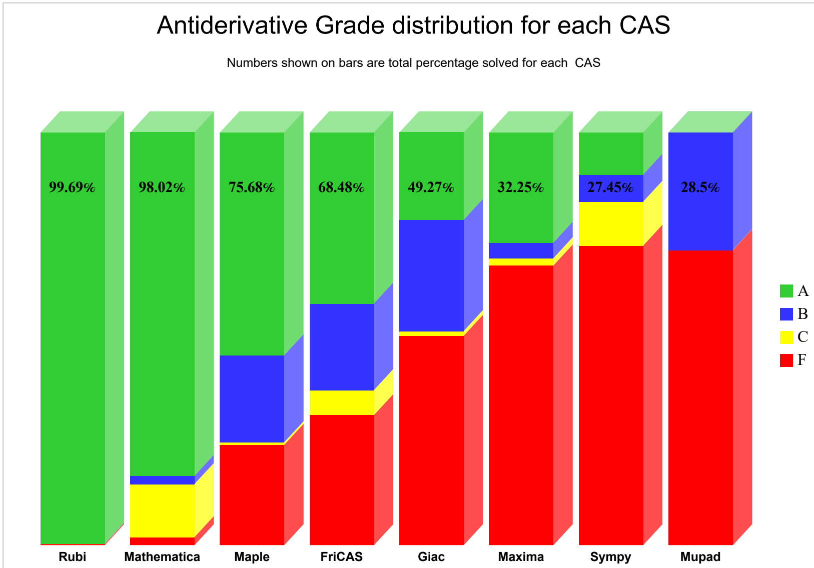
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

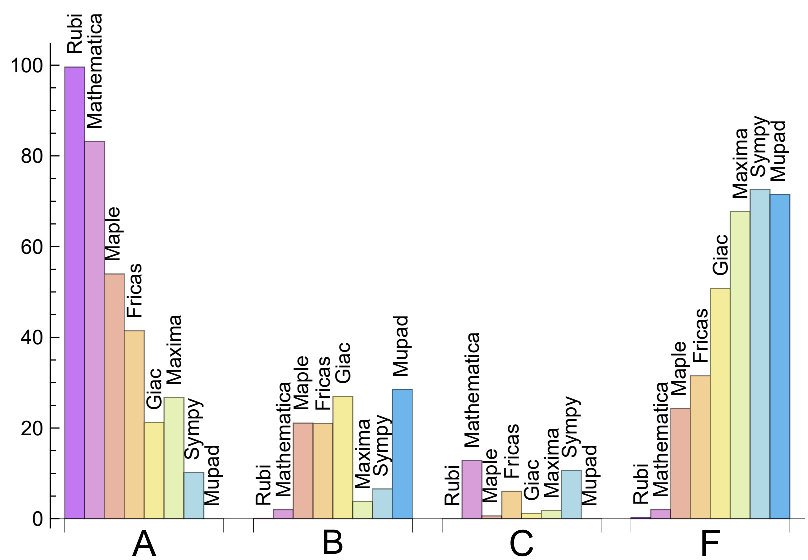
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.582	0.104	0.000	0.313
Mathematica	83.194	1.983	12.839	1.983
Maple	53.967	21.086	0.626	24.322
Fricas	41.441	20.981	6.054	31.524
Maxima	26.722	3.758	1.775	67.745
Giac	21.190	26.931	1.148	50.731
Sympy	10.230	6.576	10.647	72.547
Mupad	0.000	28.497	0.000	71.503

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Maple	230	99.57	0.43	0.00
Fricas	299	77.26	22.74	0.00
Giac	483	83.64	4.35	12.01
Maxima	646	83.90	0.46	15.63
Mupad	682	0.00	100.00	0.00
Sympy	693	72.87	25.25	1.88

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Rubi	0.32
Giac	0.58
Maple	0.75
Mathematica	3.33
Fricas	3.55
Sympy	7.19
Mupad	12.09

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	199.82	1.33	150.00	1.21
Rubi	234.11	1.00	175.00	1.00
Mathematica	526.65	1.25	152.00	0.93
Giac	764.04	3.13	303.00	1.98
Fricas	834.10	3.19	251.00	1.72
Maple	1160.25	3.07	211.00	1.26
Sympy	1676.32	8.08	313.00	2.27
Mupad	2029.94	5.61	161.00	1.28

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

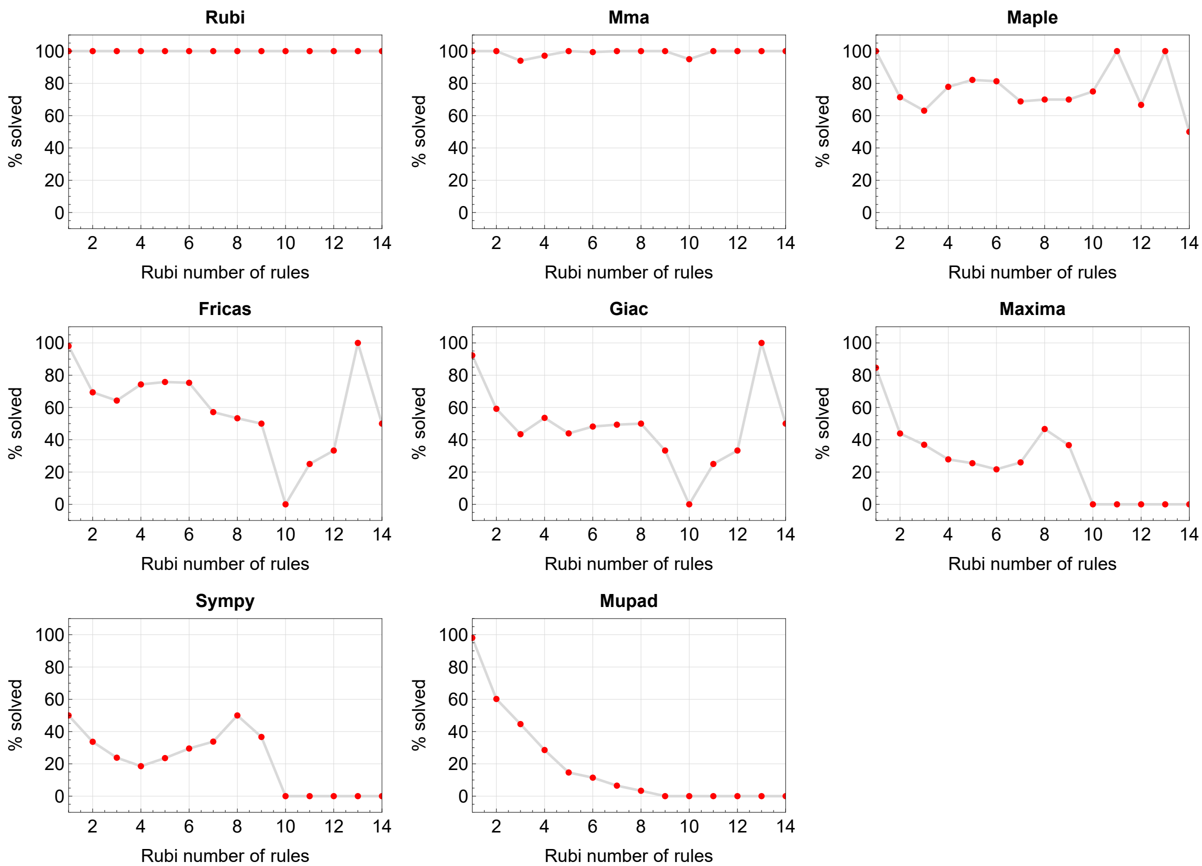


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

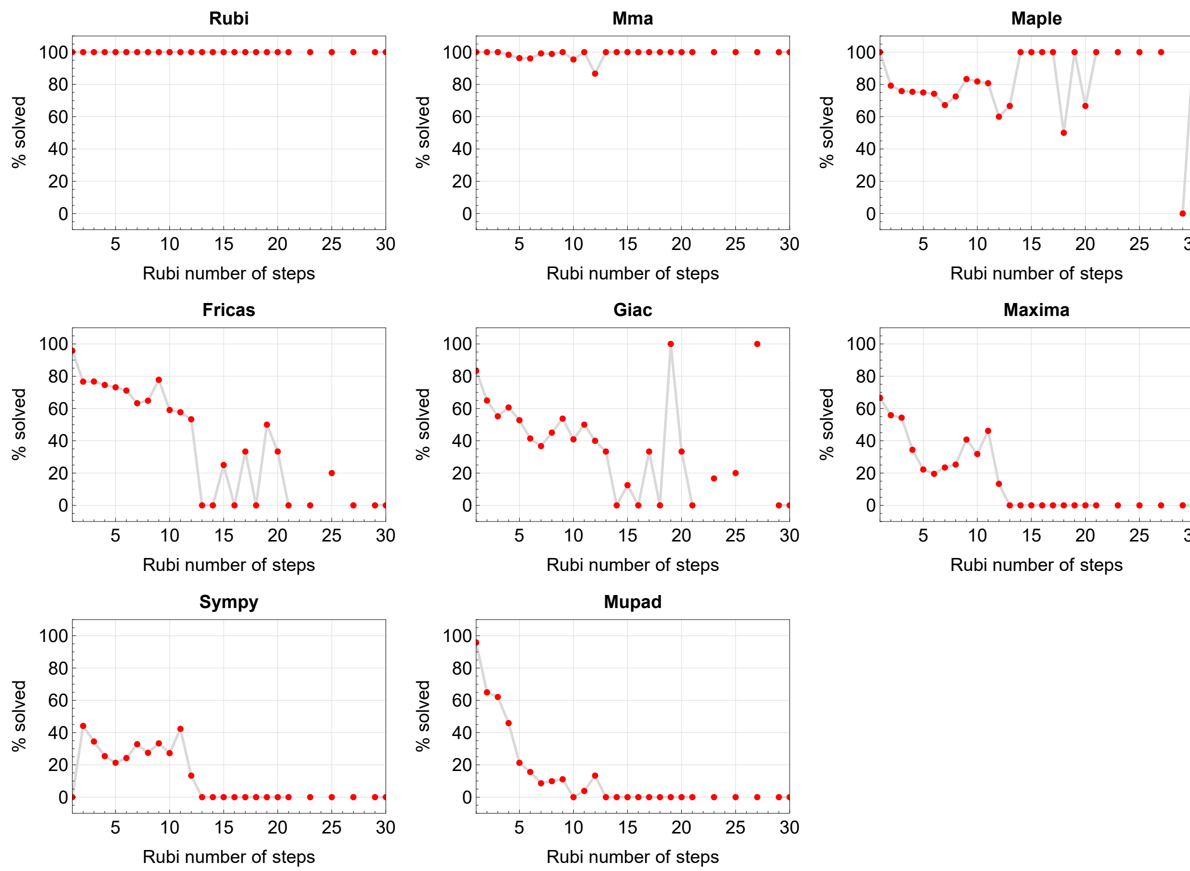


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

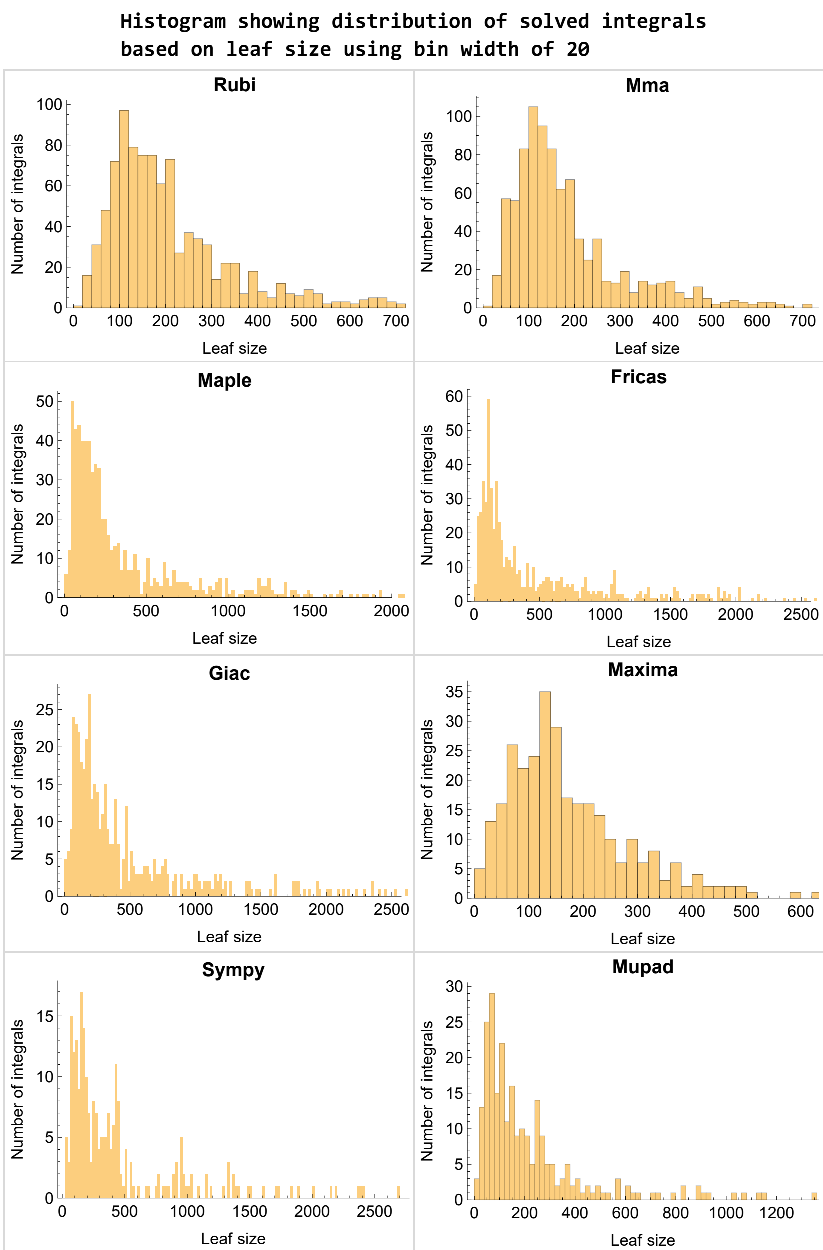


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

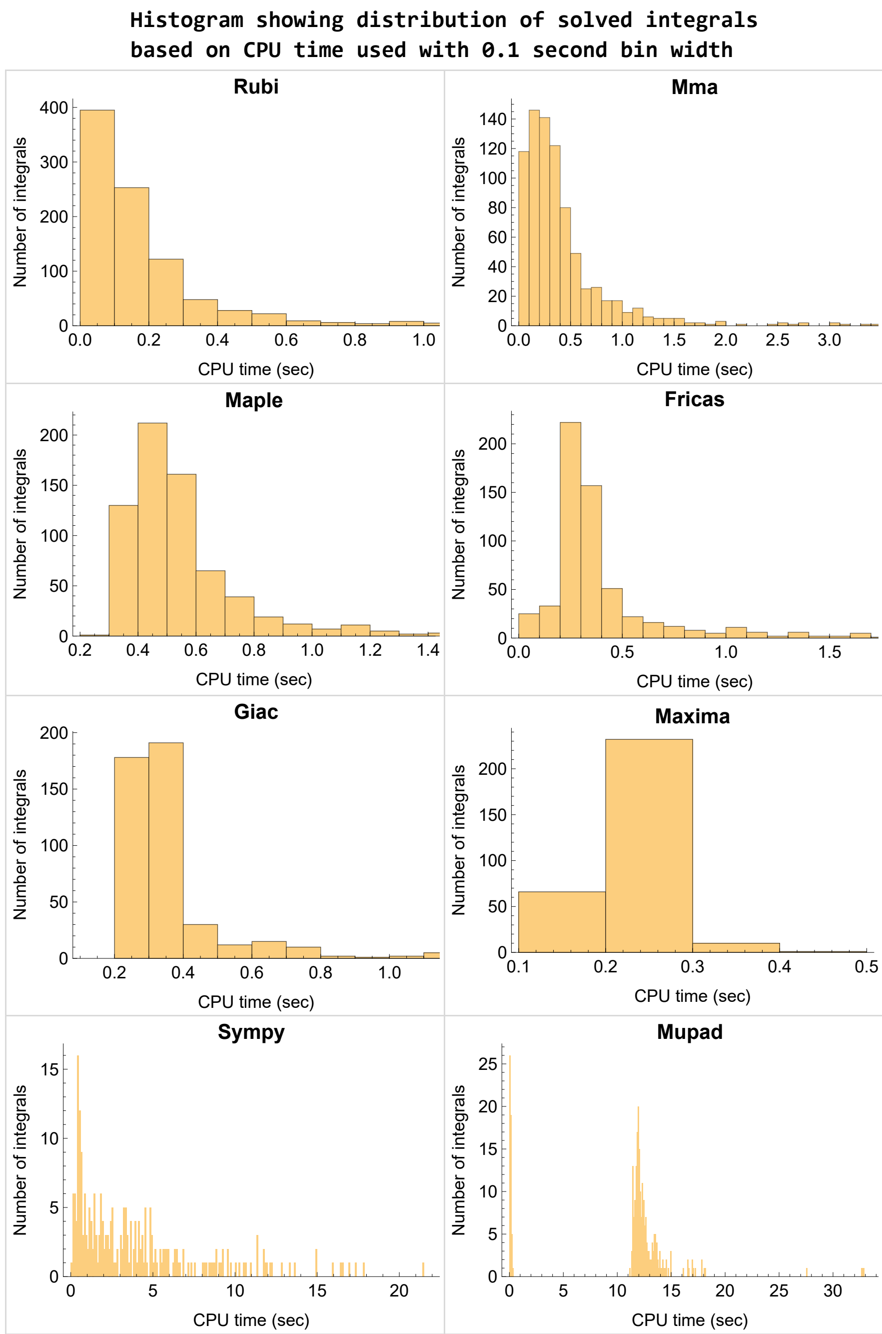


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

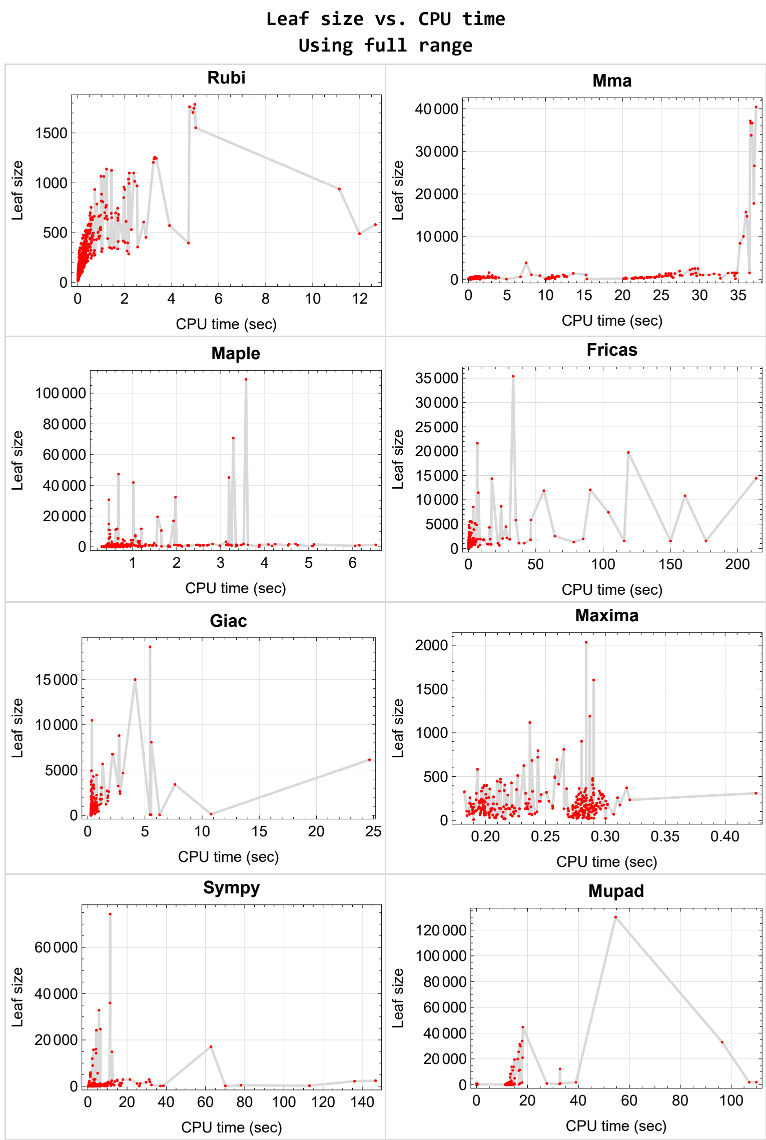


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{948, 952, 957\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{919\}$

Mathematica $\{267, 275, 276, 314, 418, 422, 429, 656, 802, 899, 907, 915\}$

Maple $\{796, 800, 801, 850, 853, 918, 919\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

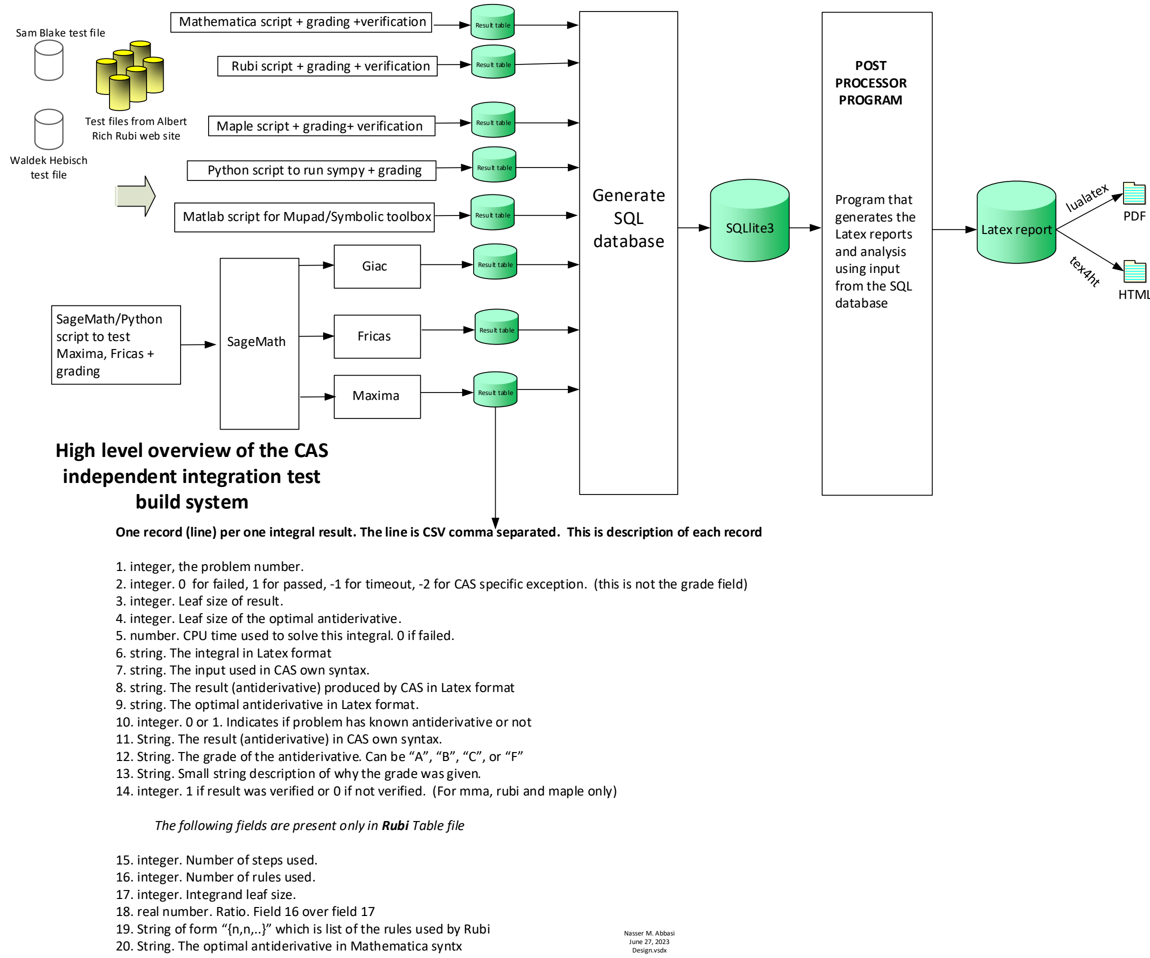
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

B grade { 833 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 269, 270, 271, 272, 273, 278, 279, 280, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 437, 438, 439, 440, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 526, 527, 530, 531, 532, 535, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 619, 620, 621, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 797, 798, 799, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 919, 920, 921, 922, 923, 924, 925, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 947, 951, 953, 956 }

B grade { 263, 268, 274, 277, 284, 285, 294, 295, 301, 304, 305, 360, 441, 442, 656, 802, 907, 918, 926 }

C grade { 267, 275, 276, 315, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 525, 528, 529, 533, 534, 536, 537, 538, 539, 540, 587, 605, 606, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 795, 796, 800, 801, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 958 }

F normal fail { 379, 380, 408, 416, 434, 435, 436, 541, 803, 804, 945, 946, 949, 950, 954, 955 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 177, 178, 179, 180, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 218, 219, 222, 224, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 332, 333, 341, 350, 352, 353, 439, 440, 448, 449, 459, 460, 471, 472, 473, 474, 475, 476, 480, 481, 482, 483, 484, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 619, 626, 627, 628, 629, 630, 634, 635, 636, 640, 641, 642, 643, 645, 648, 649, 650, 651, 653, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 708, 712, 713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 749, 750, 753, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 788, 792, 793, 808, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 840, 841, 848, 849, 854, 855, 856, 857, 862, 863, 864, 865, 866, 867, 869, 870, 871, 872, 873, 875, 876, 889, 890, 891, 892, 893, 894, 897, 898, 899, 900, 901, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 917, 919, 958 }

B grade { 45, 71, 85, 89, 97, 108, 129, 133, 136, 137, 138, 144, 145, 163, 171, 176, 181, 185, 191, 194, 204, 210, 216, 217, 220, 221, 223, 225, 320, 321, 322, 329, 330, 331, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 351, 355, 356, 357, 359, 360, 361, 437, 438, 441, 442, 443, 444, 445, 446, 447, 450, 451, 452, 453, 454, 455, 456, 457, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 477, 478, 479, 485, 486, 488, 579, 580, 581, 585, 586, 587, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 631, 632, 633, 637, 638, 639, 644, 646, 647, 652, 654, 678, 688, 698, 699, 705, 706, 707, 709, 710, 711, 720, 727, 733, 742, 751, 752, 754, 755, 756, 789, 790, 791, 797, 798, 805, 806, 807, 818, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 858, 859, 860, 861, 868, 874, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 895, 896, 902, 903, 911, 916, 918, 920, 921, 925, 926 }

C grade { 794, 795, 796, 799, 800, 801 }

F normal fail { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956 }

F(-1) timedout fail { 499 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 224, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 341, 348, 349, 350, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 548, 549, 550, 551, 552, 557, 558, 559, 560, 562, 569, 570, 572, 573, 583, 584, 589, 590, 591, 592, 593, 596, 597, 598, 599, 603, 604, 620, 621, 656, 657, 658, 659, 660, 661, 665, 666, 667, 668, 672, 673, 674, 679, 680, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 700, 701, 702, 703, 705, 706, 707, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 788, 792, 793, 794, 797, 798, 814, 815, 816, 819, 820, 821, 822, 823, 826, 827, 828, 829, 834, 835, 836, 837, 840, 842, 843, 844, 845, 857, 865 }

B grade { 18, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 131, 132, 138, 139, 140, 141, 142, 143, 144, 148, 149, 216, 221, 223, 225, 330, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 553, 554, 555, 556, 561, 563, 564, 565, 566, 567, 568, 571, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 594, 595, 600, 601, 602, 607, 608, 609, 611, 612, 613, 615, 616, 618, 619, 662, 663, 664, 669, 670, 671, 675, 676, 677, 678, 685, 686, 687, 688, 696, 697, 698, 699, 704, 708, 709, 710, 711, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 789, 790, 791, 795, 796, 799, 800, 805, 806, 807, 808, 824, 825, 830, 831, 832, 833, 838, 839, 841, 846, 847, 848, 849, 851, 852, 873, 874, 875, 880, 881, 882, 920, 921, 925, 926 }

C grade { 315, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 622, 623, 624, 625, 629, 630, 631, 632, 636, 637, 638, 639, 645, 646, 647, 648, 886, 887, 888, 889, 893, 894, 895, 896, 900, 901, 902, 903, 909, 910, 911, 912 }

F normal fail { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 655, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 802, 803, 804, 809, 810, 811, 812, 813, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 605, 606, 610, 614, 617, 626, 627, 628, 633, 634, 635, 640, 641, 642, 643, 644, 649, 650, 651, 652, 653, 654, 801, 817, 818, 850, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 890, 891, 892, 897, 898, 899, 904, 905, 906, 907, 908, 913, 914, 915, 916, 917, 918 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 137, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 202, 203, 220, 224, 321, 330, 337, 338, 351, 352, 353, 355, 356, 357, 490, 497, 504, 511, 518, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 589, 590, 591, 592, 596, 597, 598, 599, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 792, 793, 794, 797, 798, 799, 807, 808, 814, 815, 816, 819, 820, 821, 822, 826, 827, 828, 829 }

B grade { 11, 19, 20, 21, 22, 44, 45, 79, 83, 84, 85, 136, 138, 192, 193, 211, 212, 213, 214, 215, 222, 336, 359, 360, 361, 579, 580, 581, 582, 583, 805, 806, 920, 921, 925, 926 }

C grade { 102, 103, 104, 105, 106, 107, 157, 158, 159, 160, 161, 162, 198, 199, 200, 201, 604 }

F normal fail { 98, 99, 100, 101, 124, 125, 126, 133, 134, 135, 144, 145, 146, 147, 154, 155, 156, 176, 177, 178, 185, 186, 187, 191, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 322, 323, 324, 325, 331, 332, 333, 339, 340, 341, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 442, 443, 444, 445, 451, 452, 453, 454, 455, 456, 462, 463, 464, 465, 466, 467, 468, 469, 474, 475, 476, 483, 484, 485, 486, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 833, 850, 851, 852, 853, 859, 860, 861, 867, 868, 875, 876, 877, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 188, 189, 190 }

F(-2) exception fail { 316, 317, 318, 319, 320, 326, 327, 328, 329, 334, 335, 342, 343, 344, 345, 346, 347, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 457, 458, 459, 460, 461, 470, 471, 472, 473, 477, 478, 479, 480, 481, 482, 487, 488, 585, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 621, 817, 818, 823, 824, 825, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873, 874, 878, 879, 880, 881, 882 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 32, 33, 34, 35, 36, 37, 38, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 108, 109, 117, 119, 120, 121, 122, 123, 124, 151, 152, 153, 154, 157, 158, 159, 161, 162, 179, 180, 181, 182, 183, 184, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 219, 315, 322, 323, 324, 330, 332, 333, 340, 341, 437, 438, 439, 440, 442, 446, 447, 448, 449, 451, 457, 458, 459, 460, 462, 470, 471, 504, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 621, 658, 659, 660, 661, 666, 667, 668, 673, 674, 715, 771, 780, 786, 787, 792, 793, 794, 814, 815, 816, 817, 820, 821, 822, 823, 824, 828, 829, 830, 831, 833, 834, 835, 836, 837, 838, 842, 845, 874 }

B grade { 9, 10, 11, 12, 13, 14, 15, 39, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 98, 99, 100, 101, 110, 111, 112, 113, 114, 115, 116, 118, 125, 126, 156, 160, 185, 186, 187, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 220, 221, 222, 223, 224, 325, 336, 337, 338, 351, 352, 353, 355, 356, 357, 359, 360, 361, 443, 444, 445, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 468, 469, 490, 497, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 579, 580, 581, 582, 583, 585, 587, 596, 618, 619, 620, 657, 662, 663, 664, 665, 669, 670, 671, 672, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 783, 784, 785, 788, 789, 790, 791, 795, 797, 798, 799, 805, 806, 807, 808, 818, 819, 825, 826, 827, 832, 839, 840, 841, 843, 844, 846, 847, 848, 849, 860, 861, 877, 880, 881, 882, 885, 920, 921, 925, 926 }

C grade { 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178 }

F normal fail { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 342, 343, 344, 346, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 541, 542, 543, 544, 545, 546, 547, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 761, 762, 763, 764, 765, 766, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 859, 876, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-1) timedout fail { 586, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 767, 796, 800, 801, 850, 851, 852, 853, 867 }

F(-2) exception fail { 150, 155, 163, 164, 165, 166, 171, 210, 211, 218, 225, 231, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 331, 334, 335, 339, 345, 347, 441, 450, 461, 472, 473, 474, 475, 476, 605, 606, 854, 855, 856, 857, 858, 862, 863, 864, 865, 866, 868, 869, 870, 871, 872, 873, 875, 878, 879, 883 }

Mupad

A grade { }

B grade { 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 117, 118, 123, 130, 131, 132, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 175, 182, 183, 184, 192, 193, 210, 211, 212, 213, 214, 215, 220, 222, 224, 245, 315, 351, 352, 353, 355, 356, 357, 359, 360, 361, 386, 473, 480, 481, 482, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 792, 793, 794, 795, 805, 806, 807, 808, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 837, 840, 841, 844, 845, 848, 849, 920, 921, 925, 926 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 10, 11, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 541, 542, 543, 544, 545, 546, 547, 579, 580, 581, 585, 586, 587, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 788, 789, 790, 791, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 835, 838, 839, 842, 843, 846, 847, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 16, 25, 32, 33, 34, 35, 36, 37, 54, 55, 56, 57, 58, 62, 65, 66, 67, 68, 69, 70, 102, 103, 104, 105, 106, 107, 117, 157, 158, 159, 160, 161, 162, 222, 244, 245, 253, 254, 255, 262, 264, 265, 385, 386, 394, 395, 396, 401, 403, 405, 406, 448, 449, 524, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 566, 569, 570, 571, 572, 573, 576, 577, 578, 588, 590, 591, 592, 593, 596, 597, 598, 599, 600, 820, 821, 822, 823, 826, 827, 828, 829, 830 }

B grade { 19, 21, 23, 240, 241, 242, 243, 249, 250, 251, 252, 258, 259, 260, 261, 263, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 381, 382, 383, 384, 390, 391, 392, 393, 399, 400, 402, 404, 552, 553, 554, 555, 556, 563, 564, 565, 567, 568, 574, 575, 589, 805, 806, 807, 808, 814, 819, 920, 921, 925, 926 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 163, 164, 165, 166, 167, 168, 169, 170, 220, 224, 226, 227, 228, 229, 230, 231, 235, 236, 246, 247, 248, 256, 257, 266, 268, 269, 270, 271, 272, 273, 274, 306, 307, 308, 309, 310, 313, 377, 378, 387, 388, 389, 397, 398, 407, 430, 431, 432, 433 }

F normal fail { 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 225, 233, 234, 237, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 363, 364, 365, 366, 367, 368, 375, 411, 412, 413, 419, 420, 421, 428, 437, 438, 439, 440, 441, 442, 443, 450, 451, 461, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 526, 527, 528, 529, 530, 531, 532, 537, 538, 543, 544, 545, 546, 579, 580, 581, 582, 583, 584, 585, 586, 587, 594, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 679, 680, 681, 682, 683, 684, 685, 689, 690, 691, 692, 693, 694, 704, 713, 714, 715, 716, 717, 722, 723, 724, 734, 735, 736, 737, 738, 743, 744, 745, 763, 764, 783, 784, 785, 786, 787, 788, 789, 790, 795, 796, 797, 798, 800, 809, 811, 812, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 870, 871, 872, 873, 874, 875, 877, 878, 879, 880, 881, 882, 883, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 924, 927, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 945, 946, 947, 949, 950, 951, 958 }

F(-1) timedout fail { 267, 369, 370, 371, 372, 373, 374, 376, 379, 380, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 429, 434, 435, 436, 444, 445, 446, 447, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 487, 488, 525, 533, 534, 535, 536, 539, 540, 541, 547, 595, 601, 602, 604, 635, 651, 665, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 686, 687, 688, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 718, 719, 720, 721, 725, 726, 727, 728, 729, 730, 731, 732, 733, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 768, 769, 770, 771, 773, 775, 776, 777, 778, 779, 780, 781, 782, 791, 792, 793, 794, 799, 801, 802, 803, 804, 815, 816, 817, 818, 824, 825, 831, 832, 861, 867, 868, 869, 876, 884, 885, 908, 954, 955, 956, 957 }

F(-2) exception fail { 232, 238, 239, 542, 767, 772, 774, 810, 813, 923, 928, 944, 953 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	103	97	116	95	146	83	0
N.S.	1	1.00	0.78	0.73	0.88	0.72	1.11	0.63	0.00
time (sec)	N/A	0.049	0.285	0.460	0.280	0.391	0.461	0.297	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	147	141	189	138	202	130	0
N.S.	1	1.00	0.73	0.70	0.94	0.69	1.00	0.65	0.00
time (sec)	N/A	0.097	0.434	0.369	0.277	0.335	0.565	0.313	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	136	130	164	127	189	118	0
N.S.	1	1.00	0.79	0.76	0.95	0.74	1.10	0.69	0.00
time (sec)	N/A	0.067	0.395	0.352	0.283	0.329	0.529	0.303	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	125	119	139	116	173	107	0
N.S.	1	1.00	0.79	0.75	0.87	0.73	1.09	0.67	0.00
time (sec)	N/A	0.065	0.361	0.359	0.287	0.336	0.531	0.292	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	114	108	114	105	158	94	0
N.S.	1	1.00	0.98	0.93	0.98	0.91	1.36	0.81	0.00
time (sec)	N/A	0.023	0.335	0.342	0.280	0.323	0.533	0.294	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	114	108	114	105	158	94	0
N.S.	1	1.00	0.98	0.93	0.98	0.91	1.36	0.81	0.00
time (sec)	N/A	0.023	0.003	0.297	0.290	0.354	0.551	0.307	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	142	157	137	107	400	113	107
N.S.	1	1.00	1.26	1.39	1.21	0.95	3.54	1.00	0.95
time (sec)	N/A	0.060	0.384	0.336	0.287	0.319	5.955	0.308	11.738

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	143	165	143	124	389	178	114
N.S.	1	1.00	1.22	1.41	1.22	1.06	3.32	1.52	0.97
time (sec)	N/A	0.062	0.329	0.364	0.289	0.397	2.579	0.293	12.364

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	148	150	172	133	444	244	120
N.S.	1	1.00	1.22	1.24	1.42	1.10	3.67	2.02	0.99
time (sec)	N/A	0.065	0.369	0.362	0.284	0.336	3.222	0.287	12.526

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	149	137	196	129	457	285	0
N.S.	1	1.00	1.24	1.14	1.63	1.08	3.81	2.38	0.00
time (sec)	N/A	0.062	0.304	0.368	0.293	0.352	3.460	0.282	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	151	125	222	119	541	327	0
N.S.	1	1.00	1.28	1.06	1.88	1.01	4.58	2.77	0.00
time (sec)	N/A	0.065	0.281	0.383	0.283	0.306	4.588	0.297	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	107	155	98	774	388	93
N.S.	1	1.00	1.13	0.99	1.44	0.91	7.17	3.59	0.86
time (sec)	N/A	0.042	0.278	0.394	0.290	0.385	4.208	0.287	13.170

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	142	121	180	109	918	463	118
N.S.	1	1.00	0.99	0.85	1.26	0.76	6.42	3.24	0.83
time (sec)	N/A	0.059	0.293	0.402	0.279	0.313	8.065	0.295	13.786

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	153	132	205	120	1037	518	192
N.S.	1	1.00	0.89	0.77	1.19	0.70	6.03	3.01	1.12
time (sec)	N/A	0.080	0.355	0.419	0.275	0.274	8.683	0.290	14.487

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	164	143	230	131	1159	463	212
N.S.	1	1.00	0.82	0.71	1.14	0.65	5.77	2.30	1.05
time (sec)	N/A	0.103	0.325	0.395	0.284	0.299	25.087	0.294	14.951

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	83	75	93	72	129	64	112
N.S.	1	1.00	0.81	0.73	0.90	0.70	1.25	0.62	1.09
time (sec)	N/A	0.037	0.188	0.358	0.281	0.272	0.431	0.301	11.929

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	99	90	87	134	82	87
N.S.	1	1.00	1.03	1.36	1.23	1.19	1.84	1.12	1.19
time (sec)	N/A	0.028	0.205	0.362	0.281	0.268	3.265	0.287	11.780

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	55	88	104	231	0	55
N.S.	1	1.00	1.02	0.95	1.52	1.79	3.98	0.00	0.95
time (sec)	N/A	0.018	0.217	0.353	0.200	0.265	3.441	0.000	11.493

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	140	251	324	278	2004	0	0
N.S.	1	1.00	0.87	1.56	2.01	1.73	12.45	0.00	0.00
time (sec)	N/A	0.090	0.493	0.437	0.283	0.298	13.692	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	220	290	263	1821	0	0
N.S.	1	1.00	0.86	1.50	1.97	1.79	12.39	0.00	0.00
time (sec)	N/A	0.076	0.436	0.415	0.290	0.286	10.507	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	115	189	262	247	1739	0	0
N.S.	1	1.00	0.94	1.55	2.15	2.02	14.25	0.00	0.00
time (sec)	N/A	0.055	0.409	0.378	0.288	0.302	10.203	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	77	159	171	418	0	78
N.S.	1	1.00	0.98	0.92	1.89	2.04	4.98	0.00	0.93
time (sec)	N/A	0.036	0.277	0.369	0.199	0.338	7.364	0.000	11.854

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	77	134	172	337	0	78
N.S.	1	1.00	0.91	0.86	1.49	1.91	3.74	0.00	0.87
time (sec)	N/A	0.028	0.268	0.360	0.197	0.260	6.831	0.000	11.597

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	77	112	173	513	0	78
N.S.	1	1.00	0.87	0.82	1.19	1.84	5.46	0.00	0.83
time (sec)	N/A	0.032	0.276	0.352	0.190	0.267	7.106	0.000	11.441

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	77	87	172	432	0	78
N.S.	1	1.00	0.99	0.93	1.05	2.07	5.20	0.00	0.94
time (sec)	N/A	0.017	0.271	0.346	0.199	0.279	6.599	0.000	11.531

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	77	80	171	604	0	78
N.S.	1	1.00	1.02	0.96	1.00	2.14	7.55	0.00	0.98
time (sec)	N/A	0.013	0.009	0.339	0.189	0.269	7.511	0.000	11.456

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	129	182	157	244	2378	0	127
N.S.	1	1.00	1.10	1.56	1.34	2.09	20.32	0.00	1.09
time (sec)	N/A	0.066	0.353	0.348	0.188	0.278	14.974	0.000	11.973

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	134	213	189	270	2404	0	141
N.S.	1	1.00	0.88	1.39	1.24	1.76	15.71	0.00	0.92
time (sec)	N/A	0.084	0.407	0.412	0.206	0.287	12.159	0.000	12.182

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	147	244	221	291	2691	0	181
N.S.	1	1.00	0.80	1.33	1.20	1.58	14.62	0.00	0.98
time (sec)	N/A	0.104	0.468	0.388	0.197	0.329	14.955	0.000	12.280

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	99	135	239	903	0	164
N.S.	1	1.00	0.86	0.82	1.12	1.98	7.46	0.00	1.36
time (sec)	N/A	0.034	0.360	0.365	0.194	0.407	8.458	0.000	11.539

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	126	121	158	305	1401	0	202
N.S.	1	1.00	0.85	0.82	1.07	2.06	9.47	0.00	1.36
time (sec)	N/A	0.041	0.467	0.356	0.222	0.500	16.452	0.000	11.617

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	90	63	66	119	70	84
N.S.	1	1.00	1.09	1.67	1.17	1.22	2.20	1.30	1.56
time (sec)	N/A	0.023	0.184	0.374	0.274	0.277	3.399	0.293	0.090

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	114	108	165	105	182	96	0
N.S.	1	1.00	0.66	0.62	0.95	0.61	1.05	0.55	0.00
time (sec)	N/A	0.152	0.325	0.392	0.293	0.289	0.473	0.291	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	103	97	140	94	168	84	0
N.S.	1	1.00	0.72	0.67	0.97	0.65	1.17	0.58	0.00
time (sec)	N/A	0.120	0.275	0.378	0.300	0.265	0.459	0.292	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	92	86	115	83	155	73	0
N.S.	1	1.00	0.80	0.75	1.00	0.72	1.35	0.63	0.00
time (sec)	N/A	0.095	0.246	0.380	0.280	0.295	0.421	0.298	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	73	89	71	134	58	0
N.S.	1	1.00	0.96	0.88	1.07	0.86	1.61	0.70	0.00
time (sec)	N/A	0.057	0.220	0.391	0.283	0.250	0.454	0.281	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	60	62	60	112	45	0
N.S.	1	1.00	0.83	0.72	0.75	0.72	1.35	0.54	0.00
time (sec)	N/A	0.017	0.017	0.360	0.279	0.279	0.471	0.285	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	100	91	75	73	144	78	0
N.S.	1	1.00	1.52	1.38	1.14	1.11	2.18	1.18	0.00
time (sec)	N/A	0.075	0.173	0.349	0.286	0.291	2.289	0.291	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	112	93	78	79	168	126	0
N.S.	1	1.00	1.65	1.37	1.15	1.16	2.47	1.85	0.00
time (sec)	N/A	0.078	0.181	0.385	0.278	0.270	1.564	0.287	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	116	72	83	63	214	188	0
N.S.	1	1.00	1.45	0.90	1.04	0.79	2.68	2.35	0.00
time (sec)	N/A	0.073	0.237	0.371	0.277	0.279	2.416	0.284	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	99	86	108	74	303	255	0
N.S.	1	1.00	0.93	0.80	1.01	0.69	2.83	2.38	0.00
time (sec)	N/A	0.093	0.234	0.355	0.279	0.426	2.165	0.285	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	104	99	133	87	449	329	0
N.S.	1	1.00	0.74	0.71	0.95	0.62	3.21	2.35	0.00
time (sec)	N/A	0.108	0.280	0.395	0.280	0.358	4.328	0.314	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	131	110	158	98	510	385	0
N.S.	1	1.00	0.78	0.65	0.93	0.58	3.02	2.28	0.00
time (sec)	N/A	0.130	0.250	0.405	0.275	0.339	3.964	0.292	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	126	238	288	188	0	0	0
N.S.	1	1.00	0.88	1.66	2.01	1.31	0.00	0.00	0.00
time (sec)	N/A	0.172	0.427	0.418	0.284	0.356	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	324	310	172	0	0	0
N.S.	1	1.00	0.86	2.68	2.56	1.42	0.00	0.00	0.00
time (sec)	N/A	0.137	0.396	0.409	0.426	0.343	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	70	65	155	116	0	0	66
N.S.	1	1.00	0.72	0.67	1.60	1.20	0.00	0.00	0.68
time (sec)	N/A	0.114	0.303	0.382	0.273	0.404	0.000	0.000	11.604

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	70	66	131	117	0	0	67
N.S.	1	1.00	0.80	0.76	1.51	1.34	0.00	0.00	0.77
time (sec)	N/A	0.076	0.334	0.378	0.265	0.337	0.000	0.000	11.400

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	69	64	109	117	0	0	65
N.S.	1	1.00	0.78	0.72	1.22	1.31	0.00	0.00	0.73
time (sec)	N/A	0.022	0.329	0.386	0.269	0.570	0.000	0.000	11.333

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	65	78	116	0	0	66
N.S.	1	1.00	0.91	0.84	1.01	1.51	0.00	0.00	0.86
time (sec)	N/A	0.013	0.012	0.345	0.241	0.267	0.000	0.000	11.494

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	107	202	154	169	0	0	0
N.S.	1	1.00	0.91	1.73	1.32	1.44	0.00	0.00	0.00
time (sec)	N/A	0.102	0.481	0.361	0.238	0.264	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	135	250	187	195	0	0	0
N.S.	1	1.00	0.93	1.72	1.29	1.34	0.00	0.00	0.00
time (sec)	N/A	0.176	0.424	0.382	0.209	0.368	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	136	257	218	216	0	0	0
N.S.	1	1.00	0.75	1.41	1.20	1.19	0.00	0.00	0.00
time (sec)	N/A	0.238	0.473	0.398	0.199	0.267	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	161	270	243	227	0	0	0
N.S.	1	1.00	0.77	1.29	1.16	1.09	0.00	0.00	0.00
time (sec)	N/A	0.314	0.457	0.424	0.196	0.328	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	58	42	70	50	73	34	36
N.S.	1	1.00	0.72	0.52	0.86	0.62	0.90	0.42	0.44
time (sec)	N/A	0.059	0.140	0.367	0.307	0.273	0.210	0.278	11.175

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	37	56	45	60	30	31
N.S.	1	1.00	0.84	0.59	0.89	0.71	0.95	0.48	0.49
time (sec)	N/A	0.052	0.122	0.392	0.277	0.269	0.156	0.286	0.031

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	46	28	40	38	37	21	22
N.S.	1	1.00	1.12	0.68	0.98	0.93	0.90	0.51	0.54
time (sec)	N/A	0.030	0.096	0.355	0.269	0.274	0.111	0.282	0.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	25	28	33	27	19	21
N.S.	1	1.00	1.02	0.62	0.70	0.82	0.68	0.48	0.52
time (sec)	N/A	0.008	0.097	0.342	0.277	0.280	0.082	0.289	0.031

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	29	41	46	31	34	32
N.S.	1	1.00	1.62	0.91	1.28	1.44	0.97	1.06	1.00
time (sec)	N/A	0.037	0.086	0.378	0.271	0.284	2.515	0.277	11.438

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	30	42	53	51	55	35
N.S.	1	1.00	1.73	0.91	1.27	1.61	1.55	1.67	1.06
time (sec)	N/A	0.037	0.086	0.352	0.278	0.262	1.871	0.280	0.082

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	48	39	54	43	116	91	47
N.S.	1	1.00	0.94	0.76	1.06	0.84	2.27	1.78	0.92
time (sec)	N/A	0.041	0.090	0.371	0.277	0.295	3.098	0.294	11.488

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	44	68	48	128	125	67
N.S.	1	1.00	0.70	0.66	1.01	0.72	1.91	1.87	1.00
time (sec)	N/A	0.050	0.087	0.370	0.269	0.269	3.378	0.306	0.034

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	58	49	82	53	223	163	77
N.S.	1	1.00	0.65	0.55	0.92	0.60	2.51	1.83	0.87
time (sec)	N/A	0.056	0.095	0.398	0.276	0.265	5.407	0.288	0.033

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	63	54	96	58	201	199	90
N.S.	1	1.00	0.59	0.50	0.90	0.54	1.88	1.86	0.84
time (sec)	N/A	0.068	0.102	0.373	0.289	0.255	5.687	0.284	0.034

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	127	116	171	111	544	325	0
N.S.	1	1.00	0.95	0.87	1.28	0.83	4.06	2.43	0.00
time (sec)	N/A	0.148	0.431	0.404	0.288	0.279	4.416	0.298	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	202	196	282	194	303	188	0
N.S.	1	1.00	0.65	0.63	0.91	0.63	0.98	0.61	0.00
time (sec)	N/A	0.305	0.784	0.486	0.281	0.280	0.794	0.309	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	191	185	257	183	286	177	0
N.S.	1	1.00	0.68	0.66	0.91	0.65	1.02	0.63	0.00
time (sec)	N/A	0.258	0.678	0.434	0.284	0.272	0.685	0.301	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	180	174	232	172	274	165	0
N.S.	1	1.00	0.71	0.69	0.92	0.68	1.09	0.65	0.00
time (sec)	N/A	0.222	0.656	0.437	0.287	0.312	0.677	0.290	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	169	163	207	161	262	154	0
N.S.	1	1.00	0.76	0.73	0.93	0.72	1.17	0.69	0.00
time (sec)	N/A	0.212	0.622	0.444	0.280	0.291	0.641	0.297	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	158	152	182	150	241	142	0
N.S.	1	1.00	0.69	0.66	0.79	0.65	1.05	0.62	0.00
time (sec)	N/A	0.085	0.582	0.434	0.287	0.372	0.615	0.305	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	155	138	145	139	206	127	0
N.S.	1	1.00	0.82	0.73	0.77	0.74	1.10	0.68	0.00
time (sec)	N/A	0.051	0.399	0.404	0.275	0.259	0.621	0.293	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	186	353	217	151	954	161	0
N.S.	1	1.00	0.98	1.86	1.14	0.79	5.02	0.85	0.00
time (sec)	N/A	0.203	0.597	0.375	0.282	0.276	8.890	0.306	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	193	265	231	167	887	224	0
N.S.	1	1.00	1.00	1.37	1.20	0.87	4.60	1.16	0.00
time (sec)	N/A	0.199	0.590	0.430	0.280	0.276	3.247	0.297	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	203	175	241	179	887	293	0
N.S.	1	1.00	0.98	0.85	1.16	0.86	4.29	1.42	0.00
time (sec)	N/A	0.204	0.609	0.397	0.279	0.278	3.957	0.295	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	189	238	238	179	843	346	0
N.S.	1	1.00	0.90	1.13	1.13	0.85	4.01	1.65	0.00
time (sec)	N/A	0.206	0.577	0.401	0.277	0.272	4.179	0.295	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	198	223	262	180	959	409	0
N.S.	1	1.00	0.95	1.07	1.25	0.86	4.59	1.96	0.00
time (sec)	N/A	0.200	0.616	0.426	0.283	0.284	5.954	0.302	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	195	210	290	180	1182	460	0
N.S.	1	1.00	0.90	0.97	1.34	0.83	5.47	2.13	0.00
time (sec)	N/A	0.206	0.542	0.463	0.298	0.291	6.314	0.297	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	196	196	315	179	1380	526	0
N.S.	1	1.00	0.92	0.92	1.47	0.84	6.45	2.46	0.00
time (sec)	N/A	0.204	0.543	0.454	0.290	0.286	10.646	0.310	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	193	173	338	173	1513	542	0
N.S.	1	1.00	0.94	0.84	1.64	0.84	7.34	2.63	0.00
time (sec)	N/A	0.207	0.504	0.519	0.283	0.269	11.745	0.305	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	198	170	364	163	1719	584	0
N.S.	1	1.00	0.97	0.83	1.78	0.80	8.43	2.86	0.00
time (sec)	N/A	0.203	0.523	0.490	0.284	0.313	29.730	0.311	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	169	151	247	142	1889	648	0
N.S.	1	1.00	0.90	0.81	1.32	0.76	10.10	3.47	0.00
time (sec)	N/A	0.172	0.489	0.584	0.281	0.315	31.652	0.317	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	186	165	272	153	2159	731	0
N.S.	1	1.00	0.83	0.73	1.21	0.68	9.60	3.25	0.00
time (sec)	N/A	0.195	0.524	0.546	0.276	0.331	136.085	0.311	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	189	176	297	164	2397	778	0
N.S.	1	1.00	0.74	0.69	1.17	0.65	9.44	3.06	0.00
time (sec)	N/A	0.215	0.556	0.745	0.278	0.352	146.794	0.317	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	111	207	317	192	0	233	0
N.S.	1	1.00	0.64	1.19	1.82	1.10	0.00	1.34	0.00
time (sec)	N/A	0.246	0.548	0.464	0.279	0.281	0.000	0.303	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	98	195	336	177	0	211	0
N.S.	1	1.00	0.69	1.37	2.37	1.25	0.00	1.49	0.00
time (sec)	N/A	0.201	0.387	0.496	0.295	0.267	0.000	0.301	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	86	377	308	161	0	186	0
N.S.	1	1.00	0.73	3.19	2.61	1.36	0.00	1.58	0.00
time (sec)	N/A	0.144	0.366	0.400	0.296	0.281	0.000	0.300	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	53	50	154	106	0	106	49
N.S.	1	1.00	0.57	0.54	1.66	1.14	0.00	1.14	0.53
time (sec)	N/A	0.086	0.308	0.415	0.195	0.260	0.000	0.311	11.568

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	50	47	128	104	0	137	46
N.S.	1	1.00	0.58	0.55	1.49	1.21	0.00	1.59	0.53
time (sec)	N/A	0.027	0.293	0.395	0.197	0.328	0.000	0.308	11.725

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	53	50	101	106	0	165	49
N.S.	1	1.00	0.51	0.49	0.98	1.03	0.00	1.60	0.48
time (sec)	N/A	0.032	0.011	0.381	0.186	0.278	0.000	0.317	11.685

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	305	152	158	0	212	0
N.S.	1	1.00	0.88	2.68	1.33	1.39	0.00	1.86	0.00
time (sec)	N/A	0.105	0.415	0.380	0.198	0.279	0.000	0.307	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	117	208	184	184	0	304	0
N.S.	1	1.00	0.81	1.43	1.27	1.27	0.00	2.10	0.00
time (sec)	N/A	0.189	0.410	0.431	0.202	0.286	0.000	0.299	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	132	214	216	205	0	381	0
N.S.	1	1.00	0.73	1.18	1.19	1.13	0.00	2.09	0.00
time (sec)	N/A	0.231	0.444	0.408	0.195	0.287	0.000	0.314	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	103	97	125	95	0	88	0
N.S.	1	1.00	0.70	0.66	0.85	0.65	0.00	0.60	0.00
time (sec)	N/A	0.099	0.246	0.406	0.292	0.270	0.000	0.281	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	92	86	101	83	0	76	0
N.S.	1	1.00	0.78	0.73	0.86	0.70	0.00	0.64	0.00
time (sec)	N/A	0.062	0.216	0.388	0.275	0.276	0.000	0.282	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	75	77	73	0	64	0
N.S.	1	1.00	0.97	0.87	0.90	0.85	0.00	0.74	0.00
time (sec)	N/A	0.071	0.167	0.382	0.274	0.265	0.000	0.285	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	64	56	60	0	52	0
N.S.	1	1.00	1.13	1.03	0.90	0.97	0.00	0.84	0.00
time (sec)	N/A	0.024	0.161	0.402	0.275	0.291	0.000	0.289	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	49	31	52	0	36	0
N.S.	1	1.00	1.37	1.07	0.67	1.13	0.00	0.78	0.00
time (sec)	N/A	0.010	0.113	0.368	0.284	0.271	0.000	0.276	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	78	141	56	54	0	60	0
N.S.	1	1.00	1.70	3.07	1.22	1.17	0.00	1.30	0.00
time (sec)	N/A	0.040	0.130	0.376	0.282	0.279	0.000	0.297	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	73	63	0	50	0	115	0
N.S.	1	1.00	1.43	1.24	0.00	0.98	0.00	2.25	0.00
time (sec)	N/A	0.035	0.125	0.392	0.000	0.268	0.000	0.282	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	93	75	0	63	0	193	0
N.S.	1	1.00	1.13	0.91	0.00	0.77	0.00	2.35	0.00
time (sec)	N/A	0.050	0.148	0.395	0.000	0.273	0.000	0.284	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	88	0	75	0	255	0
N.S.	1	1.00	0.96	0.77	0.00	0.66	0.00	2.24	0.00
time (sec)	N/A	0.071	0.164	0.430	0.000	0.273	0.000	0.285	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	120	99	0	86	0	329	0
N.S.	1	1.00	0.84	0.69	0.00	0.60	0.00	2.30	0.00
time (sec)	N/A	0.089	0.180	0.395	0.000	0.263	0.000	0.283	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	97	174	94	167	83	0
N.S.	1	1.00	0.91	0.86	1.54	0.83	1.48	0.73	0.00
time (sec)	N/A	0.085	0.250	0.379	0.286	0.351	1.166	0.290	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	147	141	246	139	427	132	0
N.S.	1	1.00	0.73	0.70	1.22	0.69	2.12	0.66	0.00
time (sec)	N/A	0.105	0.419	0.404	0.294	0.288	1.470	0.305	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	136	130	221	127	400	120	0
N.S.	1	1.00	0.79	0.76	1.28	0.74	2.33	0.70	0.00
time (sec)	N/A	0.081	0.369	0.355	0.300	0.296	1.375	0.311	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	125	119	198	117	371	107	0
N.S.	1	1.00	0.89	0.85	1.41	0.84	2.65	0.76	0.00
time (sec)	N/A	0.098	0.345	0.387	0.289	0.332	1.421	0.312	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	114	108	176	105	343	96	0
N.S.	1	1.00	0.98	0.93	1.52	0.91	2.96	0.83	0.00
time (sec)	N/A	0.037	0.319	0.393	0.312	0.318	1.416	0.285	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	111	94	109	95	313	80	0
N.S.	1	1.00	1.11	0.94	1.09	0.95	3.13	0.80	0.00
time (sec)	N/A	0.020	0.226	0.382	0.291	0.292	1.301	0.300	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	142	297	137	107	400	114	0
N.S.	1	1.00	1.26	2.63	1.21	0.95	3.54	1.01	0.00
time (sec)	N/A	0.076	0.351	0.436	0.280	0.262	5.771	0.314	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	140	164	145	123	389	177	0
N.S.	1	1.00	1.22	1.43	1.26	1.07	3.38	1.54	0.00
time (sec)	N/A	0.077	0.356	0.413	0.288	0.291	3.157	0.304	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	149	150	150	135	444	245	0
N.S.	1	1.00	1.23	1.24	1.24	1.12	3.67	2.02	0.00
time (sec)	N/A	0.079	0.362	0.411	0.283	0.276	3.856	0.295	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	146	136	144	130	457	284	0
N.S.	1	1.00	1.22	1.13	1.20	1.08	3.81	2.37	0.00
time (sec)	N/A	0.081	0.389	0.435	0.284	0.279	4.118	0.288	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	151	126	171	119	541	328	0
N.S.	1	1.00	1.27	1.06	1.44	1.00	4.55	2.76	0.00
time (sec)	N/A	0.076	0.324	0.458	0.273	0.271	5.160	0.296	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	107	153	97	774	388	0
N.S.	1	1.00	1.13	0.99	1.42	0.90	7.17	3.59	0.00
time (sec)	N/A	0.060	0.365	0.465	0.277	0.268	5.043	0.296	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	142	121	178	108	918	463	0
N.S.	1	1.00	0.99	0.85	1.24	0.76	6.42	3.24	0.00
time (sec)	N/A	0.082	0.358	0.542	0.284	0.311	9.032	0.294	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	153	132	203	119	1037	518	0
N.S.	1	1.00	0.89	0.77	1.18	0.69	6.03	3.01	0.00
time (sec)	N/A	0.102	0.415	0.572	0.280	0.277	9.722	0.291	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	164	143	228	130	1159	463	0
N.S.	1	1.00	0.82	0.71	1.13	0.65	5.77	2.30	0.00
time (sec)	N/A	0.125	0.398	0.632	0.287	0.303	26.641	0.300	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	25	28	31	29	19	20
N.S.	1	1.00	1.37	0.93	1.04	1.15	1.07	0.70	0.74
time (sec)	N/A	0.011	0.083	0.353	0.281	0.256	1.112	0.284	0.039

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	73	87	68	74	170	125	74
N.S.	1	1.00	1.43	1.71	1.33	1.45	3.33	2.45	1.45
time (sec)	N/A	0.045	0.134	0.371	0.275	0.271	2.683	0.289	0.050

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	100	120	113	112	0	108	0
N.S.	1	1.00	0.85	1.02	0.96	0.95	0.00	0.92	0.00
time (sec)	N/A	0.068	0.262	0.355	0.283	0.268	0.000	0.308	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	89	108	86	101	0	96	0
N.S.	1	1.00	0.98	1.19	0.95	1.11	0.00	1.05	0.00
time (sec)	N/A	0.040	0.249	0.417	0.292	0.263	0.000	0.287	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	97	63	85	0	83	0
N.S.	1	1.00	0.96	1.26	0.82	1.10	0.00	1.08	0.00
time (sec)	N/A	0.063	0.198	0.386	0.286	0.266	0.000	0.305	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	65	74	40	67	0	61	0
N.S.	1	1.00	1.25	1.42	0.77	1.29	0.00	1.17	0.00
time (sec)	N/A	0.015	0.159	0.397	0.282	0.262	0.000	0.296	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	30	35	0	41	29
N.S.	1	1.00	1.00	0.94	0.97	1.13	0.00	1.32	0.94
time (sec)	N/A	0.007	0.004	0.396	0.273	0.258	0.000	0.280	11.555

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	76	88	0	62	0	87	0
N.S.	1	1.00	1.41	1.63	0.00	1.15	0.00	1.61	0.00
time (sec)	N/A	0.029	0.176	0.398	0.000	0.268	0.000	0.290	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	108	0	88	0	177	0
N.S.	1	1.00	1.09	1.33	0.00	1.09	0.00	2.19	0.00
time (sec)	N/A	0.043	0.216	0.387	0.000	0.412	0.000	0.303	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	117	0	113	0	256	0
N.S.	1	1.00	0.95	1.04	0.00	1.00	0.00	2.27	0.00
time (sec)	N/A	0.060	0.236	0.414	0.000	0.257	0.000	0.292	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	120	202	151	190	0	0	0
N.S.	1	1.00	0.94	1.58	1.18	1.48	0.00	0.00	0.00
time (sec)	N/A	0.071	0.348	0.408	0.290	0.272	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	105	186	124	175	0	0	0
N.S.	1	1.00	0.93	1.65	1.10	1.55	0.00	0.00	0.00
time (sec)	N/A	0.058	0.312	0.408	0.291	0.261	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	204	99	157	0	0	0
N.S.	1	1.00	1.03	2.29	1.11	1.76	0.00	0.00	0.00
time (sec)	N/A	0.042	0.272	0.381	0.287	0.259	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	86	103	0	0	56
N.S.	1	1.00	1.00	0.80	1.43	1.72	0.00	0.00	0.93
time (sec)	N/A	0.022	0.215	0.376	0.203	0.259	0.000	0.000	12.312

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	44	67	101	0	0	52
N.S.	1	1.00	0.97	0.76	1.16	1.74	0.00	0.00	0.90
time (sec)	N/A	0.013	0.192	0.387	0.210	0.284	0.000	0.000	12.273

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	46	65	102	0	0	56
N.S.	1	1.00	1.03	0.79	1.12	1.76	0.00	0.00	0.97
time (sec)	N/A	0.009	0.221	0.379	0.200	0.336	0.000	0.000	11.693

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	171	0	155	0	0	0
N.S.	1	1.00	1.20	1.94	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.053	0.301	0.374	0.000	0.274	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	124	198	0	181	0	0	0
N.S.	1	1.00	1.03	1.65	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.065	0.308	0.400	0.000	0.298	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	138	206	0	201	0	0	0
N.S.	1	1.00	0.91	1.36	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.082	0.335	0.400	0.000	0.287	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	140	295	289	274	0	0	0
N.S.	1	1.00	0.86	1.82	1.78	1.69	0.00	0.00	0.00
time (sec)	N/A	0.106	0.553	0.444	0.297	0.324	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	130	281	259	258	0	0	0
N.S.	1	1.00	0.88	1.90	1.75	1.74	0.00	0.00	0.00
time (sec)	N/A	0.092	0.468	0.427	0.310	0.282	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	115	450	234	241	0	0	0
N.S.	1	1.00	0.94	3.69	1.92	1.98	0.00	0.00	0.00
time (sec)	N/A	0.065	0.417	0.392	0.320	0.337	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	134	168	0	0	78
N.S.	1	1.00	0.96	0.82	1.58	1.98	0.00	0.00	0.92
time (sec)	N/A	0.048	0.312	0.432	0.213	0.275	0.000	0.000	11.743

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	110	171	0	0	78
N.S.	1	1.00	0.90	0.77	1.21	1.88	0.00	0.00	0.86
time (sec)	N/A	0.047	0.280	0.379	0.204	0.279	0.000	0.000	12.042

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	70	110	170	0	0	78
N.S.	1	1.00	0.86	0.74	1.16	1.79	0.00	0.00	0.82
time (sec)	N/A	0.032	0.289	0.374	0.205	0.290	0.000	0.000	11.854

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	90	171	0	0	78
N.S.	1	1.00	0.96	0.82	1.06	2.01	0.00	0.00	0.92
time (sec)	N/A	0.020	0.277	0.372	0.203	0.306	0.000	0.000	11.794

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	70	85	168	0	0	78
N.S.	1	1.00	1.00	0.85	1.04	2.05	0.00	0.00	0.95
time (sec)	N/A	0.015	0.011	0.388	0.196	0.289	0.000	0.000	11.710

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	255	0	237	0	0	0
N.S.	1	1.00	0.99	2.14	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.070	0.468	0.379	0.000	0.303	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	145	291	0	265	0	0	0
N.S.	1	1.00	0.94	1.89	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.094	0.430	0.442	0.000	0.409	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	160	296	0	286	0	0	0
N.S.	1	1.00	0.86	1.59	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.115	0.452	0.415	0.000	0.369	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	171	312	0	297	0	0	0
N.S.	1	1.00	0.80	1.45	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.148	0.489	0.438	0.000	0.361	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	92	133	239	0	0	161
N.S.	1	1.00	0.88	0.78	1.13	2.03	0.00	0.00	1.36
time (sec)	N/A	0.050	0.374	0.378	0.228	0.318	0.000	0.000	11.923

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	92	133	238	0	0	161
N.S.	1	1.00	0.85	0.75	1.08	1.93	0.00	0.00	1.31
time (sec)	N/A	0.041	0.386	0.384	0.220	0.351	0.000	0.000	11.764

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	97	68	75	0	0	116
N.S.	1	1.00	1.05	1.47	1.03	1.14	0.00	0.00	1.76
time (sec)	N/A	0.034	0.188	0.367	0.291	0.276	0.000	0.000	0.067

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	59	84	52	66	0	70	84
N.S.	1	1.00	1.07	1.53	0.95	1.20	0.00	1.27	1.53
time (sec)	N/A	0.055	0.166	0.367	0.286	0.279	0.000	0.302	0.067

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	53	65	33	58	0	52	57
N.S.	1	1.00	1.56	1.91	0.97	1.71	0.00	1.53	1.68
time (sec)	N/A	0.011	0.116	0.349	0.285	0.254	0.000	0.318	11.445

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	23	28	0	34	23
N.S.	1	1.00	1.00	0.85	0.88	1.08	0.00	1.31	0.88
time (sec)	N/A	0.006	0.094	0.349	0.276	0.265	0.000	0.299	11.445

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	45	58	0	52	0	74	58
N.S.	1	1.00	1.10	1.41	0.00	1.27	0.00	1.80	1.41
time (sec)	N/A	0.025	0.118	0.357	0.000	0.246	0.000	0.296	11.467

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	73	0	76	0	0	81
N.S.	1	1.00	0.89	1.14	0.00	1.19	0.00	0.00	1.27
time (sec)	N/A	0.035	0.144	0.369	0.000	0.278	0.000	0.000	11.474

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	72	94	0	97	0	213	105
N.S.	1	1.00	0.80	1.04	0.00	1.08	0.00	2.37	1.17
time (sec)	N/A	0.048	0.173	0.407	0.000	0.255	0.000	0.291	11.438

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	155	141	299	138	299	344	0
N.S.	1	1.00	0.68	0.62	1.31	0.60	1.31	1.50	0.00
time (sec)	N/A	0.208	0.269	0.421	0.298	0.271	2.140	0.339	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	136	130	275	128	340	312	0
N.S.	1	1.00	0.68	0.65	1.38	0.64	1.70	1.56	0.00
time (sec)	N/A	0.181	0.400	0.402	0.299	0.364	2.025	0.340	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	133	119	251	116	255	280	0
N.S.	1	1.00	0.78	0.70	1.47	0.68	1.49	1.64	0.00
time (sec)	N/A	0.141	0.254	0.394	0.297	0.293	1.959	0.331	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	114	108	230	106	298	248	0
N.S.	1	1.00	0.80	0.76	1.62	0.75	2.10	1.75	0.00
time (sec)	N/A	0.114	0.332	0.384	0.289	0.288	1.894	0.330	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	111	97	167	94	216	216	0
N.S.	1	1.00	0.82	0.71	1.23	0.69	1.59	1.59	0.00
time (sec)	N/A	0.035	0.179	0.388	0.282	0.315	1.832	0.330	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	83	119	84	253	184	0
N.S.	1	1.00	0.85	0.77	1.10	0.78	2.34	1.70	0.00
time (sec)	N/A	0.028	0.281	0.399	0.285	0.285	1.656	0.319	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	125	544	116	95	270	0	0
N.S.	1	1.00	1.30	5.67	1.21	0.99	2.81	0.00	0.00
time (sec)	N/A	0.104	0.297	0.393	0.281	0.270	4.817	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	130	140	126	111	330	0	0
N.S.	1	1.00	1.24	1.33	1.20	1.06	3.14	0.00	0.00
time (sec)	N/A	0.102	0.282	0.428	0.273	0.289	3.483	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	124	116	123	119	347	0	0
N.S.	1	1.00	1.13	1.05	1.12	1.08	3.15	0.00	0.00
time (sec)	N/A	0.105	0.318	0.441	0.285	0.292	4.326	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	138	113	146	106	338	0	0
N.S.	1	1.00	1.35	1.11	1.43	1.04	3.31	0.00	0.00
time (sec)	N/A	0.110	0.289	0.465	0.284	0.286	3.925	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	114	96	130	86	422	246	0
N.S.	1	1.00	1.06	0.89	1.20	0.80	3.91	2.28	0.00
time (sec)	N/A	0.096	0.293	0.455	0.287	0.264	5.176	0.323	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	131	110	155	97	660	275	0
N.S.	1	1.00	0.94	0.79	1.11	0.69	4.71	1.96	0.00
time (sec)	N/A	0.112	0.268	0.542	0.276	0.256	4.879	0.329	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	123	121	180	108	808	304	0
N.S.	1	1.00	0.73	0.72	1.07	0.64	4.78	1.80	0.00
time (sec)	N/A	0.139	0.416	0.590	0.302	0.274	9.278	0.332	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	145	132	205	119	835	333	0
N.S.	1	1.00	0.73	0.67	1.04	0.60	4.22	1.68	0.00
time (sec)	N/A	0.168	0.371	0.707	0.296	0.333	8.258	0.346	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	363	170	171	0	0	0
N.S.	1	1.00	0.85	2.95	1.38	1.39	0.00	0.00	0.00
time (sec)	N/A	0.154	0.417	0.405	0.297	0.294	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	65	157	116	0	191	66
N.S.	1	1.00	0.71	0.66	1.59	1.17	0.00	1.93	0.67
time (sec)	N/A	0.141	0.308	0.388	0.235	0.284	0.000	0.328	11.482

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	70	65	136	118	0	191	66
N.S.	1	1.00	0.79	0.73	1.53	1.33	0.00	2.15	0.74
time (sec)	N/A	0.093	0.325	0.382	0.220	0.271	0.000	0.338	11.352

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	69	64	138	116	0	177	65
N.S.	1	1.00	0.76	0.70	1.52	1.27	0.00	1.95	0.71
time (sec)	N/A	0.024	0.327	0.378	0.211	0.272	0.000	0.305	11.808

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	70	66	136	115	0	193	66
N.S.	1	1.00	0.77	0.73	1.49	1.26	0.00	2.12	0.73
time (sec)	N/A	0.020	0.323	0.384	0.219	0.273	0.000	0.319	11.850

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	107	330	0	168	0	284	0
N.S.	1	1.00	0.91	2.80	0.00	1.42	0.00	2.41	0.00
time (sec)	N/A	0.114	0.470	0.412	0.000	0.259	0.000	0.322	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	135	244	0	194	0	324	0
N.S.	1	1.00	0.92	1.67	0.00	1.33	0.00	2.22	0.00
time (sec)	N/A	0.188	0.436	0.429	0.000	0.276	0.000	0.322	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	136	251	0	215	0	360	0
N.S.	1	1.00	0.74	1.37	0.00	1.17	0.00	1.97	0.00
time (sec)	N/A	0.238	0.486	0.449	0.000	0.274	0.000	0.348	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	112	199	185	190	0	233	0
N.S.	1	1.00	0.63	1.12	1.05	1.07	0.00	1.32	0.00
time (sec)	N/A	0.283	0.469	0.467	0.302	0.289	0.000	0.308	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	99	187	160	174	0	212	0
N.S.	1	1.00	0.68	1.28	1.10	1.19	0.00	1.45	0.00
time (sec)	N/A	0.234	0.392	0.459	0.294	0.300	0.000	0.308	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	87	319	136	157	0	185	0
N.S.	1	1.00	0.72	2.66	1.13	1.31	0.00	1.54	0.00
time (sec)	N/A	0.169	0.338	0.427	0.296	0.278	0.000	0.315	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	52	49	125	104	0	106	48
N.S.	1	1.00	0.55	0.52	1.32	1.09	0.00	1.12	0.51
time (sec)	N/A	0.087	0.288	0.398	0.292	0.256	0.000	0.325	11.686

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	49	46	129	100	0	137	45
N.S.	1	1.00	0.51	0.47	1.33	1.03	0.00	1.41	0.46
time (sec)	N/A	0.029	0.277	0.399	0.296	0.271	0.000	0.321	11.833

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	52	49	128	104	0	165	48
N.S.	1	1.00	0.52	0.49	1.28	1.04	0.00	1.65	0.48
time (sec)	N/A	0.025	0.007	0.409	0.287	0.277	0.000	0.310	11.829

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	99	332	0	153	0	212	0
N.S.	1	1.00	0.86	2.89	0.00	1.33	0.00	1.84	0.00
time (sec)	N/A	0.120	0.395	0.411	0.000	0.286	0.000	0.329	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	115	199	0	181	0	304	0
N.S.	1	1.00	0.79	1.36	0.00	1.24	0.00	2.08	0.00
time (sec)	N/A	0.194	0.458	0.443	0.000	0.260	0.000	0.311	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	130	205	0	202	0	382	0
N.S.	1	1.00	0.71	1.12	0.00	1.10	0.00	2.09	0.00
time (sec)	N/A	0.248	0.446	0.448	0.000	0.274	0.000	0.332	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	130	209	0	200	0	244	0
N.S.	1	1.00	0.64	1.02	0.00	0.98	0.00	1.20	0.00
time (sec)	N/A	0.368	0.411	0.445	0.000	0.284	0.000	0.327	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	112	199	0	190	0	233	0
N.S.	1	1.00	0.70	1.24	0.00	1.19	0.00	1.46	0.00
time (sec)	N/A	0.275	0.418	0.409	0.000	0.276	0.000	0.307	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	106	186	0	174	0	211	0
N.S.	1	1.00	0.72	1.26	0.00	1.18	0.00	1.43	0.00
time (sec)	N/A	0.161	0.365	0.430	0.000	0.290	0.000	0.311	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	87	268	0	157	0	186	0
N.S.	1	1.00	0.76	2.33	0.00	1.37	0.00	1.62	0.00
time (sec)	N/A	0.098	0.335	0.413	0.000	0.288	0.000	0.308	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	52	42	125	102	0	137	46
N.S.	1	1.00	0.81	0.66	1.95	1.59	0.00	2.14	0.72
time (sec)	N/A	0.018	0.248	0.409	0.189	0.273	0.000	0.299	11.793

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	123	104	0	165	47
N.S.	1	1.00	0.76	0.64	1.84	1.55	0.00	2.46	0.70
time (sec)	N/A	0.015	0.317	0.402	0.198	0.269	0.000	0.324	11.676

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	88	415	0	153	0	212	0
N.S.	1	1.00	0.80	3.77	0.00	1.39	0.00	1.93	0.00
time (sec)	N/A	0.147	0.477	0.445	0.000	0.266	0.000	0.301	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	115	199	0	181	0	304	0
N.S.	1	1.00	0.80	1.39	0.00	1.27	0.00	2.13	0.00
time (sec)	N/A	0.207	0.475	0.461	0.000	0.264	0.000	0.309	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	116	205	0	202	0	382	0
N.S.	1	1.00	0.63	1.12	0.00	1.10	0.00	2.09	0.00
time (sec)	N/A	0.248	0.500	0.447	0.000	0.309	0.000	0.311	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	141	216	0	213	0	444	0
N.S.	1	1.00	0.67	1.03	0.00	1.01	0.00	2.11	0.00
time (sec)	N/A	0.308	0.551	0.483	0.000	0.305	0.000	0.305	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	151	164	478	156	0	151	0
N.S.	1	1.00	0.60	0.65	1.90	0.62	0.00	0.60	0.00
time (sec)	N/A	0.426	0.352	0.464	0.289	0.279	0.000	0.312	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	132	153	456	146	0	140	0
N.S.	1	1.00	0.59	0.68	2.04	0.65	0.00	0.62	0.00
time (sec)	N/A	0.348	0.449	0.455	0.289	0.279	0.000	0.307	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	129	142	407	134	0	126	0
N.S.	1	1.00	0.67	0.74	2.12	0.70	0.00	0.66	0.00
time (sec)	N/A	0.277	0.319	0.462	0.289	0.263	0.000	0.328	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	110	131	363	124	0	117	0
N.S.	1	1.00	0.60	0.72	1.99	0.68	0.00	0.64	0.00
time (sec)	N/A	0.144	0.375	0.447	0.278	0.275	0.000	0.328	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	107	119	235	111	0	103	0
N.S.	1	1.00	0.82	0.92	1.81	0.85	0.00	0.79	0.00
time (sec)	N/A	0.043	0.279	0.465	0.277	0.288	0.000	0.300	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	89	106	134	99	0	86	0
N.S.	1	1.00	0.79	0.94	1.19	0.88	0.00	0.76	0.00
time (sec)	N/A	0.029	0.326	0.435	0.269	0.263	0.000	0.305	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	115	1192	0	111	0	116	0
N.S.	1	1.00	1.29	13.39	0.00	1.25	0.00	1.30	0.00
time (sec)	N/A	0.133	0.381	0.477	0.000	0.271	0.000	0.304	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	126	133	0	127	0	189	0
N.S.	1	1.00	1.34	1.41	0.00	1.35	0.00	2.01	0.00
time (sec)	N/A	0.136	0.353	0.529	0.000	0.285	0.000	0.305	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	107	115	0	112	0	252	0
N.S.	1	1.00	0.97	1.05	0.00	1.02	0.00	2.29	0.00
time (sec)	N/A	0.141	0.335	0.595	0.000	0.277	0.000	0.319	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	131	0	123	0	318	0
N.S.	1	1.00	0.85	0.96	0.00	0.90	0.00	2.32	0.00
time (sec)	N/A	0.197	0.394	0.703	0.000	0.297	0.000	0.313	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	116	144	0	136	0	392	0
N.S.	1	1.00	0.68	0.85	0.00	0.80	0.00	2.31	0.00
time (sec)	N/A	0.246	0.446	0.742	0.000	0.266	0.000	0.330	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	141	155	0	147	0	448	0
N.S.	1	1.00	0.72	0.79	0.00	0.75	0.00	2.29	0.00
time (sec)	N/A	0.337	0.449	0.867	0.000	0.275	0.000	0.312	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	245	0	126	0	0	220
N.S.	1	1.00	0.74	2.58	0.00	1.33	0.00	0.00	2.32
time (sec)	N/A	0.090	0.256	0.378	0.000	0.278	0.000	0.000	11.455

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	50	44	153	102	0	0	287
N.S.	1	1.00	0.57	0.50	1.74	1.16	0.00	0.00	3.26
time (sec)	N/A	0.079	0.273	0.400	0.195	0.260	0.000	0.000	0.059

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	399	316	0	0	252
N.S.	1	1.00	0.66	0.63	1.91	1.51	0.00	0.00	1.21
time (sec)	N/A	0.204	0.631	0.411	0.201	0.561	0.000	0.000	12.079

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	401	317	0	0	252
N.S.	1	1.00	0.66	0.63	1.92	1.52	0.00	0.00	1.21
time (sec)	N/A	0.138	0.675	0.425	0.199	0.614	0.000	0.000	11.993

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	137	132	405	316	0	0	252
N.S.	1	1.00	0.65	0.63	1.92	1.50	0.00	0.00	1.19
time (sec)	N/A	0.075	0.593	0.403	0.216	0.677	0.000	0.000	11.916

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	137	132	393	314	0	0	242
N.S.	1	1.00	0.67	0.64	1.92	1.53	0.00	0.00	1.18
time (sec)	N/A	0.063	0.019	0.418	0.199	0.733	0.000	0.000	11.903

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	173	1328	0	432	0	0	0
N.S.	1	1.00	0.74	5.68	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.253	0.968	0.438	0.000	0.758	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	205	520	0	458	0	0	0
N.S.	1	1.00	0.76	1.92	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.437	0.921	0.434	0.000	1.116	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	81	95	0	217	0	0	0
N.S.	1	1.00	0.79	0.93	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.079	0.336	0.362	0.000	0.295	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	58	0	110	0	57	0
N.S.	1	1.00	1.21	1.49	0.00	2.82	0.00	1.46	0.00
time (sec)	N/A	0.027	0.238	0.363	0.000	0.281	0.000	0.283	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	50	57	48	92	82	65	38
N.S.	1	1.00	1.43	1.63	1.37	2.63	2.34	1.86	1.09
time (sec)	N/A	0.007	0.116	0.352	0.272	0.266	0.861	5.535	11.910

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	76	0	199	0	95	0
N.S.	1	1.00	1.00	2.17	0.00	5.69	0.00	2.71	0.00
time (sec)	N/A	0.021	1.518	0.389	0.000	0.310	0.000	5.441	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	48	41	68	90	29	65	36
N.S.	1	1.00	1.41	1.21	2.00	2.65	0.85	1.91	1.06
time (sec)	N/A	0.006	0.118	0.346	0.274	0.283	0.820	6.284	12.049

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	86	0	208	0	89	0
N.S.	1	1.00	1.00	2.53	0.00	6.12	0.00	2.62	0.00
time (sec)	N/A	0.022	1.500	0.361	0.000	0.291	0.000	5.451	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	66	82	111	148	141	54
N.S.	1	1.00	1.03	1.05	1.30	1.76	2.35	2.24	0.86
time (sec)	N/A	0.010	0.159	0.351	0.274	0.268	1.730	10.789	11.608

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	92	0	221	0	0	0
N.S.	1	1.00	0.78	1.46	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	0.021	1.401	0.364	0.000	0.331	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	199	0	0	0	502	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	2.01	0.00	0.00
time (sec)	N/A	0.266	0.877	0.000	0.000	0.000	15.984	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	174	0	0	0	432	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	2.10	0.00	0.00
time (sec)	N/A	0.143	0.741	0.000	0.000	0.000	11.942	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	121	0	0	0	366	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	2.26	0.00	0.00
time (sec)	N/A	0.056	0.629	0.000	0.000	0.000	8.752	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	61	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.019	0.338	0.000	0.000	0.000	4.297	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	243	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	1.49	0.00	0.00
time (sec)	N/A	0.092	0.654	0.000	0.000	0.000	10.034	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	173	0	0	0	182	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.136	0.642	0.000	0.000	0.000	38.527	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	245	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.741	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	199	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	1.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	116	0	0
N.S.	1	1.31	0.98	0.00	0.00	0.00	0.94	0.00	0.00
time (sec)	N/A	0.059	0.783	0.000	0.000	0.000	26.222	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	60	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.017	0.449	0.000	0.000	0.000	4.554	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	1.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	176	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	1.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	200	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	1.590	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	972	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	6.57	0.00	0.00
time (sec)	N/A	0.068	0.221	0.000	0.000	0.000	1.955	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	129	0	0	0	972	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	6.61	0.00	0.00
time (sec)	N/A	0.064	0.209	0.000	0.000	0.000	1.868	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	106	0	0	0	382	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	3.18	0.00	0.00
time (sec)	N/A	0.047	0.183	0.000	0.000	0.000	1.590	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	103	0	0	0	382	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.21	0.00	0.00
time (sec)	N/A	0.046	0.180	0.000	0.000	0.000	1.444	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	85	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.96	0.00	0.00
time (sec)	N/A	0.023	0.142	0.000	0.000	0.000	1.190	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0	78
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.99	0.00	0.94
time (sec)	N/A	0.015	0.155	0.000	0.000	0.000	1.132	0.000	13.295

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.035	0.160	0.000	0.000	0.000	3.088	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	0	82	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.037	0.176	0.000	0.000	0.000	1.787	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	106	0	0	0	83	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.042	0.185	0.000	0.000	0.000	1.871	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	159	0	0	0	2924	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	16.43	0.00	0.00
time (sec)	N/A	0.100	0.272	0.000	0.000	0.000	2.899	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	186	0	0	0	1015	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	5.49	0.00	0.00
time (sec)	N/A	0.109	0.289	0.000	0.000	0.000	2.390	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	138	0	0	0	1328	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	8.91	0.00	0.00
time (sec)	N/A	0.087	0.244	0.000	0.000	0.000	2.132	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	168	0	0	0	425	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	2.74	0.00	0.00
time (sec)	N/A	0.094	0.247	0.000	0.000	0.000	1.949	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	110	0	0	0	440	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	3.73	0.00	0.00
time (sec)	N/A	0.060	0.179	0.000	0.000	0.000	1.577	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	134	0	0	0	124	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	1.75	0.00	0.00
time (sec)	N/A	0.024	0.206	0.000	0.000	0.000	1.446	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	103	0	0	0	136	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.06	0.00	0.00
time (sec)	N/A	0.063	0.196	0.000	0.000	0.000	3.044	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	153	0	0	0	116	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.91	0.00	0.00
time (sec)	N/A	0.078	0.239	0.000	0.000	0.000	2.303	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	131	0	0	0	138	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.99	0.00	0.00
time (sec)	N/A	0.086	0.224	0.000	0.000	0.000	2.583	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	222	205	0	0	0	2966	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	13.36	0.00	0.00
time (sec)	N/A	0.124	0.421	0.000	0.000	0.000	3.684	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	219	0	0	0	2966	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	13.61	0.00	0.00
time (sec)	N/A	0.121	0.411	0.000	0.000	0.000	3.361	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	187	0	0	0	1370	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	7.10	0.00	0.00
time (sec)	N/A	0.115	0.336	0.000	0.000	0.000	2.788	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	187	0	0	0	1370	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	7.25	0.00	0.00
time (sec)	N/A	0.121	0.320	0.000	0.000	0.000	2.555	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	159	0	0	0	479	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	4.13	0.00	0.00
time (sec)	N/A	0.048	0.345	0.000	0.000	0.000	2.085	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	476	0	0
N.S.	1	1.00	2.12	0.00	0.00	0.00	6.52	0.00	0.00
time (sec)	N/A	0.017	0.288	0.000	0.000	0.000	1.847	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	169	0	0	0	178	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.04	0.00	0.00
time (sec)	N/A	0.085	0.276	0.000	0.000	0.000	3.810	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	158	0	0	0	177	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.129	0.267	0.000	0.000	0.000	2.489	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	182	0	0	0	175	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	1.05	0.00	0.00
time (sec)	N/A	0.148	0.280	0.000	0.000	0.000	3.167	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	66	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	17065	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	141.03	0.00	0.00
time (sec)	N/A	0.060	0.405	0.000	0.000	0.000	62.784	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	198	0	0	0	14895	0	0
N.S.	1	1.00	1.66	0.00	0.00	0.00	125.17	0.00	0.00
time (sec)	N/A	0.058	0.337	0.000	0.000	0.000	12.294	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	440	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	4.89	0.00	0.00
time (sec)	N/A	0.041	0.235	0.000	0.000	0.000	4.637	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	318	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	4.36	0.00	0.00
time (sec)	N/A	0.023	0.183	0.000	0.000	0.000	3.232	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	348	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	3.35	0.00	0.00
time (sec)	N/A	0.047	0.250	0.000	0.000	0.000	3.395	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	434	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	4.09	0.00	0.00
time (sec)	N/A	0.051	0.313	0.000	0.000	0.000	4.907	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	484	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	4.48	0.00	0.00
time (sec)	N/A	0.052	0.586	0.000	0.000	0.000	13.398	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	66	0	0	0	0	0	0
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	66	0	0	0	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	332	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.387	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	177	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.305	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.214	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	201	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	223	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.552	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.730	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.813	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	389	0	0	0	0	0	0
N.S.	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	245	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.450	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	202	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	328	0	0	0	0	0	0
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.369	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	280	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.507	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0	0
N.S.	1	1.00	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.668	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	446	0	0	0	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.714	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	231	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.528	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	156	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.393	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	130	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.279	0.000	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	417	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	337	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.585	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	399	0	0	0	0	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	452	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.778	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	505	0	0	0	0	0	0
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	0.898	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	194	0	0	0	257	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.97	0.00	0.00
time (sec)	N/A	0.258	0.208	0.000	0.000	0.000	9.153	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	169	0	0	0	189	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.120	0.146	0.000	0.000	0.000	5.872	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	121	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.049	0.104	0.000	0.000	0.000	3.591	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.015	0.069	0.000	0.000	0.000	1.292	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	124	0	0	0	372	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	2.28	0.00	0.00
time (sec)	N/A	0.096	0.128	0.000	0.000	0.000	5.226	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	180	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	206	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	328	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.69	0.00	0.00
time (sec)	N/A	0.035	0.119	0.000	0.000	0.000	4.563	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	90	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.238	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	68	167	0	72	0	167	201
N.S.	1	1.00	0.32	0.78	0.00	0.34	0.00	0.78	0.94
time (sec)	N/A	0.163	0.681	0.675	0.000	0.267	0.000	0.637	0.119

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	230	312	0	1104	0	0	0
N.S.	1	1.00	0.90	1.22	0.00	4.33	0.00	0.00	0.00
time (sec)	N/A	0.403	0.658	0.443	0.000	4.061	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	199	266	0	963	0	0	0
N.S.	1	1.00	0.94	1.26	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.259	0.443	0.437	0.000	4.133	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	161	227	0	776	0	0	0
N.S.	1	1.00	1.05	1.48	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.144	0.366	0.421	0.000	0.632	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	133	203	0	684	0	0	0
N.S.	1	1.00	1.05	1.60	0.00	5.39	0.00	0.00	0.00
time (sec)	N/A	0.079	0.297	0.421	0.000	0.607	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	110	184	0	574	0	0	0
N.S.	1	1.00	1.07	1.79	0.00	5.57	0.00	0.00	0.00
time (sec)	N/A	0.047	0.015	0.422	0.000	0.464	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	137	305	122	1316	0	0	0
N.S.	1	1.00	1.18	2.63	1.05	11.34	0.00	0.00	0.00
time (sec)	N/A	0.067	0.219	0.375	0.208	1.014	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	129	195	0	599	0	146	0
N.S.	1	1.00	1.23	1.86	0.00	5.70	0.00	1.39	0.00
time (sec)	N/A	0.112	0.364	0.385	0.000	0.484	0.000	0.285	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	144	212	0	726	0	230	0
N.S.	1	1.00	0.90	1.32	0.00	4.54	0.00	1.44	0.00
time (sec)	N/A	0.143	0.445	0.443	0.000	0.461	0.000	0.302	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	173	241	0	824	0	313	0
N.S.	1	1.00	0.91	1.26	0.00	4.31	0.00	1.64	0.00
time (sec)	N/A	0.161	0.606	0.454	0.000	0.451	0.000	0.292	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	215	282	0	1007	0	605	0
N.S.	1	1.00	0.78	1.03	0.00	3.68	0.00	2.21	0.00
time (sec)	N/A	0.202	0.757	0.450	0.000	0.507	0.000	0.301	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	161	222	0	1060	0	0	0
N.S.	1	1.00	0.83	1.14	0.00	5.44	0.00	0.00	0.00
time (sec)	N/A	0.317	0.512	0.443	0.000	2.393	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	137	201	0	924	0	0	0
N.S.	1	1.00	0.90	1.32	0.00	6.08	0.00	0.00	0.00
time (sec)	N/A	0.178	0.382	0.425	0.000	2.406	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	128	172	0	745	0	0	0
N.S.	1	1.00	1.17	1.58	0.00	6.83	0.00	0.00	0.00
time (sec)	N/A	0.087	0.445	0.419	0.000	0.471	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	98	151	0	631	0	0	0
N.S.	1	1.00	1.14	1.76	0.00	7.34	0.00	0.00	0.00
time (sec)	N/A	0.038	0.240	0.411	0.000	0.453	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	127	60	211	0	60	0
N.S.	1	1.00	1.19	2.35	1.11	3.91	0.00	1.11	0.00
time (sec)	N/A	0.011	0.004	0.382	0.199	0.341	0.000	0.282	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	104	158	0	634	0	0	0
N.S.	1	1.00	1.21	1.84	0.00	7.37	0.00	0.00	0.00
time (sec)	N/A	0.057	0.241	0.398	0.000	0.407	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	128	180	0	767	0	143	0
N.S.	1	1.00	1.15	1.62	0.00	6.91	0.00	1.29	0.00
time (sec)	N/A	0.063	0.329	0.425	0.000	0.438	0.000	0.282	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	198	209	0	956	0	240	0
N.S.	1	1.00	1.18	1.24	0.00	5.69	0.00	1.43	0.00
time (sec)	N/A	0.090	0.480	0.435	0.000	0.571	0.000	0.287	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	178	351	0	1525	0	0	0
N.S.	1	1.00	1.22	2.40	0.00	10.45	0.00	0.00	0.00
time (sec)	N/A	0.204	0.806	0.471	0.000	3.477	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	137	397	0	1323	0	0	0
N.S.	1	1.00	1.11	3.23	0.00	10.76	0.00	0.00	0.00
time (sec)	N/A	0.119	0.528	0.425	0.000	2.923	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	109	355	179	455	0	181	0
N.S.	1	1.00	1.15	3.74	1.88	4.79	0.00	1.91	0.00
time (sec)	N/A	0.083	0.470	0.417	0.217	0.358	0.000	0.282	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	99	335	156	425	0	168	0
N.S.	1	1.00	1.12	3.81	1.77	4.83	0.00	1.91	0.00
time (sec)	N/A	0.036	0.376	0.404	0.215	0.332	0.000	0.301	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	104	315	131	456	0	180	0
N.S.	1	1.00	1.11	3.35	1.39	4.85	0.00	1.91	0.00
time (sec)	N/A	0.031	0.045	0.348	0.204	0.321	0.000	0.281	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	363	0	1325	0	0	0
N.S.	1	1.00	0.98	2.47	0.00	9.01	0.00	0.00	0.00
time (sec)	N/A	0.092	0.703	0.368	0.000	0.471	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	178	361	0	1556	0	273	0
N.S.	1	1.00	0.92	1.86	0.00	8.02	0.00	1.41	0.00
time (sec)	N/A	0.108	0.688	0.388	0.000	0.501	0.000	0.280	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	217	407	0	1943	0	365	0
N.S.	1	1.00	0.79	1.47	0.00	7.04	0.00	1.32	0.00
time (sec)	N/A	0.150	0.765	0.453	0.000	0.696	0.000	0.303	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	251	440	0	2025	0	0	0
N.S.	1	1.00	1.03	1.80	0.00	8.30	0.00	0.00	0.00
time (sec)	N/A	0.575	1.136	0.481	0.000	25.370	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	223	420	0	1786	0	0	0
N.S.	1	1.00	1.09	2.06	0.00	8.75	0.00	0.00	0.00
time (sec)	N/A	0.351	1.461	0.452	0.000	46.200	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	173	386	0	1449	0	0	0
N.S.	1	1.00	1.08	2.41	0.00	9.06	0.00	0.00	0.00
time (sec)	N/A	0.220	0.796	0.443	0.000	5.640	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	158	369	0	1260	0	0	0
N.S.	1	1.00	1.15	2.69	0.00	9.20	0.00	0.00	0.00
time (sec)	N/A	0.112	0.851	0.424	0.000	5.491	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	100	344	0	382	0	0	0
N.S.	1	1.00	1.11	3.82	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.025	0.404	0.415	0.000	0.450	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	215	0	381	0	0	0
N.S.	1	1.00	1.11	2.36	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.024	0.037	0.398	0.000	0.453	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	153	376	0	1261	0	0	0
N.S.	1	1.00	0.85	2.10	0.00	7.04	0.00	0.00	0.00
time (sec)	N/A	0.097	0.707	0.379	0.000	0.817	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	183	395	0	1512	0	0	0
N.S.	1	1.00	0.86	1.86	0.00	7.13	0.00	0.00	0.00
time (sec)	N/A	0.110	0.913	0.392	0.000	0.671	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	224	424	0	1867	0	0	0
N.S.	1	1.00	0.84	1.58	0.00	6.97	0.00	0.00	0.00
time (sec)	N/A	0.141	1.126	0.419	0.000	1.020	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	114	320	210	368	4134	624	363
N.S.	1	1.00	0.84	2.37	1.56	2.73	30.62	4.62	2.69
time (sec)	N/A	0.056	0.107	0.368	0.200	0.301	1.258	0.289	11.633

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	146	250	2181	410	255
N.S.	1	1.00	1.07	1.91	1.43	2.45	21.38	4.02	2.50
time (sec)	N/A	0.035	0.109	0.402	0.199	0.286	0.783	0.283	11.563

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	89	148	952	237	163
N.S.	1	1.00	0.93	1.43	1.27	2.11	13.60	3.39	2.33
time (sec)	N/A	0.022	0.058	0.388	0.190	0.308	0.495	0.269	11.522

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	279	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	3.62	0.00	0.00
time (sec)	N/A	0.035	0.064	0.000	0.000	0.000	2.287	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	199	754	447	1027	14317	1750	932
N.S.	1	1.00	0.86	3.25	1.93	4.43	61.71	7.54	4.02
time (sec)	N/A	0.101	0.171	0.475	0.210	0.319	4.186	0.299	11.960

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	323	601	335	757	8940	1266	723
N.S.	1	1.00	1.75	3.25	1.81	4.09	48.32	6.84	3.91
time (sec)	N/A	0.073	0.344	0.431	0.212	0.299	2.449	0.300	11.735

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	160	420	235	519	5097	851	496
N.S.	1	1.00	1.14	3.00	1.68	3.71	36.41	6.08	3.54
time (sec)	N/A	0.049	0.162	0.420	0.192	0.306	1.436	0.277	11.625

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	1608	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	10.86	0.00	0.00
time (sec)	N/A	0.144	0.141	0.000	0.000	0.000	3.316	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	302	2232	795	2165	35984	3713	1796
N.S.	1	1.00	0.88	6.51	2.32	6.31	104.91	10.83	5.24
time (sec)	N/A	0.147	0.220	0.453	0.244	0.311	11.304	0.297	12.660

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	709	1214	625	1675	24687	2851	1459
N.S.	1	1.00	2.51	4.30	2.22	5.94	87.54	10.11	5.17
time (sec)	N/A	0.121	0.874	0.480	0.232	0.308	6.425	0.309	12.260

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	347	958	472	1244	15990	2085	1144
N.S.	1	1.00	1.56	4.30	2.12	5.58	71.70	9.35	5.13
time (sec)	N/A	0.085	0.387	0.443	0.212	0.270	4.078	0.279	12.107

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	226	0	0	0	5622	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	22.85	0.00	0.00
time (sec)	N/A	0.230	0.189	0.000	0.000	0.000	4.956	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	217	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	168	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	170	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	189	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	167	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	413	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	247	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	403	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	230	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	253	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.251	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	489	489	391	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.594	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	513	513	437	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	399	377	275	0	0	0	128	0	0
N.S.	1	0.94	0.69	0.00	0.00	0.00	0.32	0.00	0.00
time (sec)	N/A	0.496	0.130	0.000	0.000	0.000	16.572	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	150	113	0	0	0	80	0	0
N.S.	1	0.91	0.69	0.00	0.00	0.00	0.49	0.00	0.00
time (sec)	N/A	0.093	0.075	0.000	0.000	0.000	6.299	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	950	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	7.60	0.00	0.00
time (sec)	N/A	0.061	0.138	0.000	0.000	0.000	12.803	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	950	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	7.60	0.00	0.00
time (sec)	N/A	0.061	0.122	0.000	0.000	0.000	8.511	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	364	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.64	0.00	0.00
time (sec)	N/A	0.041	0.118	0.000	0.000	0.000	6.896	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	364	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.64	0.00	0.00
time (sec)	N/A	0.038	0.106	0.000	0.000	0.000	4.564	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	65	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.87	0.00	0.00
time (sec)	N/A	0.020	0.073	0.000	0.000	0.000	3.694	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	65
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	0.93
time (sec)	N/A	0.014	0.108	0.000	0.000	0.000	2.451	0.000	12.309

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	65	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	0.00
time (sec)	N/A	0.032	0.097	0.000	0.000	0.000	3.697	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	68	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.037	0.105	0.000	0.000	0.000	3.952	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	70	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.037	0.101	0.000	0.000	0.000	5.504	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	205	0	0	0	2883	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	15.34	0.00	0.00
time (sec)	N/A	0.120	0.266	0.000	0.000	0.000	17.848	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	169	156	0	0	0	986	0	0
N.S.	1	0.95	0.88	0.00	0.00	0.00	5.57	0.00	0.00
time (sec)	N/A	0.109	0.253	0.000	0.000	0.000	16.936	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	152	0	0	0	1294	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	8.68	0.00	0.00
time (sec)	N/A	0.099	0.196	0.000	0.000	0.000	9.547	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	144	139	0	0	0	400	0	0
N.S.	1	0.95	0.91	0.00	0.00	0.00	2.63	0.00	0.00
time (sec)	N/A	0.093	0.216	0.000	0.000	0.000	8.843	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	184	0	0	0	408	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	3.61	0.00	0.00
time (sec)	N/A	0.077	0.200	0.000	0.000	0.000	4.841	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0	0
N.S.	1	0.94	1.00	0.00	0.00	0.00	0.73	0.00	0.00
time (sec)	N/A	0.052	0.174	0.000	0.000	0.000	4.566	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	101	0	0	0	109	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.062	0.167	0.000	0.000	0.000	4.186	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	134	0	0	0	95	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.083	0.192	0.000	0.000	0.000	4.985	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	117	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.083	0.164	0.000	0.000	0.000	6.649	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	241	249	0	0	0	2919	0	0
N.S.	1	0.98	1.01	0.00	0.00	0.00	11.82	0.00	0.00
time (sec)	N/A	0.162	0.344	0.000	0.000	0.000	31.281	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	241	249	0	0	0	2919	0	0
N.S.	1	0.97	1.00	0.00	0.00	0.00	11.72	0.00	0.00
time (sec)	N/A	0.160	0.315	0.000	0.000	0.000	21.453	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	201	196	0	0	0	1329	0	0
N.S.	1	0.97	0.95	0.00	0.00	0.00	6.42	0.00	0.00
time (sec)	N/A	0.137	0.260	0.000	0.000	0.000	17.322	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	202	196	0	0	0	1329	0	0
N.S.	1	0.96	0.93	0.00	0.00	0.00	6.33	0.00	0.00
time (sec)	N/A	0.140	0.233	0.000	0.000	0.000	11.369	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	159	228	0	0	0	440	0	0
N.S.	1	0.95	1.37	0.00	0.00	0.00	2.63	0.00	0.00
time (sec)	N/A	0.106	0.264	0.000	0.000	0.000	9.271	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	169	223	0	0	0	437	0	0
N.S.	1	0.96	1.27	0.00	0.00	0.00	2.48	0.00	0.00
time (sec)	N/A	0.103	0.262	0.000	0.000	0.000	5.711	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	165	170	0	0	0	144	0	0
N.S.	1	0.96	0.99	0.00	0.00	0.00	0.84	0.00	0.00
time (sec)	N/A	0.096	0.239	0.000	0.000	0.000	6.361	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	0	0	143	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.90	0.00	0.00
time (sec)	N/A	0.132	0.236	0.000	0.000	0.000	5.470	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	174	0	0	0	148	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.151	0.252	0.000	0.000	0.000	8.174	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	227	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	172	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	170	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.265	0.000	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	256	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.457	0.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	392	392	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	343	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	0.588	0.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	281	277	300	0	0	0	0	0	0
N.S.	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	223	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0	0
N.S.	1	0.78	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	368	368	303	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	421	421	342	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.601	0.000	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	462	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.895	0.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	436	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.665	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	290	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	229	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	700	700	434	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	0.691	0.000	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	754	754	478	0	0	0	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	0.773	0.000	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	254	182	0	0	0	230	0	0
N.S.	1	0.92	0.66	0.00	0.00	0.00	0.83	0.00	0.00
time (sec)	N/A	0.297	0.217	0.000	0.000	0.000	113.065	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	194	158	0	0	0	168	0	0
N.S.	1	0.95	0.77	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.121	0.106	0.000	0.000	0.000	70.148	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	107	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.041	0.059	0.000	0.000	0.000	37.017	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.012	0.026	0.000	0.000	0.000	9.519	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	304	955	0	678	0	305	0
N.S.	1	1.00	0.88	2.77	0.00	1.97	0.00	0.88	0.00
time (sec)	N/A	0.271	11.128	0.681	0.000	0.380	0.000	0.363	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	245	512	0	536	0	224	0
N.S.	1	1.00	0.98	2.04	0.00	2.14	0.00	0.89	0.00
time (sec)	N/A	0.151	10.564	0.655	0.000	0.327	0.000	0.370	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	181	291	0	418	0	164	0
N.S.	1	1.00	0.87	1.41	0.00	2.02	0.00	0.79	0.00
time (sec)	N/A	0.111	0.773	0.569	0.000	0.311	0.000	0.338	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	131	0	337	0	114	0
N.S.	1	1.00	0.85	1.00	0.00	2.57	0.00	0.87	0.00
time (sec)	N/A	0.041	0.061	0.545	0.000	0.311	0.000	0.342	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	476	308	0	947	0	0	0
N.S.	1	1.00	2.83	1.83	0.00	5.64	0.00	0.00	0.00
time (sec)	N/A	0.099	1.413	0.533	0.000	0.429	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	841	708	0	355	0	220	0
N.S.	1	1.00	6.14	5.17	0.00	2.59	0.00	1.61	0.00
time (sec)	N/A	0.087	9.249	0.720	0.000	0.359	0.000	0.327	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	162	1353	0	442	0	505	0
N.S.	1	1.00	0.80	6.70	0.00	2.19	0.00	2.50	0.00
time (sec)	N/A	0.168	10.119	0.759	0.000	0.630	0.000	0.314	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	210	2059	0	558	0	912	0
N.S.	1	1.00	0.73	7.20	0.00	1.95	0.00	3.19	0.00
time (sec)	N/A	0.233	10.171	1.155	0.000	1.230	0.000	0.333	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	273	3471	0	702	0	1483	0
N.S.	1	1.00	0.70	8.92	0.00	1.80	0.00	3.81	0.00
time (sec)	N/A	0.352	10.243	1.121	0.000	8.549	0.000	0.338	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	385	1426	0	1044	0	513	0
N.S.	1	1.00	0.86	3.18	0.00	2.33	0.00	1.14	0.00
time (sec)	N/A	0.343	1.239	0.636	0.000	0.343	0.000	0.367	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	303	797	0	846	0	405	0
N.S.	1	1.00	0.86	2.26	0.00	2.40	0.00	1.15	0.00
time (sec)	N/A	0.202	0.732	0.644	0.000	0.323	0.000	0.350	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	236	483	0	676	1093	310	0
N.S.	1	1.00	0.80	1.64	0.00	2.29	3.71	1.05	0.00
time (sec)	N/A	0.183	0.518	0.549	0.000	0.311	10.979	0.355	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	180	230	0	532	751	229	0
N.S.	1	1.00	0.90	1.14	0.00	2.65	3.74	1.14	0.00
time (sec)	N/A	0.072	0.071	0.590	0.000	0.329	3.611	0.347	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	232	607	0	1327	0	0	0
N.S.	1	1.00	0.92	2.42	0.00	5.29	0.00	0.00	0.00
time (sec)	N/A	0.167	0.532	0.638	0.000	2.598	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	213	1300	0	1221	0	339	0
N.S.	1	1.00	0.89	5.42	0.00	5.09	0.00	1.41	0.00
time (sec)	N/A	0.168	0.398	0.721	0.000	1.327	0.000	0.374	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	243	2438	0	1375	0	626	0
N.S.	1	1.00	0.95	9.52	0.00	5.37	0.00	2.45	0.00
time (sec)	N/A	0.175	0.504	0.767	0.000	1.435	0.000	0.442	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	201	4330	0	558	0	1005	0
N.S.	1	1.00	0.95	20.52	0.00	2.64	0.00	4.76	0.00
time (sec)	N/A	0.139	0.343	1.011	0.000	0.956	0.000	0.341	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	247	7421	0	704	0	1618	0
N.S.	1	1.00	0.84	25.16	0.00	2.39	0.00	5.48	0.00
time (sec)	N/A	0.233	0.529	1.062	0.000	4.941	0.000	0.366	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	317	10575	0	872	0	2352	0
N.S.	1	1.00	0.80	26.77	0.00	2.21	0.00	5.95	0.00
time (sec)	N/A	0.313	0.791	1.651	0.000	14.930	0.000	0.403	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	402	16883	0	1072	0	3251	0
N.S.	1	1.00	0.81	33.90	0.00	2.15	0.00	6.53	0.00
time (sec)	N/A	0.453	1.092	1.928	0.000	37.471	0.000	0.484	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	549	1895	0	1524	0	784	0
N.S.	1	1.00	0.96	3.30	0.00	2.66	0.00	1.37	0.00
time (sec)	N/A	0.433	1.396	0.658	0.000	0.510	0.000	0.376	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	479	1080	0	1272	0	648	0
N.S.	1	1.00	1.06	2.39	0.00	2.81	0.00	1.43	0.00
time (sec)	N/A	0.246	1.200	0.662	0.000	0.671	0.000	0.376	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	388	673	0	1046	0	526	0
N.S.	1	1.00	1.02	1.77	0.00	2.75	0.00	1.38	0.00
time (sec)	N/A	0.234	0.890	0.547	0.000	0.360	0.000	0.381	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	295	327	0	844	0	415	0
N.S.	1	1.00	1.08	1.19	0.00	3.08	0.00	1.51	0.00
time (sec)	N/A	0.110	0.204	0.561	0.000	0.322	0.000	0.377	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	342	997	0	1873	0	0	0
N.S.	1	1.00	0.87	2.53	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.285	0.948	0.595	0.000	30.623	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	309	2076	0	1717	0	473	0
N.S.	1	1.00	0.88	5.90	0.00	4.88	0.00	1.34	0.00
time (sec)	N/A	0.271	1.038	0.746	0.000	10.887	0.000	0.409	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	287	3893	0	1569	0	746	0
N.S.	1	1.00	0.85	11.48	0.00	4.63	0.00	2.20	0.00
time (sec)	N/A	0.249	0.864	0.812	0.000	4.423	0.000	0.458	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	316	6850	0	1741	0	1195	0
N.S.	1	1.00	0.85	18.46	0.00	4.69	0.00	3.22	0.00
time (sec)	N/A	0.288	1.024	1.060	0.000	5.776	0.000	0.498	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	358	11685	0	1917	0	1750	0
N.S.	1	1.00	0.89	28.92	0.00	4.75	0.00	4.33	0.00
time (sec)	N/A	0.282	1.041	1.197	0.000	16.145	0.000	0.680	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	271	19539	0	872	0	2449	0
N.S.	1	1.00	0.94	67.61	0.00	3.02	0.00	8.47	0.00
time (sec)	N/A	0.202	0.711	1.566	0.000	13.959	0.000	0.395	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	404	32291	0	1072	0	3388	0
N.S.	1	1.00	1.05	83.66	0.00	2.78	0.00	8.78	0.00
time (sec)	N/A	0.327	0.912	1.973	0.000	41.437	0.000	0.510	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	500	500	497	45106	0	1300	0	4452	0
N.S.	1	1.00	0.99	90.21	0.00	2.60	0.00	8.90	0.00
time (sec)	N/A	0.405	1.217	3.187	0.000	78.301	0.000	0.775	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	572	70736	0	1550	0	5681	0
N.S.	1	1.00	0.91	112.64	0.00	2.47	0.00	9.05	0.00
time (sec)	N/A	0.557	1.539	3.286	0.000	176.453	0.000	1.301	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	298	277	514	0	758	0	228	0
N.S.	1	1.10	1.02	1.90	0.00	2.80	0.00	0.84	0.00
time (sec)	N/A	0.208	0.438	0.686	0.000	0.665	0.000	0.369	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	201	259	0	586	0	182	0
N.S.	1	1.00	1.03	1.33	0.00	3.01	0.00	0.93	0.00
time (sec)	N/A	0.210	0.277	0.697	0.000	0.402	0.000	0.366	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	130	131	0	443	0	0	0
N.S.	1	1.00	0.94	0.94	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.068	0.190	0.556	0.000	0.392	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	0	59	0	0	50
N.S.	1	1.00	0.81	0.96	0.00	1.13	0.00	0.00	0.96
time (sec)	N/A	0.013	0.010	0.575	0.000	0.340	0.000	0.000	11.710

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	131	136	0	454	0	0	0
N.S.	1	1.00	0.92	0.95	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.113	0.174	0.586	0.000	0.511	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	201	270	0	610	0	0	0
N.S.	1	1.00	0.88	1.18	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.180	0.260	0.687	0.000	0.972	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	283	545	0	792	0	0	0
N.S.	1	1.00	0.86	1.66	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.322	0.431	0.687	0.000	2.251	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	388	1923	0	2120	0	0	0
N.S.	1	1.00	0.75	3.73	0.00	4.12	0.00	0.00	0.00
time (sec)	N/A	0.385	0.833	0.869	0.000	4.284	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	300	1112	0	1782	0	0	0
N.S.	1	1.00	0.68	2.54	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.353	0.531	0.862	0.000	1.669	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	218	639	0	1466	0	0	0
N.S.	1	1.00	0.73	2.15	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.183	0.319	0.694	0.000	2.061	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	145	0	308	0	0	1071
N.S.	1	1.00	0.75	1.15	0.00	2.44	0.00	0.00	8.50
time (sec)	N/A	0.069	0.136	0.658	0.000	1.676	0.000	0.000	12.581

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	314	0	0	499
N.S.	1	1.00	0.72	1.08	0.00	2.28	0.00	0.00	3.62
time (sec)	N/A	0.057	0.132	0.536	0.000	1.866	0.000	0.000	12.477

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	306	0	0	120
N.S.	1	1.00	0.79	1.14	0.00	2.53	0.00	0.00	0.99
time (sec)	N/A	0.026	0.017	0.574	0.000	2.119	0.000	0.000	12.275

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	217	360	0	1476	0	0	0
N.S.	1	1.00	0.80	1.33	0.00	5.45	0.00	0.00	0.00
time (sec)	N/A	0.198	0.334	0.593	0.000	5.464	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	303	716	0	1812	0	0	0
N.S.	1	1.00	0.77	1.82	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.338	0.526	0.744	0.000	11.466	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	390	1354	0	2162	0	0	0
N.S.	1	1.00	0.75	2.59	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.487	0.783	0.773	0.000	28.523	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	664	664	493	2409	0	2526	0	0	0
N.S.	1	1.00	0.74	3.63	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	0.705	0.977	1.145	0.000	64.197	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	235	366	0	820	0	0	3099
N.S.	1	1.00	0.91	1.41	0.00	3.17	0.00	0.00	11.97
time (sec)	N/A	0.143	0.219	0.684	0.000	16.877	0.000	0.000	13.602

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	440	663	0	1540	0	0	11469
N.S.	1	1.00	1.29	1.94	0.00	4.52	0.00	0.00	33.63
time (sec)	N/A	0.172	0.346	0.657	0.000	115.684	0.000	0.000	16.999

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	221	169	0	33	0	0	0
N.S.	1	1.00	1.30	0.99	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.045	32.665	0.684	0.000	0.091	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	22	0	67	22
N.S.	1	1.00	1.00	0.78	0.96	0.96	0.00	2.91	0.96
time (sec)	N/A	0.011	10.025	0.581	0.286	0.317	0.000	0.299	11.905

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	347	216	0	30	0	0	0
N.S.	1	1.00	1.18	0.73	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.063	20.404	0.660	0.000	0.179	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	169	157	0	25	0	0	0
N.S.	1	1.00	1.17	1.09	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.027	20.351	0.637	0.000	0.081	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	43	0	60	0	0	0
N.S.	1	1.00	0.73	0.65	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.021	15.354	0.549	0.000	0.264	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	349	216	0	32	0	0	0
N.S.	1	1.00	1.22	0.75	0.00	0.11	0.00	0.00	0.00
time (sec)	N/A	0.064	10.297	0.634	0.000	0.077	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	185	159	0	32	0	0	0
N.S.	1	1.00	1.27	1.09	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.031	20.309	0.654	0.000	0.073	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	235	176	0	38	0	0	0
N.S.	1	1.00	1.17	0.88	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.052	21.557	0.711	0.000	0.081	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	27	27	0	173	25
N.S.	1	1.00	1.00	0.78	1.17	1.17	0.00	7.52	1.09
time (sec)	N/A	0.019	10.029	0.565	0.288	0.267	0.000	0.314	0.120

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	244	230	0	38	0	0	0
N.S.	1	1.00	0.75	0.71	0.00	0.12	0.00	0.00	0.00
time (sec)	N/A	0.079	10.333	0.645	0.000	0.077	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	176	0	0	33	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.034	20.484	180.000	0.000	0.074	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	53	57	0	65	0	0	0
N.S.	1	1.00	0.56	0.61	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.026	10.057	0.783	0.000	0.264	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	244	230	0	39	0	0	0
N.S.	1	1.00	0.76	0.71	0.00	0.12	0.00	0.00	0.00
time (sec)	N/A	0.082	10.373	0.664	0.000	0.127	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	192	173	0	38	0	0	0
N.S.	1	1.00	1.10	0.99	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.040	10.290	0.672	0.000	0.097	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	169	157	0	25	0	0	0
N.S.	1	1.00	1.19	1.11	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.035	21.426	0.681	0.000	0.180	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	17	0	18	9
N.S.	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.39
time (sec)	N/A	0.012	10.020	0.619	0.300	0.267	0.000	0.302	0.150

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	375	202	0	9	0	0	0
N.S.	1	1.00	1.48	0.80	0.00	0.04	0.00	0.00	0.00
time (sec)	N/A	0.051	21.738	0.683	0.000	0.079	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	6	0	0	0
N.S.	1	1.00	1.35	1.25	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.016	20.117	0.609	0.000	0.077	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	33	0	43	0	0	0
N.S.	1	1.00	0.69	0.79	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.018	4.921	0.583	0.000	0.278	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	400	215	0	31	0	0	0
N.S.	1	1.00	1.42	0.76	0.00	0.11	0.00	0.00	0.00
time (sec)	N/A	0.069	10.424	0.638	0.000	0.075	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	144	171	159	0	30	0	0	0
N.S.	1	0.99	1.17	1.09	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.031	10.415	0.660	0.000	0.091	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	161	157	0	38	0	0	0
N.S.	1	1.00	1.18	1.15	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.037	21.379	0.759	0.000	0.100	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	24	0	0	17
N.S.	1	1.00	1.00	0.78	0.74	1.04	0.00	0.00	0.74
time (sec)	N/A	0.015	10.023	0.571	0.286	0.251	0.000	0.000	12.328

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	402	216	0	42	0	0	0
N.S.	1	1.00	1.43	0.77	0.00	0.15	0.00	0.00	0.00
time (sec)	N/A	0.064	10.418	0.741	0.000	0.076	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	216	157	0	37	0	0	0
N.S.	1	1.00	1.58	1.15	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.022	20.267	0.715	0.000	0.121	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	81	43	0	78	0	0	0
N.S.	1	1.00	1.23	0.65	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.023	11.231	0.666	0.000	0.345	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	409	228	0	47	0	0	0
N.S.	1	1.00	1.29	0.72	0.00	0.15	0.00	0.00	0.00
time (sec)	N/A	0.082	10.489	0.750	0.000	0.119	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	170	169	0	48	0	0	0
N.S.	1	1.00	1.00	0.99	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.042	10.258	0.675	0.000	0.102	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.041	21.354	0.722	0.000	0.081	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	24	29	0	0	82
N.S.	1	1.00	1.00	0.78	1.04	1.26	0.00	0.00	3.57
time (sec)	N/A	0.014	10.037	0.664	0.290	0.263	0.000	0.000	12.444

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	409	228	0	61	0	0	0
N.S.	1	1.00	1.29	0.72	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.076	10.484	0.855	0.000	0.078	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.031	20.335	0.731	0.000	0.078	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	95	60	0	101	0	0	0
N.S.	1	1.00	0.99	0.62	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.026	10.095	0.700	0.000	0.261	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	414	240	0	62	0	0	0
N.S.	1	1.00	1.19	0.69	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.096	10.542	0.710	0.000	0.082	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	183	179	0	63	0	0	0
N.S.	1	1.00	0.90	0.88	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.052	10.426	0.681	0.000	0.080	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	75	134	88	71	84
N.S.	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.058	0.039	0.763	0.278	0.290	0.108	0.269	0.129

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	627	594	0	5507	0	1200	13879
N.S.	1	1.00	1.28	1.21	0.00	11.24	0.00	2.45	28.32
time (sec)	N/A	11.987	1.957	0.826	0.000	0.987	0.000	0.379	14.523

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	397	465	504	0	4245	0	1074	11143
N.S.	1	1.22	1.43	1.55	0.00	13.02	0.00	3.29	34.18
time (sec)	N/A	4.714	1.368	0.740	0.000	0.947	0.000	0.376	14.182

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	375	425	0	2966	0	897	8171
N.S.	1	1.00	1.19	1.34	0.00	9.39	0.00	2.84	25.86
time (sec)	N/A	2.100	0.987	0.674	0.000	0.498	0.000	0.365	13.250

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	341	326	0	1721	0	758	5664
N.S.	1	1.00	1.19	1.14	0.00	6.00	0.00	2.64	19.74
time (sec)	N/A	2.169	0.949	0.684	0.000	0.404	0.000	0.344	13.175

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	252	222	0	715	0	445	709
N.S.	1	1.00	1.27	1.12	0.00	3.61	0.00	2.25	3.58
time (sec)	N/A	0.157	0.100	0.553	0.000	0.517	0.000	0.317	12.289

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	266	291	0	2446	0	719	10894
N.S.	1	1.00	0.97	1.06	0.00	8.89	0.00	2.61	39.61
time (sec)	N/A	0.716	0.796	0.578	0.000	0.538	0.000	0.319	16.579

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	356	337	373	0	4860	0	776	19887
N.S.	1	0.97	0.92	1.01	0.00	13.21	0.00	2.11	54.04
time (sec)	N/A	2.549	1.231	0.646	0.000	6.027	0.000	0.343	16.142

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	433	489	0	7425	0	1043	33838
N.S.	1	1.00	0.82	0.92	0.00	13.98	0.00	1.96	63.73
time (sec)	N/A	2.281	1.966	0.719	0.000	104.003	0.000	0.361	17.846

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	901	762	0	14340	0	1612	31485
N.S.	1	1.00	1.39	1.17	0.00	22.06	0.00	2.48	48.44
time (sec)	N/A	1.707	3.094	0.904	0.000	17.368	0.000	0.404	16.934

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	755	680	0	11459	0	1397	25497
N.S.	1	1.00	1.30	1.17	0.00	19.72	0.00	2.40	43.88
time (sec)	N/A	12.666	2.509	0.829	0.000	7.340	0.000	0.402	16.511

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	537	542	0	8530	0	1195	19465
N.S.	1	1.00	1.22	1.23	0.00	19.34	0.00	2.71	44.14
time (sec)	N/A	1.274	1.717	0.736	0.000	3.548	0.000	0.372	14.904

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	493	473	0	5572	0	986	13841
N.S.	1	1.00	1.09	1.04	0.00	12.30	0.00	2.18	30.55
time (sec)	N/A	2.902	1.753	0.704	0.000	1.653	0.000	0.362	13.800

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	364	344	0	2770	0	797	8334
N.S.	1	1.00	1.13	1.07	0.00	8.60	0.00	2.48	25.88
time (sec)	N/A	0.716	0.275	0.638	0.000	1.111	0.000	0.331	13.556

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	371	370	0	5167	0	833	20897
N.S.	1	1.00	1.09	1.09	0.00	15.20	0.00	2.45	61.46
time (sec)	N/A	1.040	1.304	0.628	0.000	4.767	0.000	0.333	18.016

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	402	416	409	0	8653	0	890	29890
N.S.	1	1.00	1.03	1.01	0.00	21.47	0.00	2.21	74.17
time (sec)	N/A	2.010	1.587	0.740	0.000	24.310	0.000	0.359	17.202

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	560	560	0	14417	0	1129	44649
N.S.	1	1.00	0.92	0.92	0.00	23.75	0.00	1.86	73.56
time (sec)	N/A	2.805	2.736	0.913	0.000	213.706	0.000	0.383	18.134

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	290	320	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	289	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	183	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	207	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.349	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	246	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	134	155	175	176	150	190	351
N.S.	1	1.00	0.95	1.10	1.24	1.25	1.06	1.35	2.49
time (sec)	N/A	0.121	0.054	0.424	0.198	0.380	0.278	0.269	0.116

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	103	122	138	139	109	149	197
N.S.	1	1.00	0.94	1.12	1.27	1.28	1.00	1.37	1.81
time (sec)	N/A	0.088	0.033	0.440	0.196	0.364	0.225	0.265	12.025

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	73	88	97	98	70	105	127
N.S.	1	1.00	1.12	1.35	1.49	1.51	1.08	1.62	1.95
time (sec)	N/A	0.038	0.024	0.422	0.223	0.568	0.187	0.275	0.075

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	59	63	64	46	63	65
N.S.	1	1.00	0.86	1.18	1.26	1.28	0.92	1.26	1.30
time (sec)	N/A	0.021	0.016	0.377	0.195	0.280	0.130	0.285	11.930

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	82	82	76	112	84	81
N.S.	1	1.00	0.89	1.32	1.32	1.23	1.81	1.35	1.31
time (sec)	N/A	0.053	0.019	0.402	0.199	0.286	0.286	0.269	0.162

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	112	113	165	182	115	109
N.S.	1	1.00	0.95	1.30	1.31	1.92	2.12	1.34	1.27
time (sec)	N/A	0.058	0.032	0.467	0.192	0.303	0.474	0.283	12.020

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	138	149	271	185	149	100
N.S.	1	1.00	1.00	1.59	1.71	3.11	2.13	1.71	1.15
time (sec)	N/A	0.070	0.053	0.453	0.189	0.307	0.453	0.281	0.138

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	122	177	206	400	248	186	152
N.S.	1	1.00	1.08	1.57	1.82	3.54	2.19	1.65	1.35
time (sec)	N/A	0.075	0.041	0.467	0.199	0.278	0.587	0.285	12.078

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	199	236	511	282	213	180
N.S.	1	1.00	1.02	1.43	1.70	3.68	2.03	1.53	1.29
time (sec)	N/A	0.092	0.056	0.471	0.196	0.296	0.680	0.300	0.154

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	226	259	258	328	250	273	1029
N.S.	1	1.00	1.04	1.19	1.18	1.50	1.15	1.25	4.72
time (sec)	N/A	0.178	0.081	0.433	0.187	0.295	0.510	0.334	11.989

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	185	217	218	288	199	231	565
N.S.	1	1.00	1.05	1.23	1.23	1.63	1.12	1.31	3.19
time (sec)	N/A	0.151	0.074	0.440	0.197	0.328	0.460	0.362	11.853

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	154	177	182	251	162	191	316
N.S.	1	1.00	1.05	1.21	1.25	1.72	1.11	1.31	2.16
time (sec)	N/A	0.119	0.057	0.374	0.188	0.290	0.390	0.628	0.097

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	115	133	141	206	119	148	185
N.S.	1	1.00	1.07	1.24	1.32	1.93	1.11	1.38	1.73
time (sec)	N/A	0.089	0.057	0.425	0.196	0.278	0.326	0.270	0.073

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	93	104	157	94	108	116
N.S.	1	1.00	1.06	1.19	1.33	2.01	1.21	1.38	1.49
time (sec)	N/A	0.062	0.040	0.431	0.190	0.270	0.279	0.278	12.071

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	63	69	95	61	68	72
N.S.	1	1.00	0.92	1.26	1.38	1.90	1.22	1.36	1.44
time (sec)	N/A	0.038	0.030	0.424	0.187	0.270	0.187	0.265	11.898

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	91	112	114	168	182	117	111
N.S.	1	1.00	1.06	1.30	1.33	1.95	2.12	1.36	1.29
time (sec)	N/A	0.055	0.031	0.450	0.189	0.282	0.471	0.265	11.883

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	85	109	111	155	156	114	115
N.S.	1	1.00	1.15	1.47	1.50	2.09	2.11	1.54	1.55
time (sec)	N/A	0.018	0.027	0.431	0.196	0.258	0.333	0.278	12.052

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	139	180	212	417	279	201	198
N.S.	1	1.00	1.15	1.49	1.75	3.45	2.31	1.66	1.64
time (sec)	N/A	0.086	0.064	0.460	0.195	0.350	0.579	0.288	0.151

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	171	162	197	337	241	227	148
N.S.	1	1.00	1.17	1.11	1.35	2.31	1.65	1.55	1.01
time (sec)	N/A	0.102	0.064	0.458	0.195	0.366	0.628	0.273	12.059

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	195	241	298	648	376	264	274
N.S.	1	1.00	1.10	1.35	1.67	3.64	2.11	1.48	1.54
time (sec)	N/A	0.127	0.091	0.415	0.200	0.351	0.804	0.280	11.801

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	229	278	342	693	427	308	314
N.S.	1	1.00	1.09	1.32	1.63	3.30	2.03	1.47	1.50
time (sec)	N/A	0.162	0.118	0.477	0.216	0.344	0.935	0.294	0.236

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	193	216	227	336	219	226	375
N.S.	1	1.00	1.08	1.21	1.27	1.88	1.22	1.26	2.09
time (sec)	N/A	0.162	0.063	0.428	0.198	0.624	0.689	0.277	0.145

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	174	188	294	178	185	240
N.S.	1	1.00	1.05	1.17	1.26	1.97	1.19	1.24	1.61
time (sec)	N/A	0.130	0.056	0.441	0.190	0.355	0.599	0.274	0.111

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	133	149	241	151	143	161
N.S.	1	1.00	1.00	1.13	1.26	2.04	1.28	1.21	1.36
time (sec)	N/A	0.096	0.057	0.424	0.195	0.362	0.530	0.268	11.943

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	93	96	105	159	102	94	107
N.S.	1	1.00	1.15	1.19	1.30	1.96	1.26	1.16	1.32
time (sec)	N/A	0.066	0.026	0.427	0.184	0.320	0.417	0.271	11.744

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	69	81	100	83	72	80
N.S.	1	1.00	0.80	1.13	1.33	1.64	1.36	1.18	1.31
time (sec)	N/A	0.039	0.018	0.460	0.189	0.264	0.252	0.290	0.073

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	152	150	271	185	143	103
N.S.	1	1.00	1.02	1.73	1.70	3.08	2.10	1.62	1.17
time (sec)	N/A	0.066	0.053	0.414	0.199	0.272	0.456	0.270	0.140

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	140	183	211	417	277	200	198
N.S.	1	1.00	1.15	1.50	1.73	3.42	2.27	1.64	1.62
time (sec)	N/A	0.085	0.065	0.440	0.194	0.289	0.578	0.270	11.811

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	110	135	152	252	144	146	114
N.S.	1	1.00	0.87	1.06	1.20	1.98	1.13	1.15	0.90
time (sec)	N/A	0.039	0.030	0.430	0.190	0.274	0.454	0.276	0.107

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	197	245	308	662	321	277	249
N.S.	1	1.00	1.05	1.30	1.64	3.52	1.71	1.47	1.32
time (sec)	N/A	0.142	0.097	0.467	0.192	0.309	0.822	0.279	11.801

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	244	286	359	793	372	335	296
N.S.	1	1.00	1.04	1.22	1.53	3.37	1.58	1.43	1.26
time (sec)	N/A	0.178	0.107	0.494	0.204	0.353	0.926	0.269	11.944

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	301	604	1603	807	0	969	0
N.S.	1	1.00	1.12	2.25	5.96	3.00	0.00	3.60	0.00
time (sec)	N/A	0.615	3.378	1.138	0.290	0.329	0.000	0.315	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	240	481	1190	624	0	757	0
N.S.	1	1.00	1.12	2.24	5.53	2.90	0.00	3.52	0.00
time (sec)	N/A	0.423	1.131	0.895	0.287	0.402	0.000	0.306	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	157	688	903	454	0	560	0
N.S.	1	1.00	0.86	3.76	4.93	2.48	0.00	3.06	0.00
time (sec)	N/A	0.252	0.895	0.647	0.280	0.334	0.000	0.305	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	105	126	583	279	0	370	125
N.S.	1	1.00	0.72	0.87	4.02	1.92	0.00	2.55	0.86
time (sec)	N/A	0.147	0.562	0.641	0.193	0.311	0.000	0.304	12.125

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	77	80	373	183	0	276	79
N.S.	1	1.00	0.66	0.68	3.19	1.56	0.00	2.36	0.68
time (sec)	N/A	0.038	0.475	0.551	0.198	0.316	0.000	0.306	12.721

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	53	50	101	106	0	165	49
N.S.	1	1.00	0.51	0.49	0.98	1.03	0.00	1.60	0.48
time (sec)	N/A	0.032	0.010	0.441	0.199	0.299	0.000	0.294	12.659

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	225	1667	0	1767	0	642	0
N.S.	1	1.00	0.93	6.89	0.00	7.30	0.00	2.65	0.00
time (sec)	N/A	0.407	10.298	0.520	0.000	0.374	0.000	0.306	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	341	3289	0	3305	0	0	0
N.S.	1	1.00	1.10	10.58	0.00	10.63	0.00	0.00	0.00
time (sec)	N/A	1.016	10.429	0.525	0.000	0.753	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	387	6396	0	5361	0	1401	0
N.S.	1	1.00	0.97	16.07	0.00	13.47	0.00	3.52	0.00
time (sec)	N/A	2.124	10.993	0.484	0.000	2.536	0.000	0.430	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	118	111	0	499	155	101	124
N.S.	1	1.00	1.05	0.99	0.00	4.46	1.38	0.90	1.11
time (sec)	N/A	0.141	0.224	0.473	0.000	0.295	5.872	0.295	0.252

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	282	243	326	324	502	380	222
N.S.	1	1.00	1.18	1.01	1.36	1.35	2.09	1.58	0.92
time (sec)	N/A	0.234	0.176	0.460	0.182	0.287	0.998	0.282	0.127

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	177	155	197	197	311	243	159
N.S.	1	1.00	1.01	0.89	1.13	1.13	1.78	1.39	0.91
time (sec)	N/A	0.156	0.114	0.451	0.187	0.281	0.857	0.278	12.185

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	94	79	104	100	165	132	100
N.S.	1	1.00	0.83	0.70	0.92	0.88	1.46	1.17	0.88
time (sec)	N/A	0.055	0.065	0.424	0.192	0.268	0.717	0.276	0.076

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	40	53	40	70	53	44
N.S.	1	1.00	0.72	0.66	0.87	0.66	1.15	0.87	0.72
time (sec)	N/A	0.019	0.033	0.402	0.184	0.277	0.352	0.281	11.828

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	82	0	297	143	109	107
N.S.	1	1.00	0.88	0.79	0.00	2.86	1.38	1.05	1.03
time (sec)	N/A	0.086	0.194	0.684	0.000	0.290	1.789	0.268	0.112

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	126	133	138	0	539	0	151	128
N.S.	1	1.03	1.09	1.13	0.00	4.42	0.00	1.24	1.05
time (sec)	N/A	0.136	0.388	0.475	0.000	0.297	0.000	0.279	11.969

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	182	166	166	0	896	0	285	224
N.S.	1	1.02	0.93	0.93	0.00	5.03	0.00	1.60	1.26
time (sec)	N/A	0.192	0.636	0.500	0.000	0.297	0.000	0.284	12.013

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	278	247	334	333	420	465	292
N.S.	1	1.00	1.17	1.04	1.40	1.40	1.76	1.95	1.23
time (sec)	N/A	0.175	0.196	0.478	0.196	0.283	11.707	0.278	0.091

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	177	154	205	206	264	277	199
N.S.	1	1.00	1.02	0.89	1.18	1.19	1.53	1.60	1.15
time (sec)	N/A	0.131	0.143	0.477	0.197	0.260	4.815	0.269	11.824

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	86	112	110	150	137	111
N.S.	1	1.00	0.83	0.77	1.01	0.99	1.35	1.23	1.00
time (sec)	N/A	0.045	0.067	0.428	0.199	0.273	1.969	0.281	0.078

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	39	54	49	75	56	44
N.S.	1	1.00	0.73	0.66	0.92	0.83	1.27	0.95	0.75
time (sec)	N/A	0.017	0.035	0.416	0.209	0.279	0.626	0.279	0.054

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	114	112	0	492	151	114	141
N.S.	1	1.00	1.02	1.00	0.00	4.39	1.35	1.02	1.26
time (sec)	N/A	0.126	0.302	0.477	0.000	0.282	4.858	0.282	12.195

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	148	152	0	906	0	228	187
N.S.	1	1.00	1.03	1.06	0.00	6.29	0.00	1.58	1.30
time (sec)	N/A	0.172	0.475	0.480	0.000	0.316	0.000	0.277	12.675

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	230	230	0	1539	0	368	310
N.S.	1	1.00	1.07	1.07	0.00	7.19	0.00	1.72	1.45
time (sec)	N/A	0.314	0.907	0.538	0.000	0.343	0.000	0.275	12.682

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	123	306	0	336	0	165	569
N.S.	1	1.00	0.84	2.08	0.00	2.29	0.00	1.12	3.87
time (sec)	N/A	0.098	0.350	0.422	0.000	0.322	0.000	0.319	32.606

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	9	12	0	12	16
N.S.	1	1.00	1.00	0.81	0.56	0.75	0.00	0.75	1.00
time (sec)	N/A	0.008	0.040	0.442	0.190	0.277	0.000	0.281	12.481

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	438	2385	0	0	0	0	0
N.S.	1	1.00	1.07	5.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.719	1.310	0.450	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	363	1499	0	0	0	0	0
N.S.	1	1.00	1.06	4.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.448	0.900	0.454	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1387	0	1921	0	0	0
N.S.	1	1.00	0.95	5.78	0.00	8.00	0.00	0.00	0.00
time (sec)	N/A	0.230	10.305	0.405	0.000	6.811	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	361	5383	0	5816	0	0	0
N.S.	1	1.00	1.03	15.34	0.00	16.57	0.00	0.00	0.00
time (sec)	N/A	1.556	1.191	0.460	0.000	35.163	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	613	613	422	14861	0	10812	0	0	0
N.S.	1	1.00	0.69	24.24	0.00	17.64	0.00	0.00	0.00
time (sec)	N/A	2.058	1.650	0.456	0.000	160.903	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	363	2336	0	0	0	0	0
N.S.	1	1.00	1.08	6.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.700	1.017	0.455	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	577	1383	0	1913	0	0	0
N.S.	1	1.00	2.40	5.76	0.00	7.97	0.00	0.00	0.00
time (sec)	N/A	0.211	6.746	0.460	0.000	8.914	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	286	1415	0	4325	0	0	0
N.S.	1	1.00	1.24	6.15	0.00	18.80	0.00	0.00	0.00
time (sec)	N/A	0.191	0.870	0.451	0.000	15.586	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	11846	0	0	0
N.S.	1	1.00	1.08	31.01	0.00	33.46	0.00	0.00	0.00
time (sec)	N/A	0.428	0.965	0.472	0.000	56.032	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	1049	8264	0	0	0	0	0
N.S.	1	1.00	1.68	13.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.712	8.148	0.480	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	361	5383	0	5844	0	0	0
N.S.	1	1.00	1.03	15.34	0.00	16.65	0.00	0.00	0.00
time (sec)	N/A	1.357	1.266	0.462	0.000	46.421	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	12028	0	0	0
N.S.	1	1.00	1.08	31.01	0.00	33.98	0.00	0.00	0.00
time (sec)	N/A	0.533	0.929	0.458	0.000	90.510	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	543	477	30648	0	0	0	0	0
N.S.	1	0.99	0.87	55.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.956	1.809	0.458	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	305	0	129	0	375	1610
N.S.	1	1.00	0.91	4.69	0.00	1.98	0.00	5.77	24.77
time (sec)	N/A	0.034	0.075	0.431	0.000	0.303	0.000	0.785	17.832

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	82	0	193	0	266	164
N.S.	1	1.00	1.08	1.02	0.00	2.41	0.00	3.32	2.05
time (sec)	N/A	0.096	0.329	0.500	0.000	0.318	0.000	0.295	0.066

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	130	233	0	318	0	208	148
N.S.	1	1.00	1.21	2.18	0.00	2.97	0.00	1.94	1.38
time (sec)	N/A	0.160	0.542	0.561	0.000	0.392	0.000	0.300	0.298

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	130	233	0	318	0	131	148
N.S.	1	1.00	1.21	2.18	0.00	2.97	0.00	1.22	1.38
time (sec)	N/A	0.124	0.393	0.470	0.000	0.385	0.000	0.285	0.130

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	851	851	1045	1824	0	765	0	0	0
N.S.	1	1.00	1.23	2.14	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	1.958	30.019	2.815	0.000	0.108	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	635	809	1142	0	510	0	0	0
N.S.	1	1.00	1.27	1.80	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	1.265	25.889	1.397	0.000	0.093	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	610	794	0	343	0	0	0
N.S.	1	1.00	1.41	1.83	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.549	24.391	1.444	0.000	0.107	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	521	602	0	229	0	0	0
N.S.	1	1.00	1.44	1.66	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.423	23.158	0.927	0.000	0.089	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	683	683	1216	922	0	0	0	0	0
N.S.	1	1.00	1.78	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.736	29.085	2.612	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	1331	895	0	0	0	0	0
N.S.	1	1.00	2.05	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.448	26.771	0.516	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1205	1205	2526	1161	0	0	0	0	0
N.S.	1	1.00	2.10	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.220	29.362	0.891	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	872	1156	0	578	0	0	0
N.S.	1	1.00	1.31	1.74	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	1.221	27.664	2.444	0.000	0.180	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	503	665	848	0	409	0	0	0
N.S.	1	0.99	1.31	1.67	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.861	25.165	2.276	0.000	0.106	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	545	678	0	268	0	0	0
N.S.	1	1.00	1.50	1.86	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.469	24.317	1.075	0.000	0.104	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	456	584	0	208	0	0	0
N.S.	1	1.00	1.42	1.81	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.363	22.495	0.977	0.000	0.097	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1096	833	0	0	0	0	0
N.S.	1	1.00	2.32	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	24.768	0.784	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	1336	921	0	0	0	0	0
N.S.	1	1.00	1.93	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.427	26.289	1.467	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1241	1241	2197	1196	0	0	0	0	0
N.S.	1	1.00	1.77	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.246	28.723	2.721	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	527	777	882	0	445	0	0	0
N.S.	1	0.99	1.46	1.66	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.939	25.522	3.372	0.000	0.114	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	591	700	0	300	0	0	0
N.S.	1	1.00	1.44	1.71	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.592	25.092	2.094	0.000	0.105	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	464	602	0	227	0	0	0
N.S.	1	1.00	1.40	1.82	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.413	23.524	1.286	0.000	0.101	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	173	0	0	0
N.S.	1	1.00	2.16	2.91	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.146	20.378	0.440	0.000	0.104	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	300	439	0	0	0	0	0
N.S.	1	1.00	0.94	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	21.358	1.170	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	698	698	1330	922	0	0	0	0	0
N.S.	1	1.00	1.91	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.612	25.587	2.246	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1246	1246	2450	1224	0	0	0	0	0
N.S.	1	1.00	1.97	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.343	29.038	3.125	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	808	1440	948	0	0	0	0	0
N.S.	1	1.35	2.40	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.072	29.463	4.530	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	927	852	0	0	0	0	0
N.S.	1	1.00	1.98	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.650	10.873	1.373	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	619	711	0	319	0	0	0
N.S.	1	1.00	1.35	1.56	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.611	24.724	2.980	0.000	0.119	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	473	614	0	246	0	0	0
N.S.	1	1.00	1.33	1.72	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.478	23.650	2.535	0.000	0.133	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	439	520	0	180	0	0	0
N.S.	1	1.00	1.52	1.81	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.343	22.289	1.220	0.000	0.132	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	66	0	0	0
N.S.	1	1.00	1.37	1.47	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.155	21.200	0.949	0.000	0.124	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	311	235	0	0	0	0	0
N.S.	1	1.00	1.86	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	22.363	1.845	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	1349	995	0	0	0	0	0
N.S.	1	1.00	1.81	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.698	25.970	2.599	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1257	1257	2491	1192	0	0	0	0	0
N.S.	1	1.00	1.98	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.282	29.749	3.459	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	468	929	0	0	0	0	0
N.S.	1	1.00	1.21	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	23.301	2.527	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	818	1917	1079	0	0	0	0	0
N.S.	1	1.00	2.34	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.004	27.374	3.177	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	261	215	0	0	0	0	0
N.S.	1	1.00	2.37	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	21.998	1.809	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	344	401	0	0	0	0	0
N.S.	1	1.00	0.76	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	24.183	5.083	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	4	0	0	0
N.S.	1	1.00	2.06	1.12	0.00	0.08	0.00	0.00	0.00
time (sec)	N/A	0.033	34.621	1.125	0.000	0.086	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	136	170	218	193	0	613	218
N.S.	1	1.00	0.51	0.63	0.81	0.72	0.00	2.28	0.81
time (sec)	N/A	0.251	0.129	0.509	0.245	0.302	0.000	0.309	12.607

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	89	98	133	123	0	350	142
N.S.	1	1.00	0.44	0.49	0.66	0.62	0.00	1.75	0.71
time (sec)	N/A	0.141	0.072	0.533	0.235	0.312	0.000	0.288	12.504

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	53	49	65	71	0	165	88
N.S.	1	1.00	0.42	0.39	0.52	0.57	0.00	1.32	0.70
time (sec)	N/A	0.062	0.050	0.524	0.239	0.275	0.000	0.286	12.352

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	32	18	49	0	68	54
N.S.	1	1.00	0.76	0.70	0.39	1.07	0.00	1.48	1.17
time (sec)	N/A	0.013	0.013	0.529	0.216	0.284	0.000	0.279	12.361

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	77	0	252	0	126	0
N.S.	1	1.00	1.16	0.96	0.00	3.15	0.00	1.58	0.00
time (sec)	N/A	0.076	0.074	0.537	0.000	0.306	0.000	0.285	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	136	158	0	703	0	395	0
N.S.	1	1.00	0.97	1.13	0.00	5.02	0.00	2.82	0.00
time (sec)	N/A	0.114	0.243	0.548	0.000	0.308	0.000	0.369	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	163	275	0	1283	0	844	0
N.S.	1	1.00	0.77	1.29	0.00	6.02	0.00	3.96	0.00
time (sec)	N/A	0.198	0.293	0.526	0.000	0.352	0.000	0.454	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	191	440	0	2027	0	1460	0
N.S.	1	1.00	0.68	1.57	0.00	7.24	0.00	5.21	0.00
time (sec)	N/A	0.260	0.463	0.534	0.000	0.673	0.000	0.656	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	134	179	165	216	0	513	252
N.S.	1	1.00	0.52	0.70	0.64	0.84	0.00	2.00	0.98
time (sec)	N/A	0.193	0.104	0.529	0.256	0.278	0.000	0.313	12.727

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	88	108	98	147	0	298	178
N.S.	1	1.00	0.49	0.60	0.54	0.81	0.00	1.65	0.98
time (sec)	N/A	0.115	0.068	0.527	0.243	0.273	0.000	0.302	12.583

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	51	58	48	96	0	138	118
N.S.	1	1.00	0.34	0.39	0.32	0.64	0.00	0.92	0.79
time (sec)	N/A	0.097	0.047	0.533	0.228	0.285	0.000	0.292	12.387

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	42	18	74	0	70	82
N.S.	1	1.00	0.76	0.91	0.39	1.61	0.00	1.52	1.78
time (sec)	N/A	0.013	0.008	0.527	0.216	0.308	0.000	0.287	12.376

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	109	118	0	553	0	281	0
N.S.	1	1.00	0.82	0.89	0.00	4.16	0.00	2.11	0.00
time (sec)	N/A	0.112	0.101	0.545	0.000	0.294	0.000	0.385	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	141	215	0	1067	0	753	0
N.S.	1	1.00	0.70	1.06	0.00	5.28	0.00	3.73	0.00
time (sec)	N/A	0.159	0.354	0.510	0.000	0.331	0.000	0.496	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	185	369	0	1863	0	1390	0
N.S.	1	1.00	0.68	1.35	0.00	6.80	0.00	5.07	0.00
time (sec)	N/A	0.214	0.527	0.552	0.000	0.486	0.000	0.697	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	131	179	219	251	0	524	278
N.S.	1	1.00	0.55	0.75	0.92	1.05	0.00	2.19	1.16
time (sec)	N/A	0.164	0.116	0.543	0.253	0.415	0.000	0.343	12.938

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	87	108	138	180	0	303	206
N.S.	1	1.00	0.41	0.51	0.65	0.85	0.00	1.44	0.98
time (sec)	N/A	0.142	0.079	0.568	0.256	0.370	0.000	0.315	12.730

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	52	58	73	129	0	163	149
N.S.	1	1.00	0.34	0.38	0.47	0.84	0.00	1.06	0.97
time (sec)	N/A	0.088	0.054	0.533	0.237	0.310	0.000	0.303	12.512

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	28	107	0	97	110
N.S.	1	1.00	0.77	0.88	0.58	2.23	0.00	2.02	2.29
time (sec)	N/A	0.014	0.018	0.536	0.209	0.270	0.000	0.294	12.472

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	129	209	0	1015	0	683	0
N.S.	1	1.00	0.69	1.11	0.00	5.40	0.00	3.63	0.00
time (sec)	N/A	0.169	0.200	0.554	0.000	0.387	0.000	0.440	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	180	414	0	1907	0	1404	0
N.S.	1	1.00	0.67	1.54	0.00	7.12	0.00	5.24	0.00
time (sec)	N/A	0.216	0.417	0.543	0.000	0.625	0.000	0.643	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	240	660	0	2935	0	2347	0
N.S.	1	1.00	0.70	1.93	0.00	8.58	0.00	6.86	0.00
time (sec)	N/A	0.335	0.743	0.543	0.000	1.308	0.000	1.139	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	195	273	320	375	0	1107	347
N.S.	1	1.00	0.58	0.81	0.95	1.12	0.00	3.29	1.03
time (sec)	N/A	0.362	0.158	0.536	0.251	0.277	0.000	0.324	12.361

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	136	178	218	264	0	762	242
N.S.	1	1.00	0.51	0.66	0.81	0.98	0.00	2.83	0.90
time (sec)	N/A	0.233	0.113	0.541	0.246	0.308	0.000	0.304	12.292

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	106	133	173	0	476	157
N.S.	1	1.00	0.45	0.53	0.66	0.86	0.00	2.38	0.78
time (sec)	N/A	0.150	0.070	0.534	0.233	0.369	0.000	0.300	12.085

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	54	57	65	102	0	253	93
N.S.	1	1.00	0.43	0.46	0.52	0.82	0.00	2.02	0.74
time (sec)	N/A	0.059	0.043	0.529	0.221	0.371	0.000	0.281	11.983

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	40	18	57	0	91	49
N.S.	1	1.00	0.77	0.83	0.38	1.19	0.00	1.90	1.02
time (sec)	N/A	0.013	0.016	0.484	0.215	0.366	0.000	0.279	11.983

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	114	143	0	318	0	255	0
N.S.	1	1.00	0.92	1.15	0.00	2.56	0.00	2.06	0.00
time (sec)	N/A	0.112	0.105	0.549	0.000	0.380	0.000	0.364	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	151	0	562	0	315	0
N.S.	1	1.00	0.83	1.14	0.00	4.26	0.00	2.39	0.00
time (sec)	N/A	0.093	0.242	0.553	0.000	0.439	0.000	0.355	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	165	275	0	1056	0	662	0
N.S.	1	1.00	0.80	1.33	0.00	5.10	0.00	3.20	0.00
time (sec)	N/A	0.176	0.468	0.537	0.000	0.401	0.000	0.486	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	202	443	0	1732	0	1234	0
N.S.	1	1.00	0.73	1.60	0.00	6.25	0.00	4.45	0.00
time (sec)	N/A	0.227	0.717	0.534	0.000	0.803	0.000	0.663	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	234	686	0	2610	0	1955	0
N.S.	1	1.00	0.67	1.98	0.00	7.52	0.00	5.63	0.00
time (sec)	N/A	0.283	1.059	0.538	0.000	1.346	0.000	1.733	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	195	275	413	472	0	2535	445
N.S.	1	1.00	0.58	0.82	1.23	1.40	0.00	7.54	1.32
time (sec)	N/A	0.359	0.201	0.545	0.261	0.349	0.000	0.384	12.561

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	137	180	294	340	0	1778	310
N.S.	1	1.00	0.51	0.67	1.09	1.26	0.00	6.61	1.15
time (sec)	N/A	0.241	0.153	0.515	0.246	0.316	0.000	0.358	12.428

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	108	192	230	0	1145	206
N.S.	1	1.00	0.45	0.54	0.96	1.15	0.00	5.72	1.03
time (sec)	N/A	0.146	0.101	0.543	0.234	0.302	0.000	0.325	12.126

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	54	59	107	137	0	632	109
N.S.	1	1.00	0.43	0.47	0.86	1.10	0.00	5.06	0.87
time (sec)	N/A	0.067	0.064	0.575	0.225	0.296	0.000	0.298	11.946

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	43	74	0	254	62
N.S.	1	1.00	0.77	0.88	0.90	1.54	0.00	5.29	1.29
time (sec)	N/A	0.013	0.023	0.559	0.214	0.286	0.000	0.287	12.110

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	132	253	0	408	0	506	0
N.S.	1	1.00	0.74	1.41	0.00	2.28	0.00	2.83	0.00
time (sec)	N/A	0.181	0.186	0.562	0.000	0.351	0.000	0.469	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	144	296	0	444	0	605	0
N.S.	1	1.00	0.81	1.66	0.00	2.49	0.00	3.40	0.00
time (sec)	N/A	0.151	0.333	0.553	0.000	0.331	0.000	0.424	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	135	266	0	840	0	581	0
N.S.	1	1.00	0.69	1.36	0.00	4.31	0.00	2.98	0.00
time (sec)	N/A	0.152	0.462	0.550	0.000	0.343	0.000	0.476	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	201	443	0	1434	0	1087	0
N.S.	1	1.00	0.76	1.67	0.00	5.41	0.00	4.10	0.00
time (sec)	N/A	0.220	0.731	0.565	0.000	0.437	0.000	0.688	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	240	655	0	2238	0	1785	0
N.S.	1	1.00	0.72	1.96	0.00	6.68	0.00	5.33	0.00
time (sec)	N/A	0.273	1.103	0.573	0.000	0.800	0.000	1.208	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	302	945	0	3204	0	2659	0
N.S.	1	1.00	0.75	2.33	0.00	7.91	0.00	6.57	0.00
time (sec)	N/A	0.329	2.194	0.582	0.000	2.278	0.000	2.799	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	205	275	498	567	0	4287	523
N.S.	1	1.00	0.61	0.82	1.48	1.69	0.00	12.76	1.56
time (sec)	N/A	0.361	0.206	0.585	0.258	0.320	0.000	0.483	12.826

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	147	180	362	416	0	3058	379
N.S.	1	1.00	0.55	0.67	1.35	1.55	0.00	11.37	1.41
time (sec)	N/A	0.239	0.148	0.573	0.268	0.314	0.000	0.414	12.926

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	100	108	243	284	0	2010	259
N.S.	1	1.00	0.50	0.54	1.22	1.42	0.00	10.05	1.30
time (sec)	N/A	0.143	0.099	0.564	0.236	0.400	0.000	0.376	12.675

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	64	59	141	173	0	1147	134
N.S.	1	1.00	0.51	0.47	1.13	1.38	0.00	9.18	1.07
time (sec)	N/A	0.061	0.071	0.514	0.224	0.407	0.000	0.322	12.433

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	60	91	0	477	79
N.S.	1	1.00	0.77	0.88	1.25	1.90	0.00	9.94	1.65
time (sec)	N/A	0.013	0.028	0.536	0.206	0.410	0.000	0.299	12.416

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	168	421	0	587	0	853	0
N.S.	1	1.00	0.71	1.78	0.00	2.49	0.00	3.61	0.00
time (sec)	N/A	0.281	0.216	0.576	0.000	0.409	0.000	0.637	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	183	513	0	672	0	1025	0
N.S.	1	1.00	0.78	2.18	0.00	2.86	0.00	4.36	0.00
time (sec)	N/A	0.232	0.445	0.602	0.000	0.433	0.000	0.602	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	189	516	0	683	0	1038	0
N.S.	1	1.00	0.77	2.10	0.00	2.78	0.00	4.22	0.00
time (sec)	N/A	0.206	0.534	0.557	0.000	0.551	0.000	0.583	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	171	431	0	1140	0	940	0
N.S.	1	1.00	0.68	1.70	0.00	4.51	0.00	3.72	0.00
time (sec)	N/A	0.212	0.776	0.562	0.000	0.818	0.000	0.707	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	244	655	0	1862	0	1574	0
N.S.	1	1.00	0.76	2.03	0.00	5.76	0.00	4.87	0.00
time (sec)	N/A	0.295	1.212	0.551	0.000	0.548	0.000	1.746	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	301	914	0	2750	0	2407	0
N.S.	1	1.00	0.77	2.33	0.00	7.00	0.00	6.12	0.00
time (sec)	N/A	0.336	1.689	0.566	0.000	1.368	0.000	2.828	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	370	1251	0	3872	0	3412	0
N.S.	1	1.00	0.80	2.70	0.00	8.36	0.00	7.37	0.00
time (sec)	N/A	0.408	2.720	0.579	0.000	3.973	0.000	7.620	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	191	501	0	841	0	989	0
N.S.	1	1.00	0.61	1.60	0.00	2.69	0.00	3.16	0.00
time (sec)	N/A	0.338	0.396	0.571	0.000	1.023	0.000	0.644	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	156	318	0	655	0	633	0
N.S.	1	1.00	0.64	1.30	0.00	2.68	0.00	2.59	0.00
time (sec)	N/A	0.222	0.314	0.576	0.000	0.798	0.000	0.520	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	138	191	0	521	0	380	0
N.S.	1	1.00	0.82	1.13	0.00	3.08	0.00	2.25	0.00
time (sec)	N/A	0.142	0.249	0.627	0.000	0.750	0.000	0.435	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	94	102	0	343	0	161	0
N.S.	1	1.00	0.90	0.97	0.00	3.27	0.00	1.53	0.00
time (sec)	N/A	0.075	0.137	0.569	0.000	0.746	0.000	0.370	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	0	114	0	194	100
N.S.	1	1.00	0.82	0.74	0.00	1.87	0.00	3.18	1.64
time (sec)	N/A	0.040	0.046	0.531	0.000	0.345	0.000	0.306	13.992

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	61	0	288	0	676	147
N.S.	1	1.00	0.53	0.47	0.00	2.23	0.00	5.24	1.14
time (sec)	N/A	0.084	0.086	0.574	0.000	0.435	0.000	0.353	14.283

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	111	0	572	0	1612	242
N.S.	1	1.00	0.53	0.56	0.00	2.89	0.00	8.14	1.22
time (sec)	N/A	0.135	0.126	0.556	0.000	1.041	0.000	0.423	14.539

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	183	0	953	0	2971	357
N.S.	1	1.00	0.57	0.69	0.00	3.57	0.00	11.13	1.34
time (sec)	N/A	0.190	0.182	0.572	0.000	2.747	0.000	0.547	14.094

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	183	638	0	971	0	834	0
N.S.	1	1.00	0.61	2.12	0.00	3.23	0.00	2.77	0.00
time (sec)	N/A	0.278	0.462	0.550	0.000	1.018	0.000	0.627	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	142	386	0	725	0	527	0
N.S.	1	1.00	0.63	1.70	0.00	3.19	0.00	2.32	0.00
time (sec)	N/A	0.194	0.302	0.549	0.000	0.745	0.000	0.517	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	117	200	0	569	0	408	0
N.S.	1	1.00	0.73	1.24	0.00	3.53	0.00	2.53	0.00
time (sec)	N/A	0.122	0.168	0.515	0.000	0.690	0.000	0.472	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	55	0	125	0	218	147
N.S.	1	1.00	0.82	0.90	0.00	2.05	0.00	3.57	2.41
time (sec)	N/A	0.045	0.040	0.546	0.000	0.293	0.000	0.315	13.249

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	64	70	0	325	0	793	151
N.S.	1	1.00	0.52	0.56	0.00	2.62	0.00	6.40	1.22
time (sec)	N/A	0.083	0.093	0.559	0.000	0.348	0.000	0.380	13.591

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	105	120	0	649	0	2618	268
N.S.	1	1.00	0.55	0.62	0.00	3.38	0.00	13.64	1.40
time (sec)	N/A	0.134	0.130	0.555	0.000	0.420	0.000	1.841	13.978

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	150	192	0	1062	0	6137	414
N.S.	1	1.00	0.57	0.73	0.00	4.05	0.00	23.42	1.58
time (sec)	N/A	0.196	0.174	0.556	0.000	1.397	0.000	24.647	14.396

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	188	642	0	1055	0	1105	0
N.S.	1	1.00	0.65	2.22	0.00	3.65	0.00	3.82	0.00
time (sec)	N/A	0.250	0.422	0.547	0.000	0.892	0.000	0.855	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	134	333	0	755	0	685	0
N.S.	1	1.00	0.61	1.52	0.00	3.45	0.00	3.13	0.00
time (sec)	N/A	0.174	0.263	0.550	0.000	0.782	0.000	0.651	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	193	0	316	169
N.S.	1	1.00	0.83	0.87	0.00	3.06	0.00	5.02	2.68
time (sec)	N/A	0.042	0.044	0.500	0.000	0.320	0.000	0.446	12.871

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	67	72	0	318	0	552	246
N.S.	1	1.00	0.52	0.56	0.00	2.48	0.00	4.31	1.92
time (sec)	N/A	0.089	0.095	0.543	0.000	0.439	0.000	0.353	13.620

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	103	121	0	667	0	1923	255
N.S.	1	1.00	0.53	0.62	0.00	3.44	0.00	9.91	1.31
time (sec)	N/A	0.135	0.133	0.568	0.000	0.534	0.000	0.496	13.938

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	152	191	0	1065	0	4668	416
N.S.	1	1.00	0.58	0.73	0.00	4.10	0.00	17.95	1.60
time (sec)	N/A	0.195	0.188	0.588	0.000	1.032	0.000	3.080	14.247

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	235	732	0	1065	0	6752	0
N.S.	1	1.00	0.61	1.90	0.00	2.77	0.00	17.54	0.00
time (sec)	N/A	0.435	0.523	0.569	0.000	2.538	0.000	2.147	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	188	504	0	847	0	3056	0
N.S.	1	1.00	0.60	1.61	0.00	2.71	0.00	9.76	0.00
time (sec)	N/A	0.319	0.427	0.575	0.000	1.381	0.000	1.196	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	162	319	0	657	0	995	0
N.S.	1	1.00	0.67	1.32	0.00	2.73	0.00	4.13	0.00
time (sec)	N/A	0.214	0.387	0.524	0.000	1.293	0.000	0.650	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	144	188	0	516	0	409	0
N.S.	1	1.00	0.86	1.13	0.00	3.09	0.00	2.45	0.00
time (sec)	N/A	0.131	0.263	0.547	0.000	0.764	0.000	0.465	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	133	187	0	521	0	426	0
N.S.	1	1.00	0.84	1.18	0.00	3.30	0.00	2.70	0.00
time (sec)	N/A	0.116	0.187	0.568	0.000	0.724	0.000	0.520	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	53	0	169	0	311	136
N.S.	1	1.00	0.83	0.84	0.00	2.68	0.00	4.94	2.16
time (sec)	N/A	0.038	0.046	0.595	0.000	0.306	0.000	0.513	12.994

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	70	0	402	0	762	187
N.S.	1	1.00	0.53	0.54	0.00	3.12	0.00	5.91	1.45
time (sec)	N/A	0.083	0.101	0.577	0.000	0.351	0.000	0.841	13.348

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	119	0	748	0	1426	289
N.S.	1	1.00	0.53	0.60	0.00	3.78	0.00	7.20	1.46
time (sec)	N/A	0.134	0.136	0.568	0.000	0.555	0.000	0.792	13.574

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	191	0	1179	0	2280	409
N.S.	1	1.00	0.57	0.72	0.00	4.42	0.00	8.54	1.53
time (sec)	N/A	0.199	0.218	0.562	0.000	1.086	0.000	1.184	13.456

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	244	732	0	1059	0	8807	0
N.S.	1	1.00	0.64	1.92	0.00	2.77	0.00	23.05	0.00
time (sec)	N/A	0.441	0.631	0.544	0.000	1.639	0.000	2.739	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	199	504	0	847	0	3057	0
N.S.	1	1.00	0.64	1.63	0.00	2.73	0.00	9.86	0.00
time (sec)	N/A	0.317	0.517	0.566	0.000	1.006	0.000	1.206	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	165	315	0	651	0	709	0
N.S.	1	1.00	0.69	1.32	0.00	2.74	0.00	2.98	0.00
time (sec)	N/A	0.227	0.401	0.563	0.000	0.804	0.000	0.551	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	158	373	0	663	0	559	0
N.S.	1	1.00	0.71	1.68	0.00	2.99	0.00	2.52	0.00
time (sec)	N/A	0.187	0.374	0.594	0.000	1.150	0.000	0.591	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	152	321	0	685	0	686	0
N.S.	1	1.00	0.71	1.50	0.00	3.20	0.00	3.21	0.00
time (sec)	N/A	0.169	0.309	0.575	0.000	1.107	0.000	0.724	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	232	0	446	232
N.S.	1	1.00	0.83	0.87	0.00	3.68	0.00	7.08	3.68
time (sec)	N/A	0.047	0.061	0.579	0.000	0.417	0.000	0.953	12.483

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	526	0	1001	247
N.S.	1	1.00	0.53	0.77	0.00	4.08	0.00	7.76	1.91
time (sec)	N/A	0.090	0.123	0.544	0.000	0.498	0.000	0.740	12.656

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	918	0	1760	377
N.S.	1	1.00	0.53	0.85	0.00	4.64	0.00	8.89	1.90
time (sec)	N/A	0.138	0.181	0.575	0.000	1.032	0.000	1.113	13.496

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	1420	0	2715	519
N.S.	1	1.00	0.57	0.97	0.00	5.32	0.00	10.17	1.94
time (sec)	N/A	0.193	0.249	0.596	0.000	1.444	0.000	1.693	13.888

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	303	1005	0	1331	0	18597	0
N.S.	1	1.00	0.68	2.24	0.00	2.97	0.00	41.51	0.00
time (sec)	N/A	0.556	0.815	0.620	0.000	3.552	0.000	5.441	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	244	732	0	1065	0	6756	0
N.S.	1	1.00	0.65	1.95	0.00	2.83	0.00	17.97	0.00
time (sec)	N/A	0.414	0.690	0.593	0.000	1.735	0.000	2.203	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	189	498	0	837	0	1098	0
N.S.	1	1.00	0.62	1.64	0.00	2.75	0.00	3.61	0.00
time (sec)	N/A	0.302	0.566	0.578	0.000	1.030	0.000	0.695	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	199	625	0	915	0	894	0
N.S.	1	1.00	0.68	2.13	0.00	3.11	0.00	3.04	0.00
time (sec)	N/A	0.265	0.453	0.572	0.000	0.823	0.000	0.768	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	187	628	0	973	0	1178	0
N.S.	1	1.00	0.66	2.21	0.00	3.43	0.00	4.15	0.00
time (sec)	N/A	0.240	0.459	0.566	0.000	0.785	0.000	1.010	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	188	501	0	933	0	1224	0
N.S.	1	1.00	0.69	1.83	0.00	3.41	0.00	4.47	0.00
time (sec)	N/A	0.237	0.355	0.582	0.000	1.180	0.000	1.591	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	299	0	602	325
N.S.	1	1.00	0.83	1.00	0.00	4.75	0.00	9.56	5.16
time (sec)	N/A	0.047	0.075	0.579	0.000	0.442	0.000	0.778	12.820

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	639	0	1275	315
N.S.	1	1.00	0.53	0.77	0.00	4.95	0.00	9.88	2.44
time (sec)	N/A	0.098	0.174	0.575	0.000	0.599	0.000	1.163	13.083

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	115	169	0	1101	0	2167	465
N.S.	1	1.00	0.58	0.85	0.00	5.56	0.00	10.94	2.35
time (sec)	N/A	0.144	0.190	0.559	0.000	1.082	0.000	1.772	13.494

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	162	260	0	1648	0	3241	627
N.S.	1	1.00	0.61	0.97	0.00	6.17	0.00	12.14	2.35
time (sec)	N/A	0.214	0.261	0.565	0.000	1.568	0.000	2.660	13.617

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	0
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	0	0
N.S.	1	1.06	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0	0
N.S.	1	1.15	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	0
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0	0
N.S.	1	1.17	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	134	527	331	705	0	1926	615
N.S.	1	1.00	0.39	1.54	0.97	2.06	0.00	5.62	1.79
time (sec)	N/A	0.290	0.275	4.168	0.239	0.331	0.000	0.493	12.306

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	131	235	193	350	0	929	327
N.S.	1	1.00	0.53	0.96	0.78	1.42	0.00	3.78	1.33
time (sec)	N/A	0.138	0.196	2.121	0.236	0.307	0.000	0.537	12.132

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	94	145	0	347	139
N.S.	1	1.00	0.45	0.59	0.63	0.97	0.00	2.31	0.93
time (sec)	N/A	0.056	0.108	1.208	0.215	0.288	0.000	0.300	12.062

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	33	57	0	81	57
N.S.	1	1.00	0.78	1.06	0.61	1.06	0.00	1.50	1.06
time (sec)	N/A	0.012	0.010	0.783	0.203	0.292	0.000	0.300	11.773

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	82	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	84	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	64	49	66	0	106	63
N.S.	1	1.00	0.80	0.98	0.75	1.02	0.00	1.63	0.97
time (sec)	N/A	0.030	0.013	3.875	0.222	0.293	0.000	0.302	12.029

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	32	35	0	0	0
N.S.	1	1.00	0.82	0.00	0.41	0.45	0.00	0.00	0.00
time (sec)	N/A	0.093	0.025	0.000	0.207	0.291	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	222	145	0	0	0	0	0	0
N.S.	1	1.04	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	380	623	693	597	0	1789	653
N.S.	1	1.00	0.76	1.24	1.38	1.19	0.00	3.57	1.30
time (sec)	N/A	0.521	0.306	0.547	0.260	0.311	0.000	0.366	12.765

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	264	407	484	408	0	1185	438
N.S.	1	1.00	0.64	0.99	1.17	0.99	0.00	2.88	1.06
time (sec)	N/A	0.375	0.201	0.550	0.258	0.305	0.000	0.338	12.400

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	169	237	309	256	0	704	279
N.S.	1	1.00	0.53	0.74	0.96	0.80	0.00	2.19	0.87
time (sec)	N/A	0.252	0.133	0.549	0.233	0.308	0.000	0.318	12.290

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	96	113	168	141	0	346	152
N.S.	1	1.00	0.46	0.54	0.80	0.67	0.00	1.66	0.73
time (sec)	N/A	0.131	0.079	0.536	0.222	0.301	0.000	0.308	11.996

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	54	51	65	73	0	142	85
N.S.	1	1.00	0.50	0.47	0.60	0.67	0.00	1.30	0.78
time (sec)	N/A	0.038	0.021	0.521	0.221	0.294	0.000	0.280	11.914

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	140	153	0	511	0	284	0
N.S.	1	1.00	1.01	1.10	0.00	3.68	0.00	2.04	0.00
time (sec)	N/A	0.119	0.156	0.552	0.000	0.302	0.000	0.409	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	337	0	896	0	469	0
N.S.	1	1.00	0.91	1.98	0.00	5.27	0.00	2.76	0.00
time (sec)	N/A	0.143	0.444	0.560	0.000	0.318	0.000	0.406	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	200	663	0	1704	0	1071	0
N.S.	1	1.00	0.77	2.54	0.00	6.53	0.00	4.10	0.00
time (sec)	N/A	0.223	0.828	0.583	0.000	0.351	0.000	0.527	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	279	1132	0	2736	0	1879	0
N.S.	1	1.00	0.79	3.23	0.00	7.79	0.00	5.35	0.00
time (sec)	N/A	0.317	1.031	0.557	0.000	0.959	0.000	0.742	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	247	334	365	251	0	353	1768
N.S.	1	1.00	0.76	1.03	1.13	0.77	0.00	1.09	5.46
time (sec)	N/A	0.562	0.905	0.603	0.290	0.318	0.000	0.314	39.014

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	132	199	171	134	0	159	897
N.S.	1	1.00	0.80	1.20	1.03	0.81	0.00	0.96	5.40
time (sec)	N/A	0.210	0.523	0.532	0.292	0.335	0.000	0.288	27.554

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	57	67	0	60	232
N.S.	1	1.00	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.041	0.019	0.459	0.286	0.301	0.000	0.283	16.600

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	154	1759	0	4313	0	681	33018
N.S.	1	1.00	0.55	6.24	0.00	15.29	0.00	2.41	117.09
time (sec)	N/A	0.336	0.308	0.629	0.000	0.428	0.000	0.419	96.416

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	571	571	1548	41837	0	35403	0	0	0
N.S.	1	1.00	2.71	73.27	0.00	62.00	0.00	0.00	0.00
time (sec)	N/A	3.908	2.668	1.015	0.000	33.260	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	244	567	371	376	0	736	0
N.S.	1	1.00	0.88	2.05	1.34	1.36	0.00	2.67	0.00
time (sec)	N/A	0.370	0.937	0.615	0.318	0.308	0.000	0.357	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	144	346	176	204	0	391	0
N.S.	1	1.00	1.07	2.56	1.30	1.51	0.00	2.90	0.00
time (sec)	N/A	0.119	0.513	0.622	0.299	0.293	0.000	0.316	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	57	151	61	101	0	186	0
N.S.	1	1.00	1.42	3.78	1.52	2.52	0.00	4.65	0.00
time (sec)	N/A	0.031	0.331	0.453	0.281	0.306	0.000	0.295	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	443	443	456	11142	0	21628	0	0	0
N.S.	1	1.00	1.03	25.15	0.00	48.82	0.00	0.00	0.00
time (sec)	N/A	0.910	0.562	0.613	0.000	6.566	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	939	938	3830	108969	0	0	0	0	0
N.S.	1	1.00	4.08	116.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	11.132	7.473	3.576	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	0
N.S.	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	249	1786	811	2032	24206	3830	1943
N.S.	1	1.00	0.91	6.49	2.95	7.39	88.02	13.93	7.07
time (sec)	N/A	0.163	0.301	0.577	0.265	0.353	4.357	0.333	13.019

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	1029	512	1122	11946	2133	1133
N.S.	1	1.00	0.90	4.95	2.46	5.39	57.43	10.25	5.45
time (sec)	N/A	0.111	0.206	0.556	0.227	0.386	2.211	0.313	12.353

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	449	289	549	4952	1008	572
N.S.	1	1.00	0.89	3.08	1.98	3.76	33.92	6.90	3.92
time (sec)	N/A	0.069	0.156	0.500	0.219	0.331	1.157	0.284	11.958

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	147	135	218	1489	366	211
N.S.	1	1.00	0.87	1.75	1.61	2.60	17.73	4.36	2.51
time (sec)	N/A	0.035	0.114	0.469	0.234	0.316	0.581	0.273	12.083

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	93	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	106	0	0	0	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.155	0.000	0.000	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	106	0	0	0	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	227	179	0	0	0	0	0	0
N.S.	1	0.98	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	84	87	99	420	89	84
N.S.	1	1.00	1.02	1.01	1.05	1.19	5.06	1.07	1.01
time (sec)	N/A	0.066	0.035	0.484	0.202	0.295	78.093	0.266	12.098

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	177	262	255	313	0	289	266
N.S.	1	1.00	0.96	1.42	1.39	1.70	0.00	1.57	1.45
time (sec)	N/A	0.180	0.096	0.618	0.201	0.541	0.000	0.270	12.142

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	476	837	721	736	0	953	794
N.S.	1	1.00	0.90	1.58	1.36	1.39	0.00	1.79	1.50
time (sec)	N/A	0.561	0.260	0.645	0.244	2.796	0.000	0.304	13.435

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	246	244	0	0	0	388	12173
N.S.	1	1.00	1.00	0.99	0.00	0.00	0.00	1.58	49.48
time (sec)	N/A	0.276	0.209	0.796	0.000	0.000	0.000	0.292	32.761

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	644	644	710	2228	0	0	0	3400	130035
N.S.	1	1.00	1.10	3.46	0.00	0.00	0.00	5.28	201.92
time (sec)	N/A	1.155	1.428	1.488	0.000	0.000	0.000	0.301	54.584

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	412	285	429	429	695	568	283
N.S.	1	1.00	1.44	0.99	1.49	1.49	2.42	1.98	0.99
time (sec)	N/A	0.290	0.293	0.526	0.222	0.565	1.226	0.279	0.161

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	256	192	261	260	427	363	204
N.S.	1	1.00	1.21	0.91	1.23	1.23	2.01	1.71	0.96
time (sec)	N/A	0.201	0.193	0.530	0.203	0.475	1.049	0.271	0.089

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	103	129	125	209	196	125
N.S.	1	1.00	0.96	0.75	0.94	0.91	1.53	1.43	0.91
time (sec)	N/A	0.069	0.101	0.496	0.217	0.504	0.849	0.282	0.085

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	46	77	54	102	77	58
N.S.	1	1.00	0.74	0.63	1.05	0.74	1.40	1.05	0.79
time (sec)	N/A	0.026	0.039	0.477	0.224	0.470	0.451	0.277	11.826

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	104	93	0	341	160	130	117
N.S.	1	1.00	0.90	0.80	0.00	2.94	1.38	1.12	1.01
time (sec)	N/A	0.107	0.151	0.542	0.000	0.447	2.578	0.282	0.148

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	144	150	152	0	637	0	176	146
N.S.	1	1.03	1.07	1.09	0.00	4.55	0.00	1.26	1.04
time (sec)	N/A	0.168	0.432	0.552	0.000	0.494	0.000	0.279	0.247

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	210	203	201	0	1096	0	383	270
N.S.	1	1.02	0.99	0.98	0.00	5.32	0.00	1.86	1.31
time (sec)	N/A	0.230	0.696	0.566	0.000	0.494	0.000	0.283	0.295

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	406	303	437	438	568	687	394
N.S.	1	1.00	1.42	1.06	1.53	1.54	1.99	2.41	1.38
time (sec)	N/A	0.236	0.325	0.570	0.212	0.418	32.418	0.311	0.121

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	252	196	269	269	350	407	270
N.S.	1	1.00	1.20	0.93	1.28	1.28	1.67	1.94	1.29
time (sec)	N/A	0.169	0.217	0.534	0.207	0.375	11.823	0.301	11.856

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	128	107	137	135	182	195	147
N.S.	1	1.00	0.95	0.79	1.01	1.00	1.35	1.44	1.09
time (sec)	N/A	0.058	0.103	0.487	0.206	0.399	3.246	0.301	11.750

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	47	66	63	94	74	58
N.S.	1	1.00	0.76	0.66	0.93	0.89	1.32	1.04	0.82
time (sec)	N/A	0.022	0.047	0.475	0.203	0.389	1.046	0.270	0.063

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	124	122	0	540	172	124	162
N.S.	1	1.00	1.02	1.00	0.00	4.43	1.41	1.02	1.33
time (sec)	N/A	0.132	0.220	0.539	0.000	0.426	6.446	0.282	11.826

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	176	175	0	1088	0	286	218
N.S.	1	1.00	1.07	1.06	0.00	6.59	0.00	1.73	1.32
time (sec)	N/A	0.227	0.583	0.563	0.000	0.497	0.000	0.297	0.315

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	297	279	0	1883	0	472	363
N.S.	1	1.00	1.20	1.12	0.00	7.59	0.00	1.90	1.46
time (sec)	N/A	0.385	1.170	0.652	0.000	0.591	0.000	0.296	12.175

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	191	102	231	0	201	0	16	916
N.S.	1	2.10	1.12	2.54	0.00	2.21	0.00	0.18	10.07
time (sec)	N/A	0.106	0.297	0.602	0.000	0.426	0.000	0.295	14.728

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	380	0	190	833
N.S.	1	1.00	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.108	0.359	0.480	0.000	0.512	0.000	0.307	17.077

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	289	1207	0	852	0	480	0
N.S.	1	1.00	0.87	3.62	0.00	2.56	0.00	1.44	0.00
time (sec)	N/A	0.221	0.820	0.483	0.000	0.617	0.000	0.323	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	217	763	0	576	0	313	1832
N.S.	1	1.00	0.88	3.10	0.00	2.34	0.00	1.27	7.45
time (sec)	N/A	0.158	3.547	0.486	0.000	0.536	0.000	0.301	109.822

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	380	0	190	833
N.S.	1	1.00	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.084	0.056	0.479	0.000	0.476	0.000	0.305	0.003

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	131	697	0	588	0	195	0
N.S.	1	1.00	1.02	5.40	0.00	4.56	0.00	1.51	0.00
time (sec)	N/A	0.083	0.340	0.475	0.000	1.513	0.000	0.343	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	145	773	0	792	0	491	0
N.S.	1	1.00	0.91	4.83	0.00	4.95	0.00	3.07	0.00
time (sec)	N/A	0.106	0.225	0.477	0.000	3.310	0.000	0.384	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	177	210	0	353	0	1175	260
N.S.	1	1.00	0.89	1.06	0.00	1.78	0.00	5.93	1.31
time (sec)	N/A	0.126	0.185	0.497	0.000	7.211	0.000	0.419	13.213

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	301	427	0	641	0	2035	452
N.S.	1	1.00	1.07	1.52	0.00	2.28	0.00	7.24	1.61
time (sec)	N/A	0.173	0.346	0.479	0.000	22.889	0.000	0.524	13.496

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	167	834	0	580	0	263	0
N.S.	1	1.00	0.67	3.35	0.00	2.33	0.00	1.06	0.00
time (sec)	N/A	0.171	0.441	0.473	0.000	0.754	0.000	0.337	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	189	571	0	546	0	739	0
N.S.	1	1.00	0.79	2.38	0.00	2.28	0.00	3.08	0.00
time (sec)	N/A	0.145	0.455	0.444	0.000	0.422	0.000	0.370	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	163	392	0	414	0	446	1797
N.S.	1	1.00	0.93	2.23	0.00	2.35	0.00	2.53	10.21
time (sec)	N/A	0.111	10.344	0.474	0.000	0.410	0.000	0.339	107.002

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	103	247	0	308	0	146	893
N.S.	1	1.00	0.84	2.02	0.00	2.52	0.00	1.20	7.32
time (sec)	N/A	0.079	0.262	0.485	0.000	0.408	0.000	0.292	32.857

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	438	0	463	0	197	0
N.S.	1	1.00	1.13	4.06	0.00	4.29	0.00	1.82	0.00
time (sec)	N/A	0.072	0.262	0.483	0.000	0.389	0.000	0.322	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	99	601	0	665	0	220	0
N.S.	1	1.00	0.85	5.18	0.00	5.73	0.00	1.90	0.00
time (sec)	N/A	0.073	0.175	0.483	0.000	0.635	0.000	0.355	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	115	122	0	293	0	359	268
N.S.	1	1.00	0.86	0.92	0.00	2.20	0.00	2.70	2.02
time (sec)	N/A	0.081	0.131	0.477	0.000	0.898	0.000	0.380	13.362

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	185	207	0	487	0	560	389
N.S.	1	1.00	0.98	1.10	0.00	2.58	0.00	2.96	2.06
time (sec)	N/A	0.117	0.165	0.478	0.000	1.657	0.000	0.444	13.532

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	417	417	473	11686	0	0	0	0	0
N.S.	1	1.00	1.13	28.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.935	3.558	0.648	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	266	5482	0	4471	0	0	0
N.S.	1	1.00	0.93	19.24	0.00	15.69	0.00	0.00	0.00
time (sec)	N/A	0.267	10.698	0.672	0.000	27.867	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	410	5507	0	19727	0	0	0
N.S.	1	1.00	1.43	19.19	0.00	68.74	0.00	0.00	0.00
time (sec)	N/A	0.190	2.524	0.662	0.000	118.801	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	429	429	543	47351	0	0	0	0	0
N.S.	1	1.00	1.27	110.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	3.468	0.679	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	513	879	0	0	0	0	0
N.S.	1	1.00	0.96	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.984	3.040	0.891	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	316	570	0	0	0	0	0
N.S.	1	1.00	0.97	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	1.423	0.806	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	212	335	0	0	0	0	0
N.S.	1	1.00	0.97	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.878	0.723	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	175	238	0	992	0	0	0
N.S.	1	1.00	1.15	1.57	0.00	6.53	0.00	0.00	0.00
time (sec)	N/A	0.085	0.140	0.694	0.000	1.114	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	212	673	0	0	0	0	0
N.S.	1	1.00	0.93	2.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.634	0.765	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	228	1359	0	0	0	0	0
N.S.	1	1.00	0.47	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.903	0.905	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	609	2512	0	0	0	1824	0
N.S.	1	1.00	0.90	3.73	0.00	0.00	0.00	2.71	0.00
time (sec)	N/A	0.517	10.904	0.991	0.000	0.000	0.000	1.071	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	933	933	858	3777	0	0	0	8076	0
N.S.	1	1.00	0.92	4.05	0.00	0.00	0.00	8.66	0.00
time (sec)	N/A	0.724	12.448	1.162	0.000	0.000	0.000	5.559	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1098	1098	743	1409	0	0	0	0	0
N.S.	1	1.00	0.68	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.196	11.510	0.968	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	662	536	998	0	0	0	0	0
N.S.	1	1.00	0.81	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.894	10.879	0.891	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	427	735	0	0	0	0	0
N.S.	1	1.00	0.97	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	2.481	0.727	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	247	383	0	1523	0	0	0
N.S.	1	1.00	0.98	1.52	0.00	6.04	0.00	0.00	0.00
time (sec)	N/A	0.214	0.110	0.677	0.000	150.148	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	323	599	0	0	0	0	0
N.S.	1	1.00	0.66	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	10.719	0.822	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	787	787	357	900	0	0	0	0	0
N.S.	1	1.00	0.45	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.816	10.930	0.921	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1066	1066	1036	4242	0	0	0	0	0
N.S.	1	1.00	0.97	3.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.988	12.294	0.961	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	886	886	647	1248	0	0	0	0	0
N.S.	1	1.00	0.73	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.016	11.704	0.893	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	380	556	0	0	0	0	0
N.S.	1	1.00	0.88	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.797	1.975	1.095	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	247	355	0	0	0	0	0
N.S.	1	1.00	0.91	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	1.146	0.891	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	184	259	0	0	0	0	0
N.S.	1	1.00	1.05	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.695	0.802	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	142	199	0	1071	0	0	0
N.S.	1	1.00	1.08	1.52	0.00	8.18	0.00	0.00	0.00
time (sec)	N/A	0.057	0.541	0.710	0.000	21.849	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	92	157	0	343	0	71	0
N.S.	1	1.00	1.16	1.99	0.00	4.34	0.00	0.90	0.00
time (sec)	N/A	0.023	0.010	0.658	0.000	0.327	0.000	0.294	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	207	327	0	1952	0	0	0
N.S.	1	1.00	1.14	1.80	0.00	10.73	0.00	0.00	0.00
time (sec)	N/A	0.137	0.738	0.782	0.000	85.212	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	256	609	0	0	0	0	0
N.S.	1	1.00	0.75	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	10.645	0.876	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	549	1185	0	0	0	2239	0
N.S.	1	1.00	0.94	2.02	0.00	0.00	0.00	3.81	0.00
time (sec)	N/A	0.472	11.309	0.908	0.000	0.000	0.000	0.772	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	587	1026	0	0	0	0	0
N.S.	1	1.00	1.18	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	12.463	1.035	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	354	740	0	0	0	0	0
N.S.	1	1.00	0.99	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	3.950	0.836	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	244	555	0	2023	0	773	0
N.S.	1	1.00	1.02	2.31	0.00	8.43	0.00	3.22	0.00
time (sec)	N/A	0.193	1.148	0.773	0.000	3.325	0.000	0.286	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	198	445	0	1663	0	583	0
N.S.	1	1.00	1.06	2.38	0.00	8.89	0.00	3.12	0.00
time (sec)	N/A	0.086	0.779	0.700	0.000	2.947	0.000	0.286	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	190	400	0	1349	0	461	0
N.S.	1	1.00	1.23	2.58	0.00	8.70	0.00	2.97	0.00
time (sec)	N/A	0.062	0.158	0.688	0.000	0.604	0.000	0.278	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	347	815	0	0	0	0	0
N.S.	1	1.00	0.99	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	3.109	0.774	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	623	1481	0	0	0	0	0
N.S.	1	1.00	0.97	2.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	12.751	0.948	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1013	2685	0	0	0	14979	0
N.S.	1	1.00	0.95	2.52	0.00	0.00	0.00	14.08	0.00
time (sec)	N/A	1.106	15.227	1.136	0.000	0.000	0.000	4.142	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1551	1551	26600	3254	0	1741	0	0	0
N.S.	1	1.00	17.15	2.10	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	5.021	37.075	3.119	0.000	0.153	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1015	1015	15781	1936	0	1132	0	0	0
N.S.	1	1.00	15.55	1.91	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	2.424	35.974	3.522	0.000	0.136	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	8432	1229	0	726	0	0	0
N.S.	1	1.00	12.93	1.88	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.613	35.210	1.947	0.000	0.121	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	1052	892	0	481	0	0	0
N.S.	1	1.00	2.05	1.74	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.334	32.546	2.029	0.000	0.107	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	969	1470	1245	0	0	0	0	0
N.S.	1	1.27	1.92	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.524	36.442	3.880	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	934	1473	1190	0	0	0	0	0
N.S.	1	1.26	1.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.005	34.833	0.828	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1034	1705	33765	1593	0	0	0	0	0
N.S.	1	1.65	32.65	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.902	36.667	1.293	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1098	1098	17771	1845	0	1241	0	0	0
N.S.	1	1.00	16.18	1.68	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	2.384	36.997	4.703	0.000	0.123	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	755	755	10030	1272	0	828	0	0	0
N.S.	1	1.00	13.28	1.68	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	1.192	35.649	2.481	0.000	0.141	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	792	955	0	557	0	0	0
N.S.	1	1.00	1.53	1.84	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.326	29.596	2.695	0.000	0.107	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	936	811	0	417	0	0	0
N.S.	1	1.00	2.11	1.83	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.200	27.416	1.276	0.000	0.098	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	700	700	1261	1114	0	0	0	0	0
N.S.	1	1.00	1.80	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.149	31.833	1.158	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	957	1471	1208	0	0	0	0	0
N.S.	1	1.30	2.00	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.965	33.660	2.063	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1049	1747	36617	1634	0	0	0	0	0
N.S.	1	1.67	34.91	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.941	36.623	3.160	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	774	1402	1283	0	879	0	0	0
N.S.	1	1.00	1.81	1.66	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	1.222	34.650	3.617	0.000	0.190	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	567	1002	985	0	610	0	0	0
N.S.	1	1.00	1.77	1.74	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.539	31.192	4.750	0.000	0.153	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	638	821	0	448	0	0	0
N.S.	1	1.00	1.41	1.82	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.255	26.338	1.536	0.000	0.148	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	365	746	0	359	0	0	0
N.S.	1	1.00	1.94	3.97	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.037	21.598	0.635	0.000	0.158	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	379	674	0	0	0	0	0
N.S.	1	1.00	0.81	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.947	23.143	2.269	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	994	994	1502	1229	0	0	0	0	0
N.S.	1	1.00	1.51	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.182	34.192	3.474	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1786	1786	36634	1698	0	0	0	0	0
N.S.	1	1.00	20.51	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.986	36.792	4.239	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	675	675	1385	1122	0	0	0	0	0
N.S.	1	1.00	2.05	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	13.596	2.003	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	1245	0	0	0	0	0
N.S.	1	1.00	32.63	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.225	36.529	6.523	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	855	985	0	633	0	0	0
N.S.	1	1.00	1.35	1.56	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.639	34.486	6.151	0.000	0.136	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	981	834	0	471	0	0	0
N.S.	1	1.00	2.05	1.74	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.339	30.397	3.249	0.000	0.108	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	814	746	0	367	0	0	0
N.S.	1	1.00	2.07	1.90	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.137	25.686	1.810	0.000	0.099	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	128	0	0	0
N.S.	1	1.00	1.63	1.52	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.052	22.451	1.655	0.000	0.089	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	499	330	0	0	0	0	0
N.S.	1	1.00	1.78	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.731	24.038	3.344	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1037	1037	1513	1347	0	0	0	0	0
N.S.	1	1.00	1.46	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.161	34.534	3.352	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1114	1762	40396	1686	0	0	0	0	0
N.S.	1	1.58	36.26	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.759	37.301	4.552	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	950	1264	0	0	0	0	0
N.S.	1	1.00	1.72	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.971	26.224	3.229	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14762	1505	0	0	0	0	0
N.S.	1	1.00	13.12	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.441	36.106	4.085	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	475	475	1118	1463	0	0	0	0	0
N.S.	1	1.00	2.35	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	28.868	5.121	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	588	588	375	601	0	0	0	0	0
N.S.	1	1.00	0.64	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	26.553	6.056	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	198	1249	684	1381	15757	2735	1354
N.S.	1	1.00	0.90	5.68	3.11	6.28	71.62	12.43	6.15
time (sec)	N/A	0.132	0.332	0.555	0.239	0.331	2.870	0.289	13.395

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	180	503	352	613	5930	1156	602
N.S.	1	1.00	1.25	3.49	2.44	4.26	41.18	8.03	4.18
time (sec)	N/A	0.072	0.316	0.487	0.226	0.295	1.270	0.283	13.538

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	134	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	243	157	0	0	0	0	0	0
N.S.	1	0.99	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	492	4653	2034	4747	74400	10486	4871
N.S.	1	1.00	0.94	8.86	3.87	9.04	141.71	19.97	9.28
time (sec)	N/A	0.354	1.145	0.776	0.284	0.376	11.368	0.361	14.951

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	655	2149	1118	2368	32864	4936	2307
N.S.	1	1.00	2.11	6.91	3.59	7.61	105.67	15.87	7.42
time (sec)	N/A	0.229	1.106	0.643	0.237	0.339	5.649	0.324	13.680

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	265	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	261	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.683	1.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	461	461	257	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.894	1.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	91	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	110	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	152	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	251	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	252	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	156	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	149	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	150	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.173	0.000	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	274	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	376	287	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.517	0.000	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	230	171	0	0	0	0	0	0
N.S.	1	0.97	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.501	0.000	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	26	29	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.90	1.00	1.00
time (sec)	N/A	0.022	2.084	0.223	0.261	0.287	1.413	0.298	12.564

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	502	500	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	26	29	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.90	1.00	1.00
time (sec)	N/A	0.021	5.769	0.245	0.249	0.292	1.820	0.321	73.048

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	263	187	0	0	0	0	0	0
N.S.	1	0.99	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	525	523	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.07
time (sec)	N/A	0.016	2.068	0.228	0.246	0.293	0.000	0.291	12.514

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	188	148	0	0	0	0	0
N.S.	1	1.00	2.11	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	22.740	1.178	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [429] had the largest ratio of [.6999999999999999956]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.00	25	0.200
2	A	8	5	1.00	25	0.200
3	A	7	5	1.00	25	0.200
4	A	12	5	1.00	25	0.200
5	A	5	4	1.00	23	0.174
6	A	5	4	1.00	23	0.174
7	A	8	7	1.00	25	0.280
8	A	8	8	1.00	25	0.320
9	A	8	7	1.00	25	0.280
10	A	8	8	1.00	25	0.320
11	A	8	7	1.00	25	0.280
12	A	6	5	1.00	25	0.200
13	A	7	6	1.00	25	0.240
14	A	8	6	1.00	25	0.240
15	A	9	6	1.00	25	0.240
16	A	8	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	3	3	1.00	25	0.120
19	A	6	4	1.00	25	0.160
20	A	6	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	4	3	1.00	25	0.120
23	A	3	3	1.00	25	0.120
24	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	23	0.130
26	A	3	3	1.00	22	0.136
27	A	7	5	1.00	25	0.200
28	A	7	5	1.00	25	0.200
29	A	8	6	1.00	25	0.240
30	A	4	4	1.00	25	0.160
31	A	5	4	1.00	25	0.160
32	A	4	4	1.00	24	0.167
33	A	7	5	1.00	27	0.185
34	A	6	5	1.00	27	0.185
35	A	5	5	1.00	27	0.185
36	A	4	4	1.00	25	0.160
37	A	4	4	1.00	24	0.167
38	A	7	7	1.00	27	0.259
39	A	7	7	1.00	27	0.259
40	A	5	5	1.00	27	0.185
41	A	6	6	1.00	27	0.222
42	A	7	6	1.00	27	0.222
43	A	8	6	1.00	27	0.222
44	A	6	5	1.00	27	0.185
45	A	6	5	1.00	27	0.185
46	A	3	2	1.00	27	0.074
47	A	3	3	1.00	27	0.111
48	A	3	3	1.00	25	0.120
49	A	3	3	1.00	24	0.125
50	A	7	6	1.00	27	0.222
51	A	7	5	1.00	27	0.185
52	A	8	6	1.00	27	0.222
53	A	9	6	1.00	27	0.222
54	A	5	4	1.00	20	0.200
55	A	4	4	1.00	20	0.200
56	A	3	3	1.00	18	0.167
57	A	3	3	1.00	17	0.176
58	A	6	6	1.00	20	0.300
59	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	5	1.00	20	0.250
61	A	6	6	1.00	20	0.300
62	A	7	6	1.00	20	0.300
63	A	8	6	1.00	20	0.300
64	A	9	8	1.00	27	0.296
65	A	12	6	1.00	27	0.222
66	A	11	6	1.00	27	0.222
67	A	10	6	1.00	27	0.222
68	A	9	6	1.00	27	0.222
69	A	9	6	1.00	25	0.240
70	A	8	5	1.00	24	0.208
71	A	11	8	1.00	27	0.296
72	A	11	9	1.00	27	0.333
73	A	11	8	1.00	27	0.296
74	A	11	9	1.00	27	0.333
75	A	11	9	1.00	27	0.333
76	A	11	8	1.00	27	0.296
77	A	11	9	1.00	27	0.333
78	A	11	9	1.00	27	0.333
79	A	11	8	1.00	27	0.296
80	A	9	6	1.00	27	0.222
81	A	10	7	1.00	27	0.259
82	A	11	7	1.00	27	0.259
83	A	7	5	1.00	27	0.185
84	A	6	4	1.00	27	0.148
85	A	5	4	1.00	27	0.148
86	A	3	3	1.00	27	0.111
87	A	3	3	1.00	25	0.120
88	A	4	3	1.00	24	0.125
89	A	7	6	1.00	27	0.222
90	A	7	5	1.00	27	0.185
91	A	8	6	1.00	27	0.222
92	A	7	5	1.00	27	0.185
93	A	6	5	1.00	27	0.185
94	A	6	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	4	1.00	25	0.160
96	A	3	3	1.00	24	0.125
97	A	7	7	1.00	27	0.259
98	A	5	5	1.00	27	0.185
99	A	6	6	1.00	27	0.222
100	A	7	6	1.00	27	0.222
101	A	8	6	1.00	27	0.222
102	A	7	7	1.00	27	0.259
103	A	9	6	1.00	27	0.222
104	A	8	6	1.00	27	0.222
105	A	8	7	1.00	27	0.259
106	A	6	5	1.00	25	0.200
107	A	5	4	1.00	24	0.167
108	A	9	8	1.00	27	0.296
109	A	9	9	1.00	27	0.333
110	A	9	8	1.00	27	0.296
111	A	9	9	1.00	27	0.333
112	A	9	8	1.00	27	0.296
113	A	7	6	1.00	27	0.222
114	A	8	7	1.00	27	0.259
115	A	9	7	1.00	27	0.259
116	A	10	7	1.00	27	0.259
117	A	3	3	1.00	18	0.167
118	A	7	7	1.00	26	0.269
119	A	6	6	1.00	27	0.222
120	A	5	5	1.00	27	0.185
121	A	5	5	1.00	27	0.185
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	24	0.042
124	A	5	5	1.00	27	0.185
125	A	5	5	1.00	27	0.185
126	A	6	6	1.00	27	0.222
127	A	6	5	1.00	27	0.185
128	A	6	5	1.00	27	0.185
129	A	5	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	27	0.111
131	A	2	2	1.00	25	0.080
132	A	2	2	1.00	24	0.083
133	A	6	6	1.00	27	0.222
134	A	6	6	1.00	27	0.222
135	A	7	7	1.00	27	0.259
136	A	7	5	1.00	27	0.185
137	A	7	5	1.00	27	0.185
138	A	6	5	1.00	27	0.185
139	A	5	4	1.00	27	0.148
140	A	4	4	1.00	27	0.148
141	A	3	3	1.00	27	0.111
142	A	3	3	1.00	25	0.120
143	A	3	3	1.00	24	0.125
144	A	7	6	1.00	27	0.222
145	A	7	6	1.00	27	0.222
146	A	8	7	1.00	27	0.259
147	A	9	7	1.00	27	0.259
148	A	5	5	1.00	27	0.185
149	A	4	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	2	2	1.00	23	0.087
153	A	1	1	1.00	22	0.045
154	A	5	5	1.00	26	0.192
155	A	5	5	1.00	26	0.192
156	A	6	6	1.00	26	0.231
157	A	10	7	1.00	27	0.259
158	A	9	7	1.00	27	0.259
159	A	8	7	1.00	27	0.259
160	A	7	7	1.00	27	0.259
161	A	6	5	1.00	25	0.200
162	A	6	6	1.00	24	0.250
163	A	9	9	1.00	27	0.333
164	A	9	9	1.00	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	9	9	1.00	27	0.333
166	A	9	9	1.00	27	0.333
167	A	7	7	1.00	27	0.259
168	A	8	8	1.00	27	0.296
169	A	9	8	1.00	27	0.296
170	A	10	8	1.00	27	0.296
171	A	7	6	1.00	27	0.222
172	A	4	3	1.00	27	0.111
173	A	4	4	1.00	27	0.148
174	A	3	3	1.00	25	0.120
175	A	3	2	1.00	24	0.083
176	A	8	7	1.00	27	0.259
177	A	8	6	1.00	27	0.222
178	A	9	7	1.00	27	0.259
179	A	8	6	1.00	27	0.222
180	A	7	5	1.00	27	0.185
181	A	6	5	1.00	27	0.185
182	A	4	4	1.00	27	0.148
183	A	3	3	1.00	25	0.120
184	A	3	2	1.00	24	0.083
185	A	8	7	1.00	27	0.259
186	A	8	6	1.00	27	0.222
187	A	9	7	1.00	27	0.259
188	A	9	6	1.00	27	0.222
189	A	7	5	1.00	27	0.185
190	A	9	7	1.00	27	0.259
191	A	8	6	1.00	27	0.222
192	A	2	2	1.00	25	0.080
193	A	2	2	1.00	24	0.083
194	A	8	7	1.00	27	0.259
195	A	8	6	1.00	27	0.222
196	A	9	7	1.00	27	0.259
197	A	10	7	1.00	27	0.259
198	A	11	6	1.00	27	0.222
199	A	10	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	9	6	1.00	27	0.222
201	A	8	7	1.00	27	0.259
202	A	6	6	1.00	25	0.240
203	A	5	4	1.00	24	0.167
204	A	9	9	1.00	27	0.333
205	A	9	9	1.00	27	0.333
206	A	7	7	1.00	27	0.259
207	A	8	7	1.00	27	0.259
208	A	9	7	1.00	27	0.259
209	A	10	7	1.00	27	0.259
210	A	7	5	1.00	26	0.192
211	A	4	4	1.00	26	0.154
212	A	9	5	1.00	27	0.185
213	A	8	5	1.00	27	0.185
214	A	7	4	1.00	25	0.160
215	A	7	3	1.00	24	0.125
216	A	12	7	1.00	27	0.259
217	A	12	6	1.00	27	0.222
218	A	4	4	1.00	29	0.138
219	A	2	2	1.00	29	0.069
220	A	3	3	1.00	16	0.188
221	A	4	4	1.00	29	0.138
222	A	3	3	1.00	15	0.200
223	A	4	4	1.00	30	0.133
224	A	4	3	1.00	16	0.188
225	A	5	4	1.00	29	0.138
226	A	7	4	1.00	29	0.138
227	A	6	4	1.00	29	0.138
228	A	5	3	1.00	27	0.111
229	A	2	2	1.00	22	0.091
230	A	8	5	1.00	29	0.172
231	A	7	5	1.00	29	0.172
232	A	8	5	1.00	29	0.172
233	A	6	4	1.00	29	0.138
234	A	6	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	3	1.31	27	0.111
236	A	2	2	1.00	22	0.091
237	A	8	5	1.00	29	0.172
238	A	7	5	1.00	29	0.172
239	A	7	5	1.00	29	0.172
240	A	6	5	1.00	23	0.217
241	A	6	5	1.00	23	0.217
242	A	6	5	1.00	23	0.217
243	A	6	5	1.00	23	0.217
244	A	4	4	1.00	21	0.190
245	A	3	3	1.00	20	0.150
246	A	5	5	1.00	23	0.217
247	A	5	5	1.00	23	0.217
248	A	5	5	1.00	23	0.217
249	A	7	6	1.00	25	0.240
250	A	8	7	1.00	25	0.280
251	A	7	6	1.00	25	0.240
252	A	8	7	1.00	25	0.280
253	A	7	6	1.00	23	0.261
254	A	2	2	1.00	22	0.091
255	A	7	7	1.00	25	0.280
256	A	6	6	1.00	25	0.240
257	A	6	6	1.00	25	0.240
258	A	7	6	1.00	25	0.240
259	A	7	6	1.00	25	0.240
260	A	7	6	1.00	25	0.240
261	A	7	6	1.00	25	0.240
262	A	3	3	1.00	23	0.130
263	A	2	2	1.00	22	0.091
264	A	7	7	1.00	25	0.280
265	A	8	8	1.00	25	0.320
266	A	7	6	1.00	25	0.240
267	A	7	6	1.00	25	0.240
268	A	7	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	5	5	1.00	23	0.217
271	A	2	2	1.00	22	0.091
272	A	6	6	1.00	25	0.240
273	A	6	6	1.00	25	0.240
274	A	6	6	1.00	25	0.240
275	A	8	7	1.00	25	0.280
276	A	9	8	1.00	25	0.320
277	A	8	7	1.00	25	0.280
278	A	9	8	1.00	25	0.320
279	A	3	3	1.00	23	0.130
280	A	2	2	1.00	22	0.091
281	A	8	8	1.00	25	0.320
282	A	7	7	1.00	25	0.280
283	A	7	7	1.00	25	0.280
284	A	7	7	1.00	25	0.280
285	A	7	7	1.00	25	0.280
286	A	8	7	1.00	25	0.280
287	A	8	7	1.00	25	0.280
288	A	4	4	1.00	25	0.160
289	A	3	3	1.00	23	0.130
290	A	2	2	1.00	22	0.091
291	A	8	8	1.00	25	0.320
292	A	9	9	1.00	25	0.360
293	A	8	7	1.00	25	0.280
294	A	8	7	1.00	25	0.280
295	A	8	7	1.00	25	0.280
296	A	9	8	1.00	25	0.320
297	A	5	4	1.00	25	0.160
298	A	4	4	1.00	25	0.160
299	A	3	3	1.00	23	0.130
300	A	2	2	1.00	22	0.091
301	A	9	9	1.00	25	0.360
302	A	9	9	1.00	25	0.360
303	A	10	9	1.00	25	0.360
304	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	9	7	1.00	25	0.280
306	A	7	4	1.00	27	0.148
307	A	6	4	1.00	27	0.148
308	A	5	3	1.00	25	0.120
309	A	2	2	1.00	20	0.100
310	A	8	5	1.00	27	0.185
311	A	7	5	1.00	27	0.185
312	A	8	5	1.00	27	0.185
313	A	6	4	1.00	25	0.160
314	A	4	3	1.00	27	0.111
315	A	11	7	1.00	16	0.438
316	A	9	6	1.00	22	0.273
317	A	8	6	1.00	22	0.273
318	A	8	7	1.00	22	0.318
319	A	6	5	1.00	20	0.250
320	A	6	5	1.00	19	0.263
321	A	9	8	1.00	22	0.364
322	A	15	11	1.00	22	0.500
323	A	19	12	1.00	22	0.546
324	A	20	13	1.00	22	0.591
325	A	25	14	1.00	22	0.636
326	A	8	5	1.00	22	0.227
327	A	7	5	1.00	22	0.227
328	A	7	6	1.00	22	0.273
329	A	5	4	1.00	20	0.200
330	A	2	2	1.00	19	0.105
331	A	7	6	1.00	22	0.273
332	A	8	7	1.00	22	0.318
333	A	12	8	1.00	22	0.364
334	A	7	6	1.00	22	0.273
335	A	6	5	1.00	22	0.227
336	A	4	4	1.00	22	0.182
337	A	4	4	1.00	20	0.200
338	A	4	4	1.00	19	0.210
339	A	10	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
340	A	12	11	1.00	22	0.500
341	A	17	12	1.00	22	0.546
342	A	9	6	1.00	22	0.273
343	A	8	6	1.00	22	0.273
344	A	7	6	1.00	22	0.273
345	A	6	5	1.00	22	0.227
346	A	3	3	1.00	20	0.150
347	A	3	3	1.00	19	0.158
348	A	10	7	1.00	22	0.318
349	A	11	8	1.00	22	0.364
350	A	15	9	1.00	22	0.409
351	A	2	1	1.00	18	0.056
352	A	2	1	1.00	16	0.062
353	A	2	1	1.00	15	0.067
354	A	3	3	1.00	18	0.167
355	A	2	1	1.00	20	0.050
356	A	2	1	1.00	18	0.056
357	A	2	1	1.00	17	0.059
358	A	4	3	1.00	20	0.150
359	A	2	1	1.00	20	0.050
360	A	2	1	1.00	18	0.056
361	A	2	1	1.00	17	0.059
362	A	4	3	1.00	20	0.150
363	A	6	3	1.00	20	0.150
364	A	6	3	1.00	20	0.150
365	A	6	3	1.00	20	0.150
366	A	4	2	1.00	18	0.111
367	A	4	2	1.00	17	0.118
368	A	7	4	1.00	20	0.200
369	A	7	4	1.00	20	0.200
370	A	5	3	1.00	20	0.150
371	A	5	3	1.00	20	0.150
372	A	5	3	1.00	20	0.150
373	A	5	3	1.00	18	0.167
374	A	5	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	12	5	1.00	20	0.250
376	A	12	6	1.00	20	0.300
377	A	6	5	0.94	22	0.227
378	A	4	4	0.91	20	0.200
379	A	6	3	1.00	22	0.136
380	A	12	4	1.00	22	0.182
381	A	6	5	1.00	18	0.278
382	A	6	5	1.00	18	0.278
383	A	6	5	1.00	18	0.278
384	A	6	5	1.00	18	0.278
385	A	4	4	1.00	16	0.250
386	A	3	3	1.00	15	0.200
387	A	5	5	1.00	18	0.278
388	A	5	5	1.00	18	0.278
389	A	5	5	1.00	18	0.278
390	A	7	6	1.00	20	0.300
391	A	8	7	0.95	20	0.350
392	A	7	6	1.00	20	0.300
393	A	8	7	0.95	20	0.350
394	A	7	6	1.00	18	0.333
395	A	4	4	0.94	17	0.235
396	A	7	7	1.00	20	0.350
397	A	6	6	1.00	20	0.300
398	A	6	6	1.00	20	0.300
399	A	7	6	0.98	20	0.300
400	A	7	6	0.97	20	0.300
401	A	7	6	0.97	20	0.300
402	A	7	6	0.96	20	0.300
403	A	7	6	0.95	18	0.333
404	A	4	4	0.96	17	0.235
405	A	7	7	0.96	20	0.350
406	A	8	8	1.00	20	0.400
407	A	7	6	1.00	20	0.300
408	A	7	6	1.00	20	0.300
409	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	6	6	1.00	20	0.300
411	A	9	8	1.00	18	0.444
412	A	6	5	1.00	17	0.294
413	A	7	7	1.00	20	0.350
414	A	7	7	1.00	20	0.350
415	A	8	8	1.00	20	0.400
416	A	12	10	1.00	20	0.500
417	A	11	10	1.00	20	0.500
418	A	10	9	0.99	20	0.450
419	A	10	9	1.00	18	0.500
420	A	8	7	0.78	17	0.412
421	A	18	10	1.00	20	0.500
422	A	20	12	1.00	20	0.600
423	A	12	10	1.00	20	0.500
424	A	11	9	1.00	20	0.450
425	A	11	10	1.00	20	0.500
426	A	11	9	1.00	18	0.500
427	A	11	9	1.00	17	0.529
428	A	29	12	1.00	20	0.600
429	A	31	14	1.00	20	0.700
430	A	7	4	0.92	22	0.182
431	A	6	4	0.95	22	0.182
432	A	5	3	1.00	20	0.150
433	A	2	2	1.00	15	0.133
434	A	5	3	1.00	22	0.136
435	A	8	3	1.00	22	0.136
436	A	10	3	1.00	22	0.136
437	A	6	5	1.00	40	0.125
438	A	5	5	1.00	40	0.125
439	A	4	4	1.00	38	0.105
440	A	3	3	1.00	37	0.081
441	A	6	5	1.00	40	0.125
442	A	4	4	1.00	40	0.100
443	A	5	5	1.00	40	0.125
444	A	6	5	1.00	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	7	5	1.00	40	0.125
446	A	7	6	1.00	40	0.150
447	A	6	6	1.00	40	0.150
448	A	5	5	1.00	38	0.132
449	A	4	4	1.00	37	0.108
450	A	7	6	1.00	40	0.150
451	A	7	6	1.00	40	0.150
452	A	7	6	1.00	40	0.150
453	A	5	5	1.00	40	0.125
454	A	6	6	1.00	40	0.150
455	A	7	6	1.00	40	0.150
456	A	8	6	1.00	40	0.150
457	A	8	6	1.00	40	0.150
458	A	7	6	1.00	40	0.150
459	A	6	5	1.00	38	0.132
460	A	5	4	1.00	37	0.108
461	A	8	6	1.00	40	0.150
462	A	8	7	1.00	40	0.175
463	A	8	6	1.00	40	0.150
464	A	8	7	1.00	40	0.175
465	A	8	6	1.00	40	0.150
466	A	6	5	1.00	40	0.125
467	A	7	6	1.00	40	0.150
468	A	8	6	1.00	40	0.150
469	A	9	6	1.00	40	0.150
470	A	5	5	1.10	40	0.125
471	A	4	4	1.00	40	0.100
472	A	3	3	1.00	38	0.079
473	A	1	1	1.00	37	0.027
474	A	5	5	1.00	40	0.125
475	A	5	5	1.00	40	0.125
476	A	6	6	1.00	40	0.150
477	A	6	5	1.00	40	0.125
478	A	6	5	1.00	40	0.125
479	A	5	5	1.00	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	3	3	1.00	40	0.075
481	A	2	2	1.00	38	0.053
482	A	2	2	1.00	37	0.054
483	A	6	5	1.00	40	0.125
484	A	6	5	1.00	40	0.125
485	A	7	6	1.00	40	0.150
486	A	8	6	1.00	40	0.150
487	A	3	3	1.00	40	0.075
488	A	4	4	1.00	40	0.100
489	A	4	4	1.00	23	0.174
490	A	1	1	1.00	23	0.043
491	A	5	5	1.00	21	0.238
492	A	3	3	1.00	20	0.150
493	A	5	5	1.00	23	0.217
494	A	5	5	1.00	23	0.217
495	A	3	3	1.00	23	0.130
496	A	5	4	1.00	23	0.174
497	A	1	1	1.00	23	0.043
498	A	6	5	1.00	21	0.238
499	A	4	3	1.00	20	0.150
500	A	6	5	1.00	23	0.217
501	A	6	6	1.00	23	0.261
502	A	4	4	1.00	23	0.174
503	A	3	3	1.00	23	0.130
504	A	1	1	1.00	23	0.043
505	A	4	4	1.00	21	0.190
506	A	2	2	1.00	20	0.100
507	A	4	4	1.00	23	0.174
508	A	5	5	1.00	23	0.217
509	A	3	3	0.99	23	0.130
510	A	3	3	1.00	23	0.130
511	A	1	1	1.00	23	0.043
512	A	5	5	1.00	21	0.238
513	A	3	3	1.00	20	0.150
514	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	6	6	1.00	23	0.261
516	A	4	4	1.00	23	0.174
517	A	4	4	1.00	23	0.174
518	A	1	1	1.00	23	0.043
519	A	6	5	1.00	21	0.238
520	A	4	3	1.00	20	0.150
521	A	6	5	1.00	23	0.217
522	A	7	6	1.00	23	0.261
523	A	5	4	1.00	23	0.174
524	A	7	6	1.00	19	0.316
525	A	6	4	1.00	25	0.160
526	A	6	4	1.22	25	0.160
527	A	6	4	1.00	25	0.160
528	A	5	4	1.00	23	0.174
529	A	4	3	1.00	22	0.136
530	A	7	5	1.00	25	0.200
531	A	9	6	0.97	25	0.240
532	A	12	6	1.00	25	0.240
533	A	6	4	1.00	25	0.160
534	A	6	4	1.00	25	0.160
535	A	6	4	1.00	25	0.160
536	A	6	4	1.00	23	0.174
537	A	5	4	1.00	22	0.182
538	A	7	5	1.00	25	0.200
539	A	9	6	1.00	25	0.240
540	A	12	6	1.00	25	0.240
541	A	6	3	1.00	23	0.130
542	A	4	2	1.00	23	0.087
543	A	4	2	1.00	23	0.087
544	A	4	2	1.00	21	0.095
545	A	4	2	1.00	20	0.100
546	A	7	4	1.00	23	0.174
547	A	8	4	1.00	23	0.174
548	A	3	2	1.00	29	0.069
549	A	3	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
550	A	3	2	1.00	29	0.069
551	A	3	2	1.00	27	0.074
552	A	5	3	1.00	22	0.136
553	A	3	2	1.00	29	0.069
554	A	4	3	1.00	29	0.103
555	A	4	3	1.00	29	0.103
556	A	4	3	1.00	29	0.103
557	A	3	2	1.00	29	0.069
558	A	3	2	1.00	29	0.069
559	A	3	2	1.00	29	0.069
560	A	3	2	1.00	29	0.069
561	A	3	2	1.00	29	0.069
562	A	3	2	1.00	29	0.069
563	A	3	2	1.00	27	0.074
564	A	2	2	1.00	22	0.091
565	A	4	3	1.00	29	0.103
566	A	4	3	1.00	29	0.103
567	A	4	3	1.00	29	0.103
568	A	4	3	1.00	29	0.103
569	A	3	2	1.00	29	0.069
570	A	3	2	1.00	29	0.069
571	A	3	2	1.00	29	0.069
572	A	3	2	1.00	29	0.069
573	A	3	2	1.00	29	0.069
574	A	4	3	1.00	29	0.103
575	A	4	3	1.00	27	0.111
576	A	3	3	1.00	22	0.136
577	A	4	3	1.00	29	0.103
578	A	4	3	1.00	29	0.103
579	A	7	5	1.00	31	0.161
580	A	6	4	1.00	31	0.129
581	A	5	4	1.00	31	0.129
582	A	3	3	1.00	31	0.097
583	A	3	3	1.00	29	0.103
584	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	6	5	1.00	31	0.161
586	A	6	4	1.00	31	0.129
587	A	7	5	1.00	31	0.161
588	A	4	3	1.00	24	0.125
589	A	3	2	1.00	24	0.083
590	A	3	2	1.00	24	0.083
591	A	2	1	1.00	22	0.045
592	A	2	1	1.00	17	0.059
593	A	4	3	1.00	24	0.125
594	A	4	4	1.03	24	0.167
595	A	4	4	1.02	24	0.167
596	A	3	2	1.00	24	0.083
597	A	3	2	1.00	24	0.083
598	A	2	1	1.00	22	0.045
599	A	2	1	1.00	17	0.059
600	A	4	3	1.00	24	0.125
601	A	4	4	1.00	24	0.167
602	A	5	5	1.00	24	0.208
603	A	5	5	1.00	26	0.192
604	A	1	1	1.00	22	0.045
605	A	11	8	1.00	28	0.286
606	A	10	7	1.00	28	0.250
607	A	6	3	1.00	28	0.107
608	A	8	5	1.00	28	0.179
609	A	11	7	1.00	28	0.250
610	A	11	7	1.00	28	0.250
611	A	6	3	1.00	28	0.107
612	A	6	3	1.00	28	0.107
613	A	8	4	1.00	28	0.143
614	A	21	10	1.00	28	0.357
615	A	8	5	1.00	28	0.179
616	A	8	4	1.00	28	0.143
617	A	12	6	0.99	28	0.214
618	A	6	3	1.00	20	0.150
619	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	6	6	1.00	30	0.200
621	A	5	5	1.00	29	0.172
622	A	10	6	1.00	28	0.214
623	A	9	6	1.00	28	0.214
624	A	7	6	1.00	26	0.231
625	A	7	6	1.00	21	0.286
626	A	14	10	1.00	28	0.357
627	A	14	10	1.00	28	0.357
628	A	23	11	1.00	28	0.393
629	A	9	6	1.00	28	0.214
630	A	8	6	0.99	28	0.214
631	A	6	5	1.00	26	0.192
632	A	6	5	1.00	21	0.238
633	A	10	9	1.00	28	0.321
634	A	14	10	1.00	28	0.357
635	A	23	11	1.00	28	0.393
636	A	8	6	0.99	28	0.214
637	A	7	6	1.00	28	0.214
638	A	6	5	1.00	26	0.192
639	A	2	2	1.00	21	0.095
640	A	7	7	1.00	28	0.250
641	A	14	10	1.00	28	0.357
642	A	23	11	1.00	28	0.393
643	A	16	10	1.35	28	0.357
644	A	10	8	1.00	28	0.286
645	A	7	6	1.00	28	0.214
646	A	7	6	1.00	28	0.214
647	A	5	4	1.00	26	0.154
648	A	2	2	1.00	21	0.095
649	A	4	4	1.00	28	0.143
650	A	14	10	1.00	28	0.357
651	A	23	10	1.00	28	0.357
652	A	10	9	1.00	28	0.321
653	A	17	12	1.00	28	0.429
654	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	2	2	1.00	30	0.067
656	A	4	3	1.00	26	0.115
657	A	4	3	1.00	46	0.065
658	A	3	3	1.00	46	0.065
659	A	2	2	1.00	44	0.045
660	A	1	1	1.00	39	0.026
661	A	2	2	1.00	46	0.043
662	A	3	3	1.00	46	0.065
663	A	4	3	1.00	46	0.065
664	A	5	3	1.00	46	0.065
665	A	4	4	1.00	46	0.087
666	A	3	3	1.00	46	0.065
667	A	2	2	1.00	44	0.045
668	A	1	1	1.00	39	0.026
669	A	3	3	1.00	46	0.065
670	A	4	4	1.00	46	0.087
671	A	5	4	1.00	46	0.087
672	A	4	3	1.00	46	0.065
673	A	3	3	1.00	46	0.065
674	A	2	2	1.00	44	0.045
675	A	1	1	1.00	39	0.026
676	A	4	3	1.00	46	0.065
677	A	5	4	1.00	46	0.087
678	A	6	4	1.00	46	0.087
679	A	5	3	1.00	46	0.065
680	A	4	3	1.00	46	0.065
681	A	3	3	1.00	46	0.065
682	A	2	2	1.00	44	0.045
683	A	1	1	1.00	39	0.026
684	A	3	3	1.00	46	0.065
685	A	3	3	1.00	46	0.065
686	A	4	4	1.00	46	0.087
687	A	5	4	1.00	46	0.087
688	A	6	4	1.00	46	0.087
689	A	5	3	1.00	46	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	4	3	1.00	46	0.065
691	A	3	3	1.00	46	0.065
692	A	2	2	1.00	44	0.045
693	A	1	1	1.00	39	0.026
694	A	4	3	1.00	46	0.065
695	A	4	4	1.00	46	0.087
696	A	4	3	1.00	46	0.065
697	A	5	4	1.00	46	0.087
698	A	6	4	1.00	46	0.087
699	A	7	4	1.00	46	0.087
700	A	5	3	1.00	46	0.065
701	A	4	3	1.00	46	0.065
702	A	3	3	1.00	46	0.065
703	A	2	2	1.00	44	0.045
704	A	1	1	1.00	39	0.026
705	A	5	3	1.00	46	0.065
706	A	5	4	1.00	46	0.087
707	A	5	4	1.00	46	0.087
708	A	5	3	1.00	46	0.065
709	A	6	4	1.00	46	0.087
710	A	7	4	1.00	46	0.087
711	A	8	4	1.00	46	0.087
712	A	7	5	1.00	48	0.104
713	A	6	5	1.00	48	0.104
714	A	5	5	1.00	48	0.104
715	A	4	4	1.00	48	0.083
716	A	1	1	1.00	48	0.021
717	A	2	2	1.00	48	0.042
718	A	3	2	1.00	48	0.042
719	A	4	2	1.00	48	0.042
720	A	7	6	1.00	48	0.125
721	A	6	6	1.00	48	0.125
722	A	5	5	1.00	48	0.104
723	A	1	1	1.00	48	0.021
724	A	2	2	1.00	48	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	A	3	3	1.00	48	0.062
726	A	4	3	1.00	48	0.062
727	A	7	6	1.00	48	0.125
728	A	6	5	1.00	48	0.104
729	A	1	1	1.00	48	0.021
730	A	2	2	1.00	48	0.042
731	A	3	2	1.00	48	0.042
732	A	4	3	1.00	48	0.062
733	A	8	6	1.00	48	0.125
734	A	7	6	1.00	48	0.125
735	A	6	6	1.00	48	0.125
736	A	5	5	1.00	48	0.104
737	A	5	5	1.00	48	0.104
738	A	1	1	1.00	48	0.021
739	A	2	2	1.00	48	0.042
740	A	3	2	1.00	48	0.042
741	A	4	2	1.00	48	0.042
742	A	8	6	1.00	48	0.125
743	A	7	6	1.00	48	0.125
744	A	6	5	1.00	48	0.104
745	A	6	6	1.00	48	0.125
746	A	6	5	1.00	48	0.104
747	A	1	1	1.00	48	0.021
748	A	2	2	1.00	48	0.042
749	A	3	2	1.00	48	0.042
750	A	4	2	1.00	48	0.042
751	A	9	6	1.00	48	0.125
752	A	8	6	1.00	48	0.125
753	A	7	5	1.00	48	0.104
754	A	7	6	1.00	48	0.125
755	A	7	6	1.00	48	0.125
756	A	7	5	1.00	48	0.104
757	A	1	1	1.00	48	0.021
758	A	2	2	1.00	48	0.042
759	A	3	2	1.00	48	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	4	2	1.00	48	0.042
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	3	1.00	44	0.068
769	A	3	3	1.00	44	0.068
770	A	2	2	1.00	42	0.048
771	A	1	1	1.00	37	0.027
772	A	2	2	1.00	44	0.045
773	A	2	2	1.00	44	0.045
774	A	2	2	1.00	44	0.045
775	A	3	3	1.00	46	0.065
776	A	3	3	1.00	46	0.065
777	A	3	3	1.00	46	0.065
778	A	3	3	1.00	46	0.065
779	A	3	3	1.00	46	0.065
780	A	1	1	1.00	47	0.021
781	A	3	3	1.00	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	4	1.00	46	0.087
784	A	5	4	1.00	46	0.087
785	A	4	4	1.00	46	0.087
786	A	3	3	1.00	44	0.068
787	A	2	2	1.00	39	0.051
788	A	3	3	1.00	46	0.065
789	A	3	3	1.00	46	0.065
790	A	4	4	1.00	46	0.087
791	A	5	4	1.00	46	0.087
792	A	8	4	1.00	32	0.125
793	A	6	4	1.00	32	0.125
794	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
795	A	6	4	1.00	32	0.125
796	A	7	5	1.00	32	0.156
797	A	7	5	1.00	32	0.156
798	A	5	5	1.00	32	0.156
799	A	4	4	1.00	30	0.133
800	A	7	5	1.00	32	0.156
801	A	8	6	1.00	32	0.188
802	A	3	3	1.00	25	0.120
803	A	4	3	1.00	25	0.120
804	A	4	3	1.00	28	0.107
805	A	2	1	1.00	28	0.036
806	A	2	1	1.00	28	0.036
807	A	2	1	1.00	26	0.038
808	A	2	1	1.00	21	0.048
809	A	3	3	1.00	28	0.107
810	A	3	2	1.00	28	0.071
811	A	3	3	1.00	28	0.107
812	A	3	3	1.00	28	0.107
813	A	4	4	0.98	28	0.143
814	A	2	1	1.00	25	0.040
815	A	2	1	1.00	27	0.037
816	A	2	1	1.00	27	0.037
817	A	6	5	1.00	27	0.185
818	A	9	6	1.00	27	0.222
819	A	3	2	1.00	27	0.074
820	A	3	2	1.00	27	0.074
821	A	2	1	1.00	25	0.040
822	A	2	1	1.00	20	0.050
823	A	4	3	1.00	27	0.111
824	A	4	4	1.03	27	0.148
825	A	4	4	1.02	27	0.148
826	A	3	2	1.00	27	0.074
827	A	3	2	1.00	27	0.074
828	A	2	1	1.00	25	0.040
829	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
830	A	4	3	1.00	27	0.111
831	A	4	4	1.00	27	0.148
832	A	5	5	1.00	27	0.185
833	B	9	7	2.10	25	0.280
834	A	5	5	1.00	29	0.172
835	A	7	6	1.00	29	0.207
836	A	6	6	1.00	29	0.207
837	A	5	5	1.00	29	0.172
838	A	5	5	1.00	29	0.172
839	A	5	5	1.00	29	0.172
840	A	3	3	1.00	29	0.103
841	A	4	4	1.00	29	0.138
842	A	6	6	1.00	29	0.207
843	A	7	6	1.00	38	0.158
844	A	6	6	1.00	38	0.158
845	A	5	5	1.00	38	0.132
846	A	5	5	1.00	38	0.132
847	A	5	5	1.00	38	0.132
848	A	3	3	1.00	38	0.079
849	A	4	4	1.00	38	0.105
850	A	11	7	1.00	31	0.226
851	A	6	3	1.00	31	0.097
852	A	6	3	1.00	31	0.097
853	A	8	4	1.00	31	0.129
854	A	8	6	1.00	29	0.207
855	A	7	6	1.00	29	0.207
856	A	6	5	1.00	27	0.185
857	A	6	5	1.00	22	0.227
858	A	8	5	1.00	29	0.172
859	A	20	7	1.00	29	0.241
860	A	23	8	1.00	29	0.276
861	A	27	9	1.00	29	0.310
862	A	9	6	1.00	29	0.207
863	A	8	6	1.00	29	0.207
864	A	7	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
865	A	7	6	1.00	22	0.273
866	A	13	7	1.00	29	0.241
867	A	23	8	1.00	29	0.276
868	A	30	9	1.00	29	0.310
869	A	15	7	1.00	29	0.241
870	A	8	5	1.00	29	0.172
871	A	7	5	1.00	29	0.172
872	A	6	5	1.00	29	0.172
873	A	5	4	1.00	27	0.148
874	A	2	2	1.00	22	0.091
875	A	6	3	1.00	29	0.103
876	A	9	4	1.00	29	0.138
877	A	13	6	1.00	29	0.207
878	A	7	6	1.00	29	0.207
879	A	6	5	1.00	29	0.172
880	A	4	4	1.00	29	0.138
881	A	4	4	1.00	27	0.148
882	A	4	4	1.00	22	0.182
883	A	10	5	1.00	29	0.172
884	A	14	6	1.00	29	0.207
885	A	19	7	1.00	29	0.241
886	A	10	6	1.00	31	0.194
887	A	9	6	1.00	31	0.194
888	A	7	6	1.00	29	0.207
889	A	7	6	1.00	24	0.250
890	A	15	10	1.27	31	0.323
891	A	15	10	1.26	31	0.323
892	A	25	11	1.65	31	0.355
893	A	9	6	1.00	31	0.194
894	A	8	6	1.00	31	0.194
895	A	6	5	1.00	29	0.172
896	A	6	5	1.00	24	0.208
897	A	11	9	1.00	31	0.290
898	A	15	10	1.30	31	0.323
899	A	25	11	1.67	31	0.355

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	8	6	1.00	31	0.194
901	A	7	6	1.00	31	0.194
902	A	6	5	1.00	29	0.172
903	A	2	2	1.00	24	0.083
904	A	8	7	1.00	31	0.226
905	A	15	10	1.00	31	0.323
906	A	25	11	1.00	31	0.355
907	A	11	8	1.00	31	0.258
908	A	17	10	1.00	31	0.323
909	A	7	6	1.00	31	0.194
910	A	7	6	1.00	31	0.194
911	A	5	4	1.00	29	0.138
912	A	2	2	1.00	24	0.083
913	A	5	4	1.00	31	0.129
914	A	15	10	1.00	31	0.323
915	A	25	10	1.58	31	0.323
916	A	11	9	1.00	31	0.290
917	A	18	12	1.00	31	0.387
918	A	1	1	1.00	33	0.030
919	A	2	2	1.00	33	0.061
920	A	2	1	1.00	25	0.040
921	A	2	1	1.00	23	0.043
922	A	3	3	1.00	25	0.120
923	A	3	3	1.00	25	0.120
924	A	3	3	0.99	25	0.120
925	A	2	1	1.00	27	0.037
926	A	2	1	1.00	25	0.040
927	A	4	3	1.00	27	0.111
928	A	4	3	1.00	27	0.111
929	A	5	5	1.00	27	0.185
930	A	4	2	1.00	27	0.074
931	A	4	2	1.00	27	0.074
932	A	4	2	1.00	27	0.074
933	A	4	2	1.00	25	0.080
934	A	4	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
935	A	7	3	1.00	27	0.111
936	A	8	3	1.00	27	0.111
937	A	5	3	1.00	27	0.111
938	A	5	3	1.00	27	0.111
939	A	5	3	1.00	27	0.111
940	A	5	3	1.00	25	0.120
941	A	5	3	1.00	20	0.150
942	A	12	4	1.00	27	0.148
943	A	13	4	1.00	27	0.148
944	A	4	4	0.97	27	0.148
945	A	6	4	0.99	29	0.138
946	A	5	3	1.00	27	0.111
947	A	2	2	1.00	22	0.091
948	N/A	0	0	1.00	29	0.000
949	A	6	4	1.00	29	0.138
950	A	5	3	1.00	27	0.111
951	A	2	2	1.00	22	0.091
952	N/A	0	0	1.00	29	0.000
953	A	4	4	0.99	25	0.160
954	A	6	4	1.00	27	0.148
955	A	5	3	1.00	25	0.120
956	A	2	2	1.00	20	0.100
957	N/A	0	0	1.00	27	0.000
958	A	5	5	1.00	27	0.185

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx=\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/5*(-e^
2*x^2+d^2)^(5/2)/e^3+1/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 655, 201, 223, 209}

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx=\frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

```
[In] Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}+\frac{d^2\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= -\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}\\
&\quad+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\left(3d^3\right)\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}+\frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}\\
&\quad-\frac{\left(3d^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}+\frac{d^5\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}
\end{aligned}
$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{dx(d^2-e^2x^2)^{3/2}}{4e^2} + \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} - \frac{(3d^5)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{dx(d^2-e^2x^2)^{3/2}}{4e^2} + \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4) - 30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{120e^3}$$

```
[In] Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x^3 +
 24*e^4*x^4) - 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(120
*e^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(-24e^4x^4-30d\,e^3x^3+8d^2e^2x^2+15d^3ex+16d^4)\sqrt{-e^2x^2+d^2}}{120e^3}+\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$e\left(-\frac{x^2(-e^2x^2+d^2)^{\frac{3}{2}}}{5e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{3}{2}}}{15e^4}\right)+d\left(-\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}\right)$

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/120*(-24*e^4*x^4-30*d*e^3*x^3+8*d^2*e^2*x^2+15*d^3*e*x+16*d^4)/e^3*(-e^2
*x^2+d^2)^(1/2)+1/8*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(24\,e^4x^4+30\,de^3x^3-8\,d^2e^2x^2-15\,d^3ex-16\,d^4)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 + 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 - 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \begin{cases} \frac{d^5\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^3}-\frac{d^3x}{8e^2}-\frac{d^2x^2}{15e}+\frac{dx^3}{4}+\frac{ex^4}{5}\right) & \text{for } e^2\neq 0 \\ \left(\frac{dx^3}{3}+\frac{ex^4}{4}\right)\sqrt{d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((d**5*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2
) + sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**3) - d**3*x/(8*e**2) - d**2*x**2
/(15*e) + d*x**3/4 + e*x**4/5), Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)*sqrt(d
**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \frac{d^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}d^3x}{8\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}x^2}{5\,e} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^2} - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{15\,e^3}$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*d^5*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 + d
^2)*d^3*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*x^2/e - 1/4*(-e^2*x^2 + d^2)^(3/
2)*d*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \frac{d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} + \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,(4\,ex+5\,d)x-\frac{4\,d^2}{e}\right)x-\frac{15\,d^3}{e^2}\right)x-\frac{16\,d^4}{e^3}\right)$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/120*sqrt(-e^2*x^2 + d^
2)*((2*(3*(4*e*x + 5*d)*x - 4*d^2/e)*x - 15*d^3/e^2)*x - 16*d^4/e^3)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = \int x^2\,\sqrt{d^2-e^2\,x^2}\,(d+e\,x)\;dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x), x)
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-1/5040*d^3
*(315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d^
2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 201, 223, 209}

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}$$

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\text{integral} = -\frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x^3(-4d^2e-9de^2x)(d^2-e^2x^2)^{3/2}\,dx}{9e^2}$$

$$= -\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{\int x^2(27d^3e^2+32d^2e^3x)(d^2-e^2x^2)^{3/2}\,dx}{72e^4}$$

$$= -\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x(-64d^4e^3-189d^3e^4x)(d^2-e^2x^2)^{3/2}\,dx}{504e^6}$$

$$= -\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} + \frac{d^5\int (d^2-e^2x^2)^{3/2}\,dx}{16e^4}$$

$$= \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} + \frac{(3d^7)\int \sqrt{d^2-e^2x^2}\,dx}{64e^4}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} + \frac{(3d^9)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{128e^4}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} + \frac{(3d^9)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} + \frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{d^2-e^2x^2}(1024d^8+945d^7ex+512d^6e^2x^2+630d^5e^3x^3+384d^4e^4x^4-7560d^3e^5x^5-6400d^2e^6x^6+5040}{40320e^5}$$

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -1/40320*(Sqrt[d^2 - e^2*x^2]*(1024*d^8 + 945*d^7*e*x + 512*d^6*e^2*x^2 + 6
30*d^5*e^3*x^3 + 384*d^4*e^4*x^4 - 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 + 50
40*d*e^7*x^7 + 4480*e^8*x^8) + 1890*d^9*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2
- e^2*x^2])])/e^5
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(4480e^8x^8+5040d\,e^7x^7-6400d^2e^6x^6-7560d^3e^5x^5+384d^4x^4e^4+630d^5e^3x^3+512d^6e^2x^2+945d^7ex+1024d^8\right)\sqrt{-e^2x^2+d^2}}{40320e^5}+\frac{3d^9}{}$
default	$e\left(-\frac{x^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{9e^2}+\frac{4d^2\left(-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{7e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{35e^4}\right)}{9e^2}\right)+d\left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{8e^2}+\frac{3d^2\left(-\frac{x\left(-e^2x^2+d^2\right)}{6e^2}\right.}{}\right.$

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40320*(4480*e^8*x^8+5040*d*e^7*x^7-6400*d^2*e^6*x^6-7560*d^3*e^5*x^5+384
*d^4*e^4*x^4+630*d^5*e^3*x^3+512*d^6*e^2*x^2+945*d^7*e*x+1024*d^8)/e^5*(-e^
2*x^2+d^2)^(1/2)+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(4480\,e^8x^8+5040\,de^7x^7-6400\,d^2e^6x^6-7560\,d^3e^5x^5+384\,d^4e^4x^4+630}{40320\,e^5}$$

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4480*e^8*x^
8 + 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 - 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
+ 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 + 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{3d^9\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{128e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{8d^8}{315e^5}-\frac{3d^7x}{128e^4}-\frac{4d^6x^2}{315e^3}-\frac{d^5x^3}{64e^2}-\right. \\ \left(\frac{dx^5}{5}+\frac{ex^6}{6}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Piecewise((3*d**9*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*
e**4) + sqrt(d**2 - e**2*x**2)*(-8*d**8/(315*e**5) - 3*d**7*x/(128*e**4) -
4*d**6*x**2/(315*e**3) - d**5*x**3/(64*e**2) - d**4*x**4/(105*e) + 3*d**3*x
**5/16 + 10*d**2*e*x**6/63 - d*e**2*x**7/8 - e**3*x**8/9), Ne(e**2, 0)), ((
d*x**5/5 + e*x**6/6)*(d**2)**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}x^4}{9\,e}+\frac{3\,d^9\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}e^4} + \frac{3\,\sqrt{-e^2x^2+d^2}d^7x}{128\,e^4}-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}dx^3}{8\,e^2}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^5x}{64\,e^4} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x^2}{63\,e^3}-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3x}{16\,e^4}-\frac{8\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{315\,e^5}$$

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*arcsin(e^2*x/(d*sqrt(e^2)))/(
sqrt(e^2)*e^4) + 3/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d^2
)^(5/2)*d*x^3/e^2 + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x^2
+ d^2)^(5/2)*d^2*x^2/e^3 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/315*(-
e^2*x^2 + d^2)^(5/2)*d^4/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \frac{3\,d^9\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{128\,e^4|e|} -\frac{1}{40320}\sqrt{-e^2x^2+d^2}\left(\frac{1024\,d^8}{e^5}+\left(\frac{945\,d^7}{e^4}+2\left(\frac{256\,d^6}{e^3}+\left(\frac{315\,d^5}{e^2}+4\left(\frac{48\,d^4}{e}-5\left(189\,d^3+2\left(80\,d^2e-7\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/40320*sqrt(-e^2*x^2
+ d^2)*(1024*d^8/e^5 + (945*d^7/e^4 + 2*(256*d^6/e^3 + (315*d^5/e^2 + 4*(48
*d^4/e - 5*(189*d^3 + 2*(80*d^2*e - 7*(8*e^3*x + 9*d*e^2)*x)*x)*x)*x)*x)*x)
*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^4\left(d^2-e^2\,x^2\right)^{3/2}(d+e\,x)\,dx$$

```
[In] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
[Out] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}-\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

```
[Out] 1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-1/8*
x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e^4
+3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+3/128*d^6*x*(-e^2*x^2+d^2)^
(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 201, 223, 209}

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}-\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\text{integral} = -\frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{\int x^2(-3d^2e-8de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} + \frac{\int x(16d^3e^2+21d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{d^4\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{16e^3}$$

$$= \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{(3d^6)\int\sqrt{d^2-e^2x^2}\,dx}{64e^3}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{(3d^8)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{128e^3}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{(3d^8)\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{128e^3}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{d^2-e^2x^2}(256d^7+105d^6ex+128d^5e^2x^2+70d^4e^3x^3-1024d^3e^4x^4-840d^2e^5x^5+640de^6x^6+560e^7x^7)}{4480e^4}$$

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -1/4480*(Sqrt[d^2 - e^2*x^2]*(256*d^7 + 105*d^6*e*x + 128*d^5*e^2*x^2 + 70*
d^4*e^3*x^3 - 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 + 640*d*e^6*x^6 + 560*e^7*
x^7) + 210*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(560e^{7}x^{7}+640d\,e^{6}x^{6}-840d^{2}e^{5}x^{5}-1024d^{3}e^{4}x^{4}+70d^{4}e^{3}x^{3}+128d^{5}e^{2}x^{2}+105d^{6}ex+256d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{4480e^{4}}+\frac{3d^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{128e^{3}\sqrt{e^{2}}}$
default	$e\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6e^{2}}\right)}{8e^{2}}\right)+d$

[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4480*(560*e^7*x^7+640*d*e^6*x^6-840*d^2*e^5*x^5-1024*d^3*e^4*x^4+70*d^4*e^3*x^3+128*d^5*e^2*x^2+105*d^6*e*x+256*d^7)/e^4*(-e^2*x^2+d^2)^(1/2)+3/128*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(560\,e^7x^7+640\,de^6x^6-840\,d^2e^5x^5-1024\,d^3e^4x^4+70\,d^4e^3x^3+128\,d^5e^2x\right.}{4480\,e^4}$$

[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")

```
[Out] -1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 +
 640*d*e^6*x^6 - 840*d^2*e^5*x^5 - 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 + 128*
d^5*e^2*x^2 + 105*d^6*e*x + 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.10

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\begin{cases}\frac{3d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^3}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^7}{35e^4}-\frac{3d^6x}{128e^3}-\frac{d^5x^2}{35e^2}-\frac{d^4x^3}{64e}+\right. \\ \left(\frac{dx^4}{4}+\frac{ex^5}{5}\right)\left(d^2\right)^{\frac{3}{2}}\end{cases}$$

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Piecewise((3*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*
e**3) + sqrt(d**2 - e**2*x**2)*(-2*d**7/(35*e**4) - 3*d**6*x/(128*e**3) - d
**5*x**2/(35*e**2) - d**4*x**3/(64*e) + 8*d**3*x**4/35 + 3*d**2*e*x**5/16 -
 d*e**2*x**6/7 - e**3*x**7/8), Ne(e**2, 0)), ((d*x**4/4 + e*x**5/5)*(d**2)*
*(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3\,d^8\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}e^3}+\frac{3\,\sqrt{-e^2x^2+d^2}d^6x}{128\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}{7\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{16\,e^3}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{35\,e^4}$$

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^8*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^3) + 3/128*sqrt(-e^2*x^2
 + d^2)*d^6*x/e^3 - 1/8*(-e^2*x^2 + d^2)^(5/2)*x^3/e + 1/64*(-e^2*x^2 + d^2
)^(3/2)*d^4*x/e^3 - 1/7*(-e^2*x^2 + d^2)^(5/2)*d*x^2/e^2 - 1/16*(-e^2*x^2 +
 d^2)^(5/2)*d^2*x/e^3 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^3/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^3|e|}-\frac{1}{4480}\sqrt{-e^2x^2+d^2}\left(\frac{256\,d^7}{e^4}+\left(\frac{105\,d^6}{e^3}+2\left(\frac{64\,d^5}{e^2}+\left(\frac{35\,d^4}{e}-4\left(128\,d^3+5\left(21\,d^2e-2\left(7\,e^3x+8\,de^2\right)x\right)x\right)x\right)x\right)x\right)x\right)$$

[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")

[Out] 3/128*d^8*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/4480*sqrt(-e^2*x^2 + d^2)*(256*d^7/e^4 + (105*d^6/e^3 + 2*(64*d^5/e^2 + (35*d^4/e - 4*(128*d^3 + 5*(21*d^2*e - 2*(7*e^3*x + 8*d*e^2)*x)*x)*x)*x)*x)*x)

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^3\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

[In] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)

[Out] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*
x*(-e^2*x^2+d^2)^(5/2)/e^2+1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 655, 201, 223, 209}

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}$$

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx}{e^2}+\frac{d^2\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}\\
&= -\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{d\int (d^2-e^2x^2)^{5/2}\,dx}{e^2}+\frac{d^3\int (d^2-e^2x^2)^{3/2}\,dx}{e^2}\\
&= \frac{d^3x(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}\\
&\quad +\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{(5d^3)\int (d^2-e^2x^2)^{3/2}\,dx}{6e^2}+\frac{(3d^5)\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{3d^5x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}\\
&\quad +\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{(5d^5)\int \sqrt{d^2-e^2x^2}\,dx}{8e^2}+\frac{(3d^7)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}
\end{aligned}
$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)^{5/2}}{6e^2} + \frac{(d^2-e^2x^2)^{7/2}}{7e^3} - \frac{(5d^7)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^2} + \frac{(3d^7)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)^{5/2}}{6e^2} + \frac{(d^2-e^2x^2)^{7/2}}{7e^3} + \frac{3d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{(5d^7)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)^{5/2}}{6e^2} + \frac{(d^2-e^2x^2)^{7/2}}{7e^3} + \frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(96d^6+105d^5ex+48d^4e^2x^2-490d^3e^3x^3-384d^2e^4x^4+280de^5x^5+240e^6x^6)+210d^7\arctan}{1680e^3}$$

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -1/1680*(Sqrt[d^2 - e^2*x^2]*(96*d^6 + 105*d^5*e*x + 48*d^4*e^2*x^2 - 490*d
^3*e^3*x^3 - 384*d^2*e^4*x^4 + 280*d*e^5*x^5 + 240*e^6*x^6) + 210*d^7*ArcTa
n[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(240e^6x^6+280d\,e^5x^5-384d^2e^4x^4-490d^3x^3e^3+48d^4e^2x^2+105d^5ex+96d^6)\sqrt{-e^2x^2+d^2}}{1680e^3}+\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$e\left(-\frac{x^2(-e^2x^2+d^2)^{\frac{5}{2}}}{7e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{5}{2}}}{35e^4}\right)+d\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan(}{}\right.}{4}\right.}{6e^2}\right.$

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(240*e^6*x^6+280*d*e^5*x^5-384*d^2*e^4*x^4-490*d^3*e^3*x^3+48*d^4*e
^2*x^2+105*d^5*e*x+96*d^6)/e^3*(-e^2*x^2+d^2)^(1/2)+1/16*d^7/e^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(240\,e^6x^6+280\,de^5x^5-384\,d^2e^4x^4-490\,d^3e^3x^3+48\,d^4e^2x^2+105\,d^5ex+}{1680\,e^3}$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (240*e^6*x^6 +
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 - 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 + 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\begin{cases}\frac{d^7\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^6}{35e^3}-\frac{d^5x}{16e^2}-\frac{d^4x^2}{35e}+\frac{7d^3x^3}{24}+\right.\\ \left(\frac{dx^3}{3}+\frac{ex^4}{4}\right)\left(d^2\right)^{\frac{3}{2}}\end{cases}$$

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Piecewise((d**7*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**
2) + sqrt(d**2 - e**2*x**2)*(-2*d**6/(35*e**3) - d**5*x/(16*e**2) - d**4*x*
*2/(35*e) + 7*d**3*x**3/24 + 8*d**2*e*x**4/35 - d*e**2*x**5/6 - e**3*x**6/7
), Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)*(d**2)**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^7\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^2}+\frac{\sqrt{-e^2x^2+d^2}d^5x}{16\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35\,e^3}$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^7*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/16*sqrt(-e^2*x^2 +
 d^2)*d^5*x/e^2 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 + d
^2)^(5/2)*x^2/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 + d^2
)^(5/2)*d^2/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^7\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^2|e|}$$
$$-\frac{1}{1680}\sqrt{-e^2x^2+d^2}\left(\frac{96\,d^6}{e^3}+\left(\frac{105\,d^5}{e^2}+2\left(\frac{24\,d^4}{e}-\left(245\,d^3+4\left(48\,d^2e-5\left(6\,e^3x+7\,de^2\right)x\right)x\right)x\right)x\right)x\right)$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] 1/16*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/1680*sqrt(-e^2*x^2 +
d^2)*(96*d^6/e^3 + (105*d^5/e^2 + 2*(24*d^4/e - (245*d^3 + 4*(48*d^2*e - 5*
(6*e^3*x + 7*d*e^2)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x^2\,\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)\,dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
[Out] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {794, 201, 223, 209}

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}\\
&= \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}\\
&\quad+\frac{d^6\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5)+30d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(48*d^5 + 15*d^4*e*x - 96*d^3*e^2*x^2 - 70*d^2*
e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqr
t[d^2 - e^2*x^2])])/e^2
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(40e^5x^5+48d\,e^4x^4-70d^2e^3x^3-96d^3e^2x^2+15d^4ex+48d^5)\sqrt{-e^2x^2+d^2}}{240e^2}+\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$	108
default	$e\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6e^2}\right)-\frac{d(-e^2x^2+d^2)^{\frac{5}{2}}}{5e^2}$	126

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(40*e^5*x^5+48*d*e^4*x^4-70*d^2*e^3*x^3-96*d^3*e^2*x^2+15*d^4*e*x+48
*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx =$$

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5)\sqrt{-e^2x^2+d}}{240\,e^2}$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e}+\sqrt{d^2-e^2x^2}\left(-\frac{d^5}{5e^2}-\frac{d^4x}{16e}+\frac{2d^3x^2}{5}+\frac{7d^2ex^3}{24}-\frac{d}{} \right. \\ \left(\frac{dx^2}{2}+\frac{ex^3}{3}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e)
+ sqrt(d**2 - e**2*x**2)*(-d**5/(5*e**2) - d**4*x/(16*e) + 2*d**3*x**2/5 +
7*d**2*e*x**3/24 - d*e**2*x**4/5 - e**3*x**5/6), Ne(e**2, 0)), ((d*x**2/2 +
 e*x**3/3)*(d**2)**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 + d
^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5
/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|}-\frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2}+\left(\frac{15\,d^4}{e}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,e^3x+6\,de^2\right)x\right)x\right)x\right)x\right)$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d^2
)*(48*d^5/e^2 + (15*d^4/e - 2*(48*d^3 + (35*d^2*e - 4*(5*e^3*x + 6*d*e^2)*x
)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {794, 201, 223, 209}

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}\\
&= \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}\\
&\quad+\frac{d^6\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e}\\
&= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5)+30d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(48*d^5 + 15*d^4*e*x - 96*d^3*e^2*x^2 - 70*d^2*
e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqr
t[d^2 - e^2*x^2])])/e^2
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\left(40e^5x^5+48d\,e^4x^4-70d^2e^3x^3-96d^3e^2x^2+15d^4ex+48d^5\right)\sqrt{-e^2x^2+d^2}}{240e^2}+\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$	108
default	$e\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6e^2}\right)-\frac{d\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5e^2}$	126

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(40*e^5*x^5+48*d*e^4*x^4-70*d^2*e^3*x^3-96*d^3*e^2*x^2+15*d^4*e*x+48
*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+}}{240\,e^2}$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = \begin{cases} \frac{d^6\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e}+\sqrt{d^2-e^2x^2}\left(-\frac{d^5}{5e^2}-\frac{d^4x}{16e}+\frac{2d^3x^2}{5}+\frac{7d^2ex^3}{24}-\right. \\ \left(\frac{dx^2}{2}+\frac{ex^3}{3}\right)\left(d^2\right)^{\frac{3}{2}} \end{cases}$$

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e)
+ sqrt(d**2 - e**2*x**2)*(-d**5/(5*e**2) - d**4*x/(16*e) + 2*d**3*x**2/5 +
7*d**2*e*x**3/24 - d*e**2*x**4/5 - e**3*x**5/6), Ne(e**2, 0)), ((d*x**2/2 +
 e*x**3/3)*(d**2)**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] 1/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 + d
^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5
/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|}-\frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2}+\left(\frac{15\,d^4}{e}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,e^3x+6\,de^2\right)x\right)x\right)x\right)x\right)$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] 1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d^2
)*(48*d^5/e^2 + (15*d^4/e - 2*(48*d^3 + (35*d^2*e - 4*(5*e^3*x + 6*d*e^2)*x
)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=\int x\left(d^2-e^2\,x^2\right)^{3/2}(d+e\,x)\,dx$$

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x} \, dx = \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4 \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(3*e*x+8*d)*(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x} \, dx = \frac{3}{8}d^4 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4 \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2-3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2} \\
&= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4+3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4} \\
&= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{8}\left(3d^4e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + \frac{1}{2}d^5\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&\quad + \frac{1}{8}\left(3d^4e\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^5\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2} \\
&= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right) - \frac{3}{4}d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - d^3\sqrt{d^2}\log(x) + d^3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 -
(3*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/4 - d^3*Sqrt[d^2]*L
og[x] + d^3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.39

method	result
default	$e\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)+d\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2}{\ldots}\right.\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] e*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(
e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d*(1/3*(-e^2*x^2+d^
2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = -\frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \frac{1}{24}\left(6\,e^3x^3+8\,de^2x^2-15\,d^2ex-32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
[Out] -3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - 1/24*(6*e^3*x^3 + 8*d*e^2*x^2 - 15*d^2*e*x - 32*d^3)*sqr
t(-e^2*x^2 + d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$
$$+ d^2e\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2 \neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$
$$- de^2\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for } e^2 \neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right)$$
$$-e^3\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} - \frac{d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4} & \text{for } e^2 \neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + d**2*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt
(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-
e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d*
*2), True)) - d*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**
2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) - e**3
*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**
2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4
, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{3\,d^4e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}} - d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3}{8}\,\sqrt{-e^2x^2+d^2}d^2ex + \sqrt{-e^2x^2+d^2}d^3 + \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*d^4*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^4*log(2*d^2/abs(x) + 2*
sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-e
^2*x^2 + d^2)*d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^(
3/2)*d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{3\,d^4e\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} - \frac{d^4e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \frac{1}{24}\,\sqrt{-e^2x^2+d^2}\left(32\,d^3+\left(15\,d^2e-2\,\left(3\,e^3x+4\,de^2\right)x\right)x\right)$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 3/8*d^4*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^4*e*log(1/2*abs(-2*d*e - 2
*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*sqrt(-e^2*x^2 + d
^2)*(32*d^3 + (15*d^2*e - 2*(3*e^3*x + 4*d*e^2)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{3} - d^4\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right) + d^3\,\sqrt{d^2-e^2\,x^2} + \frac{e\,x\left(d^2-e^2\,x^2\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x,x)
```

```
[Out] (d*(d^2 - e^2*x^2)^(3/2))/3 - d^4*atanh((d^2 - e^2*x^2)^(1/2)/d) + d^3*(d^2
 - e^2*x^2)^(1/2) + (e*x*(d^2 - e^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2,
(e^2*x^2)/d^2))/(1 - (e^2*x^2)/d^2)^(3/2)
```

3.8 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2} dx = \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*d*e*(-3*e*x+2*d)*(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {827, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2} dx = -\frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
```

```
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{1}{2}\int\frac{\left(-2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\frac{\int\frac{4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}\\
&\quad+\left(d^4e\right)\int\frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\frac{1}{2}\left(3d^3e^2\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}\\
&\quad+\frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&\quad-\frac{1}{2}\left(3d^3e^2\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}\\
&\quad-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^4\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}\\
&\quad-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex-3de^2x^2-2e^3x^3)}{6x} + 2d^3e\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{3}{2}d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 3*d*e^2*x^2 - 2*e^3*x^3))/(6*x)
+ 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{d^3\sqrt{-e^2x^2+d^2}}{x}-\frac{e^3x^2\sqrt{-e^2x^2+d^2}}{3}+\frac{4e\,d^2\sqrt{-e^2x^2+d^2}}{3}-\frac{e\,d^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{3d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$
default	$e\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)+d\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{d^2x}-\frac{4e^2\left(\frac{x\left(-e^2x\right.}{}\right.}{}\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3*(-e^2*x^2+d^2)^(1/2)/x-1/3*e^3*x^2*(-e^2*x^2+d^2)^(1/2)+4/3*e*d^2*(-e^
2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)-3/2*d^3*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
-1/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = \frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+8\,d^3ex-(2\,e^3}{6\,x}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) + 8*d^3*e*x - (2*e^3*x^3 + 3*d*e^2*x^2 - 8*d^
2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.32

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

$$+d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

$$-de^2\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$

$$-e^3\left(\begin{cases}-\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
```

```
True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((d**2*Piecewise((log(-2*e**2
*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*lo
g(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)),
 (x*sqrt(d**2), True)) - e**3*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e*
*2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} - d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{2}\,\sqrt{-e^2x^2+d^2}de^2x + \sqrt{-e^2x^2+d^2}d^2e + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{x}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^3*e*log(2*d^2/abs(x)
 + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x + sq
rt(-e^2*x^2 + d^2)*d^2*e + 1/3*(-e^2*x^2 + d^2)^(3/2)*e - (-e^2*x^2 + d^2)^
(3/2)*d/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,|e|} + \frac{d^3e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{d^3e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{2\,x|e|} + \frac{1}{6}\,\sqrt{-e^2x^2+d^2}\left(8\,d^2e-\left(2\,e^3x+3\,de^2\right)x\right)$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/2*d^3*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^3*e^4*x/((d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))*abs(e)) - d^3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^
2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))*d^3/(x*abs(e)) + 1/6*sqrt(-e^2*x^2 + d^2)*(8*d^2*e - (2*e^3*x + 3*
d*e^2)*x)
```

Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx=\frac{e\left(d^2-e^2\,x^2\right)^{3/2}}{3}+d^2\,e\,\sqrt{d^2-e^2\,x^2}-d^3\,e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)-\frac{d^3\,\sqrt{d^2-e^2\,x^2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,\sqrt{1-\frac{e^2\,x^2}{d^2}}}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^2,x)
```

```
[Out] (e*(d^2 - e^2*x^2)^(3/2))/3 + d^2*e*(d^2 - e^2*x^2)^(1/2) - d^3*e*atanh((d^
2 - e^2*x^2)^(1/2)/d) - (d^3*(d^2 - e^2*x^2)^(1/2)*hypergeom([-3/2, -1/2],
1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(1/2))
```

3.9 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2-3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2
)^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)-3/2*d*e*(e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(-4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2-8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \frac{1}{2}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(3d^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(-\frac{\sqrt{d^2-e^2x^2}\left(d^3+2d^2ex+2de^2x^2+e^3x^3\right)}{x^2} + 6d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 3d\sqrt{d^2}e^2\log(x) - 3d\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/x^2) + 6
*d^2*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 3*d*Sqrt[d^2]*e^
2*Log[x] - 3*d*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}\,(2ex+d)}{2x^2}-\frac{3e^3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{e^3x\sqrt{-e^2x^2+d^2}}{2}+\frac{3e^2d^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-d\,e^2\sqrt{-$
default	$d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{2d^2}\right)+e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{d^2x}-\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d^2*(-e^2*x^2+d^2)^(1/2)*(2*e*x+d)/x^2-3/2*e^3*d^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*e^2*
d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d*e^2*(-e^
2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{x^3}\,dx=\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,d^2e^2x^2-}{2\,x^2}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*d^2*e^2*x^
2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d^2*e^2*x^2 - (e^3*x^3 + 2*d*e^2*x
^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.67

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$+ d^2e\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$- de^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$
$$- e^3\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)
) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)
) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*
e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piecewise((d**2*Piecewise((l
og(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2,
0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(
e**2, 0)), (x*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3\,d^2e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{2}\sqrt{-e^2x^2+d^2}e^3x - \frac{3}{2}\sqrt{-e^2x^2+d^2}de^2 - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{2\,d} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{x} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2\,dx^2}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*d^2*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d^2*e^2*log(2*d^2/
abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*e^3*x
- 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^2/d - (-e^2
*x^2 + d^2)^(3/2)*e/x - 1/2*(-e^2*x^2 + d^2)^(5/2)/(d*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{3\,d^2e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{3\,d^2e^3\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^2e^3+\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} - \frac{1}{2}\left(e^3x+2\,de^2\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,e^2}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*d^2*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 3/2*d^2*e^3*log(1/2*abs(-
2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^2*e^3
+ 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e/x)*e^4*x^2/((d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2*abs(e)) - 1/2*(e^3*x + 2*d*e^2)*sqrt(-e^2*x^2 + d^2) -
 1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x + (d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*d^2*abs(e)/(e*x^2))/e^2
```

Mupad [B] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3}\,dx = \frac{3\,d^2\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2} - \frac{d^3\,\sqrt{d^2-e^2\,x^2}}{2\,x^2} - d\,e^2\,\sqrt{d^2-e^2\,x^2} - \frac{e\,(d^2-e^2\,x^2)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^3,x)
```

```
[Out] (3*d^2*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/2 - (d^3*(d^2 - e^2*x^2)^(1/2))/
(2*x^2) - d*e^2*(d^2 - e^2*x^2)^(1/2) - (e*(d^2 - e^2*x^2)^(3/2)*hypergeom(
[-3/2, -1/2], 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(3/2))
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(-3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {825, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{\int \frac{\left(4d^3e^2+6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{\int \frac{-12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}\\
&\quad -\frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+\left(de^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}\\
&\quad -\frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&\quad +\left(de^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}\\
&\quad +de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}\\
&\quad +de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^3-3d^2ex+8de^2x^2-6e^3x^3)}{6x^3} -2de^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}\sqrt{d^2}e^3\log(x)-\frac{3}{2}\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 - 6*e^3*x^3))/(6*x^3
) - 2*d*e^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (3*Sqrt[d^2]*
e^3*Log[x])/2 - (3*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,d\left(-8e^2x^2+3dex+2d^2\right)}{6x^3}+\frac{e^4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-e^3\sqrt{-e^2x^2+d^2}+\frac{3e^3d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$
default	$e\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{2d^2}\right)+d\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3d^2x^3}\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-e^2*x^2+d^2)^(1/2)*d*(-8*e^2*x^2+3*d*e*x+2*d^2)/x^3+e^4*d/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-e^3*(-e^2*x^2+d^2)^(1/2)+3/2*e
```

```
^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = -\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+6\,de^3x^3+\left(6\,e^3x^3-8\,de^2x^2+3\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x^3}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 6*d*e^3*x^3 + (6*e^3*x^3 - 8*d*e^2*x^2
 + 3*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.81

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

$$+ d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

$$- de^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

$$- e^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x)
)/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acos
h(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**
2*x**2/d**2)), True)) - e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1
)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2
)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I
*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{de^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^4x}{d} - \frac{3}{2}\sqrt{-e^2x^2+d^2}e^3 - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2\,d^2} + \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{3\,dx} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{2\,d^2x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3\,dx^3}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] d*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d*e^3*log(2*d^2/abs(x) +
2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^4*x/d - 3/2*sqrt(
-e^2*x^2 + d^2)*e^3 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^3/d^2 + 2/3*(-e^2*x^2 +
d^2)^(3/2)*e^2/(d*x) - 1/2*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^2) - 1/3*(-e^2*x
^2 + d^2)^(5/2)/(d*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(106) = 212.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \frac{de^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\left(de^4+\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^2}{x}-\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} + \frac{3\,de^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} - \sqrt{-e^2x^2+d^2}e^3 + \frac{\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^4}{x}-\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^2}{x^2}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{x^3}}{24\,e^2|e|}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] d*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/24*(d*e^4 + 3*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))*d*e^2/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/x^
2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*abs(e)) + 3/2*d*e^4*log(1
/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - sqrt(
-e^2*x^2 + d^2)*e^3 + 1/24*(15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e^4/x
- 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^2/x^2 - (d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^3*d/x^3)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}{x^4}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4, x)
```

3.11 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5} \, dx = \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4+e^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(8*e*x+3*d)*(-e^2*x^2+
d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5} \, dx = e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{(6d^3e^2+8d^2e^3x)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4+32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + e^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + e^5\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(3de^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{d^2-e^2x^2}\left(-6d^3-8d^2ex+15de^2x^2+32e^3x^3\right)}{x^4} - 48e^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{9\sqrt{d^2}e^4\log(x)}{d} + \frac{9\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x + 15*d*e^2*x^2 + 32*e^3*x^3))/x^4
 - 48*e^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (9*Sqrt[d^2]*e^
4*Log[x])/d + (9*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d)/24
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-32 e^{3} x^{3}-15 d\,e^{2} x^{2}+8 d^{2} e x +6 d^{3}\right)}{24 x^{4}}+\frac{e^{5} \arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{3 e^{4} d \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$
default	$d\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{4 d^{2} x^{4}}-\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{2 d^{2} x^{2}}-\frac{3 e^{2}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)}{2 d^{2}}\right)}{4 d^{2}}\right)+e$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(-32*e^3*x^3-15*d*e^2*x^2+8*d^2*e*x+6*d^3)/x^4+e
^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/8*e^4*d/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = -\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+15\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2}}{24\,x^4}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 15*d*e^2*x^2 - 8*d^2*e*x -
6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.58

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$
$$+d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$
$$-de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$-e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e*
*2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*a
sin(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**
2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x
**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(
d*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{8}\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^5x}{d^2} + \frac{3\,\sqrt{-e^2x^2+d^2}e^4}{8\,d} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8\,d^3} + \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{3\,d^2x} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{8\,d^3x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{3\,d^2x^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\,dx^4}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/8*e^4*log(2*d^2/abs(x) + 2*sq
rt(-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^5*x/d^2 + 3/8*sqrt(-e
^2*x^2 + d^2)*e^4/d + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^4/d^3 + 2/3*(-e^2*x^2 +
d^2)^(3/2)*e^3/(d^2*x) + 1/8*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^2) - 1/3*(-e
^2*x^2 + d^2)^(5/2)*e/(d^2*x^3) - 1/4*(-e^2*x^2 + d^2)^(5/2)/(d*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \frac{\left(3\,e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}-\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}-\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} + \frac{e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} + \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3}-\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{192\,e^4}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*e^5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x - 24*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 120*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) + e^5*arc
sin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*
x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/192*(120*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))*e^5*abs(e)/x + 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^3*a
bs(e)/x^2 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 - 3*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)}{x^5}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5, x)
```

3.12 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6} dx = \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{3e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] -1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-3/8*e^5*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d+3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {821, 272, 43, 65, 214}

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6} dx = -\frac{3e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+e\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx\\
&= -\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}\\
&\quad +\frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{1}{8}(3e^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6}\,dx = \frac{1}{40}\left(\frac{\sqrt{d^2-e^2x^2}(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4)}{dx^5} - \frac{15e^5\log(x)}{\sqrt{d^2}} + \frac{15e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 10*d^3*e*x + 16*d^2*e^2*x^2 + 25*d*e^3*x^3
- 8*e^4*x^4))/(d*x^5) - (15*e^5*Log[x])/Sqrt[d^2] + (15*e^5*Log[Sqrt[d^2] -
 Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2])/40
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(8e^4x^4-25d\,e^3x^3-16d^2e^2x^2+10d^3ex+8d^4)}{40x^5d} - \frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4} - \frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2} - \frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{2d^2}\right)}{4d^2}\right) - \frac{(-$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-e^2*x^2+d^2)^(1/2)*(8*e^4*x^4-25*d*e^3*x^3-16*d^2*e^2*x^2+10*d^3*e*
x+8*d^4)/x^5/d-3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx=\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(8\,e^4x^4-25\,de^3x^3-16\,d^2e^2x^2+10\,d^3ex+8\right.}{40\,dx^5}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 - 25*d*e^3
*x^3 - 16*d^2*e^2*x^2 + 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 774, normalized size of antiderivative = 7.17

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx=d^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7}\end{cases}\right.$$

$$+\,d^2e\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)$$

$$-\,de^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$

$$-\,e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*Piecewise((-e*sqrt(d**2/(
e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx = -\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d}+\frac{3\,\sqrt{-e^2x^2+d^2}e^5}{8\,d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8\,d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8\,d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4\,d^2x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,dx^5}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d + 3/8*sqrt(-
e^2*x^2 + d^2)*e^5/d^2 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/d^4 + 1/8*(-e^2*x^2
 + d^2)^(5/2)*e^3/(d^4*x^2) - 1/4*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^4) - 1/5*
(-e^2*x^2 + d^2)^(5/2)/(d*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(92) = 184.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.59

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx = \frac{\left(2\,e^6+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x}-\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2}-\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3}+\right.}{320\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5d|e|} - \frac{3\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d|e|} - \frac{\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^8}{x}-\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^6}{x^2}-\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e^4}{x^3}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4e^2}{x^4}+\frac{2\left(de\right.}{}}{320\,d^5e^4|e|}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*(2*e^6 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x - 10*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 - 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/x^3 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d*abs(e)) - 3/8*e^6*log(1/2*abs(-2*d*e -
2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/320*(20*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^8/x - 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^2*d^4*e^6/x^2 - 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4*e^4/x^3 +
 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*e^2/x^4 + 2*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^5*d^4/x^5)/(d^5*e^4*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx = \frac{3\,d^2\,e\,\sqrt{d^2-e^2\,x^2}}{8\,x^4} - \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{5\,d\,x^5} - \frac{3\,e^5\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{8\,d} - \frac{5\,e\,\left(d^2-e^2\,x^2\right)^{3/2}}{8\,x^4}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^6,x)
```

```
[Out] (3*d^2*e*(d^2 - e^2*x^2)^(1/2))/(8*x^4) - (d^2 - e^2*x^2)^(5/2)/(5*d*x^5) -
 (3*e^5*atanh((d^2 - e^2*x^2)^(1/2)/d))/(8*d) - (5*e*(d^2 - e^2*x^2)^(3/2))
/(8*x^4)
```

3.13 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7} dx = \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {849, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7} dx = -\frac{e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} + \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{\int \frac{(-6d^2e-de^2x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{6d^2}$$
$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d}$$
$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$
$$= -\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d}$$
$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^6\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{32d}$$
$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{16d}$$
$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7}\,dx = \frac{\sqrt{d^2-e^2x^2}(-40d^5-48d^4ex+70d^3e^2x^2+96d^2e^3x^3-15de^4x^4-48e^5x^5)}{240d^2x^6} - \frac{\sqrt{d^2}e^6\log(x)}{16d^3} + \frac{\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^3}$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 - 48*d^4*e*x + 70*d^3*e^2*x^2 + 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 - 48*e^5*x^5))/(240*d^2*x^6) - (Sqrt[d^2]*e^6*Log[x])/(16*
d^3) + (Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(16*d^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(48 e^{5} x^{5}+15 d\,e^{4} x^{4}-96 d^{2} e^{3} x^{3}-70 d^{3} e^{2} x^{2}+48 d^{4} e x+40 d^{5}\right)}{240 x^{6} d^{2}}-\frac{e^{6} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{16 d \sqrt{d^{2}}}$
default	$d\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6 d^{2} x^{6}}+\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{4 d^{2} x^{4}}-\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{2 d^{2} x^{2}}-\frac{3 e^{2}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2}\cdots}}{x}\right)}{\sqrt{d^{2}}}\cdots\right)\right)}{2 d^{2}}\right)}{4 d^{2}}\right)}{6 d^{2}}\cdots\right)$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-e^2*x^2+d^2)^(1/2)*(48*e^5*x^5+15*d*e^4*x^4-96*d^2*e^3*x^3-70*d^3*
e^2*x^2+48*d^4*e*x+40*d^5)/x^6/d^2-1/16/d*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx=\frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(48\,e^5x^5+15\,de^4x^4-96\,d^2e^3x^3-70\,d^3e^2x^2\cdots\right.}{240\,d^2x^6}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x^5 + 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 - 70*d^3*e^2*x^2 + 48*d^4*e*x + 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.07 (sec) , antiderivative size = 918, normalized size of antiderivative = 6.42

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = d^3\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \end{cases}\right.$$

$$+d^2e\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

$$-de^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$-e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
```

```
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) - e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^2} + \frac{\sqrt{-e^2x^2+d^2}e^6}{16\,d^3} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48\,d^5} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{48\,d^5x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{24\,d^3x^4} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{5\,d^2x^5} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6\,dx^6}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d^5 + 1/48*(-e
^2*x^2 + d^2)^(5/2)*e^4/(d^5*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^
4) - 1/5*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)/(d
*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(123) = 246.

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.24

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = \frac{\left(5e^7 + \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e}{x^3} - \cdots\right.}{1920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^6 d\cdots} - \frac{e^7\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^2|e|} - \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{10}e^9|e|}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{10}e^7|e|}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{10}e^5|e|}{x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{10}e\cdots}{x^4}\cdots}{1920\,d^{12}e^6}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*(5*e^7 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^5/x - 15*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*e^3/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3*e/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e*x^4) + 120*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^3*x^5))*e^12*x^6/((d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^6*d^2*abs(e)) - 1/16*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2
+ d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) - 1/1920*(120*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^10*e^9*abs(e)/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^2*d^10*e^7*abs(e)/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^10*e^5*
abs(e)/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^10*e^3*abs(e)/x^4 +
 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^10*e*abs(e)/x^5 + 5*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^6*d^10*abs(e)/(e*x^6))/(d^12*e^6)
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = \frac{d^3\sqrt{d^2-e^2\,x^2}}{16\,x^6} - \frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{6\,x^6} - \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{16\,d\,x^6} - \frac{e\left(d^2-e^2\,x^2\right)^{5/2}}{5\,d^2\,x^5} + \frac{e^6\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,1\mathrm{i}}{16\,d^2}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^7,x)
```

```
[Out] (d^3*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (d*(d^2 - e^2*x^2)^(3/2))/(6*x^6) -
(d^2 - e^2*x^2)^(5/2)/(16*d*x^6) + (e^6*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*
1i)/(16*d^2) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5)
```

3.14 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] -1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/16*e^
7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {849, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} -\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}+\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{\int \frac{(-7d^2e-2de^2x)(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{\int \frac{(12d^3e^2+7d^2e^3x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^3\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^3\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d^2}$$

$$= -\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^5\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d^2}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^7\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{32d^2}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^5\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{16d^2}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{d^2-e^2x^2}(-240d^6-280d^5ex+384d^4e^2x^2+490d^3e^3x^3-48d^2e^4x^4-105de}{1680d^3x^7}$$
$$-\frac{\sqrt{d^2}e^7\log(x)}{16d^4}+\frac{\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^4}$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 - 280*d^5*e*x + 384*d^4*e^2*x^2 + 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 - 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) - (Sqr
t[d^2]*e^7*Log[x])/(16*d^4) + (Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^2
*x^2]])/(16*d^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(96e^6x^6+105d\,e^5x^5+48d^2e^4x^4-490d^3x^3e^3-384d^4e^2x^2+280d^5ex+240d^6)}{1680x^7d^3}-\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d^2\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2}}{x}\right.}{\sqrt{d^2}}\right.\right.}{2d^2}\right.}{4d^2}\right.}{6d^2}\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(-e^2*x^2+d^2)^(1/2)*(96*e^6*x^6+105*d*e^5*x^5+48*d^2*e^4*x^4-490*d
^3*e^3*x^3-384*d^4*e^2*x^2+280*d^5*e*x+240*d^6)/x^7/d^3-1/16/d^2*e^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(96\,e^6x^6+105\,de^5x^5+48\,d^2e^4x^4-490\,d^3e^3\right.}{1680\,d^3x^7}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (96*e^6*x^6 + 105*
d*e^5*x^5 + 48*d^2*e^4*x^4 - 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 + 280*d^5*e*
x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.68 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.03

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
```

```
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3}+\frac{\sqrt{-e^2x^2+d^2}e^7}{16\,d^4} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{48\,d^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{48\,d^6x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{24\,d^4x^4} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{35\,d^3x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{6\,d^2x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{7\,dx^7}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^7/d^4 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^7/d^6 + 1/48*(-e
^2*x^2 + d^2)^(5/2)*e^5/(d^6*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^
4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)
*e/(d^2*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)/(d*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(148) = 296$.

Time = 0.29 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.01

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx = \frac{\left(15\,e^8 + \frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^6}{x} - \frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^4}{x^2} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x^3}\right.}{13440\left(de+\sqrt{-\right.}}$$
$$-\frac{e^8\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^3|e|}$$
$$-\frac{\frac{315\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{12}}{x} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{18}e^{10}}{x^2} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{18}e^8}{x^3} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{18}e}{x^4}}{13440\,d^{21}e^6|e|}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/13440*(15*e^8 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^6/x - 21*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^4/x^2 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3*e^2/x^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/x^4 - 105*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^2*x^5) + 315*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^6/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^3*
abs(e)) - 1/16*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*abs(x)))/(d^3*abs(e)) - 1/13440*(315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d
^18*e^12/x - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^18*e^10/x^2 - 105*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^18*e^8/x^3 - 105*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^4*d^18*e^6/x^4 - 21*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5
*d^18*e^4/x^5 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^18*e^2/x^6 + 15*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18/x^7)/(d^21*e^6*abs(e))
```

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx &= \frac{8\,d\,e^2\sqrt{d^2-e^2\,x^2}}{35\,x^5} - \frac{d^3\sqrt{d^2-e^2\,x^2}}{7\,x^7} \\ &- \frac{e^4\sqrt{d^2-e^2\,x^2}}{35\,d\,x^3} - \frac{2\,e^6\sqrt{d^2-e^2\,x^2}}{35\,d^3\,x} - \frac{e\,(d^2-e^2\,x^2)^{3/2}}{6\,x^6} \\ &+ \frac{d^2\,e\sqrt{d^2-e^2\,x^2}}{16\,x^6} - \frac{e\,(d^2-e^2\,x^2)^{5/2}}{16\,d^2\,x^6} + \frac{e^7\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,1\mathrm{i}}{16\,d^3}\end{aligned}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^8,x)
```

```
[Out] (e^7*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*1i)/(16*d^3) - (d^3*(d^2 - e^2*x^2)
^(1/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(3/2))/(6*x^6) - (e^4*(d^2 - e^2*x^2)^
(1/2))/(35*d*x^3) - (2*e^6*(d^2 - e^2*x^2)^(1/2))/(35*d^3*x) + (8*d*e^2*(d^
2 - e^2*x^2)^(1/2))/(35*x^5) + (d^2*e*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (e*
(d^2 - e^2*x^2)^(5/2))/(16*d^2*x^6)
```

3.15 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^9}\,dx = \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{3e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

```
[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/7*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-2/35*e^
3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {849, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^9}\,dx = -\frac{3e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

p])

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{\int \frac{(-8d^2e-3de^2x)(d^2-e^2x^2)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} + \frac{\int \frac{(21d^3e^2+16d^2e^3x)(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{\int \frac{(-96d^4e^3-21d^3e^4x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{16d^3}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{32d^3}$$

$$= -\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{(3e^6)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{128d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{(3e^8)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{256d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{(3e^6)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{128d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{\sqrt{d^2-e^2x^2}(-560d^7-640d^6ex+840d^5e^2x^2+1024d^4e^3x^3-70d^3e^4x^4-12}{4480d^4x^8}$$

$$-\frac{3\sqrt{d^2}e^8\log(x)}{128d^5}+\frac{3\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d^5}$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 - 640*d^6*e*x + 840*d^5*e^2*x^2 + 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 - 128*d^2*e^5*x^5 - 105*d*e^6*x^6 - 256*e^7*x^7))/(
4480*d^4*x^8) - (3*Sqrt[d^2]*e^8*Log[x])/(128*d^5) + (3*Sqrt[d^2]*e^8*Log[S
qrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*d^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(256e^7x^7+105d\,e^6x^6+128d^2e^5x^5+70d^3e^4x^4-1024d^4e^3x^3-840d^5e^2x^2+640d^6ex+560d^7)}{4480x^8d^4}-\frac{3e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2}}{x}\right.}{128d^3\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{7d^2x^7}-\frac{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}{35d^4x^5}\right)+d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{8d^2x^8}+\frac{3e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2}{}\right.}{}\right)}{}\right)}{}\right.$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4480*(-e^2*x^2+d^2)^(1/2)*(256*e^7*x^7+105*d*e^6*x^6+128*d^2*e^5*x^5+70*
d^3*e^4*x^4-1024*d^4*e^3*x^3-840*d^5*e^2*x^2+640*d^6*e*x+560*d^7)/x^8/d^4-3
/128/d^3*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(256\,e^7x^7+105\,de^6x^6+128\,d^2e^5x^5+70\,d^3e\right.}{4480\,d^4x^8}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^7*x^7 + 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 + 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 - 840*d^5*
e^2*x^2 + 640*d^6*e*x + 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.09 (sec) , antiderivative size = 1159, normalized size of antiderivative = 5.77

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
```

```
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d^4} + \frac{3\,\sqrt{-e^2x^2+d^2}e^8}{128\,d^5} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{128\,d^7} + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{128\,d^7x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{64\,d^5x^4} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{35\,d^4x^5} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{16\,d^3x^6} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{7\,d^2x^7} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{8\,dx^8}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 1/128*(-e^2*x^2 + d^2)^(3/2)*e^8/d^7 + 1/128
*(-e^2*x^2 + d^2)^(5/2)*e^6/(d^7*x^2) - 1/64*(-e^2*x^2 + d^2)^(5/2)*e^4/(d^
5*x^4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^5) - 1/16*(-e^2*x^2 + d^2)^
(5/2)*e^2/(d^3*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^7) - 1/8*(-e^2*x^
2 + d^2)^(5/2)/(d*x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(173) = 346$.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{\left(35\,e^9 + \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^3}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right.}{x^4}}{71680\left(de+\sqrt{-e^2x^2+d^2}|\right.}$$
$$- \frac{3\,e^9\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d^4|e|}$$
$$- \frac{\frac{1680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{28}e^{13}|e|}{x} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{28}e^9|e|}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{28}e^7|e|}{x^4} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e\right.}{x^5}}{71680\,d^{32}e^8}$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/71680*(35*e^9 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x - 112*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^3*e^3/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^4*e/x^4 - 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) + 1680*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^8*d^4*abs(e)) - 3/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-e
^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) - 1/71680*(1680*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))*d^28*e^13*abs(e)/x - 560*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*d^28*e^9*abs(e)/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^4*d^28*e^7*abs(e)/x^4 - 112*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^28*e^5
*abs(e)/x^5 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^28*e*abs(e)/x^7 +
35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^28*abs(e)/(e*x^8))/(d^32*e^8)
```

Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = \frac{3\,d^3\,\sqrt{d^2-e^2\,x^2}}{128\,x^8} - \frac{11\,d\,\left(d^2-e^2\,x^2\right)^{3/2}}{128\,x^8}$$
$$- \frac{11\,\left(d^2-e^2\,x^2\right)^{5/2}}{128\,d\,x^8} + \frac{3\,\left(d^2-e^2\,x^2\right)^{7/2}}{128\,d^3\,x^8} + \frac{8\,e^3\,\sqrt{d^2-e^2\,x^2}}{35\,x^5} - \frac{e^5\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,x^3}$$
$$- \frac{2\,e^7\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,x} - \frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{7\,x^7} + \frac{e^8\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1\mathrm{i}}{d}\right)\,3\mathrm{i}}{128\,d^4}$$

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^9,x)
```

```
[Out] (3*d^3*(d^2 - e^2*x^2)^(1/2))/(128*x^8) - (11*d*(d^2 - e^2*x^2)^(3/2))/(128
*x^8) - (11*(d^2 - e^2*x^2)^(5/2))/(128*d*x^8) + (3*(d^2 - e^2*x^2)^(7/2))/
```

```
(128*d^3*x^8) + (8*e^3*(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (e^8*atan(((d^2 -
e^2*x^2)^(1/2)*1i)/d)*3i)/(128*d^4) - (e^5*(d^2 - e^2*x^2)^(1/2))/(35*d^2*x
^3) - (2*e^7*(d^2 - e^2*x^2)^(1/2))/(35*d^4*x) - (d^2*e*(d^2 - e^2*x^2)^(1/
2))/(7*x^7)
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] 1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d
^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 655, 201, 223, 209}

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}$$

```
[In] Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}+\frac{d^2\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{d\int \sqrt{d^2-e^2x^2}\,dx}{e^2}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}\\
&\quad -\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}+\frac{d^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}\\
&\quad +\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{d^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{(-4d^2-3dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{6e^3} - \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
[Out] ((-4*d^2 - 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*e^3) - (d^3*ArcTan[
(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{(2e^2x^2+3dex+4d^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$	75
default	$e\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+d\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)$	107

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(2*e^2*x^2+3*d*e*x+4*d^2)/e^3*(-e^2*x^2+d^2)^(1/2)+1/2*d^3/e^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^2x^2+3\,dex+4\,d^2)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{d^3\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{2d^2}{3e^3}-\frac{dx}{2e^2}-\frac{x^2}{3e}\right) & \text{for } e^2\neq 0 \\ \frac{\frac{dx^3}{3}+\frac{ex^4}{4}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((d**3*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(2*e**2
) + sqrt(d**2 - e**2*x**2)*(-2*d**2/(3*e**3) - d*x/(2*e**2) - x**2/(3*e)),
Ne(e**2, 0)), ((d*x**3/3 + e*x**4/4)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}x^2}{3\,e} + \frac{d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^2} - \frac{2\,\sqrt{-e^2x^2+d^2}d^2}{3\,e^3}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2/e + 1/2*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt
(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 - 2/3*sqrt(-e^2*x^2 + d^2)*d^
2/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|} - \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e}+\frac{3\,d}{e^2}\right)+\frac{4\,d^2}{e^3}\right)$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/6*sqrt(-e^2*x^2 + d^2)
*(x*(2*x/e + 3*d/e^2) + 4*d^2/e^3)
```

Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{d\,x^3}{3\,\sqrt{d^2}} & \text{if } e=0 \\ -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3} - \frac{d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} - \frac{d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if } e\neq 0 \end{cases}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (d*x^3)/(3*(d^2)^(1/2)), e ~= 0, - ((d^2 - e^2*x^2)^(1/2)
*(2*d^2 + e^2*x^2))/(3*e^3) - (d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)
^(1/2)))/(2*(-e^2)^(3/2)) - (d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.17 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+d*(e*x+d)/e^3/(-e^2*x^2+d^2)^(1/2)+
(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 655, 223, 209, 651}

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}$$

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx}{e^2} \\
&= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} \\
&= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2} \\
&= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{(-2d+ex)\sqrt{d^2-e^2x^2}}{e^3(-d+ex)} + \frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(-d + e*x)) + (2*d*ArcTan[(e*x)/(Sq
rt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
risch	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right)}}{e^{4} \left(x-\frac{d}{e}\right)}$	99
default	$e\left(-\frac{x^{2}}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{2 d^{2}}{e^{4} \sqrt{-e^{2} x^{2}+d^{2}}}\right)+d\left(\frac{x}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}\right)$	103

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] (-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))-d/e^4/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{2\,dex-2\,d^{2}+2\,(dex-d^{2})\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\sqrt{-e^{2}x^{2}+d^{2}}(ex-2\,d)}{e^{4}x-de^{3}}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] (2*d*e*x - 2*d^2 + 2*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(e^4*x - d*e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = d\left(\begin{cases} \frac{i\operatorname{acosh}\left(\frac{ex}{d}\right)}{e^{3}}-\frac{ix}{de^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^{3}}+\frac{x}{de^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{2d^{2}}{e^{4}\sqrt{d^{2}-e^{2}x^{2}}}-\frac{x^{2}}{e^{2}\sqrt{d^{2}-e^{2}x^{2}}} & \text{for}\ e\neq 0 \\ \frac{x^{4}}{4(d^{2})^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2*x**2
/d**2)), True)) + e*Piecewise((2*d**2/(e**4*sqrt(d**2 - e**2*x**2)) - x**2/
(e**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{x^2}{\sqrt{-e^2x^2+d^2}\,e}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^2}-\frac{d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}\,e^2}+\frac{2\,d^2}{\sqrt{-e^2x^2+d^2}\,e^3}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(-e^2*x^2 + d^2)*e) + d*x/(sqrt(-e^2*x^2 + d^2)*e^2) - d*arcsin(e
^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 2*d^2/(sqrt(-e^2*x^2 + d^2)*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|}+\frac{\sqrt{-e^2x^2+d^2}}{e^3}+\frac{2\,d}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^3 + 2*
d/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{2\,d^2-e^2\,x^2}{e^3\,\sqrt{d^2-e^2\,x^2}}+\frac{d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{(-e^2)^{3/2}}+\frac{d\,x}{e^2\,\sqrt{d^2-e^2\,x^2}}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x)
[Out] (2*d^2 - e^2*x^2)/(e^3*(d^2 - e^2*x^2)^(1/2)) + (d*log(x*(-e^2)^(1/2) + (d^
2 - e^2*x^2)^(1/2)))/(-e^2)^(3/2) + (d*x)/(e^2*(d^2 - e^2*x^2)^(1/2))
```

3.18 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx=\frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)-2/3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {810, 12, 267}

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx=\frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{2d^2ex}{(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2} \\ &= \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{2\int \frac{x}{(d^2-e^2x^2)^{3/2}}\,dx}{3e} \\ &= \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^2+2dex+e^2x^2)}{3de^3(d-ex)^2(d+ex)}$$

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^2 + 2*d*e*x + e^2*x^2))/(3*d*e^3*(d - e*x)^2*(d
+ e*x))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{(-ex+d)(ex+d)^2(-e^2x^2-2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{5}{2}}}$	55
trager	$-\frac{(-e^2x^2-2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3d\,e^3(-ex+d)^2(ex+d)}$	57
default	$e\left(\frac{x^2}{e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{2d^2}{3e^4(-e^2x^2+d^2)^{\frac{3}{2}}}\right)+d\left(\frac{x}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}\right)$	120

[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = -\frac{2\,e^3x^3-2\,de^2x^2-2\,d^2ex+2\,d^3-(e^2x^2+2\,dex-2\,d^2)\sqrt{-e^2x^2+d^2}}{3\,(de^6x^3-d^2e^5x^2-d^3e^4x+d^4e^3)}$$

[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] -1/3*(2*e^3*x^3 - 2*d*e^2*x^2 - 2*d^2*e*x + 2*d^3 - (e^2*x^2 + 2*d*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - d^2*e^5*x^2 - d^3*e^4*x + d^4*e^3)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.98

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = d\left(\begin{cases}\frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right) + e\left(\begin{cases}\frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}}-\frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0\\ \frac{x^4}{4(d^2)^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)

```
[Out] d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt(1
 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e
*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*sqrt(
d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e
**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e} + \frac{dx}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{x}{3\,\sqrt{-e^2x^2+d^2}de^2}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^2+2\,d\,e\,x+e^2\,x^2\right)}{3\,d\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^2}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 2*d^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)*(d -
 e*x)^2)
```

3.19 $\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

```
[Out] 1/5*x^6*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(7*e*x+6*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(35*e*x+2
4*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e^8
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {833, 794, 223, 209}

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=-\frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
```

```
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3+7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5+35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4} \\
&= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7+105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}
\end{aligned}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{(7d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^7}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{(7d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^7}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-15de^5x^5-15e^6x^6\right)}{(d-ex)^3(d+ex)^2} + 210d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{30e^8}$$

```
[In] Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^3*x^3
 + 176*d^2*e^4*x^4 - 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^3*(d + e*x)^2)
+ 210*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(30*e^8)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

method	result
default	$e\left(-\frac{x^{7}}{2e^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{7d^{2}\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{\frac{x^{3}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}}{e^{2}}}{e^{2}}\right)}{2e^{2}}\right)+d\left(-\frac{}{e^{2}(-}\right.$
risch	$\frac{(ex+2d)\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{8}}-\frac{7d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{7}\sqrt{e^{2}}}-\frac{7d^{3}\sqrt{-\left(x-\frac{d}{e}\right)^{2}e^{2}-2de\left(x-\frac{d}{e}\right)}}{15e^{10}\left(x-\frac{d}{e}\right)^{2}}-\frac{773d^{2}\sqrt{-\left(x-\frac{d}{e}\right)^{2}e^{2}-2de\left(x-\frac{d}{e}\right)}}{240e^{9}\left(x-\frac{d}{e}\right)}+\frac{d^{3}}{}$

```
[In] int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/2*x^7/e^2/(-e^2*x^2+d^2)^(5/2)+7/2*d^2/e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2
)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)
^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d*(
-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2
/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{96\,d^2e^5x^5-96\,d^3e^4x^4-192\,d^4e^3x^3+192\,d^5e^2x^2+96\,d^6ex-96\,d^7+210\,(d^2e^5x^5-d^3}{}$$

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 - 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 + 192*d^5*e^2*x^2 +
 96*d^6*e*x - 96*d^7 + 210*(d^2*e^5*x^5 - d^3*e^4*x^4 - 2*d^4*e^3*x^3 + 2*d
^5*e^2*x^2 + d^6*e*x - d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15
*e^6*x^6 + 15*d*e^5*x^5 - 176*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 - 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 - d*e^12*x^4 - 2*d^2
*e^11*x^3 + 2*d^3*e^10*x^2 + d^4*e^9*x - d^5*e^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(144) = 288$.

Time = 13.69 (sec) , antiderivative size = 2004, normalized size of antiderivative = 12.45

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**4*
e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x**4/(
5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x*
*2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8*sqrt(
d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**
4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True)) + e*P
iecewise((210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*s
qrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) +
60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**7*sqrt(-1 + e**2*x**
2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 210*I*d
**6*e*x/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 420*I*d
**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*sqrt(-1
+ e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e*
*13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*pi*d**5*e**2*x**2*sqrt(-1 + e**2*
x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sq
rt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 490*
I*d**4*e**3*x**3/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x
**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2))
+ 210*I*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9
*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2)
+ 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**3*e**4*x**4*sqrt(-
1 + e**2*x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**1
1*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2
)) - 322*I*d**2*e**5*x**5/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)) + 30*I*e**7*x**7/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**2/d**2)*asi
n(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1
 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 105*d**6*e
*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**
```

```
2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 210*d**5*e**2*x*
*2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d*
*2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1
- e**2*x**2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e**2*x**2/d*
*2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1
- e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d
)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2
*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 161*d**2*e**5*x**
5/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2
*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**7*x**7/(30*
d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/
d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.01

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{7\,d^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right)}{30\,e} - \frac{dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{7\,d^2x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{6\,e^3} + \frac{6\,d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} - \frac{8\,d^5x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6} + \frac{16\,d^7}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8} + \frac{14\,d^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7} - \frac{49\,d^2x}{30\sqrt{-e^2x^2+d^2}\,e^7} - \frac{7\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}\,e^7}$$

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^7/((-e^2*x^2 + d^2)^(5/2)*e) + 7/30*d^2*x*(15*x^4/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d
^2)^(5/2)*e^6))/e - d*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/6*d^2*x*(3*x^2/(
(-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^3 + 6*d
^3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) - 8*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6
) + 16/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e^8) + 14/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^7) - 49/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^7) - 7/2*d^2*arcsin(e^2*x/(
d*sqrt(e^2)))/(sqrt(e^2)*e^7)
```

Giac [F]

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(ex+d)x^7}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*x^7/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{x^7\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.20 $\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}} dx = \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] 1/5*x^5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(6*e*x+5*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(8*e*x+5*d)/e^6/(-
e^2*x^2+d^2)^(1/2)+16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {833, 655, 223, 209}

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}} dx = -\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x^4\left(5d^3+6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x^2\left(15d^5+24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^7+48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^6}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}\\
&\quad +\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^6}
\end{aligned}
$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3+38de^4x^4-15e^5x^5\right)}{(d-ex)^3(d+ex)^2}+30d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^7}$$

```
[In] Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e^3*x^
3 + 38*d*e^4*x^4 - 15*e^5*x^5))/((d - e*x)^3*(d + e*x)^2) + 30*d*ArcTan[(e*
x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^7)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.50

method	result
default	$e\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right)+d\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\right.$
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^7}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^6\sqrt{e^2}}-\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{24e^9\left(x+\frac{d}{e}\right)^2}+\frac{25d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48e^8\left(x+\frac{d}{e}\right)}-\frac{23d^2\sqrt{-\left(x-\frac{d}{e}\right.}}{60e^9(}$

```
[In] int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] e*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*
d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)
)))+d*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(
3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.79

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{48\,de^5x^5 - 48\,d^2e^4x^4 - 96\,d^3e^3x^3 + 96\,d^4e^2x^2 + 48\,d^5ex - 48\,d^6 + 30\,(de^5x^5 - d^2e^4x^4}{15}$$

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(48*d*e^5*x^5 - 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 + 96*d^4*e^2*x^2 + 48*
d^5*e*x - 48*d^6 + 30*(d*e^5*x^5 - d^2*e^4*x^4 - 2*d^3*e^3*x^3 + 2*d^4*e^2*
x^2 + d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x^
5 - 38*d*e^4*x^4 - 52*d^2*e^3*x^3 + 87*d^3*e^2*x^2 + 33*d^4*e*x - 48*d^5)*s
qrt(-e^2*x^2 + d^2))/(e^12*x^5 - d*e^11*x^4 - 2*d^2*e^10*x^3 + 2*d^3*e^9*x^
2 + d^4*e^8*x - d^5*e^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 1821, normalized size of antiderivative = 12.39

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) +
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1 + e**2*x**2
/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1
+ e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*d**4*
e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60*I*d**3*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*
x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*
sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)
/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2
*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*d**2*e**3*x
**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*d*e**4*x
**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*x**
2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqr
```

```
t(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(30*d
**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/
d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x**5/(30*d**
5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d*
*2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2
*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*
sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e**2*x**2/d*
*2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 -
 e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/
(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x*
*2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e**3*x**3/(1
5*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2
/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x**4*sqrt(1
- e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d
**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2
/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**
9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d*
*2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2))
 - 40*d**4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*
e**4*x**4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4
*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) +
5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.97

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,dx\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e^2} + \frac{6\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{8\,d^4x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{16\,d^6}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7} + \frac{4\,d^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{7\,dx}{15\,\sqrt{-e^2x^2+d^2}e^6} - \frac{d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^6}$$

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*d*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/((-e^2*x^2 + d^2)^
(5/2)*e) - 1/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 +
 d^2)^(3/2)*e^4))/e^2 + 6*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^3) - 8*d^4*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^5) + 16/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^7) + 4/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^6) - 7/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) -
 d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^6)
```

Giac [F]

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^6}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*x^6/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^6\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.21 $\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*x^4*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(5*e*x+4*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(15*e*x+8*d)/e^6/(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {833, 792, 223, 209}

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{x^4(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^2(4d+5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x^3\left(4d^3+5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x\left(8d^5+15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(8d^4+7d^3ex-27d^2e^2x^2-8de^3x^3+23e^4x^4)}{(d-ex)^3(d+ex)^2}+30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^6}$$

```
[In] Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 + 2
3*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d
^2 - e^2*x^2]])])/(15*e^6)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.55

method	result
default	$e\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+d\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2\right.}\right.}{}\right.$

```
[In] int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)
-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-e^2*x^2+d^2)^(1/2)))))+d*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/
e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(108) = 216.

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.02

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5+30\,(e^5x^5-de^4x^4-2\,d^2e^3x}{15\,(e^{11}x^5-de^{1}}$$

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 + 30*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^4*e*
```

```
x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e^4*x^4 - 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 + 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 -
 d*e^10*x^4 - 2*d^2*e^9*x^3 + 2*d^3*e^8*x^2 + d^4*e^7*x - d^5*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.

Time = 10.20 (sec) , antiderivative size = 1739, normalized size of antiderivative = 14.25

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**
2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**2*
e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**
2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(15*d
**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2)
+ 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2))
, True)) + e*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(3
0*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x*
*2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1
+ e**2*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) -
 30*I*d**4*e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60
*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt
(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*
e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2
*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqr
t(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*
d**2*e**3*x**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30
*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1
 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**
11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 +
 e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x
**5/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sq
rt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*
e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e
```

```
**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x*
*4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*a
sin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(
1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e
**3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1
- e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x
**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d
**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1
- e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) -
30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*
x**2/d**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.15

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,ex\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e} + \frac{dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} + \frac{4\,d^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{7\,x}{15\,\sqrt{-e^2x^2+d^2}e^5} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^5}$$

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*x^
2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + d*x^4/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 4/3*d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8/15*d^5
/((-e^2*x^2 + d^2)^(5/2)*e^6) + 4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 7
/15*x/(sqrt(-e^2*x^2 + d^2)*e^5) - arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e
^5)
```

Giac [F]

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^5}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*x^5/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.22 $\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}}-\frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^4*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2)+
4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {819, 272, 45}

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e} \\
&= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}\,dx, x, x^2\right)}{5e} \\
&= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\text{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)\,dx, x, x^2\right)}{5e} \\
&= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4)}{15de^5(d-ex)^3(d+ex)^2}$$

```
[In] Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3
*e^4*x^4))/(15*d*e^5*(d - e*x)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{(-ex+d)(ex+d)^2(3e^4x^4+12d\,e^3x^3-12d^2e^2x^2-8d^3ex+8d^4)}{15d\,e^5(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(3e^4x^4+12d\,e^3x^3-12d^2e^2x^2-8d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{15e^5d(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+d\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\cdots\right)}{\cdots}\right.$

[In] int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(e*x+d)^2*(3*e^4*x^4+12*d*e^3*x^3-12*d^2*e^2*x^2-8*d^3*e*x+8*d^4)/d/e^5/(-e^2*x^2+d^2)^(7/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(72) = 144.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.04

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5-(3\,e^4x^4+12\,de^3x^3-12\,d^2e\cdots}{15\,(de^{10}x^5-d^2e^9x^4-2\,d^3e^8x^3+2\,d^4e^7x^2+d^5e^6x-d^6\cdots}$$

[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x - 8*d^5 - (3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 - 8*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 - d^2*e^9*x^4 - 2*d^3*e^8*x^3 + 2*d^4*e^7*x^2 + d^5*e^6*x - d^6*e^5)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.36 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.98

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} - \frac{20d^2e^2x^2}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} + \frac{}{15d^4e^6} \\ \frac{x^6}{6(d^2)^{\frac{7}{2}}} \end{cases}\right)$$

```
[In] integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2
)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*
d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2
)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*
x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e*
*4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2
- e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(
d**2)**(7/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(72) = 144.

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,d^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{3\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{dx}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} + \frac{x}{5\,\sqrt{-e^2x^2+d^2}de^4}$$

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4
/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^3*x/((-e^2*x^2 + d^2)^(5/2
)*e^4) + 8/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 1/10*d*x/((-e^2*x^2 + d^2)
^(3/2)*e^4) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

Giac [F]

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^4}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*x^4/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(8\,d^4-8\,d^3\,e\,x-12\,d^2\,e^2\,x^2+12\,d\,e^3\,x^3+3\,e^4\,x^4)}{15\,d\,e^5\,(d+e\,x)^2\,(d-e\,x)^3}$$

```
[In] int((x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 8*d^3*e*x))/(15*d*e^5*(d + e*x)^2*(d - e*x)^3)
```

3.23 $\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\, dx = \frac{x^2(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d+3ex}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(-3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {833, 792, 197}

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\, dx = \frac{x^2(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d+3ex}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3+3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3} \\
&= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^3(d+ex)^2}$$

```
[In] Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(-3e^4x^4+3d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)}{15d^2e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-3e^4x^4+3d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^2e^4(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+d\Big(\frac{}{3e}$

```
[In] int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*
d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5+(3\,e^4x^4-3\,de^3x^3+3\,d^2e^2x^2+2\,d^3ex-2\,d^4)\sqrt{-}}{15\,(d^2e^9x^5-d^3e^8x^4-2\,d^4e^7x^3+2\,d^5e^6x^2+d^6e^5x-d^7e^4)}$$

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 + (3*e^4*x^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 - d^3*e^8*x^4 - 2*d^4*e^7*x^3 + 2*d^5*e^6*x
^2 + d^6*e^5*x - d^7*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(78) = 156$.

Time = 6.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.74

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15 \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} \end{cases}\right) + e\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**
*2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x
**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e*
*2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)
**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) -
 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2
/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{2\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e^3}$$

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2)
 - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d^2)
*d^2*e^3)
```

Giac [F]

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*x^3/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2-3\,d\,e^3\,x^3+3\,e^4\,x^4)}{15\,d^2\,e^4\,(d+e\,x)^2\,(d-e\,x)^3}$$

```
[In] int((x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^2*(d - e*x)^3)
```

3.24 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3\,(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-2/15*(-e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {810, 792, 197}

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3\,(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x(2d^2e-2de^2x)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15de^2} \\
&= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4\right)}{15d^3e^3(d-ex)^3(d+ex)^2}$$

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*
e^4*x^4))/(15*d^3*e^3*(d - e*x)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(2e^4x^4-2d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)}{15d^3e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(2e^4x^4-2d\,e^3x^3-3d^2e^2x^2-2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^3e^3(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+d\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e}}}{d^2}\right.}{4e^2}\right.$

[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5-(2\,e^4x^4-2\,de^3x^3-3\,d^2e^2x^2-2\,d^3ex+2\,d^4)\sqrt{-e}}{15\,(d^3e^8x^5-d^4e^7x^4-2\,d^5e^6x^3+2\,d^6e^5x^2+d^7e^4x-d^8e^3)}$$

[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x - 2*d^5 - (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*x^5 - d^4*e^7*x^4 - 2*d^5*e^6*x^3 + 2*d^6*e^5*x^2 + d^7*e^4*x - d^8*e^3)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.11 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.46

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} \cdots} \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4 \cdots} \end{cases}\right. \cdots$$
$$+e\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq \cdots \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} & \text{otherw}\cdots \end{cases}\right. \cdots$$

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d**2) -
 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 3
0*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**
2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*
sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (
x**4/(4*(d**2)**(7/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{2\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2} - \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3e^2}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e^2) -
 2/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e
^2) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d*e^3*x^3 - 2*e^4*x^4 - 2*d^4 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^2*(d - e*x)^3)
```

3.25 $\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)-2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {792, 198, 197}

$$\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e} \\
&= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e} \\
&= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(3d^4-3d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4)}{15d^4e^2(d-ex)^3(d+ex)^2}$$

```
[In] Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2(-2e^4x^4+2d\,e^3x^3+3d^2e^2x^2-3d^3ex+3d^4)}{15d^4e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$	77
trager	$\frac{(-2e^4x^4+2d\,e^3x^3+3d^2e^2x^2-3d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^4(-ex+d)^3(ex+d)^2e^2}$	79
default	$e\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)+\frac{d}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}$	120

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*d
^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{3\,e^5x^5-3\,de^4x^4-6\,d^2e^3x^3+6\,d^3e^2x^2+3\,d^4ex-3\,d^5+(2\,e^4x^4-2\,de^3x^3-3\,d^2e^2x^2}{15\,(d^4e^7x^5-d^5e^6x^4-2\,d^6e^5x^3+2\,d^7e^4x^2+d^8e^3x-d^9e}$$

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 + (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 - d^5*e^6*x^4 - 2*d^6*e^5*x^3 + 2*d^7*e^4*x^
2 + d^8*e^3*x - d^9*e^2)
```

Sympy [A] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 432, normalized size of antiderivative = 5.20

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases}\frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0\\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$

$$+e\left(\begin{cases}-\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}\\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}\end{cases}\right.$$

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt
(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/
(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 +
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x
**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**
9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1
5*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 -
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4
*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e} - \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^4e}$$

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

Giac [F]

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)x}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*x/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-3\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

```
[In] int((x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 + 2*d*e^3*x^3 + 3*d^2*e^2*x^2 - 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^2*(d - e*x)^3)
```

3.26 $\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{d+ex}{5de(d^2-e^2x^2)^{5/2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {653, 198, 197}

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{d+ex}{5de(d^2-e^2x^2)^{5/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\ &= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\ &= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int\frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4)}{15d^5e(d-ex)^3(d+ex)^2}$$

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8
*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2(8e^4x^4-8d\,e^3x^3-12d^2e^2x^2+12d^3ex+3d^4)}{15d^5e(-e^2x^2+d^2)^{\frac{7}{2}}}$	77
trager	$\frac{(8e^4x^4-8d\,e^3x^3-12d^2e^2x^2+12d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^5(-ex+d)^3(ex+d)^2e}$	79
default	$d\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+\frac{1}{5e(-e^2x^2+d^2)^{\frac{5}{2}}}$	90

[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{3\,e^5x^5-3\,de^4x^4-6\,d^2e^3x^3+6\,d^3e^2x^2+3\,d^4ex-3\,d^5-(8\,e^4x^4-8\,de^3x^3-12\,d^2e^2x^2}{15\,(d^5e^6x^5-d^6e^5x^4-2\,d^7e^4x^3+2\,d^8e^3x^2+d^9e^2x-d^{10}}$$

[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x - 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e^3*x^2 + d^9*e^2*x - d^10*e)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.51 (sec) , antiderivative size = 604, normalized size of antiderivative = 7.55

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{20id^2e^2x^3}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2}{}}} \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4} \end{cases}\right.$$
$$+e\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x
**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e,
0)), (x**2/(2*(d**2)**(7/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\sqrt{-e^2x^2+d^2}\,d^5}$$

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{ex+d}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

```
[In] int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

3.27 $\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 1/5*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*(4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(8*e*x+15*d)/d^6/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {837, 12, 272, 65, 214}

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{5d^3e^2+4d^2e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^5e^4+8d^4e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\int \frac{15d^7e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}
\end{aligned}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^5e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(23d^4-8d^3ex-27d^2e^2x^2+7de^3x^3+8e^4x^4\right)}{(d-ex)^3(d+ex)^2}-15\sqrt{d^2}\log(x)+15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2\right.}}{15d^7}$$

```
[In] Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3
+ 8*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) - 15*Sqrt[d^2]*Log[x] + 15*Sqrt[d^2
]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

method	result
default	$e\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{1}{d^2\sqrt{-e^2x^2+d\right.}}}{d^2}\right.$

```
[In] int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2
/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/
d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.09

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{23\,e^5x^5-23\,de^4x^4-46\,d^2e^3x^3+46\,d^3e^2x^2+23\,d^4ex-23\,d^5+15\left(e^5x^5-de^4x^4-2\right.}{15\left(d^6e^5x^5-\right.}$$

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 - 23*d*e^4*x^4 - 46*d^2*e^3*x^3 + 46*d^3*e^2*x^2 + 23*d^4*
e*x - 23*d^5 + 15*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^
4*e*x - d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 + 7*d*e^3*x^3
- 27*d^2*e^2*x^2 - 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 -
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 + 2*d^9*e^2*x^2 + d^10*e*x - d^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.97 (sec) , antiderivative size = 2378, normalized size of antiderivative = 20.32

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**
2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6)
 + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x*
*2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d
**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**1
1*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*lo
g(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6
*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*sqrt(-1 + e*
```

```
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e
**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x**4*log(e
*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2 - 9
0*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e**2*x**2/d**2)/(-
30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30
*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e*
*2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1),
 (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d*
*9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**2)/(-30*d**13
+ 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 30*d**6*log
(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e*
*4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d**13 + 90*d**11*e**2*x**2
 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**4*e**2*x**2*sqrt(1 - e**2
*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e
**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*log(sqr
t(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x
**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*d**2*e**4*x**4*sqrt(1
- e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*
d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x**4*l
og(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*
e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4*x**4/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e*
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*I*pi*e**6*x
**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6
), True)) + e*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 +
e**2*x**2/d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2)
- 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2
*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*
x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4
*sqrt(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2
/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqr
t(1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 3
0*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2
*x**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex}{5(-e^2x^2+d^2)^{\frac{5}{2}}d^2} + \frac{1}{5(-e^2x^2+d^2)^{\frac{5}{2}}d} + \frac{4\,ex}{15(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{1}{3(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{8\,ex}{15\sqrt{-e^2x^2+d^2}\,d^6} - \frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^6} + \frac{1}{\sqrt{-e^2x^2+d^2}\,d^5}$$

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*d) + 4/1
5*e*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/1
5*e*x/(sqrt(-e^2*x^2 + d^2)*d^6) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2
)*d/abs(x))/d^6 + 1/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}x}\,dx$$

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{d^2-e^2\,x^2}{3\,d^3}+\frac{(d^2-e^2\,x^2)^2}{d^5}+\frac{1}{5\,d}}{(d^2-e^2\,x^2)^{5/2}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^6} + \frac{e\,x\,(15\,d^4-20\,d^2\,e^2\,x^2+8\,e^4\,x^4)}{15\,d^6\,(d^2-e^2\,x^2)^{5/2}}$$

```
[In] int((d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] ((d^2 - e^2*x^2)/(3*d^3) + (d^2 - e^2*x^2)^2/d^5 + 1/(5*d))/(d^2 - e^2*x^2)
^(5/2) - atanh((d^2 - e^2*x^2)^(1/2)/d)/d^6 + (e*x*(15*d^4 + 8*e^4*x^4 - 20
*d^2*e^2*x^2))/(15*d^6*(d^2 - e^2*x^2)^(5/2))
```

3.28 $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5d^2x(d^2-e^2x^2)^{5/2}}+\frac{6d+5ex}{15d^4x(d^2-e^2x^2)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] 1/5*(e*x+d)/d^2/x/(-e^2*x^2+d^2)^(5/2)+1/15*(5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2
)^(3/2)-e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(5*e*x+8*d)/d^6/x/(-e^2*x
^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {837, 821, 272, 65, 214}

$$\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx = -\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{d+ex}{5d^2x(d^2-e^2x^2)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\text{integral} = \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{6d^3e^2+5d^2e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{24d^5e^4+15d^4e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{\int \frac{48d^7e^6+15d^6e^7x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^6e}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^5-38d^4ex-52d^3e^2x^2+87d^2e^3x^3+33de^4x^4-48e^5x^5\right)}{x(-d+ex)^3(d+ex)^2}+30e\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^7}$$

```
[In] Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^5 - 38*d^4*e*x - 52*d^3*e^2*x^2 + 87*d^2*e^3*x^
3 + 33*d*e^4*x^4 - 48*e^5*x^5))/(x*(-d + e*x)^3*(d + e*x)^2) + 30*e*ArcTanh
[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^7)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.39

method	result
default	$e\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)+d\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\right.}{}\right.$
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7x}-\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^6\sqrt{d^2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{24d^6e\left(x+\frac{d}{e}\right)^2}-\frac{23\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48d^7\left(x+\frac{d}{e}\right)}+\frac{17\sqrt{-\left(x-\right.}}{60d}$

```
[In] int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(
1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x))))+d*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2
/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e
^2*x^2+d^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.76

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{23\,e^6x^6-23\,de^5x^5-46\,d^2e^4x^4+46\,d^3e^3x^3+23\,d^4e^2x^2-23\,d^5ex+15\,(e^6x^6-de^5x}{1}$$

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^6*x^6 - 23*d*e^5*x^5 - 46*d^2*e^4*x^4 + 46*d^3*e^3*x^3 + 23*d^4*
e^2*x^2 - 23*d^5*e*x + 15*(e^6*x^6 - d*e^5*x^5 - 2*d^2*e^4*x^4 + 2*d^3*e^3*
x^3 + d^4*e^2*x^2 - d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x
^5 - 33*d*e^4*x^4 - 87*d^2*e^3*x^3 + 52*d^3*e^2*x^2 + 38*d^4*e*x - 15*d^5)*
sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 - d^8*e^4*x^5 - 2*d^9*e^3*x^4 + 2*d^10*e
^2*x^3 + d^11*e*x^2 - d^12*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.16 (sec) , antiderivative size = 2404, normalized size of antiderivative = 15.71

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/
(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d*
*8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 +
 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e*
*5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**
10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) +
1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6),
 True)) + e*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**
2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*
e**6*x**6) + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e
**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2
*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x
**4 + 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13
 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e*
*2*x**2*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30
*d**7*e**6*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11*
e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*sq
rt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 +
90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*
x**4*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d*
*7*e**6*x**6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e**2*x**
2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*
x**6) - 30*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e
**4*x**4 + 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e**2*x**2/d
```

```
**2) > 1), (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x*
*2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**2)/(
-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 3
0*d**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 -
90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d**13 + 90*d**11
*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**4*e**2*x**2*sqr
t(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x*
*2*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d
**9*e**4*x**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*d**2*e**4*x*
*4*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*
x**4 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e
**4*x**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4*x**4/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x
**6*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
 + 30*d**7*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30
*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*I
*pi*e**6*x**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{6\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3} + \frac{e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2} + \frac{8\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5} + \frac{e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4} - \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx} + \frac{16\,e^2x}{5\sqrt{-e^2x^2+d^2}d^7} - \frac{e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^7} + \frac{e}{\sqrt{-e^2x^2+d^2}d^6}$$

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 1/5*e/((-e^2*x^2 + d^2)^(5/2)*d^2)
 + 8/5*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 1/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^4) - 1/((-e^2*x^2 + d^2)^(5/2)*d*x) + 16/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^7
) - e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 + e/(sqrt(-e^
2*x^2 + d^2)*d^6)
```

Giac [F]

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{ex+d}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x^2}\,dx$$

[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")

[Out] integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x^2), x)

Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{e}{5\,d^2}+\frac{e\,\left(d^2-e^2\,x^2\right)^2}{d^6}+\frac{e\,\left(d^2-e^2\,x^2\right)}{3\,d^4}}{\left(d^2-e^2\,x^2\right)^{5/2}} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^7} - \frac{d^6-6\,d^4\,e^2\,x^2+8\,d^2\,e^4\,x^4-\frac{16\,e^6\,x^6}{5}}{d^7\,x\,\left(d^2-e^2\,x^2\right)^{5/2}}$$

[In] int((d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x)

[Out] (e/(5*d^2) + (e*(d^2 - e^2*x^2)^2)/d^6 + (e*(d^2 - e^2*x^2))/(3*d^4))/(d^2 - e^2*x^2)^(5/2) - (e*atanh((d^2 - e^2*x^2)^(1/2)/d))/d^7 - (d^6 - (16*e^6*x^6)/5 - 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4)/(d^7*x*(d^2 - e^2*x^2)^(5/2))

3.29 $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

```
[Out] 1/5*(e*x+d)/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+1/15*(6*e*x+7*d)/d^4/x^2/(-e^2*x^2
+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(24*e*x+35*d)/
d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2-16/5*e*(-e^2*
x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {837, 849, 821, 272, 65, 214}

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = -\frac{7e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} + \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
```

```
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\text{integral} = \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{7d^3e^2+6d^2e^3x}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{35d^5e^4+24d^4e^5x}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{\int \frac{105d^7e^6+48d^6e^7x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{\int \frac{-96d^8e^7-105d^7e^8x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^{14}e^6}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} + \frac{(7e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^7}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} + \frac{(7e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^7}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^7}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.80

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(15d^6+15d^5ex-176d^4e^2x^2-4d^3e^3x^3+249d^2e^4x^4-9de^5x^5-96e^6x^6)}{x^2(-d+ex)^3(d+ex)^2}+210e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x\cdots}{\cdots}\right)}{30d^8}$$

[In] Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]

[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^6 + 15*d^5*e*x - 176*d^4*e^2*x^2 - 4*d^3*e^3*x^3 + 249*d^2*e^4*x^4 - 9*d*e^5*x^5 - 96*e^6*x^6))/(x^2*(-d + e*x)^3*(d + e*x)^2) + 210*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(30*d^8)

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.33

method	result
default	$d\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)}{2d^2}\right)+\cdots$
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(2ex+d)}{2d^8x^2}-\frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^7\sqrt{d^2}}+\frac{29e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48d^8\left(x+\frac{d}{e}\right)}+\frac{11\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)\cdots}}{30d^7\left(x-\frac{d}{e}\right)^2}\cdots$

[In] int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] d*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.58

$$\int \frac{d+ex}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{116\,e^7x^7-116\,de^6x^6-232\,d^2e^5x^5+232\,d^3e^4x^4+116\,d^4e^3x^3-116\,d^5e^2x^2+105\,(e$$

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 - 116*d*e^6*x^6 - 232*d^2*e^5*x^5 + 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 - 116*d^5*e^2*x^2 + 105*(e^7*x^7 - d*e^6*x^6 - 2*d^2*e^5*x^5 +
 2*d^3*e^4*x^4 + d^4*e^3*x^3 - d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) - (96*e^6*x^6 + 9*d*e^5*x^5 - 249*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 - 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 - d^9*e^
4*x^6 - 2*d^10*e^3*x^5 + 2*d^11*e^2*x^4 + d^12*e*x^3 - d^13*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.95 (sec) , antiderivative size = 2691, normalized size of antiderivative = 14.62

$$\int \frac{d+ex}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**1
3*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*I*d**6*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d*
*11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)
/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6
*x**8) + 210*d**6*e**2*x**2*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**
4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**6*e**2*x**2*asin(d/
(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**
9*e**6*x**8) + 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x*
*2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d
**4*e**4*x**4*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 1
80*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*log(e*x/d)/(-6
0*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**
8) + 630*I*d**4*e**4*x**4*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x*
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**2*e**6*x**6*sqrt(-
1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*
x**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**
```

15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) +
630*d**2*e**6*x**6*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d
11*e4*x**6 + 60*d**9*e**6*x**8) - 630*I*d**2*e**6*x**6*asin(d/(e*x))/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x*
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(e*x/d)/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 210*I*e**8*x**8*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 -
180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), Abs(e**2*x**2/d**2) > 1), (30*d*
*8*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**
11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*d**6*e**2*x**2*sqrt(1 - e**2*x**2/d
2)/(-60*d15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*
e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d
13*e2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 210*d**6*e**2*x
2*log(sqrt(1 - e2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4
- 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*I*pi*d**6*e**2*x**2/(-60*
d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8)
+ 490*d**4*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*
e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d**4*e**4*x**4*l
og(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x
6 + 60*d9*e**6*x**8) - 630*d**4*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2)
+ 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*
e**6*x**8) + 315*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*e**2*x**4
- 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 +
60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**15*x**
2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 630*d*
*2*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*
e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e**6*x
6/(-60*d15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e
6*x8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e
2*x4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(sqr
t(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**1
1*e**4*x**6 + 60*d**9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2 + 180
*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) + e*Pie
cewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2
- 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2/(e**
2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e
6*x6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**
12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*sqrt(d
2/(e2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 +
5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/(e**2
*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e*
*6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d
12*e2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e**5*x*

```
*4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e*
*4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) + 1)/(-
5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{6\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4} + \frac{7\,e^2}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} + \frac{8\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6} + \frac{7\,e^2}{6\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} - \frac{e}{(-e^2x^2+d^2)^{\frac{5}{2}}d^2x} + \frac{16\,e^3x}{5\,\sqrt{-e^2x^2+d^2}d^8} - \frac{7\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^8} + \frac{7\,e^2}{2\,\sqrt{-e^2x^2+d^2}d^7} - \frac{1}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}dx^2}$$

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/10*e^2/((-e^2*x^2 + d^2)^(5/2)*d
^3) + 8/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/6*e^2/((-e^2*x^2 + d^2)^(3
/2)*d^5) - e/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 16/5*e^3*x/(sqrt(-e^2*x^2 + d
^2)*d^8) - 7/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^8
+ 7/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^7) - 1/2/((-e^2*x^2 + d^2)^(5/2)*d*x^2)
```

Giac [F]

$$\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}x^3}\,dx$$

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/((-e^2*x^2 + d^2)^(7/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{161\,e^2}{30\,d^3\,(d^2-e^2\,x^2)^{5/2}} - \frac{1}{2\,d\,x^2\,(d^2-e^2\,x^2)^{5/2}} - \frac{7\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2\,d^8} - \frac{49\,e^4\,x^2}{6\,d^5\,(d^2-e^2\,x^2)^{5/2}} + \frac{7\,e^6\,x^4}{2\,d^7\,(d^2-e^2\,x^2)^{5/2}} - \frac{e\,(5\,d^6-30\,d^4\,e^2\,x^2+40\,d^2\,e^4\,x^4-16\,e^6\,x^6)}{5\,d^8\,x\,(d^2-e^2\,x^2)^{5/2}}$$

```
[In] int((d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (161*e^2)/(30*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(2*d*x^2*(d^2 - e^2*x^2)^(5/2)
) - (7*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/(2*d^8) - (49*e^4*x^2)/(6*d^5*(d
^2 - e^2*x^2)^(5/2)) + (7*e^6*x^4)/(2*d^7*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d^
6 - 16*e^6*x^6 - 30*d^4*e^2*x^2 + 40*d^2*e^4*x^4))/(5*d^8*x*(d^2 - e^2*x^2)
^(5/2))
```

3.30 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx=\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/7*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)-2/35*(-2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {810, 792, 198, 197}

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx=\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x(2d^2e-4de^2x)}{(d^2-e^2x^2)^{7/2}}\,dx}{7d^2e^2} \\
&= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{35de^2} \\
&= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{105d^3e^2} \\
&= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6\right)}{105d^5e^3(d-ex)^4(d+ex)^3}$$

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3
- 20*d^2*e^4*x^4 - 8*d*e^5*x^5 + 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^4*(d +
e*x)^3)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2\left(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6\right)}{105d^5e^3\left(-e^2x^2+d^2\right)^{\frac{9}{2}}}$
trager	$-\frac{\left(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6\right)\sqrt{-e^2x^2+d^2}}{105d^5(-ex+d)^4(ex+d)^3e^3}$
default	$e\left(\frac{x^2}{5e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{2d^2}{35e^4(-e^2x^2+d^2)^{\frac{7}{2}}}\right)+d\left(\frac{x}{6e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{d^2\left(\frac{x}{7d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{\cdots}{15d^2(\cdots}\right)}{\cdots}}{\cdots}\right)}{6e^2}\right.$

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^3
*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(107) = 214$.

Time = 0.41 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.98

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = -\frac{6\,e^7x^7 - 6\,de^6x^6 - 18\,d^2e^5x^5 + 18\,d^3e^4x^4 + 18\,d^4e^3x^3 - 18\,d^5e^2x^2 - 6\,d^6ex + 6\,d^7 - (8\,e^6x^6 - 8\,de^5x^5 - 2\ldots}{105\,(d^5e^{10}x^7 - d^6e^9x^6 - 3\,d^7e^8x^5 + 3\,d^8e^7x^4 + 3\,d^9e^6x^3 - 3\,d^{10}e^5\ldots}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(6*e^7*x^7 - 6*d*e^6*x^6 - 18*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 18*d^4*
e^3*x^3 - 18*d^5*e^2*x^2 - 6*d^6*e*x + 6*d^7 - (8*e^6*x^6 - 8*d*e^5*x^5 - 2
0*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x - 6*d^6)*sqrt(-
e^2*x^2 + d^2))/(d^5*e^10*x^7 - d^6*e^9*x^6 - 3*d^7*e^8*x^5 + 3*d^8*e^7*x^4
 + 3*d^9*e^6*x^3 - 3*d^10*e^5*x^2 - d^11*e^4*x + d^12*e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.46 (sec) , antiderivative size = 903, normalized size of antiderivative = 7.46

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = d\left(\begin{cases} \frac{35id^4x^3}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{\ldots}{-105d^{1}\ldots} \\ -\frac{35d^4x^3}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\ldots}{-105d^{13}\sqrt{1\ldots}} \end{cases}\right) + e\left(\begin{cases} \frac{2d^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}} - \frac{7e^2x^2\ldots}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-10\ldots} \\ \frac{x^4}{4(d^2)^{\frac{9}{2}}} \end{cases}\right)$$

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

```
[Out] d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**
11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*
x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e**2
*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt(-1 +
e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*
e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqrt(1 - e**
2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4
```

```
*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2
)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**
2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**
2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x**7/(-105*d**13
*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) -
315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e
**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e**4*sqrt(d**2 - e*
*2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 105*d**2*e**8*x**4*s
qrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x**2)) - 7*e**2*x**2
/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**
2*x**2) - 105*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d*
*2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(9/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{x^2}{5\,(-e^2x^2+d^2)^{\frac{7}{2}}e} + \frac{dx}{7\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2} - \frac{2\,d^2}{35\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3} - \frac{x}{35\,(-e^2x^2+d^2)^{\frac{5}{2}}de^2} - \frac{4\,x}{105\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2} - \frac{8\,x}{105\,\sqrt{-e^2x^2+d^2}d^5e^2}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2) -
 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d*e
^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d
^2)*d^5*e^2)
```

Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.36

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,(d-e\,x)^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}-\frac{3\,x}{70\,d\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}+\frac{4\,x}{105\,d^3\,e^2}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d - e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(2/
(35*e^3) - (3*x)/(70*d*e^2)))/((d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^
(1/2)*(1/(56*d^2*e^3) + (4*x)/(105*d^3*e^2)))/((d + e*x)^2*(d - e*x)^2) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

3.31 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}} \, dx = \frac{x^2(d+ex)}{9de(d^2-e^2x^2)^{9/2}} - \frac{2(d-3ex)}{63de^3(d^2-e^2x^2)^{7/2}} - \frac{2x}{105d^3e^2(d^2-e^2x^2)^{5/2}} - \frac{8x}{315d^5e^2(d^2-e^2x^2)^{3/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/9*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(9/2)-2/63*(-3*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(7/2)-2/105*x/d^3/e^2/(-e^2*x^2+d^2)^(5/2)-8/315*x/d^5/e^2/(-e^2*x^2+d^2
)^(3/2)-16/315*x/d^7/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {810, 792, 198, 197}

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}} \, dx = \frac{x^2(d+ex)}{9de(d^2-e^2x^2)^{9/2}} - \frac{2(d-3ex)}{63de^3(d^2-e^2x^2)^{7/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}} - \frac{8x}{315d^5e^2(d^2-e^2x^2)^{3/2}} - \frac{2x}{105d^3e^2(d^2-e^2x^2)^{5/2}}$$

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{x(2d^2e-6de^2x)}{(d^2-e^2x^2)^{9/2}}\,dx}{9d^2e^2} \\
&= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{7/2}}\,dx}{21de^2} \\
&= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{105d^3e^2} \\
&= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{16\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{315d^5e^2} \\
&= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6}{315d^7e^3(d-ex)^5(d+ex)^4}$$

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^
3 - 70*d^4*e^4*x^4 - 56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^
8*x^8))/(315*d^7*e^3*(d - e*x)^5*(d + e*x)^4)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(16e^8x^8-16d\,e^7x^7-56d^2e^6x^6+56d^3e^5x^5+70d^4x^4e^4-70d^5e^3x^3-35d^6e^2x^2-10d^7ex+10d^8)}{315d^7e^3(-e^2x^2+d^2)^{\frac{11}{2}}}$
trager	$-\frac{(16e^8x^8-16d\,e^7x^7-56d^2e^6x^6+56d^3e^5x^5+70d^4x^4e^4-70d^5e^3x^3-35d^6e^2x^2-10d^7ex+10d^8)\sqrt{-e^2x^2+d^2}}{315d^7(-ex+d)^5(ex+d)^4e^3}$
default	$e\left(\frac{x^2}{7e^2(-e^2x^2+d^2)^{\frac{9}{2}}}-\frac{2d^2}{63e^4(-e^2x^2+d^2)^{\frac{9}{2}}}\right)+d\left(\frac{x}{8e^2(-e^2x^2+d^2)^{\frac{9}{2}}}-\frac{d^2\left(\frac{x}{9d^2\left(-e^2x^2+d^2\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{8\left(\frac{}{35d^2(\cdots}\right.}{}}{}\right.}{}\right.$

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(-e*x+d)*(e*x+d)^2*(16*e^8*x^8-16*d*e^7*x^7-56*d^2*e^6*x^6+56*d^3*e^
5*x^5+70*d^4*e^4*x^4-70*d^5*e^3*x^3-35*d^6*e^2*x^2-10*d^7*e*x+10*d^8)/d^7/e
^3/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(130) = 260.

Time = 0.50 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.06

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{10\,e^9x^9-10\,de^8x^8-40\,d^2e^7x^7+40\,d^3e^6x^6+60\,d^4e^5x^5-60\,d^5e^4x^4-40\,d^6e^3x^3+40\,d^7e^2x^2+10\,d^8ex-}{315\,(d^7e^{12}x^9-d^8e^{11}x^8-4\,d^9e^{10}x^7+4\,d^{10}e^9x^6+}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/315*(10*e^9*x^9 - 10*d*e^8*x^8 - 40*d^2*e^7*x^7 + 40*d^3*e^6*x^6 + 60*d^
4*e^5*x^5 - 60*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 40*d^7*e^2*x^2 + 10*d^8*e*x -
 10*d^9 - (16*e^8*x^8 - 16*d*e^7*x^7 - 56*d^2*e^6*x^6 + 56*d^3*e^5*x^5 + 70
*d^4*e^4*x^4 - 70*d^5*e^3*x^3 - 35*d^6*e^2*x^2 - 10*d^7*e*x + 10*d^8)*sqrt(
-e^2*x^2 + d^2))/(d^7*e^12*x^9 - d^8*e^11*x^8 - 4*d^9*e^10*x^7 + 4*d^10*e^9
*x^6 + 6*d^11*e^8*x^5 - 6*d^12*e^7*x^4 - 4*d^13*e^6*x^3 + 4*d^14*e^5*x^2 +
d^15*e^4*x - d^16*e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.45 (sec) , antiderivative size = 1401, normalized size of antiderivative = 9.47

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

```
[Out] d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d
**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*
e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 18
90*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1
 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 72*I*d
**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
```

```
 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-
1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*x**2/d**2)
- 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 3
15*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (10
5*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt
(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260
*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**
2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 12
60*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 -
 e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9
*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) + 72*d**2*e**4*x**7/(315*d**17*sqrt(1
- e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d*
*13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2
*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) - 16*e**6*x**9/(
315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**
2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x
**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((-2*d**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*
d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2
*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d*
*2 - e**2*x**2)) + 9*e**2*x**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*d
**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2*
x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d**
2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(11/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx &= \frac{x^2}{7\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e}+\frac{dx}{9\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^2}\\ &\quad-\frac{2\,d^2}{63\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^3}-\frac{x}{63\left(-e^2x^2+d^2\right)^{\frac{7}{2}}de^2}-\frac{2\,x}{105\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^2}\\ &\quad-\frac{8\,x}{315\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5e^2}-\frac{16\,x}{315\sqrt{-e^2x^2+d^2}d^7e^2}\end{aligned}$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/7*x^2/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*d*x/((-e^2*x^2 + d^2)^(9/2)*e^2) -
 2/63*d^2/((-e^2*x^2 + d^2)^(9/2)*e^3) - 1/63*x/((-e^2*x^2 + d^2)^(7/2)*d*e
^2) - 2/105*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^2) - 8/315*x/((-e^2*x^2 + d^2)^
(3/2)*d^5*e^2) - 16/315*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^2)
```

Giac [F]

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)x^2}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{144\,d^3\,e^3\,(d-e\,x)^5} \\ &- \frac{\sqrt{d^2-e^2x^2}\,\left(\frac{1}{252\,e^3}-\frac{17\,x}{252\,d\,e^2}\right)}{(d+e\,x)^4\,(d-e\,x)^4} - \frac{\sqrt{d^2-e^2x^2}\,\left(\frac{5}{144\,d^2\,e^3}+\frac{131\,x}{5040\,d^3\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3} \\ &- \frac{8\,x\,\sqrt{d^2-e^2x^2}}{315\,d^5\,e^2\,(d+e\,x)^2\,(d-e\,x)^2} - \frac{16\,x\,\sqrt{d^2-e^2x^2}}{315\,d^7\,e^2\,(d+e\,x)\,(d-e\,x)}\end{aligned}$$

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(144*d^3*e^3*(d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(1
/(252*e^3) - (17*x)/(252*d*e^2)))/((d + e*x)^4*(d - e*x)^4) - ((d^2 - e^2*x
^2)^(1/2)*(5/(144*d^2*e^3) + (131*x)/(5040*d^3*e^2)))/((d + e*x)^3*(d - e*x
)^3) - (8*x*(d^2 - e^2*x^2)^(1/2))/(315*d^5*e^2*(d + e*x)^2*(d - e*x)^2) -
(16*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*e^2*(d + e*x)*(d - e*x))
```

3.32 $\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\arcsin(ax)}{a^3}$$

```
[Out] -arcsin(a*x)/a^3+(a*x-1)/a^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {811, 655, 222, 651}

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{\arcsin(ax)}{a^3} - \frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[In] Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1-ax}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

```
[In] Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (2*ArcTan[(a*x)/(-1 + Sqrt
[1 - a^2*x^2])])/a^3
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

method	result	size
default	$-a\left(-\frac{x^{2}}{a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{2}{a^{4}\sqrt{-a^{2}x^{2}+1}}\right)+\frac{x}{a^{2}\sqrt{-a^{2}x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}$	90
risch	$\frac{a^{2}x^{2}-1}{a^{3}\sqrt{-a^{2}x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}+2\left(x+\frac{1}{a}\right)a}}{a^{4}\left(x+\frac{1}{a}\right)}$	92
meijerg	$-\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(-4a^{2}x^{2}+8\right)}{4\sqrt{-a^{2}x^{2}+1}}}{a^{3}\sqrt{\pi}}-\frac{\frac{\sqrt{\pi}\,x\left(-a^{2}\right)^{\frac{3}{2}}}{a^{2}\sqrt{-a^{2}x^{2}+1}}-\frac{\sqrt{\pi}\left(-a^{2}\right)^{\frac{3}{2}}\arcsin\left(ax\right)}{a^{3}}}{a^{2}\sqrt{\pi}\,\sqrt{-a^{2}}}$	105

```
[In] int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -a*(-x^2/a^2/(-a^2*x^2+1)^(1/2)+2/a^4/(-a^2*x^2+1)^(1/2))+x/a^2/(-a^2*x^2+1
)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.20

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = -a\left(\begin{cases}\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^6x^2-a^4}-\frac{2\sqrt{-a^2x^2+1}}{a^6x^2-a^4} & \text{for}\ a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right) + \begin{cases}-\frac{ix}{a^2\sqrt{a^2x^2-1}}+\frac{i\operatorname{acosh}(ax)}{a^3} & \text{for}\ |a^2x^2|>1\\ \frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a*Piecewise((a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**6*x**2 - a**4) - 2*sqrt(-a
**2*x**2 + 1)/(a**6*x**2 - a**4), Ne(a, 0)), (x**4/4, True)) + Piecewise((-
I*x/(a**2*sqrt(a**2*x**2 - 1)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1), (x
/(a**2*sqrt(-a**2*x**2 + 1)) - asin(a*x)/a**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{x^2}{\sqrt{-a^2x^2+1}a}+\frac{x}{\sqrt{-a^2x^2+1}a^2}-\frac{\arcsin\left(ax\right)}{a^3}-\frac{2}{\sqrt{-a^2x^2+1}a^3}$$

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3 -
 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx = -\frac{\arcsin\left(ax\right)\operatorname{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

```
[In] int(-(x^2*(a*x - 1))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 173

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} + \frac{11d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

```
[Out] 11/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-8/15*d^3*x^2*(-e^2*x^2+d^2)^
(1/2)/e^3-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-2/5*d*x^4*(-e^2*x^2+d^2)^(
1/2)/e-1/6*x^5*(-e^2*x^2+d^2)^(1/2)-1/240*d^4*(165*e*x+256*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1823, 847, 794, 223, 209}

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = \frac{11d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} - \frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

```
[In] Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = -\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^4(-11d^2e^2-12de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2}$$

$$= -\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^3(48d^3e^3+55d^2e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{30e^4}$$

$$= -\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2(-165d^4e^4-192d^3e^5x)}{\sqrt{d^2-e^2x^2}}\,dx}{120e^6}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x(384d^5e^5+495d^4e^6x)}{\sqrt{d^2-e^2x^2}}\,dx}{360e^8}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} + \frac{(11d^6)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^4}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} + \frac{(11d^6)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^4}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} + \frac{11d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(256d^5+165d^4ex+128d^3e^2x^2+110d^2e^3x^3+96de^4x^4+40e^5x^5)+330d^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{240e^5}$$

```
[In] Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(256*d^5 + 165*d^4*e*x + 128*d^3*e^2*x^2 + 110*
d^2*e^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 330*d^6*ArcTan[(e*x)/(Sqrt[d^2]
- Sqrt[d^2 - e^2*x^2])])/e^5
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(40 e^{5} x^{5}+96 d\,e^{4} x^{4}+110 d^{2} e^{3} x^{3}+128 d^{3} e^{2} x^{2}+165 d^{4} e x +256 d^{5}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{240 e^{5}}+\frac{11 d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 e^{4} \sqrt{e^{2}}}$
default	$e^{2} \left(-\frac{x^{5} \sqrt{-e^{2} x^{2}+d^{2}}}{6 e^{2}}+\frac{5 d^{2} \left(-\frac{x^{3} \sqrt{-e^{2} x^{2}+d^{2}}}{4 e^{2}}+\frac{3 d^{2} \left(-\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2 e^{2}}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 e^{2} \sqrt{e^{2}}}\right)}{4 e^{2}}\right)}{6 e^{2}}\right)+d^{2} \left(-\frac{x^{3} \sqrt{-e^{2} x^{2}+d}}{4 e^{2}}\right.$

[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/240*(40*e^5*x^5+96*d*e^4*x^4+110*d^2*e^3*x^3+128*d^3*e^2*x^2+165*d^4*e*x +256*d^5)/e^5*(-e^2*x^2+d^2)^(1/2)+11/16*d^6/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{330\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+96\,de^4x^4+110\,d^2e^3x^3+128\,d^3e^2x^2+165\,d^4ex+256\,d^5\right)\sqrt{}}{240\,e^5}$$

[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/240*(330*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 9 6*d*e^4*x^4 + 110*d^2*e^3*x^3 + 128*d^3*e^2*x^2 + 165*d^4*e*x + 256*d^5)*sq rt(-e^2*x^2 + d^2))/e^5

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{11d^6\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e^4} + \sqrt{d^2-e^2x^2}\left(-\frac{16d^5}{15e^5}-\frac{11d^4x}{16e^4}-\frac{8d^3x^2}{15e^3}-\frac{11d^2x^3}{24e^2}-\frac{2dx^4}{5e}-\frac{x^5}{6}\right) \\ \frac{\frac{d^2x^5}{5}+\frac{dex^6}{3}+\frac{e^2x^7}{7}}{\sqrt{d^2}} \end{cases}$$

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((11*d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*
e**4) + sqrt(d**2 - e**2*x**2)*(-16*d**5/(15*e**5) - 11*d**4*x/(16*e**4) -
8*d**3*x**2/(15*e**3) - 11*d**2*x**3/(24*e**2) - 2*d*x**4/(5*e) - x**5/6),
Ne(e**2, 0)), ((d**2*x**5/5 + d*e*x**6/3 + e**2*x**7/7)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.95

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}x^5 - \frac{2\sqrt{-e^2x^2+d^2}dx^4}{5\,e} - \frac{11\sqrt{-e^2x^2+d^2}d^2x^3}{24\,e^2} - \frac{8\sqrt{-e^2x^2+d^2}d^3x^2}{15\,e^3} + \frac{11\,d^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^4} - \frac{11\sqrt{-e^2x^2+d^2}d^4x}{16\,e^4} - \frac{16\sqrt{-e^2x^2+d^2}d^5}{15\,e^5}$$

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*x^5 - 2/5*sqrt(-e^2*x^2 + d^2)*d*x^4/e - 11/24*sq
rt(-e^2*x^2 + d^2)*d^2*x^3/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*d^3*x^2/e^3 + 11
/16*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) - 11/16*sqrt(-e^2*x^2 +
 d^2)*d^4*x/e^4 - 16/15*sqrt(-e^2*x^2 + d^2)*d^5/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{11\,d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^4|e|} - \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(4\left(5\,x+\frac{12\,d}{e}\right)x+\frac{55\,d^2}{e^2}\right)x+\frac{64\,d^3}{e^3}\right)x+\frac{165\,d^4}{e^4}\right)x+\frac{256\,d^5}{e^5}\right)$$

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 11/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/240*sqrt(-e^2*x^2 +
d^2)*((2*((4*(5*x + 12*d/e)*x + 55*d^2/e^2)*x + 64*d^3/e^3)*x + 165*d^4/e^4
)*x + 256*d^5/e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

```
[Out] 3/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/
2)/e^2-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/20*d
^3*(5*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1823, 847, 794, 223, 209}

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4} - \frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

```
[In] Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{\int \frac{x^3\left(-9d^2e^2-10de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\ &= -\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{\int \frac{x^2\left(30d^3e^3+36d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}\end{aligned}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x(-72d^4e^4-90d^3e^5x)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{(3d^5)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{4e^3}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{(3d^5)\operatorname{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{4e^3}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(24d^4+15d^3ex+12d^2e^2x^2+10de^3x^3+4e^4x^4)+30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{20e^4}$$

```
[In] Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/20*(Sqrt[d^2 - e^2*x^2]*(24*d^4 + 15*d^3*e*x + 12*d^2*e^2*x^2 + 10*d*e^3
*x^3 + 4*e^4*x^4) + 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])
/e^4
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{(4e^4x^4+10d\,e^3x^3+12d^2e^2x^2+15d^3ex+24d^4)\sqrt{-e^2x^2+d^2}}{20e^4}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4e^3\sqrt{e^2}}$
default	$e^2\left(-\frac{x^4\sqrt{-e^2x^2+d^2}}{5e^2}+\frac{4d^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)}{5e^2}\right)+d^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+2de$

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(4*e^4*x^4+10*d*e^3*x^3+12*d^2*e^2*x^2+15*d^3*e*x+24*d^4)/e^4*(-e^2*x
^2+d^2)^(1/2)+3/4*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(4\,e^4x^4+10\,de^3x^3+12\,d^2e^2x^2+15\,d^3ex+24\,d^4)\sqrt{-e^2x^2+d^2}}{20\,e^4}$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^4*x^4 + 10*d
*e^3*x^3 + 12*d^2*e^2*x^2 + 15*d^3*e*x + 24*d^4)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{3d^5\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{4e^3}+\sqrt{d^2-e^2x^2}\left(-\frac{6d^4}{5e^4}-\frac{3d^3x}{4e^3}-\frac{3d^2x^2}{5e^2}-\frac{dx^3}{2e}-\frac{x^4}{5}\right) & \text{for } e^2\neq \\ \frac{\frac{d^2x^4}{4}+\frac{2dex^5}{5}+\frac{e^2x^6}{6}}{\sqrt{d^2}} & \text{otherwis} \end{cases}$$

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((3*d**5*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(4*e*
*3) + sqrt(d**2 - e**2*x**2)*(-6*d**4/(5*e**4) - 3*d**3*x/(4*e**3) - 3*d**2
*x**2/(5*e**2) - d*x**3/(2*e) - x**4/5), Ne(e**2, 0)), ((d**2*x**4/4 + 2*d*
e*x**5/5 + e**2*x**6/6)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}\sqrt{-e^2x^2+d^2}x^4 - \frac{\sqrt{-e^2x^2+d^2}dx^3}{2\,e} - \frac{3\sqrt{-e^2x^2+d^2}d^2x^2}{5\,e^2} + \frac{3\,d^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{4\sqrt{e^2}e^3} - \frac{3\sqrt{-e^2x^2+d^2}d^3x}{4\,e^3} - \frac{6\sqrt{-e^2x^2+d^2}d^4}{5\,e^4}$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*x^4 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x^3/e - 3/5*sqrt
(-e^2*x^2 + d^2)*d^2*x^2/e^2 + 3/4*d^5*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^
2)*e^3) - 3/4*sqrt(-e^2*x^2 + d^2)*d^3*x/e^3 - 6/5*sqrt(-e^2*x^2 + d^2)*d^4
/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{4\,e^3|e|} - \frac{1}{20}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(2\,x+\frac{5\,d}{e}\right)x+\frac{6\,d^2}{e^2}\right)x+\frac{15\,d^3}{e^3}\right)x+\frac{24\,d^4}{e^4}\right)$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/20*sqrt(-e^2*x^2 + d^2
)*((2*((2*x + 5*d/e)*x + 6*d^2/e^2)*x + 15*d^3/e^3)*x + 24*d^4/e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=\int \frac{x^3\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}} \,d x$$

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
[Out] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] 7/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2/3*d*x^2*(-e^2*x^2+d^2)^(1/2)
/e-1/4*x^3*(-e^2*x^2+d^2)^(1/2)-1/24*d^2*(21*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)
/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1823, 847, 794, 223, 209}

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=\frac{7d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}$$

```
[In] Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{\int \frac{x^2(-7d^2e^2-8de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\ &= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{\int \frac{x(16d^3e^3+21d^2e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}\end{aligned}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{(7d^4)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{(7d^4)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(32d^3+21d^2ex+16de^2x^2+6e^3x^3)+42d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}$$

```
[In] Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)
 + 42*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(6e^3x^3+16d\,e^2x^2+21d^2ex+32d^3)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{7d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$e^2\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d}}\right.}{2e^2\sqrt{e^2}}\right.$

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*e^3*x^3+16*d*e^2*x^2+21*d^2*e*x+32*d^3)/e^3*(-e^2*x^2+d^2)^(1/2)+7
/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{42\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(6\,e^3x^3+16\,de^2x^2+21\,d^2ex+32\,d^3)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(42*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 16*d
*e^2*x^2 + 21*d^2*e*x + 32*d^3)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{7d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{4d^3}{3e^3}-\frac{7d^2x}{8e^2}-\frac{2dx^2}{3e}-\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{\frac{d^2x^3}{3}+\frac{dex^4}{2}+\frac{e^2x^5}{5}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((7*d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e*
*2) + sqrt(d**2 - e**2*x**2)*(-4*d**3/(3*e**3) - 7*d**2*x/(8*e**2) - 2*d*x*
*2/(3*e) - x**3/4), Ne(e**2, 0)), ((d**2*x**3/3 + d*e*x**4/2 + e**2*x**5/5)
/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}\sqrt{-e^2x^2+d^2}x^3 - \frac{2\sqrt{-e^2x^2+d^2}dx^2}{3\,e} + \frac{7\,d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} - \frac{7\sqrt{-e^2x^2+d^2}d^2x}{8\,e^2} - \frac{4\sqrt{-e^2x^2+d^2}d^3}{3\,e^3}$$

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*x^3 - 2/3*sqrt(-e^2*x^2 + d^2)*d*x^2/e + 7/8*d^4*
arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 7/8*sqrt(-e^2*x^2 + d^2)*d^2*
x/e^2 - 4/3*sqrt(-e^2*x^2 + d^2)*d^3/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{7\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} - \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x+\frac{8\,d}{e}\right)x+\frac{21\,d^2}{e^2}\right)x+\frac{32\,d^3}{e^3}\right)$$

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/24*sqrt(-e^2*x^2 + d^2
)*((2*(3*x + 8*d/e)*x + 21*d^2/e^2)*x + 32*d^3/e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2\,{(d+e\,x)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}+\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/3*x^2*(-e^2*x^2+d^2)^(1/2)-1/3*d
*(3*e*x+5*d)*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1823, 794, 223, 209}

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

```
[In] Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*(x^2*Sqrt[d^2 - e^2*x^2]) - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e
^2) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{\int \frac{x(-5d^2e^2-6de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$
\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(5d^2+3dex+e^2x^2)+6d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^2}
$$

```
[In] Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(5*d^2 + 3*d*e*x + e^2*x^2) + 6*d^3*ArcTan[(e*x)/
(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^2
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result	si
risch	$-\frac{\left(e^{2} x^{2}+3 d e x+5 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}+\frac{d^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e \sqrt{e^{2}}}$	7
default	$e^{2}\left(-\frac{x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}-\frac{2 d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{4}}\right)-\frac{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{e^{2}}+2 d e\left(-\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2 e^{2}}+\frac{d^{2} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 e^{2} \sqrt{e^{2}}}\right)$	1

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e^2*x^2+3*d*e*x+5*d^2)/e^2*(-e^2*x^2+d^2)^(1/2)+d^3/e/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(e^2x^2+3\,dex+5\,d^2)\sqrt{-e^2x^2+d^2}}{3\,e^2}$$

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^2*x^2 + 3*d*e*x
+ 5*d^2)*sqrt(-e^2*x^2 + d^2))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{d^3\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{e}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^2}{3e^2}-\frac{dx}{e}-\frac{x^2}{3}\right) & \text{for}\ e^2\neq 0 \\ \frac{\frac{d^2x^2}{2}+\frac{2dex^3}{3}+\frac{e^2x^4}{4}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((d**3*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x
**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/e + sqr
t(d**2 - e**2*x**2)*(-5*d**2/(3*e**2) - d*x/e - x**2/3), Ne(e**2, 0)), ((d*
*2*x**2/2 + 2*d*e*x**3/3 + e**2*x**4/4)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}\sqrt{-e^2x^2+d^2}x^2 + \frac{d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e} - \frac{\sqrt{-e^2x^2+d^2}dx}{e} - \frac{5\sqrt{-e^2x^2+d^2}d^2}{3\,e^2}$$

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2 + d^3*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*
e) - sqrt(-e^2*x^2 + d^2)*d*x/e - 5/3*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e|e|} - \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(\left(x+\frac{3\,d}{e}\right)x+\frac{5\,d^2}{e^2}\right)$$

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/3*sqrt(-e^2*x^2 + d^2)*((x +
 3*d/e)*x + 5*d^2/e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-3/2*d*(-e^2*x^2+d^2)^(1/2)/e-1/2
*(e*x+d)*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {685, 655, 223, 209}

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx = \frac{3d^2 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e} - \frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(4d+ex)\sqrt{d^2-e^2x^2}+6d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
[Out] -1/2*((4*d + e*x)*Sqrt[d^2 - e^2*x^2] + 6*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqr
t[d^2 - e^2*x^2])])/e
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{(ex+4d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$	60
default	$\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{2d\sqrt{-e^2x^2+d^2}}{e}$	113

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(e*x+4*d)/e*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+4\,d)}{2\,e}$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x + 4*d))/e
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{3d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d}{e}-\frac{x}{2}\right) & \text{for}\ e^2\neq 0 \\ \frac{\begin{cases} d^2x & \text{for}\ e=0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((3*d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + s
qrt(d**2 - e**2*x**2)*(-2*d/e - x/2), Ne(e**2, 0)), (Piecewise((d**2*x, Eq(
e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}x - \frac{2\,\sqrt{-e^2x^2+d^2}d}{e}$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*x
- 2*sqrt(-e^2*x^2 + d^2)*d/e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(x+\frac{4\,d}{e}\right)$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*sqrt(-e^2*x^2 + d^2)*(x +
4*d/e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -\sqrt{d^2-e^2x^2}+2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1823, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = 2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\sqrt{d^2-e^2x^2}$$

```
[In] Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\sqrt{d^2-e^2x^2} - \frac{\int \frac{-d^2e^2-2de^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{e^2} \\
&= -\sqrt{d^2-e^2x^2} + d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (2de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx
\end{aligned}
$$

$$= -\sqrt{d^2-e^2x^2} + \frac{1}{2}d^2\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (2de)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\sqrt{d^2-e^2x^2} + 2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}$$

$$= -\sqrt{d^2-e^2x^2} + 2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -\sqrt{d^2-e^2x^2} - 4d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2}\log(x) + \sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] - 4*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]
- Sqrt[d^2]*Log[x] + Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

method	result	size
default	$-\sqrt{-e^2x^2+d^2} - \frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} + \frac{2de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$	91

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2
)^(1/2))/x)+2*d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -4\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \sqrt{-e^2x^2+d^2}$$

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d*log(-(d - sqrt(-e^2*x^2
+ d^2))/x) - sqrt(-e^2*x^2 + d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\wedge e^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{for}\ e^2\neq 0 \\ \frac{x}{\sqrt{d^2}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{for}\ e^2\neq 0 \\ \frac{x^2}{2\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e
*x))/d, True)) + 2*d*e*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
 e**2*x**2))/sqrt(-e**2), Ne(d**2, 0) & Ne(e**2, 0)), (x*log(x)/sqrt(-e**2*
x**2), Ne(e**2, 0)), (x/sqrt(d**2), True)) + e**2*Piecewise((-sqrt(d**2 - e
**2*x**2)/e**2, Ne(e**2, 0)), (x**2/(2*sqrt(d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,de\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - d\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \sqrt{-e^2x^2+d^2}$$

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*d*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d*log(2*d^2/abs(x) + 2*sqrt(-
e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,de\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{de\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \sqrt{-e^2x^2+d^2}$$

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d*e*log(1/2*abs(-2*d*e - 2*sqrt(
-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = \int \frac{{\left(d+e\,x\right)}^2}{x\,\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)), x)
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 68

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{x} + e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1821, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{\sqrt{d^2-e^2x^2}}{x}$$

```
[In] Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d^2-e^2x^2}}{x} - \frac{\int \frac{-2d^3e-d^2e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{x} + (2de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{x} + (de)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + e^2\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{x} + e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{(2d)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{x} + e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{x} - 2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{2\sqrt{d^2}e\log(x)}{d} + \frac{2\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

```
[In] Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) - 2*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2
])] - (2*Sqrt[d^2]*e*Log[x])/d + (2*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 -
e^2*x^2]])/d
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}}{x} - \frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	93
risch	$\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}}{x} - \frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	93

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-(-e^2*x^2+d^2)^(
1/2)/x-2*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-2\,ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}}{x}$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*e*x*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right) + 2de\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\wedge e^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{for}\ e^2\neq 0\\ \frac{x}{\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piecewise((-aco
sh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e*x))/d, True)) + e**
2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e*
*2), Ne(d**2, 0) & Ne(e**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Ne(e**2, 0)),
(x/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - 2\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{\sqrt{-e^2x^2+d^2}}{x}$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 2*e*log(2*d^2/abs(x) + 2*sqrt(-
e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{2\,e^2\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,x|e|}$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*e^4*x/((d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))*abs(e)) - 2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*ab
s(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(x*abs
(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{e^2\,\ln\left(x\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{\sqrt{d^2-e^2\,x^2}}{x} - \frac{2\,d\,e\,\ln\left(\frac{\sqrt{d^2}+\sqrt{d^2-e^2\,x^2}}{x}\right)}{\sqrt{d^2}} & \text{if } e^2<0 \\ \int \frac{e^2}{\sqrt{d^2-e^2\,x^2}} + \frac{d^2}{x^2\,\sqrt{d^2-e^2\,x^2}} + \frac{2\,d\,e}{x\,\sqrt{d^2-e^2\,x^2}}\,dx & \text{if } \neg e^2<0 \end{cases}$$

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] piecewise(e^2 < 0, - (d^2 - e^2*x^2)^(1/2)/x + (e^2*log(x*(-e^2)^(1/2) + (d
^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (2*d*e*log(((d^2)^(1/2) + (d^2 - e^2*x
^2)^(1/2))/x))/(d^2)^(1/2), ~e^2 < 0, int(e^2/(d^2 - e^2*x^2)^(1/2) + d^2/(
x^2*(d^2 - e^2*x^2)^(1/2)) + (2*d*e)/(x*(d^2 - e^2*x^2)^(1/2)), x))
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-1/2*(-e^2*x^2+d^2)^(1/2)/x^2-2*e
*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1821, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

```
[In] Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/2*Sqrt[d^2 - e^2*x^2]/x^2 - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{\int \frac{-4d^3e-3d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{2}\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{4}\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+4ex)\sqrt{d^2-e^2x^2}-3e^2x^2\log\left(d\left(-d-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)\right)+3e^2x^2\log\left(d-\sqrt{-e^2}x+\sqrt{d^2-e^2 \cdots}\right.}{2dx^2}$$

[In] Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]

[Out] -1/2*((d + 4*e*x)*Sqrt[d^2 - e^2*x^2] - 3*e^2*x^2*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] + 3*e^2*x^2*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(d*x^2)

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4ex+d)}{2d\,x^2}-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$	72
default	$-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)-\frac{2e\sqrt{-e^2x^2+d^2}}{dx}$	139

[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(4*e*x+d)/d/x^2-3/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}(4\,ex+d)}{2\,dx^2}$$

[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(3*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - sqrt(-e^2*x^2 + d^2)*(4*e*x + d))/(d*x^2)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*
x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x)
)/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) +
 e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(
e*x))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d} - \frac{2\,\sqrt{-e^2x^2+d^2}e}{dx} - \frac{\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 2*sqrt(-e^
2*x^2 + d^2)*e/(d*x) - 1/2*sqrt(-e^2*x^2 + d^2)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^3 + \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d|e|} - \frac{3\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d|e|} - \frac{\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de|e|}{x} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d|e|}{ex^2}}{8\,d^2e^2}$$

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(e^3 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x)*e^4*x^2/((d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*d*abs(e)) - 3/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/8*(8*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))*d*e*abs(e)/x + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*ab
s(e)/(e*x^2))/(d^2*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^3\,\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)), x)
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/3*(-e^2*x^2+d^2)^(1/2)/x^3-e*(-e
^2*x^2+d^2)^(1/2)/d/x^2-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1821, 849, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx=-\frac{e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}$$

```
[In] Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/3*Sqrt[d^2 - e^2*x^2]/x^3 - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{\int \frac{-6d^3e-5d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}+\frac{\int \frac{10d^4e^2+6d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{e^3\int\frac{1}{x\sqrt{d^2-e^2x^2}}dx}{d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{e^3\text{Subst}\left(\int\frac{1}{x\sqrt{d^2-e^2x}}dx, x, x^2\right)}{2d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}dx, x, \sqrt{d^2-e^2x^2}\right)}{d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int\frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(d^2+3dex+5e^2x^2)}{x^3} + 3\sqrt{d^2}e^3\log(x) - 3\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^3}$$

```
[In] Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/3*((d*Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3 + 3*Sqrt[d^2]
*e^3*Log[x] - 3*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^3
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(5e^2x^2+3dex+d^2)}{3x^3d^2} - \frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}$
default	$-\frac{e^2\sqrt{-e^2x^2+d^2}}{d^2x} + d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3} - \frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right) + 2de\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2} - \frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)$

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(1/2)*(5*e^2*x^2+3*d*e*x+d^2)/x^3/d^2-1/d*e^3/(d^2)^(1/
2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(5\,e^2x^2+3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{3\,d^2x^3}$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (5*e^2*x^2 + 3*d*e*x +
d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.83

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise}\end{cases}\right) + 2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d
**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) -
e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e
**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**
2) - 1)/d**2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)
/d**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^2} - \frac{5\sqrt{-e^2x^2+d^2}e^2}{3\,d^2x} - \frac{\sqrt{-e^2x^2+d^2}e}{dx^2} - \frac{\sqrt{-e^2x^2+d^2}}{3\,x^3}$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 5/3*sqrt(-e^
2*x^2 + d^2)*e^2/(d^2*x) - sqrt(-e^2*x^2 + d^2)*e/(d*x^2) - 1/3*sqrt(-e^2*x
^2 + d^2)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(95) = 190.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^4+\frac{6\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x}+\frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2|e|} - \frac{e^4\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{\frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^4}{x}+\frac{6\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^2}{x^2}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{x^3}}{24\,d^6e^2|e|}$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(e^4 + 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 21*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
*d^2*abs(e)) - e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*abs(x)))/(d^2*abs(e)) - 1/24*(21*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e
^4/x + 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*e^2/x^2 + (d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3*d^4/x^3)/(d^6*e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^4\,\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)), x)
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

```
[Out] -7/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/4*(-e^2*x^2+d^2)^(1/2)/x^4-2
/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-7/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-4/3*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1821, 849, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{7e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

```
[In] Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/4*Sqrt[d^2 - e^2*x^2]/x^4 - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{\int \frac{-8d^3e-7d^2e^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{\int \frac{21d^4e^2+16d^3e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{\int \frac{-32d^5e^3-21d^4e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{16d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{(7e^2)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{8d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3-16d^2ex-21de^2x^2-32e^3x^3)}{24d^3x^4} + \frac{7e^4\text{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^3}$$

```
[In] Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 16*d^2*e*x - 21*d*e^2*x^2 - 32*e^3*x^3))/(24
*d^3*x^4) + (7*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*d^
3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(32 e^{3} x^{3}+21 d\, e^{2} x^{2}+16 d^{2} e x +6 d^{3}\right)}{24 d^{3} x^{4}}-\frac{7 e^{4} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 d^{2} \sqrt{d^{2}}}$
default	$d^{2} \left(-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{4 d^{2} x^{4}}+\frac{3 e^{2} \left(-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{2 d^{2} x^{2}}-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{2} \sqrt{d^{2}}}\right)}{4 d^{2}}\right)+e^{2} \left(-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{2 d^{2} x^{2}}-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{2} \sqrt{d^{2}}}\right.$

[In] int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(32*e^3*x^3+21*d*e^2*x^2+16*d^2*e*x+6*d^3)/d^3/x
^4-7/8/d^2*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{21\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(32\,e^3x^3+21\,de^2x^2+16\,d^2ex+6\,d^3)\sqrt{-e^2x^2+d^2}}{24\,d^3x^4}$$

[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] 1/24*(21*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 21*d*e^
2*x^2 + 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*x^4)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$$
$$= d^2\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right)$$
$$+ 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$
$$+ e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*s
qrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) -
 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))
/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*
x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/
(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2
) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1
), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{7\,e^4\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d^3} - \frac{4\sqrt{-e^2x^2+d^2}\,e^3}{3\,d^3x} - \frac{7\sqrt{-e^2x^2+d^2}\,e^2}{8\,d^2x^2} - \frac{2\sqrt{-e^2x^2+d^2}\,e}{3\,dx^3} - \frac{\sqrt{-e^2x^2+d^2}}{4\,x^4}$$

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 4/3*sqrt
(-e^2*x^2 + d^2)*e^3/(d^3*x) - 7/8*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^2) - 2/3
*sqrt(-e^2*x^2 + d^2)*e/(d*x^3) - 1/4*sqrt(-e^2*x^2 + d^2)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(120) = 240.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(3\,e^5+\frac{16\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{48\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}+\frac{144\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^3|e|} - \frac{7\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d^3|e|} - \frac{\frac{144\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^9e^5|e|}{x}+\frac{48\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^9e^3|e|}{x^2}+\frac{16\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^9e|e|}{x^3}+\frac{3\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^9|e|}{ex^4}}{192\,d^{12}e^4}$$

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(3*e^5 + 16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 48*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 + 144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*abs(e)) - 7/
8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^
3*abs(e)) - 1/192*(144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^9*e^5*abs(e)/x
 + 48*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^9*e^3*abs(e)/x^2 + 16*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^9*e*abs(e)/x^3 + 3*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^4*d^9*abs(e)/(e*x^4))/(d^12*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = \int \frac{{\left(d+e\,x\right)}^2}{x^5\,\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)), x)
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 169

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

```
[Out] -3/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/5*(-e^2*x^2+d^2)^(1/2)/x^5-1
/2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/4*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1821, 849, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}$$

```
[In] Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*Sqrt[d^2 - e^2*x^2]/x^5 - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{\int \frac{-10d^3e-9d^2e^2x}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} + \frac{\int \frac{36d^4e^2+30d^3e^3x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{\int \frac{-90d^5e^3-72d^4e^4x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} + \frac{\int \frac{144d^6e^4+90d^5e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{120d^8}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{(3e^5)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{4d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{(3e^5)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{8d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{(3e^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{4d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-4d^4-10d^3ex-12d^2e^2x^2-15de^3x^3-24e^4x^4)}{20d^4x^5} - \frac{3\sqrt{d^2}e^5\log(x)}{4d^5} + \frac{3\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{4d^5}$$

```
[In] Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^4 - 10*d^3*e*x - 12*d^2*e^2*x^2 - 15*d*e^3*x^3 -
 24*e^4*x^4))/(20*d^4*x^5) - (3*Sqrt[d^2]*e^5*Log[x])/(4*d^5) + (3*Sqrt[d^2
]*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(4*d^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(24e^4x^4+15d\,e^3x^3+12d^2e^2x^2+10d^3ex+4d^4)}{20x^5d^4}-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4d^3\sqrt{d^2}}$
default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{5d^2x^5}+\frac{4e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)}{5d^2}\right)+e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)+2de\Bigg(-\frac{\sqrt{-e}}{4}$

```
[In] int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(-e^2*x^2+d^2)^(1/2)*(24*e^4*x^4+15*d*e^3*x^3+12*d^2*e^2*x^2+10*d^3*e
*x+4*d^4)/x^5/d^4-3/4/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(24\,e^4x^4+15\,de^3x^3+12\,d^2e^2x^2+10\,d^3ex+4\,d^4)\sqrt{-e^2x^2+d^2}}{20\,d^4x^5}$$

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (24*e^4*x^4 + 15*d*e^
3*x^3 + 12*d^2*e^2*x^2 + 10*d^3*e*x + 4*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*x^5
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.02

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$$
$$= d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4} - \frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2} - \frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4} - \frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2} - \frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases}\right)$$
$$+ 2de\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right)$$
$$+ e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(15
*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(5*d*
*2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) - 8*I*e**5*s
qrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-1/(4*e*x**
5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(e*x))/(8*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
 + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e**3/(8*d**4*x*sqrt(
-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**5), True)) + e**2*Pi
ecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**4), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{4\,d^4} - \frac{6\,\sqrt{-e^2x^2+d^2}e^4}{5\,d^4x} - \frac{3\,\sqrt{-e^2x^2+d^2}e^3}{4\,d^3x^2} - \frac{3\,\sqrt{-e^2x^2+d^2}e^2}{5\,d^2x^3} - \frac{\sqrt{-e^2x^2+d^2}e}{2\,dx^4} - \frac{\sqrt{-e^2x^2+d^2}}{5\,x^5}$$

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] -3/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 - 6/5*sqrt
(-e^2*x^2 + d^2)*e^4/(d^4*x) - 3/4*sqrt(-e^2*x^2 + d^2)*e^3/(d^3*x^2) - 3/5
*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^3) - 1/2*sqrt(-e^2*x^2 + d^2)*e/(d*x^4) -
1/5*sqrt(-e^2*x^2 + d^2)/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(145) = 290.

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^6+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2}+\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3}+\frac{110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^2x^4}\right)e^{10}x^5}{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5d^4|e|} - \frac{3\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{4\,d^4|e|} - \frac{\frac{110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{16}e^8}{x}+\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{16}e^6}{x^2}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{16}e^4}{x^3}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{16}e^2}{x^4}+}{160\,d^{20}e^4|e|}$$

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] 1/160*(e^6 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x + 15*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/x^3 + 110*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^4*abs(e)) - 3/4*e^6*log(1/2*abs(-2*d*e -
 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) - 1/160*(110*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^16*e^8/x + 40*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^2*d^16*e^6/x^2 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^16*e^
4/x^3 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^16*e^2/x^4 + (d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^5*d^16/x^5)/(d^20*e^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{x^6\,\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)), x)
```

3.44 $\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*d^4*(e*x+d)^2/e^6/(-e^2*x^2+d^2)^(5/2)-22/15*d^3*(e*x+d)/e^6/(-e^2*x^2+
d^2)^(3/2)-2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+2/15*d*(23*e*x+30*d)/e^
6/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1649, 1828, 655, 223, 209}

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} + \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d} \\ &= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{\frac{16d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2} \end{aligned}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{30d^5}{e^5}+\frac{15d^4x}{e^4}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-56d^4+82d^3ex+32d^2e^2x^2-76de^3x^3+15e^4x^4\right)}{15e^6(-d+ex)^3(d+ex)} + \frac{2d\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^9}$$

```
[In] Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-56*d^4 + 82*d^3*e*x + 32*d^2*e^2*x^2 - 76*d*e^3*x^3
+ 15*e^4*x^4))/(15*e^6*(-d + e*x)^3*(d + e*x)) + (2*d*(-e^2)^(3/2)*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^9
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.66

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^6}-\frac{2d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8e^7\left(x+\frac{d}{e}\right)}-\frac{41d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{60e^8\left(x-\frac{d}{e}\right)^2}-\frac{383d\sqrt{-\left(x-\right.}}{120}$
default	$e^2\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right)+d^2\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$

[In] int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] (-e^2*x^2+d^2)^(1/2)/e^6-2*d/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/8*d/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-41/60*d^2/e^8/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-383/120*d/e^7/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/10*d^3/e^9/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{56\,de^4x^4-112\,d^2e^3x^3+112\,d^4ex-56\,d^5+60\left(de^4x^4-2\,d^2e^3x^3+2\,d^4ex-d^5\right)\mathrm{arcta}}{15\left(e^{10}x^4-2\,de^9x\right.}$$

[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(56*d*e^4*x^4 - 112*d^2*e^3*x^3 + 112*d^4*e*x - 56*d^5 + 60*(d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^4*x^4 - 76*d*e^3*x^3 + 32*d^2*e^2*x^2 + 82*d^3*e*x - 56*d^4)*sqrt(-e^2*x^2 + d^2))/(e^10*x^4 - 2*d*e^9*x^3 + 2*d^3*e^7*x - d^4*e^6)

Sympy [F]

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5(d+ex)^2}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.01

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2}{15}\,dex\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{2\,dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{3\,e} + \frac{7\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{28\,d^4x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{56\,d^6}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} + \frac{8\,d^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{14\,dx}{15\,\sqrt{-e^2x^2+d^2}e^5} - \frac{2\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^5}$$

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 2/15*d*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/(-e^2*x^2 + d^2)
^(5/2) - 2/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d
^2)^(3/2)*e^4))/e + 7*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 28/3*d^4*x^2/(
(-e^2*x^2 + d^2)^(5/2)*e^4) + 56/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e^6) + 8/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 14/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^5)
- 2*d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^5)
```

Giac [F]

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^5}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*x^5/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.45 $\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/5*d^3*(e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)-17/15*d^2*(e*x+d)/e^5/(-e^2*x^2+
d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(13*e*x+15*d)/e^5/(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1649, 1828, 12, 223, 209}

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} + \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

```
[In] Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^
5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2])
 - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{\frac{11d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}
\end{aligned}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(16d^3-17d^2ex-22de^2x^2+26e^3x^3\right)}{(d-ex)^3(d+ex)} + 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^5}$$

```
[In] Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3))/((
d - e*x)^3*(d + e*x)) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])
/(15*e^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(107) = 214.

Time = 0.41 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.68

method	result
default	$e^2\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}} - \frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right) + d^2\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{3d^2\left(\frac{\ldots}{4e^2(-\ldots}\right.}{\ldots}\right.$

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/
2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))))+d^2*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2
*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2
)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
)+2*d*e*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)
^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{16\,e^4x^4 - 32\,de^3x^3 + 32\,d^3ex - 16\,d^4 + 30\,(e^4x^4 - 2\,de^3x^3 + 2\,d^3ex - d^4)\arctan\left(-\ldots\right.}{15\,(e^9x^4 - 2\,de^8x^3 + 2\,d^3e^6\ldots}$$

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*e^4*x^4 - 32*d*e^3*x^3 + 32*d^3*e*x - 16*d^4 + 30*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (26*e
^3*x^3 - 22*d*e^2*x^2 - 17*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^4
 - 2*d*e^8*x^3 + 2*d^3*e^6*x - d^4*e^5)
```

Sympy [F]

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.

Time = 0.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.56

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}e^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)+\frac{2\,dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2}-\frac{8\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3}-\frac{3\,d^4x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{16\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5}+\frac{11\,d^2x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}-\frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}e^4}-\frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^4}$$

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*
x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 2*d*x^4/((-e^
2*x^2 + d^2)^(5/2)*e) + 1/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8/3*d^3*
x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 16/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^5) + 11/30*d^2*x/((-e^2*x^2 + d^2)^(3
/2)*e^4) - 4/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e^2*x/(d*sqrt(e^2)))/
(sqrt(e^2)*e^4)
```

Giac [F]

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^4}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*x^4/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.46 $\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}} \, dx = \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(e*x+d)/e^4/(-e^2*x^2+d^2)
^(3/2)+1/5*(2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1649, 651}

$$\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}} \, dx = \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}\\ &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}\\ &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(2d^3-4d^2ex+de^2x^2+2e^3x^3\right)}{5de^4(d-ex)^3(d+ex)}$$

```
[In] Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3))/(5*d*e^4*
(d - e*x)^3*(d + e*x))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(2e^3x^3+d\,e^2x^2-4d^2ex+2d^3)}{5d\,e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(2e^3x^3+d\,e^2x^2-4d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d\,e^4(-ex+d)^3(ex+d)}$
default	$e^2\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+d^2\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)$

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x^
2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-(2\,e^3x^3+de^2x^2-4\,d^2ex+2\,d^3)\sqrt{-e^2x^2+d^2}}{5\,(de^8x^4-2\,d^2e^7x^3+2\,d^4e^5x-d^5e^4)}$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 + d*e^2*x^2 -
 4*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 - 2*d^2*e^7*x^3 + 2*d^
4*e^5*x - d^5*e^4)
```

Sympy [F]

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dx^3}{(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{3\,d^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{2\,d^4}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{dx}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{2\,x}{5\,\sqrt{-e^2x^2+d^2}de^3}$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*d^
4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 2/5
*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)
```

Giac [F]

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*x^3/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3-4\,d^2\,e\,x+d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 + d*e^2*x^2 - 4*d^2*e*x))/(5*d*e^
4*(d + e*x)*(d - e*x)^3)
```

3.47 $\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^(5/2)-7/15*(e*x+d)/e^3/(-e^2*x^2+d^2)^(3
/2)+1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1649, 792, 197}

$$\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} + \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
- e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d} \\
&= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15e^2} \\
&= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-4d^3+8d^2ex-2de^2x^2+e^3x^3)}{15d^2e^3(d-ex)^3(d+ex)}$$

```
[In] Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*
e^3*(d - e*x)^3*(d + e*x))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

method	result
gosper	$-\frac{(-ex+d)(ex+d)^{3}\left(-e^{3}x^{3}+2d\,e^{2}x^{2}-8d^{2}ex+4d^{3}\right)}{15d^{2}e^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-e^{3}x^{3}+2d\,e^{2}x^{2}-8d^{2}ex+4d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{2}e^{3}(-ex+d)^{3}(ex+d)}$
default	$e^{2}\left(\frac{x^{3}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3d^{2}\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2}\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)}{4e^{2}}\right)}{2e^{2}}\right)+d^{2}\left(\cdots\right.$

[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-e^2*x^2+d^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{4\,e^4x^4-8\,de^3x^3+8\,d^3ex-4\,d^4+(e^3x^3-2\,de^2x^2+8\,d^2ex-4\,d^3)\sqrt{-e^2x^2+d^2}}{15\,(d^2e^7x^4-2\,d^3e^6x^3+2\,d^5e^4x-d^6e^3)}$$

[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15*(4*e^4*x^4 - 8*d*e^3*x^3 + 8*d^3*e*x - 4*d^4 + (e^3*x^3 - 2*d*e^2*x^2 + 8*d^2*e*x - 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 - 2*d^3*e^6*x^3 + 2*d^5*e^4*x - d^6*e^3)

Sympy [F]

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2\,dx^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,d^3}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2} + \frac{x}{15\,\sqrt{-e^2x^2+d^2}d^2e^2}$$

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 1/2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 1/1
0*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e^3
) + 1/30*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*
e^2)
```

Giac [F]

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*x^2/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3-8\,d^2\,e\,x+2\,d\,e^2\,x^2-e^3\,x^3\right)}{15\,d^2\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(4*d^3 - e^3*x^3 + 2*d*e^2*x^2 - 8*d^2*e*x))/(15*d^
2*e^3*(d + e*x)*(d - e*x)^3)
```

3.48 $\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*(e*x+d)/d/e^2/(-e^2*x^2+d^2)^(3
/2)-4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {803, 653, 197}

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 -
e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2\int\frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int\frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(d^3-2d^2ex+8de^2x^2-4e^3x^3\right)}{15d^3e^2(d-ex)^3(d+ex)}$$

```
[In] Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)^3*(d + e*x))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)}{15d^3e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)\sqrt{-e^2x^2+d^2}}{15d^3(-ex+d)^3e^2(ex+d)}$
default	$e^2\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+\frac{d^2}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+2de\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.}{}\right.$

[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e^2*x^2+d^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e^4x^4-2\,de^3x^3+2\,d^3ex-d^4+(4\,e^3x^3-8\,de^2x^2+2\,d^2ex-d^3)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^6x^4-2\,d^4e^5x^3+2\,d^6e^3x-d^7e^2)}$$

[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4 + (4*e^3*x^3 - 8*d*e^2*x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 - 2*d^4*e^5*x^3 + 2*d^6*e^3*x - d^7*e^2)

Sympy [F]

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2\,x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}de} - \frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}d^3e}$$

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(-e^2*x^2 + d^2)^(5/2) + 2/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*
d^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e) - 4/
15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

Giac [F]

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2x}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*x/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(d^3-2\,d^2\,e\,x+8\,d\,e^2\,x^2-4\,e^3\,x^3)}{15\,d^3\,e^2\,(d+e\,x)\,(d-e\,x)^3}$$

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 - 4*e^3*x^3 + 8*d*e^2*x^2 - 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)*(d - e*x)^3)
```

3.49 $\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}+\frac{x}{5d^2\,(d^2-e^2x^2)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*(e*x+d)/e/(-e^2*x^2+d^2)^(5/2)+1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^4
/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {667, 198, 197}

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{x}{5d^2\,(d^2-e^2x^2)^{3/2}}+\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{3}{5}\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int\frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(2d^3+d^2ex-4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)^3(d+ex)}$$

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e*
(d - e*x)^3*(d + e*x))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)}{5d^4e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d^4(-ex+d)^3e(ex+d)}$
default	$d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^2\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\cdots}{15d^2(\cdots}\right.}{\cdots}\right.$

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x^
2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3)\sqrt{-e^2x^2+d^2}}{5\,(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e)}$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^2
 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 + 2*
d^7*e^2*x - d^8*e)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{2\,d}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e}+\frac{x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}+\frac{2\,x}{5\,\sqrt{-e^2x^2+d^2}d^4}$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((-
e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3)}{5\,d^4\,e\,(d+e\,x)\,(d-e\,x)^3}$$

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)*(d - e*x)^3)
```

3.50 $\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 2/5*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3/2
)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(16*e*x+15*d)/d^5/(-e^2*x^2+d^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1819, 837, 12, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2-8dex}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{-15d^4e^2-16d^3e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^4}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$
$$+ \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^4e^2}$$

$$= \frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(26d^3-22d^2ex-17de^2x^2+16e^3x^3)}{(d-ex)^3(d+ex)} + 30\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

```
[In] Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 - 22*d^2*e*x - 17*d*e^2*x^2 + 16*e^3*x^3))/((
d - e*x)^3*(d + e*x)) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(15*d^5)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

method	result
default	$\frac{1}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+d^2\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)+2de\left(\frac{}{5d^2(-}\right.$

[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/5/(-e^2*x^2+d^2)^(5/2)+d^2*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+2*d*e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{26\,e^4x^4-52\,de^3x^3+52\,d^3ex-26\,d^4+15\left(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{}}{}\right.}{15\left(d^5e^4x^4-2\,d^6e^3x^3+2\,d^8e\right.}$$

[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(26*e^4*x^4 - 52*d*e^3*x^3 + 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^3*x^3 - 17*d*e^2*x^2 - 22*d^2*e*x + 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 - 2*d^6*e^3*x^3 + 2*d^8*e*x - d^9)

Sympy [F]

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(d+ex\right)^2}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2\,ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{8\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} + \frac{16\,ex}{15\sqrt{-e^2x^2+d^2}d^5} - \frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^5} + \frac{1}{\sqrt{-e^2x^2+d^2}d^4}$$

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 2/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d) + 2/5/(-e^2*x^2 + d^2)^(5/2) + 8/15*e*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^2) + 16/15*e*x
/(sqrt(-e^2*x^2 + d^2)*d^5) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x))/d^5 + 1/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Giac [F]

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(ex+d)^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x}\,dx$$

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d+e\,x)^2}{x\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.51 $\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 2/5*e*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e*(13*e*x+10*d)/d^4/(-e^2*x^2+d
^2)^(3/2)-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e*(41*e*x+30*d)/d^6/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1819, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\ &= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}\end{aligned}$$

$$= \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(2e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d^5}$$

$$= \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^5e}$$

$$= \frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(15d^4-76d^3ex+32d^2e^2x^2+82de^3x^3-56e^4x^4)}{x(-d+ex)^3(d+ex)} - 30\sqrt{d^2}e\log(x) + 30\sqrt{d^2}e\log\left(\sqrt{d^2}-\right.}{15d^7}$$

```
[In] Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 - 76*d^3*e*x + 32*d^2*e^2*x^2 + 82*d*e^3*x^
3 - 56*e^4*x^4))/(x*(-d + e*x)^3*(d + e*x)) - 30*Sqrt[d^2]*e*Log[x] + 30*Sq
rt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^6x}-\frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}+\frac{29\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{60d^5e\left(x-\frac{d}{e}\right)^2}-\frac{313\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{120d^6\left(x-\frac{d}{e}\right)}-\frac{\sqrt{-}}{}$
default	$e^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^2\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{}{15}\right.}{}\right.$

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^6/x-2/d^5*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)+29/60/d^5/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^
(1/2)-313/120/d^6/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/8/d^6/(x+d
/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/10/d^4/e^2/(x-d/e)^3*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{46\,e^5x^5-92\,de^4x^4+92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5-2\,de^4x^4+2\,d^3e^2x^2-d^4ex\right)}{15\left(d^6e^4x^5-2\,d^7\right.}$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(46*e^5*x^5 - 92*d*e^4*x^4 + 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^5
 - 2*d*e^4*x^4 + 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) - (56*e^4*x^4 - 82*d*e^3*x^3 - 32*d^2*e^2*x^2 + 76*d^3*e*x - 15*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^6*e^4*x^5 - 2*d^7*e^3*x^4 + 2*d^9*e*x^2 - d^10*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x^2 (-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{7\,e^2x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} + \frac{2\,e}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d} \\ &+ \frac{28\,e^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{2\,e}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} - \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}x} \\ &+ \frac{56\,e^2x}{15\,\sqrt{-e^2x^2+d^2}d^6} - \frac{2\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^6} + \frac{2\,e}{\sqrt{-e^2x^2+d^2}d^5}\end{aligned}$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 7/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 2/5*e/((-e^2*x^2 + d^2)^(5/2)*d) +
 28/15*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 2/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^3) - 1/((-e^2*x^2 + d^2)^(5/2)*x) + 56/15*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^6)
 - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^6 + 2*e/(sqrt(
-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}\,dx$$

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^2}{x^2\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.52 $\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

```
[Out] 2/5*e^2*(e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(6*e*x+5*d)/d^5/(-e^2*x^2+
d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(11*e*x+10*d
)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2-2*e*(-e^2*x^2+d
^2)^(1/2)/d^7/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{9e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
```

```
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{8e^3x^3}{d}}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{36e^3x^3}{d}}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{60d^3e+135d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} + \frac{\left(9e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} + \frac{\left(9e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(5d^5+10d^4ex-94d^3e^2x^2+58d^2e^3x^3+83de^4x^4-64e^5x^5)}{x^2(-d+ex)^3(d+ex)}+90e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{10d^7}$$

```
[In] Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 + 10*d^4*e*x - 94*d^3*e^2*x^2 + 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 - 64*e^5*x^5))/(x^2*(-d + e*x)^3*(d + e*x)) + 90*e^2*ArcTan
h[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4ex+d)}{2d^7x^2}-\frac{9e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^6\sqrt{d^2}}+\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8d^7\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{10d^5e\left(x-\frac{d}{e}\right)^3}+$
default	$e^2\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)+d^2\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.$

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(4*e*x+d)/d^7/x^2-9/2/d^6*e^2/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/8/d^7*e/(x+d/e)*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2)-1/10/d^5/e/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(
1/2)+13/20/d^6/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-181/40/d^7*e/
(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{54\,e^6x^6-108\,de^5x^5+108\,d^3e^3x^3-54\,d^4e^2x^2+45\,(e^6x^6-2\,de^5x^5+2\,d^3e^3x^3-d^4e}{10\,(d^7e^4x^6}$$

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 - 108*d*e^5*x^5 + 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 - 2*d*e^5*x^5 + 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) - (64*e^5*x^5 - 83*d*e^4*x^4 - 58*d^2*e^3*x^3 + 94*d^3*e^2*x^2
- 10*d^4*e*x - 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 - 2*d^8*e^3*x^5 +
2*d^10*e*x^3 - d^11*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{x^3\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{12\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} + \frac{9\,e^2}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} \\ &+ \frac{16\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} + \frac{3\,e^2}{2\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} - \frac{2\,e}{(-e^2x^2+d^2)^{\frac{5}{2}}dx} + \frac{32\,e^3x}{5\,\sqrt{-e^2x^2+d^2}d^7} \\ &- \frac{9\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^7} + \frac{9\,e^2}{2\,\sqrt{-e^2x^2+d^2}d^6} - \frac{1}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}x^2}\end{aligned}$$

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 12/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 9/10*e^2/((-e^2*x^2 + d^2)^(5/2)*
d^2) + 16/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 3/2*e^2/((-e^2*x^2 + d^2)^
(3/2)*d^4) - 2*e/((-e^2*x^2 + d^2)^(5/2)*d*x) + 32/5*e^3*x/(sqrt(-e^2*x^2 +
 d^2)*d^7) - 9/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^
7 + 9/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^6) - 1/2/((-e^2*x^2 + d^2)^(5/2)*x^2)
```

Giac [F]

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}x^3}\,dx$$

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^2}{x^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.53 $\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

```
[Out] 2/5*e^3*(e*x+d)/d^4/(-e^2*x^2+d^2)^(5/2)+1/15*e^3*(23*e*x+20*d)/d^6/(-e^2*x
^2+d^2)^(3/2)-7*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+2/15*e^3*(53*e*x+45
*d)/d^8/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^6/x^3-e*(-e^2*x^2+d
^2)^(1/2)/d^7/x^2-14/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
```

```
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{10e^3x^3}{d}-\frac{8e^4x^4}{d^2}}{x^4(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{60e^3x^3}{d}+\frac{46e^4x^4}{d^2}}{x^4(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2-\frac{90e^3x^3}{d}}{x^4\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{\int \frac{90d^3e+210d^2e^2x+270de^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{45d^8}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{\int \frac{-420d^4e^2-630d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{90d^{10}}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} + \frac{(7e^3)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^7}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} + \frac{(7e^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^7}$$

$$= \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} - \frac{(7e)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^7}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} - \frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^6+5d^5ex+40d^4e^2x^2-246d^3e^3x^3+122d^2e^4x^4+247de^5x^5-176e^6x^6\right)}{x^3(-d+ex)^3(d+ex)} - 105\sqrt{d^2}e^3\log(x) +}{15d^9}$$

```
[In] Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(5*d^6 + 5*d^5*e*x + 40*d^4*e^2*x^2 - 246*d^3*e^3*x
^3 + 122*d^2*e^4*x^4 + 247*d*e^5*x^5 - 176*e^6*x^6))/(x^3*(-d + e*x)^3*(d +
 e*x)) - 105*Sqrt[d^2]*e^3*Log[x] + 105*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[
d^2 - e^2*x^2]])/(15*d^9)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\left(14e^2x^2+3dex+d^2\right)}{3d^8x^3} - \frac{7e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^7\sqrt{d^2}} - \frac{e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8d^8\left(x+\frac{d}{e}\right)} - \frac{833e^2\sqrt{-\left(x-\frac{d}{e}\right)^2 e}}{120d^8\left(x-\right.}$
default	$e^2\left(-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{d^2}\right) + d^2\left(-\frac{1}{3d^2x^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right.$

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(1/2)*(14*e^2*x^2+3*d*e*x+d^2)/d^8/x^3-7*e^3/d^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/8*e^2/d^8/(x+d/e)*
```

```
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-833/120*e^2/d^8/(x-d/e)*(-(x-d/e)^2*e^
2-2*d*e*(x-d/e))^(1/2)+49/60/d^7*e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))
^(1/2)-1/10/d^6/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}} \, dx = \frac{116 e^7x^7 - 232 de^6x^6 + 232 d^3e^4x^4 - 116 d^4e^3x^3 + 105 (e^7x^7 - 2 de^6x^6 + 2 d^3e^4x^4 -}{1}$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(116*e^7*x^7 - 232*d*e^6*x^6 + 232*d^3*e^4*x^4 - 116*d^4*e^3*x^3 + 105
*(e^7*x^7 - 2*d*e^6*x^6 + 2*d^3*e^4*x^4 - d^4*e^3*x^3)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (176*e^6*x^6 - 247*d*e^5*x^5 - 122*d^2*e^4*x^4 + 246*d^3*e
^3*x^3 - 40*d^4*e^2*x^2 - 5*d^5*e*x - 5*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^4
*x^7 - 2*d^9*e^3*x^6 + 2*d^11*e*x^4 - d^12*x^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}} \, dx = \int \frac{(d+ex)^2}{x^4 \left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}} \, dx$$

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.16

$$\begin{aligned} \int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}} \, dx = {} & \frac{22 e^4 x}{5 (-e^2x^2+d^2)^{\frac{5}{2}} d^4} + \frac{7 e^3}{5 (-e^2x^2+d^2)^{\frac{5}{2}} d^3} \\ & + \frac{88 e^4 x}{15 (-e^2x^2+d^2)^{\frac{3}{2}} d^6} + \frac{7 e^3}{3 (-e^2x^2+d^2)^{\frac{3}{2}} d^5} - \frac{11 e^2}{3 (-e^2x^2+d^2)^{\frac{5}{2}} d^2 x} \\ & + \frac{176 e^4 x}{15 \sqrt{-e^2x^2+d^2} d^8} - \frac{7 e^3 \log\left(\frac{2 d^2}{|x|} + \frac{2 \sqrt{-e^2x^2+d^2} d}{|x|}\right)}{d^8} \\ & + \frac{7 e^3}{\sqrt{-e^2x^2+d^2} d^7} - \frac{e}{(-e^2x^2+d^2)^{\frac{5}{2}} d x^2} - \frac{1}{3 (-e^2x^2+d^2)^{\frac{5}{2}} x^3} \end{aligned}$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 22/5*e^4*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/5*e^3/((-e^2*x^2 + d^2)^(5/2)*d
^3) + 88/15*e^4*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/3*e^3/((-e^2*x^2 + d^2)^
(3/2)*d^5) - 11/3*e^2/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 176/15*e^4*x/(sqrt(-
e^2*x^2 + d^2)*d^8) - 7*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs
(x))/d^8 + 7*e^3/(sqrt(-e^2*x^2 + d^2)*d^7) - e/((-e^2*x^2 + d^2)^(5/2)*d*x
^2) - 1/3/((-e^2*x^2 + d^2)^(5/2)*x^3)
```

Giac [F]

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{\left(ex+d\right)^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x^4}\,dx$$

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{\left(d+e\,x\right)^2}{x^4\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)), x)
```

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2} -\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3\arcsin(x)}{4}$$

```
[Out] 3/4*arcsin(x)-3/5*x^2*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)-1/5*x^4*(-x^2+1
)^(1/2)-3/20*(8+5*x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1823, 847, 794, 222}

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\arcsin(x)}{4}-\frac{3}{5}\sqrt{1-x^2}x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2} -\frac{1}{5}\sqrt{1-x^2}x^4-\frac{1}{2}\sqrt{1-x^2}x^3$$

```
[In] Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{5}\int\frac{(-9-10x)x^3}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}+\frac{1}{20}\int\frac{x^2(30+36x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{60}\int\frac{(-72-90x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{20}\sqrt{1-x^2}(-24-15x-12x^2-10x^3-4x^4)+\frac{3}{2}\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-24 - 15*x - 12*x^2 - 10*x^3 - 4*x^4))/20 + (3*ArcTan[x/(-1
 + Sqrt[1 - x^2])])/2
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{(4x^4+10x^3+12x^2+15x+24)(x^2-1)}{20\sqrt{-x^2+1}}+\frac{3\arcsin(x)}{4}$	42
trager	$\left(-\frac{1}{5}x^4-\frac{1}{2}x^3-\frac{3}{5}x^2-\frac{3}{4}x-\frac{6}{5}\right)\sqrt{-x^2+1}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{4}$	59
default	$-\frac{x^4\sqrt{-x^2+1}}{5}-\frac{3x^2\sqrt{-x^2+1}}{5}-\frac{6\sqrt{-x^2+1}}{5}-\frac{x^3\sqrt{-x^2+1}}{2}-\frac{3x\sqrt{-x^2+1}}{4}+\frac{3\arcsin(x)}{4}$	71
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^2+8\right)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x\left(10x^2+15\right)\sqrt{-x^2+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{4}\right)}{\sqrt{\pi}}-\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6x^4+8x^2+16\right)\sqrt{-x^2+1}}{15}}{2\sqrt{\pi}}$	109

```
[In] int(x^3*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*(4*x^4+10*x^3+12*x^2+15*x+24)*(x^2-1)/(-x^2+1)^(1/2)+3/4*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{20}\left(4\,x^4+10\,x^3+12\,x^2+15\,x+24\right)\sqrt{-x^2+1} - \frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^4\sqrt{1-x^2}}{5} - \frac{x^3\sqrt{1-x^2}}{2} - \frac{3x^2\sqrt{1-x^2}}{5} - \frac{3x\sqrt{1-x^2}}{4} - \frac{6\sqrt{1-x^2}}{5} + \frac{3\operatorname{asin}(x)}{4}$$

```
[In] integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{5}\sqrt{-x^2+1}x^4 - \frac{1}{2}\sqrt{-x^2+1}x^3 - \frac{3}{5}\sqrt{-x^2+1}x^2 - \frac{3}{4}\sqrt{-x^2+1}x - \frac{6}{5}\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2 -
 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{20}\left((2\left((2x+5)x+6\right)x+15)x+24\right)\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.44

$$\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\,\mathrm{asin}(x)}{4} - \sqrt{1-x^2}\left(\frac{x^4}{5} + \frac{x^3}{2} + \frac{3\,x^2}{5} + \frac{3\,x}{4} + \frac{6}{5}\right)$$

```
[In] int((x^3*(x + 1)^2)/(1 - x^2)^(1/2),x)
[Out] (3*asin(x))/4 - (1 - x^2)^(1/2)*((3*x)/4 + (3*x^2)/5 + x^3/2 + x^4/5 + 6/5)
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7\arcsin(x)}{8}$$

```
[Out] 7/8*arcsin(x)-2/3*x^2*(-x^2+1)^(1/2)-1/4*x^3*(-x^2+1)^(1/2)-1/24*(32+21*x)*
(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1823, 847, 794, 222}

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{7\arcsin(x)}{8}-\frac{2}{3}\sqrt{1-x^2}x^2-\frac{1}{24}(21x+32)\sqrt{1-x^2}-\frac{1}{4}\sqrt{1-x^2}x^3$$

```
[In] Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{4}\int\frac{(-7-8x)x^2}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}+\frac{1}{12}\int\frac{x(16+21x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\int\frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{24}\sqrt{1-x^2}\left(-32-21x-16x^2-6x^3\right)+\frac{7}{4}\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-32 - 21*x - 16*x^2 - 6*x^3))/24 + (7*ArcTan[x/(-1 + Sqrt[1
 - x^2])])/4
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(6x^3+16x^2+21x+32)(x^2-1)}{24\sqrt{-x^2+1}}+\frac{7\arcsin(x)}{8}$	37
trager	$\left(-\frac{1}{4}x^3-\frac{2}{3}x^2-\frac{7}{8}x-\frac{4}{3}\right)\sqrt{-x^2+1}+\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{8}$	54
default	$-\frac{x^3\sqrt{-x^2+1}}{4}-\frac{7x\sqrt{-x^2+1}}{8}+\frac{7\arcsin(x)}{8}-\frac{2x^2\sqrt{-x^2+1}}{3}-\frac{4\sqrt{-x^2+1}}{3}$	57
meijerg	$\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x\,(10x^2+15)\sqrt{-x^2+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{4}\right)}{2\sqrt{\pi}}$	102

```
[In] int(x^2*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(6*x^3+16*x^2+21*x+32)*(x^2-1)/(-x^2+1)^(1/2)+7/8*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{24}\left(6\,x^3+16\,x^2+21\,x+32\right)\sqrt{-x^2+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2 +
 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^3\sqrt{1-x^2}}{4} - \frac{2x^2\sqrt{1-x^2}}{3} - \frac{7x\sqrt{1-x^2}}{8} - \frac{4\sqrt{1-x^2}}{3} + \frac{7\operatorname{asin}(x)}{8}$$

```
[In] integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 - 4
*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{4}\sqrt{-x^2+1}x^3 - \frac{2}{3}\sqrt{-x^2+1}x^2 - \frac{7}{8}\sqrt{-x^2+1}x - \frac{4}{3}\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x - 4
/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.48

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{24}\left((2(3x+8)x+21)x+32\right)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.49

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{7\,\mathrm{asin}(x)}{8} - \sqrt{1-x^2}\left(\frac{x^3}{4}+\frac{2\,x^2}{3}+\frac{7\,x}{8}+\frac{4}{3}\right)$$

```
[In] int((x^2*(x + 1)^2)/(1 - x^2)^(1/2),x)
[Out] (7*asin(x))/8 - (1 - x^2)^(1/2)*((7*x)/8 + (2*x^2)/3 + x^3/4 + 4/3)
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\arcsin(x)$$

```
[Out] arcsin(x)-1/3*x^2*(-x^2+1)^(1/2)-1/3*(5+3*x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1823, 794, 222}

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = \arcsin(x)-\frac{1}{3}\sqrt{1-x^2}x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}$$

```
[In] Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/3*(x^2*Sqrt[1 - x^2]) - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}\int \frac{(-5-6x)x}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}(5+3x)\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}(5+3x)\sqrt{1-x^2} + \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$
\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{3}\sqrt{1-x^2}\left(-5-3x-x^2\right) + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)
$$

```
[In] Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-5 - 3*x - x^2))/3 + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{(x^2+3x+5)(x^2-1)}{3\sqrt{-x^2+1}}+\arcsin(x)$	28
default	$-\frac{5\sqrt{-x^2+1}}{3}-\frac{x^2\sqrt{-x^2+1}}{3}-x\sqrt{-x^2+1}+\arcsin(x)$	41
trager	$\left(-\frac{1}{3}x^2-x-\frac{5}{3}\right)\sqrt{-x^2+1}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	48
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}+\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}$	90

```
[In] int(x*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+3*x+5)*(x^2-1)/(-x^2+1)^(1/2)+arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(x^2+3\,x+5\right)\sqrt{-x^2+1}-2\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x^2\sqrt{1-x^2}}{3}-x\sqrt{1-x^2}-\frac{5\sqrt{1-x^2}}{3}+\operatorname{asin}(x)$$

```
[In] integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{-x^2+1}x^2 - \sqrt{-x^2+1}x - \frac{5}{3}\sqrt{-x^2+1} + \arcsin(x)$$

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^2+1} + \arcsin(x)$$

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = \mathrm{asin}(x) - \sqrt{1-x^2}\left(\frac{x^2}{3} + x + \frac{5}{3}\right)$$

```
[In] int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
[Out] asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3\arcsin(x)}{2}$$

```
[Out] 3/2*arcsin(x)-3/2*(-x^2+1)^(1/2)-1/2*(1+x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {685, 655, 222}

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\arcsin(x)}{2}-\frac{1}{2}\sqrt{1-x^2}(x+1)-\frac{3\sqrt{1-x^2}}{2}$$

```
[In] Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int\frac{1+x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int\frac{(1+x)^2}{\sqrt{1-x^2}}\,dx=\frac{1}{2}(-4-x)\sqrt{1-x^2}-3\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
[Out] ((-4 - x)*Sqrt[1 - x^2])/2 - 3*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{(4+x)(x^2-1)}{2\sqrt{-x^2+1}}+\frac{3\arcsin(x)}{2}$	25
default	$\frac{3\arcsin(x)}{2}-\frac{x\sqrt{-x^2+1}}{2}-2\sqrt{-x^2+1}$	29
trager	$\left(-2-\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	44
meijerg	$\arcsin(x)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{\sqrt{\pi}}+\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	60

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4+x)*(x^2-1)/(-x^2+1)^(1/2)+3/2*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}(x+4) - 3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{x\sqrt{1-x^2}}{2} - 2\sqrt{1-x^2} + \frac{3\operatorname{asin}(x)}{2}$$

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}(x+4)+\frac{3}{2}\arcsin{(x)}$$

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = \frac{3\,\mathrm{asin}(x)}{2}-\left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

```
[In] int((x + 1)^2/(1 - x^2)^(1/2),x)
[Out] (3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}+2\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] 2*arcsin(x)-arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1823, 858, 222, 272, 65, 212}

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = 2\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)-\sqrt{1-x^2}$$

```
[In] Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\sqrt{1-x^2} - \int \frac{-1-2x}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + 4\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) - \log(x) + \log\left(-1+\sqrt{1-x^2}\right)$$

```
[In] Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2] + 4*ArcTan[x/(-1 + Sqrt[1 - x^2])] - Log[x] + Log[-1 + Sqrt[
1 - x^2]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$2\arcsin(x) - \sqrt{-x^2+1} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	29
trager	$-\sqrt{-x^2+1} + \ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - 2\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)x + \sqrt{-x^2+1}\right)$	56
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2\sqrt{\pi}} + 2\arcsin(x) - \frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}$	73

```
[In] int((1+x)^2/x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsin(x)-(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} - 4\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} + 2\operatorname{asin}(x)$$

```
[In] integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), T
rue)) + 2*asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 2\arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + 2\arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx = 2\,\mathrm{asin}(x) + \ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \sqrt{1-x^2}$$

```
[In] int((x + 1)^2/(x*(1 - x^2)^(1/2)),x)
[Out] 2*asin(x) + log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}+\arcsin(x)-2\mathrm{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] arcsin(x)-2*arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 858, 222, 272, 65, 212}

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \arcsin(x)-2\mathrm{arctanh}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{x}$$

```
[In] Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-x^2}}{x} - \int \frac{-2-x}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + 2\int \frac{1}{x\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{x}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-2\log(x)+2\log\left(-1+\sqrt{1-x^2}\right)$$

```
[In] Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
[Out] -(Sqrt[1 - x^2]/x) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])] - 2*Log[x] + 2*Log[-1
 + Sqrt[1 - x^2]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\arcsin(x)-\frac{\sqrt{-x^2+1}}{x}-2\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	30
risch	$\frac{x^2-1}{x\sqrt{-x^2+1}}+\arcsin(x)-2\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	34
meijerg	$-\frac{\sqrt{-x^2+1}}{x}+\frac{-2\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{\sqrt{\pi}}+\arcsin(x)$	59
trager	$-\frac{\sqrt{-x^2+1}}{x}+2\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+1}\right)$	61

```
[In] int((1+x)^2/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx=-\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-2\,x\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\sqrt{-x^2+1}}{x}$$

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + sq
rt(-x^2 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}\right) + \operatorname{asin}(x)$$

```
[In] integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
+ 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin(x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}}{x} + \arcsin(x) - 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2x} + \arcsin(x) + 2\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*log
(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) + 2\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right) - \frac{\sqrt{1-x^2}}{x}$$

```
[In] int((x + 1)^2/(x^2*(1 - x^2)^(1/2)),x)
[Out] asin(x) + 2*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)/x
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\text{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -3/2*arctanh((-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2-2*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1821, 821, 272, 65, 212}

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = -\frac{3}{2}\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}$$

```
[In] Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -1/2*Sqrt[1 - x^2]/x^2 - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-x^2}}{2x^2} - \frac{1}{2}\int \frac{-4-3x}{x^2\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{2x^2} - \frac{2\sqrt{1-x^2}}{x} + \frac{3}{2}\int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{2x^2} - \frac{2\sqrt{1-x^2}}{x} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{2x^2} - \frac{2\sqrt{1-x^2}}{x} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{2x^2} - \frac{2\sqrt{1-x^2}}{x} - \frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{(-1-4x)\sqrt{1-x^2}}{2x^2} - \frac{3\log(x)}{2} + \frac{3}{2}\log\left(-1+\sqrt{1-x^2}\right)$$

```
[In] Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
[Out] ((-1 - 4*x)*Sqrt[1 - x^2])/(2*x^2) - (3*Log[x])/2 + (3*Log[-1 + Sqrt[1 - x^
2]])/2
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
trager	$-\frac{(1+4x)\sqrt{-x^2+1}}{2x^2}+\frac{3\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{2}$
risch	$\frac{4x^3+x^2-4x-1}{2x^2\sqrt{-x^2+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}$
default	$-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}-\frac{\sqrt{-x^2+1}}{2x^2}-\frac{2\sqrt{-x^2+1}}{x}$
meijerg	$-\frac{-\frac{\sqrt{\pi}\left(-4x^2+8\right)}{8x^2}+\frac{\sqrt{\pi}\sqrt{-x^2+1}}{x^2}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)-\frac{(1-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}+\frac{\sqrt{\pi}}{x^2}}{2\sqrt{\pi}}-\frac{2\sqrt{-x^2+1}}{x}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+}{2}$

```
[In] int((1+x)^2/x^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(1+4*x)/x^2*(-x^2+1)^(1/2)+3/2*ln(((-x^2+1)^(1/2)-1)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}(4\,x+1)}{2\,x^2}$$

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = 2\left(\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} + \frac{1}{2x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True)
) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**3*sqrt(
-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2)/(2*x)
, True)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}}{x} - \frac{\sqrt{-x^2+1}}{2\,x^2} - \frac{3}{2}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/abs
(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.78

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2} - \frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2} + \frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
+ 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) - 1
)/abs(x))
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx = \frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2} - \frac{2\,\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{2\,x^2}$$

```
[In] int((x + 1)^2/(x^3*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/2 - (2*(1 - x^2)^(1/2))/x - (1 -
 x^2)^(1/2)/(2*x^2)
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arctanh((-x^2+1)^(1/2))-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-5/3*(-x^
2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 849, 821, 272, 65, 212}

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\operatorname{arctanh}\left(\sqrt{1-x^2}\right)-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\frac{\sqrt{1-x^2}}{3x^3}$$

```
[In] Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] -1/3*Sqrt[1 - x^2]/x^3 - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{1}{3}\int \frac{-6-5x}{x^3\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{6}\int \frac{10+6x}{x^2\sqrt{1-x^2}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{1-x^2}}{3x^3} - \frac{\sqrt{1-x^2}}{x^2} - \frac{5\sqrt{1-x^2}}{3x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{3x^3} - \frac{\sqrt{1-x^2}}{x^2} - \frac{5\sqrt{1-x^2}}{3x} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{3x^3} - \frac{\sqrt{1-x^2}}{x^2} - \frac{5\sqrt{1-x^2}}{3x} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{3x^3} - \frac{\sqrt{1-x^2}}{x^2} - \frac{5\sqrt{1-x^2}}{3x} - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$
\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \frac{(-1-3x-5x^2)\sqrt{1-x^2}}{3x^3} - \log(x) + \log\left(-1+\sqrt{1-x^2}\right)
$$

```
[In] Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
[Out] ((-1 - 3*x - 5*x^2)*Sqrt[1 - x^2])/(3*x^3) - Log[x] + Log[-1 + Sqrt[1 - x^2
]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	siz
trager	$-\frac{(5x^2+3x+1)\sqrt{-x^2+1}}{3x^3} - \ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)$	44
risch	$\frac{5x^4+3x^3-4x^2-3x-1}{3x^3\sqrt{-x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	48
default	$-\frac{5\sqrt{-x^2+1}}{3x} - \frac{\sqrt{-x^2+1}}{3x^3} - \frac{\sqrt{-x^2+1}}{x^2} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	56
meijerg	$-\frac{(2x^2+1)\sqrt{-x^2+1}}{3x^3} - \frac{-\frac{\sqrt{\pi}\,(-4x^2+8)}{8x^2} + \frac{\sqrt{\pi}\,\sqrt{-x^2+1}}{x^2} + \sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right) - \frac{(1-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2} + \frac{\sqrt{\pi}}{x^2}}{\sqrt{\pi}} - \frac{\sqrt{-x^2+1}}{x}$	11

```
[In] int((1+x)^2/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(5*x^2+3*x+1)/x^3*(-x^2+1)^(1/2)-ln(((-x^2+1)^(1/2)+1)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \frac{3\,x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (5\,x^2+3\,x+1)\sqrt{-x^2+1}}{3\,x^3}$$

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))/
x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} + \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} + \frac{1}{2x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x,
True)) + 2*Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**
3*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2
)/(2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{5\,\sqrt{-x^2+1}}{3\,x} - \frac{\sqrt{-x^2+1}}{x^2} - \frac{\sqrt{-x^2+1}}{3\,x^3} - \log\left(\frac{2\,\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = -\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3} - \frac{7\left(\sqrt{-x^2+1}-1\right)}{8\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{24\,x^3} + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(s
qrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1
)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/abs(x
))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx = \ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \sqrt{1-x^2}\left(\frac{2}{3\,x} + \frac{1}{3\,x^3}\right) - \frac{\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{x^2}$$

```
[In] int((x + 1)^2/(x^4*(1 - x^2)^(1/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)*(2/(3*x) + 1/(3*x^
3)) - (1 - x^2)^(1/2)/x - (1 - x^2)^(1/2)/x^2
```

3.62 $\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\text{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -7/8*arctanh((-x^2+1)^(1/2))-1/4*(-x^2+1)^(1/2)/x^4-2/3*(-x^2+1)^(1/2)/x^3-
7/8*(-x^2+1)^(1/2)/x^2-4/3*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 849, 821, 272, 65, 212}

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = -\frac{7}{8}\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}$$

```
[In] Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -1/4*Sqrt[1 - x^2]/x^4 - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x
^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{1}{4}\int \frac{-8-7x}{x^4\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}+\frac{1}{12}\int \frac{21+16x}{x^3\sqrt{1-x^2}}\,dx\end{aligned}$$

$$= -\frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{1}{24}\int \frac{-32-21x}{x^2\sqrt{1-x^2}}\,dx$$

$$= -\frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{4\sqrt{1-x^2}}{3x} + \frac{7}{8}\int \frac{1}{x\sqrt{1-x^2}}\,dx$$

$$= -\frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{4\sqrt{1-x^2}}{3x} + \frac{7}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{4\sqrt{1-x^2}}{3x} - \frac{7}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)$$

$$= -\frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{4\sqrt{1-x^2}}{3x} - \frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{\sqrt{1-x^2}(-6-16x-21x^2-32x^3)}{24x^4} - \frac{7\log(x)}{8} + \frac{7}{8}\log\left(-1+\sqrt{1-x^2}\right)$$

```
[In] Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
[Out] (Sqrt[1 - x^2]*(-6 - 16*x - 21*x^2 - 32*x^3))/(24*x^4) - (7*Log[x])/8 + (7*
Log[-1 + Sqrt[1 - x^2]])/8
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

method	result
trager	$-\frac{(32x^3+21x^2+16x+6)\sqrt{-x^2+1}}{24x^4} - \frac{7\ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)}{8}$
risch	$\frac{32x^5+21x^4-16x^3-15x^2-16x-6}{24x^4\sqrt{-x^2+1}} - \frac{7\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}$
default	$-\frac{\sqrt{-x^2+1}}{4x^4} - \frac{7\sqrt{-x^2+1}}{8x^2} - \frac{7\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8} - \frac{2\sqrt{-x^2+1}}{3x^3} - \frac{4\sqrt{-x^2+1}}{3x}$
meijerg	$\frac{\frac{\sqrt{\pi}\left(-7x^4+8x^2+8\right)}{16x^4} - \frac{\sqrt{\pi}\left(12x^2+8\right)\sqrt{-x^2+1}}{16x^4} - \frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)}{4} + \frac{3\left(\frac{7}{6}-2\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{8} - \frac{\sqrt{\pi}}{2x^4} - \frac{\sqrt{\pi}}{2x^2}}{2\sqrt{\pi}} - \frac{2(2x^2+1)\sqrt{-x^2+1}}{3x^3} -$

```
[In] int((1+x)^2/x^5/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/24*(32*x^3+21*x^2+16*x+6)/x^4*(-x^2+1)^(1/2)-7/8*ln(((-x^2+1)^(1/2)+1)/x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{21\,x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (32\,x^3+21\,x^2+16\,x+6)\sqrt{-x^2+1}}{24\,x^4}$$

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sqr
t(-x^2 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.51

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = 2\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} + \frac{1}{2x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise} \end{cases} + \begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
< 1))) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**3*
sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2)/
(2*x), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1
/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x**2) >
 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x
**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = -\frac{4\sqrt{-x^2+1}}{3\,x} - \frac{7\sqrt{-x^2+1}}{8\,x^2} - \frac{2\sqrt{-x^2+1}}{3\,x^3} - \frac{\sqrt{-x^2+1}}{4\,x^4} - \frac{7}{8}\,\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 - 1
/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.83

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - 3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4} - \frac{3\left(\sqrt{-x^2+1}-1\right)}{4\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{12\,x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^4}{64\,x^4} + \frac{7}{8}\,\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 + 1
) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/x^3
 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx = \frac{7\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{8} - \sqrt{1-x^2}\left(\frac{4}{3\,x}+\frac{2}{3\,x^3}\right) - \sqrt{1-x^2}\left(\frac{3}{8\,x^2}+\frac{1}{4\,x^4}\right) - \frac{\sqrt{1-x^2}}{2\,x^2}$$

```
[In] int((x + 1)^2/(x^5*(1 - x^2)^(1/2)),x)
```

```
[Out] (7*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/8 - (1 - x^2)^(1/2)*(4/(3*x) + 2
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(8*x^2) + 1/(4*x^4)) - (1 - x^2)^(1/2)/(2*x^
2)
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\text{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -3/4*arctanh((-x^2+1)^(1/2))-1/5*(-x^2+1)^(1/2)/x^5-1/2*(-x^2+1)^(1/2)/x^4-
3/5*(-x^2+1)^(1/2)/x^3-3/4*(-x^2+1)^(1/2)/x^2-6/5*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 849, 821, 272, 65, 212}

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{3}{4}\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}$$

```
[In] Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -1/5*Sqrt[1 - x^2]/x^5 - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
 x^2]])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{1}{5}\int \frac{-10-9x}{x^5\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}+\frac{1}{20}\int \frac{36+30x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{1}{60}\int \frac{-90-72x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}+\frac{1}{120}\int \frac{144+90x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}\\
&\quad -\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}\\
&\quad -\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx &= \frac{\sqrt{1-x^2}(-4-10x-12x^2-15x^3-24x^4)}{20x^5}\\
&\quad -\frac{3\log(x)}{4}+\frac{3}{4}\log\left(-1+\sqrt{1-x^2}\right)
\end{aligned}
$$

```
[In] Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*(-4 - 10*x - 12*x^2 - 15*x^3 - 24*x^4))/(20*x^5) - (3*Log[x]
)/4 + (3*Log[-1 + Sqrt[1 - x^2]])/4
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

method	result
trager	$-\frac{\left(24x^{4}+15x^{3}+12x^{2}+10x+4\right)\sqrt{-x^{2}+1}}{20x^{5}}+\frac{3\ln\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)}{4}$
risch	$\frac{24x^{6}+15x^{5}-12x^{4}-5x^{3}-8x^{2}-10x-4}{20x^{5}\sqrt{-x^{2}+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)}{4}$
default	$-\frac{\sqrt{-x^{2}+1}}{5x^{5}}-\frac{3\sqrt{-x^{2}+1}}{5x^{3}}-\frac{6\sqrt{-x^{2}+1}}{5x}-\frac{\sqrt{-x^{2}+1}}{2x^{4}}-\frac{3\sqrt{-x^{2}+1}}{4x^{2}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)}{4}$
meijerg	$-\frac{\left(\frac{8}{3}x^{4}+\frac{4}{3}x^{2}+1\right)\sqrt{-x^{2}+1}}{5x^{5}}+\frac{\frac{\sqrt{\pi}\left(-7x^{4}+8x^{2}+8\right)}{16x^{4}}-\frac{\sqrt{\pi}\left(12x^{2}+8\right)\sqrt{-x^{2}+1}}{16x^{4}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^{4}}}{\sqrt{\pi}}$

```
[In] int((1+x)^2/x^6/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(24*x^4+15*x^3+12*x^2+10*x+4)/x^5*(-x^2+1)^(1/2)+3/4*ln(((-x^2+1)^(1/
2)-1)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{(1+x)^{2}}{x^{6}\sqrt{1-x^{2}}}\,dx=\frac{15\,x^{5}\log\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(24\,x^{4}+15\,x^{3}+12\,x^{2}+10\,x+4\right)\sqrt{-x^{2}+1}}{20\,x^{5}}$$

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*x
 + 4)*sqrt(-x^2 + 1))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.88

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} + \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{2(1-x^2)^{\frac{3}{2}}}{3x^3} - \frac{(1-x^2)^{\frac{5}{2}}}{5x^5} & \text{for } x > -1 \wedge x < 1 \end{cases} + 2\left(\begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 - x
**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/8 +
 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/
x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{6\sqrt{-x^2+1}}{5\,x} - \frac{3\sqrt{-x^2+1}}{4\,x^2} - \frac{3\sqrt{-x^2+1}}{5\,x^3} - \frac{\sqrt{-x^2+1}}{2\,x^4} - \frac{\sqrt{-x^2+1}}{5\,x^5} - \frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 - 1
/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$$
$$= -\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - \frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4} - 1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5}$$
$$- \frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2} - \frac{3\left(\sqrt{-x^2+1}-1\right)^3}{32\,x^3}$$
$$+ \frac{\left(\sqrt{-x^2+1}-1\right)^4}{32\,x^4} - \frac{\left(\sqrt{-x^2+1}-1\right)^5}{160\,x^5} + \frac{3}{4}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*(
sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2
+ 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2
 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/16
0*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = \frac{3\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{4} - \sqrt{1-x^2}\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)$$
$$- \sqrt{1-x^2}\left(\frac{3}{4\,x^2}+\frac{1}{2\,x^4}\right) - \sqrt{1-x^2}\left(\frac{8}{15\,x}+\frac{4}{15\,x^3}+\frac{1}{5\,x^5}\right)$$

```
[In] int((x + 1)^2/(x^6*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/4 - (1 - x^2)^(1/2)*(2/(3*x) + 1
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(4*x^2) + 1/(2*x^4)) - (1 - x^2)^(1/2)*(8/(1
5*x) + 4/(15*x^3) + 1/(5*x^5))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{8}e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-e*(-e^2*x^2+d^2)^(3/2)/x^3-e^4*arctan(e*x/(
-e^2*x^2+d^2)^(1/2))+13/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/8*e^2*(8*e*
x+13*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1821, 825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = e^4\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) + \frac{13}{8}e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3}$$

```
[In] Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/8*(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{\int \frac{\sqrt{d^2-e^2x^2}\left(-12d^4e-13d^3e^2x-4d^2e^3x^2\right)}{x^4}\,dx}{4d^2} \\
&= -\frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} + \frac{\int \frac{\left(39d^5e^2+12d^4e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{12d^4} \\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - \frac{\int \frac{78d^7e^4+48d^6e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{48d^6} \\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} \\
&\quad - \frac{1}{8}\left(13de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} \\
&\quad - \frac{1}{16}\left(13de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&\quad - e^5\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} \\
&\quad - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{8}\left(13de^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} \\
&\quad - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$
\begin{aligned}
\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = &-\frac{d\sqrt{d^2-e^2x^2}(2d^2+8dex+11e^2x^2)}{8x^4} \\
&-\frac{13}{4}e^4\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right) \\
&+e\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/8*(d*Sqrt[d^2 - e^2*x^2]*(2*d^2 + 8*d*e*x + 11*e^2*x^2))/x^4 - (13*e^4*A
rcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/4 + e*(-e^2)^(3/2)*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(11e^2x^2+8dex+2d^2)d}{8x^4}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{13e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{4d^2x^4}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}\right)}{4d^2}\right)+e^3\left(-\frac{(-e^2x^2+d^2)}{d^2x}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(1/2)*(11*e^2*x^2+8*d*e*x+2*d^2)*d/x^4-e^5/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+13/8*e^4*d/(d^2)^(1/2)*ln((2*d^2
+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = \frac{16\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(11\,de^2x^2+8\,d^2ex+2\,d^3)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(16*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (11*d*e^2*x^2 + 8*d^2*e*x + 2*d^3)*sqrt(-e
^2*x^2 + d^2))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.06

$$\begin{aligned}\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx &= d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right) \\ &+ 3d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right) \\ &+ 3de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right) \\ &+ e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + 3*d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x)
+ e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e*
*2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2
/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e*
*2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2
*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = -\frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{13}{8}\,e^4\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{13\,\sqrt{-e^2x^2+d^2}e^4}{8\,d} - \frac{\sqrt{-e^2x^2+d^2}e^3}{x} - \frac{13\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{4\,x^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 13/8*e^4*log(2*d^2/abs(x) + 2*
sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 13/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^
2*x^2 + d^2)*e^3/x - 13/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) - (-e^2*x^2 +
d^2)^(3/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(118) = 236.

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.43

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = \frac{\left(e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{64\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} - \frac{e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{13\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} - \frac{\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{64\,e^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(e^5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 24*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*e/x^2 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e*
x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - e^5*arcsin(e
*x/d)*sgn(d)*sgn(e)/abs(e) + 13/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2
+ d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/64*(8*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))*e^5*abs(e)/x + 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^3*abs(e)/x
^2 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 + (d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,{(d+e\,x)}^3}{x^5}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5, x)
```

3.65 $\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 310

$$\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6} + \frac{35d^{14}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}$$

```
[Out] 35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+7/768*d^8*x*(-e^2*x^2+d^2)^(5/2)/e^
5-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-7/48*d^4*x^3*(-e^2*x^2+d^2)^(7/
2)/e^3-31/143*d^3*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-7/24*d^2*x^5*(-e^2*x^2+d^2)^
(7/2)/e-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-1/1
153152*d^6*(63063*e*x+31744*d)*(-e^2*x^2+d^2)^(7/2)/e^6+35/2048*d^14*arctan
(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+35/2048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{1823, 847, 794, 201, 223, 209\}$

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{35d^{14}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}$$
$$-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}+\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}$$
$$+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}$$
$$-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}$$

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^5\left(d^2-e^2x^2\right)^{5/2}\left(-14d^3e^2-49d^2e^3x-42de^4x^2\right)\,dx}{14e^2}\\
&= -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^5\left(434d^3e^4+637d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{182e^4}\\
&= -\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}\\
&\quad -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(-3185d^4e^5-5208d^3e^6x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{2184e^6}\\
&= -\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}\\
&\quad -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^3\left(20832d^5e^6+35035d^4e^7x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{24024e^8}\\
&= -\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}\\
&\quad -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^2\left(-105105d^6e^7-208320d^5e^8x\right)\left(d^2-e^2x^2\right)^{5/2}\,d}{240240e^{10}}
\end{aligned}$$

$$= -\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x(416640d^7e^8+945945d^6e^9x)\left(d^2-e^2x^2\right)^{5/2}\,dx}{2162160e^{12}}$$

$$= -\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6} + \frac{(7d^8)\int (d^2 -}{128}$$

$$= \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6} + \frac{(35d^{10})\int (d^2}{76}$$

$$= \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6} + \frac{(35d^{12})\int \sqrt{d}}{1024}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6} + \frac{(35d^{14})\int \frac{}{\sqrt{d^2}}}{2048e}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6(d^2-e^2x^2)^{7/2} - \frac{1}{14}ex^7(d^2-e^2x^2)^{7/2} - \frac{d^6(31744d+63063ex)(d^2-e^2x^2)^{7/2}}{1153152e^6} + \frac{(35d^{14})\,\mathrm{Sub}}{}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6(d^2-e^2x^2)^{7/2} - \frac{1}{14}ex^7(d^2-e^2x^2)^{7/2} - \frac{d^6(31744d+63063ex)(d^2-e^2x^2)^{7/2}}{1153152e^6} + \frac{35d^{14}\tan^{-1}}{204}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.65

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(507904d^{13}+315315d^{12}ex+253952d^{11}e^2x^2+210210d^{10}e^3x^3+190464d^9e^4x^4+168168d^8e^5x^5}{}$$

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -1/18450432*(Sqrt[d^2 - e^2*x^2]*(507904*d^13 + 315315*d^12*e*x + 253952*d^
11*e^2*x^2 + 210210*d^10*e^3*x^3 + 190464*d^9*e^4*x^4 + 168168*d^8*e^5*x^5
- 2916352*d^7*e^6*x^6 - 7763184*d^6*e^7*x^7 - 2551808*d^5*e^8*x^8 + 9499776
*d^4*e^9*x^9 + 8773632*d^3*e^10*x^10 - 1427712*d^2*e^11*x^11 - 4257792*d*e^
12*x^12 - 1317888*e^13*x^13) + 630630*d^14*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d
^2 - e^2*x^2])])/e^6
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\left(-1317888e^{13}x^{13}-4257792d\,e^{12}x^{12}-1427712d^{2}e^{11}x^{11}+8773632d^{3}e^{10}x^{10}+9499776d^{4}e^{9}x^{9}-2551808d^{5}e^{8}x^{8}-7763184d^{6}e^{7}x^{7}-2\cdots\right.}{18450432e\cdots}$ $\left(d^{2}\left(-\frac{x^{5}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{12e^{2}}+\frac{5d^{2}\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{10e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{\left(d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\cdots\right.}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}\right)}{12e^{2}\cdots}\right.\right.$

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18450432*(-1317888*e^13*x^13-4257792*d*e^12*x^12-1427712*d^2*e^11*x^11+8
773632*d^3*e^10*x^10+9499776*d^4*e^9*x^9-2551808*d^5*e^8*x^8-7763184*d^6*e^
7*x^7-2916352*d^7*e^6*x^6+168168*d^8*e^5*x^5+190464*d^9*e^4*x^4+210210*d^10
*e^3*x^3+253952*d^11*e^2*x^2+315315*d^12*e*x+507904*d^13)/e^6*(-e^2*x^2+d^2
)^(1/2)+35/2048*d^14/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.63

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{630630\,d^{14}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(1317888\,e^{13}x^{13}+4257792\,de^{12}x^{12}+1427712\,d^2e^{11}x^{11}-8773632\,d^3e}{}$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/18450432*(630630*d^14*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (13178
88*e^13*x^13 + 4257792*d*e^12*x^12 + 1427712*d^2*e^11*x^11 - 8773632*d^3*e^
10*x^10 - 9499776*d^4*e^9*x^9 + 2551808*d^5*e^8*x^8 + 7763184*d^6*e^7*x^7 +
 2916352*d^7*e^6*x^6 - 168168*d^8*e^5*x^5 - 190464*d^9*e^4*x^4 - 210210*d^1
0*e^3*x^3 - 253952*d^11*e^2*x^2 - 315315*d^12*e*x - 507904*d^13)*sqrt(-e^2*
x^2 + d^2))/e^6
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{35d^{14}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2048e^5}+\sqrt{d^2-e^2x^2}\left(-\frac{248d^{13}}{9009e^6}-\frac{35d^{12}x}{2048e^5}-\frac{124d^{11}x^2}{9009e^4}-\right. \\ \left(\frac{d^3x^6}{6}+\frac{3d^2ex^7}{7}+\frac{3de^2x^8}{8}+\frac{e^3x^9}{9}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Piecewise((35*d**14*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(20
48*e**5) + sqrt(d**2 - e**2*x**2)*(-248*d**13/(9009*e**6) - 35*d**12*x/(204
8*e**5) - 124*d**11*x**2/(9009*e**4) - 35*d**10*x**3/(3072*e**3) - 31*d**9*
x**4/(3003*e**2) - 7*d**8*x**5/(768*e) + 1424*d**7*x**6/9009 + 377*d**6*e*x
**7/896 + 178*d**5*e**2*x**8/1287 - 173*d**4*e**3*x**9/336 - 68*d**3*e**4*x
**10/143 + 13*d**2*e**5*x**11/168 + 3*d*e**6*x**12/13 + e**7*x**13/14), Ne(
e**2, 0)), ((d**3*x**6/6 + 3*d**2*e*x**7/7 + 3*d*e**2*x**8/8 + e**3*x**9/9)
*(d**2)**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.91

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{14}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^7 - \frac{3}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^6 - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24\,e} + \frac{35\,d^{14}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2048\,\sqrt{e^2}e^5} + \frac{35\,\sqrt{-e^2x^2+d^2}d^{12}x}{2048\,e^5} - \frac{31\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^4}{143\,e^2} + \frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{10}x}{3072\,e^5} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^3}{48\,e^3} + \frac{7\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8x}{768\,e^5} - \frac{124\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x^2}{1287\,e^4} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6x}{128\,e^5} - \frac{248\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7}{9009\,e^6}$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*d*x^6 - 7/
24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcsin(e^2*x/(d*sqrt(e^2
)))/(sqrt(e^2)*e^5) + 35/2048*sqrt(-e^2*x^2 + d^2)*d^12*x/e^5 - 31/143*(-e^
2*x^2 + d^2)^(7/2)*d^3*x^4/e^2 + 35/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/e^5
- 7/48*(-e^2*x^2 + d^2)^(7/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/2)*d^
8*x/e^5 - 124/1287*(-e^2*x^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x^2 + d
^2)^(7/2)*d^6*x/e^5 - 248/9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.61

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{35\,d^{14}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2048\,e^5|e|}$$
$$-\frac{1}{18450432}\left(\frac{507904\,d^{13}}{e^6}+\left(\frac{315315\,d^{12}}{e^5}+2\left(\frac{126976\,d^{11}}{e^4}+\left(\frac{105105\,d^{10}}{e^3}+4\left(\frac{23808\,d^9}{e^2}+\left(\frac{21021\,d^8}{e}-2\,(1\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2048*d^14*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) - 1/18450432*(507904*
d^13/e^6 + (315315*d^12/e^5 + 2*(126976*d^11/e^4 + (105105*d^10/e^3 + 4*(23
808*d^9/e^2 + (21021*d^8/e - 2*(182272*d^7 + (485199*d^6*e + 8*(19936*d^5*e
^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*e^7*x +
 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^5\left(d^2-e^2\,x^2\right)^{5/2}(d+e\,x)^3\,dx$$

```
[In] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.66 $\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 281

$$\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4}+\frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4}-\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3}-\frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2}-\frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}+\frac{27d^{13}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}$$

```
[Out] 9/512*d^9*x*(-e^2*x^2+d^2)^(3/2)/e^4+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4-2
0/143*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^
2-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-1/4*d*x^5*(-e^2*x^2+d^2)^(7/2)-1/13
*e*x^6*(-e^2*x^2+d^2)^(7/2)-1/320320*d^5*(27027*e*x+12800*d)*(-e^2*x^2+d^2)
^(7/2)/e^5+27/1024*d^13*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+27/1024*d^11*x
*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{1823, 847, 794, 201, 223, 209\}$

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{27d^{13}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e} + \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4} + \frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} - \frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3} - \frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}$$

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^4\left(d^2-e^2x^2\right)^{5/2}\left(-13d^3e^2-45d^2e^3x-39de^4x^2\right)\,dx}{13e^2} \\
&= -\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^4\left(351d^3e^4+540d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{156e^4} \\
&= -\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e} - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} \\
&\quad - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^3\left(-2160d^4e^5-3861d^3e^6x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{1716e^6} \\
&= -\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2} - \frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e} - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} \\
&\quad - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^2\left(11583d^5e^6+21600d^4e^7x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{17160e^8} \\
&= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3} - \frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2} - \frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e} \\
&\quad - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x\left(-43200d^6e^7-104247d^5e^8x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{154440e^{10}}
\end{aligned}$$

$$= -\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{(27d^7)\int (d^2 -}{320e}$$

$$= \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{(9d^9)\int (d^2 - e}{128e}$$

$$= \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3}$$
$$- \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{(27d^{11})\int \sqrt{d^2}}{512e}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4}$$
$$- \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{(27d^{13})\int \frac{1}{\sqrt{d^2-}}}{1024e^4}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4}$$
$$- \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{(27d^{13})\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4}$$
$$- \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$
$$-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5} + \frac{27d^{13}\tan^{-1}\left(\right.}{1024e}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{\sqrt{d^2-e^2x^2}(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+\cdots}{}$$

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-204800*d^12 - 135135*d^11*e*x - 102400*d^10*e^2*x^2
- 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d^7*e^5*x^5 + 2498560*d^6*
e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8 - 2690688*d^3*e^9*x^9 +
430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 394240*e^12*x^12) - 270270*d^1
3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(5125120*e^5)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(-394240e^{12}x^{12}-1281280d\,e^{11}x^{11}-430080d^{2}e^{10}x^{10}+2690688d^{3}e^{9}x^{9}+2938880d^{4}e^{8}x^{8}-816816d^{5}e^{7}x^{7}-2498560d^{6}e^{6}x^{6}-952952d^{7}\right.}{5125120e^{5}}$
default	$e^{3}\left(-\frac{x^{6}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{13e^{2}}+\frac{6d^{2}\left(-\frac{x^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{11e^{2}}+\frac{4d^{2}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}\right)}{11e^{2}}\right)}{13e^{2}}\right)+d^{3}\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right.}{10e^{2}}\right.$

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5125120*(-394240*e^12*x^12-1281280*d*e^11*x^11-430080*d^2*e^10*x^10+2690
688*d^3*e^9*x^9+2938880*d^4*e^8*x^8-816816*d^5*e^7*x^7-2498560*d^6*e^6*x^6-
952952*d^7*e^5*x^5+76800*d^8*e^4*x^4+90090*d^9*e^3*x^3+102400*d^10*e^2*x^2+
135135*d^11*e*x+204800*d^12)/e^5*(-e^2*x^2+d^2)^(1/2)+27/1024*d^13/e^4/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.65

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{270270\,d^{13}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(394240\,e^{12}x^{12}+1281280\,de^{11}x^{11}+430080\,d^2e^{10}x^{10}-2690688\,d^3e^9}{}$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5125120*(270270*d^13*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (394240
*e^12*x^12 + 1281280*d*e^11*x^11 + 430080*d^2*e^10*x^10 - 2690688*d^3*e^9*x
^9 - 2938880*d^4*e^8*x^8 + 816816*d^5*e^7*x^7 + 2498560*d^6*e^6*x^6 + 95295
2*d^7*e^5*x^5 - 76800*d^8*e^4*x^4 - 90090*d^9*e^3*x^3 - 102400*d^10*e^2*x^2
 - 135135*d^11*e*x - 204800*d^12)*sqrt(-e^2*x^2 + d^2))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{27d^{13}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{1024e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{40d^{12}}{1001e^5}-\frac{27d^{11}x}{1024e^4}-\frac{20d^{10}x^2}{1001e^3}-\right. \\ \left(\frac{d^3x^5}{5}+\frac{d^2ex^6}{2}+\frac{3de^2x^7}{7}+\frac{e^3x^8}{8}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Piecewise((27*d**13*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(10
24*e**4) + sqrt(d**2 - e**2*x**2)*(-40*d**12/(1001*e**5) - 27*d**11*x/(1024
```

```
*e**4) - 20*d**10*x**2/(1001*e**3) - 9*d**9*x**3/(512*e**2) - 15*d**8*x**4/
(1001*e) + 119*d**7*x**5/640 + 488*d**6*e*x**6/1001 + 51*d**5*e**2*x**7/320
 - 82*d**4*e**3*x**8/143 - 21*d**3*e**4*x**9/40 + 12*d**2*e**5*x**10/143 +
d*e**6*x**11/4 + e**7*x**12/13), Ne(e**2, 0)), ((d**3*x**5/5 + d**2*e*x**6/
2 + 3*d*e**2*x**7/7 + e**3*x**8/8)*(d**2)**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.91

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=-\frac{1}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^6-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^5+\frac{27\,d^{13}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{1024\sqrt{e^2}e^4}+\frac{27\sqrt{-e^2x^2+d^2}d^{11}x}{1024\,e^4}-\frac{45\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^4}{143\,e}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9x}{512\,e^4}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^3}{40\,e^2}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^7x}{640\,e^4}-\frac{20\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^2}{143\,e^3}-\frac{27\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x}{320\,e^4}-\frac{40\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6}{1001\,e^5}$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/13*(-e^2*x^2 + d^2)^(7/2)*e*x^6 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d*x^5 + 27/
1024*d^13*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 27/1024*sqrt(-e^2*x
^2 + d^2)*d^11*x/e^4 - 45/143*(-e^2*x^2 + d^2)^(7/2)*d^2*x^4/e + 9/512*(-e^
2*x^2 + d^2)^(3/2)*d^9*x/e^4 - 9/40*(-e^2*x^2 + d^2)^(7/2)*d^3*x^3/e^2 + 9/
640*(-e^2*x^2 + d^2)^(5/2)*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^(7/2)*d^4*x^
2/e^3 - 27/320*(-e^2*x^2 + d^2)^(7/2)*d^5*x/e^4 - 40/1001*(-e^2*x^2 + d^2)^
(7/2)*d^6/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.63

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{27\,d^{13}\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{1024\,e^4|e|}-\frac{1}{5125120}\left(\frac{204800\,d^{12}}{e^5}+\left(\frac{135135\,d^{11}}{e^4}+2\left(\frac{51200\,d^{10}}{e^3}+\left(\frac{45045\,d^9}{e^2}+4\left(\frac{9600\,d^8}{e}-\left(119119\,d^7+2\,(15616\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 27/1024*d^13*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - 1/5125120*(204800*d
^12/e^5 + (135135*d^11/e^4 + 2*(51200*d^10/e^3 + (45045*d^9/e^2 + 4*(9600*d
^8/e - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^3 +
(3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*e^7*x + 13*d*e^6)*x)*x)*x)*x)*x)*x)*
x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^4\left(d^2-e^2\,x^2\right)^{5/2}(d+e\,x)^3\,dx$$

```
[In] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.67 $\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 252

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3}$$
$$+\frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}-\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}-\frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}+\frac{41d^{12}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}$$

```
[Out] 41/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e
^3-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/
2)/e-3/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-1/2217
60*d^4*(28413*e*x+14720*d)*(-e^2*x^2+d^2)^(7/2)/e^4+41/1024*d^12*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^4+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1823, 847, 794, 201, 223, 209}

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{41d^{12}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}+\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3}+\frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}$$

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^3\left(d^2-e^2x^2\right)^{5/2}\left(-12d^3e^2-41d^2e^3x-36de^4x^2\right)\,dx}{12e^2}\\
&= -\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^3\left(276d^3e^4+451d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{132e^4}\\
&= -\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}\\
&\quad -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^2\left(-1353d^4e^5-2760d^3e^6x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{1320e^6}\\
&= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}\\
&\quad -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x\left(5520d^5e^6+12177d^4e^7x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{11880e^8}\\
&= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}\\
&\quad -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}+\frac{\left(41d^6\right)\int\left(d^2-e^2x^2\right)^{5/2}\,dx}{320e^3}
\end{aligned}
$$

$$= \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4} + \frac{(41d^8)\int (d^2}{3}$$

$$= \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4} + \frac{(41d^{10})\int \sqrt{}}{51}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}$$
$$- \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4} + \frac{(41d^{12})\int \frac{}{\sqrt{d}}}{1024}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}$$
$$- \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4} + \frac{(41d^{12})\,\mathrm{Sub}}{}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}$$
$$- \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e}$$
$$-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4} + \frac{41d^{12}\tan^{-1}}{102}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.71

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+}{}$$

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-235520*d^11 - 142065*d^10*e*x - 117760*d^9*e^2*x^2 -
 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*d^6*e^5*x^5 + 665600*d^5*
```

```
e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^8 + 325248*d^2*e^9*x^9 +
967680*d*e^10*x^10 + 295680*e^11*x^11) - 284130*d^12*ArcTan[(e*x)/(Sqrt[d^2
] - Sqrt[d^2 - e^2*x^2])])/(3548160*e^4)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(-295680e^{11}x^{11}-967680d\,e^{10}x^{10}-325248d^{2}e^{9}x^{9}+2078720d^{3}e^{8}x^{8}+2295216d^{4}e^{7}x^{7}-665600d^{5}e^{6}x^{6}-2053128d^{6}e^{5}x^{5}-798720d^{7}e \cdots\right.}{3548160e^{4}}$
default	$e^{3}\left(-\frac{x^{5}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{12e^{2}}+\frac{5d^{2}\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{10e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x \cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{8e^{2}}\right)}{10e^{2}}\right)}{12e^{2}} \cdots\right.$

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3548160*(-295680*e^11*x^11-967680*d*e^10*x^10-325248*d^2*e^9*x^9+2078720
*d^3*e^8*x^8+2295216*d^4*e^7*x^7-665600*d^5*e^6*x^6-2053128*d^6*e^5*x^5-798
720*d^7*e^4*x^4+94710*d^8*e^3*x^3+117760*d^9*e^2*x^2+142065*d^10*e*x+235520
*d^11)/e^4*(-e^2*x^2+d^2)^(1/2)+41/1024*d^12/e^3/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.68

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{284130\,d^{12}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(295680\,e^{11}x^{11}+967680\,de^{10}x^{10}+325248\,d^2e^9x^9-2078720\,d^3e^8x^8-$$

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3548160*(284130*d^12*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (295680
*e^11*x^11 + 967680*d*e^10*x^10 + 325248*d^2*e^9*x^9 - 2078720*d^3*e^8*x^8
- 2295216*d^4*e^7*x^7 + 665600*d^5*e^6*x^6 + 2053128*d^6*e^5*x^5 + 798720*d
^7*e^4*x^4 - 94710*d^8*e^3*x^3 - 117760*d^9*e^2*x^2 - 142065*d^10*e*x - 235
520*d^11)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{41d^{12}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{1024e^3}+\sqrt{d^2-e^2x^2}\left(-\frac{46d^{11}}{693e^4}-\frac{41d^{10}x}{1024e^3}-\frac{23d^9x^2}{693e^2}-\frac{41d}{15} \right. \\ \left(\frac{d^3x^4}{4}+\frac{3d^2ex^5}{5}+\frac{de^2x^6}{2}+\frac{e^3x^7}{7}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Piecewise((41*d**12*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(10
24*e**3) + sqrt(d**2 - e**2*x**2)*(-46*d**11/(693*e**4) - 41*d**10*x/(1024*
```

```
e**3) - 23*d**9*x**2/(693*e**2) - 41*d**8*x**3/(1536*e) + 52*d**7*x**4/231
+ 1111*d**6*e*x**5/1920 + 130*d**5*e**2*x**6/693 - 207*d**4*e**3*x**7/320 -
 58*d**3*e**4*x**8/99 + 11*d**2*e**5*x**9/120 + 3*d*e**6*x**10/11 + e**7*x*
*11/12), Ne(e**2, 0)), ((d**3*x**4/4 + 3*d**2*e*x**5/5 + d*e**2*x**6/2 + e*
*3*x**7/7)*(d**2)**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{12}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^5 + \frac{41\,d^{12}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{1024\sqrt{e^2}e^3} + \frac{41\sqrt{-e^2x^2+d^2}d^{10}x}{1024\,e^3} - \frac{3}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^4 + \frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536\,e^3} - \frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^3}{120\,e} + \frac{41\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^6x}{1920\,e^3} - \frac{23\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^2}{99\,e^2} - \frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x}{320\,e^3} - \frac{46\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5}{693\,e^4}$$

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-e^2*x^2 + d^2)^(7/2)*e*x^5 + 41/1024*d^12*arcsin(e^2*x/(d*sqrt(e^2)
))/(sqrt(e^2)*e^3) + 41/1024*sqrt(-e^2*x^2 + d^2)*d^10*x/e^3 - 3/11*(-e^2*x
^2 + d^2)^(7/2)*d*x^4 + 41/1536*(-e^2*x^2 + d^2)^(3/2)*d^8*x/e^3 - 41/120*(
-e^2*x^2 + d^2)^(7/2)*d^2*x^3/e + 41/1920*(-e^2*x^2 + d^2)^(5/2)*d^6*x/e^3
- 23/99*(-e^2*x^2 + d^2)^(7/2)*d^3*x^2/e^2 - 41/320*(-e^2*x^2 + d^2)^(7/2)*
d^4*x/e^3 - 46/693*(-e^2*x^2 + d^2)^(7/2)*d^5/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.65

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{41\,d^{12}\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{1024\,e^3|e|} - \frac{1}{3548160}\left(\frac{235520\,d^{11}}{e^4} + \left(\frac{142065\,d^{10}}{e^3} + 2\left(\frac{58880\,d^9}{e^2} + \left(\frac{47355\,d^8}{e} - 4\left(99840\,d^7 + \left(256641\,d^6e + 2\left(4160\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 41/1024*d^12*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/3548160*(235520*d
^11/e^4 + (142065*d^10/e^3 + 2*(58880*d^9/e^2 + (47355*d^8/e - 4*(99840*d^7
```

```
 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4 -
3*(121*d^2*e^5 + 10*(11*e^7*x + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqr
t(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^3\left(d^2-e^2\,x^2\right)^{5/2}(d+e\,x)^3\,dx$$

```
[In] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.68 $\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 223

$$\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{19d^{11}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}$$

```
[Out] 19/384*d^7*x*(-e^2*x^2+d^2)^(3/2)/e^2+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2
-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-1/11*
e*x^4*(-e^2*x^2+d^2)^(7/2)-1/55440*d^3*(13167*e*x+5920*d)*(-e^2*x^2+d^2)^(7
/2)/e^3+19/256*d^11*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+19/256*d^9*x*(-e^2
*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1823, 847, 794, 201, 223, 209}

$$\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{19d^{11}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} + \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{d^3(5920d+13167e}{5544}$$

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = -\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2(d^2-e^2x^2)^{5/2}\left(-11d^3e^2-37d^2e^3x-33de^4x^2\right)\,dx}{11e^2}$$

$$= -\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^2(209d^3e^4+370d^2e^5x)\left(d^2-e^2x^2\right)^{5/2}\,dx}{110e^4}$$

$$= -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x(-740d^4e^5-1881d^3e^6x)\left(d^2-e^2x^2\right)^{5/2}\,dx}{990e^6}$$

$$= -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{(19d^5)\int\left(d^2-e^2x^2\right)^{5/2}\,dx}{80e^2}$$

$$= \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{(19d^7)\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{96e^2}$$

$$= \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{(19d^9)\int\sqrt{d^2}}{128}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{(19d^{11})\int\frac{}{\sqrt{d^2}}}{256e^2}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3} + \frac{(19d^{11})\,\text{Subs}}{}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e}$$
$$- \frac{3}{10}dx^3(d^2-e^2x^2)^{7/2} - \frac{1}{11}ex^4(d^2-e^2x^2)^{7/2} - \frac{d^3(5920d+13167ex)(d^2-e^2x^2)^{7/2}}{55440e^3} + \frac{19d^{11}\tan^{-1}\left(\right.}{256e^3}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.76

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+2014}{}$$

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-94720*d^10 - 65835*d^9*e*x - 47360*d^8*e^2*x^2 + 251
790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*e^5*x^5 - 657920*d^4*e^6*
x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266112*d*e^9*x^9 + 80640*e^1
0*x^10) - 131670*d^11*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(887
040*e^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-80640e^{10}x^{10}-266112d\,e^{9}x^{9}-89600d^{2}e^{8}x^{8}+587664d^{3}e^{7}x^{7}+657920d^{4}e^{6}x^{6}-201432d^{5}e^{5}x^{5}-629760d^{6}e^{4}x^{4}-251790d^{7}e^{3}x^{3}+47}{887040e^{3}}$
default	$e^{3}\left(-\frac{x^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{11e^{2}}+\frac{4d^{2}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}\right)}{11e^{2}}\right)+d^{3}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}\right.}{}\right.$

[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/887040*(-80640*e^10*x^10-266112*d*e^9*x^9-89600*d^2*e^8*x^8+587664*d^3*e^7*x^7+657920*d^4*e^6*x^6-201432*d^5*e^5*x^5-629760*d^6*e^4*x^4-251790*d^7*e^3*x^3+47360*d^8*e^2*x^2+65835*d^9*e*x+94720*d^10)/e^3*(-e^2*x^2+d^2)^(1/2

```
)+19/256*d^11/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{131670\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(80640\,e^{10}x^{10}+266112\,de^9x^9+89600\,d^2e^8x^8-587664\,d^3e^7x^7-6579}{}$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/887040*(131670*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (80640*e
^10*x^10 + 266112*d*e^9*x^9 + 89600*d^2*e^8*x^8 - 587664*d^3*e^7*x^7 - 6579
20*d^4*e^6*x^6 + 201432*d^5*e^5*x^5 + 629760*d^6*e^4*x^4 + 251790*d^7*e^3*x
^3 - 47360*d^8*e^2*x^2 - 65835*d^9*e*x - 94720*d^10)*sqrt(-e^2*x^2 + d^2))/
e^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.17

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \begin{cases} \frac{19d^{11}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{256e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{74d^{10}}{693e^3}-\frac{19d^9x}{256e^2}-\frac{37d^8x^2}{693e}+\frac{109}{3}\right. \\ \left(\frac{d^3x^3}{3}+\frac{3d^2ex^4}{4}+\frac{3de^2x^5}{5}+\frac{e^3x^6}{6}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Piecewise((19*d**11*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(25
6*e**2) + sqrt(d**2 - e**2*x**2)*(-74*d**10/(693*e**3) - 19*d**9*x/(256*e**
2) - 37*d**8*x**2/(693*e) + 109*d**7*x**3/384 + 164*d**6*e*x**4/231 + 109*d
**5*e**2*x**5/480 - 514*d**4*e**3*x**6/693 - 53*d**3*e**4*x**7/80 + 10*d**2
*e**5*x**8/99 + 3*d*e**6*x**9/10 + e**7*x**10/11), Ne(e**2, 0)), ((d**3*x**
3/3 + 3*d**2*e*x**4/4 + 3*d*e**2*x**5/5 + e**3*x**6/6)*(d**2)**(5/2), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.93

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{19\,d^{11}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{256\sqrt{e^2}e^2}+\frac{19\sqrt{-e^2x^2+d^2}d^9x}{256\,e^2}-\frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^4+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384\,e^2}-\frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^3+\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x}{480\,e^2}-\frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^2}{99\,e}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x}{80\,e^2}-\frac{74\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4}{693\,e^3}$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 19/256*d^11*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 19/256*sqrt(-e^2*
x^2 + d^2)*d^9*x/e^2 - 1/11*(-e^2*x^2 + d^2)^(7/2)*e*x^4 + 19/384*(-e^2*x^2
 + d^2)^(3/2)*d^7*x/e^2 - 3/10*(-e^2*x^2 + d^2)^(7/2)*d*x^3 + 19/480*(-e^2*
x^2 + d^2)^(5/2)*d^5*x/e^2 - 37/99*(-e^2*x^2 + d^2)^(7/2)*d^2*x^2/e - 19/80
*(-e^2*x^2 + d^2)^(7/2)*d^3*x/e^2 - 74/693*(-e^2*x^2 + d^2)^(7/2)*d^4/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{19\,d^{11}\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{256\,e^2|e|}-\frac{1}{887040}\left(\frac{94720\,d^{10}}{e^3}+\left(\frac{65835\,d^9}{e^2}+2\left(\frac{23680\,d^8}{e}-\left(125895\,d^7+4\left(78720\,d^6e+\left(25179\,d^5e^2-2\left(41120\,d^4\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/887040*(94720*d^10
/e^3 + (65835*d^9/e^2 + 2*(23680*d^8/e - (125895*d^7 + 4*(78720*d^6*e + (25
179*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(10*e
^7*x + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x^2\left(d^2-e^2\,x^2\right)^{5/2}(d+e\,x)^3\,dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.69 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 230

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e}+\frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e}-\frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^{10}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}$$

```
[Out] 11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+11/160*d^4*x*(-e^2*x^2+d^2)^(5/2)/e-33/
560*d^3*(-e^2*x^2+d^2)^(7/2)/e^2-11/240*d^2*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e^
2-1/30*d*(e*x+d)^2*(-e^2*x^2+d^2)^(7/2)/e^2-1/10*(e*x+d)^3*(-e^2*x^2+d^2)^(
7/2)/e^2+33/256*d^10*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+33/256*d^8*x*(-e^
2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {809, 685, 655, 201, 223, 209}

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \frac{33d^{10}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e}+\frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e}-\frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2}$$

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
```

```
+ a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\text{integral} = -\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(3d)\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}\,dx}{10e}$$

$$=-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(11d^2)\int (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}\,dx}{30e}$$

$$=-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33d^3)\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx}{80e}$$

$$=-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33d^4)\int \left(d^2-e^2x^2\right)^{5/2}\,dx}{80e}$$

$$=\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(11d^6)\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{32e}$$

$$=\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33d^8)\int \sqrt{d^2-e^2x^2}\,dx}{128e}$$

$$=\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33d^{10})\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{256e}$$

$$=\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33d^{10})\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{256e}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} - \frac{11d^2(d+ex)\,(d^2-e^2x^2)^{7/2}}{240e^2} - \frac{d(d+ex)^2\,(d^2-e^2x^2)^{7/2}}{30e^2} - \frac{(d+ex)^3\,(d^2-e^2x^2)^{7/2}}{10e^2} + \frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

$$\int x(d+ex)^3\,(d^2-e^2x^2)^{5/2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5x}{2688}$$

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6400*d^9 - 3465*d^8*e*x + 10240*d^7*e^2*x^2 + 24570*
d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5 - 20480*d^3*e^6*x^6 + 30
24*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) - 6930*d^10*ArcTan[(e*x)/(S
qrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(26880*e^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(-2688e^{9}x^{9}-8960d\,e^{8}x^{8}-3024d^{2}e^{7}x^{7}+20480d^{3}e^{6}x^{6}+23352d^{4}e^{5}x^{5}-7680d^{5}e^{4}x^{4}-24570d^{6}e^{3}x^{3}-10240x^{2}d^{7}e^{2}+3465x\,d^{8}e+6400\right.}{26880e^{2}}$
default	$e^{3}\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{10e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{}{\sqrt{}}\right.}{2\sqrt{}}\right.}{4}\right.}{6}\right.}{8e^{2}}\right.}{10e^{2}}\right.$

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/26880*(-2688*e^9*x^9-8960*d*e^8*x^8-3024*d^2*e^7*x^7+20480*d^3*e^6*x^6+2
3352*d^4*e^5*x^5-7680*d^5*e^4*x^4-24570*d^6*e^3*x^3-10240*d^7*e^2*x^2+3465*
d^8*e*x+6400*d^9)/e^2*(-e^2*x^2+d^2)^(1/2)+33/256*d^10/e/(e^2)^(1/2)*arctan
```

((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = -\frac{6930 d^{10} \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - (2688 e^9x^9 + 8960 de^8x^8 + 3024 d^2e^7x^7 - 20480 d^3e^6x^6 - 23352 d^4e^5x^5 +}{26880 e^2}$$

[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] -1/26880*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2688*e^9*x^9 + 8960*d*e^8*x^8 + 3024*d^2*e^7*x^7 - 20480*d^3*e^6*x^6 - 23352*d^4*e^5*x^5 + 7680*d^5*e^4*x^4 + 24570*d^6*e^3*x^3 + 10240*d^7*e^2*x^2 - 3465*d^8*e*x - 6400*d^9)*sqrt(-e^2*x^2 + d^2))/e^2

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = \begin{cases} \frac{33d^{10}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{256e} + \sqrt{d^2-e^2x^2}\left(-\frac{5d^9}{21e^2} - \frac{33d^8x}{256e} + \frac{8d^7x^2}{21} + \frac{117d^6}{128} \right. \\ \left(\frac{d^3x^2}{2} + d^2ex^3 + \frac{3de^2x^4}{4} + \frac{e^3x^5}{5}\right)\left(d^2\right)^{\frac{5}{2}} \end{cases}$$

[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)

[Out] Piecewise((33*d**10*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(256*e) + sqrt(d**2 - e**2*x**2)*(-5*d**9/(21*e**2) - 33*d**8*x/(256*e) + 8*d**7*x**2/21 + 117*d**6*e*x**3/128 + 2*d**5*e**2*x**4/7 - 139*d**4*e**3*x**5/160 - 16*d**3*e**4*x**6/21 + 9*d**2*e**5*x**7/80 + d*e**6*x**8/3 + e**7*x**9/10), Ne(e**2, 0)), ((d**3*x**2/2 + d**2*e*x**3 + 3*d*e**2*x**4/4 + e**3*x**5/5)*(d**2)**(5/2), True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{33\,d^{10}\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{256\,\sqrt{e^2}e}+\frac{33\,\sqrt{-e^2x^2+d^2}d^8x}{256\,e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x}{128\,e}-\frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^3+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x}{160\,e}-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^2-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x}{80\,e}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3}{21\,e^2}$$

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 33/256*d^10*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e) + 33/256*sqrt(-e^2*x^
2 + d^2)*d^8*x/e + 11/128*(-e^2*x^2 + d^2)^(3/2)*d^6*x/e - 1/10*(-e^2*x^2 +
 d^2)^(7/2)*e*x^3 + 11/160*(-e^2*x^2 + d^2)^(5/2)*d^4*x/e - 1/3*(-e^2*x^2 +
 d^2)^(7/2)*d*x^2 - 33/80*(-e^2*x^2 + d^2)^(7/2)*d^2*x/e - 5/21*(-e^2*x^2 +
 d^2)^(7/2)*d^3/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.62

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{33\,d^{10}\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{256\,e|e|}-\frac{1}{26880}\left(\frac{6400\,d^9}{e^2}+\left(\frac{3465\,d^8}{e}-2\left(5120\,d^7+\left(12285\,d^6e+4\left(960\,d^5e^2-\left(2919\,d^4e^3+2\left(1280\,d^3e^4-7\left(27\right.\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 33/256*d^10*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/26880*(6400*d^9/e^2
+ (3465*d^8/e - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e^3
 + 2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*e^7*x + 10*d*e^6)*x)*x)*x)*x)*x)*
x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int x\left(d^2-e^2\,x^2\right)^{5/2}(d+e\,x)^3\,dx$$

```
[In] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \frac{55}{128} d^7 x \sqrt{d^2-e^2x^2} + \frac{55}{192} d^5 x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48} d^3 x \left(d^2-e^2x^2\right)^{5/2} - \frac{11 d^2 \left(d^2-e^2x^2\right)^{7/2}}{56 e} - \frac{11 d (d+ex) \left(d^2-e^2x^2\right)^{7/2}}{72 e} - \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9 e} + \frac{55 d^9 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128 e}$$

```
[Out] 55/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)-11/56*d^
2*(-e^2*x^2+d^2)^(7/2)/e-11/72*d*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e-1/9*(e*x+d)
^2*(-e^2*x^2+d^2)^(7/2)/e+55/128*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+55/
128*d^7*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {685, 655, 201, 223, 209}

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \frac{55 d^9 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128 e} - \frac{11 d^2 \left(d^2-e^2x^2\right)^{7/2}}{56 e} - \frac{11 d (d+ex) \left(d^2-e^2x^2\right)^{7/2}}{72 e} - \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9 e} + \frac{55}{128} d^7 x \sqrt{d^2-e^2x^2} + \frac{55}{192} d^5 x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48} d^3 x \left(d^2-e^2x^2\right)^{5/2}$$

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e} + \frac{1}{9}(11d)\int (d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx$$

$$= -\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e} - \frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e} + \frac{1}{8}\left(11d^2\right)\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$$

$$= -\frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{1}{8}(11d^3)\int (d^2-e^2x^2)^{5/2}\,dx$$

$$= \frac{11}{48}d^3x(d^2-e^2x^2)^{5/2} - \frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{1}{48}(55d^5)\int (d^2-e^2x^2)^{3/2}\,dx$$

$$= \frac{55}{192}d^5x(d^2-e^2x^2)^{3/2} + \frac{11}{48}d^3x(d^2-e^2x^2)^{5/2} - \frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{1}{64}(55d^7)\int \sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x(d^2-e^2x^2)^{3/2} + \frac{11}{48}d^3x(d^2-e^2x^2)^{5/2} - \frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{1}{128}(55d^9)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x(d^2-e^2x^2)^{3/2} + \frac{11}{48}d^3x(d^2-e^2x^2)^{5/2} - \frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{1}{128}(55d^9)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x(d^2-e^2x^2)^{3/2} + \frac{11}{48}d^3x(d^2-e^2x^2)^{5/2} - \frac{11d^2(d^2-e^2x^2)^{7/2}}{56e} - \frac{11d(d+ex)(d^2-e^2x^2)^{7/2}}{72e} - \frac{(d+ex)^2(d^2-e^2x^2)^{7/2}}{9e} + \frac{55d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int (d+ex)^3(d^2-e^2x^2)^{5/2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5}{8064e} - \frac{55d^9\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{128\sqrt{-e^2}}$$

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3712*d^8 + 4599*d^7*e*x + 10240*d^6*e^2*x^2 + 3066*d
^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 + 1024*d^2*e^6*x^6 + 3024*
d*e^7*x^7 + 896*e^8*x^8))/(8064*e) - (55*d^9*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
 - e^2*x^2]])/(128*Sqrt[-e^2])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(-896 e^{8} x^{8}-3024 d\, e^{7} x^{7}-1024 d^{2} e^{6} x^{6}+7224 d^{3} e^{5} x^{5}+8448 d^{4} x^{4} e^{4}-3066 d^{5} e^{3} x^{3}-10240 d^{6} e^{2} x^{2}-4599 d^{7} e x+3712 d^{8}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{8064 e}+$
default	$d^{3}\left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2}\left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2}\left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6}\right)+e^{3}\left(-\frac{x^{2}\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{9 e^{2}}-\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8064*(-896*e^8*x^8-3024*d*e^7*x^7-1024*d^2*e^6*x^6+7224*d^3*e^5*x^5+8448
*d^4*e^4*x^4-3066*d^5*e^3*x^3-10240*d^6*e^2*x^2-4599*d^7*e*x+3712*d^8)/e*(-
e^2*x^2+d^2)^(1/2)+55/128*d^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx =$$
$$-\frac{6930\, d^9 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - (896\, e^8x^8 + 3024\, de^7x^7 + 1024\, d^2e^6x^6 - 7224\, d^3e^5x^5 - 8448\, d^4e^4x^4 + 30}{8064\, e}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(6930*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (896*e^8*x^8
+ 3024*d*e^7*x^7 + 1024*d^2*e^6*x^6 - 7224*d^3*e^5*x^5 - 8448*d^4*e^4*x^4 +
 3066*d^5*e^3*x^3 + 10240*d^6*e^2*x^2 + 4599*d^7*e*x - 3712*d^8)*sqrt(-e^2*
x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

$$\int (d+ex)^3 \left(d^2 -e^2x^2\right)^{5/2} \, dx = \begin{cases} \frac{55d^9 \left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\: d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{128} + \sqrt{d^2-e^2x^2}\left(-\frac{29d^8}{63e} + \frac{73d^7x}{128} + \frac{80d^6ex^2}{63} + \frac{73d}{} \right. \\ \left(d^2\right)^{\frac{5}{2}} \left(\begin{cases} d^3x & \text{for}\: e = 0 \\ \frac{(d+ex)^4}{4e} & \text{otherwise} \end{cases}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Piecewise((55*d**9*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/128
+ sqrt(d**2 - e**2*x**2)*(-29*d**8/(63*e) + 73*d**7*x/128 + 80*d**6*e*x**2/
63 + 73*d**5*e**2*x**3/192 - 22*d**4*e**3*x**4/21 - 43*d**3*e**4*x**5/48 +
8*d**2*e**5*x**6/63 + 3*d*e**6*x**7/8 + e**7*x**8/9), Ne(e**2, 0)), ((d**2)
**(5/2)*Piecewise((d**3*x, Eq(e, 0)), ((d + e*x)**4/(4*e), True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{55\,d^9\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\sqrt{e^2}}+\frac{55}{128}\sqrt{-e^2x^2+d^2}d^7x+\frac{55}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{11}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x-\frac{1}{9}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^2-\frac{3}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63\,e}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 55/128*d^9*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 55/128*sqrt(-e^2*x^2 + d
^2)*d^7*x + 55/192*(-e^2*x^2 + d^2)^(3/2)*d^5*x + 11/48*(-e^2*x^2 + d^2)^(5
/2)*d^3*x - 1/9*(-e^2*x^2 + d^2)^(7/2)*e*x^2 - 3/8*(-e^2*x^2 + d^2)^(7/2)*d
*x - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{55\,d^9\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,|e|}-\frac{1}{8064}\left(\frac{3712\,d^8}{e}-\left(4599\,d^7+2\left(5120\,d^6e+\left(1533\,d^5e^2-4\left(1056\,d^4e^3+\left(903\,d^3e^4-2\left(64\,d^2e^5+7\left(8\,e^7x+\right.\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 55/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/8064*(3712*d^8/e - (4599*
d^7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*(6
4*d^2*e^5 + 7*(8*e^7*x + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2
)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(d^2-e^2\,x^2\right)^{5/2} (d+e\,x)^3 \, dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.71 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 190

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{125}{128}d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/192*d^4*(125*e*x+64*d)*(-e^2*x^2+d^2)^(3/2)+1/240*d^2*(125*e*x+48*d)*(-e^
2*x^2+d^2)^(5/2)-3/7*d*(-e^2*x^2+d^2)^(7/2)-1/8*e*x*(-e^2*x^2+d^2)^(7/2)+12
5/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^8*arctanh((-e^2*x^2+d^2)^(1/2)
/d)+1/128*d^6*(125*e*x+128*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1823, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx=\frac{125}{128}d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = -\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-8d^3e^2-25d^2e^3x-24de^4x^2\right)}{x}\,dx}{8e^2}$$

$$= -\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{\int \frac{\left(56d^3e^4+175d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{56e^4}$$

$$= \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} - \frac{\int \frac{\left(-336d^5e^6-875d^4e^7x\right)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx}{336e^6}$$

$$= \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{\int \frac{\left(1344d^7e^8+2625d^6e^9x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{1344e^8}$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} - \frac{\int \frac{-2688d^9e^{10}-2625d^8e^{11}x}{x\sqrt{d^2-e^2x^2}}\,dx}{2688e^{10}}$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + d^9\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{128}\left(125d^8e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{1}{2}d^9\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + \frac{1}{128}\left(125d^8e\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{125}{128}d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^9\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{125}{128}d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = \frac{\sqrt{d^2-e^2x^2}(14848d^7+27195d^6ex+7424d^5e^2x^2-17710d^4e^3x^3-14592d^3}{13440} - \frac{125}{64}d^8\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - d^7\sqrt{d^2}\log(x) + d^7\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 - (125*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])
/64 - d^7*Sqrt[d^2]*Log[x] + d^7*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x
^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(164) = 328.

Time = 0.38 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.86

method	result
default	$e^{3}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{8e^{2}}\right)+d^{3}$

[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x,method=_RETURNVERBOSE)

[Out] e^3*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2
)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d^3*(1/5*
(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2
)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+3*d^2
e(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*
(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2)))))-3/7*d*(-e^2*x^2+d^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = -\frac{125}{64}\,d^8 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^8 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \frac{1}{13440}\left(1680\,e^7x^7 + 5760\,de^6x^6 + 1960\,d^2e^5x^5 - 14592\,d^3e^4x^4 - 17710\,d^4e^3x^3 + 7424\,d^5e^2x^2 + 27195\,d^6ex\right.$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -125/64*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^8*log(-(d - sqrt(
-e^2*x^2 + d^2))/x) + 1/13440*(1680*e^7*x^7 + 5760*d*e^6*x^6 + 1960*d^2*e^5
*x^5 - 14592*d^3*e^4*x^4 - 17710*d^4*e^3*x^3 + 7424*d^5*e^2*x^2 + 27195*d^6
*e*x + 14848*d^7)*sqrt(-e^2*x^2 + d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.89 (sec) , antiderivative size = 954, normalized size of antiderivative = 5.02

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

```
[Out] d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + 3*d**6*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sq
rt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt
(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(
d**2), True)) + d**5*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2)
+ x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) -
 5*d**4*e**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(
d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), T
rue))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 -
e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) - 5*d**3*e**4*Piecew
ise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*
x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e**2, 0)), (x**4*sqrt(d
```

```
**2)/4, True)) + d**2*e**5*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqr
t(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(
-e**2*x**2), True))/(16*e**4) - d**4*x*sqrt(d**2 - e**2*x**2)/(16*e**4) - d
**2*x**3*sqrt(d**2 - e**2*x**2)/(24*e**2) + x**5*sqrt(d**2 - e**2*x**2)/6,
Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) + 3*d*e**6*Piecewise((-8*d**6*sqrt
(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**
4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**
2)/7, Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) + e**7*Piecewise((5*d**8*Pie
cewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2),
Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*e**6) - 5*d**6*x*sqrt
(d**2 - e**2*x**2)/(128*e**6) - 5*d**4*x**3*sqrt(d**2 - e**2*x**2)/(192*e**
4) - d**2*x**5*sqrt(d**2 - e**2*x**2)/(48*e**2) + x**7*sqrt(d**2 - e**2*x**
2)/8, Ne(e**2, 0)), (x**7*sqrt(d**2)/7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = \frac{125\,d^8 e \arcsin\left(\frac{e^2 x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}} - d^8 \log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{125}{128}\,\sqrt{-e^2x^2+d^2}d^6ex + \sqrt{-e^2x^2+d^2}d^7 + \frac{125}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4ex + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5 + \frac{25}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2ex + \frac{1}{5}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3 - \frac{1}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex - \frac{3}{7}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 125/128*d^8*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^8*log(2*d^2/abs(x)
+ 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*e*x +
 sqrt(-e^2*x^2 + d^2)*d^7 + 125/192*(-e^2*x^2 + d^2)^(3/2)*d^4*e*x + 1/3*(-
e^2*x^2 + d^2)^(3/2)*d^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*e*x + 1/5*(-e^2
*x^2 + d^2)^(5/2)*d^3 - 1/8*(-e^2*x^2 + d^2)^(7/2)*e*x - 3/7*(-e^2*x^2 + d^
2)^(7/2)*d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x}\,dx = \frac{125\,d^8e\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{128\,|e|}$$
$$-\frac{d^8e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|}$$
$$+\frac{1}{13440}\left(14848\,d^7+\left(27195\,d^6e+2\left(3712\,d^5e^2-\left(8855\,d^4e^3+4\left(1824\,d^3e^4-5\left(49\,d^2e^5+6\left(7\,e^7x+24\,de^6\right)\right.\right.\right.\right.\right.\right.$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
[Out] 125/128*d^8*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^8*e*log(1/2*abs(-2*d*e
 - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/13440*(14848*d^7
 + (27195*d^6*e + 2*(3712*d^5*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49
*d^2*e^5 + 6*(7*e^7*x + 24*d*e^6)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x, x)
```

3.72 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx = \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{15}{16}d^7e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 3d^7e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/8*d^3*e*(-5*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)+1/10*d*e*(-5*e*x+6*d)*(-e^2*x^2
+d^2)^(5/2)-1/7*e*(-e^2*x^2+d^2)^(7/2)-d*(-e^2*x^2+d^2)^(7/2)/x-15/16*d^7*e
*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-3*d^7*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3
/16*d^5*e*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1821, 1823, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx = -\frac{15}{16}d^7e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 3d^7e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{d(d^2-e^2x^2)^{7/2}}{x} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} + \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-3d^4e+3d^3e^2x-d^2e^3x^2)}{x}\,dx}{d^2} \\
&= -\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(21d^4e^3-21d^3e^4x)(d^2-e^2x^2)^{5/2}}{x}\,dx}{7d^2e^2} \\
&= \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} \\
&\quad - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{\int \frac{(-126d^6e^5+105d^5e^6x)(d^2-e^2x^2)^{3/2}}{x}\,dx}{42d^2e^4} \\
&= \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} \\
&\quad - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(504d^8e^7-315d^7e^8x)\sqrt{d^2-e^2x^2}}{x}\,dx}{168d^2e^6}
\end{aligned}
$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{d(d^2-e^2x^2)^{7/2}}{x}-\frac{\int \frac{-1008d^{10}e^9+315d^9e^{10}x}{x\sqrt{d^2-e^2x^2}}\,dx}{336d^2e^8}$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{d(d^2-e^2x^2)^{7/2}}{x} + \left(3d^8e\right)\int\frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\frac{1}{16}\left(15d^7e^2\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{d(d^2-e^2x^2)^{7/2}}{x} +\frac{1}{2}\left(3d^8e\right)\text{Subst}\left(\int\frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-\frac{1}{16}\left(15d^7e^2\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{d(d^2-e^2x^2)^{7/2}}{x} -\frac{15}{16}d^7e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{\left(3d^8\right)\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{d(d^2-e^2x^2)^{7/2}}{x} -\frac{15}{16}d^7e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-560d^7+2496d^6ex+525d^5e^2x^2-992d^4e^3x^3-770d^3e^4x^4+96}{560x}$$

$$+\frac{15}{8}d^7e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)-3d^6\sqrt{d^2}e\log(x)+3d^6\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 2496*d^6*e*x + 525*d^5*e^2*x^2 - 992*d^4*e
^3*x^3 - 770*d^3*e^4*x^4 + 96*d^2*e^5*x^5 + 280*d*e^6*x^6 + 80*e^7*x^7))/(5
60*x) + (15*d^7*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/8 - 3*d^
6*Sqrt[d^2]*e*Log[x] + 3*d^6*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2
]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{d^7\sqrt{-e^2x^2+d^2}}{x}+\frac{e^7x^6\sqrt{-e^2x^2+d^2}}{7}+\frac{6e^5d^2x^4\sqrt{-e^2x^2+d^2}}{35}-\frac{62e^3d^4x^2\sqrt{-e^2x^2+d^2}}{35}+\frac{156e\,d^6\sqrt{-e^2x^2+d^2}}{35}-\frac{3e\,d^8\ln\left(\frac{2d^2}{}\right.}{}$
default	$-\frac{e(-e^2x^2+d^2)^{\frac{7}{2}}}{7}+d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}}{\sqrt{-e^2x}}\right)}{2\sqrt{e^2}}\right.}{4}\right.}{6}\right.}{d^2}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^7*(-e^2*x^2+d^2)^(1/2)/x+1/7*e^7*x^6*(-e^2*x^2+d^2)^(1/2)+6/35*e^5*d^2*x
^4*(-e^2*x^2+d^2)^(1/2)-62/35*e^3*d^4*x^2*(-e^2*x^2+d^2)^(1/2)+156/35*e*d^6
*(-e^2*x^2+d^2)^(1/2)-3*e*d^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)+1/2*d*e^6*x^5*(-e^2*x^2+d^2)^(1/2)-11/8*d^3*e^4*x^3*(-e^2*x
^2+d^2)^(1/2)+15/16*d^5*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/16*d^7*e^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^2}\,dx = \frac{1050\,d^7ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1680\,d^7ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2496$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/560*(1050*d^7*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1680*d^7*e*
x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2496*d^7*e*x + (80*e^7*x^7 + 280*d*e
^6*x^6 + 96*d^2*e^5*x^5 - 770*d^3*e^4*x^4 - 992*d^4*e^3*x^3 + 525*d^5*e^2*x
^2 + 2496*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.60

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d**6*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acos
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((d**2*Piecewise((log(-2
*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)),
(x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2,
 0)), (x*sqrt(d**2), True)) - 5*d**4*e**3*Piecewise((-d**2*sqrt(d**2 - e**2
*x**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d
```

```
**2)/2, True)) - 5*d**3*e**4*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*s
qrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqr
t(-e**2*x**2), True))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x
**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + d*
*2*e**5*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e**2, 0))
, (x**4*sqrt(d**2)/4, True)) + 3*d*e**6*Piecewise((d**6*Piecewise((log(-2*e
**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x
*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) - d**4*x*sqrt(d**2 - e**2*x**2)/
(16*e**4) - d**2*x**3*sqrt(d**2 - e**2*x**2)/(24*e**2) + x**5*sqrt(d**2 - e
**2*x**2)/6, Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) + e**7*Piecewise((-8*
d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)
/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 -
 e**2*x**2)/7, Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = -\frac{15\,d^7e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\,\sqrt{e^2}} - 3\,d^7e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{15}{16}\,\sqrt{-e^2x^2+d^2}d^5e^2x + 3\,\sqrt{-e^2x^2+d^2}d^6e - \frac{5}{8}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2x + \left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4e + \frac{1}{2}\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2x + \frac{3}{5}\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e - \frac{1}{7}\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{x}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -15/16*d^7*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3*d^7*e*log(2*d^2/ab
s(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 15/16*sqrt(-e^2*x^2 + d^2)*d^5*e^
2*x + 3*sqrt(-e^2*x^2 + d^2)*d^6*e - 5/8*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2*x +
 (-e^2*x^2 + d^2)^(3/2)*d^4*e + 1/2*(-e^2*x^2 + d^2)^(5/2)*d*e^2*x + 3/5*(-
e^2*x^2 + d^2)^(5/2)*d^2*e - 1/7*(-e^2*x^2 + d^2)^(7/2)*e - (-e^2*x^2 + d^2
)^(5/2)*d^3/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = -\frac{15\,d^7e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,|e|} + \frac{d^7e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} - \frac{3\,d^7e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^7}{2\,x|e|} + \frac{1}{560}\left(2496\,d^6e+\left(525\,d^5e^2-2\left(496\,d^4e^3+\left(385\,d^3e^4-4\left(12\,d^2e^5+5\left(2\,e^7x+7\,de^6\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*d^7*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^7*e^4*x/((d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))*abs(e)) - 3*d^7*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))*d^7/(x*abs(e)) + 1/560*(2496*d^6*e + (525*d^5*e^2 - 2*(496*d^4
*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*e^7*x + 7*d*e^6)*x)*x)*x)*x)*x)*
sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^2}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2, x)
```

3.73 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 207

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^3}\,dx = \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} - \frac{85}{16}d^6e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^6e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/24*d^2*e^2*(-85*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+1/30*e^2*(-85*e*x+3*d)*(-e^
2*x^2+d^2)^(5/2)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2-3*e*(-e^2*x^2+d^2)^(7/2)/x-
85/16*d^6*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^6*e^2*arctanh((-e^2*x^
2+d^2)^(1/2)/d)+1/16*d^4*e^2*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1821, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^3}\,dx = -\frac{85}{16}d^6e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^6e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} + \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-6d^4e-d^3e^2x-2d^2e^3x^2)}{x^2}\,dx}{2d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(d^5e^2-34d^4e^3x)(d^2-e^2x^2)^{5/2}}{x}\,dx}{2d^4} \\
&= \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} \\
&\quad - \frac{3e(d^2-e^2x^2)^{7/2}}{x} - \frac{\int \frac{(-6d^7e^4+170d^6e^5x)(d^2-e^2x^2)^{3/2}}{x}\,dx}{12d^4e^2} \\
&= \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} \\
&\quad - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(24d^9e^6-510d^8e^7x)\sqrt{d^2-e^2x^2}}{x}\,dx}{48d^4e^4} \\
&= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} \\
&\quad - \frac{\int \frac{-48d^{11}e^8+510d^{10}e^9x}{x\sqrt{d^2-e^2x^2}}\,dx}{96d^4e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{d(d^2-e^2x^2)^{7/2}}{2x^2}-\frac{3e(d^2-e^2x^2)^{7/2}}{x} \\
&\quad+\frac{1}{2}\left(d^7e^2\right)\int\frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\frac{1}{16}\left(85d^6e^3\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{d(d^2-e^2x^2)^{7/2}}{2x^2}-\frac{3e(d^2-e^2x^2)^{7/2}}{x} \\
&\quad+\frac{1}{4}\left(d^7e^2\right)\text{Subst}\left(\int\frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-\frac{1}{16}\left(85d^6e^3\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{d(d^2-e^2x^2)^{7/2}}{2x^2}-\frac{3e(d^2-e^2x^2)^{7/2}}{x} \\
&\quad-\frac{85}{16}d^6e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}d^7\text{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right) \\
&= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{d(d^2-e^2x^2)^{7/2}}{2x^2}-\frac{3e(d^2-e^2x^2)^{7/2}}{x} \\
&\quad-\frac{85}{16}d^6e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}d^6e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx=\frac{\sqrt{d^2-e^2x^2}(-120d^7-720d^6ex+544d^5e^2x^2-645d^4e^3x^3-448d^3e^4x^4+50\ldots}{240x^2} \\
&+\frac{85}{8}d^6e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) \\
&-\frac{1}{2}d^5\sqrt{d^2}e^2\log(x)+\frac{1}{2}d^5\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^7 - 720*d^6*e*x + 544*d^5*e^2*x^2 - 645*d^4*e^
3*x^3 - 448*d^3*e^4*x^4 + 50*d^2*e^5*x^5 + 144*d*e^6*x^6 + 40*e^7*x^7))/(24
0*x^2) + (85*d^6*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/8 - (
d^5*Sqrt[d^2]*e^2*Log[x])/2 + (d^5*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-40 e^{7} x^{7}-144 d\, e^{6} x^{6}-50 d^{2} e^{5} x^{5}+448 d^{3} e^{4} x^{4}+645 d^{4} e^{3} x^{3}-544 d^{5} e^{2} x^{2}+720 d^{6} e x +120 d^{7}\right)}{240 x^{2}}-\frac{85 d^{6} e^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 \sqrt{e^{2}}}$
default	$e^{3} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6}\right)+d^{3} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{2 d^{2} x^{2}}-$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-e^2*x^2+d^2)^(1/2)*(-40*e^7*x^7-144*d*e^6*x^6-50*d^2*e^5*x^5+448*d
^3*e^4*x^4+645*d^4*e^3*x^3-544*d^5*e^2*x^2+720*d^6*e*x+120*d^7)/x^2-85/16*d
^6*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^7*e^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^3} \, dx = \frac{2550\, d^6 e^2 x^2 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 120\, d^6 e^2 x^2 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 544$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/240*(2550*d^6*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 120*d^6
*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 544*d^6*e^2*x^2 + (40*e^7*x^7
```

```
+ 144*d*e^6*x^6 + 50*d^2*e^5*x^5 - 448*d^3*e^4*x^4 - 645*d^4*e^3*x^3 + 544
*d^5*e^2*x^2 - 720*d^6*e*x - 120*d^7)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.29

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh
(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2
*x**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2)
 - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*
x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x))
 + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**4*e**3*Piecewise((d**2*
Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2
), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*
x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 5*d**3*e**4*Piecewise((-d**2
*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2,
0)), (x**2*sqrt(d**2)/2, True)) + d**2*e**5*Piecewise((d**4*Piecewise((log(
-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0))
, (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) - d**2*x*sqrt(d**2 - e**2*x**
2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(d**2)
/3, True)) + 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) -
 d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5
, Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) + e**7*Piecewise((d**6*Piecewise
((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**
2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) - d**4*x*sqrt(d**2 - e
**2*x**2)/(16*e**4) - d**2*x**3*sqrt(d**2 - e**2*x**2)/(24*e**2) + x**5*sqr
t(d**2 - e**2*x**2)/6, Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3}\,dx = -\frac{85\,d^6e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}} - \frac{1}{2}\,d^6e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{85}{16}\sqrt{-e^2x^2+d^2}d^4e^3x + \frac{1}{2}\sqrt{-e^2x^2+d^2}d^5e^2 - \frac{85}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3x + \frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2 + \frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x + \frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2 - \frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e}{x} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{2\,x^2}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -85/16*d^6*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*d^6*e^2*log(2*d^
2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 85/16*sqrt(-e^2*x^2 + d^2)*d^
4*e^3*x + 1/2*sqrt(-e^2*x^2 + d^2)*d^5*e^2 - 85/24*(-e^2*x^2 + d^2)^(3/2)*d
^2*e^3*x + 1/6*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2 + 1/6*(-e^2*x^2 + d^2)^(5/2)*
e^3*x + 1/10*(-e^2*x^2 + d^2)^(5/2)*d*e^2 - 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e/
x - 1/2*(-e^2*x^2 + d^2)^(7/2)*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3}\,dx = -\frac{85\,d^6e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,|e|} - \frac{d^6e^3\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^6e^3+\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} + \frac{1}{240}\left(544\,d^5e^2-\left(645\,d^4e^3+2\left(224\,d^3e^4-\left(25\,d^2e^5+4\left(5\,e^7x+18\,de^6\right)x\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,e^2}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -85/16*d^6*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*d^6*e^3*log(1/2*abs
(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^6*e^
```

```
3 + 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e/x)*e^4*x^2/((d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*abs(e)) + 1/240*(544*d^5*e^2 - (645*d^4*e^3 + 2*(22
4*d^3*e^4 - (25*d^2*e^5 + 4*(5*e^7*x + 18*d*e^6)*x)*x)*x)*x)*sqrt(-e^2*x^2
+ d^2) - 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs(e)/x + (d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^3} \, dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^3} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3, x)
```

3.74 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{25}{8}d^5e^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{2}d^5e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*d*e^3*(25*e*x+26*d)*(-e^2*x^2+d^2)^(3/2)-1/30*e^2*(39*e*x+50*d)*(-e^2
*x^2+d^2)^(5/2)/x-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-3/2*e*(-e^2*x^2+d^2)^(7/2)
/x^2-25/8*d^5*e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+13/2*d^5*e^3*arctanh((-e
^2*x^2+d^2)^(1/2)/d)-1/8*d^3*e^3*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1821, 827, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = -\frac{25}{8}d^5e^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{2}d^5e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] -1/8*(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2]) - (d*e^3*(26*d + 25*e*x)
*(d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(3
0*x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x
^2) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-9d^4e-5d^3e^2x-3d^2e^3x^2)}{x^3}\,dx}{3d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} + \frac{\int \frac{(10d^5e^2-39d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^2}\,dx}{6d^4} \\
&= -\frac{e^2(50d+39ex)(d^2-e^2x^2)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} \\
&\quad - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{\int \frac{(78d^6e^3+100d^5e^4x)(d^2-e^2x^2)^{3/2}}{x}\,dx}{12d^4}
\end{aligned}$$

$$= -\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} + \frac{\int \frac{(-312d^8e^5-300d^7e^6x)\sqrt{d^2-e^2x^2}}{x}\,dx}{48d^4e^2}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{\int \frac{624d^{10}e^7+300d^9e^8x}{x\sqrt{d^2-e^2x^2}}\,dx}{96d^4e^4}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{1}{2}\left(13d^6e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{8}\left(25d^5e^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{1}{4}\left(13d^6e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \frac{1}{8}\left(25d^5e^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{25}{8}d^5e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(13d^6e\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{25}{8}d^5e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{13}{2}d^5e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4} \, dx = \frac{\sqrt{d^2-e^2x^2}(-40d^7-180d^6ex-80d^5e^2x^2-656d^4e^3x^3-345d^3e^4x^4+32d^2e}{120x^3}$$

$$-13d^5e^3\text{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{25}{8}d^5\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 180*d^6*e*x - 80*d^5*e^2*x^2 - 656*d^4*e^3*
x^3 - 345*d^3*e^4*x^4 + 32*d^2*e^5*x^5 + 90*d*e^6*x^6 + 24*e^7*x^7))/(120*x
^3) - 13*d^5*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (25*d^
5*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{d^5\sqrt{-e^2x^2+d^2}\,(4e^2x^2+9dex+2d^2)}{6x^3}+\frac{e^7x^4\sqrt{-e^2x^2+d^2}}{5}+\frac{4e^5d^2x^2\sqrt{-e^2x^2+d^2}}{15}-\frac{82e^3d^4\sqrt{-e^2x^2+d^2}}{15}-\frac{25e^4d^5\arctan\left(\frac{-}{\sqrt{}}\right.}{8\sqrt{e^2}}$
default	$e^3\left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)\right)+d^3\left(-\frac{(-}{}\right.$

[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/6*d^5*(-e^2*x^2+d^2)^(1/2)*(4*e^2*x^2+9*d*e*x+2*d^2)/x^3+1/5*e^7*x^4*(-e^2*x^2+d^2)^(1/2)+4/15*e^5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-82/15*e^3*d^4*(-e^2*x^2+d^2)^(1/2)-25/8*e^4*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2

```
)^(1/2))+13/2*e^3*d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)+3/4*e^6*d*x^3*(-e^2*x^2+d^2)^(1/2)-23/8*e^4*d^3*x*(-e^2*x^2+d^2)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = \frac{750\,d^5e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 780\,d^5e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 656\,d}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/120*(750*d^5*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 780*d^5*
e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 656*d^5*e^3*x^3 + (24*e^7*x^7
+ 90*d*e^6*x^6 + 32*d^2*e^5*x^5 - 345*d^3*e^4*x^4 - 656*d^4*e^3*x^3 - 80*d^
5*e^2*x^2 - 180*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 843, normalized size of antiderivative = 4.01

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*
x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(
2*d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e
*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1
- e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e*
*2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(
d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**3*e**4*Piecewis
e((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sq
```

```
rt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2
 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) + d**2*e**5*Piecewise(
(-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(
e**2, 0)), (x**2*sqrt(d**2)/2, True)) + 3*d*e**6*Piecewise((d**4*Piecewise(
(log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2
, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) - d**2*x*sqrt(d**2 - e**
2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)), (x**3*sqrt(
d**2)/3, True)) + e**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
- d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/
5, Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx = -\frac{25\,d^5e^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}} + \frac{13}{2}\,d^5e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{25}{8}\sqrt{-e^2x^2+d^2}d^3e^4x - \frac{13}{2}\sqrt{-e^2x^2+d^2}d^4e^3 - \frac{25}{12}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4x - \frac{13}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3 - \frac{13}{10}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3 - \frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}{3\,x} - \frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2\,x^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{3\,x^3}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -25/8*d^5*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 13/2*d^5*e^3*log(2*d^
2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 25/8*sqrt(-e^2*x^2 + d^2)*d^3
*e^4*x - 13/2*sqrt(-e^2*x^2 + d^2)*d^4*e^3 - 25/12*(-e^2*x^2 + d^2)^(3/2)*d
*e^4*x - 13/6*(-e^2*x^2 + d^2)^(3/2)*d^2*e^3 - 13/10*(-e^2*x^2 + d^2)^(5/2)
*e^3 - 5/3*(-e^2*x^2 + d^2)^(5/2)*d*e^2/x - 3/2*(-e^2*x^2 + d^2)^(7/2)*e/x^
2 - 1/3*(-e^2*x^2 + d^2)^(7/2)*d/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx = -\frac{25\,d^5e^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} + \frac{13\,d^5e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^5e^4+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e^2}{x}+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} - \frac{1}{120}\left(656\,d^4e^3+\left(345\,d^3e^4-2\left(16\,d^2e^5+3\left(4\,e^7x+15\,de^6\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e^4}{x}+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5e^2}{x^2}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^5}{x^3}}{24\,e^2|e|}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -25/8*d^5*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 13/2*d^5*e^4*log(1/2*abs
(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*(d^5*e
^4 + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^5*e^2/x + 9*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2*d^5/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
*abs(e)) - 1/120*(656*d^4*e^3 + (345*d^3*e^4 - 2*(16*d^2*e^5 + 3*(4*e^7*x +
 15*d*e^6)*x)*x)*x)*sqrt(-e^2*x^2 + d^2) - 1/24*(9*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))*d^5*e^4/x + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*e^2/x^2
 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^5/x^3)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^4}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4, x)
```

3.75 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5} dx = -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)(d^2-e^2x^2)^{3/2}}{8x} - \frac{3e^2(3d+2ex)(d^2-e^2x^2)^{5/2}}{8x^2} - \frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} + \frac{45}{8}d^4e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{45}{8}d^4e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 15/8*d*e^3*(-e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x-3/8*e^2*(2*e*x+3*d)*(-e^2*x^2+
d^2)^(5/2)/x^2-1/4*d*(-e^2*x^2+d^2)^(7/2)/x^4-e*(-e^2*x^2+d^2)^(7/2)/x^3+45
/8*d^4*e^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+45/8*d^4*e^4*arctanh((-e^2*x^2+
d^2)^(1/2)/d)-45/8*d^2*e^4*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1821, 827, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5} dx = \frac{45}{8}d^4e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{45}{8}d^4e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3e^2(3d+2ex)(d^2-e^2x^2)^{5/2}}{8x^2} - \frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} - \frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)(d^2-e^2x^2)^{3/2}}{8x}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-12d^4e-9d^3e^2x-4d^2e^3x^2)}{x^4}\,dx}{4d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} + \frac{\int \frac{(27d^5e^2-36d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^3}\,dx}{12d^4} \\
&= -\frac{3e^2(3d+2ex)(d^2-e^2x^2)^{5/2}}{8x^2} - \frac{d(d^2-e^2x^2)^{7/2}}{4x^4} \\
&\quad - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} - \frac{5\int \frac{(144d^6e^3+216d^5e^4x)(d^2-e^2x^2)^{3/2}}{x^2}\,dx}{192d^4}
\end{aligned}
$$

$$= \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{5\int \frac{\left(-432d^7e^4+864d^6e^5x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{384d^4}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - \frac{5\int \frac{864d^9e^6-864d^8e^7x}{x\sqrt{d^2-e^2x^2}}\,dx}{768d^4e^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - \frac{1}{8}\left(45d^5e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{8}\left(45d^4e^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - \frac{1}{16}\left(45d^5e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + \frac{1}{8}\left(45d^4e^5\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{45}{8}d^4e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{8}\left(45d^5e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{45}{8}d^4e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{45}{8}d^4e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5} dx = \frac{1}{8} \left(\frac{\sqrt{d^2-e^2x^2}(-2d^7-8d^6ex-3d^5e^2x^2+48d^4e^3x^3-48d^3e^4x^4+3d^2e^5x^5}{x^4} - 90d^4e^4 \arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 45d^3\sqrt{d^2}e^4 \log(x) - 45d^3\sqrt{d^2}e^4 \log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right) \right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^7 - 8*d^6*e*x - 3*d^5*e^2*x^2 + 48*d^4*e^3*x^3
- 48*d^3*e^4*x^4 + 3*d^2*e^5*x^5 + 8*d*e^6*x^6 + 2*e^7*x^7))/x^4 - 90*d^4*e
^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 45*d^3*Sqrt[d^2]*e^4*L
og[x] - 45*d^3*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/8
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{d^{4} \sqrt{-e^{2} x^{2}+d^{2}}\, \left(-48 e^{3} x^{3}+3 d\, e^{2} x^{2}+8 d^{2} e x +2 d^{3}\right)}{8 x^{4}}+\frac{e^{7} x^{3} \sqrt{-e^{2} x^{2}+d^{2}}}{4}+\frac{3 e^{5} d^{2} x \sqrt{-e^{2} x^{2}+d^{2}}}{8}+\frac{45 e^{5} d^{4} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{8 \sqrt{e^{2}}}$
default	$d^{3} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{4 d^{2} x^{4}}-\frac{3 e^{2} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{2 d^{2} x^{2}}-\frac{5 e^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x}}{x}\right.}{\sqrt{d^{2}}}\right.\right.\right.}{2 d^{2}}\right.}{4 d^{2}}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d^4*(-e^2*x^2+d^2)^(1/2)*(-48*e^3*x^3+3*d*e^2*x^2+8*d^2*e*x+2*d^3)/x^4
+1/4*e^7*x^3*(-e^2*x^2+d^2)^(1/2)+3/8*e^5*d^2*x*(-e^2*x^2+d^2)^(1/2)+45/8*e
^5*d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+45/8*e^4*d^5/
```

```
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e^6*d*x^2*(-e^
2*x^2+d^2)^(1/2)-6*e^4*d^3*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx =$$
$$-\frac{90\,d^4e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+45\,d^4e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+48\,d^4e^4x^4-\left(2\,e^7x^7+8\,de^6x^6+3\,d^2e\right.}{8\,x^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(90*d^4*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 45*d^4*e^4
*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 48*d^4*e^4*x^4 - (2*e^7*x^7 + 8*d
*e^6*x^6 + 3*d^2*e^5*x^5 - 48*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 3*d^5*e^2*x^2
- 8*d^6*e*x - 2*d^7)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.95 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.59

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

```
[Out] d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x)
 + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3
*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*Piecewise((I*d/(x*sqrt(-1 + e
**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2))
```

```
, Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d
) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((d*
*2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**
2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x*
*2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) +
 d**2*e**5*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d*
*2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tru
e))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) + 3
*d*e**6*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*sqrt(d**2 -
 e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) + e**7*Piecewise((d
**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-
e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) - d**2*x*s
qrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4, Ne(e**2, 0)
), (x**3*sqrt(d**2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5}\,dx = \frac{45\,d^4e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}} + \frac{45}{8}\,d^4e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{45}{8}\sqrt{-e^2x^2+d^2}d^2e^5x - \frac{45}{8}\sqrt{-e^2x^2+d^2}d^3e^4 + \frac{15}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x - \frac{15}{8}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4 - \frac{9\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4}{8\,d} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3}{x} - \frac{9\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{8\,dx^2} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e}{x^3} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{4\,x^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 45/8*d^4*e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 45/8*d^4*e^4*log(2*d^2
/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 45/8*sqrt(-e^2*x^2 + d^2)*d^2*
e^5*x - 45/8*sqrt(-e^2*x^2 + d^2)*d^3*e^4 + 15/4*(-e^2*x^2 + d^2)^(3/2)*e^5
*x - 15/8*(-e^2*x^2 + d^2)^(3/2)*d*e^4 - 9/8*(-e^2*x^2 + d^2)^(5/2)*e^4/d +
 3*(-e^2*x^2 + d^2)^(5/2)*e^3/x - 9/8*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^2) -
(-e^2*x^2 + d^2)^(7/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(183) = 366.

Time = 0.30 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5}\,dx = \frac{45\,d^4e^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} + \frac{45\,d^4e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} + \frac{\left(d^4e^5+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^3}{x}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e}{x^2}-\frac{184\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{ex^3}\right)e^8x^4}{64\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} - \frac{1}{8}\left(48\,d^3e^4-\left(3\,d^2e^5+2\left(e^7x+4\,de^6\right)x\right)x\right)\sqrt{-e^2x^2+d^2} + \frac{\frac{184\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^5|e|}{x}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^3|e|}{x^2}-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e|e|}{x^3}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4|e|}{ex^4}}{64\,e^4}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 45/8*d^4*e^5*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 45/8*d^4*e^5*log(1/2*abs(
-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/64*(d^4*e^
5 + 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^3/x + 8*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^2*d^4*e/x^2 - 184*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^
4/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - 1/8*(48
*d^3*e^4 - (3*d^2*e^5 + 2*(e^7*x + 4*d*e^6)*x)*x)*sqrt(-e^2*x^2 + d^2) + 1/
64*(184*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^5*abs(e)/x - 8*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*d^4*e^3*abs(e)/x^2 - 8*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*d^4*e*abs(e)/x^3 - (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*
abs(e)/(e*x^4))/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^5}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^5}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5, x)
```

3.76 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} - \frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} + \frac{13}{2}d^3e^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{25}{8}d^3e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/24*d*e^3*(-52*e*x+25*d)*(-e^2*x^2+d^2)^(3/2)/x^2-1/60*e^2*(25*e*x+52*d)*(
-e^2*x^2+d^2)^(5/2)/x^3-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5-3/4*e*(-e^2*x^2+d^2)
^(7/2)/x^4+13/2*d^3*e^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-25/8*d^3*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*e^4*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1821, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{13}{2}d^3e^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{25}{8}d^3e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} + \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-15d^4e-13d^3e^2x-5d^2e^3x^2)}{x^5}\,dx}{5d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} + \frac{\int \frac{(52d^5e^2-25d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^4}\,dx}{20d^4} \\
&= -\frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} - \frac{d(d^2-e^2x^2)^{7/2}}{5x^5} \\
&\quad - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{\int \frac{(150d^6e^3+312d^5e^4x)(d^2-e^2x^2)^{3/2}}{x^3}\,dx}{72d^4}
\end{aligned}$$

$$= \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{\int \frac{\left(-1248d^7e^4+600d^6e^5x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{192d^4}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{\int \frac{-1200d^8e^5-2496d^7e^6x}{x\sqrt{d^2-e^2x^2}}\,dx}{384d^4}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{1}{8}\left(25d^4e^5\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{2}\left(13d^3e^6\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{1}{16}\left(25d^4e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + \frac{1}{2}\left(13d^3e^6\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{13}{2}d^3e^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(25d^4e^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{13}{2}d^3e^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{25}{8}d^3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{\sqrt{d^2-e^2x^2}(-24d^7-90d^6ex-32d^5e^2x^2+345d^4e^3x^3+656d^3e^4x^4+80d^2e^5}{120x^5}$$

$$-13d^3e^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)$$

$$-\frac{25}{8}\left(d^2\right)^{3/2}e^5\log(x)+\frac{25}{8}\left(d^2\right)^{3/2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^7 - 90*d^6*e*x - 32*d^5*e^2*x^2 + 345*d^4*e^3*x
^3 + 656*d^3*e^4*x^4 + 80*d^2*e^5*x^5 + 180*d*e^6*x^6 + 40*e^7*x^7))/(120*x
^5) - 13*d^3*e^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (25*(d^2
)^(3/2)*e^5*Log[x])/8 + (25*(d^2)^(3/2)*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*
x^2]])/8
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{d^3\sqrt{-e^2x^2+d^2}\left(-656e^4x^4-345d\,e^3x^3+32d^2e^2x^2+90d^3ex+24d^4\right)}{120x^5}+\frac{e^7x^2\sqrt{-e^2x^2+d^2}}{3}+\frac{2e^5d^2\sqrt{-e^2x^2+d^2}}{3}+\frac{13e^6d^3\arctan}{2}$
default	$d^3\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{5d^2x^5}-\frac{2e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2}}{2}\right.}{}\right.}{}\right)}{d^2}\right)}{3d^2}\right)}{5d^2}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*d^3*(-e^2*x^2+d^2)^(1/2)*(-656*e^4*x^4-345*d*e^3*x^3+32*d^2*e^2*x^2+
90*d^3*e*x+24*d^4)/x^5+1/3*e^7*x^2*(-e^2*x^2+d^2)^(1/2)+2/3*e^5*d^2*(-e^2*x
^2+d^2)^(1/2)+13/2*e^6*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^
(1/2))-25/8*e^5*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2
))/x)+3/2*e^6*d*x*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx = -\frac{1560\,d^3e^5x^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-375\,d^3e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-80\,d^3e^5x^5-(40\,e^7x^7+180\,de^6x^6+}{120\,x^5}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(1560*d^3*e^5*x^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 375*d^
3*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 80*d^3*e^5*x^5 - (40*e^7*x^7
 + 180*d*e^6*x^6 + 80*d^2*e^5*x^5 + 656*d^3*e^4*x^4 + 345*d^4*e^3*x^3 - 32*
d^5*e^2*x^2 - 90*d^6*e*x - 24*d^7)*sqrt(-e^2*x^2 + d^2))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 1182, normalized size of antiderivative = 5.47

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

```
[Out] d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
```

```
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)
) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecewise((-e*sqrt(d*
*2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**4*e**3*Piecewise((-e
*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e*
*2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**3*e
**4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**
2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
- e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), T
rue)) + d**2*e**5*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acos
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + 3*d*e**6*Piecewise((d**2*Piecewise((log(-2*
e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (
x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2,
0)), (x*sqrt(d**2), True)) + e**7*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(
3*e**2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{13\,d^3e^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} - \frac{25}{8}\,d^3e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{13}{2}\sqrt{-e^2x^2+d^2}de^6x + \frac{25}{8}\sqrt{-e^2x^2+d^2}d^2e^5 + \frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x}{3\,d} + \frac{25}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5 + \frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{8\,d^2} + \frac{52\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{15\,dx} + \frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{8\,d^2x^2} - \frac{13\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{15\,dx^3} - \frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{4\,x^4} - \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{5\,x^5}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 13/2*d^3*e^6*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 25/8*d^3*e^5*log(2*d^2
/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 13/2*sqrt(-e^2*x^2 + d^2)*d*e^
6*x + 25/8*sqrt(-e^2*x^2 + d^2)*d^2*e^5 + 13/3*(-e^2*x^2 + d^2)^(3/2)*e^6*x
/d + 25/24*(-e^2*x^2 + d^2)^(3/2)*e^5 + 5/8*(-e^2*x^2 + d^2)^(5/2)*e^5/d^2
+ 52/15*(-e^2*x^2 + d^2)^(5/2)*e^4/(d*x) + 5/8*(-e^2*x^2 + d^2)^(7/2)*e^3/(
d^2*x^2) - 13/15*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^3) - 3/4*(-e^2*x^2 + d^2)^
(7/2)*e/x^4 - 1/5*(-e^2*x^2 + d^2)^(7/2)*d/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = \frac{13\,d^3e^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|}$$

$$+\frac{\left(6\,d^3e^6+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3e^4}{x}+\frac{50\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^3e^2}{x^2}-\frac{600\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^3}{x^3}-\frac{2580\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d}{e^2x^4}\right.}{960\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5|e|}$$

$$-\frac{25\,d^3e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|}+\frac{1}{6}\left(4\,d^2e^5+\left(2\,e^7x+9\,de^6\right)x\right)\sqrt{-e^2x^2+d^2}$$

$$+\frac{\frac{2580\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3e^8}{x}+\frac{600\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^3e^6}{x^2}-\frac{50\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^3e^4}{x^3}-\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^3e^2}{x^4}-\frac{6\,(d}{}}{960\,e^4|e|}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 13/2*d^3*e^6*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/960*(6*d^3*e^6 + 45*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*e^4/x + 50*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^2*d^3*e^2/x^2 - 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3/x^3
- 2580*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3/(e^2*x^4))*e^10*x^5/((d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^5*abs(e)) - 25/8*d^3*e^6*log(1/2*abs(-2*d*e
- 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/6*(4*d^2*e^5 + (2
*e^7*x + 9*d*e^6)*x)*sqrt(-e^2*x^2 + d^2) + 1/960*(2580*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^3*e^8/x + 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^3*
e^6/x^2 - 50*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*e^4/x^3 - 45*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*e^2/x^4 - 6*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^5*d^3/x^5)/(e^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^6} \, dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)^3}{x^6} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6, x)
```

3.77 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7} dx$

Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7} dx = -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{1}{2}d^2e^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d^2e^6\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/48*d*e^3*(85*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/120*e^2*(12*e*x+85*d)*(-
e^2*x^2+d^2)^(5/2)/x^4-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-3/5*e*(-e^2*x^2+d^2)^
(7/2)/x^5-1/2*d^2*e^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-85/16*d^2*e^6*arctan
h((-e^2*x^2+d^2)^(1/2)/d)-1/16*d*e^5*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1821, 827, 825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} \, dx = -\frac{1}{2}d^2e^6 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d^2e^6 \text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] -1/16*(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/x + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-18d^4e-17d^3e^2x-6d^2e^3x^2)}{x^6}\,dx}{6d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} + \frac{\int \frac{(85d^5e^2-6d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^5}\,dx}{30d^4}$$

$$= -\frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{\int \frac{(48d^6e^3+340d^5e^4x)(d^2-e^2x^2)^{3/2}}{x^4}\,dx}{96d^4}$$

$$= \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} + \frac{\int \frac{(192d^8e^5+2040d^7e^6x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{384d^6}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{\int \frac{-4080d^9e^6+384d^8e^7x}{x\sqrt{d^2-e^2x^2}}\,dx}{768d^6}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} + \frac{1}{16}(85d^3e^6)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}(d^2e^7)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} + \frac{1}{32}(85d^3e^6)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \frac{1}{2}(d^2e^7)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)(d^2-e^2x^2)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)(d^2-e^2x^2)^{5/2}}{120x^4} - \frac{d(d^2-e^2x^2)^{7/2}}{6x^6} - \frac{3e(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{1}{2}d^2e^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{16}(85d^3e^4)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{1}{2}d^2e^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d^2e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{d^2-e^2x^2}(-40d^7-144d^6ex-50d^5e^2x^2+448d^4e^3x^3+645d^3e^4x^4-544d^2}{240x^6} + d^2e^6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{85}{16}d\sqrt{d^2}e^6\log(x) + \frac{85}{16}d\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 144*d^6*e*x - 50*d^5*e^2*x^2 + 448*d^4*e^3*
x^3 + 645*d^3*e^4*x^4 - 544*d^2*e^5*x^5 + 720*d*e^6*x^6 + 120*e^7*x^7))/(24
0*x^6) + d^2*e^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (85*d*Sq
rt[d^2]*e^6*Log[x])/16 + (85*d*Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 - e^2
*x^2]])/16
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}\, \left(544 e^{5} x^{5}-645 d\, e^{4} x^{4}-448 d^{2} e^{3} x^{3}+50 d^{3} e^{2} x^{2}+144 d^{4} e x+40 d^{5}\right)}{240 x^{6}}+\frac{e^{7} x \sqrt{-e^{2} x^{2}+d^{2}}}{2}-\frac{e^{7} d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}$
default	$d^{3}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{6 d^{2} x^{6}}-\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{4 d^{2} x^{4}}-\frac{3 e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{2 d^{2} x^{2}}-\frac{5 e^{2}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2} x^{2}+d^{2}}-\right.\right.\right.}{2 d^{2}}\right)}{4 d^{2}}\right)}{6 d^{2}}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*d^2*(-e^2*x^2+d^2)^(1/2)*(544*e^5*x^5-645*d*e^4*x^4-448*d^2*e^3*x^3+
50*d^3*e^2*x^2+144*d^4*e*x+40*d^5)/x^6+1/2*e^7*x*(-e^2*x^2+d^2)^(1/2)-1/2*e
^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-85/16*e^6*d^3
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+3*e^6*d*(-e^2
*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^7}\,dx = \frac{240\,d^2e^6x^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1275\,d^2e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+720}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*d^2*e^6*x^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1275*d^2
*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 720*d^2*e^6*x^6 + (120*e^7*x^
7 + 720*d*e^6*x^6 - 544*d^2*e^5*x^5 + 645*d^3*e^4*x^4 + 448*d^4*e^3*x^3 - 5
0*d^5*e^2*x^2 - 144*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 1380, normalized size of antiderivative = 6.45

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^7}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
```

```
[Out] d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise((3*I*d**3*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*s
qrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
```

```
*2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*
e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e*
*4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) +
 d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*
x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e
*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))
/(8*d**3), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3
*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x*
*2) + 1)/(3*d**2), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2)
 - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**
2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) + d**2*e**5*Piecewise((I*d/(x*sq
rt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x*
*2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*a
sin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewi
se((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**
2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e
**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), Tr
ue)) + e**7*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d
**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tr
ue))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^7}\,dx &= -\frac{d^2e^7\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} \\ &\quad -\frac{85}{16}\,d^2e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)-\frac{1}{2}\sqrt{-e^2x^2+d^2}e^7x \\ &\quad +\frac{85}{16}\sqrt{-e^2x^2+d^2}de^6-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^7x}{3\,d^2}+\frac{85\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6}{48\,d} \\ &\quad +\frac{17\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6}{16\,d^3}-\frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5}{15\,d^2x}+\frac{17\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{16\,d^3x^2} \\ &\quad +\frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{15\,d^2x^3}-\frac{17\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{24\,dx^4}-\frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{5\,x^5}-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{6\,x^6}\end{aligned}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

[Out] -1/2*d^2*e^7*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 85/16*d^2*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*e^7*x + 85/16*sqrt(-e^2*x^2 + d^2)*d*e^6 - 1/3*(-e^2*x^2 + d^2)^(3/2)*e^7*x/d^2 + 85/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d + 17/16*(-e^2*x^2 + d^2)^(5/2)*e^6/d^3 - 4/15*(-e^2*x^2 + d^2)^(5/2)*e^5/(d^2*x) + 17/16*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^2) + 1/15*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^3) - 17/24*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^4) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(7/2)*d/x^6

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(186) = 372.

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^7} dx = \frac{\left(5 d^2 e^7 + \frac{36 \left(de+\sqrt{-e^2x^2+d^2}|e|\right) d^2 e^5}{x} + \frac{45 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^2 e^3}{x^2} - \frac{340 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 \cdots}{x^3} \cdots\right)}{1920 \left(de+\sqrt{-e^2x^2+\cdots}\right)\cdots} - \frac{d^2 e^7 \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{2|e|} - \frac{85 d^2 e^7 \log\left(\frac{\left|-2 de - 2\sqrt{-e^2x^2+d^2}|e|\right|}{2 e^2 |x|}\right)}{16|e|} + \frac{1}{2}\left(e^7 x + 6 de^6\right)\sqrt{-e^2x^2+d^2} - \frac{\frac{1800 \left(de+\sqrt{-e^2x^2+d^2}|e|\right) d^2 e^9 |e|}{x} - \frac{1215 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^2 e^7 |e|}{x^2} - \frac{340 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^2 e^5 |e|}{x^3} + \frac{45 \left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^2 e \cdots}{x^4} \cdots}{1920 e^6}$$

[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")

[Out] 1/1920*(5*d^2*e^7 + 36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e^5/x + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*e^3/x^2 - 340*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*e/x^3 - 1215*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^2/(e*x^4) + 1800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^2/(e^3*x^5))*e^12*x^6/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*abs(e)) - 1/2*d^2*e^7*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 85/16*d^2*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/2*(e^7*x + 6*d*e^6)*sqrt(-e^2*x^2 + d^2) - 1/1920*(1800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e^9*abs(e)/x - 1215*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*e^7*abs(e)/x^2 - 340*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*e^5*abs(e)/x^3 + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^2*e^3*abs(e)/x^4 + 36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^2*e*abs(e)/x^5 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^2*abs(e)/(e*x^6))/e^6

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)^3}{x^7}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7, x)
```

3.78 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx=-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-3de^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{15}{16}de^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/16*e^4*(5*e*x+16*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/40*e^2*(5*e*x+24*d)*(-e^2*
x^2+d^2)^(5/2)/x^5-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7-1/2*e*(-e^2*x^2+d^2)^(7/2
)/x^6-3*d*e^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-15/16*d*e^7*arctanh((-e^2*x^
2+d^2)^(1/2)/d)-3/16*e^6*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1821, 825, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx = -3de^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{15}{16}de^7\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) -\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}-\frac{e(d^2-e^2x^2)^{7/2}}{2x^6}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-21d^4e-21d^3e^2x-7d^2e^3x^2)}{x^7}\,dx}{7d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} + \frac{\int \frac{(126d^5e^2+21d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} - \frac{\int \frac{(1008d^7e^4+210d^6e^5x)(d^2-e^2x^2)^{3/2}}{x^4}\,dx}{336d^6}$$

$$= \frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} + \frac{\int \frac{(4032d^9e^6+1260d^8e^7x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{1344d^8}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} - \frac{\int \frac{-2520d^{10}e^7+8064d^9e^8x}{x\sqrt{d^2-e^2x^2}}\,dx}{2688d^8}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} + \frac{1}{16}(15d^2e^7)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - (3de^8)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} + \frac{1}{32}(15d^2e^7)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - (3de^8)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} - 3de^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{16}(15d^2e^5)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e(d^2-e^2x^2)^{7/2}}{2x^6} - 3de^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{15}{16}de^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{d^2-e^2x^2}(-80d^7-280d^6ex-96d^5e^2x^2+770d^4e^3x^3+992d^3e^4x^4-525d^2}{560x^7} + 6de^7\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{15}{16}\sqrt{d^2}e^7\log(x) + \frac{15}{16}\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-80*d^7 - 280*d^6*e*x - 96*d^5*e^2*x^2 + 770*d^4*e^3*
x^3 + 992*d^3*e^4*x^4 - 525*d^2*e^5*x^5 - 2496*d*e^6*x^6 + 560*e^7*x^7))/(5
60*x^7) + 6*d*e^7*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (15*Sqr
t[d^2]*e^7*Log[x])/16 + (15*Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^
2]])/16
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(-560e^{7}x^{7}+2496d\,e^{6}x^{6}+525d^{2}e^{5}x^{5}-992d^{3}e^{4}x^{4}-770d^{4}e^{3}x^{3}+96d^{5}e^{2}x^{2}+280d^{6}ex+80d^{7}\right)}{560x^{7}}-\frac{3d\,e^{8}\arctan\left(\frac{\sqrt{e^{2}}}{\sqrt{-e^{2}x^{2}}}\right.}{\sqrt{e^{2}}}$
default	$e^{3}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{4d^{2}x^{4}}-\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}}}{x}\right.}{\sqrt{d^{2}}}\right.\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*(-e^2*x^2+d^2)^(1/2)*(-560*e^7*x^7+2496*d*e^6*x^6+525*d^2*e^5*x^5-99
2*d^3*e^4*x^4-770*d^4*e^3*x^3+96*d^5*e^2*x^2+280*d^6*e*x+80*d^7)/x^7-3*d*e^
8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-15/16*d^2*e^7/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8}\,dx = \frac{3360\,de^7x^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+525\,de^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+560\,d}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/560*(3360*d*e^7*x^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 525*d*e^7
*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 560*d*e^7*x^7 + (560*e^7*x^7 - 24
96*d*e^6*x^6 - 525*d^2*e^5*x^5 + 992*d^3*e^4*x^4 + 770*d^4*e^3*x^3 - 96*d^5
*e^2*x^2 - 280*d^6*e*x - 80*d^7)*sqrt(-e^2*x^2 + d^2))/x^7
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.75 (sec) , antiderivative size = 1513, normalized size of antiderivative = 7.34

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + 3*d**6*e
*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5)
, Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)
) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
```

```
 I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*
d**4*e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x
**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)
) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*
x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1
), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + d**2*e**5*Piecewise((-e*sqrt(d**2/(e**2*x**2) -
1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(
2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**2*asin(d/(e*x))/(2*d), True)) + 3*d*e**6*Piecewise((I*d/(x*sqrt(-
1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d
**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(
e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((d**2
/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*
x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2
) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^8}\,dx = -\frac{3\,de^8\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{15}{16}\,de^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3\sqrt{-e^2x^2+d^2}e^8x}{d} + \frac{15}{16}\sqrt{-e^2x^2+d^2}e^7 - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^8x}{d^3} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^7}{16\,d^2} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7}{16\,d^4} - \frac{8\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6}{5\,d^3x} + \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{16\,d^4x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{5\,d^3x^3} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{8\,d^2x^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{5\,dx^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e}{2\,x^6} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,x^7}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -3*d*e^8*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 15/16*d*e^7*log(2*d^2/abs(
x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3*sqrt(-e^2*x^2 + d^2)*e^8*x/d + 15
/16*sqrt(-e^2*x^2 + d^2)*e^7 - 2*(-e^2*x^2 + d^2)^(3/2)*e^8*x/d^3 + 5/16*(-
e^2*x^2 + d^2)^(3/2)*e^7/d^2 + 3/16*(-e^2*x^2 + d^2)^(5/2)*e^7/d^4 - 8/5*(-
e^2*x^2 + d^2)^(5/2)*e^6/(d^3*x) + 3/16*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^2
) + 2/5*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(7/2)*e
^3/(d^2*x^4) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^5) - 1/2*(-e^2*x^2 + d^2
)^(7/2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(7/2)*d/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(180) = 360$.

Time = 0.31 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.63

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8}\,dx = \frac{\left(5\,de^8 + \frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^6}{x} + \frac{49\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^4}{x^2} - \frac{245\left(de+\sqrt{-e^2x^2+d^2}|e\right.}{x^3}\right.}{4480\,(de+}$$
$$- \frac{3\,de^8 \arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{15\,de^8 \log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,|e|} + \sqrt{-e^2x^2+d^2}e^7$$
$$- \frac{\frac{9065\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^{12}}{x} + \frac{455\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^{10}}{x^2} - \frac{875\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3de^8}{x^3} - \frac{245\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4de^6}{x^4} + \frac{49\,(}{}}{4480\,e^6|e|}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/4480*(5*d*e^8 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e^6/x + 49*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^4/x^2 - 245*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^3*d*e^2/x^3 - 875*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/x^4 + 4
55*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d/(e^2*x^5) + 9065*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^6*d/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^7*abs(e)) - 3*d*e^8*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 15/16*d*e^8
*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) +
 sqrt(-e^2*x^2 + d^2)*e^7 - 1/4480*(9065*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)*d*e^12/x + 455*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^10/x^2 - 875*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*e^8/x^3 - 245*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^4*d*e^6/x^4 + 49*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d*e^4/x^
5 + 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d*e^2/x^6 + 5*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^7*d/x^7)/(e^6*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8} \, dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^8} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8, x)
```

3.79 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 204

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9}\,dx = -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} - e^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{125}{128}e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/192*e^4*(64*e*x+125*d)*(-e^2*x^2+d^2)^(3/2)/x^4-1/240*e^2*(48*e*x+125*d)*
(-e^2*x^2+d^2)^(5/2)/x^6-1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-3/7*e*(-e^2*x^2+d^2
)^(7/2)/x^7-e^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+125/128*e^8*arctanh((-e^2*
x^2+d^2)^(1/2)/d)-1/128*e^6*(128*e*x+125*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1821, 825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9} \, dx = e^8\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) + \frac{125}{128}e^8\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] -1/128*(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-24d^4e-25d^3e^2x-8d^2e^3x^2)}{x^8}\,dx}{8d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} + \frac{\int \frac{(175d^5e^2+56d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^7}\,dx}{56d^4} \\
&= -\frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{8x^8} \\
&\quad - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{\int \frac{(1750d^7e^4+672d^6e^5x)(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{672d^6}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} \\
&\quad - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} + \frac{\int \frac{\left(10500d^9e^6+5376d^8e^7x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{5376d^8} \\
&= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} \\
&\quad - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} \\
&\quad - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{\int \frac{21000d^{11}e^8+21504d^{10}e^9x}{x\sqrt{d^2-e^2x^2}}\,dx}{21504d^{10}} \\
&= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} \\
&\quad - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} \\
&\quad - \frac{1}{128}\left(125de^8\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^9\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} \\
&\quad - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} \\
&\quad - \frac{1}{256}\left(125de^8\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e^9\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} \\
&\quad - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} \\
&\quad - e^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{128}\left(125de^6\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} \\
&\quad - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} \\
&\quad - e^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{125}{128}e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{d^2-e^2x^2}(-1680d^7 - 5760d^6ex - 1960d^5e^2x^2 + 14592d^4e^3x^3 + 17710d^3e^4}{13440x^8}$$

$$+ 2e^8 \arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{125\sqrt{d^2}e^8\log(x)}{128d} - \frac{125\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d}$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-1680*d^7 - 5760*d^6*e*x - 1960*d^5*e^2*x^2 + 14592*d
^4*e^3*x^3 + 17710*d^3*e^4*x^4 - 7424*d^2*e^5*x^5 - 27195*d*e^6*x^6 - 14848
*e^7*x^7))/(13440*x^8) + 2*e^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2
])] + (125*Sqrt[d^2]*e^8*Log[x])/(128*d) - (125*Sqrt[d^2]*e^8*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/(128*d)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(14848e^7x^7+27195d\,e^6x^6+7424d^2e^5x^5-17710d^3e^4x^4-14592d^4e^3x^3+1960d^5e^2x^2+5760d^6ex+1680d^7)}{13440x^8}-\frac{e^9\,\mathrm{arcta}}{}$
default	$e^3\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{5d^2x^5}-\frac{2e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2}}{2}\right.}{}\right.}{}\right.}{d^2}\right.}{3d^2}\right.}{5d^2}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13440*(-e^2*x^2+d^2)^(1/2)*(14848*e^7*x^7+27195*d*e^6*x^6+7424*d^2*e^5*x
^5-17710*d^3*e^4*x^4-14592*d^4*e^3*x^3+1960*d^5*e^2*x^2+5760*d^6*e*x+1680*d
^7)/x^8-e^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+125/128*
e^8*d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9} dx = \frac{26880\, e^8 x^8 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 13125\, e^8 x^8 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (148$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/13440*(26880*e^8*x^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13125*e^
8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (14848*e^7*x^7 + 27195*d*e^6*x^6
 + 7424*d^2*e^5*x^5 - 17710*d^3*e^4*x^4 - 14592*d^4*e^3*x^3 + 1960*d^5*e^2*
x^2 + 5760*d^6*e*x + 1680*d^7)*sqrt(-e^2*x^2 + d^2))/x^8
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.73 (sec) , antiderivative size = 1719, normalized size of antiderivative = 8.43

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9} dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

```
[Out] d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6)
 + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*
x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*
```

```
e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**
6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*ac
osh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**4*e**3
*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*
x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15
*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), True)) - 5*d**3*e**4*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**
2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x*
*2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
 - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e**5*Piecewise((-e*sqrt(d**
2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs
(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**
3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-e*sqr
t(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(
-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**7*Piecewi
se((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt
(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2
/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(178) = 356$.

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = -\frac{e^9 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{125}{128}e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{\sqrt{-e^2x^2+d^2}e^9x}{d^2} - \frac{125\sqrt{-e^2x^2+d^2}e^8}{128\,d} - \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^9x}{3\,d^4} - \frac{125\,(-e^2x^2+d^2)^{\frac{3}{2}}e^8}{384\,d^3} - \frac{25\,(-e^2x^2+d^2)^{\frac{5}{2}}e^8}{128\,d^5} - \frac{8\,(-e^2x^2+d^2)^{\frac{5}{2}}e^7}{15\,d^4x} - \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^6}{128\,d^5x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{15\,d^4x^3} + \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{192\,d^3x^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{5\,d^2x^5} - \frac{25\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{48\,dx^6} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{7\,x^7} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{8\,x^8}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] -e^9*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 125/128*e^8*log(2*d^2/abs(x) +
 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)*e^9*x/d^2 - 125/12
8*sqrt(-e^2*x^2 + d^2)*e^8/d - 2/3*(-e^2*x^2 + d^2)^(3/2)*e^9*x/d^4 - 125/3
84*(-e^2*x^2 + d^2)^(3/2)*e^8/d^3 - 25/128*(-e^2*x^2 + d^2)^(5/2)*e^8/d^5 -
 8/15*(-e^2*x^2 + d^2)^(5/2)*e^7/(d^4*x) - 25/128*(-e^2*x^2 + d^2)^(7/2)*e^
6/(d^5*x^2) + 2/15*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^3) + 25/192*(-e^2*x^2
+ d^2)^(7/2)*e^4/(d^3*x^4) - 1/5*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^5) - 25/
48*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^6) - 3/7*(-e^2*x^2 + d^2)^(7/2)*e/x^7 -
1/8*(-e^2*x^2 + d^2)^(7/2)*d/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(178) = 356.

Time = 0.31 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = \frac{\left(105\,e^9 + \frac{720\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} + \frac{1120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^5}{x^2} - \frac{3696\left(de+\sqrt{-e^2x^2+d^2}\right.}{x^3}\right.}{} - \frac{e^9\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{|e|} + \frac{125\,e^9\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,|e|} - \frac{122640\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^{13}|e|}{x} + \frac{77280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^{11}|e|}{x^2} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^9|e|}{x^3} - \frac{14280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4e}{x^4}$$

2150

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/215040*(105*e^9 + 720*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x + 1120*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^5/x^2 - 3696*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*e^3/x^3 - 14280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e/x^4 -
 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) + 77280*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^6/(e^3*x^6) + 122640*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*abs(e)) - e^
9*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 125/128*e^9*log(1/2*abs(-2*d*e - 2*s
qrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/215040*(122640*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*e^13*abs(e)/x + 77280*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^2*e^11*abs(e)/x^2 - 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e
^9*abs(e)/x^3 - 14280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^7*abs(e)/x^4
- 3696*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e^5*abs(e)/x^5 + 1120*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^6*e^3*abs(e)/x^6 + 720*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^7*e*abs(e)/x^7 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*abs(
e)/(e*x^8))/e^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^9}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9, x)
```

3.80 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx=-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4}-\frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}-\frac{3e(d^2-e^2x^2)^{7/2}}{8x^8}-\frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7}+\frac{55e^9\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}$$

```
[Out] 55/192*e^5*(-e^2*x^2+d^2)^(3/2)/x^4-11/48*e^3*(-e^2*x^2+d^2)^(5/2)/x^6-1/9*
d*(-e^2*x^2+d^2)^(7/2)/x^9-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-29/63*e^2*(-e^2*x
^2+d^2)^(7/2)/d/x^7+55/128*e^9*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-55/128*e^7
*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1821, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx=\frac{55e^9\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}-\frac{3e(d^2-e^2x^2)^{7/2}}{8x^8}-\frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7}-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4}-\frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-27d^4e-29d^3e^2x-9d^2e^3x^2\right)}{x^9}\,dx}{9d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}+\frac{\int \frac{\left(232d^5e^2+99d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx}{72d^4}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{1}{8}\left(11e^3\right)\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx\\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}\\
&\quad+\frac{1}{16}\left(11e^3\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{5/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}\\
&\quad-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{1}{96}\left(55e^5\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}\\
&\quad-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{1}{128}\left(55e^7\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}\\
&\quad-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{1}{256}\left(55e^9\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}\\
&\quad-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}\\
&\quad+\frac{1}{128}\left(55e^7\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}\\
&\quad-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{55e^9\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-896d^8-3024d^7ex-1024d^6e^2x^2+7224d^5e^3x^3+8448d^4e^4x^4}{8064dx^9}$$
$$+\frac{55e^9\log(x)}{128\sqrt{d^2}}-\frac{55e^9\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128\sqrt{d^2}}$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-896*d^8 - 3024*d^7*e*x - 1024*d^6*e^2*x^2 + 7224*d^5
*e^3*x^3 + 8448*d^4*e^4*x^4 - 3066*d^3*e^5*x^5 - 10240*d^2*e^6*x^6 - 4599*d
*e^7*x^7 + 3712*e^8*x^8))/(8064*d*x^9) + (55*e^9*Log[x])/(128*Sqrt[d^2]) -
(55*e^9*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*Sqrt[d^2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(-3712e^{8}x^{8}+4599d\,e^{7}x^{7}+10240d^{2}e^{6}x^{6}+3066d^{3}e^{5}x^{5}-8448d^{4}x^{4}e^{4}-7224d^{5}e^{3}x^{3}+1024d^{6}e^{2}x^{2}+3024d^{7}ex+896d^{8}\right)}{8064x^{9}d}+$
default	$e^{3}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{6d^{2}x^{6}}-\frac{e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{4d^{2}x^{4}}-\frac{3e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{2}x^{2}}-\frac{5e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}}{}\right.\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.}{6d^{2}}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8064*(-e^2*x^2+d^2)^(1/2)*(-3712*e^8*x^8+4599*d*e^7*x^7+10240*d^2*e^6*x^
6+3066*d^3*e^5*x^5-8448*d^4*e^4*x^4-7224*d^5*e^3*x^3+1024*d^6*e^2*x^2+3024*
d^7*e*x+896*d^8)/x^9/d+55/128*e^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{3465\,e^9x^9\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (3712\,e^8x^8 - 4599\,de^7x^7 - 10240\,d^2e^6x^6 - 3066\,d^3e^5x^5 + 8448\,d^4e^4x^4 + 7}{8064\,dx^9}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*e^9*x^9*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (3712*e^8*x^8 -
4599*d*e^7*x^7 - 10240*d^2*e^6*x^6 - 3066*d^3*e^5*x^5 + 8448*d^4*e^4*x^4 +
7224*d^5*e^3*x^3 - 1024*d^6*e^2*x^2 - 3024*d^7*e*x - 896*d^8)*sqrt(-e^2*x^2
 + d^2))/(d*x^9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.65 (sec) , antiderivative size = 1889, normalized size of antiderivative = 10.10

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d*
*2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**
2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x
**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt
(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2)
+ 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e*
*2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*
x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x*
```

```
*2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d
/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(
-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1))
 - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**2*Piecewise((-e*sqrt
(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*
x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x*
*4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*d**4*e**3*Piecewise((-d**2/(6
*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) -
1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x)
)/(16*d**5), True)) - 5*d**3*e**4*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**
2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqr
t(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1
 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2
*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + d**2*e**5*Piecewise(
(-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*
x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*
x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2
*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + 3
*d*e**6*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/
(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e*
*2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) + e**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x
))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x*
*2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2
*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{55\,e^9\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d} - \frac{55\sqrt{-e^2x^2+d^2}e^9}{128\,d^2} - \frac{55\,(-e^2x^2+d^2)^{\frac{3}{2}}e^9}{384\,d^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{5}{2}}e^9}{128\,d^6} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^7}{128\,d^6x^2} + \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{192\,d^4x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{48\,d^2x^6} - \frac{29\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{63\,dx^7} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{8\,x^8} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{9\,x^9}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 55/128*e^9*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 55/128*s
qrt(-e^2*x^2 + d^2)*e^9/d^2 - 55/384*(-e^2*x^2 + d^2)^(3/2)*e^9/d^4 - 11/12
8*(-e^2*x^2 + d^2)^(5/2)*e^9/d^6 - 11/128*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x
^2) + 11/192*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^4) - 11/48*(-e^2*x^2 + d^2)^
(7/2)*e^3/(d^2*x^6) - 29/63*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^7) - 3/8*(-e^2*
x^2 + d^2)^(7/2)*e/x^8 - 1/9*(-e^2*x^2 + d^2)^(7/2)*d/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(159) = 318.

Time = 0.32 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.47

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{\left(28\,e^{10} + \frac{189\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^8}{x} + \frac{324\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^6}{x^2} - \frac{672\left(de+\sqrt{-e^2x^2+d\right.}{x^3}\right.}{} + \frac{55\,e^{10}\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d|e|} + \frac{\frac{16632\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^8e^{16}}{x} - \frac{18144\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^8e^{14}}{x^2} - \frac{9744\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^8e^{12}}{x^3} + \frac{1512\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{x^4}}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/129024*(28*e^10 + 189*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^8/x + 324*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^6/x^2 - 672*(d*e + sqrt(-e^2*x^2 + d^2
```

```
)*abs(e))^3*e^4/x^3 - 3024*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^2/x^4 -
1512*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/x^5 + 9744*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^6/(e^2*x^6) + 18144*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e
^4*x^7) - 16632*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^6*x^8))*e^18*x^9/(
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d*abs(e)) + 55/128*e^10*log(1/2*abs(-
2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) + 1/129024*
(16632*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^8*e^16/x - 18144*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2*d^8*e^14/x^2 - 9744*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3*d^8*e^12/x^3 + 1512*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^8*e^10/
x^4 + 3024*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^8*e^8/x^5 + 672*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^6*d^8*e^6/x^6 - 324*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^7*d^8*e^4/x^7 - 189*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^8*e^2/
x^8 - 28*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d^8/x^9)/(d^9*e^8*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{10}}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^{10}}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10, x)
```

3.81 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} -\frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6}-\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{e(d^2-e^2x^2)^{7/2}}{3x^9} -\frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8}-\frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7}+\frac{33e^{10}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}$$

```
[Out] 11/128*e^6*(-e^2*x^2+d^2)^(3/2)/d/x^4-11/160*e^4*(-e^2*x^2+d^2)^(5/2)/d/x^6
-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10-1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-33/80*e^2*
(-e^2*x^2+d^2)^(7/2)/d/x^8-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7+33/256*e^1
0*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-33/256*e^8*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1821, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{33e^{10}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}-\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} -\frac{e(d^2-e^2x^2)^{7/2}}{3x^9}-\frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8}-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} +\frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4}-\frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6}-\frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-30d^4e-33d^3e^2x-10d^2e^3x^2)}{x^{10}}\,dx}{10d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} + \frac{\int \frac{(297d^5e^2+150d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^9}\,dx}{90d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{\int \frac{(-1200d^6e^3-297d^5e^4x)(d^2-e^2x^2)^{5/2}}{x^8}\,dx}{720d^6}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} + \frac{(33e^4)\int \frac{(d^2-e^2x^2)^{5/2}}{x^7}\,dx}{80d}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} + \frac{(33e^4)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{5/2}}{x^4}\,dx, x, x^2\right)}{160d}$$

$$= -\frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6} - \frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} - \frac{(11e^6)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{64d}$$

$$= \frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} - \frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6} - \frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} + \frac{(33e^8)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{256d}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} - \frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6} - \frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} - \frac{(33e^{10})\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{512d}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} - \frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6} - \frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} + \frac{(33e^8)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{256d}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4} - \frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6} - \frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{e(d^2-e^2x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8} - \frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7} + \frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2688d^9 - 8960d^8ex - 3024d^7e^2x^2 + 20480d^6e^3x^3 + 23352d^5e^4}{26880d} + \frac{33\sqrt{d^2}e^{10}\log(x)}{256d^3} - \frac{33\sqrt{d^2}e^{10}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{256d^3}$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2688*d^9 - 8960*d^8*e*x - 3024*d^7*e^2*x^2 + 20480*d
^6*e^3*x^3 + 23352*d^5*e^4*x^4 - 7680*d^4*e^5*x^5 - 24570*d^3*e^6*x^6 - 102
40*d^2*e^7*x^7 + 3465*d*e^8*x^8 + 6400*e^9*x^9))/(26880*d^2*x^10) + (33*Sqr
t[d^2]*e^10*Log[x])/(256*d^3) - (33*Sqrt[d^2]*e^10*Log[Sqrt[d^2] - Sqrt[d^2
 - e^2*x^2]])/(256*d^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-6400 e^{9} x^{9}-3465 d\,e^{8} x^{8}+10240 d^{2} e^{7} x^{7}+24570 d^{3} e^{6} x^{6}+7680 d^{4} e^{5} x^{5}-23352 d^{5} e^{4} x^{4}-20480 d^{6} e^{3} x^{3}+3024 x^{2} d^{7} e^{2}+8960\right.}{26880 d^{2} x^{10}}$
default	$-\frac{e^{3}\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{7 d^{2} x^{7}}+d^{3}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{10 d^{2} x^{10}}+\frac{3 e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{8 d^{2} x^{8}}+\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{6 d^{2} x^{6}}-\frac{e^{2}\left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{4 d^{2} x^{4}}-\frac{3 e^{2}\left(-\frac{\left(-e^{2} x^{2}\right.}{2 d^{2}}\right.}{}\right)}{}\right)}{}\right)}{}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/26880*(-e^2*x^2+d^2)^(1/2)*(-6400*e^9*x^9-3465*d*e^8*x^8+10240*d^2*e^7*x
^7+24570*d^3*e^6*x^6+7680*d^4*e^5*x^5-23352*d^5*e^4*x^4-20480*d^6*e^3*x^3+3
024*d^7*e^2*x^2+8960*d^8*e*x+2688*d^9)/d^2/x^10+33/256/d*e^10/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx = -\frac{3465\,e^{10}x^{10}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (6400\,e^9x^9 + 3465\,de^8x^8 - 10240\,d^2e^7x^7 - 24570\,d^3e^6x^6 - 7680\,d^4e^5x^5}{26880\,d^2x^{10}}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(3465*e^10*x^10*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^9*x^9
 + 3465*d*e^8*x^8 - 10240*d^2*e^7*x^7 - 24570*d^3*e^6*x^6 - 7680*d^4*e^5*x^
5 + 23352*d^5*e^4*x^4 + 20480*d^6*e^3*x^3 - 3024*d^7*e^2*x^2 - 8960*d^8*e*x
 - 2688*d^9)*sqrt(-e^2*x^2 + d^2))/(d^2*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 136.08 (sec) , antiderivative size = 2159, normalized size of antiderivative = 9.60

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2) -
 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*d**
6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**
2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 7
*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768*d**6*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(e**2*x*
*2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e*Piecewise
```

```
((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**2) - 1)/
(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4) + 8*e**7
*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2)
 - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8
), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)
) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**
2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5
*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*
d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2)
 + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**
5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*a
sin(d/(e*x))/(128*d**7), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e
**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**
2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e
**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**6), True)) - 5*d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqr
t(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/
(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2
*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e*
*5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5)
, True)) + d**2*e**5*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/
d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)
/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(
-15*d**3*x**5 + 15*d*e**2*x**7), True)) + 3*d*e**6*Piecewise((-d**2/(4*e*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
- 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/
(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise(
(-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(
3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{33\,e^{10}\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{256\,d^2} - \frac{33\sqrt{-e^2x^2+d^2}e^{10}}{256\,d^3} - \frac{11\,(-e^2x^2+d^2)^{\frac{3}{2}}e^{10}}{256\,d^5} - \frac{33\,(-e^2x^2+d^2)^{\frac{5}{2}}e^{10}}{1280\,d^7} - \frac{33\,(-e^2x^2+d^2)^{\frac{7}{2}}e^8}{1280\,d^7x^2} + \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^6}{640\,d^5x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{160\,d^3x^6} - \frac{5\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{21\,d^2x^7} - \frac{33\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{80\,dx^8} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}e}{3\,x^9} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{10\,x^{10}}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 33/256*e^10*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 33/25
6*sqrt(-e^2*x^2 + d^2)*e^10/d^3 - 11/256*(-e^2*x^2 + d^2)^(3/2)*e^10/d^5 -
33/1280*(-e^2*x^2 + d^2)^(5/2)*e^10/d^7 - 33/1280*(-e^2*x^2 + d^2)^(7/2)*e^
8/(d^7*x^2) + 11/640*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^4) - 11/160*(-e^2*x^
2 + d^2)^(7/2)*e^4/(d^3*x^6) - 5/21*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^7) -
33/80*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^8) - 1/3*(-e^2*x^2 + d^2)^(7/2)*e/x^9
 - 1/10*(-e^2*x^2 + d^2)^(7/2)*d/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(193) = 386.

Time = 0.31 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \frac{\left(42\,e^{11} + \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^9}{x} + \frac{525\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^7}{x^2} - \frac{600\left(de+\sqrt{-e^2x^2+d\right.}}{x^3}\right.}{} + \frac{33\,e^{11}\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{256\,d^2|e|} + \frac{\frac{31920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{17}|e|}{x} - \frac{10500\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{18}e^{15}|e|}{x^2} - \frac{16800\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{18}e^{13}|e|}{x^3} - \frac{5880\left(de+\sqrt{-e^{2}\right.}}{}}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/430080*(42*e^11 + 280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^9/x + 525*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^7/x^2 - 600*(d*e + sqrt(-e^2*x^2 + d^2
```

```
)*abs(e))^3*e^5/x^3 - 3570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^3/x^4 -
3360*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e/x^5 + 5880*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^6/(e*x^6) + 16800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e
^3*x^7) + 10500*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^5*x^8) - 31920*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^7*x^9))*e^20*x^10/((d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^10*d^2*abs(e)) + 33/256*e^11*log(1/2*abs(-2*d*e - 2*sqrt(
-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/430080*(31920*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))*d^18*e^17*abs(e)/x - 10500*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2*d^18*e^15*abs(e)/x^2 - 16800*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^3*d^18*e^13*abs(e)/x^3 - 5880*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^4*d^18*e^11*abs(e)/x^4 + 3360*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^18*e
^9*abs(e)/x^5 + 3570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^18*e^7*abs(e)/
x^6 + 600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18*e^5*abs(e)/x^7 - 525*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^18*e^3*abs(e)/x^8 - 280*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^9*d^18*e*abs(e)/x^9 - 42*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^10*d^18*abs(e)/(e*x^10))/(d^20*e^10)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)^3}{x^{11}}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11, x)
```

3.82 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx=-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}+\frac{19e^{11}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}$$

```
[Out] 19/384*e^7*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-19/480*e^5*(-e^2*x^2+d^2)^(5/2)/d^2
/x^6-1/11*d*(-e^2*x^2+d^2)^(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10-37/9
9*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-74/
693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x^7+19/256*e^11*arctanh((-e^2*x^2+d^2)^(1/
2)/d)/d^3-19/256*e^9*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1821, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx=\frac{19e^{11}\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9}-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-33d^4e-37d^3e^2x-11d^2e^3x^2)}{x^{11}}\,dx}{11d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} + \frac{\int \frac{(370d^5e^2+209d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^{10}}\,dx}{110d^4} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} \\
&\quad - \frac{\int \frac{(-1881d^6e^3-740d^5e^4x)(d^2-e^2x^2)^{5/2}}{x^9}\,dx}{990d^6} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} \\
&\quad - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} + \frac{\int \frac{(5920d^7e^4+1881d^6e^5x)(d^2-e^2x^2)^{5/2}}{x^8}\,dx}{7920d^8} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} \\
&\quad - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{(19e^5)\int \frac{(d^2-e^2x^2)^{5/2}}{x^7}\,dx}{80d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} \\
&\quad - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{(19e^5)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{5/2}}{x^4}\,dx, x, x^2\right)}{160d^2}
\end{aligned}
$$

$$= -\frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} - \frac{(19e^7)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{192d^2}$$

$$= \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{(19e^9)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{256d^2}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} - \frac{(19e^{11})\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{512d^2}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{(19e^9)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{256d^2}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8} - \frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7} + \frac{19e^{11}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}} \, dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(-80640d^{10}-266112d^9ex-89600d^8e^2x^2+587664d^7e^3x^3+657920d^6e^4x^4-201432d^5e^5x^5-\right.}{x^{11}}}{}$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(-80640*d^10 - 266112*d^9*e*x - 89600*d^8*e^2*x^2 +
 587664*d^7*e^3*x^3 + 657920*d^6*e^4*x^4 - 201432*d^5*e^5*x^5 - 629760*d^4*
e^6*x^6 - 251790*d^3*e^7*x^7 + 47360*d^2*e^8*x^8 + 65835*d*e^9*x^9 + 94720*
e^10*x^10))/x^11 + 65835*Sqrt[d^2]*e^11*Log[x] - 65835*Sqrt[d^2]*e^11*Log[S
qrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(887040*d^4)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-94720e^{10}x^{10}-65835d\,e^9x^9-47360d^2e^8x^8+251790d^3e^7x^7+629760d^4e^6x^6+201432d^5e^5x^5-657920d^6e^4x^4-587664d}{887040x^{11}d^3}$
default	$e^3\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8d^2x^8}+\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6d^2x^6}-\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}\right.\right.}{}\right.}{4d^2}\right)}{6d^2}\right)}{8d^2}\right.$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/887040*(-e^2*x^2+d^2)^(1/2)*(-94720*e^10*x^10-65835*d*e^9*x^9-47360*d^2*
e^8*x^8+251790*d^3*e^7*x^7+629760*d^4*e^6*x^6+201432*d^5*e^5*x^5-657920*d^6
*e^4*x^4-587664*d^7*e^3*x^3+89600*d^8*e^2*x^2+266112*d^9*e*x+80640*d^10)/x^
11/d^3+19/256/d^2*e^11/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}} \, dx =$$

$$-\frac{65835\, e^{11}x^{11} \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (94720\, e^{10}x^{10} + 65835\, de^9x^9 + 47360\, d^2e^8x^8 - 251790\, d^3e^7x^7 - 62976}{8}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/887040*(65835*e^11*x^11*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (94720*e^10
*x^10 + 65835*d*e^9*x^9 + 47360*d^2*e^8*x^8 - 251790*d^3*e^7*x^7 - 629760*d
^4*e^6*x^6 - 201432*d^5*e^5*x^5 + 657920*d^6*e^4*x^4 + 587664*d^7*e^3*x^3 -
 89600*d^8*e^2*x^2 - 266112*d^9*e*x - 80640*d^10)*sqrt(-e^2*x^2 + d^2))/(d^
3*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 146.79 (sec) , antiderivative size = 2397, normalized size of antiderivative = 9.44

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}} \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/(
e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d**
4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*sqr
t(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x**2)
- 1)/(3465*d**10), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x**8) + 8*I*
e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt(-d**2/(e*
*2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(346
```

5*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**10), True))
 + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/
(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(
768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e
2*x2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqr
t(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768
*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + d**5*e**2*P
iecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**
2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4)
+ 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e*
*2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) +
 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2
/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(
315*d**8), True)) - 5*d**4*e**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**
5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2)
 - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e
*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(19
2*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d*
*2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) -
5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(
d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x
4) + 4*e5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2
/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/
(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**
4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-
d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e**5*Piecewise((-d**2/(6*e*
x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d*
*2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2))
 > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)
) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(
16*d**5), True)) + 3*d*e**6*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-1
5*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 -
e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**

```
2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**
2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/
d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**7*Piecewise((-d**2/(4*e
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1))
 - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3
), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1
)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx = \frac{19\,e^{11}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{256\,d^3} - \frac{19\,\sqrt{-e^2x^2+d^2}e^{11}}{256\,d^4} - \frac{19\,(-e^2x^2+d^2)^{\frac{3}{2}}e^{11}}{768\,d^6} - \frac{19\,(-e^2x^2+d^2)^{\frac{5}{2}}e^{11}}{1280\,d^8} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^9}{1280\,d^8x^2} + \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^7}{1920\,d^6x^4} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^5}{480\,d^4x^6} - \frac{74\,(-e^2x^2+d^2)^{\frac{7}{2}}e^4}{693\,d^3x^7} - \frac{19\,(-e^2x^2+d^2)^{\frac{7}{2}}e^3}{80\,d^2x^8} - \frac{37\,(-e^2x^2+d^2)^{\frac{7}{2}}e^2}{99\,dx^9} - \frac{3\,(-e^2x^2+d^2)^{\frac{7}{2}}e}{10\,x^{10}} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{11\,x^{11}}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 19/256*e^11*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 19/25
6*sqrt(-e^2*x^2 + d^2)*e^11/d^4 - 19/768*(-e^2*x^2 + d^2)^(3/2)*e^11/d^6 -
19/1280*(-e^2*x^2 + d^2)^(5/2)*e^11/d^8 - 19/1280*(-e^2*x^2 + d^2)^(7/2)*e^
9/(d^8*x^2) + 19/1920*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x^4) - 19/480*(-e^2*x
^2 + d^2)^(7/2)*e^5/(d^4*x^6) - 74/693*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^7)
 - 19/80*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^8) - 37/99*(-e^2*x^2 + d^2)^(7/2
)*e^2/(d*x^9) - 3/10*(-e^2*x^2 + d^2)^(7/2)*e/x^10 - 1/11*(-e^2*x^2 + d^2)^
(7/2)*d/x^11
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(218) = 436.

Time = 0.32 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}}\,dx = \frac{\left(630\,e^{12} + \frac{4158\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^{10}}{x} + \frac{8470\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e^8}{x^2} - \frac{3465\left(de+\sqrt{-e^2x\right.}}{x^3}\right.}{} + \frac{19\,e^{12}\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{256\,d^3|e|} + \frac{\frac{568260\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{30}e^{20}}{x} - \frac{152460\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^{30}e^{18}}{x^2} - \frac{244860\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^{30}e^{16}}{x^3} - \frac{138600\left(de+\sqrt{-e^2x^2+\right.}}{x^4}}{}$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/14192640*(630*e^12 + 4158*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^10/x + 84
70*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^8/x^2 - 3465*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^3*e^6/x^3 - 40590*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*e^
4/x^4 - 57750*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*e^2/x^5 + 6930*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^6/x^6 + 138600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^7/(e^2*x^7) + 244860*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^4*x^8) +
152460*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^6*x^9) - 568260*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^10/(e^8*x^10))*e^22*x^11/((d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^11*d^3*abs(e)) + 19/256*e^12*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x
^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*abs(e)) + 1/14192640*(568260*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))*d^30*e^20/x - 152460*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^2*d^30*e^18/x^2 - 244860*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3
0*e^16/x^3 - 138600*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^30*e^14/x^4 - 6
930*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^30*e^12/x^5 + 57750*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^6*d^30*e^10/x^6 + 40590*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^7*d^30*e^8/x^7 + 3465*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^30*
e^6/x^8 - 8470*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9*d^30*e^4/x^9 - 4158*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^10*d^30*e^2/x^10 - 630*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^11*d^30/x^11)/(d^33*e^10*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)^3}{x^{12}}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12, x)
```

3.83 $\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 174

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/5*d^4*(e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(e*x+d)^2/e^6/(-e^2*x^
2+d^2)^(3/2)-13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(e*x+
d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*x*(-e^2*x^2+d^
2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1649, 1829, 655, 223, 209}

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
```

```
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}}{\sqrt{d^2-e^2x^2}}\,dx}{30d^3e^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(13d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^5}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(13d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.64

$$\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(304d^4-717d^3ex+479d^2e^2x^2-45de^3x^3-15e^4x^4\right)}{(d-ex)^3} + 390d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{30e^6}$$

```
[In] Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^2 - 45*d*e^3*x
^3 - 15*e^4*x^4))/(d - e*x)^3 + 390*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2
- e^2*x^2])])/(30*e^6)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}-\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}-\frac{d^4\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^9\left(x-\frac{d}{e}\right)^3}-\frac{23d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15e^8\left(x-\frac{d}{e}\right)^2}-$
default	$e^3\left(-\frac{x^7}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7d^2\left(\frac{x^5}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)}{2e^2}\right)+d^3\Bigg(\frac{}{e^2}$

```
[In] int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*x+6*d)/e^6*(-e^2*x^2+d^2)^(1/2)-13/2*d^2/e^5/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*d^4/e^9/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e
*(x-d/e))^(1/2)-23/15*d^3/e^8/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2
)-127/15*d^2/e^7/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{304\,d^2e^3x^3-912\,d^3e^2x^2+912\,d^4ex-304\,d^5+390\,(d^2e^3x^3-3\,d^3e^2x^2+3\,d^4ex-d^5)}{30\,(e^9x^3-3\,}$$

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 - 912*d^3*e^2*x^2 + 912*d^4*e*x - 304*d^5 + 390*(d^2*
e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) + (15*e^4*x^4 + 45*d*e^3*x^3 - 479*d^2*e^2*x^2 + 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 - 3*d*e^8*x^2 + 3*d^2*e^7*x - d^3*e^6
)
```

Sympy [F]

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(152) = 304.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= -\frac{ex^7}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} \\ &+ \frac{13}{30}\,d^2ex\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) \\ &- \frac{3\,dx^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{13\,d^2x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{6\,e} \\ &+ \frac{19\,d^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{76\,d^5x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{152\,d^7}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} \\ &+ \frac{26\,d^4x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{91\,d^2x}{30\,\sqrt{-e^2x^2+d^2}e^5} - \frac{13\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^5}\end{aligned}$$

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] -1/2*e*x^7/(-e^2*x^2 + d^2)^(5/2) + 13/30*d^2*e*x*(15*x^4/((-e^2*x^2 + d^2)
^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
d^2)^(5/2)*e^6)) - 3*d*x^6/(-e^2*x^2 + d^2)^(5/2) - 13/6*d^2*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 19*d^3*x^
4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 76/3*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 152/15*d^7/((-e^2*x^2 + d^2)^(5/2)*e^6) + 26/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^5) - 91/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 13/2*d^2*arcsin(e^2*x/
(d*sqrt(e^2)))/(sqrt(e^2)*e^5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.34

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^5}+\frac{6\,d}{e^6}\right) - \frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^5|e|} + \frac{2\left(107\,d^2 - \frac{445\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x} + \frac{665\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2} - \frac{405\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3} + \frac{90\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^2}{e^8x^4}\right)}{15\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-e^2*x^2 + d^2)*(x/e^5 + 6*d/e^6) - 13/2*d^2*arcsin(e*x/d)*sgn(d)*
sgn(e)/(e^5*abs(e)) + 2/15*(107*d^2 - 445*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))*d^2/(e^2*x) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2) -
405*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 90*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^5*((d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^5\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.84 $\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/5*d^3*(e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)-6/5*d^2*(e*x+d)^2/e^5/(-e^2*x^2+
d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+24/5*d*(e*x+d)/e^5/(-e^
2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1649, 655, 223, 209}

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} + \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

```
[In] Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{(d+ex)\left(\frac{27d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{\frac{45d^4}{e^4}+\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}\\
&= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}\\
&\quad+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{(3d)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(24d^3-57d^2ex+39de^2x^2-5e^3x^3\right)}{(d-ex)^3} + 30d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{5e^5}$$

```
[In] Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5*e^3*x^3))/(d
- e*x)^3 + 30*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(5*e^5)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.37

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^5} - \frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}} - \frac{d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^8\left(x-\frac{d}{e}\right)^3} - \frac{6d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^7\left(x-\frac{d}{e}\right)^2} - \frac{24d\sqrt{-\left(x-\frac{d}{e}\right.\cdots}}{5e^6\left(\cdots\right.}$
default	$e^3\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right) + d^3\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}\cdots\right.$

```
[In] int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))-1/5*d^3/e^8/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-6/5
*d^2/e^7/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-24/5*d/e^6/(x-d/e)*
(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.25

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{24\,de^3x^3 - 72\,d^2e^2x^2 + 72\,d^3ex - 24\,d^4 + 30\,(de^3x^3 - 3\,d^2e^2x^2 + 3\,d^3ex - d^4)\arctan}{5\,(e^8x^3 - 3\,de^7x^2 + 3\,d^2e}$$

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*d*e^3*x^3 - 72*d^2*e^2*x^2 + 72*d^3*e*x - 24*d^4 + 30*(d*e^3*x^3 -
3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
+ (5*e^3*x^3 - 39*d*e^2*x^2 + 57*d^2*e*x - 24*d^3)*sqrt(-e^2*x^2 + d^2))/(e
^8*x^3 - 3*d*e^7*x^2 + 3*d^2*e^6*x - d^3*e^5)
```

Sympy [F]

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(126) = 252.

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.37

$$\begin{aligned}\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{1}{5}\,de^2x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right)\\
&- \frac{ex^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - dx\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)\\
&+ \frac{9\,d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^3x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{12\,d^4x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{3\,d^5x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4}\\
&+ \frac{24\,d^6}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^5} + \frac{9\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{6\,dx}{5\,\sqrt{-e^2x^2+d^2}e^4} - \frac{3\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^4}\end{aligned}$$

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 + d
^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^
2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^5*
x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5) + 9/
10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2)*e^4)
- 3*d*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{3\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|}+\frac{\sqrt{-e^2x^2+d^2}}{e^5} + \frac{2\left(19\,d-\frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x}+\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2}-\frac{70\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{5\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^5 +
2/5*(19*d - 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 120*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) - 70*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8*x^4
))/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^4\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.85 $\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(e*x+d)^2/e^4/(-e^2*x^2+
d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(e*x+d)/e^4/(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1649, 792, 223, 209}

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3} \\
&= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3} \\
&= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(22d^2-51dex+32e^2x^2)}{(d-ex)^3}+30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

[In] Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]

[Out] ((Sqrt[d^2 - e^2*x^2]*(22*d^2 - 51*d*e*x + 32*e^2*x^2))/(d - e*x)^3 + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(104) = 208.

Time = 0.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.19

method	result
default	$e^3\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+d^3\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d}{15e^4(-e^2x}\right.$

[In] int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] e^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+d^3*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+3*d*e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+3*d^2*e*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{22\,e^3x^3 - 66\,de^2x^2 + 66\,d^2ex - 22\,d^3 + 30\,(e^3x^3 - 3\,de^2x^2 + 3\,d^2ex - d^3)\arctan\left(-\frac{d-}{}\right.}{15\,(e^7x^3 - 3\,de^6x^2 + 3\,d^2e^5x - d^3e}$$

[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/15*(22*e^3*x^3 - 66*d*e^2*x^2 + 66*d^2*e*x - 22*d^3 + 30*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (32*e^2*x^2 - 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 - 3*d*e^6*x^2 + 3*d^2*e^5*x - d^3*e^4)

Sympy [F]

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

[In] integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(104) = 208.

Time = 0.30 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.61

$$\begin{aligned}\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{1}{15}\,e^3x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right)\\&\quad - \frac{1}{3}\,ex\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{3\,dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}}\\&\quad + \frac{3\,d^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{11\,d^3x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{9\,d^4x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3}\\&\quad + \frac{22\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{17\,d^2x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}e^3} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^3}\end{aligned}$$

[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

```
[Out] 1/15*e^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 3*d*x^4/(-e
^2*x^2 + d^2)^(5/2) + 3/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - 11/3*d^3*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2
2/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^4) + 17/30*d^2*x/((-e^2*x^2 + d^2)^(3/2)
*e^3) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e^2*x/(d*sqrt(e^2)))/(sq
rt(e^2)*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} - \frac{2\left(\frac{95\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{145\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 22\right)}{15\,e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 2/15*(95*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))/(e^2*x) - 145*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2)
 + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) - 15*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 22)/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3\,{(d+e\,x)}^3}{{(d^2-e^2\,x^2)}^{7/2}} \,dx$$

```
[In] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.86 $\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 93

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d*(e*x+d)^3/e^3/(-e^2*x^2+d^2)^(5/2)-8/15*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^
(3/2)+7/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1649, 803, 651}

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{\left(\frac{3d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}\\
&= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(2d^2-6dex+7e^2x^2\right)}{15de^3(d-ex)^3}$$

```
[In] Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
trager	$\frac{\left(7 e^{2} x^{2}-6 d e x+2 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{15 d\, e^{3} \left(-e x+d\right)^{3}}$
gosper	$\frac{\left(-e x+d\right) \left(e x+d\right)^{4} \left(7 e^{2} x^{2}-6 d e x+2 d^{2}\right)}{15 d\, e^{3} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}$
default	$e^{3} \left(\frac{x^{4}}{e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4 d^{2} \left(\frac{x^{2}}{3 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2 d^{2}}{15 e^{4} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)+d^{3} \left(\frac{x}{4 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2} \left(\frac{x}{5 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}+\right.}{}\right.$

```
[In] int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e*x+d)^3*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^3x^3-6\,de^2x^2+6\,d^2ex-2\,d^3-(7\,e^2x^2-6\,dex+2\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(de^6x^3-3\,d^2e^5x^2+3\,d^3e^4x-d^4e^3)}$$

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] 1/15*(2*e^3*x^3 - 6*d*e^2*x^2 + 6*d^2*e*x - 2*d^3 - (7*e^2*x^2 - 6*d*e*x +
2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - 3*d^2*e^5*x^2 + 3*d^3*e^4*x - d^4
*e^3)
```

Sympy [F]

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,dx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{d^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{7\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{2\,d^4}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{7\,dx}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2} + \frac{7\,x}{15\,\sqrt{-e^2x^2+d^2}de^2}$$

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*x
^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 2
/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2
) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{4\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} - 1\right)}{15\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] -4/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 10*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e^4*x^2) - 1)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(2\,d^2-6\,d\,e\,x+7\,e^2\,x^2)}{15\,d\,e^3\,(d-e\,x)^3}$$

```
[In] int((x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 - 6*d*e*x))/(15*d*e^3*(d - e*x)^3
)
```

3.87 $\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^3/e^2/(-e^2*x^2+d^2)^(5/2)-2/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)
-1/5*x/d^2/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {803, 667, 197}

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3\int\frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}\\ &= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int\frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{5e}\\ &= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

$$\int\frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}(d^2-3dex+e^2x^2)}{5d^2e^2(d-ex)^3}$$

```
[In] Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + e^2*x^2))/(d^2*e^2*(d - e*x)^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result
trager	$-\frac{(e^2x^2-3dex+d^2)\sqrt{-e^2x^2+d^2}}{5d^2(-ex+d)^3e^2}$
gosper	$-\frac{(-ex+d)(ex+d)^4(e^2x^2-3dex+d^2)}{5d^2e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+\frac{}{5e^2(-}$

```
[In] int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(e^2*x^2-3*d*e*x+d^2)/d^2/(-e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-(e^2x^2-3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{5\,(d^2e^5x^3-3\,d^3e^4x^2+3\,d^4e^3x-d^5e^2)}$$

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 - 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 - 3*d^3*e^4*x^2 + 3*d^4*e^3*x - d^5*e^2)
```

Sympy [F]

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{dx^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{d^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e}$$

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^3/(-e^2*x^2 + d^2)^(5/2) + d*x^2/(-e^2*x^2 + d^2)^(5/2) + 3/10*d^2*
x/((-e^2*x^2 + d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/10*
x/((-e^2*x^2 + d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.59

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - 1\right)}{5\,d^2e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 5*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2/(e^4*x^2) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*
x^3) - 1)/(d^2*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e)
)
```

Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2-3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,\left(d-e\,x\right)^3}$$

```
[In] int((x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 - 3*d*e*x))/(5*d^2*e^2*(d - e*x)^3)
```

3.88 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {669, 673, 665}

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
trager	$\frac{(2e^2x^2-6dex+7d^2)\sqrt{-e^2x^2+d^2}}{15d^3(-ex+d)^3 e}$
gosper	$\frac{(-ex+d)(ex+d)^4(2e^2x^2-6dex+7d^2)}{15d^3e(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$d^3\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+3d\,e$

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/(-e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-(2\,e^2x^2-6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e)}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{4\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7\,d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 40*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/
(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 7)/(d^3*((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2)}{15\,d^3\,e\,(d-e\,x)^3}$$

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

3.89 $\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/2)
-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(22*e*x+15*d)/d^4/(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1819, 837, 12, 272, 65, 214}

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}+\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-11d^2ex}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2-22d^4e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(32d^2-51dex+22e^2x^2\right)}{(d-ex)^3} - 15\sqrt{d^2}\log(x) + 15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

```
[In] Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(32*d^2 - 51*d*e*x + 22*e^2*x^2))/(d - e*x)^3 - 15*
Sqrt[d^2]*Log[x] + 15*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d
^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(100) = 200.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.68

method	result
default	$e^{3}\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2}\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)}{4e^{2}}\right)+d^{3}\left(\frac{1}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{}{3d^{2}}\right.$

```
[In] int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)
)))+d^3*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1
/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x))))+3*d^2*e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2
*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+3/5*d/(-e
^2*x^2+d^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{32\,e^3x^3-96\,de^2x^2+96\,d^2ex-32\,d^3+15\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\log\left(-\frac{d-}{}\right.}{15\left(d^4e^3x^3-3\,d^5e^2x^2+3\,d^6ex-\right.}$$

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 - 96*d*e^2*x^2 + 96*d^2*e*x - 32*d^3 + 15*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (22*e^2*x^2
- 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 - 3*d^5*e^2*x^2 + 3
*d^6*e*x - d^7)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{\left(d+ex\right)^3}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4\,ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{11\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} + \frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{22\,ex}{15\sqrt{-e^2x^2+d^2}d^4} - \frac{\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d^4} + \frac{1}{\sqrt{-e^2x^2+d^2}d^3}$$

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 4/5*e*x/(-e^2*x^2 + d^2)^(5/2) + 4/5*d/(-e^2*x^2 + d^2)^(5/2) + 11/15*e*x/(
(-e^2*x^2 + d^2)^(3/2)*d^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d) + 22/15*e*x/(s
qrt(-e^2*x^2 + d^2)*d^4) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(
x))/d^4 + 1/(sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} + \frac{2\left(32\,e - \frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex} + \frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2} - \frac{135\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{15\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*a
bs(e)) + 2/15*(32*e - 115*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 185*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) - 135*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3/(e^5*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^7*x
^4))/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d+e\,x)^3}{x\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

```
[In] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.90 $\int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}}\,dx=\frac{4e(d+ex)}{5d(d^2-e^2x^2)^{5/2}}+\frac{e(5d+7ex)}{5d^3(d^2-e^2x^2)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 4/5*e*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/5*e*(7*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(
3/2)-3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/5*e*(19*e*x+15*d)/d^5/(-e^2*
x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1819, 821, 272, 65, 214}

$$\int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}}\,dx=-\frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{4e(d+ex)}{5d(d^2-e^2x^2)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}+\frac{e(5d+7ex)}{5d^3(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-16de^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+42de^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^3-45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{(3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{(3e)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^4e}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(5d^3-39d^2ex+57de^2x^2-24e^3x^3\right)}{x(-d+ex)^3}-15\sqrt{d^2}e\log(x)+15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{5d^6}$$

```
[In] Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(5*d^3 - 39*d^2*e*x + 57*d*e^2*x^2 - 24*e^3*x^3))/(
x*(-d + e*x)^3) - 15*Sqrt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2]])/(5*d^6)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}-\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}+\frac{4\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5d^4e\left(x-\frac{d}{e}\right)^2}-\frac{19\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5d^5\left(x-\frac{d}{e}\right)}-\frac{\sqrt{-(x}}{}$
default	$\frac{e}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+d^3\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{d^2}\right)+3d\,e^2\Big($

```
[In] int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^5/x-3/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)+4/5/d^4/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1
/2)-19/5/d^5/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/5/d^3/e^2/(x-d/
e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{24\,e^4x^4-72\,de^3x^3+72\,d^2e^2x^2-24\,d^3ex+15\left(e^4x^4-3\,de^3x^3+3\,d^2e^2x^2-d^3ex\right)}{5\left(d^5e^3x^4-3\,d^6e^2x^3+\right.}$$

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*e^4*x^4 - 72*d*e^3*x^3 + 72*d^2*e^2*x^2 - 24*d^3*e*x + 15*(e^4*x^4
- 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 - (24*e^3*x^3 - 57*d*e^2*x^2 + 39*d^2*e*x - 5*d^3)*sqrt(-e^2*x^2 + d^2))/(
d^5*e^3*x^4 - 3*d^6*e^2*x^3 + 3*d^7*e*x^2 - d^8*x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{(d+ex)^3}{x^2\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{9\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4\,e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{12\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}-\frac{d}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{24\,e^2x}{5\sqrt{-e^2x^2+d^2}\,d^5}-\frac{3\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^5}+\frac{3\,e}{\sqrt{-e^2x^2+d^2}\,d^4}$$

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 9/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d) + 4/5*e/(-e^2*x^2 + d^2)^(5/2) + 12/5*
e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + e/((-e^2*x^2 + d^2)^(3/2)*d^2) - d/((-
e^2*x^2 + d^2)^(5/2)*x) + 24/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^5) - 3*e*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 + 3*e/(sqrt(-e^2*x^2 + d
^2)*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(129) = 258.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^5|e|}-\frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^5x|e|}-\frac{\left(5\,e^2-\frac{121\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x}+\frac{410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2}-\frac{610\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3}+\frac{425\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4}-125\,(de+\right.}{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d
^5*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^5*x*abs(e)) - 1/10*
(5*e^2 - 121*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 410*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2/(e^2*x^2) - 610*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(
e^4*x^3) + 425*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) - 125*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^3}{x^2\,{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

```
[In] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] 4/5*e^2*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(31*e*x+25*d)/d^4/(-e^2*x
^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(107*e*
x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-3*e*(-e^2
*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
```

```
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-20de^2x^2-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+75de^2x^2+62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex-90de^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{90d^4e+195d^3e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{\left(13e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^5}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{\left(13e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d^5}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} - \frac{13\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^5}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} - \frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^3}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(15d^4+45d^3ex-479d^2e^2x^2+717de^3x^3-304e^4x^4)}{x^2(-d+ex)^3} - 195\sqrt{d^2}e^2\log(x) + 195\sqrt{d^2}e^2\log(}{30d^7}$$

```
[In] Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 + 45*d^3*e*x - 479*d^2*e^2*x^2 + 717*d*e^3*
x^3 - 304*e^4*x^4))/(x^2*(-d + e*x)^3) - 195*Sqrt[d^2]*e^2*Log[x] + 195*Sqr
t[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(30*d^7)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(6ex+d)}{2d^6x^2} - \frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}} - \frac{107e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15d^6\left(x-\frac{d}{e}\right)} + \frac{17\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\right.}}{15d^5\left(x-\frac{d}{e}\right)^2}$
default	$e^3\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right) + d^3\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\right.}{}\right.$

```
[In] int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(6*e*x+d)/d^6/x^2-13/2/d^5*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-107/15/d^6*e/(x-d/e)*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)+17/15/d^5/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e)
)^(1/2)-1/5/d^4/e/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{254\,e^5x^5-762\,de^4x^4+762\,d^2e^3x^3-254\,d^3e^2x^2+195\,(e^5x^5-3\,de^4x^4+3\,d^2e^3x^3-}{30\,(d^6e^3x^5-}$$

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 - 762*d*e^4*x^4 + 762*d^2*e^3*x^3 - 254*d^3*e^2*x^2 + 195
*(e^5*x^5 - 3*d*e^4*x^4 + 3*d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (304*e^4*x^4 - 717*d*e^3*x^3 + 479*d^2*e^2*x^2 - 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 - 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 - d^9*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{x^3\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex)^3}{x^3 (d^2-e^2x^2)^{7/2}}\,dx &= \frac{19\,e^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}+\frac{13\,e^2}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}d} \\ &+\frac{76\,e^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}+\frac{13\,e^2}{6\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}-\frac{3\,e}{(-e^2x^2+d^2)^{\frac{5}{2}}x}+\frac{152\,e^3x}{15\,\sqrt{-e^2x^2+d^2}d^6} \\ &-\frac{13\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^6}+\frac{13\,e^2}{2\,\sqrt{-e^2x^2+d^2}d^5}-\frac{d}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}x^2}\end{aligned}$$

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 19/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 13/10*e^2/((-e^2*x^2 + d^2)^(5/2)
*d) + 76/15*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 13/6*e^2/((-e^2*x^2 + d^2)
```

```
^(3/2)*d^3) - 3*e/((-e^2*x^2 + d^2)^(5/2)*x) + 152/15*e^3*x/(sqrt(-e^2*x^2
+ d^2)*d^6) - 13/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/
d^6 + 13/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^5) - 1/2*d/((-e^2*x^2 + d^2)^(5/2)*x
^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = -\frac{13\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^6|e|} - \frac{\left(15\,e^3+\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}-\frac{2782\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}+\frac{9410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3}-\frac{13645\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}+\cdots\right)}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e\cdots} - \frac{\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,d^{12}e^2}$$

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))
/(d^6*abs(e)) - 1/120*(15*e^3 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x
 - 2782*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) + 9410*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 13645*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^4/(e^5*x^4) + 9285*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5) - 258
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d^6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1
)^5*abs(e)) - 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs(e)/x +
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/(d^12*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^3}{x^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

```
[Out] 3/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+4/15*d^2*x^2*(-e^2*x^2+d^2)^(1
/2)/e^3-1/4*d*x^3*(-e^2*x^2+d^2)^(1/2)/e^2+1/5*x^4*(-e^2*x^2+d^2)^(1/2)/e+1
/120*d^3*(-45*e*x+64*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 847, 794, 223, 209}

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}$$

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x^3(4d^2e-5de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2} \\
&= -\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{\int \frac{x^2(15d^3e^2-16d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x(32d^4e^3-45d^3e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} \\
&\quad + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^4} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} \\
&\quad + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^4} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} \\
&\quad + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$
\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx
= \frac{\sqrt{d^2-e^2x^2}(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4)-90d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{120e^5}
$$

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3 +
 24*e^4*x^4) - 90*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(120
*e^5)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
risch	$\frac{(24e^4x^4-30d\,e^3x^3+32d^2e^2x^2-45d^3ex+64d^4)\sqrt{-e^2x^2+d^2}}{120e^5}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^4\sqrt{e^2}}$
default	$\frac{-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{5e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{15e^4}}{e}-\frac{d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3e^5}-\frac{d^3\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^4}-\frac{d\left(-\frac{x\left(-e^2x^2+d^2\right)}{4e^2}\right.}{}$

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(24*e^4*x^4-30*d*e^3*x^3+32*d^2*e^2*x^2-45*d^3*e*x+64*d^4)/e^5*(-e^2*
x^2+d^2)^(1/2)+3/8*d^5/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{90\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4-30\,de^3x^3+32\,d^2e^2x^2-45\,d^3ex+64\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^5}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(90*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 - 30
*d*e^3*x^3 + 32*d^2*e^2*x^2 - 45*d^3*e*x + 64*d^4)*sqrt(-e^2*x^2 + d^2))/e^
5
```

Sympy [F]

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^4\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^5}-\frac{5\,\sqrt{-e^2x^2+d^2}d^3x}{8\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e^3}+\frac{\sqrt{-e^2x^2+d^2}d^4}{e^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15\,e^5}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*d^5*arcsin(e*x/d)/e^5 - 5/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^4 - 1/5*(-e^2*
x^2 + d^2)^(3/2)*x^2/e^3 + sqrt(-e^2*x^2 + d^2)*d^4/e^5 + 1/4*(-e^2*x^2 + d
^2)^(3/2)*d*x/e^4 - 7/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{3\,d^5 \arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,e^4|e|} + \frac{1}{120}\,\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x\left(\frac{4\,x}{e}-\frac{5\,d}{e^2}\right)+\frac{16\,d^2}{e^3}\right)x-\frac{45\,d^3}{e^4}\right)x+\frac{64\,d^4}{e^5}\right)$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] 3/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/120*sqrt(-e^2*x^2 + d^
2)*((2*(3*x*(4*x/e - 5*d/e^2) + 16*d^2/e^3)*x - 45*d^3/e^4)*x + 64*d^4/e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{3d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

```
[Out] -3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/3*d*x^2*(-e^2*x^2+d^2)^(1/2
)/e^2+1/4*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/24*d^2*(-9*e*x+16*d)*(-e^2*x^2+d^2)^
(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 847, 794, 223, 209}

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{3d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} +\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}$$

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -1/3*(d*x^2*Sqrt[d^2 - e^2*x^2])/e^2 + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{\int \frac{x^2(3d^2e-4de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} + \frac{\int \frac{x(8d^3e^2-9d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{(3d^4)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^3}
\end{aligned}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{(3d^4)\operatorname{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(-16d^3+9d^2ex-8de^2x^2+6e^3x^3)+18d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^4}$$

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) + 18*d
^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(24*e^4)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(-6e^3x^3+8d\,e^2x^2-9d^2ex+16d^3)\sqrt{-e^2x^2+d^2}}{24e^4}-\frac{3d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^3\sqrt{e^2}}$
default	$\frac{-\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}}{e}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^3}+\frac{d(-e^2x^2+d}{3e^4}$

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-6*e^3*x^3+8*d*e^2*x^2-9*d^2*e*x+16*d^3)/e^4*(-e^2*x^2+d^2)^(1/2)-3/
8*d^4/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{18\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-8\,de^2x^2+9\,d^2ex-16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(18*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 8*d*e
^2*x^2 + 9*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [F]

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{3\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^4}+\frac{5\,\sqrt{-e^2x^2+d^2}d^2x}{8\,e^3}-\frac{\sqrt{-e^2x^2+d^2}d^3}{e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x}{4\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(e*x/d)/e^4 + 5/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^3 - sqrt(-e^2
*x^2 + d^2)*d^3/e^4 - 1/4*(-e^2*x^2 + d^2)^(3/2)*x/e^3 + 1/3*(-e^2*x^2 + d^
2)^(3/2)*d/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{3\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^3|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\,x\left(\frac{3\,x}{e}-\frac{4\,d}{e^2}\right)+\frac{9\,d^2}{e^3}\right)x-\frac{16\,d^3}{e^4}\right)$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] -3/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 1/24*sqrt(-e^2*x^2 + d^
2)*((2*x*(3*x/e - 4*d/e^2) + 9*d^2/e^3)*x - 16*d^3/e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.94 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+
1/2*d*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1653, 12, 799, 794, 223, 209}

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}+\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{\int \frac{3de^3x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}\\ &= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{d\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{e}\\ &= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\end{aligned}$$

$$= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{d^3 \int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}$$

$$= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{d^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

$$= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(4d^2-3dex+2e^2x^2)}{6e^3} - \frac{d^3 \arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2))/(6*e^3) - (d^3*ArcTan[(
e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

method	result
risch	$\frac{(2e^2x^2-3dex+4d^2)\sqrt{-e^2x^2+d^2}}{6e^3} + \frac{d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^3} - \frac{d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^2} + \frac{d^2 \Bigg(\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de \arctan\Bigg(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2d\cdots}}}{\sqrt{e^2}}}{e^3}$

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*e^2*x^2-3*d*e*x+4*d^2)/e^3*(-e^2*x^2+d^2)^(1/2)+1/2*d^3/e^2/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(2\,e^2x^2-3\,dex+4\,d^2)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2*e^2*x^2 - 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}d^2}{e^3}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] 1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 + sqrt(-e^2*x^
2 + d^2)*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|} + \frac{1}{6}\,\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e}-\frac{3\,d}{e^2}\right)+\frac{4\,d^2}{e^3}\right)$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] 1/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/6*sqrt(-e^2*x^2 + d^2)
*(x*(2*x/e - 3*d/e^2) + 4*d^2/e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

```
[Out] -1/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/2*(-e*x+2*d)*(-e^2*x^2+d^2)
^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {799, 794, 223, 209}

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2} - \frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}$$

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -1/2*((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/e^2 - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{de}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{(-2d+ex)\sqrt{d^2-e^2x^2}}{2e^2} + \frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2*ArcTan[(e*x)/(Sqrt[d^2] -
 Sqrt[d^2 - e^2*x^2])])/e^2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^2}-\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e\sqrt{e^2}}$	64
default	$\frac{\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}}{e}-\frac{d\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^2}$	135

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
[Out] -1/2*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^2-1/2*d^2/e/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{2\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-2\,d)}{2\,e^2}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] 1/2*(2*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2)
*(e*x - 2*d))/e^2
```

Sympy [F]

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^2} + \frac{\sqrt{-e^2x^2+d^2}x}{2\,e} - \frac{\sqrt{-e^2x^2+d^2}d}{e^2}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] -1/2*d^2*arcsin(e*x/d)/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*x/e - sqrt(-e^2*x^2 +
 d^2)*d/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e|e|} + \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e}-\frac{2\,d}{e^2}\right)$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] -1/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) + 1/2*sqrt(-e^2*x^2 + d^2)*
(x/e - 2*d/e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
[Out] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {679, 223, 209}

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{e} + d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{\sqrt{d^2-e^2x^2}}{e} + d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{\sqrt{d^2-e^2x^2}}{e} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$
\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e} - \frac{d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}
$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
[Out] Sqrt[d^2 - e^2*x^2]/e - (d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt
[-e^2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$	49
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{e}$	78

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/e+d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{2\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\sqrt{-e^2x^2+d^2}}{e}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - sqrt(-e^2*x^2 + d^2))/e
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{\sqrt{-e^2x^2+d^2}}{e}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] d*arcsin(e*x/d)/e + sqrt(-e^2*x^2 + d^2)/e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|}+\frac{\sqrt{-e^2x^2+d^2}}{e}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + sqrt(-e^2*x^2 + d^2)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(d + e*x), x)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 46

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)} \, dx = -\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -arctan(e*x/(-e^2*x^2+d^2)^(1/2))-arctanh((-e^2*x^2+d^2)^(1/2)/d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 858, 223, 209, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)} \, dx = -\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= d\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}
\end{aligned}
$$

$$= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = 2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2}\left(-\log(x)+\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)}{d}$$

[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]

[Out] 2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (Sqrt[d^2]*(-Log[x] + Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(42) = 84.

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

method	result	size
default	$\frac{\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}}{d}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{d}$	141

[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/d*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))-1/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = 2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
[Out] 2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + log(-(d - sqrt(-e^2*x^2 + d^2
))/x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = -\frac{e\left(\frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{d\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{e}\right)}{d}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
[Out] -e*(d*arcsin(e*x/d)/e + d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x
))/e)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = -\frac{e\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
[Out] -e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*
x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 51

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{dx} - e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{dx} - \frac{1}{2}e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\log(x)}{\sqrt{d^2}}-\frac{e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*Log[x])/Sqrt[d^2] - (e*Log[Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2]])/Sqrt[d^2]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{dx}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}}{d}-\frac{e\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^2}+\frac{e\left(\sqrt{-(x \ldots\right.}{\ldots}$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
[Out] -(-e^2*x^2+d^2)^(1/2)/d/x+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=-\frac{ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}}{dx}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
[Out] -(e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/(d*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^2\left(d+ex\right)}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(47)=94$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx=\frac{e^4x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d|e|}+\frac{e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d|e|}-\frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,dx|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
[Out] 1/2*e^4*x/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*abs(e)) + e^2*log(1/2*abs(
-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/2*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))/(d*x*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

```
[Out] -1/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/2*(-e^2*x^2+d^2)^(1/2)/d/x^2
+e*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 849, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=-\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[d^2 - e^2*x^2]/(d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{\int \frac{2d^2e-de^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}
\end{aligned}
$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$$
$$= -\frac{d(d-2ex)\sqrt{d^2-e^2x^2} + \sqrt{d^2}e^2x^2\log(x) - \sqrt{d^2}e^2x^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^3x^2}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
[Out] -1/2*(d*(d - 2*e*x)*Sqrt[d^2 - e^2*x^2] + Sqrt[d^2]*e^2*x^2*Log[x] - Sqrt[d
^2]*e^2*x^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(d^3*x^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^2x^2} - \frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2} - \frac{e^2\left(\sqrt{-e^2x^2+d^2} - \frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}}{d} + \frac{e^2\left(\sqrt{-e^2x^2+d^2} - \frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^3} - \frac{e \ldots}{\ldots}$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d*e^2/(d^2)^(1/2)*ln((2*d^
2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx = \frac{e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(2\,ex-d)}{2\,d^2x^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
[Out] 1/2*(e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*(2*e
*x - d))/(d^2*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^3(d+ex)}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(72) = 144.

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx = \frac{\left(e^3-\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|} - \frac{e^3\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^2|e|} + \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,d^4e^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/8*(e^3 - 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x)*e^4*x^2/((d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*d^2*abs(e)) - 1/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/8*(4*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x - (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
2*d^2*abs(e)/(e*x^2))/(d^4*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,\left(d+e\,x\right)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

```
[Out] 1/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/3*(-e^2*x^2+d^2)^(1/2)/d/x^3+
1/2*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-2/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 849, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \frac{e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/3*Sqrt[d^2 - e^2*x^2]/(d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2
*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(2*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{3d^2e-2de^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}$$

$$
\begin{aligned}
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} + \frac{\int \frac{4d^3e^2-3d^2e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$
\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \frac{(-2d^2+3dex-4e^2x^2)\sqrt{d^2-e^2x^2}}{6d^3x^3} + \frac{\sqrt{d^2}e^3\log(x)}{2d^4} - \frac{\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^4}
$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*d^3*x^3) + (Sqrt[d^
2]*e^3*Log[x])/(2*d^4) - (Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]
])/(2*d^4)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4e^2x^2-3dex+2d^2)}{6d^3x^3}+\frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3d^3x^3}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}\right)}{d^3}-\frac{e^3\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}}{}\right.}{\sqrt{d^2}}\right.}{d^4}$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-e^2*x^2+d^2)^(1/2)*(4*e^2*x^2-3*d*e*x+2*d^2)/d^3/x^3+1/2/d^2*e^3/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = -\frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(4\,e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,d^3x^3}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (4*e^2*x^2 - 3*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \frac{\left(e^4 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x} + \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^3|e|} + \frac{e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^3|e|} - \frac{\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e^4}{x} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6e^2}{x^2} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^6}{x^3}}{24\,d^9e^2|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/24*(e^4 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 9*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/x^2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*
d^3*abs(e)) + 1/2*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(
e^2*abs(x)))/(d^3*abs(e)) - 1/24*(9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6
*e^4/x - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*e^2/x^2 + (d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^3*d^6/x^3)/(d^9*e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,\left(d+e\,x\right)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

```
[Out] -3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/4*(-e^2*x^2+d^2)^(1/2)/d/x^4
+1/3*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2+2/
3*e^3*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 849, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx=-\frac{3e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[d^2 - e^2*x^2]/(d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3
*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*
x) - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{\int \frac{4d^2e-3de^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}+\frac{\int \frac{9d^3e^2-8d^2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{\int \frac{16d^4e^3-9d^3e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}+\frac{(3e^4)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}\\
&\quad+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}+\frac{(3e^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{16d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}\\
&\quad+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{8d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex-9de^2x^2+16e^3x^3)}{24d^4x^4} - \frac{3\sqrt{d^2}e^4\log(x)}{8d^5} + \frac{3\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{8d^5}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3))/(24*d
^4*x^4) - (3*Sqrt[d^2]*e^4*Log[x])/(8*d^5) + (3*Sqrt[d^2]*e^4*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/(8*d^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-16e^3x^3+9d\,e^2x^2-8d^2ex+6d^3)}{24d^4x^4}-\frac{3e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^3\sqrt{d^2}}$
default	$\frac{-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{4d^2x^4}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}\right)}{4d^2}}{d}+\frac{e^4\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{}}{x}\right)}{\sqrt{d^2}}\right.}{d^5}$

[In] int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(-16*e^3*x^3+9*d*e^2*x^2-8*d^2*e*x+6*d^3)/d^4/x^4-3/8/d^3*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \frac{9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(16\,e^3x^3-9\,de^2x^2+8\,d^2ex-6\,d^3)\sqrt{-e^2x^2+d^2}}{24\,d^4x^4}$$

[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")

[Out] 1/24*(9*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 - 9*d*e^2*x^2 + 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*x^4)

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{-(-d+ex)\,(d+ex)}}{x^5\,(d+ex)}\,dx$$

[In] integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)

[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^5}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(123) = 246$.

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \frac{\left(3\,e^5-\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x}+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2}-\frac{72\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4|e|} - \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d^4|e|} + \frac{\frac{72\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{12}e^5|e|}{x}-\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{12}e^3|e|}{x^2}+\frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{12}e|e|}{x^3}-\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{12}|e|}{ex^4}}{192\,d^{16}e^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*(3*e^5 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 24*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 72*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*abs(e)) - 3/8*
e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*
abs(e)) + 1/192*(72*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^12*e^5*abs(e)/x -
 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^12*e^3*abs(e)/x^2 + 8*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^3*d^12*e*abs(e)/x^3 - 3*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^4*d^12*abs(e)/(e*x^4))/(d^16*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2 - e^2x^2}}{x^5(d + ex)} \, dx = \int \frac{\sqrt{d^2 - e^2\,x^2}}{x^5\,\left(d + e\,x\right)} \, dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex} \, dx = \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] 1/12*d*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)/e^3-1/5*(-e^2*x^2+d^2)^(5/2)/e^3+1
/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/
e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1653, 12, 799, 794, 201, 223, 209}

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex} \, dx = \frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
```

```
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{\int \frac{5de^3x(d^2-e^2x^2)^{3/2}}{d+ex}\,dx}{5e^4}\\
&= -\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{d\int \frac{x(d^2-e^2x^2)^{3/2}}{d+ex}\,dx}{e}\\
&= -\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{\int x(d^2e-de^2x)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= \frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3}-\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^3\int\sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3}-\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3}\\
&\quad -\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3}-\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int\frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx=\frac{\sqrt{d^2-e^2x^2}(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4)-30d^5\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d}}\right.}{120e^3}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*e^2*x^2 + 30*d*e^3*x^3 -
24*e^4*x^4) - 30*d^5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(120*
e^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(-24e^4x^4+30d\,e^3x^3+8d^2e^2x^2-15d^3ex+16d^4)\sqrt{-e^2x^2+d^2}}{120e^3}+\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5e^3}-\frac{d\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{e^2}+\frac{d^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+de\right.}{}$

```
[In] int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-24*e^4*x^4+30*d*e^3*x^3+8*d^2*e^2*x^2-15*d^3*e*x+16*d^4)/e^3*(-e^2*
x^2+d^2)^(1/2)+1/8*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx =$$
$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(24\,e^4x^4-30\,de^3x^3-8\,d^2e^2x^2+15\,d^3ex-16\,d^4)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 + 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = d\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } \\ \frac{x^3\sqrt{d^2}}{3} & \text{othe} \end{cases}\right) - e\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2 \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

```
[Out] d*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2
*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e*
*2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x*
*3*sqrt(d**2)/3, True)) - e*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*
e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, Tr
ue))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.54

$$\begin{aligned} \int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = & -\frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^3} - \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3} \\ & + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{2\,e^2} - \frac{3\,\sqrt{-e^2x^2+d^2}d^3x}{8\,e^2} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{e^3} \\ & - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{3\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^3} \end{aligned}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*I*d^5*arcsin(e*x/d + 2)/e^3 - 3/8*d^5*arcsin(e*x/d)/e^3 + 1/2*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^3*x/e^2 - 3/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 + sq
rt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^2
+ 1/3*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3 - 1/5*(-e^2*x^2 + d^2)^(5/2)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \frac{d^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,e^2|e|} - \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,(4\,ex-5\,d)x-\frac{4\,d^2}{e}\right)x+\frac{15\,d^3}{e^2}\right)x-\frac{16\,d^4}{e^3}\right)$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
[Out] 1/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - 1/120*sqrt(-e^2*x^2 + d^
2)*((2*(3*(4*e*x - 5*d)*x - 4*d^2/e)*x + 15*d^3/e^2)*x - 16*d^4/e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{d+e\,x}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x), x)
```

3.103 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 201

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}+\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}+\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}+\frac{d^3(128d-315ex)\,(d^2-e^2x^2)^{5/2}}{5040e^5}+\frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2+1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e+1/5040*d^3
*(-315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 847, 794, 201, 223, 209}

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}+\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)\,(d^2-e^2x^2)^{5/2}}{5040e^5}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx \\
&= \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x^3(4d^2e-9de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2} \\
&= -\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{\int x^2(27d^3e^2-32d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{72e^4} \\
&= \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} \\
&\quad - \frac{\int x(64d^4e^3-189d^3e^4x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{504e^6} \\
&= \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} \\
&\quad + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5} + \frac{d^5\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{16e^4} \\
&= \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} \\
&\quad + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5} + \frac{(3d^7)\int\sqrt{d^2-e^2x^2}\,dx}{64e^4} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} \\
&\quad + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5} + \frac{(3d^9)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{128e^4} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} \\
&\quad - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5} \\
&\quad + \frac{(3d^9)\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{128e^4} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} \\
&\quad + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5} + \frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5}{40320e^5}$$

[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]

[Out] (Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e^3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^7*x^7 + 4480*e^8*x^8) - 1890*d^9*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(40320*e^5)

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(4480e^8x^8-5040d\,e^7x^7-6400d^2e^6x^6+7560d^3e^5x^5+384d^4x^4e^4-630d^5e^3x^3+512d^6e^2x^2-945d^7ex+1024d^8)\sqrt{-e^2x^2+d^2}}{40320e^5}+\frac{3d^9\,\mathrm{arcta}}{}$
default	$\frac{-\frac{x^2(-e^2x^2+d^2)^{\frac{7}{2}}}{9e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{7}{2}}}{63e^4}}{e}-\frac{d^2(-e^2x^2+d^2)^{\frac{7}{2}}}{7e^5}-\frac{d^3\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\,\mathrm{a}}{}\right)}{4}\right)}{6}\right)}{e^4}$

[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/40320*(4480*e^8*x^8-5040*d*e^7*x^7-6400*d^2*e^6*x^6+7560*d^3*e^5*x^5+384*d^4*e^4*x^4-630*d^5*e^3*x^3+512*d^6*e^2*x^2-945*d^7*e*x+1024*d^8)/e^5*(-e^2

```
*x^2+d^2)^(1/2)+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(4480\,e^8x^8-5040\,de^7x^7-6400\,d^2e^6x^6+7560\,d^3e^5x^5+384\,d^4e^4x^4-63}{40320\,e^5}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (4480*e^8*x^
8 - 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 + 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
- 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 - 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.12

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x}{6}\right. \\ \frac{x^5\sqrt{d^2}}{5}\end{cases}\right.$$

$$-d^2e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

$$-de^2\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq \\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwi}\end{cases}\right.$$

$$+e^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{16d^8}{315e^8}-\frac{8d^6x^2}{315e^6}-\frac{2d^4x^4}{105e^4}-\frac{d^2x^6}{63e^2}+\frac{x^8}{9}\right) & \text{for } e^2\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(1
6*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) +
 x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - d**2*e*Piecewise((sqrt
(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4
/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) - d*e**2*Pie
cewise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x*
*2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*e**
6) + sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(192*e**4)
- d**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7, True)) +
 e**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-16*d**8/(315*e**8) - 8*d**6*x**2/
(315*e**6) - 2*d**4*x**4/(105*e**4) - d**2*x**6/(63*e**2) + x**8/9), Ne(e**
2, 0)), (x**8*sqrt(d**2)/8, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = &-\frac{3i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5}-\frac{45\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e^5}\\&+\frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^7x}{8\,e^4}-\frac{45\sqrt{-e^2x^2+d^2}d^7x}{128\,e^4}+\frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^8}{4\,e^5}\\&+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^5x}{64\,e^4}-\frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3x}{16\,e^4}-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}{9\,e^3}\\&+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{5\,e^5}+\frac{(-e^2x^2+d^2)^{\frac{7}{2}}dx}{8\,e^4}-\frac{11\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2}{63\,e^5}\end{aligned}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^9*arcsin(e*x/d + 2)/e^5 - 45/128*d^9*arcsin(e*x/d)/e^5 + 3/8*sqrt(
e^2*x^2 + 4*d*e*x + 3*d^2)*d^7*x/e^4 - 45/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^
4 + 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^8/e^5 + 1/64*(-e^2*x^2 + d^2)^(3/
2)*d^5*x/e^4 - 3/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 1/9*(-e^2*x^2 + d^2)
^(7/2)*x^2/e^3 + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5 + 1/8*(-e^2*x^2 + d^2)^
(7/2)*d*x/e^4 - 11/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3\,d^9\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^4|e|}$$
$$+\frac{1}{40320}\sqrt{-e^2x^2+d^2}\left(\frac{1024\,d^8}{e^5}-\left(\frac{945\,d^7}{e^4}-2\left(\frac{256\,d^6}{e^3}-\left(\frac{315\,d^5}{e^2}-4\left(\frac{48\,d^4}{e}+5\left(189\,d^3-2\left(80\,d^2e-7\left(\right.\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/40320*sqrt(-e^2*x^2
+ d^2)*(1024*d^8/e^5 - (945*d^7/e^4 - 2*(256*d^6/e^3 - (315*d^5/e^2 - 4*(48
*d^4/e + 5*(189*d^3 - 2*(80*d^2*e - 7*(8*e^3*x - 9*d*e^2)*x)*x)*x)*x)*x)*x)
*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.104 $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}+\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d-35ex)\,(d^2-e^2x^2)^{5/2}}{560e^4}-\frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

```
[Out] -1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2+1/8
*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e
^4-3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/128*d^6*x*(-e^2*x^2+d^2
)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 847, 794, 201, 223, 209}

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=-\frac{3d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}+\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d-35ex)\,(d^2-e^2x^2)^{5/2}}{560e^4}-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int x^3(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{\int x^2(3d^2e-8de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} + \frac{\int x(16d^3e^2-21d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{d^4\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{16e^3}$$

$$= -\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{(3d^6)\int\sqrt{d^2-e^2x^2}\,dx}{64e^3}$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{(3d^8)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{128e^3}$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{(3d^8)\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{128e^3}$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x}{4480e^4}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7) +
210*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(4480*e^4)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(-560e^7x^7+640d\,e^6x^6+840d^2e^5x^5-1024d^3e^4x^4-70d^4e^3x^3+128d^5e^2x^2-105d^6ex+256d^7)\sqrt{-e^2x^2+d^2}}{4480e^4}-\frac{3d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128e^3\sqrt{e^2}}$
default	$\frac{-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8e^2}}{e}+\frac{d^2\left(\frac{x\left(-e^2x^2\right.}{6}\right.}{}$

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4480*(-560*e^7*x^7+640*d*e^6*x^6+840*d^2*e^5*x^5-1024*d^3*e^4*x^4-70*d^4
*e^3*x^3+128*d^5*e^2*x^2-105*d^6*e*x+256*d^7)/e^4*(-e^2*x^2+d^2)^(1/2)-3/12
8*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(560\,e^7x^7-640\,de^6x^6-840\,d^2e^5x^5+1024\,d^3e^4x^4+\dots}{4480\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 -
640*d*e^6*x^6 - 840*d^2*e^5*x^5 + 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 - 128*d
^5*e^2*x^2 + 105*d^6*e*x - 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.33

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for}\ e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for}\ e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for}\ e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for}\ e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e
**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) - d**2*e*Piecewise
((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqr
```

```
t(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) + sqrt
(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6), Ne(e
**2, 0)), (x**5*sqrt(d**2)/5, True)) - d*e**2*Piecewise((sqrt(d**2 - e**2*x
**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/(35*e**2) + x
**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((5*d**8*P
iecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2)
, Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*e**6) + sqrt(d**2 -
 e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(192*e**4) - d**2*x**5/(48*
e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^4} + \frac{45\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^4} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^6x}{8\,e^3} + \frac{45\sqrt{-e^2x^2+d^2}d^6x}{128\,e^3} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^7}{4\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^4x}{64\,e^3} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x}{16\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{5\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x}{8\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^8*arcsin(e*x/d + 2)/e^4 + 45/128*d^8*arcsin(e*x/d)/e^4 - 3/8*sqrt(e
^2*x^2 + 4*d*e*x + 3*d^2)*d^6*x/e^3 + 45/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^3
 - 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^7/e^4 - 1/64*(-e^2*x^2 + d^2)^(3/2
)*d^4*x/e^3 + 3/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 1/5*(-e^2*x^2 + d^2)^
(5/2)*d^3/e^4 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^3 + 1/7*(-e^2*x^2 + d^2)^(7/
2)*d/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,e^3|e|} - \frac{1}{4480}\sqrt{-e^2x^2+d^2}\left(\frac{256\,d^7}{e^4} - \left(\frac{105\,d^6}{e^3} - 2\left(\frac{64\,d^5}{e^2} - \left(\frac{35\,d^4}{e} + 4\,(128\,d^3 - 5\,(21\,d^2e - 2\,(7\,e^3x - 8\,de^2)x\right.\right.\right.\right.$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/128*d^8*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 1/4480*sqrt(-e^2*x^2
+ d^2)*(256*d^7/e^4 - (105*d^6/e^3 - 2*(64*d^5/e^2 - (35*d^4/e + 4*(128*d^3
 - 5*(21*d^2*e - 2*(7*e^3*x - 8*d*e^2)*x)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=\int \frac{x^3\,(d^2-e^2\,x^2)^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.105 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} + \frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/30*d*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2
)/e^3-1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1653, 12, 799, 794, 201, 223, 209}

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3} -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
```

```
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{\int \frac{7de^3x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{7e^4}\\
&= -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{d\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{e}\\
&= -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}-\frac{\int x(d^2e-de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}\\
&= \frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^3\int (d^2-e^2x^2)^{3/2}\,dx}{6e^2}\\
&= \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^5\int \sqrt{d^2-e^2x^2}\,dx}{8e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}\\
&\quad -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}\\
&\quad -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}\\
&\quad -\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+2}{1680e^3}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*x
^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) - 210*d^7*ArcTan[(e*x)/
(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(1680*e^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
risch	$\frac{(240e^6x^6-280d\,e^5x^5-384d^2e^4x^4+490d^3x^3e^3+48d^4e^2x^2-105d^5ex+96d^6)\sqrt{-e^2x^2+d^2}}{1680e^3}+\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7e^3}-\frac{d\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{e^2}+\frac{d^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)\right.}{}\right.}{}$

[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/1680*(240*e^6*x^6-280*d*e^5*x^5-384*d^2*e^4*x^4+490*d^3*e^3*x^3+48*d^4*e^2*x^2-105*d^5*e*x+96*d^6)/e^3*(-e^2*x^2+d^2)^(1/2)+1/16*d^7/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx =$$
$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(240\,e^6x^6-280\,de^5x^5-384\,d^2e^4x^4+490\,d^3e^3x^3+48\,d^4e^2x^2-105\,d^5ex+}{1680\,e^3}$$

[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")

[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (240*e^6*x^6 - 280*d*e^5*x^5 - 384*d^2*e^4*x^4 + 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 - 105*d^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.65

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} + \sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} \frac{d^6\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{16e^4} + \sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0 \\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8
*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)),
(x**3*sqrt(d**2)/3, True)) - d**2*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d
**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**
2)/4, True)) - d*e**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e*
*2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2
*x**2), True))/(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2
*x**3/(24*e**2) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) + e**3*
Piecewise((sqrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e*
*4) - d**2*x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True
))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.41

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3} - \frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{16\,e^3} + \frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{8\,e^2} - \frac{5\,\sqrt{-e^2x^2+d^2}d^5x}{16\,e^2} + \frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{4\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{24\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}dx}{6\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{5\,e^3} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^7*arcsin(e*x/d + 2)/e^3 - 5/16*d^7*arcsin(e*x/d)/e^3 + 3/8*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^5*x/e^2 - 5/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2 +
3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e^3 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d
^3*x/e^2 - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)*
d^2/e^3 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{d^7\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^2|e|} + \frac{1}{1680}\,\sqrt{-e^2x^2+d^2}\left(\frac{96\,d^6}{e^3} - \left(\frac{105\,d^5}{e^2} - 2\left(\frac{24\,d^4}{e} + \left(245\,d^3 - 4\left(48\,d^2e - 5\left(6\,e^3x - 7\,de^2\right)x\right)x\right)x\right)x\right)x\right)$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/16*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/1680*sqrt(-e^2*x^2 +
d^2)*(96*d^6/e^3 - (105*d^5/e^2 - 2*(24*d^4/e + (245*d^3 - 4*(48*d^2*e - 5*
(6*e^3*x - 7*d*e^2)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx=\int \frac{x^2\,(d^2-e^2\,x^2)^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.106 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] -1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e
^2-1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/16*d^4*x*(-e^2*x^2+d^2)^
(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {799, 794, 201, 223, 209}

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}-\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -1/16*(d^4*x*Sqrt[d^2 - e^2*x^2])/e - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int x(d^2e-de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{de}\\
&= -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}\\
&= -\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}\\
&= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}
\end{aligned}$$

$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^6\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5)+30d^6 \text{arc}}{240e^2}$$

[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]

[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x^3 - 48*d*e^4*x^4 + 40*e^5*x^5) + 30*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(240*e^2)

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{(-40e^5x^5+48d\,e^4x^4+70d^2e^3x^3-96d^3e^2x^2-15d^4ex+48d^5)\sqrt{-e^2x^2+d^2}}{240e^2} - \frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$
default	$\frac{\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}}{e} - \frac{d\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\cdots\right.\right.}{\cdots}$

[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/240*(-40*e^5*x^5+48*d*e^4*x^4+70*d^2*e^3*x^3-96*d^3*e^2*x^2-15*d^4*e*x+48*d^5)/e^2*(-e^2*x^2+d^2)^(1/2)-1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5-48\,de^4x^4-70\,d^2e^3x^3+96\,d^3e^2x^2+15\,d^4\right.}{240\,e^2}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 48*
d*e^4*x^4 - 70*d^2*e^3*x^3 + 96*d^3*e^2*x^2 + 15*d^4*e*x - 48*d^5)*sqrt(-e^
2*x^2 + d^2))/e^2
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.96

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
0)), (x**2*sqrt(d**2)/2, True)) - d**2*e*Piecewise((d**4*Piecewise((log(-2*
e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (
```

```
x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*
x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) - d*e**2*Pie
cewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x
**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((d**6*Pie
cewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2),
Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) + sqrt(d**2 - e*
*2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6), Ne(e**2, 0)),
(x**5*sqrt(d**2)/5, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.52

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^2} + \frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^2} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{8\,e} + \frac{5\sqrt{-e^2x^2+d^2}d^4x}{16\,e} - \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{4\,e^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24\,e} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6\,e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,e^2}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^6*arcsin(e*x/d + 2)/e^2 + 5/16*d^6*arcsin(e*x/d)/e^2 - 3/8*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^4*x/e + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e - 3/4*s
qrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e^2 - 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/
e + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e|e|} - \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\frac{48\,d^5}{e^2} - \left(\frac{15\,d^4}{e} + 2\left(48\,d^3 - \left(35\,d^2e - 4\left(5\,e^3x - 6\,de^2\right)x\right)x\right)x\right)x\right)$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 1/240*sqrt(-e^2*x^2 + d^
2)*(48*d^5/e^2 - (15*d^4/e + 2*(48*d^3 - (35*d^2*e - 4*(5*e^3*x - 6*d*e^2)*
x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.107 $\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} \, dx = \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] 1/4*d*x*(-e^2*x^2+d^2)^(3/2)+1/5*(-e^2*x^2+d^2)^(5/2)/e+3/8*d^5*arctan(e*x/
(-e^2*x^2+d^2)^(1/2))/e+3/8*d^3*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {679, 201, 223, 209}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} \, dx = \frac{3d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{3}{8}d^3x\sqrt{d^2-e^2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d^2-e^2x^2)^{5/2}}{5e} + d\int (d^2-e^2x^2)^{3/2}\,dx \\
&= \frac{1}{4}dx(d^2-e^2x^2)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{1}{4}(3d^3)\int \sqrt{d^2-e^2x^2}\,dx \\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx(d^2-e^2x^2)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{1}{8}(3d^5)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx(d^2-e^2x^2)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} \\
&\quad + \frac{1}{8}(3d^5)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx(d^2-e^2x^2)^{3/2} + \frac{(d^2-e^2x^2)^{5/2}}{5e} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4)}{40e} - \frac{3d^5\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4))/(40*e) - (3*d^5*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*
Sqrt[-e^2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(8e^4x^4-10d\,e^3x^3-16d^2e^2x^2+25d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{40e}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right.}}{4e^2}\right.}{4}\right.}{e}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(8*e^4*x^4-10*d*e^3*x^3-16*d^2*e^2*x^2+25*d^3*e*x+8*d^4)/e*(-e^2*x^2+d
^2)^(1/2)+3/8*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(8\,e^4x^4-10\,de^3x^3-16\,d^2e^2x^2+25\,d^3ex+8\,d^4)\sqrt{-e^2x^2+d^2}}{40\,e}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
[Out] -1/40*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (8*e^4*x^4 - 10*d
*e^3*x^3 - 16*d^2*e^2*x^2 + 25*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = d^3\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right) - d^2e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) - de^2\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) + e^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
[Out] d**3*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2
```

```
+ x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - d**2*e*
Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2, 0)),
(x**2*sqrt(d**2)/2, True)) - d*e**2*Piecewise((d**4*Piecewise((log(-2*e**2*
x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log
(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*
e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + e**3*Piecewise(
(sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5),
 Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{3}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{4\,e}+\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,e}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^5*arcsin(e*x/d + 2)/e + 3/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
+ 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*
x + 1/5*(-e^2*x^2 + d^2)^(5/2)/e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} + \frac{1}{40}\sqrt{-e^2x^2+d^2}\left(\frac{8\,d^4}{e}+\left(25\,d^3-2\left(8\,d^2e-\left(4\,e^3x-5\,de^2\right)x\right)x\right)x\right)$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/40*sqrt(-e^2*x^2 + d^2)*(8*d
^4/e + (25*d^3 - 2*(8*d^2*e - (4*e^3*x - 5*d*e^2)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{d + ex}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{d + e\,x}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x), x)
```

3.108 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)} \, dx = \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)-3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(-3*e*x+8*d)*(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {864, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)} \, dx = -\frac{3}{8}d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx \\
&= \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2+3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2} \\
&= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4-3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4} \\
&= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{8}\left(3d^4e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad + \frac{1}{2}d^5\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&\quad - \frac{1}{8}\left(3d^4e\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^5\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2} \\
&= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} \\
&\quad - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right) + \frac{3}{4}d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - d^3\sqrt{d^2}\log(x) + d^3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 +
(3*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/4 - d^3*Sqrt[d^2]*L
og[x] + d^3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(99) = 198.

Time = 0.44 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.63

method	result
default	$\frac{\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{d} - \frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\right.}{}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+
d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
)))-1/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*
e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \frac{1}{24}\left(6\,e^3x^3-8\,de^2x^2-15\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + 1/24*(6*e^3*x^3 - 8*d*e^2*x^2 - 15*d^2*e*x + 32*d^3)*sqrt
(-e^2*x^2 + d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\,\text{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\,\text{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for } e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} \frac{d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8e^2} - \frac{d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4} & \text{for } e^2\neq 0 \\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - d**2*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt
(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-
e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d*
*2), True)) - d*e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**
2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True)) + e**3
*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*
x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**
2) - d**2*x*sqrt(d**2 - e**2*x**2)/(8*e**2) + x**3*sqrt(d**2 - e**2*x**2)/4
, Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = -\frac{3\,d^4e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}} - d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{3}{8}\,\sqrt{-e^2x^2+d^2}d^2ex + \sqrt{-e^2x^2+d^2}d^3 - \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex + \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^4*log(2*d^2/abs(x) + 2
*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-
e^2*x^2 + d^2)*d^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^
(3/2)*d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = -\frac{3\,d^4e\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,|e|} - \frac{d^4e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \frac{1}{24}\,\sqrt{-e^2x^2+d^2}\left(32\,d^3-\left(15\,d^2e-2\left(3\,e^3x-4\,de^2\right)x\right)x\right)$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d^4*e*log(1/2*abs(-2*d*e -
2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/24*sqrt(-e^2*x^2 +
d^2)*(32*d^3 - (15*d^2*e - 2*(3*e^3*x - 4*d*e^2)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx=\int \frac{(d^2-e^2\,x^2)^{5/2}}{x\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.109 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)} dx$

Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)} dx = -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*d*e*(3*e*x+2*d)*(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {864, 827, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)} dx = -\frac{3}{2}d^3e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)(d^2-e^2x^2)^{3/2}}{3x}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2]) - ((3*d + e*x)*(d^2 - e^2*x^2)
^(3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTa
nh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
```

```
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx \\
&= -\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{-4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2} \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} \\
&\quad - \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} \\
&\quad - \frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&\quad - \frac{1}{2}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)
\end{aligned}
$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{d^4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3-8d^2ex-3de^2x^2+2e^3x^3)}{6x} + 3d^3e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \left(d^2\right)^{3/2}e\log(x) - \left(d^2\right)^{3/2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x - 3*d*e^2*x^2 + 2*e^3*x^3))/(6*x)
+ 3*d^3*e*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + (d^2)^(3/2)*e*L
og[x] - (d^2)^(3/2)*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{d^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{x}+\frac{e^{3}x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}-\frac{4e\,d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3}+\frac{e\,d^{4}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}-\frac{3d^{3}e^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}$
default	$\frac{-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{d^{2}x}-\frac{6e^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{d^{2}}}{d}-\frac{e\left(\frac{\left(-e^{2}x^{2}+d\right.}{5}\right.}{}$

[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -d^3*(-e^2*x^2+d^2)^(1/2)/x+1/3*e^3*x^2*(-e^2*x^2+d^2)^(1/2)-4/3*e*d^2*(-e^2*x^2+d^2)^(1/2)+e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/2*d^3*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-8\,d^3ex+(2\,e^3x^3-3}{6\,x}$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")

[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 6*d^3*e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 8*d^3*e*x + (2*e^3*x^3 - 3*d*e^2*x^2 - 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/x

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.38

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = d^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\,\text{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\,\text{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$- d^2e\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\,\text{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\,\text{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$
$$- de^2\left(\begin{cases} \frac{d^2\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise} \end{cases}\right)$$
$$+ e^3\left(\begin{cases} -\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for}\ e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((d**2*Piecewise((log(-2*e**2
*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*lo
g(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)),
 (x*sqrt(d**2), True)) + e**3*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e*
*2) + x**2*sqrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} + d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}de^2x - \sqrt{-e^2x^2+d^2}d^2e - \frac{1}{3}\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e - \frac{\sqrt{-e^2x^2+d^2}d^3}{x}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + d^3*e*log(2*d^2/abs(x)
 + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x - sq
rt(-e^2*x^2 + d^2)*d^2*e - 1/3*(-e^2*x^2 + d^2)^(3/2)*e - sqrt(-e^2*x^2 + d
^2)*d^3/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = -\frac{3\,d^3e^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,|e|} + \frac{d^3e^4x}{2\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)|e|} + \frac{d^3e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{2\,x|e|} - \frac{1}{6}\,\sqrt{-e^2x^2+d^2}\left(8\,d^2e-\left(2\,e^3x-3\,de^2\right)x\right)$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/2*d^3*e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*d^3*e^4*x/((d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))*abs(e)) + d^3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^
2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))*d^3/(x*abs(e)) - 1/6*sqrt(-e^2*x^2 + d^2)*(8*d^2*e - (2*e^3*x - 3*
d*e^2)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^2\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.110 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2+3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3/2*d*e*(-e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {864, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3}{2}d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2+8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + \frac{1}{2}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(3d^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}(-d^3+2d^2ex-2de^2x^2+e^3x^3)}{x^2} - 6d^2e^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 3d\sqrt{d^2}e^2\log(x) - 3d\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 - 6*d
^2*e^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 3*d*Sqrt[d^2]*e^2*
Log[x] - 3*d*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2x^2}+\frac{3e^3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{e^3x\sqrt{-e^2x^2+d^2}}{2}+\frac{3e^2d^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-d\,e^2\sqrt{}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)\right)}{2d^2}}{d}+\frac{e^2\left(\frac{\left(-e^2x^2+d^2\right)}{5}\right.}{}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d^2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/x^2+3/2*e^3*d^2/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/2*e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*e^2
*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d*e^2*(-e
^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^3(d + ex)}\,dx =$$

$$-\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 2\,d^2e^2x^2 - (e^3x^3 - 2\,de^2x^2 + 2\,d^2ex - }{2\,x^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*d^2*e^2*x
^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2*d^2*e^2*x^2 - (e^3*x^3 - 2*d*e^2*
x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.67

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases} \frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$

[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)
) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)
) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*
e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piecewise((d**2*Piecewise((l
og(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2,
0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(
e**2, 0)), (x*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3\,d^2e^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} + \frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}e^3x - \frac{3}{2}\,\sqrt{-e^2x^2+d^2}de^2 + \frac{\sqrt{-e^2x^2+d^2}d^2e}{x} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{2\,x^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 3/2*d^2*e^2*log(2*d^2/a
bs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^3*x -
 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2 + sqrt(-e^2*x^2 + d^2)*d^2*e/x - 1/2*(-e^2*
x^2 + d^2)^(3/2)*d/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(106) = 212.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3\,d^2e^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{3\,d^2e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \frac{\left(d^2e^3-\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e}{x}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2|e|} + \frac{1}{2}\left(e^3x-2\,de^2\right)\sqrt{-e^2x^2+d^2} + \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2|e|}{ex^2}}{8\,e^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/2*d^2*e^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 3/2*d^2*e^3*log(1/2*abs(-2
*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/8*(d^2*e^3 -
 4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e/x)*e^4*x^2/((d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2*abs(e)) + 1/2*(e^3*x - 2*d*e^2)*sqrt(-e^2*x^2 + d^2) +
1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*abs(e)/x - (d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d^2*abs(e)/(e*x^2))/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{(d^2-e^2\,x^2)^{5/2}}{x^3\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.111 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(-3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {864, 825, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = de^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx \\
&= -\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{\int \frac{\left(4d^3e^2-6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2} \\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{\int \frac{12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2} \\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} \\
&\quad +\frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+\left(de^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} \\
&\quad +\frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) \\
&\quad +\left(de^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)
\end{aligned}$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx = \frac{1}{6}\left(\frac{\sqrt{d^2-e^2x^2}\left(-2d^3+3d^2ex+8de^2x^2+6e^3x^3\right)}{x^3} - 12de^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - 9\sqrt{d^2}e^3\log(x) + 9\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^3 + 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))/x^3 -
 12*d*e^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - 9*Sqrt[d^2]*e^3
*Log[x] + 9*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,d(-8e^2x^2-3dex+2d^2)}{6x^3}+\frac{e^4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^3\sqrt{-e^2x^2+d^2}-\frac{3e^3d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{d^2}\right)}{3d^2}}{d}$

[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/6*(-e^2*x^2+d^2)^(1/2)*d*(-8*e^2*x^2-3*d*e*x+2*d^2)/x^3+e^4*d/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+e^3*(-e^2*x^2+d^2)^(1/2)-3/2*e
^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx =$$
$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-6\,de^3x^3-(6\,e^3x^3+8\,de^2x^2+3\,d^2ex-2}{6\,x^3}$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 6*d*e^3*x^3 - (6*e^3*x^3 + 8*d*e^2*x^2
 + 3*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.81

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$
$$-d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$
$$-de^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$
$$+e^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x)
)/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acos
h(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**
2*x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1
)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2
)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I
*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{de^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3}{2}\sqrt{-e^2x^2+d^2}e^3 + \frac{\sqrt{-e^2x^2+d^2}de^2}{x} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{2\,x^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{3\,x^3}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] d*e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/2*d*e^3*log(2*d^2/abs(x) +
2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/2*sqrt(-e^2*x^2 + d^2)*e^3 + sqrt(-e^2
*x^2 + d^2)*d*e^2/x + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/x^2 - 1/3*(-e^2*x^2 + d^
2)^(3/2)*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.37

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \frac{de^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\left(de^4 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^2}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{x^2}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3|e|} - \frac{3\,de^4\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,|e|} + \sqrt{-e^2x^2+d^2}e^3 + \frac{\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de^4}{x} + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2de^2}{x^2} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{x^3}}{24\,e^2|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] d*e^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/24*(d*e^4 - 3*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))*d*e^2/x - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/x^
2)*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*abs(e)) - 3/2*d*e^4*log(1
/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + sqrt(
-e^2*x^2 + d^2)*e^3 + 1/24*(15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e^4/x
+ 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*e^2/x^2 - (d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^3*d/x^3)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d + ex)}\,dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}}{x^4\,(d + e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(-4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4-e^4*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(-8*e*x+3*d)*(-e^2*x^
2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {864, 825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = e^4\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) - \frac{3}{8}e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx \\
&= -\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{\left(6d^3e^2-8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2} \\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4-32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4} \\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} \\
&\quad + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} \\
&\quad + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&\quad - e^5\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} \\
&\quad - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(3de^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} \\
&\quad - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.27

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{1}{24}\left(\frac{\sqrt{d^2-e^2x^2}(-6d^3+8d^2ex+15de^2x^2-32e^3x^3)}{x^4} + 48e^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{9\sqrt{d^2}e^4\log(x)}{d} + \frac{9\sqrt{d^2}e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/x^4
 + 48*e^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (9*Sqrt[d^2]*e^
4*Log[x])/d + (9*Sqrt[d^2]*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d)/24
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(32e^3x^3-15d\,e^2x^2-8d^2ex+6d^3)}{24x^4}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{3e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)\right)}{2d^2}\right)}{4d^2}}{d}$

[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(32*e^3*x^3-15*d*e^2*x^2-8*d^2*e*x+6*d^3)/x^4-e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/8*e^4*d/(d^2)^(1

```
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(32\,e^3x^3-15\,de^2x^2-}{24\,x^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 - 15*d*e^2*x^2 - 8*d^2*e*x + 6
*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.16 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.55

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)$$
$$-d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$
$$-de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$
$$+e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e*
*2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*a
sin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**
2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x
**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(
d*sqrt(1 - e**2*x**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = -\frac{e^5\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3}{8}e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{3\sqrt{-e^2x^2+d^2}e^4}{8\,d} - \frac{\sqrt{-e^2x^2+d^2}e^3}{x} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{4\,x^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] -e^5*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 3/8*e^4*log(2*d^2/abs(x) + 2*s
qrt(-e^2*x^2 + d^2)*d/abs(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^2*
x^2 + d^2)*e^3/x + 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) + 1/3*(-e^2*x^2 +
 d^2)^(3/2)*e/x^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(105) = 210.

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.76

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{\left(3\,e^5 - \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x} - \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e}{x^2} + \frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3}\right)e^8x^4}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|} - \frac{e^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{|e|} - \frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,|e|} - \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5|e|}{x} - \frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3|e|}{x^2} - \frac{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e|e|}{x^3} + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4|e|}{ex^4}}{192\,e^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*(3*e^5 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x - 24*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 + 120*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e*x^3))*e^8*x^4/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)) - e^5*arc
sin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*
x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) - 1/192*(120*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))*e^5*abs(e)/x - 24*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^3*a
bs(e)/x^2 - 8*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*e*abs(e)/x^3 + 3*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^4*abs(e)/(e*x^4))/e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^5\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.113 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{3e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] 1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5+3/8*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)/d-3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{3e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx\\ &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx\end{aligned}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{1}{8}(3e^3)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{1}{16}(3e^5)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)$$

$$= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{1}{8}(3e^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{1}{40}\left(\frac{\sqrt{d^2-e^2x^2}(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4)}{dx^5} + \frac{15e^5\log(x)}{\sqrt{d^2}} - \frac{15e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3
- 8*e^4*x^4))/(d*x^5) + (15*e^5*Log[x])/Sqrt[d^2] - (15*e^5*Log[Sqrt[d^2] -
 Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2])/40
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(8 e^{4} x^{4}+25 d\, e^{3} x^{3}-16 d^{2} e^{2} x^{2}-10 d^{3} e x +8 d^{4}\right)}{40 x^{5} d}+\frac{3 e^{5} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$	107
default	Expression too large to display	1100

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-e^2*x^2+d^2)^(1/2)*(8*e^4*x^4+25*d*e^3*x^3-16*d^2*e^2*x^2-10*d^3*e*
x+8*d^4)/x^5/d+3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx = -\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^4x^4+25\,de^3x^3-16\,d^2e^2x^2-10\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,dx^5}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 + 25*d*e^
3*x^3 - 16*d^2*e^2*x^2 - 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 774, normalized size of antiderivative = 7.17

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = d^3\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$
$$- d^2e\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$
$$- de^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$
$$+ e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*Piecewise((-e*sqrt(d**2/(
e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
```

```
1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d} - \frac{3\,\sqrt{-e^2x^2+d^2}e^5}{8\,d^2} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{5\,dx^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{4\,x^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{5\,x^5}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 3/8*sqrt(-e
^2*x^2 + d^2)*e^5/d^2 - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^2) + 1/5*(-e^
2*x^2 + d^2)^(3/2)*e^2/(d*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e/x^4 - 1/5*(-e
^2*x^2 + d^2)^(3/2)*d/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.59

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{\left(2\,e^6 - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \frac{20\left(de+\sqrt{-\right.}{e^2}\right.}{320\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5 d|e|} + \frac{3\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{8\,d|e|} - \frac{\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^8}{x} + \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^6}{x^2} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4e^4}{x^3} - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^4e^2}{x^4} + \frac{2\left(de+\right.}{}}{320\,d^5e^4|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/320*(2*e^6 - 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x - 10*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/x^3 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4))*e^10*x^5/((d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d*abs(e)) + 3/8*e^6*log(1/2*abs(-2*d*e -
2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)) - 1/320*(20*(d*e +
```

```
sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^8/x + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^2*d^4*e^6/x^2 - 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4*e^4/x^3 -
 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*e^2/x^4 + 2*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^5*d^4/x^5)/(d^5*e^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^6\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.114 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6+1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = -\frac{e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{\left(6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^6\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{32d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5)}{240d^2x^6} - \frac{\sqrt{d^2}e^6\log(x)}{16d^3} + \frac{\sqrt{d^2}e^6\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^3}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5))/(240*d^2*x^6) - (Sqrt[d^2]*e^6*Log[x])/(16*
d^3) + (Sqrt[d^2]*e^6*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(16*d^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-48e^5x^5+15d\,e^4x^4+96d^2e^3x^3-70d^3e^2x^2-48d^4ex+40d^5)}{240x^6d^2}-\frac{e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d\sqrt{d^2}}$	121
default	Expression too large to display	1300

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-e^2*x^2+d^2)^(1/2)*(-48*e^5*x^5+15*d*e^4*x^4+96*d^2*e^3*x^3-70*d^3
*e^2*x^2-48*d^4*e*x+40*d^5)/x^6/d^2-1/16/d*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(48\,e^5x^5-15\,de^4x^4-96\,d^2e^3x^3+70\,d^3e^2x^2+48\,d^4ex}{240\,d^2x^6}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*x^5 - 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 + 70*d^3*e^2*x^2 + 48*d^4*e*x - 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.03 (sec) , antiderivative size = 918, normalized size of antiderivative = 6.42

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = d^3\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{fo} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{o} \end{cases}\right.$$

$$-d^2e\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

$$-de^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

$$+e^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
```

```
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = -\frac{e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^2} + \frac{\sqrt{-e^2x^2+d^2}e^6}{16\,d^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{16\,d^3x^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{5\,d^2x^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^4} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{5\,x^5} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{6\,x^6}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^2) - 1/
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^3) + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*
x^4) + 1/5*(-e^2*x^2 + d^2)^(3/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(3/2)*d/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(123) = 246$.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.24

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{\left(5\,e^7 - \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^5}{x} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^3}{x^2} + \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e}{x^3} - \frac{15\left(de+\right.}{}\right.}{1920\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^6d^2|e|} - \frac{e^7\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^2|e|} + \frac{\frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{10}e^9|e|}{x} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{10}e^7|e|}{x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{10}e^5|e|}{x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{10}}{x^4}}{1920\,d^{12}e^6}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/1920*(5*e^7 - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^5/x - 15*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*e^3/x^2 + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^3*e/x^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e*x^4) - 120*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^3*x^5))*e^12*x^6/((d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^6*d^2*abs(e)) - 1/16*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2
+ d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) + 1/1920*(120*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^10*e^9*abs(e)/x + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^2*d^10*e^7*abs(e)/x^2 - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^10*e^5*
abs(e)/x^3 + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^10*e^3*abs(e)/x^4 +
 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^10*e*abs(e)/x^5 - 5*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^6*d^10*abs(e)/(e*x^6))/(d^12*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx=\int \frac{(d^2-e^2\,x^2)^{5/2}}{x^7\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.115 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}+\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] 1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7+1/6*e*
(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5+1/16*e^7
*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] -1/16*(e^5*Sqrt[d^2 - e^2*x^2])/(d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24
*d^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*
```

```
d^2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{\int \frac{(7d^2e-2de^2x)(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{\int \frac{(12d^3e^2-7d^2e^3x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^3\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^3\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d^2}$$

$$= \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^5\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d^2}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^7\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{32d^2}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^5\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{16d^2}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5-96}{1680d^3x^7}$$
$$+\frac{\sqrt{d^2}e^7\log(x)}{16d^4}-\frac{\sqrt{d^2}e^7\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{16d^4}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) + (Sqr
t[d^2]*e^7*Log[x])/(16*d^4) - (Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^2
*x^2]])/(16*d^4)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(96e^6x^6-105d\,e^5x^5+48d^2e^4x^4+490d^3x^3e^3-384d^4e^2x^2-280d^5ex+240d^6)}{1680x^7d^3}+\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d^2\sqrt{d^2}}$	132
default	Expression too large to display	132

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(-e^2*x^2+d^2)^(1/2)*(96*e^6*x^6-105*d*e^5*x^5+48*d^2*e^4*x^4+490*d
^3*e^3*x^3-384*d^4*e^2*x^2-280*d^5*e*x+240*d^6)/x^7/d^3+1/16/d^2*e^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx =$$
$$-\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(96\,e^6x^6-105\,de^5x^5+48\,d^2e^4x^4+490\,d^3e^3x^3-384\,d^4e^2x^2-280\,d^5ex+2}{1680\,d^3x^7}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (96*e^6*x^6 - 105
*d*e^5*x^5 + 48*d^2*e^4*x^4 + 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 - 280*d^5*e
*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.72 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.03

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^8(d + ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.18

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3} - \frac{\sqrt{-e^2x^2+d^2}e^7}{16\,d^4} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{16\,d^4x^2} + \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{35\,d^3x^3} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^4} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{35\,dx^5} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{6\,x^6} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{7\,x^7}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 1/16*sqr
t(-e^2*x^2 + d^2)*e^7/d^4 - 1/16*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^2) + 2/3
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
2*x^4) + 3/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^5) + 1/6*(-e^2*x^2 + d^2)^(3/
2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(3/2)*d/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(148) = 296.

Time = 0.29 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{\left(15\,e^8 - \frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^6}{x} - \frac{21\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^4}{x^2} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^2}{x^3} - \frac{105\,(\cdots}{\cdots}\right.}{13440\,\left(de+\sqrt{-e^2x^2+d^2\cdots}\right.} + \frac{e^8\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{16\,d^3|e|} - \frac{\frac{315\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{18}e^{12}}{x} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{18}e^{10}}{x^2} - \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{18}e^8}{x^3} + \frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{18}e^6}{x^4}\cdots}{13440\,d^{21}e^6|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*(15*e^8 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^6/x - 21*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*e^4/x^2 + 105*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3*e^2/x^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/x^4 + 105*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^2*x^5) + 315*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^6/(e^4*x^6))*e^14*x^7/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^3*
```

```
abs(e)) + 1/16*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*abs(x)))/(d^3*abs(e)) - 1/13440*(315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d
^18*e^12/x + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^18*e^10/x^2 - 105*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^18*e^8/x^3 + 105*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^4*d^18*e^6/x^4 - 21*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5
*d^18*e^4/x^5 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d^18*e^2/x^6 + 15*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^18/x^7)/(d^21*e^6*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{(d^2-e^2\,x^2)^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.116 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 201

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{3e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

```
[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8+1/7*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6+2/35*e^
3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = -\frac{3e^8\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*
```

```
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{(8d^2e-3de^2x)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{(21d^3e^2-16d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{(96d^4e^3-21d^3e^4x)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{16d^3}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{32d^3}$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} - \frac{(3e^6)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{128d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{(3e^8)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{256d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} - \frac{(3e^6)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{128d^3}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{\sqrt{d^2-e^2x^2}(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5 -}{4480d^4x^8} - \frac{3\sqrt{d^2}e^8\log(x)}{128d^5} + \frac{3\sqrt{d^2}e^8\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{128d^5}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7))/(
4480*d^4*x^8) - (3*Sqrt[d^2]*e^8*Log[x])/(128*d^5) + (3*Sqrt[d^2]*e^8*Log[S
qrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(128*d^5)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-256e^7x^7+105d\,e^6x^6-128d^2e^5x^5+70d^3e^4x^4+1024d^4e^3x^3-840d^5e^2x^2-640d^6ex+560d^7)}{4480x^8d^4} - \frac{3e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{}}{x}\right.}{128d^3\sqrt{d}}$
default	Expression too large to display

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4480*(-e^2*x^2+d^2)^(1/2)*(-256*e^7*x^7+105*d*e^6*x^6-128*d^2*e^5*x^5+70
*d^3*e^4*x^4+1024*d^4*e^3*x^3-840*d^5*e^2*x^2-640*d^6*e*x+560*d^7)/x^8/d^4-
3/128/d^3*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(256\,e^7x^7-105\,de^6x^6+128\,d^2e^5x^5-70\,d^3e^4x^4-1024}{4480\,d^4x^8}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (256*e^7*x^7 - 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 - 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 + 840*d^5*
e^2*x^2 + 640*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.64 (sec) , antiderivative size = 1159, normalized size of antiderivative = 5.77

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
```

```
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = -\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{128\,d^4}+\frac{3\,\sqrt{-e^2x^2+d^2}e^8}{128\,d^5} + \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6}{128\,d^5x^2}-\frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{35\,d^4x^3}+\frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{64\,d^3x^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{35\,d^2x^5}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{16\,dx^6}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{7\,x^7}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}d}{8\,x^8}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 3/128*(-e^2*x^2 + d^2)^(3/2)*e^6/(d^5*x^2) -
 2/35*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^3) + 3/64*(-e^2*x^2 + d^2)^(3/2)*e^
4/(d^3*x^4) - 3/35*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^5) + 1/16*(-e^2*x^2 +
d^2)^(3/2)*e^2/(d*x^6) + 1/7*(-e^2*x^2 + d^2)^(3/2)*e/x^7 - 1/8*(-e^2*x^2 +
 d^2)^(3/2)*d/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(173) = 346$.

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{\left(35\,e^9 - \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^7}{x} + \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3e^3}{x^3} - \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4e}{x^4} + \frac{560\left(\cdots\right.}{\cdots}\right.}{71680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^8d^4|e|}$$
$$- \frac{3\,e^9\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{128\,d^4|e|}$$
$$+ \frac{\frac{1680\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{28}e^{13}|e|}{x} - \frac{560\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{28}e^9|e|}{x^3} + \frac{280\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{28}e^7|e|}{x^4} - \frac{112\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5\cdots}{x^5}}{71680\,d^{32}e^8}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/71680*(35*e^9 - 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^7/x + 112*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^3*e^3/x^3 - 280*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^4*e/x^4 + 560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e*x^5) - 1680*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e^5*x^7))*e^16*x^8/((d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^8*d^4*abs(e)) - 3/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-e
^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e)) + 1/71680*(1680*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))*d^28*e^13*abs(e)/x - 560*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^3*d^28*e^9*abs(e)/x^3 + 280*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^4*d^28*e^7*abs(e)/x^4 - 112*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^28*e^5
*abs(e)/x^5 + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^28*e*abs(e)/x^7 -
35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^28*abs(e)/(e*x^8))/(d^32*e^8)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^9\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{\arcsin(x)}{2}$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {799, 794, 222}

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = -\frac{\arcsin(x)}{2}-\frac{1}{2}\sqrt{1-x^2}(2-x)$$

```
[In] Int[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}(-2+x)\sqrt{1-x^2}+\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
[Out] ((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arcsin(x)}{2}-\sqrt{-(1+x)^2+2+2x}$	34
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45

```
[In] int(x*(-x^2+1)^(1/2)/(1+x),x,method=_RETURNVERBOSE)
[Out] -1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \begin{cases} \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate(x*(-x**2+1)**(1/2)/(1+x),x)
[Out] Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x <
 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}(x-2) - \frac{1}{2}\,\arcsin\left(x\right)$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \left(\frac{x}{2}-1\right)\,\sqrt{1-x^2} - \frac{\mathrm{asin}(x)}{2}$$

```
[In] int((x*(1 - x^2)^(1/2))/(x + 1),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.118 $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\arcsin(ax) - a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arcsin(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))-(-a*x+1)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {864, 827, 858, 222, 272, 65, 214}

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = -a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\sqrt{1-a^2x^2}(1-ax)}{x} - a\arcsin(ax)$$

```
[In] Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx$$

$$= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - \frac{1}{2}\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - a^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$

$$= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}$$

$$= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = \frac{(-1+ax)\sqrt{1-a^2x^2}}{x} - 2a\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right) - a\log(x) + a\log\left(-1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x - 2*a*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2]
)] - a*Log[x] + a*Log[-1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

method	result
risch	$\frac{a^2x^2-1}{x\sqrt{-a^2x^2+1}} - \frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}} + a\sqrt{-a^2x^2+1} - a\ \text{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$
default	$-\frac{(-a^2x^2+1)^{\frac{5}{2}}}{x} - 4a^2\left(\frac{x(-a^2x^2+1)^{\frac{3}{2}}}{4} + \frac{3x\sqrt{-a^2x^2+1}}{8} + \frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}\right) + a\left(\frac{(-a^2x^2+1)^{\frac{3}{2}}}{3} + \sqrt{-a^2x^2}\right.$

```
[In] int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (a^2*x^2-1)/x/(-a^2*x^2+1)^(1/2)-a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))+a*(-a^2*x^2+1)^(1/2)-a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = \frac{2\,ax\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+ax+\sqrt{-a^2x^2+1}(ax-1)}{x}$$

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = a\left(\begin{cases} i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1 \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

```
[Out] a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**2 -
 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**
2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = -a\arcsin(ax) - a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \sqrt{-a^2x^2+1}\,a - \frac{\sqrt{-a^2x^2+1}}{x}$$

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
[Out] -a*arcsin(a*x) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*
x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = \frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{a^2\arcsin(ax)\operatorname{sgn}(a)}{|a|} - \frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \sqrt{-a^2x^2+1}\,a - \frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a)
/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))
/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs
(a))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx = a\sqrt{1-a^2x^2} - \frac{\sqrt{1-a^2x^2}}{x} - \frac{a^2\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} + a\operatorname{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(-(1 - a^2*x^2)^(3/2)/(x^2*(a*x - 1)),x)
[Out] a*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a*(1 - a^2*x^2)^(1/2) - (1 - a^2*x^2)^(
1/2)/x - (a^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{3d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -3/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+x^3*(-e*x+d)/e^2/(-e^2*x^2+d^
2)^(1/2)-4/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/6*d*(-9*e*x+16*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {864, 833, 847, 794, 223, 209}

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{3d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}$$

```
[In] Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^4(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x^2(3d^3-4d^2ex)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} + \frac{\int \frac{x(8d^4e-9d^3e^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{(3d^3)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-16d^3-7d^2ex+de^2x^2-2e^3x^3)}{6e^5(d+ex)} + \frac{3d^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[In] Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(6*e^5*
(d + e*x)) + (3*d^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^5
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{(2e^2x^2-3dex+10d^2)\sqrt{-e^2x^2+d^2}}{6e^5}-\frac{3d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^4\sqrt{e^2}}-\frac{d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^6\left(x+\frac{d}{e}\right)}$
default	$\frac{-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}}{e}-\frac{d^2\sqrt{-e^2x^2+d^2}}{e^5}-\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{d\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right.}{e^2}$

[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(2*e^2*x^2-3*d*e*x+10*d^2)/e^5*(-e^2*x^2+d^2)^(1/2)-3/2*d^3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d^3/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{16\,d^3ex+16\,d^4-18\,(d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^3x^3-de^2x^2+7\,d^2ex+16\,d^3)\sqrt{-e^2x^2+}}{6\,(e^6x+de^5)}$$

[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/6*(16*d^3*e*x + 16*d^4 - 18*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^3*x^3 - d*e^2*x^2 + 7*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^6*x + d*e^5)

Sympy [F]

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)

[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d^3}{e^6x+de^5} - \frac{\sqrt{-e^2x^2+d^2}x^2}{3\,e^3} - \frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^5} + \frac{\sqrt{-e^2x^2+d^2}dx}{2\,e^4} - \frac{5\,\sqrt{-e^2x^2+d^2}d^2}{3\,e^5}$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*d^3/(e^6*x + d*e^5) - 1/3*sqrt(-e^2*x^2 + d^2)*x^2/e^
3 - 3/2*d^3*arcsin(e*x/d)/e^5 + 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^4 - 5/3*sqrt
(-e^2*x^2 + d^2)*d^2/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{2\,x}{e^3}-\frac{3\,d}{e^4}\right)+\frac{10\,d^2}{e^5}\right) - \frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^4|e|} + \frac{2\,d^3}{e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*(x*(2*x/e^3 - 3*d/e^4) + 10*d^2/e^5) - 3/2*d^3*ar
csin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 2*d^3/(e^4*((d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2
)^(1/2)+1/2*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 794, 223, 209}

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}$$

```
[In] Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x(2d^3-3d^2ex)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{(3d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(4d^2+dex-e^2x^2)}{2e^4(d+ex)} - \frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[In] Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2))/(2*e^4*(d + e*x)) - (3*d^2*
ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result	siz
risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^4}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^3\sqrt{e^2}}+\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$	10
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{d\sqrt{-e^2x^2+d^2}}{e^4}+\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$	15

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-e*x+2*d)/e^4*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^2/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{4\,d^2ex+4\,d^3-6\,(d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(e^2x^2-dex-4\,d^2)\sqrt{-e^2x^2+d^2}}{2\,(e^5x+de^4)}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*d^2*e*x + 4*d^3 - 6*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^2}{e^5x+de^4} + \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^4} - \frac{\sqrt{-e^2x^2+d^2}x}{2\,e^3} + \frac{\sqrt{-e^2x^2+d^2}d}{e^4}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(-e^2*x^2 + d^2)*d^2/(e^5*x + d*e^4) + 3/2*d^2*arcsin(e*x/d)/e^4 - 1/2*
sqrt(-e^2*x^2 + d^2)*x/e^3 + sqrt(-e^2*x^2 + d^2)*d/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^3}-\frac{2\,d}{e^4}\right) + \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^3|e|} - \frac{2\,d^2}{e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-e^2*x^2 + d^2)*(x/e^3 - 2*d/e^4) + 3/2*d^2*arcsin(e*x/d)*sgn(d)*
sgn(e)/(e^3*abs(e)) - 2*d^2/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*
x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-(-e^2*x^2+d^2)^(1/2)/e^3-d*(-e^2*x^
2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1653, 12, 807, 223, 209}

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}$$

```
[In] Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{de^3x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{(-2d-ex)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} + \frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((-2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) + (2*d*ArcTan[(e*x)/(Sqr
t[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{-e^2x^2+d^2}}{e^3}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}-\frac{d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4\left(x+\frac{d}{e}\right)}$	97
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^3}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}-\frac{d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4\left(x+\frac{d}{e}\right)}$	97

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))-d/e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,dex+2\,d^2-2\,(dex+d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+2\,d)}{e^4x+de^3}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -(2*d*e*x + 2*d^2 - 2*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(e^4*x + d*e^3)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d}{e^4x+de^3} - \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^3} - \frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-e^2*x^2 + d^2)*d/(e^4*x + d*e^3) - d*arcsin(e*x/d)/e^3 - sqrt(-e^2*x
^2 + d^2)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e^3} + \frac{2\,d}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] -d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - sqrt(-e^2*x^2 + d^2)/e^3 + 2*
d/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 52

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 223, 209}

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

```
[In] Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}\\ &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}\\ &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}-\frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[In] Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) - (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2
 - e^2*x^2])])/e^2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}{e^3\left(x+\frac{d}{e}\right)}$	74

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/e^3/(x+d/e)*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{ex-2\,(ex+d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d+\sqrt{-e^2x^2+d^2}}{e^3x+de^2}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] (e*x - 2*(e*x + d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d + sqrt(-e^
2*x^2 + d^2))/(e^3*x + d*e^2)
```

Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}}{e^3x+de^2}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^2}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(-e^2*x^2 + d^2)/(e^3*x + d*e^2) + arcsin(e*x/d)/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e|e|} - \frac{2}{e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) - 2/(e*((d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {665}

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{-ex+d}{de\sqrt{-e^2x^2+d^2}}$	29
trager	$-\frac{\sqrt{-e^2x^2+d^2}}{de(ex+d)}$	30
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^2d\left(x+\frac{d}{e}\right)}$	46

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{ex+d+\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2}{d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] 2/(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,(d+e\,x)}$$

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] -(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+(-e^2*x^2+d^2)^(1/2)/d^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {871, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[In] Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\int \frac{de^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{de^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}}{d+ex} - \sqrt{d^2}\log(x) + \sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d^3}$$

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2])/(d + e*x) - Sqrt[d^2]*Log[x] + Sqrt[d^2]*Log[Sqrt[
d^2] - Sqrt[d^2 - e^2*x^2]])/d^3
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

method	result	size
default	$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e\,d^2\left(x+\frac{d}{e}\right)}$	88

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/e/d^2/(
x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{ex + (ex+d)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + d + \sqrt{-e^2x^2+d^2}}{d^2ex+d^3}$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + (e*x + d)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + d + sqrt(-e^2*x^2 + d
^2))/(d^2*e*x + d^3)
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{2\,e}{d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] -e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*a
bs(e)) - 2*e/(d^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-2*(-e^2*x^2+d^2)^(1/2)/d^3/x+(-e^2*x^
2+d^2)^(1/2)/d^2/x/(e*x+d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {871, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}$$

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{\int \frac{-2de^2+e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2} \\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e}
\end{aligned}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{-\frac{d(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)} + \sqrt{d^2}e\log(x) - \sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d^4}$$

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((d*(d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) + Sqrt[d^2]*e*Log[x]
- Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^4
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\sqrt{-e^2x^2+d^2}}{d^3x} + \frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^3\left(x+\frac{d}{e}\right)}$	108
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^3x} + \frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^3\left(x+\frac{d}{e}\right)}$	108

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^3/x+e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-1/d^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^2x^2+dex+(e^2x^2+dex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(2\,ex+d)}{d^3ex^2+d^4x}$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e^2*x^2 + d*e*x + (e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) +
sqrt(-e^2*x^2 + d^2)*(2*e*x + d))/(d^3*e*x^2 + d^4*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\sqrt{-(-d+ex)(d+ex)}(d+ex)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^3|e|} + \frac{\left(e^2+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x}\right)e^2x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^3x|e|}$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*
abs(e)) + 1/2*(e^2 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x)*e^2*x/((d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*x) + 1)*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^3*x*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-3/2*(-e^2*x^2+d^2)^(1/2)/d^3/x
^2+2*e*(-e^2*x^2+d^2)^(1/2)/d^4/x+(-e^2*x^2+d^2)^(1/2)/d^2/x^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {871, 849, 821, 272, 65, 214}

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{\int \frac{-3de^2+2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\int \frac{-4d^2e^3+3de^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-d^2+dex+4e^2x^2)}{x^2(d+ex)} - 3\sqrt{d^2}e^2\log(x) + 3\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{2d^5}$$

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(-d^2 + d*e*x + 4*e^2*x^2))/(x^2*(d + e*x)) - 3*Sqr
t[d^2]*e^2*Log[x] + 3*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(
2*d^5)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-2 e x +d \right)}{2 d^{4} x^{2}}-\frac{3 e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{3} \sqrt{d^{2}}}+\frac{e \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}{d^{4} \left(x +\frac{d}{e}\right)}$
default	$\frac{-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{2 d^{2} x^{2}}-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{2} \sqrt{d^{2}}}}{d}-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{3} \sqrt{d^{2}}}+\frac{e \sqrt{-e^{2} x^{2}+d^{2}}}{d^{4} x}+\frac{e \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}{d^{4} \left(x +\frac{d}{e}\right)}$

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^4/x^2-3/2*e^2/d^3/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2\,e^3x^3+2\,de^2x^2+3\,(e^3x^3+de^2x^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(4\,e^2x^2+dex-d^2)\sqrt{-e^2x^2+d^2}}{2\,(d^4ex^3+d^5x^2)}$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^3*x^3 + 2*d*e^2*x^2 + 3*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (4*e^2*x^2 + d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^3
 + d^5*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(101) = 202.

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\left(e^3 - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x} - \frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{3\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^4|e|} + \frac{\frac{4\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^4|e|}{ex^2}}{8\,d^8e^2}$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] 1/8*(e^3 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x - 20*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e*x^2))*e^4*x^2/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
2*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 3/2*e^3*l
og(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*abs(e
)) + 1/8*(4*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e*abs(e)/x - (d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*d^4*abs(e)/(e*x^2))/(d^8*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.127 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 128

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/3*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-5/2*d^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^6-1/3*x^2*(-5*e*x+4*d)/e^4/(-e^2*x^2+d^2)^(1/2)-1/6*(-15*e*x+16*d
)*(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 794, 223, 209}

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{5d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^5(d-ex)}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^3(4d^3-5d^2ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$
= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}
$$

$$
= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{(5d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^5}
$$

$$
= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{(5d^2)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}
$$

$$
= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$
\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{6e^6(-d+ex)(d+ex)^2} + \frac{5d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^6}
$$

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*e
^4*x^4))/(6*e^6*(-d + e*x)*(d + e*x)^2) + (5*d^2*ArcTan[(e*x)/(Sqrt[d^2] -
Sqrt[d^2 - e^2*x^2])])/e^6
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^6} - \frac{5d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}} + \frac{d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{6e^8\left(x+\frac{d}{e}\right)^2} - \frac{25d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12e^7\left(x+\frac{d}{e}\right)} -$
default	$\frac{-\frac{x^3}{2e^2\sqrt{-e^2x^2+d^2}}+\frac{3d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{2e^2}}{e} + \frac{d^2x}{e^5\sqrt{-e^2x^2+d^2}} + \frac{d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{e^3}$

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e*x+2*d)/e^6*(-e^2*x^2+d^2)^(1/2)-5/2*d^2/e^5/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/6*d^3/e^8/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)-25/12*d^2/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
-1/4*d^2/e^7/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.48

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{16\,d^2e^3x^3+16\,d^3e^2x^2-16\,d^4ex-16\,d^5-30\left(d^2e^3x^3+d^3e^2x^2-d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(3}{6\left(e^9x^3+de^8x^2-d^2e^7x-d^3e^6\right)}$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 - 16*d^4*e*x - 16*d^5 - 30*(d^2*e^3*x
^3 + d^3*e^2*x^2 - d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 - (3*e^4*x^4 - 3*d*e^3*x^3 - 23*d^2*e^2*x^2 + d^3*e*x + 16*d^4)*sqrt(-e^2*
x^2 + d^2))/(e^9*x^3 + d*e^8*x^2 - d^2*e^7*x - d^3*e^6)
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{d^4}{3\left(\sqrt{-e^2x^2+d^2}e^7x+\sqrt{-e^2x^2+d^2}de^6\right)} \\ &\quad -\frac{x^3}{2\sqrt{-e^2x^2+d^2}e^3}+\frac{dx^2}{\sqrt{-e^2x^2+d^2}e^4} \\ &\quad +\frac{17\,d^2x}{6\sqrt{-e^2x^2+d^2}e^5}-\frac{5\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6}-\frac{3\,d^3}{\sqrt{-e^2x^2+d^2}e^6}\end{aligned}$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*d^4/(sqrt(-e^2*x^2 + d^2)*e^7*x + sqrt(-e^2*x^2 + d^2)*d*e^6) - 1/2*x^3
/(sqrt(-e^2*x^2 + d^2)*e^3) + d*x^2/(sqrt(-e^2*x^2 + d^2)*e^4) + 17/6*d^2*x
/(sqrt(-e^2*x^2 + d^2)*e^5) - 5/2*d^2*arcsin(e*x/d)/e^6 - 3*d^3/(sqrt(-e^2*
x^2 + d^2)*e^6)
```

Giac [F]

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.128 $\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{x^3(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/3*x^3*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)+d*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^5-1/3*x*(-4*e*x+3*d)/e^4/(-e^2*x^2+d^2)^(1/2)+8/3*(-e^2*x^2+d^2)^(1/2)/
e^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 655, 223, 209}

$$\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{x^3(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(d-ex)}{(d^2-e^2x^2)^{5/2}}\,dx \\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{\int \frac{3d^5-8d^4ex}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}
\end{aligned}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^3+5d^2ex-7de^2x^2-3e^3x^3\right)}{(d-ex)(d+ex)^2} - 6d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^5}$$

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d -
e*x)*(d + e*x)^2) - 6*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(3
*e^5)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.65

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^5} + \frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}} + \frac{19d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12e^6\left(x+\frac{d}{e}\right)} - \frac{d\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{4e^6\left(x-\frac{d}{e}\right)} -$
default	$\frac{-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\frac{2d^2}{e^4\sqrt{-e^2x^2+d^2}}}{e} + \frac{d^2}{e^5\sqrt{-e^2x^2+d^2}} - \frac{dx}{e^4\sqrt{-e^2x^2+d^2}} - \frac{d\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}} - \frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{e^2} +$

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/e^5+d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))+19/12*d/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/4*d/e
^6/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/6*d^2/e^7/(x+d/e)^2*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{8\,de^3x^3 + 8\,d^2e^2x^2 - 8\,d^3ex - 8\,d^4 - 6\left(de^3x^3 + d^2e^2x^2 - d^3ex - d^4\right)\arctan\Big(}{3\left(e^8x^3 + de^7x^2 - d^2e^6\right.}$$

[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*(8*d*e^3*x^3 + 8*d^2*e^2*x^2 - 8*d^3*e*x - 8*d^4 - 6*(d*e^3*x^3 + d^2*e^2*x^2 - d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*x^3 + 7*d*e^2*x^2 - 5*d^2*e*x - 8*d^3)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + d*e^7*x^2 - d^2*e^6*x - d^3*e^5)

Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)

[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{d^3}{3\left(\sqrt{-e^2x^2+d^2}e^6x + \sqrt{-e^2x^2+d^2}de^5\right)} - \frac{x^2}{\sqrt{-e^2x^2+d^2}e^3} - \frac{4\,dx}{3\sqrt{-e^2x^2+d^2}e^4} + \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^5} + \frac{3\,d^2}{\sqrt{-e^2x^2+d^2}e^5}$$

[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")

[Out] -1/3*d^3/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt(-e^2*x^2 + d^2)*d*e^5) - x^2/(sqrt(-e^2*x^2 + d^2)*e^3) - 4/3*d*x/(sqrt(-e^2*x^2 + d^2)*e^4) + d*arcsin(e*x/d)/e^5 + 3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5)

Giac [F]

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.129 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/3*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/
e^4+1/3*(3*e*x-2*d)/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 792, 223, 209}

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}\left(-2d^2+dex+4e^2x^2\right)}{(d-ex)(d+ex)^2}+6\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^4}$$

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2)
 + 6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(3*e^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(79) = 158.

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

method	result
default	$\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e}+\frac{x}{\sqrt{-e^2x^2+d^2}\,e^3}-\frac{d}{e^4\sqrt{-e^2x^2+d^2}}-\frac{d^3\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-}{3e\,d^3\sqrt{-}}\right.}{e^4}$

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/e*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^
2*x^2+d^2)^(1/2)))+1/(-e^2*x^2+d^2)^(1/2)/e^3*x-d/e^4/(-e^2*x^2+d^2)^(1/2)-
d^3/e^4*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-
2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=$$
$$-\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3-6\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^2x^2+dex-\right.}{3\left(e^7x^3+de^6x^2-d^2e^5x-d^3e^4\right)}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 - 6*(e^3*x^3 + d*e^2*x^2
- d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^2*x^2 + d
```

```
*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + d*e^6*x^2 - d^2*e^5*x - d^3*
e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{d^2}{3\left(\sqrt{-e^2x^2+d^2}e^5x+\sqrt{-e^2x^2+d^2}de^4\right)} + \frac{4\,x}{3\sqrt{-e^2x^2+d^2}e^3} - \frac{\arcsin\left(\frac{ex}{d}\right)}{e^4} - \frac{d}{\sqrt{-e^2x^2+d^2}e^4}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] 1/3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2*x^2 + d^2)*d*e^4) + 4/3*x/(
sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4 - d/(sqrt(-e^2*x^2 + d^2)*e^4
)
```

Giac [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)}\,dx$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^3}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx = \frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/3/e^3/(-e^2*x^2+d^2)^(1/2)-1/3*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {869, 12, 267}

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}} dx = \frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{2dx}{(d^2-e^2x^2)^{3/2}}\,dx}{3de}\\ &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{x}{(d^2-e^2x^2)^{3/2}}\,dx}{3e}\\ &= \frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^2+2dex-e^2x^2)}{3de^3(d-ex)(d+ex)^2}$$

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{(-ex+d)(-e^2x^2+2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{3}{2}}}$	48
trager	$\frac{(-e^2x^2+2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3d\,e^3(ex+d)^2(-ex+d)}$	57
default	$\frac{1}{e^3\sqrt{-e^2x^2+d^2}}-\frac{x}{d\,e^2\sqrt{-e^2x^2+d^2}}+\frac{d^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{e^3}$	149

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3+(e^2x^2-2\,dex-2\,d^2)\sqrt{-e^2x^2+d^2}}{3\,(de^6x^3+d^2e^5x^2-d^3e^4x-d^4e^3)}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 + (e^2*x^2 - 2*d*e*x - 2*d
^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + d^2*e^5*x^2 - d^3*e^4*x - d^4*e^3)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{d}{3\left(\sqrt{-e^2x^2+d^2}e^4x+\sqrt{-e^2x^2+d^2}de^3\right)} - \frac{x}{3\sqrt{-e^2x^2+d^2}de^2} + \frac{1}{\sqrt{-e^2x^2+d^2}e^3}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*d/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x^2 + d^2)*d*e^3) - 1/3*x/(s
qrt(-e^2*x^2 + d^2)*d*e^2) + 1/(sqrt(-e^2*x^2 + d^2)*e^3)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+2\,d\,e\,x-e^2\,x^2\right)}{3\,d\,e^3\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 - e^2*x^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)^2*(d
 - e*x))
```

3.131 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x/d^2/e/(-e^2*x^2+d^2)^(1/2)+1/3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {807, 197}

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{3e}\\ &= \frac{x}{3d^2e\sqrt{d^2-e^2x^2}} + \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(d^2+dex+e^2x^2)}{3d^2e^2(d-ex)(d+ex)^2}$$

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{(-ex+d)(e^2x^2+dex+d^2)}{3d^2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}$	44
trager	$\frac{(e^2x^2+dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^2e^2(ex+d)^2(-ex+d)}$	53
default	$\frac{x}{d^2e\sqrt{-e^2x^2+d^2}} - \frac{d\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}} - \frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{e^2}$	129

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{e^3x^3+de^2x^2-d^2ex-d^3-\left(e^2x^2+dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2e^5x^3+d^3e^4x^2-d^4e^3x-d^5e^2\right)}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 - (e^2*x^2 + d*e*x + d^2)*sqrt(-e^
2*x^2 + d^2))/(d^2*e^5*x^3 + d^3*e^4*x^2 - d^4*e^3*x - d^5*e^2)
```

Sympy [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3\left(\sqrt{-e^2x^2+d^2}e^3x+\sqrt{-e^2x^2+d^2}de^2\right)}+\frac{x}{3\sqrt{-e^2x^2+d^2}d^2e}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] 1/3/(sqrt(-e^2*x^2 + d^2)*e^3*x + sqrt(-e^2*x^2 + d^2)*d*e^2) + 1/3*x/(sqrt
(-e^2*x^2 + d^2)*d^2*e)
```

Giac [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+d\,e\,x+e^2\,x^2\right)}{3\,d^2\,e^2\,\left(d+e\,x\right)^2\,\left(d-e\,x\right)}$$

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + d*e*x))/(3*d^2*e^2*(d + e*x)^2*(d -
 e*x))
```

3.132 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)-1/3/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 197}

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
```

```
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{3d}\\ &= \frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-d^2+2dex+2e^2x^2)}{3d^3e(d-ex)(d+ex)^2}$$

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 2*d*e*x + 2*e^2*x^2))/(3*d^3*e*(d - e*x)*(d +
e*x)^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{(-ex+d)(-2e^2x^2-2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	46
trager	$-\frac{(-2e^2x^2-2dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^3(ex+d)^2e(-ex+d)}$	55
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{e}$	104

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = -\frac{e^3x^3+de^2x^2-d^2ex-d^3+(2\,e^2x^2+2\,dex-d^2)\sqrt{-e^2x^2+d^2}}{3\,(d^3e^4x^3+d^4e^3x^2-d^5e^2x-d^6e)}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-(-d+ex)\,(d+ex))^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = -\frac{1}{3\,\left(\sqrt{-e^2x^2+d^2}de^2x+\sqrt{-e^2x^2+d^2}d^2e\right)}+\frac{2\,x}{3\,\sqrt{-e^2x^2+d^2}d^3}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] -1/3/(sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e) + 2/3*x/(s
qrt(-e^2*x^2 + d^2)*d^3)
```

Giac [F]

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(-d^2+2\,d\,e\,x+2\,e^2\,x^2)}{3\,d^3\,e\,(d+e\,x)^2\,(d-e\,x)}$$

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(d
 - e*x))
```

3.133 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/3*(-2*e*x+3*d)/d^4/(-e^2*x^2+d^2)^(1
/2)+1/3/d^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {871, 837, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-3de^2+2e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{3d^3e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d(4d^2+dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{(d-ex)(d+ex)^2} - 3\sqrt{d^2}\log(x) + 3\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^5}$$

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((d*(4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^2
) - 3*Sqrt[d^2]*Log[x] + 3*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/
(3*d^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d}-\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{d}$	171

[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))-1/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{4\,e^3x^3+4\,de^2x^2-4\,d^2ex-4\,d^3+3\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{-}}{}\right.}{3\left(d^4e^3x^3+d^5e^2x^2-d^6ex-d^7\right.}$$

[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*(4*e^3*x^3 + 4*d*e^2*x^2 - 4*d^2*e*x - 4*d^3 + 3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (2*e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + d^5*e^2*x^2 - d^6*e*x - d^7)

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)

[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.134 $\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/3*(-3*e*x+4*d)/d^4/x/(-e^2*x^2+d^2)
^(1/2)+1/3/d^2/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-8/3*(-e^2*x^2+d^2)^(1/2)/d^5/
x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {871, 837, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
```

```
, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-4de^2+3e^3x}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-8d^3e^4+3d^2e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^4e}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(3d^3+7d^2ex-5de^2x^2-8e^3x^3)}{x(-d+ex)(d+ex)^2} + 3\sqrt{d^2}e\log(x) - 3\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-\right.}{3d^6}$$

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(
-d + e*x)*(d + e*x)^2) + 3*Sqrt[d^2]*e*Log[x] - 3*Sqrt[d^2]*e*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/(3*d^6)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{6d^4e\left(x+\frac{d}{e}\right)^2}-\frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12d^5\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-\left(x-\right.}}{4}$
default	$\frac{-\frac{1}{d^2x\sqrt{-e^2x^2+d^2}}+\frac{2e^2x}{d^4\sqrt{-e^2x^2+d^2}}}{d}-\frac{e\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{d^2}+\frac{e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right.}}\right.}{d^2}$

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^5/x+1/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-1/6/d^4/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)-17/12/d^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/4/d^5/(x-d/e)*
(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{4\,e^4x^4+4\,de^3x^3-4\,d^2e^2x^2-4\,d^3ex+3\,\left(e^4x^4+de^3x^3-d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^3x^3+\right.}{3\,\left(d^5e^3x^4+d^6e^2x^3-d^7ex^2-d^8x\right)}$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*e^4*x^4 + 4*d*e^3*x^3 - 4*d^2*e^2*x^2 - 4*d^3*e*x + 3*(e^4*x^4 + d*
e^3*x^3 - d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^
3*x^3 + 5*d*e^2*x^2 - 7*d^2*e*x - 3*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^4
 + d^6*e^2*x^3 - d^7*e*x^2 - d^8*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(-(-d+ex)\,(d+ex))^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.135 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] -5/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/3*(-4*e*x+5*d)/d^4/x^2/(-e^2
*x^2+d^2)^(1/2)+1/3/d^2/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-5/2*(-e^2*x^2+d^2)
^(1/2)/d^5/x^2+8/3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{5e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-5de^2+4e^3x}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-16d^4e^5+15d^3e^6x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^8e^4}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}\\
&\quad - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{(5e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^5}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}\\
&\quad + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{(5e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^5}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}\\
&\quad + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^5}\\
&= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}\\
&\quad - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}\left(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4\right)}{x^2(-d+ex)(d+ex)^2}-15\sqrt{d^2}e^2\log(x)+15\sqrt{d^2}e^2\log\left(\right.}{6d^7}$$

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 1
6*e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) - 15*Sqrt[d^2]*e^2*Log[x] + 15*Sqr
t[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(6*d^7)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^6x^2}-\frac{5e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}}+\frac{23e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12d^6\left(x+\frac{d}{e}\right)}-\frac{e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{4d^6\left(x-\frac{d}{e}\right)}$
default	$\frac{-\frac{1}{2d^2x^2\sqrt{-e^2x^2+d^2}}+\frac{3e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{2d^2}}{d}+\frac{e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{d^3}-$

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^6/x^2-5/2/d^5*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+23/12/d^6*e/(x+d/e)*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)-1/4/d^6*e/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(
1/2)+1/6/d^5/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{14\,e^5x^5+14\,de^4x^4-14\,d^2e^3x^3-14\,d^3e^2x^2+15\,(e^5x^5+de^4x^4-d^2e^3x^3-}{6\,(d^6e^3x^5}$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(14*e^5*x^5 + 14*d*e^4*x^4 - 14*d^2*e^3*x^3 - 14*d^3*e^2*x^2 + 15*(e^5*
x^5 + d*e^4*x^4 - d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (16*e^4*x^4 + d*e^3*x^3 - 23*d^2*e^2*x^2 - 3*d^3*e*x + 3*d^4)*sqrt(-
e^2*x^2 + d^2))/(d^6*e^3*x^5 + d^7*e^2*x^4 - d^8*e*x^3 - d^9*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.136 $\int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 162

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

```
[Out] 1/5*x^6*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(-7*e*x+6*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(-35*e*
x+24*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e
^8
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 794, 223, 209}

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{7d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
```

```
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^7(d-ex)}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{x^6(d-ex)}{5e^2\,(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{x^5\,(6d^3-7d^2ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{x^3(24d^5-35d^4ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x(48d^7-105d^6ex)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{(7d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^7}$$

$$= \frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{(7d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^7}$$

$$= \frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6)}{(d-ex)^2(d+ex)^3} - 210d^2\arctan\left(\frac{}{\sqrt{}}\right.}{30e^8}$$

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3
 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3)
- 210*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(30*e^8)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(142) = 284$.

Time = 0.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

method	result
risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^8}+\frac{7d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^7\sqrt{e^2}}-\frac{7d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^{10}\left(x+\frac{d}{e}\right)^2}+\frac{773d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240e^9\left(x+\frac{d}{e}\right)}+$
default	$\frac{-\frac{x^5}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{5d^2\left(\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{-\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}\right)}{2e^2}}{e}+\frac{d^6\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^7}$

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-e*x+2*d)/e^8*(-e^2*x^2+d^2)^(1/2)+7/2*d^2/e^7/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-7/15*d^3/e^10/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)+773/240*d^2/e^9/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)+1/24*d^3/e^10/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+31/48*d^2/
e^9/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+1/20*d^4/e^11/(x+d/e)^3*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{96\,d^2e^5x^5+96\,d^3e^4x^4-192\,d^4e^3x^3-192\,d^5e^2x^2+96\,d^6ex+96\,d^7-210\,(d$$

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 + 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 - 192*d^5*e^2*x^2 +
 96*d^6*e*x + 96*d^7 - 210*(d^2*e^5*x^5 + d^3*e^4*x^4 - 2*d^4*e^3*x^3 - 2*d
^5*e^2*x^2 + d^6*e*x + d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15
*e^6*x^6 - 15*d*e^5*x^5 - 176*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 + 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 + d*e^12*x^4 - 2*d^2
*e^11*x^3 - 2*d^3*e^10*x^2 + d^4*e^9*x + d^5*e^8)
```

Sympy [F]

$$\int \frac{x^7}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^7}{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(143) = 286.

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int \frac{x^7}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \frac{d^6}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^9x+(-e^2x^2+d^2)^{\frac{3}{2}}de^8\right)} - \frac{x^5}{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{dx^4}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4} + \frac{25\,d^2x^3}{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5} - \frac{65\,d^3x^2}{6\,(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{164\,d^4x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^7} - \frac{7\,dx^2}{6\,\sqrt{-e^2x^2+d^2}e^6} + \frac{53\,d^5}{6\,(-e^2x^2+d^2)^{\frac{3}{2}}e^8} + \frac{229\,d^2x}{30\,\sqrt{-e^2x^2+d^2}e^7} + \frac{7\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^8} - \frac{14\,d^3}{3\,\sqrt{-e^2x^2+d^2}e^8} - \frac{7\,\sqrt{-e^2x^2+d^2}d}{6\,e^8}$$

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] 1/5*d^6/((-e^2*x^2 + d^2)^(3/2)*e^9*x + (-e^2*x^2 + d^2)^(3/2)*d*e^8) - 1/2
*x^5/((-e^2*x^2 + d^2)^(3/2)*e^3) + d*x^4/((-e^2*x^2 + d^2)^(3/2)*e^4) + 25
/2*d^2*x^3/((-e^2*x^2 + d^2)^(3/2)*e^5) - 65/6*d^3*x^2/((-e^2*x^2 + d^2)^(3
/2)*e^6) - 164/15*d^4*x/((-e^2*x^2 + d^2)^(3/2)*e^7) - 7/6*d*x^2/(sqrt(-e^2
*x^2 + d^2)*e^6) + 53/6*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8) + 229/30*d^2*x/(sq
rt(-e^2*x^2 + d^2)*e^7) + 7/2*d^2*arcsin(e*x/d)/e^8 - 14/3*d^3/(sqrt(-e^2*x
^2 + d^2)*e^8) - 7/6*sqrt(-e^2*x^2 + d^2)*d/e^8
```

Giac [F]

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(x^7/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.137 $\int \frac{x^6}{(d+ex)(d^2-e^2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} \, dx = \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] 1/5*x^5*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(-6*e*x+5*d)/e^4/(-e^2*x
^2+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(-8*e*x+5*d)/e^6
/(-e^2*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 655, 223, 209}

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} \, dx = -\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^6(d-ex)}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4(5d^3-6d^2ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2(15d^5-24d^4ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7-48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-48d^5-33d^4ex+87d^3e^2x^2+52d^2e^3x^3-38de^4x^4-15e^5x^5)}{15e^7(-d+ex)^2(d+ex)^3} + \frac{2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[In] Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 33*d^4*e*x + 87*d^3*e^2*x^2 + 52*d^2*e^3*x^
3 - 38*d*e^4*x^4 - 15*e^5*x^5))/(15*e^7*(-d + e*x)^2*(d + e*x)^3) + (2*d*Ar
cTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(130) = 260.

Time = 0.43 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{e^{7}}-\frac{d\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{6}\sqrt{e^{2}}}+\frac{23d^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{60e^{9}\left(x+\frac{d}{e}\right)^{2}}-\frac{493d\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{240e^{8}\left(x+\frac{d}{e}\right)}+\frac{d^{2}\sqrt{-(}}{}$
default	$\frac{-\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{4d^{2}\left(\frac{x^{2}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{2d^{2}}{3e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}\right)}{e^{2}}}{e}+\frac{d^{2}\left(\frac{x^{2}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{2d^{2}}{3e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}\right)}{e^{3}}+\frac{d^{4}}{3e^{7}(-e^{2}x^{2}+d^{2})}$

```
[In] int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^7-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))+23/60*d^2/e^9/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-49
3/240*d/e^8/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24*d^2/e^9/(x-d/
e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+25/48*d/e^8/(x-d/e)*(-(x-d/e)^2*e
^2-2*d*e*(x-d/e))^(1/2)-1/20*d^3/e^10/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.74

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{48\,de^5x^5+48\,d^2e^4x^4-96\,d^3e^3x^3-96\,d^4e^2x^2+48\,d^5ex+48\,d^6-30\left(de^5x^5+d^2e^4x^4-2\,d^3e^3x^3-2\,d^4e^2x\right.}{15\left(e^{12}x^5+de^{11}x^4-2\right.}$$

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(48*d*e^5*x^5 + 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 - 96*d^4*e^2*x^2 + 48
*d^5*e*x + 48*d^6 - 30*(d*e^5*x^5 + d^2*e^4*x^4 - 2*d^3*e^3*x^3 - 2*d^4*e^2
*x^2 + d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x
^5 + 38*d*e^4*x^4 - 52*d^2*e^3*x^3 - 87*d^3*e^2*x^2 + 33*d^4*e*x + 48*d^5)*
sqrt(-e^2*x^2 + d^2))/(e^12*x^5 + d*e^11*x^4 - 2*d^2*e^10*x^3 - 2*d^3*e^9*x
^2 + d^4*e^8*x + d^5*e^7)
```

Sympy [F]

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.75

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{d^5}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^7\right)} - \frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{5\,dx^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4} + \frac{20\,d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5} + \frac{64\,d^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6} + \frac{x^2}{3\sqrt{-e^2x^2+d^2}e^5} - \frac{14\,d^4}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7} - \frac{52\,dx}{15\sqrt{-e^2x^2+d^2}e^6} - \frac{d\arcsin\left(\frac{ex}{d}\right)}{e^7} + \frac{4\,d^2}{3\sqrt{-e^2x^2+d^2}e^7} + \frac{\sqrt{-e^2x^2+d^2}}{3\,e^7}$$

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8*x + (-e^2*x^2 + d^2)^(3/2)*d*e^7) - x^
4/((-e^2*x^2 + d^2)^(3/2)*e^3) - 5*d*x^3/((-e^2*x^2 + d^2)^(3/2)*e^4) + 20/
3*d^2*x^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 64/15*d^3*x/((-e^2*x^2 + d^2)^(3/2
)*e^6) + 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*e^5) - 14/3*d^4/((-e^2*x^2 + d^2)^(3
/2)*e^7) - 52/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) - d*arcsin(e*x/d)/e^7 + 4/3
*d^2/(sqrt(-e^2*x^2 + d^2)*e^7) + 1/3*sqrt(-e^2*x^2 + d^2)/e^7
```

Giac [F]

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(ex+d\right)}\,dx$$

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(x^6/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\int \frac{x^6}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.138 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(-5*e*x+4*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(-15*e*x+8*d)/e^6/(
-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 792, 223, 209}

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^5(d-ex)}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{x^3(4d^3-5d^2ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{x(8d^5-15d^4ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4)}{(d-ex)^2(d+ex)^3} - 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^6}$$

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 2
3*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) - 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d
^2 - e^2*x^2])])/(15*e^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(108) = 216.

Time = 0.39 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.69

method	result
default	$\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}} - \frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e} + \frac{d^4\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^5} + \frac{d^2\left(\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \cdots\right.}{\cdots}$

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d^4/e^5*(1/3*x/
d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))+d^2/e^3*(1/2*x/e^2
/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4
/(-e^2*x^2+d^2)^(1/2)))-d/e^2*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e
```

```
^2*x^2+d^2)^(3/2))-1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)-d^5/e^6*(-1/5/d/e/(x+d/
e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*(x+d/e)*e^2+2*d*e
)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+
2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.98

$$\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{8\,e^5x^5 + 8\,de^4x^4 - 16\,d^2e^3x^3 - 16\,d^3e^2x^2 + 8\,d^4ex + 8\,d^5 - 30\,(e^5x^5 + de^4x^4}{15\,(e}$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*x
 + 8*d^5 - 30*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*
x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (23*e^4*x^4 + 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 - 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 +
 d*e^10*x^4 - 2*d^2*e^9*x^3 - 2*d^3*e^8*x^2 + d^4*e^7*x + d^5*e^6)
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^5}{(-(-d+ex)(d+ex))^{\frac{5}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx &= \frac{d^4}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^7x + (-e^2x^2+d^2)^{\frac{3}{2}}de^6\right)}\\ &+ \frac{x^3}{(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{8\,dx^2}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{4\,d^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^5}\\ &- \frac{x^2}{3\,\sqrt{-e^2x^2+d^2}de^4} + \frac{2\,d^3}{(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}e^5}\\ &+ \frac{\arcsin\left(\frac{ex}{d}\right)}{e^6} - \frac{4\,d}{3\,\sqrt{-e^2x^2+d^2}e^6} - \frac{\sqrt{-e^2x^2+d^2}}{3\,de^6}\end{aligned}$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^4/((-e^2*x^2 + d^2)^(3/2)*e^7*x + (-e^2*x^2 + d^2)^(3/2)*d*e^6) + x^3
/((-e^2*x^2 + d^2)^(3/2)*e^3) - 8/3*d*x^2/((-e^2*x^2 + d^2)^(3/2)*e^4) - 4/
15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*d*e^4
) + 2*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6) - 8/15*x/(sqrt(-e^2*x^2 + d^2)*e^5)
+ arcsin(e*x/d)/e^6 - 4/3*d/(sqrt(-e^2*x^2 + d^2)*e^6) - 1/3*sqrt(-e^2*x^2
+ d^2)/(d*e^6)
```

Giac [F]

$$\int \frac{x^5}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^5}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^5}{(d^2-e^2\,x^2)^{5/2}\,(d+e\,x)}\,dx$$

```
[In] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.139 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/5*x^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2
)-4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {864, 819, 272, 45}

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/5*(x^4*(d - e*x))/(d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e
^2*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(d-ex)}{(d^2-e^2x^2)^{7/2}}\,dx\\
&= -\frac{x^4(d-ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{4\int \frac{x^3}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}\\
&= -\frac{x^4(d-ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{2\text{Subst}\left(\int \frac{x}{(d^2-e^2x)^{5/2}}\,dx,x,x^2\right)}{5e}\\
&= -\frac{x^4(d-ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{2\text{Subst}\left(\int \left(\frac{d^2}{e^2(d^2-e^2x)^{5/2}}-\frac{1}{e^2(d^2-e^2x)^{3/2}}\right)\,dx,x,x^2\right)}{5e}\\
&= -\frac{x^4(d-ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-8d^4-8d^3ex+12d^2e^2x^2+12de^3x^3-3e^4x^4)}{15de^5(d-ex)^2(d+ex)^3}$$

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 8*d^3*e*x + 12*d^2*e^2*x^2 + 12*d*e^3*x^3 - 3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

method	result
gosper	$-\frac{(-ex+d)\left(3e^4x^4-12d\,e^3x^3-12d^2e^2x^2+8d^3ex+8d^4\right)}{15d\,e^5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(3e^4x^4-12d\,e^3x^3-12d^2e^2x^2+8d^3ex+8d^4\right)\sqrt{-e^2x^2+d^2}}{15e^5d(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}}{e}+\frac{d^2}{3e^5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d^3\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^4}-\frac{d\left(\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\right.}{}$

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e^5/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.98

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=-\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5+(3\,e^4x^4-12\,de^3x^3-12\,d^2e^2x^2+8\,d^3ex+8\,d^4}{15\,(de^{10}x^5+d^2e^9x^4-2\,d^3e^8x^3-2\,d^4e^7x^2+d^5e^6x+d^6e^5)}$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*
x + 8*d^5 + (3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2 + 8*d^3*e*x + 8*d^4)
*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 + d^2*e^9*x^4 - 2*d^3*e^8*x^3 - 2*d^4*e^
7*x^2 + d^5*e^6*x + d^6*e^5)
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{d^3}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^5\right)} + \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{2\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4} - \frac{d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5} + \frac{x}{5\sqrt{-e^2x^2+d^2}de^4}$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6*x + (-e^2*x^2 + d^2)^(3/2)*d*e^5) + x^
2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

Giac [F]

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4+8\,d^3\,e\,x-12\,d^2\,e^2\,x^2-12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

```
[In] int(x^4/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2
+ 8*d^3*e*x))/(15*d*e^5*(d + e*x)^3*(d - e*x)^2)
```

3.140 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)-1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {864, 833, 792, 197}

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx\\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x(2d^3-3d^2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}\\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
gosper	$-\frac{(-ex+d)\left(-3e^4x^4-3d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)}{15d^2e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(-3e^4x^4-3d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15d^2e^4(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}}{e}+\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^3}-\frac{d}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-$

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/e
^4/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5-(3\,e^4x^4+3\,de^3x^3+3\,d^2e^2x^2-2\,d^3ex-2\,d^4)\sqrt{-e\cdots}}{15\,(d^2e^9x^5+d^3e^8x^4-2\,d^4e^7x^3-2\,d^5e^6x^2+d^6e^5x+d^7e^4)}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
+ 2*d^5 - (3*e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 + d^3*e^8*x^4 - 2*d^4*e^7*x^3 - 2*d^5*e^6*x
^2 + d^6*e^5*x + d^7*e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^3}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{d^2}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}e^5x+(-e^2x^2+d^2)^{\frac{3}{2}}de^4\right)} + \frac{2x}{5(-e^2x^2+d^2)^{\frac{3}{2}}e^3} - \frac{d}{3(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e^3}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5*x + (-e^2*x^2 + d^2)^(3/2)*d*e^4) + 2/5
*x/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*d/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*
x/(sqrt(-e^2*x^2 + d^2)*d^2*e^3)
```

Giac [F]

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{x^3}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4-2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^3*(d - e*x)^2)
```

3.141 $\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = -\frac{x^2}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/5*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {869, 792, 197}

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = -\frac{x^2}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/5*x^2/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d
^2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x(2d+2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de} \\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2} \\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{(-ex+d)(2e^4x^4+2d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4)}{15d^3e^3(-e^2x^2+d^2)^{\frac{5}{2}}}$
trager	$\frac{(2e^4x^4+2d\,e^3x^3-3d^2e^2x^2+2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^3e^3(ex+d)^3(-ex+d)^2}$
default	$\frac{1}{3e^3(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^2}+\frac{d^2\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2(}{6d^2e^2\left(-\left(x+\right.\right.}\right.}{}\right.}{e^3}$

[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(2*e^4*x^4+2*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^3/e^3
/(-e^2*x^2+d^2)^(5/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{2e^5x^5+2de^4x^4-4d^2e^3x^3-4d^3e^2x^2+2d^4ex+2d^5+(2e^4x^4+2de^3x^3-}{15(d^3e^8x^5+d^4e^7x^4-2d^5e^6x^3-2d^6e^5x^2+d^7e}$$

[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] 1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x +
 2*d^5 + (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2*d^3*e*x + 2*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^3*e^8*x^5 + d^4*e^7*x^4 - 2*d^5*e^6*x^3 - 2*d^6*e^5*x^
2 + d^7*e^4*x + d^8*e^3)

Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{d}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^3\right)} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2} + \frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{2\,x}{15\sqrt{-e^2x^2+d^2}d^3e^2}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*d/((-e^2*x^2 + d^2)^(3/2)*e^4*x + (-e^2*x^2 + d^2)^(3/2)*d*e^3) - 1/15
*x/((-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/15
*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4+2\,d^3\,e\,x-3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3+2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4 + 2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^3*(d - e*x)^2)
```

3.142 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)+1/5/e^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 198, 197}

$$\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e} \\
&= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e} \\
&= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$
\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4)}{15d^4e^2(d-ex)^2(d+ex)^3}
$$

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

method	result
gosper	$\frac{(-ex+d)\left(-2e^4x^4-2d\,e^3x^3+3d^2e^2x^2+3d^3ex+3d^4\right)}{15d^4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$\frac{\left(-2e^4x^4-2d\,e^3x^3+3d^2e^2x^2+3d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^4(ex+d)^3(-ex+d)^2e^2}$
default	$\frac{\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}}{e}-\frac{d\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{}{3e^2d^4\sqrt{}}\right)}{5d}\right)}{e^2}$

[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(-2*e^4*x^4-2*d*e^3*x^3+3*d^2*e^2*x^2+3*d^3*e*x+3*d^4)/d^4/e^2/(-e^2*x^2+d^2)^(5/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5-\left(2\,e^4x^4+2\,de^3x^3-\right.}{15\left(d^4e^7x^5+d^5e^6x^4-2\,d^6e^5x^3-2\,d^7e^4x^2+d^8e^3\right.}$$

[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] 1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x + 3*d^5 - (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 3*d^3*e*x - 3*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*e^7*x^5 + d^5*e^6*x^4 - 2*d^6*e^5*x^3 - 2*d^7*e^4*x^2 + d^8*e^3*x + d^9*e^2)

Sympy [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)

[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2\right)} + \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^4e}$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5/((-e^2*x^2 + d^2)^(3/2)*e^3*x + (-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^2*e) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

Giac [F]

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+3\,d^3\,e\,x+3\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

```
[In] int(x/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 - 2*d*e^3*x^3 + 3*d^2*e^2*x^2 + 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^3*(d - e*x)^2)
```

3.143 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)-1/5/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {673, 198, 197}

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int\frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(-3d^4+12d^3ex+12d^2e^2x^2-8de^3x^3-8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 12*d^3*e*x + 12*d^2*e^2*x^2 - 8*d*e^3*x^3 -
8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(-ex+d)\left(8e^{4}x^{4}+8d\,e^{3}x^{3}-12d^{2}e^{2}x^{2}-12d^{3}ex+3d^{4}\right)}{15d^{5}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}$	70
trager	$-\frac{\left(8e^{4}x^{4}+8d\,e^{3}x^{3}-12d^{2}e^{2}x^{2}-12d^{3}ex+3d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{5}(ex+d)^{3}(-ex+d)^{2}e}$	79
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\frac{d}{e}\right)e^{2}+2de}{6d^{2}e^{2}\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2\left(x+\frac{d}{e}\right)e^{2}+2de}{3e^{2}d^{4}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e}$	164

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/
e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{-}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*e^3
*x^2 + d^9*e^2*x + d^10*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{5}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = -\frac{1}{5\left((-e^2x^2+d^2)^{\frac{3}{2}}de^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^2e\right)} + \frac{4\,x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}d^5}$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] -1/5/((-e^2*x^2 + d^2)^(3/2)*d*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^2*e) + 4/15
*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

```
[In] int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

3.144 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 1/15*(-4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/(e*x+d)/(-e^2*x^2+d^2)^(
3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(-8*e*x+15*d)/d^6/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {871, 837, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-5de^2+4e^3x}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^5e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^6}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^5}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^5e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4)}{(d-ex)^2(d+ex)^3} + 30\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2
 - e^2*x^2])/d])/(15*d^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.

Time = 0.38 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.14

method	result
default	$\frac{\frac{1}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d} - \frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2\left(x+\frac{d}{e}\right.}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e\right.}\right.}{}}{d}$

[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))-1/d*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(105) = 210.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.99

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{23\,e^5x^5+23\,de^4x^4-46\,d^2e^3x^3-46\,d^3e^2x^2+23\,d^4ex+23\,d^5+15\,(e^5x^5+d}{15\,(d}$$

[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] 1/15*(23*e^5*x^5 + 23*d*e^4*x^4 - 46*d^2*e^3*x^3 - 46*d^3*e^2*x^2 + 23*d^4*e*x + 23*d^5 + 15*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x + d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 - 7*d*e^3*x^3 - 27*d^2*e^2*x^2 + 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 + d^7*e^4*x^4 - 2*d^8*e^3*x^3 - 2*d^9*e^2*x^2 + d^10*e*x + d^11)

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.145 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] 1/15*(-5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x/(e*x+d)/(-e^2*x^2+d^
2)^(3/2)+e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(-5*e*x+8*d)/d^6/x/(-e^2
*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {871, 837, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
```

, 0]

Rubi steps

$$\text{integral} = \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-6de^2+5e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-24d^3e^4+15d^2e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-48d^5e^6+15d^4e^7x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^6}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^6}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^6}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^6e}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(15d^5+38d^4ex-52d^3e^2x^2-87d^2e^3x^3+33de^4x^4+48e^5x^5)}{x(d-ex)^2(d+ex)^3}-15\sqrt{d^2}e\log(x)+15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^8}$$

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((d*Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d^
2*e^3*x^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) - 15*Sq
rt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(136) = 272.

Time = 0.44 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7x}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^6\sqrt{d^2}}-\frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{60d^6e\left(x+\frac{d}{e}\right)^2}-\frac{413\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240d^7\left(x+\frac{d}{e}\right)}+$
default	$\frac{-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{d^2}}{d}-$

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^7/x+1/d^6*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-17/60/d^6/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-413/240/d^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24/d^6/e/(
x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-23/48/d^7/(x-d/e)*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)-1/20/d^5/e^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{23\,e^6x^6+23\,de^5x^5-46\,d^2e^4x^4-46\,d^3e^3x^3+23\,d^4e^2x^2+23\,d^5ex+15\left(e^6x^6+de^5x^5-2\,d^2e^4x^4-2\,d^3e^3x^3\cdots\right.}{15\left(d^7e^5x^6+d^8e^4x^5-2\,d\cdots\right.}$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(23*e^6*x^6 + 23*d*e^5*x^5 - 46*d^2*e^4*x^4 - 46*d^3*e^3*x^3 + 23*d^4
*e^2*x^2 + 23*d^5*e*x + 15*(e^6*x^6 + d*e^5*x^5 - 2*d^2*e^4*x^4 - 2*d^3*e^3
*x^3 + d^4*e^2*x^2 + d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*
x^5 + 33*d*e^4*x^4 - 87*d^2*e^3*x^3 - 52*d^3*e^2*x^2 + 38*d^4*e*x + 15*d^5)
*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 + d^8*e^4*x^5 - 2*d^9*e^3*x^4 - 2*d^10*
e^2*x^3 + d^11*e*x^2 + d^12*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.146 $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{7d-6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

```
[Out] 1/15*(-6*e*x+7*d)/d^4/x^2/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^2/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(-24*e*x+35*d
)/d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2+16/5*e*(-e^
2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = -\frac{7e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} + \frac{1}{5d^2x^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d-6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
```

```
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-7de^2+6e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-35d^3e^4+24d^2e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4} \\
&= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-105d^5e^6+48d^4e^7x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^6} \\
&= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{\int \frac{-96d^6e^7+105d^5e^8x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^{12}e^6} \\
&= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} \\
&\quad - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} + \frac{\left(7e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^7} \\
&= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} \\
&\quad - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} + \frac{\left(7e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^7}
\end{aligned}
$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^7}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6)}{x^2(d-ex)^2(d+ex)^3} - 105\sqrt{d^2}e^2}{30d^9}$$

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3
*x^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d + e
*x)^3) - 105*Sqrt[d^2]*e^2*Log[x] + 105*Sqrt[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[
d^2 - e^2*x^2]])/(30*d^9)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^8x^2} - \frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^7\sqrt{d^2}} - \frac{29e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{48d^8\left(x-\frac{d}{e}\right)} + \frac{11\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}{30d^7\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{1}{2d^2x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{5e^2\left(\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}} - \frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}\right)}{2d^2}}{d} + \frac{e^2\left(\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{1}{d^2\sqrt{-e^2}}\right.}{}$

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e*x+d)/d^8/x^2-7/2/d^7*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-29/48/d^8*e/(x-d/e)*(-(x-d/e)^2*
```

```
e^2-2*d*e*(x-d/e))^(1/2)+11/30/d^7/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)+673/240/d^8*e/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/20/d^6/
e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24/d^7/(x-d/e)^2*(-(x-d/
e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{116\,e^7x^7+116\,de^6x^6-232\,d^2e^5x^5-232\,d^3e^4x^4+116\,d^4e^3x^3+116\,d^5e^2x^2}{}$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 + 116*d*e^6*x^6 - 232*d^2*e^5*x^5 - 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 + 116*d^5*e^2*x^2 + 105*(e^7*x^7 + d*e^6*x^6 - 2*d^2*e^5*x^5 -
 2*d^3*e^4*x^4 + d^4*e^3*x^3 + d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) + (96*e^6*x^6 - 9*d*e^5*x^5 - 249*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 + 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 + d^9*e^
4*x^6 - 2*d^10*e^3*x^5 - 2*d^11*e^2*x^4 + d^12*e*x^3 + d^13*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\,\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 215

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3} + \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x} + \frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}$$

```
[Out] 1/15*(-7*e*x+8*d)/d^4/x^3/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^3/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)+7/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^9+1/15*(-35*e*x+48*d
)/d^6/x^3/(-e^2*x^2+d^2)^(1/2)-64/15*(-e^2*x^2+d^2)^(1/2)/d^7/x^3+7/2*e*(-e
^2*x^2+d^2)^(1/2)/d^8/x^2-128/15*e^2*(-e^2*x^2+d^2)^(1/2)/d^9/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{7e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x} + \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} + \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{-8de^2+7e^3x}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{-48d^3e^4+35d^2e^5x}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}\\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-192d^5e^6+105d^4e^7x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^6}\\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{\int \frac{-315d^6e^7+384d^5e^8x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{45d^{12}e^6}\\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}\\
&\quad-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{\int \frac{-768d^7e^8+315d^6e^9x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{90d^{14}e^6}\\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}\\
&\quad-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}-\frac{\left(7e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} \\
&\quad-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}-\frac{(7e^3)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d^8} \\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} \\
&\quad-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{(7e)\mathrm{Subst}\left(\int\frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^8} \\
&= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} \\
&\quad-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$
\int\frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}\left(10d^7-5d^6ex+75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5+151de^6x^6+256e^7x^7\right)}{x^3(d-ex)^2(d+ex)^3}-105\sqrt{d^2}e^3\log(x)+105\sqrt{d^2}e^3\log}{30d^{10}}
$$

```
[In] Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/30*((d*Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d^
4*e^3*x^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x^7
))/(x^3*(d - e*x)^2*(d + e*x)^3) - 105*Sqrt[d^2]*e^3*Log[x] + 105*Sqrt[d^2]
*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^10
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(22e^2x^2-3dex+2d^2)}{6d^9x^3}+\frac{7e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^8\sqrt{d^2}}-\frac{331e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{80d^9\left(x+\frac{d}{e}\right)}-\frac{35e^2\sqrt{-\left(x-\frac{d}{e}\right)^2}}{48d^9\left(\right.}$
default	$\frac{-\frac{1}{3d^2x^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2e^2\left(-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{d^2}\right)}{d^2}}{d}+\frac{e^2\left(-\frac{1}{d^2x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4e^2\left(\frac{}{3d^2}\right.}{}\right.}{d}$

```
[In] int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-e^2*x^2+d^2)^(1/2)*(22*e^2*x^2-3*d*e*x+2*d^2)/d^9/x^3+7/2*e^3/d^8/(d
^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-331/80*e^2/d^9/(
x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-35/48*e^2/d^9/(x-d/e)*(-(x-d/e)
^2*e^2-2*d*e*(x-d/e))^(1/2)-9/20/d^8*e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)-1/20/d^7/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/24/d^8
*e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{116\,e^8x^8+116\,de^7x^7-232\,d^2e^6x^6-232\,d^3e^5x^5+116\,d^4e^4x^4+116\,d^5e^3x^3+105\left(e^8x^8+de^7x^7-2\,d^2e^6x^6\right.}{30\,(d^9}$$

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(116*e^8*x^8 + 116*d*e^7*x^7 - 232*d^2*e^6*x^6 - 232*d^3*e^5*x^5 + 11
6*d^4*e^4*x^4 + 116*d^5*e^3*x^3 + 105*(e^8*x^8 + d*e^7*x^7 - 2*d^2*e^6*x^6
- 2*d^3*e^5*x^5 + d^4*e^4*x^4 + d^5*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (256*e^7*x^7 + 151*d*e^6*x^6 - 489*d^2*e^5*x^5 - 244*d^3*e^4*x^4 + 2
36*d^4*e^3*x^3 + 75*d^5*e^2*x^2 - 5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 + d^2))
/(d^9*e^5*x^8 + d^10*e^4*x^7 - 2*d^11*e^3*x^6 - 2*d^12*e^2*x^5 + d^13*e*x^4
 + d^14*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^4\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.148 $\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{7/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{7/2}} dx = \frac{x^2(d-ex)}{7e^2(d^2-e^2x^2)^{7/2}} - \frac{2d-3ex}{35e^4(d^2-e^2x^2)^{5/2}} - \frac{x}{35d^2e^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/7*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(7/2)+1/35*(3*e*x-2*d)/e^4/(-e^2*x^2+d^2)^(5/2)-1/35*x/d^2/e^3/(-e^2*x^2+d^2)^(3/2)-2/35*x/d^4/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 792, 198, 197}

$$\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{7/2}} dx = \frac{x^2(d-ex)}{7e^2(d^2-e^2x^2)^{7/2}} - \frac{2d-3ex}{35e^4(d^2-e^2x^2)^{5/2}} - \frac{x}{35d^2e^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2 - e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^3(d-ex)}{(d^2-e^2x^2)^{9/2}}\,dx$$

$$= \frac{x^2(d-ex)}{7e^2\,(d^2-e^2x^2)^{7/2}} - \frac{\int \frac{x(2d^3-3d^2ex)}{(d^2-e^2x^2)^{7/2}}\,dx}{7d^2e^2}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35e^3}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{35d^2e^3}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^6-2d^5ex+5d^4e^2x^2+5d^3e^3x^3+5d^2e^4x^4-2de^5x^5-2e^6x^6)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^6 - 2*d^5*e*x + 5*d^4*e^2*x^2 + 5*d^3*e^3*x^3 +
5*d^2*e^4*x^4 - 2*d*e^5*x^5 - 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*x)
^4)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
gosper	$-\frac{(-ex+d)(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3x^3e^3-5d^4e^2x^2+2d^5ex+2d^6)}{35d^4e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3x^3e^3-5d^4e^2x^2+2d^5ex+2d^6)\sqrt{-e^2x^2+d^2}}{35d^4e^4(ex+d)^4(-ex+d)^3}$
default	$\frac{\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}}{e}+\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\cdots}{d^2}\right.}{e^3}$

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(-e*x+d)*(2*e^6*x^6+2*d*e^5*x^5-5*d^2*e^4*x^4-5*d^3*e^3*x^3-5*d^4*e^2
*x^2+2*d^5*e*x+2*d^6)/d^4/e^4/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(103) = 206$.

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.03

$$\int \frac{x^3}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{2\,e^7x^7+2\,de^6x^6-6\,d^2e^5x^5-6\,d^3e^4x^4+6\,d^4e^3x^3+6\,d^5e^2x^2-2\,d^6ex-2\,d^7-(2\,e^6x^6+2\,de^5x^5-5\,d^2e^4x}{35\,(d^4e^{11}x^7+d^5e^{10}x^6-3\,d^6e^9x^5-3\,d^7e^8x^4+3\,d^8e^7x^3+3\,d^9e^6x^2-}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(2*e^7*x^7 + 2*d*e^6*x^6 - 6*d^2*e^5*x^5 - 6*d^3*e^4*x^4 + 6*d^4*e^3*
x^3 + 6*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (2*e^6*x^6 + 2*d*e^5*x^5 - 5*d^2*
e^4*x^4 - 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 + 2*d^5*e*x + 2*d^6)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^11*x^7 + d^5*e^10*x^6 - 3*d^6*e^9*x^5 - 3*d^7*e^8*x^4 + 3*d^
8*e^7*x^3 + 3*d^9*e^6*x^2 - d^10*e^5*x - d^11*e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2}{7\left((-e^2x^2+d^2)^{\frac{5}{2}}e^5x+(-e^2x^2+d^2)^{\frac{5}{2}}de^4\right)}$$
$$+\frac{8\,x}{35\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3}-\frac{d}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4}-\frac{x}{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^3}-\frac{2\,x}{35\,\sqrt{-e^2x^2+d^2}d^4e^3}$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/7*d^2/((-e^2*x^2 + d^2)^(5/2)*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d*e^4) + 8/3
5*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/3
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^3) - 2/35*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^3
)
```

Giac [F]

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{56\,d\,e^4\,(d+e\,x)^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d\,e^4}+\frac{x}{35\,d^2\,e^3}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2\,d}{35\,e^4}-\frac{11\,x}{70\,e^3}\right)}{(d+e\,x)^3\,(d-e\,x)^3} - \frac{2\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e^3\,(d+e\,x)\,(d-e\,x)}$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d*e^4*(d + e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(1/(5
6*d*e^4) + x/(35*d^2*e^3)))/((d + e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1
/2)*((2*d)/(35*e^4) - (11*x)/(70*e^3)))/((d + e*x)^3*(d - e*x)^3) - (2*x*(d
^2 - e^2*x^2)^(1/2))/(35*d^4*e^3*(d + e*x)*(d - e*x))
```

3.149 $\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/7*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+2/35*(2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {869, 792, 198, 197}

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -1/7*x^2/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*
(d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/
(105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{x(2d+4ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7de}\\
&= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}\\
&= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{105d^3e^2}\\
&= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6}{105d^5e^3(d-ex)^3(d+ex)^4}$$

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{(-ex+d)\left(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3x^3e^3-15d^4e^2x^2+6d^5ex+6d^6\right)}{105d^5e^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$\frac{\left(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3x^3e^3-15d^4e^2x^2+6d^5ex+6d^6\right)\sqrt{-e^2x^2+d^2}}{105d^5(ex+d)^4(-ex+d)^3e^3}$
default	$\frac{1}{5e^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{e^2}+\frac{d^2\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)}\right.}{}$

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^4
*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(107) = 214.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.93

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{6\,e^7x^7+6\,de^6x^6-18\,d^2e^5x^5-18\,d^3e^4x^4+18\,d^4e^3x^3+18\,d^5e^2x^2-6\,d^6ex - \cdots}{105\,(d^5e^{10}x^7+d^6e^9x^6-3\,d^7e^8x^5 - \cdots}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*e^7*x^7 + 6*d*e^6*x^6 - 18*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 18*d^4*e
^3*x^3 + 18*d^5*e^2*x^2 - 6*d^6*e*x - 6*d^7 + (8*e^6*x^6 + 8*d*e^5*x^5 - 20
*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 - 6*d^5*e*x - 6*d^6)*sqrt(-e
^2*x^2 + d^2))/(d^5*e^10*x^7 + d^6*e^9*x^6 - 3*d^7*e^8*x^5 - 3*d^8*e^7*x^4
+ 3*d^9*e^6*x^3 + 3*d^10*e^5*x^2 - d^11*e^4*x - d^12*e^3)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{x^2}{(d+ex)\,(d^2-e^2x^2)^{7/2}}\,dx &= -\frac{d}{7\left((-e^2x^2+d^2)^{\frac{5}{2}}e^4x+(-e^2x^2+d^2)^{\frac{5}{2}}de^3\right)} \\ &\quad - \frac{x}{35\,(-e^2x^2+d^2)^{\frac{5}{2}}de^2}+\frac{1}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} \\ &\quad - \frac{4\,x}{105\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2}-\frac{8\,x}{105\,\sqrt{-e^2x^2+d^2}d^5e^2}\end{aligned}$$

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/7*d/((-e^2*x^2 + d^2)^(5/2)*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d*e^3) - 1/35
*x/((-e^2*x^2 + d^2)^(5/2)*d*e^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e^3) - 4/10
5*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d^2)*d^5*e^
2)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")

[Out] integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)

Mupad [B] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}-\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}+\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d+e\,x\right)}^4}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)

[Out] ((d^2 - e^2*x^2)^(1/2)*(1/(56*d^2*e^3) - (4*x)/(105*d^3*e^2)))/((d + e*x)^2
*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*(2/(35*e^3) + (3*x)/(70*d*e^2)))/((d
 + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d + e*x)^4) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\arcsin(ax)}{2a^4}$$

```
[Out] 3/2*arcsin(a*x)/a^4+x^2*(-a*x+1)/a^2/(-a^2*x^2+1)^(1/2)+1/2*(-3*a*x+4)*(-a^
2*x^2+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {864, 833, 794, 222}

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{3\arcsin(ax)}{2a^4} + \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

```
[In] Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(1-ax)}{(1-a^2x^2)^{3/2}}\,dx \\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2-3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3} \\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\sin^{-1}(ax)}{2a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}(4+ax-a^2x^2)}{2a^4(1+ax)} + \frac{3\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^4}$$

```
[In] Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] (Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2))/(2*a^4*(1 + a*x)) + (3*ArcTan[(a*x)
/(-1 + Sqrt[1 - a^2*x^2])])/a^4
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{(ax-2)(a^2x^2-1)}{2a^4\sqrt{-a^2x^2+1}} + \frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^3\sqrt{a^2}} + \frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2+2\left(x+\frac{1}{a}\right)a}}{a^5\left(x+\frac{1}{a}\right)}$	97
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^3\sqrt{a^2}} + \frac{-\frac{x\sqrt{-a^2x^2+1}}{2a^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}}{a} + \frac{\sqrt{-a^2x^2+1}}{a^4} + \frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2+2\left(x+\frac{1}{a}\right)a}}{a^5\left(x+\frac{1}{a}\right)}$	134

```
[In] int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x-2)*(a^2*x^2-1)/a^4/(-a^2*x^2+1)^(1/2)+3/2/a^3/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{4\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-(a^2x^2-ax-4)\sqrt{-a^2x^2+1}+4}{2\,(a^5x+a^4)}$$

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
- a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

Sympy [F]

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

```
[In] integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-a^2x^2+1}}{a^5x+a^4} - \frac{\sqrt{-a^2x^2+1}x}{2\,a^3} + \frac{3\,\arcsin(ax)}{2\,a^4} + \frac{\sqrt{-a^2x^2+1}}{a^4}$$

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcsi
n(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,\sqrt{-a^2}} - \frac{\left(\frac{1}{a^2\,\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a^3}\right)\,\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

```
[In] int(x^3/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
[Out] (3*asinh(x*(-a^2)^(1/2)))/(2*a^3*(-a^2)^(1/2)) - ((1/(a^2*(-a^2)^(1/2)) + (
x*(-a^2)^(1/2))/(2*a^3))*(1 - a^2*x^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^
(1/2)/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\arcsin(ax)}{a^3}$$

```
[Out] -arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)/a^3-(-a^2*x^2+1)^(1/2)/a^3/(a*x+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1653, 12, 807, 222}

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\arcsin(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(ax+1)} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[In] Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{a^3x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a^4}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

```
[In] Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (2*ArcTan[(a*x)/(-1 + Sqrt
[1 - a^2*x^2])])/a^3
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}}{a^{3}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\, x}{\sqrt{-a^{2} x^{2}+1}}\right)}{a^{2} \sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a}}{a^{4}\left(x+\frac{1}{a}\right)}$	84
risch	$\frac{a^{2} x^{2}-1}{a^{3} \sqrt{-a^{2} x^{2}+1}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\, x}{\sqrt{-a^{2} x^{2}+1}}\right)}{a^{2} \sqrt{a^{2}}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2}+2\left(x+\frac{1}{a}\right) a}}{a^{4}\left(x+\frac{1}{a}\right)}$	92

```
[In] int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a^2*x^2+1)^(1/2)/a^3-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [F]

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

```
[In] integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3} - \frac{\arcsin{(ax)}}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3}$$

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a^
3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\arcsin{(ax)}\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3}$$

```
[In] int(x^2/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\arcsin(ax)}{a^2}$$

```
[Out] arcsin(a*x)/a^2+(-a^2*x^2+1)^(1/2)/a^2/(a*x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {807, 222}

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx=\frac{\arcsin(ax)}{a^2}+\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}$$

```
[In] Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
```

```
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a} \\ &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)} + \frac{\sin^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)} + \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^2}$$

```
[In] Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + (2*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2]
)])/a^2
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a\sqrt{a^2}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^3\left(x+\frac{1}{a}\right)}$	65

```
[In] int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3/(x+1/a)*(-(x
+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}+1}{a^3x+a^2}$$

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 +
 1) + 1)/(a^3*x + a^2)
```

Sympy [F]

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x}{\sqrt{-\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

```
[In] integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{-a^2x^2+1}}{a^3x+a^2} + \frac{\arcsin{(ax)}}{a^2}$$

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin{(ax)}\,\mathrm{sgn}(a)}{a|a|} - \frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{1}{a^2\sqrt{1-a^2\,x^2}} - \frac{x}{a\sqrt{1-a^2\,x^2}} - \frac{\mathrm{asinh}\left(x\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3}$$

[In] int(x/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)

[Out] 1/(a^2*(1 - a^2*x^2)^(1/2)) - x/(a*(1 - a^2*x^2)^(1/2)) - (asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/a^3

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/a/(a*x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {665}

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

```
[In] Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

```
[In] Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{ax-1}{a\sqrt{-a^2x^2+1}}$	22
trager	$-\frac{\sqrt{-a^2x^2+1}}{a(ax+1)}$	25
default	$-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2+2\left(x+\frac{1}{a}\right)a}}{a^2\left(x+\frac{1}{a}\right)}$	36

```
[In] int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*x-1)/a/(-a^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

Sympy [F]

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

```
[In] integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \frac{2}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2\,x^2}}{x\,a^2+a}$$

```
[In] int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
[Out] -(1 - a^2*x^2)^(1/2)/(a + a^2*x)
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{1-ax} - \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arctanh((-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {871, 12, 272, 65, 214}

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{1-ax} - \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[In] Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{-1+ax} - \log(x) + \log\left(-1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(-1 + a*x)) - Log[x] + Log[-1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}$	58

```
[In] int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(1/(-a^2*x^2+1)^(1/2))-1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{ax + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1} - 1}{ax-1}$$

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*x - 1)
```

Sympy [F]

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx$$

```
[In] integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] -Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x}\,dx$$

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 2
*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a\sqrt{1-a^2x^2}}{\sqrt{-a^2}\left(\frac{a}{\sqrt{-a^2}}+x\sqrt{-a^2}\right)} - \operatorname{atanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[In] int(-1/(x*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
[Out] (a*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*(a/(-a^2)^(1/2) + x*(-a^2)^(1/2))) -
atanh((1 - a^2*x^2)^(1/2))
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a\,\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-2*(-a^2*x^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x/
(-a*x+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {871, 821, 272, 65, 214}

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = -a\,\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)}$$

```
[In] Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\int \frac{-2a^2-a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{(-1+2ax)\sqrt{1-a^2x^2}}{x(-1+ax)} - a\log(x) + a\log\left(-1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(((-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x))) - a*Log[x] + a*Log[-1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}}{x} - a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}$	73
risch	$\frac{a^2x^2-1}{x\sqrt{-a^2x^2+1}} - a\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}\right)$	84

```
[In] int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a^2*x^2+1)^(1/2)/x-a*arctanh(1/(-a^2*x^2+1)^(1/2))-1/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^2x^2 - ax + (a^2x^2-ax)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}(2\,ax-1)}{ax^2-x}$$

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx$$

```
[In] integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] -Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^2}\,dx$$

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^2\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}} - \frac{\sqrt{1-a^2x^2}}{x} - a\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[In] int(-1/(x^2*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
[Out] (a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/x - a*atanh((1 - a^2*x^2)^(1/2))
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3}{2}a^2\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -3/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-3/2*(-a^2*x^2+1)^(1/2)/x^2-2*a*(-a^2*x
^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x^2/(-a*x+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {871, 849, 821, 272, 65, 214}

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{3}{2}a^2\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}$$

```
[In] Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{\int \frac{-3a^2-2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{\int \frac{4a^3+3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{2}\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$
\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}\left(\frac{(1+ax-4a^2x^2)\sqrt{1-a^2x^2}}{x^2(-1+ax)} - 3a^2\log(x) + 3a^2\log\left(-1+\sqrt{1-a^2x^2}\right)\right)
$$

```
[In] Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(x^2*(-1 + a*x)) - 3*a^2*Log[x]
+ 3*a^2*Log[-1 + Sqrt[1 - a^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{\sqrt{-a^{2}x^{2}+1}}{2x^{2}}-\frac{3a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}-\frac{a\sqrt{-a^{2}x^{2}+1}}{x}-\frac{a\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}$	94
risch	$\frac{2a^{3}x^{3}+a^{2}x^{2}-2ax-1}{2x^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{a^{2}\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)-\frac{2\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}\right)}{2}$	102

```
[In] int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-a^2*x^2+1)^(1/2)/x^2-3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*x
^2+1)^(1/2)/x-a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{2\,a^3x^3-2\,a^2x^2+3\,(a^3x^3-a^2x^2)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-(4\,a^2x^2-ax-1)\sqrt{-a^2x^2+1}}{2\,(ax^3-x^2)}$$

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\int \frac{1}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx$$

```
[In] integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] -Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \int -\frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^3}\,dx$$

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.37

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|} - \frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2
))/a^2
```

Mupad [B] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{a^3\sqrt{1-a^2\,x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}} - \frac{a\sqrt{1-a^2\,x^2}}{x} - \frac{\sqrt{1-a^2\,x^2}}{2\,x^2} + \frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{2}$$

```
[In] int(-1/(x^3*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a*
(1 - a^2*x^2)^(1/2))/x + (a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2
)^(1/2)/a)*(-a^2)^(1/2))
```

3.157 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6(d^2-e^2x^2)^{3/2} - \frac{d^5(256d-315ex)(d^2-e^2x^2)^{3/2}}{2016e^6} - \frac{5d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}$$

```
[Out] -4/21*d^4*x^2*(-e^2*x^2+d^2)^(3/2)/e^4+5/24*d^3*x^3*(-e^2*x^2+d^2)^(3/2)/e^
3-5/21*d^2*x^4*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d*x^5*(-e^2*x^2+d^2)^(3/2)/e-1/
9*x^6*(-e^2*x^2+d^2)^(3/2)-1/2016*d^5*(-315*e*x+256*d)*(-e^2*x^2+d^2)^(3/2)
/e^6-5/64*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6-5/64*d^7*x*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6} - \frac{1}{9}x^6(d^2-e^2x^2)^{3/2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} - \frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{d^5(256d-315ex)(d^2-e^2x^2)^{3/2}}{2016e^6} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3}$$

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5(d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^5(-15d^2e^2+18de^3x)\sqrt{d^2-e^2x^2}\,dx}{9e^2} \\
&= \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x^4(-90d^3e^3+120d^2e^4x)\sqrt{d^2-e^2x^2}\,dx}{72e^4} \\
&= -\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} \\
&\quad -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^3(-480d^4e^4+630d^3e^5x)\sqrt{d^2-e^2x^2}\,dx}{504e^6} \\
&= \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3}-\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} \\
&\quad -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x^2(-1890d^5e^5+2880d^4e^6x)\sqrt{d^2-e^2x^2}\,dx}{3024e^8} \\
&= -\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4}+\frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} \\
&\quad -\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} \\
&\quad -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x(-5760d^6e^6+9450d^5e^7x)\sqrt{d^2-e^2x^2}\,dx}{15120e^{10}} \\
&= -\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4}+\frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3}-\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} \\
&\quad -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6}-\frac{(5d^7)\int\sqrt{d^2-e^2x^2}\,dx}{32e^5}
\end{aligned}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6} - \frac{(5d^9)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{64e^5}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6} - \frac{(5d^9)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{64e^5}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6} - \frac{5d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.68

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5}{40}$$

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*
e^3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*
x^7 + 448*e^8*x^8) - 315*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^
2*x^2]])/(4032*e^7)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(-448e^{8}x^{8}+1008d\,e^{7}x^{7}-512d^{2}e^{6}x^{6}-168d^{3}e^{5}x^{5}+192d^{4}x^{4}e^{4}-210d^{5}e^{3}x^{3}+256d^{6}e^{2}x^{2}-315d^{7}ex+512d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{4032e^{6}}-\frac{5d^{9}\,\mathrm{arcta}}{}$
default	$\frac{-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{63e^{4}}}{e^{2}}-\frac{4d^{3}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{e^{5}}$

[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/4032*(-448*e^8*x^8+1008*d*e^7*x^7-512*d^2*e^6*x^6-168*d^3*e^5*x^5+192*d^4*e^4*x^4-210*d^5*e^3*x^3+256*d^6*e^2*x^2-315*d^7*e*x+512*d^8)/e^6*(-e^2*x^2+d^2)^(1/2)-5/64*d^9/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx=\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(448\,e^8x^8-1008\,de^7x^7+512\,d^2e^6x^6+168\,d^3e^5x^5\right.}{4032\,e^6}$$

[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

```
[Out] 1/4032*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (448*e^8*x^8 -
1008*d*e^7*x^7 + 512*d^2*e^6*x^6 + 168*d^3*e^5*x^5 - 192*d^4*e^4*x^4 + 210*
d^5*e^3*x^3 - 256*d^6*e^2*x^2 + 315*d^7*e*x - 512*d^8)*sqrt(-e^2*x^2 + d^2)
)/e^6
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{16d^8}{315e^8}-\frac{8d^6x^2}{315e^6}-\frac{2d^4x^4}{105e^4}-\frac{d^2x^6}{63e^2}+\frac{x^8}{9}\right) & \text{for } e^2\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(1
05*e**4) - d**2*x**4/(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6,
 True)) - 2*d*e*Piecewise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*
sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**
2), True))/(128*e**6) + sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d*
*4*x**3/(192*e**4) - d**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqr
t(d**2)/7, True)) + e**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-16*d**8/(315*e
**8) - 8*d**6*x**2/(315*e**6) - 2*d**4*x**4/(105*e**4) - d**2*x**6/(63*e**2
) + x**8/9), Ne(e**2, 0)), (x**8*sqrt(d**2)/8, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.31

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^5}{4\,(e^7x+de^6)} - \frac{5i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^6} - \frac{85\,d^9\arcsin\left(\frac{ex}{d}\right)}{64\,e^6} + \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^7x}{4\,e^5} - \frac{85\,\sqrt{-e^2x^2+d^2}d^7x}{64\,e^5} + \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^8}{2\,e^6} + \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5x}{96\,e^5} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6}{12\,e^6} - \frac{17\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3x}{24\,e^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x^2}{9\,e^4} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{e^6} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}dx}{4\,e^5} - \frac{29\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2}{63\,e^6}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^7*x + d*e^6) - 5/4*I*d^9*arcsin(e*x/d +
2)/e^6 - 85/64*d^9*arcsin(e*x/d)/e^6 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*
d^7*x/e^5 - 85/64*sqrt(-e^2*x^2 + d^2)*d^7*x/e^5 + 5/2*sqrt(e^2*x^2 + 4*d*e
*x + 3*d^2)*d^8/e^6 + 35/96*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^5 - 5/12*(-e^2*x
^2 + d^2)^(3/2)*d^6/e^6 - 17/24*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^5 - 1/9*(-e^
2*x^2 + d^2)^(7/2)*x^2/e^4 + (-e^2*x^2 + d^2)^(5/2)*d^4/e^6 + 1/4*(-e^2*x^2
 + d^2)^(7/2)*d*x/e^5 - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^6
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.50

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(161280\,d^{10}e^{10}\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(315\,d^{10}e^{10}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{17}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right.\right.}{}\right.}{}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1032192*(161280*d^10*e^10*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d)
)*sgn(e) + (315*d^10*e^10*(2*d/(e*x + d) - 1)^(17/2)*sgn(1/(e*x + d))*sgn(e
) - 18774*d^10*e^10*(2*d/(e*x + d) - 1)^(15/2)*sgn(1/(e*x + d))*sgn(e) + 10
458*d^10*e^10*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) - 68958*d^
10*e^10*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 8192*d^10*e^10
```

```
*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 32418*d^10*e^10*(2*d/(
e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 10458*d^10*e^10*(2*d/(e*x + d
) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 2730*d^10*e^10*(2*d/(e*x + d) - 1)^(
3/2)*sgn(1/(e*x + d))*sgn(e) - 315*d^10*e^10*sqrt(2*d/(e*x + d) - 1)*sgn(1/
(e*x + d))*sgn(e))*(e*x + d)^9/d^9)*abs(e)/(d*e^17)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2 - e^2x^2)^{5/2}}{(d + ex)^2}\,dx = \int \frac{x^5\,\left(d^2 - e^2\,x^2\right)^{5/2}}{\left(d + e\,x\right)^2}\,dx$$

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 200

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5(d^2-e^2x^2)^{3/2} + \frac{d^4(1024d-1365ex)(d^2-e^2x^2)^{3/2}}{6720e^5} + \frac{13d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

```
[Out] 8/35*d^3*x^2*(-e^2*x^2+d^2)^(3/2)/e^3-13/48*d^2*x^3*(-e^2*x^2+d^2)^(3/2)/e^
2+2/7*d*x^4*(-e^2*x^2+d^2)^(3/2)/e-1/8*x^5*(-e^2*x^2+d^2)^(3/2)+1/6720*d^4*
(-1365*e*x+1024*d)*(-e^2*x^2+d^2)^(3/2)/e^5+13/128*d^8*arctan(e*x/(-e^2*x^2
+d^2)^(1/2))/e^5+13/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{13d^8\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5} - \frac{1}{8}x^5(d^2-e^2x^2)^{3/2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^4(1024d-1365ex)(d^2-e^2x^2)^{3/2}}{6720e^5} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4(d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^4(-13d^2e^2+16de^3x)\sqrt{d^2-e^2x^2}\,dx}{8e^2} \\
&= \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^3(-64d^3e^3+91d^2e^4x)\sqrt{d^2-e^2x^2}\,dx}{56e^4} \\
&= -\frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} \\
&\quad - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2(-273d^4e^4+384d^3e^5x)\sqrt{d^2-e^2x^2}\,dx}{336e^6} \\
&= \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} \\
&\quad - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x(-768d^5e^5+1365d^4e^6x)\sqrt{d^2-e^2x^2}\,dx}{1680e^8} \\
&= \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} \\
&\quad - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5} + \frac{(13d^6)\int\sqrt{d^2-e^2x^2}\,dx}{64e^4} \\
&= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} \\
&\quad - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5} + \frac{(13d^8)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{128e^4}
\end{aligned}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5(d^2-e^2x^2)^{3/2} + \frac{d^4(1024d-1365ex)(d^2-e^2x^2)^{3/2}}{6720e^5} + \frac{(13d^8)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5(d^2-e^2x^2)^{3/2} + \frac{d^4(1024d-1365ex)(d^2-e^2x^2)^{3/2}}{6720e^5} + \frac{13d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e}{13440e^5}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4*
e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*x^
7) - 2730*d^8*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(13440*e^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(1680e^7x^7-3840d\,e^6x^6+1960d^2e^5x^5+768d^3e^4x^4-910d^4e^3x^3+1024d^5e^2x^2-1365d^6ex+2048d^7)\sqrt{-e^2x^2+d^2}}{13440e^5}+\frac{13d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128e^4\sqrt{e^2}}$
default	$\frac{-\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8e^2}}{e^2}+\frac{3d^2\left(\frac{x(-e^2x^2+\ldots}{6}\right.\ldots}{\ldots}$

[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/13440*(1680*e^7*x^7-3840*d*e^6*x^6+1960*d^2*e^5*x^5+768*d^3*e^4*x^4-910*d^4*e^3*x^3+1024*d^5*e^2*x^2-1365*d^6*e*x+2048*d^7)/e^5*(-e^2*x^2+d^2)^(1/2)+13/128*d^8/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx =$$
$$-\frac{2730\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(1680\,e^7x^7-3840\,de^6x^6+1960\,d^2e^5x^5+768\,d^3e^4x^4-910\,d^4e^3x^3+1024\ldots}{13440\,e^5}$$

[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] -1/13440*(2730*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1680*e^7*x^7 - 3840*d*e^6*x^6 + 1960*d^2*e^5*x^5 + 768*d^3*e^4*x^4 - 910*d^4*e^3*x^3 + 1024*d^5*e^2*x^2 - 1365*d^6*e*x + 2048*d^7)*sqrt(-e^2*x^2 + d^2))/e^5

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.70

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\frac{5d^8\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{128e^6}+\sqrt{d^2-e^2x^2}\left(-\frac{5d^6x}{128e^6}-\frac{5d^4x^3}{192e^4}-\frac{d^2x^5}{48e^2}+\frac{x^7}{8}\right) & \text{for } e^2\neq 0\\ \frac{x^7\sqrt{d^2}}{7} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(1
6*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) +
 x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - 2*d*e*Piecewise((sqrt(
d**2 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/
(35*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True)) + e**2*Piecew
ise((5*d**8*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2)
)/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(128*e**6)
+ sqrt(d**2 - e**2*x**2)*(-5*d**6*x/(128*e**6) - 5*d**4*x**3/(192*e**4) - d
**2*x**5/(48*e**2) + x**7/8), Ne(e**2, 0)), (x**7*sqrt(d**2)/7, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.38

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{4\,(e^6x+de^5)} + \frac{7i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5} + \frac{125\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^5} - \frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}d^6x}{8\,e^4} + \frac{125\sqrt{-e^2x^2+d^2}d^6x}{128\,e^4} - \frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}d^7}{4\,e^5} - \frac{67\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4x}{192\,e^4} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{12\,e^5} + \frac{25\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2x}{48\,e^4} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{5\,e^5} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}x}{8\,e^4} + \frac{2\,(-e^2x^2+d^2)^{\frac{7}{2}}d}{7\,e^5}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^6*x + d*e^5) + 7/8*I*d^8*arcsin(e*x/d + 2
)/e^5 + 125/128*d^8*arcsin(e*x/d)/e^5 - 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)
*d^6*x/e^4 + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^4 - 7/4*sqrt(e^2*x^2 + 4*
d*e*x + 3*d^2)*d^7/e^5 - 67/192*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^4 + 5/12*(-e
^2*x^2 + d^2)^(3/2)*d^5/e^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^4 - 4/5*
(-e^2*x^2 + d^2)^(5/2)*d^3/e^5 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^4 + 2/7*(-e
^2*x^2 + d^2)^(7/2)*d/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.56

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(349440\,d^9e^9\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(1365\,d^9e^9\left(\frac{2\,d}{ex+d}-1\right)^{\frac{15}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) - 61215\,d^9e^9\left(\frac{2\,d}{ex+d}-1\right)^{\frac{13}{2}}\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/1720320*(349440*d^9*e^9*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))
*sgn(e) + (1365*d^9*e^9*(2*d/(e*x + d) - 1)^(15/2)*sgn(1/(e*x + d))*sgn(e)
- 61215*d^9*e^9*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) + 20517*
d^9*e^9*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 141159*d^9*e^9
*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 34969*d^9*e^9*(2*d/(e*
```

```
x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 34853*d^9*e^9*(2*d/(e*x + d) -
1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 10465*d^9*e^9*(2*d/(e*x + d) - 1)^(3/2)*
sgn(1/(e*x + d))*sgn(e) - 1365*d^9*e^9*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x +
 d))*sgn(e))*(e*x + d)^8/d^8)*abs(e)/(d*e^15)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx=\int \frac{x^4\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.159 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4} - \frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

```
[Out] -11/35*d^2*x^2*(-e^2*x^2+d^2)^(3/2)/e^2+1/3*d*x^3*(-e^2*x^2+d^2)^(3/2)/e-1/
7*x^4*(-e^2*x^2+d^2)^(3/2)-1/420*d^3*(-105*e*x+88*d)*(-e^2*x^2+d^2)^(3/2)/e
^4-1/8*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/8*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4} - \frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -1/8*(d^5*x*Sqrt[d^2 - e^2*x^2])/e^3 - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3(d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^3\left(-11d^2e^2+14de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{7e^2} \\
&= \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x^2\left(-42d^3e^3+66d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{42e^4} \\
&= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} \\
&\quad -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x\left(-132d^4e^4+210d^3e^5x\right)\sqrt{d^2-e^2x^2}\,dx}{210e^6} \\
&= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} \\
&\quad -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{d^5\int\sqrt{d^2-e^2x^2}\,dx}{4e^3} \\
&= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} \\
&\quad -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{d^7\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^3} \\
&= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} \\
&\quad -\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{d^7\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} \\
&= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} \\
&\quad -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5 -}{840e^5}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^
3*x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(840*e^5)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(-120e^6x^6+280d\,e^5x^5-144d^2e^4x^4-70d^3x^3e^3+88d^4e^2x^2-105d^5ex+176d^6\right)\sqrt{-e^2x^2+d^2}}{840e^4}-\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^3\sqrt{e^2}}$
default	$-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7e^4}-\frac{2d\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{e^3}-\frac{d^3\left(-(x \cdots\right.}{\cdots}$

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/840*(-120*e^6*x^6+280*d*e^5*x^5-144*d^2*e^4*x^4-70*d^3*e^3*x^3+88*d^4*e^
2*x^2-105*d^5*e*x+176*d^6)/e^4*(-e^2*x^2+d^2)^(1/2)-1/8*d^7/e^3/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.68

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(120\,e^6x^6-280\,de^5x^5+144\,d^2e^4x^4+70\,d^3e^3x^3-8}{840\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (120*e^6*x^6 - 2
80*d*e^5*x^5 + 144*d^2*e^4*x^4 + 70*d^3*e^3*x^3 - 88*d^4*e^2*x^2 + 105*d^5*
e*x - 176*d^6)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.49

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{8d^6}{105e^6}-\frac{4d^4x^2}{105e^4}-\frac{d^2x^4}{35e^2}+\frac{x^6}{7}\right) & \text{for } e^2\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e
**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) - 2*d*e*Piecewise(
(d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt
(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**4) + sqrt(
d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**5/6), Ne(e*
*2, 0)), (x**5*sqrt(d**2)/5, True)) + e**2*Piecewise((sqrt(d**2 - e**2*x**2
)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/(35*e**2) + x**6
/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.47

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{4\,(e^5x+de^4)} - \frac{i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4} - \frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{8\,e^4} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{2\,e^3} - \frac{5\sqrt{-e^2x^2+d^2}d^5x}{8\,e^3} + \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{e^4} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{3\,e^3} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{12\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}dx}{3\,e^3} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{5\,e^4} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^4}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^5*x + d*e^4) - 1/2*I*d^7*arcsin(e*x/d +
2)/e^4 - 5/8*d^7*arcsin(e*x/d)/e^4 + 1/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
5*x/e^3 - 5/8*sqrt(-e^2*x^2 + d^2)*d^5*x/e^3 + sqrt(e^2*x^2 + 4*d*e*x + 3*d
^2)*d^6/e^4 + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^3 - 5/12*(-e^2*x^2 + d^2)^
(3/2)*d^4/e^4 - 1/3*(-e^2*x^2 + d^2)^(5/2)*d*x/e^3 + 3/5*(-e^2*x^2 + d^2)^(
5/2)*d^2/e^4 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.64

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(13440\,d^8e^8\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) + \frac{\left(105\,d^8e^8\left(\frac{2\,d}{ex+d}-1\right)^{\frac{13}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sg}\right.}{}\right.}{}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/53760*(13440*d^8*e^8*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn
(e) + (105*d^8*e^8*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e) - 378
0*d^8*e^8*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 189*d^8*e^8*
(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 4992*d^8*e^8*(2*d/(e*x
+ d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 1981*d^8*e^8*(2*d/(e*x + d) - 1)^
(5/2)*sgn(1/(e*x + d))*sgn(e) - 700*d^8*e^8*(2*d/(e*x + d) - 1)^(3/2)*sgn(1
/(e*x + d))*sgn(e) - 105*d^8*e^8*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*s
gn(e))*(e*x + d)^7/d^7)*abs(e)/(d*e^13)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx=\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.160 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2(d^2-e^2x^2)^{3/2}}{5e} - \frac{1}{6}x^3(d^2-e^2x^2)^{3/2}+\frac{d^2(32d-45ex)\,(d^2-e^2x^2)^{3/2}}{120e^3}+\frac{3d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 2/5*d*x^2*(-e^2*x^2+d^2)^(3/2)/e-1/6*x^3*(-e^2*x^2+d^2)^(3/2)+1/120*d^2*(-4
5*e*x+32*d)*(-e^2*x^2+d^2)^(3/2)/e^3+3/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))/e^3+3/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{3d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{2dx^2(d^2-e^2x^2)^{3/2}}{5e} - \frac{1}{6}x^3(d^2-e^2x^2)^{3/2}+\frac{d^2(32d-45ex)\,(d^2-e^2x^2)^{3/2}}{120e^3}+\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = \int x^2(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2\left(-9d^2e^2+12de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{6e^2}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x\left(-24d^3e^3+45d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{30e^4}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + \frac{\left(3d^4\right)\int\sqrt{d^2-e^2x^2}\,dx}{8e^2}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + \frac{\left(3d^6\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^2}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + \frac{\left(3d^6\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + \frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5)-90d^6\,\mathrm{arct}}{240e^3}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^3
 - 96*d*e^4*x^4 + 40*e^5*x^5) - 90*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 -
 e^2*x^2])])/(240*e^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
risch	$\frac{(40e^5x^5-96de^4x^4+50d^2e^3x^3+32d^3e^2x^2-45d^4ex+64d^5)\sqrt{-e^2x^2+d^2}}{240e^3}+\frac{3d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$
default	$\frac{\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}}{e^2}+\frac{d^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(40*e^5*x^5-96*d*e^4*x^4+50*d^2*e^3*x^3+32*d^3*e^2*x^2-45*d^4*e*x+64*
d^5)/e^3*(-e^2*x^2+d^2)^(1/2)+3/16*d^6/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{90\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(40\,e^5x^5-96\,de^4x^4+50\,d^2e^3x^3+32\,d^3e^2x^2-45\,d^4ex+64\,d^5)\sqrt{-e^2x^2+}}{240\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(90*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^5*x^5 - 96
*d*e^4*x^4 + 50*d^2*e^3*x^3 + 32*d^3*e^2*x^2 - 45*d^4*e*x + 64*d^5)*sqrt(-e
^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.10

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for}\\ \frac{x^3\sqrt{d^2}}{3} & \text{oth}\end{cases}\right.$$
$$-2de\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8
*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)),
(x**3*sqrt(d**2)/3, True)) - 2*d*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d*
```

```
*4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2
)/4, True)) + e**2*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)
*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x*
*2), True))/(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x*
*3/(24*e**2) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.62

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3}+\frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{4\left(e^4x+de^3\right)} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{8\,e^2}+\frac{5\sqrt{-e^2x^2+d^2}d^4x}{16\,e^2}-\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{4\,e^3} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e^2}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}{12\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^3}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*I*d^6*arcsin(e*x/d + 2)/e^3 + 5/16*d^6*arcsin(e*x/d)/e^3 + 1/4*(-e^2*x^
2 + d^2)^(5/2)*d^2/(e^4*x + d*e^3) - 1/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
4*x/e^2 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x
+ 3*d^2)*d^5/e^3 - 7/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e^2 + 5/12*(-e^2*x^2 +
 d^2)^(3/2)*d^3/e^3 + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e^2 - 2/5*(-e^2*x^2 + d^
2)^(5/2)*d/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(122) = 244.

Time = 0.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.75

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(2880\,d^7e^7\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)+\frac{\left(45\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{11}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)-1025\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/7680*(2880*d^7*e^7*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(
e) + (45*d^7*e^7*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) - 1025*
```

```
d^7*e^7*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 174*d^7*e^7*(2*
d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 594*d^7*e^7*(2*d/(e*x + d)
 - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 255*d^7*e^7*(2*d/(e*x + d) - 1)^(3/2)
*sgn(1/(e*x + d))*sgn(e) - 45*d^7*e^7*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x +
d))*sgn(e))*(e*x + d)^6/d^6)*abs(e)/(d*e^11)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x^2\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.161 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

```
[Out] -1/6*d*x*(-e^2*x^2+d^2)^(3/2)/e-2/15*(-e^2*x^2+d^2)^(5/2)/e^2-1/3*(-e^2*x^2
+d^2)^(7/2)/e^2/(e*x+d)^2-1/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/4*
d^3*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 679, 201, 223, 209}

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = -\frac{d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2} - \frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{d^3x\sqrt{d^2-e^2x^2}}{4e}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -1/4*(d^3*x*Sqrt[d^2 - e^2*x^2])/e - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{2\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{3e} \\
&= -\frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{(2d)\int (d^2-e^2x^2)^{3/2}\,dx}{3e} \\
&= -\frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{2e}
\end{aligned}
$$

$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{4e}$$

$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{4e}$$

$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx(d^2-e^2x^2)^{3/2}}{6e} - \frac{2(d^2-e^2x^2)^{5/2}}{15e^2} - \frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{e\sqrt{d^2-e^2x^2}(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4) - 15d^5\sqrt{-e^2}\log\left(-\sqrt{\right.}}{60e^3}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^
3 + 12*e^4*x^4) - 15*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^
2]])/(60*e^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(-12e^{4}x^{4}+30d\,e^{3}x^{3}-16d^{2}e^{2}x^{2}-15d^{3}ex+28d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{60e^{2}}-\frac{d^{5}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{4e\sqrt{e^{2}}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^{2}+2de\right)\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^{2}}+\frac{3d^{2}\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^{2}+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\right.}}{4e^{2}}\right.}{4}\right.}{e^{2}}$

[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/60*(-12*e^4*x^4+30*d*e^3*x^3-16*d^2*e^2*x^2-15*d^3*e*x+28*d^4)/e^2*(-e^2*x^2+d^2)^(1/2)-1/4*d^5/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(12\,e^4x^4-30\,de^3x^3+16\,d^2e^2x^2+15\,d^3ex-28\,d^4)\cdot}{60\,e^2}$$

[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] 1/60*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (12*e^4*x^4 - 30*d*e^3*x^3 + 16*d^2*e^2*x^2 + 15*d^3*e*x - 28*d^4)*sqrt(-e^2*x^2 + d^2))/e^2

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
0)), (x**2*sqrt(d**2)/2, True)) - 2*d*e*Piecewise((d**4*Piecewise((log(-2*e
**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x
*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x
/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + e**2*Piecew
ise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4
/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^2} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{4\,e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{4\,(e^3x+de^2)} - \frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{2\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{12\,e^2} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^2}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*d^5*arcsin(e*x/d + 2)/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
/e - 1/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x + d*e^2) - 1/2*sqrt(e^2*x^2 + 4*d*
```

```
e*x + 3*d^2)*d^4/e^2 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e - 5/12*(-e^2*x^2 +
d^2)^(3/2)*d^2/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\left(480\,d^6e^6\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+\frac{\left(15\,d^6e^6\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-2\right.}{}\right.}{}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/960*(480*d^6*e^6*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(e)
+ (15*d^6*e^6*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 250*d^6*e
^6*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 128*d^6*e^6*(2*d/(e*
x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 70*d^6*e^6*(2*d/(e*x + d) - 1)^
(3/2)*sgn(1/(e*x + d))*sgn(e) - 15*d^6*e^6*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e
*x + d))*sgn(e))*(e*x + d)^5/d^5)*abs(e)/(d*e^9)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.162 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{5}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e} + \frac{5d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] 5/12*d*(-e^2*x^2+d^2)^(3/2)/e+1/4*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/e+5/8*d^4*a
rctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {669, 685, 655, 201, 223, 209}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = \frac{5d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e} + \frac{5}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e} + \frac{1}{4}(5d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e} + \frac{1}{4}\left(5d^2\right)\int \sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e} + \frac{1}{8}\left(5d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e} + \frac{1}{8}\left(5d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{5d(d^2-e^2x^2)^{3/2}}{12e} + \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e} + \frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(16d^3+9d^2ex-16de^2x^2+6e^3x^3) - 30d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3) - 30*d
^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(24*e)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

method	result
risch	$\frac{(6e^3x^3-16d\,e^2x^2+9d^2ex+16d^3)\sqrt{-e^2x^2+d^2}}{24e} + \frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2} + \frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5} + de\left(-\frac{\left(-2\left(x+\frac{d}{e}\right)e^2+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2} + \frac{3d^2\left(-\frac{\left(-2\left(x+\frac{d}{e}\right.\right.}{}\right.}{}\right.\right.}{3d}}{e^2}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*e^3*x^3-16*d*e^2*x^2+9*d^2*e*x+16*d^3)/e*(-e^2*x^2+d^2)^(1/2)+5/8*d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{30\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(6\,e^3x^3-16\,de^2x^2+9\,d^2ex+16\,d^3)\sqrt{-e^2x^2+d^2}}{24\,e}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(30*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (6*e^3*x^3 - 16*d
*e^2*x^2 + 9*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for}\ e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for}\ e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2
+ x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 2*d*e*P
iecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2, 0)), (
x**2*sqrt(d**2)/2, True)) + e**2*Piecewise((d**4*Piecewise((log(-2*e**2*x +
 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)
/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(-d**2*x/(8*e**
2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e} + \frac{5}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^2x + \frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^3}{4\,e} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4\,(e^2x+de)} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{12\,e}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(e*x/d + 2)/e + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^2*x
+ 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e + 1/4*(-e^2*x^2 + d^2)^(5/2)/(e
^2*x + d*e) + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.70

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(240\,d^5e^5\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) + \frac{\left(15\,d^5e^5\left(\frac{2\,d}{ex+d}-1\right)^{\frac{7}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) - 73\,d^5e^5\left(\frac{2\,d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sg}\right.}{}\right.}{192\,de^7}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/192*(240*d^5*e^5*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(e)
 + (15*d^5*e^5*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 73*d^5*e
^5*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 55*d^5*e^5*(2*d/(e*x
 + d) - 1)^(3/2)*sgn(1/(e*x + d))*sgn(e) - 15*d^5*e^5*sqrt(2*d/(e*x + d) -
1)*sgn(1/(e*x + d))*sgn(e))*(e*x + d)^4/d^4)*abs(e)/(d*e^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d + ex)^2}\,dx = \int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{\left(d + e\,x\right)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2, x)
```

3.163 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2} dx = d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}(d^2-e^2x^2)^{3/2} - d^3 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3 \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)-d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^3*arctanh(
(-e^2*x^2+d^2)^(1/2)/d)+d*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {866, 1823, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2} dx = d^3\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) - d^3 \operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}(d^2-e^2x^2)^{3/2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-3d^2e^2+6de^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{3e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{6d^4e^4-6d^3e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{6e^4}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + d^4\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \left(d^3e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^4\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \left(d^3e\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \frac{1}{3}\sqrt{d^2-e^2x^2}\left(2d^2-3dex+e^2x^2\right) + 2d^3\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{d^3e\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 3*d*e*x + e^2*x^2))/3 + 2*d^3*ArcTanh[(Sqrt[-
e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (d^3*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
- e^2*x^2]])/Sqrt[-e^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(86) = 172$.

Time = 0.39 (sec) , antiderivative size = 544, normalized size of antiderivative = 5.67

method	result
default	$\frac{\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5}+d^2\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)\right)}{d^2}-\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+5e\left(\cdots\right.}{\cdots}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^
2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x))))-1/e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d
*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)
/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d
*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-1/d^2*(1/5*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)
```

```
^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = 2\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + d^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \frac{1}{3}\left(e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 2*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^3*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) + 1/3*(e^2*x^2 - 3*d*e*x + 2*d^2)*sqrt(-e^2*x^2 + d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.81

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0 \\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}-\frac{d^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^2\sqrt{d^2-e^2x^2}}{3} & \text{for } e^2\neq 0 \\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
```

```
*2) + 1), True)) - 2*d*e*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(
-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e
**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**
2), True)) + e**2*Piecewise((-d**2*sqrt(d**2 - e**2*x**2)/(3*e**2) + x**2*s
qrt(d**2 - e**2*x**2)/3, Ne(e**2, 0)), (x**2*sqrt(d**2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = -\frac{d^3e\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - d^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) - \sqrt{-e^2x^2+d^2}dex + \sqrt{-e^2x^2+d^2}d^2 - \frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -d^3*e*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - d^3*log(2*d^2/abs(x) + 2*sqr
t(-e^2*x^2 + d^2)*d/abs(x)) - sqrt(-e^2*x^2 + d^2)*d*e*x + sqrt(-e^2*x^2 +
d^2)*d^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2), x)
```

3.164 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2} dx = -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(-e^2*x^2+d^2)^(3/2)/x-1/2*d^2*e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+2*d^2*e*
arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1821, 829, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2} dx = -\frac{1}{2}d^2e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] -1/2*(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2]) - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}-\frac{\int \frac{\left(2d^3e+d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{d^2}\\
&= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}+\frac{\int \frac{-4d^5e^3-d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2e^2}\\
&= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}\\
&\quad-\left(2d^3e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\frac{1}{2}\left(d^2e^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}-\left(d^3e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&\quad-\frac{1}{2}\left(d^2e^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}\\
&\quad-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} \\
&\quad - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx &= \frac{\sqrt{d^2-e^2x^2}(-2d^2-4dex+e^2x^2)}{2x} \\
&\quad + d^2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + 2d\sqrt{d^2}e\log(x) - 2d\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)
\end{aligned}
$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^2 - 4*d*e*x + e^2*x^2))/(2*x) + d^2*e*ArcTan[(e*
x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] + 2*d*Sqrt[d^2]*e*Log[x] - 2*d*Sqrt[d
^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}}{x}+\frac{\sqrt{-e^2x^2+d^2}\,e^2x}{2}-\frac{e^2d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{2e\,d^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-2ed\sqrt{-e^2x^2+}$
default	$\frac{-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{d^2x}-\frac{6e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{d^2}}{d^2}-\frac{2e\left(\frac{(-e^2x^2+d^2}{5}\right.}{}$

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2*(-e^2*x^2+d^2)^(1/2)/x+1/2*(-e^2*x^2+d^2)^(1/2)*e^2*x-1/2*e^2*d^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2*e*d^3/(d^2)^(1/2)*ln((
2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-2*e*d*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \frac{2\,d^2ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-4\,d^2ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-4\,d^2ex+(e^2x^2-4\,de\cdots}{2\,x}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 4*d^2*e*x*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - 4*d^2*e*x + (e^2*x^2 - 4*d*e*x - 2*d^2)*sqrt
(-e^2*x^2 + d^2))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\,\text{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\,\text{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$
$$-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\text{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\text{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for}\ e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d
/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + e**2*Piecewise((d**2*Piecewise((log(-2*e**2*x
```

```
+ 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x
)/sqrt(-e**2*x**2), True))/2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x
*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = -\frac{d^2e^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + 2\,d^2e\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{1}{2}\sqrt{-e^2x^2+d^2}e^2x - 2\sqrt{-e^2x^2+d^2}de - \frac{\sqrt{-e^2x^2+d^2}d^2}{x}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*e^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 2*d^2*e*log(2*d^2/abs(
x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2*x - 2*
sqrt(-e^2*x^2 + d^2)*d*e - sqrt(-e^2*x^2 + d^2)*d^2/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^2\,{\left(d+e\,x\right)}^2} \,d x$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2), x)
```

3.165 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(3/2)/x^2+2*d*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*
d*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/
x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1821, 827, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = 2de^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{\int \frac{(4d^3e-d^2e^2x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{2d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{\int \frac{2d^4e^2+8d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{2}(d^2e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (2de^3)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{4}(d^2e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (2de^3)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = -\frac{(d^2-4dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{2x^2} + de^2\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right) + 2de\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/2*((d^2 - 4*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/x^2 + d*e^2*ArcTanh[
(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + 2*d*e*Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-2 e^{2} x^{2}-4 d e x +d^{2}\right)}{2 x^{2}}+\frac{2 d \,e^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{d^{2} e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}}$
default	$\frac{-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{2 d^{2} x^{2}}-\frac{5 e^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5}+d^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)\right)}{2 d^{2}}}{d^{2}}+\frac{3 e^{2} \left(\frac{\left(-e^{2} x^{2}+\right.}{5}\right.}{}$

[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-2*e^2*x^2-4*d*e*x+d^2)/x^2+2*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx =$$
$$-\frac{8\,de^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-de^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,de^2x^2-(2\,e^2x^2+4\,dex-d^2)\sqrt{-e^2x^2+}}{2\,x^2}$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")

[Out] -1/2*(8*d*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d*e^2*x^2 - (2*e^2*x^2 + 4*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/x^2

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.15

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$
$$-2de\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*
x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-
d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x*
*2/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) -
 d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) >
 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x
/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \frac{2\,de^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}-\frac{1}{2}\,de^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)$$
$$+\frac{1}{2}\sqrt{-e^2x^2+d^2}e^2+\frac{2\sqrt{-e^2x^2+d^2}de}{x}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2\,x^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 2*d*e^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*d*e^2*log(2*d^2/abs(x)
+ 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2 + 2*sqrt(
-e^2*x^2 + d^2)*d*e/x - 1/2*(-e^2*x^2 + d^2)^(3/2)/x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^3\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2), x)
```

3.166 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} - e^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/x^3-e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-e^3*arct
anh((-e^2*x^2+d^2)^(1/2)/d)+e*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1821, 825, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = e^3\left(-\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) - e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}-\frac{\int \frac{\left(6d^3e-3d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{3d^2} \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\frac{\int \frac{12d^5e^3-12d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{12d^4} \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\left(de^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-e^4\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\frac{1}{2}\left(de^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) \\
&\quad -e^4\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} \\
&\quad -e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-(de)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right) \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}-e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.35

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{(-d^2+3dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{3x^3} + 2e^3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) - \frac{\sqrt{d^2}e^3\log(x)}{d} + \frac{\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] ((-d^2 + 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*x^3) + 2*e^3*ArcTan[(
e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])] - (Sqrt[d^2]*e^3*Log[x])/d + (Sqrt[
d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(2e^2x^2-3dex+d^2)}{3x^3}-\frac{e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{e^3d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	113
default	Expression too large to display	983

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(1/2)*(2*e^2*x^2-3*d*e*x+d^2)/x^3-e^4/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-e^3*d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(
1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \frac{6\,e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(2\,e^2x^2-3\,dex+d^2)\sqrt{}}{3\,x^3}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*e^3*x^3*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^2*x^2 - 3*d*e*x + d^2)*sqrt(-e^2*x^2 +
d^2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.31

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))
/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d
), True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = -\frac{e^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right) + \frac{\sqrt{-e^2x^2+d^2}e^3}{d} - \frac{\sqrt{-e^2x^2+d^2}e^2}{x} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{dx^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3\,x^3}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -e^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - e^3*log(2*d^2/abs(x) + 2*sqrt(
-e^2*x^2 + d^2)*d/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^3/d - sqrt(-e^2*x^2 + d^
2)*e^2/x + (-e^2*x^2 + d^2)^(3/2)*e/(d*x^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)/x^
3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: abs(s
ageVARe)*(1/3*(12*sageVARe^2*sqrt(2*sageVARd*sageVARe*(sageVARe*sageVARx+sa
geVARd)^-1/sageVARe-1)*(2*sageVARd*sageVARe*(sageVARe*sageVARx+sageVARd)^-1
/sageVARe-1)^2*s
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^4\,(d+e\,x)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2), x)
```

3.167 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}+\frac{5e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(3/2)/x^4+2/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^3+5/8*e^4*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d-5/8*e^2*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1821, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{5e^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}\,dx \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{\int \frac{(8d^3e-5d^2e^2x)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{4d^2} \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{1}{4}(5e^2)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{1}{8}(5e^2)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} \\
&\quad - \frac{1}{16}(5e^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} \\
&\quad + \frac{1}{8}(5e^2)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$
\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^3+16d^2ex-9de^2x^2-16e^3x^3)}{24dx^4} + \frac{5e^4\log(x)}{8\sqrt{d^2}} - \frac{5e^4\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{8\sqrt{d^2}}
$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 16*d^2*e*x - 9*d*e^2*x^2 - 16*e^3*x^3))/(24*
d*x^4) + (5*e^4*Log[x])/(8*Sqrt[d^2]) - (5*e^4*Log[Sqrt[d^2] - Sqrt[d^2 - e
^2*x^2]])/(8*Sqrt[d^2])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(16 e^{3} x^{3}+9 d \,e^{2} x^{2}-16 d^{2} e x +6 d^{3}\right)}{24 x^{4} d}+\frac{5 e^{4} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$	96
default	Expression too large to display	1153

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(16*e^3*x^3+9*d*e^2*x^2-16*d^2*e*x+6*d^3)/x^4/d+
5/8*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = -\frac{15\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(16\,e^3x^3+9\,de^2x^2-16\,d^2ex+6\,d^3)\sqrt{-e^2x^2+d^2}}{24\,dx^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")
[Out] -1/24*(15*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 + 9*d*e^
2*x^2 - 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.91

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right) - 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*
acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin
(d/(e*x))/(2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{5\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}-\frac{5\sqrt{-e^2x^2+d^2}\,e^4}{8\,d^2} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,d^2x^2}+\frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,dx^3}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{4\,x^4}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 5/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 5/8*sqrt(-e
^2*x^2 + d^2)*e^4/d^2 - 5/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^2) + 2/3*(-e^
2*x^2 + d^2)^(3/2)*e/(d*x^3) - 1/4*(-e^2*x^2 + d^2)^(3/2)/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.28

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \frac{1}{192}\left(\frac{120\,e^3\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d} - \frac{120\,e^3\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)}{d}\right.$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/192*(120*e^3*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/d -
 120*e^3*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(e)/d +
4*(15*e^3*log(2) - 30*e^3*log(I + 1) + 32*I*e^3)*sgn(1/(e*x + d))*sgn(e)/d
- (15*e^3*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) + 73*e^3*(2*d/(
e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 55*e^3*(2*d/(e*x + d) - 1)^(3
/2)*sgn(1/(e*x + d))*sgn(e) + 15*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d
))*sgn(e))/(d*(d/(e*x + d) - 1)^4))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^5\,{\left(d+e\,x\right)}^2} \,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2), x)
```

3.168 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/x^5+1/2*e*(-e^2*x^2+d^2)^(3/2)/d/x^4-7/15*e^2*(-e
^2*x^2+d^2)^(3/2)/d^2/x^3-1/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/4*e
^3*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = -\frac{e^5\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} + \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} - \frac{\int \frac{(10d^3e-7d^2e^2x)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{5d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} + \frac{\int \frac{(28d^4e^2-10d^3e^3x)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{20d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{2d}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{4d}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} + \frac{e^5\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{8d}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{4d}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{\sqrt{d^2-e^2x^2}(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4)}{60d^2x^5} - \frac{\sqrt{d^2}e^5\log(x)}{4d^3} + \frac{\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{4d^3}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4))/(60*d^2*x^5) - (Sqrt[d^2]*e^5*Log[x])/(4*d^3) + (Sqrt[d^2]*e
^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(4*d^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-28e^4x^4+15d\,e^3x^3+16d^2e^2x^2-30d^3ex+12d^4)}{60x^5d^2}-\frac{e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4d\sqrt{d^2}}$	110
default	Expression too large to display	1349

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-e^2*x^2+d^2)^(1/2)*(-28*e^4*x^4+15*d*e^3*x^3+16*d^2*e^2*x^2-30*d^3*
e*x+12*d^4)/x^5/d^2-1/4/d*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(28\,e^4x^4-15\,de^3x^3-16\,d^2e^2x^2+30\,d^3ex-12\,d^4)\sqrt{-}}{60\,d^2x^5}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (28*e^4*x^4 - 15*d*e^
3*x^3 - 16*d^2*e^2*x^2 + 30*d^3*e*x - 12*d^4)*sqrt(-e^2*x^2 + d^2))/(d^2*x^
5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.88 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.71

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = d^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*
x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2
)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqr
t(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(
e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(
-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = -\frac{e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{4\,d^2}+\frac{\sqrt{-e^2x^2+d^2}e^5}{4\,d^3} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^2}-\frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{15\,d^2x^3}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{2\,dx^4}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{5\,x^5}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/4*sqrt
(-e^2*x^2 + d^2)*e^5/d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^2) - 7/15*
(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^3) + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^4)
 - 1/5*(-e^2*x^2 + d^2)^(3/2)/x^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.96

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = -\frac{1}{960}\left(\frac{240\,e^4\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^2}-\frac{240\,e^4\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^2}+\right.$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/960*(240*e^4*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/d^
2 - 240*e^4*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(e)/d
^2 + 8*(15*e^4*log(2) - 30*e^4*log(I + 1) + 56*I*e^4)*sgn(1/(e*x + d))*sgn(
e)/d^2 - (15*e^4*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) + 250*e^
4*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 128*e^4*(2*d/(e*x + d
) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) + 70*e^4*(2*d/(e*x + d) - 1)^(3/2)*sgn
(1/(e*x + d))*sgn(e) - 15*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(
e))/(d^2*(d/(e*x + d) - 1)^5))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2), x)
```

3.169 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 169

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{3e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] -1/6*(-e^2*x^2+d^2)^(3/2)/x^6+2/5*e*(-e^2*x^2+d^2)^(3/2)/d/x^5-3/8*e^2*(-e^
2*x^2+d^2)^(3/2)/d^2/x^4+4/15*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^3+3/16*e^6*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-3/16*e^4*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \frac{3e^6\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} - \frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
```

```
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} - \frac{\int \frac{(12d^3e-9d^2e^2x)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} + \frac{\int \frac{(45d^4e^2-24d^3e^3x)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{30d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} - \frac{\int \frac{(96d^5e^3-45d^4e^4x)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{120d^6}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{(3e^4)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{(3e^4)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d^2}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} - \frac{(3e^6)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{32d^2}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{(3e^4)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{16d^2}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{\sqrt{d^2-e^2x^2}(40d^5-96d^4ex+50d^3e^2x^2+32d^2e^3x^3-45de^4x^4+64e^5x^5)+90e^6x^6\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{240d^3x^6}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*
e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 90*e^6*x^6*ArcTanh[(Sqrt[-e^2]*x - S
qrt[d^2 - e^2*x^2])/d])/(d^3*x^6)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(64e^5x^5-45d\,e^4x^4+32d^2e^3x^3+50d^3e^2x^2-96d^4ex+40d^5)}{240d^3x^6}+\frac{3e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d^2\sqrt{d^2}}$	121
default	Expression too large to display	1550

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-e^2*x^2+d^2)^(1/2)*(64*e^5*x^5-45*d*e^4*x^4+32*d^2*e^3*x^3+50*d^3*
e^2*x^2-96*d^4*e*x+40*d^5)/d^3/x^6+3/16/d^2*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^
2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = -\frac{45\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(64\,e^5x^5-45\,de^4x^4+32\,d^2e^3x^3+50\,d^3e^2x^2-96\,d^4ex+40\,d^5)\sqrt{-e^2x^2+d^2}}{240\,d^3x^6}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(45*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (64*e^5*x^5 - 45*d*
e^4*x^4 + 32*d^2*e^3*x^3 + 50*d^3*e^2*x^2 - 96*d^4*e*x + 40*d^5)*sqrt(-e^2*
x^2 + d^2))/(d^3*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.28 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.78

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} \end{cases}\right. - 2de\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
```

```
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e*
*2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqr
t(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sq
rt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \frac{3\,e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^3} - \frac{3\sqrt{-e^2x^2+d^2}e^6}{16\,d^4} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{16\,d^4x^2} + \frac{4\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{15\,d^3x^3} - \frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{8\,d^2x^4} + \frac{2\,(-e^2x^2+d^2)^{\frac{3}{2}}e}{5\,dx^5} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{6\,x^6}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/16*sqr
t(-e^2*x^2 + d^2)*e^6/d^4 - 3/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^2) + 4/1
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^3) - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^
2*x^4) + 2/5*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^5) - 1/6*(-e^2*x^2 + d^2)^(3/2)/
x^6
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \frac{1}{7680}\left(\frac{1440\,e^5\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^3} - \frac{1440\,e^5\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-\right.\right.}{d^3}\right.$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*(1440*e^5*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)/d
^3 - 1440*e^5*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(e)
/d^3 + 16*(45*e^5*log(2) - 90*e^5*log(I + 1) + 128*I*e^5)*sgn(1/(e*x + d))*
sgn(e)/d^3 - (45*e^5*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 1
025*e^5*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 174*e^5*(2*d/(e
*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) + 594*e^5*(2*d/(e*x + d) - 1)^(5
/2)*sgn(1/(e*x + d))*sgn(e) - 255*e^5*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x
+ d))*sgn(e) + 45*e^5*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))/(d^3
*(d/(e*x + d) - 1)^6))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^7\,{\left(d+e\,x\right)}^2} \,d x$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2), x)
```

3.170 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 198

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx=\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}-\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

```
[Out] -1/7*(-e^2*x^2+d^2)^(3/2)/x^7+1/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^6-11/35*e^2*(-
e^2*x^2+d^2)^(3/2)/d^2/x^5+1/4*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-22/105*e^4*
(-e^2*x^2+d^2)^(3/2)/d^4/x^3-1/8*e^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/
8*e^5*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx=-\frac{e^7\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}+\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
```

```
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}-\frac{\int \frac{\left(14d^3e-11d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^7}\,dx}{7d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}+\frac{\int \frac{\left(66d^4e^2-42d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{42d^4} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}-\frac{\int \frac{\left(210d^5e^3-132d^4e^4x\right)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{210d^6} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} \\
&\quad+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}+\frac{\int \frac{\left(528d^6e^4-210d^5e^5x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{840d^8} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} \\
&\quad+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}-\frac{e^5\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{4d^3} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} \\
&\quad+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}-\frac{e^5\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)}{8d^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} \\
&\quad + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3} + \frac{e^7\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{16d^3} \\
&= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} \\
&\quad + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3} - \frac{e^5\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{8d^3} \\
&= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} \\
&\quad + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3} - \frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$
\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de^5x^5+176e^6x^6)}{x^7} - 105\sqrt{d^2}e^7\log(x)}{840d^5}
$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*
e^3*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6))/x^7 - 105*Sqrt[d^2
]*e^7*Log[x] + 105*Sqrt[d^2]*e^7*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(840
*d^5)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-176e^6x^6+105d\,e^5x^5-88d^2e^4x^4+70d^3x^3e^3+144d^4e^2x^2-280d^5ex+120d^6)}{840x^7d^4} - \frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^3\sqrt{d^2}}$
default	Expression too large to display

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/840*(-e^2*x^2+d^2)^(1/2)*(-176*e^6*x^6+105*d*e^5*x^5-88*d^2*e^4*x^4+70*d
^3*e^3*x^3+144*d^4*e^2*x^2-280*d^5*e*x+120*d^6)/x^7/d^4-1/8/d^3*e^7/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + (176\,e^6x^6 - 105\,de^5x^5 + 88\,d^2e^4x^4 - 70\,d^3e^3x^3 - 144\,d}{840\,d^4x^7}$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")

[Out] 1/840*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (176*e^6*x^6 - 105*d*e^5*x^5 + 88*d^2*e^4*x^4 - 70*d^3*e^3*x^3 - 144*d^4*e^2*x^2 + 280*d^5*e*x - 120*d^6)*sqrt(-e^2*x^2 + d^2))/(d^4*x^7)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.26 (sec) , antiderivative size = 835, normalized size of antiderivative = 4.22

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4} + \frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2} + \frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4} + \frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2} + \frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise} \end{cases}\right)$$
$$-2de\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise} \end{cases}\right)$$
$$+e^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)

[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 2*d*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**

```
5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), A
bs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) -
 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*
e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((3*I*d**3*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**
2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (
3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x
**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*s
qrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqr
t(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = -\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^4}+\frac{\sqrt{-e^2x^2+d^2}e^7}{8\,d^5} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^5}{8\,d^5x^2}-\frac{22\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4}{105\,d^4x^3}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^4} - \frac{11\,(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{35\,d^2x^5}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{3\,dx^6}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{7\,x^7}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 1/8*sqrt
(-e^2*x^2 + d^2)*e^7/d^5 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^5*x^2) - 22/10
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
3*x^4) - 11/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^5) + 1/3*(-e^2*x^2 + d^2)^
(3/2)*e/(d*x^6) - 1/7*(-e^2*x^2 + d^2)^(3/2)/x^7
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.68

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = -\frac{1}{53760}\left(\frac{6720\,e^6\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{d^4}-\frac{6720\,e^6\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e}{d^4}\right.$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/53760*(6720*e^6*log(sqrt(2*d/(e*x + d) - 1) + 1)*sgn(1/(e*x + d))*sgn(e)
/d^4 - 6720*e^6*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))*sgn(1/(e*x + d))*sgn(
e)/d^4 + 32*(105*e^6*log(2) - 210*e^6*log(I + 1) + 352*I*e^6)*sgn(1/(e*x +
d))*sgn(e)/d^4 - (105*e^6*(2*d/(e*x + d) - 1)^(13/2)*sgn(1/(e*x + d))*sgn(e
) + 3780*e^6*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 189*e^6*(
2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) + 4992*e^6*(2*d/(e*x + d)
- 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 1981*e^6*(2*d/(e*x + d) - 1)^(5/2)*sgn
(1/(e*x + d))*sgn(e) + 700*e^6*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))*s
gn(e) - 105*e^6*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))/(d^4*(d/(e
*x + d) - 1)^7))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^8\,{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2), x)
```

3.171 $\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)+17/15*d^2*(-e*x+d)/e^5/(-e^2*x
^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-2/15*(-13*e*x+15*d)/e^5/
(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1828, 12, 223, 209}

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/5*(d^3*(d - e*x)^2)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^
2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(d-ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{(d^2-e^2x^2)^{3/2}}dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}dx}{15d^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}}dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-16d^3-17d^2ex+22de^2x^2+26e^3x^3)}{(d-ex)(d+ex)^3} + 30\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^5}$$

```
[In] Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 17*d^2*e*x + 22*d*e^2*x^2 + 26*e^3*x^3))/(
(d - e*x)*(d + e*x)^3) + 30*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]
)/(15*e^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(109) = 218.

Time = 0.40 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.95

method	result
default	$\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}+\frac{3x}{\sqrt{-e^2x^2+d^2}\,e^4}-\frac{2d}{e^5\sqrt{-e^2x^2+d^2}}+\frac{d^4\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\right.}{3}\right.}{}$

```
[In] int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2)))+3/(-e^2*x^2+d^2)^(1/2)/e^4*x-2*d/e^5/(-e^2*x^2+d^2)^(1
/2)+d^4/e^6*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/
d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*(x+d
/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-4/e^5*d^3*(-1/3/d/e/(
x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*(x+d/e)*e^2+2*d*e
)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{16\,e^4x^4+32\,de^3x^3-32\,d^3ex-16\,d^4-30\left(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(26\,e^3x\right.}{15\left(e^9x^4+2\,de^8x^3-2\,d^3e^6x-d^4e^5\right)}$$

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(16*e^4*x^4 + 32*d*e^3*x^3 - 32*d^3*e*x - 16*d^4 - 30*(e^4*x^4 + 2*d*
e^3*x^3 - 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (26*
e^3*x^3 + 22*d*e^2*x^2 - 17*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^
4 + 2*d*e^8*x^3 - 2*d^3*e^6*x - d^4*e^5)
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

```
[In] integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{d^3}{5\left(\sqrt{-e^2x^2+d^2}e^7x^2+2\sqrt{-e^2x^2+d^2}de^6x+\sqrt{-e^2x^2+d^2}d^2e^5\right)} + \frac{17\,d^2}{15\left(\sqrt{-e^2x^2+d^2}e^6x+\sqrt{-e^2x^2+d^2}de^5\right)} + \frac{26\,x}{15\sqrt{-e^2x^2+d^2}e^4} - \frac{\arcsin\left(\frac{ex}{d}\right)}{e^5} - \frac{2\,d}{\sqrt{-e^2x^2+d^2}e^5}$$

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] -1/5*d^3/(sqrt(-e^2*x^2 + d^2)*e^7*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^6*x + s
qrt(-e^2*x^2 + d^2)*d^2*e^5) + 17/15*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt
(-e^2*x^2 + d^2)*d*e^5) + 26/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*x/d
)/e^5 - 2*d/(sqrt(-e^2*x^2 + d^2)*e^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: 1/abs
(sageVARe)*(1/32768*(-20480/3*sageVARe^16*sqrt(2*sageVARd*sageVARe*(sageVAR
e*sageVARx+sageVARd)^-1/sageVARe-1)*(2*sageVARd*sageVARe*(sageVARe*sageVARx
+sageVARd)^-1/sa
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4}{(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)^2}\,dx$$

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.172 $\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{x^3}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = \frac{d^2(d-ex)^2}{5e^4 (d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4 (d^2-e^2x^2)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(-e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(-e*x+d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*(-2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {866, 1649, 651}

$$\int \frac{x^3}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = \frac{d^2(d-ex)^2}{5e^4 (d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4 (d^2-e^2x^2)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(d-ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx \\
&= \frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)\left(-\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d} \\
&= \frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{-\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^3+4d^2ex+de^2x^2-2e^3x^3)}{5de^4(d-ex)(d+ex)^3}$$

```
[In] Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(-ex+d)\left(-2e^{3}x^{3}+d\,e^{2}x^{2}+4d^{2}ex+2d^{3}\right)}{5(ex+d)d\,e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}$
trager	$\frac{\left(-2e^{3}x^{3}+d\,e^{2}x^{2}+4d^{2}ex+2d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{5d\,e^{4}(ex+d)^{3}(-ex+d)}$
default	$\frac{1}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}-\frac{2x}{d\,e^{3}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{d^{3}\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{\cdots}{3e\,\cdots}\cdots\right.}{5d}\cdots\right.}{e^{5}}\cdots$

[In] int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^2+d^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3-de^2x^2-4\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2\cdots}}{5\left(de^8x^4+2\,d^2e^7x^3-2\,d^4e^5x-d^5e^4\right)}$$

[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")

[Out] 1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 - d*e^2*x^2 - 4*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 + 2*d^2*e^7*x^3 - 2*d^4*e^5*x - d^5*e^4)

Sympy [F]

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

[In] integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)

[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{d^2}{5\left(\sqrt{-e^2x^2+d^2}e^6x^2+2\sqrt{-e^2x^2+d^2}de^5x+\sqrt{-e^2x^2+d^2}d^2e^4\right)} - \frac{4\,d}{5\left(\sqrt{-e^2x^2+d^2}e^5x+\sqrt{-e^2x^2+d^2}de^4\right)} - \frac{2\,x}{5\sqrt{-e^2x^2+d^2}de^3} + \frac{1}{\sqrt{-e^2x^2+d^2}e^4}$$

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^5*x + sq
rt(-e^2*x^2 + d^2)*d^2*e^4) - 4/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2
*x^2 + d^2)*d*e^4) - 2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3) + 1/(sqrt(-e^2*x^2
+ d^2)*e^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.93

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\frac{16i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{de^3} - \frac{5}{de^3\sqrt{\frac{2\,d}{ex+d}-1}\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{d^4e^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 - 5\,d^4e^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 + \cdots}{d^5e^{15}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{40\,|e|}$$

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*(16*I*sgn(1/(e*x + d))*sgn(e)/(d*e^3) - 5/(d*e^3*sqrt(2*d/(e*x + d) -
 1)*sgn(1/(e*x + d))*sgn(e)) - (d^4*e^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e
*x + d))^4*sgn(e)^4 - 5*d^4*e^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))
^4*sgn(e)^4 + 15*d^4*e^12*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)
^4)/(d^5*e^15*sgn(1/(e*x + d))^5*sgn(e)^5))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+4\,d^2\,e\,x+d\,e^2\,x^2-2\,e^3\,x^3\right)}{5\,d\,e^4\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 - 2*e^3*x^3 + d*e^2*x^2 + 4*d^2*e*x))/(5*d*e^
4*(d + e*x)^3*(d - e*x))
```

3.173 $\int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)-1/5*d/e^3/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2
)+7/15/e^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {866, 1649, 792, 197}

$$\int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d(d-ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} + \frac{7(d-ex)}{15e^3(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/5*(d*(d - e*x)^2)/(e^3*(d^2 - e^2*x^2)^(5/2)) + (7*(d - e*x))/(15*e^3*(d
^2 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}-\frac{5dx}{e}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2} \\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(4d^3+8d^2ex+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}$$

```
[In] Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{(-ex+d)\left(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3\right)}{15(ex+d)d^2e^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$
trager	$\frac{\left(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3\right)\sqrt{-e^2x^2+d^2}}{15d^2e^3(ex+d)^3(-ex+d)}$
default	$\frac{x}{d^2e^2\sqrt{-e^2x^2+d^2}}+\frac{d^2\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}\right.}{5d}\right.}{e^4}$

```
[In] int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*x
^2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{4\,e^4x^4+8\,de^3x^3-8\,d^3ex-4\,d^4-\left(e^3x^3+2\,de^2x^2+8\,d^2ex+4\,d^3\right)\sqrt{-e^2x^2}}{15\left(d^2e^7x^4+2\,d^3e^6x^3-2\,d^5e^4x-d^6e^3\right)}$$

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] 1/15*(4*e^4*x^4 + 8*d*e^3*x^3 - 8*d^3*e*x - 4*d^4 - (e^3*x^3 + 2*d*e^2*x^2
+ 8*d^2*e*x + 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 + 2*d^3*e^6*x^3 - 2
*d^5*e^4*x - d^6*e^3)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^2}{\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}} (d+ex)^2}\,dx$$

```
[In] integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{d}{5\left(\sqrt{-e^2x^2+d^2}e^5x^2 + 2\sqrt{-e^2x^2+d^2}de^4x + \sqrt{-e^2x^2+d^2}d^2e^3\right)} + \frac{7}{15\left(\sqrt{-e^2x^2+d^2}e^4x + \sqrt{-e^2x^2+d^2}de^3\right)} + \frac{x}{15\sqrt{-e^2x^2+d^2}d^2e^2}$$

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] -1/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^4*x + sqr
t(-e^2*x^2 + d^2)*d^2*e^3) + 7/15/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x
^2 + d^2)*d*e^3) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.15

$$\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{-\frac{8i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^2e^2} - \frac{15}{d^2e^2\sqrt{\frac{2d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} + \frac{3d^8e^8\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 - 5d^8e^8\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e}{d^{10}e^{10}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{120\,|e|}$$

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] -1/120*(-8*I*sgn(1/(e*x + d))*sgn(e)/(d^2*e^2) - 15/(d^2*e^2*sqrt(2*d/(e*x
+ d) - 1)*sgn(1/(e*x + d))*sgn(e)) + (3*d^8*e^8*(2*d/(e*x + d) - 1)^(5/2)*s
gn(1/(e*x + d))^4*sgn(e)^4 - 5*d^8*e^8*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x
 + d))^4*sgn(e)^4 - 15*d^8*e^8*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*s
gn(e)^4)/(d^10*e^10*sgn(1/(e*x + d))^5*sgn(e)^5))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 11.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3+8\,d^2\,e\,x+2\,d\,e^2\,x^2+e^3\,x^3\right)}{15\,d^2\,e^3\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(4*d^3 + e^3*x^3 + 2*d*e^2*x^2 + 8*d^2*e*x))/(15*d^2
*e^3*(d + e*x)^3*(d - e*x))
```

3.174 $\int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)+1/5/e^2/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-2/
15/d/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 673, 197}

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx}{5e}\\
&= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{4\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15de}\\
&= \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{x}{(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(d^3+2d^2ex+8de^2x^2+4e^3x^3)}{15d^3e^2(d-ex)(d+ex)^3}$$

```
[In] Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result
gosper	$\frac{(-ex+d)\left(4e^{3}x^{3}+8d\,e^{2}x^{2}+2d^{2}ex+d^{3}\right)}{15(ex+d)d^{3}e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}$
trager	$\frac{\left(4e^{3}x^{3}+8d\,e^{2}x^{2}+2d^{2}ex+d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}(ex+d)^{3}e^{2}(-ex+d)}$
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^{2}+2de}{3e\,d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}}{e^{2}}-\frac{d\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{}{3de\left(x+\frac{d}{e}\right.}\right.}{}\right.}{}$

[In] int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*x^2+d^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{e^4x^4+2\,de^3x^3-2\,d^3ex-d^4-\left(4\,e^3x^3+8\,de^2x^2+2\,d^2ex+d^3\right)\sqrt{-e^2x^2+}}{15\left(d^3e^6x^4+2\,d^4e^5x^3-2\,d^6e^3x-d^7e^2\right)}$$

[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")

[Out] 1/15*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4 - (4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 + 2*d^4*e^5*x^3 - 2*d^6*e^3*x - d^7*e^2)

Sympy [F]

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

[In] integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)

[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{1}{5\left(\sqrt{-e^2x^2+d^2}e^4x^2+2\sqrt{-e^2x^2+d^2}de^3x+\sqrt{-e^2x^2+d^2}d^2e^2\right)} - \frac{2}{15\left(\sqrt{-e^2x^2+d^2}de^3x+\sqrt{-e^2x^2+d^2}d^2e^2\right)} + \frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}d^3e}$$

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5/(sqrt(-e^2*x^2 + d^2)*e^4*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-
e^2*x^2 + d^2)*d^2*e^2) - 2/15/(sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-e^2*x^
2 + d^2)*d^2*e^2) + 4/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.95

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx =$$

$$-\frac{-\frac{32i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{d^3} - \frac{15}{d^3\sqrt{\frac{2\,d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{3\,d^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4+5\,d^{12}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4-15}{d^{15}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^5\mathrm{sgn}(e)^5}}{120\,e|e|}$$

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/120*(-32*I*sgn(1/(e*x + d))*sgn(e)/d^3 - 15/(d^3*sqrt(2*d/(e*x + d) - 1)
*sgn(1/(e*x + d))*sgn(e)) - (3*d^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x +
d))^4*sgn(e)^4 + 5*d^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)
^4 - 15*d^12*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^15*sgn
(1/(e*x + d))^5*sgn(e)^5))/(e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{x}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3+2\,d^2\,e\,x+8\,d\,e^2\,x^2+4\,e^3\,x^3\right)}{15\,d^3\,e^2\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 + 4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)^3*(d - e*x))
```

3.175 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*x/d^4/(-e^2*x^2+d^2)^(1/2)-1/5/d/e/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-1/5/d
^2/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 197}

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{3\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx}{5d} \\ &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{5d^2} \\ &= \frac{2x}{5d^4\sqrt{d^2-e^2x^2}} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{1}{(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-2d^3+d^2ex+4de^2x^2+2e^3x^3)}{5d^4e(d-ex)(d+ex)^3}$$

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{(-ex+d)\left(-2e^3x^3-4d\,e^2x^2-d^2ex+2d^3\right)}{5(ex+d)d^4e(-e^2x^2+d^2)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-2e^3x^3-4d\,e^2x^2-d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5d^4(ex+d)^3e(-ex+d)}$	68
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2\left(x+\frac{d}{e}\right)e^2+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e^2}$	156

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x^
2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = -\frac{2 e^4x^4 + 4 de^3x^3 - 4 d^3ex - 2 d^4 + (2 e^3x^3 + 4 de^2x^2 + d^2ex - 2 d^3)\sqrt{-e^2x^2 + d^2}}{5 (d^4e^5x^4 + 2 d^5e^4x^3 - 2 d^7e^2x - d^8e)}$$

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] -1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x^
2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 - 2
*d^7*e^2*x - d^8*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{3}{2}} (d+ex)^2} dx$$

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = -\frac{1}{5 \left(\sqrt{-e^2x^2+d^2}de^3x^2 + 2\sqrt{-e^2x^2+d^2}d^2e^2x + \sqrt{-e^2x^2+d^2}d^3e\right)} - \frac{1}{5 \left(\sqrt{-e^2x^2+d^2}d^2e^2x + \sqrt{-e^2x^2+d^2}d^3e\right)} + \frac{2x}{5\sqrt{-e^2x^2+d^2}d^4}$$

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(-e^2*x^2 + d^2)*d*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + s
qrt(-e^2*x^2 + d^2)*d^3*e) - 1/5/(sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + sqrt(-e^
2*x^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.12

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{e^3\left(\frac{5}{d^4e^3\sqrt{\frac{2d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{d^{16}e^{12}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^4\operatorname{sgn}(e)^4+5\,d^{16}e^{12}\left(\frac{2d}{ex+d}-\right.}{d^{20}e^{15}\operatorname{sgn}}\right.}{40\,|e|}$$

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(e^3*(5/(d^4*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e)) - (d
^16*e^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 5*d^16*e^1
2*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*d^16*e^12*sqrt
(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^20*e^15*sgn(1/(e*x + d)
)^5*sgn(e)^5)) + 16*I*sgn(1/(e*x + d))*sgn(e)/d^4)/abs(e)
```

Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^3+d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)^3*(d - e*x))
```

3.176 $\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{5d-8ex}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 2/5*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(-16*e*x+15*d)/d^5/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{5d-8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2+16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^4} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^4e^2} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(26d^3+22d^2ex-17de^2x^2-16e^3x^3)}{(d-ex)(d+ex)^3}+30\text{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

```
[In] Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((
d - e*x)*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(15*d^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(104) = 208.

Time = 0.41 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.80

method	result
default	$\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}-\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\right.}{5d}}{ed}$

```
[In] int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x))-1/e/d*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/
2)-1/3/e/d^3*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-
1/d^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*
(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.42

$$\int \frac{1}{x(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{26\,e^4x^4+52\,de^3x^3-52\,d^3ex-26\,d^4+15\,(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4)}{15\,(d^5e^4x^4+2\,d^6e}$$

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 + 52*d*e^3*x^3 - 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 + 2*d*e
^3*x^3 - 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3
+ 17*d*e^2*x^2 - 22*d^2*e*x - 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 +
2*d^6*e^3*x^3 - 2*d^8*e*x - d^9)
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

```
[In] integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)^2x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.41

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\left(e^4\left(\frac{120\log\left(\sqrt{\frac{2d}{ex+d}-1}+1\right)}{d^5e^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{120\log\left(\left|\sqrt{\frac{2d}{ex+d}-1}-1\right|\right)}{d^5e^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{15}{d^5e^4\sqrt{\frac{2d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{3d^{20}e^{16}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{ex+d}\right)^4\mathrm{sgn}(e)^4 \cdots}{\cdots}\right.\right.}{12\cdots}$$

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] -1/120*(e^4*(120*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^5*e^4*sgn(1/(e*x + d))
*sgn(e)) - 120*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^5*e^4*sgn(1/(e*x +
d))*sgn(e)) - 15/(d^5*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))
- (3*d^20*e^16*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 25*d
^20*e^16*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 165*d^20*e
^16*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^25*e^20*sgn(1/(
e*x + d))^5*sgn(e)^5)) + 4*(15*log(2) - 30*log(I + 1) + 32*I)*sgn(1/(e*x +
d))*sgn(e)/d^5)*e/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.177 $\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = -\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4(d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] -2/5*e*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(-13*e*x+10*d)/d^4/(-e^2*x^
2+d^2)^(3/2)+2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/15*e*(-41*e*x+30*d)/
d^6/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1819, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{2e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} - \frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{e(10d-13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2+10dex-8e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^2-30dex+26e^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{(2e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d^5}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^5e}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(15d^4+76d^3ex+32d^2e^2x^2-82de^3x^3-56e^4x^4)}{x(-d+ex)(d+ex)^3}+30\sqrt{d^2}e\log(x)-30\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^7}$$

```
[In] Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x^
3 - 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) + 30*Sqrt[d^2]*e*Log[x] - 30*Sq
rt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^6x}+\frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}-\frac{29\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{60d^5e\left(x+\frac{d}{e}\right)^2}-\frac{313\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{120d^6\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-\ldots}}{\ldots}$
default	$\frac{-\frac{1}{d^2x\sqrt{-e^2x^2+d^2}}+\frac{2e^2x}{d^4\sqrt{-e^2x^2+d^2}}}{d^2}-\frac{2e\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{d^3}+\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\ldots}{\ldots}$

```
[In] int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^6/x+2/d^5*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-29/60/d^5/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-313/120/d^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/10/d^4/e^2
/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/8/d^6/(x-d/e)*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx =$$

$$-\frac{46\,e^5x^5+92\,de^4x^4-92\,d^3e^2x^2-46\,d^4ex+30\,(e^5x^5+2\,de^4x^4-2\,d^3e^2x^2-d^4ex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\ldots}{15\,(d^6e^4x^5+2\,d^7e^3x^4-2\,d^9ex^2-d^{10}x)}$$

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(46*e^5*x^5 + 92*d*e^4*x^4 - 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^
5 + 2*d*e^4*x^4 - 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/
x) + (56*e^4*x^4 + 82*d*e^3*x^3 - 32*d^2*e^2*x^2 - 76*d^3*e*x - 15*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^6*e^4*x^5 + 2*d^7*e^3*x^4 - 2*d^9*e*x^2 - d^10*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.22

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{e^7\left(\frac{240\log\left(\sqrt{\frac{2d}{ex+d}-1}+1\right)}{d^6e^5\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} - \frac{240\log\left(\left|\sqrt{\frac{2d}{ex+d}-1}-1\right|\right)}{d^6e^5\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)} + \frac{30\left(\frac{17d}{ex+d}-9\right)}{\left(\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}-\sqrt{\frac{2d}{ex+d}-1}\right)d^6e^5\mathrm{sgn}}\right.}{}$$

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(e^7*(240*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^6*e^5*sgn(1/(e*x + d))*
sgn(e)) - 240*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^6*e^5*sgn(1/(e*x + d
))*sgn(e)) + 30*(17*d/(e*x + d) - 9)/(((2*d/(e*x + d) - 1)^(3/2) - sqrt(2*d
/(e*x + d) - 1))*d^6*e^5*sgn(1/(e*x + d))*sgn(e)) - (3*d^24*e^20*(2*d/(e*x
+ d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 35*d^24*e^20*(2*d/(e*x + d) -
 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 345*d^24*e^20*sqrt(2*d/(e*x + d) -
1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^30*e^25*sgn(1/(e*x + d))^5*sgn(e)^5)) +
8*(15*e^2*log(2) - 30*e^2*log(I + 1) + 56*I*e^2)*sgn(1/(e*x + d))*sgn(e)/d^
6)/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^2\,(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(d^2-e^2\,x^2)^{3/2}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.178 $\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

```
[Out] 2/5*e^2*(-e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(-6*e*x+5*d)/d^5/(-e^2*x^
2+d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(-11*e*x+1
0*d)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2+2*e*(-e^2*x^
2+d^2)^(1/2)/d^7/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=-\frac{9e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
```

```
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^2+10dex-10e^2x^2+\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^2-30dex+45e^2x^2-\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^2+30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad +\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{\int \frac{-60d^3e+135d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} \\
&\quad -\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}+\frac{\left(9e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^6} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} \\
&\quad -\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}+\frac{\left(9e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d^6} \\
&= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} \\
&\quad -\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^6}
\end{aligned}$$

$$= \frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(5d^5-10d^4ex-94d^3e^2x^2-58d^2e^3x^3+83de^4x^4+64e^5x^5)}{x^2(-d+ex)(d+ex)^3} + 90e^2\text{arctanh}\left(\frac{\sqrt{-e^2}x-\cdots}{\cdots}\right)}{10d^7}$$

```
[In] Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 90*e^2*ArcTan
h[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-4ex+d)}{2d^7x^2} - \frac{9e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^6\sqrt{d^2}} + \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{10d^5e\left(x+\frac{d}{e}\right)^3} + \frac{13\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{20d^6\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{1}{2d^2x^2\sqrt{-e^2x^2+d^2}} + \frac{3e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}} - \frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{2d^2}}{d^2} + \frac{3e^2\left(\frac{1}{d^2\sqrt{-e^2x^2+d^2}} - \frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^2\sqrt{d^2}}\right)}{d^4}$

```
[In] int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-4*e*x+d)/d^7/x^2-9/2/d^6*e^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/10/d^5/e/(x+d/e)^3*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)+13/20/d^6/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)+181/40/d^7*e/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/8/d^7*e
/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{54\,e^6x^6+108\,de^5x^5-108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6+2\,de^5x^5-2\,d^3e\right.}{1(}$$

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 + 108*d*e^5*x^5 - 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 + 2*d*e^5*x^5 - 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) + (64*e^5*x^5 + 83*d*e^4*x^4 - 58*d^2*e^3*x^3 - 94*d^3*e^2*x^2
- 10*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 + 2*d^8*e^3*x^5 -
2*d^10*e*x^3 - d^11*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^3), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx =$$

$$-\frac{e^9\left(\frac{180\,\log\left(\sqrt{\frac{2\,d}{ex+d}-1}+1\right)}{d^7e^6\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}-\frac{180\,\log\left(\left|\sqrt{\frac{2\,d}{ex+d}-1}-1\right|\right)}{d^7e^6\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}-\frac{5}{d^7e^6\sqrt{\frac{2\,d}{ex+d}-1}\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}+\frac{10\left(5\left(\frac{2\,d}{ex+d}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{2\,d}{ex+d}-1}\right)}{d^7e^6\left(\frac{d}{ex+d}-1\right)^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}-\frac{d^2 \cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*(e^9*(180*log(sqrt(2*d/(e*x + d) - 1) + 1)/(d^7*e^6*sgn(1/(e*x + d))*
sgn(e)) - 180*log(abs(sqrt(2*d/(e*x + d) - 1) - 1))/(d^7*e^6*sgn(1/(e*x + d
))*sgn(e)) - 5/(d^7*e^6*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e)) +
10*(5*(2*d/(e*x + d) - 1)^(3/2) - 3*sqrt(2*d/(e*x + d) - 1))/(d^7*e^6*(d/(e
*x + d) - 1)^2*sgn(1/(e*x + d))*sgn(e)) - (d^28*e^24*(2*d/(e*x + d) - 1)^(5
/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*d^28*e^24*(2*d/(e*x + d) - 1)^(3/2)*sg
n(1/(e*x + d))^4*sgn(e)^4 + 195*d^28*e^24*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*
x + d))^4*sgn(e)^4)/(d^35*e^30*sgn(1/(e*x + d))^5*sgn(e)^5)) + 2*(45*e^3*lo
g(2) - 90*e^3*log(I + 1) + 128*I*e^3)*sgn(1/(e*x + d))*sgn(e)/d^7)/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/5*d^4*(-e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(-e*x+d)^2/e^6/(-e^2*
x^2+d^2)^(3/2)+13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(-e
*x+d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6-1/2*x*(-e^2*x^2
+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
```

```
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^5(d-ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d-ex)\left(-\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}}{\sqrt{d^2-e^2x^2}}\,dx}{30d^3e^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{(13d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^5}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{(13d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4)}{30e^6(d+ex)^3} - \frac{13d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

[In] Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]

[Out] (Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x^3 - 15*e^4*x^4))/(30*e^6*(d + e*x)^3) - (13*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^6

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(-ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}+\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}+\frac{d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^9\left(x+\frac{d}{e}\right)^3}-\frac{23d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^8\left(x+\frac{d}{e}\right)^2}+$
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e^3}+\frac{6d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{3d\sqrt{-e^2x^2+d^2}}{e^6}+\frac{5d^4\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\right.}{e^7}$

[In] int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(-e*x+6*d)/e^6*(-e^2*x^2+d^2)^(1/2)+13/2*d^2/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*d^4/e^9/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-23/15*d^3/e^8/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+127/15*d^2/e^7/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{304\,d^2e^3x^3 + 912\,d^3e^2x^2 + 912\,d^4ex + 304\,d^5 - 390\,(d^2e^3x^3 + 3\,d^3e^2x^2 + 3\,d^4ex + d^5)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d}}{ex}\right.}{30\,(e^9x^3 + 3\,de^8x^2 + 3\,d^2e^7x + d^3e^6}$$

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 + 912*d^3*e^2*x^2 + 912*d^4*e*x + 304*d^5 - 390*(d^2*
e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (15*e^4*x^4 - 45*d*e^3*x^3 - 479*d^2*e^2*x^2 - 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
)
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

```
[In] integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^4}{5\,(e^9x^3 + 3\,de^8x^2 + 3\,d^2e^7x + d^3e^6)} - \frac{23\,\sqrt{-e^2x^2+d^2}d^3}{15\,(e^8x^2 + 2\,de^7x + d^2e^6)} + \frac{127\,\sqrt{-e^2x^2+d^2}d^2}{15\,(e^7x + de^6)} + \frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6} - \frac{\sqrt{-e^2x^2+d^2}x}{2\,e^5} + \frac{3\,\sqrt{-e^2x^2+d^2}d}{e^6}$$

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^4/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
) - 23/15*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 127/15
*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x + d*e^6) + 13/2*d^2*arcsin(e*x/d)/e^6 - 1/
2*sqrt(-e^2*x^2 + d^2)*x/e^5 + 3*sqrt(-e^2*x^2 + d^2)*d/e^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^5}-\frac{6\,d}{e^6}\right)+\frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^5|e|} - \frac{2\left(107\,d^2+\frac{445\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x}+\frac{665\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}+\frac{405\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3}+\frac{90\left(de+\sqrt{-e^2x^2+d^2}|e\right.}{e^8x^4}\right.}{15\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-e^2*x^2 + d^2)*(x/e^5 - 6*d/e^6) + 13/2*d^2*arcsin(e*x/d)*sgn(d)
*sgn(e)/(e^5*abs(e)) - 2/15*(107*d^2 + 445*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))*d^2/(e^2*x) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2) +
 405*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 90*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^5*((d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)+6/5*d^2*(-e*x+d)^2/e^5/(-e^2*x
^2+d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-24/5*d*(-e*x+d)/e^5/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {866, 1649, 655, 223, 209}

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(d^3*(d - e*x)^3)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5
*e^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(d-ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(\frac{27d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}-\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-24d^3-57d^2ex-39de^2x^2-5e^3x^3)}{5e^5(d+ex)^3} + \frac{6d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[In] Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^3 - 57*d^2*e*x - 39*d*e^2*x^2 - 5*e^3*x^3))/(5*
e^5*(d + e*x)^3) + (6*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^
5
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{24d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^6\left(x+\frac{d}{e}\right)}+\frac{6d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^7\left(x+\frac{d}{e}\right)^2}-\frac{d^3\sqrt{-\left(x\right.}}{}$
default	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}+\frac{d^4\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+}}{3d^2\left(x+\right.}\right.}{5d}\right.}{e^7}$

```
[In] int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-24/5*d/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+6/5*d
^2/e^7/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/5*d^3/e^8/(x+d/e)^3
*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{24\,de^3x^3+72\,d^2e^2x^2+72\,d^3ex+24\,d^4-30\,(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+}{5\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)}$$

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*d*e^3*x^3 + 72*d^2*e^2*x^2 + 72*d^3*e*x + 24*d^4 - 30*(d*e^3*x^3 +
 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 + (5*e^3*x^3 + 39*d*e^2*x^2 + 57*d^2*e*x + 24*d^3)*sqrt(-e^2*x^2 + d^2))/(
e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d^3}{5\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)} + \frac{6\,\sqrt{-e^2x^2+d^2}d^2}{5\,(e^7x^2+2\,de^6x+d^2e^5)} - \frac{24\,\sqrt{-e^2x^2+d^2}d}{5\,(e^6x+de^5)} - \frac{3\,d\arcsin\left(\frac{ex}{d}\right)}{e^5} - \frac{\sqrt{-e^2x^2+d^2}}{e^5}$$

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^
5) + 6/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 24/5*sq
rt(-e^2*x^2 + d^2)*d/(e^6*x + d*e^5) - 3*d*arcsin(e*x/d)/e^5 - sqrt(-e^2*x^
2 + d^2)/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.45

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{3\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e^5} + \frac{2\left(19\,d + \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} + \frac{70\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{5\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) - sqrt(-e^2*x^2 + d^2)/e^5 +
2/5*(19*d + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 120*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) + 70*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8*x^4
))/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)^3}\,dx$$

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(-e*x+d)^2/e^4/(-e^2*x^
2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(-e*x+d)/e^4/(-e^2*
x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {866, 1649, 792, 223, 209}

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(d-ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\left(-\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(22d^2+51dex+32e^2x^2)}{15e^4(d+ex)^3} - \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[In] Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(15*e^4*(d + e*x)^3)
 - (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(106) = 212.

Time = 0.43 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.66

method	result
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}} + \frac{3d^2\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{e^5} + \frac{3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)} - \frac{d^3\left(-\right.}{}$

```
[In] int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3/e^5*d^2*(-1/
3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+3/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)-d^3/e^6*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/
5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d
/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{22\,e^3x^3+66\,de^2x^2+66\,d^2ex+22\,d^3-30\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(32\,e^2}{15\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}$$

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 + 66*d*e^2*x^2 + 66*d^2*e*x + 22*d^3 - 30*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (32*e
^2*x^2 + 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + 3*d*e^6*x^2 +
3*d^2*e^5*x + d^3*e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

```
[In] integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}d^2}{5\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)} - \frac{13\sqrt{-e^2x^2+d^2}d}{15\,(e^6x^2+2\,de^5x+d^2e^4)} + \frac{32\sqrt{-e^2x^2+d^2}}{15\,(e^5x+de^4)} + \frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}$$

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4
) - 13/15*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 32/15*sq
rt(-e^2*x^2 + d^2)/(e^5*x + d*e^4) + arcsin(e*x/d)/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} - \frac{2\left(\frac{95\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{145\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + 22\right)}{15\,e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) - 2/15*(95*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))/(e^2*x) + 145*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2)
+ 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) + 15*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^4/(e^8*x^4) + 22)/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.182 $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)^3+8/15*(-e^2*x^2+d^2)^(1/2)/e^3/(e*
x+d)^2-7/15*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1653, 807, 673, 665}

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

```
[In] Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(1
5*e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{2d^2e^2+de^3x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{(7d)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}+\frac{7\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{(-2d^2-6dex-7e^2x^2)\sqrt{d^2-e^2x^2}}{15de^3(d+ex)^3}$$

```
[In] Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((-2*d^2 - 6*d*e*x - 7*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

method	result
trager	$-\frac{(7e^2x^2+6dex+2d^2)\sqrt{-e^2x^2+d^2}}{15d\,e^3(ex+d)^3}$
gosper	$-\frac{(-ex+d)(7e^2x^2+6dex+2d^2)}{15(ex+d)^2d\,e^3\sqrt{-e^2x^2+d^2}}$
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}+\frac{d^2\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{5d}\right)}{e^5}$

```
[In] int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(7*e^2*x^2+6*d*e*x+2*d^2)/d/e^3/(e*x+d)^3*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,e^3x^3+6\,de^2x^2+6\,d^2ex+2\,d^3+(7\,e^2x^2+6\,dex+2\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)}$$

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x + 2*d^3 + (7*e^2*x^2 + 6*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^
4*e^3)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

[In] integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)

[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}d}{5\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)} + \frac{8\sqrt{-e^2x^2+d^2}}{15\left(e^5x^2+2\,de^4x+d^2e^3\right)} - \frac{7\sqrt{-e^2x^2+d^2}}{15\left(de^4x+d^2e^3\right)}$$

[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) + 8/15*sqrt(-e^2*x^2 + d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 7/15*sqrt(-e^2*x^2 + d^2)/(d*e^4*x + d^2*e^3)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + 1\right)}{15\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} + 1\right)^5|e|}$$

[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 4/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 1)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))

Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d+e\,x\right)}^3}$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 + 6*d*e*x))/(15*d*e^3*(d + e*x)^
3)
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)^3-1/5*(-e^2*x^2+d^2)^(1/2)/d/e^2/(e*x+
d)^2-1/5*(-e^2*x^2+d^2)^(1/2)/d^2/e^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 673, 665}

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} + \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

```
[In] Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}+\frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{5de}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(d^2+3dex+e^2x^2)}{5d^2e^2(d+ex)^3}$$

```
[In] Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(d^2*e^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

method	result
trager	$-\frac{\left(e^{2} x^{2}+3 d e x+d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{5 d^{2} \left(e x+d\right)^{3} e^{2}}$
gosper	$-\frac{\left(-e x+d\right) \left(e^{2} x^{2}+3 d e x+d^{2}\right)}{5 \left(e x+d\right)^{2} d^{2} e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}$
default	$\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d e \left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d^{2} \left(x+\frac{d}{e}\right)}}{e^{3}}-\frac{d \left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{5 d e \left(x+\frac{d}{e}\right)^{3}}+\frac{2 e \left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d e \left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right.}}{3 \cdots}\right.}{5 d}\right.}{e^{4}}$

[In] int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/5*(e^2*x^2+3*d*e*x+d^2)/d^2/(e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3+(e^2x^2+3\,dex+d^2)\sqrt{-e^2x^2+d^2}}{5\,(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2)}$$

[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/5*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 + (e^2*x^2 + 3*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2)

Sympy [F]

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

[In] integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)

[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}}{5\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}}{5\,(de^4x^2+2\,d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}}{5\,(d^2e^3x+d^3e^2)}$$

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) -
1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/5*sqrt(-e^
2*x^2 + d^2)/(d^2*e^3*x + d^3*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + 1\right)}{5\,d^2e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 5*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2/(e^4*x^2) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*
x^3) + 1)/(d^2*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e)
)
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d+e\,x\right)}^3}$$

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + 3*d*e*x))/(5*d^2*e^2*(d + e*x)^3)
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)^3-2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(e*
x+d)^2-2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 665}

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^
2*e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}+\frac{2\int\frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}+\frac{2\int\frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.52

$$\int\frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=\frac{(-7d^2-6dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)^3}$$

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((-7*d^2 - 6*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

method	result	size
trager	$-\frac{(2e^2x^2+6dex+7d^2)\sqrt{-e^2x^2+d^2}}{15d^3(ex+d)^3e}$	49
gosper	$-\frac{(-ex+d)(2e^2x^2+6dex+7d^2)}{15(ex+d)^2d^3e\sqrt{-e^2x^2+d^2}}$	55
default	$\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{5d}}{e^3}$	145

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(2*e^2*x^2+6*d*e*x+7*d^2)/d^3/(e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{7\,e^3x^3+21\,de^2x^2+21\,d^2ex+7\,d^3+(2\,e^2x^2+6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e)}$$

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*x
+ d^6*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{5\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{15\,(d^2e^3x^2+2\,d^3e^2x+d^4e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{15\,(d^3e^2x+d^4e)}$$

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e)
 - 2/15*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/15*sqr
t(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.65

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 40*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(
e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 7)/(d^3*((d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d+e\,x\right)}^3}$$

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)^
3)
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}+\frac{5d-11ex}{15d^2(d^2-e^2x^2)^{3/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(-11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/
2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(-22*e*x+15*d)/d^4/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}+\frac{5d-11ex}{15d^2(d^2-e^2x^2)^{3/2}}+\frac{4(d-ex)}{5(d^2-e^2x^2)^{5/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2+22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(32d^2+51dex+22e^2x^2)}{(d+ex)^3}-15\sqrt{d^2}\log(x)+15\sqrt{d^2}\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

```
[In] Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 - 15*
Sqrt[d^2]*Log[x] + 15*Sqrt[d^2]*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(15*d
^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(101) = 202.

Time = 0.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.89

method	result
default	$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}}-\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}}{e\,d^2}+\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e\,d^4\left(x+\frac{d}{e}\right)}-$

```
[In] int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/e/d^2
*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)-1/e^2/d*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d
^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{32\,e^3x^3+96\,de^2x^2+96\,d^2ex+32\,d^3+15\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(22\,e^2x^2}{15\,(d^4e^3x^3+3\,d^5e^2x^2+3\,d^6ex+d^7)}$$

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 + 96*d*e^2*x^2 + 96*d^2*e*x + 32*d^3 + 15*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (22*e^2*x^2
+ 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + 3*d^5*e^2*x^2 + 3
*d^6*e*x + d^7)
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(102) = 204.

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.84

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} - \frac{2\left(32\,e+\frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex}+\frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2}+\frac{135\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3}+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{15\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^4*a
bs(e)) - 2/15*(32*e + 115*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 185*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) + 135*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3/(e^5*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^7*x
^4))/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] -4/5*e*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)-1/5*e*(-7*e*x+5*d)/d^3/(-e^2*x^2+d^2
)^(3/2)+3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5-1/5*e*(-19*e*x+15*d)/d^5/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1819, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{3e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} - \frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^3+15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^3-45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^3+45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{(3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}\\
&\quad-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{(3e)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^4}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}\\
&\quad-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^4e}\\
&= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(5d^3+39d^2ex+57de^2x^2+24e^3x^3)}{x(d+ex)^3} - 15\sqrt{d^2}e\log(x) + 15\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{5d^6}$$

```
[In] Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((d*Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^
3))/(x*(d + e*x)^3) - 15*Sqrt[d^2]*e*Log[x] + 15*Sqrt[d^2]*e*Log[Sqrt[d^2]
- Sqrt[d^2 - e^2*x^2]])/d^6
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}+\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}-\frac{4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^4e\left(x+\frac{d}{e}\right)^2}-\frac{19\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^5\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-\left(x+\cdots\right.}}{5d\cdots}$
default	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}+\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}+\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2\cdots}}{3d^2\left(x+\frac{d}{\cdots}\right.}\cdots\right.}{5d}}{e\,d^2}\cdots$

```
[In] int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^5/x+3/d^4*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-4/5/d^4/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)-19/5/d^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/5/d^3/e^2/(x+d/
e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{24\,e^4x^4+72\,de^3x^3+72\,d^2e^2x^2+24\,d^3ex+15\,(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\cdots}{5\,(d^5e^3x^4+3\,d^6e^2x^3+3\,d^7ex^2+d^8x)}$$

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/5*(24*e^4*x^4 + 72*d*e^3*x^3 + 72*d^2*e^2*x^2 + 24*d^3*e*x + 15*(e^4*x^4
 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) + (24*e^3*x^3 + 57*d*e^2*x^2 + 39*d^2*e*x + 5*d^3)*sqrt(-e^2*x^2 + d^2))/
(d^5*e^3*x^4 + 3*d^6*e^2*x^3 + 3*d^7*e*x^2 + d^8*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(131) = 262.

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^5|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^5x|e|} + \frac{\left(5\,e^2 + \frac{121\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x} + \frac{410\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2} + \frac{610\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3} + \frac{425\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4} + \frac{125\,(\cdots}{\cdots}\right.}{10\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^
5*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^5*x*abs(e)) + 1/10*(
5*e^2 + 121*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 410*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^2/(e^2*x^2) + 610*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e
^4*x^3) + 425*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) + 125*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{4e^2(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{e^2(25d-31ex)}{15d^4(d^2-e^2x^2)^{3/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] 4/5*e^2*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(-31*e*x+25*d)/d^4/(-e^2
*x^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(-107
*e*x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2+3*e*(-
e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{13e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}+\frac{4e^2(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d-31ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-20de^2x^2+16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+75de^2x^2-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex-90de^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-90d^4e+195d^3e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(13e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^5}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(13e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^5}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^5}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{d\sqrt{d^2-e^2x^2}(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4)}{x^2(d+ex)^3}-195\sqrt{d^2}e^2\log(x)+195\sqrt{d^2}e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{30d^7}$$

```
[In] Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((d*Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3
*x^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) - 195*Sqrt[d^2]*e^2*Log[x] + 195*Sqr
t[d^2]*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/(30*d^7)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-6ex+d)}{2d^6x^2}-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}}+\frac{107e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^6\left(x+\frac{d}{e}\right)}+\frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}{15d^5\left(x+\frac{d}{e}\right)^2}$
default	$\frac{-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}}{d^3}-\frac{6e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^5\sqrt{d^2}}+\frac{3e\sqrt{-e^2x^2+d^2}}{d^6x}-\frac{3e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}$

```
[In] int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-6*e*x+d)/d^6/x^2-13/2/d^5*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+107/15/d^6*e/(x+d/e)*(-(x+d/e)^
```

```
2*e^2+2*d*e*(x+d/e))^(1/2)+17/15/d^5/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2)+1/5/d^4/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{254\,e^5x^5 + 762\,de^4x^4 + 762\,d^2e^3x^3 + 254\,d^3e^2x^2 + 195\,(e^5x^5 + 3\,de^4x^4 + 3\,d^2e^3x^3 + d^3e^2x^2)\log\left(-\frac{d-\sqrt{-e^2}}{x}\right.}{30\,(d^6e^3x^5 + 3\,d^7e^2x^4 + 3\,d^8ex^3 +}$$

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 + 762*d*e^4*x^4 + 762*d^2*e^3*x^3 + 254*d^3*e^2*x^2 + 195
*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (304*e^4*x^4 + 717*d*e^3*x^3 + 479*d^2*e^2*x^2 + 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 + d^9*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(162) = 324.

Time = 0.33 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{13\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^6|e|} + \frac{\left(15\,e^3-\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x}-\frac{2782\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}-\frac{9410\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3}-\frac{13645\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}-\cdots\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|\cdots} + \frac{\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^6e|e|}{x}-\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^6|e|}{ex^2}}{8\,d^{12}e^2}$$

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -13/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))
/(d^6*abs(e)) + 1/120*(15*e^3 - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x
 - 2782*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) - 9410*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 13645*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^4/(e^5*x^4) - 9285*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5) - 258
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2*d^6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1
)^5*abs(e)) + 1/8*(12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^6*e*abs(e)/x -
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^6*abs(e)/(e*x^2))/(d^12*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 204

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}+\frac{18d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*d^4*(-e*x+d)^4/e^6/(-e^2*x^2+d^2)^(5/2)-8/5*d^3*(-e*x+d)^3/e^6/(-e^2*x^
2+d^2)^(3/2)+18*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+10*d^2*(-e*x+d)^2/
e^6/(-e^2*x^2+d^2)^(1/2)+59/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^6-2*d*x*(-e^2*x^2+
d^2)^(1/2)/e^5+1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{18d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
```

```
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^5(d-ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(-\frac{60d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)\left(-\frac{240d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{\int \frac{\frac{720d^6}{e^3}-\frac{885d^5x}{e^2}+\frac{180d^4x^2}{e}}{\sqrt{d^2-e^2x^2}}\,dx}{45d^3e^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{\int \frac{-\frac{1620d^6}{e}+1770d^5x}{\sqrt{d^2-e^2x^2}}\,dx}{90d^3e^4}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{(18d^3)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{(18d^3)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6(d^2-e^2x^2)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{18d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.64

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5)}{15e^6(d+ex)^3} + \frac{18d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^7}$$

[In] Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]

[Out] (Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^3 *x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(15*e^6*(d + e*x)^3) + (18*d^3*Sqrt[-e^2] *Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(e^2x^2-6dex+29d^2)\sqrt{-e^2x^2+d^2}}{3e^6}+\frac{18d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{2d^5\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^9\left(x+\frac{d}{e}\right)^3}-\frac{17d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}{5e^8\left(x+\frac{d}{e}\right)^2}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^6}-\frac{4d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^5}-\frac{d^5\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right.}{15d^2\left(x+\frac{d}{e}\right)^3}\right.}{e^9}$

[In] int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] 1/3*(e^2*x^2-6*d*e*x+29*d^2)/e^6*(-e^2*x^2+d^2)^(1/2)+18/e^5*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2/5/e^9*d^5/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-17/5/e^8*d^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+108/5/e^7*d^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$
$$= \frac{424\,d^3e^3x^3 + 1272\,d^4e^2x^2 + 1272\,d^5ex + 424\,d^6 - 540\,(d^3e^3x^3 + 3\,d^4e^2x^2 + 3\,d^5ex + d^6)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)\cdots}{15\,(e^9x^3 + 3\,de^8x^2 + 3\,d^2e^7x + \cdots)}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(424*d^3*e^3*x^3 + 1272*d^4*e^2*x^2 + 1272*d^5*e*x + 424*d^6 - 540*(d^
3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (5*e^5*x^5 - 15*d*e^4*x^4 + 70*d^2*e^3*x^3 + 674*d^3*e^2*x^2
+ 1002*d^4*e*x + 424*d^5)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*
d^2*e^7*x + d^3*e^6)
```

Sympy [F]

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$$
$$= \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(x\left(\frac{x}{e^4}-\frac{6\,d}{e^5}\right)+\frac{29\,d^2}{e^6}\right)+\frac{18\,d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^5|e|}$$
$$-\frac{2\left(93\,d^3+\frac{385\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^3}{e^2x}+\frac{575\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^3}{e^4x^2}+\frac{355\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^3}{e^6x^3}+\frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}\right.}{5\,e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-e^2*x^2 + d^2)*(x*(x/e^4 - 6*d/e^5) + 29*d^2/e^6) + 18*d^3*arcsin
(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) - 2/5*(93*d^3 + 385*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))*d^3/(e^2*x) + 575*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^3
/(e^4*x^2) + 355*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3/(e^6*x^3) + 80*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3/(e^8*x^4))/(e^5*((d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\int \frac{x^5\,\sqrt{d^2-e^2\,x^2}}{{(d+e\,x)}^4}\,dx$$

```
[In] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^4/e^5/(-e^2*x^2+d^2)^(5/2)+19/15*d^2*(-e*x+d)^3/e^5/(-e^2
*x^2+d^2)^(3/2)-19/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-6*d*(-e*x+d)^
2/e^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e*x+20*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {866, 1649, 794, 223, 209}

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}}$$

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/5*(d^3*(d - e*x)^4)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(
15*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2])
```

```
 - ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(2*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(d-ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(\frac{45d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{135d^4}{e^4}-\frac{15d^3x}{e^3}\right)(d-ex)}{\sqrt{d^2-e^2x^2}}dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(19d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^4}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(19d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx = \frac{\sqrt{d^2-e^2x^2}(-448d^4-1059d^3ex-713d^2e^2x^2-75de^3x^3+15e^4x^4)}{30e^5(d+ex)^3} + \frac{19d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 1059*d^3*e*x - 713*d^2*e^2*x^2 - 75*d*e^3*
x^3 + 15*e^4*x^4))/(30*e^5*(d + e*x)^3) + (19*d^2*ArcTan[(e*x)/(Sqrt[d^2] -
 Sqrt[d^2 - e^2*x^2])])/e^5
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(-ex+8d)\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{5}}-\frac{19d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{4}\sqrt{e^{2}}}-\frac{2d^{4}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{5e^{8}\left(x+\frac{d}{e}\right)^{3}}+\frac{41d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{15e^{7}\left(x+\frac{d}{e}\right)^{2}}$
default	$\frac{\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}}{e^{4}}+\frac{d^{4}\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^{4}}-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^{2}\left(x+\frac{d}{e}\right)^{3}}\right)}{e^{8}}+\frac{6d^{2}\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}\right.}{de}\right.}{}$

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e*x+8*d)/e^5*(-e^2*x^2+d^2)^(1/2)-19/2/e^4*d^2/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-2/5/e^8*d^4/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)+41/15/e^7*d^3/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)-199/15/e^6*d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{448\,d^2e^3x^3+1344\,d^3e^2x^2+1344\,d^4ex+448\,d^5-570\,(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{30\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(448*d^2*e^3*x^3 + 1344*d^3*e^2*x^2 + 1344*d^4*e*x + 448*d^5 - 570*(d
^2*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*e^4*x^4 - 75*d*e^3*x^3 - 713*d^2*e^2*x^2 - 1059*d^3*e*x
- 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3
*e^5)
```

Sympy [F]

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\int \frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\text{Timed out}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.46

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{x}{e^4}-\frac{8\,d}{e^5}\right)-\frac{19\,d^2\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^4|e|}+\frac{2\left(164\,d^2+\frac{685\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x}+\frac{1025\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}+\frac{615\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3}+\frac{135\left(de+\sqrt{-e^2x^2+d^2}\right.}{e^8x^4}\right.}{15\,e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
[Out] 1/2*sqrt(-e^2*x^2 + d^2)*(x/e^4 - 8*d/e^5) - 19/2*d^2*arcsin(e*x/d)*sgn(d)*
sgn(e)/(e^4*abs(e)) + 2/15*(164*d^2 + 685*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))*d^2/(e^2*x) + 1025*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2) +
 615*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 135*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4))/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^4-14/15*d*(-e^2*x^2+d^2)^(3/2)/e^4
/(e*x+d)^3-(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^2+4*d*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^4+8*d*(-e^2*x^2+d^2)^(1/2)/e^4/(e*x+d)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1653, 1651, 673, 665, 677, 223, 209}

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} + \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
```

```
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
```

```
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{\int \frac{\sqrt{d^2-e^2x^2}(2d^3e^2+5d^2e^3x+4de^4x^2)}{(d+ex)^4}\,dx}{e^5}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{\int \left(\frac{d^3e^2\sqrt{d^2-e^2x^2}}{(d+ex)^4} - \frac{3d^2e^2\sqrt{d^2-e^2x^2}}{(d+ex)^3} + \frac{4de^2\sqrt{d^2-e^2x^2}}{(d+ex)^2}\right)dx}{e^5}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{(4d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^3} + \frac{(3d^2)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^3} - \frac{d^3\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{d(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{(4d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3} - \frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{(4d)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(94d^3+222d^2ex+149de^2x^2+15e^3x^3)}{15e^4(d+ex)^3} + \frac{4d\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^5}$$

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/(
15*e^4*(d + e*x)^3) + (4*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^5
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^4}+\frac{4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{104d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^5\left(x+\frac{d}{e}\right)}-\frac{31d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^6\left(x+\frac{d}{e}\right)^2}+\frac{2d^3\sqrt{-\left(x-\right.}}{5\ldots}$
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{e^4}-\frac{d^3\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right.}{15d^2\left(x+\frac{d}{e}\right)^3}\ldots\right.}{e^7}$

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-e^2*x^2+d^2)^(1/2)+4*d/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))+104/15*d/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-31
/15/e^6*d^2/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5/e^7*d^3/(x+d
/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{94\,de^3x^3+282\,d^2e^2x^2+282\,d^3ex+94\,d^4-120\,(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{15\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*d*e^3*x^3 + 282*d^2*e^2*x^2 + 282*d^3*e*x + 94*d^4 - 120*(d*e^3*x^
3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + (15*e^3*x^3 + 149*d*e^2*x^2 + 222*d^2*e*x + 94*d^3)*sqrt(-e^2*x^2 + d
^2))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [F]

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.43

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{4\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e^4} - \frac{2\left(79\,d + \frac{335\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{505\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} + \frac{285\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{15\,e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 4*d*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + sqrt(-e^2*x^2 + d^2)/e^4 - 2
/15*(79*d + 335*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 505*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) + 285*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^3*d/(e^6*x^3) + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8*x
^4))/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4} \,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x+d)^4+3/5*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x
+d)^3-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+
d)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1651, 673, 665, 677, 223, 209}

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} - \frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3} + \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2} \right) dx$$

$$
= \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}
$$

$$
= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4} + \frac{2(d^2-e^2x^2)^{3/2}}{3e^3(d+ex)^3} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e^2}
$$

$$
= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4} + \frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}
$$

$$
= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4} + \frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$
\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{(-8d^2-19dex-13e^2x^2)\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} + \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}
$$

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
[Out] ((-8*d^2 - 19*d*e*x - 13*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3)
+ (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(103) = 206.

Time = 0.41 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.33

method	result
default	$\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{de\left(x+\frac{d}{e}\right)^2}-\frac{e\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d}}{e^4}+\frac{d^2\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.\right.}{5de\left(x+\frac{d}{e}\right)^4}\right.}{}$

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] 1/e^4*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2))))+d^2/e^6*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2
```

```
*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
)+2/3/e^6/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{8\,e^3x^3+24\,de^2x^2+24\,d^2ex+8\,d^3-10\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(13\,e^2x}{5\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
[Out] -1/5*(8*e^3*x^3 + 24*d*e^2*x^2 + 24*d^2*e*x + 8*d^3 - 10*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (13*e^2
*x^2 + 19*d*e*x + 8*d^2)*sqrt(-e^2*x^2 + d^2))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

Sympy [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^2\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}\,x^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)*x^2/(e*x + d)^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.62

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} + \frac{2\left(\frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{25\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+8\right)}{5\,e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 2/5*(35*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))/(e^2*x) + 55*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) +
 25*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) + 5*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^4/(e^8*x^4) + 8)/(e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(3/2)/e^2/(e*x+d)^4-4/15*(-e^2*x^2+d^2)^(3/2)/d/e^2/(e*x
+d)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {807, 665}

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}+\frac{4\int\frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e}\\ &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int\frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx=\frac{\sqrt{d^2-e^2x^2}\left(-d^2-3dex+4e^2x^2\right)}{15de^2(d+ex)^3}$$

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 - 3*d*e*x + 4*e^2*x^2))/(15*d*e^2*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(-ex+d)(4ex+d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3 d\, e^2}$	42
trager	$-\frac{\left(-4e^2x^2+3dex+d^2\right)\sqrt{-e^2x^2+d^2}}{15d(ex+d)^3 e^2}$	47
default	$-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^5d\left(x+\frac{d}{e}\right)^3}-\frac{d\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^5}$	141

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] -1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d/e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3-\left(4\,e^2x^2-3\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{15\,\left(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2\right)}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 - (4*e^2*x^2 - 3*d*e*x - d^2
)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2)
```

Sympy [F]

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{2\,\sqrt{-e^2x^2+d^2}d}{5\,\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)} - \frac{11\,\sqrt{-e^2x^2+d^2}}{15\,\left(e^4x^2+2\,de^3x+d^2e^2\right)} + \frac{4\,\sqrt{-e^2x^2+d^2}}{15\,\left(de^3x+d^2e^2\right)}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(-e^2*x^2 + d^2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
- 11/15*sqrt(-e^2*x^2 + d^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 4/15*sqrt(-e
^2*x^2 + d^2)/(d*e^3*x + d^2*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.14

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + 1\right)}{15\,de\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 2/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 5*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^
6*x^3) + 1)/(d*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e)
)
```

Mupad [B] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x-4\,e^2\,x^2\right)}{15\,d\,e^2\,{\left(d+e\,x\right)}^3}$$

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 - 4*e^2*x^2 + 3*d*e*x))/(15*d*e^2*(d + e*x)^3)
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4} - \frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/d/e/(e*x+d)^4-1/15*(-e^2*x^2+d^2)^(3/2)/d^2/e/(e*
x+d)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 665}

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx = -\frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -1/5*(d^2 - e^2*x^2)^(3/2)/(d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^
2*e*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4} + \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5d} \\ &= -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4} - \frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(-4d^2+3dex+e^2x^2)}{15d^2e(d+ex)^3}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(-ex+d)(ex+4d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3d^2e}$	43
trager	$-\frac{(-e^2x^2-3dex+4d^2)\sqrt{-e^2x^2+d^2}}{15d^2(ex+d)^3e}$	49
default	$\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}}{e^4}$	93

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] -1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{4\,e^3x^3+12\,de^2x^2+12\,d^2ex+4\,d^3-(e^2x^2+3\,dex-4\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e)}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^3*x^3 + 12*d*e^2*x^2 + 12*d^2*e*x + 4*d^3 - (e^2*x^2 + 3*d*e*x -
 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x +
d^5*e)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(59) = 118.

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{2\sqrt{-e^2x^2+d^2}}{5\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{\sqrt{-e^2x^2+d^2}}{15\,(de^3x^2+2\,d^2e^2x+d^3e)} + \frac{\sqrt{-e^2x^2+d^2}}{15\,(d^2e^2x+d^3e)}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-e^2*x^2 + d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 1
/15*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) + 1/15*sqrt(-e^2
*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(59) = 118$.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{25\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+4\right)}{15\,d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 2/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 25*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e
^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 4)/(d^2*((d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.68 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-4\,d^2+3\,d\,e\,x+e^2\,x^2\right)}{15\,d^2\,e\,{\left(d+e\,x\right)}^3}$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x)^4,x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 4*d^2 + 3*d*e*x))/(15*d^2*e*(d + e*x)^3)
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] 8/5*d*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/5*e*x/d/(-e^2*x^2+d^2)^(3/2)-arctanh(
(-e^2*x^2+d^2)^(1/2)/d)/d^3+1/5*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x\,(d^2-e^2x^2)^{7/2}}\,dx \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^4+12d^3ex+5d^2e^2x^2}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^4-24d^3ex}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^4} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^6e^2}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^8e^2} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e^2} \\
&= \frac{8d(d-ex)}{5\,(d^2-e^2x^2)^{5/2}} - \frac{4ex}{5d\,(d^2-e^2x^2)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\,(13d^2+19dex+8e^2x^2)}{(d+ex)^3} + 10\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{5d^3}
$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 10*Arc
Tanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(5*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(96) = 192.

Time = 0.44 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.77

method	result
default	$\frac{\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}}{d^4}-\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}}{e^3d}-\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+\cdots\right.}{de\left(x+\cdots\right.}}{}$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x))-1/e^3/d*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-1/e/d^3*(
-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2))))-1/d^4*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/3/e^3
/d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \frac{13\,e^3x^3+39\,de^2x^2+39\,d^2ex+13\,d^3+5\,(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(8\,e^2x^2+\cdots}{5\,(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6)}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(13*e^3*x^3 + 39*d*e^2*x^2 + 39*d^2*e*x + 13*d^3 + 5*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^2*x^2 + 1
9*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^
5*e*x + d^6)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{\left(ex+d\right)^4x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = -\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^3|e|} - \frac{2\left(13\,e+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{ex}+\frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^3x^2}+\frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^5x^3}+\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^7x^4}\right)}{5\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
[Out] -e*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^3*a
bs(e)) - 2/5*(13*e + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e*x) + 75*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^3*x^2) + 55*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^3/(e^5*x^3) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^7*x^4))
/(d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x\,{(d+e\,x)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4), x)
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx=-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -8/5*e*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/15*e*(-8*e*x+5*d)/d^2/(-e^2*x^2+d^2)
^(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/15*e*(-79*e*x+60*d)/d^4/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1819, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx=\frac{4e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^2 (d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-27d^2e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+64d^2e^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(4e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(2e)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d^3}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e}$$

$$= -\frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(15d^3+149d^2ex+222de^2x^2+94e^3x^3)}{x(d+ex)^3}-60\sqrt{d^2}e\log(x)+60\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{15d^5}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] -1/15*((d*Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^
3*x^3))/(x*(d + e*x)^3) - 60*Sqrt[d^2]*e*Log[x] + 60*Sqrt[d^2]*e*Log[Sqrt[d
^2] - Sqrt[d^2 - e^2*x^2]])/d^5
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^4x}+\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}}-\frac{19\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e\,d^3\left(x+\frac{d}{e}\right)^2}-\frac{79\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^4\left(x+\frac{d}{e}\right)}-\frac{2\sqrt{-}}{}$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{d^2}}{d^4}-\frac{4e\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{d^5}+\frac{-\frac{(-(}{}}{}$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^4/x+4*e/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-19/15/e/d^3/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-79/15/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2/5/e^2/d^2/(x
+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = -\frac{104\,e^4x^4+312\,de^3x^3+312\,d^2e^2x^2+104\,d^3ex+60\,(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex)\log\left(-\frac{d-\sqrt{-e^2x^2+d}}{x}\right.}{15\,(d^4e^3x^4+3\,d^5e^2x^3+3\,d^6ex^2+d^7x)}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(104*e^4*x^4 + 312*d*e^3*x^3 + 312*d^2*e^2*x^2 + 104*d^3*e*x + 60*(e^
4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (94*e^3*x^3 + 222*d*e^2*x^2 + 149*d^2*e*x + 15*d^3)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^3*x^4 + 3*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^2\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{\left(ex+d\right)^4x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \frac{4\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^4|e|} - \frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,d^4x|e|} + \frac{\left(15\,e^2 + \frac{491\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x} + \frac{1690\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^2x^2} + \frac{2570\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^4x^3} + \frac{1815\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^6x^4} + \cdots}{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 4*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^
4*abs(e)) - 1/2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(d^4*x*abs(e)) + 1/30*(
15*e^2 + 491*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/x + 1690*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2/(e^2*x^2) + 2570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/(e^4*x^3) + 1815*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^6*x^4) + 555*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^8*x^5))*e^2*x/((d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4), x)
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

```
[Out] 8/5*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+4/15*e^2*(-13*e*x+10*d)/d^3/(-e^2*x
^2+d^2)^(3/2)-19/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*e^2*(-164*e
*x+135*d)/d^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^4/x^2+4*e*(-e
^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = -\frac{19e^2\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5} + \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
```

```
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^3 (d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+32de^3x^3}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-104de^3x^3}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{\int \frac{-120d^5e+285d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(19e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^4}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(19e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^4}$$

$$= \frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^4}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4\right)}{x^2(d+ex)^3} + 570e^2\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^5}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*
x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 570*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqr
t[d^2 - e^2*x^2])/d])/(30*d^5)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-8ex+d)}{2d^5x^2} - \frac{19e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^4\sqrt{d^2}} + \frac{164e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^5\left(x+\frac{d}{e}\right)} + \frac{2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\cdots\right.}}{5e\,d^3\left(x+\frac{d}{e}\right)^3}\cdots$
default	$\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2} - \frac{e^2\left(\sqrt{-e^2x^2+d^2} - \frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}}{d^4} - \frac{4e\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x} - \frac{2e^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2} + \frac{d^2\arctan\left(\frac{\sqrt{e\cdots}}{\sqrt{-e^2\cdots}}\right)}{2\sqrt{e^2}}\cdots\right)}{d^2}\cdots\right)}{d^5}\cdots$

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-8*e*x+d)/d^5/x^2-19/2/d^4*e^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+164/15/d^5*e/(x+d/e)*(-(x+d/e)^
```

```
2*e^2+2*d*e*(x+d/e))^(1/2)+2/5/e/d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2)+29/15/d^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \frac{398\,e^5x^5+1194\,de^4x^4+1194\,d^2e^3x^3+398\,d^3e^2x^2+285\,(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2)\log\left(-\frac{d-\sqrt{-e}}{x}\right.}{30\,(d^5e^3x^5+3\,d^6e^2x^4+3\,d^7ex^3+}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/30*(398*e^5*x^5 + 1194*d*e^4*x^4 + 1194*d^2*e^3*x^3 + 398*d^3*e^2*x^2 + 2
85*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + (448*e^4*x^4 + 1059*d*e^3*x^3 + 713*d^2*e^2*x^2 + 75*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d^7*e
*x^3 + d^8*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^3\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}}{\left(ex+d\right)^4x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(162) = 324$.

Time = 0.31 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = -\frac{19\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^5|e|} + \frac{\left(15\,e^3 - \frac{165\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x} - \frac{4234\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2} - \frac{14330\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^3x^3} - \frac{20965\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^5x^4}\right.}{120\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)} + \frac{\frac{16\,\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^5e|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^5|e|}{ex^2}}{8\,d^{10}e^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -19/2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))
/(d^5*abs(e)) + 1/120*(15*e^3 - 165*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x
 - 4234*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e*x^2) - 14330*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^3/(e^3*x^3) - 20965*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^4/(e^5*x^4) - 14385*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^7*x^5) - 4
080*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^9*x^6))*e^4*x^2/((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2*d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) +
 1)^5*abs(e)) + 1/8*(16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^5*e*abs(e)/x
- (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*abs(e)/(e*x^2))/(d^10*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4), x)
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{18e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] -8/5*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-4/5*e^3*(-6*e*x+5*d)/d^4/(-e^2*x
^2+d^2)^(3/2)+18*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/5*e^3*(-93*e*x+8
0*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^4/x^3+2*e*(-e^2*x^
2+d^2)^(1/2)/d^5/x^2-29/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = \frac{18e^3\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}-\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}-\frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}}$$

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^4 (d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+40de^3x^3-32e^4x^4}{x^4(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-180de^3x^3+144e^4x^4}{x^4(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2+240de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{\int \frac{-180d^5e+435d^4e^2x-720d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{45d^8}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{\int \frac{-870d^6e^2+1620d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{90d^{10}}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{(18e^3)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{(9e^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d^5}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{(18e)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^5}$$

$$= -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(5d^5-15d^4ex+70d^3e^2x^2+674d^2e^3x^3+1002de^4x^4+424e^5x^5)}{x^3(d+ex)^3} - 270\sqrt{d^2}e^3\log(x) + 270\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2}\right.}{15d^7}$$

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/15*((d*Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d^
2*e^3*x^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) - 270*Sqrt[d^2
]*e^3*Log[x] + 270*Sqrt[d^2]*e^3*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^7
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,(29e^{2}x^{2}-6dex+d^{2})}{3d^{6}x^{3}}+\frac{18e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{d^{5}\sqrt{d^{2}}}-\frac{93e^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{5d^{6}\left(x+\frac{d}{e}\right)}-\frac{13e\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}}}{5d^{5}\left(x+\right.}$
default	$-\frac{(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}{3d^{6}x^{3}}-\frac{4e\left(-\frac{(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}{2d^{2}x^{2}}-\frac{e^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)}{2d^{2}}\right)}{d^{5}}-\frac{20e^{3}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(2d^{2}+\right.}{}\right.}{d^{7}}$

[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] -1/3*(-e^2*x^2+d^2)^(1/2)*(29*e^2*x^2-6*d*e*x+d^2)/d^6/x^3+18/d^5*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-93/5/d^6*e^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-13/5/d^5*e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2/5/d^4/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = -\frac{324\,e^6x^6+972\,de^5x^5+972\,d^2e^4x^4+324\,d^3e^3x^3+270\,(e^6x^6+3\,de^5x^5+3\,d^2e^4x^4+d^3e^3x^3)\log\left(-\frac{d-\sqrt{-e}}{}\right.}{15\,(d^6e^3x^6+3\,d^7e^2x^5+3\,d}$$

[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")

[Out] -1/15*(324*e^6*x^6 + 972*d*e^5*x^5 + 972*d^2*e^4*x^4 + 324*d^3*e^3*x^3 + 270*(e^6*x^6 + 3*d*e^5*x^5 + 3*d^2*e^4*x^4 + d^3*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (424*e^5*x^5 + 1002*d*e^4*x^4 + 674*d^2*e^3*x^3 + 70*d^3*e^2*x^2 - 15*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^6 + 3*d^7*e^2*x^5 + 3*d^8*e*x^4 + d^9*x^3)

Sympy [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{-e^2x^2+d^2}}{\left(ex+d\right)^4x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(187) = 374$.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\frac{18\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^6|e|}+\frac{\left(5\,e^4-\frac{35\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x}+\frac{335\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2}+\frac{7559\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^2x^3}+\frac{25195\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^4x^4}+\cdots\right.}{120\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^6\left(\frac{de+\sqrt{-e^2x\cdots}}{e^2x\cdots}\right.}-\frac{\frac{117\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{12}e^4}{x}-\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{12}e^2}{x^2}+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{12}}{x^3}}{24\,d^{18}e^2|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
[Out] 18*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d
^6*abs(e)) + 1/120*(5*e^4 - 35*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x +
335*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/x^2 + 7559*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3/(e^2*x^3) + 25195*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^
4*x^4) + 36035*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^6*x^5) + 24225*(d*e
```

```
 + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^8*x^6) + 6585*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^7/(e^10*x^7))*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^
6*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e)) - 1/24*(117*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^12*e^4/x - 12*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^2*d^12*e^2/x^2 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^12/x^3
)/(d^18*e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4), x)
```

3.198 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 252

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}+\frac{65d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}$$

```
[Out] 65/4*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+d^4*(-e*x+d)^4/e^6/(-e^2*x^2+
d^2)^(1/2)+515/21*d^6*(-e^2*x^2+d^2)^(1/2)/e^6-49/4*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^5+121/21*d^4*x^2*(-e^2*x^2+d^2)^(1/2)/e^4-17/6*d^3*x^3*(-e^2*x^2+d^2)
^(1/2)/e^3+11/7*d^2*x^4*(-e^2*x^2+d^2)^(1/2)/e^2-2/3*d*x^5*(-e^2*x^2+d^2)^(
1/2)/e+1/7*x^6*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{65d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}$$

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^5(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{d^4x}{e^4}-\frac{d^3x^2}{e^3}+\frac{d^2x^3}{e^2}-\frac{dx^4}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{28d^8}{e^3}-\frac{91d^7x}{e^2}+\frac{112d^6x^2}{e}-77d^5x^3+56d^4ex^4-55d^3e^2x^5+28d^2e^3x^6}{\sqrt{d^2-e^2x^2}}\,dx}{7de^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-\frac{168d^8}{e}+546d^7x-672d^6ex^2+462d^5e^2x^3-476d^4e^3x^4+330d^3e^4x^5}{\sqrt{d^2-e^2x^2}}\,dx}{42de^4}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{840d^8e-2730d^7e^2x+3360d^6e^3x^2-3630d^5e^4x^3+2380d^4e^5x^4}{\sqrt{d^2-e^2x^2}}\,dx}{210de^6}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-3360d^8e^3+10920d^7e^4x-20580d^6e^5x^2+14520d^5e^6x^3}{\sqrt{d^2-e^2x^2}}\,dx}{840de^8}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{10080d^8e^5-61800d^7e^6x+61740d^6e^7x^2}{\sqrt{d^2-e^2x^2}}\,dx}{2520de^{10}}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-81900d^8e^7+123600d^7e^8x}{\sqrt{d^2-e^2x^2}}\,dx}{5040de^{12}}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{(65d^7)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{4e^5}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{(65d^7)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{4e^5}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{65d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{e\sqrt{d^2-e^2x^2}(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7)}{d+ex} + 1365d^7\sqrt{}}{84e^7}$$

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4
*e^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(
d + e*x) + 1365*d^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/
(84*e^7)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(12e^6x^6-56d\,e^5x^5+132d^2e^4x^4-238d^3x^3e^3+400d^4e^2x^2-693d^5ex+1472d^6)\sqrt{-e^2x^2+d^2}}{84e^6}+\frac{65d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4e^5\sqrt{e^2}}+\frac{8d^7\sqrt{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*(12*e^6*x^6-56*d*e^5*x^5+132*d^2*e^4*x^4-238*d^3*e^3*x^3+400*d^4*e^2*x
^2-693*d^5*e*x+1472*d^6)/e^6*(-e^2*x^2+d^2)^(1/2)+65/4*d^7/e^5/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+8*d^7/e^7/(x+d/e)*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{2144\,d^7ex+2144\,d^8-2730\,(d^7ex+d^8)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(12\,e^7x^7-44\,de\cdots}{84\cdots}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2144*d^7*e*x + 2144*d^8 - 2730*(d^7*e*x + d^8)*arctan(-(d - sqrt(-e^2
*x^2 + d^2))/(e*x)) + (12*e^7*x^7 - 44*d*e^6*x^6 + 76*d^2*e^5*x^5 - 106*d^3
*e^4*x^4 + 162*d^4*e^3*x^3 - 293*d^5*e^2*x^2 + 779*d^6*e*x + 2144*d^7)*sqrt
(-e^2*x^2 + d^2))/(e^7*x + d*e^6)
```

Sympy [F]

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^5(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.90

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^5}{2\,(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6)} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6}{2\,(e^8x^2+2\,de^7x+d^2e^6)} + \frac{15\,\sqrt{-e^2x^2+d^2}d^7}{e^7x+de^6} + \frac{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{3\,(e^8x^2+2\,de^7x+d^2e^6)} + \frac{25\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{6\,(e^7x+de^6)} - \frac{5\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{2\,(e^7x+de^6)} + \frac{5i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^6} + \frac{75\,d^7\arcsin\left(\frac{ex}{d}\right)}{4\,e^6} - \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x}{2\,e^5} - \frac{5\,\sqrt{-e^2x^2+d^2}d^5x}{4\,e^5} - \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{e^6} + \frac{25\,\sqrt{-e^2x^2+d^2}d^6}{2\,e^6} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3x}{3\,e^5} - \frac{25\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{6\,e^6} - \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}dx}{3\,e^5} + \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{e^6} - \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{7\,e^6}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*
e^6) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^6/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 15*
sqrt(-e^2*x^2 + d^2)*d^7/(e^7*x + d*e^6) + 5/3*(-e^2*x^2 + d^2)^(5/2)*d^4/(
e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 25/6*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x + d
*e^6) - 5/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x + d*e^6) + 5/2*I*d^7*arcsin(e
*x/d + 2)/e^6 + 75/4*d^7*arcsin(e*x/d)/e^6 - 5/2*sqrt(e^2*x^2 + 4*d*e*x + 3
*d^2)*d^5*x/e^5 - 5/4*sqrt(-e^2*x^2 + d^2)*d^5*x/e^5 - 5*sqrt(e^2*x^2 + 4*d
*e*x + 3*d^2)*d^6/e^6 + 25/2*sqrt(-e^2*x^2 + d^2)*d^6/e^6 + 5/3*(-e^2*x^2 +
 d^2)^(3/2)*d^3*x/e^5 - 25/6*(-e^2*x^2 + d^2)^(3/2)*d^4/e^6 - 2/3*(-e^2*x^2
 + d^2)^(5/2)*d*x/e^5 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2/e^6 - 1/7*(-e^2*x^2 +
d^2)^(7/2)/e^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{65\,d^7\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{4\,e^5|e|} + \frac{1}{84}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(2\left(\left(3\,x-\frac{14\,d}{e}\right)x+\frac{33\,d^2}{e^2}\right)x-\frac{119\,d^3}{e^3}\right)x+\frac{200\,d^4}{e^4}\right)x-\frac{693\,d^5}{e^5}\right)x+\frac{1472\,d^6}{e^6}\right) - \frac{16\,d^7}{e^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 65/4*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^5*abs(e)) + 1/84*sqrt(-e^2*x^2 + d^
2)*((2*((2*((3*x - 14*d/e)*x + 33*d^2/e^2)*x - 119*d^3/e^3)*x + 200*d^4/e^4
)*x - 693*d^5/e^5)*x + 1472*d^6/e^6) - 16*d^7/(e^5*((d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^5\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.199 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{239d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

```
[Out] -239/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-d^3*(-e*x+d)^4/e^5/(-e^2*x
^2+d^2)^(1/2)-337/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5+175/16*d^4*x*(-e^2*x^2+d^
2)^(1/2)/e^4-71/15*d^3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3+47/24*d^2*x^3*(-e^2*x^2
+d^2)^(1/2)/e^2-4/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e+1/6*x^5*(-e^2*x^2+d^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{239d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
```

```
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{d^3x}{e^3}+\frac{d^2x^2}{e^2}-\frac{dx^3}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-\frac{24d^7}{e^2}+\frac{78d^6x}{e}-96d^5x^2+66d^4ex^3-47d^3e^2x^4+24d^2e^3x^5}{\sqrt{d^2-e^2x^2}}\,dx}{6de^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{120d^7-390d^6ex+480d^5e^2x^2-426d^4e^3x^3+235d^3e^4x^4}{\sqrt{d^2-e^2x^2}}\,dx}{30de^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-480d^7e^2+1560d^6e^3x-2625d^5e^4x^2+1704d^4e^5x^3}{\sqrt{d^2-e^2x^2}}\,dx}{120de^6}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{1440d^7e^4-8088d^6e^5x+7875d^5e^6x^2}{\sqrt{d^2-e^2x^2}}\,dx}{360de^8}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-10755d^7e^6+16176d^6e^7x}{\sqrt{d^2-e^2x^2}}\,dx}{720de^{10}}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{(239d^6)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{(239d^6)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{239d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6)}{d+ex} + 7170d^6\,\text{arcta}}{240e^5}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3
*e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6))/(d + e*x) + 7170*
d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(240*e^5)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(-40e^5x^5+192d\,e^4x^4-470d^2e^3x^3+896d^3e^2x^2-1665d^4ex+3712d^5)\sqrt{-e^2x^2+d^2}}{240e^5} - \frac{239d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^4\sqrt{e^2}} - \frac{8d^6\sqrt{-(x}}{}$
default	Expression too large to display

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-40*e^5*x^5+192*d*e^4*x^4-470*d^2*e^3*x^3+896*d^3*e^2*x^2-1665*d^4*
e*x+3712*d^5)/e^5*(-e^2*x^2+d^2)^(1/2)-239/16*d^6/e^4/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^6/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.65

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{5632\,d^6ex+5632\,d^7-7170\,(d^6ex+d^7)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(40\,e^6x^6-152\,de^5x^5+278\,d^2e^4x^4-426}{240\,(e^6x+de^5)}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
[Out] -1/240*(5632*d^6*e*x + 5632*d^7 - 7170*(d^6*e*x + d^7)*arctan(-(d - sqrt(-e
^2*x^2 + d^2))/(e*x)) - (40*e^6*x^6 - 152*d*e^5*x^5 + 278*d^2*e^4*x^4 - 426
*d^3*e^3*x^3 + 769*d^4*e^2*x^2 - 2047*d^5*e*x - 5632*d^6)*sqrt(-e^2*x^2 + d
^2))/(e^6*x + d*e^5)
```

Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^4}{2\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5}{2\,(e^7x^2+2\,de^6x+d^2e^5)} - \frac{15\,\sqrt{-e^2x^2+d^2}d^6}{e^6x+de^5} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{3\,(e^7x^2+2\,de^6x+d^2e^5)} \\ &- \frac{10\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{3\,(e^6x+de^5)} + \frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{2\,(e^6x+de^5)} - \frac{9i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^5} \\ &- \frac{275\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^5} + \frac{9\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x}{4\,e^4} + \frac{5\,\sqrt{-e^2x^2+d^2}d^4x}{16\,e^4} \\ &+ \frac{9\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{2\,e^5} - \frac{10\,\sqrt{-e^2x^2+d^2}d^5}{e^5} - \frac{19\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24\,e^4} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,e^5} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6\,e^4} - \frac{4\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,e^5}\end{aligned}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e
^5) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 15*s
qrt(-e^2*x^2 + d^2)*d^6/(e^6*x + d*e^5) - 4/3*(-e^2*x^2 + d^2)^(5/2)*d^3/(e
^7*x^2 + 2*d*e^6*x + d^2*e^5) - 10/3*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x + d*
e^5) + 3/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x + d*e^5) - 9/4*I*d^6*arcsin(e*
x/d + 2)/e^5 - 275/16*d^6*arcsin(e*x/d)/e^5 + 9/4*sqrt(e^2*x^2 + 4*d*e*x +
3*d^2)*d^4*x/e^4 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 + 9/2*sqrt(e^2*x^2 +
 4*d*e*x + 3*d^2)*d^5/e^5 - 10*sqrt(-e^2*x^2 + d^2)*d^5/e^5 - 19/24*(-e^2*x
^2 + d^2)^(3/2)*d^2*x/e^4 + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/e^5 + 1/6*(-e^2*
x^2 + d^2)^(5/2)*x/e^4 - 4/5*(-e^2*x^2 + d^2)^(5/2)*d/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{239\,d^6\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^4|e|} + \frac{1}{240}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(4\left(5\,x-\frac{24\,d}{e}\right)x+\frac{235\,d^2}{e^2}\right)x-\frac{448\,d^3}{e^3}\right)x+\frac{1665\,d^4}{e^4}\right)x-\frac{3712\,d^5}{e^5}\right) + \frac{16\,d^6}{e^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -239/16*d^6*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^4*abs(e)) + 1/240*sqrt(-e^2*x^2
+ d^2)*((2*((4*(5*x - 24*d/e)*x + 235*d^2/e^2)*x - 448*d^3/e^3)*x + 1665*d^
4/e^4)*x - 3712*d^5/e^5) + 16*d^6/(e^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.200 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 192

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{27d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

```
[Out] 27/2*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+d^2*(-e*x+d)^4/e^4/(-e^2*x^2+
d^2)^(1/2)+101/5*d^4*(-e^2*x^2+d^2)^(1/2)/e^4-19/2*d^3*x*(-e^2*x^2+d^2)^(1/
2)/e^3+18/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-d*x^3*(-e^2*x^2+d^2)^(1/2)/e+1
/5*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{27d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
```

```
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^3}{e^3}+\frac{d^2x}{e^2}-\frac{dx^2}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{20d^6}{e}-65d^5x+80d^4ex^2-54d^3e^2x^3+20d^2e^3x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5de^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-80d^6e+260d^5e^2x-380d^4e^3x^2+216d^3e^4x^3}{\sqrt{d^2-e^2x^2}}\,dx}{20de^4}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{240d^6e^3-1212d^5e^4x+1140d^4e^5x^2}{\sqrt{d^2-e^2x^2}}\,dx}{60de^6}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-1620d^6e^5+2424d^5e^6x}{\sqrt{d^2-e^2x^2}}\,dx}{120de^8}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{(27d^5)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{(27d^5)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{27d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{e\sqrt{d^2-e^2x^2}(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5)}{d+ex}+135d^5\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\right.}{10e^5}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3
*x^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*Sqrt[-e^2]*Log[-(Sqrt[
-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(10*e^5)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(2e^4x^4-10d\,e^3x^3+26d^2e^2x^2-55d^3ex+132d^4)\sqrt{-e^2x^2+d^2}}{10e^4}+\frac{27d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^3\sqrt{e^2}}+\frac{8d^5\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$
default	Expression too large to display

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(2*e^4*x^4-10*d*e^3*x^3+26*d^2*e^2*x^2-55*d^3*e*x+132*d^4)/e^4*(-e^2*x
^2+d^2)^(1/2)+27/2*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^
(1/2))+8*d^5/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{212\,d^5ex+212\,d^6-270\,(d^5ex+d^6)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^5x^5-8\,de^4x^4+}{10\,(e^5x+de^4)}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/10*(212*d^5*e*x + 212*d^6 - 270*(d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^5*x^5 - 8*d*e^4*x^4 + 16*d^2*e^3*x^3 - 29*d^3*e^2*x
^2 + 77*d^4*e*x + 212*d^5)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)

[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.12

$$\begin{aligned}
\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^3}{2\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)} \\
&\quad -\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4}{2\,(e^6x^2+2\,de^5x+d^2e^4)}+\frac{15\,\sqrt{-e^2x^2+d^2}d^5}{e^5x+de^4}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{e^6x^2+2\,de^5x+d^2e^4} \\
&\quad +\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,(e^5x+de^4)}-\frac{3\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{4\,(e^5x+de^4)}+\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4} \\
&\quad +\frac{15\,d^5\arcsin\left(\frac{ex}{d}\right)}{e^4}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{2\,e^3}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{e^4} \\
&\quad +\frac{15\,\sqrt{-e^2x^2+d^2}d^4}{2\,e^4}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4\,e^3}-\frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{4\,e^4}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5\,e^4}
\end{aligned}$$

[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")

[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 15*sqrt(-e^2*x^2 + d^2)*d^5/(e^5*x + d*e^4) + (-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x + d*e^4) - 3/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x + d*e^4) + 3/2*I*d^5*arcsin(e*x/d + 2)/e^4 + 15*d^5*arcsin(e*x/d)/e^4 - 3/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x/e^3 - 3*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^4 + 15/2*sqrt(-e^2*x^2 + d^2)*d^4/e^4 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^3 - 5/4*(-e^2*x^2 + d^2)^(3/2)*d^2/e^4 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^4

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{27\,d^5\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^3|e|} + \frac{1}{10}\sqrt{-e^2x^2+d^2}\left(\left(2\left(\left(x-\frac{5\,d}{e}\right)x+\frac{13\,d^2}{e^2}\right)x-\frac{55\,d^3}{e^3}\right)x+\frac{132\,d^4}{e^4}\right) - \frac{16\,d^5}{e^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 27/2*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 1/10*sqrt(-e^2*x^2 + d^
2)*((2*((x - 5*d/e)*x + 13*d^2/e^2)*x - 55*d^3/e^3)*x + 132*d^4/e^4) - 16*d
^5/(e^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.201 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -95/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d*(-e*x+d)^4/e^3/(-e^2*x^2+d
^2)^(1/2)-95/8*d^3*(-e^2*x^2+d^2)^(1/2)/e^3-95/24*d^2*(-e*x+d)*(-e^2*x^2+d^
2)^(1/2)/e^3-19/12*d*(-e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/4*(-e*x+d)^3*(-e
^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1649, 809, 685, 655, 223, 209}

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{95d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
```

```
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{4d^2}{e^2}-\frac{dx}{e}\right)(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(19d)\int \frac{(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(95d^2)\int \frac{(d-ex)^2}{\sqrt{d^2-e^2x^2}}\,dx}{12e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(95d^3)\int \frac{d-ex}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(95d^4)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(95d^4)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-448d^4-163d^3ex+61d^2e^2x^2-26de^3x^3+6e^4x^4)}{d+ex}+570d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 163*d^3*e*x + 61*d^2*e^2*x^2 - 26*d*e^3*x
^3 + 6*e^4*x^4))/(d + e*x) + 570*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e
^2*x^2])])/(24*e^3)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{(-6e^3x^3+32d\,e^2x^2-93d^2ex+256d^3)\sqrt{-e^2x^2+d^2}}{24e^3}-\frac{95d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}-\frac{8d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4\left(x+\frac{d}{e}\right)}$	131
default	Expression too large to display	890

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-6*e^3*x^3+32*d*e^2*x^2-93*d^2*e*x+256*d^3)/e^3*(-e^2*x^2+d^2)^(1/2)
-95/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^4/
e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx =$$

$$-\frac{448\,d^4ex+448\,d^5-570\,(d^4ex+d^5)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(6\,e^4x^4-26\,de^3x^3+61\,d^2e^2x^2-163\,d^3ex}{24\,(e^4x+de^3)}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(448*d^4*e*x + 448*d^5 - 570*(d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) - (6*e^4*x^4 - 26*d*e^3*x^3 + 61*d^2*e^2*x^2 - 163*d^3*e*
x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3)
```

Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d^2}{2\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} \\ &+ \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3}{2\,(e^5x^2+2\,de^4x+d^2e^3)} - \frac{15\,\sqrt{-e^2x^2+d^2}d^4}{e^4x+de^3} \\ &- \frac{2\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{3\,(e^5x^2+2\,de^4x+d^2e^3)} - \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{3\,(e^4x+de^3)} - \frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3} \\ &- \frac{25\,d^4\arcsin\left(\frac{ex}{d}\right)}{2\,e^3} + \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4\,(e^4x+de^3)} + \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^2x}{8\,e^2} \\ &+ \frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}d^3}{4\,e^3} - \frac{5\,\sqrt{-e^2x^2+d^2}d^3}{e^3} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{12\,e^3}\end{aligned}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e
^3) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 15*s
qrt(-e^2*x^2 + d^2)*d^4/(e^4*x + d*e^3) - 2/3*(-e^2*x^2 + d^2)^(5/2)*d/(e^5
*x^2 + 2*d*e^4*x + d^2*e^3) - 5/3*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x + d*e^3
) - 5/8*I*d^4*arcsin(e*x/d + 2)/e^3 - 25/2*d^4*arcsin(e*x/d)/e^3 + 1/4*(-e^
2*x^2 + d^2)^(5/2)/(e^4*x + d*e^3) + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
2*x/e^2 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e^3 - 5*sqrt(-e^2*x^2 + d
^2)*d^3/e^3 + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.64

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{95\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,x-\frac{16\,d}{e}\right)x+\frac{93\,d^2}{e^2}\right)x-\frac{256\,d^3}{e^3}\right) + \frac{16\,d^4}{e^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -95/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 1/24*sqrt(-e^2*x^2 + d
^2)*((2*(3*x - 16*d/e)*x + 93*d^2/e^2)*x - 256*d^3/e^3) + 16*d^4/(e^2*((d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.202 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{10d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] 20/3*(-e^2*x^2+d^2)^(3/2)/e^2+8*(-e^2*x^2+d^2)^(5/2)/e^2/(e*x+d)^2+(-e^2*x^
2+d^2)^(7/2)/e^2/(e*x+d)^4+10*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+10*d
*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {807, 677, 679, 201, 223, 209}

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx = \frac{10d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{10dx\sqrt{d^2-e^2x^2}}{e}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{4\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx}{e}$$

$$= \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{20\int \frac{(d^2-e^2x^2)^{3/2}}{d+ex}\,dx}{e}$$

$$= \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(20d)\int \sqrt{d^2-e^2x^2}\,dx}{e}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(10d^3)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(10d^3)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\sqrt{d^2-e^2x^2}(47d^3+17d^2ex-5de^2x^2+e^3x^3)}{3e^2(d+ex)} + \frac{10d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^3}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(47*d^3 + 17*d^2*e*x - 5*d*e^2*x^2 + e^3*x^3))/(3*e^2*
(d + e*x)) + (10*d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])
/e^3
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(e^{2} x^{2}-6 d e x+23 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}+\frac{10 d^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e \sqrt{e^{2}}}+\frac{8 d^{3} \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e^{3} \left(x+\frac{d}{e}\right)}$

[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] 1/3*(e^2*x^2-6*d*e*x+23*d^2)/e^2*(-e^2*x^2+d^2)^(1/2)+10*d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+8*d^3/e^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{47\,d^3ex+47\,d^4-60\,(d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(e^3x^3-5\,de^2x^2+17\,d^2e}{3\,(e^3x+de^2)}$$

[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")

[Out] 1/3*(47*d^3*e*x + 47*d^4 - 60*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^3*x^3 - 5*d*e^2*x^2 + 17*d^2*e*x + 47*d^3)*sqrt(-e^2*x^2 + d^2))/(e^3*x + d*e^2)

Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)

[Out] Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx =& -\frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{2\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} \\ &- \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{2\,(e^4x^2+2\,de^3x+d^2e^2)} + \frac{15\,\sqrt{-e^2x^2+d^2}d^3}{e^3x+de^2} + \frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)}{e^2} \\ &+ \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{3\,(e^4x^2+2\,de^3x+d^2e^2)} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{6\,(e^3x+de^2)} + \frac{5\,\sqrt{-e^2x^2+d^2}d^2}{2\,e^2}\end{aligned}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^
2) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 15*sq
rt(-e^2*x^2 + d^2)*d^3/(e^3*x + d*e^2) + 10*d^3*arcsin(e*x/d)/e^2 + 1/3*(-e
^2*x^2 + d^2)^(5/2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 5/6*(-e^2*x^2 + d^2)^
(3/2)*d/(e^3*x + d*e^2) + 5/2*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{e|e|} + \frac{1}{3}\sqrt{-e^2x^2+d^2}\left(\left(x-\frac{6\,d}{e}\right)x+\frac{23\,d^2}{e^2}\right) - \frac{16\,d^3}{e\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 10*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e*abs(e)) + 1/3*sqrt(-e^2*x^2 + d^2)*((
x - 6*d/e)*x + 23*d^2/e^2) - 16*d^3/(e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.203 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] -5/2*(-e^2*x^2+d^2)^(3/2)/e/(e*x+d)-2*(-e^2*x^2+d^2)^(5/2)/e/(e*x+d)^3-15/2
*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-15/2*d*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {677, 679, 223, 209}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - 5\int \frac{(d^2-e^2x^2)^{3/2}}{(d+ex)^2}\,dx \\
&= -\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx \\
&= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} \\
&\quad - \frac{1}{2}(15d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx=\frac{\sqrt{d^2-e^2x^2}(-24d^2-7dex+e^2x^2)}{2e(d+ex)}+\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^2 - 7*d*e*x + e^2*x^2))/(2*e*(d + e*x)) + (15*d
^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(-ex+8d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{15d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{8d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^2\left(x+\frac{d}{e}\right)}$
default	$-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^4}-\frac{3e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^3}+\frac{4e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$

[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] -1/2*(-e*x+8*d)/e*(-e^2*x^2+d^2)^(1/2)-15/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^2/e^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{24\,d^2ex+24\,d^3-30\,(d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(e^2x^2-7\,dex-24\,d^2)\sqrt{-e^2x^2+d^2}}{2\,(e^2x+de)}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^2*e*x + 24*d^3 - 30*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) - (e^2*x^2 - 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x +
 d*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{15\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{2\,(e^3x^2+2\,de^2x+d^2e)}-\frac{15\,\sqrt{-e^2x^2+d^2}d^2}{e^2x+de}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -15/2*d^2*arcsin(e*x/d)/e + 1/2*(-e^2*x^2 + d^2)^(5/2)/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d/(e^3*x^2 + 2*d*e^2
*x + d^2*e) - 15*sqrt(-e^2*x^2 + d^2)*d^2/(e^2*x + d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{15\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(x-\frac{8\,d}{e}\right) + \frac{16\,d^2}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -15/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*sqrt(-e^2*x^2 + d^2)*(x
- 8*d/e) + 16*d^2/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x)
```

3.204 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2} + 4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 4*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)+8*d*
(-e*x+d)/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {866, 1819, 1823, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = 4d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4-4d^3ex+d^2e^2x^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{\int \frac{d^4e^2+4d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (4de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{1}{2}d^2\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)\\
&\quad + (4de)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{(9d+ex)\sqrt{d^2-e^2x^2}}{d+ex} + 2d\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{4de\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] ((9*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d
- Sqrt[d^2 - e^2*x^2]/d] - (4*d*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/Sqrt[-e^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(81) = 162$.

Time = 0.48 (sec) , antiderivative size = 1192, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	1192

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^
2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x))))-1/e^3/d*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*
(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e
)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2))))))))-1/e/d^3*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/
8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(
-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^
2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
)-1/d^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*
e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-1/e^2/d^2*(1/d/e
/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/
```

```
e))^(5/2)+d*e*(-1/8*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(3/2)+3/4*d^2*(-1/4*(-2*(x+d/e)*e^2+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{9\,dex + 9\,d^2 - 8\,(dex + d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (dex + d^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right.}{ex+d}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] (9*d*e*x + 9*d^2 - 8*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (d*e*x + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)
*(e*x + 9*d))/(e*x + d)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x\left(d+ex\right)^{4}}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^{4}x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{4\,de\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{de\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|} + \sqrt{-e^2x^2+d^2} - \frac{16\,de}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 4*d*e*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - d*e*log(1/2*abs(-2*d*e - 2*sqrt(
-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + sqrt(-e^2*x^2 + d^2) - 16*d*
e/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4), x)
```

3.205 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 94

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x} - e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-8*e
*(-e*x+d)/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {866, 1819, 1821, 858, 223, 209, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = -e\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex+d^2e^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}+\frac{\int \frac{-4d^5e-d^4e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4} \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(4de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(2de)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) \\
&\quad -e^2\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) \\
&\quad +\frac{(4d)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e} \\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \frac{(-d-9ex)\sqrt{d^2-e^2x^2}}{x(d+ex)} + 2e\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right) + \frac{4\sqrt{d^2}e\log(x)}{d} - \frac{4\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] ((-d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x)) + 2*e*ArcTan[(e*x)/(Sqrt[d
^2] - Sqrt[d^2 - e^2*x^2])] + (4*Sqrt[d^2]*e*Log[x])/d - (4*Sqrt[d^2]*e*Log
[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{x}-\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{4de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{8\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{x+\frac{d}{e}}$	133
default	Expression too large to display	1322

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/x-e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))+4*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-
8/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = -\frac{8\,e^2x^2+8\,dex-2\,(e^2x^2+dex)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+4\,(e^2x^2+dex)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+}}{ex^2+dx}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -(8*e^2*x^2 + 8*d*e*x - 2*(e^2*x^2 + d*e*x)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) + 4*(e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(
-e^2*x^2 + d^2)*(9*e*x + d))/(e*x^2 + d*x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx=\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^2\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx=\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx=-\frac{e^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|}+\frac{4\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{|e|}+\frac{\left(e^2+\frac{33\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{x}\right)e^2x}{2\left(de+\sqrt{-e^2x^2+d^2}|e|\right)\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}-\frac{de+\sqrt{-e^2x^2+d^2}|e|}{2\,x|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
[Out] -e^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 4*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/abs(e) + 1/2*(e^2 + 33*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))/x)*e^2*x/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*((d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 1/2*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))/(x*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^2\,{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

3.206 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4} dx = \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -15/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d+8*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)
^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/x^2+4*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4} dx = -\frac{15e^2\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d} + \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{\sqrt{d^2-e^2x^2}}{2x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{\int \frac{-8d^5e+15d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{2}\left(15e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{4}\left(15e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{15}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}(-d^2+7dex+24e^2x^2)}{dx^2(d+ex)} - \frac{15e^2\log(x)}{\sqrt{d^2}}+\frac{15e^2\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2}}\right)$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(d*x^2*(d + e*x)) - (1
5*e^2*Log[x])/Sqrt[d^2] + (15*e^2*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/Sqr
t[d^2])/2
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,(-8ex+d)}{2d\,x^{2}}-\frac{15e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}+\frac{8e\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2de\left(x+\frac{d}{e}\right)}}{d\left(x+\frac{d}{e}\right)}$	115
default	Expression too large to display	1461

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*(-8*e*x+d)/d/x^2-15/2*e^2/(d^2)^(1/2)*ln((2*d^2+2
*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+8*e/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{16\,e^3x^3+16\,de^2x^2+15\,(e^3x^3+de^2x^2)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(24\,e^2x^2+7\,dex-d^2)\sqrt{}}{2\,(dex^3+d^2x^2)}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
[Out] 1/2*(16*e^3*x^3 + 16*d*e^2*x^2 + 15*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (24*e^2*x^2 + 7*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d*e
*x^3 + d^2*x^2)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^3\left(d+ex\right)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(99) = 198$.

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{\left(e^3 - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e}{x} - \frac{144\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{ex^2}\right)e^4x^2}{8\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} - \frac{15\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d|e|} + \frac{\frac{16\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de|e|}{x} - \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d|e|}{ex^2}}{8\,d^2e^2}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
[Out] 1/8*(e^3 - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e/x - 144*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e*x^2))*e^4*x^2/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^2*d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) - 15/2*e^3*
log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d*abs(e)
) + 1/8*(16*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e*abs(e)/x - (d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*d*abs(e)/(e*x^2))/(d^2*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^3\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

3.207 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4} dx = -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{10e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] 10*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-8*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2
)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/x^3+2*e*(-e^2*x^2+d^2)^(1/2)/d/x^2-23/3*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4} dx = \frac{10e^3\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^4 (d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{\int \frac{-12d^5e+23d^4e^2x-24d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^4}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\int \frac{-46d^6e^2+60d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^6}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{(10e^3)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{(5e^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{(10e)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(d^3-5d^2ex+17de^2x^2+47e^3x^3)}{x^3(d+ex)}-30\sqrt{d^2}e^3\log(x)+30\sqrt{d^2}e^3\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{3d^3}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
[Out] -1/3*((d*Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))
/(x^3*(d + e*x)) - 30*Sqrt[d^2]*e^3*Log[x] + 30*Sqrt[d^2]*e^3*Log[Sqrt[d^2]
 - Sqrt[d^2 - e^2*x^2]])/d^3
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(23e^2x^2-6dex+d^2)}{3x^3d^2}+\frac{10e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}-\frac{8e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^2\left(x+\frac{d}{e}\right)}$	131
default	Expression too large to display	1626

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(-e^2*x^2+d^2)^(1/2)*(23*e^2*x^2-6*d*e*x+d^2)/x^3/d^2+10*e^3/d/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-8*e^2/d^2/(x+d/e)*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = -\frac{24\,e^4x^4+24\,de^3x^3+30\,(e^4x^4+de^3x^3)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(47\,e^3x^3+17\,de^2x^2-5\,d^2ex+d^3)\sqrt{-e^2x^2}}{3\,(d^2ex^4+d^3x^3)}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*e^4*x^4 + 24*d*e^3*x^3 + 30*(e^4*x^4 + d*e^3*x^3)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (47*e^3*x^3 + 17*d*e^2*x^2 - 5*d^2*e*x + d^3)*sqrt(-e^
2*x^2 + d^2))/(d^2*e*x^4 + d^3*x^3)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x^4(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex+d)^4x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.32

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \frac{\left(e^4 - \frac{11\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2}{x} + \frac{81\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{x^2} + \frac{477\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^2x^3}\right)e^6x^3}{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|} + \frac{10\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{d^2|e|} - \frac{\frac{93\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^4e^4}{x} - \frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^4e^2}{x^2} + \frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^4}{x^3}}{24\,d^6e^2|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

[Out] 1/24*(e^4 - 11*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2/x + 81*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/x^2 + 477*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^2*x^3))*e^6*x^3/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e)) + 10*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)))/(d^2*abs(e)) - 1/24*(93*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*e^4/x - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*e^2/x^2 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4/x^3)/(d^6*e^2*abs(e))

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^4\,{\left(d+e\,x\right)}^4}\,dx$$

[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x)

[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4), x)

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 170

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4} dx = \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{95e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

```
[Out] -95/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+8*e^4*(-e*x+d)/d^3/(-e^2*x^2+
d^2)^(1/2)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4+4/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-31
/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2+32/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4} dx = -\frac{95e^4\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
```

```
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x^5\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{\int \frac{-16d^5e+31d^4e^2x-32d^3e^3x^2+32d^2e^4x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^4} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{-93d^6e^2+128d^5e^3x-96d^4e^4x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^6} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{\int \frac{-256d^7e^3+285d^6e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^8} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} \\
&\quad - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(95e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} \\
&\quad + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(95e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{16d^2} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} \\
&\quad + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{\left(95e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{8d^2} \\
&= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} \\
&\quad + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.68

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4)}{x^4(d+ex)}+570e^4\operatorname{arctanh}\left(\frac{\sqrt{-e^2}x-\sqrt{d^2-e^2x^2}}{d}\right)}{24d^3}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
 + 448*e^4*x^4))/(x^4*(d + e*x)) + 570*e^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2
 - e^2*x^2])/d])/(24*d^3)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.85

method	result	s
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-256e^3x^3+93d\,e^2x^2-32d^2ex+6d^3)}{24d^3x^4}-\frac{95e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^2\sqrt{d^2}}+\frac{8e^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^3\left(x+\frac{d}{e}\right)}$	1
default	Expression too large to display	1

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-e^2*x^2+d^2)^(1/2)*(-256*e^3*x^3+93*d*e^2*x^2-32*d^2*e*x+6*d^3)/d^3
/x^4-95/8/d^2*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)+8/d^3*e^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \frac{192\,e^5x^5+192\,de^4x^4+285\,(e^5x^5+de^4x^4)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(448\,e^4x^4+163\,de^3}{24\,(d^3ex^5+d^4x^4)}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(192*e^5*x^5 + 192*d*e^4*x^4 + 285*(e^5*x^5 + d*e^4*x^4)*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 163*d*e^3*x^3 - 61*d^2*e^2*x^2 + 26*
d^3*e*x - 6*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e*x^5 + d^4*x^4)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x^5(d+ex)^4}\,dx$$

[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^5}\,dx$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^5), x)

Giac [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 392 vs. $2(149) = 298$.

Time = 0.33 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.31

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \frac{\left(3e^5 - \frac{29\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3}{x} + \frac{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 e}{x^2} - \frac{864\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{ex^3} - \frac{4128\left(de+\right.}{}\right.}{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$
$$-\frac{95e^5\log\left(\frac{\left|-2\,de-2\sqrt{-e^2x^2+d^2}|e|\right|}{2e^2|x|}\right)}{8d^3|e|}$$
$$+\frac{\frac{1056\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^9e^5|e|}{x} - \frac{192\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2 d^9e^3|e|}{x^2} + \frac{32\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3 d^9e|e|}{x^3} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4 d^9|e|}{ex^4}}{192d^{12}e^4}$$

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")

[Out] 1/192*(3*e^5 - 29*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3/x + 160*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2*e/x^2 - 864*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^3/(e*x^3) - 4128*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^3*x^4))*e^8*x^4
/((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*((d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))/(e^2*x) + 1)*abs(e)) - 95/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2

```
+ d^2)*abs(e))/(e^2*abs(x)))/(d^3*abs(e)) + 1/192*(1056*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*d^9*e^5*abs(e)/x - 192*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^2*d^9*e^3*abs(e)/x^2 + 32*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^9*e*abs(
e)/x^3 - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^9*abs(e)/(e*x^4))/(d^12*
e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^5\,{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 196

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{27e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

```
[Out] 27/2*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-8*e^5*(-e*x+d)/d^4/(-e^2*x^2+d
^2)^(1/2)-1/5*(-e^2*x^2+d^2)^(1/2)/x^5+e*(-e^2*x^2+d^2)^(1/2)/d/x^4-13/5*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3+11/2*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-66/5*e
^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \frac{27e^5\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
```

```
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^6 (d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4+\frac{8e^5x^5}{d}}{x^6\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{\int \frac{-20d^5e+39d^4e^2x-40d^3e^3x^2+40d^2e^4x^3-40de^5x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^4}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{\int \frac{-156d^6e^2+220d^5e^3x-160d^4e^4x^2+160d^3e^5x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^6}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{\int \frac{-660d^7e^3+792d^6e^4x-480d^5e^5x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^8}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{\int \frac{-1584d^8e^4+1620d^7e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{120d^{10}}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{(27e^5)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^3}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{(27e^5)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^3}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{(27e^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^3}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = -\frac{\frac{d\sqrt{d^2-e^2x^2}(2d^5-8d^4ex+16d^3e^2x^2-29d^2e^3x^3+77de^4x^4+212e^5x^5)}{x^5(d+ex)} - 135\sqrt{d^2}e^5\log(x) + 135\sqrt{d^2}e^5\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{10d^5}$$

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] -1/10*((d*Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2*
e^3*x^3 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) - 135*Sqrt[d^2]*e^5*
Log[x] + 135*Sqrt[d^2]*e^5*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^5
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(132e^4x^4-55d\,e^3x^3+26d^2e^2x^2-10d^3ex+2d^4)}{10x^5d^4} + \frac{27e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^3\sqrt{d^2}} - \frac{8e^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^4\left(x+\frac{d}{e}\right)}$
default	Expression too large to display

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(-e^2*x^2+d^2)^(1/2)*(132*e^4*x^4-55*d*e^3*x^3+26*d^2*e^2*x^2-10*d^3*
e*x+2*d^4)/x^5/d^4+27/2/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)-8/d^4*e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^6(d+ex)^4}\, dx =$$
$$-\frac{80\, e^6x^6 + 80\, de^5x^5 + 135\,(e^6x^6 + de^5x^5)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + (212\, e^5x^5 + 77\, de^4x^4 - 29\, d^2e^3x^3 + 16\, d^3e^2x}{10\,(d^4ex^6 + d^5x^5)}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/10*(80*e^6*x^6 + 80*d*e^5*x^5 + 135*(e^6*x^6 + d*e^5*x^5)*log(-(d - sqrt
(-e^2*x^2 + d^2))/x) + (212*e^5*x^5 + 77*d*e^4*x^4 - 29*d^2*e^3*x^3 + 16*d^
3*e^2*x^2 - 8*d^4*e*x + 2*d^5)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^6 + d^5*x^5)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^6(d+ex)^4}\, dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^6\,(d+ex)^4}\, dx$$

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^6(d+ex)^4}\, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex+d)^4x^6}\, dx$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(173) = 346$.

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.29

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \frac{\left(e^6 - \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^4}{x} + \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2e^2}{x^2} - \frac{185\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{x^3} + \frac{870\left(de+\sqrt{\cdots}\right.}{\cdots}\cdots\right.}{160\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^5 d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+\cdots\right.}$$

$$+ \frac{27\,e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-e^2x^2+d^2}|e|\right|}{2\,e^2|x|}\right)}{2\,d^4|e|}$$

$$- \frac{\frac{1110\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^{16}e^8}{x} - \frac{240\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^{16}e^6}{x^2} + \frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^{16}e^4}{x^3} - \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^{16}e^2}{x^4}\cdots}{160\,d^{20}e^4|e|}$$

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/160*(e^6 - 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^4/x + 45*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^2*e^2/x^2 - 185*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/x^3 + 870*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^2*x^4) + 3670*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^5/(e^4*x^5))*e^10*x^5/((d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^5*d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
 + 27/2*e^6*log(1/2*abs(-2*d*e - 2*sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*abs(x)
))/(d^4*abs(e)) - 1/160*(1110*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^16*e^8/
x - 240*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^16*e^6/x^2 + 55*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^3*d^16*e^4/x^3 - 10*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^4*d^16*e^2/x^4 + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d^16/x^5)/(d^2
0*e^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^{5/2}}{x^6\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)} + \frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4} - \frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3} - \frac{\arcsin(ax)}{a^3}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^4-3/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
-arcsin(a*x)/a^3+2*(-a^2*x^2+1)^(1/2)/a^3/(-a*x+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1651, 673, 665, 677, 222}

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = -\frac{\arcsin(ax)}{a^3} - \frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3} + \frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4} + \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^2}\right)\,dx\\
&= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}-\frac{2(1-a^2x^2)^{3/2}}{3a^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}-\frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \frac{\left(-8+19ax-13a^2x^2\right)\sqrt{1-a^2x^2}}{5a^3(-1+ax)^3} - \frac{2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a^3}$$

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] ((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(5*a^3*(-1 + a*x)^3) - (2*Ar
cTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2])])/a^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(85) = 170.

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

method	result
default	$\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a\left(x-\frac{1}{a}\right)^2}+a\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}\right)}{a^4}+\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{1}{a}\right)^4}-}{a^6}$

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+a*((-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(
x-1/a))^(1/2))))+1/a^6*(1/5/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-
1/15/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2))+2/3/a^6/(x-1/a)^3*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$$
$$= \frac{8\,a^3x^3-24\,a^2x^2+24\,ax+10\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13\,a^2x^2-19\,ax+8\right)\sqrt{-}}{5\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(ax-1)^4}\,dx$$

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \frac{\sqrt{-a^2x^2+1}x^2}{(ax-1)^4}\,dx$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.32

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}-\frac{13\,\sqrt{1-a^2\,x^2}}{5\,\left(a\,\sqrt{-a^2}-a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,x^2-2\,a^4\,x+a^3\right)}$$

```
[In] int((x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^4,x)
```

```
[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5 - 2*a^6*x + a^7*x^2)) - asinh(x*(-a^2)
^(1/2))/(a^2*(-a^2)^(1/2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*(-a
^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2) - a^4*x^3*(-a^2)^
(1/2))) - (13*(1 - a^2*x^2)^(1/2))/(5*(a*(-a^2)^(1/2) - a^2*x*(-a^2)^(1/2))
*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2))
```

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5} - \frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4} + \frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^5-12/35*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)
^4+23/105*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1653, 807, 673, 665}

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3} - \frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4} + \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}$$

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1-a^2x^2)^{3/2}}{a^3(1-ax)^4}+\frac{\int \frac{(4a^2-3a^3x)\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{a^4}\\
&= \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}-\frac{(1-a^2x^2)^{3/2}}{a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{7a^2}\\
&= \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}-\frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{35a^2}\\
&= \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}-\frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4}+\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{\sqrt{1-a^2x^2}(2-8ax+13a^2x^2+23a^3x^3)}{105a^3(-1+ax)^4}$$

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{\sqrt{-a^2x^2+1}\,(23a^2x^2-10ax+2)(ax+1)}{105(ax-1)^4a^3}$
trager	$\frac{(23a^3x^3+13a^2x^2-8ax+2)\sqrt{-a^2x^2+1}}{105(ax-1)^4a^3}$
default	$-\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{7a\left(x-\frac{1}{a}\right)^5}-\frac{2a\left(\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{1}{a}\right)^4}-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{1}{a}\right)^3}\right)}{7}}{a^7}-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a^6\left(x-\frac{1}{a}\right)^3}-$

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+(23\,a^3x^3+13\,a^2x^2-8\,ax+2)\sqrt{-a^2x^2+1}+2}{105\,(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3)}$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4
*a^4*x + a^3)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = -\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx$$

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
[Out] -Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3
 - 10*a**2*x**2 + 5*a*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\sqrt{-a^2x^2+1}}{7\,(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3)} + \frac{29\sqrt{-a^2x^2+1}}{35\,(a^6x^3-3\,a^5x^2+3\,a^4x-a^3)} + \frac{82\sqrt{-a^2x^2+1}}{105\,(a^5x^2-2\,a^4x+a^3)} + \frac{23\sqrt{-a^2x^2+1}}{105\,(a^4x-a^3)}$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*sqr
t(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*
x - a^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.26

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx = \frac{2\sqrt{1-a^2x^2}}{7\left(a^7x^4-4a^6x^3+6a^5x^2-4a^4x+a^3\right)} + \frac{4\sqrt{1-a^2x^2}}{3\left(a^5x^2-2a^4x+a^3\right)} + \frac{4a\sqrt{1-a^2x^2}}{35\left(a^6x^2-2a^5x+a^4\right)} + \frac{29\sqrt{1-a^2x^2}}{35\sqrt{-a^2}\left(a\sqrt{-a^2}-3a^2x\sqrt{-a^2}+3a^3x^2\sqrt{-a^2}-a^4x^3\sqrt{-a^2}\right)} + \frac{23\sqrt{1-a^2x^2}}{105\left(a\sqrt{-a^2}-a^2x\sqrt{-a^2}\right)\sqrt{-a^2}} - \frac{2a^2\sqrt{1-a^2x^2}}{3\left(a^7x^2-2a^6x+a^5\right)}$$

```
[In] int(-(x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^5,x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^3 - 4*a^4*x + 6*a^5*x^2 - 4*a^6*x^3 + a^7*x^4
)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2)) + (4*a*(1 - a^2*
x^2)^(1/2))/(35*(a^4 - 2*a^5*x + a^6*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(35*(
-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2)
 - a^4*x^3*(-a^2)^(1/2))) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*(-a^2)^(1/2) -
 a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^5 -
2*a^6*x + a^7*x^2))
```

3.212 $\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\begin{aligned}\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = & -\frac{24x}{5005d^3e^3 (d^2-e^2x^2)^{5/2}} \\ & + \frac{d^2}{13e^4(d+ex)^4 (d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 (d^2-e^2x^2)^{5/2}} \\ & + \frac{21}{143e^4(d+ex)^2 (d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex) (d^2-e^2x^2)^{5/2}} \\ & - \frac{32x}{5005d^5e^3 (d^2-e^2x^2)^{3/2}} - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

```
[Out] -24/5005*x/d^3/e^3/(-e^2*x^2+d^2)^(5/2)+1/13*d^2/e^4/(e*x+d)^4/(-e^2*x^2+d^
2)^(5/2)-30/143*d/e^4/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)+21/143/e^4/(e*x+d)^2/(
-e^2*x^2+d^2)^(5/2)+4/1001/d/e^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2)-32/5005*x/d^5
/e^3/(-e^2*x^2+d^2)^(3/2)-64/5005*x/d^7/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

= {1653, 807, 673, 198, 197}

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2}{13e^4(d+ex)^4 (d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 (d^2-e^2x^2)^{5/2}} + \frac{21}{143e^4(d+ex)^2 (d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex) (d^2-e^2x^2)^{5/2}} - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}} - \frac{32x}{5005d^5e^3 (d^2-e^2x^2)^{3/2}} - \frac{24x}{5005d^3e^3 (d^2-e^2x^2)^{5/2}}$$

```
[In] Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
```

```
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{2d^3e^2-3d^2e^3x-12de^4x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx}{7e^5}\\
&=-\frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{-20d^3e^6+36d^2e^7x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx}{56e^9}\\
&=\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\left(9d^2\right)\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx}{182e^3}\\
&=\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad+\frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{(36d)\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx}{1001e^3}\\
&=\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4\int \frac{1}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx}{143e^3}\\
&=\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad+\frac{4}{1001de^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{24\int \frac{1}{(d^2-e^2x^2)^{7/2}}\,dx}{1001de^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad +\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{96\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5005d^3e^3} \\
&= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad -\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad +\frac{4}{1001de^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{32x}{5005d^5e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{64\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{5005d^5e^3} \\
&= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad -\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad +\frac{4}{1001de^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{32x}{5005d^5e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$
\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d}{5005d^7e^4(d-ex)^3(d+e}
$$

```
[In] Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result	s
gosper	$\frac{(-ex+d)\left(-64e^9x^9-256d\,e^8x^8-224d^2e^7x^7+384d^3e^6x^6+776d^4e^5x^5+160d^5e^4x^4-540d^6e^3x^3+315x^2d^7e^2+360x\,d^8e+90d^9\right)}{5005(ex+d)^3d^7e^4\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$	1
trager	$\frac{\left(-64e^9x^9-256d\,e^8x^8-224d^2e^7x^7+384d^3e^6x^6+776d^4e^5x^5+160d^5e^4x^4-540d^6e^3x^3+315x^2d^7e^2+360x\,d^8e+90d^9\right)\sqrt{-e^2x^2+d^2}}{5005d^7(ex+d)^7e^4(-ex+d)^3}$	1
default	Expression too large to display	1

```
[In] int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5005*(-e*x+d)*(-64*e^9*x^9-256*d*e^8*x^8-224*d^2*e^7*x^7+384*d^3*e^6*x^6+
776*d^4*e^5*x^5+160*d^5*e^4*x^4-540*d^6*e^3*x^3+315*d^7*e^2*x^2+360*d^8*e*x
+90*d^9)/(e*x+d)^3/d^7/e^4/(-e^2*x^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = \frac{90 e^{10}x^{10} + 360 de^9x^9 + 270 d^2e^8x^8 - 720 d^3e^7x^7 - 1260 d^4e^6x^6 + 1260 d^6e^4x}{5005 (}$$

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5005*(90*e^10*x^10 + 360*d*e^9*x^9 + 270*d^2*e^8*x^8 - 720*d^3*e^7*x^7 -
1260*d^4*e^6*x^6 + 1260*d^6*e^4*x^4 + 720*d^7*e^3*x^3 - 270*d^8*e^2*x^2 - 3
60*d^9*e*x - 90*d^10 + (64*e^9*x^9 + 256*d*e^8*x^8 + 224*d^2*e^7*x^7 - 384*
d^3*e^6*x^6 - 776*d^4*e^5*x^5 - 160*d^5*e^4*x^4 + 540*d^6*e^3*x^3 - 315*d^7
*e^2*x^2 - 360*d^8*e*x - 90*d^9)*sqrt(-e^2*x^2 + d^2))/(d^7*e^14*x^10 + 4*d
^8*e^13*x^9 + 3*d^9*e^12*x^8 - 8*d^10*e^11*x^7 - 14*d^11*e^10*x^6 + 14*d^13
*e^8*x^4 + 8*d^14*e^7*x^3 - 3*d^15*e^6*x^2 - 4*d^16*e^5*x - d^17*e^4)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = \int \frac{x^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}} (d+ex)^4} \, dx$$

```
[In] integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(181) = 362$.

Time = 0.20 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}e^8x^4+4(-e^2x^2+d^2)^{\frac{5}{2}}de^7x^3+6(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^6x^2+\cdots\right)} - \frac{30\,d}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}e^7x^3+3(-e^2x^2+d^2)^{\frac{5}{2}}de^6x^2+3(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^5x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^4\right)} + \frac{21}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}e^6x^2+2(-e^2x^2+d^2)^{\frac{5}{2}}de^5x+(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4\right)} + \frac{4}{1001\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x+(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4\right)} - \frac{24\,x}{5005\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3} - \frac{32\,x}{5005\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5e^3} - \frac{64\,x}{5005\sqrt{-e^2x^2+d^2}\,d^7e^3}$$

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*d^2/((-e^2*x^2 + d^2)^(5/2)*e^8*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^7*x
^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^6*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^
5*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^4) - 30/143*d/((-e^2*x^2 + d^2)^(5/2)*e^
7*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e
^5*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^4) + 21/143/((-e^2*x^2 + d^2)^(5/2)*e^6
*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) +
 4/1001/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) -
 24/5005*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 32/5005*x/((-e^2*x^2 + d^2)^(
3/2)*d^5*e^3) - 64/5005*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^3)
```

Giac [F]

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^3}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{107}{4004\,d^2\,e^4}-\frac{1139\,x}{80080\,d^3\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{23}{32032\,d^4\,e^4}+\frac{32\,x}{5005\,d^5\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}}{104\,d\,e^4\,{\left(d+e\,x\right)}^7} - \frac{27\,\sqrt{d^2-e^2\,x^2}}{2288\,d^2\,e^4\,{\left(d+e\,x\right)}^6} - \frac{15\,\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^4\,{\left(d+e\,x\right)}^5} + \frac{23\,\sqrt{d^2-e^2\,x^2}}{32032\,d^4\,e^4\,{\left(d+e\,x\right)}^4} - \frac{64\,x\,\sqrt{d^2-e^2\,x^2}}{5005\,d^7\,e^3\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(107/(4004*d^2*e^4) - (1139*x)/(80080*d^3*e^3)))/((d
 + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(23/(32032*d^4*e^4) + (32*x
)/(5005*d^5*e^3)))/((d + e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(104*d
*e^4*(d + e*x)^7) - (27*(d^2 - e^2*x^2)^(1/2))/(2288*d^2*e^4*(d + e*x)^6) -
 (15*(d^2 - e^2*x^2)^(1/2))/(2288*d^3*e^4*(d + e*x)^5) + (23*(d^2 - e^2*x^2
)^(1/2))/(32032*d^4*e^4*(d + e*x)^4) - (64*x*(d^2 - e^2*x^2)^(1/2))/(5005*d
^7*e^3*(d + e*x)*(d - e*x))
```

3.213 $\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\begin{aligned}\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx &= \frac{14x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} \\ &- \frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ &- \frac{7}{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{7}{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}} \\ &+ \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}}\end{aligned}$$

```
[Out] 14/2145*x/d^4/e^2/(-e^2*x^2+d^2)^(5/2)-1/13*d/e^3/(e*x+d)^4/(-e^2*x^2+d^2)^
(5/2)+17/143/e^3/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-7/1287/d/e^3/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-7/1287/d^2/e^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+56/6435*x/d^6/
e^2/(-e^2*x^2+d^2)^(3/2)+112/6435*x/d^8/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

= {1653, 807, 673, 198, 197}

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = -\frac{d}{13e^3(d+ex)^4 (d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3 (d^2-e^2x^2)^{5/2}} - \frac{7}{1287de^3(d+ex)^2 (d^2-e^2x^2)^{5/2}} - \frac{7}{1287d^2e^3(d+ex) (d^2-e^2x^2)^{5/2}} + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}} + \frac{56x}{6435d^6e^2 (d^2-e^2x^2)^{3/2}} + \frac{14x}{2145d^4e^2 (d^2-e^2x^2)^{5/2}}$$

```
[In] Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
```

```
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{3d^2e^2-5de^3x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{8e^4}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{(7d)\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{104e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{49\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx}{1287de^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287d^2e^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{14\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{429d^2e^2}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287d^2e^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{56\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{2145d^4e^2}$$

$$= \frac{14x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} - \frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{7}{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{7}{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{112\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{6435d^6e^2}$$

$$= \frac{14x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} - \frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{7}{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{7}{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358}{6435d^8e^3(d-ex)^3(d+}$$

```
[In] Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result	s
gosper	$\frac{(-ex+d)(112e^9x^9+448d\,e^8x^8+392d^2e^7x^7-672d^3e^6x^6-1358d^4e^5x^5-280d^5e^4x^4+945d^6e^3x^3+700x^2d^7e^2+800x\,d^8e+200d^9)}{6435(ex+d)^3d^8e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$	1
trager	$\frac{(112e^9x^9+448d\,e^8x^8+392d^2e^7x^7-672d^3e^6x^6-1358d^4e^5x^5-280d^5e^4x^4+945d^6e^3x^3+700x^2d^7e^2+800x\,d^8e+200d^9)\sqrt{-e^2x^2+d^2}}{6435d^8(ex+d)^7(-ex+d)^3e^3}$	1
default	Expression too large to display	9

```
[In] int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

[Out] 1/6435*(-e*x+d)*(112*e^9*x^9+448*d*e^8*x^8+392*d^2*e^7*x^7-672*d^3*e^6*x^6-1358*d^4*e^5*x^5-280*d^5*e^4*x^4+945*d^6*e^3*x^3+700*d^7*e^2*x^2+800*d^8*e*x+200*d^9)/(e*x+d)^3/d^8/e^3/(-e^2*x^2+d^2)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{200\,e^{10}x^{10} + 800\,de^9x^9 + 600\,d^2e^8x^8 - 1600\,d^3e^7x^7 - 2800\,d^4e^6x^6 + 2800\,d^6 \cdots}{6 \cdots}$$

[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/6435*(200*e^10*x^10 + 800*d*e^9*x^9 + 600*d^2*e^8*x^8 - 1600*d^3*e^7*x^7 - 2800*d^4*e^6*x^6 + 2800*d^6*e^4*x^4 + 1600*d^7*e^3*x^3 - 600*d^8*e^2*x^2 - 800*d^9*e*x - 200*d^10 - (112*e^9*x^9 + 448*d*e^8*x^8 + 392*d^2*e^7*x^7 - 672*d^3*e^6*x^6 - 1358*d^4*e^5*x^5 - 280*d^5*e^4*x^4 + 945*d^6*e^3*x^3 + 700*d^7*e^2*x^2 + 800*d^8*e*x + 200*d^9)*sqrt(-e^2*x^2 + d^2))/(d^8*e^13*x^10 + 4*d^9*e^12*x^9 + 3*d^10*e^11*x^8 - 8*d^11*e^10*x^7 - 14*d^12*e^9*x^6 + 14*d^14*e^7*x^4 + 8*d^15*e^6*x^3 - 3*d^16*e^5*x^2 - 4*d^17*e^4*x - d^18*e^3)

Sympy [F]

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

[In] integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(181) = 362.

Time = 0.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.92

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{d}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}e^7x^4+4(-e^2x^2+d^2)^{\frac{5}{2}}de^6x^3+6(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^5x^2+4(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^4x+(-e^2x\right.}$$

$$+\frac{17}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}e^6x^3+3(-e^2x^2+d^2)^{\frac{5}{2}}de^5x^2+3(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3\right)}$$

$$-\frac{7}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x^2+2(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3\right)}$$

$$-\frac{7}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3\right)}+\frac{14\,x}{2145\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2}$$

$$+\frac{56\,x}{6435\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6e^2}+\frac{112\,x}{6435\,\sqrt{-e^2x^2+d^2}d^8e^2}$$

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13*d/((-e^2*x^2 + d^2)^(5/2)*e^7*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^
3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^5*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^4
*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^3) + 17/143/((-e^2*x^2 + d^2)^(5/2)*e^6*x
^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 7/1287/((-e^2*x^2 + d^2)^(5/2)*d*e^5*
x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3)
- 7/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3
) + 14/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2) + 56/6435*x/((-e^2*x^2 + d^2
)^(3/2)*d^6*e^2) + 112/6435*x/(sqrt(-e^2*x^2 + d^2)*d^8*e^2)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x^2}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(d+ex)^4\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{227}{6864\,d^3\,e^3}-\frac{353\,x}{17160\,d^4\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{353}{41184\,d^5\,e^3}-\frac{56\,x}{6435\,d^6\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^2\,e^3\,{\left(d+e\,x\right)}^7} + \frac{\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^3\,{\left(d+e\,x\right)}^6} + \frac{37\,\sqrt{d^2-e^2\,x^2}}{5148\,d^4\,e^3\,{\left(d+e\,x\right)}^5} + \frac{353\,\sqrt{d^2-e^2\,x^2}}{41184\,d^5\,e^3\,{\left(d+e\,x\right)}^4} + \frac{112\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^8\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(227/(6864*d^3*e^3) - (353*x)/(17160*d^4*e^2)))/((d
+ e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(353/(41184*d^5*e^3) - (56*x
)/(6435*d^6*e^2)))/((d + e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(104*d
^2*e^3*(d + e*x)^7) + (d^2 - e^2*x^2)^(1/2)/(2288*d^3*e^3*(d + e*x)^6) + (3
7*(d^2 - e^2*x^2)^(1/2))/(5148*d^4*e^3*(d + e*x)^5) + (353*(d^2 - e^2*x^2)^
(1/2))/(41184*d^5*e^3*(d + e*x)^4) + (112*x*(d^2 - e^2*x^2)^(1/2))/(6435*d^
8*e^2*(d + e*x)*(d - e*x))
```

3.214 $\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}$$

```
[Out] 64/2145*x/d^5/e/(-e^2*x^2+d^2)^(5/2)+1/13/e^2/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2
)-4/143/d/e^2/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-32/1287/d^2/e^2/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-32/1287/d^3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+256/6435*x/d^
7/e/(-e^2*x^2+d^2)^(3/2)+512/6435*x/d^9/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {807, 673, 198, 197}

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx}{13e}\\ &= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{32\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx}{143de}\end{aligned}$$

$$= \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{224\int \frac{1}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx}{1287d^2e}$$

$$= \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{64\int \frac{1}{(d^2-e^2x^2)^{7/2}}\,dx}{429d^3e}$$

$$= \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} + \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{256\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{2145d^5e}$$

$$= \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} + \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{256x}{6435d^7e(d^2-e^2x^2)^{3/2}} + \frac{512\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{6435d^7e}$$

$$= \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} + \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{256x}{6435d^7e(d^2-e^2x^2)^{3/2}} + \frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-62}{6435d^9e^2(d-ex)^3(d}$$

```
[In] Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{(-ex+d)\left(-512e^{9}x^{9}-2048d\,e^{8}x^{8}-1792d^{2}e^{7}x^{7}+3072d^{3}e^{6}x^{6}+6208d^{4}e^{5}x^{5}+1280d^{5}e^{4}x^{4}-4320d^{6}e^{3}x^{3}-3200x^{2}d^{7}e^{2}+20x\,d^{8}e+5d^{9}\right)}{6435(ex+d)^{3}d^{9}e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-512e^{9}x^{9}-2048d\,e^{8}x^{8}-1792d^{2}e^{7}x^{7}+3072d^{3}e^{6}x^{6}+6208d^{4}e^{5}x^{5}+1280d^{5}e^{4}x^{4}-4320d^{6}e^{3}x^{3}-3200x^{2}d^{7}e^{2}+20x\,d^{8}e+5d^{9}\right)\sqrt{-e^{2}x^{2}+}}{6435d^{9}(ex+d)^{7}(-ex+d)^{3}e^{2}}$

```
[In] int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6435*(-e*x+d)*(-512*e^9*x^9-2048*d*e^8*x^8-1792*d^2*e^7*x^7+3072*d^3*e^6
*x^6+6208*d^4*e^5*x^5+1280*d^5*e^4*x^4-4320*d^6*e^3*x^3-3200*d^7*e^2*x^2+20
*d^8*e*x+5*d^9)/(e*x+d)^3/d^9/e^2/(-e^2*x^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.50

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{5\,e^{10}x^{10}+20\,de^9x^9+15\,d^2e^8x^8-40\,d^3e^7x^7-70\,d^4e^6x^6+70\,d^6e^4x^4+40\,d^7e^3x^3-15\,d^8e^2x^2-20\,d^9ex-}{6435\,(d^9e^{12}x^{10}+4\,d^{10}e^{11}x^9+3\,d^{11}e^{10}x}$$

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6435*(5*e^10*x^10 + 20*d*e^9*x^9 + 15*d^2*e^8*x^8 - 40*d^3*e^7*x^7 - 70*
d^4*e^6*x^6 + 70*d^6*e^4*x^4 + 40*d^7*e^3*x^3 - 15*d^8*e^2*x^2 - 20*d^9*e*x
 - 5*d^10 + (512*e^9*x^9 + 2048*d*e^8*x^8 + 1792*d^2*e^7*x^7 - 3072*d^3*e^6
*x^6 - 6208*d^4*e^5*x^5 - 1280*d^5*e^4*x^4 + 4320*d^6*e^3*x^3 + 3200*d^7*e^
2*x^2 - 20*d^8*e*x - 5*d^9)*sqrt(-e^2*x^2 + d^2))/(d^9*e^12*x^10 + 4*d^10*e
^11*x^9 + 3*d^11*e^10*x^8 - 8*d^12*e^9*x^7 - 14*d^13*e^8*x^6 + 14*d^15*e^6*
x^4 + 8*d^16*e^5*x^3 - 3*d^17*e^4*x^2 - 4*d^18*e^3*x - d^19*e^2)
```

Sympy [F]

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)^4}\,dx$$

```
[In] integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(183) = 366.

Time = 0.22 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.92

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}e^6x^4+4\,(-e^2x^2+d^2)^{\frac{5}{2}}de^5x^3+6\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2+\right.}$$
$$-\frac{4}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x^3+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)}$$
$$-\frac{32}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^2+2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)}$$
$$-\frac{32}{1287\left((-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2\right)}+\frac{64\,x}{2145\,(-e^2x^2+d^2)^{\frac{5}{2}}d^5e}$$
$$+\frac{256\,x}{6435\,(-e^2x^2+d^2)^{\frac{3}{2}}d^7e}+\frac{512\,x}{6435\,\sqrt{-e^2x^2+d^2}d^9e}$$

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13/((-e^2*x^2 + d^2)^(5/2)*e^6*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3 +
 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x
+ (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 4/143/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3
 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*
x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e
^4*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^
2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4
*e^2) + 64/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^5*e) + 256/6435*x/((-e^2*x^2 +
d^2)^(3/2)*d^7*e) + 512/6435*x/(sqrt(-e^2*x^2 + d^2)*d^9*e)
```

Giac [F]

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{x}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

$$\int \frac{x}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{41}{41184\,d^6\,e^2}+\frac{256\,x}{6435\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{47}{1716\,d^4\,e^2}-\frac{1369\,x}{34320\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} + \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^3\,e^2\,{\left(d+e\,x\right)}^7} + \frac{25\,\sqrt{d^2-e^2\,x^2}}{2288\,d^4\,e^2\,{\left(d+e\,x\right)}^6} + \frac{125\,\sqrt{d^2-e^2\,x^2}}{20592\,d^5\,e^2\,{\left(d+e\,x\right)}^5} - \frac{41\,\sqrt{d^2-e^2\,x^2}}{41184\,d^6\,e^2\,{\left(d+e\,x\right)}^4} + \frac{512\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^9\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

```
[In] int(x/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(41/(41184*d^6*e^2) + (256*x)/(6435*d^7*e)))/((d + e
*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(47/(1716*d^4*e^2) - (1369*x)/(
34320*d^5*e)))/((d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(104*d^3*e
^2*(d + e*x)^7) + (25*(d^2 - e^2*x^2)^(1/2))/(2288*d^4*e^2*(d + e*x)^6) + (
125*(d^2 - e^2*x^2)^(1/2))/(20592*d^5*e^2*(d + e*x)^5) - (41*(d^2 - e^2*x^2
)^(1/2))/(41184*d^6*e^2*(d + e*x)^4) + (512*x*(d^2 - e^2*x^2)^(1/2))/(6435*
d^9*e*(d + e*x)*(d - e*x))
```

3.215 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}$$

```
[Out] 48/715*x/d^6/(-e^2*x^2+d^2)^(5/2)-1/13/d/e/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2)-9
/143/d^2/e/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-8/143/d^3/e/(e*x+d)^2/(-e^2*x^2+d
^2)^(5/2)-8/143/d^4/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+64/715*x/d^8/(-e^2*x^2+d
^2)^(3/2)+128/715*x/d^10/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {673, 198, 197}

$$\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{9\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13d}\\
&= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{72\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143d^2}\\
&= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad -\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{56\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143d^3}\\
&= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad -\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\\
&\quad -\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{48\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143d^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{192\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{715d^6} \\
&= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}} + \frac{128\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{715d^8} \\
&= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} \\
&\quad - \frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}} + \frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

$$
\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552}{715d^{10}e(d-ex)^3(d+e}
$$

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(-ex+d)\left(-128e^9x^9-512d\,e^8x^8-448d^2e^7x^7+768d^3e^6x^6+1552d^4e^5x^5+320d^5e^4x^4-1080d^6e^3x^3-800x^2d^7e^2+5x\,d^8e+180d^9\right)}{715(ex+d)^3d^{10}e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-128e^9x^9-512d\,e^8x^8-448d^2e^7x^7+768d^3e^6x^6+1552d^4e^5x^5+320d^5e^4x^4-1080d^6e^3x^3-800x^2d^7e^2+5x\,d^8e+180d^9\right)\sqrt{-e^2x^2+d^2}}{715d^{10}(ex+d)^7(-ex+d)^3e}$
default	$-\frac{1}{13de\left(x+\frac{d}{e}\right)^4\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{9e\left(-\frac{1}{11de\left(x+\frac{d}{e}\right)^3\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{8e\left(-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right.}\right.}{}\right)}{}$

[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^6
+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*e*

```
x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = -\frac{180\,e^{10}x^{10}+720\,de^9x^9+540\,d^2e^8x^8-1440\,d^3e^7x^7-2520\,d^4e^6x^6+2520\,d^6e^4x^4+1440\,d^7e^3x^3-540\,d^8e^2x^2}{715\,(d^{10}e^{11}x^{10}+4\,d^{11}e^{10}x^9+3\,d}$$

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
- 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^2
- 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^7 -
 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^3 +
800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^11*x^1
0 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x^6 + 1
4*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^20*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)^4}\,dx$$

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(177) = 354.

Time = 0.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.92

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = -\frac{1}{13\left((-e^2x^2+d^2)^{\frac{5}{2}}de^5x^4+4(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^3+6(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+4(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)} - \frac{9}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^4x^3+3(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+3(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)} - \frac{8}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^3e^3x^2+2(-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)} - \frac{8}{143\left((-e^2x^2+d^2)^{\frac{5}{2}}d^4e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^5e\right)} + \frac{48\,x}{715\,(-e^2x^2+d^2)^{\frac{5}{2}}d^6} + \frac{64\,x}{715\,(-e^2x^2+d^2)^{\frac{3}{2}}d^8} + \frac{128\,x}{715\,\sqrt{-e^2x^2+d^2}\,d^{10}}$$

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^4*e
^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 9/143/((-e^2*x^2 + d^2)^(5/2)*d^2*e^
4*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^4
*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^3*
e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e
) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e)
 + 48/715*x/((-e^2*x^2 + d^2)^(5/2)*d^6) + 64/715*x/((-e^2*x^2 + d^2)^(3/2)
*d^8) + 128/715*x/(sqrt(-e^2*x^2 + d^2)*d^10)
```

Giac [F]

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4}\,dx$$

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4), x)
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{715\,d^8}+\frac{189}{4576\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1139\,x}{5720\,d^6}-\frac{427}{2288\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{104\,d^4\,e\,{\left(d+e\,x\right)}^7} - \frac{51\,\sqrt{d^2-e^2\,x^2}}{2288\,d^5\,e\,{\left(d+e\,x\right)}^6} - \frac{19\,\sqrt{d^2-e^2\,x^2}}{572\,d^6\,e\,{\left(d+e\,x\right)}^5} - \frac{189\,\sqrt{d^2-e^2\,x^2}}{4576\,d^7\,e\,{\left(d+e\,x\right)}^4} + \frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{715\,d^{10}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

```
[In] int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*((64*x)/(715*d^8) + 189/(4576*d^7*e)))/((d + e*x)^2*
(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((1139*x)/(5720*d^6) - 427/(2288*d^5*
e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(104*d^4*e*(d + e*x)
^7) - (51*(d^2 - e^2*x^2)^(1/2))/(2288*d^5*e*(d + e*x)^6) - (19*(d^2 - e^2*
x^2)^(1/2))/(572*d^6*e*(d + e*x)^5) - (189*(d^2 - e^2*x^2)^(1/2))/(4576*d^7
*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(715*d^10*(d + e*x)*(d - e*
x))
```

3.216 $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 234

$$\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}$$

```
[Out] 8/13*d*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/13*e*x/d/(-e^2*x^2+d^2)^(11/2)+1/11
7*(-40*e*x+13*d)/d^3/(-e^2*x^2+d^2)^(9/2)+1/819*(-320*e*x+117*d)/d^5/(-e^2*
x^2+d^2)^(7/2)+1/1365*(-640*e*x+273*d)/d^7/(-e^2*x^2+d^2)^(5/2)+1/819*(-512
*e*x+273*d)/d^9/(-e^2*x^2+d^2)^(3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^11+1
/819*(-1024*e*x+819*d)/d^11/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}}$$

```
[In] Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
```

```
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}\,dx \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+44d^3ex+13d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-440d^3ex}{x\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{\int \frac{1287d^6e^2-3520d^5e^3x}{x\left(d^2-e^2x^2\right)^{9/2}}\,dx}{1287d^8e^2} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} \\
&\quad + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} + \frac{\int \frac{9009d^8e^4-21120d^7e^5x}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx}{9009d^{12}e^4} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} \\
&\quad + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} + \frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{45045d^{10}e^6-84480d^9e^7x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{45045d^{16}e^6} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} \\
&\quad + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} + \frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}} + \frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad + \frac{\int \frac{135135d^{12}e^8-168960d^{11}e^9x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{135135d^{20}e^8}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} \\
&\quad + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} + \frac{\int \frac{135135d^{14}e^{10}}{x\sqrt{d^2-e^2x^2}}\,dx}{135135d^{24}e^{10}} \\
&= \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} \\
&\quad + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^{10}} \\
&= \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} \\
&\quad + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} \\
&\quad + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^{10}} \\
&= \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} \\
&\quad + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} \\
&\quad + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^{10}e^2} \\
&= \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} \\
&\quad + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

$$
\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x}{(d-ex)^3(d+ex)^7}}{4095d^{11}}
$$

```
[In] Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*
d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 1
540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x)^7
) + 8190*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(4095*d^11)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. 2(204) = 408.

Time = 0.44 (sec) , antiderivative size = 1328, normalized size of antiderivative = 5.68

method	result	size
default	Expression too large to display	1328

```
[In] int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d
^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x))))-1/e^3/d*(-1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+
7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/
10*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/
d^2*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/
2)-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
)))))-1/e/d^3*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*
e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(
-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*
(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1
/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))-
1/d^4*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10
*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^
2*(-1/6*(-2*(x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
-1/3/e^2/d^4*(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
-1/e^2/d^2*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e
/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*(x+d/e)*
e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*(
x+d/e)*e^2+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*
(-2*(x+d/e)*e^2+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(205) = 410.

Time = 0.76 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.85

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{9839\,e^{10}x^{10} + 39356\,de^9x^9 + 29517\,d^2e^8x^8 - 78712\,d^3e^7x^7 - 137746\,d^4e^6x}{}$$

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4095*(9839*e^10*x^10 + 39356*d*e^9*x^9 + 29517*d^2*e^8*x^8 - 78712*d^3*e^
7*x^7 - 137746*d^4*e^6*x^6 + 137746*d^6*e^4*x^4 + 78712*d^7*e^3*x^3 - 29517
*d^8*e^2*x^2 - 39356*d^9*e*x - 9839*d^10 + 4095*(e^10*x^10 + 4*d*e^9*x^9 +
3*d^2*e^8*x^8 - 8*d^3*e^7*x^7 - 14*d^4*e^6*x^6 + 14*d^6*e^4*x^4 + 8*d^7*e^3
*x^3 - 3*d^8*e^2*x^2 - 4*d^9*e*x - d^10)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 + (5120*e^9*x^9 + 16385*d*e^8*x^8 + 1540*d^2*e^7*x^7 - 45735*d^3*e^6*x^6 -
 40240*d^4*e^5*x^5 + 34156*d^5*e^4*x^4 + 56304*d^6*e^3*x^3 + 4466*d^7*e^2*x
^2 - 22976*d^8*e*x - 9839*d^9)*sqrt(-e^2*x^2 + d^2))/(d^11*e^10*x^10 + 4*d^
12*e^9*x^9 + 3*d^13*e^8*x^8 - 8*d^14*e^7*x^7 - 14*d^15*e^6*x^6 + 14*d^17*e^
4*x^4 + 8*d^18*e^3*x^3 - 3*d^19*e^2*x^2 - 4*d^20*e*x - d^21)
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

```
[In] integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^4x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{7/2}\left(d+e\,x\right)^4}\,dx$$

```
[In] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.217 $\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 271

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx=-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}+\frac{4e\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}$$

```
[Out] -8/13*e*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/143*e*(-24*e*x+13*d)/d^2/(-e^2*x^2
+d^2)^(11/2)-1/1287*e*(-1103*e*x+572*d)/d^4/(-e^2*x^2+d^2)^(9/2)-1/9009*e*(
-10111*e*x+5148*d)/d^6/(-e^2*x^2+d^2)^(7/2)-1/15015*e*(-23225*e*x+12012*d)/
d^8/(-e^2*x^2+d^2)^(5/2)-1/9009*e*(-21583*e*x+12012*d)/d^10/(-e^2*x^2+d^2)^
(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^12-1/9009*e*(-52175*e*x+36036*d
)/d^12/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^12/x
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1819, 821, 272, 65, 214}

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e\,\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}}$$

```
[In] Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+52d^3ex-83d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-572d^3ex+960d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{-1287d^4+5148d^3ex-8824d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{9/2}}\,dx}{1287d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} + \frac{\int \frac{9009d^4-36036d^3ex+60666d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{9009d^8}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-45045d^4+180180d^3ex-278700d^2e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{45045d^{10}}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{135135d^4-540540d^3ex+647490d^2e^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{135135d^{12}}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-135135d^4+540540d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{135135d^{14}}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} - \frac{(4e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^{11}}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} - \frac{(2e)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{d^{11}}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} + \frac{4\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^{11}e}$$

$$= -\frac{8e(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4e(13d-24ex)}{143d^2(d^2-e^2x^2)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4(d^2-e^2x^2)^{9/2}} - \frac{e(5148d-10111ex)}{9009d^6(d^2-e^2x^2)^{7/2}} - \frac{e(12012d-23225ex)}{15015d^8(d^2-e^2x^2)^{5/2}} - \frac{e(12012d-21583ex)}{9009d^{10}(d^2-e^2x^2)^{3/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} + \frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(45045d^{10}+546316d^9ex+1014094d^8e^2x^2-700504d^7e^3x^3-3}{}$$
$$+\frac{4\sqrt{d^2}e\log(x)}{d^{13}} - \frac{4\sqrt{d^2}e\log\left(\sqrt{d^2}-\sqrt{d^2-e^2x^2}\right)}{d^{13}}$$

```
[In] Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 - 7
00504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d^4
*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9 - 3
05920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) + (4*Sqrt[d^2]*e*
Log[x])/d^13 - (4*Sqrt[d^2]*e*Log[Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]])/d^13
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(239) = 478.

Time = 0.43 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^{12}x} - \frac{1257577\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{1921920d^{10}e^2\left(x+\frac{d}{e}\right)^3} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{104d^6e^6\left(x+\frac{d}{e}\right)^7} - \frac{103\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{2288d^7e^5\left(x+\frac{d}{e}\right)^6} - \frac{665}{}$
default	Expression too large to display

```
[In] int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^12/x-1257577/1921920/d^10/e^2/(x+d/e)^3*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)-1/104/d^6/e^6/(x+d/e)^7*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)-103/2288/d^7/e^5/(x+d/e)^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
-665/5148/d^8/e^4/(x+d/e)^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-86917/2882
88/d^9/e^3/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-17417683/11531520
```

```
/d^11/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+59/3840/d^11/e/(x-d/
e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-569/3840/d^12/(x-d/e)*(-(x-d/e)^2
*e^2-2*d*e*(x-d/e))^(1/2)-65075293/11531520/d^12/(x+d/e)*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)+4/d^11*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)-1/640/d^10/e^2/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx = -\frac{366136 e^{11}x^{11} + 1464544 de^{10}x^{10} + 1098408 d^2e^9x^9 - 2929088 d^3e^8x^8 - 5125904 d^4e^7x^7 + 5125904 d^6e^5x^5}{}$$

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45045*(366136*e^11*x^11 + 1464544*d*e^10*x^10 + 1098408*d^2*e^9*x^9 - 29
29088*d^3*e^8*x^8 - 5125904*d^4*e^7*x^7 + 5125904*d^6*e^5*x^5 + 2929088*d^7
*e^4*x^4 - 1098408*d^8*e^3*x^3 - 1464544*d^9*e^2*x^2 - 366136*d^10*e*x + 18
0180*(e^11*x^11 + 4*d*e^10*x^10 + 3*d^2*e^9*x^9 - 8*d^3*e^8*x^8 - 14*d^4*e^
7*x^7 + 14*d^6*e^5*x^5 + 8*d^7*e^4*x^4 - 3*d^8*e^3*x^3 - 4*d^9*e^2*x^2 - d^
10*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (305920*e^10*x^10 + 1043500*d*
e^9*x^9 + 350000*d^2*e^8*x^8 - 2496180*d^3*e^7*x^7 - 2748320*d^4*e^6*x^6 +
1301264*d^5*e^5*x^5 + 3157776*d^6*e^4*x^4 + 700504*d^7*e^3*x^3 - 1014094*d^
8*e^2*x^2 - 546316*d^9*e*x - 45045*d^10)*sqrt(-e^2*x^2 + d^2))/(d^12*e^10*x
^11 + 4*d^13*e^9*x^10 + 3*d^14*e^8*x^9 - 8*d^15*e^7*x^8 - 14*d^16*e^6*x^7 +
 14*d^18*e^4*x^5 + 8*d^19*e^3*x^4 - 3*d^20*e^2*x^3 - 4*d^21*e*x^2 - d^22*x)
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^4 (d^2-e^2x^2)^{7/2}} dx = \int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}} \left(d+ex\right)^4} dx$$

```
[In] integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{x^2\left(d^2-e^2\,x^2\right)^{7/2}(d+e\,x)^4}\,dx$$

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2} dx = -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -c^2*(-a^2*x^2+1)^(3/2)/x/(-a*c*x+c)^(3/2)+a*arctanh(c^(1/2)*(-a^2*x^2+1)^(
1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-a*c*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {893, 879, 889, 214}

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2} dx = a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}$$

```
[In] Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}(ac)\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - (a^3c^2)\,\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \frac{\sqrt{c-acx}\left((1+2ax)\sqrt{1-a^2x^2}+ax\sqrt{-1+ax}\arctan\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)\right)}{x(-1+ax)}$$

```
[In] Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a*c*x]*((1 + 2*a*x)*Sqrt[1 - a^2*x^2] + a*x*Sqrt[-1 + a*x]*ArcTan
[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]]))/(x*(-1 + a*x))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)acx+2ax\sqrt{c(ax+1)}\,\sqrt{c}+\sqrt{c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{c(ax+1)}\,x\sqrt{c}}$	95
risch	$\frac{(2a^2x^2+3ax+1)\sqrt{-\frac{(-a^2x^2+1)c}{ax-1}}\,(ax-1)c}{x\sqrt{c(ax+1)}\,\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{acx+c}}{\sqrt{c}}\right)\sqrt{-\frac{(-a^2x^2+1)c}{ax-1}}\,(ax-1)}{\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}$	147

```
[In] int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+2*a*x*(c*(a*x+1))^(1/2)*c^(1/2)+
(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c
*(a*x+1))^(1/2)/x/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \left[\frac{(a^2x^2-ax)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2\,c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(2\,ax+1)}{2\,(ax^2-x)}, \frac{(a^2x^2-\right.$$

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt
(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

Sympy [F]

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)}}{x^2}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{x^2}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = \int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2}\,dx$$

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2,x)
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2, x)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {889, 214}

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[In] Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\text{integral} = (2a^2c^2)\,\text{Subst}\left(\int \frac{1}{-a^2c + a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$
$$= -2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{c-acx}\arctan\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)}{\sqrt{-1+ax}}$$

```
[In] Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
[Out] (-2*Sqrt[c - a*c*x]*ArcTan[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]])/Sqrt[-1 + a*x
]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)}{(ax-1)\sqrt{c(ax+1)}}$	58

```
[In] int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)*c^(1/2)*a
rctanh((c*(a*x+1))^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2\,c}{ax^2-x}\right),\ -2\,\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sq
rt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

Sympy [F]

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

```
[In] integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{-acx+c}}{\sqrt{-a^2x^2+1}x}\,dx$$

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = -\frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}\right)}{|c|}$$

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c^3*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/(sqrt(-c)*c) - arctan(sqrt(a*c*x +
 c)/sqrt(-c))/(sqrt(-c)*c))/abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

```
[In] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}} dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}} dx = \sqrt{x}\sqrt{1-ax} + \frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}} dx = \frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} + \sqrt{x}\sqrt{1-ax}$$

```
[In] Int[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{x}\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx\\ &= \sqrt{x}\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1-ax} + \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[1 - a*x] + (2*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/
Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

method	result	size
meijerg	$\frac{\sqrt{-a}\left(-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{-a}\,\sqrt{-ax+1}-\frac{2\sqrt{\pi}\,\sqrt{-a}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}\right)}{2\sqrt{\pi}\,a}$	57
default	$\sqrt{x}\,\sqrt{-ax+1}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	62
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{(-ax+1)x}}{\sqrt{-x(ax-1)}\,\sqrt{-ax+1}}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	88

```
[In] int((-a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-a)^(1/2)/Pi^(1/2)/a*(-2*Pi^(1/2)*x^(1/2)*(-a)^(1/2)*(-a*x+1)^(1/2)-2*
Pi^(1/2)*(-a)^(1/2)/a^(1/2)*arcsin(a^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \left[\frac{2\sqrt{-ax+1}a\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{2\,a}, \frac{\sqrt{-ax+1}a\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{a}\right.$$

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*s
qrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(-a
*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \begin{cases} i\sqrt{x}\sqrt{ax-1}-\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{for } |ax|>1 \\ -\frac{ax^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{\sqrt{x}}{\sqrt{-ax+1}}+\frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate((-a*x+1)**(1/2)/x**(1/2),x)
[Out] Piecewise((I*sqrt(x)*sqrt(a*x - 1) - I*acosh(sqrt(a)*sqrt(x))/sqrt(a), Abs(
a*x) > 1), (-a*x**(3/2)/sqrt(-a*x + 1) + sqrt(x)/sqrt(-a*x + 1) + asin(sqrt
(a)*sqrt(x))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{a}} + \frac{\sqrt{-ax+1}}{\left(a-\frac{ax-1}{x}\right)\sqrt{x}}$$

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))/sqrt(a) + sqrt(-a*x + 1)/((a - (a
*x - 1)/x)*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.

Time = 5.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \frac{a\left(\frac{\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}} + \frac{\sqrt{(ax-1)a+a}\sqrt{-ax+1}}{a}\right)}{|a|}$$

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] a*(log(abs(-sqrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) + sq
rt((a*x - 1)*a + a)*sqrt(-a*x + 1)/a)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1-a\,x} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{1-a\,x}-1}\right)}{\sqrt{a}}$$

```
[In] int((1 - a*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(1 - a*x)^(1/2) + (2*atan((a^(1/2)*x^(1/2))/((1 - a*x)^(1/2) - 1)))
/a^(1/2)
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 35

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \sqrt{x}\sqrt{1-ax} + \frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {862, 52, 56, 222}

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} + \sqrt{x}\sqrt{1-ax}$$

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \sqrt{x}\sqrt{1-ax} + \frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(25) = 50$.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

method	result	size
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{x}\left(2\sqrt{a}\,\sqrt{-x(ax-1)}+\arctan\left(\frac{2ax-1}{2\sqrt{a}\,\sqrt{-x(ax-1)}}\right)\right)}{2\sqrt{ax+1}\,\sqrt{-x(ax-1)}\,\sqrt{a}}$	76
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{\sqrt{-x(ax-1)}\,\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{2\sqrt{a}\,\sqrt{x}\,\sqrt{-a^2x^2+1}}$	132

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)+
arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 5.69

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$$
$$=\left[\frac{4\sqrt{-a^2x^2+1}\sqrt{ax+1}a\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{4(a^2x+a)},\frac{2\sqrt{-a^2x^2}}{}\right.$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*log
(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sqrt
(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(a*
x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{ax+1}}\,dx$$

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\sqrt{ax+1}\sqrt{x}}\,dx$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.

Time = 5.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \frac{a\left(\frac{\sqrt{2}-\log\left(\left|-\sqrt{2}\sqrt{-a}+\sqrt{-a}\right|\right)}{\sqrt{-a}} + \frac{\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}} + \frac{\sqrt{(ax-1)a+a}\sqrt{-ax+1}}{a}\right)}{|a|}$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
[Out] a*((sqrt(2) - log(abs(-sqrt(2)*sqrt(-a) + sqrt(-a))))/sqrt(-a) + log(abs(-s
qrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) + sqrt((a*x - 1)*
a + a)*sqrt(-a*x + 1)/a)/abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{a\,x+1}}\,dx$$

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)),x)
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)), x)
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}} \, dx = \sqrt{x}\sqrt{1+ax} + \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}} \, dx = \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} + \sqrt{x}\sqrt{ax+1}$$

```
[In] Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \sqrt{x}\sqrt{1+ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx \\ &= \sqrt{x}\sqrt{1+ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right) \\ &= \sqrt{x}\sqrt{1+ax} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{1+ax} + \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1+ax}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[1 + a*x] + (2*ArcTanh[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 + a*x])])
/Sqrt[a]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{a}\,\sqrt{x}\,\sqrt{ax+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{a}\,\sqrt{x}\right)}{2\sqrt{a}\,\sqrt{\pi}}$	41
default	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57
risch	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57

```
[In] int((a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)*a^(1/2)*x^(1/2)*(a*x+1)^(1/2)-2*Pi^(1/2)
*arcsinh(a^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \left[\frac{2\sqrt{ax+1}a\sqrt{x}+\sqrt{a}\log\left(2\,ax+2\sqrt{ax+1}\sqrt{a}\sqrt{x}+1\right)}{2\,a}, \frac{\sqrt{ax+1}a\sqrt{x}-\sqrt{-a}\arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{a}\right]$$

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt(
a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x + 1
)*sqrt(-a)/(a*sqrt(x))))/a]
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{ax+1}+\frac{\operatorname{asinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[In] integrate((a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = -\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax+1}}{\sqrt{x}}}{\sqrt{a}+\frac{\sqrt{ax+1}}{\sqrt{x}}}\right)}{2\sqrt{a}}-\frac{\sqrt{ax+1}}{\left(a-\frac{ax+1}{x}\right)\sqrt{x}}$$

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(sqrt(a) - sqrt(a*x + 1)/sqrt(x))/(sqrt(a) + sqrt(a*x + 1)/sqrt(x
)))/sqrt(a) - sqrt(a*x + 1)/((a - (a*x + 1)/x)*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.

Time = 6.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = -\frac{a\left(\frac{\log\left(\left|-\sqrt{ax+1}\sqrt{a}+\sqrt{(ax+1)a-a}\right|\right)}{\sqrt{a}} - \frac{\sqrt{(ax+1)a-a}\sqrt{ax+1}}{a}\right)}{|a|}$$

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -a*(log(abs(-sqrt(a*x + 1)*sqrt(a) + sqrt((a*x + 1)*a - a)))/sqrt(a) - sqrt
((a*x + 1)*a - a)*sqrt(a*x + 1)/a)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{a\,x+1} + \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{a\,x+1}-1}\right)}{\sqrt{a}}$$

```
[In] int((a*x + 1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x + 1)^(1/2) + (2*atanh((a^(1/2)*x^(1/2))/((a*x + 1)^(1/2) - 1))
)/a^(1/2)
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 34

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \sqrt{x}\sqrt{1+ax} + \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {862, 52, 56, 221}

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} + \sqrt{x}\sqrt{ax+1}$$

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx \\
&= \sqrt{x}\sqrt{1+ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx \\
&= \sqrt{x}\sqrt{1+ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1+ax} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \sqrt{x}\sqrt{1+ax} + \frac{\operatorname{arcsinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(24) = 48$.

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{x}\, \sqrt{-a x +1}\, \left(2 \sqrt{\left(a x +1\right) x}\, \sqrt{a}+\ln \left(\frac{2 \sqrt{\left(a x +1\right) x}\, \sqrt{a}+2 a x +1}{2 \sqrt{a}}\right)\right)}{2 \left(a x -1\right) \sqrt{\left(a x +1\right) x}\, \sqrt{a}}$	86
risch	$-\frac{\left(a x +1\right) \sqrt{x}\, \sqrt{\frac{\left(-a^{2} x^{2}+1\right) x \left(-a x +1\right)}{\left(a x -1\right)^{2}}}\, \left(a x -1\right)}{\sqrt{\left(a x +1\right) x}\, \sqrt{-a^{2} x^{2}+1}\, \sqrt{-a x +1}}-\frac{\ln \left(\frac{\frac{1}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+x}\right) \sqrt{\frac{\left(-a^{2} x^{2}+1\right) x \left(-a x +1\right)}{\left(a x -1\right)^{2}}}\, \left(a x -1\right)}{2 \sqrt{a}\, \sqrt{-a^{2} x^{2}+1}\, \sqrt{x}\, \sqrt{-a x +1}}$	153

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)
^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(24) = 48$.

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 6.12

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \left[-\frac{4\,\sqrt{-a^{2} x^{2} + 1} \sqrt{-a x + 1} a \sqrt{x} - {\left(a x - 1\right)} \sqrt{a} \log\left(-\frac{8\,a^{3} x^{3} - 4\,\sqrt{-a^{2} x^{2} + 1} {\left(2\,a x + 1\right)} \sqrt{-a x + 1} \sqrt{a} \sqrt{x} - 7\,a x - 1}{a x - 1}\right)}{4\,{\left(a^{2} x - a\right)}}, -\frac{2\,\sqrt{-a^{2} x^{2} + 1} \sqrt{-a x + 1} a \sqrt{x} - {\left(a x - 1\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a^{2} x^{2} + 1} \sqrt{-a x + 1} \sqrt{-a} \sqrt{x}}{2\,a^{2} x^{2} - a x - 1}\right)}{2\,{\left(a^{2} x - a\right)}}\right]$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*sqr
t(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(-
a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{-ax+1}}\,dx$$

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}}{\sqrt{-ax+1}\sqrt{x}}\,dx$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(24) = 48$.

Time = 5.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = -\frac{a\left(\frac{\sqrt{2}-\log\left(\left|-\sqrt{2}\sqrt{a}+\sqrt{a}\right|\right)}{\sqrt{a}} + \frac{\log\left(\left|-\sqrt{ax+1}\sqrt{a}+\sqrt{(ax+1)a-a}\right|\right)}{\sqrt{a}} - \frac{\sqrt{(ax+1)a-a}\sqrt{ax+1}}{a}\right)}{|a|}$$

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -a*((sqrt(2) - log(abs(-sqrt(2)*sqrt(a) + sqrt(a))))/sqrt(a) + log(abs(-sqr
t(a*x + 1)*sqrt(a) + sqrt((a*x + 1)*a - a)))/sqrt(a) - sqrt((a*x + 1)*a - a
)*sqrt(a*x + 1)/a)/abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx = \int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{1-a\,x}}\,dx$$

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)),x)
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)), x)
```

3.224 $\int \sqrt{x}\sqrt{1-ax}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \sqrt{x}\sqrt{1-ax}\,dx=-\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\int \sqrt{x}\sqrt{1-ax}\,dx=\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[In] Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[1 - a*x])/a + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{1}{4}\int\frac{\sqrt{x}}{\sqrt{1-ax}}\,dx\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\int\frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)}{4a}\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int\sqrt{x}\sqrt{1-ax}\,dx=\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(-1+2ax)+2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{4a^{3/2}}$$

```
[In] Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + 2*ArcTan[(Sqrt[a]*Sqrt[x])/(-
1 + Sqrt[1 - a*x])])/(4*a^(3/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}(-6ax+3)\sqrt{-ax+1}}{6a}-\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{2a^{\frac{3}{2}}}}{2\sqrt{-a}\,\sqrt{\pi}\,a}$	66
default	$-\frac{\sqrt{x}\,(-ax+1)^{\frac{3}{2}}}{2a}+\frac{\sqrt{x}\,\sqrt{-ax+1}+\frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}}{4a}$	84
risch	$-\frac{(2ax-1)\sqrt{x}\,(ax-1)\sqrt{(-ax+1)x}}{4a\sqrt{-x(ax-1)}\,\sqrt{-ax+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^{2}+x}}\right)\sqrt{(-ax+1)x}}{8a^{\frac{3}{2}}\sqrt{x}\,\sqrt{-ax+1}}$	97

```
[In] int(x^(1/2)*(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(-a)^(1/2)/Pi^(1/2)/a*(1/6*Pi^(1/2)*x^(1/2)*(-a)^(3/2)*(-6*a*x+3)/a*(-a
*x+1)^(1/2)-1/2*Pi^(1/2)*(-a)^(3/2)/a^(3/2)*arcsin(a^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.76

$$\int \sqrt{x}\sqrt{1-ax}\,dx$$
$$=\left[\frac{2\,(2\,a^{2}x-a)\sqrt{-ax+1}\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\,\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{8\,a^{2}},\frac{(2\,a^{2}x-a)\sqrt{-ax+1}\sqrt{x}-\sqrt{}}{4\,a^{2}}\right.$$

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] [1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt
(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*sq
rt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}} - \frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for } |ax| > 1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}} + \frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
+ I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Ab
s(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)
) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \frac{\frac{\sqrt{-ax+1}a}{\sqrt{x}} - \frac{(-ax+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(a^3 - \frac{2(ax-1)a^2}{x} + \frac{(ax-1)^2 a}{x^2}\right)} - \frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a*x + 1)*a/sqrt(x) - (-a*x + 1)^(3/2)/x^(3/2))/(a^3 - 2*(a*x - 1
)*a^2/x + (a*x - 1)^2*a/x^2) - 1/4*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))
/a^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(43) = 86$.

Time = 10.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24

$$\int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \frac{\frac{\left(\sqrt{(ax-1)a+a}(2ax+3)\sqrt{-ax+1} - \frac{3a\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}}\right)|a|}{a^2} + \frac{4\left(\frac{a\log\left(\left|-\sqrt{-ax+1}\sqrt{-a}+\sqrt{(ax-1)a+a}\right|\right)}{\sqrt{-a}} - \sqrt{(ax-1)a+a}\sqrt{-ax}\right.}{a^2}}{4a}$$

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt((a*x - 1)*a + a)*(2*a*x + 3)*sqrt(-a*x + 1) - 3*a*log(abs(-sqrt(
-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a))*abs(a)/a^2 + 4*(a*lo
g(abs(-sqrt(-a*x + 1)*sqrt(-a) + sqrt((a*x - 1)*a + a)))/sqrt(-a) - sqrt((a
*x - 1)*a + a)*sqrt(-a*x + 1))*abs(a)/a^2)/a
```

Mupad [B] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \sqrt{x}\left(\frac{x}{2}-\frac{1}{4\,a}\right)\sqrt{1-a\,x}-\frac{\ln\left(2\sqrt{-a}\,\sqrt{x}\,\sqrt{1-a\,x}-2\,a\,x+1\right)}{8\,{\left(-a\right)}^{3/2}}$$

```
[In] int(x^(1/2)*(1 - a*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(4*a))*(1 - a*x)^(1/2) - log(2*(-a)^(1/2)*x^(1/2)*(1 - a*x
)^(1/2) - 2*a*x + 1)/(8*(-a)^(3/2))
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 63

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx=-\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {862, 52, 56, 222}

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx=\frac{\arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[In] Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[1 - a*x])/a + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{x}\sqrt{1-ax}\,dx \\
&= \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx \\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a} \\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a} \\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(-1+2ax) + \arcsin\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

```
[In] Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(43) = 86$.

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{-a^{2} x^{2}+1}\,\left(4 a^{\frac{3}{2}} x \sqrt{-x \left(a x -1\right)}-2 \sqrt{a}\,\sqrt{-x \left(a x -1\right)}+\arctan\left(\frac{2 a x -1}{2 \sqrt{a}\,\sqrt{-x \left(a x -1\right)}}\right)\right)}{8 a^{\frac{3}{2}} \sqrt{a x +1}\,\sqrt{-x \left(a x -1\right)}}$	92
risch	$-\frac{\left(2 a x -1\right) \sqrt{x}\,\left(a x -1\right) \sqrt{\frac{x \left(-a^{2} x^{2}+1\right)}{a x +1}}\,\sqrt{a x +1}}{4 a \sqrt{-x \left(a x -1\right)}\,\sqrt{-a^{2} x^{2}+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x -\frac{1}{2 a}\right)}{\sqrt{-a \,x^{2}+x}}\right) \sqrt{\frac{x \left(-a^{2} x^{2}+1\right)}{a x +1}}\,\sqrt{a x +1}}{8 a^{\frac{3}{2}} \sqrt{x}\,\sqrt{-a^{2} x^{2}+1}}$	141

```
[In] int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*a^(3/2)*x*(-x*(a*x-1))^(1/2)-2*a^
(1/2)*(-x*(a*x-1))^(1/2)+arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/
(a*x+1)^(1/2)/(-x*(a*x-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \left[\frac{4\sqrt{-a^2x^2+1}(2\,a^2x-a)\sqrt{ax+1}\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8\,a^3x^3-4\,\sqrt{-a^2x^2+1}(2\,ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7\,ax+1}{ax+1}\right)}{16\,(a^3x+a^2)}\right.$$

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)
*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*
sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^2*x
^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sq
rt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^3
*x + a^2)]
```

Sympy [F]

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{x}\sqrt{-(ax-1)(ax+1)}}{\sqrt{ax+1}}\,dx$$

```
[In] integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{-a^2x^2+1}\sqrt{x}}{\sqrt{ax+1}}\,dx$$

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \frac{\sqrt{x}\,\sqrt{1-a^2\,x^2}}{\sqrt{a\,x+1}}\,dx$$

```
[In] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2),x)
[Out] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2), x)
```

3.226 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{d^7(11+4m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6e(29+4m)(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(9+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(7/2)/g^2/(9+m)+d^7*(11+4*m)*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1
/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8+m)/(1-e^2*x^2/d^2)^(1/2)
+d^6*e*(29+4*m)*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2
)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(9+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1823, 822, 372, 371}

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx = -\frac{e(d^2-e^2x^2)^{7/2}(gx)^{m+2}}{g^2(m+9)}-\frac{3d(d^2-e^2x^2)^{7/2}(gx)^{m+1}}{g(m+8)} + \frac{d^7(4m+11)\sqrt{d^2-e^2x^2}(gx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^6e(4m+29)\sqrt{d^2-e^2x^2}(gx)^{m+2}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+9)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) - (e*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m)) + (d^7*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^
2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^6*e*(29 + 4*m)*(g*x)^(2 +
m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x
^2)/d^2])/(g^2*(2 + m)*(9 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} \\
&\quad -\frac{\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\left(-d^3e^2(9+m)-d^2e^3(29+4m)x-3de^4(9+m)x^2\right)dx}{e^2(9+m)} \\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} \\
&\quad +\frac{\int (gx)^m\left(d^3e^4(9+m)(11+4m)+d^2e^5(8+m)(29+4m)x\right)\left(d^2-e^2x^2\right)^{5/2}dx}{e^4(8+m)(9+m)} \\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} \\
&\quad +\frac{\left(d^3(11+4m)\right)\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}dx}{8+m} \\
&\quad +\frac{\left(d^2e(29+4m)\right)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}dx}{g(9+m)} \\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} \\
&\quad +\frac{\left(d^7(11+4m)\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&\quad +\frac{\left(d^6e(29+4m)\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{g(9+m)\sqrt{1-\frac{e^2x^2}{d^2}}}
\end{aligned}
$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{d^7(11+4m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^6e(29+4m)(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(9+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.80

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m} + ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2}\right.}{2+m}\right.\right.}{\sqrt{1-}}$$

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*((d^3*Hypergeometric2F1[-5/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[-5/2, (
2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[
-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F
1[-5/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/Sqrt[1 - (e^2*x^2
)/d^2]
```

Maple [F]

$$\int (gx)^m\,(ex+d)^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx$$

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x
^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.98 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.01

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^8g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3d^7eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{d^6e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\\frac{m}{2}+\frac{5}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{5d^5e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\\frac{m}{2}+3\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}-\frac{5d^4e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\\frac{m}{2}+\frac{7}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{d^3e^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\\frac{m}{2}+4\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+4\right)}+\frac{3d^2e^6g^mx^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\\frac{m}{2}+\frac{9}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}+\frac{de^7g^mx^{m+8}\Gamma\left(\frac{m}{2}+4\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+4\\\frac{m}{2}+5\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+5\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**8*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**7*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_po
lar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + d**6*e**2*g**m*x**(m + 3)*gamma(m/2
```

```
+ 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/(2*gamma(m/2 + 5/2)) - 5*d**5*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hype
r((-1/2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m
/2 + 3)) - 5*d**4*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 +
5/2), (m/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2))
+ d**3*e**5*g**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4)) + 3*d**2*e**6*g**m*
x**(m + 7)*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2)) + d*e**7*g**m*x**(m + 8)*gam
ma(m/2 + 4)*hyper((-1/2, m/2 + 4), (m/2 + 5,), e**2*x**2*exp_polar(2*I*pi)/
d**2)/(2*gamma(m/2 + 5))
```

Maxima [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^3\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3, x)
```

3.227 $\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = -\frac{(gx)^{1+m} \left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} + \frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{2d^5e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)+d^6*(9+2*m)*(g*x)^(1+m)*hypergeom
([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8
+m)/(1-e^2*x^2/d^2)^(1/2)+2*d^5*e*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+
1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used

= {1823, 822, 372, 371}

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} dx = -\frac{\left(d^2-e^2x^2\right)^{7/2} (gx)^{m+1}}{g(m+8)} + \frac{d^6(2m+9)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{2d^5e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d^6*(9 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^5*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} \\
&\quad -\frac{\int (gx)^m\left(-d^2e^2(9+2m)-2de^3(8+m)x\right)\left(d^2-e^2x^2\right)^{5/2}\,dx}{e^2(8+m)} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g} \\
&\quad +\frac{\left(d^2(9+2m)\right)\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\,dx}{8+m} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{\left(2d^5e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&\quad +\frac{\left(d^6(9+2m)\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} \\
&\quad +\frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&\quad +\frac{2d^5e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m\right.}{(1+}$$

[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]

[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometric2F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])

Maple [F]

$$\int (gx)^m (ex+d)^2 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)

[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)

Fricas [F]

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^2(gx)^m dx$$

[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")

[Out] integral((e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^2 + 2*d^5*e*x + d^6)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.94 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.10

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^7g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^6eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^5e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} - \frac{2d^4e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)} - \frac{d^3e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{d^2e^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+4\right)} + \frac{de^6g^mx^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**6*e*g**m*x**(
m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/gamma(m/2 + 2) - d**5*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2)
*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(
```

```
2*gamma(m/2 + 5/2)) - 2*d**4*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-1/
2, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) -
 d**3*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 +
 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**2*e**5*
g**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 4) + d*e**6*g**m*x**(m + 7)*gamma(m/2
+ 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/(2*gamma(m/2 + 9/2))
```

Maxima [F]

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^2(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^2(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(d^2-e^2\,x^2\right)^{5/2}\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^2 \,d x$$

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2, x)
```

3.228 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 162

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^5*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)+d^4*e*(g*x)^(2+m)*hypergeom([-
5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {822, 372, 371}

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx = \frac{d^5\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^4e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = d\int (gx)^m \left(d^2-e^2x^2\right)^{5/2}\,dx+\frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g}$$

$$=\frac{\left(d^5\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{\left(d^4e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$=\frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d(2+m)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{Hype}\right.}{(1+m)(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d*(2 + m)*Hypergeometric2F1[-5/2, (1 +
m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[-5/2, (2 +
m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]

$$\int (gx)^m\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx$$

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

Fricas [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
[Out] integral((e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x + d
^5)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.75 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.26

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\frac{d^6g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^5eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^4e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{d^3e^3g^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)}+\frac{d^2e^4g^mx^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{de^5g^mx^{m+6}\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+4\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**6*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**5*e*g**m*x**(
m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) - d**4*e**2*g**m*x**(m + 3)*gamma(m/2 +
3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/gamma(m/2 + 5/2) - d**3*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-1/2,
 m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) + d
**2*e**4*g**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7
/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d*e**5*g**m*
x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_
polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4))
```

Maxima [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx=\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,\left(d+e\,x\right)\,dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x), x)
```

3.229 $\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx = \frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^4*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {372, 371}

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx = \frac{d^4\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(d^4\sqrt{d^2-e^2x^2}\right)\int(gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$= \frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int(gx)^m\left(d^2-e^2x^2\right)^{5/2}dx = \frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2},1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, 1 + (
1 + m)/2, (e^2*x^2)/d^2])/((1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]

$$\int(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx$$

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

Fricas [F]

$$\int (gx)^m \left(d^2 - e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
[Out] integral((e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int (gx)^m \left(d^2 - e^2x^2\right)^{5/2} dx = \frac{d^5 g^m x^{m+1} \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2),x)
[Out] d**5*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int (gx)^m \left(d^2 - e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m \left(d^2 - e^2x^2\right)^{5/2} dx = \int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx = \int \left(d^2-e^2\,x^2\right)^{5/2} (g\,x)^m \, dx$$

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m, x)
```

3.230 $\int \frac{(gx)^m(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx=\frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^3*(g*x)^(1+m)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)-d^2*e*(g*x)^(2+m)*hypergeom([-
3/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {906, 83, 127, 372, 371}

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx=\frac{d^3\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] && !RationalQ[
p] && !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d^2-e^2x^2}\int (gx)^m(d-ex)^{5/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\left(d\sqrt{d^2-e^2x^2}\right)\int (gx)^m(d-ex)^{3/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}(d-ex)^{3/2}(d+ex)^{3/2}\,dx}{g\sqrt{d-ex}\sqrt{d+ex}}$$

$$= d\int (gx)^m\left(d^2-e^2x^2\right)^{3/2}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}\,dx}{g}$$

$$= \frac{\left(d^3\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{\left(d^2e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$= \frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{d^2x(gx)^m\sqrt{d^2-e^2x^2}\left(-e(1+m)x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1+\frac{m}{2},2+\frac{m}{2},\frac{e^2}{d}\right.\right.}{(1+m)(2+m)\sqrt{1-}}$$

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^2*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(-(e*(1 + m)*x*Hypergeometric2F1[-3/2, 1
 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[-3/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{ex+d}dx$$

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{ex+d}\,dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")

[Out] integral((e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2)*(g*x)^m
, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.49

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \frac{d^4g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} - \frac{d^3eg^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^2e^2g^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{de^3g^m x^{m+4}\Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)

[Out] d**4*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**3*e*g**m*x**(
m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) - d**2*e**2*g**m*x**(m + 3)*gamma(m/2 +
3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/(2*gamma(m/2 + 5/2)) + d*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-1/2
, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3)
)

Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex + d} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex + d} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{d + ex} \, dx = \int \frac{(d^2 - e^2 x^2)^{5/2} (g x)^m}{d + e x} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x),x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x), x)
```

3.231 $\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 204

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx = -\frac{(gx)^{1+m}(d^2-e^2x^2)^{3/2}}{g(4+m)} + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2de(gx)^{2+m}\sqrt{d^2-e^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(3/2)/g/(4+m)+d^2*(5+2*m)*(g*x)^(1+m)*hypergeom
([-1/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(4
+m)/(1-e^2*x^2/d^2)^(1/2)-2*d*e*(g*x)^(2+m)*hypergeom([-1/2, 1+1/2*m],[2+1/
2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

= {866, 1823, 822, 372, 371}

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{2de\sqrt{d^2-e^2x^2}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^2(2m+5)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{\left(d^2-e^2x^2\right)^{3/2}(gx)^{m+1}}{g(m+4)}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d^2*(5 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(4 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*d*e*(g*x)^
(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
```

```
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (gx)^m (d-ex)^2 \sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\int (gx)^m \left(-d^2e^2(5+2m)+2de^3(4+m)x\right)\sqrt{d^2-e^2x^2}\,dx}{e^2(4+m)} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{(2de)\int (gx)^{1+m}\sqrt{d^2-e^2x^2}\,dx}{g} \\
&\quad + \frac{\left(d^2(5+2m)\right)\int (gx)^m\sqrt{d^2-e^2x^2}\,dx}{4+m} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\left(2de\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&\quad + \frac{\left(d^2(5+2m)\sqrt{d^2-e^2x^2}\right)\int (gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} \\
&\quad + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}} \\
&\quad - \frac{2de(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.85

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right.\right.}{}$$

[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]

[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeome tric2F1[-1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome tric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])

Maple [F]

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^2}dx$$

[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)

[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(gx)^m}{(ex+d)^2}\,dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.89

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \frac{d^3g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} - \frac{d^2eg^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{de^2g^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
[Out] d**3*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**2*e*g**m*x**(
m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/gamma(m/2 + 2) + d*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2)*hy
per((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*g
amma(m/2 + 5/2))
```

Maxima [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,6,1,0,0]%%%}+%%%{-3,[0,4,1,0,0]%%%}+%%%{-3,[0,2,1,0,0]%
%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2, x)
```

3.232 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)}+\frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(2+m)\sqrt{d^2-e^2x^2}} - \frac{d^2e(11+4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(3+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1/2)/g/(2+m)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(1/2)/g^2/(3+m)+d^3*(5+4*m)*(g*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2
*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/g/(1+m)/(2+m)/(-e^2*x^2+d^2)^(1/2)-d
^2*e*(11+4*m)*(g*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(
1-e^2*x^2/d^2)^(1/2)/g^2/(2+m)/(3+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

= {866, 1823, 822, 372, 371}

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx = -\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}} + \frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)} - \frac{3d\sqrt{d^2-e^2x^2}(gx)^{m+1}}{g(m+2)} + \frac{d^3(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (e*(g*x)^(2 + m)*Sqr
t[d^2 - e^2*x^2])/(g^2*(3 + m)) + (d^3*(5 + 4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g
*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]) - (d^2*e*(11 + 4*m)*(g*x)^(2 + m)*Sqr
t[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)
/d^2])/(g^2*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
```

```
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(gx)^m(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} - \frac{\int \frac{(gx)^m\left(-d^3e^2(3+m)+d^2e^3(11+4m)x-3de^4(3+m)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2(3+m)} \\
&= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} \\
&\quad + \frac{\int \frac{(gx)^m\left(d^3e^4(3+m)(5+4m)-d^2e^5(2+m)(11+4m)x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^4(2+m)(3+m)} \\
&= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} \\
&\quad + \frac{\left(d^3(5+4m)\right)\int \frac{(gx)^m}{\sqrt{d^2-e^2x^2}}\,dx}{2+m} - \frac{\left(d^2e(11+4m)\right)\int \frac{(gx)^{1+m}}{\sqrt{d^2-e^2x^2}}\,dx}{g(3+m)} \\
&= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} \\
&\quad + \frac{\left(d^3(5+4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{(2+m)\sqrt{d^2-e^2x^2}} \\
&\quad - \frac{\left(d^2e(11+4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{g(3+m)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(2+m)\sqrt{d^2-e^2x^2}} - \frac{d^2e(11+4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(3+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx = \frac{x(gx)^m\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^3(24+26m+9m^2+m^3)\,\text{Hypergeometric2F1}\right.}{}$$

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*(d^3*(24 + 26*m + 9*
m^2 + m^3)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*
(1 + m)*x*(3*d^2*(12 + 7*m + m^2)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2] + e*(2 + m)*x*(-3*d*(4 + m)*Hypergeometric2F1[1/2, (3 +
m)/2, (5 + m)/2, (e^2*x^2)/d^2] + e*(3 + m)*x*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, (e^2*x^2)/d^2]))))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(d - e*
x)*(d + e*x))
```

Maple [F]

$$\int \frac{(gx)^m (-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^3}dx$$

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}(gx)^m}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e*x + d),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d+ex)^3} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^{5/2}}{(d+ex)^3} \, dx = \int \frac{(d^2 - e^2\,x^2)^{5/2}\,(g\,x)^m}{(d+e\,x)^3} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3, x)
```

3.233 $\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 213

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{4(gx)^{1+m}(d+ex)}{5g(d^2-e^2x^2)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g^2(2+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/5*(g*x)^(1+m)*(e*x+d)/g/(-e^2*x^2+d^2)^(5/2)+1/5*(1-4*m)*(g*x)^(1+m)*hype
rgeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^3/g
/(1+m)/(-e^2*x^2+d^2)^(1/2)+1/5*e*(7-4*m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used

= {1820, 822, 372, 371}

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{4(d+ex)(gx)^{m+1}}{5g\,(d^2-e^2x^2)^{5/2}} + \frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + ((1 - 4*m)*(g*x)^
(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2
, (e^2*x^2)/d^2])/(5*d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g*x
)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
```

```
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^3(1-4m)-d^2e(7-4m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{5}(d(1-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx + \frac{(e(7-4m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5g}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left((1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\left(e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^4g\sqrt{d^2-e^2x^2}}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m} + ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{2+m}{2}\right.}{2+m}\right.\right.}{d^6}$$

```
[In] Integrate[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[7/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[7/2, (2
+ m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[7/
2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[7
/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*Sqrt[d^2 - e^2*x
^2])
```

Maple [F]

$$\int \frac{(gx)^m (ex+d)^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

[In] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)

[Out] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)

Fricas [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^5*x^5 - 3*d*e^4*x^4 + 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 - 3*d^4*e*x + d^5), x)

Sympy [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((g*x)**m*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)

Maxima [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)

Giac [F]

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^3(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.234 $\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 216

$$\begin{aligned}\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}\\&+\frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}}\\&+\frac{2e(3-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{5d^5g^2(2+m)\sqrt{d^2-e^2x^2}}\end{aligned}$$

```
[Out] 2/5*(g*x)^(1+m)*(e*x+d)/d/g/(-e^2*x^2+d^2)^(5/2)+1/5*(3-2*m)*(g*x)^(1+m)*hy
pergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4
/g/(1+m)/(-e^2*x^2+d^2)^(1/2)+2/5*e*(3-m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^5/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

= {1820, 822, 372, 371}

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d+ex)(gx)^{m+1}}{5dg\,(d^2-e^2x^2)^{5/2}} + \frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + ((3 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
```

```
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(gx)^m\left(-d^2(3-2m)-2de(3-m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}\\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{(2e(3-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5dg}-\frac{1}{5}(-3+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{\left(2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^5g\sqrt{d^2-e^2x^2}}\\
&\quad -\frac{\left((-3+2m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^4\sqrt{d^2-e^2x^2}}\\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}}\\
&\quad +\frac{2e(3-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(2+m)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

$$\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1\cdots\right.}{d^6(\cdots}$$

```
[In] Integrate[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[7
/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperge
ometric2F1[7/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeo
metric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^6*(1 + m)*(2 + m)
*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]

$$\int \frac{(gx)^m (ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

[In] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)

[Out] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)

Fricas [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^6*x^6 - 2*d*e^5*x^5 - d^2*e^4*x^4 + 4*d^3*e^3*x^3 - d^4*e^2*x^2 - 2*d^5*e*x + d^6), x)

Sympy [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m (d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)

Maxima [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)

Giac [F]

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.235 $\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(gx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-4+m),\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{dg(1+m)\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(gx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([1, -2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d/g/(1+m)/(-e^
2*x^2+d^2)^(5/2)+e*(g*x)^(2+m)*hypergeom([1, -3/2+1/2*m],[2+1/2*m],e^2*x^2/
d^2)/d^2/g^2/(2+m)/(-e^2*x^2+d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {822, 372, 371}

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(g*x)^(2
```

```
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^6*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = d\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx + \frac{e\int \frac{(gx)^{1+m}}{(d^2-e^2x^2)^{7/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^5\sqrt{d^2-e^2x^2}} + \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d(2+m)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{Hy}\right.}{d^6(1+m)(2+m)\sqrt{d^2-e^2x^2}}$$

```
[In] Integrate[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d*(2 + m)*Hypergeometric2F1[7/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[7/2, (2 + m)
/2, (4 + m)/2, (e^2*x^2)/d^2]))/(d^6*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]

$$\int \frac{(gx)^m (ex+d)}{(-e^2x^2+d^2)^{\frac{7}{2}}} dx$$

```
[In] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^7*x^7 - d*e^6*x^6 - 3*d^2*e^5*x^5
+ 3*d^3*e^4*x^4 + 3*d^4*e^3*x^3 - 3*d^5*e^2*x^2 - d^6*e*x + d^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{7}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^6\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{eg^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}\frac{7}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+2\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
[Out] g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*d**6*gamma(m/2 + 3/2)) + e*g**m*x**(m + 2)
*gamma(m/2 + 1)*hyper((7/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I*p
i)/d**2)/(2*d**7*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Giac [F]

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.236 $\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([7/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^6/g/(1+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {372, 371}

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6\sqrt{d^2-e^2x^2}}$$
$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int\frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{d^6(1+m)\sqrt{d^2-e^2x^2}}$$

```
[In] Integrate[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1
 + m)/2, (e^2*x^2)/d^2])/(d^6*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]

$$\int\frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}dx$$

```
[In] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^8*x^8 - 4*d^2*e^6*x^6 + 6*d^4*e^4*
x^4 - 4*d^6*e^2*x^2 + d^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

```
[In] integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2
*x**2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Giac [F]

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((g*x)^m/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((g*x)^m/(d^2 - e^2*x^2)^(7/2), x)
```

3.237 $\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^7/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-e*(g*x)^(2+m)*hypergeom([9/2, 1
+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^8/g^2/(2+m)/(-e^2*x^
2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {906, 83, 127, 372, 371}

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}} - \frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] && !RationalQ[
p] && !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d-ex}\sqrt{d+ex}\right)\int\frac{(gx)^m}{(d-ex)^{7/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}}$$

$$=\frac{\left(d\sqrt{d-ex}\sqrt{d+ex}\right)\int\frac{(gx)^m}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}}-\frac{\left(e\sqrt{d-ex}\sqrt{d+ex}\right)\int\frac{(gx)^{1+m}}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{g\sqrt{d^2-e^2x^2}}$$

$$=d\int\frac{(gx)^m}{(d^2-e^2x^2)^{9/2}}\,dx-\frac{e\int\frac{(gx)^{1+m}}{(d^2-e^2x^2)^{9/2}}\,dx}{g}$$

$$=\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^7\sqrt{d^2-e^2x^2}}-\frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int\frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^8g\sqrt{d^2-e^2x^2}}$$

$$=\frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}}-\frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int\frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(-e(1+m)x\,\text{Hypergeometric2F1}\left(\frac{9}{2},1+\frac{m}{2},2+\frac{m}{2},\frac{e^2x^2}{d^2}\right)\right.}{d^8(1+m)(2+m)\sqrt{d^2-}}$$

```
[In] Integrate[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(-(e*(1 + m)*x*Hypergeometric2F1[9/2, 1
+ m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[9/2, (1 + m)/
2, (3 + m)/2, (e^2*x^2)/d^2]))/(d^8*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]

$$\int\frac{(gx)^m}{(ex+d)\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}dx$$

```
[In] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^9*x^9 + d*e^8*x^8 - 4*d^2*e^7*x^7 - 4*d^3*e^6*x^6 + 6*d^4*e^5*x^5 + 6*d^5*e^4*x^4 - 4*d^6*e^3*x^3 - 4*d^7*e^2*x^2 + d^8*e*x + d^9), x)

Sympy [F]

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)}\,dx$$

[In] integrate((g*x)**m/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)

Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)

Giac [F]

$$\int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")

[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(g\,x)^m}{\left(d^2-e^2\,x^2\right)^{7/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)), x)
```

3.238 $\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}} - \frac{2e(7-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{9d^9g^2(2+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/9*(g*x)^(1+m)*(-e*x+d)/d/g/(-e^2*x^2+d^2)^(9/2)+1/9*(7-2*m)*(g*x)^(1+m)*h
ypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^
8/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-2/9*e*(7-m)*(g*x)^(2+m)*hypergeom([9/2, 1+1/
2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^9/g^2/(2+m)/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

= {866, 1820, 822, 372, 371}

$$\int \frac{(gx)^m}{(d+ex)^2\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d-ex)(gx)^{m+1}}{9dg\,(d^2-e^2x^2)^{9/2}} - \frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{9}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + ((7 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(9*d^8*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(9*d^9*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(gx)^m(d-ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{(gx)^m\left(-d^2(7-2m)+2de(7-m)x\right)}{(d^2-e^2x^2)^{9/2}}\,dx}{9d^2} \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{(2e(7-m))\int \frac{(gx)^{1+m}}{(d^2-e^2x^2)^{9/2}}\,dx}{9dg} - \frac{1}{9}(-7+2m)\int \frac{(gx)^m}{(d^2-e^2x^2)^{9/2}}\,dx \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\left(2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{9d^9g\sqrt{d^2-e^2x^2}} \\
&\quad - \frac{\left((-7+2m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{9d^8\sqrt{d^2-e^2x^2}} \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}} \\
&\quad - \frac{2e(7-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(2+m)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.81

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}} \, dx = \frac{x(gx)^m \sqrt{1-\frac{e^2x^2}{d^2}} \left(d^2(6+5m+m^2) \text{Hypergeometric2F1}\left(\frac{11}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \frac{e^2}{d}\right.\right.}{}$$

```
[In] Integrate[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[1
1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[11/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperg
eometric2F1[11/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^10*(1 + m)*(2
+ m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]

$$\int \frac{(gx)^m}{(ex+d)^2 (-e^2x^2+d^2)^{\frac{7}{2}}} dx$$

```
[In] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}} \, dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}} (ex+d)^2} \, dx$$

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^10*x^10 + 2*d*e^9*x^9 - 3*d^2*e^8*
x^8 - 8*d^3*e^7*x^7 + 2*d^4*e^6*x^6 + 12*d^5*e^5*x^5 + 2*d^6*e^4*x^4 - 8*d^
7*e^3*x^3 - 3*d^8*e^2*x^2 + 2*d^9*e*x + d^10), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^2}\,dx$$

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^2}\,dx$$

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^2}\,dx$$

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2), x)
```

3.239 $\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 214

$$\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{4(gx)^{1+m}(d-ex)}{11g(d^2-e^2x^2)^{11/2}} + \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(25-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{2+m}{2},\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(2+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/11*(g*x)^(1+m)*(-e*x+d)/g/(-e^2*x^2+d^2)^(11/2)+1/11*(7-4*m)*(g*x)^(1+m)*
hypergeom([11/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/
d^9/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-1/11*e*(25-4*m)*(g*x)^(2+m)*hypergeom([11/
2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^10/g^2/(2+m)/(-e
^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used

= {866, 1820, 822, 372, 371}

$$\int \frac{(gx)^m}{(d+ex)^3\,(d^2-e^2x^2)^{7/2}}\,dx = \frac{4(d-ex)(gx)^{m+1}}{11g\,(d^2-e^2x^2)^{11/2}} - \frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{m+2}{2},\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{11}{2},\frac{m+1}{2},\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[In] Int[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + ((7 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m)
*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2, (
4 + m)/2, (e^2*x^2)/d^2])/(11*d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(gx)^m(d-ex)^3}{(d^2-e^2x^2)^{13/2}}\,dx$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} - \frac{\int \frac{(gx)^m\left(-d^3(7-4m)+d^2e(25-4m)x\right)}{(d^2-e^2x^2)^{11/2}}\,dx}{11d^2}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{1}{11}(d(7-4m))\int \frac{(gx)^m}{(d^2-e^2x^2)^{11/2}}\,dx - \frac{(e(25-4m))\int \frac{(gx)^{1+m}}{(d^2-e^2x^2)^{11/2}}\,dx}{11g}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{\left((7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}\,dx}{11d^9\sqrt{d^2-e^2x^2}} - \frac{\left(e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}\,dx}{11d^{10}g\sqrt{d^2-e^2x^2}}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(25-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{11}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}} \, dx = \frac{x(gx)^m \sqrt{1-\frac{e^2x^2}{d^2}} \left(\frac{d^3 \text{Hypergeometric2F1}\left(\frac{13}{2},\frac{1+m}{2},\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m} + ex\left(-\frac{3d^2 \text{Hypergeometri}}{} \right.\right.}{}$$

```
[In] Integrate[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[13/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[13/2,
(2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1
[13/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2
F1[13/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^12*Sqrt[d^2 -
 e^2*x^2])
```

Maple [F]

$$\int \frac{(gx)^m}{(ex+d)^3 (-e^2x^2+d^2)^{\frac{7}{2}}} dx$$

```
[In] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}} \, dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^11*x^11 + 3*d*e^10*x^10 - d^2*e^9*
x^9 - 11*d^3*e^8*x^8 - 6*d^4*e^7*x^7 + 14*d^5*e^6*x^6 + 14*d^6*e^5*x^5 - 6*
d^7*e^4*x^4 - 11*d^8*e^3*x^3 - d^9*e^2*x^2 + 3*d^10*e*x + d^11), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^3}\,dx$$

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3), x)
```

3.240 $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^5(d^2-e^2x^2)^{1+p}}{2e^6(1+p)}+\frac{d^3(d^2-e^2x^2)^{2+p}}{e^6(2+p)}-\frac{d(d^2-e^2x^2)^{3+p}}{2e^6(3+p)}+\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^5*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)+d^3*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
1/2*d*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/7*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([
7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 272, 45, 372, 371}

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)-\frac{d(d^2-e^2x^2)^{p+3}}{2e^6(p+3)}-\frac{d^5(d^2-e^2x^2)^{p+1}}{2e^6(p+1)}+\frac{d^3(d^2-e^2x^2)^{p+2}}{e^6(p+2)}$$

```
[In] Int[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^5*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2
+ p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int x^5\left(d^2-e^2x^2\right)^p\,dx+e\int x^6\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)\\ &\quad+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\end{aligned}$$

$$= \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\text{Subst}\left(\int\left(\frac{d^4(d^2-e^2x)^p}{e^4}-\frac{2d^2(d^2-e^2x)^{1+p}}{e^4}+\frac{(d^2-e^2x)^{2+p}}{e^4}\right)dx,x,x^2\right)$$

$$= -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{2e^6(1+p)}+\frac{d^3\left(d^2-e^2x^2\right)^{2+p}}{e^6(2+p)}-\frac{d\left(d^2-e^2x^2\right)^{3+p}}{2e^6(3+p)} + \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{\left(d^2-e^2x^2\right)^p\left(-\frac{7d\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(1+p)x^2+e^4(2+3p+p^2)x^4\right)}{(1+p)(2+p)(3+p)}+2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\right.\right.}{14e^6}$$

```
[In] Integrate[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-7*d*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 +
e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (2*e^7*x^7*Hypergeome
tric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F]

$$\int x^5(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x^6 + d*x^5)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(121) = 242$.

Time = 1.95 (sec) , antiderivative size = 972, normalized size of antiderivative = 6.57

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$
$$= d\left(\begin{cases} \frac{x^6(d^2)^p}{6} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(\frac{d}{e}\right.}{4d^4e^6-8d^2e^8x^2+} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{e^4x^4}{-2d^2e^6+2e^8x^2} \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6} - \frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6} - \frac{d^2x^2}{2e^4} - \frac{x^4}{4e^2} \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \cdots \end{cases}\right.$$
$$+ \frac{d^{2p}ex^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

```
[In] integrate(x**5*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 +
 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e*
*8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d*
*2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e*
*4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e
**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(
p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)
**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(
d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e
**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
 + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
```

```
*2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e*x**7*hyper((7/2,
 -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Maxima [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^6*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*((p^2 + 3*p
+ 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*d^6)*(-e^2*x^2 +
 d^2)^p*d/((p^3 + 6*p^2 + 11*p + 6)*e^6)
```

Giac [F]

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^5\,\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)\,dx$$

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.241 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{2e^5(1+p)}+\frac{d^2\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^5(3+p)}+\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^4*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+d^2*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+1/5*d*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 372, 371, 272, 45}

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+\frac{d^2\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^5(p+3)}-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}$$

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^4*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2
+ p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2
 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*
x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int x^4(d^2-e^2x^2)^p\,dx + e\int x^5(d^2-e^2x^2)^p\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int x^2(d^2-e^2x)^p\,dx, x, x^2\right)\\ &\quad + \left(d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\end{aligned}$$

$$= \frac{1}{5} d x^5 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right) + \frac{1}{2} e \text{Subst}\left(\int \left(\frac{d^4 (d^2 - e^2 x)^p}{e^4} - \frac{2 d^2 (d^2 - e^2 x)^{1+p}}{e^4} + \frac{(d^2 - e^2 x)^{2+p}}{e^4}\right) dx, x, x^2\right)$$

$$= -\frac{d^4 (d^2 - e^2 x^2)^{1+p}}{2 e^5 (1+p)} + \frac{d^2 (d^2 - e^2 x^2)^{2+p}}{e^5 (2+p)} - \frac{(d^2 - e^2 x^2)^{3+p}}{2 e^5 (3+p)} + \frac{1}{5} d x^5 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int x^4 (d + e x) (d^2 - e^2 x^2)^p \, dx = \frac{1}{10} (d^2 - e^2 x^2)^p \left(-\frac{5 (d^2 - e^2 x^2) (2 d^4 + 2 d^2 e^2 (1+p) x^2 + e^4 (2 + 3p + p^2) x^4)}{e^5 (1+p)(2+p)(3+p)} + 2 d x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2}, -p, \frac{7}{2}, \frac{e^2 x^2}{d^2}\right)\right)$$

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 + e^
4*(2 + 3*p + p^2)*x^4))/(e^5*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeome
tric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/10
```

Maple [F]

$$\int x^4 (e x + d) \left(-e^2 x^2 + d^2\right)^p dx$$

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e*x^5 + d*x^4)*(-e^2*x^2 + d^2)^p, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. 2(119) = 238.

Time = 1.87 (sec) , antiderivative size = 972, normalized size of antiderivative = 6.61

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{dd^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$
$$+e\left(\begin{cases}\frac{x^6(d^2)^p}{6}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(\right.}{4d^4e^6-8d^2e^8x^2\cdots}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2}\\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2}\\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\cdots\end{cases}\right.$$

[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**p,x)

[Out] d*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/ 5 + e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d** 4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e** 6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x* *2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e **8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d **2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8 *x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e** 8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e** 8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(- 2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)

```
+ e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/
(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
 Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 2
2*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x
**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x*
*4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
 + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Maxima [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.242 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^3(d^2-e^2x^2)^{1+p}}{2e^4(1+p)}+\frac{d(d^2-e^2x^2)^{2+p}}{2e^4(2+p)}+\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)+1/2*d*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p
)+1/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2
*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 272, 45, 372, 371}

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+\frac{d(d^2-e^2x^2)^{p+2}}{2e^4(p+2)}-\frac{d^3(d^2-e^2x^2)^{p+1}}{2e^4(p+1)}$$

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^3*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 +
p))/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7
/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int x^3(d^2-e^2x^2)^p\,dx + e\int x^4(d^2-e^2x^2)^p\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int x(d^2-e^2x)^p\,dx, x, x^2\right) + \left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$

$$= \frac{1}{5}ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x)^p}{e^2}-\frac{(d^2-e^2x)^{1+p}}{e^2}\right)dx, x, x^2\right)$$

$$= -\frac{d^3(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} + \frac{d(d^2-e^2x^2)^{2+p}}{2e^4(2+p)} + \frac{1}{5}ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{5d(d^2-e^2x^2)(d^2+e^2(1+p)x^2)}{(1+p)(2+p)} + 2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)\right)}{10e^4}$$

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/((1 + p)
*(2 + p)) + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/(10*e^4)
```

Maple [F]

$$\int x^3(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x^4 + d*x^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(97) = 194.

Time = 1.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.18

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$$
$$=d\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$
$$+\frac{d^{2p}ex^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-
2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (
-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2),
Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
- d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
+ e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**
4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d*
*(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

Maxima [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^4*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*(e^4*(p + 1)
*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d/((p^2 + 3*p + 2)*e^4)
```

Giac [F]

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^3\,\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)\;dx$$

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.243 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^2*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)+
1/3*d*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 372, 371, 272, 45}

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}$$

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^2*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/
(2*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2,
(e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int x^2\left(d^2-e^2x^2\right)^p\,dx+e\int x^3\left(d^2-e^2x^2\right)^p\,dx\\
&= \frac{1}{2}e\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&= \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\\
&\quad+\frac{1}{2}e\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x)^p}{e^2}-\frac{(d^2-e^2x)^{1+p}}{e^2}\right)dx,x,x^2\right)\\
&= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(-\frac{3(d^2-e^2x^2)\left(d^2+e^2(1+p)x^2\right)}{e^3(1+p)(2+p)}+2dx^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/(e^3*(1 +
p)*(2 + p)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/6
```

Maple [F]

$$\int x^2(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)\left(-e^2x^2+d^2\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x^3 + d*x^2)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(95) = 190.

Time = 1.44 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.21

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{dd^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + e\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**
2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*
e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)
), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**
2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
 e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True))
```

Maxima [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x^2\,\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)\,dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.244 $\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d(d^2-e^2x^2)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(p+1)/e^2/(p+1)+1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom(
[3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {778, 267, 372, 371}

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)-\frac{d(d^2-e^2x^2)^{p+1}}{2e^2(p+1)}$$

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int x\left(d^2-e^2x^2\right)^p\,dx+e\int x^2\left(d^2-e^2x^2\right)^p\,dx\\ &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\ &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=-\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int x(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + d*x)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=d\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}ex^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**p,x)
[Out] d*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2
)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tr
ue)) + d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)
/d**2)/3
```

Maxima [F]

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^2*e^(p*log(e*x + d) + p*log(-e*x + d)), x) - 1/2*(-e^2*x^2 +
d^2)^(p + 1)*d/(e^2*(p + 1))
```

Giac [F]

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int x\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.245 $\int (d+ex) \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (d+ex) \left(d^2-e^2x^2\right)^p \, dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)} + dx\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(p+1)/e/(p+1)+d*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],
[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {655, 252, 251}

$$\int (d+ex) \left(d^2-e^2x^2\right)^p \, dx = dx\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

```
[In] Int[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{1+p}}{2e(1+p)}+d\int (d^2-e^2x^2)^p\,dx\\
&= -\frac{(d^2-e^2x^2)^{1+p}}{2e(1+p)}+\left(d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&= -\frac{(d^2-e^2x^2)^{1+p}}{2e(1+p)}+dx(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx = -\frac{(d^2-e^2x^2)^{1+p}}{2e(1+p)}+dx(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

```
[In] Integrate[(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Maple [F]

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p dx$$

[In] int((e*x+d)*(-e^2*x^2+d^2)^p,x)

[Out] int((e*x+d)*(-e^2*x^2+d^2)^p,x)

Fricas [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p dx$$

[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p, x)

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx = dd^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + e\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p,x)

[Out] d*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e
*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)
(p + 1)/(p + 1), Ne(p, -1)), (log(d2 - e**2*x**2), True))/(2*e**2), Tru
e))

Maxima [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

Giac [F]

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx=\frac{d\,x\,\left(d^2-e^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^p}-\frac{\left(d^2-e^2\,x^2\right)^{p+1}}{2\,e\,\left(p+1\right)}$$

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] (d*x*(d^2 - e^2*x^2)^p*hypergeom([1/2, -p], 3/2, (e^2*x^2)/d^2))/(1 - (e^2*
x^2)/d^2)^p - (d^2 - e^2*x^2)^(p + 1)/(2*e*(p + 1))
```

3.246 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x} dx = ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

```
[Out] e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2
)^p)-1/2*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/d/(p+
1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 272, 67, 252, 251}

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x} dx = ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{(d^2-e^2x^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(d^2-e^2x^2)^p}{x}\,dx + e\int \left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$

$$= ex(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{1+p} {}_2F_1\left(1, 1+p; 2+p; 1 - \frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x} dx = ex(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{1+p} \text{Hypergeometric2F1}\left(1, 1+p, 2+p, 1 - \frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Maple [F]

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x} dx$$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x} dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x} dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx=-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x,x)
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2,), e**2*x
**2*exp_polar(2*I*pi)/d**2)
```

Maxima [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx=\int \frac{(ex+d)(-e^2x^2+d^2)^p}{x}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx=\int \frac{(ex+d)(-e^2x^2+d^2)^p}{x}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx=\int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)}{x}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x, x)
```

3.247 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

```
[Out] -d*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/d
^2/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 372, 371, 272, 67}

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(d^2-e^2x^2)^p}{x^2}\,dx + e\int \frac{(d^2-e^2x^2)^p}{x}\,dx$$

$$= \frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,dx$$

$$= -\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Maple [F]

$$\int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}dx$$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{dd^{2p}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)
[Out] -d*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x -
 e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx=\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)}{x^2}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2, x)
```

3.248 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^3(1+p)}$$

```
[Out] -e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^(p+1)*hypergeom([2, p+1],[2+p],1-e^2*x^2/d^2)
/d^3/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 272, 67, 372, 371}

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(2,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(d^2-e^2x^2)^p}{x^3}\,dx + e\int \frac{(d^2-e^2x^2)^p}{x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,dx$$

$$= -\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2-e^2x^2)^{1+p}{}_2F_1\left(2,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = \frac{1}{2}e\left(d^2-e^2x^2\right)^p\left(-\frac{2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{e(-d^2+e^2x^2)\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1+p)}\right)$$

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
[Out] (e*(d^2 - e^2*x^2)^p*((-2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/
(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*Hypergeometric2F1[2, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2])/(d^3*(1 + p))))/2
```

Maple [F]

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}dx$$

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^3}\,dx = -\frac{de^{2p}x^{2p-2}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p,1-p \\ 2-p \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(2-p\right)} - \frac{d^{2p}e\,{}_2F_1\left(\begin{matrix} -\frac{1}{2},-p \\ \frac{1}{2} \end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}$$

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p
,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - d**(2*p)*e*hyper((-1/2, -p), (1/2,
), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
```

Maxima [F]

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\,(d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)}{x^3}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3, x)
```

3.249 $\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=-\frac{d^6(d^2-e^2x^2)^{1+p}}{e^6(1+p)}+\frac{5d^4(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{2d^2(d^2-e^2x^2)^{3+p}}{e^6(3+p)}+\frac{(d^2-e^2x^2)^{4+p}}{2e^6(4+p)}+\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -d^6*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)+5/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
2*d^2*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/2*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7
*d*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^
2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1666, 457, 78, 12, 372, 371}

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)-\frac{2d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)}+\frac{(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}$$

```
[In] Int[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (5*d^4*(d^2 - e^2*x^2)^(2
+ p))/(2*e^6*(2 + p)) - (2*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^
2 - e^2*x^2)^(4 + p)/(2*e^6*(4 + p)) + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int 2dex^6\left(d^2-e^2x^2\right)^p\,dx + \int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx, x, x^2\right) + (2de)\int x^6\left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6(d^2-e^2x)^p}{e^4} - \frac{5d^4(d^2-e^2x)^{1+p}}{e^4} + \frac{4d^2(d^2-e^2x)^{2+p}}{e^4} - \frac{(d^2-e^2x)^{3+p}}{e^4}\right)dx, x, x^2\right)$$

$$+ \left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$

$$= -\frac{d^6(d^2-e^2x^2)^{1+p}}{e^6(1+p)} + \frac{5d^4(d^2-e^2x^2)^{2+p}}{2e^6(2+p)} - \frac{2d^2(d^2-e^2x^2)^{3+p}}{e^6(3+p)} + \frac{(d^2-e^2x^2)^{4+p}}{2e^6(4+p)} + \frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{(d^2-e^2x^2)^p\left(-\frac{14d^6(d^2-e^2x^2)}{1+p} + \frac{35d^4(d^2-e^2x^2)^2}{2+p} - \frac{28d^2(d^2-e^2x^2)^3}{3+p} + \frac{7(d^2-e^2x^2)^4}{4+p} + 4de^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hyperge}\ldots\right.}{14e^6}$$

```
[In] Integrate[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-14*d^6*(d^2 - e^2*x^2))/(1 + p) + (35*d^4*(d^2 - e^2*
x^2)^2)/(2 + p) - (28*d^2*(d^2 - e^2*x^2)^3)/(3 + p) + (7*(d^2 - e^2*x^2)^4
)/(4 + p) + (4*d*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F]

$$\int x^5 (ex+d)^2 \left(-e^2 x^2+d^2\right)^p dx$$

[In] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

[Out] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

Fricas [F]

$$\int x^5 (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^5 \, dx$$

[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(-e^2*x^2 + d^2)^p, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(150) = 300.

Time = 2.90 (sec) , antiderivative size = 2924, normalized size of antiderivative = 16.43

$$\int x^5 (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \text{Too large to display}$$

[In] integrate(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)

[Out] d**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x*

*2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
+ e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**
6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6
*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p*
*3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 2*d*d**(2*p)*e*x**7*hype
r((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e**2*Piecewise((
x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-
12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 1
1*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*
x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2
 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-1
2*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 27
*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e
10*x2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e
 + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x
6) - 18*d2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**1
2*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e +
x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6
), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*
e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**1
2*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4
*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*
x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
- 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12
*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
, Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6
*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**1
0*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*
d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**
4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2),
Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d*
*4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8
*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**
8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20
*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(
d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*
p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
8*p3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2
 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +

```
48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e*
*8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 -
 e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 +
 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)
**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11
*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
*2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4
 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True))
```

Maxima [F]

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2\left(-e^2x^2+d^2\right)^px^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^2
*x^7 + 2*d*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Giac [F]

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2\left(-e^2x^2+d^2\right)^px^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^5\,\left(d^2-e^2\,x^2\right)^p\,{\left(d+e\,x\right)}^2\,dx$$

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.250 $\int x^4(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int x^4(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^5(d^2-e^2x^2)^{1+p}}{e^5(1+p)} - \frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p} + \frac{2d^3(d^2-e^2x^2)^{2+p}}{e^5(2+p)} - \frac{d(d^2-e^2x^2)^{3+p}}{e^5(3+p)} + \frac{2d^2(6+p)x^5(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)-x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)+2*d^3*
(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-d*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+2/5*d^2*(6+
p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-
e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\int x^4(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{2d^2(p+6)x^5(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)} - \frac{x^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{d(d^2-e^2x^2)^{p+3}}{e^5(p+3)} - \frac{d^5(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{2d^3(d^2-e^2x^2)^{p+2}}{e^5(p+2)}$$

```
[In] Int[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p))) - (x^5*(d^2 - e^2*x^2)^(1 +
p))/(7 + 2*p) + (2*d^3*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d*(d^2 - e
^2*x^2)^(3 + p))/(e^5*(3 + p)) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)
^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2dex^5\left(d^2-e^2x^2\right)^p\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= -\frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p}+(2de)\int x^5\left(d^2-e^2x^2\right)^p\,dx+\frac{(2d^2(6+p))\int x^4(d^2-e^2x^2)^p\,dx}{7+2p}\\
&= -\frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p}+(de)\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2(6+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{7+2p}\\
&= -\frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{2d^2(6+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(7+2p)}\\
&\quad+(de)\text{Subst}\left(\int\left(\frac{d^4(d^2-e^2x)^p}{e^4}-\frac{2d^2(d^2-e^2x)^{1+p}}{e^4}+\frac{(d^2-e^2x)^{2+p}}{e^4}\right)dx,x,x^2\right)\\
&= -\frac{d^5(d^2-e^2x^2)^{1+p}}{e^5(1+p)}-\frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{2d^3(d^2-e^2x^2)^{2+p}}{e^5(2+p)}-\frac{d(d^2-e^2x^2)^{3+p}}{e^5(3+p)}\\
&\quad+\frac{2d^2(6+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(7+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{35}\left(d^2-e^2x^2\right)^p\left(-\frac{35d^5(d^2-e^2x^2)}{e^5(1+p)}+\frac{70d^3(d^2-e^2x^2)^2}{e^5(2+p)}-\frac{35d(d^2-e^2x^2)^3}{e^5(3+p)}+7d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+5e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)\right)$$

```
[In] Integrate[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-35*d^5*(d^2 - e^2*x^2))/(e^5*(1 + p)) + (70*d^3*(d^2
- e^2*x^2)^2)/(e^5*(2 + p)) - (35*d*(d^2 - e^2*x^2)^3)/(e^5*(3 + p)) + (7*d
^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^
p + (5*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^
2)/d^2)^p))/35
```

Maple [F]

$$\int x^4(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
[Out] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(153) = 306$.

Time = 2.39 (sec) , antiderivative size = 1015, normalized size of antiderivative = 5.49

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \frac{d^2d^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

$$+2de\left(\begin{cases}\frac{x^6(d^2)^p}{6} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2}{4d^4e^6-8d^2} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2} \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2} \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}\end{cases}\right.$$

$$+\frac{d^{2p}e^2x^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

```
[In] integrate(x**4*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 2*d*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)
/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*
e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*
d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8
*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/
e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4
*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p
**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e*
*6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 -
 e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e*
```

```
*4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 1
2*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 +
 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e**2*x*
*7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Maxima [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^px^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^px^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.251 $\int x^3(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int x^3(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^4(d^2-e^2x^2)^{1+p}}{e^4(1+p)} + \frac{3d^2(d^2-e^2x^2)^{2+p}}{2e^4(2+p)} - \frac{(d^2-e^2x^2)^{3+p}}{2e^4(3+p)} + \frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -d^4*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)+3/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2/5*d*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([
5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1666, 457, 78, 12, 372, 371}

$$\int x^3(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right) + \frac{3d^2(d^2-e^2x^2)^{p+2}}{2e^4(p+2)} - \frac{(d^2-e^2x^2)^{p+3}}{2e^4(p+3)} - \frac{d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)}$$

```
[In] Int[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (3*d^2*(d^2 - e^2*x^2)^(2
+ p))/(2*e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^4*(3 + p)) + (2*d*e*x^
5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 -
 (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2dex^4\left(d^2-e^2x^2\right)^p\,dx + \int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx, x, x^2\right) + (2de)\int x^4\left(d^2-e^2x^2\right)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^4(d^2-e^2x)^p}{e^2} - \frac{3d^2(d^2-e^2x)^{1+p}}{e^2} + \frac{(d^2-e^2x)^{2+p}}{e^2}\right)dx, x, x^2\right)\\
&\quad + \left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&= -\frac{d^4(d^2-e^2x^2)^{1+p}}{e^4(1+p)} + \frac{3d^2(d^2-e^2x^2)^{2+p}}{2e^4(2+p)} - \frac{(d^2-e^2x^2)^{3+p}}{2e^4(3+p)}\\
&\quad + \frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{(d^2-e^2x^2)^p\left(-\frac{5(d^2-e^2x^2)\left(d^4(5+p)+d^2e^2(5+6p+p^2)x^2+e^4(2+3p+p^2)x^4\right)}{(1+p)(2+p)(3+p)} + 4de^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\right.}{10e^4}$$

```
[In] Integrate[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(d^4*(5 + p) + d^2*e^2*(5 + 6*p + p
^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (4*d*e^5*x^
5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(
10*e^4)
```

Maple [F]

$$\int x^3(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(124) = 248.

Time = 2.13 (sec) , antiderivative size = 1328, normalized size of antiderivative = 8.91

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x**3*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
[Out] d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/5 + e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2
*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**
8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d
/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(
4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*
```

```
p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e
**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2
- e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e
**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(
2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Maxima [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^2/((p^2 +
3*p + 2)*e^4) + integrate((e^2*x^5 + 2*d*e*x^4)*e^(p*log(e*x + d) + p*log(-
e*x + d)), x)
```

Giac [F]

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^3\,\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^2\,dx$$

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.252 $\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^3(d^2-e^2x^2)^{1+p}}{e^3(1+p)} - \frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p} + \frac{d(d^2-e^2x^2)^{2+p}}{e^3(2+p)} + \frac{2d^2(4+p)x^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(5+2p)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)-x^3*(-e^2*x^2+d^2)^(p+1)/(5+2*p)+d*(-e^
2*x^2+d^2)^(2+p)/e^3/(2+p)+2/3*d^2*(4+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/
2, -p],[5/2],e^2*x^2/d^2)/(5+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{2d^2(p+4)x^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)} - \frac{x^3(d^2-e^2x^2)^{p+1}}{2p+5} + \frac{d(d^2-e^2x^2)^{p+2}}{e^3(p+2)} - \frac{d^3(d^2-e^2x^2)^{p+1}}{e^3(p+1)}$$

```
[In] Int[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p))) - (x^3*(d^2 - e^2*x^2)^(1 +
p))/(5 + 2*p) + (d*(d^2 - e^2*x^2)^(2 + p))/(e^3*(2 + p)) + (2*d^2*(4 + p)*
x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5
 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int 2dex^3\left(d^2-e^2x^2\right)^p\,dx+\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+(2de)\int x^3\left(d^2-e^2x^2\right)^p\,dx+\frac{\left(2d^2(4+p)\right)\int x^2\left(d^2-e^2x^2\right)^p\,dx}{5+2p}\\
&=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+(de)\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2(4+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{5+2p}\\
&=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(5+2p)}\\
&\quad+(de)\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^p}{e^2}-\frac{\left(d^2-e^2x\right)^{1+p}}{e^2}\right)dx,x,x^2\right)\\
&=-\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{d\left(d^2-e^2x^2\right)^{2+p}}{e^3(2+p)}\\
&\quad+\frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(5+2p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-15d\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(d^2+e^2(1+p)x^2\right)+5d^2e^3\left(2+3p+p^2\right)x^3\,\text{Hyper}\right.}{15e^3(1+p)(2+p}$$

```
[In] Integrate[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*(-15*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(d^2 + e^2*
(1 + p)*x^2) + 5*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2] + 3*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2]))/(15*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int x^2(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

[In] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

[Out] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

Fricas [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(-e^2*x^2 + d^2)^p, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(128) = 256$.

Time = 1.95 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.74

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\frac{d^2d^{2p}x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+2de\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}e^2x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

[In] integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)

[Out] d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2

```
) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e*
*4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq
(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2
/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/5
```

Maxima [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x^2\,\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^2\,dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.253 $\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^2(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)$$

```
[Out] -d^2*(-e^2*x^2+d^2)^(p+1)/e^2/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^2/(2+p)+2/3*
d*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2
/d^2)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1666, 455, 45, 12, 372, 371}

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^2(p+2)}$$

```
[In] Int[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^2*(2 + p)) + (2*d*e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int 2dex^2\left(d^2-e^2x^2\right)^p\,dx+\int x\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(d^2-e^2x^2\right)^p\,dx\end{aligned}$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(2d^2\left(d^2-e^2x\right)^p-\left(d^2-e^2x\right)^{1+p}\right)dx,x,x^2\right)$$
$$+\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p dx$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^2(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^2(2+p)}$$
$$+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{3\left(d^2-e^2x^2\right)\left(d^2(3+p)+e^2(1+p)x^2\right)}{(1+p)(2+p)}+4de^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)}{6e^2}$$

```
[In] Integrate[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2*(3 + p) + e^2*(1 + p)*x^2))/((
1 + p)*(2 + p)) + (4*d*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^
2])/(1 - (e^2*x^2)/d^2)^p))/(6*e^2)
```

Maple [F]

$$\int x(ex+d)^2\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
[Out] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2\left(-e^2x^2+d^2\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.73

$$\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$$

$$= d^2 \left(\begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases} \right) + \frac{2dd^{2p}ex^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

$$+ e^2 \left(\begin{cases} \frac{x^4 (d^2)^p}{4} & \text{for } e = 0 \\ -\frac{d^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p = -2 \\ -\frac{d^2 \log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases} \right)$$

```
[In] integrate(x*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)/3 + e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-
d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e*
*4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4), True))
```

Maxima [F]

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2(-e^2x^2+d^2)^px\,dx$$

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(p + 1)*d^2/(e^2*(p + 1)) + integrate((e^2*x^3 + 2*d*
e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Giac [F]

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^2(-e^2x^2+d^2)^px\,dx$$

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int x\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.254 $\int (d+ex)^2 (d^2-e^2x^2)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int (d+ex)^2 (d^2-e^2x^2)^p \, dx = -\frac{2^{2+p} d \left(1+\frac{ex}{d}\right)^{-1-p} (d^2-e^2x^2)^{1+p} \text{Hypergeometric2F1}\left(-2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

```
[Out] -2^(2+p)*d*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -2-p],[2+p
],1/2*(-e*x+d)/d)/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {692, 71}

$$\int (d+ex)^2 (d^2-e^2x^2)^p \, dx = -\frac{d 2^{p+2} \left(\frac{ex}{d}+1\right)^{-p-1} (d^2-e^2x^2)^{p+1} \text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[In] Int[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(d(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{2+p}\,dx\\ &= -\frac{2^{2+p}d\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-3d\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p+3d^2e(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right.\right.}{3e(1+p)}$$

```
[In] Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-3*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p + 3*d^2*e*(1
 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + e^3*(1 + p)*x^3*Hy
pergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]))/(3*e*(1 + p)*(1 - (e^2*x^2)/
d^2)^p)
```

Maple [F]

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p dx$$

[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)

[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)

Fricas [F]

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p \, dx$$

[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p, x)

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.75

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = d^2 d^{2p} x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 2de \left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + \frac{d^{2p}e^2x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p,x)

[Out] d**2*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 2*d*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3

Maxima [F]

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p \,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

Giac [F]

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx = \int (ex+d)^2 \left(-e^2x^2+d^2\right)^p \,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \,dx = \int \left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^2 \,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.255 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\begin{aligned}\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}\\ &\quad+2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\\ &\quad-\frac{\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}\end{aligned}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(p+1)/(p+1)+2*d*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p
],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^(p+1)*hypergeom
([1, p+1],[2+p],1-e^2*x^2/d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1666, 457, 81, 67, 12, 252, 251}

$$\begin{aligned}\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx &= 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\\ &\quad-\frac{\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2(p+1)}\end{aligned}$$

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(1 + p) + (2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
 p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2de\left(d^2-e^2x^2\right)^p\,dx+\int \frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)+(2de)\int \left(d^2-e^2x^2\right)^p\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}+\frac{1}{2}d^2\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)\\
&\quad+\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}+2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)\\
&\quad-\frac{\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx\\
&= \frac{1}{2}\left(d^2-e^2x^2\right)^p\left(4dex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\right.\\
&\qquad\left.-\frac{\left(d^2-e^2x^2\right)\left(1+\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)\right)}{1+p}\right)
\end{aligned}$$

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)*(1 + Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2]))/(1 + p)))/2
```

Maple [F]

$$\int \frac{(ex+d)^2 (-e^2x^2+d^2)^p}{x} dx$$

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x} dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} dx = -\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p,-p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 2dd^{2p}ex{}_2F_1\left(\begin{matrix} \frac{1}{2},-p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^2\left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 2*d*d**(2*p)*e*x*hyper((1/2, -p), (3/2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*Piecewise((x**2*(d**2)**p/2, Eq(e
**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (lo
g(d**2 - e**2*x**2), True))/(2*e**2), True))
```

Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x} \, dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x} \, dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x, x)
```

3.256 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x} - 2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{e\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{d(1+p)}$$

```
[Out] -(-e^2*x^2+d^2)^(p+1)/x-2*e^2*p*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2
],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1
],[2+p],1-e^2*x^2/d^2)/d/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1821, 778, 272, 67, 252, 251}

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{e\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{d(p+1)} - \frac{\left(d^2-e^2x^2\right)^{p+1}}{x}$$

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{1+p}}{x} - \frac{\int \frac{(-2d^3e+2d^2e^2px)(d^2-e^2x^2)^p}{x}\,dx}{d^2}$$
$$= -\frac{(d^2-e^2x^2)^{1+p}}{x} + (2de)\int \frac{(d^2-e^2x^2)^p}{x}\,dx - (2e^2p)\int (d^2-e^2x^2)^p\,dx$$
$$= -\frac{(d^2-e^2x^2)^{1+p}}{x} + (de)\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) - \left(2e^2p(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx$$
$$= -\frac{(d^2-e^2x^2)^{1+p}}{x} - 2e^2px(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{e(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1, 1+p; 2+p; 1-\frac{e^2x^2}{d^2}\right)}{d(1+p)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-d^3(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, -p, \frac{1}{2}, \frac{e^2x^2}{d^2}\right) + ex\left(de(1+p)x\,\text{Hypergeo}\right.\right.}{d(1+}$$

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-(d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2
)/d^2]) + e*x*(d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
 - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 - (e^2*x^2)/d^2])))/(d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}dx$$

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^2, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d^2d^{2p}{}_2F_1\left(\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{dee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{\Gamma(1-p)} + d^{2p}e^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**2,x)

[Out] -d**2*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - d*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/gamma(1 - p) + d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)

Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")

[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)

Giac [F]

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x^2}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.257 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2} - \frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(1-p)\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(p+1)/x^2-2*d*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[
1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d^2)^p)-1/2*e^2*(1-p)*(-e^2*x^2+d^2)^(p+1)*
hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/d^2/(p+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1821, 778, 372, 371, 272, 67}

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)} - \frac{\left(d^2-e^2x^2\right)^{p+1}}{2x^2}$$

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/x^2 - (2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^
2])/(2*d^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{\int \frac{(-4d^3e-2d^2e^2(1-p)x)(d^2-e^2x^2)^p}{x^2}\,dx}{2d^2} \\
&= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} + (2de)\int \frac{(d^2-e^2x^2)^p}{x^2}\,dx + \left(e^2(1-p)\right)\int \frac{(d^2-e^2x^2)^p}{x}\,dx \\
&= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} + \frac{1}{2}\left(e^2(1-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) \\
&\quad + \left(2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,dx \\
&= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} \\
&\quad - \frac{e^2(1-p)(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = \frac{e(d^2-e^2x^2)^p\left(-\frac{4d^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{e(-d^2+e^2x^2)\left(\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)+\text{H}\right.}{1+p}\right.}{2d^2}$$

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-4*d^3*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*(Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e
^2*x^2)/d^2]))/(1 + p)))/(2*d^2)
```

Maple [F]

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}dx$$

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx = -\frac{d^2e^{2p}x^{2p-2}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)} - \frac{2dd^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{e^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}$$

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**3,x)
[Out] -d**2*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2
- p,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - 2*d*d**(2*p)*e*hyper((-1/2, -p)
, (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e**2*e**(2*p)*x**(2*p)*exp(
I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(1 -
p))
```

Maxima [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^2}{x^3}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.258 $\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\begin{aligned}\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx &= -\frac{2d^7\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)} - \frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p} \\ &\quad + \frac{11d^5\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)} - \frac{5d^3\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)} + \frac{3d\left(d^2-e^2x^2\right)^{4+p}}{2e^6(4+p)} \\ &\quad + \frac{2d^2e(17+3p)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7(9+2p)}\end{aligned}$$

```
[Out] -2*d^7*(-e^2*x^2+d^2)^(p+1)/e^6/(p+1)-e*x^7*(-e^2*x^2+d^2)^(p+1)/(9+2*p)+11
/2*d^5*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-5*d^3*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+
3/2*d*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7*d^2*e*(17+3*p)*x^7*(-e^2*x^2+d^2)^
p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/(9+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{1666, 457, 78, 470, 372, 371\}$

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{2d^2e(3p+17)x^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7(2p+9)}$$
$$-\frac{ex^7(d^2-e^2x^2)^{p+1}}{2p+9}+\frac{3d(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{2d^7(d^2-e^2x^2)^{p+1}}{e^6(p+1)}$$
$$+\frac{11d^5(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}-\frac{5d^3(d^2-e^2x^2)^{p+3}}{e^6(p+3)}$$

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1
+ p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3
*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*
e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^6\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= -\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2e(17+3p)\right)\int x^6\left(d^2-e^2x^2\right)^p\,dx}{9+2p}\\
&= -\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^7\left(d^2-e^2x\right)^p}{e^4}-\frac{11d^5\left(d^2-e^2x\right)^{1+p}}{e^4}\right.\right.\\
&\qquad\left.\left.+\frac{10d^3\left(d^2-e^2x\right)^{2+p}}{e^4}-\frac{3d\left(d^2-e^2x\right)^{3+p}}{e^4}\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2e(17+3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{9+2p}\\
&= -\frac{2d^7\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{11d^5\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{5d^3\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)}\\
&\quad+\frac{3d\left(d^2-e^2x^2\right)^{4+p}}{2e^6(4+p)}+\frac{2d^2e(17+3p)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(9+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{252d^7\left(d^2-e^2x^2\right)}{1+p}+\frac{693d^5\left(d^2-e^2x^2\right)^2}{2+p}+\frac{189d\left(d^2-e^2x^2\right)^4}{4+p}-\frac{630\left(d^3-de^2x^2\right)^3}{3+p}+54d^2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hyp}\right.}{126e^6}$$

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-252*d^7*(d^2 - e^2*x^2))/(1 + p) + (693*d^5*(d^2 - e^
2*x^2)^2)/(2 + p) + (189*d*(d^2 - e^2*x^2)^4)/(4 + p) - (630*(d^3 - d*e^2*x
^2)^3)/(3 + p) + (54*d^2*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/
d^2])/(1 - (e^2*x^2)/d^2)^p + (14*e^9*x^9*Hypergeometric2F1[9/2, -p, 11/2,
(e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(126*e^6)
```

Maple [F]

$$\int x^5(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(190) = 380$.

Time = 3.68 (sec) , antiderivative size = 2966, normalized size of antiderivative = 13.36

$$\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \text{Too large to display}$$

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2
*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2
*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) +
 e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(
2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x*
*2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
+ e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**
6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6
*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p*
*3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d**2*d**(2*p)*e*x**7*h
yper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + 3*d*e**2*Piec
ewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 +
 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e
+ x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x*
*6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12
*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**
10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e +
 x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**
6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*
x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36
*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*
log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*
```

e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)) + d**(2*p)*e**3*x**9*hyper((9/2, -p), (11/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/9

Maxima [F]

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^3
*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Giac [F]

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^5\,\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^3\,dx$$

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.259 $\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{2d^6\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)}-\frac{3d^2\left(d^2-e^2x^2\right)^{3+p}}{e^5(3+p)}+\frac{\left(d^2-e^2x^2\right)^{4+p}}{2e^5(4+p)}+\frac{2d^3(11+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)-3*d*x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)+
9/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-3*d^2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)
+1/2*(-e^2*x^2+d^2)^(4+p)/e^5/(4+p)+2/5*d^3*(11+p)*x^5*(-e^2*x^2+d^2)^p*hyp
ergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{1666, 470, 372, 371, 457, 78\}$

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$= -\frac{3dx^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{3d^2(d^2-e^2x^2)^{p+3}}{e^5(p+3)} + \frac{(d^2-e^2x^2)^{p+4}}{2e^5(p+4)} - \frac{2d^6(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{9d^4(d^2-e^2x^2)^{p+2}}{2e^5(p+2)} + \frac{2d^3(p+11)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}$$

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(
1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^
2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(
4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4 \left(d^2-e^2x^2\right)^p \left(d^3+3de^2x^2\right)\,dx+\int x^5 \left(d^2-e^2x^2\right)^p \left(3d^2e+e^3x^2\right)\,dx \\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x^2 \left(d^2-e^2x\right)^p \left(3d^2e+e^3x\right)\,dx,x,x^2\right) \\
&\quad+\frac{\left(2d^3(11+p)\right)\int x^4\left(d^2-e^2x^2\right)^p\,dx}{7+2p} \\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{4d^6\left(d^2-e^2x\right)^p}{e^3}-\frac{9d^4\left(d^2-e^2x\right)^{1+p}}{e^3}\right.\right. \\
&\quad\left.\left.+\frac{6d^2\left(d^2-e^2x\right)^{2+p}}{e^3}-\frac{\left(d^2-e^2x\right)^{3+p}}{e^3}\right)\,dx,x,x^2\right) \\
&\quad+\frac{\left(2d^3(11+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{7+2p} \\
&= -\frac{2d^6\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)}-\frac{3d^2\left(d^2-e^2x^2\right)^{3+p}}{e^5(3+p)} \\
&\quad+\frac{\left(d^2-e^2x^2\right)^{4+p}}{2e^5(4+p)}+\frac{2d^3(11+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(7+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\frac{1}{70}\left(d^2-e^2x^2\right)^p\left(-\frac{35(d^2-e^2x^2)\left(6d^6(5+p)+6d^4e^2(5+6p+p^2)x^2+3d^2e^4(10+17p+8p^2+p^3)x^4+e^6(6+\right.}{e^5(1+p)(2+p)(3+p)(4+p)}+14d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)+30de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)\right)$$

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*(d^2 - e^2*x^2)*(6*d^6*(5 + p) + 6*d^4*e^2*(5 + 6*
p + p^2)*x^2 + 3*d^2*e^4*(10 + 17*p + 8*p^2 + p^3)*x^4 + e^6*(6 + 11*p + 6*
p^2 + p^3)*x^6))/(e^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyperg
eometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (30*d*e^2
*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p)
)/70
```

Maple [F]

$$\int x^4(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(185) = 370$.

Time = 3.36 (sec) , antiderivative size = 2966, normalized size of antiderivative = 13.61

$$\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \text{Too large to display}$$

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 3*d**2*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d*
*2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) -
 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d
**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(
-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4
/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**
2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2
*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
+ 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
*2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**
3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d*d**(2*p)*
e**2*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e*
*3*Piecewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*l
og(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e
**14*x**6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x*
*4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d
**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*lo
g(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e*
*14*x**6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2
*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e*
*8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**
```

```
4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**
4 + 12*e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2
- 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x
**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e
**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10
*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x
**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x
**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
4*e**12*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*
x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*
e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*
x**2) - 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2
*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e*
*10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3
*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4
*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)
/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p,
-1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*
p**2 + 100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*
e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e*
*4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
*2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e*
*8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p
**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**
8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*
p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 1
00*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 +
20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**
2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +
 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**
3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**
p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)
)
```

Maxima [F]

$$\int x^4(d+ex)^3 (d^2-e^2x^2)^p \, dx = \int (ex+d)^3 (-e^2x^2+d^2)^p x^4 \, dx$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex)^3 (d^2-e^2x^2)^p \, dx = \int (ex+d)^3 (-e^2x^2+d^2)^p x^4 \, dx$$

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3 (d^2-e^2x^2)^p \, dx = \int x^4 (d^2-e^2 x^2)^p (d+e x)^3 \, dx$$

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.260 $\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{2d^5\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{7d^3\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{3d\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}+\frac{2d^2e(13+3p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(7+2p)}$$

```
[Out] -2*d^5*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)-e*x^5*(-e^2*x^2+d^2)^(p+1)/(7+2*p)+7/
2*d^3*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-3/2*d*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2
/5*d^2*e*(13+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^
2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1666, 457, 78, 470, 372, 371}

$$\int x^3(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{3d\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}-\frac{2d^5\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{7d^3\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}$$

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1
+ p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d
^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= -\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2e(13+3p)\right)\int x^4\left(d^2-e^2x^2\right)^p\,dx}{7+2p}\\
&= -\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^p}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{1+p}}{e^2}+\frac{3d\left(d^2-e^2x\right)^{2+p}}{e^2}\right)dx,x,x^2\right)\\
&\quad+\frac{\left(2d^2e(13+3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{7+2p}\\
&= -\frac{2d^5\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{7d^3\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{3d\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}\\
&\quad+\frac{2d^2e(13+3p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(7+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{35d\left(d^2-e^2x^2\right)\left(d^4(9+p)+d^2e^2\left(9+10p+p^2\right)x^2+3e^4\left(2+3p+p^2\right)x^4\right)}{(1+p)(2+p)(3+p)}+42d^2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometr}\right.}{70e^4}$$

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d*(d^2 - e^2*x^2)*(d^4*(9 + p) + d^2*e^2*(9 + 10*p
 + p^2)*x^2 + 3*e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (42*d
^2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d
^2)^p + (10*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e
^2*x^2)/d^2)^p))/(70*e^4)
```

Maple [F]

$$\int x^3(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)

[Out] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)

Fricas [F]

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^3\,dx$$

[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(-e^2*x^2 + d^2)^p
, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(165) = 330$.

Time = 2.79 (sec) , antiderivative size = 1370, normalized size of antiderivative = 7.10

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\text{Too large to display}$$

[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)

[Out] d**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
6*x2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 3*d**2*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/5 + 3*d*e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-
d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 -
 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**

```
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(
-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x*
*2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/
(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4
*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) -
 x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4
*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) -
 d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**
2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)
*e**3*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Maxima [F]

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^3\left(-e^2x^2+d^2\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^3/((p^2 +
3*p + 2)*e^4) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*e^(p*log(e*
x + d) + p*log(-e*x + d)), x)
```

Giac [F]

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^3\left(-e^2x^2+d^2\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^3\,dx$$

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.261 $\int x^2(d+ex)^3 (d^2-e^2x^2)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int x^2(d+ex)^3 (d^2-e^2x^2)^p \, dx = -\frac{2d^4(d^2-e^2x^2)^{1+p}}{e^3(1+p)} - \frac{3dx^3(d^2-e^2x^2)^{1+p}}{5+2p} + \frac{5d^2(d^2-e^2x^2)^{2+p}}{2e^3(2+p)} - \frac{(d^2-e^2x^2)^{3+p}}{2e^3(3+p)} + \frac{2d^3(7+p)x^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(5+2p)}$$

```
[Out] -2*d^4*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)-3*d*x^3*(-e^2*x^2+d^2)^(p+1)/(5+2*p)+
5/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)-1/2*(-e^2*x^2+d^2)^(3+p)/e^3/(3+p)+2
/3*d^3*(7+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/(5
+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 470, 372, 371, 457, 78}

$$\int x^2(d+ex)^3 (d^2-e^2x^2)^p \, dx = -\frac{3dx^3(d^2-e^2x^2)^{p+1}}{2p+5} + \frac{5d^2(d^2-e^2x^2)^{p+2}}{2e^3(p+2)} - \frac{(d^2-e^2x^2)^{p+3}}{2e^3(p+3)} - \frac{2d^4(d^2-e^2x^2)^{p+1}}{e^3(p+1)} + \frac{2d^3(p+7)x^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3(2p+5)}$$

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(
1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2
- e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/
d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^3(7+p)\right)\int x^2\left(d^2-e^2x^2\right)^p\,dx}{5+2p}\\
&= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^4\left(d^2-e^2x\right)^p}{e}-\frac{5d^2\left(d^2-e^2x\right)^{1+p}}{e}+\frac{\left(d^2-e^2x\right)^{2+p}}{e}\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^3(7+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{5+2p}\\
&= -\frac{2d^4\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{5d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}\\
&\quad-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^3(3+p)}+\frac{2d^3(7+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(5+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx\\
&= \frac{1}{30}\left(d^2-e^2x^2\right)^p\left(-\frac{15\left(d^2-e^2x^2\right)\left(d^4(11+3p)+d^2e^2\left(11+14p+3p^2\right)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{e^3(1+p)(2+p)(3+p)}\right.\\
&\quad+10d^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\\
&\quad\left.+18de^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)\right)
\end{aligned}$$

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-15*(d^2 - e^2*x^2)*(d^4*(11 + 3*p) + d^2*e^2*(11 + 14
*p + 3*p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/(e^3*(1 + p)*(2 + p)*(3 + p)) +
 (10*d^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)
/d^2)^p + (18*d*e^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p))/30
```

Maple [F]

$$\int x^2(ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(160) = 320$.

Time = 2.55 (sec) , antiderivative size = 1370, normalized size of antiderivative = 7.25

$$\int x^2(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \text{Too large to display}$$

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 3*d**2*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2),
 Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x
**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
```

```
4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4
), True)) + 3*d*d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/5 + e**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**
4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*lo
g(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e
 + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*
e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/
e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)),
 (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*
e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2
*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*
*2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*
e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e
**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Maxima [F]

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int(ex+d)^3\left(-e^2x^2+d^2\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^2\left(d^2-e^2x^2\right)^p(d+e\,x)^3\,dx$$

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.262 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} - \frac{3\ 2^{3+p} d^3 \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e^2(1+p)(5+2p)}$$

```
[Out] -(e*x+d)^3*(-e^2*x^2+d^2)^(p+1)/e^2/(5+2*p)-3*2^(3+p)*d^3*(1+e*x/d)^(-1-p)*
(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -3-p],[2+p],1/2*(-e*x+d)/d)/e^2/(2*p^2
+7*p+5)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {809, 692, 71}

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)} - \frac{3d^3 2^{p+3} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1} \text{Hypergeometric2F1}\left(-p-3,p+1,p+2,\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}$$

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^3
*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}+\frac{(3d)\int(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx}{e(5+2p)}\\
&= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}\\
&\quad+\frac{\left(3d^3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{3+p}\,dx}{e(5+2p)}\\
&= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}\\
&\quad-\frac{3\ 2^{3+p}d^3\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\ {}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e^2(1+p)(5+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.37

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$$
$$= \frac{\left(d^2-e^2x^2\right)^p \left(-\frac{5d(d^2-e^2x^2)\left(d^2(5+p)+3e^2(1+p)x^2\right)}{(1+p)(2+p)} + 10d^2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right.\right.}{10e^2}$$

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2*(5 + p) + 3*e^2*(1 + p)*x^2)
)/((1 + p)*(2 + p)) + (10*d^2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*
x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/
2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^2)
```

Maple [F]

$$\int x(ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x \, dx$$

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(-e^2*x^2 + d^2)^p,
x)
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.13

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$

$$= d^3\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+d^2d^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

$$+3de^2\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

$$+\frac{d^{2p}e^3x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar
(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*lo
g(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4
+ 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2
*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 +
6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/5
```

Maxima [F]

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3(-e^2x^2+d^2)^p x \, dx$$

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(p + 1)*d^3/(e^2*(p + 1)) + integrate((e^3*x^4 + 3*d*
e^2*x^3 + 3*d^2*e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Giac [F]

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3(-e^2x^2+d^2)^p x \, dx$$

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int x \left(d^2-e^2 x^2\right)^p (d+e x)^3 \, dx$$

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.263 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{3+p} d^2 \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

```
[Out] -2^(3+p)*d^2*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -3-p],[2
+p],1/2*(-e*x+d)/d)/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {692, 71}

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = -\frac{d^2 2^{p+3} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(-p-3,p+1,p+2,\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(d^2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{3+p}dx\\ &= -\frac{2^{3+p}d^2\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 155 vs. $2(73) = 146$.

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \frac{1}{2}\left(d^2-e^2x^2\right)^p\left(\frac{\left(-d^2+e^2x^2\right)\left(d^2(7+3p)+e^2(1+p)x^2\right)}{e(1+p)(2+p)} + 2d^3x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) + 2de^2x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(((-d^2 + e^2*x^2)*(d^2*(7 + 3*p) + e^2*(1 + p)*x^2))/(e
*(1 + p)*(2 + p)) + (2*d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p + (2*d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2
*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/2
```

Maple [F]

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(58) = 116$.

Time = 1.85 (sec) , antiderivative size = 476, normalized size of antiderivative = 6.52

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = d^3 d^{2p} x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3d^2e \left(\begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + dd^{2p}e^2x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^3 \left(\begin{cases} \frac{x^4 (d^2)^p}{4} & \text{for } e=0 \\ -\frac{d^2 \log(-\frac{d}{e}+x)}{-2d^2e^4+2e^6x^2} - \frac{d^2 \log(\frac{d}{e}+x)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log(-\frac{d}{e}+x)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log(\frac{d}{e}+x)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2 \\ -\frac{d^2 \log(-\frac{d}{e}+x)}{2e^4} - \frac{d^2 \log(\frac{d}{e}+x)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p=-1 \\ -\frac{d^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p,x)
[Out] d**3*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 3*d**2*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 -
```

```
e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*
e**2), True)) + d*d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp
_polar(2*I*pi)/d**2) + e**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*
log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e +
 x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/
(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4), True))
```

Maxima [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p \, dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

Giac [F]

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p \, dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p\,{\left(d+e\,x\right)}^3 \,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.264 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx &= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)}-\frac{ex(d^2-e^2x^2)^{1+p}}{3+2p}\\&\quad+\frac{2d^2e(5+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{3+2p}\\&\quad-\frac{d(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}\end{aligned}$$

```
[Out] -3/2*d*(-e^2*x^2+d^2)^(p+1)/(p+1)-e*x*(-e^2*x^2+d^2)^(p+1)/(3+2*p)+2*d^2*e*
(5+3*p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/(3+2*p)/(
(1-e^2*x^2/d^2)^p)-1/2*d*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^
2*x^2/d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{1666, 457, 81, 67, 396, 252, 251\}$

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx$$
$$= \frac{2d^2e(3p+5)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{2p+3}$$
$$- \frac{d(d^2-e^2x^2)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}$$
$$- \frac{ex(d^2-e^2x^2)^{p+1}}{2p+3} - \frac{3d(d^2-e^2x^2)^{p+1}}{2(p+1)}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (-3*d*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (e*x*(d^2 - e^2*x^2)^(1 + p))/
(3 + 2*p) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d^2-e^2x^2)^p\,(d^3+3de^2x^2)}{x}\,dx + \int (d^2-e^2x^2)^p\left(3d^2e+e^3x^2\right)\,dx \\
&= -\frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{1}{2}\text{Subst}\left(\int \frac{(d^2-e^2x)^p\,(d^3+3de^2x)}{x}\,dx, x, x^2\right) \\
&\quad + \frac{(2d^2e(5+3p))\int (d^2-e^2x^2)^p\,dx}{3+2p} \\
&= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{1}{2}d^3\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) \\
&\quad + \frac{\left(2d^2e(5+3p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{3+2p}
\end{aligned}$$

$$= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{2d^2e(5+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3+2p} - \frac{d(d^2-e^2x^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx = \frac{1}{6}(d^2-e^2x^2)^p\left(-\frac{9d(d^2-e^2x^2)}{1+p} + 18d^2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{3d(d^2-e^2x^2)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{1+p} + 2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)\right)$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-9*d*(d^2 - e^2*x^2))/(1 + p) + (18*d^2*e*x*Hypergeome
tric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*d*(d^2 - e
^2*x^2)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p) + (2
*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2
)^p))/6
```

Maple [F]

$$\int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x}\,dx$$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x}\,dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = -\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 3d^2d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3de^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(d^2-e^2x^2) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}e^3x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d**2*d**(2*p)*e*x*hyper((1/2, -p), (3/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**2*(d**2)**p/
2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**3*x**3*hy
per((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x} \, dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x} \, dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x} \, dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x, x)
```

3.265 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\begin{aligned}\int \frac{(d+ex)^3\,(d^2-e^2x^2)^p}{x^2}\,dx &= -\frac{e(d^2-e^2x^2)^{1+p}}{2(1+p)}-\frac{d(d^2-e^2x^2)^{1+p}}{x}\\ &\quad+2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\\ &\quad-\frac{3e(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}\end{aligned}$$

```
[Out] -1/2*e*(-e^2*x^2+d^2)^(p+1)/(p+1)-d*(-e^2*x^2+d^2)^(p+1)/x+2*d*e^2*(1-p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-3/2*e*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{1821, 1666, 457, 81, 67, 12, 252, 251\}$

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2}\,dx$$
$$= 2de^2(1-p)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) - \frac{3e(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2(p+1)} - \frac{e(d^2-e^2x^2)^{p+1}}{2(p+1)} - \frac{d(d^2-e^2x^2)^{p+1}}{x}$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -1/2*(e*(d^2 - e^2*x^2)^(1 + p))/(1 + p) - (d*(d^2 - e^2*x^2)^(1 + p))/x +
(2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x
^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{1+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^p(-3d^4e-2d^3e^2(1-p)x-d^2e^3x^2)}{x}\,dx}{d^2} \\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{x} - \frac{\int -2d^3e^2(1-p)\left(d^2-e^2x^2\right)^p\,dx}{d^2} - \frac{\int \frac{(d^2-e^2x^2)^p(-3d^4e-d^2e^3x^2)}{x}\,dx}{d^2} \\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{x} - \frac{\operatorname{Subst}\left(\int \frac{(d^2-e^2x)^p(-3d^4e-d^2e^3x)}{x}\,dx, x, x^2\right)}{2d^2} \\
&\quad + \left(2de^2(1-p)\right)\int \left(d^2-e^2x^2\right)^p\,dx
\end{aligned}
$$

$$= -\frac{e(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{d(d^2-e^2x^2)^{1+p}}{x} + \frac{1}{2}(3d^2e)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right)$$
$$+ \left(2de^2(1-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^p dx$$
$$= -\frac{e(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{d(d^2-e^2x^2)^{1+p}}{x}$$
$$+ 2de^2(1-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$
$$- \frac{3e(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2(1+p)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^p}{x^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-2d^3(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+ex\left(6de(1+p)x\,\text{Hyperg}\right.\right.}{2}$$

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-2*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^
2)/d^2] + e*x*(6*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(1 + 3*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{(ex+d)^3\,(-e^2x^2+d^2)^p}{x^2}dx$$

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2}\,dx$$

[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")

[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^2, x)

Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^3\,(d^2-e^2x^2)^p}{x^2}\,dx = -\frac{d^3d^{2p}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{3d^2ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 3dd^{2p}e^2x{}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right) + e^3\left(\begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2=0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2-e^2x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right)$$

[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**2,x)

[Out] -d**3*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d**2*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d*d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True))

Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2} dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^2} dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^2} dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x^2} dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.266 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\begin{aligned}&\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} dx\\ &= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{3e(d^2-e^2x^2)^{1+p}}{x}\\ &\quad - 2e^3(1+3p)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\\ &\quad - \frac{e^2(3-p)(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}\end{aligned}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(p+1)/x^2-3*e*(-e^2*x^2+d^2)^(p+1)/x-2*e^3*(1+3*p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([1, p+1],[2+p],1-e^2*x^2/d^2)
/d/(p+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

$= \{1821, 778, 272, 67, 252, 251\}$

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx$$
$$= -\frac{e^2(3-p)(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)} - \frac{3e(d^2-e^2x^2)^{p+1}}{x} - \frac{d(d^2-e^2x^2)^{p+1}}{2x^2} - 2e^3(3p+1)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)$$

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/x^2 - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2
*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{\int \frac{(d^2-e^2x^2)^p(-6d^4e-2d^3e^2(3-p)x-2d^2e^3x^2)}{x^2}\,dx}{2d^2}\\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{3e(d^2-e^2x^2)^{1+p}}{x} + \frac{\int \frac{(2d^5e^2(3-p)-4d^4e^3(1+3p)x)(d^2-e^2x^2)^p}{x}\,dx}{2d^4}\\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{3e(d^2-e^2x^2)^{1+p}}{x}\\
&\quad + \left(de^2(3-p)\right)\int \frac{(d^2-e^2x^2)^p}{x}\,dx - \left(2e^3(1+3p)\right)\int (d^2-e^2x^2)^p\,dx\\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{3e(d^2-e^2x^2)^{1+p}}{x}\\
&\quad + \frac{1}{2}\left(de^2(3-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right)\\
&\quad - \left(2e^3(1+3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx\\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{3e(d^2-e^2x^2)^{1+p}}{x}\\
&\quad - 2e^3(1+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)\\
&\quad - \frac{e^2(3-p)\left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} \, dx = \frac{e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-6d^3(1+p) \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+ex\left(2de(1+p)x \text{Hyper}\right.\right.}{}$$

[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]

[Out] (e*(d^2 - e^2*x^2)^p*(-6*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2] + e*x*(2*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(3*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)

Maple [F]

$$\int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x^3} dx$$

[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)

[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)

Fricas [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3} \, dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3} \, dx$$

[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")

[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^3, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = -\frac{d^3e^{2p}x^{2p-2}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)} - \frac{3d^2d^{2p}e_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{3de^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + d^{2p}e^3x_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p - 2)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2
- p,), d**2/(e**2*x**2))/(2*gamma(2 - p)) - 3*d**2*d**(2*p)*e*hyper((-1/2,
-p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d*e**2*e**(2*p)*x**(2*
p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gam
ma(1 - p)) + d**(2*p)*e**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2
*I*pi)/d**2)
```

Maxima [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x^3}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.267 $\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{d^4(d^2-e^2x^2)^p}{2e^5p} - \frac{d^2(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{(d^2-e^2x^2)^{2+p}}{2e^5(2+p)} + \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d}$$

```
[Out] 1/2*d^4*(-e^2*x^2+d^2)^p/e^5/p-d^2*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+1/2*(-e^2
*x^2+d^2)^(2+p)/e^5/(2+p)+1/5*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/
2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 372, 371, 272, 45}

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{d^2(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{(d^2-e^2x^2)^{p+2}}{2e^5(p+2)} + \frac{d^4(d^2-e^2x^2)^p}{2e^5p}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^p)/(2*e^5*p) - (d^2*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 +
 p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^5*(2 + p)) + (x^5*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= d\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^5\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d}$$

$$= \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\text{Subst}\left(\int\left(\frac{d^4(d^2-e^2x)^{-1+p}}{e^4}-\frac{2d^2(d^2-e^2x)^p}{e^4}+\frac{(d^2-e^2x)^{1+p}}{e^4}\right)dx,x,x^2\right)$$

$$= \frac{d^4(d^2-e^2x^2)^p}{2e^5p} - \frac{d^2(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{(d^2-e^2x^2)^{2+p}}{2e^5(2+p)} + \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(5,-p,1-p,6,\frac{ex}{d},-\frac{ex}{d}\right)}{5d}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 1 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{ex+d}dx$$

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{x^4\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.268 $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex} dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex} dx = -\frac{d^3(d^2-e^2x^2)^p}{2e^4p} + \frac{d(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2}$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^p/e^4/p+1/2*d*(-e^2*x^2+d^2)^(p+1)/e^4/(p+1)-1/5*e*
x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/2],e^2*x^2/d^2)/d^2/((1-e^2*x^
2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 272, 45, 372, 371}

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex} dx = -\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},1-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d(d^2-e^2x^2)^{p+1}}{2e^4(p+1)} - \frac{d^3(d^2-e^2x^2)^p}{2e^4p}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d^3*(d^2 - e^2*x^2)^p)/(e^4*p) + (d*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(
1 + p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*
x^2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx\\
&= d\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx, x, x^2\right)\\
&\quad - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\
&= -\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}\\
&\quad + \frac{1}{2}d\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x)^{-1+p}}{e^2}-\frac{(d^2-e^2x)^p}{e^2}\right)dx, x, x^2\right)\\
&= -\frac{d^3(d^2-e^2x^2)^p}{2e^4p}+\frac{d(d^2-e^2x^2)^{1+p}}{2e^4(1+p)}-\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$
\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx
= \frac{\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(6d^2e(1+p)x\left(1+\frac{ex}{d}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+2e^3(\right.}{}
$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(6*d^2*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/2
, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1 + p)*x^3*(1 + (e*x)/d)^p*Hypergeometri
c2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 3*d*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e
^2*x^2)/d^2)^p) + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + d*(d - e*x)*(2 - (2*e
^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(
6*e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{ex+d}dx$$

[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)

[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)

Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^px^3}{ex+d}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 62.78 (sec) , antiderivative size = 17065, normalized size of antiderivative = 141.03

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \text{Too large to display}$$

[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)

[Out] Piecewise((-3*0**p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*p*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**5*d**(2*p + 3)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1)

```
- 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p -
1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p
*d**5*d**(2*p + 3)*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma
(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)*e*p*x*gamma(-p - 1/2)*g
amma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamm
a(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
+ 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)
*e*x*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p +
 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-
p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*
0**p*d**3*d**(2*p + 3)*e**2*p*x**2*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*ga
mma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) +
 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p + 3)*
e**2*p*x**2*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5
*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*g
amma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*acoth(d
/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(
p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamm
a(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) +
 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5
*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*g
amma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*log(d**2/
(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*g
amma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p +
1)) - 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*log(d**2/(e**2*x**2) - 1)*gamma(-p
 - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*
e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamm
a(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**
(2*p + 3)*e**2*x**2*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e*
*4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1
) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamm
a(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*gamma(-p - 1
/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 4*0**p*d**2*d**(2*p
 + 3)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1
/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p
*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma
(p + 1)) + 4*0**p*d**2*d**(2*p + 3)*e**3*x**3*gamma(-p - 1/2)*gamma(p + 1)/
```

```
(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*
gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**
6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p + 3)*e**4*p*x**4*ga
mma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6
*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2
)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*
d**(2*p + 3)*e**4*x**4*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-
p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e
**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*
gamma(p + 1)) + 2*0**p*d**(2*p + 3)*e**5*p*x**5*gamma(-p - 1/2)*gamma(p + 1
)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2
)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e
**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p + 3)*e**5*x**5*gamm
a(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d
**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*
gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(
2*p + 3)*(-1 + e**2*x**2/d**2)**(p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2
)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2
)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e
**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(2*p + 3)*gamma(p)*gamma
(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) +
 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*d**5*e**4*e**(2*p - 1)*
p**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3
/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) +
3*d**5*e**4*e**(2*p - 1)*p*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2
)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*g
amma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*
d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p -
 1/2)*gamma(p + 1)) - 3*d**3*d**(2*p + 3)*e**2*p*x**2*(-1 + e**2*x**2/d**2)
**(p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1
/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p
*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma
(p + 1)) + 3*d**3*d**(2*p + 3)*e**2*x**2*gamma(p)*gamma(-p - 1/2)/(-6*d**5*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p +
 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*ga
mma(-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p**2*x**2*x**(2*p +
3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,
), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*
e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamm
a(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**
(2*p - 1)*p*x**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1
 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p -
```

```
1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*
p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamm
a(p + 1)), (Abs(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2)) > 1)), (-3*0*
*p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(
-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*p*log(-d**2/
(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*
x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(
p + 1)) + 6*0**p*d**5*d**(2*p + 3)*p*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p
 + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p -
 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d*
*3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**5*d**(2*p + 3)*log(d
**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*
x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(
p + 1)) + 3*0**p*d**5*d**(2*p + 3)*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/
2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*
gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p +
 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**5*d**(2*p
+ 3)*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p -
 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gam
ma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)/(-
6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*ga
mma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*
x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)*e*x*gamma(-p
- 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e
**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma
(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(
2*p + 3)*e**2*p*x**2*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6
*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gam
ma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x
**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*lo
g(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma
(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2
)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*atanh(d/(e*x))*gamma
(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d*
*5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*
d**(2*p + 3)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma
(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2
```

```
)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*log(d**2/(e**2*x**2))*
gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) -
 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1
/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*
d**3*d**(2*p + 3)*e**2*x**2*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamm
a(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-
p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6
*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p + 3)*e*
*2*x**2*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-
p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e
**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*
gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*gamma(-p - 1/2)*gamma(p
+ 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p -
1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**
3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 4*0**p*d**2*d**(2*p + 3)*e**3*p
*x**3*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
+ 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(
-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 4
*0**p*d**2*d**(2*p + 3)*e**3*x**3*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma
(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p + 3)*e**4*p*x**4*gamma(-p - 1/2
)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*g
amma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p +
1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p + 3)
*e**4*x**4*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gam
ma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*g
amma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)
) + 2*0**p*d**(2*p + 3)*e**5*p*x**5*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e
**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p +
1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gam
ma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p + 3)*e**5*x**5*gamma(-p - 1/2)*
gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gam
ma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(2*p + 3)*(-1
 + e**2*x**2/d**2)**(p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e
**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p +
1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gam
ma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(2*p + 3)*gamma(p)*gamma(-p - 1/2)/(
-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*d**5*e**4*e**(2*p - 1)*p**2*x**(2*p
 + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1
/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d*
*5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*g
```

```
amma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*d**5*e**4*
e**(2*p - 1)*p*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 -
 p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*
x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(
p + 1)) - 3*d**3*d**(2*p + 3)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**(p + 1)*ex
p(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
+ 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(
-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3
*d**3*d**(2*p + 3)*e**2*x**2*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma
(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2
)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p**2*x**2*x**(2*p + 3)*exp(I*pi*
p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**
2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-
p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6
*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p*
x**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3
/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)), A
bs(e**2*x**2/d**2) > 1), (-3*0**p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2))
*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
- 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p -
1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p
*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)
/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)
*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e*
*6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*p*acoth(d/
(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) -
3*0**p*d**5*d**(2*p + 3)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)
/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)
*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e*
*6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*log(d**2/(
e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2
)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x
**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p
 + 1)) + 6*0**p*d**5*d**(2*p + 3)*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p +
1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/
2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*
e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)*e*p*x*ga
mma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6
*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2
```

```
)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d*
*4*d**(2*p + 3)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p -
 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gam
ma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*log(d**2/(e**2*x**2))*gam
ma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*
d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)
*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**
3*d**(2*p + 3)*e**2*p*x**2*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(
p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p
- 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d
**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p + 3)*e**2
*p*x**2*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-
p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e
**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*
gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*gamma(-p - 1/2)*gamma(
p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p
- 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d
**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2
*x**2*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*ga
mma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d
**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p -
1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*log(d**2/(e**2*x**2
) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) -
6*0**p*d**3*d**(2*p + 3)*e**2*x**2*acoth(d/(e*x))*gamma(-p - 1/2)*gamma(p +
 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1
/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*x*
*2*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1
) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p
- 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 4*0*
*p*d**2*d**(2*p + 3)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4
*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1)
+ 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(
-p - 1/2)*gamma(p + 1)) + 4*0**p*d**2*d**(2*p + 3)*e**3*x**3*gamma(-p - 1/2
)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*g
amma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p +
1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p + 3)
*e**4*p*x**4*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*g
amma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p +
1)) - 3*0**p*d*d**(2*p + 3)*e**4*x**4*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5
*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p
```

+ 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*g
amma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p + 3)*e**5*p*x**5*gamma(-p - 1
/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p + 3)
*e**5*x**5*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gam
ma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*g
amma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)
) + 3*d**5*d**(2*p + 3)*(1 - e**2*x**2/d**2)**(p + 1)*gamma(p)*gamma(-p - 1
/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1
/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(2*p + 3)*gamma(p)*gam
ma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gam
ma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*d**5*e**4*e**(2*p - 1
)*p**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p -
 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma
(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gam
ma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1))
+ 3*d**5*e**4*e**(2*p - 1)*p*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3
/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p
*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) +
6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p
 - 1/2)*gamma(p + 1)) + 3*d**3*d**(2*p + 3)*e**2*p*x**2*(1 - e**2*x**2/d**2
)**(p + 1)*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) +
3*d**3*d**(2*p + 3)*e**2*x**2*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamm
a(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**
3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/
2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p**2*x**2*x**(2*p + 3)*exp(I*pi
*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e*
*2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(
-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) +
6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p
*x**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p -
3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(
p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamm
a(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)),
Abs(d**2/(e**2*x**2)) > 1), (-3*0**p*d**5*d**(2*p + 3)*p*log(d**2/(e**2*x**
2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p +
1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p
 - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0
p*d5*d**(2*p + 3)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p
+ 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p -

1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**
3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*p*atan
h(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gam
ma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*g
amma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)
) - 3*0**p*d**5*d**(2*p + 3)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p
+ 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p -
1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**
3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**5*d**(2*p + 3)*log(-d
2/(e2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p
- 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**
6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*ga
mma(p + 1)) + 6*0**p*d**5*d**(2*p + 3)*atanh(d/(e*x))*gamma(-p - 1/2)*gamma
(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p
 - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*
d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**4*d**(2*p + 3)*e*p
*x*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1
) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p
- 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0*
*p*d**4*d**(2*p + 3)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma
(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3
*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2
)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*log(d**2/(e**2*x**2)
)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
 - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p -
 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**
p*d**3*d**(2*p + 3)*e**2*p*x**2*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*
gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gam
ma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p + 3
)*e**2*p*x**2*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*g
amma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*
d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p -
 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*e**2*p*x**2*gamma(-p - 1/2)*
gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gam
ma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3
)*e**2*x**2*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1)
 + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma
(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p + 3)*e**2*x**2*log(-d**2/(e*
*2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*
gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**
2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p +
 1)) - 6*0**p*d**3*d**(2*p + 3)*e**2*x**2*atanh(d/(e*x))*gamma(-p - 1/2)*ga

```
mma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) +
 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p + 3)*
e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamm
a(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*ga
mma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1))
 + 4*0**p*d**2*d**(2*p + 3)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p + 1)/(-6*d*
*5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(
p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2
*gamma(-p - 1/2)*gamma(p + 1)) + 4*0**p*d**2*d**(2*p + 3)*e**3*x**3*gamma(-
p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5
*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gam
ma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2
*p + 3)*e**4*p*x**4*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p -
 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gam
ma(p + 1)) - 3*0**p*d*d**(2*p + 3)*e**4*x**4*gamma(-p - 1/2)*gamma(p + 1)/(
-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6
*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p + 3)*e**5*p*x**5*gamma
(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d*
*5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*g
amma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2
*p + 3)*e**5*x**5*gamma(-p - 1/2)*gamma(p + 1)/(-6*d**5*e**4*p*gamma(-p - 1
/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p
*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma
(p + 1)) + 3*d**5*d**(2*p + 3)*(1 - e**2*x**2/d**2)**(p + 1)*gamma(p)*gamma
(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma
(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) +
 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**5*d**(2*p + 3)*gamma
(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e
**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma
(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) + 3*d**5*e**4*e**(
2*p - 1)*p**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 -
p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2
)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x
**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p
 + 1)) + 3*d**5*e**4*e**(2*p - 1)*p*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma
(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5
*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*g
amma(-p - 1/2)*gamma(p + 1)) + 3*d**3*d**(2*p + 3)*e**2*p*x**2*(1 - e**2*x*
*2/d**2)**(p + 1)*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4*p*gamma(-p - 1/2)*
gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**
2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p +
```

```
1)) + 3*d**3*d**(2*p + 3)*e**2*x**2*gamma(p)*gamma(-p - 1/2)/(-6*d**5*e**4
*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1)
+ 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(
-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p - 1)*p**2*x**2*x**(2*p + 3)*e
xp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d
**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)*gamma(p + 1) - 6*d**5*e**4
*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x**2*gamma(-p - 1/2)*gamma(p
+ 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*e**6*e**(2*p
 - 1)*p*x**2*x**(2*p + 3)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p
, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(-6*d**5*e**4*p*gamma(-p - 1/2)
*gamma(p + 1) - 6*d**5*e**4*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*p*x*
*2*gamma(-p - 1/2)*gamma(p + 1) + 6*d**3*e**6*x**2*gamma(-p - 1/2)*gamma(p
+ 1)), True))
```

Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.269 $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{d^2(d^2-e^2x^2)^p}{2e^3p} - \frac{(d^2-e^2x^2)^{1+p}}{2e^3(1+p)} + \frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^p/e^3/p-1/2*(-e^2*x^2+d^2)^(p+1)/e^3/(p+1)+1/3*x^3*(
-e^2*x^2+d^2)^p*hypergeom([3/2, 1-p],[5/2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^
p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 372, 371, 272, 45}

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d} + \frac{d^2(d^2-e^2x^2)^p}{2e^3p} - \frac{(d^2-e^2x^2)^{p+1}}{2e^3(p+1)}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^p)/(2*e^3*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^3*(1 + p))
 + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2]
)/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int x^2(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= d\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d}$$

$$= \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d} - \frac{1}{2}e\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^{-1+p}}{e^2}-\frac{\left(d^2-e^2x\right)^p}{e^2}\right)dx,x,x^2\right)$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.66

$$\int \frac{x^2\left(d^2-e^2x^2\right)^p}{d+ex}\,dx =$$

$$-\frac{\left(1+\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\left(1+\frac{ex}{d}\right)^p\left(-e^2x^2\left(1-\frac{e^2x^2}{d^2}\right)^p+d^2\left(-1+\left(1-\frac{e^2x^2}{d^2}\right)^p\right)\right)+2de(\cdots\right.}{\cdots}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*((d^2 - e^2*x^2)^p*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e^2*x^2)/d^2)^p)
+ d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + 2*d*e*(1 + p)*x*(1 + (e*x)/d)^p*Hyper
geometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + d*(d - e*x)*(2 - (2*e^2*x^2)/d^
2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^3*(1 + p)
*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^2\left(-e^2x^2+d^2\right)^p}{ex+d}dx$$

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.29 (sec) , antiderivative size = 14895, normalized size of antiderivative = 125.17

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*d**4*d**(2*p + 2)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*ga
mma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(
1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*
d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*p*log(
d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(
p + 1)) - 2*0**p*d**4*d**(2*p + 2)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p
+ 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 -
p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e
**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**4*d**(2*p + 2)*log(d**2/(e*
*2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(
p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma
(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0*
*p*d**4*d**(2*p + 2)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/
(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*ga
mma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x
**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d**4*d**(2*p + 2)*acoth(d/(e*x))*
gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2
*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*
gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*
d**(2*p + 2)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 -
p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x
**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p +
 1)) + 2*0**p*d**3*d**(2*p + 2)*e*x*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e*
```

```
*3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1)
+ 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1
/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*p*x**2*log(d**2/(e**2*x
**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p +
1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d
**2*d**(2*p + 2)*e**2*p*x**2*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma
(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**
2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*p*
x**2*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 -
 p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*
x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p
+ 1)) - 0**p*d**2*d**(2*p + 2)*e**2*p*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*
d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(
p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*
gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*x**2*log(d**2/(e
**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma
(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamm
a(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0
**p*d**2*d**(2*p + 2)*e**2*x**2*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*ga
mma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(
1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*
d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2
*x**2*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
- p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p
*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p
 + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d
**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p
 + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*g
amma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p + 2)*e**3*p*x**3*gamma(1/2 -
 p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*
gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p + 2)*e
**3*x**3*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p
 + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(
1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**
p*d**(2*p + 2)*e**4*p*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamm
a(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*
e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*g
amma(p + 1)) + 0**p*d**(2*p + 2)*e**4*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*
d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(
p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*
gamma(1/2 - p)*gamma(p + 1)) - d**5*e**3*e**(2*p - 2)*p**2*x**(2*p + 1)*exp
(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2
```

```
/(e**2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gam
ma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) +
 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - d**5*e**3*e**(2*p - 2)*p*x
**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (
1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) -
2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)
*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**4*d**(2*
p + 2)*(-1 + e**2*x**2/d**2)**(p + 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(
-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gam
ma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x*
*2*gamma(1/2 - p)*gamma(p + 1)) + d**4*d**(2*p + 2)*gamma(p)*gamma(1/2 - p)
/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
x**2*gamma(1/2 - p)*gamma(p + 1)) + d**3*e**5*e**(2*p - 2)*p**2*x**2*x**(2*
p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 -
 p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**
4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamm
a(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**3*e**5*e**(2*
p - 2)*p*x**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 -
p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*
gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2
*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)
) + d**2*d**(2*p + 2)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**(p + 1)*exp(I*pi*p
)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d
**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*ga
mma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - d**2*d**(2*p +
 2)*e**2*x**2*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(
p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma
(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)), (Ab
s(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2)) > 1)), (0**p*d**4*d**(2*p +
 2)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gam
ma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*
gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/
2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e*
*3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p
+ 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d**4*d**(2*p
+ 2)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(
p + 1)) + 0**p*d**4*d**(2*p + 2)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma
(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**
2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*log(d**2/
(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)
```

```
*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**
2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1
)) - 2*0**p*d**4*d**(2*p + 2)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-
2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamm
a(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**
2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**(2*p + 2)*e*p*x*gamma(1/2 -
 p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*
gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**(2*p + 2
)*e*x*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p +
1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d
**2*d**(2*p + 2)*e**2*p*x**2*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p +
 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p
)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e*
*5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)*e**2*p*x**2*l
og(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(
1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e*
*5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gam
ma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*p*x**2*acoth(d/(e*x))*gamma(1/2
- p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3
*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p +
1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2)
*e**2*p*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gam
ma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*ga
mma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) -
 0**p*d**2*d**(2*p + 2)*e**2*x**2*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamm
a(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/
2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d*
*2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)*e**2*x**
2*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gam
ma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*
gamma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*x**2*acoth(d/(e*x))*gamma(1/2
 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**
3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p +
 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2
)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamm
a(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gam
ma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) -
2*0**p*d*d**(2*p + 2)*e**3*p*x**3*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3
*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) +
2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2
 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p + 2)*e**3*x**3*gamma(1/2 - p)*gamma(
p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2
```

```
- p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p + 2)*e**4*p*x**4*gam
ma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d*
*4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gam
ma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p +
2)*e**4*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gam
ma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*ga
mma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) -
 d**5*e**3*e**(2*p - 2)*p**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1
/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*
gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d
**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 -
p)*gamma(p + 1)) - d**5*e**3*e**(2*p - 2)*p*x**(2*p + 1)*exp(I*pi*p)*gamma(
p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(
-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gam
ma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x*
*2*gamma(1/2 - p)*gamma(p + 1)) - d**4*d**(2*p + 2)*(1 - e**2*x**2/d**2)**(
p + 1)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1)
 - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 -
p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**4*d**(
2*p + 2)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1
) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
- p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**3*e*
*5*e**(2*p - 2)*p**2*x**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)
*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*gam
ma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*
gamma(p + 1)) + d**3*e**5*e**(2*p - 2)*p*x**2*x**(2*p + 1)*exp(I*pi*p)*gamm
a(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))
/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
x**2*gamma(1/2 - p)*gamma(p + 1)) - d**2*d**(2*p + 2)*e**2*p*x**2*(1 - e**2
*x**2/d**2)**(p + 1)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)
*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**
2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1
)) - d**2*d**(2*p + 2)*e**2*x**2*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**
2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)
*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (0**p*d**4*d**(2*p + 2)*p*log(d
**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)
*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**
2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1
)) - 0**p*d**4*d**(2*p + 2)*p*log(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gam
ma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1
/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d
```

```
**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d**4*d**(2*p + 2)*p*ata
nh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamm
a(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gam
ma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) +
0**p*d**4*d**(2*p + 2)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-
2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamm
a(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**
2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*log(-d**2/(e**2*x**
2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p
+ 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1
/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0*
*p*d**4*d**(2*p + 2)*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1)
+ 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1
/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**(2*p + 2)*e*p*x*gamma(1/2 - p)*gamma
(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**
2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**(2*p + 2)*e*x*gam
ma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d*
*4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gam
ma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2
*p + 2)*e**2*p*x**2*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d
**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p
 + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*g
amma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)*e**2*p*x**2*log(-d**2/
(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)
*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**
2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1
)) + 2*0**p*d**2*d**(2*p + 2)*e**2*p*x**2*atanh(d/(e*x))*gamma(1/2 - p)*gam
ma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1
/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d
**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*p*
x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1
) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
- p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d*
*2*d**(2*p + 2)*e**2*x**2*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)
/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)*e**2*x**2*log(-d
**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
- p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p
*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p
 + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*x**2*atanh(d/(e*x))*gamma(1/2 - p)*g
amma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma
(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2
```

```
*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*
x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1
) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
- p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*
d*d**(2*p + 2)*e**3*p*x**3*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamm
a(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*
e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*g
amma(p + 1)) - 2*0**p*d*d**(2*p + 2)*e**3*x**3*gamma(1/2 - p)*gamma(p + 1)/
(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*ga
mma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x
**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p + 2)*e**4*p*x**4*gamma(1/2
- p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3
*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p +
1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p + 2)*e**4
*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p +
1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2
 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - d**5*e
**3*e**(2*p - 2)*p**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyp
er((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3*p*gamma(1
/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**
5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamm
a(p + 1)) - d**5*e**3*e**(2*p - 2)*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamm
a(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4
*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p +
1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamm
a(1/2 - p)*gamma(p + 1)) + d**4*d**(2*p + 2)*(-1 + e**2*x**2/d**2)**(p + 1)
*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p
 + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(
1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**
4*d**(2*p + 2)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma
(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamm
a(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d
**3*e**5*e**(2*p - 2)*p**2*x**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p
- 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*e**3
*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) +
2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2
 - p)*gamma(p + 1)) + d**3*e**5*e**(2*p - 2)*p*x**2*x**(2*p + 1)*exp(I*pi*p
)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*
x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2
- p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2
*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + d**2*d**(2*p + 2)*e**2*p*x**2*(-1
 + e**2*x**2/d**2)**(p + 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1)
+ 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1
/2 - p)*gamma(p + 1)) - d**2*d**(2*p + 2)*e**2*x**2*gamma(p)*gamma(1/2 - p)
```

```
/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
x**2*gamma(1/2 - p)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (0**p*d**4*d**
(2*p + 2)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3
*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) +
2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2
 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*p*log(-d**2/(e**2*x**2) + 1)*g
amma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*
d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d**4*d
**(2*p + 2)*p*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**
2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)
*gamma(p + 1)) + 0**p*d**4*d**(2*p + 2)*log(d**2/(e**2*x**2))*gamma(1/2 - p
)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*ga
mma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1)
+ 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**4*d**(2*p + 2)*lo
g(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(
1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e*
*5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gam
ma(p + 1)) - 2*0**p*d**4*d**(2*p + 2)*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p
 + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 -
 p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*
e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**(2*p + 2)*e*p*x*gam
ma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d*
*4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gam
ma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**3*d**
(2*p + 2)*e*x*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*ga
mma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*g
amma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1))
- 0**p*d**2*d**(2*p + 2)*e**2*p*x**2*log(d**2/(e**2*x**2))*gamma(1/2 - p)*g
amma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma
(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2
*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)*e**2*
p*x**2*log(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3
*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) +
2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2
 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*p*x**2*atanh(d/(e*x))*g
amma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*
d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*g
amma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**
(2*p + 2)*e**2*p*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(
p + 1)) - 0**p*d**2*d**(2*p + 2)*e**2*x**2*log(d**2/(e**2*x**2))*gamma(1/2
```

```
- p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3
*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p +
1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p + 2)
*e**2*x**2*log(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*
e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma
(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p + 2)*e**2*x**2*atanh(d/(e*x))
*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) -
2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)
*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d
**(2*p + 2)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2
 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*
p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(
p + 1)) - 2*0**p*d*d**(2*p + 2)*e**3*p*x**3*gamma(1/2 - p)*gamma(p + 1)/(-2
*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma
(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2
*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p + 2)*e**3*x**3*gamma(1/2 -
 p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*
gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p + 2)*e**4*
p*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p +
 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/
2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) + 0**p*
d**(2*p + 2)*e**4*x**4*gamma(1/2 - p)*gamma(p + 1)/(-2*d**4*e**3*p*gamma(1/
2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5
*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma
(p + 1)) - d**5*e**3*e**(2*p - 2)*p**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d*
*4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p
+ 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*ga
mma(1/2 - p)*gamma(p + 1)) - d**5*e**3*e**(2*p - 2)*p*x**(2*p + 1)*exp(I*pi
*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**
2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/
2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d*
*2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - d**4*d**(2*p + 2)*(1 - e**2*x**
2/d**2)**(p + 1)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*gam
ma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*ga
mma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) +
 d**4*d**(2*p + 2)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamma(1/2 - p)*g
amma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*
gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1))
 + d**3*e**5*e**(2*p - 2)*p**2*x**2*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma
(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(-2*d**4*
e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1
) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma
```

```
(1/2 - p)*gamma(p + 1)) + d**3*e**5*e**(2*p - 2)*p*x**2*x**(2*p + 1)*exp(I*
pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e
**2*x**2))/(-2*d**4*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(
1/2 - p)*gamma(p + 1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*
d**2*e**5*x**2*gamma(1/2 - p)*gamma(p + 1)) - d**2*d**(2*p + 2)*e**2*p*x**2
*(1 - e**2*x**2/d**2)**(p + 1)*gamma(p)*gamma(1/2 - p)/(-2*d**4*e**3*p*gamm
a(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*
e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamma(1/2 - p)*g
amma(p + 1)) - d**2*d**(2*p + 2)*e**2*x**2*gamma(p)*gamma(1/2 - p)/(-2*d**4
*e**3*p*gamma(1/2 - p)*gamma(p + 1) - 2*d**4*e**3*gamma(1/2 - p)*gamma(p +
1) + 2*d**2*e**5*p*x**2*gamma(1/2 - p)*gamma(p + 1) + 2*d**2*e**5*x**2*gamm
a(1/2 - p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{x^2\,(d^2-e^2\,x^2)^p}{d+e\,x}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.270 $\int \frac{x(d^2-e^2x^2)^p}{d+ex} dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex} dx = -\frac{d(d^2-e^2x^2)^p}{2e^2p} - \frac{ex^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^p/e^2/p-1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 1-
p],[5/2],e^2*x^2/d^2)/d^2/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {799, 778, 267, 372, 371}

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex} dx = -\frac{ex^3(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},1-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2} - \frac{d(d^2-e^2x^2)^p}{2e^2p}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^p)/(e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometri
c2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int x(d^2e-de^2x)\left(d^2-e^2x^2\right)^{-1+p}\,dx}{de}\\
&= d\int x\left(d^2-e^2x^2\right)^{-1+p}\,dx-e\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx\\
&= -\frac{d(d^2-e^2x^2)^p}{2e^2p}-\frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\
&= -\frac{d(d^2-e^2x^2)^p}{2e^2p}-\frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$$
$$= \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+(d\right.}{e^2(1+p)}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^2*(1 + p)*(1 + (
e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{ex+d}dx$$

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{ex+d}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 440, normalized size of antiderivative = 4.89

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$$
$$= \begin{cases} \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2e^2} - \frac{0^p d^{2p+1}\operatorname{acoth}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p+1}x}{de} - \frac{e^{2p-1}px^{2p+1}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-\\ \frac{1}{2}-\end{matrix}\right.}{2\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} \\ \frac{0^p d^{2p+1}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d^{2p+1}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2e^2} - \frac{0^p d^{2p+1}\operatorname{atanh}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p+1}x}{de} - \frac{e^{2p-1}px^{2p+1}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,\\ \frac{1}{2}\end{matrix}\right.}{2\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} \end{cases}$$

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*d**(2*p + 1)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d**(2*p
+ 1)*log(d**2/(e**2*x**2) - 1)/(2*e**2) - 0**p*d**(2*p + 1)*acoth(d/(e*x))/
e**2 + 0**p*d**(2*p + 1)*x/(d*e) - e**(2*p - 1)*p*x**(2*p + 1)*exp(I*pi*p)*
gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x*
*2))/(2*gamma(1/2 - p)*gamma(p + 1)) - d**(2*p + 1)*x**2*gamma(p)*gamma(1 -
 p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*
gamma(-p)*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (0**p*d**(2*p + 1)*log
(d**2/(e**2*x**2))/(2*e**2) - 0**p*d**(2*p + 1)*log(-d**2/(e**2*x**2) + 1)/
(2*e**2) - 0**p*d**(2*p + 1)*atanh(d/(e*x))/e**2 + 0**p*d**(2*p + 1)*x/(d*e
) - e**(2*p - 1)*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper(
(1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*gamma(1/2 - p)*gamma(p
+ 1)) - d**(2*p + 1)*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{ex+d}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

Giac [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p x}{ex + d} \, dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{x\,(d^2 - e^2\,x^2)^p}{d + e\,x} \, dx$$

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.271 $\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

```
[Out] -2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([1-p, p+1],[2+p],
1/2*(-e*x+d)/d)/d^2/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {692, 71}

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = -\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\text{integral} = \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}\,dx}{d^2}$$
$$= -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\ {}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$$
$$= -\frac{2^{-1+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{de(1+p)}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x),x]
[Out] -((2^(-1 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{ex+d}dx$$

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx=\int \frac{(-e^2x^2+d^2)^p}{ex+d}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.36

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = \begin{cases} \frac{0^p\log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e}+\frac{0^p\,\mathrm{acoth}\left(\frac{ex}{d}\right)}{e}+\frac{de^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)}+\frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1\\ 2,2\end{matrix}\right.}{2d^2\Gamma(-p)\Gamma(p+1)} \\ \frac{0^p\log\left(1-\frac{e^2x^2}{d^2}\right)}{2e}+\frac{0^p\,\mathrm{atanh}\left(\frac{ex}{d}\right)}{e}+\frac{de^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)}+\frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-\\ 2,2\end{matrix}\right.}{2d^2\Gamma(-p)\Gamma(p+1)}\end{cases}$$

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*e*
*(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p,
 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)) +
d**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x**2/d*
*2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e + d*e**
(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p,
1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)) + d
**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{ex + d} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{ex + d} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{d + e\,x} \, dx$$

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x), x)
```

3.272 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx=-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

```
[Out] -e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 1-p],[3/2],e^2*x^2/d^2)/d^2/((1-e^2*x
^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1, p],[p+1],1-e^2*x^2/d^2)/d/p
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 272, 67, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx=-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},1-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2])
/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx$$
$$= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx - e\int \left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$=\frac{1}{2}d\text{Subst}\left(\int\frac{(d^2-e^2x)^{-1+p}}{x}\,dx,x,x^2\right)-\frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}$$

$$=-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

$$\int\frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$$
$$=\frac{2^{-1+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2+\right.\right.}{d^2p(1+p)}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*(1 + p)*(1/2 + (e*x)/(2*d
))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^2*p*(1 + p)*(1 -
d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int\frac{(-e^2x^2+d^2)^p}{x\,(ex+d)}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

Fricas [F]

$$\int\frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx=\int\frac{(-e^2x^2+d^2)^p}{(ex+d)x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^2 + d*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.35

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = \begin{cases} -\frac{0^p d^{2p-1}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2} - 0^p d^{2p-1}\operatorname{acoth}\left(\frac{d}{ex}\right) + \frac{de^{2p-2}px^{2p-2}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)\Gamma(p+1)} - \frac{e^{2p-1}px^{2p-1}e^{i\pi p}\Gamma \\ -\frac{0^p d^{2p-1}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2} - 0^p d^{2p-1}\operatorname{atanh}\left(\frac{d}{ex}\right) + \frac{de^{2p-2}px^{2p-2}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(2-p)\Gamma(p+1)} - \frac{e^{2p-1}px^{2p-1}e^{i\pi}}{} \end{cases}$$

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p - 1)*log(d**2/(e**2*x**2) - 1)/2 - 0**p*d**(2*p -
1)*acoth(d/(e*x)) + d*e**(2*p - 2)*p*x**(2*p - 2)*exp(I*pi*p)*gamma(p)*gamm
a(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p)*
gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2
- p)*hyper((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*gamma(3/2 - p
)*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (-0**p*d**(2*p - 1)*log(-d**2/
(e**2*x**2) + 1)/2 - 0**p*d**(2*p - 1)*atanh(d/(e*x)) + d*e**(2*p - 2)*p*x*
*(2*p - 2)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,)
, d**2/(e**2*x**2))/(2*gamma(2 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p
- 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,)
, d**2/(e**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)), x)
```

3.273 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = -\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{dx} + \frac{e(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

```
[Out] -(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d/x/((1-e^2*x^2/
d^2)^p)+1/2*e*(-e^2*x^2+d^2)^p*hypergeom([1, p],[p+1],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 372, 371, 272, 67}

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \frac{e(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^2p} - \frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{dx}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1
 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx\\ &= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx - e\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx\end{aligned}$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x}\,dx,x,x^2\right)\right) + \frac{\left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d}$$

$$= -\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\ {}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx} + \frac{e(d^2-e^2x^2)^p\ {}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{2d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{2^pe(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-\cdots}{2d}\right)}{1+p} \cdots\right.}{2d^3}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-2*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[1 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (d*e*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(2*d^3)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2\,(ex+d)}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^3 + d*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.09

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx = \begin{cases} -\frac{0^p d d^{2p-2}}{x} - \frac{0^p d^{2p-2} e \log\left(\frac{e^2x^2}{d^2}\right)}{2} + \frac{0^p d^{2p-2} e \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2} + 0^p d^{2p-2} e \operatorname{acoth}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-3} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right){}_2F_1\left(\ldots\right)}{2\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+\ldots} \\ -\frac{0^p d d^{2p-2}}{x} - \frac{0^p d^{2p-2} e \log\left(\frac{e^2x^2}{d^2}\right)}{2} + \frac{0^p d^{2p-2} e \log\left(1-\frac{e^2x^2}{d^2}\right)}{2} + 0^p d^{2p-2} e \operatorname{atanh}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-3} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right){}_2F_1\left(1\ldots\right)}{2\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+1)\ldots} \end{cases}$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d),x)
[Out] Piecewise((-0**p*d*d**(2*p - 2)/x - 0**p*d**(2*p - 2)*e*log(e**2*x**2/d**2)
/2 + 0**p*d**(2*p - 2)*e*log(-1 + e**2*x**2/d**2)/2 + 0**p*d**(2*p - 2)*e*a
coth(e*x/d) + d*e**(2*p - 2)*p*x**(2*p - 3)*exp(I*pi*p)*gamma(p)*gamma(3/2
- p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e**2*x**2))/(2*gamma(5/2 - p
)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p - 2)*exp(I*pi*p)*gamma(p)*gamma(1
- p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*gamma(2 - p)*gamm
a(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*d*d**(2*p - 2)/x - 0**p*d**(2*p
 - 2)*e*log(e**2*x**2/d**2)/2 + 0**p*d**(2*p - 2)*e*log(1 - e**2*x**2/d**2)
/2 + 0**p*d**(2*p - 2)*e*atanh(e*x/d) + d*e**(2*p - 2)*p*x**(2*p - 3)*exp(I
*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e*
*2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p - 2)*exp
(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x
**2))/(2*gamma(2 - p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d+ex)}\,dx = \int \frac{(d^2 - e^2\,x^2)^p}{x^2\,(d+e\,x)}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)), x)
```

3.274 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

```
[Out] e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d^2/x/((1-e^2*x
^2/d^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^p*hypergeom([2, p],[p+1],1-e^2*x^2/d^2)/d
^3/p
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {864, 778, 272, 67, 372, 371}

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},1-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(2,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x^3}\,dx$$

$$= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^3}\,dx - e\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx$$

$$= \frac{1}{2} d\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d^2}$$

$$= \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\ {}_2F_1\left(2,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(108) = 216.

Time = 0.59 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.03

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \frac{(d^2-e^2x^2)^p\left(\frac{2d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}\right.\right.}{2d^4}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((2*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + ((d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d^
2/(e^2*x^2)])/((-1 + p)*x^2) + e^2*(((2 - (2*d^2)/(e^2*x^2))^p*(d - e*x)*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (d*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/p))/(1 - d^2/(e
^2*x^2))^p))/(2*d^4)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3\,(ex+d)}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^4 + d*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.40 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.48

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx = \begin{cases} -\frac{0^p d^2 d^{2p-3}}{2x^2} + \frac{0^p d d^{2p-3} e}{x} + \frac{0^p d^{2p-3} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2} - \frac{0^p d^{2p-3} e^2 \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2} - 0^p d^{2p-3} e^2 \operatorname{acoth}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-4} e^{i\pi p}\Gamma(\ldots}{\ldots} \\ -\frac{0^p d^2 d^{2p-3}}{2x^2} + \frac{0^p d d^{2p-3} e}{x} + \frac{0^p d^{2p-3} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2} - \frac{0^p d^{2p-3} e^2 \log\left(1-\frac{e^2x^2}{d^2}\right)}{2} - 0^p d^{2p-3} e^2 \operatorname{atanh}\left(\frac{ex}{d}\right) + \frac{d e^{2p-2} p x^{2p-4} e^{i\pi p}\Gamma(\ldots}{\ldots} \end{cases}$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d),x)
[Out] Piecewise((-0**p*d**2*d**(2*p - 3)/(2*x**2) + 0**p*d*d**(2*p - 3)*e/x + 0**
p*d**(2*p - 3)*e**2*log(e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*log(-1 +
 e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*acoth(e*x/d) + d*e**(2*p - 2)*p
*x**(2*p - 4)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 -
p,), d**2/(e**2*x**2))/(2*gamma(3 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2
*p - 3)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 -
p,), d**2/(e**2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)), Abs(e**2*x**2/d**2)
 > 1), (-0**p*d**2*d**(2*p - 3)/(2*x**2) + 0**p*d*d**(2*p - 3)*e/x + 0**p*d
**(2*p - 3)*e**2*log(e**2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*log(1 - e**
2*x**2/d**2)/2 - 0**p*d**(2*p - 3)*e**2*atanh(e*x/d) + d*e**(2*p - 2)*p*x**
(2*p - 4)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,),
 d**2/(e**2*x**2))/(2*gamma(3 - p)*gamma(p + 1)) - e**(2*p - 1)*p*x**(2*p -
 3)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,),
 d**2/(e**2*x**2))/(2*gamma(5/2 - p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^3} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^3} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^3(d + ex)} \, dx = \int \frac{(d^2 - e^2 \, x^2)^p}{x^3 \, (d + e \, x)} \, dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)), x)
```

3.275 $\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{d^6(d^2-e^2x^2)^{-1+p}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2d^2(d^2-e^2x^2)^{1+p}}{e^6(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{2ex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

```
[Out] d^6*(-e^2*x^2+d^2)^(-1+p)/e^6/(1-p)+5/2*d^4*(-e^2*x^2+d^2)^p/e^6/p-2*d^2*(-
e^2*x^2+d^2)^(p+1)/e^6/(p+1)+1/2*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-2/7*e*x^7*(
-e^2*x^2+d^2)^p*hypergeom([7/2, 2-p],[9/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2
)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1666, 457, 78, 12, 372, 371}

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = -\frac{2d^2(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}+\frac{d^6(d^2-e^2x^2)^{p-1}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{7}{2},2-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right)}{7d^3}$$

```
[In] Int[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p)/(2
*e^6*p) - (2*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2)^(
2 + p)/(2*e^6*(2 + p)) - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2,
2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^6\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^5\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)\,dx,x,x^2\right)-(2de)\int x^6\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6(d^2-e^2x)^{-2+p}}{e^4}-\frac{5d^4(d^2-e^2x)^{-1+p}}{e^4}+\frac{4d^2(d^2-e^2x)^p}{e^4}-\frac{(d^2-e^2x)^{1+p}}{e^4}\right)dx,x,x^2\right)\\
&\quad-\frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^3}\\
&= \frac{d^6(d^2-e^2x^2)^{-1+p}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2d^2(d^2-e^2x^2)^{1+p}}{e^6(1+p)}\\
&\quad+\frac{(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{2ex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},2-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7d^3}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int\frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\frac{x^6(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(6,-p,2-p,7,\frac{ex}{d},-\frac{ex}{d}\right)}{6d^2}$$

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
[Out] (x^6*(d - e*x)^p*(d + e*x)^p*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)]
)/(6*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^5(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

```
[In] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2} dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2} dx$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^5/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2} dx = \int \frac{x^5(-(-d+ex)(d+ex))^p}{(d+ex)^2} dx$$

```
[In] integrate(x**5*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2} dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2} dx$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^5}{(ex+d)^2}\,dx$$

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^5\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.276 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = -\frac{d^5(d^2-e^2x^2)^{-1+p}}{e^5(1-p)} - \frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p} - \frac{2d^3(d^2-e^2x^2)^p}{e^5p} + \frac{d(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-x^5*(-e^2*x^2+d^2)^(-1+p)/(3+2*p)-2*d^
3*(-e^2*x^2+d^2)^p/e^5/p+d*(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+2/5*(4+p)*x^5*(-e
^2*x^2+d^2)^p*hypergeom([5/2, 2-p],[7/2],e^2*x^2/d^2)/d^2/(3+2*p)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {866, 1666, 470, 372, 371, 12, 272, 45}

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)} - \frac{x^5(d^2-e^2x^2)^{p-1}}{2p+3} + \frac{d(d^2-e^2x^2)^{p+1}}{e^5(p+1)} - \frac{d^5(d^2-e^2x^2)^{p-1}}{e^5(1-p)} - \frac{2d^3(d^2-e^2x^2)^p}{e^5p}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(-1 + p))/(e^5*(1 - p))) - (x^5*(d^2 - e^2*x^2)^(-1
+ p))/(3 + 2*p) - (2*d^3*(d^2 - e^2*x^2)^p)/(e^5*p) + (d*(d^2 - e^2*x^2)^(1
 + p))/(e^5*(1 + p)) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5
/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^5\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^4\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx\\
&= -\frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p}-(2de)\int x^5\left(d^2-e^2x^2\right)^{-2+p}\,dx+\frac{(2d^2(4+p))\int x^4(d^2-e^2x^2)^{-2+p}\,dx}{3+2p}\\
&= -\frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p}-(de)\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\,dx,x,x^2\right)\\
&\quad+\frac{\left(2(4+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^2(3+2p)}\\
&= -\frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p}+\frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)}\\
&\quad-(de)\text{Subst}\left(\int\left(\frac{d^4(d^2-e^2x)^{-2+p}}{e^4}-\frac{2d^2(d^2-e^2x)^{-1+p}}{e^4}+\frac{(d^2-e^2x)^p}{e^4}\right)dx,x,x^2\right)\\
&= -\frac{d^5(d^2-e^2x^2)^{-1+p}}{e^5(1-p)}-\frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p}-\frac{2d^3(d^2-e^2x^2)^p}{e^5p}+\frac{d(d^2-e^2x^2)^{1+p}}{e^5(1+p)}\\
&\quad+\frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.36

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{AppellF1}\left(5,-p,2-p,6,\frac{ex}{d},-\frac{ex}{d}\right)}{5d^2}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 2 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^2}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^2} dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^2} dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d + ex)^2} dx = \int \frac{x^4 (d^2 - e^2 x^2)^p}{(d + e x)^2} dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.277 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{d^4(d^2-e^2x^2)^{-1+p}}{e^4(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{2ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

```
[Out] d^4*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)+3/2*d^2*(-e^2*x^2+d^2)^p/e^4/p-1/2*(-e^
2*x^2+d^2)^(p+1)/e^4/(p+1)-2/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 2-p],
[7/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1666, 457, 78, 12, 372, 371}

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4(p+1)}+\frac{d^4(d^2-e^2x^2)^{p-1}}{e^4(1-p)} - \frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},2-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(2
*e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*(1 + p)) - (2*e*x^5*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^2*x^2
)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^4\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^3\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)\,dx,x,x^2\right)-(2de)\int x^4\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^4(d^2-e^2x)^{-2+p}}{e^2}-\frac{3d^2(d^2-e^2x)^{-1+p}}{e^2}+\frac{(d^2-e^2x)^p}{e^2}\right)\,dx,x,x^2\right)\\
&\quad-\frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^3}\\
&= \frac{d^4(d^2-e^2x^2)^{-1+p}}{e^4(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4(1+p)}\\
&\quad-\frac{2ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(150) = 300.

Time = 0.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.21

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$$

$$= \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p-2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^p+2e^2x^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\left(1\right.\right.}{}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*d^2*(1/2 + (e*x)/(2*d))^p - 2*d^2*(1/2 + (
e*x)/(2*d))^p*(1 - (e^2*x^2)/d^2)^p + 2*e^2*x^2*(1/2 + (e*x)/(2*d))^p*(1 -
```

```
(e^2*x^2)/d^2)^p - 8*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[
1/2, -p, 3/2, (e^2*x^2)/d^2] - 6*d*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d^2*(1 - (e^2*x^2)/d^2)^p
*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - d*e*x*(1 - (e^2*
x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^4*
(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx=\int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.278 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx = -\frac{d^3(d^2-e^2x^2)^{-1+p}}{e^3(1-p)} - \frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p} - \frac{d(d^2-e^2x^2)^p}{e^3p} + \frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},2-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(-1+p)/e^3/(1-p)-x^3*(-e^2*x^2+d^2)^(-1+p)/(1+2*p)-d*(-
e^2*x^2+d^2)^p/e^3/p+2/3*(2+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 2-p],[5
/2],e^2*x^2/d^2)/d^2/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {866, 1666, 470, 372, 371, 12, 272, 45}

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx = \frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\text{Hypergeometric2F1}\left(\frac{3}{2},2-p,\frac{5}{2},\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)} - \frac{x^3(d^2-e^2x^2)^{p-1}}{2p+1} - \frac{d(d^2-e^2x^2)^p}{e^3p} - \frac{d^3(d^2-e^2x^2)^{p-1}}{e^3(1-p)}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(-1 + p))/(e^3*(1 - p))) - (x^3*(d^2 - e^2*x^2)^(-1
+ p))/(1 + 2*p) - (d*(d^2 - e^2*x^2)^p)/(e^3*p) + (2*(2 + p)*x^3*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 + 2*p)
*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx \\
&= \int -2dex^3\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^2\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx \\
&= -\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}-(2de)\int x^3\left(d^2-e^2x^2\right)^{-2+p}\,dx+\frac{(2d^2(2+p))\int x^2(d^2-e^2x^2)^{-2+p}\,dx}{1+2p} \\
&= -\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}-(de)\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\,dx,x,x^2\right) \\
&\quad+\frac{\left(2(2+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^2(1+2p)} \\
&= -\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}+\frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)} \\
&\quad-(de)\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x)^{-2+p}}{e^2}-\frac{(d^2-e^2x)^{-1+p}}{e^2}\right)dx,x,x^2\right) \\
&= -\frac{d^3(d^2-e^2x^2)^{-1+p}}{e^3(1-p)}-\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}-\frac{d(d^2-e^2x^2)^p}{e^3p} \\
&\quad+\frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)+(d\right.}{}$$

[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]

[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(4*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(4*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(e^3*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)

Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)

[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)

Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)

Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x^2\,(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.279 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2} dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2} - \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^2(1-p^2)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(1-p)/(e*x+d)^2-2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(p+1)*hypergeom([1-p, p+1],[2+p],1/2*(-e*x+d)/d)/d^2/e^2/(-p^2+1
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {807, 692, 71}

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2} dx = \frac{(d^2-e^2x^2)^{p+1}}{2e^2(1-p)(d+ex)^2} - \frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e^2(1-p^2)}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p^2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}+\frac{\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx}{e(1-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}\\
&\quad+\frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}\,dx}{d^2e(1-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}-\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^2\,(1-p^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$$
$$=\frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeometr}\right.}{de^2(1+p)}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^2}dx$$

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2 - e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{x\,(d^2 - e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.280 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx = -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

```
[Out] -2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 2-p],[2+p],
1/2*(-e*x+d)/d)/d^3/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {692, 71}

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2} dx = -\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{d^3}\\ &= -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = -\frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
[Out] -((2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2}dx$$

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{(d+ex)^2}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{{(d+e\,x)}^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^2, x)
```

3.281 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^{-1+p}}{1-p} - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

```
[Out] (-e^2*x^2+d^2)^(-1+p)/(1-p)-2*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p],[3/
2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1,
p],[p+1],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {866, 1666, 457, 80, 67, 12, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx = -\frac{(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{e^2x^2}{d^2}\right)}{2d^2p} + \frac{(d^2-e^2x^2)^{p-1}}{1-p} - \frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^3}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (d^2 - e^2*x^2)^(-1 + p)/(1 - p) - (2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^3*(1 - (e^2*x^2)/d^2)^p) - ((d^2 - e
^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx\\
&= \int -2de\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int \frac{\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)}{x}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)-(2de)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p}+\frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx,x,x^2\right)\\
&\quad-\frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^3}\\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p}-\frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}\\
&\quad-\frac{\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.57

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$$
$$=\frac{2^{-2+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(2p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2-\right.\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + p*(1 - d^2/(e^2*x^2))^p*(
d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*(1 +
 p)*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))
/(d^3*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^2}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} dx = \int \frac{(-(-d + ex)(d + ex))^p}{x(d + ex)^2} dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x} dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x} dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x(d + ex)^2} dx = \int \frac{(d^2 - e^2 x^2)^p}{x (d + e x)^2} dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2), x)
```

3.282 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2} \, dx = -\frac{(d^2-e^2x^2)^{-1+p}}{x} + \frac{2e^2(2-p)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{e(d^2-e^2x^2)^{-1+p} \text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}$$

```
[Out] -(-e^2*x^2+d^2)^(-1+p)/x+2*e^2*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p
],[3/2],e^2*x^2/d^2)/d^4/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(-1+p)*hyperg
eom([1, -1+p],[p],1-e^2*x^2/d^2)/d/(1-p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1821, 778, 272, 67, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2} \, dx = -\frac{e(d^2-e^2x^2)^{p-1} \text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d(1-p)} - \frac{(d^2-e^2x^2)^{p-1}}{x} + \frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p \text{Hypergeometric2F1}\left(\frac{1}{2},2-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] -((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) - (e
*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2
])/(d*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - \frac{\int \frac{\left(2d^3e-2d^2e^2(2-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx}{d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx + \left(2e^2(2-p)\right)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (de)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) \\
&\quad + \frac{\left(2e^2(2-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^4} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} + \frac{2e^2(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4} \\
&\quad - \frac{e(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(1,-1+p;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.63

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{4d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2+p\right.}{1+p}\right.}{4\cdots}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d
+ e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) - (4*d*e*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p
*(1 - d^2/(e^2*x^2))^p)))/(4*d^4)
```

Maple [F]

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{2} \left(e x +d \right)^{2}}d x$$

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx=\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^2\left(d+ex\right)^2}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x^2} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2 x^2} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^2} \, dx = \int \frac{(d^2 - e^2 x^2)^p}{x^2 (d + e x)^2} \, dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.283 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x} + \frac{e^2(3-p)\,(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(-1+p)/x^2+2*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 2-p],[
1/2],e^2*x^2/d^2)/d^3/x/((1-e^2*x^2/d^2)^p)+1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(-
1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/(1-p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \frac{e^2(3-p)\,(d^2-e^2x^2)^{p-1}\,\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)} - \frac{(d^2-e^2x^2)^{p-1}}{2x^2} + \frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^3x}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(-1 + p)/x^2 + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (e^2*(3
 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2
)/d^2])/(2*d^2*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\,(d^2-e^2x^2)^{-2+p}}{x^3}\,dx \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} - \frac{\int \frac{(4d^3e-2d^2e^2(3-p)x)\,(d^2-e^2x^2)^{-2+p}}{x^2}\,dx}{2d^2} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} - (2de)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^2}\,dx + \left(e^2(3-p)\right)\int \frac{(d^2-e^2x^2)^{-2+p}}{x}\,dx \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} + \frac{1}{2}\left(e^2(3-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) \\
&\quad - \frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}}{x^2}\,dx}{d^3} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{2x^2} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x} \\
&\quad + \frac{e^2(3-p)\,(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(1,-1+p;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.98

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \frac{(d^2-e^2x^2)^p\left(\frac{8d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{2d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2} + \cdots\right.}{\cdots}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2*d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d
^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d*e^2*Hypergeometric2F1[-p,
 -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^5)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3(ex+d)^2}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x^3(d+ex)^2}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^3\,(d+e\,x)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2), x)
```

3.284 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx &= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3}\\ &\quad -\frac{2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^4x}\\ &\quad -\frac{e^3(d^2-e^2x^2)^{-1+p}\,\text{Hypergeometric2F1}\left(2,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}\end{aligned}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(-1+p)/x^3-2/3*e^2*(4-p)*(-e^2*x^2+d^2)^p*hypergeom([-1
/2, 2-p],[1/2],e^2*x^2/d^2)/d^4/x/((1-e^2*x^2/d^2)^p)-e^3*(-e^2*x^2+d^2)^(-
1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^3/(1-p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{866, 1821, 778, 272, 67, 372, 371\}$

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = -\frac{(d^2-e^2x^2)^{p-1}}{3x^3} - \frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p \,\text{Hypergeometric2F1}\left(-\frac{1}{2},2-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^4x} - \frac{e^3(d^2-e^2x^2)^{p-1}\,\text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] -1/3*(d^2 - e^2*x^2)^(-1 + p)/x^3 - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2)^
p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^2
*x^2)/d^2])/(d^3*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^2\,(d^2-e^2x^2)^{-2+p}}{x^4}\,dx\\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - \frac{\int \frac{(6d^3e-2d^2e^2(4-p)x)\,(d^2-e^2x^2)^{-2+p}}{x^3}\,dx}{3d^2}\\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - (2de)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^3}\,dx + \frac{1}{3}\left(2e^2(4-p)\right)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^2}\,dx\\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - (de)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x^2}\,dx, x, x^2\right)\\
&\quad + \frac{\left(2e^2(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}}{x^2}\,dx}{3d^4}\\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - \frac{2e^2(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}\\
&\quad - \frac{e^3(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(2,-1+p;p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 334 vs. 2(145) = 290.

Time = 0.81 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-\frac{4d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{36d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (36*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1/
2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (12*d^3*e*Hypergeometric2F1[
1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3
*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1
[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (24*d*e
^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^
p)))/(12*d^6)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4\,(ex+d)^2}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^6 + 2*d*e*x^5 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^4\,(d+ex)^2}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^2}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2), x)
```

3.285 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2} dx = -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} + \frac{2e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3} + \frac{e^4(5-p)\,(d^2-e^2x^2)^{-1+p} \text{Hypergeometric2F1}\left(2,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(-1+p)/x^4+2/3*e*(-e^2*x^2+d^2)^p*hypergeom([-3/2, 2-p]
,[-1/2],e^2*x^2/d^2)/d^3/x^3/((1-e^2*x^2/d^2)^p)+1/4*e^4*(5-p)*(-e^2*x^2+d^
2)^(-1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^4/(1-p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2} dx = -\frac{(d^2-e^2x^2)^{p-1}}{4x^4} + \frac{e^4(5-p)\,(d^2-e^2x^2)^{p-1} \text{Hypergeometric2F1}\left(2,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)} + \frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p} (d^2-e^2x^2)^p \text{Hypergeometric2F1}\left(-\frac{3}{2},2-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] -1/4*(d^2 - e^2*x^2)^(-1 + p)/x^4 + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2])/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (e
^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^
2*x^2)/d^2])/(4*d^4*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^5}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} - \frac{\int \frac{\left(8d^3e-2d^2e^2(5-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx}{4d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx + \frac{1}{2}\left(e^2(5-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} + \frac{1}{4}\left(e^2(5-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}}{x^4}\,dx}{d^3}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3} + \frac{e^4(5-p)\left(d^2-e^2x^2\right)^{-1+p}{}_2F_1\left(2,-1+p;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(145) = 290.

Time = 0.90 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.68

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^2}\,dx = \frac{\left(d^2-e^2x^2\right)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3} + \frac{48d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (48*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (18*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (3*2^p*e^4*(d
 - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(
1 + (e*x)/d)^p) + (30*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]
)/(p*(1 - d^2/(e^2*x^2))^p)))/(12*d^7)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^5\,(ex+d)^2}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2x^5}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^7 + 2*d*e*x^6 + d^2*x^5), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^5\,(d+ex)^2}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^5} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^5} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d + ex)^2} \, dx = \int \frac{(d^2 - e^2 x^2)^p}{x^5 (d + e x)^2} \, dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2), x)
```

3.286 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx &= -\frac{2d^6(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p}\\ &\quad+\frac{9d^4(d^2-e^2x^2)^{-1+p}}{2e^5(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{e^5p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^5(1+p)}\\ &\quad+\frac{2(8+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3(1+2p)}\end{aligned}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(-2+p)/e^5/(2-p)-3*d*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p
)+9/2*d^4*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)+3*d^2*(-e^2*x^2+d^2)^p/e^5/p-1/2*
(-e^2*x^2+d^2)^(p+1)/e^5/(p+1)+2/5*(8+p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, 3-p],[7/2],e^2*x^2/d^2)/d^3/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{866, 1666, 470, 372, 371, 457, 78\}$

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{3dx^5(d^2-e^2x^2)^{p-2}}{2p+1}+\frac{3d^2(d^2-e^2x^2)^p}{e^5p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^5(p+1)} -\frac{2d^6(d^2-e^2x^2)^{p-2}}{e^5(2-p)}+\frac{9d^4(d^2-e^2x^2)^{p-1}}{2e^5(1-p)} +\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^
(-2 + p))/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(2*e^5*(1 - p)) + (3
*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^5*(1 + p)) +
 (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\
&= \int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)\,dx+\int x^5\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)\,dx\\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-3+p}\left(-3d^2e-e^3x\right)\,dx,x,x^2\right)\\
&\quad+\frac{\left(2d^3(8+p)\right)\int x^4\left(d^2-e^2x^2\right)^{-3+p}\,dx}{1+2p}
\end{aligned}$$

$$= -\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4d^6(d^2-e^2x)^{-3+p}}{e^3} + \frac{9d^4(d^2-e^2x)^{-2+p}}{e^3} - \frac{6d^2(d^2-e^2x)^{-1+p}}{e^3} + \frac{(d^2-e^2x)^p}{e^3}\right) dx, x, x^2\right) + \frac{\left(2(8+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}dx}{d^3(1+2p)}$$

$$= -\frac{2d^6(d^2-e^2x^2)^{-2+p}}{e^5(2-p)} - \frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{9d^4(d^2-e^2x^2)^{-1+p}}{2e^5(1-p)} + \frac{3d^2(d^2-e^2x^2)^p}{e^5p} - \frac{(d^2-e^2x^2)^{1+p}}{2e^5(1+p)} + \frac{2(8+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3(1+2p)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.11

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(24de(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)\right.}{}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d^2 - e^2*x^2)^p*(24*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyp
ergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)
^p*(4*d*(1/2 + (e*x)/(2*d))^p + 4*e*x*(1/2 + (e*x)/(2*d))^p + 24*d*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 8*d*Hypergeometric2F1[2 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*Hypergeometric2F1[3 - p, 1 + p, 2 +
p, (d - e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p))
```

Maple [F]

$$\int \frac{x^4\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3}dx$$

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^4\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.287 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 194

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3} dx = \frac{2d^5(d^2-e^2x^2)^{-2+p}}{e^4(2-p)} + \frac{ex^5(d^2-e^2x^2)^{-2+p}}{1+2p} - \frac{7d^3(d^2-e^2x^2)^{-1+p}}{2e^4(1-p)} - \frac{3d(d^2-e^2x^2)^p}{2e^4p} - \frac{2e(4+3p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(1+2p)}$$

```
[Out] 2*d^5*(-e^2*x^2+d^2)^(-2+p)/e^4/(2-p)+e*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p)-7
/2*d^3*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)-3/2*d*(-e^2*x^2+d^2)^p/e^4/p-2/5*e*(
4+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 3-p],[7/2],e^2*x^2/d^2)/d^4/(1+
2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1666, 457, 78, 470, 372, 371}

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$$
$$=\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}-\frac{3d(d^2-e^2x^2)^p}{2e^4p}+\frac{2d^5(d^2-e^2x^2)^{p-2}}{e^4(2-p)}$$
$$-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},3-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}$$
$$-\frac{7d^3(d^2-e^2x^2)^{p-1}}{2e^4(1-p)}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^(-2
 + p))/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^(-1 + p))/(2*e^4*(1 - p)) - (3*d*
(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\
&= \int x^3\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)\,dx+\int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)\,dx\\
&= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)\,dx,x,x^2\right)\\
&\quad-\frac{\left(2d^2e(4+3p)\right)\int x^4\left(d^2-e^2x^2\right)^{-3+p}\,dx}{1+2p}
\end{aligned}$$

$$= \frac{ex^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{4d^5(d^2-e^2x)^{-3+p}}{e^2} - \frac{7d^3(d^2-e^2x)^{-2+p}}{e^2} + \frac{3d(d^2-e^2x)^{-1+p}}{e^2}\right) dx, x, x^2\right) - \frac{\left(2e(4+3p)\,(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right) \int x^4 \left(1-\frac{e^2x^2}{d^2}\right)^{-3+p} dx}{d^4(1+2p)}$$

$$= \frac{2d^5(d^2-e^2x^2)^{-2+p}}{e^4(2-p)} + \frac{ex^5(d^2-e^2x^2)^{-2+p}}{1+2p} - \frac{7d^3(d^2-e^2x^2)^{-1+p}}{2e^4(1-p)} - \frac{3d(d^2-e^2x^2)^p}{2e^4p} - \frac{2e(4+3p)x^5(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, 3-p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4(1+2p)}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3} dx = \frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(8e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2x^2}{d^2}\right) + (d\right.}{}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(8*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(1
2*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 6*Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)])))/(e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^
2)^p)
```

Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^3} dx$$

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)

[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)

Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)

Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^3}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.288 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=-\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}+\frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(1+p)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(p+1)/e^3/(2-p)/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(p+1)/e^
3/p/(e*x+d)^2+2^(-3+p)*(4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeo
m([p+1, 2-p],[2+p],1/2*(-e*x+d)/d)/d^2/e^3/p/(-p^2+p+2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1653, 807, 692, 71}

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx =\frac{2^{p-3}(p+4)(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{2e^3p(d+ex)^2}-\frac{d(d^2-e^2x^2)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^3*(2 - p)*(d + e*x)^3) - (d^2 - e^2*x^2
)^(1 + p)/(2*e^3*p*(d + e*x)^2) + (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1 - p
```

```
)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/(d^2*e^3*(2 - p)*p*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2} - \frac{\int \frac{(2d^2e^2+2de^3(1+p)x)(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{2e^4p}$$

$$= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3} - \frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2} - \frac{(d(4+p))\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx}{2e^2(2-p)p}$$

$$= -\frac{d(d^2 - e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3} - \frac{(d^2 - e^2x^2)^{1+p}}{2e^3p(d+ex)^2} - \frac{\left((4+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}dx}{2d^2e^2(2-p)p}$$

$$= -\frac{d(d^2 - e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3} - \frac{(d^2 - e^2x^2)^{1+p}}{2e^3p(d+ex)^2} + \frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(1+p)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)-4\,\text{Hypergeon}\right.}{de^3(1+p)}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e
^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^3}\,dx$$

[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)

[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)

Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^3}\,dx$$

[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)

Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^3}\,dx$$

[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx=\int \frac{x^2\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.289 $\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3} - \frac{3\ 2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(2-p)/(e*x+d)^3-3*2^(-3+p)*(1+e*x/d)^(-1-p)*(-
e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 2-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^2/(-p^2
+p+2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {807, 692, 71}

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3} - \frac{3\ 2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}+\frac{3\int\frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx}{2e(2-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}\\
&\quad+\frac{\left(3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{2d^3e(2-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeometr}\right.}{d^2e^2(1+p)}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
[Out] (2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^2*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^3}dx$$

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^3}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{x\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.290 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3} \, dx = -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

```
[Out] -2^(-3+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[2+p],
1/2*(-e*x+d)/d)/d^4/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3} \, dx = -\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}dx}{d^4}\\ &= -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int\frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = -\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3}dx$$

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^p}{(d + ex)^3} \, dx = \int \frac{(d^2 - e^2\,x^2)^p}{(d + e\,x)^3} \, dx$$

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^3, x)
```

3.291 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx = \frac{2d(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} - \frac{2e(4-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)} + \frac{(d^2-e^2x^2)^{-1+p}\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}$$

```
[Out] 2*d*(-e^2*x^2+d^2)^(-2+p)/(2-p)-e*x*(-e^2*x^2+d^2)^(-2+p)/(3-2*p)-2*e*(4-3*
p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^4/(3-2*p)/(
(1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*
x^2/d^2)/d/(1-p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{866, 1666, 457, 80, 67, 396, 252, 251\}$

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{(d^2-e^2x^2)^{p-1}\,\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)} - \frac{ex(d^2-e^2x^2)^{p-2}}{3-2p} + \frac{2d(d^2-e^2x^2)^{p-2}}{2-p} - \frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2*d*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (e*x*(d^2 - e^2*x^2)^(-2 + p))/(3
- 2*p) - (2*e*(4 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3
/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2
)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d*(1 - p)
)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^3 \left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx \\
&= \int \frac{\left(d^2-e^2x^2\right)^{-3+p} \left(d^3+3de^2x^2\right)}{x}\,dx + \int \left(d^2-e^2x^2\right)^{-3+p} \left(-3d^2e-e^3x^2\right)\,dx \\
&= -\frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p} \left(d^3+3de^2x\right)}{x}\,dx, x, x^2\right) \\
&\quad - \frac{(2d^2e(4-3p)) \int \left(d^2-e^2x^2\right)^{-3+p}\,dx}{3-2p}
\end{aligned}
$$

$$=\frac{2d(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p}+\frac{1}{2}d\text{Subst}\left(\int\frac{(d^2-e^2x)^{-2+p}}{x}\,dx,x,x^2\right)$$
$$-\frac{\left(2e(4-3p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}dx}{d^4(3-2p)}$$
$$=\frac{2d(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p}$$
$$-\frac{2e(4-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$
$$+\frac{(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(1,-1+p;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.87

$$\int\frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$$
$$=\frac{2^{-3+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2-\right.\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(4*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*p*(1
 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
 - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)] + 4*d*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^
2/(e^2*x^2)] + 4*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p,
 d^2/(e^2*x^2)]))/(d^4*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x(ex+d)^3}dx$$

[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)

[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x(d+ex)^3}\,dx$$

[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx=\int \frac{(d^2-e^2\,x^2)^p}{x\,(d+e\,x)^3}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3), x)
```

3.292 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{d(d^2-e^2x^2)^{-2+p}}{x} + \frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5} - \frac{3e(d^2-e^2x^2)^{-1+p}\text{Hypergeometric2F1}\left(1,-1+p,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}$$

```
[Out] -2*e*(-e^2*x^2+d^2)^(-2+p)/(2-p)-d*(-e^2*x^2+d^2)^(-2+p)/x+2*e^2*(4-p)*x*(-
e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^5/((1-e^2*x^2/d^2)
^p)-3/2*e*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/
(1-p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{866, 1821, 1666, 457, 80, 67, 12, 252, 251\}$

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$$
$$= -\frac{3e(d^2-e^2x^2)^{p-1}\,\text{Hypergeometric2F1}\left(1,p-1,p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)} - \frac{2e(d^2-e^2x^2)^{p-2}}{2-p} - \frac{d(d^2-e^2x^2)^{p-2}}{x} + \frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] (-2*e*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (d*(d^2 - e^2*x^2)^(-2 + p))/x +
(2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*
x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p) - (3*e*(d^2 - e^2*x^2)^(-1 + p)*Hype
rgeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^3 (d^2-e^2x^2)^{-3+p}}{x^2} \, dx$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e-2d^3e^2(4-p)x+d^2e^3x^2)}{x} \, dx}{d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int -2d^3e^2(4-p)\left(d^2-e^2x^2\right)^{-3+p}\,dx}{d^2} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e+d^2e^3x^2)}{x}\,dx}{d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}(3d^4e+d^2e^3x)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(4-p)\right)\int \left(d^2-e^2x^2\right)^{-3+p}\,dx$$

$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{1}{2}(3e)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}\,dx}{d^5}$$

$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} + \frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\ {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{d^5} - \frac{3e(d^2-e^2x^2)^{-1+p}\ {}_2F_1\left(1, -1+p; p; 1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.69

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{8d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{3\ 2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,1+p,2\right.}{1+p}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (3*2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1
 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^(2 + p
)*e*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (12*d*e*Hypergeometric
2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^5)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^2\,(ex+d)^3}dx$$

[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)

[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^2}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)
, x)

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^2\,(d+ex)^3}\,dx$$

[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^2}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx=\int \frac{(d^2-e^2\,x^2)^p}{x^2\,(d+e\,x)^3}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.293 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx &= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x} \\ &\quad -\frac{2e^3(8-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6} \\ &\quad +\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{-2+p}\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}\end{aligned}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(-2+p)/x^2+3*e*(-e^2*x^2+d^2)^(-2+p)/x-2*e^3*(8-3*p)*
x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^6/((1-e^2*x^2/
d^2)^p)+1/2*e^2*(6-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^
2*x^2/d^2)/d/(2-p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

= {866, 1821, 778, 272, 67, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$$
$$= \frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)} + \frac{3e(d^2-e^2x^2)^{p-2}}{x} - \frac{d(d^2-e^2x^2)^{p-2}}{2x^2} - \frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},3-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(-2 + p))/x^2 + (3*e*(d^2 - e^2*x^2)^(-2 + p))/x -
(2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^
2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p)
)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(6d^4e-2d^3e^2(6-p)x+2d^2e^3x^2\right)}{x^2}\,dx}{2d^2} \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x}+\frac{\int \frac{\left(2d^5e^2(6-p)-4d^4e^3(8-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx}{2d^4} \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x} \\
&\quad -\left(2e^3(8-3p)\right)\int\left(d^2-e^2x^2\right)^{-3+p}\,dx+\left(de^2(6-p)\right)\int\frac{\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x} \\
&\quad +\frac{1}{2}\left(de^2(6-p)\right)\text{Subst}\left(\int\frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx,x,x^2\right) \\
&\quad -\frac{\left(2e^3(8-3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}\,dx}{d^6}
\end{aligned}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{3e(d^2-e^2x^2)^{-2+p}}{x} - \frac{2e^3(8-3p)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6} + \frac{e^2(6-p)\left(d^2-e^2x^2\right)^{-2+p} {}_2F_1\left(1,-2+p;-1+p;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.97

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^p \left(\frac{24d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x} + \frac{4d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2} + \right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((24*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (4*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(3 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hy
pergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (24*d*e^2*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 -
d^2/(e^2*x^2))^p)))/(8*d^6)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3\,(ex+d)^3}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)
, x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx=\int \frac{(-(-d+ex)(d+ex))^p}{x^3(d+ex)^3}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^3\,(d+e\,x)^3}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3), x)
```

3.294 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} - \frac{2e^2(8-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^5x} - \frac{e^3(10-3p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}$$

```
[Out] -1/3*d*(-e^2*x^2+d^2)^(-2+p)/x^3+3/2*e*(-e^2*x^2+d^2)^(-2+p)/x^2-2/3*e^2*(8
-p)*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^5/x/((1-e^2
*x^2/d^2)^p)-1/2*e^3*(10-3*p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1
+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {866, 1821, 778, 372, 371, 272, 67}

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$$
$$= \frac{3e(d^2-e^2x^2)^{p-2}}{2x^2} - \frac{d(d^2-e^2x^2)^{p-2}}{3x^3}$$
$$- \frac{e^3(10-3p)\,(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}$$
$$- \frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^5x}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] -1/3*(d*(d^2 - e^2*x^2)^(-2 + p))/x^3 + (3*e*(d^2 - e^2*x^2)^(-2 + p))/(2*x
^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2,
(e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^
2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2
*d^2*(2 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^4}\,dx \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(9d^4e-2d^3e^2(8-p)x+3d^2e^3x^2\right)}{x^3}\,dx}{3d^2} \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{\int \frac{\left(4d^5e^2(8-p)-6d^4e^3(10-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx}{6d^4} \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2}-\left(e^3(10-3p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx \\
&\quad+\frac{1}{3}\left(2de^2(8-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx \\
&= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} \\
&\quad-\frac{1}{2}\left(e^3(10-3p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx,x,x^2\right) \\
&\quad+\frac{\left(2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}}{x^2}\,dx}{3d^5}
\end{aligned}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} + \frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} - \frac{2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^5x} - \frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{-2+p}{}_2F_1\left(1,-2+p;-1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 393 vs. $2(179) = 358$.

Time = 0.63 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.20

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{8d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{144d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (144*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (36*d^3*e*Hypergeometric2F1
[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (
15*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x
)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeom
etric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) +
 (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*
d)])/((1 + p)*(1 + (e*x)/d)^p) - (120*d*e^3*Hypergeometric2F1[-p, -p, 1 - p
, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(24*d^7)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4\,(ex+d)^3}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^3x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)
, x)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^4\,(d+ex)^3}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^3x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex+d)^3x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^3}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3), x)
```

3.295 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3} dx = -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4}+\frac{e(d^2-e^2x^2)^{-2+p}}{x^3} + \frac{2e^3(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^6x} + \frac{e^4(10-p)\,(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(2,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(-2+p)/x^4+e*(-e^2*x^2+d^2)^(-2+p)/x^3+2*e^3*(4-p)*(-
e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^6/x/((1-e^2*x^2/d
^2)^p)+1/4*e^4*(10-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([2, -2+p],[-1+p],1-e^
2*x^2/d^2)/d^3/(2-p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{866, 1821, 778, 272, 67, 372, 371\}$

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx$$
$$= -\frac{d(d^2-e^2x^2)^{p-2}}{4x^4} + \frac{e(d^2-e^2x^2)^{p-2}}{x^3} + \frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{d^6x} + \frac{e^4(10-p)(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(2,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] -1/4*(d*(d^2 - e^2*x^2)^(-2 + p))/x^4 + (e*(d^2 - e^2*x^2)^(-2 + p))/x^3 +
(2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x
^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 -
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d-ex)^3 \left(d^2-e^2x^2\right)^{-3+p}}{x^5}\,dx \\
&= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(12d^4e-2d^3e^2(10-p)x+4d^2e^3x^2\right)}{x^4}\,dx}{4d^2} \\
&= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{\int \frac{\left(6d^5e^2(10-p)-24d^4e^3(4-p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx}{12d^4} \\
&= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} - \left(2e^3(4-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx \\
&\quad + \frac{1}{2}\left(de^2(10-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx \\
&= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} \\
&\quad + \frac{1}{4}\left(de^2(10-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x^2}\,dx, x, x^2\right) \\
&\quad - \frac{\left(2e^3(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}}{x^2}\,dx}{d^6}
\end{aligned}
$$

$$= -\frac{d(d^2 - e^2x^2)^{-2+p}}{4x^4} + \frac{e(d^2 - e^2x^2)^{-2+p}}{x^3} + \frac{2e^3(4-p)\left(d^2 - e^2x^2\right)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, 3-p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{d^6x} + \frac{e^4(10-p)\left(d^2 - e^2x^2\right)^{-2+p} {}_2F_1\left(2, -2+p; -1+p; 1 - \frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 446 vs. 2(174) = 348.

Time = 0.71 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.56

$$\int \frac{(d^2 - e^2x^2)^p}{x^5(d+ex)^3} \, dx = \frac{(d^2 - e^2x^2)^p \left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{3}{2}, -p, -\frac{1}{2}, \frac{e^2x^2}{d^2}\right)}{x^3} + \frac{80d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2}, -p, \frac{1}{2}, \frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (80*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (24*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (4*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (5*2^(1 + p)*
e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (2^p*e^4*(d - e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (60*d*e^4*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^8)
```

Maple [F]

$$\int \frac{(-e^2x^2 + d^2)^p}{x^5 (ex+d)^3} dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^5}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)
, x)

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx=\int \frac{(-(-d+ex)(d+ex))^p}{x^5(d+ex)^3}\,dx$$

[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**3), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^5}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^5}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^5\,(d+e\,x)^3}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3), x)
```

3.296 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 265

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx &= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}\\ &\quad+\frac{10d^5(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{8d^3(d^2-e^2x^2)^{-1+p}}{e^5(1-p)}-\frac{2d(d^2-e^2x^2)^p}{e^5p}\\ &\quad-\frac{4(16+15p+p^2)\,x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4(1-4p^2)}\end{aligned}$$

```
[Out] -4*d^7*(-e^2*x^2+d^2)^(-3+p)/e^5/(3-p)+d^2*(13+12*p)*x^5*(-e^2*x^2+d^2)^(-3
+p)/(-4*p^2+1)-e^2*x^7*(-e^2*x^2+d^2)^(-3+p)/(1+2*p)+10*d^5*(-e^2*x^2+d^2)^
(-2+p)/e^5/(2-p)-8*d^3*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-2*d*(-e^2*x^2+d^2)^p
/e^5/p-4/5*(p^2+15*p+16)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 4-p],[7/2],e^
2*x^2/d^2)/d^4/(-4*p^2+1)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{866, 1666, 1281, 470, 372, 371, 457, 78\}$

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$$
$$= \frac{d^2(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-4p^2} - \frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1} - \frac{2d(d^2-e^2x^2)^p}{e^5p} - \frac{4d^7(d^2-e^2x^2)^{p-3}}{e^5(3-p)} + \frac{10d^5(d^2-e^2x^2)^{p-2}}{e^5(2-p)} - \frac{4(p^2+15p+16)\,x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{5}{2},4-p,\frac{7}{2},\frac{e^2x^2}{d^2}\right)}{5d^4\,(1-4p^2)} - \frac{8d^3(d^2-e^2x^2)^{p-1}}{e^5(1-p)}$$

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (-4*d^7*(d^2 - e^2*x^2)^(-3 + p))/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2
 - e^2*x^2)^(-3 + p))/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 +
 2*p) + (10*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (8*d^3*(d^2 - e^2
*x^2)^(-1 + p))/(e^5*(1 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 +
15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int x^4(d-ex)^4 \left(d^2-e^2x^2\right)^{-4+p} \, dx$$

$$= \int x^5 \left(d^2-e^2x^2\right)^{-4+p} \left(-4d^3e-4de^3x^2\right) dx + \int x^4 \left(d^2-e^2x^2\right)^{-4+p} \left(d^4+6d^2e^2x^2+e^4x^4\right) dx$$

$$= -\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int x^2 \left(d^2-e^2x\right)^{-4+p} \left(-4d^3e-4de^3x\right) dx, x, x^2\right) - \frac{\int x^4(d^2-e^2x^2)^{-4+p} \left(-d^4e^2(1+2p)-d^2e^4(13+12p)x^2\right) dx}{e^2(1+2p)}$$

$$= \frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2} - \frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int \left(-\frac{8d^7(d^2-e^2x)^{-4+p}}{e^3} + \frac{20d^5(d^2-e^2x)^{-3+p}}{e^3} - \frac{16d^3(d^2-e^2x)^{-2+p}}{e^3} + \frac{4d(d^2-e^2x)^{-1+p}}{e^3}\right) dx, x, x^2\right) - \frac{(4d^4(16+15p+p^2)) \int x^4(d^2-e^2x^2)^{-4+p}\, dx}{1-4p^2}$$

$$= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2} - \frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p} + \frac{10d^5(d^2-e^2x^2)^{-2+p}}{e^5(2-p)} - \frac{8d^3(d^2-e^2x^2)^{-1+p}}{e^5(1-p)} - \frac{2d(d^2-e^2x^2)^p}{e^5p} - \frac{\left(4(16+15p+p^2)\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right) \int x^4 \left(1-\frac{e^2x^2}{d^2}\right)^{-4+p} dx}{d^4\left(1-4p^2\right)}$$

$$= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2} - \frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p} + \frac{10d^5(d^2-e^2x^2)^{-2+p}}{e^5(2-p)} - \frac{8d^3(d^2-e^2x^2)^{-1+p}}{e^5(1-p)} - \frac{2d(d^2-e^2x^2)^p}{e^5p} - \frac{4\left(16+15p+p^2\right)x^5(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2},4-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4\left(1-4p^2\right)}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.87

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\, dx$$

$$= \frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(16e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right) + \right.}{}$$

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(16*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(
32*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 24*Hypergeomet
ric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*Hypergeometric2F1[3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2 + d^2)^p x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2 + d^2)^p x^4}{(ex+d)^4}\,dx$$

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d^2 - e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^4\,(d^2 - e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.297 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\,(d^2-e^2x^2)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2} + \frac{3\,2^{-2+p}(2+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(1+p)}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^(p+1)/e^4/(3-p)/(e*x+d)^4-d*(1+2*p)*(-e^2*x^2+d^2)^(
p+1)/e^4/(1-2*p)/p/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(p+1)/e^4/p/(e*x+d)^2+3*2^(
-2+p)*(2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[2+p
],1/2*(-e*x+d)/d)/d^2/e^4/p/(2*p^3-5*p^2-4*p+3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1653, 807, 692, 71}

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{3\,2^{p-2}(p+2)\,(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)} - \frac{(d^2-e^2x^2)^{p+1}}{2e^4p(d+ex)^2} - \frac{d(2p+1)\,(d^2-e^2x^2)^{p+1}}{e^4(1-2p)p(d+ex)^3} + \frac{d^2(d^2-e^2x^2)^{p+1}}{2e^4(3-p)(d+ex)^4}$$

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d
^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 +
 p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d
^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{(d^2-e^2x^2)^p(2d^3e^2+2d^2e^3(2+p)x+2de^4(1+2p)x^2)}{(d+ex)^4}\,dx}{2e^5p}$$

$$= -\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{\left(8d^3e^6(1+p)+2d^2e^7\left(4+3p+2p^2\right)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx}{2e^9(1-2p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6d^2(2+p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e^3(1-2p)(3-p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6(2+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^2e^3(1-2p)(3-p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} + \frac{3\,2^{-2+p}(2+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(1+p)}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\left(-8\,\text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{d-ex}{2d}\right)+12\,\text{Hypergeom}\right.}{}$$

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-8*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + 12*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] - 6*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hy
pergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e^4*(1 + p)*(1 +
 (e*x)/d)^p)
```

Maple [F]

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)

[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)

Fricas [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^3(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)

[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)

Maxima [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)

Giac [F]

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^3}{(ex+d)^4}\,dx$$

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^3\,(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.298 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx = -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} - \frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(1+p)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(p+1)/e^3/(3-p)/(e*x+d)^4+(-e^2*x^2+d^2)^(p+1)/e^3/(1
-2*p)/(e*x+d)^3-2^(-3+p)*(7+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hyperg
eom([p+1, 3-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^3/(2*p^3-5*p^2-4*p+3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1653, 807, 692, 71}

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx = \frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3} - \frac{d(d^2-e^2x^2)^{p+1}}{2e^3(3-p)(d+ex)^4} - \frac{2^{p-3}(p+7)(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}$$

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2
)^(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(
```

```
-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\int \frac{(3d^2e^2+2de^3(1+p)x)(d^2-e^2x^2)^p}{(d+ex)^4}\,dx}{e^4(1-2p)} \\ &= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{(d(7+p))\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{e^2(1-2p)(3-p)}\end{aligned}$$

$$= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\left((7+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^3e^2(1-2p)(3-p)}$$

$$= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} - \frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(1+p)}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = -\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{d-ex}{2d}\right)-4\,\text{Hypergeon}\ldots\right.}{d^2e^3(1+p)}$$

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[2 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d^2
*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x^2(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x^2}{(ex+d)^4}\,dx$$

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.299 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4} dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(p+1)/e^2/(3-p)/(e*x+d)^4-2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(p+1)*hypergeom([p+1, 3-p],[2+p],1/2*(-e*x+d)/d)/d^4/e^2/(-p^2+2
*p+3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {807, 692, 71}

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4} dx = \frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4} - \frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4}+\frac{2\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e(3-p)}\\
&= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4}\\
&\quad+\frac{\left(2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^4e(3-p)}\\
&= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4}-\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$$
$$= \frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{d-ex}{2d}\right)+\text{Hypergeometr}\right.}{d^3e^2(1+p)}$$

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[3 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^3*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^4}\,dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^
3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \int \frac{x(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^4} \, dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^4} \, dx$$

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{x\,(d^2 - e^2\,x^2)^p}{(d + e\,x)^4} \, dx$$

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.300 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(4-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^5e(1+p)}$$

```
[Out] -2^(-4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, 4-p],[2+p],
1/2*(-e*x+d)/d)/d^5/e/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = -\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(4-p,p+1,p+2,\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int(d-ex)^p\left(1+\frac{ex}{d}\right)^{-4+p}dx}{d^5}\\ &= -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^5e(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int\frac{(d^2-e^2x^2)^p}{(d+ex)^4}dx = -\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(4-p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[4 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]

$$\int\frac{(-e^2x^2+d^2)^p}{(ex+d)^4}dx$$

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-(-d + ex)(d + ex))^p}{(d + ex)^4} \, dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{(d^2 - e^2x^2)^p}{(d + ex)^4} \, dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} \, dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^4, x)
```

3.301 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4} dx = \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)} + \frac{(d^2-e^2x^2)^{-2+p}\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}$$

```
[Out] 4*d^2*(-e^2*x^2+d^2)^(-3+p)/(3-p)-4*d*e*x*(-e^2*x^2+d^2)^(-3+p)/(5-2*p)-1/2
*(-e^2*x^2+d^2)^(-2+p)/(2-p)-8*e*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 4
-p],[3/2],e^2*x^2/d^2)/d^5/(5-2*p)/((1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(
-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/(2-p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used

= {866, 1666, 1265, 965, 80, 67, 396, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2(2-p)} - \frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p} + \frac{4d^2(d^2-e^2x^2)^{p-3}}{3-p} - \frac{(d^2-e^2x^2)^{p-2}}{2(2-p)} - \frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] (4*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (4*d*e*x*(d^2 - e^2*x^2)^(-3 + p
))/(5 - 2*p) - (d^2 - e^2*x^2)^(-2 + p)/(2*(2 - p)) - (8*e*(2 - p)*x*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*
p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F1[1,
-2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*(2 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] || !IntegerQ[m])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4 (d^2-e^2x^2)^{-4+p}}{x}\,dx$$

$$= \int (d^2-e^2x^2)^{-4+p}\left(-4d^3e-4de^3x^2\right)\,dx+\int \frac{(d^2-e^2x^2)^{-4+p}(d^4+6d^2e^2x^2+e^4x^4)}{x}\,dx$$

$$= -\frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(d^4+6d^2e^2x+e^4x^2)}{x}\,dx,x,x^2\right) - \frac{(8d^3e(2-p))\int (d^2-e^2x^2)^{-4+p}\,dx}{5-2p}$$

$$= -\frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(-d^4e^4(2-p)-7d^2e^6(2-p)x)}{x}\,dx,x,x^2\right)}{2e^4(2-p)} - \frac{\left(8e(2-p)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-4+p}\,dx}{d^5(5-2p)}$$

$$= \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)} + \frac{1}{2}d^2\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}}{x}\,dx,x,x^2\right)$$

$$= \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)} + \frac{(d^2-e^2x^2)^{-2+p}{}_2F_1\left(1,-2+p;-1+p;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 417 vs. $2(204) = 408$.

Time = 0.42 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.04

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$$
$$= \frac{2^{-4+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(8p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,\text{Hypergeometric2F1}\left(1-p,1+p,2+\right.\right.}{}$$

[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]

[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(8*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 4*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 2*e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*d*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)] + 8*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^5*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^4}dx$$

[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)

[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*e*x^2 + d^4*x), x)

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x\,(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4), x)
```

3.302 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 207

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{x}+\frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p}+\frac{4e^2(16-9p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}-\frac{2e(d^2-e^2x^2)^{-2+p}\,\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}$$

```
[Out] -4*d*e*(-e^2*x^2+d^2)^(-3+p)/(3-p)-d^2*(-e^2*x^2+d^2)^(-3+p)/x+e^2*x*(-e^2*
x^2+d^2)^(-3+p)/(5-2*p)+4*e^2*(p^2-9*p+16)*x*(-e^2*x^2+d^2)^p*hypergeom([1/
2, 4-p],[3/2],e^2*x^2/d^2)/d^6/(5-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(-e^2*x^2+d^
2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d/(2-p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{866, 1821, 1666, 457, 80, 67, 396, 252, 251\}$

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$$
$$= -\frac{2e(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{d(2-p)} + \frac{e^2x(d^2-e^2x^2)^{p-3}}{5-2p} - \frac{4de(d^2-e^2x^2)^{p-3}}{3-p} - \frac{d^2(d^2-e^2x^2)^{p-3}}{x} + \frac{4e^2(p^2-9p+16)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] (-4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2*(d^2 - e^2*x^2)^(-3 + p))/
x + (e^2*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(
5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^(-2 + p)*Hypergeomet
ric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4 \left(d^2-e^2x^2\right)^{-4+p}}{x^2}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e-d^4e^2(13-2p)x+4d^3e^3x^2-d^2e^4x^3)}{x}\,dx}{d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e+4d^3e^3x^2)}{x}\,dx}{d^2} - \frac{\int (d^2-e^2x^2)^{-4+p}(-d^4e^2(13-2p)-d^2e^4x^2)\,dx}{d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(4d^5e+4d^3e^3x)}{x}\,dx,x,x^2\right)}{2d^2} + \frac{(4d^2e^2(16-9p+p^2))\int (d^2-e^2x^2)^{-4+p}\,dx}{5-2p}$$

$$= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - (2de)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}}{x}\,dx,x,x^2\right) + \frac{\left(4e^2(16-9p+p^2)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-4+p}dx}{d^6(5-2p)}$$

$$= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} + \frac{4e^2(16-9p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)} - \frac{2e(d^2-e^2x^2)^{-2+p}{}_2F_1\left(1,-2+p;-1+p;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.63

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(-16d^2p(1+p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)+2^{5+p}epx(-d+ex)(1\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

[Out] ((d^2 - e^2*x^2)^p*((-16*d^2*p*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2^(5 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (3*2^(2 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (2^(2 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (2^p*e*p*x*(-d + e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p - (32*d*e*(1 + p)*x*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(1 - d^2/(e^2*x^2))^p))/(16*d^6*p*(1 + p)*x)

Maple [F]

$$\int \frac{(-e^2x^2 + d^2)^p}{x^2 (ex + d)^4} dx$$

[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)

[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)

Fricas [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^4} dx = \int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4 x^2} dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*e*x^3 + d^4*x^2), x)

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^p}{x^2(d + ex)^4} dx = \int \frac{(-(-d + ex)(d + ex))^p}{x^2 (d + ex)^4} dx$$

[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^2}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^2\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4), x)
```

3.303 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx &= \frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} \\
&\quad - \frac{8e^3(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7} \\
&\quad + \frac{e^2(10-p)\left(d^2-e^2x^2\right)^{-2+p}\text{Hypergeometric2F1}\left(1,-2+p,-1+p,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}
\end{aligned}
$$

```
[Out] 1/2*e^2*(11-p)*(-e^2*x^2+d^2)^(-3+p)/(3-p)-1/2*d^2*(-e^2*x^2+d^2)^(-3+p)/x^
2+4*d*e*(-e^2*x^2+d^2)^(-3+p)/x-8*e^3*(4-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1
/2, 4-p],[3/2],e^2*x^2/d^2)/d^7/((1-e^2*x^2/d^2)^p)+1/2*e^2*(10-p)*(-e^2*x^
2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {866, 1821, 1666, 457, 80, 67, 12, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$$
$$= \frac{e^2(10-p)\,(d^2-e^2x^2)^{p-2}\,\text{Hypergeometric2F1}\left(1,p-2,p-1,1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)} + \frac{e^2(11-p)\,(d^2-e^2x^2)^{p-3}}{2(3-p)} + \frac{4de(d^2-e^2x^2)^{p-3}}{x} - \frac{d^2(d^2-e^2x^2)^{p-3}}{2x^2} - \frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{d^7}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] (e^2*(11 - p)*(d^2 - e^2*x^2)^(-3 + p))/(2*(3 - p)) - (d^2*(d^2 - e^2*x^2)^
(-3 + p))/(2*x^2) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1
 - (e^2*x^2)/d^2)^p) + (e^2*(10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometri
c2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4 \left(d^2-e^2x^2\right)^{-4+p}}{x^3}\,dx$$

$$=-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}-\frac{\int\frac{(d^2-e^2x^2)^{-4+p}(8d^5e-2d^4e^2(10-p)x+8d^3e^3x^2-2d^2e^4x^3)}{x^2}\,dx}{2d^2}$$

$$=-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}+\frac{\int\frac{(d^2-e^2x^2)^{-4+p}(2d^6e^2(10-p)-16d^5e^3(4-p)x+2d^4e^4x^2)}{x}\,dx}{2d^4}$$

$$=-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}+\frac{\int-16d^5e^3(4-p)\left(d^2-e^2x^2\right)^{-4+p}\,dx}{2d^4}+\frac{\int\frac{(d^2-e^2x^2)^{-4+p}(2d^6e^2(10-p)+2d^4e^4x^2)}{x}\,dx}{2d^4}$$

$$=-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}+\frac{\text{Subst}\left(\int\frac{(d^2-e^2x)^{-4+p}(2d^6e^2(10-p)+2d^4e^4x)}{x}\,dx,x,x^2\right)}{4d^4}-\left(8de^3(4-p)\right)\int\left(d^2-e^2x^2\right)^{-4+p}\,dx$$

$$=\frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}+\frac{1}{2}\left(e^2(10-p)\right)\text{Subst}\left(\int\frac{(d^2-e^2x)^{-3+p}}{x}\,dx,x,x^2\right)-\frac{\left(8e^3(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-4+p}\,dx}{d^7}$$

$$=\frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}-\frac{8e^3(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^7}+\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{-2+p}{}_2F_1\left(1,-2+p;-1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.89

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$$
$$= \frac{(d^2-e^2x^2)^p\left(\frac{64d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}+\frac{8d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (8*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (5*2^(4 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(3 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (80*d*e^2*Hypergeometric2F1[-p
, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(16*d^7)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^3\,(ex+d)^4}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*
e*x^4 + d^4*x^3), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^p}{x^3\,(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^3}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^3\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4), x)
```

3.304 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4} dx = -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3}+\frac{2de(d^2-e^2x^2)^{-3+p}}{x^2}-\frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} + \frac{4e^4(48-17p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{3d^8} - \frac{2e^3(5-p)\,(d^2-e^2x^2)^{-3+p}\,\text{Hypergeometric2F1}\left(1,-3+p,-2+p,1-\frac{e^2x^2}{d^2}\right)}{d(3-p)}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(-3+p)/x^3+2*d*e*(-e^2*x^2+d^2)^(-3+p)/x^2-1/3*e^2*
(27-2*p)*(-e^2*x^2+d^2)^(-3+p)/x+4/3*e^4*(p^2-17*p+48)*x*(-e^2*x^2+d^2)^p*h
ypergeom([1/2, 4-p],[3/2],e^2*x^2/d^2)/d^8/((1-e^2*x^2/d^2)^p)-2*e^3*(5-p)*
(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d/(3-p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {866, 1821, 778, 272, 67, 252, 251}

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$$
$$= -\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{p-3}}{3x} + \frac{2de(d^2-e^2x^2)^{p-3}}{x^2} - \frac{d^2(d^2-e^2x^2)^{p-3}}{3x^3}$$
$$- \frac{2e^3(5-p)\left(d^2-e^2x^2\right)^{p-3}\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{d(3-p)}$$
$$+ \frac{4e^4(p^2-17p+48)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},4-p,\frac{3}{2},\frac{e^2x^2}{d^2}\right)}{3d^8}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/3*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^3 + (2*d*e*(d^2 - e^2*x^2)^(-3 + p))/
x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^(-3 + p))/(3*x) + (4*e^4*(48 - 17*p +
 p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2]
)/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^(-3 + p)*H
ypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^4}\,dx$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(12d^5e-d^4e^2(27-2p)x+12d^3e^3x^2-3d^2e^4x^3\right)}{x^3}\,dx}{3d^2}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} + \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(2d^6e^2(27-2p)-24d^5e^3(5-p)x+6d^4e^4x^2\right)}{x^2}\,dx}{6d^4}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} - \frac{\int \frac{\left(24d^7e^3(5-p)-8d^6e^4\left(48-17p+p^2\right)x\right)\left(d^2-e^2x^2\right)^{-4+p}}{x}\,dx}{6d^6}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} - \left(4de^3(5-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-4+p}}{x}\,dx + \frac{1}{3}\left(4e^4\left(48-17p+p^2\right)\right)\int \left(d^2-e^2x^2\right)^{-4+p}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)(d^2-e^2x^2)^{-3+p}}{3x} - (2de^3(5-p))\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}}{x}\,dx, x, x^2\right) + \frac{\left(4e^4(48-17p+p^2)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-4+p}dx}{3d^8}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)(d^2-e^2x^2)^{-3+p}}{3x} + \frac{4e^4(48-17p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3d^8} - \frac{2e^3(5-p)(d^2-e^2x^2)^{-3+p}{}_2F_1\left(1,-3+p;-2+p;1-\frac{e^2x^2}{d^2}\right)}{d(3-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 452 vs. 2(210) = 420.

Time = 0.78 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.15

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^p\left(-\frac{16d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3} - \frac{480d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2]])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (480*d^2*e^2*Hypergeometric2F1[-1/2, -p,
1/2, (e^2*x^2)/d^2]])/(x*(1 - (e^2*x^2)/d^2)^p) - (96*d^3*e*Hypergeometric2F
1[1 - p, -p, 2 - p, d^2/(e^2*x^2)]])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
(15*2^(5 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(3 + p)*e^3*(-d + e*x)*Hyperge
ometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p)
 + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometr
ic2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (4
80*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*
x^2))^p)))/(48*d^8)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^4(ex+d)^4}dx$$

[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)

[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^4}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^8 + 4*d*e^3*x^7 + 6*d^2*e^2*x^6 + 4*d^3*
e*x^5 + d^4*x^4), x)

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x^4(d+ex)^4}\,dx$$

[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**4), x)

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^4}\,dx$$

[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^4}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4), x)
```

3.305 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4} dx = -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4}+\frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3}-\frac{e^2(17-p)(d^2-e^2x^2)^{-3+p}}{4x^2}+\frac{8e^3(6-p)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}+\frac{e^4(70-21p+p^2)(d^2-e^2x^2)^{-3+p}\text{Hypergeometric2F1}\left(1,-3+p,-2+p,1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}$$

```
[Out] -1/4*d^2*(-e^2*x^2+d^2)^(-3+p)/x^4+4/3*d*e*(-e^2*x^2+d^2)^(-3+p)/x^3-1/4*e^
2*(17-p)*(-e^2*x^2+d^2)^(-3+p)/x^2+8/3*e^3*(6-p)*(-e^2*x^2+d^2)^p*hypergeom
([-1/2, 4-p],[1/2],e^2*x^2/d^2)/d^7/x/((1-e^2*x^2/d^2)^p)+1/4*e^4*(p^2-21*p
+70)*(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d^2/(3
-p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

= {866, 1821, 778, 372, 371, 272, 67}

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$$
$$= -\frac{e^2(17-p)\,(d^2-e^2x^2)^{p-3}}{4x^2} - \frac{d^2(d^2-e^2x^2)^{p-3}}{4x^4} + \frac{4de(d^2-e^2x^2)^{p-3}}{3x^3} + \frac{e^4(p^2-21p+70)\,(d^2-e^2x^2)^{p-3}\,\text{Hypergeometric2F1}\left(1,p-3,p-2,1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)} + \frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] -1/4*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^4 + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/
(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^(-3 + p))/(4*x^2) + (8*e^3*(6 - p)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^
7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(-3 + p
)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x^5}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(16d^5e-2d^4e^2(17-p)x+16d^3e^3x^2-4d^2e^4x^3)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(6d^6e^2(17-p)-32d^5e^3(6-p)x+12d^4e^4x^2)}{x^3}\,dx}{12d^4}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} - \frac{\int \frac{(64d^7e^3(6-p)-12d^6e^4(70-21p+p^2)x)\,(d^2-e^2x^2)^{-4+p}}{x^2}\,dx}{24d^6}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} - \frac{1}{3}\left(8de^3(6-p)\right)\int \frac{(d^2-e^2x^2)^{-4+p}}{x^2}\,dx + \frac{1}{2}\left(e^4(70-21p+p^2)\right)\int \frac{(d^2-e^2x^2)^{-4+p}}{x}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{1}{4}\big(e^4\big(70-21p+p^2\big)\big)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}}{x}\,dx, x, x^2\right) - \frac{\left(8e^3(6-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-4+p}}{x^2}\,dx}{3d^7}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{8e^3(6-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},4-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^7x} + \frac{e^4(70-21p+p^2)\,(d^2-e^2x^2)^{-3+p}\,{}_2F_1\left(1,-3+p;-2+p;1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 505 vs. $2(216) = 432$.

Time = 0.90 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.34

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$$

$$= \frac{(d^2-e^2x^2)^p\left(\frac{64d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x^3} + \frac{960d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},\frac{e^2x^2}{d^2}\right)}{x}\right.}{}$$

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (960*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (240*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2
) + (105*2^(3 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
- e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (24*d^5*Hypergeometric2F1[2 - p,
 -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (45*2^(2
 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1
[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*
e^4*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (840*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^
2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(48*d^9)
```

Maple [F]

$$\int \frac{(-e^2x^2+d^2)^p}{x^5(ex+d)^4}dx$$

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^9 + 4*d*e^3*x^8 + 6*d^2*e^2*x^7 + 4*d^3*
e*x^6 + d^4*x^5), x)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(-(-d+ex)(d+ex))^p}{x^5(d+ex)^4}\,dx$$

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

Giac [F]

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^5}\,dx$$

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(d^2-e^2\,x^2)^p}{x^5\,(d+e\,x)^4}\,dx$$

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4), x)
```

3.306 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 264

$$\begin{aligned}\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}\\ &+\frac{2d^3(3+2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)}\\ &+\frac{2d^2e(7+2m+3p)(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(4+m+2p)}\end{aligned}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(p+1)/g/(3+m+2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d
^2)^(p+1)/g^2/(4+m+2*p)+2*d^3*(3+2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hyperg
eom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3+m+2*p)/((1-e^2*x^2/
d^2)^p)+2*d^2*e*(7+2*m+3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1+1
/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/(4+m+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1823, 822, 372, 371}

$$\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^p dx$$
$$= \frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}$$
$$-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}-\frac{3d(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$
$$+\frac{2d^3(2m+p+3)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2p+3)}$$

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p)) + (2*d^3*(3 + 2*m + p)*
(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e
*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/
2, -p, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(4 + m + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} - \frac{\int (gx)^m\left(d^2-e^2x^2\right)^p\left(-d^3e^2(4+m+2p)-2d^2e^3(7+2m+3p)x-3de^4(4+m+2p)x^2\right)dx}{e^2(4+m+2p)}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\int (gx)^m\left(2d^3e^4(3+2m+p)(4+m+2p)+2d^2e^5(3+m+2p)(7+2m+3p)x\right)\left(d^2-e^2x^2\right)^p dx}{e^4(3+m+2p)(4+m+2p)}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\left(2d^3(3+2m+p)\right)\int (gx)^m\left(d^2-e^2x^2\right)^p dx}{3+m+2p} + \frac{\left(2d^2e(7+2m+3p)\right)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p dx}{g(4+m+2p)}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\left(2d^3(3+2m+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{3+m+2p} + \frac{\left(2d^2e(7+2m+3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^p dx}{g(4+m+2p)}$$

$$= -\frac{3d(gx)^{1+m}(d^2-e^2x^2)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}(d^2-e^2x^2)^{1+p}}{g^2(4+m+2p)} + \frac{2d^3(3+2m+p)(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)} + \frac{2d^2e(7+2m+3p)(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)(4+m+2p)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$$
$$= x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m} + ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{2+m} + ex\left(\frac{3d\,\text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},\frac{e^2x^2}{d^2}\right)}{3+m} + \frac{ex\,\text{Hypergeometric2F1}\left(\frac{4+m}{2},-p,\frac{6+m}{2},\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)$$

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p,
(4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2,
 -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2,
 -p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(1 - (e^2*x^2)/d^2)^p
```

Maple [F]

$$\int (gx)^m\,(ex+d)^3\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \int (ex+d)^3 \left(-e^2x^2+d^2\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p*(g*x)
^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97

$$\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx = \frac{d^3 d^{2p} g^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3d^2 d^{2p} e g^m x^{m+2} \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3d d^{2p} e^2 g^m x^{m+3} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{d^{2p} e^3 g^m x^{m+4} \Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2
+ 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**2*d*
*(2*p)*e*g**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + 3*d*d**(2*p)*e**2*g**m
*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2)) + d**(2*p)*e**3*g**m*x**(m +
4)*gamma(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*
pi)/d**2)/(2*gamma(m/2 + 3))
```

Maxima [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^3\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)^3\,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3, x)
```

3.307 $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=-\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}+\frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)}+\frac{2de(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(p+1)/g/(3+m+2*p)+2*d^2*(2+m+p)*(g*x)^(1+m)*(-e
^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3
+m+2*p)/((1-e^2*x^2/d^2)^p)+2*d*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-
p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1823, 822, 372, 371}

$$\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$$
$$= \frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}$$
$$+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2p+3)}$$
$$-\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d^2*(2 +
m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3
+ m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (
2*d*e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{\int (gx)^m\left(-2d^2e^2(2+m+p)-2de^3(3+m+2p)x\right)\left(d^2-e^2x^2\right)^p\,dx}{e^2(3+m+2p)} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g} + \frac{\left(2d^2(2+m+p)\right)\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx}{3+m+2p} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{g} \\
&\quad + \frac{\left(2d^2(2+m+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{3+m+2p} \\
&= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} \\
&\quad + \frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)} \\
&\quad + \frac{2de(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m\right.}{(1+m}$$

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, -p, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric
2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometric2F
1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*(1 -
 (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int (gx)^m\,(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)^2(-e^2x^2+d^2)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \frac{d^2d^{2p}g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{dd^{2p}eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{d^{2p}e^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2
+ 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d*d**(2*p
)*e*g**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x*
*2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 2) + d**(2*p)*e**2*g**m*x**(m + 3)*g
amma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/(2*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^2\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)^2\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)^2\,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2, x)
```

3.308 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}$$

```
[Out] d*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^
2/d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom
([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {822, 372, 371}

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{g^2(m+2)} + \frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])
/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int (gx)^m \left(d^2-e^2x^2\right)^p\,dx+\frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g}\\
&= \left(d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&\quad+\frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx}{g}\\
&= \frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}\\
&\quad+\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(2+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)+e(1+m)x\,\text{Hype}\right.}{(1+m)(2+m)}$$

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3
 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int (gx)^m\,(ex+d)\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = \frac{dd^{2p}g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^{2p}eg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
[Out] d*d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3
/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e*g
**m*x**(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*ex
p_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

Maxima [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx=\int \left(d^2-e^2\,x^2\right)^p\,{\left(g\,x\right)}^m\,\left(d+e\,x\right)\,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x), x)
```

3.309 $\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{1+m} \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/
d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {372, 371}

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{m+1} \left(d^2-e^2x^2\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2
, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$

$$=\frac{(gx)^{1+m}\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (gx)^m\,(d^2-e^2x^2)^p\,dx$$
$$=\frac{x(gx)^m\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}$$

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2
, (e^2*x^2)/d^2])/((1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int (gx)^m\left(-e^2x^2+d^2\right)^p dx$$

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p (gx)^m \, dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \frac{d^{2p}g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)

[Out] d**(2*p)*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))

Maxima [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p (gx)^m \, dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)

Giac [F]

$$\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p (gx)^m \, dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="giac")

[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(d^2 - e^2x^2\right)^p \, dx = \int \left(d^2 - e^2\,x^2\right)^p (g\,x)^m \, dx$$

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m, x)
```

3.310 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},1-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},1-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2
/d^2)/d/g/(1+m)/((1-e^2*x^2/d^2)^p)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeo
m([1-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^2/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {906, 83, 127, 372, 371}

$$\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{(gx)^{m+1}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},1-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{dg(m+1)} - \frac{e(gx)^{m+2}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},1-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m
)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x^2)
/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{-1+p}\,dx$$

$$= \left(d(d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^{-1+p}(d+ex)^{-1+p}\,dx - \frac{\left(e(d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^{1+m}(d-ex)^{-1+p}(d+ex)^{-1+p}\,dx}{g}$$

$$= d\int (gx)^m\left(d^2-e^2x^2\right)^{-1+p}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}\,dx}{g}$$

$$= \frac{\left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d} - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2g}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.76

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$$
$$= \frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-e(1+m)x\,\text{Hypergeometric2F1}\left(1+\frac{m}{2},1-p,2+\frac{m}{2},\frac{e^2x^2}{d^2}\right)+d(2+m\right.}{d^2(1+m)(2+m)}$$

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(-(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, 1 -
p, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p,
 (3 + m)/2, (e^2*x^2)/d^2]))/(d^2*(1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]

$$\int \frac{(gx)^m (-e^2x^2+d^2)^p}{ex+d}dx$$

[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)

[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx = \int \frac{(-e^2x^2+d^2)^p(gx)^m}{ex+d}\,dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.28

$$\begin{aligned}
&\int \frac{(gx)^m (d^2-e^2x^2)^p}{d+ex}\,dx \\
&= -\frac{0^p d^{1-m} d^{m+2p} e^{-m-1} e^{m-1} g^m m x^{m-1} \Phi\left(\frac{d^2}{e^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} \\
&+ \frac{0^p d^{1-m} d^{m+2p} e^{-m-1} e^{m-1} g^m x^{m-1} \Phi\left(\frac{d^2}{e^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} \\
&+ \frac{0^p d^{-m} d^{m+2p} e^{m} e^{-m-1} g^m m x^{m} \Phi\left(\frac{d^2}{e^2x^2},1,\frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4\Gamma\left(1-\frac{m}{2}\right)} \\
&+ \frac{d e^{2p-2} g^m p x^{m+2p-1} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}1-p,-\frac{m}{2}-p+\frac{1}{2}\\ -\frac{m}{2}-p+\frac{3}{2}\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)} \\
&- \frac{e^{2p-1} g^m p x^{m+2p} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p\right) {}_2F_1\left(\begin{matrix}1-p,-\frac{m}{2}-p\\ -\frac{m}{2}-p+1\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+1\right)}
\end{aligned}$$

[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d),x)

```
[Out] -0**p*d**(1 - m)*d**(m + 2*p)*e**(-m - 1)*e**(m - 1)*g**m*m*x**(m - 1)*lerc
hphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2))
+ 0**p*d**(1 - m)*d**(m + 2*p)*e**(-m - 1)*e**(m - 1)*g**m*x**(m - 1)*lerch
phi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2)) +
 0**p*d**(m + 2*p)*e**m*e**(-m - 1)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2),
1, m*exp_polar(I*pi)/2)*gamma(-m/2)/(4*d**m*gamma(1 - m/2)) + d*e**(2*p - 2
)*g**m*p*x**(m + 2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyper(
(1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), d**2/(e**2*x**2))/(2*gamma(p +
1)*gamma(-m/2 - p + 3/2)) - e**(2*p - 1)*g**m*p*x**(m + 2*p)*exp(I*pi*p)*ga
mma(p)*gamma(-m/2 - p)*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), d**2/(e**2
*x**2))/(2*gamma(p + 1)*gamma(-m/2 - p + 1))
```

Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{d + ex} \, dx = \int \frac{(d^2 - e^2\,x^2)^p\,(g\,x)^m}{d + e\,x} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x),x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x), x)
```

3.311 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{(gx)^{1+m}(d^2-e^2x^2)^{-1+p}}{g(1-m-2p)} - \frac{2(m+p)(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},2-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^2g(1+m)(1-m-2p)} - \frac{2e(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},2-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^3g^2(2+m)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^(-1+p)/g/(1-m-2*p)-2*(m+p)*(g*x)^(1+m)*(-e^2*x^2
+d^2)^p*hypergeom([2-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^2/g/(1+m)/(1-
m-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([2-p,
 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^3/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {866, 1823, 822, 372, 371}

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx$$
$$=-\frac{2(m+p)(gx)^{m+1}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},2-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^2g(m+1)(-m-2p+1)}$$
$$+\frac{(gx)^{m+1}(d^2-e^2x^2)^{p-1}}{g(-m-2p+1)}$$
$$-\frac{2e(gx)^{m+2}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},2-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^3g^2(m+2)}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]

[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*(m + p)*(g*
x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2,
 (e^2*x^2)/d^2])/(d^2*g*(1 + m)*(1 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e
*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m
)/2, (e^2*x^2)/d^2])/(d^3*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = \int (gx)^m (d-ex)^2 \left(d^2-e^2x^2\right)^{-2+p} \, dx$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} + \frac{\int (gx)^m \left(-2d^2e^2(m+p)-2de^3(1-m-2p)x\right)\left(d^2-e^2x^2\right)^{-2+p} \, dx}{e^2(1-m-2p)}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p} \, dx}{g} - \frac{\left(2d^2(m+p)\right)\int (gx)^m \left(d^2-e^2x^2\right)^{-2+p} \, dx}{1-m-2p}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p} \, dx}{d^3g} - \frac{\left(2(m+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p} \, dx}{d^2(1-m-2p)}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{2(m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},2-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^2g(1+m)(1-m-2p)} - \frac{2e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},2-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^3g^2(2+m)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{x(gx)^m (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(d^2(6+5m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},2-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)-e(1+\right.}{d^4(1-}$$

[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]

[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeometric2F1[(3 + m)/2, 2 - p, (5 + m)/2, (e^2*x^2)/d^2])))/(d^4*(1 + m)*(2 + m)*(3 + m)*(1 - (e^2*x^2)/d^2)^p)

Maple [F]

$$\int \frac{(gx)^m \left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2}dx$$

[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)

[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{\left(ex+d\right)^2}\,dx$$

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)

Sympy [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(gx)^m (-(-d+ex)(d+ex))^p}{(d+ex)^2} \, dx$$

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^2} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex+d)^2} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2 - e^2x^2)^p}{(d+ex)^2} \, dx = \int \frac{(d^2 - e^2 x^2)^p (g x)^m}{(d + e x)^2} \, dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2, x)
```

3.312 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 275

$$\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{3d(gx)^{1+m}(d^2-e^2x^2)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}(d^2-e^2x^2)^{-2+p}}{g^2(2-m-2p)} - \frac{2(2m+p)(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},3-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{d^3g(1+m)(3-m-2p)} - \frac{2e(2-2m-3p)(gx)^{2+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},3-p,\frac{4+m}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(2+m)(2-m-2p)}$$

```
[Out] 3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(-2+p)/g/(3-m-2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d
^2)^(-2+p)/g^2/(2-m-2*p)-2*(2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([
3-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^3/g/(1+m)/(3-m-2*p)/((1-e^2*x^2/
d^2)^p)-2*e*(2-2*m-3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([3-p, 1+1/2*
m],[2+1/2*m],e^2*x^2/d^2)/d^4/g^2/(2+m)/(2-m-2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{866, 1823, 822, 372, 371\}$

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3}\,dx = -\frac{e(gx)^{m+2}(d^2-e^2x^2)^{p-2}}{g^2(-m-2p+2)} + \frac{3d(gx)^{m+1}(d^2-e^2x^2)^{p-2}}{g(-m-2p+3)} - \frac{2e(-2m-3p+2)(gx)^{m+2}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},3-p,\frac{m+4}{2},\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)(-m-2p+2)} - \frac{2(2m+p)(gx)^{m+1}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},3-p,\frac{m+3}{2},\frac{e^2x^2}{d^2}\right)}{d^3g(m+1)(-m-2p+3)}$$

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p)) - (2*(2*m + p)*(g*x)^(
1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^
2*x^2)/d^2])/(d^3*g*(1 + m)*(3 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(2
- 2*m - 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3
 - p, (4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(2 - m - 2*p)*(1 - (e^2*x
^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (gx)^m(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx \\
&= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} \\
&\quad +\frac{\int (gx)^m\left(d^2-e^2x^2\right)^{-3+p}\left(d^3e^2(2-m-2p)-2d^2e^3(2-2m-3p)x+3de^4(2-m-2p)x^2\right)\,dx}{e^2(2-m-2p)} \\
&= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} \\
&\quad +\frac{\int (gx)^m\left(-2d^3e^4(2-m-2p)(2m+p)-2d^2e^5(2-2m-3p)(3-m-2p)x\right)\left(d^2-e^2x^2\right)^{-3+p}\,dx}{e^4(2-m-2p)(3-m-2p)} \\
&= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} \\
&\quad -\frac{\left(2d^2e(2-2m-3p)\right)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-3+p}\,dx}{g(2-m-2p)} \\
&\quad -\frac{\left(2d^3(2m+p)\right)\int (gx)^m\left(d^2-e^2x^2\right)^{-3+p}\,dx}{3-m-2p} \\
&= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} \\
&\quad -\frac{\left(2e(2-2m-3p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}\,dx}{d^4g(2-m-2p)} \\
&\quad -\frac{\left(2(2m+p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-3+p}\,dx}{d^3(3-m-2p)}
\end{aligned}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)}$$
$$- \frac{2(2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},3-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^3g(1+m)(3-m-2p)}$$
$$- \frac{2e(2-2m-3p)(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},3-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^4g^2(2+m)(2-m-2p)}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$$
$$= \frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},3-p,\frac{3+m}{2},\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(-\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},3-p\right.}{2+m}\right.\right.}{d^6}$$

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 +
 m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[(2 + m)/2,
3 - p, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3
+ m)/2, 3 - p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2F1[
(4 + m)/2, 3 - p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*(1 - (e^2*x^2
)/d^2)^p)
```

Maple [F]

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}dx$$

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^
3), x)
```

Sympy [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(gx)^m (-(-d+ex)(d+ex))^p}{(d+ex)^3} \, dx$$

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3} \, dx = \int \frac{(-e^2x^2+d^2)^p (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int \frac{(d^2-e^2\,x^2)^p\,(g\,x)^m}{(d+e\,x)^3}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3, x)
```

3.313 $\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \frac{(gx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},1-p,\frac{3+m}{2},a^2x^2\right)}{g(1+m)} - \frac{a(gx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},1-p,\frac{4+m}{2},a^2x^2\right)}{g^2(2+m)}$$

```
[Out] (g*x)^(1+m)*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/g/(1+m)-a*(g*x)
^(2+m)*hypergeom([1-p, 1+1/2*m],[2+1/2*m],a^2*x^2)/g^2/(2+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {904, 83, 126, 371}

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \frac{(gx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},1-p,\frac{m+3}{2},a^2x^2\right)}{g(m+1)} - \frac{a(gx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{m+2}{2},1-p,\frac{m+4}{2},a^2x^2\right)}{g^2(m+2)}$$

```
[In] Int[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g*
(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2,
a^2*x^2])/(g^2*(2 + m))
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 904

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (gx)^m(1-ax)^p(1+ax)^{-1+p}\,dx\\
&= -\frac{a\int (gx)^{1+m}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx}{g} + \int (gx)^m(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx\\
&= -\frac{a\int (gx)^{1+m}\left(1-a^2x^2\right)^{-1+p}\,dx}{g} + \int (gx)^m\left(1-a^2x^2\right)^{-1+p}\,dx\\
&= \frac{(gx)^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{g(1+m)} - \frac{a(gx)^{2+m}\,{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};a^2x^2\right)}{g^2(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax} dx = x(gx)^m \left(-\frac{ax \text{Hypergeometric2F1}\left(1+\frac{m}{2}, 1-p, 2+\frac{m}{2}, a^2x^2\right)}{2+m} + \frac{\text{Hypergeometric2F1}\left(\frac{1+m}{2}, 1-p, \frac{3+m}{2}, a^2x^2\right)}{1+m}\right)$$

```
[In] Integrate[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
[Out] x*(g*x)^m*(-((a*x*Hypergeometric2F1[1 + m/2, 1 - p, 2 + m/2, a^2*x^2])/(2 +
 m)) + Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [F]

$$\int \frac{(gx)^m (-a^2x^2+1)^p}{ax+1} dx$$

```
[In] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
[Out] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

Fricas [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax} dx = \int \frac{(-a^2x^2+1)^p (gx)^m}{ax+1} dx$$

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="fricas")
[Out] integral((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.69

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax} dx = \frac{0^p a^m a^{-m-1} g^m m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{me^{i\pi}}{2}\right) \Gamma\left(-\frac{m}{2}\right)}{4\Gamma\left(1-\frac{m}{2}\right)} - \frac{0^p a^{-m-1} a^{m-1} g^m m x^{m-1} \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right) \Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p a^{-m-1} a^{m-1} g^m x^{m-1} \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right) \Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{a^{2p-2} g^m p x^{m+2p-1} e^{i\pi p} \Gamma(p) \Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} 1-p, -\frac{m}{2}-p+\frac{1}{2} \\ -\frac{m}{2}-p+\frac{3}{2} \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)} - \frac{a^{2p-1} g^m p x^{m+2p} e^{i\pi p} \Gamma(p) \Gamma\left(-\frac{m}{2}-p\right) {}_2F_1\left(\left.\begin{matrix} 1-p, -\frac{m}{2}-p \\ -\frac{m}{2}-p+1 \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+1\right)}$$

```
[In] integrate((g*x)**m*(-a**2*x**2+1)**p/(a*x+1),x)
```

```
[Out] 0**p*a**m*a**(-m - 1)*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, m*exp_polar(I*
pi)/2)*gamma(-m/2)/(4*gamma(1 - m/2)) - 0**p*a**(-m - 1)*a**(m - 1)*g**m*m*
x**(m - 1)*lerchphi(1/(a**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(
3/2 - m/2)) + 0**p*a**(-m - 1)*a**(m - 1)*g**m*x**(m - 1)*lerchphi(1/(a**2*
x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*gamma(3/2 - m/2)) + a**(2*p - 2)*g
**m*p*x**(m + 2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyper((1
- p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), 1/(a**2*x**2))/(2*gamma(p + 1)*gam
ma(-m/2 - p + 3/2)) - a**(2*p - 1)*g**m*p*x**(m + 2*p)*exp(I*pi*p)*gamma(p)
*gamma(-m/2 - p)*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), 1/(a**2*x**2))/(
2*gamma(p + 1)*gamma(-m/2 - p + 1))
```

Maxima [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax} dx = \int \frac{(-a^2x^2+1)^p (gx)^m}{ax+1} dx$$

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Giac [F]

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(-a^2x^2+1)^p (gx)^m}{ax+1}\,dx$$

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="giac")
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (1-a^2x^2)^p}{1+ax}\,dx = \int \frac{(g\,x)^m\,(1-a^2\,x^2)^p}{a\,x+1}\,dx$$

```
[In] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1),x)
[Out] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1), x)
```

3.314 $\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{1+m}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(1+\frac{ex}{d}\right)^{-n-p} \left(d^2-e^2x^2\right)^p \text{AppellF1}\left(1+m,-p,-n-p,2+m,\frac{ex}{d},-\frac{ex}{d}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(e*x+d)^n*(1+e*x/d)^(-n-p)*(-e^2*x^2+d^2)^p*AppellF1(1+m,-p,-n-
p,2+m,e*x/d,-e*x/d)/g/(1+m)/((1-e*x/d)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {906, 140, 138}

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \frac{(gx)^{m+1}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(\frac{ex}{d}+1\right)^{-n-p} \text{AppellF1}\left(m+1,-p,-n-p,m+2,\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

```
[In] Int[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*AppellF
1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d)^
p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{n+p}\,dx\\
&= \left((d+ex)^{-p}\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d+ex)^{n+p}\left(1-\frac{ex}{d}\right)^p\,dx\\
&= \left((d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m\left(1-\frac{ex}{d}\right)^p\left(1+\frac{ex}{d}\right)^{n+p}\,dx\\
&= \frac{(gx)^{1+m}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p F_1\left(1+m;-p,-n-p;2+m;\frac{ex}{d},-\frac{ex}{d}\right)}{g(1+m)}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$
\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx
$$

$$
= \frac{x(gx)^m(d-ex)^p\left(\frac{d-ex}{d}\right)^{-p}(d+ex)^{n+p}\left(\frac{d+ex}{d}\right)^{-n-p}\text{AppellF1}\left(1+m,-p,-n-p,2+m,\frac{ex}{d},-\frac{ex}{d}\right)}{1+m}
$$

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d - e*x)^p*(d + e*x)^(n + p)*((d + e*x)/d)^(-n - p)*AppellF1[1
+ m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/((1 + m)*((d - e*x)/d)^p)
```

Maple [F]

$$\int (gx)^m (ex+d)^n \left(-e^2x^2+d^2\right)^p dx$$

```
[In] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p(ex+d)^n(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

Sympy [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int (gx)^m \left(-(-d+ex)\,(d+ex)\right)^p (d+ex)^n \, dx$$

```
[In] integrate((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p,x)
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p*(d + e*x)**n, x)
```

Maxima [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p(ex+d)^n(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \,dx = \int \left(-e^2x^2+d^2\right)^p(ex+d)^n(gx)^m \,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^n \left(d^2-e^2x^2\right)^p \,dx = \int \left(d^2-e^2\,x^2\right)^p\,(g\,x)^m\,(d+e\,x)^n \,dx$$

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n, x)
```

3.315 $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 214

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x} + \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

```
[Out] 2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)^(1/2)+arctan((-2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))/(
2+2*2^(1/2))^(1/2)-arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2)
)^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

$= \{839, 841, 1183, 648, 632, 210, 642\}$

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} + 2\sqrt{x+1}$$
$$+ \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)$$
$$- \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)$$

```
[In] Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\sqrt{1+x}+\int \frac{-1+x}{\sqrt{1+x}\,(1+x^2)}\,dx\\
&= 2\sqrt{1+x}+2\text{Subst}\left(\int \frac{-2+x^2}{2-2x^2+x^4}\,dx,x,\sqrt{1+x}\right)\\
&= 2\sqrt{1+x}+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}-\left(-2-\sqrt{2}\right)x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}+\left(-2-\sqrt{2}\right)x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}
\end{aligned}$$

$$= 2\sqrt{1+x} - \frac{1}{2}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$- \frac{1}{2}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$+ \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$- \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\sqrt{1+x} + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

$$- \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

$$+ \sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}\right)$$

$$+ \sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}\right)$$

$$= 2\sqrt{1+x} + \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)$$

$$- \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)$$

$$+ \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

$$- \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x} - \sqrt{-1+i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right) - \sqrt{-1-i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

```
[In] Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[-1 + I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] - Sqrt[-1
 - I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

method	result
derivativedivides	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\frac{\ln\left(1+x+\sqrt{2}+\right.}{}$
default	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\frac{\ln\left(1+x+\sqrt{2}+\right.}{}$
risch	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\frac{\ln\left(1+x+\sqrt{2}+\right.}{}$
trager	$2\sqrt{1+x}-2\,\mathrm{RootOf}\left(128\_Z^4-16\_Z^2+1\right)\ln\left(\frac{256\,\mathrm{RootOf}\left(128\_Z^4-16\_Z^2+1\right)^5 x-16\,\mathrm{RootOf}}{}\right.$

```
[In] int(x*(1+x)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1
/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/
2))^(1/2))*(2+2*2^(1/2))^(1/2)+(-2^(1/2)+1)/(-2+2*2^(1/2))^(1/2)*arctan((2*
(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.34

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = -\frac{1}{2}\sqrt{i+1}\log\left(\sqrt{i+1}+\sqrt{x+1}\right)+\frac{1}{2}\sqrt{i+1}\log\left(-\sqrt{i+1}+\sqrt{x+1}\right) - \frac{1}{2}\sqrt{-i+1}\log\left(\sqrt{-i+1}+\sqrt{x+1}\right) + \frac{1}{2}\sqrt{-i+1}\log\left(-\sqrt{-i+1}+\sqrt{x+1}\right)+2\sqrt{x+1}$$

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(I + 1)*log(sqrt(I + 1) + sqrt(x + 1)) + 1/2*sqrt(I + 1)*log(-sqrt
(I + 1) + sqrt(x + 1)) - 1/2*sqrt(-I + 1)*log(sqrt(-I + 1) + sqrt(x + 1)) +
 1/2*sqrt(-I + 1)*log(-sqrt(-I + 1) + sqrt(x + 1)) + 2*sqrt(x + 1)
```

Sympy [F]

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = \int \frac{x\sqrt{x+1}}{x^2+1}\,dx$$

```
[In] integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(x*sqrt(x + 1)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = \int \frac{\sqrt{x+1}x}{x^2+1}\,dx$$

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = -\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) + \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) + 2\sqrt{x+1}$$

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/2*sqrt(2*sqrt(2) - 2)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/4*s
qrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2)
+ 1) + 1/4*sqrt(2*sqrt(2) + 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) +
 x + sqrt(2) + 1) + 2*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{x+1} + \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} - 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} + 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)$$

```
[In] int((x*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] 2*(x + 1)^(1/2) + atanh((x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (x + 1)
^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1
/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/
8 - 2^(1/2)/8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((x + 1)^(1/2)/(4*
(1/8 - 2^(1/2)/8)^(1/2)) + (x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(
1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(
8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8
)^(1/2))
```

3.316 $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5}+\frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a+cx^2)^{3/2}}{20ce^3}+\frac{(d+ex)^2\,(a+cx^2)^{3/2}}{5ce^3} - \frac{d(8c^2d^4+4acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6} - \frac{d^4\sqrt{cd^2+ae^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^6}$$

```
[Out] 1/60*(-8*a*e^2+47*c*d^2)*(c*x^2+a)^(3/2)/c^2/e^3-13/20*d*(e*x+d)*(c*x^2+a)^
(3/2)/c/e^3+1/5*(e*x+d)^2*(c*x^2+a)^(3/2)/c/e^3-1/8*d*(-a^2*e^4+4*a*c*d^2*e
^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^6-d^4*arctanh((-
c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^6+1/8
*d*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^5
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used

$= \{1668, 829, 858, 223, 212, 739\}$

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)}{8c^{3/2}e^6} - \frac{d^4\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^6} + \frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3} + \frac{d\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{8ce^5} - \frac{13d(a+cx^2)^{3/2}\left(d+ex\right)}{20ce^3} + \frac{\left(a+cx^2\right)^{3/2}\left(d+ex\right)^2}{5ce^3}$$

```
[In] Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3}+\frac{\int \frac{\sqrt{a+cx^2}\left(-2ad^2e^2-de\left(3cd^2+4ae^2\right)x-e^2\left(11cd^2+2ae^2\right)x^2-13cde^3x^3\right)}{d+ex}\,dx}{5ce^4}\\
&= -\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3}+\frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3}\\
&\quad+\frac{\int \frac{\sqrt{a+cx^2}\left(5acd^2e^5+3cde^4\left(9cd^2-ae^2\right)x+ce^5\left(47cd^2-8ae^2\right)x^2\right)}{d+ex}\,dx}{20c^2e^7}\\
&= \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3}-\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3}\\
&\quad+\frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3}+\frac{\int \frac{\left(15ac^2d^2e^7-15c^2de^6\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{60c^3e^9}\\
&= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5}+\frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3}\\
&\quad-\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3}+\frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3}\\
&\quad+\frac{\int \frac{15ac^3d^2e^7\left(4cd^2+ae^2\right)-15c^3de^6\left(8c^2d^4+4acd^2e^2-a^2e^4\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{120c^4e^{11}}
\end{aligned}$$

$$= \frac{d(8cd^3 - e(4cd^2 - ae^2)x)\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2 - 8ae^2)(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)(a+cx^2)^{3/2}}{20ce^3} + \frac{(d+ex)^2(a+cx^2)^{3/2}}{5ce^3} + \frac{(d^4(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}dx}{e^6} - \frac{(d(8c^2d^4 + 4acd^2e^2 - a^2e^4))\int \frac{1}{\sqrt{a+cx^2}}dx}{8ce^6}$$

$$= \frac{d(8cd^3 - e(4cd^2 - ae^2)x)\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2 - 8ae^2)(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)(a+cx^2)^{3/2}}{20ce^3} + \frac{(d+ex)^2(a+cx^2)^{3/2}}{5ce^3} - \frac{(d^4(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^6} - \frac{(d(8c^2d^4 + 4acd^2e^2 - a^2e^4))\,\text{Subst}\left(\int \frac{1}{1-cx^2}dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{8ce^6}$$

$$= \frac{d(8cd^3 - e(4cd^2 - ae^2)x)\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2 - 8ae^2)(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)(a+cx^2)^{3/2}}{20ce^3} + \frac{(d+ex)^2(a+cx^2)^{3/2}}{5ce^3} - \frac{d(8c^2d^4 + 4acd^2e^2 - a^2e^4)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6} - \frac{d^4\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e\sqrt{a+cx^2}(-16a^2e^4 + ace^2(40d^2 - 15dex + 8e^2x^2) + 2c^2(60d^4 - 30d^3ex + 20d^2e^2x^2 - 15de^3x^3 + 12e^4x^4))}{12}$$

```
[In] Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) +
 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
+ 240*c^2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + 15*Sqrt[c]*d*(8*c^2*d^4 + 4*a*c*d^2*e^2
- a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(120*c^2*e^6)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(-24e^4c^2x^4+30d\,e^3c^2x^3-8ac\,e^4x^2-40c^2d^2e^2x^2+15acd\,e^3x+60c^2d^3ex+16a^2e^4-40ac\,d^2e^2-120c^2d^4)\sqrt{c\,x^2+a}}{120c^2e^5}+\frac{d\left(\frac{(a^2e^4-4a\cdots}{\cdots}\cdots\right)}{\cdots}$
default	$\frac{\frac{x^2(c\,x^2+a)^{\frac{3}{2}}}{5c}-\frac{2a(c\,x^2+a)^{\frac{3}{2}}}{15c^2}}{e}+\frac{d^2(c\,x^2+a)^{\frac{3}{2}}}{3e^3c}-\frac{d^3\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{e^4}-\frac{d\left(\frac{x(c\,x^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln(x\sqrt{\cdots}}{\cdots}\right)}{4c}\cdots\right)}{e^2}\cdots$

```
[In] int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(-24*c^2*e^4*x^4+30*c^2*d*e^3*x^3-8*a*c*e^4*x^2-40*c^2*d^2*e^2*x^2+1
5*a*c*d*e^3*x+60*c^2*d^3*e*x+16*a^2*e^4-40*a*c*d^2*e^2-120*c^2*d^4)*(c*x^2+
a)^(1/2)/c^2/e^5+1/8*d/e^5/c*((a^2*e^4-4*a*c*d^2*e^2-8*c^2*d^4)/e*ln(x*c^(1
/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*d^3*(a*e^2+c*d^2)*c/e^2/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 4.06 (sec) , antiderivative size = 1104, normalized size of antiderivative = 4.33

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$
$$=\left[\frac{120\sqrt{cd^2+ae^2}c^2d^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-15\,(8\,c^2d^5+4\,acd^3e^2\cdots}{\cdots}\right.$$
$$-\frac{240\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+15\,(8\,c^2d^5+4\,acd^3e^2-a^2de^4)\sqrt{c}\log\left(-2\,cx^2\cdots\right.}{\cdots}$$
$$-\frac{120\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-15\,(8\,c^2d^5+4\,acd^3e^2-a^2de^4)\sqrt{-c}\arctan\left(\cdots\right.}{\cdots}$$

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^
2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(24*c^
2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5
+ 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(
c*x^2 + a))/(c^2*e^6), -1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log
(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(24*c^2*e^5*x^4 - 30*c^2*d
*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 +
 a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6
), 1/120*(60*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24*c^2*e^5*x^4 - 3
0*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^
2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(
c^2*e^6), -1/120*(120*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - (24*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e +
40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d
^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6)]
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

```
[In] integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^4\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int((x^4*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.317 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 211

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^4}-\frac{7d(a+cx^2)^{3/2}}{12ce^2}+\frac{(d+ex)\,(a+cx^2)^{3/2}}{4ce^2}+\frac{(8c^2d^4+4acd^2e^2-a^2e^4)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}+\frac{d^3\sqrt{cd^2+ae^2}\,\mathrm{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5}$$

```
[Out] -7/12*d*(c*x^2+a)^(3/2)/c/e^2+1/4*(e*x+d)*(c*x^2+a)^(3/2)/c/e^2+1/8*(-a^2*e
^4+4*a*c*d^2*e^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5+
d^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)
^(1/2)/e^5-1/8*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1668, 829, 858, 223, 212, 739}

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{ae^2+cd^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5} - \frac{\sqrt{a+cx^2}(8cd^3-ex(4cd^2-ae^2))}{8ce^4} - \frac{7d(a+cx^2)^{3/2}}{12ce^2} + \frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}$$

```
[In] Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(c*e^4) - (7*d*(a
+ c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*
c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8
*c^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2}+\frac{\int \frac{\sqrt{a+cx^2}\left(-ade^2-e\left(3cd^2+ae^2\right)x-7cde^2x^2\right)}{d+ex}\,dx}{4ce^3}\\
&= -\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2}+\frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2}+\frac{\int \frac{\left(-3acde^4+3ce^3\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{12c^2e^5}\\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4}-\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2}\\
&\quad+\frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2}+\frac{\int \frac{-3ac^2de^4\left(4cd^2+ae^2\right)+3c^2e^3\left(8c^2d^4+4acd^2e^2-a^2e^4\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{24c^3e^7}\\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4}-\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2}+\frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2}\\
&\quad-\frac{\left(d^3\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^5}+\frac{\left(8c^2d^4+4acd^2e^2-a^2e^4\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8ce^5}
\end{aligned}$$

$$= -\frac{(8cd^3 - e(4cd^2 - ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^4} - \frac{7d(a+cx^2)^{3/2}}{12ce^2} + \frac{(d+ex)\,(a+cx^2)^{3/2}}{4ce^2} + \frac{(d^3(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^5} + \frac{(8c^2d^4 + 4acd^2e^2 - a^2e^4)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{8ce^5}$$

$$= -\frac{(8cd^3 - e(4cd^2 - ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^4} - \frac{7d(a+cx^2)^{3/2}}{12ce^2} + \frac{(d+ex)\,(a+cx^2)^{3/2}}{4ce^2} + \frac{(8c^2d^4 + 4acd^2e^2 - a^2e^4)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{c}e\sqrt{a+cx^2}(ae^2(-8d+3ex) + c(-24d^3 + 12d^2ex - 8de^2x^2 + 6e^3x^3)) - 48c^{3/2}d^3\sqrt{-cd^2-ae^2}\arctan\Big(}{24c^{3/2}e^5}$$

```
[In] Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3*e*x) + c*(-24*d^3 + 12*d^2*e*x
- 8*d*e^2*x^2 + 6*e^3*x^3)) - 48*c^(3/2)*d^3*Sqrt[-(c*d^2) - a*e^2]*ArcTan[
(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - 3*(8*c^2*
d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(24*c^(
3/2)*e^5)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\left(-6c\,x^{3}e^{3}+8cd\,e^{2}x^{2}-3a\,e^{3}x-12c\,d^{2}ex+8ad\,e^{2}+24c\,d^{3}\right)\sqrt{c\,x^{2}+a}}{24c\,e^{4}}-\frac{\frac{\left(a^{2}e^{4}-4ac\,d^{2}e^{2}-8c^{2}d^{4}\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e\sqrt{c}}-\frac{8d^{3}\left(e^{2}a+c\,d^{2}\right)c\ln\left(\right.}{}}{}$
default	$\frac{\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4c}}{e}+\frac{d^{2}\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{e^{3}}-\frac{d\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{3c\,e^{2}}-\frac{d^{3}\left(\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2}{}}\right.}{}$

```
[In] int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-6*c*e^3*x^3+8*c*d*e^2*x^2-3*a*e^3*x-12*c*d^2*e*x+8*a*d*e^2+24*c*d^3
)*(c*x^2+a)^(1/2)/c/e^4-1/8/c/e^4*((a^2*e^4-4*a*c*d^2*e^2-8*c^2*d^4)/e*ln(x
*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*d^3*(a*e^2+c*d^2)*c/e^2/((a*e^2+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1
/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 4.13 (sec) , antiderivative size = 963, normalized size of antiderivative = 4.56

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \left[\frac{24\sqrt{cd^2+ae^2}c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(8\,c^2d^4+4\,acd^2e^2-a\right.}{}\right.$$

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e
^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*
x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c
*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/48*(48*sqrt(-c*d^2 - a*e^2)*c^2*d^3*
arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^
```

```
2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqr
t(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*x^3 - 8
*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*x
)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^
2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(24*sqrt(-c*d^2 -
a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*e^4*x^3 -
8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^2*e^5)]
```

Sympy [F]

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx=\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

```
[In] integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
[Out] Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^3\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int((x^3*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.318 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d(2cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4} - \frac{d^2\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4}$$

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/e^4/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1
/2))*(a*e^2+c*d^2)^(1/2)/e^4+1/2*d*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1668, 12, 829, 858, 223, 212, 739}

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{d^2\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4} - \frac{d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+2cd^2)}{2\sqrt{c}e^4}+\frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3}+\frac{(a+cx^2)^{3/2}}{3ce}$$

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
```

```
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
```

```
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+cx^2)^{3/2}}{3ce} + \frac{\int -\frac{3cdex\sqrt{a+cx^2}}{d+ex}\,dx}{3ce^2}\\
&= \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d\int \frac{-acde+c(2cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce}\\
&\quad + \frac{(d^2(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4} - \frac{(d(2cd^2+ae^2))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2e^4}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce}\\
&\quad - \frac{(d^2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}\\
&\quad - \frac{(d(2cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2e^4}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}\\
&\quad - \frac{d^2\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e\sqrt{a+cx^2}(6cd^2+2ae^2-3cdex+2ce^2x^2)+12cd^2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+3\sqrt{c}d(2cd^2+\cdots}{6ce^4}$$

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(6*c*d^2 + 2*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2) + 12*c*d^2
*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt
[-(c*d^2) - a*e^2]] + 3*Sqrt[c]*d*(2*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/(6*c*e^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(2ce^2x^2-3cdex+2e^2a+6cd^2)\sqrt{cx^2+a}}{6ce^3}-\frac{d\left(\frac{(e^2a+2cd^2)\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e\sqrt{c}}+\frac{2d(e^2a+cd^2)\ln\left(\frac{\frac{2e^2a+2cd^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+cd^2}{e^2}}\cdots}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+cd^2}{e^2}}}\right)}{2e^3}$
default	$\frac{(cx^2+a)^{\frac{3}{2}}}{3ce}-\frac{d\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{e^2}+\frac{d^2\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+cd^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c-\cdots}\right)}{e}\cdots\right)}{\cdots}$

```
[In] int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*e^2*x^2-3*c*d*e*x+2*a*e^2+6*c*d^2)*(c*x^2+a)^(1/2)/c/e^3-1/2*d/e^3
*((a*e^2+2*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+2*d*(a*e^2+c*d^2)
/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 776, normalized size of antiderivative = 5.07

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \left[\frac{6\sqrt{cd^2+ae^2}cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+3\,(2\,cd^3+ade^2)\sqrt{c}\log\left(-\right.}{12\,ce^4}\right.$$
$$-\frac{12\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(2\,cd^3+ade^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}\right.}{12\,ce^4}$$
$$-\frac{6\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(2\,cd^3+ade^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(2\,ce^3x^2}{6\,ce^4}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c
*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -1/12*(12*sqrt(-c*d^2 - a*e^2)*
c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6
*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), 1/6*(3*sqrt(c*d^2 + a*e^2)*c*
d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2
*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -
1/6*(6*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^
3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e^3*x^2 - 3
*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.319 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx=-\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{(2cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3}+\frac{d\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3}$$

```
[Out] 1/2*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^(1/2)+d*arctan
h((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^3
-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {829, 858, 223, 212, 739}

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(ae^2+2cd^2)}{2\sqrt{c}e^3}+\frac{d\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/2*((2*d - e*x)*Sqrt[a + c*x^2])/e^2 + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{\int \frac{-acde+c(2cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^2}$$

$$=-\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}-\frac{(d(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3}+\frac{(2cd^2+ae^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2e^3}$$

$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{(d(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{(2cd^2+ae^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2e^3}$$

$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3} + \frac{d\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e(-2d+ex)\sqrt{a+cx^2} - 4d\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) - \frac{(2cd^2+ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{2e^3}$$

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*(-2*d + e*x)*Sqrt[a + c*x^2] - 4*d*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c
]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - ((2*c*d^2 + a*e^
2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*e^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-ex+2d)\sqrt{c\,x^2+a}}{2e^2} + \frac{\frac{\left(e^2a+2c\,d^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}} + \frac{2d\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2e^2}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2} + \frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}}{e} - \frac{d\left(\sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2a+c\,d^2}{e^2}} - \frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}} + \sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2a+c\,d^2}{e^2}}\right)}{e}\right)}{e^2}$

```
[In] int(x*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2+1/2/e^2*((a*e^2+2*c*d^2)/e*ln(x*c^(1/2)
+(c*x^2+a)^(1/2))/c^(1/2)+2*d*(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.39

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \left[\frac{2\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(2\,cd^2+ae^2)\sqrt{c}\log\left(-2\,c\right.}{4\,ce^3}\right.$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c
*e^3), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (2*c
*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(sqrt(c*d^2 + a*e^2)*c*d*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) -
(2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*e^2*x -
2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d*arctan(s
qrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3)]
```

Sympy [F]

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$

```
[In] integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{x\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x),x)
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.320 $\int \frac{\sqrt{a+cx^2}}{d+ex} dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\sqrt{a+cx^2}}{d+ex} dx = \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/e^2-arctanh((-c*d*x+a*e)/(a*e
^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^2+(c*x^2+a)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {749, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+cx^2}}{d+ex} dx = -\frac{\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2} - \frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\sqrt{a+cx^2}}{e}$$

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a+cx^2}}{e}+\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\sqrt{a+cx^2}}{e}+\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx-\frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{e}+\left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\
&\quad-\frac{(cd)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{e}-\frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{e\sqrt{a+cx^2}+2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+\sqrt{c}d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{e^2}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] + 2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e
*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2]])/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(89) = 178.

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.79

method	result
risch	$\frac{\sqrt{c\,x^2+a}}{e}-\frac{\frac{\sqrt{c}\,d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e}-\frac{\left(-e^2a-c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{e}$
default	$\frac{\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}\right)}{e}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}}{}\right.}{}}{e}$

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/e-1/e*(1/e*c^(1/2)*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-(-a*e^2-
c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^
2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.57

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$
$$= \left[\frac{\sqrt{c}d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,\sqrt{cx^2+a}e+\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x}{e^2x^2+2\,dex}\right.}{2\,e^2}\right.$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*
x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(sqrt(
c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*e
- 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqrt(-c)*d*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x), x)
```

3.321 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx=\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}+\frac{\sqrt{cd^2+ae^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/e+arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
e^2+c*d^2)^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {910, 272, 65, 214, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx=\frac{\sqrt{ae^2+cd^2}\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}$$

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
```

```
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} + \frac{a\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \left(\frac{cd}{e} + \frac{ae}{d}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} - \left(-\frac{cd}{e} - \frac{ae}{d}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e} + \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right) + \sqrt{c}d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{de}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
[Out] -((2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/
Sqrt[-(c*d^2) - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])
/Sqrt[a]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(d*e))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(98) = 196$.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.63

method	result
default	$\frac{\sqrt{c x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{d}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c}{e^2}}\right.}{e}}{}$

```
[In] int((c*x^2+a)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))-1/d*(((
x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/e*c^(1/2)*d*ln((-1/e*
c*d+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2
))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*
c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2
+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 1.01 (sec) , antiderivative size = 1316, normalized size of antiderivative = 11.34

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*e
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt
(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/
(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*lo
g(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^2)*log
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*
e), 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a
)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a)
+ 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
```

```
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e),
1/2*(2*sqrt(-a)*e*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2
 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sq
rt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt(-a)*e*arctan(sqrt(-a)/
sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*e*arctan(sqrt
(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) + 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -(sqr
t(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(-a)*e*arctan(sqrt(-a)/sqr
t(c*x^2 + a)) - sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx=\int \frac{\sqrt{a+cx^2}}{x\,(d+ex)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{e\left(\frac{\sqrt{a+\frac{cd^2}{e^2}}\operatorname{arsinh}\left(\frac{2\,cdx}{e\sqrt{\frac{ac}{e^2}}\,|2\,ex+2\,d|}-\frac{2\,a}{\sqrt{\frac{ac}{e^2}}\,|2\,ex+2\,d|}\right)}{e}+\frac{\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ac}\,|x|}\right)}{e}-\frac{cd\operatorname{arsinh}\left(\frac{cx}{e\sqrt{\frac{ac}{e^2}}}\right)}{e^3\sqrt{\frac{c}{e^2}}}\right)}{d}$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
[Out] -e*(sqrt(a + c*d^2/e^2)*arcsinh(2*c*d*x/(e*sqrt(a*c/e^2)*abs(2*e*x + 2*d))
- 2*a/(sqrt(a*c/e^2)*abs(2*e*x + 2*d)))/e + sqrt(a)*arcsinh(a/(sqrt(a*c)*ab
s(x)))/e - c*d*arcsinh(c*x/(e*sqrt(a*c/e^2)))/(e^3*sqrt(c/e^2)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.322 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = -\frac{\sqrt{a+cx^2}}{dx}-\frac{\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {975, 283, 223, 212, 272, 52, 65, 214, 749, 858, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = -\frac{\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{a+cx^2}}{dx^2}-\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}\right)dx\\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d}-\frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2}+\frac{e^2\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^2}\\
&= \frac{e\sqrt{a+cx^2}}{d^2}-\frac{\sqrt{a+cx^2}}{dx}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d}-\frac{e\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d^2}+\frac{e\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{a+cx^2}}{dx}-\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d}+\frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{d}\\
&\quad -\frac{(ae)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^2}+\left(c+\frac{ae^2}{d^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx
\end{aligned}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d}$$
$$- \frac{(ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2}$$
$$+ \left(-c-\frac{ae^2}{d^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)$$
$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$$
$$= \frac{-d\sqrt{a+cx^2}+2\sqrt{-cd^2-ae^2}x\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)+\sqrt{a}ex\log(x)-\sqrt{a}ex\log\left(-\sqrt{a}+\sqrt{a+cx^2}\right)}{d^2x}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
[Out] (-(d*Sqrt[a + c*x^2]) + 2*Sqrt[-(c*d^2) - a*e^2]*x*ArcTan[(Sqrt[-(c*d^2) -
a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^2])] + Sqrt[a]*e*x*Log[x] - S
qrt[a]*e*x*Log[-Sqrt[a] + Sqrt[a + c*x^2]])/(d^2*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(91) = 182.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{\sqrt{c\,x^2+a}}{dx}+\frac{\frac{\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{de\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{d}$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2c\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{a}}{d}-\frac{e\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^2}+\frac{e\left(\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2}{}\right.}{}$

```
[In] int((c*x^2+a)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+a)^(1/2)/d/x+1/d*(1/d*a^(1/2)*e*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/
x)-(a*e^2+c*d^2)/d/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*
c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2
+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 599, normalized size of antiderivative = 5.70

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$$
$$= \left[\frac{\sqrt{a}ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{cd^2+ae^2}x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2}}{e^2x^2+2dex+d^2}\right.}{2d^2x}\right.$$
$$-\frac{2\sqrt{-a}ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-\sqrt{cd^2+ae^2}x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)}{2d^2x}$$
$$\left.-\frac{\sqrt{-a}ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+\sqrt{-cd^2-ae^2}x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+\sqrt{cx^2+a}d}{d^2x}\right]$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*e*x*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqr
t(c*d^2 + a*e^2)*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*e*x*log(-(c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*x*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sqrt(-a)*
e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*x*log((2*a*c*d*e
*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*x^2
+ a)*d)/(d^2*x), -(sqrt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(-c*
d^2 - a*e^2)*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + sqrt(c*x^2 + a)*d)/(d^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^2}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = -\frac{2\,ae\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^2} + \frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)d} + \frac{2\,(cd^2+ae^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^2}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
[Out] -2*a*e*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^2) + 2*a
*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*arct
an(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/(sq
rt(-c*d^2 - a*e^2)*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.323 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3} - \frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\sqrt{a}e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

```
[Out] -1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-e^2*arctanh((c*x^2+a)^(1/
2)/a^(1/2))*a^(1/2)/d^3+e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a
)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2
)/d^2/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {975, 272, 43, 65, 214, 283, 223, 212, 52, 749, 858, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{a}e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3} - \frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2
+ a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3
```

```
- (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^3} dx}{d} - \frac{e \int \frac{\sqrt{a+cx^2}}{x^2} dx}{d^2} + \frac{e^2 \int \frac{\sqrt{a+cx^2}}{x} dx}{d^3} - \frac{e^3 \int \frac{\sqrt{a+cx^2}}{d+ex} dx}{d^3}$$

$$= -\frac{e^2\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2} dx, x, x^2\right)}{2d} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2} + \frac{e^2\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d^3} - \frac{e^2 \int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}} dx}{d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{4d} + \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2} - \frac{(ce)\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d^3} - \frac{(e(cd^2+ae^2)) \int \frac{1}{(d+ex)\sqrt{a+cx^2}} dx}{d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(ce)\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd^3} + \frac{(e(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2} dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{cd^2+ae^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3} - \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)} dx$$

$$= \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{x^2} - 4e\sqrt{-cd^2-ae^2} \arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \frac{2(cd^2+2ae^2)\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 - 4*e*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(
Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (2*(c*d^2
+ 2*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/(2*d^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\sqrt{c x^2+a}\,(-2ex+d)}{2d^2x^2}-\frac{-\frac{\left(-2e^2a-c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{2\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2c}{}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2d^2}$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}+\frac{c\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{2a}}{d}+\frac{e^2\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^3}-\frac{e\left(-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2c\left(\frac{x\sqrt{c\,x^2}}{2}\right.}{}\right.}{d}$

```
[In] int((c*x^2+a)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(-(-2*a*e^2-c*d^2)/d/a^(1/2
)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*(a*e^2+c*d^2)/d/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \left[\frac{2\sqrt{cd^2+ae^2}aex^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+2\,ae^2)\sqrt{a}x^2\log\left(\right.}{4\,ad^3x^2}\right.$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
```

```
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*
x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*
x^2 + a))/(a*d^3*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(s
qrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^
2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a
*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)x^3}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$$
$$= -\frac{2\,(cd^2e+ae^3)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^3} + \frac{(cd^2+2\,ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^3}$$
$$+ \frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd+2\,a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2d^2}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arctan
(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x - sq
rt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqr
t(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^
2 + a))^2 - a)^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^3\,(d+e\,x)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.324 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 191

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \frac{e\sqrt{a+cx^2}}{2d^2x^2}-\frac{e^2\sqrt{a+cx^2}}{d^3x}-\frac{(a+cx^2)^{3/2}}{3adx^3}-\frac{e^2\sqrt{cd^2+ae^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4}+\frac{c\,e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}+\frac{\sqrt{a}e^3\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}$$

```
[Out] -1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/2*c*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a
^(1/2)+e^3*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^4-e^2*arctanh((-c*d*x
+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^4+1/2*e*(c
*x^2+a)^(1/2)/d^2/x^2-e^2*(c*x^2+a)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {975, 270, 272, 43, 65, 214, 283, 223, 212, 52, 749, 858, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \frac{\sqrt{a}e^3\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}+\frac{c\,e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}-\frac{e^2\sqrt{ae^2+cd^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4}-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}-\frac{(a+cx^2)^{3/2}}{3adx^3}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+cx^2}}{dx^4} - \frac{e\sqrt{a+cx^2}}{d^2x^3} + \frac{e^2\sqrt{a+cx^2}}{d^3x^2} - \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^4}$$

$$= \frac{e^3\sqrt{a+cx^2}}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d^2} + \frac{(ce^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^4} + \frac{e^3\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^4}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{(ce)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^2} - \frac{(ce^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(ce^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^4} + \frac{(e^2(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^4}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} + \frac{\sqrt{c}e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d^2} - \frac{(ce^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^4} - \frac{(e^2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^4}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4} + \frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2} + \frac{\sqrt{a}e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = -\frac{\frac{d\sqrt{a+cx^2}(2ad^2-3adex+2cd^2x^2+6ae^2x^2)}{ax^3} - 12e^2\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \frac{6e(cd^2+2ae^2)\text{arctanh}\left(\sqrt{}\right.}{\sqrt{a}}}{6d^4}$$

[In] Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]

[Out] -1/6*((d*Sqrt[a + c*x^2]*(2*a*d^2 - 3*a*d*e*x + 2*c*d^2*x^2 + 6*a*e^2*x^2))/(a*x^3) - 12*e^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (6*e*(c*d^2 + 2*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^4

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(6a\,e^2x^2+2c\,d^2x^2-3adex+2a\,d^2)}{6d^3x^3a}+\frac{e\left(-\frac{(-2e^2a-c\,d^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{2(e^2a+c\,d^2)\ln\left(\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}\right.}{d}\right.}{2d^3}$
default	$-\frac{(c\,x^2+a)^{\frac{3}{2}}}{3ad\,x^3}+\frac{e^2\left(-\frac{(c\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2c\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{a}\right)}{d^3}-\frac{e^3\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^4}-\frac{e\left(\right.}{}$

[In] int((c*x^2+a)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/6*(c*x^2+a)^(1/2)*(6*a*e^2*x^2+2*c*d^2*x^2-3*a*d*e*x+2*a*d^2)/d^3/x^3/a+1/2*e/d^3*(-(-2*a*e^2-c*d^2)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*(a*e^2+c*d^2)/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 824, normalized size of antiderivative = 4.31

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \left[\frac{6\sqrt{cd^2+ae^2}ae^2x^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+3\,(cd^2e+2\,ae^3)\sqrt{a}x^3\log}{12\,ad^4x^3}\right.$$

$$-\frac{12\sqrt{-cd^2-ae^2}ae^2x^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(cd^2e+2\,ae^3)\sqrt{a}x^3\log\left(-\frac{cx^2+2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right.}{12\,ad^4x^3}$$

$$-\frac{6\sqrt{-cd^2-ae^2}ae^2x^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+3\,(cd^2e+2\,ae^3)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-(3\,ad}{6\,ad^4x^3}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(a)*x^3*l
og(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/12*(12*sqr
t(-c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c*d^2*e + 2*
a*e^3)*sqrt(a)*x^3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4
*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)
*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 +
 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/6*(6*sqrt(-c*d^2 - a*e^2)
*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d
^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)*x
^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3
*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^4\,(d+ex)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^4}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \frac{2\,(cd^2e^2+ae^4)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^4} - \frac{(cd^2e+2\,ae^3)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^4} - \frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^5cde-6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4c^{\frac{3}{2}}d^2-6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4a\sqrt{c}e^2+12\left(\sqrt{c}x-\sqrt{c\right.}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^3d^3}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)
*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 + 12*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*
e - 2*a^2*c^(3/2)*d^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + a))^
2 - a)^3*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^4\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = -\frac{\sqrt{a+cx^2}}{4dx^4}-\frac{c\sqrt{a+cx^2}}{8adx^2}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}+\frac{e^3\sqrt{a+cx^2}}{d^4x}+\frac{e(a+cx^2)^{3/2}}{3ad^2x^3}+\frac{e^3\sqrt{cd^2+ae^2}\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^5}+\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}-\frac{ce^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3}-\frac{\sqrt{a}e^4\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3+1/8*c^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^
(3/2)/d-1/2*c*e^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-e^4*arctanh(
(c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^5+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)
^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^5-1/4*(c*x^2+a)^(1/2)/d/x^4-1
/8*c*(c*x^2+a)^(1/2)/a/d/x^2-1/2*e^2*(c*x^2+a)^(1/2)/d^3/x^2+e^3*(c*x^2+a)^
(1/2)/d^4/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules

used = {975, 272, 43, 44, 65, 214, 270, 283, 223, 212, 52, 749, 858, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}e^4\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5} - \frac{ce^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3} + \frac{e^3\sqrt{ae^2+cd^2}\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{\sqrt{a+cx^2}}{4dx^4}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[a + c*x^2]/(d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[
a + c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3
/2))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c
*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]
])/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3)
 - (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{a+cx^2}}{dx^5}-\frac{e\sqrt{a+cx^2}}{d^2x^4}+\frac{e^2\sqrt{a+cx^2}}{d^3x^3}-\frac{e^3\sqrt{a+cx^2}}{d^4x^2}+\frac{e^4\sqrt{a+cx^2}}{d^5x}-\frac{e^5\sqrt{a+cx^2}}{d^5(d+ex)}\right)dx\\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x^5}\,dx}{d}-\frac{e\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d^2}+\frac{e^2\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^3}-\frac{e^3\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^4}+\frac{e^4\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^5}-\frac{e^5\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^5}\\
&= -\frac{e^4\sqrt{a+cx^2}}{d^5}+\frac{e^3\sqrt{a+cx^2}}{d^4x}+\frac{e(a+cx^2)^{3/2}}{3ad^2x^3}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx,x,x^2\right)}{2d}+\frac{e^2\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx,x,x^2\right)}{2d^3}-\frac{(ce^3)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^4}\\
&\quad+\frac{e^4\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d^5}-\frac{e^4\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^5}
\end{aligned}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{c\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{8d} + \frac{(ce^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^3} + \frac{(ce^3)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^4} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^4} + \frac{(ae^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^5} - \frac{(e^3(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^5}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} - \frac{\sqrt{c}e^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^4} - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{16ad} + \frac{e^2\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d^3} + \frac{(ce^3)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^4} + \frac{(ae^4)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^5} + \frac{(e^3(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^5}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^5} - \frac{ce^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3} - \frac{\sqrt{a}e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5} - \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{8ad}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^5} + \frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{ce^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^3} - \frac{\sqrt{a}e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$$
$$= \frac{\sqrt{a}\left(d\sqrt{a+cx^2}(cd^2x^2(-3d+8ex)+a(-6d^3+8d^2ex-12de^2x^2+24e^3x^3))-48ae^3\sqrt{-cd^2-ae^2}x^4\arctan\right.}{24a^{3/2}d^5x^4}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a]*(d*Sqrt[a + c*x^2]*(c*d^2*x^2*(-3*d + 8*e*x) + a*(-6*d^3 + 8*d^2*e
*x - 12*d*e^2*x^2 + 24*e^3*x^3)) - 48*a*e^3*Sqrt[-(c*d^2) - a*e^2]*x^4*ArcT
an[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]) - 6*(c^
2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2]
)/Sqrt[a]])/(24*a^(3/2)*d^5*x^4)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,\left(-24a\,e^3x^3-8c\,d^2e\,x^3+12ad\,e^2x^2+3c\,d^3x^2-8a\,d^2ex+6a\,d^3\right)}{24d^4x^4a}-\frac{-\frac{\left(-8a^2e^4-4ac\,d^2e^2+c^2d^4\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}-\frac{8a\,e^2(}{}}{}$
default	$\frac{-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{4a\,x^4}-\frac{c\left(-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}+\frac{c\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}}{d}+\frac{e^4\left(\sqrt{c\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)\right)}{d^5}+\frac{e^2\left(-\frac{(c}{}\right.}{}$

```
[In] int((c*x^2+a)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*x^2+a)^(1/2)*(-24*a*e^3*x^3-8*c*d^2*e*x^3+12*a*d*e^2*x^2+3*c*d^3*x
^2-8*a*d^2*e*x+6*a*d^3)/d^4/x^4/a-1/8/d^4/a*(-(-8*a^2*e^4-4*a*c*d^2*e^2+c^2
*d^4)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-8*a*e^2*(a*e^2+c*d^2)
/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 1007, normalized size of antiderivative = 3.68

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$$
$$= \left[\frac{24\sqrt{cd^2+ae^2}a^2e^3x^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(c^2d^4-4\,acd^2e^2-\right.}{} \right.$$

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a
^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4
+ 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/48*(48*sqrt(-c*d^2
- a*e^2)*a^2*e^3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2
*e^2 - 8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3
*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/24*(12*sq
rt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt
(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*
(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2
 + a))/(a^2*d^5*x^4), 1/24*(24*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt(-a)*x^4*a
rctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e
 + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^
2*d^5*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^5\,(d+ex)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)x^5}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(229) = 458.

Time = 0.30 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = -\frac{2\,(cd^2e^3+ae^5)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^5} - \frac{(c^2d^4-4\,acd^2e^2-8\,a^2e^4)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}ad^5} + \frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^7c^2d^3+12\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^7acde^2-24\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^6ac^{\frac{3}{2}}d^2e-24\left(\sqrt{c}x-\sqrt{\right.}\right.}{}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c*d
^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 + 12*(sqrt(c)*x
 - sqrt(c*x^2 + a))^7*a*c*d*e^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(3
/2)*d^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 + 21*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2
```

```
*c*d*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 72*(sqrt(
c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*e^3 + 21*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2 - 8*(sqrt(
c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d^2*e - 72*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^4*sqrt(c)*e^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d^3 + 12*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^4*c*d*e^2 + 8*a^4*c^(3/2)*d^2*e + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx=\int \frac{\sqrt{c\,x^2+a}}{x^5\,\left(d+e\,x\right)} \,d x$$

```
[In] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.326 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d(2cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} - \frac{d^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4\sqrt{cd^2+ae^2}}$$

```
[Out] -1/2*d*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4-d^4*
arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^4/(a*e^2+c*d^2)
^(1/2)+1/6*(-4*a*e^2+11*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3-7/6*d*(e*x+d)*(c*x^2
+a)^(1/2)/c/e^3+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1668, 858, 223, 212, 739}

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd^2-ae^2)}{2c^{3/2}e^4} - \frac{d^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}} + \frac{\sqrt{a+cx^2}(11cd^2-4ae^2)}{6c^2e^3} - \frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}$$

```
[In] Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}+\frac{\int \frac{-2ad^2e^2-de(cd^2+4ae^2)x-e^2(5cd^2+2ae^2)x^2-7cde^3x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4}\\
&=-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}+\frac{\int \frac{3acd^2e^5+cde^4(5cd^2-ae^2)x+ce^5(11cd^2-4ae^2)x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{6c^2e^7}\\
&=\frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3}-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}\\
&\quad+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}+\frac{\int \frac{3ac^2d^2e^7-3c^2de^6(2cd^2-ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{6c^3e^9}\\
&=\frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3}-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}\\
&\quad+\frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}-\frac{(d(2cd^2-ae^2))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^4}\\
&=\frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3}-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}\\
&\quad+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}-\frac{d^4\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}\\
&\quad-\frac{(d(2cd^2-ae^2))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2ce^4}\\
&=\frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3}-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}\\
&\quad-\frac{d(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.83

$$
\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}(-4ae^2+c(6d^2-3dex+2e^2x^2))}{c^2}-\frac{12d^4\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{3d(2cd^2-ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{6e^4}
$$

```
[In] Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
[Out] ((e*Sqrt[a + c*x^2]*(-4*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c^2 - (12
*d^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]
)/Sqrt[-(c*d^2) - a*e^2] + (3*d*(2*c*d^2 - a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a
 + c*x^2]])/c^(3/2))/(6*e^4)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{(-2c\,e^2x^2+3cdex+4e^2a-6c\,d^2)\sqrt{c\,x^2+a}}{6c^2e^3}+\frac{d\left(\frac{(e^2a-2c\,d^2)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}}-\frac{2c\,d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{(}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}\right.}{2e^3c}$
default	$\frac{\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}}{e}+\frac{d^2\sqrt{c\,x^2+a}}{e^3c}-\frac{d^3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^4\sqrt{c}}-\frac{d\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^2}-\frac{d^4\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}}{}\right.}{}$

[In] int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6*(-2*c*e^2*x^2+3*c*d*e*x+4*a*e^2-6*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3+1/2*d/e^3/c*((a*e^2-2*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-2*c*d^3/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [A] (verification not implemented)

none

Time = 2.39 (sec) , antiderivative size = 1060, normalized size of antiderivative = 5.44

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{6\sqrt{cd^2+ae^2}c^2d^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\,(2\,c^2d^5+acd^3e^2-a^2d}{}\right.$$
$$-\frac{12\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+3\,(2\,c^2d^5+acd^3e^2-a^2de^4)\sqrt{c}\log\left(-2\,cx^2-2\right.}{12\,(c^3d^2}$$
$$-\frac{6\sqrt{-cd^2-ae^2}c^2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\,(2\,c^2d^5+acd^3e^2-a^2de^4)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right.}{6\,(c^3d^2e^4+ac^2e^6}$$

[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x

```
^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d*e^
4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*d^4*e
 + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e
^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/12*(12*sq
rt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(2*c^2*d^5 + a
*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(6*c^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^
5)*x^2 - 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c
^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c^2*d^5 + a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*d^4*e + 2*a*
c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2 + a*
c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/6*(6*sqrt(-c*d^2
 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^
2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*d^4*e +
 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2
 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6)]
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)),x)

[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)), x)

3.327 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{(2cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\sqrt{cd^2+ae^2}}$$

```
[Out] 1/2*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^3+d^3*arc
tanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^3/(a*e^2+c*d^2)^(1
/2)-3/2*d*(c*x^2+a)^(1/2)/c/e^2+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1668, 858, 223, 212, 739}

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd^2-ae^2)}{2c^{3/2}e^3}+\frac{d^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

```
[In] Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
```

```
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-ade^2-e(cd^2+ae^2)x-3cde^2x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-acde^4+ce^3(2cd^2-ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(2cd^2-ae^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{d^3\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{(2cd^2-ae^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\sqrt{cd^2+ae^2}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{e(-2d+ex)\sqrt{a+cx^2}}{c} + \frac{4d^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} + \frac{(-2cd^2+ae^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{2e^3}$$

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
[Out] ((e*(-2*d + e*x)*Sqrt[a + c*x^2])/c + (4*d^3*ArcTan[(Sqrt[c]*(d + e*x) - e*
Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + ((-2*c*d
^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/c^(3/2))/(2*e^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{(-ex+2d)\sqrt{c\,x^2+a}}{2c\,e^2} - \frac{\frac{\left(e^2a-2c\,d^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}} - \frac{2c\,d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2a+\cdots}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}}{2e^2c}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2c} - \frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e} + \frac{d^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^3\sqrt{c}} - \frac{d\sqrt{c\,x^2+a}}{c\,e^2} + \frac{d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\cdots}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/c/e^2-1/2/e^2/c*((a*e^2-2*c*d^2)/e*ln(x*c^(
1/2)+(c*x^2+a)^(1/2))/c^(1/2)-2*c*d^3/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2
/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 2.41 (sec) , antiderivative size = 924, normalized size of antiderivative = 6.08

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \left[\frac{2\sqrt{cd^2+ae^2}c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-(2\,c^2d^4+acd^2e^2-a^2e^4}{4\,(c^3d^2e^3+ac^2}\right.$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sq
rt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*
a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^
2*e^5), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
 (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 + a*e^2)*c^2
*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c
*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c^2*d^3*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2
*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.328 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx=\frac{\sqrt{a+cx^2}}{ce}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}-\frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(1/2)+(c*x^2+a)^(1/2
)/c/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1668, 12, 858, 223, 212, 739}

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

```
[In] Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a+cx^2}}{ce} - \frac{\int \frac{cdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2}
\end{aligned}$$

$$= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{d^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}}{c} - \frac{2d^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{\sqrt{-cd^2-ae^2}} + \frac{2d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+cx^2}}\right)}{\sqrt{c}}}{e^2}$$

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2])/c - (2*d^2*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*
(d + e*x) - d*Sqrt[a + c*x^2])])/Sqrt[-(c*d^2) - a*e^2] + (2*d*ArcTanh[(Sqr
t[c]*x)/(Sqrt[a] - Sqrt[a + c*x^2])])/Sqrt[c])/e^2
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\sqrt{c\,x^2+a}}{ce} - \frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}} - \frac{d^2\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	172
risch	$\frac{\sqrt{c\,x^2+a}}{ce} - \frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}} - \frac{d^2\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	172

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c/e-d/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/((a
*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*
d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 745, normalized size of antiderivative = 6.83

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \left[\frac{\sqrt{cd^2+ae^2}cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^3+ade^2)\sqrt{c}\log\left(-2\,cx\right.}{2\,(c^2d^2e^2+ace^4)} - \frac{2\,\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-\right.}{2\,(c^2d^2e^2+ace^4)} - \frac{\sqrt{-cd^2-ae^2}cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(cd^2e+ae^3)}{c^2d^2e^2+ace^4}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2
*d^2*e^2 + a*c*e^4), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2
- a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*
e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4),
 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 +
 a*c*e^4), -(sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*
d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d^2*e + a*e
^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.329 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e} + \frac{d\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e/c^(1/2)+d*arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 223, 212, 739}

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

```
[In] Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \frac{d\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e} + \frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\frac{2d\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} - \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e}$$

```
[In] Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
[Out] ((2*d*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]
])/Sqrt[-(c*d^2) - a*e^2] - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/Sqrt[c])/e
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(74) = 148$.

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{e^{2}\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}$	151

```
[In] int(x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 631, normalized size of antiderivative = 7.34

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+ae^2)\sqrt{c}\log\left(-2\,cx^2-\right.}{2\,(c^2d^2e+ace^3)}\right.$$

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(2*sqrt(-c*d^2 - a*
e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2
 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x
^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(sqrt(c*d
^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2
*d*e*x + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)))/(c^2*d^2*e + a*c*e^3), (sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(c^
2*d^2*e + a*c*e^3)]
```

Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

[In] integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.330 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}$$

```
[Out] -arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {739, 212}

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
[Out] (-2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])
/Sqrt[-(c*d^2) - a*e^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(48) = 96.

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.35

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	127

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.91

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}}, -\frac{\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)}{cd^2+ae^2}\right]$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
)/(c*d^2 + a*e^2)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}e}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*abs(e*x + d
)))/(sqrt(a + c*d^2/e^2)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.331 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+e*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {975, 272, 65, 214, 739, 212}

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd}
\end{aligned}
$$

$$= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\left(\frac{e\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} + \frac{\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
[Out] (2*((e*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/Sqrt[-(c*d^2) - a*e^2] + ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]]
/Sqrt[a]))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(74) = 148$.

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.84

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}} + \frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	158

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 634, normalized size of antiderivative = 7.37

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2+ae^2)\sqrt{a}\log\left(-\frac{cx^2-2\,\sqrt{}}{}\right.}{2\,(acd^3+a^2de^2)}\right.$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(2*sqrt(-c*d^2
- a*e^2)*a*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c
*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(
sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2
*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*
x^2 + a)))/(a*c*d^3 + a^2*d*e^2), (sqrt(-c*d^2 - a*e^2)*a*e*arctan(sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))
/(a*c*d^3 + a^2*d*e^2)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.332 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-e^2*arctanh((-c*d*x+a*e)/(a*
e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)-(c*x^2+a)^(1/2)/a
/d/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {975, 270, 272, 65, 214, 739, 212}

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx=-\frac{e^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}
\end{aligned}
$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{e^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\frac{d\sqrt{a+cx^2}}{ax} + \frac{2e^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} + \frac{2e\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((d*Sqrt[a + c*x^2])/(a*x) + (2*e^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
+ c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + (2*e*ArcTanh[(S
qrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.62

method	result	size
default	$-\frac{\sqrt{c\,x^2+a}}{adx}+\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^2\sqrt{a}}-\frac{e\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	180
risch	$-\frac{\sqrt{c\,x^2+a}}{adx}+\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^2\sqrt{a}}-\frac{e\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$	180

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d/x+e/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-
e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 767, normalized size of antiderivative = 6.91

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{\sqrt{cd^2+ae^2}ae^2x\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+(cd^2e+ae^3)\sqrt{a}x\log\left(-\right.}{2\,(acd^4+a^2d^2e^2)x}\right.$$
$$-\frac{2\,\sqrt{-cd^2-ae^2}ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^2e+ae^3)\sqrt{a}x\log\left(-\frac{cx^2+2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2}{2\,(acd^4+a^2d^2e^2)x}$$
$$-\frac{\sqrt{-cd^2-ae^2}ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(cd^2e+ae^3)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+(cd^3+ad}{(acd^4+a^2d^2e^2)x}$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
(2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2
+ 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 +
a))/((a*c*d^4 + a^2*d^2*e^2)*x), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e^2*x*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*
c*d^4 + a^2*d^2*e^2)*x), 1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e
^3)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*d*e^2)*sqrt(
c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -(sqrt(-c*d^2 - a*e^2)*a*e^2*x*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(-a)*x*arctan(sqrt(-a)/s
qrt(c*x^2 + a)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^
2)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = 2\,c\left(\frac{e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}cd^2} - \frac{e\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}cd^2} + \frac{1}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}d}\right)$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*c*(e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
- a*e^2))/(sqrt(-c*d^2 - a*e^2)*c*d^2) - e*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ a))/sqrt(-a))/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)
*sqrt(c)*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.333 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}+\frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

```
[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-e^2*arctanh((c*x^2+a)^(1/2
)/a^(1/2))/d^3/a^(1/2)+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+
a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(
1/2)/a/d^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {975, 272, 44, 65, 214, 270, 739, 212}

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \frac{c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 +
```

```
a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^3\sqrt{a+cx^2}}-\frac{e}{d^2x^2\sqrt{a+cx^2}}+\frac{e^2}{d^3x\sqrt{a+cx^2}}-\frac{e^3}{d^3(d+ex)\sqrt{a+cx^2}}\right)dx\\
&= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d}-\frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2}+\frac{e^2\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3}-\frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3}\\
&= \frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}\\
&\quad +\frac{e^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^3}+\frac{e^3\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^3}\\
&= -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}\\
&\quad -\frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{4ad}+\frac{e^2\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd^3}\\
&= -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}\\
&\quad -\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{2ad}\\
&= -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}\\
&\quad +\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$$
$$= \frac{\sqrt{a}\left(d(cd^2+ae^2)\,(-d+2ex)\sqrt{a+cx^2}-4ae^3\sqrt{-cd^2-ae^2}x^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)\right)-2(c^2d^4-acd}{2a^{3/2}d^3\left(cd^2+ae^2\right)x^2}$$

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a]*(d*(c*d^2 + a*e^2)*(-d + 2*e*x)*Sqrt[a + c*x^2] - 4*a*e^3*Sqrt[-(c
*d^2) - a*e^2]*x^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*
d^2) - a*e^2]]) - 2*(c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*x^2*ArcTanh[(Sqrt[c
]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(2*a^(3/2)*d^3*(c*d^2 + a*e^2)*x^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2a\,d^2x^2}+\frac{\frac{2e^2a\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{e^2a+c\,d^2}{e^2}}}+\frac{\left(-2e^2a+c\,d^2\right)\ln\left(\frac{2a+2}{}\right.}{d\sqrt{a}}}{2a\,d^2}$
default	$\frac{-\frac{\sqrt{c\,x^2+a}}{2a\,x^2}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{e^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^3\sqrt{a}}+\frac{e\sqrt{c\,x^2+a}}{a\,d^2x}+\frac{e^2\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{(}}{x+\frac{d}{e}}\right.}{d^3\sqrt{\frac{e^2a+c\,d^2}{e^2}}}$

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a/d^2/x^2+1/2/a/d^2*(2*e^2*a/d/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)
^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/d*
(-2*a*e^2+c*d^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 956, normalized size of antiderivative = 5.69

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \left[\frac{2\sqrt{cd^2+ae^2}a^2e^3x^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-(c^2d^4-acd^2e^2-2\,a^2e}{4\,(a^2cd^5+a^3d^3e^2)x^2}\right.$$

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4
)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*
d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d
^5 + a^3*d^3*e^2)*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(a)*x^2*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^4 + a^2*d^2*e^2 -
2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^
2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^
4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 -
 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x
^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x
^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2
 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = -c^{\frac{3}{2}}\left(\frac{2\,e^3\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,c^{\frac{3}{2}}d^3}+\frac{(cd^2-2\,ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3}{}\right.$$

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c^(3/2)*(2*e^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) - ((s
qrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)*x -
sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.334 $\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{a(ae+cdx)}{c^2(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2(cd^2+ae^2)^{3/2}}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^2-d^4*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(3/2)+a*(c*d*x+a*e)/
c^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)+(c*x^2+a)^(1/2)/c^2/e
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1661, 1668, 858, 223, 212, 739}

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}} + \frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{\sqrt{a+cx^2}}{c^2e}$$

```
[In] Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
```

```
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{\frac{a^2d^2}{cd^2+ae^2}-ax^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{\int \frac{\frac{a^2cd^2e^2}{cd^2+ae^2}+acdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac^2e^2} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^2} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2\left(cd^2+ae^2\right)} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^2} \\
&\quad - \frac{d^4\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2\left(cd^2+ae^2\right)} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{\frac{e\left(2a^2e^2+c^2d^2x^2+ac\left(d^2+dex+e^2x^2\right)\right)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{2d^4\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}} + \frac{2d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+cx^2}}\right)}{c^{3/2}}}{e^2}$$

```
[In] Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((e*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(c^2*(c*d^2 +
a*e^2)*Sqrt[a + c*x^2]) + (2*d^4*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]
*(d + e*x) - d*Sqrt[a + c*x^2])])/(-(c*d^2) - a*e^2)^(3/2) + (2*d*ArcTanh[(
Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + c*x^2])])/c^(3/2))/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(130) = 260$.

Time = 0.47 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.40

method	result
risch	$\frac{\sqrt{c x^2+a}}{c^2 e} - \frac{d \ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{c^{\frac{3}{2}} e^2} + \frac{c d^4 \ln\left(\frac{\frac{2e^2 a+2c d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{e^2 a+c d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{e^2 a+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\left(e\sqrt{-ac}+cd\right)\left(e\sqrt{-ac}-cd\right)\sqrt{\frac{e^2 a+c d^2}{e^2}}} + \frac{a\sqrt{\left(x\right.}}{2}$
default	$\frac{\frac{x^2}{c\sqrt{c x^2+a}}+\frac{2a}{c^2\sqrt{c x^2+a}}}{e} - \frac{d^2}{e^3 c\sqrt{c x^2+a}} - \frac{d^3 x}{e^4 a\sqrt{c x^2+a}} - \frac{d\left(-\frac{x}{c\sqrt{c x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{c^{\frac{3}{2}}}\right)}{e^2} + \frac{d^4\left(\frac{e^2}{\left(e^2 a+c d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2c}{}}}\right.}{}$

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c^2/e-1/c^(3/2)/e^2*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+c/e^3*d
^4/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/2/c^2*a/(e*(-a*c)^(1
/2)+c*d)/(x-(-a*c)^(1/2)/c)*((x-(-a*c)^(1/2)/c)^2*c+2*(-a*c)^(1/2)*(x-(-a*c
)^(1/2)/c))^(1/2)-1/2/c^2*a/(e*(-a*c)^(1/2)-c*d)/(x+(-a*c)^(1/2)/c)*((x+(-a
*c)^(1/2)/c)^2*c-2*(-a*c)^(1/2)*(x+(-a*c)^(1/2)/c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(131) = 262$.

Time = 3.48 (sec) , antiderivative size = 1525, normalized size of antiderivative = 10.45

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2
 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^4*e + 3*a^2*c
*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c
^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^
```

```
2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)*x^2), -
1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^
3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c
^4*d^2*e^4 + a^2*c^3*e^6)*x^2), 1/2*(2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d
*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log(
(2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(
a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 +
a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d
^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 +
 a^2*c^3*e^6)*x^2), -((c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d
^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c
^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
 a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*x^2)]
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx = \int \frac{x^4}{(c\,x^2+a)^{3/2}\,(d+e\,x)}\,dx$$

```
[In] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.335 $\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{d^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e+d^3*arctanh((-c*d*x+a*e)/(a*e^
2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(3/2)+a*(-e*x+d)/c/(a*e^2+c
*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 858, 223, 212, 739}

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{d^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

```
[In] Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{\int \frac{-\frac{a^2de}{cd^2+ae^2}-ax}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}\\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce}-\frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e\left(cd^2+ae^2\right)}\\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{ce}+\frac{d^3\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)}
\end{aligned}$$

$$= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{2d^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{e\left(-cd^2-ae^2\right)^{3/2}} - \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}e}$$

```
[In] Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d^3*ArcTan[(Sqrt[c]*
(d + e*x) - e*Sqrt[a + c*x^2]])/Sqrt[-(c*d^2) - a*e^2]])/(e*(-(c*d^2) - a*e^
2)^(3/2)) - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(c^(3/2)*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(109) = 218.

Time = 0.42 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.23

method	result
default	$\frac{-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}}{e}+\frac{d^2x}{e^3a\sqrt{c\,x^2+a}}+\frac{d}{e^2c\sqrt{c\,x^2+a}}-\frac{d^3\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{\cdots}{\left(e^2a+c\,d^2\right)\left(\cdots\right.}\right.}{\cdots}$

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+d^2/e^3*
x/a/(c*x^2+a)^(1/2)+d/e^2/c/(c*x^2+a)^(1/2)-d^3/e^4*(1/(a*e^2+c*d^2)*e^2/((
x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d^2)*(
2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*c-2/e
*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(112) = 224$.

Time = 2.92 (sec) , antiderivative size = 1323, normalized size of antiderivative = 10.76

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^3*e + a^2*c*d*e^3
 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3
*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2),
1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*
e^2 + a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x -
a) + 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x
^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*
c^4*d^2*e^3 + a^2*c^3*e^5)*x^2), -1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2
*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2
 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*
c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(
a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^
3 + a^2*c^3*e^5)*x^2), ((c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) + (a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(
c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2
*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \int \frac{x^3}{(a+cx^2)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{x^3}{(c\,x^2+a)^{3/2}\,(d+e\,x)}\,dx$$

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)),x)
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.336 $\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{ae+cdx}{c(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(-c*d*x-a*e)/c/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1661, 12, 739, 212}

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{d^2\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{ae+cdx}{c\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

```
[In] Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{acd^2}{\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\frac{-ae-cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{2d^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}$$

```
[In] Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*e) - c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*d^2*ArcTan[(Sqrt[
-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^2])])/(-(c*d^2) -
a*e^2)^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(88) = 176.

Time = 0.42 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.74

method	result
default	$-\frac{1}{ec\sqrt{c\,x^2+a}}-\frac{dx}{e^2a\sqrt{c\,x^2+a}}+\frac{d^2\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-}}$

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/c/(c*x^2+a)^(1/2)-d/e^2*x/a/(c*x^2+a)^(1/2)+d^2/e^3*(1/(a*e^2+c*d^2)*e
^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d
^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*
c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)
^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(88) = 176$.

Time = 0.36 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.79

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = \left[\frac{(c^2d^2x^2+acd^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx\cdots}{e^2x^2+2\,dex+d^2}\right)\cdots}{2\,(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+\cdots}\right.$$
$$-\frac{(c^2d^2x^2+acd^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(acd^2e+a^2e^3+(c^2d^3+acde^2)x)\sqrt{cx^2+\cdots}}{ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4)x^2}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
 a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2), -((c^2*d^2*x^2
 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx = \int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(88) = 176$.

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx &= \frac{cd^3x}{\sqrt{cx^2+a}acd^2e^2+\sqrt{cx^2+a}a^2e^4} \\ &\quad+\frac{d^2}{\sqrt{cx^2+a}cd^2e+\sqrt{cx^2+a}ae^3}-\frac{dx}{\sqrt{cx^2+a}ae^2} \\ &\quad+\frac{d^2\,\mathrm{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3}-\frac{1}{\sqrt{cx^2+a}ce}\end{aligned}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^3*x/(sqrt(c*x^2 + a)*a*c*d^2*e^2 + sqrt(c*x^2 + a)*a^2*e^4) + d^2/(sqrt
(c*x^2 + a)*c*d^2*e + sqrt(c*x^2 + a)*a*e^3) - d*x/(sqrt(c*x^2 + a)*a*e^2)
+ d^2*arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*abs(e
*x + d)))/((a + c*d^2/e^2)^(3/2)*e^3) - 1/(sqrt(c*x^2 + a)*c*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.91

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} - \frac{\frac{(c^2d^3+acde^2)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{x^2}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.337 $\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = -\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{de\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] d*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)+(e*x-d)/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {837, 12, 739, 212}

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{de\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{d-ex}{\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

```
[In] Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{\int \frac{acde}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac\,(cd^2+ae^2)} \\
&= -\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{(de)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2} \\
&= -\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{(de)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2} \\
&= -\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{-d+ex}{(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{2de\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

```
[In] Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d*e*ArcTan[(Sqrt[c]*(d +
e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(80) = 160.

Time = 0.40 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.81

method	result
default	$\frac{x}{ea\sqrt{c\,x^2+a}} - \frac{d\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}} \cdots\right.}{e^2}$

```
[In] int(x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*x/a/(c*x^2+a)^(1/2)-d/e^2*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x+
d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)/(
4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*
d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+
c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d
*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(81) = 162$.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.83

$$\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx = \Bigg[\frac{(cdex^2+ade)\sqrt{cd^2+ae^2}\log\Big(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)}{e^2x^2+2\,dex+d^2}}{2\,(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+(c^3d^4+2\,ac^2}$$

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^3 + a*d*e^2 - (c*d^2*
e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^
3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2), ((c*d*e*x^2 + a*d*e)*sqrt(-c*d^2
 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^
2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 - (c*d^2*e + a*e
^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx = \int \frac{x}{(a+cx^2)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{cd^2x}{\sqrt{cx^2+a}acd^2e+\sqrt{cx^2+a}a^2e^3} - \frac{d}{\sqrt{cx^2+a}cd^2+\sqrt{cx^2+a}ae^2} + \frac{x}{\sqrt{cx^2+a}ae} - \frac{d\,\mathrm{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^2}$$

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^2*x/(sqrt(c*x^2 + a)*a*c*d^2*e + sqrt(c*x^2 + a)*a^2*e^3) - d/(sqrt(c*
x^2 + a)*c*d^2 + sqrt(c*x^2 + a)*a*e^2) + x/(sqrt(c*x^2 + a)*a*e) - d*arcsi
nh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x + d)) - a/(sqrt(a*c/e^2)*abs(e*x + d)))/(
(a + c*d^2/e^2)^(3/2)*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{2\,de\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} + \frac{\frac{(cd^2e+ae^3)x}{c^2d^4+2\,acd^2e^2+a^2e^4} - \frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d*e*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))/sqrt(c*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{x}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.338 $\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{ae+cdx}{a(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{e^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {755, 12, 739, 212}

$$\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{ae+cdx}{a\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{e^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}\\ &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\ &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\ &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{2e^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}$$

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*e^2*ArcTan[(Sqrt[c]*
(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^
(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(86) = 172.

Time = 0.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.35

method	result
default	$\frac{\frac{e^2}{(e^2a+c\,d^2)\sqrt{(x+\frac{d}{e})^2c-\frac{2cd(x+\frac{d}{e})}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd(2c(x+\frac{d}{e})-\frac{2cd}{e})}{(e^2a+c\,d^2)\left(\frac{4c(e^2a+c\,d^2)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{(x+\frac{d}{e})^2c-\frac{2cd(x+\frac{d}{e})}{e}+\frac{e^2a+c\,d^2}{e^2}}}-\frac{e^2\ln\left(\frac{2e^2a+2c\,d^2}{e^2}\right.}{}}{e}$

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1
/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^
2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c
*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(87) = 174.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \left[\frac{(ace^2x^2+a^2e^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-a}{e^2x^2+2\,dex+d^2}\right.}{2\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2}\right.$$

$$-\frac{(ace^2x^2+a^2e^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(acd^2e+a^2e^3+(c^2d^3+acde^2)x)\sqrt{cx^2+a}}{a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4)x^2}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^2*x^2
 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{1}{(a+cx^2)^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = \frac{cdx}{\sqrt{cx^2+a}acd^2+\sqrt{cx^2+a}a^2e^2} + \frac{1}{\frac{\sqrt{cx^2+a}cd^2}{e}+\sqrt{cx^2+a}ae} + \frac{\operatorname{arsinh}\left(\frac{cdx}{e\sqrt{\frac{ac}{e^2}}|ex+d|}-\frac{a}{\sqrt{\frac{ac}{e^2}}|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
[Out] c*d*x/(sqrt(c*x^2 + a)*a*c*d^2 + sqrt(c*x^2 + a)*a^2*e^2) + 1/(sqrt(c*x^2 +
 a)*c*d^2/e + sqrt(c*x^2 + a)*a*e) + arcsinh(c*d*x/(e*sqrt(a*c/e^2)*abs(e*x
 + d)) - a/(sqrt(a*c/e^2)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int \frac{1}{(d+ex)\,(a+cx^2)^{3/2}}\,dx = -\frac{2\,e^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}} + \frac{\frac{(c^2d^3+acde^2)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
[Out] -2*e^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c^2*d^3 + a*c*d*e^2)*x/(a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2*e + a^2*e^3)/(a*c^2*d^4 +
2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.339 $\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx = \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{e^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d(cd^2+ae^2)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^
2)^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)-e
*(c*d*x+a*e)/a/d/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {975, 272, 53, 65, 214, 755, 12, 739, 212}

$$\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{e^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} - \frac{e(ae+cdx)}{ad\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{1}{ad\sqrt{a+cx^2}}$$

```
[In] Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx\,(a+cx^2)^{3/2}}-\frac{e}{d(d+ex)\,(a+cx^2)^{3/2}}\right)dx\\
&= \frac{\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d}-\frac{e\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx}{d}\\
&= -\frac{e(ae+cdx)}{ad\,(cd^2+ae^2)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d}-\frac{e\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad\,(cd^2+ae^2)}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\,(cd^2+ae^2)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2ad}-\frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d\,(cd^2+ae^2)}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\,(cd^2+ae^2)\sqrt{a+cx^2}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{acd}+\frac{e^3\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d\,(cd^2+ae^2)}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\,(cd^2+ae^2)\sqrt{a+cx^2}}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\,(cd^2+ae^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{c(d-ex)}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{2e^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{d\left(-cd^2-ae^2\right)^{3/2}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

[In] Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]

[Out] (c*(d - e*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*e^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d*(-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3/2)*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(131) = 262.

Time = 0.37 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.47

method	result
default	$\frac{\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{d}-\frac{\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\frac{2ecd\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(e^2a+c\,d^2\right)\left(\frac{4c\left(e^2a+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{\left(x+\frac{d}{e}\right)^2c-\cdots}}}{\cdots}$

[In] int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))-1/d*(1/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*e*c*d/(a*e^2+c*d^2)*(2*c*(x+d/e)-2/e*c*d)/(4*c*(a*e^2+c*d^2)/e^2-4/e^2*c^2*d^2)/((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(132) = 264$.

Time = 0.47 (sec) , antiderivative size = 1325, normalized size of antiderivative = 9.01

$$\int \frac{1}{x(d+ex)\,(a+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2
*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4
*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^
2), 1/2*(2*(a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*
sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2), 1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(c*x^2 + a)) + (a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 +
a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d
^3*e^2 + a^4*c*d*e^4)*x^2), ((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c
*x^2 + a)) + (a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sq
rt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 +
2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(c\,x^2+a\right)^{3/2}(d+e\,x)}\,dx$$

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.340 $\int \frac{1}{x^2(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

```
[Out] -e^4*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c
*d^2)^(3/2)+e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a
)^(1/2)-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+e^2*(c*d*x+a*e)
/a/d^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {975, 277, 197, 272, 53, 65, 214, 755, 12, 739, 212}

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}-\frac{e^4\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

```
[In] Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}}-\frac{e}{d^2x\left(a+cx^2\right)^{3/2}}+\frac{e^2}{d^2(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx\\
&= \frac{\int \frac{1}{x^2(a+cx^2)^{3/2}}\,dx}{d}-\frac{e\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d^2}+\frac{e^2\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx}{d^2}\\
&= -\frac{1}{adx\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{(2c)\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{ad}\\
&\quad -\frac{e\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d^2}+\frac{e^2\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad^2\left(cd^2+ae^2\right)}
\end{aligned}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad^2} + \frac{e^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2\left(cd^2+ae^2\right)}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd^2} - \frac{e^4\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{-\frac{d\left(a^2e^2+2c^2d^2x^2+ac\left(d^2+dex+e^2x^2\right)\right)}{a^2\left(cd^2+ae^2\right)x\sqrt{a+cx^2}} + \frac{2e^4\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}} - \frac{2e\text{arctanh}\left(\frac{\sqrt{c}x-\cdots}{\cdots}\right)}{a^{3/2}}}{d^2}$$

```
[In] Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((d*(a^2*e^2 + 2*c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(a^2*(c*d^2
+ a*e^2)*x*Sqrt[a + c*x^2])) + (2*e^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
+ c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) - (2*e*ArcTanh[
(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(174) = 348.

Time = 0.39 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{\sqrt{c\,x^2+a}}{a^2 d x}+\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}} d^2}-\frac{c\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2 c+2\sqrt{-ac}\,\left(x-\frac{\sqrt{-ac}}{c}\right)}}{2a^2\left(e\sqrt{-ac}+cd\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}+\frac{c\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^2 c-2\sqrt{-ac}\,\left(x+\frac{\sqrt{-ac}}{c}\right)}}{2a^2\left(e\sqrt{-ac}-cd\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}+$
default	$\frac{-\frac{1}{ax\sqrt{c\,x^2+a}}-\frac{2cx}{a^2\sqrt{c\,x^2+a}}}{d}-\frac{e\left(\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{d^2}+\frac{e\left(\frac{e^2}{\left(e^2a+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}+\right.}{}$

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/d*(c*x^2+a)^(1/2)/x+1/a^(3/2)/d^2*e*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2
))/x)-1/2/a^2*c/(e*(-a*c)^(1/2)+c*d)/(x-(-a*c)^(1/2)/c)*((x-(-a*c)^(1/2)/c)
^2*c+2*(-a*c)^(1/2)*(x-(-a*c)^(1/2)/c))^(1/2)+1/2/a^2*c/(e*(-a*c)^(1/2)-c*d
)/(x+(-a*c)^(1/2)/c)*((x+(-a*c)^(1/2)/c)^2*c-2*(-a*c)^(1/2)*(x+(-a*c)^(1/2)
/c))^(1/2)+1/d^2*c*e^3/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/((a*e^2+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(175) = 350$.

Time = 0.50 (sec) , antiderivative size = 1556, normalized size of antiderivative = 8.02

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + ((c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*s
qrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^5 +
 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*
x^2 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a
^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*
d^2*e^4)*x), -1/2*(2*(a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 +
 (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(
```

```
c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4
 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d
^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e
^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -1/2*(2*((c^3*d
^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
a^3*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c*e^4*x^3 + a^
3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^
2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 +
(2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d^2*
e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -((a^2*c*e^4*x^3 +
 a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + ((c^3*d^4*
e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3
*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c^2*d^5 + 2*a^2*c*d
^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c
^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4
*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x
)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(a+cx^2)^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\,(a+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{2\,e^4\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^4+ad^2e^2\right)\sqrt{-cd^2-ae^2}} - \frac{\frac{\left(ac^3d^3+a^2c^2de^2\right)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}} - \frac{2\,e\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ad^2} + \frac{2\,\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)ad}$$

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*e^4*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - ((a*c^3*d^3 + a^2*c^2*d
*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) + (a^2*c^2*d^2*e + a^3*c*
e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqrt(c*x^2 + a) - 2*e*arcta
n(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^2) + 2*sqrt(c)/(((
sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.341 $\int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 276

$$\begin{aligned}\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx =& -\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}\\ &-\frac{1}{2adx^2\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}}\\ &+\frac{e^5\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\left(cd^2+ae^2\right)^{3/2}}+\frac{3c\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}\end{aligned}$$

```
[Out] e^5*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*
d^2)^(3/2)+3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(5/2)/d-e^2*arctanh((c*
x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^3-3/2*c/a^2/d/(c*x^2+a)^(1/2)+e^2/a/d^3/(c*
x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^2+a)^(1/2)+e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/
a^2/d^2/(c*x^2+a)^(1/2)-e^3*(c*d*x+a*e)/a/d^3/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules

used = {975, 272, 44, 53, 65, 214, 277, 197, 755, 12, 739, 212}

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{e^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3} + \frac{3c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{3c}{2a^2d\sqrt{a+cx^2}} + \frac{e^5\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^{3/2}} + \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{1}{2adx^2\sqrt{a+cx^2}}$$

```
[In] Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-3*c)/(2*a^2*d*Sqrt[a + c*x^2]) + e^2/(a*d^3*Sqrt[a + c*x^2]) - 1/(2*a*d*x
^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt
[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])
+ (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(
c*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d
) - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx^3(a+cx^2)^{3/2}} - \frac{e}{d^2x^2(a+cx^2)^{3/2}} + \frac{e^2}{d^3x(a+cx^2)^{3/2}} - \frac{e^3}{d^3(d+ex)(a+cx^2)^{3/2}}\right) dx$$

$$= \frac{\int \frac{1}{x^3(a+cx^2)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x^2(a+cx^2)^{3/2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx}{d^3}$$

$$= \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x^2(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} + \frac{(2ce)\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{ad^2} + \frac{e^2\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d^3} - \frac{e^3\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad^3(cd^2+ae^2)}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} - \frac{1}{2adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{(3c)\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{4ad} + \frac{e^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad^3} - \frac{e^5\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3(cd^2+ae^2)}$$

$$= -\frac{3c}{2a^2d\sqrt{a+cx^2}} + \frac{e^2}{ad^3\sqrt{a+cx^2}} - \frac{1}{2adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4a^2d} + \frac{e^2\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd^3} + \frac{e^5\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^3(cd^2+ae^2)}$$

$$
\begin{aligned}
&= -\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}-\frac{1}{2adx^2\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}} \\
&\quad +\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\left(cd^2+ae^2\right)^{3/2}} \\
&\quad -\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}-\frac{3\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{2a^2d} \\
&= -\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}-\frac{1}{2adx^2\sqrt{a+cx^2}} \\
&\quad +\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}} \\
&\quad +\frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\left(cd^2+ae^2\right)^{3/2}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.79

$$
\int\frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=
$$

$$
-\frac{\frac{d\left(c^2d^2x^2(3d-4ex)+a^2e^2(d-2ex)+ac\left(d^3-2d^2ex+de^2x^2-2e^3x^3\right)\right)}{a^2\left(cd^2+ae^2\right)x^2\sqrt{a+cx^2}}+\frac{4e^5\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}+\frac{2\left(3cd^2-2ae^2\right)\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{5/2}}}{2d^3}
$$

```
[In] Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -1/2*((d*(c^2*d^2*x^2*(3*d - 4*e*x) + a^2*e^2*(d - 2*e*x) + a*c*(d^3 - 2*d^
2*e*x + d*e^2*x^2 - 2*e^3*x^3)))/(a^2*(c*d^2 + a*e^2)*x^2*Sqrt[a + c*x^2])
+ (4*e^5*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e
^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(3*c*d^2 - 2*a*e^2)*ArcTanh[(Sqrt[c]*x
- Sqrt[a + c*x^2])/Sqrt[a]])/a^(5/2))/d^3
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2a^2d^2x^2}-\frac{-\frac{\left(-2e^2a+3c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d\sqrt{a}}+\frac{c^2d^2\sqrt{\left(x+\frac{\sqrt{-ac}}{c}\right)^2c-2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}}{(e\sqrt{-ac}-cd)\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}+\frac{c^2d^2\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2c+2 \ldots}}{(e\sqrt{-ac}+cd)\sqrt{-ac} \ldots}}{2a^2 \ldots}$
default	$\frac{-\frac{1}{2a\,x^2\sqrt{c\,x^2+a}}-\frac{3c\left(\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}}{d}+\frac{e^2\left(\frac{1}{a\sqrt{c\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{d^3}-\frac{e\left(-\frac{1}{ax\sqrt{c\,x^2+a}}-\frac{2cx}{a^2\sqrt{c\,x^2 \ldots}} \ldots\right)}{d^2} \ldots$

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a^2/d^2/x^2-1/2/a^2/d^2*(-(-2*a*e^2+3*c*d^2
)/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+c^2*d^2/(e*(-a*c)^(1/2)-c
*d)/(-a*c)^(1/2)/(x+(-a*c)^(1/2)/c)*((x+(-a*c)^(1/2)/c)^2*c-2*(-a*c)^(1/2)*
(x+(-a*c)^(1/2)/c))^(1/2)+c^2*d^2/(e*(-a*c)^(1/2)+c*d)/(-a*c)^(1/2)/(x-(-a*
c)^(1/2)/c)*((x-(-a*c)^(1/2)/c)^2*c+2*(-a*c)^(1/2)*(x-(-a*c)^(1/2)/c))^(1/2
)+2*c*a^2*e^4/(e*(-a*c)^(1/2)+c*d)/(e*(-a*c)^(1/2)-c*d)/d/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 1943, normalized size of antiderivative = 7.04

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - ((3*c^4*d^6 + 4*
a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c
^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*
e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3*d^
6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*d^3
*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2 + a
^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2), 1/4
*(4*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
```

```
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e
^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*
x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sq
rt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6
+ 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^
3*c*d*e^5)*x^3 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*
(a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3
*d^7 + 2*a^4*c^2*d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*
e^2 + a^6*d^3*e^4)*x^2), -1/2*(((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*
e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 -
 2*a^4*e^6)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^3*c*e^5*x^4
 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d
^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3
*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*
d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2
+ a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2),
1/2*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^
6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2
 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 +
(3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e +
2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*
d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4
)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3), x)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{2\,e^5\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^5+ad^3e^2)\sqrt{-cd^2-ae^2}} + \frac{\frac{(a^2c^3d^2e+a^3c^2e^3)x}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4} - \frac{a^2c^3d^3+a^3c^2de^2}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4}}{\sqrt{cx^2+a}} - \frac{(3\,cd^2-2\,ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2d^3} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd+2\,a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2a^2d^2}$$

[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")

[Out] -2*e^5*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) + ((a^2*c^3*d^2*e + a^3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4) - (a^2*c^3*d^3 + a^3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/sqrt(c*x^2 + a) - (3*c*d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.342 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 244

$$\begin{aligned}\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)}\\&\quad-\frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4}\\&\quad-\frac{d(4cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}\\&\quad-\frac{d^4(4cd^2+5ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5(cd^2+ae^2)^{3/2}}\end{aligned}$$

```
[Out] -d*(-a*e^2+4*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5-d^4*(5*a
*e^2+4*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^5
/(a*e^2+c*d^2)^(3/2)+1/3*(-2*a*e^2+13*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d^5*(c
*x^2+a)^(1/2)/e^4/(a*e^2+c*d^2)/(e*x+d)-5/3*d*(e*x+d)*(c*x^2+a)^(1/2)/c/e^4
+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used

$= \{1665, 1668, 858, 223, 212, 739\}$

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(4cd^2-ae^2)}{c^{3/2}e^5} - \frac{d^4(5ae^2+4cd^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5\left(ae^2+cd^2\right)^{3/2}} + \frac{\sqrt{a+cx^2}(13cd^2-2ae^2)}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)} - \frac{5d\sqrt{a+cx^2}(d+ex)}{3ce^4} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^4}$$

```
[In] Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-\frac{ad^4}{e^3}+\frac{d^3(cd^2+ae^2)x}{e^4}-\frac{d^2(cd^2+ae^2)x^2}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^3-\frac{(cd^2+ae^2)x^4}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2} \\
&= \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} \\
&\quad - \frac{\int \frac{-ad^2e(cd^2-2ae^2)+4d(cd^2+ae^2)^2x+2e(cd^2+ae^2)^2x^2+10cde^2(cd^2+ae^2)x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4(cd^2+ae^2)} \\
&= \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} \\
&\quad - \frac{\int \frac{-6acd^2e^4(2cd^2+ae^2)-2cde^3(cd^2+ae^2)^2x-2ce^4(13cd^2-2ae^2)(cd^2+ae^2)x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{6c^2e^7(cd^2+ae^2)} \\
&= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} \\
&\quad + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-6ac^2d^2e^6(2cd^2+ae^2)+6c^2de^5(4cd^2-ae^2)(cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{6c^3e^9(cd^2+ae^2)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} \\
&\quad + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{(d(4cd^2-ae^2))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^5} \\
&\quad + \frac{(d^4(4cd^2+5ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^5(cd^2+ae^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} \\
&\quad + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{(d(4cd^2-ae^2))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^5} \\
&\quad - \frac{(d^4(4cd^2+5ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^5(cd^2+ae^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} \\
&\quad + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{d(4cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5} \\
&\quad - \frac{d^4(4cd^2+5ae^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^5(cd^2+ae^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.03

$$
\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx
$$

$$
= \frac{\frac{e\sqrt{a+cx^2}(-2a^2e^4(d+ex)+ace^2(7d^3+4d^2ex-2de^2x^2+e^3x^3)+c^2d^2(12d^3+6d^2ex-2de^2x^2+e^3x^3))}{c^2(cd^2+ae^2)(d+ex)} + \frac{6d^4(4cd^2+5ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+c}}{\sqrt{-cd^2-ae^2}}\right.}{(-cd^2-ae^2)^{3/2}}}{3e^5}
$$

```
[In] Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(-2*a^2*e^4*(d + e*x) + a*c*e^2*(7*d^3 + 4*d^2*e*x - 2*
d*e^2*x^2 + e^3*x^3) + c^2*d^2*(12*d^3 + 6*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)
))/(c^2*(c*d^2 + a*e^2)*(d + e*x)) + (6*d^4*(4*c*d^2 + 5*a*e^2)*ArcTan[(Sqr
t[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*
e^2)^(3/2) + (3*(4*c*d^3 - a*d*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/c^
(3/2))/(3*e^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(218) = 436$.

Time = 0.48 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{(-c e^2 x^2+3cdex+2e^2 a-9c d^2)\sqrt{c x^2+a}}{3c^2 e^4}+\frac{d\left(\frac{(e^2 a-4c d^2)\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{e\sqrt{c}}-\frac{5c d^3 \ln\left(\frac{\frac{2e^2 a+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2 a+c d^2}{e^2}}\sqrt{(\cdots}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2 a+c d^2}{e^2}}}\cdots\right)}{\cdots}$
default	$\frac{\frac{x^2\sqrt{c x^2+a}}{3c}-\frac{2a\sqrt{c x^2+a}}{3c^2}}{e^2}-\frac{4d^3\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{e^5\sqrt{c}}-\frac{2d\left(\frac{x\sqrt{c x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^3}+\frac{3d^2\sqrt{c x^2+a}}{e^4 c}-\frac{d^5\left(-\frac{e^2\sqrt{(x+\cdots}}{\cdots}\cdots\right)}{\cdots}$

```
[In] int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-c*e^2*x^2+3*c*d*e*x+2*a*e^2-9*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d/e^4/c
*((a*e^2-4*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-5*c*d^3/e^2/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^
2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e
))-c*d^4/e^3*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*
e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2
/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(219) = 438$.

Time = 25.37 (sec) , antiderivative size = 2025, normalized size of antiderivative = 8.30

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c
^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(c)*log(-2
*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
 a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(12*c^3*d^7*e +
 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*
d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*
x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt
(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
+ 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/6*(6*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e
^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(
c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*c^3*d^7*e + 19*a
*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e
^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*x^2 +
 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt(c*x^
2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6 + 2*a
*c^3*d^2*e^8 + a^2*c^2*e^10)*x), 1/6*(6*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^
2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^
5 - a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2
)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sq
rt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))
 + 2*(12*c^3*d^7*e + 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^
3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3
*e^5 + a^2*c*d*e^7)*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^
6 - a^3*e^8)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d
*e^9 + (c^4*d^4*e^6 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/3*(3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^
2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2
*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c
*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 -
 a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (12*c^3*d^7*e
+ 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2
*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)
*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqr
t(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x)]
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^5}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.343 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}+\frac{(6cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{d^3(3cd^2+4ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4(cd^2+ae^2)^{3/2}}$$

```
[Out] 1/2*(-a*e^2+6*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4+d^3*(4*
a*e^2+3*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^
4/(a*e^2+c*d^2)^(3/2)-5/2*d*(c*x^2+a)^(1/2)/c/e^3-d^4*(c*x^2+a)^(1/2)/e^3/(
a*e^2+c*d^2)/(e*x+d)+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1665, 1668, 858, 223, 212, 739}

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(6cd^2-ae^2)}{2c^{3/2}e^4}+\frac{d^3(4ae^2+3cd^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\frac{\sqrt{a+cx^2}(d+ex)}{2ce^3}$$

```
[In] Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}-\frac{\int\frac{\frac{ad^3}{e^2}-\frac{d^2(cd^2+ae^2)x}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^2-\frac{(cd^2+ae^2)x^3}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}-\frac{\int\frac{ade(3cd^2+ae^2)-(c^2d^4-a^2e^4)x+5cde(cd^2+ae^2)x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3(cd^2+ae^2)}\\
&= -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}\\
&\quad-\frac{\int\frac{acde^3(3cd^2+ae^2)-ce^2(6cd^2-ae^2)(cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5(cd^2+ae^2)}\\
&= -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}\\
&\quad+\frac{(6cd^2-ae^2)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^4}-\frac{(d^3(3cd^2+4ae^2))\int\frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4(cd^2+ae^2)}\\
&= -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}\\
&\quad+\frac{(6cd^2-ae^2)\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2ce^4}\\
&\quad+\frac{(d^3(3cd^2+4ae^2))\,\text{Subst}\left(\int\frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4(cd^2+ae^2)}\\
&= -\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3(cd^2+ae^2)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}\\
&\quad+\frac{(6cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{d^3(3cd^2+4ae^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^4(cd^2+ae^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$

$$= \frac{\frac{e\sqrt{a+cx^2}(cd^2(-6d^2-3dex+e^2x^2)+ae^2(-4d^2-3dex+e^2x^2))}{c(cd^2+ae^2)(d+ex)} - \frac{4d^3(3cd^2+4ae^2)\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2(6cd^2-ae^2)\text{arctanh}(\cdots}{c^{3/2}}}{2e^4}$$

[In] Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]

[Out] ((e*Sqrt[a + c*x^2]*(c*d^2*(-6*d^2 - 3*d*e*x + e^2*x^2) + a*e^2*(-4*d^2 - 3 *d*e*x + e^2*x^2)))/(c*(c*d^2 + a*e^2)*(d + e*x)) - (4*d^3*(3*c*d^2 + 4*a*e ^2)*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^2])])/(-(c*d^2) - a*e^2)^(3/2) + (2*(6*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/(- Sqrt[a] + Sqrt[a + c*x^2])])/c^(3/2))/(2*e^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(180) = 360.

Time = 0.45 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{(-ex+4d)\sqrt{c\,x^2+a}}{2c\,e^3} - \frac{\frac{(e^2a-6c\,d^2)\ln(x\sqrt{c}+\sqrt{c\,x^2+a})}{e\sqrt{c}} - \frac{8c\,d^3\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2} - \frac{2cd(x+\frac{d}{e})}{e} + 2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{(x+\frac{d}{e})^2c - \frac{2cd(x+\frac{d}{e})}{e} + \frac{e^2a \cdots}{}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}\cdots}{\cdots}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2c} - \frac{a\ln(x\sqrt{c}+\sqrt{c\,x^2+a})}{2c^{\frac{3}{2}}}}{e^2} + \frac{3d^2\ln(x\sqrt{c}+\sqrt{c\,x^2+a})}{e^4\sqrt{c}} - \frac{2d\sqrt{c\,x^2+a}}{c\,e^3} + \frac{d^4\left(-\frac{e^2\sqrt{(x+\frac{d}{e})^2c - \frac{2cd(x+\frac{d}{e})}{e} + \frac{e^2a+c\,d^2}{e^2}}}{(e^2a+c\,d^2)(x+\frac{d}{e})} - \frac{ecd\ln(\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-e*x+4*d)*(c*x^2+a)^(1/2)/c/e^3-1/2/c/e^3*((a*e^2-6*c*d^2)/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-8*c*d^3/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2 /e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-2*c*d^4/e^3*(-1/(a*e^2+c*

```
d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*
d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(
x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^
2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(181) = 362$.

Time = 46.20 (sec) , antiderivative size = 1786, normalized size of antiderivative = 8.75

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3
*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (
3*c^3*d^5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^3*d^6*e + 10*
a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^
7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a)
)/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d
^2*e^7 + a^2*c^2*e^9)*x), 1/4*(4*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*
e + 4*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*
e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a
^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5
*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*
a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^
9)*x), -1/2*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 +
(6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^
5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (6*c^3*d^6*e + 10*a*c^2*d^4*
e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3
*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5
*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a
^2*c^2*e^9)*x), 1/2*(2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*e + 4*a*c^
2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e
```

```
)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (6*c^3*d
^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*e + 11*a*c
^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(
c*x^2 + a)) - (6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*
e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 +
 a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*
d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x)]
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^4}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.344 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a+cx^2}}{ce^2}+\frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)}-\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}-\frac{d^2\left(2cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] -d^2*(3*a*e^2+2*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(
1/2))/e^3/(a*e^2+c*d^2)^(3/2)-2*d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^
(1/2)+(c*x^2+a)^(1/2)/c/e^2+d^3*(c*x^2+a)^(1/2)/e^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1665, 1668, 858, 223, 212, 739}

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\left(3ae^2+2cd^2\right)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}+\frac{\sqrt{a+cx^2}}{ce^2}$$

```
[In] Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
```

```
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-\frac{ad^2}{e}+d\left(a+\frac{cd^2}{e^2}\right)x-\frac{(cd^2+ae^2)x^2}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-acd^2e+2cd(cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2(cd^2+ae^2)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{(2d)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(d^2(2cd^2+3ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3(cd^2+ae^2)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^3} - \frac{(d^2(2cd^2+3ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3(cd^2+ae^2)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3} - \frac{d^2(2cd^2+3ae^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3(cd^2+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\frac{e\sqrt{a+cx^2}(ae^2(d+ex)+cd^2(2d+ex))}{c(cd^2+ae^2)(d+ex)} + \frac{2d^2(2cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2d\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e^3}$$

```
[In] Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(a*e^2*(d + e*x) + c*d^2*(2*d + e*x)))/(c*(c*d^2 + a*e^
2)*(d + e*x)) + (2*d^2*(2*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sq
rt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*d*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/e^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(144) = 288$.

Time = 0.44 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.41

method	result
risch	$\frac{\sqrt{c x^2+a}}{c e^2}-\frac{2 d \ln\left(x \sqrt{c}+\sqrt{c x^2+a}\right)}{e^3 \sqrt{c}}+\frac{d^3 \sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c d^2}{e^2}}}{e^3 (e^2 a+c d^2) \left(x+\frac{d}{e}\right)}+\frac{d^4 c \ln\left(\frac{\frac{2 e^2 a+2 c d^2}{e^2}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{e^2 a+c d^2}{e^2}} \sqrt{\left(x+\cdots\right.}}{x+\frac{d}{e}}\right.}{e^4 (e^2 a+c d^2) \sqrt{\frac{e^2 a+c\cdots}{e^2}}}$
default	$\frac{\sqrt{c x^2+a}}{c e^2}-\frac{2 d \ln\left(x \sqrt{c}+\sqrt{c x^2+a}\right)}{e^3 \sqrt{c}}-\frac{d^3 \left(-\frac{e^2 \sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^2 a+c d^2}{e^2}}}{\left(e^2 a+c d^2\right) \left(x+\frac{d}{e}\right)}-\frac{e c d \ln\left(\frac{\frac{2 e^2 a+2 c d^2}{e^2}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{e^2 a+c d^2}{e^2}} \sqrt{\left(x\cdots\right.}}{x+\frac{d}{e}}\right.}{\left(e^2 a+c d^2\right) \sqrt{\frac{e^2 a+c\cdots}{e^2}}}\right.}{e^5}$

```
[In] int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c/e^2-2*d/e^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^3
/(a*e^2+c*d^2)/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2
)+d^4/e^4*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(
a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-3/e^4*d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c
-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(145) = 290$.

Time = 5.64 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.06

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3
 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*(2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d
^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*
```

```
c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*
a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(
sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^
2*d^2 + a*c*e^2)*x^2)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*
e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - (2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 +
2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5
 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), 1/2*(4*(c^
2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^
5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2*d^5 + 3*a*c*d^3*
e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*d^5*e
+ 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sq
rt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4
+ 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^
4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a
^2*d*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^5*e + 3
*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(
c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2
*a*c^2*d^2*e^6 + a^2*c*e^8)*x)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^3}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.345 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}+\frac{d(cd^2+2ae^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] d*(2*a*e^2+c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))
/e^2/(a*e^2+c*d^2)^(3/2)+arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2
*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1665, 858, 223, 212, 739}

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d(2ae^2+cd^2)\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}$$

```
[In] Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} - \frac{\int \frac{ad-\frac{\left(cd^2+ae^2\right)x}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2} + \frac{d\left(2a+\frac{cd^2}{e^2}\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\left(-\frac{de\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{2(cd^2+2ae^2)\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+cx^2}}\right)}{(-cd^2-ae^2)^{3/2}}\right) + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}}}{e^2}$$

```
[In] Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*(-((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (2*(c*d^2 + 2*a*
e^2)*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + c*x^
2])])/(-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a
 + c*x^2])])/Sqrt[c])/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(123) = 246$.

Time = 0.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.69

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}} + \frac{d^2\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)} - \frac{ecd\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\right.}{e}}}{x+\frac{d}{e}}\right.}{\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a+c\,d^2}{e^2}}}\right.}{e^4}$

```
[In] int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^2/e^4*(-1/(a*e^2+c*d^2)*e^2/(
x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c
```

```
*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e)))+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/
e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e
^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(124) = 248$.

Time = 5.49 (sec) , antiderivative size = 1260, normalized size of antiderivative = 9.20

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a
^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2*d
^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c
^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a
^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), 1/2*(2*(c^2*d^
4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*
e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 +
2*a*c^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)
*x), -1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*
e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4 +
2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c
*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^
4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*
d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), ((c^2*d^4 + 2*a*c*d
^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) - (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4
*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d
*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.346 $\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)} - \frac{ae\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -a*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+d*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {821, 739, 212}

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\sqrt{a+cx^2}}{(d+ex)\,(ae^2+cd^2)} - \frac{ae\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

```
[In] Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(ae)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{(ae)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{2ae\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

```
[In] Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + (2*a*e*ArcTan[(Sqrt[c]*(d
 + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3
/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(82) = 164.

Time = 0.42 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.82

method	result
default	$-\frac{\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{e^2a+c\,d^2}{e^2}}}-\frac{d\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\right.}{}\right.}{}$

```
[In] int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*
((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))-d/e^3*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+
d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)
*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/
e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(83) = 166.

Time = 0.45 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.24

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$

$$=\left[\frac{(ae^2x+ade)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\,(cd^3+ade^2)\sqrt{c}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x)}\right.$$

$$\left.-\frac{(ae^2x+ade)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^3+ade^2)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x}\right]$$

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*e^2*x + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((a*e^2*x + a*d*e)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
```

2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]

Sympy [F]

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

[In] integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)

[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F]

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx=\int \frac{x}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.347 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{cd\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -c*d*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)-e*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {745, 739, 212}

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{cd\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)}$$

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}+\frac{(cd)\int\frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\ &= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{(cd)\text{Subst}\left(\int\frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\ &= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int\frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx=-\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}+\frac{2cd\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (2*c*d*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)
^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(83) = 166$.

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

method	result	size
default	$\frac{-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2} c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^{2} a+c\,d^{2}}{e^{2}}}}{\left(e^{2} a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{e c d \ln\left(\frac{\frac{2 e^{2} a+2 c\,d^{2}}{e^{2}}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2} c-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+\frac{e^{2} a+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(e^{2} a+c\,d^{2}\right)\sqrt{\frac{e^{2} a+c\,d^{2}}{e^{2}}}}}{e^{2}}$	215

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d
^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c
*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*
(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(84) = 168$.

Time = 0.45 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.19

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{(cdex+cd^2)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-(2\,c^2d^2+ace^2)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\,(cd^2e+ae^3)\sqrt{c\ldots}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x)}\right.$$
$$\left.-\frac{(cdex+cd^2)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(cd^2e+ae^3)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x}\right]$$

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.348 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{e^2\sqrt{a+cx^2}}{d\,(cd^2+ae^2)\,(d+ex)} + \frac{ce\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}} + \frac{e\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

```
[Out] c*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)+e*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)+e^2*(c*x^2+a)^
(1/2)/d/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {975, 272, 65, 214, 745, 739, 212}

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{e\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}} + \frac{ce\,\text{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{e^2\sqrt{a+cx^2}}{d(d+ex)\,(ae^2+cd^2)}$$

```
[In] Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
```

```
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{d^2x\sqrt{a+cx^2}}-\frac{e}{d(d+ex)^2\sqrt{a+cx^2}}-\frac{e}{d^2(d+ex)\sqrt{a+cx^2}}\right)dx\\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}-\frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}-\frac{e\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d}\\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^2}\\
&\quad+\frac{e\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2}-\frac{(ce)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd^2}+\frac{(ce)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}\\
&\quad+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$
\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{e\left(\frac{de\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{2(2cd^2+ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}\right)+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}
$$

```
[In] Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (2*(2*c*d^2 + a*e^2
)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(
-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]
])/Sqrt[a])/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(159) = 318.

Time = 0.38 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{d^{2}\sqrt{a}}-\frac{-\frac{e^{2}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a+c\,d^{2}}{e^{2}}}}{\left(e^{2}a+c\,d^{2}\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^{2}a+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}}}{x+\frac{d}{e}}\right.}{\left(e^{2}a+c\,d^{2}\right)\sqrt{\frac{e^{2}a+c\,d^{2}}{e^{2}}}}}{ed}$

```
[In] int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-1/e/d*(-1/(a*e^2+c*d^2
)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(
a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d
/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/
e^2)^(1/2))/(x+d/e)))+1/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e
^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)
+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 1261, normalized size of antiderivative = 7.04

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 +
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
 a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 +
 a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), 1/2*(2*(2*
a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*
d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*
d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*
```

```
d^2*e^5)*x), 1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*c
*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a
*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3
*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), ((2*a*c*d^3*e
+ a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*
a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c*
d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^
3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x}\,dx$$

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.349 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2(cd^2+ae^2)(d+ex)} - \frac{ce^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d(cd^2+ae^2)^{3/2}} - \frac{2e^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{2e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

```
[Out] -c*e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c
*d^2)^(3/2)+2*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-2*e^2*arctanh(
(-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-(
c*x^2+a)^(1/2)/a/d^2/x-e^3*(c*x^2+a)^(1/2)/d^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {975, 270, 272, 65, 214, 745, 739, 212}

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{2e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} - \frac{ce^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} - \frac{2e^2\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}} - \frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)(ae^2+cd^2)} - \frac{\sqrt{a+cx^2}}{ad^2x}$$

[In] Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]

[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{d^2x^2\sqrt{a+cx^2}} - \frac{2e}{d^3x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)^2\sqrt{a+cx^2}} + \frac{2e^2}{d^3(d+ex)\sqrt{a+cx^2}}\right) dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(2e^2)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3} + \frac{e^2\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2(cd^2+ae^2)(d+ex)} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{d^3} - \frac{(2e^2)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^3} + \frac{(ce^2)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d(cd^2+ae^2)}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2(cd^2+ae^2)(d+ex)} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{(2e)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^3} - \frac{(ce^2)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d(cd^2+ae^2)}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2(cd^2+ae^2)(d+ex)} - \frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d(cd^2+ae^2)^{3/2}} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$= \frac{-\frac{d\sqrt{a+cx^2}(cd^2(d+ex)+ae^2(d+2ex))}{a(cd^2+ae^2)x(d+ex)} + \frac{2e^2(3cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} - \frac{4e\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^3}$$

```
[In] Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2]*(c*d^2*(d + e*x) + a*e^2*(d + 2*e*x)))/(a*(c*d^2 + a*
e^2)*x*(d + e*x))) + (2*e^2*(3*c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) -
 e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) - (4*
e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(190) = 380.

Time = 0.39 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{c\,x^2+a}}{a\,d^2 x}+\frac{2e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^3\sqrt{a}}+\frac{-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x\cdots\right.}}{x+\frac{d}{e}}\cdots\right)}{\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a+c\cdots}{e^2}}}\cdots}{d^2}$
risch	$-\frac{\sqrt{c\,x^2+a}}{a\,d^2 x}+\frac{2e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^3\sqrt{a}}-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2 c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{d^2\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ec\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\cdots}}{x+\frac{d}{e}}\cdots\right)}{d\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a\cdots}{}}}\cdots$

```
[In] int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d^2/x+2/d^3*e/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))
/x)+1/d^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2
+c*d^2)/e^2)^(1/2)-e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*
c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))-2*e/d^3/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*
((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 1512, normalized size of antiderivative = 7.13

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2
 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) + 2*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d
^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5
*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e + 2*a^2
*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*d^4*e^4)
*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e
^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^2*d^4*e^2 + 2*a*c*
d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)
*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x), -1/2*(4*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a
^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*
x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^
2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c
^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^
2 + a^3*d^4*e^4)*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^
2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 2*((c^
2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.350 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 268

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)}+\frac{ce^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}+\frac{3e^3\text{arctanh}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4}$$

```
[Out] c*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+
c*d^2)^(3/2)+1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-3*e^2*arcta
nh((c*x^2+a)^(1/2)/a^(1/2))/d^4/a^(1/2)+3*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^4/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/
d^2/x^2+2*e*(c*x^2+a)^(1/2)/a/d^3/x+e^4*(c*x^2+a)^(1/2)/d^3/(a*e^2+c*d^2)/(
e*x+d)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used

$= \{975, 272, 44, 65, 214, 270, 745, 739, 212\}$

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2} - \frac{3e^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4} + \frac{ce^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}} + \frac{3e^3\operatorname{arctanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}} + \frac{2e\sqrt{a+cx^2}}{ad^3x} - \frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}$$

```
[In] Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*S
qrt[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (
3*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sq
rt[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) -
 (3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{d^2x^3\sqrt{a+cx^2}} - \frac{2e}{d^3x^2\sqrt{a+cx^2}} + \frac{3e^2}{d^4x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)^2\sqrt{a+cx^2}} - \frac{3e^3}{d^4(d+ex)\sqrt{a+cx^2}}\right) dx$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(3e^2)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^4} - \frac{(3e^3)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^4} - \frac{e^3\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^4} + \frac{(3e^3)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^4} - \frac{(ce^3)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2(cd^2+ae^2)}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}} - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4ad^2} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^4} + \frac{(ce^3)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2(cd^2+ae^2)}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2(cd^2+ae^2)^{3/2}} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2ad^2}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2(cd^2+ae^2)^{3/2}} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{\frac{d\sqrt{a+cx^2}\left(cd^2(-d^2+3dex+4e^2x^2)+ae^2(-d^2+3dex+6e^2x^2)\right)}{a(cd^2+ae^2)x^2(d+ex)} - \frac{4e^3(4cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2(-cd^2+6ae^2)\text{arctanh}\Big(}{a^{3/2}}}{2d^4}$$

```
[In] Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((d*Sqrt[a + c*x^2]*(c*d^2*(-d^2 + 3*d*e*x + 4*e^2*x^2) + a*e^2*(-d^2 + 3*d
*e*x + 6*e^2*x^2)))/(a*(c*d^2 + a*e^2)*x^2*(d + e*x)) - (4*e^3*(4*c*d^2 + 3
```

```
*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^
2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(-(c*d^2) + 6*a*e^2)*ArcTanh[(Sqrt[c]*x
- Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/(2*d^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-4ex+d)}{2a\,d^3x^2}-\frac{2ae\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{ecd\ln\left(\frac{\frac{2e^2a+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd}{}}}{x+\frac{d}{e}}\right)}{\left(e^2a+c\,d^2\right)\sqrt{\frac{e^2a+c\,d^2}{e^2}}}\right.}{}$
default	$\frac{-\frac{\sqrt{c\,x^2+a}}{2a\,x^2}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d^2}-\frac{3e^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{d^4\sqrt{a}}+\frac{2e\sqrt{c\,x^2+a}}{a\,d^3x}-\frac{e\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2c-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a+c\,d^2}{e^2}}}{\left(e^2a+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\right.}{}$

```
[In] int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+a)^(1/2)*(-4*e*x+d)/a/d^3/x^2-1/2/d^3/a*(2*a*e*(-1/(a*e^2+c*d^2
)*e^2/(x+d/e)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-e*c*d/(
a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2/e*c*d*(x+d
/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+d/e)+(a*e^2+c*d^2)/
e^2)^(1/2))/(x+d/e)))-6*e^2*a/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^
2)/e^2-2/e*c*d*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2/e*c*d*(x+
d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-(-6*a*e^2+c*d^2)/d/a^(1/2)*ln((2*a+
2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 1867, normalized size of antiderivative = 6.97

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5
)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*
```

```
d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(
e^2*x^2 + 2*d*e*x + d^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^
5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*
d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*
d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5
)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3
+ (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), 1/4*(4*((4*a^2*c*d^2*
e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^
2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^
4 - 6*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2
+ 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a
^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4
*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), -1/2*(((c^3
*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4
*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*arctan(sqrt(
-a)/sqrt(c*x^2 + a)) - ((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^
3 + 3*a^3*d*e^5)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a
^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a
*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d
^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2
+ a^4*d^5*e^4)*x^2), 1/2*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d
^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2
)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c
^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4
 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e
 + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a
^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*
e^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.351 $\int x^2(a+bx)^n \left(c+dx^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 135

$$\int x^2(a+bx)^n \left(c+dx^2\right) \, dx = \frac{a^2(b^2c+a^2d)\,(a+bx)^{1+n}}{b^5(1+n)} - \frac{2a(b^2c+2a^2d)\,(a+bx)^{2+n}}{b^5(2+n)} + \frac{(b^2c+6a^2d)\,(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad(a+bx)^{4+n}}{b^5(4+n)} + \frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

```
[Out] a^2*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^5/(1+n)-2*a*(2*a^2*d+b^2*c)*(b*x+a)^(2+n)
/b^5/(2+n)+(6*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/
(4+n)+d*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {962}

$$\int x^2(a+bx)^n \left(c+dx^2\right) \, dx = \frac{a^2(a^2d+b^2c)\,(a+bx)^{n+1}}{b^5(n+1)} - \frac{2a(2a^2d+b^2c)\,(a+bx)^{n+2}}{b^5(n+2)} + \frac{(6a^2d+b^2c)\,(a+bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a+bx)^{n+4}}{b^5(n+4)} + \frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(a^2b^2c+a^4d)\,(a+bx)^n}{b^4}-\frac{2(ab^2c+2a^3d)\,(a+bx)^{1+n}}{b^4}\right.\\&\qquad\left.+\frac{(b^2c+6a^2d)\,(a+bx)^{2+n}}{b^4}-\frac{4ad(a+bx)^{3+n}}{b^4}+\frac{d(a+bx)^{4+n}}{b^4}\right)dx\\&=\frac{a^2(b^2c+a^2d)\,(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a(b^2c+2a^2d)\,(a+bx)^{2+n}}{b^5(2+n)}\\&\quad+\frac{(b^2c+6a^2d)\,(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{4+n}}{b^5(4+n)}+\frac{d(a+bx)^{5+n}}{b^5(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^2(a+bx)^n\left(c+dx^2\right)dx = \frac{(a+bx)^{1+n}\left(\frac{a^2b^2c+a^4d}{1+n}-\frac{2a(b^2c+2a^2d)(a+bx)}{2+n}+\frac{(b^2c+6a^2d)(a+bx)^2}{3+n}-\frac{4ad(a+bx)^3}{4+n}+\frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
[Out] ((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(a
 + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*
x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(135) = 270.

Time = 0.37 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.37

method	result
norman	$\frac{d\,x^5 e^{n\ln(bx+a)}}{5+n}+\frac{nad\,x^4 e^{n\ln(bx+a)}}{b(n^2+9n+20)}+\frac{(b^2cn^2+9b^2cn+12a^2d+20b^2c)an\,x^2e^{n\ln(bx+a)}}{b^3(n^4+14n^3+71n^2+154n+120)}+\frac{2a^3(b^2cn^2+9b^2cn+12a^2d+20b^2c}{b^5(n^5+15n^4+85n^3+225n^2+27}$
gosper	$\frac{(bx+a)^{1+n}(b^4dn^4x^4+10b^4dn^3x^4-4ab^3dn^3x^3+b^4cn^4x^2+35b^4dn^2x^4-24ab^3dn^2x^3+12b^4cn^3x^2+50b^4dnx^4+12a^2b^2dn^2x^2-}{}$
risch	$\frac{(b^5dn^4x^5+ab^4dn^4x^4+10b^5dn^3x^5+6ab^4dn^3x^4+b^5cn^4x^3+35b^5dn^2x^5-4a^2b^3dn^3x^3+ab^4cn^4x^2+11ab^4dn^2x^4+12b^5cn^3x^3+}{}$
parallelrisch	$\frac{x^3(bx+a)^nb^5cn^4+50x^5(bx+a)^nb^5dn+12x^3(bx+a)^nb^5cn^3+49x^3(bx+a)^nb^5cn^2+78x^3(bx+a)^nb^5cn+2(bx+a)^na^3b^2cn^2+18(ba}{}$

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(5+n)*x^5*exp(n*ln(b*x+a))+n*a/b*d/(n^2+9*n+20)*x^4*exp(n*ln(b*x+a))+(b^2
*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)*a/b^3*n/(n^4+14*n^3+71*n^2+154*n+120)*x
^2*exp(n*ln(b*x+a))+2*a^3*(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/b^5/(n^5+
15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(b*x+a))-(-b^2*c*n^2+4*a^2*d*n-9*b
^2*c*n-20*b^2*c)/b^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(b*x+a))-2/b^4*n*a^2*
(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/(n^5+15*n^4+85*n^3+225*n^2+274*n+12
0)*x*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.73

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx = \frac{(2\,a^3b^2cn^2+18\,a^3b^2cn+40\,a^3b^2c+24\,a^5d+(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d)x^5+(ab^4dn^4}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 + 6
*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c +
4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c -
 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3
*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2
*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^
5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4134 vs. 2(122) = 244.

Time = 1.26 (sec) , antiderivative size = 4134, normalized size of antiderivative = 30.62

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) - a**2*b**2*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**
7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*a*b**3*c*x/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**
3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b
**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 6*b**4*c*x**2/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d*lo
g(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 22
*a**4*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a*
*3*b*d*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 54*a**3*b*d*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**
8*x**3) - a**2*b**2*c/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 36*a**2*b**2*d*x**2*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*
a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d*x**2/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*a*b**3*c*x/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d*x**3*log(a/b + x)/(3*a**3*b
**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*b**4*c*x**2/(3*a**3*
b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + 3*b**4*d*x**4/(3*a**3
*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3), Eq(n, -4)), (12*a**4*
d*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2
*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5
 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*
b**7*x**2) + 2*a**2*b**2*c*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*
x**2) + 3*a**2*b**2*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b*
```

*2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*
c*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x/(2
*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a
*b**6*x + 2*b**7*x**2) + 2*b**4*c*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6
*x + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), E
q(n, -3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*
b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**
2*b**2*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**2*b**2*c/(3*a*b**5 + 3*b
6*x) + 6*a2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) - 6*a*b**3*c*x*log(a/b +
x)/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c
*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2))
, (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a*
*2*d*x**2/(2*b**3) - a*c*x/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 2*a**3*b**2*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 40*a**3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2
*a**2*b**3*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) - 40*a**2*b**3*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
5*n3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**
2*b**3*d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x**2*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 10*a*b**4*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 29*a*b**4*c*n**2*x**2*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 20*a*b**4*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**
4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b
5*n4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**
4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b
5*n2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n*

```
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 b**5*c*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**5*c*n**3*x**3*(a + b*x)**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) + 49*b**5*c*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5
*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 78*b**5*c*n*x**3*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 40*b**5*c*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 10*b**5*d*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) + 50*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**
5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.56

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$$
$$= \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^nc}{(n^3+6\,n^2+11\,n+6)b^3} + \frac{((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+1}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n +
24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n
)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)
^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(135) = 270.

Time = 0.29 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.62

$$\int x^2(a+bx)^n\left(c+dx^2\right)\,dx = \frac{(bx+a)^n b^5 d n^4 x^5 + (bx+a)^n a b^4 d n^4 x^4 + 10\,(bx+a)^n b^5 d n^3 x^5 + (bx+a)^n b^5 c n^4 x^3 + 6\,(bx+a)^n a b^4 d n^3 x^4 +}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 +
35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a)^n
*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4*d*n
^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 + 49*(
b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x + a)^n
*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*n^3*x +
 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 + 78*(b*
x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a)^n*a^2*b
^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^2
+ 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b*x + a)^n*a
^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*b^2*c*n + 40
*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*b
^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.69

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)\,dx = (a+b\,x)^n\Bigg(&\frac{2\,a^3\,(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\\&+\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}\\&+\frac{x^3\,\left(n^2+3\,n+2\right)\,\left(-4\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\\&-\frac{2\,a^2\,n\,x\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\\&+\frac{a\,n\,x^2\,\left(n+1\right)\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\\&+\frac{a\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\Bigg)\end{aligned}$$

```
[In] int(x^2*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((2*a^3*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^5*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d*x^5*(50*n + 35*n^2 + 10*
n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^3*(3*
n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 4*a^2*d*n + 9*b^2*c*n))/(b^2*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (2*a^2*n*x*(12*a^2*d + 20*b^2*c +
 b^2*c*n^2 + 9*b^2*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (a*n*x^2*(n + 1)*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2
 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.352 $\int x(a+bx)^n\left(c+dx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=-\frac{a(b^2c+a^2d)\,(a+bx)^{1+n}}{b^4(1+n)}+\frac{(b^2c+3a^2d)\,(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] -a*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+(3*a^2*d+b^2*c)*(b*x+a)^(2+n)/b^4/
(2+n)-3*a*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {786}

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=-\frac{a(a^2d+b^2c)\,(a+bx)^{n+1}}{b^4(n+1)}+\frac{(3a^2d+b^2c)\,(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a(-b^2c-a^2d)(a+bx)^n}{b^3} + \frac{(b^2c+3a^2d)(a+bx)^{1+n}}{b^3} - \frac{3ad(a+bx)^{2+n}}{b^3} + \frac{d(a+bx)^{3+n}}{b^3}\right) dx$$

$$= -\frac{a(b^2c+a^2d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{(b^2c+3a^2d)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int x(a+bx)^n (c+dx^2) \, dx$$
$$= \frac{(a+bx)^{1+n}(-6a^3d+6a^2bd(1+n)x+b^3(3+4n+n^2)x(c(4+n)+d(2+n)x^2)-ab^2(c(12+7n+n^2)\cdot}{b^4(1+n)(2+n)(3+n)(4+n)}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x*
(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n + n^
2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.91

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-b^3cn^3x-11b^3dnx^3+9ab^2dnx^2-8b^3cn^2x-6x^3b^3d-6a^2bdnx+ab^2cn^2+}{b^4(n^4+10n^3+35n^2+50n+24)}$
norman	$\frac{dx^4 e^{n\ln(bx+a)}}{4+n}+\frac{na(b^2cn^2+7b^2cn+6a^2d+12b^2c)x e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{nadx^3 e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a^2(b^2cn^2+7b^2cn+6a^2d+12b^2c)e}{b^4(n^4+10n^3+35n^2+50n+}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-b^4cn^3x^2-11b^4dnx^4+3a^2b^2dn^2x^2-ab^3cn^3x-2ab^3dnx^3-8b^4cn^2x^2-}{(3+n)(4+n)(}$
parallelrisch	$\frac{-(bx+a)^n a^3b^2cn^2-7(bx+a)^n a^3b^2cn-12(bx+a)^n a^3b^2c+12x(bx+a)^n a^2b^3cn+11x^4(bx+a)^n ab^4dn+3x^3(bx+a)^n a^2b^3dn^2+x}{}$

```
[In] int(x*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(b*x+a)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(-b^3*d*n^3*x^3-6*b^3*d*n^
2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*
```

```
n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^
2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a^3*d+12*a*b^2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx = -\frac{(a^2b^2cn^2+7\,a^2b^2cn+12\,a^2b^2c+6\,a^4d-(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d)x^4-(ab^3dn^3+3\,ab^3dn^2+\cdots}{b^4n^4+\cdots}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*b
^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b
^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*b^4
*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c + a^
3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24
*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(90) = 180$.

Time = 0.78 (sec) , antiderivative size = 2181, normalized size of antiderivative = 21.38

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6
*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*
log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
+ 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*
x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b +
 x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*
```

```
b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) -
 a*b**2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*log(a/
b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*x/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3
*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c/(2*a*b**4 + 2*
b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**
3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2
*b**2) + c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n*
*4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a
 + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
- a**2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 7*a**2*b**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**
4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d
*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*
n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*x*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2
*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b
**4) + 12*a*b**3*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*c*n**3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 1
0*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3n+2)b^2}+\frac{((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4)(bx+a)^nd}{(n^4+10n^3+35n^2+50n+24)b^4}$$

[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")

[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(102) = 204.

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.02

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx$$
$$=\frac{(bx+a)^nb^4dn^3x^4+(bx+a)^nab^3dn^3x^3+6(bx+a)^nb^4dn^2x^4+(bx+a)^nb^4cn^3x^2+3(bx+a)^nab^3dn^2x^3+}{}$$

[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")

[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^4*c*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n*b^4*c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 + 12*(b*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b^4*c*x^2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x + a)^n*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)

Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.50

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=(a+b\,x)^n\left(\frac{d\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{a^2\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{x^2\,\left(n+1\right)\,\left(-3\,d\,a^2\,n+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

```
[In] int(x*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
 + 24) - (a^2*(6*a^2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^4*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24)) + (x^2*(n + 1)*(12*b^2*c + b^2*c*n^2 - 3*a^2*d*n
 + 7*b^2*c*n))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x*(6*a^2*d
+ 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.353 $\int (a+bx)^n (c+dx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (a+bx)^n (c+dx^2)\,dx=\frac{(b^2c+a^2d)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}$$

```
[Out] (a^2*d+b^2*c)*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+
a)^(3+n)/b^3/(3+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {711}

$$\int (a+bx)^n (c+dx^2)\,dx=\frac{(a^2d+b^2c)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(\frac{(b^2c+a^2d)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right)dx\\&=\frac{(b^2c+a^2d)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (a+bx)^n \left(c+dx^2\right) dx = \frac{(a+bx)^{1+n} \left(2a^2d-2abd(1+n)x+b^2(2+n)\left(c(3+n)+d(1+n)x^2\right)\right)}{b^3(1+n)(2+n)(3+n)}$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^2dn^2x^2+3b^2dnx^2-2abdnx+b^2cn^2+2dx^2b^2-2abdx+5b^2cn+2a^2d+6b^2c\right)}{b^3(n^3+6n^2+11n+6)}$
risch	$\frac{\left(b^3dn^2x^3+ab^2dn^2x^2+3b^3dnx^3+ab^2dnx^2+b^3cn^2x+2x^3b^3d-2a^2bdnx+ab^2cn^2+5b^3cnx+5ab^2cn+6b^3cx+2da^3+6ab^2c\right)(b}{(2+n)(3+n)(1+n)b^3}$
norman	$\frac{dx^3e^{n\ln(bx+a)}}{3+n}+\frac{a\left(b^2cn^2+5b^2cn+2a^2d+6b^2c\right)e^{n\ln(bx+a)}}{b^3(n^3+6n^2+11n+6)}+\frac{adnx^2e^{n\ln(bx+a)}}{b(n^2+5n+6)}-\frac{\left(-b^2cn^2+2a^2dn-5b^2cn-6b^2c\right)xe^{n\ln}}{b^2(n^3+6n^2+11n+6)}$
parallelrisch	$\frac{x^3(bx+a)^nb^3dn^2+3x^3(bx+a)^nb^3dn+x^2(bx+a)^nab^2dn^2+2x^3(bx+a)^nb^3d+x^2(bx+a)^nab^2dn+x(bx+a)^nb^3cn^2-2x(bx+a)^n}{b^3(n^3+6n^2+11n+6}$

```
[In] int((b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(b*x+a)^(1+n)/(n^3+6*n^2+11*n+6)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d
*n*x+b^2*c*n^2+2*b^2*d*x^2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int (a+bx)^n \left(c+dx^2\right) dx = \frac{(ab^2cn^2+5\,ab^2cn+6\,ab^2c+2\,a^3d+(b^3dn^2+3\,b^3dn+2\,b^3d)x^3+(ab^2dn^2+ab^2dn)x^2+(b^3cn^2+6\,b^3c+}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n +
 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b
```

```
^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(61) = 122$.

Time = 0.50 (sec) , antiderivative size = 952, normalized size of antiderivative = 13.60

$$\int (a+bx)^n\left(c+dx^2\right)\,dx$$

$$= \begin{cases} a^n\left(cx+\frac{dx^3}{3}\right) \\ \frac{2a^2d\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2d}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{b^2c}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2dx^2\log\left(\frac{a}{b}\right.}{2a^2b^3+4ab^4x+} \\ -\frac{2a^2d\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2d}{ab^3+b^4x}-\frac{2abdx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{b^2c}{ab^3+b^4x}+\frac{b^2dx^2}{ab^3+b^4x} \\ \frac{a^2d\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{adx}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx^2}{2b} \\ \frac{2a^3d(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bdnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2cn^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5ab^2cn(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6ab^2c(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6} \end{cases}$$

```
[In] integrate((b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*
b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*d*log(a/b + x)/(a*b**3 + b**
4*x) - 2*a**2*d/(a*b**3 + b**4*x) - 2*a*b*d*x*log(a/b + x)/(a*b**3 + b**4*x
) - b**2*c/(a*b**3 + b**4*x) + b**2*d*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (
a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 + c*log(a/b + x)/b + d*x**2/(2*b), Eq
(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6
*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*
b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*
n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b*
*3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + 3*b**3*d*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
 11*b**3*n + 6*b**3) + 2*b**3*d*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
+ 11*b**3*n + 6*b**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int (a+bx)^n\left(c+dx^2\right)\,dx = \frac{(bx+a)^{n+1}c}{b(n+1)} + \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n d}{(n^3+6\,n^2+11\,n+6)b^3}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^
2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*d/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.39

$$\int (a+bx)^n\left(c+dx^2\right)\,dx = \frac{(bx+a)^n b^3dn^2x^3+(bx+a)^n ab^2dn^2x^2+3\,(bx+a)^n b^3dnx^3+(bx+a)^n b^3cn^2x+(bx+a)^n ab^2dnx^2+2\,(b}{}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b^
3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x +
a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(b*x
 + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x + 6*(
b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n +
 6*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.33

$$\int (a+bx)^n\left(c+dx^2\right)\,dx = (a+b\,x)^n\left(\frac{d\,x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6} + \frac{x\,(-2\,d\,a^2\,b\,n+c\,b^3\,n^2+5\,c\,b^3\,n+6\,c\,b^3)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,(2\,d\,a^2+c\,b^2\,n^2+5\,c\,b^2\,n+6\,c\,b^2)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,d\,n\,x^2\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

```
[In] int((c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*b^3*c
 + b^3*c*n^2 + 5*b^3*c*n - 2*a^2*b*d*n))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (
a*(2*a^2*d + 6*b^2*c + b^2*c*n^2 + 5*b^2*c*n))/(b^3*(11*n + 6*n^2 + n^3 + 6
)) + (a*d*n*x^2*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6)))
```

3.354 $\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx = -\frac{ad(a+bx)^{1+n}}{b^2(1+n)}+\frac{d(a+bx)^{2+n}}{b^2(2+n)} - \frac{c(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -a*d*(b*x+a)^(1+n)/b^2/(1+n)+d*(b*x+a)^(2+n)/b^2/(2+n)-c*(b*x+a)^(1+n)*hype
rgeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {966, 81, 67}

$$\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx = -\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)} - \frac{c(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^2))/x,x]
```

```
[Out] -((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(a+bx)^{2+n}}{b^2(2+n)} + \frac{\int \frac{(a+bx)^n\left(b^2c(2+n)-abd(2+n)x\right)}{x}\,dx}{b^2(2+n)}\\ &= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} + c\int \frac{(a+bx)^n}{x}\,dx\\ &= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} - \frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^n\left(c+dx^2\right)}{x}\,dx = -\frac{(a+bx)^{1+n}\left(ad(a-b(1+n)x)+b^2c(2+n)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)\right)}{ab^2(1+n)(2+n)}$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^2))/x,x]
[Out] -(((a + b*x)^(1 + n)*(a*d*(a - b*(1 + n)*x) + b^2*c*(2 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/(a*b^2*(1 + n)*(2 + n)))
```

Maple [F]

$$\int \frac{(bx+a)^n (d\,x^2+c)}{x}dx$$

[In] int((b*x+a)^n*(d*x^2+c)/x,x)

[Out] int((b*x+a)^n*(d*x^2+c)/x,x)

Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^2)}{x}\,dx = \int \frac{(dx^2+c)(bx+a)^n}{x}\,dx$$

[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="fricas")

[Out] integral((d*x^2 + c)*(b*x + a)^n/x, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(61) = 122$.

Time = 2.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx)^n (c+dx^2)}{x}\,dx = d\left(\begin{cases} \frac{a^n x^2}{2} & \text{for } b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } n=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } n=-1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}\right) - \frac{b^{n+1}cn\left(\frac{a}{b}+x\right)^{n+1}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)} - \frac{b^{n+1}c\left(\frac{a}{b}+x\right)^{n+1}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

[In] integrate((b*x+a)**n*(d*x**2+c)/x,x)

[Out] d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/

```
(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b**(n + 1)*c*n*(a/b + x)**(n + 1)
*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b**(n + 1)*c
*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n +
 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^2)}{x} dx = \int \frac{(dx^2+c)(bx+a)^n}{x} dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^2)}{x} dx = \int \frac{(dx^2+c)(bx+a)^n}{x} dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^2)}{x} dx = \int \frac{(d\,x^2+c)\,(a+b\,x)^n}{x} dx$$

```
[In] int(((c + d*x^2)*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)*(a + b*x)^n)/x, x)
```

3.355 $\int x^2(a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 232

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^2\right)^2 \, dx &= \frac{a^2(b^2c+a^2d)^2 (a+bx)^{1+n}}{b^7(1+n)} \\ &- \frac{2a(b^2c+a^2d)(b^2c+3a^2d)(a+bx)^{2+n}}{b^7(2+n)} \\ &+ \frac{(b^4c^2+12a^2b^2cd+15a^4d^2)(a+bx)^{3+n}}{b^7(3+n)} \\ &- \frac{4ad(2b^2c+5a^2d)(a+bx)^{4+n}}{b^7(4+n)} \\ &+ \frac{d(2b^2c+15a^2d)(a+bx)^{5+n}}{b^7(5+n)} \\ &- \frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^2(a+bx)^{7+n}}{b^7(7+n)}\end{aligned}$$

```
[Out] a^2*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^7/(1+n)-2*a*(a^2*d+b^2*c)*(3*a^2*d+b^2*
c)*(b*x+a)^(2+n)/b^7/(2+n)+(15*a^4*d^2+12*a^2*b^2*c*d+b^4*c^2)*(b*x+a)^(3+n
)/b^7/(3+n)-4*a*d*(5*a^2*d+2*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+d*(15*a^2*d+2*b
^2*c)*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(
7+n)/b^7/(7+n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {962}

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx=\frac{a^2(a^2d+b^2c)^2\,(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a(a^2d+b^2c)\,(3a^2d+b^2c)\,(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad(5a^2d+2b^2c)\,(a+bx)^{n+4}}{b^7(n+4)}+\frac{d(15a^2d+2b^2c)\,(a+bx)^{n+5}}{b^7(n+5)}+\frac{(15a^4d^2+12a^2b^2cd+b^4c^2)\,(a+bx)^{n+3}}{b^7(n+3)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\text{integral}=\int\left(\frac{\left(ab^2c+a^3d\right)^2(a+bx)^n}{b^6}+\frac{2a(-b^2c-3a^2d)\,(b^2c+a^2d)\,(a+bx)^{1+n}}{b^6}+\frac{(b^4c^2+12a^2b^2cd+15a^4d^2)\,(a+bx)^{2+n}}{b^6}-\frac{4ad(2b^2c+5a^2d)\,(a+bx)^{3+n}}{b^6}+\frac{d(2b^2c+15a^2d)\,(a+bx)^{4+n}}{b^6}-\frac{6ad^2(a+bx)^{5+n}}{b^6}+\frac{d^2(a+bx)^{6+n}}{b^6}\right)dx$$

$$= \frac{a^2(b^2c+a^2d)^2(a+bx)^{1+n}}{b^7(1+n)} - \frac{2a(b^2c+a^2d)(b^2c+3a^2d)(a+bx)^{2+n}}{b^7(2+n)} + \frac{(b^4c^2+12a^2b^2cd+15a^4d^2)(a+bx)^{3+n}}{b^7(3+n)} - \frac{4ad(2b^2c+5a^2d)(a+bx)^{4+n}}{b^7(4+n)} + \frac{d(2b^2c+15a^2d)(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^2(a+bx)^{7+n}}{b^7(7+n)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$$

$$= \frac{(a+bx)^{1+n}\left(\frac{(ab^2c+a^3d)^2}{1+n} - \frac{2a(b^2c+a^2d)(b^2c+3a^2d)(a+bx)}{2+n} + \frac{(b^4c^2+12a^2b^2cd+15a^4d^2)(a+bx)^2}{3+n} - \frac{4ad(2b^2c+5a^2d)(a+bx)^3}{4+n} + d\right.}{b^7}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^2
*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^2)
*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n) + (
d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n)
 + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(232) = 464.

Time = 0.48 (sec) , antiderivative size = 754, normalized size of antiderivative = 3.25

method	result
norman	$\frac{d^2x^7e^{n\ln(bx+a)}}{7+n} + \frac{n\,d^2a\,x^6e^{n\ln(bx+a)}}{b(n^2+13n+42)} + \frac{(b^4c^2n^4+22b^4c^2n^3+24a^2b^2cd\,n^2+179b^4c^2n^2+312a^2b^2cdn+638b^4c^2n+360d^2a^4+}{b^5(n^6+27n^5+295n^4+1665n^3+5104n^2+8028n+}$
gosper	$\frac{(bx+a)^{1+n}(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+2b^6cd\,n^6x^4+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+46b^6cd\,n^5x^4+735b^6d^2n^3x^6+3}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(7+n)*x^7*exp(n*ln(b*x+a))+n*d^2/b*a/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))
+(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b^2
*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)*a/b^5*n/(n^6
+27*n^5+295*n^4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))+2*a^3*(
b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b^2*c
```

*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp(n*ln(b*x+a))-(-b^4*c^2*n^4+8*a^2*b^2*c*d*n^3-22*b^4*c^2*n^3+104*a^2*b^2*c*d*n^2-179*b^4*c^2*n^2+120*a^4*d^2*n+336*a^2*b^2*c*d*n-638*b^4*c^2*n-840*b^4*c^2)/b^4/(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))-2*d*(-b^2*c*n^2+3*a^2*d*n-13*b^2*c*n-42*b^2*c)/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x+a))-2/b^6*n*a^2*(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b^2*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))+2*(b^2*c*n^2+13*b^2*c*n+15*a^2*d+42*b^2*c)*a/b^3*d*n/(n^4+22*n^3+179*n^2+638*n+840)*x^4*exp(n*ln(b*x+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(232) = 464$.

Time = 0.32 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.43

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx = \frac{(2\,a^3b^4c^2n^4+44\,a^3b^4c^2n^3+1680\,a^3b^4c^2+2016\,a^5b^2cd+720\,a^7d^2+(b^7d^2n^6+21\,b^7d^2n^5+175\,b^7d^2n^4+73}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*c
*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*b^
7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d
^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^
6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23*b^7*
c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(185*b^7
*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 + 36*(67
*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*n^5 + (1
31*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d^2)*n^3 +
 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^3*b^4*d^2)
*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*c*d)*n^5 +
(247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b^5*c*d - 120
*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*b^3*d^2)*n^2
+ 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 + 2*(179*a^3*b^
4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*n^5 + 3*(67*a*b
^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*c*d)*n^3 + 2*(73
9*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*(35*a*b^6*c^2 + 4
2*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c^2 + 156*a^5*b^2*c
*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a^2*b^5*c^2 + 24*a^4
*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d)*n^2 + 24*(35*a^2*b^5*
c^2 + 42*a^4*b^3*c*d + 15*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
```

```
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n
 + 5040*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14317 vs. 2(218) = 436.

Time = 4.19 (sec) , antiderivative size = 14317, normalized size of antiderivative = 61.71

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)
[Out] Piecewise((a**n*(c**2*x**3/3 + 2*c*d*x**5/5 + d**2*x**7/7), Eq(b, 0)), (60*
a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x
 + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*
b**12*x**5 + 60*b**13*x**6) - 4*a**4*b**2*c*d/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a*
*6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900
*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*
x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10
*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 24*a**3*b
**3*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*
b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200
*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**
4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a
*b**12*x**5 + 60*b**13*x**6) - a**2*b**4*c**2/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) - 60*a**2*b**4*c*d*x**2/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x
)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b*
*4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6
*a*b**5*c**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*
```

```
a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) -
 80*a*b**5*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) + 360*a*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**
8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360
*a*b**12*x**5 + 60*b**13*x**6) - 15*b**6*c**2*x**2/(60*a**6*b**7 + 360*a**5
*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 +
 360*a*b**12*x**5 + 60*b**13*x**6) - 60*b**6*c*d*x**4/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-180*a
**6*d**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2
+ 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 411*a**6*d**2/(
30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 +
 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**5*b*d**2*x*log(a/b + x)/(30*a**
5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a
*b**11*x**4 + 30*b**12*x**5) - 1875*a**5*b*d**2*x/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 12*a**4*b**2*c*d/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b*
*9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a*
*4*b**2*d**2*x**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 3300*a
**4*b**2*d**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 3
00*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*a**3*b**3*c*d*x
/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3
 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a**3*b**3*d**2*x**3*log(a/b + x
)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**
3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 2700*a**3*b**3*d**2*x**3/(30*a**5*b
**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b*
*11*x**4 + 30*b**12*x**5) - a**2*b**4*c**2/(30*a**5*b**7 + 150*a**4*b**8*x
+ 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x*
*5) - 120*a**2*b**4*c*d*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*
a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 5*a*b**5*c**2*x/(30*a
**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150
*a*b**11*x**4 + 30*b**12*x**5) - 120*a*b**5*c*d*x**3/(30*a**5*b**7 + 150*a*
*4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 3
0*b**12*x**5) - 180*a*b**5*d**2*x**5*log(a/b + x)/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
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12*x5) - 10*b**6*c**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b **9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*b** 6*c*d*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2* b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) + 30*b**6*d**2*x**6/(30*a**5 *b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a* b**11*x**4 + 30*b**12*x**5), Eq(n, -6)), (180*a**6*d**2*log(a/b + x)/(12*a* *4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x **4) + 375*a**6*d**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 4 8*a*b**10*x**3 + 12*b**11*x**4) + 720*a**5*b*d**2*x*log(a/b + x)/(12*a**4*b **7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 1320*a**5*b*d**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 24*a**4*b**2*c*d*log(a/b + x)/(12*a**4*b **7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 50*a**4*b**2*c*d/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48 *a*b**10*x**3 + 12*b**11*x**4) + 1080*a**4*b**2*d**2*x**2*log(a/b + x)/(12* a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11 *x**4) + 1620*a**4*b**2*d**2*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2* b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 96*a**3*b**3*c*d*x*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 176*a**3*b**3*c*d*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a* *2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3*l og(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10 *x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3/(12*a**4*b**7 + 48*a**3*b* *8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - a**2*b**4*c** 2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12 *b**11*x**4) + 144*a**2*b**4*c*d*x**2*log(a/b + x)/(12*a**4*b**7 + 48*a**3* b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 216*a**2*b* *4*c*d*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10 *x**3 + 12*b**11*x**4) + 180*a**2*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - 4*a*b**5*c**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b **10*x**3 + 12*b**11*x**4) + 96*a*b**5*c*d*x**3*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 9 6*a*b**5*c*d*x**3/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a *b**10*x**3 + 12*b**11*x**4) - 36*a*b**5*d**2*x**5/(12*a**4*b**7 + 48*a**3* b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - 6*b**6*c**2 *x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 24*b**6*c*d*x**4*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b* *8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 6*b**6*d**2*x **6/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b + x)/(3*a**3*b**7 + 9*a* *2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*d**2/(3*a**3*b**7 + 9* a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 270*a**5*b *d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 24*a

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**4*b**2*c*d*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*
b**10*x**3) - 44*a**4*b**2*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a*
*2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3
*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**3*b**3*c*d*x*
log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) -
 108*a**3*b**3*c*d*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x +
 9*a*b**9*x**2 + 3*b**10*x**3) - a**2*b**4*c**2/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d*x**2*log(a/b + x)/(3*a
**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d
*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**
2*b**4*d**2*x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**
3) - 3*a*b**5*c**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 24*a*b**5*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a
*b**9*x**2 + 3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 3*b**6*c**2*x**2/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*b**6*c*d*x**4/(3*a**3*b**7 + 9*a
**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 +
9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*lo
g(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*
b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b
**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8
*x + 4*b**9*x**2) + 48*a**4*b**2*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
 + 4*b**9*x**2) + 72*a**4*b**2*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2)
 + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 20*a**
3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*a**2*b**4*c**
2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 6*a**2*b**4*c**2/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 48*a**2*b**4*c*d*x**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/(4*a**2
*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x*log(a/b + x)/(4*a**2*b*
*7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 16*a*b**5*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2
) - 2*a*b**5*d**2*x**5/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*
*2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*d*
x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d**2*x**6/(4*a**2*b**7
 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-180*a**6*d**2*log(a/b + x)/(30*
a*b**7 + 30*b**8*x) - 180*a**6*d**2/(30*a*b**7 + 30*b**8*x) - 180*a**5*b*d*
*2*x*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d*log(a/b + x)/
(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d/(30*a*b**7 + 30*b**8*x) + 90*a*
*4*b**2*d**2*x**2/(30*a*b**7 + 30*b**8*x) - 240*a**3*b**3*c*d*x*log(a/b + x
)/(30*a*b**7 + 30*b**8*x) - 30*a**3*b**3*d**2*x**3/(30*a*b**7 + 30*b**8*x)
- 60*a**2*b**4*c**2*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 60*a**2*b**4*c**
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2/(30*a*b**7 + 30*b**8*x) + 120*a**2*b**4*c*d*x**2/(30*a*b**7 + 30*b**8*x) + 15*a**2*b**4*d**2*x**4/(30*a*b**7 + 30*b**8*x) - 60*a*b**5*c**2*x*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 40*a*b**5*c*d*x**3/(30*a*b**7 + 30*b**8*x) - 9*a*b**5*d**2*x**5/(30*a*b**7 + 30*b**8*x) + 30*b**6*c**2*x**2/(30*a*b**7 + 30*b**8*x) + 20*b**6*c*d*x**4/(30*a*b**7 + 30*b**8*x) + 6*b**6*d**2*x**6 /(30*a*b**7 + 30*b**8*x), Eq(n, -2)), (a**6*d**2*log(a/b + x)/b**7 - a**5*d **2*x/b**6 + 2*a**4*c*d*log(a/b + x)/b**5 + a**4*d**2*x**2/(2*b**5) - 2*a** 3*c*d*x/b**4 - a**3*d**2*x**3/(3*b**4) + a**2*c**2*log(a/b + x)/b**3 + a**2 *c*d*x**2/b**3 + a**2*d**2*x**4/(4*b**3) - a*c**2*x/b**2 - 2*a*c*d*x**3/(3* b**2) - a*d**2*x**5/(5*b**2) + c**2*x**2/(2*b) + c*d*x**4/(2*b) + d**2*x**6 /(6*b), Eq(n, -1)), (720*a**7*d**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068* b**7*n + 5040*b**7) - 720*a**6*b*d**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7 *n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 48*a**5*b**2*c*d*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b* *7*n**2 + 13068*b**7*n + 5040*b**7) + 624*a**5*b**2*c*d*n*(a + b*x)**n/(b** 7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1 3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2016*a**5*b**2*c*d*(a + b*x)** n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n* *3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b**2*d**2*n**2* x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n** 4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5 *b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b** 7) - 48*a**4*b**3*c*d*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b **7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 624*a**4*b**3*c*d*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7* n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2016*a**4*b**3*c*d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b* *7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7 *n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4*b**3*d**2*n* *2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7* n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 240*a **4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040* b**7) + 2*a**3*b**4*c**2*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322* b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7* n + 5040*b**7) + 44*a**3*b**4*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n **6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1 3068*b**7*n + 5040*b**7) + 358*a**3*b**4*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b* *7*n**2 + 13068*b**7*n + 5040*b**7) + 1276*a**3*b**4*c**2*n*(a + b*x)**n/(b

7*n7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*a**3*b**4*c**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 24*a**3*b**4*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 336*a**3*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1320*a**3*b**4*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1008*a**3*b**4*c*d*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2*a**2*b**5*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 44*a**2*b**5*c**2*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 358*a**2*b**5*c**2*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1276*a**2*b**5*c**2*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1680*a**2*b**5*c**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 8*a**2*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 128*a**2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 664*a**2*b**5*c*d*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1216*a**2*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 672*a**2*b**5*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7

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*n + 5040*b**7) - 60*a**2*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x*
*5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b
**5*d**2*n*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 19
60*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + a*b**6*c**2*n**6*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 23*a*b**6*c**2*n**5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 201*a*b**6*c**2*n**4*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 817*a*b**6*c**2*n**3*x**2*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1478*a*b**6*c**2*n**2*x**
2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 840*a*b**6*
c**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2
*a*b**6*c*d*n**6*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 38*a*b**6*c*d*n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 262*a*b**6*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 802*a*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13
132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1080*a*b**6*c*d*n**2*x**4*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 504*a*b**6*c*d*n*x
**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*d*
*2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2*n**2*x**6*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
```

3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 120*a*b**6*d**2*n*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*c**2*n**6*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 25*b**7*
c**2*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + 247*b**7*c**2*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1219*b**7*c**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 3112*b**7*c**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 3796*b**7*c**2*n*x**3*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*b**7*c**2*x**3*(a + b*x)
n/(b7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**5*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 67
69*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 46*b**7*c*d*n*
*5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 414*b
7*c*d*n4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1850*b**7*c*d*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
7*n5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) + 4288*b**7*c*d*n**2*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
6 + 322*b7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 4824*b**7*c*d*n*x**5*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 2016*b**7*c*d*x**5*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n/(
b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n**
4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
4 + 6769*b7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*b*
*7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13

```
068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.93

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx = \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^nc^2}{(n^3+6\,n^2+11\,n+6)b^3} + \frac{2\,((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5} + \frac{((n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b^7x^7+(n^6+15\,n^5+85\,n^4+225\,n^3+274\,n^2}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 +
6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. 2(232) = 464.

Time = 0.30 (sec) , antiderivative size = 1750, normalized size of antiderivative = 7.54

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6*d
```

^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*
x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^5 +
85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (b*x +
 a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x + a)^n*
b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^
6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^6*c^2*n^
6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d*n^5*x^3
+ 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 +
1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n^3*x^5 + 27
4*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^7 + 23*(b*x
+ a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 128*(b*x + a)^
n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 180*(b*x + a)^n
*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 300*(b*x + a)^n*a
^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*
d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^n*a*b^6*c^2*n^4*x
^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^n*b^7*c^2*n^3*x^3
- 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^2*n^3*x^3
 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*n^2*x^4
 + 4824*(b*x + a)^n*b^7*c*d*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 - 44*
(b*x + a)^n*a^2*b^5*c^2*n^4*x + 817*(b*x + a)^n*a*b^6*c^2*n^3*x^2 + 336*(b*
x + a)^n*a^3*b^4*c*d*n^3*x^2 + 3112*(b*x + a)^n*b^7*c^2*n^2*x^3 - 1216*(b*x
 + a)^n*a^2*b^5*c*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 504*(b*
x + a)^n*a*b^6*c*d*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n*x^4 + 2016*(b*x +
a)^n*b^7*c*d*x^5 + 2*(b*x + a)^n*a^3*b^4*c^2*n^4 - 358*(b*x + a)^n*a^2*b^5*
c^2*n^3*x - 48*(b*x + a)^n*a^4*b^3*c*d*n^3*x + 1478*(b*x + a)^n*a*b^6*c^2*n
^2*x^2 + 1320*(b*x + a)^n*a^3*b^4*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2
*n^2*x^2 + 3796*(b*x + a)^n*b^7*c^2*n*x^3 - 672*(b*x + a)^n*a^2*b^5*c*d*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 44*(b*x + a)^n*a^3*b^4*c^2*n^3 - 1
276*(b*x + a)^n*a^2*b^5*c^2*n^2*x - 624*(b*x + a)^n*a^4*b^3*c*d*n^2*x + 840
*(b*x + a)^n*a*b^6*c^2*n*x^2 + 1008*(b*x + a)^n*a^3*b^4*c*d*n*x^2 + 360*(b*
x + a)^n*a^5*b^2*d^2*n*x^2 + 1680*(b*x + a)^n*b^7*c^2*x^3 + 358*(b*x + a)^n
*a^3*b^4*c^2*n^2 + 48*(b*x + a)^n*a^5*b^2*c*d*n^2 - 1680*(b*x + a)^n*a^2*b^
5*c^2*n*x - 2016*(b*x + a)^n*a^4*b^3*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 1276*(b*x + a)^n*a^3*b^4*c^2*n + 624*(b*x + a)^n*a^5*b^2*c*d*n + 1680*(
b*x + a)^n*a^3*b^4*c^2 + 2016*(b*x + a)^n*a^5*b^2*c*d + 720*(b*x + a)^n*a^7
*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 1
3132*b^7*n^2 + 13068*b^7*n + 5040*b^7)

Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.02

$$\int x^2(a+bx)^n \left(c+dx^2\right)^2 \, dx$$
$$= \frac{2\,a^3\,(a+b\,x)^n\,(360\,a^4\,d^2+24\,a^2\,b^2\,c\,d\,n^2+312\,a^2\,b^2\,c\,d\,n+1008\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c^2\,n^3+179\,b^4\,c}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{d^2\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$
$$+\frac{x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,\left(-120\,a^4\,d^2\,n-8\,a^2\,b^2\,c\,d\,n^3-104\,a^2\,b^2\,c\,d\,n^2-336\,a^2\,b^2\,c\,d\,n+b^4\,c^2\,n^4+\right.}{b^4\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\right.}$$
$$-\frac{2\,a^2\,n\,x\,(a+b\,x)^n\,(360\,a^4\,d^2+24\,a^2\,b^2\,c\,d\,n^2+312\,a^2\,b^2\,c\,d\,n+1008\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c^2\,n^3+1}{b^6\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+50\right.}$$
$$+\frac{2\,d\,x^5\,(a+b\,x)^n\,(-3\,d\,a^2\,n+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{a\,d^2\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{a\,n\,x^2\,(n+1)\,(a+b\,x)^n\,(360\,a^4\,d^2+24\,a^2\,b^2\,c\,d\,n^2+312\,a^2\,b^2\,c\,d\,n+1008\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c^2}{b^5\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+\right.}$$
$$+\frac{2\,a\,d\,n\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,(15\,d\,a^2+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)}{b^3\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

```
[In] int(x^2*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2
*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*n
+ 24*a^2*b^2*c*d*n^2))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 32
2*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 7
35*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)
*(840*b^4*c^2 - 120*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^
2*n^3 + b^4*c^2*n^4 - 336*a^2*b^2*c*d*n - 104*a^2*b^2*c*d*n^2 - 8*a^2*b^2*c
*d*n^3))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6
 + n^7 + 5040)) - (2*a^2*n*x*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b
^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*
d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^6*(13068*n + 13132*n^2 + 67
69*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^5*(a + b*x)^n*
(42*b^2*c + b^2*c*n^2 - 3*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^
6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(360*a^4*d^2 + 840*b^
4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 10
```

```
08*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n
*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(15*a^2*d + 42*b^2*c + b^2*c*n^2
+ 13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
28*n^6 + n^7 + 5040))
```

3.356 $\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 185

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = -\frac{a(b^2c+a^2d)^2 (a+bx)^{1+n}}{b^6(1+n)} + \frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6(2+n)} - \frac{2ad(3b^2c+5a^2d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{2d(b^2c+5a^2d)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)}$$

```
[Out] -a*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^6/(1+n)+(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b
*x+a)^(2+n)/b^6/(2+n)-2*a*d*(5*a^2*d+3*b^2*c)*(b*x+a)^(3+n)/b^6/(3+n)+2*d*(
5*a^2*d+b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*
(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used

$= \{786\}$

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx = -\frac{a(a^2d+b^2c)^2(a+bx)^{n+1}}{b^6(n+1)} + \frac{(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+2}}{b^6(n+2)} - \frac{2ad(5a^2d+3b^2c)(a+bx)^{n+3}}{b^6(n+3)} + \frac{2d(5a^2d+b^2c)(a+bx)^{n+4}}{b^6(n+4)} - \frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a+bx)^{n+6}}{b^6(n+6)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int\left(-\frac{a(b^2c+a^2d)^2(a+bx)^n}{b^5} + \frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{1+n}}{b^5} - \frac{2ad(3b^2c+5a^2d)(a+bx)^{2+n}}{b^5} + \frac{2d(b^2c+5a^2d)(a+bx)^{3+n}}{b^5} - \frac{5ad^2(a+bx)^{4+n}}{b^5} + \frac{d^2(a+bx)^{5+n}}{b^5}\right)dx$$

$$= -\frac{a(b^2c+a^2d)^2(a+bx)^{1+n}}{b^6(1+n)} + \frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6(2+n)} - \frac{2ad(3b^2c+5a^2d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{2d(b^2c+5a^2d)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.75

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx$$

$$=\frac{(a+bx)^{1+n}\left(b^4(1+n)(2+n)(3+n)(4+n)(5+n)(a+bx)\left(c+dx^2\right)^2-a(6+n)\left(b^4(1+n)(2+n)(3+\right.\right.}{}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(a + b*x)*(
c + d*x^2)^2 - a*(6 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2
 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*
(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 +
 n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2))) + 4*(1 + n)*(a + b*x)*((b
^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n)
 + d*(2 + n)*x^2)) - a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b
^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(185) = 370$.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.25

method	result
norman	$\frac{d^2x^6 e^{n\ln(bx+a)}}{6+n}+\frac{na\left(b^4c^2n^4+18b^4c^2n^3+12a^2b^2cd\,n^2+119b^4c^2n^2+132a^2b^2cdn+342b^4c^2n+120d^2a^4+360a^2b^2cd+360b^4c^2\right)}{b^5\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}$
gosper	$-\frac{(bx+a)^{1+n}\left(-b^5d^2n^5x^5-15b^5d^2n^4x^5+5a\,b^4d^2n^4x^4-2b^5cd\,n^5x^3-85b^5d^2n^3x^5+50a\,b^4d^2n^3x^4-34b^5cd\,n^4x^3-225b^5d^2n^2x^5\right.}{}$
risch	$-\frac{\left(-b^6d^2n^5x^6-a\,b^5d^2n^5x^5-15b^6d^2n^4x^6-10a\,b^5d^2n^4x^5-2b^6cd\,n^5x^4-85b^6d^2n^3x^6+5a^2b^4d^2n^4x^4-2a\,b^5cd\,n^5x^3-35a\,b^5d^2n\right.}{}$
parallelrisch	Expression too large to display

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(6+n)*x^6*exp(n*ln(b*x+a))+1/b^5*n*a*(b^4*c^2*n^4+18*b^4*c^2*n^3+12*a^2
*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c*d*n+342*b^4*c^2*n+120*a^4*d^2+36
0*a^2*b^2*c*d+360*b^4*c^2)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
*x*exp(n*ln(b*x+a))+n*d^2/b*a/(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-a^2*(b^4*c
^2*n^4+18*b^4*c^2*n^3+12*a^2*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c*d*n+
342*b^4*c^2*n+120*a^4*d^2+360*a^2*b^2*c*d+360*b^4*c^2)/b^6/(n^6+21*n^5+175*
n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(b*x+a))-(-b^4*c^2*n^4+6*a^2*b^2*c
*d*n^3-18*b^4*c^2*n^3+66*a^2*b^2*c*d*n^2-119*b^4*c^2*n^2+60*a^4*d^2*n+180*a
```

```
^2*b^2*c*d*n-342*b^4*c^2*n-360*b^4*c^2)/b^4/(n^5+20*n^4+155*n^3+580*n^2+104
4*n+720)*x^2*exp(n*ln(b*x+a))-d*(-2*b^2*c*n^2+5*a^2*d*n-22*b^2*c*n-60*b^2*c
)/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))+2*(b^2*c*n^2+11*b^2*c*n+10
*a^2*d+30*b^2*c)*a/b^3*d*n/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(185) = 370.

Time = 0.30 (sec) , antiderivative size = 757, normalized size of antiderivative = 4.09

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx = -\frac{(a^2b^4c^2n^4+18\,a^2b^4c^2n^3+360\,a^2b^4c^2+360\,a^4b^2cd+120\,a^6d^2-(b^6d^2n^5+15\,b^6d^2n^4+85\,b^6d^2n^3+225\,b^6}{}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
+ 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^
2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^
4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*
n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*
a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*
a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d
 + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5
*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*
c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 7
2*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 +
 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c
^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2
 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b
^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5
 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8940 vs. 2(170) = 340.

Time = 2.45 (sec) , antiderivative size = 8940, normalized size of antiderivative = 48.32

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a*
*5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d
**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d
**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b + x)/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*
d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x
**4 + 60*b**11*x**5) - 15*b**5*c**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
60*b**5*c*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600
*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d**2*x**5*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d**2*lo
g(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4) - 125*a**5*d**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d**2*x*log(a/b
+ x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 440*a**4*b*d**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*a**3*b**2*c*d/(12*a**4*b**
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
360*a**3*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*d**2*x**2/(1
2*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**1
0*x**4) - 24*a**2*b**3*c*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*
x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d**2*x**3*log(a/b +
x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
```

*b**10*x**4) - 240*a**2*b**3*d**2*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a*b**4*c**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 36*a*b**4*c*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*b**5*c**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 24*b**5*c*d*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*d**2*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d**2*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*d**2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*a**3*b**2*c*d*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 22*a**3*b**2*c*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a**2*b**3*c*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 54*a**2*b**3*c*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d**2*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - a*b**4*c**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*d**2*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 3*b**5*c**2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*b**5*c*d*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*d**2*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*d**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5*d**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d**2*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a**3*b**2*c*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 54*a**3*b**2*c*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d**2*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 3*a*b**4*c**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*d**2*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 6*b**5*c**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 12*b**5*c*d*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d

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**2*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*
*2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5*d**2/(12*a*b**6 + 12*b**7
*x) + 60*a**4*b*d**2*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*
c*d*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*c*d/(12*a*b**6 + 12
*b**7*x) - 30*a**3*b**2*d**2*x**2/(12*a*b**6 + 12*b**7*x) + 72*a**2*b**3*c*
d*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d**2*x**3/(12*a*b**
6 + 12*b**7*x) + 12*a*b**4*c**2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 12*a
*b**4*c**2/(12*a*b**6 + 12*b**7*x) - 36*a*b**4*c*d*x**2/(12*a*b**6 + 12*b**
7*x) - 5*a*b**4*d**2*x**4/(12*a*b**6 + 12*b**7*x) + 12*b**5*c**2*x*log(a/b
+ x)/(12*a*b**6 + 12*b**7*x) + 12*b**5*c*d*x**3/(12*a*b**6 + 12*b**7*x) + 3
*b**5*d**2*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d**2*log(a/b +
x)/b**6 + a**4*d**2*x/b**5 - 2*a**3*c*d*log(a/b + x)/b**4 - a**3*d**2*x**2/
(2*b**4) + 2*a**2*c*d*x/b**3 + a**2*d**2*x**3/(3*b**3) - a*c**2*log(a/b + x
)/b**2 - a*c*d*x**2/b**2 - a*d**2*x**4/(4*b**2) + c**2*x/b + 2*c*d*x**3/(3*
b) + d**2*x**5/(5*b), Eq(n, -1)), (-120*a**6*d**2*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 120*a**5*b*d**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
- 12*a**4*b**2*c*d*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 132*a**4*b
**2*c*d*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a**4*b**2*c*d*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n**2*x**2*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n*x**2*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 12*a**3*b**3*c*d*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 132*a**3*b**3*c*d*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) +
 360*a**3*b**3*c*d*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**
3*d**2*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*d**2
*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*d**2*n*x**
3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - a**2*b**4*c**2*n**4*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 18*a**2*b**4*c**2*n**3*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 119*a**2*b**4*c**2*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) - 342*a**2*b**4*c**2*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
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**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a
**2*b**4*c**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*
b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 6*a**2*b**4*c*d*n**4
*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**
3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 72*a**2*b**4*c*d*n**3*x**2*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 246*a**2*b**4*c*d*n**2*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) - 180*a**2*b**4*c*d*n*x**2*(a + b*x)**n/(b*
*6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) - 5*a**2*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 30*a**2*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) - 55*a**2*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) - 30*a**2*b**4*d**2*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
**5*c**2*n**5*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*a*b**5*c**2*n**
4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 119*a*b**5*c**2*n**3*x*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 342*a*b**5*c**2*n**2*x*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 360*a*b**5*c**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 2*a*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 28
*a*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 130*a*b**5*c
*d*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 224*a*b**5*c*d*n**2*x
**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a*b**5*c*d*n*x**3*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + a*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 10*a*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 35*a*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
 + 50*a*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b
**5*d**2*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
```

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5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*c**2*n**5*x**
2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 19*b**6*c**2*n**4*x**2*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 137*b**6*c**2*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 461*b**6*c**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 702*b**6*c**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 3
60*b**6*c**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 2*b**6*c*d*n**5*
x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 34*b**6*c*d*n**4*x**4*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 214*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 614*b**6*c*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 792*b**6*c*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 36
0*b**6*c*d*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d**2*n**5*x**
6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d**2*n**4*x**6*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 225*b**6*d**2*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 274*b**6*d**2*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12
0*b**6*d**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.81

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc^2}{(n^2+3n+2)b^2}+\frac{2\left((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^ncd}{(n^4+10n^3+35n^2+50n+24)b^4}+\frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4+\cdots}{(n^6+21n^5+175n^4+735\cdots}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*
b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^
3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^
2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^
5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(185) = 370$.

Time = 0.30 (sec) , antiderivative size = 1266, normalized size of antiderivative = 6.84

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a)
^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5*d
^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^5*x
^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4 + 3
5*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b*x +
a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x + a)^n*b
^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n*a*b^5*
d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2*n^5*x +
 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^2 + 130*(
b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3 + 614*(b*
x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 + 24*(b*x + a
)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a)^n*a*b^5*c^
2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2*b^4*c*d*n^3*
x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*b^3*d^2*n^2*x^
```

```
3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n*x^4 - (b*x
 + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x + 12*(b*x + a)^n*
a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*(b*x + a)^n*a^2*b
^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120*(b*x + a)^n*a*b^5
*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x + a)^n*b^6*c*d*x^4
 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*a*b^5*c^2*n^2*x + 132*(
b*x + a)^n*a^3*b^3*c*d*n^2*x + 702*(b*x + a)^n*b^6*c^2*n*x^2 - 180*(b*x + a
)^n*a^2*b^4*c*d*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n*x^2 - 119*(b*x + a)^n*
a^2*b^4*c^2*n^2 - 12*(b*x + a)^n*a^4*b^2*c*d*n^2 + 360*(b*x + a)^n*a*b^5*c^
2*n*x + 360*(b*x + a)^n*a^3*b^3*c*d*n*x + 120*(b*x + a)^n*a^5*b*d^2*n*x + 3
60*(b*x + a)^n*b^6*c^2*x^2 - 342*(b*x + a)^n*a^2*b^4*c^2*n - 132*(b*x + a)^
n*a^4*b^2*c*d*n - 360*(b*x + a)^n*a^2*b^4*c^2 - 360*(b*x + a)^n*a^4*b^2*c*d
 - 120*(b*x + a)^n*a^6*d^2)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n
^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.91

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{d^2\,x^6\,{\left(a+b\,x\right)}^n\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}$$
$$-\frac{a^2\,{\left(a+b\,x\right)}^n\,\left(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+132\,a^2\,b^2\,c\,d\,n+360\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3+119\,b^4\,c^2\right.}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$
$$+\frac{x^2\,\left(n+1\right)\,{\left(a+b\,x\right)}^n\,\left(-60\,a^4\,d^2\,n-6\,a^2\,b^2\,c\,d\,n^3-66\,a^2\,b^2\,c\,d\,n^2-180\,a^2\,b^2\,c\,d\,n+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3\right.}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$
$$+\frac{d\,x^4\,{\left(a+b\,x\right)}^n\,\left(-5\,d\,a^2\,n+2\,c\,b^2\,n^2+22\,c\,b^2\,n+60\,c\,b^2\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$
$$+\frac{a\,n\,x\,{\left(a+b\,x\right)}^n\,\left(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+132\,a^2\,b^2\,c\,d\,n+360\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+18\,b^4\,c^2\,n^3+119\,b\right.}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$
$$+\frac{a\,d^2\,n\,x^5\,{\left(a+b\,x\right)}^n\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$
$$+\frac{2\,a\,d\,n\,x^3\,{\left(a+b\,x\right)}^n\,\left(n^2+3\,n+2\right)\,\left(10\,d\,a^2+c\,b^2\,n^2+11\,c\,b^2\,n+30\,c\,b^2\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$

```
[In] int(x*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (a^2*(a + b*x)^n*
(120*a^4*d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n
^3 + b^4*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2
))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^
2*(n + 1)*(a + b*x)^n*(360*b^4*c^2 - 60*a^4*d^2*n + 342*b^4*c^2*n + 119*b^4
*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 - 180*a^2*b^2*c*d*n - 66*a^2*b^2*c*
d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
```

```
1*n^5 + n^6 + 720)) + (d*x^4*(a + b*x)^n*(60*b^2*c + 2*b^2*c*n^2 - 5*a^2*d*
n + 22*b^2*c*n)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3
 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x*(a + b*x)^n*(120*a^4*d^2 + 360*b
^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 + 3
60*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(1764*n + 16
24*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d^2*n*x^5*(a + b*x)^
n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(10
*a^2*d + 30*b^2*c + b^2*c*n^2 + 11*b^2*c*n))/(b^3*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

3.357 $\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{\left(b^2c+a^2d\right)^2 (a+bx)^{1+n}}{b^5(1+n)} - \frac{4ad(b^2c+a^2d)\,(a+bx)^{2+n}}{b^5(2+n)} + \frac{2d(b^2c+3a^2d)\,(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad^2(a+bx)^{4+n}}{b^5(4+n)} + \frac{d^2(a+bx)^{5+n}}{b^5(5+n)}$$

```
[Out] (a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^5/(1+n)-4*a*d*(a^2*d+b^2*c)*(b*x+a)^(2+n)/b
^5/(2+n)+2*d*(3*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d^2*(b*x+a)^(4+n)/
b^5/(4+n)+d^2*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {711}

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \frac{\left(a^2d+b^2c\right)^2 (a+bx)^{n+1}}{b^5(n+1)} - \frac{4ad(a^2d+b^2c)\,(a+bx)^{n+2}}{b^5(n+2)} + \frac{2d(3a^2d+b^2c)\,(a+bx)^{n+3}}{b^5(n+3)} - \frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)} + \frac{d^2(a+bx)^{n+5}}{b^5(n+5)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(b^2c+a^2d)^2\,(a+bx)^n}{b^4} - \frac{4ad(b^2c+a^2d)\,(a+bx)^{1+n}}{b^4} \right. \\
&\qquad \left. + \frac{2d(b^2c+3a^2d)\,(a+bx)^{2+n}}{b^4} - \frac{4ad^2(a+bx)^{3+n}}{b^4} + \frac{d^2(a+bx)^{4+n}}{b^4}\right) dx \\
&= \frac{(b^2c+a^2d)^2\,(a+bx)^{1+n}}{b^5(1+n)} - \frac{4ad(b^2c+a^2d)\,(a+bx)^{2+n}}{b^5(2+n)} \\
&\quad + \frac{2d(b^2c+3a^2d)\,(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad^2(a+bx)^{4+n}}{b^5(4+n)} + \frac{d^2(a+bx)^{5+n}}{b^5(5+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$$
$$= \frac{(a+bx)^{1+n}\left(\left(c+dx^2\right)^2 + \frac{4(b^2c+a^2d)\left(2a^2d-2abd(1+n)x+b^2(2+n)\left(c(3+n)+d(1+n)x^2\right)\right)}{b^4(1+n)(2+n)(3+n)} - \frac{4ad(a+bx)\left(2a^2d-2abd(2+n)x+b^2(3+n)\right.}{b^4(2+n)(3+n)(4+n}}{b(5+n)}$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^2 + (4*(b^2*c + a^2*d)*(2*a^2*d - 2*a*b*d*(
1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)))/(b^4*(1 + n)*(2 + n)*(
3 + n)) - (4*a*d*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4
 + n) + d*(2 + n)*x^2)))/(b^4*(2 + n)*(3 + n)*(4 + n))))/(b*(5 + n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(140) = 280$.

Time = 0.42 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.00

method	result
gosper	$\frac{(bx+a)^{1+n}(b^4d^2n^4x^4+10b^4d^2n^3x^4-4a\,b^3d^2n^3x^3+2b^4cd\,n^4x^2+35b^4d^2n^2x^4-24a\,b^3d^2n^2x^3+24b^4cd\,n^3x^2+50b^4d^2n\,x^4+12a^2}{}$
norman	$\frac{d^2x^5e^{n\ln(bx+a)}}{5+n}+\frac{a(b^4c^2n^4+14b^4c^2n^3+4a^2b^2cd\,n^2+71b^4c^2n^2+36a^2b^2cdn+154b^4c^2n+24d^2a^4+80a^2b^2cd+120b^4c^2)e^{n\ln(b}}{b^5(n^5+15n^4+85n^3+225n^2+274n+120)}$
risch	$\frac{(b^5d^2n^4x^5+a\,b^4d^2n^4x^4+10b^5d^2n^3x^5+6a\,b^4d^2n^3x^4+2b^5cd\,n^4x^3+35b^5d^2n^2x^5-4a^2b^3d^2n^3x^3+2a\,b^4cd\,n^4x^2+11a\,b^4d^2n^2x^4+}{}$
parallelrisch	$\frac{-36x(bx+a)^na^3b^3cd\,n^2-80x(bx+a)^na^3b^3cdn-24x(bx+a)^na^5b\,d^2n+154x(bx+a)^na\,b^5c^2n+4(bx+a)^na^4b^2cd\,n^2+36(bx+a)^n}{}$

```
[In] int((b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(b*x+a)^(1+n)/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*(b^4*d^2*n^4*x^4+
10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4*c*d*n^4*x^2+35*b^4*d^2*n^2*x^4
-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*b^4*d^2*n*x^4+12*a^2*b^2*d^2*n^
2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b^4*c^2*n^4+98*b^4*c*d*n^2*x^2+2
4*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*c*d*n^2*x-24*a*b^3*d^2*x^3+14*b
^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*x+4*a^2*b^2*c*d*n^2+24*a^2*b^2*
d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*b^4*c*d*x^2-24*a^3*b*d^2*x+36*a
^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24*a^4*d^2+80*a^2*b^2*c*d+120*b^4
*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(140) = 280$.

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.71

$$\int (a+bx)^n\left(c+dx^2\right)^2\,dx$$
$$=\frac{(ab^4c^2n^4+14\,ab^4c^2n^3+120\,ab^4c^2+80\,a^3b^2cd+24\,a^5d^2+(b^5d^2n^4+10\,b^5d^2n^3+35\,b^5d^2n^2+50\,b^5d^2n+}{}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^5
*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24*b
^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*b^4
*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)*n^3
 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n)*x^3
 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c*d*n^3
 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*d^2)*n)
```

```
*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5*c^2 + 2
*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(7
7*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5097 vs. 2(128) = 256.

Time = 1.44 (sec) , antiderivative size = 5097, normalized size of antiderivative = 36.41

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**4*
d**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d**2/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d**2*x*log(a
/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 88*a**3*b*d**2*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 2*a**2*b**2*c*d/(12*a**4*b**
5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 7
2*a**2*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2
*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d**2*x**2/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) - 8*a*b**3*c*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b
**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) +
 48*a*b**3*d**2*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) - 3*b**4*c**2/(12*a**4*b**5 + 48*a**3*b**6*x
+ 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*b**4*c*d*x**2/(12
*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*
x**4) + 12*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d**2
*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 22*a**4*d**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 36*a**3*b*d**2*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2
 + 3*b**8*x**3) - 54*a**3*b*d**2*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*
x**2 + 3*b**8*x**3) - 2*a**2*b**2*c*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b*
*7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**5 +
 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2/(3*a
**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 6*a*b**3*c*d*x/(3
*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d**2*
x**3*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
```

3) - b**4*c**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3)
- 6*b**4*c*d*x**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) + 3*b**4*d**2*x**4/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3), Eq(n, -4)), (12*a**4*d**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
2*b**7*x**2) + 18*a**4*d**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a
3*b*d2*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**
3*b*d**2*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a**2*b**2*c*d*log(a
/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*a**2*b**2*c*d/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d**2*x**2*log(a/b + x)/(2*a
2*b5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x*log(a/b + x)/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x/(2*a**2*b**5 + 4*a*b**6*x
 + 2*b**7*x**2) - 4*a*b**3*d**2*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) - b**4*c**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*b**4*c*d*x**2*l
og(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d**2*x**4/(2*a*
*2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d**2*log(a/b + x
)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d**2/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*
*2*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d*log(a/b + x)/(3*
a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d
2*x2/(3*a*b**5 + 3*b**6*x) - 12*a*b**3*c*d*x*log(a/b + x)/(3*a*b**5 + 3
*b**6*x) - 2*a*b**3*d**2*x**3/(3*a*b**5 + 3*b**6*x) - 3*b**4*c**2/(3*a*b**5
 + 3*b**6*x) + 6*b**4*c*d*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d**2*x**4/(3*a*
b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d**2*log(a/b + x)/b**5 - a**3*d**2*x/b*
*4 + 2*a**2*c*d*log(a/b + x)/b**3 + a**2*d**2*x**2/(2*b**3) - 2*a*c*d*x/b**
2 - a*d**2*x**3/(3*b**2) + c**2*log(a/b + x)/b + c*d*x**2/b + d**2*x**4/(4*
b), Eq(n, -1)), (24*a**5*d**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
5*n3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d**2*n*x*(a +
 b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**
5*n + 120*b**5) + 4*a**3*b**2*c*d*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 36*a**3*b**2*c
*d*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*a**3*b**2*c*d*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b
2*d2*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n*x**2*(a + b*x)
n/(b5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) - 4*a**2*b**3*c*d*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 36*a**2*b**3*c*d*
n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) - 80*a**2*b**3*c*d*n*x*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a
2*b3*d**2*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n*
*3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d**2*n**2*x**3*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) - 8*a**2*b**3*d**2*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b
5*n4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c

```
**2*n**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n
**2 + 274*b**5*n + 120*b**5) + 14*a*b**4*c**2*n**3*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 71
*a*b**4*c**2*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 154*a*b**4*c**2*n*(a + b*x)**n/(b**
5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) + 120*a*b**4*c**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*a*b**4*c*d*n**4*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 20*a*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 58*a*b**4*c*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 40*a*b**4*c*d*n*x**2*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*
b**4*d**2*n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n**3*x**4*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 11*a*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n
*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
 + 274*b**5*n + 120*b**5) + b**5*c**2*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 14*b**5*
c**2*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 71*b**5*c**2*n**2*x*(a + b*x)**n/(b**5*n*
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 154*b**5*c**2*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 120*b**5*c**2*x*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*c*d*n**3*x**3*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 98*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 156*b**5*c*d*n*
x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*b**5*c*d*x**3*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d**2
*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 35*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d**2*n*x**5*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 24*b**5*d**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 8
5*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.68

$$\int (a+bx)^n \left(c+dx^2\right)^2 dx$$
$$= \frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n cd}{(n^3+6n^2+11n+6)b^3}$$
$$+ \frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+1}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c*d/((n^3 + 6*n^2 + 11*n + 6)*
b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2
 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^
2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d^2/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(140) = 280.

Time = 0.28 (sec) , antiderivative size = 851, normalized size of antiderivative = 6.08

$$\int (a+bx)^n \left(c+dx^2\right)^2 dx$$
$$= \frac{(bx+a)^n b^5 d^2 n^4 x^5 + (bx+a)^n ab^4 d^2 n^4 x^4 + 10(bx+a)^n b^5 d^2 n^3 x^5 + 2(bx+a)^n b^5 cdn^4 x^3 + 6(bx+a)^n ab^4 d}{}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a)
^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d^
2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4*x^
2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3 + 11
*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x + a)^n
*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^5*c*d*
n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d^2*n*x^
4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*x + a)^n
*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*a*b^4*c*d
*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b^5*c*d*n*x
^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*n^3 + 71*(b
```

```
*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40*(b*x + a)^n
*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x + a)^n*b^5*c*
d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*c*d*n^2 + 154*
(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 24*(b*x + a)^n*a
^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n*a^3*b^2*c*d*n +
 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 80*(b*x + a)^n*a^3
*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.54

$$\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$$
$$= (a+b\,x)^n \left(\frac{a\,(24\,a^4\,d^2+4\,a^2\,b^2\,c\,d\,n^2+36\,a^2\,b^2\,c\,d\,n+80\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+14\,b^4\,c^2\,n^3+71\,b^4\,c^2\,n^2+15}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{d^2\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} + \frac{x\,(-24\,a^4\,b\,d^2\,n-4\,a^2\,b^3\,c\,d\,n^3-36\,a^2\,b^3\,c\,d\,n^2-80\,a^2\,b^3\,c\,d\,n+b^5\,c^2\,n^4+14\,b^5\,c^2\,n^3+71\,b^5\,c^2\,n^2+154}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{2\,d\,x^3\,(n^2+3\,n+2)\,\left(-2\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{a\,d^2\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{2\,a\,d\,n\,x^2\,(n+1)\,\left(6\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

```
[In] int((c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((a*(24*a^4*d^2 + 120*b^4*c^2 + 154*b^4*c^2*n + 71*b^4*c^2*n^2
+ 14*b^4*c^2*n^3 + b^4*c^2*n^4 + 80*a^2*b^2*c*d + 36*a^2*b^2*c*d*n + 4*a^2*
b^2*c*d*n^2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d^2*
x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4
 + n^5 + 120) + (x*(120*b^5*c^2 + 154*b^5*c^2*n + 71*b^5*c^2*n^2 + 14*b^5*c
^2*n^3 + b^5*c^2*n^4 - 24*a^4*b*d^2*n - 80*a^2*b^3*c*d*n - 36*a^2*b^3*c*d*n
^2 - 4*a^2*b^3*c*d*n^3))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (2*d*x^3*(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 2*a^2*d*n + 9*b^2*c*
n))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d^2*n*x^4*(1
1*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (2*a*d*n*x^2*(n + 1)*(6*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.358 $\int \frac{(a+bx)^n(c+dx^2)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{(a+bx)^n(c+dx^2)^2}{x}\,dx = -\frac{ad(2b^2c+a^2d)(a+bx)^{1+n}}{b^4(1+n)}+\frac{d(2b^2c+3a^2d)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)}+\frac{d^2(a+bx)^{4+n}}{b^4(4+n)} - \frac{c^2(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -a*d*(a^2*d+2*b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+d*(3*a^2*d+2*b^2*c)*(b*x+a)^(2
+n)/b^4/(2+n)-3*a*d^2*(b*x+a)^(3+n)/b^4/(3+n)+d^2*(b*x+a)^(4+n)/b^4/(4+n)-c
^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {966, 1634, 67}

$$\int \frac{(a+bx)^n(c+dx^2)^2}{x}\,dx = -\frac{ad(a^2d+2b^2c)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d(3a^2d+2b^2c)(a+bx)^{n+2}}{b^4(n+2)} - \frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)} - \frac{c^2(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] -((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(b^
4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \frac{(a+bx)^n\left(b^4c^2(4+n)-a^3bd^2(4+n)x+b^2d\left(2b^2c-3a^2d\right)(4+n)x^2-3ab^3d^2(4+n)x^3\right)}{x}\,dx}{b^4(4+n)}$$

$$= \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \left(-abd\left(2b^2c+a^2d\right)(4+n)(a+bx)^n + \frac{\left(4b^4c^2+b^4c^2n\right)(a+bx)^n}{x} + bd\left(2b^2c+3a^2d\right)(4+n)(a+bx)^{1+n}\right.}{b^4(4+n)}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + c^2\int \frac{(a+bx)^n}{x}\,dx$$

$$= -\frac{ad(2b^2c+a^2d)\,(a+bx)^{1+n}}{b^4(1+n)} + \frac{d(2b^2c+3a^2d)\,(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} - \frac{c^2(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^n\,(c+dx^2)^2}{x}\,dx = (a+bx)^{1+n}\left(-\frac{ad(2b^2c+a^2d)}{b^4(1+n)} + \frac{d(2b^2c+3a^2d)\,(a+bx)}{b^4(2+n)} - \frac{3ad^2(a+bx)^2}{b^4(3+n)} + \frac{d^2(a+bx)^3}{b^4(4+n)} - \frac{c^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^2)/x,x]
[Out] (a + b*x)^(1 + n)*(-((a*d*(2*b^2*c + a^2*d))/(b^4*(1 + n))) + (d*(2*b^2*c +
 3*a^2*d)*(a + b*x))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^2)/(b^4*(3 + n)) +
(d^2*(a + b*x)^3)/(b^4*(4 + n)) - (c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*x)/a])/(a + a*n))
```

Maple [F]

$$\int \frac{(bx+a)^n\,(d\,x^2+c)^2}{x}dx$$

```
[In] int((b*x+a)^n*(d*x^2+c)^2/x,x)
[Out] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n\,(c+dx^2)^2}{x}\,dx = \int \frac{(dx^2+c)^2(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="fricas")
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(129) = 258.

Time = 3.32 (sec) , antiderivative size = 1608, normalized size of antiderivative = 10.86

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2/x,x)
```

```
[Out] 2*c*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x)
+ a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a
*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b*
*2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**
2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)
**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + d**2*Piecewise((a**n*x**4/4,
Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x
**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2
 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x +
18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)/(6*a**3*b**4 +
 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*x**3*log(a/b +
 x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4
)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log(a/b + x)/(2*a**
2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b**4 + 4*a*b**5*x
+ 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq(n, -3)
), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(2*a*b**4 + 2*b**5*x
) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*x**2/(2*a*b**4
 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*log(a/b
+ x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b), Eq(n, -1)), (-6*a**
4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b
**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n*x**2*(a + b*
x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b
**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b*
*4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3*(a + b*x)**n/(b**4
*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + b**4*n**3*x**4
*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**
4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*
```

```
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4*(a + b*x)**n/(b**4
*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True)) - b**(n
+ 1)*c**2*n*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(
a*gamma(n + 2)) - b**(n + 1)*c**2*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1,
 n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} dx = \int \frac{(dx^2+c)^2(bx+a)^n}{x} dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} dx = \int \frac{(dx^2+c)^2(bx+a)^n}{x} dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^2)^2}{x} dx = \int \frac{(d\,x^2+c)^2\,(a+b\,x)^n}{x} dx$$

```
[In] int(((c + d*x^2)^2*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)^2*(a + b*x)^n)/x, x)
```

3.359 $\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 343

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\frac{a^2(b^2c+a^2d)^3\,(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a(b^2c+a^2d)^2\,(b^2c+4a^2d)\,(a+bx)^{2+n}}{b^9(2+n)}+\frac{(b^2c+a^2d)\,(b^4c^2+17a^2b^2cd+28a^4d^2)\,(a+bx)^{3+n}}{b^9(3+n)}-\frac{4ad(3b^4c^2+15a^2b^2cd+14a^4d^2)\,(a+bx)^{4+n}}{b^9(4+n)}+\frac{d(3b^4c^2+45a^2b^2cd+70a^4d^2)\,(a+bx)^{5+n}}{b^9(5+n)}-\frac{2ad^2(9b^2c+28a^2d)\,(a+bx)^{6+n}}{b^9(6+n)}+\frac{d^2(3b^2c+28a^2d)\,(a+bx)^{7+n}}{b^9(7+n)}-\frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)}+\frac{d^3(a+bx)^{9+n}}{b^9(9+n)}$$

```
[Out] a^2*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(a^2*d+b^2*c)^2*(4*a^2*d+b^
2*c)*(b*x+a)^(2+n)/b^9/(2+n)+(a^2*d+b^2*c)*(28*a^4*d^2+17*a^2*b^2*c*d+b^4*c
^2)*(b*x+a)^(3+n)/b^9/(3+n)-4*a*d*(14*a^4*d^2+15*a^2*b^2*c*d+3*b^4*c^2)*(b*
x+a)^(4+n)/b^9/(4+n)+d*(70*a^4*d^2+45*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(5+n)/
b^9/(5+n)-2*a*d^2*(28*a^2*d+9*b^2*c)*(b*x+a)^(6+n)/b^9/(6+n)+d^2*(28*a^2*d+
3*b^2*c)*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a
)^(9+n)/b^9/(9+n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {962}

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=-\frac{2ad^2(28a^2d+9b^2c)\,(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2(28a^2d+3b^2c)\,(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2(a^2d+b^2c)^3\,(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a(a^2d+b^2c)^2\,(4a^2d+b^2c)\,(a+bx)^{n+2}}{b^9(n+2)}+\frac{(a^2d+b^2c)\,(28a^4d^2+17a^2b^2cd+b^4c^2)\,(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad(14a^4d^2+15a^2b^2cd+3b^4c^2)\,(a+bx)^{n+4}}{b^9(n+4)}+\frac{d(70a^4d^2+45a^2b^2cd+3b^4c^2)\,(a+bx)^{n+5}}{b^9(n+5)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2(b^2c+a^2d)^3(a+bx)^n}{b^8} - \frac{2a(b^2c+a^2d)^2(b^2c+4a^2d)(a+bx)^{1+n}}{b^8} + \frac{(b^2c+a^2d)(b^4c^2+17a^2b^2cd+28a^4d^2)(a+bx)^{2+n}}{b^8} - \frac{4ad(3b^4c^2+15a^2b^2cd+14a^4d^2)(a+bx)^{3+n}}{b^8} + \frac{d(3b^4c^2+45a^2b^2cd+70a^4d^2)(a+bx)^{4+n}}{b^8} - \frac{2ad^2(9b^2c+28a^2d)(a+bx)^{5+n}}{b^8} + \frac{d^2(3b^2c+28a^2d)(a+bx)^{6+n}}{b^8} - \frac{8ad^3(a+bx)^{7+n}}{b^8} + \frac{d^3(a+bx)^{8+n}}{b^8}\right) dx$$

$$= \frac{a^2(b^2c+a^2d)^3(a+bx)^{1+n}}{b^9(1+n)} - \frac{2a(b^2c+a^2d)^2(b^2c+4a^2d)(a+bx)^{2+n}}{b^9(2+n)} + \frac{(b^2c+a^2d)(b^4c^2+17a^2b^2cd+28a^4d^2)(a+bx)^{3+n}}{b^9(3+n)} - \frac{4ad(3b^4c^2+15a^2b^2cd+14a^4d^2)(a+bx)^{4+n}}{b^9(4+n)} + \frac{d(3b^4c^2+45a^2b^2cd+70a^4d^2)(a+bx)^{5+n}}{b^9(5+n)} - \frac{2ad^2(9b^2c+28a^2d)(a+bx)^{6+n}}{b^9(6+n)} + \frac{d^2(3b^2c+28a^2d)(a+bx)^{7+n}}{b^9(7+n)} - \frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^3(a+bx)^{9+n}}{b^9(9+n)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3 dx = \frac{(a+bx)^{1+n}\left(\frac{a^2(b^2c+a^2d)^3}{1+n} - \frac{2a(b^2c+a^2d)^2(b^2c+4a^2d)(a+bx)}{2+n} + \frac{(b^2c+a^2d)(b^4c^2+17a^2b^2cd+28a^4d^2)(a+bx)^2}{3+n} - \frac{4ad(3b^4c^2+15a^2b^2c}{4+}\right.}{}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)^
2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a^2
*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^
2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d +
 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)
^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d^3*(a
+ b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. 2(343) = 686.

Time = 0.45 (sec) , antiderivative size = 2232, normalized size of antiderivative = 6.51

method	result	size
gosper	Expression too large to display	2232
risch	Expression too large to display	2558
parallelrisch	Expression too large to display	3685

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^9*(b*x+a)^(1+n)/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+72368
0*n^3+1172700*n^2+1026576*n+362880)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8*a
*b^7*d^3*n^7*x^7+3*b^8*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n^6*
x^7+114*b^8*c*d^2*n^7*x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-18*a*
b^7*c*d^2*n^7*x^5-2576*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*c*d^2
*n^6*x^6+22449*b^8*d^3*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d^2*n^6
*x^5-15680*a*b^7*d^3*n^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n^5*x^6+
67284*b^8*d^3*n^3*x^8-336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*x^4+9800
*a^2*b^6*d^3*n^4*x^6-12*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x^5-54152*
a*b^7*d^3*n^3*x^7+b^8*c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^8*c*d^2*n^
4*x^6+118124*b^8*d^3*n^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b^6*c*d^2*n^
5*x^4+41160*a^2*b^6*d^3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a*b^7*c*d^2*n
^4*x^5-105056*a*b^7*d^3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c^2*d*n^5*x^4+
246876*b^8*c*d^2*n^3*x^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^3*n^4*x^4-360*
a^3*b^5*c*d^2*n^5*x^3-28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c^2*d*n^6*x^2+24
930*a^2*b^6*c*d^2*n^4*x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7*c^3*n^7*x-6312*
a*b^7*c^2*d*n^5*x^3-183942*a*b^7*c*d^2*n^3*x^5-104544*a*b^7*d^3*n*x^7+744*b
^8*c^3*n^6*x^2+98319*b^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n^2*x^6+40320*b^8*d
^3*x^8+16800*a^4*b^4*d^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*x^3-75600*a^3*b^5*d
^3*n^2*x^5+1188*a^2*b^6*c^2*d*n^5*x^2+122850*a^2*b^6*c*d^2*n^3*x^4+98784*a^
2*b^6*d^3*n*x^6-80*a*b^7*c^3*n^6*x-47952*a*b^7*c^2*d*n^4*x^3-377604*a*b^7*c
*d^2*n^2*x^5-40320*a*b^7*d^3*x^7+7218*b^8*c^3*n^5*x^2+316380*b^8*c^2*d*n^3*
x^4+417744*b^8*c*d^2*n*x^6-6720*a^5*b^3*d^3*n^3*x^3+1080*a^4*b^4*c*d^2*n^4*
x^2+58800*a^4*b^4*d^3*n^2*x^4-72*a^3*b^5*c^2*d*n^5*x-66600*a^3*b^5*c*d^2*n^
3*x^3-92064*a^3*b^5*d^3*n*x^5+2*a^2*b^6*c^3*n^6+15372*a^2*b^6*c^2*d*n^4*x^2
+305460*a^2*b^6*c*d^2*n^2*x^4+40320*a^2*b^6*d^3*x^6-1328*a*b^7*c^3*n^5*x-20
1468*a*b^7*c^2*d*n^3*x^3-391824*a*b^7*c*d^2*n*x^5+41619*b^8*c^3*n^4*x^2+589
140*b^8*c^2*d*n^2*x^4+155520*b^8*c*d^2*x^6-40320*a^5*b^3*d^3*n^2*x^3+21600*
a^4*b^4*c*d^2*n^3*x^2+84000*a^4*b^4*d^3*n*x^4-2232*a^3*b^5*c^2*d*n^4*x-2250
00*a^3*b^5*c*d^2*n^2*x^3-40320*a^3*b^5*d^3*x^5+78*a^2*b^6*c^3*n^5+97740*a^2
*b^6*c^2*d*n^3*x^2+360720*a^2*b^6*c*d^2*n*x^4-11780*a*b^7*c^3*n^4*x-459648*
a*b^7*c^2*d*n^2*x^3-155520*a*b^7*c*d^2*x^5+144468*b^8*c^3*n^3*x^2+572400*b^
8*c^2*d*n*x^4+20160*a^6*b^2*d^3*n^2*x^2-2160*a^5*b^3*c*d^2*n^3*x-73920*a^5*
```

```
b^3*d^3*n*x^3+72*a^4*b^4*c^2*d*n^4+135000*a^4*b^4*c*d^2*n^2*x^2+40320*a^4*b
^4*d^3*x^4-26280*a^3*b^5*c^2*d*n^3*x-321840*a^3*b^5*c*d^2*n*x^3+1250*a^2*b^
6*c^3*n^4+311184*a^2*b^6*c^2*d*n^2*x^2+155520*a^2*b^6*c*d^2*x^4-59678*a*b^7
*c^3*n^3*x-517968*a*b^7*c^2*d*n*x^3+290276*b^8*c^3*n^2*x^2+217728*b^8*c^2*d
*x^4+60480*a^6*b^2*d^3*n*x^2-38880*a^5*b^3*c*d^2*n^2*x-40320*a^5*b^3*d^3*x^
3+2160*a^4*b^4*c^2*d*n^3+270000*a^4*b^4*c*d^2*n*x^2-142920*a^3*b^5*c^2*d*n^
2*x-155520*a^3*b^5*c*d^2*x^3+10530*a^2*b^6*c^3*n^3+445392*a^2*b^6*c^2*d*n*x
^2-169580*a*b^7*c^3*n^2*x-217728*a*b^7*c^2*d*x^3+301872*b^8*c^3*n*x^2-40320
*a^7*b*d^3*n*x+2160*a^6*b^2*c*d^2*n^2+40320*a^6*b^2*d^3*x^2-192240*a^5*b^3*
c*d^2*n*x+24120*a^4*b^4*c^2*d*n^2+155520*a^4*b^4*c*d^2*x^2-336528*a^3*b^5*c
^2*d*n*x+49148*a^2*b^6*c^3*n^2+217728*a^2*b^6*c^2*d*x^2-241392*a*b^7*c^3*n*
x+120960*b^8*c^3*x^2-40320*a^7*b*d^3*x+36720*a^6*b^2*c*d^2*n-155520*a^5*b^3
*c*d^2*x+118800*a^4*b^4*c^2*d*n-217728*a^3*b^5*c^2*d*x+120432*a^2*b^6*c^3*n
-120960*a*b^7*c^3*x+40320*a^8*d^3+155520*a^6*b^2*c*d^2+217728*a^4*b^4*c^2*d
+120960*a^2*b^6*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2165 vs. $2(343) = 686$.

Time = 0.31 (sec) , antiderivative size = 2165, normalized size of antiderivative = 6.31

$$\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*b
^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d^3
*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^9*d
^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^
8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 676
9*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^
3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a^2*b^
7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9*c*d^2 -
 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4 + 28*(881
7*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)
*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 +
96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b
^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c*d^2 + 191
8*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)*x^6 + 3*(b^9*c
^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*d^2)*n^7 + 2*(33
5*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2
 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b
^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^
```

```
3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 + 48*(3
975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 + 2*(625*a^3*b^6
*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8 + 36*a*b^8*c^2*d*n^7 + 2*
(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6 + 6*(666*a*b^8*c^2*d + 115*a^3*b^6
*c*d^2)*n^5 + (16789*a*b^8*c^2*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^
4 + 6*(6384*a*b^8*c^2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10
791*a*b^8*c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^3*b^6*c^3
+ 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3 + 6*(7*b^9*c^3 - 2*a
^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b^7*c^2*d)*n^6 + 6*(1203*b^9*c^
3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*c*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^
2*b^7*c^2*d - 2400*a^4*b^5*c*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^
2*d - 3750*a^4*b^5*c*d^2 - 560*a^6*b^3*d^3)*n^3 + 4*(72569*b^9*c^3 - 37116*
a^2*b^7*c^2*d - 22500*a^4*b^5*c*d^2 - 5040*a^6*b^3*d^3)*n^2 + 48*(6289*b^9*
c^3 - 1512*a^2*b^7*c^2*d - 1080*a^4*b^5*c*d^2 - 280*a^6*b^3*d^3)*n)*x^3 + 4
*(12287*a^3*b^6*c^3 + 6030*a^5*b^4*c^2*d + 540*a^7*b^2*c*d^2)*n^2 + (a*b^8*
c^3*n^8 + 40*a*b^8*c^3*n^7 + 4*(166*a*b^8*c^3 + 9*a^3*b^6*c^2*d)*n^6 + 62*(
95*a*b^8*c^3 + 18*a^3*b^6*c^2*d)*n^5 + (29839*a*b^8*c^3 + 13140*a^3*b^6*c^2
*d + 1080*a^5*b^4*c*d^2)*n^4 + 10*(8479*a*b^8*c^3 + 7146*a^3*b^6*c^2*d + 19
44*a^5*b^4*c*d^2)*n^3 + 24*(5029*a*b^8*c^3 + 7011*a^3*b^6*c^2*d + 4005*a^5*
b^4*c*d^2 + 840*a^7*b^2*d^3)*n^2 + 576*(105*a*b^8*c^3 + 189*a^3*b^6*c^2*d +
 135*a^5*b^4*c*d^2 + 35*a^7*b^2*d^3)*n)*x^2 + 48*(2509*a^3*b^6*c^3 + 2475*a
^5*b^4*c^2*d + 765*a^7*b^2*c*d^2)*n - 2*(a^2*b^7*c^3*n^7 + 39*a^2*b^7*c^3*n
^6 + (625*a^2*b^7*c^3 + 36*a^4*b^5*c^2*d)*n^5 + 135*(39*a^2*b^7*c^3 + 8*a^4
*b^5*c^2*d)*n^4 + 2*(12287*a^2*b^7*c^3 + 6030*a^4*b^5*c^2*d + 540*a^6*b^3*c
*d^2)*n^3 + 24*(2509*a^2*b^7*c^3 + 2475*a^4*b^5*c^2*d + 765*a^6*b^3*c*d^2)*
n^2 + 576*(105*a^2*b^7*c^3 + 189*a^4*b^5*c^2*d + 135*a^6*b^3*c*d^2 + 35*a^8
*b*d^3)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^
6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 102
6576*b^9*n + 362880*b^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35984 vs. 2(328) = 656.

Time = 11.30 (sec) , antiderivative size = 35984, normalized size of antiderivative = 104.91

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**3/3 + 3*c**2*d*x**5/5 + 3*c*d**2*x**7/7 + d**3*x**
9/9), Eq(b, 0)), (840*a**8*d**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**
10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**
4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840
```

*b**17*x**8) + 2283*a**8*d**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a*
*6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*
b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) +
6720*a**7*b*d**3*x*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*
a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**
3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8)
+ 17424*a**7*b*d**3*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 45*a**6*
b**2*c*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47
040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520
*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d*
*3*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*
x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**
5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*a**
6*b**2*d**3*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 +
23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 360*a**5*b**3
*c*d**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4704
0*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a
2*b15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**5*b**3*d**3
*x**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x*
*2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5
+ 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 107408*a**5
*b**3*d**3*x**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
+ 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2
3520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 9*a**4*b**4*c*
*2*d/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**
5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b
15*x6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 1260*a**4*b**4*c*d**2*x**
2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b
12*x3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**1
5*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 58800*a**4*b**4*d**3*x**4*lo
g(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 470
40*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*
a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 122500*a**4*b**4*d*
*3*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*
a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**
2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 72*a**3*b**5*c**2*d*x/
(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**
12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*
x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 2520*a**3*b**5*c*d**2*x**3/(84
0*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*
x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**
6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**3*b**5*d**3*x**5*log(a/b

```
 + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a*
*5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*
b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 86240*a**3*b**5*d**3*x**
5/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b
**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**1
5*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 5*a**2*b**6*c**3/(840*a**8*b
**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 5
8800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720
*a*b**16*x**7 + 840*b**17*x**8) - 252*a**2*b**6*c**2*d*x**2/(840*a**8*b**9
+ 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800
*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b
**16*x**7 + 840*b**17*x**8) - 3150*a**2*b**6*c*d**2*x**4/(840*a**8*b**9 + 6
720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a*
*4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**1
6*x**7 + 840*b**17*x**8) + 23520*a**2*b**6*d**3*x**6*log(a/b + x)/(840*a**8
*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 67
20*a*b**16*x**7 + 840*b**17*x**8) + 35280*a**2*b**6*d**3*x**6/(840*a**8*b**
9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 588
00*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a
*b**16*x**7 + 840*b**17*x**8) - 40*a*b**7*c**3*x/(840*a**8*b**9 + 6720*a**7
*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13
*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 +
 840*b**17*x**8) - 504*a*b**7*c**2*d*x**3/(840*a**8*b**9 + 6720*a**7*b**10*
x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 +
 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b*
*17*x**8) - 2520*a*b**7*c*d**2*x**5/(840*a**8*b**9 + 6720*a**7*b**10*x + 23
520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040
*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x*
*8) + 6720*a*b**7*d**3*x**7*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x
 + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 +
47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**
17*x**8) + 6720*a*b**7*d**3*x**7/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520
*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a*
*3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8)
 - 140*b**8*c**3*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 630*b**8
*c**2*d*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4
7040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 1260*b**8*c*d**2*
x**6/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**
5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b
**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 840*b**8*d**3*x**8*log(a/
b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a
```

5*b12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2
*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8), Eq(n, -9)), (-840*a**8*d
3*log(a/b + x)/(105*a7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 +
3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a
*b**15*x**6 + 105*b**16*x**7) - 2178*a**8*d**3/(105*a**7*b**9 + 735*a**6*b*
*10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4
+ 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a**7*b*d
3*x*log(a/b + x)/(105*a7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2
+ 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735
*a*b**15*x**6 + 105*b**16*x**7) - 14406*a**7*b*d**3*x/(105*a**7*b**9 + 735*
a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**1
3*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 45*a**
6*b**2*c*d**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 36
75*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b*
*15*x**6 + 105*b**16*x**7) - 17640*a**6*b**2*d**3*x**2*log(a/b + x)/(105*a*
*7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 +
3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*
x**7) - 40278*a**6*b**2*d**3*x**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*
a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b
14*x5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 315*a**5*b**3*c*d**2*x/(10
5*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**
3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b*
*16*x**7) - 29400*a**5*b**3*d**3*x**3*log(a/b + x)/(105*a**7*b**9 + 735*a**
6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x
4 + 2205*a2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 61250*a**
5*b**3*d**3*x**3/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 +
3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a
*b**15*x**6 + 105*b**16*x**7) - 3*a**4*b**4*c**2*d/(105*a**7*b**9 + 735*a**
6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x
4 + 2205*a2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 945*a**4*
b**4*c*d**2*x**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 +
3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a
*b**15*x**6 + 105*b**16*x**7) - 29400*a**4*b**4*d**3*x**4*log(a/b + x)/(105
*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3
+ 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**
16*x**7) - 53900*a**4*b**4*d**3*x**4/(105*a**7*b**9 + 735*a**6*b**10*x + 22
05*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**
2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 21*a**3*b**5*c**2*d*x/(
105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x
3 + 3675*a3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*
b**16*x**7) - 1575*a**3*b**5*c*d**2*x**3/(105*a**7*b**9 + 735*a**6*b**10*x
+ 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205
*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 17640*a**3*b**5*d**
3*x**5*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**
2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 73

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5*a*b**15*x**6 + 105*b**16*x**7) - 26460*a**3*b**5*d**3*x**5/(105*a**7*b**9
 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a*
*3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) -
 a**2*b**6*c**3/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 +
3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*
b**15*x**6 + 105*b**16*x**7) - 63*a**2*b**6*c**2*d*x**2/(105*a**7*b**9 + 73
5*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b*
*13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 1575
*a**2*b**6*c*d**2*x**4/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*
x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 +
 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a**2*b**6*d**3*x**6*log(a/b + x)
/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12
*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 10
5*b**16*x**7) - 5880*a**2*b**6*d**3*x**6/(105*a**7*b**9 + 735*a**6*b**10*x
+ 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205
*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 7*a*b**7*c**3*x/(10
5*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**
3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b*
*16*x**7) - 105*a*b**7*c**2*d*x**3/(105*a**7*b**9 + 735*a**6*b**10*x + 2205
*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*
b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 945*a*b**7*c*d**2*x**5/(1
05*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x*
*3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b
**16*x**7) - 840*a*b**7*d**3*x**7*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b*
*10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4
+ 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 21*b**8*c**3*
x**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b
**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6
+ 105*b**16*x**7) - 105*b**8*c**2*d*x**4/(105*a**7*b**9 + 735*a**6*b**10*x
+ 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205
*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 315*b**8*c*d**2*x**
6/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**1
2*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 1
05*b**16*x**7) + 105*b**8*d**3*x**8/(105*a**7*b**9 + 735*a**6*b**10*x + 220
5*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2
*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7), Eq(n, -8)), (1680*a**8*d*
*3*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 12
00*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6
) + 4116*a**8*d**3/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 +
 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x
**6) + 10080*a**7*b*d**3*x*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x +
900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b*
*14*x**5 + 60*b**15*x**6) + 23016*a**7*b*d**3*x/(60*a**6*b**9 + 360*a**5*b*
*10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 +
360*a*b**14*x**5 + 60*b**15*x**6) + 180*a**6*b**2*c*d**2*log(a/b + x)/(60*a
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**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 +
900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 441*a**6*b**2*c*d
**2/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**1
2*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 25200*a*
*6*b**2*d**3*x**2*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*
b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5
+ 60*b**15*x**6) + 52500*a**6*b**2*d**3*x**2/(60*a**6*b**9 + 360*a**5*b**10
*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360
*a*b**14*x**5 + 60*b**15*x**6) + 1080*a**5*b**3*c*d**2*x*log(a/b + x)/(60*a
**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 +
900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 2466*a**5*b**3*c*
d**2*x/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b
**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 33600
*a**5*b**3*d**3*x**3*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a*
*4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x*
*5 + 60*b**15*x**6) + 61600*a**5*b**3*d**3*x**3/(60*a**6*b**9 + 360*a**5*b*
*10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 +
360*a*b**14*x**5 + 60*b**15*x**6) - 6*a**4*b**4*c**2*d/(60*a**6*b**9 + 360*
a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*
x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 2700*a**4*b**4*c*d**2*x**2*log(a
/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*
b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 5625
*a**4*b**4*c*d**2*x**2/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x*
*2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**
15*x**6) + 25200*a**4*b**4*d**3*x**4*log(a/b + x)/(60*a**6*b**9 + 360*a**5*
b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4
+ 360*a*b**14*x**5 + 60*b**15*x**6) + 37800*a**4*b**4*d**3*x**4/(60*a**6*b*
*9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a*
*2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 36*a**3*b**5*c**2*d*x/(
60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**
3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 3600*a**3*b**
5*c*d**2*x**3*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**1
1*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60
*b**15*x**6) + 6600*a**3*b**5*c*d**2*x**3/(60*a**6*b**9 + 360*a**5*b**10*x
+ 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*
b**14*x**5 + 60*b**15*x**6) + 10080*a**3*b**5*d**3*x**5*log(a/b + x)/(60*a*
*6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 9
00*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 10080*a**3*b**5*d*
*3*x**5/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*
b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - a**2
*b**6*c**3/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a*
*3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 9
0*a**2*b**6*c**2*d*x**2/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x
**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b*
*15*x**6) + 2700*a**2*b**6*c*d**2*x**4*log(a/b + x)/(60*a**6*b**9 + 360*a**
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5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**
4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 4050*a**2*b**6*c*d**2*x**4/(60*a**6
*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900
*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 1680*a**2*b**6*d**3*
x**6*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 +
1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x*
*6) - 6*a*b**7*c**3*x/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**
2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**1
5*x**6) - 120*a*b**7*c**2*d*x**3/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**
4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**
5 + 60*b**15*x**6) + 1080*a*b**7*c*d**2*x**5*log(a/b + x)/(60*a**6*b**9 + 3
60*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**
13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 1080*a*b**7*c*d**2*x**5/(60*a
6*b9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 +
900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 240*a*b**7*d**3*x
7/(60*a6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**1
2*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 15*b**8*
c**3*x**2/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**
3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 90
*b**8*c**2*d*x**4/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 +
1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x*
*6) + 180*b**8*c*d**2*x**6*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x +
900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b*
*14*x**5 + 60*b**15*x**6) + 30*b**8*d**3*x**8/(60*a**6*b**9 + 360*a**5*b**1
0*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 36
0*a*b**14*x**5 + 60*b**15*x**6), Eq(n, -7)), (-1680*a**8*d**3*log(a/b + x)/
(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**
3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 3836*a**8*d**3/(30*a**5*b**9 + 150*
a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4
 + 30*b**14*x**5) - 8400*a**7*b*d**3*x*log(a/b + x)/(30*a**5*b**9 + 150*a**
4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 +
30*b**14*x**5) - 17500*a**7*b*d**3*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300
*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5)
- 540*a**6*b**2*c*d**2*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*
a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) -
 1233*a**6*b**2*c*d**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x*
*2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 16800*a**6*b
2*d3*x**2*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**1
1*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 30800*a*
*6*b**2*d**3*x**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 +
300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 2700*a**5*b**3*c*
d**2*x*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2
+ 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 5625*a**5*b**3*
c*d**2*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*
b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 16800*a**5*b**3*d**3*x**3*

log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 25200*a**5*b**3*d**3*x**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 18*a**4*b**4*c**2*d/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 5400*a**4*b**4*c*d**2*x**2*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 9900*a**4*b**4*c*d**2*x**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 8400*a**4*b**4*d**3*x**4*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 8400*a**4*b**4*d**3*x**4/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 90*a**3*b**5*c**2*d*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 5400*a**3*b**5*c*d**2*x**3*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 8100*a**3*b**5*c*d**2*x**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 1680*a**3*b**5*d**3*x**5*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - a**2*b**6*c**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 180*a**2*b**6*c**2*d*x**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 2700*a**2*b**6*c*d**2*x**4*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 2700*a**2*b**6*c*d**2*x**4/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 280*a**2*b**6*d**3*x**6/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 5*a*b**7*c**3*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 180*a*b**7*c**2*d*x**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 540*a*b**7*c*d**2*x**5*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 40*a*b**7*d**3*x**7/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 10*b**8*c**3*x**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 90*b**8*c**2*d*x**4/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 90*b**8*c*d**2*x**6/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 10*b**8*d**3*x**8/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5

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), Eq(n, -6)), (840*a**8*d**3*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x
+ 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 1750*a**8*d**3/(1
2*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b
**13*x**4) + 3360*a**7*b*d**3*x*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*
x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 6160*a**7*b*d**
3*x/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3
+ 12*b**13*x**4) + 540*a**6*b**2*c*d**2*log(a/b + x)/(12*a**4*b**9 + 48*a**
3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 1125*a*
*6*b**2*c*d**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*
b**12*x**3 + 12*b**13*x**4) + 5040*a**6*b**2*d**3*x**2*log(a/b + x)/(12*a**
4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*
x**4) + 7560*a**6*b**2*d**3*x**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*
b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 2160*a**5*b**3*c*d**2*x*log
(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12
*x**3 + 12*b**13*x**4) + 3960*a**5*b**3*c*d**2*x/(12*a**4*b**9 + 48*a**3*b*
*10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*b
**3*d**3*x**3*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*
x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*b**3*d**3*x**3/(12*a**4
*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x
**4) + 36*a**4*b**4*c**2*d*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 7
2*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 75*a**4*b**4*c**2*d/
(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12
*b**13*x**4) + 3240*a**4*b**4*c*d**2*x**2*log(a/b + x)/(12*a**4*b**9 + 48*a
**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 4860*
a**4*b**4*c*d**2*x**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2
+ 48*a*b**12*x**3 + 12*b**13*x**4) + 840*a**4*b**4*d**3*x**4*log(a/b + x)/(
12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*
b**13*x**4) + 144*a**3*b**5*c**2*d*x*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b
**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 264*a**3*b
**5*c**2*d*x/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b*
*12*x**3 + 12*b**13*x**4) + 2160*a**3*b**5*c*d**2*x**3*log(a/b + x)/(12*a**
4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*
x**4) + 2160*a**3*b**5*c*d**2*x**3/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**
2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 168*a**3*b**5*d**3*x**5/(
12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*
b**13*x**4) - a**2*b**6*c**3/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**1
1*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 216*a**2*b**6*c**2*d*x**2*log(a
/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x
**3 + 12*b**13*x**4) + 324*a**2*b**6*c**2*d*x**2/(12*a**4*b**9 + 48*a**3*b*
*10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 540*a**2*b*
*6*c*d**2*x**4*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11
*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 28*a**2*b**6*d**3*x**6/(12*a**4*
b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x*
*4) - 4*a*b**7*c**3*x/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2
+ 48*a*b**12*x**3 + 12*b**13*x**4) + 144*a*b**7*c**2*d*x**3*log(a/b + x)/(1
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2*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b
13*x4) + 144*a*b**7*c**2*d*x**3/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a*
*2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 108*a*b**7*c*d**2*x**5/(
12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*
b**13*x**4) - 8*a*b**7*d**3*x**7/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*
b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 6*b**8*c**3*x**2/(12*a**4*b
9 + 48*a3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**
4) + 36*b**8*c**2*d*x**4*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*
a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 18*b**8*c*d**2*x**6/(1
2*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b
13*x4) + 3*b**8*d**3*x**8/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**
11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4), Eq(n, -5)), (-840*a**8*d**3*log
(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3
) - 1540*a**8*d**3/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b
12*x3) - 2520*a**7*b*d**3*x*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*
x + 45*a*b**11*x**2 + 15*b**12*x**3) - 3780*a**7*b*d**3*x/(15*a**3*b**9 + 4
5*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 900*a**6*b**2*c*d**2*lo
g(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**
3) - 1650*a**6*b**2*c*d**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**
2 + 15*b**12*x**3) - 2520*a**6*b**2*d**3*x**2*log(a/b + x)/(15*a**3*b**9 +
45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2520*a**6*b**2*d**3*x*
*2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 270
0*a**5*b**3*c*d**2*x*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b*
*11*x**2 + 15*b**12*x**3) - 4050*a**5*b**3*c*d**2*x/(15*a**3*b**9 + 45*a**2
*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 840*a**5*b**3*d**3*x**3*log(a
/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3)
- 180*a**4*b**4*c**2*d*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*
b**11*x**2 + 15*b**12*x**3) - 330*a**4*b**4*c**2*d/(15*a**3*b**9 + 45*a**2*
b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2700*a**4*b**4*c*d**2*x**2*log
(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3
) - 2700*a**4*b**4*c*d**2*x**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11
*x**2 + 15*b**12*x**3) + 210*a**4*b**4*d**3*x**4/(15*a**3*b**9 + 45*a**2*b*
*10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 540*a**3*b**5*c**2*d*x*log(a/b +
x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 81
0*a**3*b**5*c**2*d*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15
*b**12*x**3) - 900*a**3*b**5*c*d**2*x**3*log(a/b + x)/(15*a**3*b**9 + 45*a*
*2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 42*a**3*b**5*d**3*x**5/(15*
a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 5*a**2*b**
6*c**3/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) -
540*a**2*b**6*c**2*d*x**2*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 4
5*a*b**11*x**2 + 15*b**12*x**3) - 540*a**2*b**6*c**2*d*x**2/(15*a**3*b**9 +
45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 225*a**2*b**6*c*d**2*
x**4/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 1
4*a**2*b**6*d**3*x**6/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 1
5*b**12*x**3) - 15*a*b**7*c**3*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**

11*x**2 + 15*b**12*x**3) - 180*a*b**7*c**2*d*x**3*log(a/b + x)/(15*a**3*b**
9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 45*a*b**7*c*d**2*x
5/(15*a3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 6*
a*b**7*d**3*x**7/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**
12*x**3) - 15*b**8*c**3*x**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x
2 + 15*b12*x**3) + 45*b**8*c**2*d*x**4/(15*a**3*b**9 + 45*a**2*b**10*x
+ 45*a*b**11*x**2 + 15*b**12*x**3) + 15*b**8*c*d**2*x**6/(15*a**3*b**9 + 45
*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 3*b**8*d**3*x**8/(15*a**
3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3), Eq(n, -4)), (1
680*a**8*d**3*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) +
 2520*a**8*d**3/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 3360*a**7*
b*d**3*x*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 3360
*a**7*b*d**3*x/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 2700*a**6*b
2*c*d2*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 40
50*a**6*b**2*c*d**2/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 1680*a
6*b2*d**3*x**2*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x*
*2) + 5400*a**5*b**3*c*d**2*x*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10*x +
60*b**11*x**2) + 5400*a**5*b**3*c*d**2*x/(60*a**2*b**9 + 120*a*b**10*x + 60
*b**11*x**2) - 560*a**5*b**3*d**3*x**3/(60*a**2*b**9 + 120*a*b**10*x + 60*b
11*x2) + 1080*a**4*b**4*c**2*d*log(a/b + x)/(60*a**2*b**9 + 120*a*b**10
*x + 60*b**11*x**2) + 1620*a**4*b**4*c**2*d/(60*a**2*b**9 + 120*a*b**10*x +
 60*b**11*x**2) + 2700*a**4*b**4*c*d**2*x**2*log(a/b + x)/(60*a**2*b**9 + 1
20*a*b**10*x + 60*b**11*x**2) + 140*a**4*b**4*d**3*x**4/(60*a**2*b**9 + 120
*a*b**10*x + 60*b**11*x**2) + 2160*a**3*b**5*c**2*d*x*log(a/b + x)/(60*a**2
*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 2160*a**3*b**5*c**2*d*x/(60*a**2*b
9 + 120*a*b10*x + 60*b**11*x**2) - 900*a**3*b**5*c*d**2*x**3/(60*a**2*b
9 + 120*a*b10*x + 60*b**11*x**2) - 56*a**3*b**5*d**3*x**5/(60*a**2*b**9
 + 120*a*b**10*x + 60*b**11*x**2) + 60*a**2*b**6*c**3*log(a/b + x)/(60*a**2
*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 90*a**2*b**6*c**3/(60*a**2*b**9 +
120*a*b**10*x + 60*b**11*x**2) + 1080*a**2*b**6*c**2*d*x**2*log(a/b + x)/(6
0*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 225*a**2*b**6*c*d**2*x**4/(6
0*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 28*a**2*b**6*d**3*x**6/(60*a
2*b9 + 120*a*b**10*x + 60*b**11*x**2) + 120*a*b**7*c**3*x*log(a/b + x)/
(60*a**2*b**9 + 120*a*b**10*x + 60*b**11*x**2) + 120*a*b**7*c**3*x/(60*a**2
*b**9 + 120*a*b**10*x + 60*b**11*x**2) - 360*a*b**7*c**2*d*x**3/(60*a**2*b*
*9 + 120*a*b**10*x + 60*b**11*x**2) - 90*a*b**7*c*d**2*x**5/(60*a**2*b**9 +
 120*a*b**10*x + 60*b**11*x**2) - 16*a*b**7*d**3*x**7/(60*a**2*b**9 + 120*a
*b**10*x + 60*b**11*x**2) + 60*b**8*c**3*x**2*log(a/b + x)/(60*a**2*b**9 +
120*a*b**10*x + 60*b**11*x**2) + 90*b**8*c**2*d*x**4/(60*a**2*b**9 + 120*a*
b**10*x + 60*b**11*x**2) + 45*b**8*c*d**2*x**6/(60*a**2*b**9 + 120*a*b**10*
x + 60*b**11*x**2) + 10*b**8*d**3*x**8/(60*a**2*b**9 + 120*a*b**10*x + 60*b
11*x2), Eq(n, -3)), (-1680*a**8*d**3*log(a/b + x)/(210*a*b**9 + 210*b**
10*x) - 1680*a**8*d**3/(210*a*b**9 + 210*b**10*x) - 1680*a**7*b*d**3*x*log(
a/b + x)/(210*a*b**9 + 210*b**10*x) - 3780*a**6*b**2*c*d**2*log(a/b + x)/(2
10*a*b**9 + 210*b**10*x) - 3780*a**6*b**2*c*d**2/(210*a*b**9 + 210*b**10*x)

+ 840*a**6*b**2*d**3*x**2/(210*a*b**9 + 210*b**10*x) - 3780*a**5*b**3*c*d**2*x*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 280*a**5*b**3*d**3*x**3/(210*a*b**9 + 210*b**10*x) - 2520*a**4*b**4*c**2*d*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 2520*a**4*b**4*c**2*d/(210*a*b**9 + 210*b**10*x) + 1890*a**4*b**4*c*d**2*x**2/(210*a*b**9 + 210*b**10*x) + 140*a**4*b**4*d**3*x**4/(210*a*b**9 + 210*b**10*x) - 2520*a**3*b**5*c**2*d*x*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 630*a**3*b**5*c*d**2*x**3/(210*a*b**9 + 210*b**10*x) - 84*a**3*b**5*d**3*x**5/(210*a*b**9 + 210*b**10*x) - 420*a**2*b**6*c**3*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 420*a**2*b**6*c**3/(210*a*b**9 + 210*b**10*x) + 1260*a**2*b**6*c**2*d*x**2/(210*a*b**9 + 210*b**10*x) + 315*a**2*b**6*c*d**2*x**4/(210*a*b**9 + 210*b**10*x) + 56*a**2*b**6*d**3*x**6/(210*a*b**9 + 210*b**10*x) - 420*a*b**7*c**3*x*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 420*a*b**7*c**2*d*x**3/(210*a*b**9 + 210*b**10*x) - 189*a*b**7*c*d**2*x**5/(210*a*b**9 + 210*b**10*x) - 40*a*b**7*d**3*x**7/(210*a*b**9 + 210*b**10*x) + 210*b**8*c**3*x**2/(210*a*b**9 + 210*b**10*x) + 210*b**8*c**2*d*x**4/(210*a*b**9 + 210*b**10*x) + 126*b**8*c*d**2*x**6/(210*a*b**9 + 210*b**10*x) + 30*b**8*d**3*x**8/(210*a*b**9 + 210*b**10*x), Eq(n, -2)), (a**8*d**3*log(a/b + x)/b**9 - a**7*d**3*x/b**8 + 3*a**6*c*d**2*log(a/b + x)/b**7 + a**6*d**3*x**2/(2*b**7) - 3*a**5*c*d**2*x/b**6 - a**5*d**3*x**3/(3*b**6) + 3*a**4*c**2*d*log(a/b + x)/b**5 + 3*a**4*c*d**2*x**2/(2*b**5) + a**4*d**3*x**4/(4*b**5) - 3*a**3*c**2*d*x/b**4 - a**3*c*d**2*x**3/b**4 - a**3*d**3*x**5/(5*b**4) + a**2*c**3*log(a/b + x)/b**3 + 3*a**2*c**2*d*x**2/(2*b**3) + 3*a**2*c*d**2*x**4/(4*b**3) + a**2*d**3*x**6/(6*b**3) - a*c**3*x/b**2 - a*c**2*d*x**3/b**2 - 3*a*c*d**2*x**5/(5*b**2) - a*d**3*x**7/(7*b**2) + c**3*x**2/(2*b) + 3*c**2*d*x**4/(4*b) + c*d**2*x**6/(2*b) + d**3*x**8/(8*b), Eq(n, -1)), (40320*a**9*d**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 40320*a**8*b*d**3*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 2160*a**7*b**2*c*d**2*n**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 36720*a**7*b**2*c*d**2*n*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 155520*a**7*b**2*c*d**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 20160*a**7*b**2*d**3*n**2*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 20160*a**7*b**2*d**3*n*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 2160*a**6*b**3*c*d**2*n**3*x*(a + b*x)**n/(b**9*n*

*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) - 36720*a**6*b**3*c*d**2*n**2*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*
n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4
+ 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 15
5520*a**6*b**3*c*d**2*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9
*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n
3 + 1172700*b9*n**2 + 1026576*b**9*n + 362880*b**9) - 6720*a**6*b**3*d*
*3*n**3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*
b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700
*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 20160*a**6*b**3*d**3*n**2*x**3
*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) - 13440*a**6*b**3*d**3*n*x**3*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
+ 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
+ 362880*b**9) + 72*a**5*b**4*c**2*d*n**4*(a + b*x)**n/(b**9*n**9 + 45*b**
9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**
4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
2160*a**5*b**4*c**2*d*n**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 24120*a**5*b**4*
c**2*d*n**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
9*n6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 118800*a**5*b**4*c**2*d*n*(a +
b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*
b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10265
76*b**9*n + 362880*b**9) + 217728*a**5*b**4*c**2*d*(a + b*x)**n/(b**9*n**9
+ 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) + 1080*a**5*b**4*c*d**2*n**4*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 194
40*a**5*b**4*c*d**2*n**3*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*
b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b*
*9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 96120*a**5*b*
*4*c*d**2*n**2*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
+ 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 77760*a**5*b**4*c*d**2*
n*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
6 + 63273*b9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) + 1680*a**5*b**4*d**3*n**4*x**4*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
9*n5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) + 10080*a**5*b**4*d**3*n**3*x**4*(a + b*x)**n/(b**9

*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2
69325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3
62880*b**9) + 18480*a**5*b**4*d**3*n**2*x**4*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
+ 10080*a**5*b**4*d**3*n*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*
b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b*
*9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 72*a**4*b**5*
c**2*d*n**5*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 2160*a**4*b**5*c**2*d*n**4*x*
(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6
3273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) - 24120*a**4*b**5*c**2*d*n**3*x*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) - 118800*a**4*b**5*c**2*d*n**2*x*(a + b*x)**n/(b**9*n**9 +
 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b
9*n4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b
9) - 217728*a4*b**5*c**2*d*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7236
80*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 360*a**4
*b**5*c*d**2*n**5*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n*
*7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3
 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 7200*a**4*b**5*c*d**
2*n**4*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
9*n6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 45000*a**4*b**5*c*d**2*n**3*x**
3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
+ 1026576*b**9*n + 362880*b**9) - 90000*a**4*b**5*c*d**2*n**2*x**3*(a + b*x
)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**
9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*
b**9*n + 362880*b**9) - 51840*a**4*b**5*c*d**2*n*x**3*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) - 336*a**4*b**5*d**3*n**5*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 336
0*a**4*b**5*d**3*n**4*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 11760*a**4*b**5*
d**3*n**3*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 16800*a**4*b**5*d**3*n**2*x*

```
*5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
 + 1026576*b**9*n + 362880*b**9) - 8064*a**4*b**5*d**3*n*x**5*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) + 2*a**3*b**6*c**3*n**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*
n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4
+ 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 78
*a**3*b**6*c**3*n**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 1250*a**3*b**6*c**3*n*
*4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
 + 1026576*b**9*n + 362880*b**9) + 10530*a**3*b**6*c**3*n**3*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
 + 362880*b**9) + 49148*a**3*b**6*c**3*n**2*(a + b*x)**n/(b**9*n**9 + 45*b*
*9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n*
*4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
 120432*a**3*b**6*c**3*n*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*
n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n*
*3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 120960*a**3*b**6*c
**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**
2 + 1026576*b**9*n + 362880*b**9) + 36*a**3*b**6*c**2*d*n**6*x**2*(a + b*x)
**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9
*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b
**9*n + 362880*b**9) + 1116*a**3*b**6*c**2*d*n**5*x**2*(a + b*x)**n/(b**9*n
**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 13140*a**3*b**6*c**2*d*n**4*x**2*(a + b*x)**n/(b**9*n**9 + 45*b
**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
+ 71460*a**3*b**6*c**2*d*n**3*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7236
80*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 168264*a
**3*b**6*c**2*d*n**2*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9
*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n
**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 108864*a**3*b**6*
c**2*d*n*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 90*a**3*b**6*c*d**2*n**6*x**4
*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
 1026576*b**9*n + 362880*b**9) + 2070*a**3*b**6*c*d**2*n**5*x**4*(a + b*x)*
```

*n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*
n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b*
*9*n + 362880*b**9) + 16650*a**3*b**6*c*d**2*n**4*x**4*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 56250*a**3*b**6*c*d**2*n**3*x**4*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
+ 80460*a**3*b**6*c*d**2*n**2*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7236
80*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 38880*a*
*3*b**6*c*d**2*n*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**
7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3
+ 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 56*a**3*b**6*d**3*n**
6*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
6 + 63273*b9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) + 840*a**3*b**6*d**3*n**5*x**6*(a + b*
x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b*
*9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576
*b**9*n + 362880*b**9) + 4760*a**3*b**6*d**3*n**4*x**6*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 12600*a**3*b**6*d**3*n**3*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**
9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**
4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
15344*a**3*b**6*d**3*n**2*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
9*n3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 6720*a**3*b*
*6*d**3*n*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 2*a**2*b**7*c**3*n**7*x*(a +
b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) - 78*a**2*b**7*c**3*n**6*x*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) - 1250*a**2*b**7*c**3*n**5*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72
3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 10530*
a**2*b**7*c**3*n**4*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**
7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3
+ 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 49148*a**2*b**7*c**3*
n**3*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
6 + 63273*b9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) - 120432*a**2*b**7*c**3*n**2*x*(a + b*

x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b*
*9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576
*b**9*n + 362880*b**9) - 120960*a**2*b**7*c**3*n*x*(a + b*x)**n/(b**9*n**9
+ 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) - 12*a**2*b**7*c**2*d*n**7*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 396*a
2*b7*c**2*d*n**6*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9
*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n
3 + 1172700*b9*n**2 + 1026576*b**9*n + 362880*b**9) - 5124*a**2*b**7*c*
*2*d*n**5*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 32580*a**2*b**7*c**2*d*n**4*
x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) - 103728*a**2*b**7*c**2*d*n**3*x**3*(a +
b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) - 148464*a**2*b**7*c**2*d*n**2*x**3*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) - 72576*a**2*b**7*c**2*d*n*x**3*(a + b*x)**n/(b**9*n**9 +
45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b*
*9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b*
*9) - 18*a**2*b**7*c*d**2*n**7*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723
680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 486*a**
2*b**7*c*d**2*n**6*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 4986*a**2*b**7*c*d*
*2*n**5*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*
b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700
*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 24570*a**2*b**7*c*d**2*n**4*x*
5(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
+ 1026576*b**9*n + 362880*b**9) - 61092*a**2*b**7*c*d**2*n**3*x**5*(a + b*
x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b*
*9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576
*b**9*n + 362880*b**9) - 72144*a**2*b**7*c*d**2*n**2*x**5*(a + b*x)**n/(b**
9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) - 31104*a**2*b**7*c*d**2*n*x**5*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)

- 8*a**2*b**7*d**3*n**7*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b
9*n7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**
9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 168*a**2*b**7*
d**3*n**6*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 1400*a**2*b**7*d**3*n**5*x**
7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
+ 1026576*b**9*n + 362880*b**9) - 5880*a**2*b**7*d**3*n**4*x**7*(a + b*x)**
n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n
5 + 269325*b9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**
9*n + 362880*b**9) - 12992*a**2*b**7*d**3*n**3*x**7*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) - 14112*a**2*b**7*d**3*n**2*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 576
0*a**2*b**7*d**3*n*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + a*b**8*c**3*n**8*x*
2(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
 + 1026576*b**9*n + 362880*b**9) + 40*a*b**8*c**3*n**7*x**2*(a + b*x)**n/(b
9*n9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
+ 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
+ 362880*b**9) + 664*a*b**8*c**3*n**6*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**
9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**
4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
5890*a*b**8*c**3*n**5*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 29839*a*b**8*c**
3*n**4*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
9*n6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 84790*a*b**8*c**3*n**3*x**2*(a
+ b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6327
3*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102
6576*b**9*n + 362880*b**9) + 120696*a*b**8*c**3*n**2*x**2*(a + b*x)**n/(b**
9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) + 60480*a*b**8*c**3*n*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 3*a
*b**8*c**2*d*n**8*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n*
*7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3
 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 108*a*b**8*c**2*d*n*

*7*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*
n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9
*n**2 + 1026576*b**9*n + 362880*b**9) + 1578*a*b**8*c**2*d*n**6*x**4*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
9*n5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) + 11988*a*b**8*c**2*d*n**5*x**4*(a + b*x)**n/(b**9*
n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26
9325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36
2880*b**9) + 50367*a*b**8*c**2*d*n**4*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**
9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**
4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
114912*a*b**8*c**2*d*n**3*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
9*n3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 129492*a*b**
8*c**2*d*n**2*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +
9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1
172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 54432*a*b**8*c**2*d*n*x*
4(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
+ 1026576*b**9*n + 362880*b**9) + 3*a*b**8*c*d**2*n**8*x**6*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
+ 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
+ 362880*b**9) + 96*a*b**8*c*d**2*n**7*x**6*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
+ 1236*a*b**8*c*d**2*n**6*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
9*n3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 8250*a*b**8*
c*d**2*n**5*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9
450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117
2700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 30657*a*b**8*c*d**2*n**4*x
6*(a + b*x)n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**
2 + 1026576*b**9*n + 362880*b**9) + 62934*a*b**8*c*d**2*n**3*x**6*(a + b*x)
n/(b9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9
*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b
9*n + 362880*b9) + 65304*a*b**8*c*d**2*n**2*x**6*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) + 25920*a*b**8*c*d**2*n*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72
3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + a*b**8
*d**3*n**8*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 94
50*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172
700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 28*a*b**8*d**3*n**7*x**8*(a

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 + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) + 322*a*b**8*d**3*n**6*x**8*(a + b*x)**n/(b**9*
n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26
9325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36
2880*b**9) + 1960*a*b**8*d**3*n**5*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 676
9*a*b**8*d**3*n**4*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 13132*a*b**8*d**3*n
**3*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9
*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**
9*n**2 + 1026576*b**9*n + 362880*b**9) + 13068*a*b**8*d**3*n**2*x**8*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) + 5040*a*b**8*d**3*n*x**8*(a + b*x)**n/(b**9*n**9 +
 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b
**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b
**9) + b**9*c**3*n**8*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 42*b**9*c**3*n**
7*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) + 744*b**9*c**3*n**6*x**3*(a + b*x)**n
/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n*
*5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9
*n + 362880*b**9) + 7218*b**9*c**3*n**5*x**3*(a + b*x)**n/(b**9*n**9 + 45*b
**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
 + 41619*b**9*c**3*n**4*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b*
*9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9
*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 144468*b**9*c**
3*n**3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 290276*b**9*c**3*n**2*x**3*(a +
 b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) + 301872*b**9*c**3*n*x**3*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) + 120960*b**9*c**3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 3*b**9*c**2*
d*n**8*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
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**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 120*b**9*c**2*d*n**7*x**5*(a +
b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*
b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10265
76*b**9*n + 362880*b**9) + 2010*b**9*c**2*d*n**6*x**5*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) + 18300*b**9*c**2*d*n**5*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n*
*8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 9831
9*b**9*c**2*d*n**4*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 316380*b**9*c**2*d*
n**3*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**
9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b*
*9*n**2 + 1026576*b**9*n + 362880*b**9) + 589140*b**9*c**2*d*n**2*x**5*(a +
 b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) + 572400*b**9*c**2*d*n*x**5*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) + 217728*b**9*c**2*d*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7236
80*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 3*b**9*c
*d**2*n**8*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 94
50*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172
700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 114*b**9*c*d**2*n**7*x**7*(
a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63
273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1
026576*b**9*n + 362880*b**9) + 1812*b**9*c*d**2*n**6*x**7*(a + b*x)**n/(b**
9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) + 15666*b**9*c*d**2*n**5*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**
9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**
4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
80157*b**9*c*d**2*n**4*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b*
*9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9
*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 246876*b**9*c*d
**2*n**3*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 442908*b**9*c*d**2*n**2*x**7*
(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6
3273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) + 417744*b**9*c*d**2*n*x**7*(a + b*x)**n/(b**
9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
```

```
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) + 155520*b**9*c*d**2*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n*
*8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + b**9
*d**3*n**8*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 94
50*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172
700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 36*b**9*d**3*n**7*x**9*(a +
 b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) + 546*b**9*d**3*n**6*x**9*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) + 4536*b**9*d**3*n**5*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72368
0*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 22449*b**
9*d**3*n**4*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9
450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117
2700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 67284*b**9*d**3*n**3*x**9*
(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6
3273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) + 118124*b**9*d**3*n**2*x**9*(a + b*x)**n/(b*
*9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
 362880*b**9) + 109584*b**9*d**3*n*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 403
20*b**9*d**3*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +
9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11
72700*b**9*n**2 + 1026576*b**9*n + 362880*b**9), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(343) = 686$.

Time = 0.24 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.32

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$$
$$=\frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^nc^3}{(n^3+6n^2+11n+6)b^3}$$
$$+\frac{3((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$
$$+\frac{3((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n}{}$$
$$+\frac{((n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^9x^9+(n^8+}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^
6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b
^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 -
720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*
n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^
6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x
^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764
*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x
^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*
n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)
*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n
^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3713 vs. $2(343) = 686$.

Time = 0.30 (sec) , antiderivative size = 3713, normalized size of antiderivative = 10.83

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b^8
*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d^2*
n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3*n^7
*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^5*x^9
 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x^6 + 1
812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*x^7 + 1
960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x^9 + 3*(
b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5 - 18*(b*
x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6*x^6 + 56*
(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^5*x^7 - 140
0*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*n^4*x^8 + 67
284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3 + 108*(b*x +
a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5 - 486*(b*x +
a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^5*x^6 + 840*(b*
x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*n^4*x^7 - 5880*(
b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^3*n^3*x^8 + 1181
24*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^8*x^2 + 42*(b*x +
a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*x^3 + 1578*(b*x + a
)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d^2*n^6*x^4 + 18300*(b*x
 + a)^n*b^9*c^2*d*n^5*x^5 - 4986*(b*x + a)^n*a^2*b^7*c*d^2*n^5*x^5 - 336*(b
*x + a)^n*a^4*b^5*d^3*n^5*x^5 + 30657*(b*x + a)^n*a*b^8*c*d^2*n^4*x^6 + 476
0*(b*x + a)^n*a^3*b^6*d^3*n^4*x^6 + 246876*(b*x + a)^n*b^9*c*d^2*n^3*x^7 -
12992*(b*x + a)^n*a^2*b^7*d^3*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^3*n^2*x^8
 + 109584*(b*x + a)^n*b^9*d^3*n*x^9 + 40*(b*x + a)^n*a*b^8*c^3*n^7*x^2 + 74
4*(b*x + a)^n*b^9*c^3*n^6*x^3 - 396*(b*x + a)^n*a^2*b^7*c^2*d*n^6*x^3 + 119
88*(b*x + a)^n*a*b^8*c^2*d*n^5*x^4 + 2070*(b*x + a)^n*a^3*b^6*c*d^2*n^5*x^4
 + 98319*(b*x + a)^n*b^9*c^2*d*n^4*x^5 - 24570*(b*x + a)^n*a^2*b^7*c*d^2*n^
4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^3*n^4*x^5 + 62934*(b*x + a)^n*a*b^8*c*d^
2*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^3*n^3*x^6 + 442908*(b*x + a)^n*b^9*
c*d^2*n^2*x^7 - 14112*(b*x + a)^n*a^2*b^7*d^3*n^2*x^7 + 5040*(b*x + a)^n*a*
b^8*d^3*n*x^8 + 40320*(b*x + a)^n*b^9*d^3*x^9 - 2*(b*x + a)^n*a^2*b^7*c^3*n
^7*x + 664*(b*x + a)^n*a*b^8*c^3*n^6*x^2 + 36*(b*x + a)^n*a^3*b^6*c^2*d*n^6
*x^2 + 7218*(b*x + a)^n*b^9*c^3*n^5*x^3 - 5124*(b*x + a)^n*a^2*b^7*c^2*d*n^
5*x^3 - 360*(b*x + a)^n*a^4*b^5*c*d^2*n^5*x^3 + 50367*(b*x + a)^n*a*b^8*c^2
```

*d*n^4*x^4 + 16650*(b*x + a)^n*a^3*b^6*c*d^2*n^4*x^4 + 1680*(b*x + a)^n*a^5
*b^4*d^3*n^4*x^4 + 316380*(b*x + a)^n*b^9*c^2*d*n^3*x^5 - 61092*(b*x + a)^n
*a^2*b^7*c*d^2*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^3*n^3*x^5 + 65304*(b*x
 + a)^n*a*b^8*c*d^2*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^3*n^2*x^6 + 41774
4*(b*x + a)^n*b^9*c*d^2*n*x^7 - 5760*(b*x + a)^n*a^2*b^7*d^3*n*x^7 - 78*(b*
x + a)^n*a^2*b^7*c^3*n^6*x + 5890*(b*x + a)^n*a*b^8*c^3*n^5*x^2 + 1116*(b*x
 + a)^n*a^3*b^6*c^2*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^3*n^4*x^3 - 32580*(
b*x + a)^n*a^2*b^7*c^2*d*n^4*x^3 - 7200*(b*x + a)^n*a^4*b^5*c*d^2*n^4*x^3 +
 114912*(b*x + a)^n*a*b^8*c^2*d*n^3*x^4 + 56250*(b*x + a)^n*a^3*b^6*c*d^2*n
^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n^3*x^4 + 589140*(b*x + a)^n*b^9*c^2
*d*n^2*x^5 - 72144*(b*x + a)^n*a^2*b^7*c*d^2*n^2*x^5 - 16800*(b*x + a)^n*a^
4*b^5*d^3*n^2*x^5 + 25920*(b*x + a)^n*a*b^8*c*d^2*n*x^6 + 6720*(b*x + a)^n*
a^3*b^6*d^3*n*x^6 + 155520*(b*x + a)^n*b^9*c*d^2*x^7 + 2*(b*x + a)^n*a^3*b^
6*c^3*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^3*n^5*x - 72*(b*x + a)^n*a^4*b^5*c^2
*d*n^5*x + 29839*(b*x + a)^n*a*b^8*c^3*n^4*x^2 + 13140*(b*x + a)^n*a^3*b^6*
c^2*d*n^4*x^2 + 1080*(b*x + a)^n*a^5*b^4*c*d^2*n^4*x^2 + 144468*(b*x + a)^n
*b^9*c^3*n^3*x^3 - 103728*(b*x + a)^n*a^2*b^7*c^2*d*n^3*x^3 - 45000*(b*x +
a)^n*a^4*b^5*c*d^2*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^3*n^3*x^3 + 129492*
(b*x + a)^n*a*b^8*c^2*d*n^2*x^4 + 80460*(b*x + a)^n*a^3*b^6*c*d^2*n^2*x^4 +
 18480*(b*x + a)^n*a^5*b^4*d^3*n^2*x^4 + 572400*(b*x + a)^n*b^9*c^2*d*n*x^5
 - 31104*(b*x + a)^n*a^2*b^7*c*d^2*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^3*n*x
^5 + 78*(b*x + a)^n*a^3*b^6*c^3*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^3*n^4*x -
 2160*(b*x + a)^n*a^4*b^5*c^2*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^3*n^3*x^2
 + 71460*(b*x + a)^n*a^3*b^6*c^2*d*n^3*x^2 + 19440*(b*x + a)^n*a^5*b^4*c*d^
2*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^3*n^2*x^3 - 148464*(b*x + a)^n*a^2*b^7
*c^2*d*n^2*x^3 - 90000*(b*x + a)^n*a^4*b^5*c*d^2*n^2*x^3 - 20160*(b*x + a)^
n*a^6*b^3*d^3*n^2*x^3 + 54432*(b*x + a)^n*a*b^8*c^2*d*n*x^4 + 38880*(b*x +
a)^n*a^3*b^6*c*d^2*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n*x^4 + 217728*(b*
x + a)^n*b^9*c^2*d*x^5 + 1250*(b*x + a)^n*a^3*b^6*c^3*n^4 + 72*(b*x + a)^n*
a^5*b^4*c^2*d*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^3*n^3*x - 24120*(b*x + a)^n
*a^4*b^5*c^2*d*n^3*x - 2160*(b*x + a)^n*a^6*b^3*c*d^2*n^3*x + 120696*(b*x +
 a)^n*a*b^8*c^3*n^2*x^2 + 168264*(b*x + a)^n*a^3*b^6*c^2*d*n^2*x^2 + 96120*
(b*x + a)^n*a^5*b^4*c*d^2*n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n^2*x^2 +
 301872*(b*x + a)^n*b^9*c^3*n*x^3 - 72576*(b*x + a)^n*a^2*b^7*c^2*d*n*x^3 -
 51840*(b*x + a)^n*a^4*b^5*c*d^2*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d^3*n*x^
3 + 10530*(b*x + a)^n*a^3*b^6*c^3*n^3 + 2160*(b*x + a)^n*a^5*b^4*c^2*d*n^3
- 120432*(b*x + a)^n*a^2*b^7*c^3*n^2*x - 118800*(b*x + a)^n*a^4*b^5*c^2*d*n
^2*x - 36720*(b*x + a)^n*a^6*b^3*c*d^2*n^2*x + 60480*(b*x + a)^n*a*b^8*c^3*
n*x^2 + 108864*(b*x + a)^n*a^3*b^6*c^2*d*n*x^2 + 77760*(b*x + a)^n*a^5*b^4*
c*d^2*n*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n*x^2 + 120960*(b*x + a)^n*b^9*
c^3*x^3 + 49148*(b*x + a)^n*a^3*b^6*c^3*n^2 + 24120*(b*x + a)^n*a^5*b^4*c^2
*d*n^2 + 2160*(b*x + a)^n*a^7*b^2*c*d^2*n^2 - 120960*(b*x + a)^n*a^2*b^7*c^
3*n*x - 217728*(b*x + a)^n*a^4*b^5*c^2*d*n*x - 155520*(b*x + a)^n*a^6*b^3*c
*d^2*n*x - 40320*(b*x + a)^n*a^8*b*d^3*n*x + 120432*(b*x + a)^n*a^3*b^6*c^3
n + 118800(b*x + a)^n*a^5*b^4*c^2*d*n + 36720*(b*x + a)^n*a^7*b^2*c*d^2*n

```
 + 120960*(b*x + a)^n*a^3*b^6*c^3 + 217728*(b*x + a)^n*a^5*b^4*c^2*d + 1555
20*(b*x + a)^n*a^7*b^2*c*d^2 + 40320*(b*x + a)^n*a^9*d^3)/(b^9*n^9 + 45*b^9
*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680
*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 1796, normalized size of antiderivative = 5.24

$$\int x^2(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] int(x^2*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574
*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^
3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n
+ 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2
+ 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^9*(1026576*n + 1172700
*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
n^9 + 362880)) - (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(6720*a^6*d^3*n - 60480*b
^6*c^3 - 60216*b^6*c^3*n - 24574*b^6*c^3*n^2 - 5265*b^6*c^3*n^3 - 625*b^6*c
^3*n^4 - 39*b^6*c^3*n^5 - b^6*c^3*n^6 + 36288*a^2*b^4*c^2*d*n + 25920*a^4*b
^2*c*d^2*n + 19800*a^2*b^4*c^2*d*n^2 + 6120*a^4*b^2*c*d^2*n^2 + 4020*a^2*b^
4*c^2*d*n^3 + 360*a^4*b^2*c*d^2*n^3 + 360*a^2*b^4*c^2*d*n^4 + 12*a^2*b^4*c^
2*d*n^5))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n
^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*d*x^5*(a + b*x)^n*(5
0*n + 35*n^2 + 10*n^3 + n^4 + 24)*(3024*b^4*c^2 - 112*a^4*d^2*n + 1650*b^4*
c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 - 432*a^2*b^2*c*d*n
- 102*a^2*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^
6*c^3*n + 24574*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c
^3*n^5 + b^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a
^2*b^4*c^2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4
*b^2*c*d^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^8*(1026
576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*
n^7 + 45*n^8 + n^9 + 362880)) + (d^2*x^7*(a + b*x)^n*(216*b^2*c + 3*b^2*c*n
^2 - 8*a^2*d*n + 51*b^2*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^
5 + n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 6
3273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*n*x^2*(n + 1)*
(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574*b^6*c^
```

3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^7*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*d^2*n*x^6*(a + b*x)^n*(56*a^2*d + 216*b^2*c + 3*b^2*c*n^2 + 51*b^2*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(560*a^4*d^2 + 3024*b^4*c^2 + 1650*b^4*c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 + 2160*a^2*b^2*c*d + 510*a^2*b^2*c*d*n + 30*a^2*b^2*c*d*n^2))/(b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))

3.360 $\int x(a+bx)^n (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 282

$$\int x(a+bx)^n (c+dx^2)^3 \, dx = -\frac{a(b^2c+a^2d)^3 (a+bx)^{1+n}}{b^8(1+n)} + \frac{(b^2c+a^2d)^2 (b^2c+7a^2d) (a+bx)^{2+n}}{b^8(2+n)} - \frac{3ad(b^2c+a^2d) (3b^2c+7a^2d) (a+bx)^{3+n}}{b^8(3+n)} + \frac{d(3b^4c^2+30a^2b^2cd+35a^4d^2) (a+bx)^{4+n}}{b^8(4+n)} - \frac{5ad^2(3b^2c+7a^2d) (a+bx)^{5+n}}{b^8(5+n)} + \frac{3d^2(b^2c+7a^2d) (a+bx)^{6+n}}{b^8(6+n)} - \frac{7ad^3(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^3(a+bx)^{8+n}}{b^8(8+n)}$$

```
[Out] -a*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^8/(1+n)+(a^2*d+b^2*c)^2*(7*a^2*d+b^2*c)*
(b*x+a)^(2+n)/b^8/(2+n)-3*a*d*(a^2*d+b^2*c)*(7*a^2*d+3*b^2*c)*(b*x+a)^(3+n)
/b^8/(3+n)+d*(35*a^4*d^2+30*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(4+n)/b^8/(4+n)-
5*a*d^2*(7*a^2*d+3*b^2*c)*(b*x+a)^(5+n)/b^8/(5+n)+3*d^2*(7*a^2*d+b^2*c)*(b*
x+a)^(6+n)/b^8/(6+n)-7*a*d^3*(b*x+a)^(7+n)/b^8/(7+n)+d^3*(b*x+a)^(8+n)/b^8/
(8+n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {786}

$$\int x(a+bx)^n\left(c+dx^2\right)^3\,dx = -\frac{5ad^2(7a^2d+3b^2c)\,(a+bx)^{n+5}}{b^8(n+5)} + \frac{3d^2(7a^2d+b^2c)\,(a+bx)^{n+6}}{b^8(n+6)} - \frac{a(a^2d+b^2c)^3\,(a+bx)^{n+1}}{b^8(n+1)} + \frac{(a^2d+b^2c)^2\,(7a^2d+b^2c)\,(a+bx)^{n+2}}{b^8(n+2)} - \frac{3ad(a^2d+b^2c)\,(7a^2d+3b^2c)\,(a+bx)^{n+3}}{b^8(n+3)} + \frac{d(35a^4d^2+30a^2b^2cd+3b^4c^2)\,(a+bx)^{n+4}}{b^8(n+4)} - \frac{7ad^3(a+bx)^{n+7}}{b^8(n+7)} + \frac{d^3(a+bx)^{n+8}}{b^8(n+8)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{a(b^2c+a^2d)^3\,(a+bx)^n}{b^7} + \frac{(b^2c+a^2d)^2\,(b^2c+7a^2d)\,(a+bx)^{1+n}}{b^7} + \frac{3ad(-3b^2c-7a^2d)\,(b^2c+a^2d)\,(a+bx)^{2+n}}{b^7} + \frac{d(3b^4c^2+30a^2b^2cd+35a^4d^2)\,(a+bx)^{3+n}}{b^7} - \frac{5ad^2(3b^2c+7a^2d)\,(a+bx)^{4+n}}{b^7} + \frac{3d^2(b^2c+7a^2d)\,(a+bx)^{5+n}}{b^7} - \frac{7ad^3(a+bx)^{6+n}}{b^7} + \frac{d^3(a+bx)^{7+n}}{b^7}\right) dx$$

$$= -\frac{a(b^2c+a^2d)^3 (a+bx)^{1+n}}{b^8(1+n)} + \frac{(b^2c+a^2d)^2 (b^2c+7a^2d) (a+bx)^{2+n}}{b^8(2+n)} - \frac{3ad(b^2c+a^2d) (3b^2c+7a^2d) (a+bx)^{3+n}}{b^8(3+n)} + \frac{d(3b^4c^2+30a^2b^2cd+35a^4d^2) (a+bx)^{4+n}}{b^8(4+n)} - \frac{5ad^2(3b^2c+7a^2d) (a+bx)^{5+n}}{b^8(5+n)} + \frac{3d^2(b^2c+7a^2d) (a+bx)^{6+n}}{b^8(6+n)} - \frac{7ad^3(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^3(a+bx)^{8+n}}{b^8(8+n)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 709 vs. $2(282) = 564$.

Time = 0.87 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.51

$$\int x(a+bx)^n (c+dx^2)^3 \, dx$$
$$= \frac{(a+bx)^{1+n} \left(b^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)(a+bx) (c+dx^2)^3 - a(8+n) \left(b^6(1+n\right.\right.}{}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7
+ n)*(a + b*x)*(c + d*x^2)^3 - a*(8 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*(5 + n)*(6 + n)*(c + d*x^2)^3 + 6*(b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n)*(
2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d -
 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 +
n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 +
 n)*x^2))) - 6*a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(
c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2
*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))) + 6*(1 + n)*
(a + b*x)*((b^2*c + a^2*d)*(7 + n)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(c
+ d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(
3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*
a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) - a*d*(2 + n)*(
a + b*x)*(b^4*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^
2*d)*(6 + n)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 +
 n)*x^2)) - 4*a*d*(3 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(4 + n)*x + b^2*(5 +
 n)*(c*(6 + n) + d*(4 + n)*x^2))))))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(
5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. 2(282) = 564.

Time = 0.48 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.30

method	result	size
norman	Expression too large to display	1214
gosper	Expression too large to display	1639
risch	Expression too large to display	1955
parallelrisch	Expression too large to display	2917

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^2
*b^4*c^2*d*n^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360*a
^4*b^2*c*d^2*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c*d^
2*n+19188*a^2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*d^2+
30240*a^2*b^4*c^2*d+20160*b^6*c^3)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+6
7284*n^3+118124*n^2+109584*n+40320)*x*exp(n*ln(b*x+a))+n/b*d^3*a/(n^2+15*n+
56)*x^7*exp(n*ln(b*x+a))-a^2*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^2*b^4*c^2*d*n
^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360*a^4*b^2*c*d^2
*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c*d^2*n+19188*a^
2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*d^2+30240*a^2*b^
4*c^2*d+20160*b^6*c^3)/b^8/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3
+118124*n^2+109584*n+40320)*exp(n*ln(b*x+a))-(-b^6*c^3*n^6+9*a^2*b^4*c^2*d*
n^5-33*b^6*c^3*n^5+234*a^2*b^4*c^2*d*n^4-445*b^6*c^3*n^4+180*a^4*b^2*c*d^2*
n^3+2259*a^2*b^4*c^2*d*n^3-3135*b^6*c^3*n^3+2700*a^4*b^2*c*d^2*n^2+9594*a^2
*b^4*c^2*d*n^2-12154*b^6*c^3*n^2+2520*a^6*d^3*n+10080*a^4*b^2*c*d^2*n+15120
*a^2*b^4*c^2*d*n-24552*b^6*c^3*n-20160*b^6*c^3)/b^6/(n^7+35*n^6+511*n^5+402
5*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp(n*ln(b*x+a))-(-3*b^2*c*n^2
+7*a^2*d*n-45*b^2*c*n-168*b^2*c)/b^2*d^2/(n^3+21*n^2+146*n+336)*x^6*exp(n*l
n(b*x+a))-3*(-b^4*c^2*n^4+5*a^2*b^2*c*d*n^3-26*b^4*c^2*n^3+75*a^2*b^2*c*d*n
^2-251*b^4*c^2*n^2+70*a^4*d^2*n+280*a^2*b^2*c*d*n-1066*b^4*c^2*n-1680*b^4*c
^2)*d/b^4/(n^5+30*n^4+355*n^3+2070*n^2+5944*n+6720)*x^4*exp(n*ln(b*x+a))+3*
(b^2*c*n^2+15*b^2*c*n+14*a^2*d+56*b^2*c)*n*d^2/b^3*a/(n^4+26*n^3+251*n^2+10
66*n+1680)*x^5*exp(n*ln(b*x+a))+3*(b^4*c^2*n^4+26*b^4*c^2*n^3+20*a^2*b^2*c*
d*n^2+251*b^4*c^2*n^2+300*a^2*b^2*c*d*n+1066*b^4*c^2*n+280*a^4*d^2+1120*a^2
*b^2*c*d+1680*b^4*c^2)*a/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+245
52*n+20160)*x^3*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(282) = 564$.

Time = 0.31 (sec) , antiderivative size = 1675, normalized size of antiderivative = 5.94

$$\int x(a+bx)^n (c+dx^2)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4*
c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^6
+ 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*n^2
 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*n^6
+ 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*b^7*d
^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 + (90*b^
8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^5 + 5*(
1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2*b^6*d^3
)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c*d^2 - 35
*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 + (241*a*b^7
*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d^3)*n^4 + 2
*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2 + 35*a^3*b^
5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a^2*b^6*c^3 +
18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d + (32*b^8*c^2*d
- 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)*n^5 + (2864*b^
8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*b^8*c^2*d - 2535
*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d - 1765*a^2*b^6*c
*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^2*b^6*c*d^2 - 35*a
^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c^2*d)*n^3 - 3*(a*b^
7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d + 20*a^3*b^5*c*d^2)*n^5
 + (1871*a*b^7*c^2*d + 360*a^3*b^5*c*d^2)*n^4 + 20*(269*a*b^7*c^2*d + 103*a
^3*b^5*c*d^2 + 14*a^5*b^3*d^3)*n^3 + 4*(1793*a*b^7*c^2*d + 990*a^3*b^5*c*d^
2 + 210*a^5*b^3*d^3)*n^2 + 560*(6*a*b^7*c^2*d + 4*a^3*b^5*c*d^2 + a^5*b^3*d
^3)*n)*x^3 + 2*(6077*a^2*b^6*c^3 + 2259*a^4*b^4*c^2*d + 180*a^6*b^2*c*d^2)*
n^2 - (b^8*c^3*n^7 + 20160*b^8*c^3 + (34*b^8*c^3 - 9*a^2*b^6*c^2*d)*n^6 + (
478*b^8*c^3 - 243*a^2*b^6*c^2*d)*n^5 + (3580*b^8*c^3 - 2493*a^2*b^6*c^2*d -
 180*a^4*b^4*c*d^2)*n^4 + (15289*b^8*c^3 - 11853*a^2*b^6*c^2*d - 2880*a^4*b
^4*c*d^2)*n^3 + 2*(18353*b^8*c^3 - 12357*a^2*b^6*c^2*d - 6390*a^4*b^4*c*d^2
 - 1260*a^6*b^2*d^3)*n^2 + 72*(621*b^8*c^3 - 210*a^2*b^6*c^2*d - 140*a^4*b^
4*c*d^2 - 35*a^6*b^2*d^3)*n)*x^2 + 36*(682*a^2*b^6*c^3 + 533*a^4*b^4*c^2*d
+ 150*a^6*b^2*c*d^2)*n - (a*b^7*c^3*n^7 + 33*a*b^7*c^3*n^6 + (445*a*b^7*c^3
 + 18*a^3*b^5*c^2*d)*n^5 + 3*(1045*a*b^7*c^3 + 156*a^3*b^5*c^2*d)*n^4 + 2*(
6077*a*b^7*c^3 + 2259*a^3*b^5*c^2*d + 180*a^5*b^3*c*d^2)*n^3 + 36*(682*a*b^
7*c^3 + 533*a^3*b^5*c^2*d + 150*a^5*b^3*c*d^2)*n^2 + 5040*(4*a*b^7*c^3 + 6*
a^3*b^5*c^2*d + 4*a^5*b^3*c*d^2 + a^7*b*d^3)*n)*x)*(b*x + a)^n/(b^8*n^8 + 3
```

```
6*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 11
8124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24687 vs. $2(265) = 530$.

Time = 6.42 (sec) , antiderivative size = 24687, normalized size of antiderivative = 87.54

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**4/4 + c*d**2*x**6/2 + d**3*x**8/
8), Eq(b, 0)), (420*a**7*d**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*
x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 +
8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**3
/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**1
1*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 +
 420*b**15*x**7) + 2940*a**6*b*d**3*x*log(a/b + x)/(420*a**7*b**8 + 2940*a*
*6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12
*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a
**6*b*d**3*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 147
00*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*
b**14*x**6 + 420*b**15*x**7) - 30*a**5*b**2*c*d**2/(420*a**7*b**8 + 2940*a*
*6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12
*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a
**5*b**2*d**3*x**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 20139*a**5*b**2*d**3*x**2
/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**1
1*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 +
 420*b**15*x**7) - 210*a**4*b**3*c*d**2*x/(420*a**7*b**8 + 2940*a**6*b**9*x
 + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8
820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 14700*a**4*b**3
*d**3*x**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10
*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**
5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 30625*a**4*b**3*d**3*x**3/(420*a*
*7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 +
 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**
15*x**7) - 9*a**3*b**4*c**2*d/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5
*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**
13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 630*a**3*b**4*c*d**2*x**2/(
420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*
x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 4
```

20*b**15*x**7) + 14700*a**3*b**4*d**3*x**4*log(a/b + x)/(420*a**7*b**8 + 29
40*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*
b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 2
6950*a**3*b**4*d**3*x**4/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**1
0*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x*
*5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 63*a**2*b**5*c**2*d*x/(420*a**7*
b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14
700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*
x**7) - 1050*a**2*b**5*c*d**2*x**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820
*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a**2*b**5*d**3*x*
*5*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 +
14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940
*a*b**14*x**6 + 420*b**15*x**7) + 13230*a**2*b**5*d**3*x**5/(420*a**7*b**8
+ 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a
3*b12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7)
 - 10*a*b**6*c**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2
+ 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 29
40*a*b**14*x**6 + 420*b**15*x**7) - 189*a*b**6*c**2*d*x**2/(420*a**7*b**8 +
 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a*
*3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7)
- 1050*a*b**6*c*d**2*x**4/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**
10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x
5 + 2940*a*b14*x**6 + 420*b**15*x**7) + 2940*a*b**6*d**3*x**6*log(a/b +
 x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
11*x3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**
6 + 420*b**15*x**7) + 2940*a*b**6*d**3*x**6/(420*a**7*b**8 + 2940*a**6*b**9
*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 +
 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 70*b**7*c**3*
x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**
11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6
+ 420*b**15*x**7) - 315*b**7*c**2*d*x**3/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 630*b**7*c*d**2*
x**5/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*
b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x*
*6 + 420*b**15*x**7) + 420*b**7*d**3*x**7*log(a/b + x)/(420*a**7*b**8 + 294
0*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b
12*x4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7), Eq(
n, -8)), (-420*a**7*d**3*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900
*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13
*x**5 + 60*b**14*x**6) - 1029*a**7*d**3/(60*a**6*b**8 + 360*a**5*b**9*x + 9
00*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**
13*x**5 + 60*b**14*x**6) - 2520*a**6*b*d**3*x*log(a/b + x)/(60*a**6*b**8 +
360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**

12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 5754*a**6*b*d**3*x/(60*a**6*b
8 + 360*a5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a*
*2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 30*a**5*b**2*c*d**2/(60
*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 +
 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 6300*a**5*b**2*d
3*x2*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2
 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14
*x**6) - 13125*a**5*b**2*d**3*x**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a*
*4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x*
*5 + 60*b**14*x**6) - 180*a**4*b**3*c*d**2*x/(60*a**6*b**8 + 360*a**5*b**9*
x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*
a*b**13*x**5 + 60*b**14*x**6) - 8400*a**4*b**3*d**3*x**3*log(a/b + x)/(60*a
6*b8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 9
00*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 15400*a**4*b**3*d*
*3*x**3/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b
11*x3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 3*a**
3*b**4*c**2*d/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*
a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) -
 450*a**3*b**4*c*d**2*x**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10
*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*
b**14*x**6) - 6300*a**3*b**4*d**3*x**4*log(a/b + x)/(60*a**6*b**8 + 360*a**
5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4
 + 360*a*b**13*x**5 + 60*b**14*x**6) - 9450*a**3*b**4*d**3*x**4/(60*a**6*b*
*8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**
2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 18*a**2*b**5*c**2*d*x/(6
0*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3
+ 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 600*a**2*b**5*c
*d**2*x**3/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**
3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 25
20*a**2*b**5*d**3*x**5*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a
4*b10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x
5 + 60*b14*x**6) - 2520*a**2*b**5*d**3*x**5/(60*a**6*b**8 + 360*a**5*b*
*9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 3
60*a*b**13*x**5 + 60*b**14*x**6) - 2*a*b**6*c**3/(60*a**6*b**8 + 360*a**5*b
9*x + 900*a4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 +
360*a*b**13*x**5 + 60*b**14*x**6) - 45*a*b**6*c**2*d*x**2/(60*a**6*b**8 + 3
60*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**1
2*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 450*a*b**6*c*d**2*x**4/(60*a**
6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900
*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 420*a*b**6*d**3*x**6
*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*
a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) -
 12*b**7*c**3*x/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 120
0*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6)
 - 60*b**7*c**2*d*x**3/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**

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2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**1
4*x**6) - 180*b**7*c*d**2*x**5/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b
**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 +
 60*b**14*x**6) + 60*b**7*d**3*x**7/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a
**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x
**5 + 60*b**14*x**6), Eq(n, -7)), (420*a**7*d**3*log(a/b + x)/(20*a**5*b**8
 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**1
2*x**4 + 20*b**13*x**5) + 959*a**7*d**3/(20*a**5*b**8 + 100*a**4*b**9*x + 2
00*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5
) + 2100*a**6*b*d**3*x*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a
**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) +
4375*a**6*b*d**3*x/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 +
200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 60*a**5*b**2*c*d*
*2*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200
*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 137*a**5*b**2*c*d**2
/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**
3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 4200*a**5*b**2*d**3*x**2*log(a/b +
x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x
**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 7700*a**5*b**2*d**3*x**2/(20*a**5
*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a
*b**12*x**4 + 20*b**13*x**5) + 300*a**4*b**3*c*d**2*x*log(a/b + x)/(20*a**5
*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a
*b**12*x**4 + 20*b**13*x**5) + 625*a**4*b**3*c*d**2*x/(20*a**5*b**8 + 100*a
**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 +
 20*b**13*x**5) + 4200*a**4*b**3*d**3*x**3*log(a/b + x)/(20*a**5*b**8 + 100
*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4
 + 20*b**13*x**5) + 6300*a**4*b**3*d**3*x**3/(20*a**5*b**8 + 100*a**4*b**9*
x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13
*x**5) - 3*a**3*b**4*c**2*d/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**1
0*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 600*a**3
*b**4*c*d**2*x**2*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b
**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 1100*
a**3*b**4*c*d**2*x**2/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2
 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 2100*a**3*b**4
*d**3*x**4*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x*
*2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 2100*a**3*b*
*4*d**3*x**4/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a*
*2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 15*a**2*b**5*c**2*d*x/(
20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3
+ 100*a*b**12*x**4 + 20*b**13*x**5) + 600*a**2*b**5*c*d**2*x**3*log(a/b + x
)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x*
*3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 900*a**2*b**5*c*d**2*x**3/(20*a**5
*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a
*b**12*x**4 + 20*b**13*x**5) + 420*a**2*b**5*d**3*x**5*log(a/b + x)/(20*a**
5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*
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a*b**12*x**4 + 20*b**13*x**5) - a*b**6*c**3/(20*a**5*b**8 + 100*a**4*b**9*x
 + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*
x**5) - 30*a*b**6*c**2*d*x**2/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b*
*10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 300*a*
b**6*c*d**2*x**4*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b*
*10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 300*a*
b**6*c*d**2*x**4/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 20
0*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 70*a*b**6*d**3*x**6
/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**
3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 5*b**7*c**3*x/(20*a**5*b**8 + 100*a
**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 +
 20*b**13*x**5) - 30*b**7*c**2*d*x**3/(20*a**5*b**8 + 100*a**4*b**9*x + 200
*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5)
+ 60*b**7*c*d**2*x**5*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a*
*3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 1
0*b**7*d**3*x**7/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 20
0*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5), Eq(n, -6)), (-420*a*
*7*d**3*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 +
48*a*b**11*x**3 + 12*b**12*x**4) - 875*a**7*d**3/(12*a**4*b**8 + 48*a**3*b*
*9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 1680*a**6*b*
d**3*x*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 4
8*a*b**11*x**3 + 12*b**12*x**4) - 3080*a**6*b*d**3*x/(12*a**4*b**8 + 48*a**
3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 180*a**5
*b**2*c*d**2*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x*
*2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 375*a**5*b**2*c*d**2/(12*a**4*b**8
+ 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) -
2520*a**5*b**2*d**3*x**2*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a
**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 3780*a**5*b**2*d**3*x**
2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 1
2*b**12*x**4) - 720*a**4*b**3*c*d**2*x*log(a/b + x)/(12*a**4*b**8 + 48*a**3
*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 1320*a**4
*b**3*c*d**2*x/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b
**11*x**3 + 12*b**12*x**4) - 1680*a**4*b**3*d**3*x**3*log(a/b + x)/(12*a**4
*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x*
*4) - 1680*a**4*b**3*d**3*x**3/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**
10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 9*a**3*b**4*c**2*d/(12*a**4*b*
*8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4)
 - 1080*a**3*b**4*c*d**2*x**2*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x +
 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 1620*a**3*b**4*c*d
**2*x**2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x
**3 + 12*b**12*x**4) - 420*a**3*b**4*d**3*x**4*log(a/b + x)/(12*a**4*b**8 +
 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 3
6*a**2*b**5*c**2*d*x/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 +
48*a*b**11*x**3 + 12*b**12*x**4) - 720*a**2*b**5*c*d**2*x**3*log(a/b + x)/(
12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b
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**12*x**4) - 720*a**2*b**5*c*d**2*x**3/(12*a**4*b**8 + 48*a**3*b**9*x + 72*
a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 84*a**2*b**5*d**3*x**5
/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12
*b**12*x**4) - a*b**6*c**3/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x
**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 54*a*b**6*c**2*d*x**2/(12*a**4*b**
8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4)
- 180*a*b**6*c*d**2*x**4*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a
**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 14*a*b**6*d**3*x**6/(12
*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**
12*x**4) - 4*b**7*c**3*x/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**
2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 36*b**7*c**2*d*x**3/(12*a**4*b**8 +
48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 36
*b**7*c*d**2*x**5/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*
a*b**11*x**3 + 12*b**12*x**4) + 4*b**7*d**3*x**7/(12*a**4*b**8 + 48*a**3*b*
*9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4), Eq(n, -5)), (
420*a**7*d**3*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2
 + 12*b**11*x**3) + 770*a**7*d**3/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**
10*x**2 + 12*b**11*x**3) + 1260*a**6*b*d**3*x*log(a/b + x)/(12*a**3*b**8 +
36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 1890*a**6*b*d**3*x/(12*
a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 360*a**5*b*
*2*c*d**2*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 1
2*b**11*x**3) + 660*a**5*b**2*c*d**2/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*
b**10*x**2 + 12*b**11*x**3) + 1260*a**5*b**2*d**3*x**2*log(a/b + x)/(12*a**
3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 1260*a**5*b**2
*d**3*x**2/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3
) + 1080*a**4*b**3*c*d**2*x*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 3
6*a*b**10*x**2 + 12*b**11*x**3) + 1620*a**4*b**3*c*d**2*x/(12*a**3*b**8 + 3
6*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 420*a**4*b**3*d**3*x**3*
log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x*
*3) + 36*a**3*b**4*c**2*d*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*
a*b**10*x**2 + 12*b**11*x**3) + 66*a**3*b**4*c**2*d/(12*a**3*b**8 + 36*a**2
*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 1080*a**3*b**4*c*d**2*x**2*log
(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3)
 + 1080*a**3*b**4*c*d**2*x**2/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x
**2 + 12*b**11*x**3) - 105*a**3*b**4*d**3*x**4/(12*a**3*b**8 + 36*a**2*b**9
*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 108*a**2*b**5*c**2*d*x*log(a/b + x)
/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 162*a*
*2*b**5*c**2*d*x/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**1
1*x**3) + 360*a**2*b**5*c*d**2*x**3*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b*
*9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 21*a**2*b**5*d**3*x**5/(12*a**3*b
**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 2*a*b**6*c**3/(12
*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 108*a*b**6
*c**2*d*x**2*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2
+ 12*b**11*x**3) + 108*a*b**6*c**2*d*x**2/(12*a**3*b**8 + 36*a**2*b**9*x +
36*a*b**10*x**2 + 12*b**11*x**3) - 90*a*b**6*c*d**2*x**4/(12*a**3*b**8 + 36
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*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 7*a*b**6*d**3*x**6/(12*a*
*3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 6*b**7*c**3*x
/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 36*b**
7*c**2*d*x**3*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2
 + 12*b**11*x**3) + 18*b**7*c*d**2*x**5/(12*a**3*b**8 + 36*a**2*b**9*x + 36
*a*b**10*x**2 + 12*b**11*x**3) + 3*b**7*d**3*x**7/(12*a**3*b**8 + 36*a**2*b
**9*x + 36*a*b**10*x**2 + 12*b**11*x**3), Eq(n, -4)), (-420*a**7*d**3*log(a
/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 630*a**7*d**3/(20*a*
*2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 840*a**6*b*d**3*x*log(a/b + x)/(20
*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 840*a**6*b*d**3*x/(20*a**2*b**8
 + 40*a*b**9*x + 20*b**10*x**2) - 600*a**5*b**2*c*d**2*log(a/b + x)/(20*a**
2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 900*a**5*b**2*c*d**2/(20*a**2*b**8
+ 40*a*b**9*x + 20*b**10*x**2) - 420*a**5*b**2*d**3*x**2*log(a/b + x)/(20*a
**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 1200*a**4*b**3*c*d**2*x*log(a/b +
 x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 1200*a**4*b**3*c*d**2*x/
(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 140*a**4*b**3*d**3*x**3/(20*
a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 180*a**3*b**4*c**2*d*log(a/b + x
)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 270*a**3*b**4*c**2*d/(20*a
**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 600*a**3*b**4*c*d**2*x**2*log(a/b
 + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 35*a**3*b**4*d**3*x**4
/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 360*a**2*b**5*c**2*d*x*log(
a/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 360*a**2*b**5*c**2*
d*x/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 200*a**2*b**5*c*d**2*x**
3/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 14*a**2*b**5*d**3*x**5/(20
*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 10*a*b**6*c**3/(20*a**2*b**8 +
40*a*b**9*x + 20*b**10*x**2) - 180*a*b**6*c**2*d*x**2*log(a/b + x)/(20*a**2
*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 50*a*b**6*c*d**2*x**4/(20*a**2*b**8
+ 40*a*b**9*x + 20*b**10*x**2) - 7*a*b**6*d**3*x**6/(20*a**2*b**8 + 40*a*b*
*9*x + 20*b**10*x**2) - 20*b**7*c**3*x/(20*a**2*b**8 + 40*a*b**9*x + 20*b**
10*x**2) + 60*b**7*c**2*d*x**3/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2)
 + 20*b**7*c*d**2*x**5/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 4*b**
7*d**3*x**7/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2), Eq(n, -3)), (420*
a**7*d**3*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 420*a**7*d**3/(60*a*b**8 +
 60*b**9*x) + 420*a**6*b*d**3*x*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 900*
a**5*b**2*c*d**2*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 900*a**5*b**2*c*d**
2/(60*a*b**8 + 60*b**9*x) - 210*a**5*b**2*d**3*x**2/(60*a*b**8 + 60*b**9*x)
 + 900*a**4*b**3*c*d**2*x*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 70*a**4*b*
*3*d**3*x**3/(60*a*b**8 + 60*b**9*x) + 540*a**3*b**4*c**2*d*log(a/b + x)/(6
0*a*b**8 + 60*b**9*x) + 540*a**3*b**4*c**2*d/(60*a*b**8 + 60*b**9*x) - 450*
a**3*b**4*c*d**2*x**2/(60*a*b**8 + 60*b**9*x) - 35*a**3*b**4*d**3*x**4/(60*
a*b**8 + 60*b**9*x) + 540*a**2*b**5*c**2*d*x*log(a/b + x)/(60*a*b**8 + 60*b
**9*x) + 150*a**2*b**5*c*d**2*x**3/(60*a*b**8 + 60*b**9*x) + 21*a**2*b**5*d
**3*x**5/(60*a*b**8 + 60*b**9*x) + 60*a*b**6*c**3*log(a/b + x)/(60*a*b**8 +
 60*b**9*x) + 60*a*b**6*c**3/(60*a*b**8 + 60*b**9*x) - 270*a*b**6*c**2*d*x*
*2/(60*a*b**8 + 60*b**9*x) - 75*a*b**6*c*d**2*x**4/(60*a*b**8 + 60*b**9*x)
```

- 14*a*b**6*d**3*x**6/(60*a*b**8 + 60*b**9*x) + 60*b**7*c**3*x*log(a/b + x)
/(60*a*b**8 + 60*b**9*x) + 90*b**7*c**2*d*x**3/(60*a*b**8 + 60*b**9*x) + 45
*b**7*c*d**2*x**5/(60*a*b**8 + 60*b**9*x) + 10*b**7*d**3*x**7/(60*a*b**8 +
60*b**9*x), Eq(n, -2)), (-a**7*d**3*log(a/b + x)/b**8 + a**6*d**3*x/b**7 -
3*a**5*c*d**2*log(a/b + x)/b**6 - a**5*d**3*x**2/(2*b**6) + 3*a**4*c*d**2*x
/b**5 + a**4*d**3*x**3/(3*b**5) - 3*a**3*c**2*d*log(a/b + x)/b**4 - 3*a**3*
c*d**2*x**2/(2*b**4) - a**3*d**3*x**4/(4*b**4) + 3*a**2*c**2*d*x/b**3 + a**
2*c*d**2*x**3/b**3 + a**2*d**3*x**5/(5*b**3) - a*c**3*log(a/b + x)/b**2 - 3
*a*c**2*d*x**2/(2*b**2) - 3*a*c*d**2*x**4/(4*b**2) - a*d**3*x**6/(6*b**2) +
 c**3*x/b + c**2*d*x**3/b + 3*c*d**2*x**5/(5*b) + d**3*x**7/(7*b), Eq(n, -1
)), (-5040*a**8*d**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
109584*b**8*n + 40320*b**8) + 5040*a**7*b*d**3*n*x*(a + b*x)**n/(b**8*n**8
+ 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b
8*n3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 360*a**6*b**2*c
*d**2*n**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b*
*8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**
8*n + 40320*b**8) - 5400*a**6*b**2*c*d**2*n*(a + b*x)**n/(b**8*n**8 + 36*b*
*8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**
3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 20160*a**6*b**2*c*d**2
*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 4032
0*b**8) - 2520*a**6*b**2*d**3*n**2*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n
7 + 546*b8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 +
118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2520*a**6*b**2*d**3*n*x**2
*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 4032
0*b**8) + 360*a**5*b**3*c*d**2*n**3*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**
7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11
8124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 5400*a**5*b**3*c*d**2*n**2*x
*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 4032
0*b**8) + 20160*a**5*b**3*c*d**2*n*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118
124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 840*a**5*b**3*d**3*n**3*x**3*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) + 2520*a**5*b**3*d**3*n**2*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n*
*7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1
18124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1680*a**5*b**3*d**3*n*x**3*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) - 18*a**4*b**4*c**2*d*n**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124
*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 468*a**4*b**4*c**2*d*n**3*(a + b

*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b **8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 4518*a**4*b**4*c**2*d*n**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546* b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b** 8*n**2 + 109584*b**8*n + 40320*b**8) - 19188*a**4*b**4*c**2*d*n*(a + b*x)** n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n **4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 30 240*a**4*b**4*c**2*d*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 180*a**4*b**4*c*d**2*n**4*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2880* a**4*b**4*c*d**2*n**3*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b** 8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n **2 + 109584*b**8*n + 40320*b**8) - 12780*a**4*b**4*c*d**2*n**2*x**2*(a + b *x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b **8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 10080*a**4*b**4*c*d**2*n*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 5 46*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124* b**8*n**2 + 109584*b**8*n + 40320*b**8) - 210*a**4*b**4*d**3*n**4*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449 *b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b** 8) - 1260*a**4*b**4*d**3*n**3*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812 4*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2310*a**4*b**4*d**3*n**2*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22 449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320* b**8) - 1260*a**4*b**4*d**3*n*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812 4*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 18*a**3*b**5*c**2*d*n**5*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449 *b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b** 8) + 468*a**3*b**5*c**2*d*n**4*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 5 46*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124* b**8*n**2 + 109584*b**8*n + 40320*b**8) + 4518*a**3*b**5*c**2*d*n**3*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449 *b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b** 8) + 19188*a**3*b**5*c**2*d*n**2*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812 4*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 30240*a**3*b**5*c**2*d*n*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449 *b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b** 8) + 60*a**3*b**5*c*d**2*n**5*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812 4*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1080*a**3*b**5*c*d**2*n**4*x**3 *(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 6180*a**3*b**5*c*d**2*n**3*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 11880*a**3*b**5*c*d**2*n**2*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 6720*a**3*b**5*c*d**2*n*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 42*a**3*b**5*d**3*n**5*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 420*a**3*b**5*d**3*n**4*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1470*a**3*b**5*d**3*n**3*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 2100*a**3*b**5*d**3*n**2*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1008*a**3*b**5*d**3*n*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - a**2*b**6*c**3*n**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 33*a**2*b**6*c**3*n**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 445*a**2*b**6*c**3*n**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 3135*a**2*b**6*c**3*n**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 12154*a**2*b**6*c**3*n**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 24552*a**2*b**6*c**3*n*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 20160*a**2*b**6*c**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 9*a**2*b**6*c**2*d*n**6*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 243*a**2*b**6*c**2*d*n**5*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2493*a**2*b**6*c**2*d*n**4*x**2*(a + b*x)**n/(b**8*n

8 + 36*b8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 672
84*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 11853*a**2*
b**6*c**2*d*n**3*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**
6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
 109584*b**8*n + 40320*b**8) - 24714*a**2*b**6*c**2*d*n**2*x**2*(a + b*x)**
n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n
4 + 67284*b8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 15
120*a**2*b**6*c**2*d*n*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b*
*8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*
n**2 + 109584*b**8*n + 40320*b**8) - 15*a**2*b**6*c*d**2*n**6*x**4*(a + b*x
)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**
8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) -
 315*a**2*b**6*c*d**2*n**5*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 54
6*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b
8*n2 + 109584*b**8*n + 40320*b**8) - 2355*a**2*b**6*c*d**2*n**4*x**4*(a
 + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 224
49*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b
8) - 7605*a2*b**6*c*d**2*n**3*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n*
*7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1
18124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 10590*a**2*b**6*c*d**2*n**2
*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n*
*5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
 40320*b**8) - 5040*a**2*b**6*c*d**2*n*x**4*(a + b*x)**n/(b**8*n**8 + 36*b*
*8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**
3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 7*a**2*b**6*d**3*n**6*
x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**
5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
40320*b**8) - 105*a**2*b**6*d**3*n**5*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**
8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3
 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 595*a**2*b**6*d**3*n**4
*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n*
*5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
 40320*b**8) - 1575*a**2*b**6*d**3*n**3*x**6*(a + b*x)**n/(b**8*n**8 + 36*b
8*n7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n*
*3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 1918*a**2*b**6*d**3*n
2*x6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8
*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*
n + 40320*b**8) - 840*a**2*b**6*d**3*n*x**6*(a + b*x)**n/(b**8*n**8 + 36*b*
*8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**
3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + a*b**7*c**3*n**7*x*(a
+ b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2244
9*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b*
*8) + 33*a*b**7*c**3*n**6*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b*
*8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*
n**2 + 109584*b**8*n + 40320*b**8) + 445*a*b**7*c**3*n**5*x*(a + b*x)**n/(b

8*n8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4
+ 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 3135*a
*b**7*c**3*n**4*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) + 12154*a*b**7*c**3*n**3*x*(a + b*x)**n/(b**8*n**8
 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*
b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 24552*a*b**7*c
3*n2*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b*
*8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**
8*n + 40320*b**8) + 20160*a*b**7*c**3*n*x*(a + b*x)**n/(b**8*n**8 + 36*b**8
*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3
+ 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 3*a*b**7*c**2*d*n**7*x**
3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 87*a*b**7*c**2*d*n**6*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**
7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11
8124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 993*a*b**7*c**2*d*n**5*x**3*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) + 5613*a*b**7*c**2*d*n**4*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**
7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11
8124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 16140*a*b**7*c**2*d*n**3*x**
3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 21516*a*b**7*c**2*d*n**2*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*
n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 +
 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 10080*a*b**7*c**2*d*n*x**
3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 3*a*b**7*c*d**2*n**7*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118
124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 75*a*b**7*c*d**2*n**6*x**5*(a
 + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 224
49*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b
8) + 723*a*b7*c*d**2*n**5*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812
4*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 3405*a*b**7*c*d**2*n**4*x**5*(a
 + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 224
49*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b
8) + 8202*a*b7*c*d**2*n**3*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
+ 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1181
24*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 9480*a*b**7*c*d**2*n**2*x**5*(
a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22
449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*
b**8) + 4032*a*b**7*c*d**2*n*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +

546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124
*b**8*n**2 + 109584*b**8*n + 40320*b**8) + a*b**7*d**3*n**7*x**7*(a + b*x)*
*n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*
n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 2
1*a*b**7*d**3*n**6*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n
6 + 4536*b8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2
 + 109584*b**8*n + 40320*b**8) + 175*a*b**7*d**3*n**5*x**7*(a + b*x)**n/(b*
*8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 +
 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 735*a*b
7*d3*n**4*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10
9584*b**8*n + 40320*b**8) + 1624*a*b**7*d**3*n**3*x**7*(a + b*x)**n/(b**8*n
8 + 36*b8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 672
84*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1764*a*b**7
*d**3*n**2*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 45
36*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10958
4*b**8*n + 40320*b**8) + 720*a*b**7*d**3*n*x**7*(a + b*x)**n/(b**8*n**8 + 3
6*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8
*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + b**8*c**3*n**7*x**
2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 34*b**8*c**3*n**6*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124
*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 478*b**8*c**3*n**5*x**2*(a + b*x
)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**
8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) +
 3580*b**8*c**3*n**4*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8
*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n*
*2 + 109584*b**8*n + 40320*b**8) + 15289*b**8*c**3*n**3*x**2*(a + b*x)**n/(
b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4
 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 36706
*b**8*c**3*n**2*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
109584*b**8*n + 40320*b**8) + 44712*b**8*c**3*n*x**2*(a + b*x)**n/(b**8*n**
8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284
*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 20160*b**8*c*
*3*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*
n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n
 + 40320*b**8) + 3*b**8*c**2*d*n**7*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*
n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 +
 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 96*b**8*c**2*d*n**6*x**4*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) + 1254*b**8*c**2*d*n**5*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1181

24*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 8592*b**8*c**2*d*n**4*x**4*(a
+ b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2244
9*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b*
*8) + 32979*b**8*c**2*d*n**3*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124
*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 69936*b**8*c**2*d*n**2*x**4*(a +
b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449
*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**
8) + 74628*b**8*c**2*d*n*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*
b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**
8*n**2 + 109584*b**8*n + 40320*b**8) + 30240*b**8*c**2*d*x**4*(a + b*x)**n/
(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**
4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 3*b*
*8*c*d**2*n**7*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
+ 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1
09584*b**8*n + 40320*b**8) + 90*b**8*c*d**2*n**6*x**6*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1098*b**8*c*
d**2*n**5*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 453
6*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584
*b**8*n + 40320*b**8) + 7020*b**8*c*d**2*n**4*x**6*(a + b*x)**n/(b**8*n**8
+ 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b
8*n3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 25227*b**8*c*d*
*2*n**3*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*
b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b
8*n + 40320*b8) + 50490*b**8*c*d**2*n**2*x**6*(a + b*x)**n/(b**8*n**8 +
36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b*
*8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 51432*b**8*c*d**
2*n*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8
*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*
n + 40320*b**8) + 20160*b**8*c*d**2*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*
n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 +
118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + b**8*d**3*n**7*x**8*(a +
b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*
b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8
) + 28*b**8*d**3*n**6*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**
8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n
2 + 109584*b8*n + 40320*b**8) + 322*b**8*d**3*n**5*x**8*(a + b*x)**n/(b
8*n8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4
+ 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1960*b
8*d3*n**4*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10
9584*b**8*n + 40320*b**8) + 6769*b**8*d**3*n**3*x**8*(a + b*x)**n/(b**8*n**
8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284
*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 13132*b**8*d*

```
*3*n**2*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*
b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b
**8*n + 40320*b**8) + 13068*b**8*d**3*n*x**8*(a + b*x)**n/(b**8*n**8 + 36*b
**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n*
*3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 5040*b**8*d**3*x**8*(
a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22
449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*
b**8), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(282) = 564$.

Time = 0.23 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.22

$$\int x(a+bx)^n\left(c+dx^2\right)^3\,dx = \frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n c^3}{(n^2+3\,n+2)b^2} + \frac{3\left((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n c^2}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4} + \frac{3\left((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5x^5-5\,(n^4\right.}{(n^6+21\,n^5+175\,n^4+7} + \frac{\left((n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)b^8x^8+(n^7+21\,n^6+175\,n^5+\right.}{}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 +
 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60
*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/((n^6 +
 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7 + 28*n^
6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (
n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 -
 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^
5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^
2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)
*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n^8 + 36*n^7 +
546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)
*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2851 vs. $2(282) = 564$.

Time = 0.31 (sec) , antiderivative size = 2851, normalized size of antiderivative = 10.11

$$\int x(a+bx)^n \left(c+dx^2\right)^3 dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b^7
*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d^2*
n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*n^6*
x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4*x^8
+ 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^5 + 10
98*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x^6 + 73
5*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8 + 3*(b*x
 + a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 15*(b*x +
a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5 + 42*(b*x
+ a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6 - 595*(b*x
+ a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7 + 13132*(b*
x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b*x + a)^n*a*b
^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(b*x + a)^n*a^2
*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 + 420*(b*x + a)^n
*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6 - 1575*(b*x + a)
^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^7 + 13068*(b*x +
a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*x + a)^n*b^8*c^3*n
^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x + a)^n*a*b^7*c^2*d*
n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 8592*(b*x + a)^n*b^8*c^2*d
*n^4*x^4 - 2355*(b*x + a)^n*a^2*b^6*c*d^2*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4
*d^3*n^4*x^4 + 8202*(b*x + a)^n*a*b^7*c*d^2*n^3*x^5 + 1470*(b*x + a)^n*a^3*
b^5*d^3*n^3*x^5 + 50490*(b*x + a)^n*b^8*c*d^2*n^2*x^6 - 1918*(b*x + a)^n*a^
2*b^6*d^3*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^3*n*x^7 + 5040*(b*x + a)^n*b^8*
d^3*x^8 + 33*(b*x + a)^n*a*b^7*c^3*n^6*x + 478*(b*x + a)^n*b^8*c^3*n^5*x^2
- 243*(b*x + a)^n*a^2*b^6*c^2*d*n^5*x^2 + 5613*(b*x + a)^n*a*b^7*c^2*d*n^4*
x^3 + 1080*(b*x + a)^n*a^3*b^5*c*d^2*n^4*x^3 + 32979*(b*x + a)^n*b^8*c^2*d*
n^3*x^4 - 7605*(b*x + a)^n*a^2*b^6*c*d^2*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4
*d^3*n^3*x^4 + 9480*(b*x + a)^n*a*b^7*c*d^2*n^2*x^5 + 2100*(b*x + a)^n*a^3*
b^5*d^3*n^2*x^5 + 51432*(b*x + a)^n*b^8*c*d^2*n*x^6 - 840*(b*x + a)^n*a^2*b
^6*d^3*n*x^6 - (b*x + a)^n*a^2*b^6*c^3*n^6 + 445*(b*x + a)^n*a*b^7*c^3*n^5*
x + 18*(b*x + a)^n*a^3*b^5*c^2*d*n^5*x + 3580*(b*x + a)^n*b^8*c^3*n^4*x^2 -
 2493*(b*x + a)^n*a^2*b^6*c^2*d*n^4*x^2 - 180*(b*x + a)^n*a^4*b^4*c*d^2*n^4
*x^2 + 16140*(b*x + a)^n*a*b^7*c^2*d*n^3*x^3 + 6180*(b*x + a)^n*a^3*b^5*c*d
^2*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^3*n^3*x^3 + 69936*(b*x + a)^n*b^8*c^
```

```
2*d*n^2*x^4 - 10590*(b*x + a)^n*a^2*b^6*c*d^2*n^2*x^4 - 2310*(b*x + a)^n*a^
4*b^4*d^3*n^2*x^4 + 4032*(b*x + a)^n*a*b^7*c*d^2*n*x^5 + 1008*(b*x + a)^n*a
^3*b^5*d^3*n*x^5 + 20160*(b*x + a)^n*b^8*c*d^2*x^6 - 33*(b*x + a)^n*a^2*b^6
*c^3*n^5 + 3135*(b*x + a)^n*a*b^7*c^3*n^4*x + 468*(b*x + a)^n*a^3*b^5*c^2*d
*n^4*x + 15289*(b*x + a)^n*b^8*c^3*n^3*x^2 - 11853*(b*x + a)^n*a^2*b^6*c^2*
d*n^3*x^2 - 2880*(b*x + a)^n*a^4*b^4*c*d^2*n^3*x^2 + 21516*(b*x + a)^n*a*b^
7*c^2*d*n^2*x^3 + 11880*(b*x + a)^n*a^3*b^5*c*d^2*n^2*x^3 + 2520*(b*x + a)^
n*a^5*b^3*d^3*n^2*x^3 + 74628*(b*x + a)^n*b^8*c^2*d*n*x^4 - 5040*(b*x + a)^
n*a^2*b^6*c*d^2*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n*x^4 - 445*(b*x + a)^
n*a^2*b^6*c^3*n^4 - 18*(b*x + a)^n*a^4*b^4*c^2*d*n^4 + 12154*(b*x + a)^n*a*
b^7*c^3*n^3*x + 4518*(b*x + a)^n*a^3*b^5*c^2*d*n^3*x + 360*(b*x + a)^n*a^5*
b^3*c*d^2*n^3*x + 36706*(b*x + a)^n*b^8*c^3*n^2*x^2 - 24714*(b*x + a)^n*a^2
*b^6*c^2*d*n^2*x^2 - 12780*(b*x + a)^n*a^4*b^4*c*d^2*n^2*x^2 - 2520*(b*x +
a)^n*a^6*b^2*d^3*n^2*x^2 + 10080*(b*x + a)^n*a*b^7*c^2*d*n*x^3 + 6720*(b*x
+ a)^n*a^3*b^5*c*d^2*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^3*n*x^3 + 30240*(b*
x + a)^n*b^8*c^2*d*x^4 - 3135*(b*x + a)^n*a^2*b^6*c^3*n^3 - 468*(b*x + a)^n
*a^4*b^4*c^2*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^3*n^2*x + 19188*(b*x + a)^n*
a^3*b^5*c^2*d*n^2*x + 5400*(b*x + a)^n*a^5*b^3*c*d^2*n^2*x + 44712*(b*x + a
)^n*b^8*c^3*n*x^2 - 15120*(b*x + a)^n*a^2*b^6*c^2*d*n*x^2 - 10080*(b*x + a)
^n*a^4*b^4*c*d^2*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n*x^2 - 12154*(b*x +
a)^n*a^2*b^6*c^3*n^2 - 4518*(b*x + a)^n*a^4*b^4*c^2*d*n^2 - 360*(b*x + a)^n
*a^6*b^2*c*d^2*n^2 + 20160*(b*x + a)^n*a*b^7*c^3*n*x + 30240*(b*x + a)^n*a^
3*b^5*c^2*d*n*x + 20160*(b*x + a)^n*a^5*b^3*c*d^2*n*x + 5040*(b*x + a)^n*a^
7*b*d^3*n*x + 20160*(b*x + a)^n*b^8*c^3*x^2 - 24552*(b*x + a)^n*a^2*b^6*c^3
*n - 19188*(b*x + a)^n*a^4*b^4*c^2*d*n - 5400*(b*x + a)^n*a^6*b^2*c*d^2*n -
 20160*(b*x + a)^n*a^2*b^6*c^3 - 30240*(b*x + a)^n*a^4*b^4*c^2*d - 20160*(b
*x + a)^n*a^6*b^2*c*d^2 - 5040*(b*x + a)^n*a^8*d^3)/(b^8*n^8 + 36*b^8*n^7 +
 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^
2 + 109584*b^8*n + 40320*b^8)
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 1459, normalized size of antiderivative = 5.17

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] int(x*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^3 +
20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 + 44
5*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 20160*
a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b^4
```

```
*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c^2
*d*n^4))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 5
46*n^6 + 36*n^7 + n^8 + 40320)) - (x^2*(n + 1)*(a + b*x)^n*(2520*a^6*d^3*n
- 20160*b^6*c^3 - 24552*b^6*c^3*n - 12154*b^6*c^3*n^2 - 3135*b^6*c^3*n^3 -
445*b^6*c^3*n^4 - 33*b^6*c^3*n^5 - b^6*c^3*n^6 + 15120*a^2*b^4*c^2*d*n + 10
080*a^4*b^2*c*d^2*n + 9594*a^2*b^4*c^2*d*n^2 + 2700*a^4*b^2*c*d^2*n^2 + 225
9*a^2*b^4*c^2*d*n^3 + 180*a^4*b^2*c*d^2*n^3 + 234*a^2*b^4*c^2*d*n^4 + 9*a^2
*b^4*c^2*d*n^5))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (d^2*x^6*(a + b*x)^n*(168*b^2*c +
 3*b^2*c*n^2 - 7*a^2*d*n + 45*b^2*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5
 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 +
n^3 + 6)*(1680*b^4*c^2 - 70*a^4*d^2*n + 1066*b^4*c^2*n + 251*b^4*c^2*n^2 +
26*b^4*c^2*n^3 + b^4*c^2*n^4 - 280*a^2*b^2*c*d*n - 75*a^2*b^2*c*d*n^2 - 5*a
^2*b^2*c*d*n^3))/(b^4*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)^n*(5040*a^6*d^3
+ 20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 +
445*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 2016
0*a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b
^4*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c
^2*d*n^4))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 +
 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^3*n*x^7*(a + b*x)^n*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 +
 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a
*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(280*a^4*d^2 + 1680*b^4*c^2 + 1066*b^4
*c^2*n + 251*b^4*c^2*n^2 + 26*b^4*c^2*n^3 + b^4*c^2*n^4 + 1120*a^2*b^2*c*d
+ 300*a^2*b^2*c*d*n + 20*a^2*b^2*c*d*n^2))/(b^5*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a*d^
2*n*x^5*(a + b*x)^n*(14*a^2*d + 56*b^2*c + b^2*c*n^2 + 15*b^2*c*n)*(50*n +
35*n^2 + 10*n^3 + n^4 + 24))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 2244
9*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

3.361 $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 223

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \frac{\left(b^2c+a^2d\right)^3 (a+bx)^{1+n}}{b^7(1+n)} - \frac{6ad\left(b^2c+a^2d\right)^2 (a+bx)^{2+n}}{b^7(2+n)} + \frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{3+n}}{b^7(3+n)} - \frac{4ad^2\left(3b^2c+5a^2d\right)(a+bx)^{4+n}}{b^7(4+n)} + \frac{3d^2\left(b^2c+5a^2d\right)(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^3(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^3(a+bx)^{7+n}}{b^7(7+n)}$$

```
[Out] (a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^7/(1+n)-6*a*d*(a^2*d+b^2*c)^2*(b*x+a)^(2+n)
/b^7/(2+n)+3*d*(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^7/(3+n)-4*a*d^
2*(5*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+3*d^2*(5*a^2*d+b^2*c)*(b*x+a)^(
5+n)/b^7/(5+n)-6*a*d^3*(b*x+a)^(6+n)/b^7/(6+n)+d^3*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used

$= \{711\}$

$$\int (a+bx)^n \left(c+dx^2\right)^3 dx = -\frac{4ad^2(5a^2d+3b^2c)(a+bx)^{n+4}}{b^7(n+4)} + \frac{3d^2(5a^2d+b^2c)(a+bx)^{n+5}}{b^7(n+5)} + \frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)} - \frac{6ad\left(a^2d+b^2c\right)^2(a+bx)^{n+2}}{b^7(n+2)} + \frac{3d(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+3}}{b^7(n+3)} - \frac{6ad^3(a+bx)^{n+6}}{b^7(n+6)} + \frac{d^3(a+bx)^{n+7}}{b^7(n+7)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\left(b^2c+a^2d\right)^3(a+bx)^n}{b^6} - \frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6} + \frac{3d(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6} - \frac{4ad^2(3b^2c+5a^2d)(a+bx)^{3+n}}{b^6} + \frac{3d^2(b^2c+5a^2d)(a+bx)^{4+n}}{b^6} - \frac{6ad^3(a+bx)^{5+n}}{b^6} + \frac{d^3(a+bx)^{6+n}}{b^6}\right) dx \\
&= \frac{\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^7(1+n)} - \frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{2+n}}{b^7(2+n)} + \frac{3d(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{3+n}}{b^7(3+n)} - \frac{4ad^2(3b^2c+5a^2d)(a+bx)^{4+n}}{b^7(4+n)} + \frac{3d^2(b^2c+5a^2d)(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^3(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^3(a+bx)^{7+n}}{b^7(7+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$
$$= \frac{(a+bx)^{1+n}\left(\left(c+dx^2\right)^3+\frac{6\left(\left(b^2c+a^2d\right)(6+n)\left(b^4(1+n)(2+n)(3+n)(4+n)\left(c+dx^2\right)^2+4\left(b^2c+a^2d\right)(4+n)\left(2a^2d-2abd(1+n)x+b^2(2+n\right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^3 + (6*((b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n
)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*
d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1
 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(
2 + n)*x^2))) - a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*
(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^
2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))))/(b*(7 + n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(223) = 446.

Time = 0.44 (sec) , antiderivative size = 958, normalized size of antiderivative = 4.30

method	result
norman	$\frac{d^3x^7e^{n\ln(bx+a)}}{7+n}+\frac{a\left(b^6c^3n^6+27b^6c^3n^5+6a^2b^4c^2d\,n^4+295b^6c^3n^4+132a^2b^4c^2d\,n^3+1665b^6c^3n^3+72a^4b^2c\,d^2n^2+1074a^2b^4c^2\right.}{b^7\left(n^7+28n^6+322n^5+19\right.}$
gosper	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^3*n^6+27*b^6*c^3*n^5+6*a^2*b^4*c^2*
d*n^4+295*b^6*c^3*n^4+132*a^2*b^4*c^2*d*n^3+1665*b^6*c^3*n^3+72*a^4*b^2*c*d
^2*n^2+1074*a^2*b^4*c^2*d*n^2+5104*b^6*c^3*n^2+936*a^4*b^2*c*d^2*n+3828*a^2
*b^4*c^2*d*n+8028*b^6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4*c^2
*d+5040*b^6*c^3)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*
n+5040)*exp(n*ln(b*x+a))+a*d^3*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6
*c^3*n^6+6*a^2*b^4*c^2*d*n^5-27*b^6*c^3*n^5+132*a^2*b^4*c^2*d*n^4-295*b^6*c
^3*n^4+72*a^4*b^2*c*d^2*n^3+1074*a^2*b^4*c^2*d*n^3-1665*b^6*c^3*n^3+936*a^4
```

```
*b^2*c*d^2*n^2+3828*a^2*b^4*c^2*d*n^2-5104*b^6*c^3*n^2+720*a^6*d^3*n+3024*a
^4*b^2*c*d^2*n+5040*a^2*b^4*c^2*d*n-8028*b^6*c^3*n-5040*b^6*c^3)/b^6/(n^7+2
8*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))-
3*(-b^2*c*n^2+2*a^2*d*n-13*b^2*c*n-42*b^2*c)/b^2*d^2/(n^3+18*n^2+107*n+210)
*x^5*exp(n*ln(b*x+a))-3*(-b^4*c^2*n^4+4*a^2*b^2*c*d*n^3-22*b^4*c^2*n^3+52*a
^2*b^2*c*d*n^2-179*b^4*c^2*n^2+40*a^4*d^2*n+168*a^2*b^2*c*d*n-638*b^4*c^2*n
-840*b^4*c^2)/b^4*d/(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(
b*x+a))+3*(b^2*c*n^2+13*b^2*c*n+10*a^2*d+42*b^2*c)*d^2*a/b^3*n/(n^4+22*n^3+
179*n^2+638*n+840)*x^4*exp(n*ln(b*x+a))+3*(b^4*c^2*n^4+22*b^4*c^2*n^3+12*a^
2*b^2*c*d*n^2+179*b^4*c^2*n^2+156*a^2*b^2*c*d*n+638*b^4*c^2*n+120*a^4*d^2+5
04*a^2*b^2*c*d+840*b^4*c^2)*d*a/b^5*n/(n^6+27*n^5+295*n^4+1665*n^3+5104*n^2
+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(223) = 446$.

Time = 0.27 (sec) , antiderivative size = 1244, normalized size of antiderivative = 5.58

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3
024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7*d
^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d^3)
*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6*d^3
*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1008*b
^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a^2*b^5
*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^2 - 25*
a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295*a*b^6*c
^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^5 + (131*
a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4*d^3)*n^3
+ 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2 + 5*a^3*b^4
*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^7*c^2*d*n^6
+ 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247*b^7*c^2*d -
64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 - 40*a^4*b^3*d^
3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^3)*n^2 + 4*(949
*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*(2552*a*b^6*c^3
+ 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2*d*n^6 + 23*a*b^6
*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (817*a*b^6*c^2*d +
168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^4*c*d^2 + 60*a^5*b^
2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 + 5*a^5*b^2*d^3)*n)*x^2
+ 12*(669*a*b^6*c^3 + 319*a^3*b^4*c^2*d + 78*a^5*b^2*c*d^2)*n + (b^7*c^3*n^
6 + 5040*b^7*c^3 + 3*(9*b^7*c^3 - 2*a^2*b^5*c^2*d)*n^5 + (295*b^7*c^3 - 132
```

```
*a^2*b^5*c^2*d)*n^4 + 3*(555*b^7*c^3 - 358*a^2*b^5*c^2*d - 24*a^4*b^3*c*d^2
)*n^3 + 4*(1276*b^7*c^3 - 957*a^2*b^5*c^2*d - 234*a^4*b^3*c*d^2)*n^2 + 36*(
223*b^7*c^3 - 140*a^2*b^5*c^2*d - 84*a^4*b^3*c*d^2 - 20*a^6*b*d^3)*n)*x)*(b
*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3
 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15990 vs. $2(207) = 414$.

Time = 4.08 (sec) , antiderivative size = 15990, normalized size of antiderivative = 71.70

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), Eq(
b, 0)), (60*a**6*d**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + 147*a**6*d**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + 360*a**5*b*d**3*x*log(a/b + x)/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
+ 360*a*b**12*x**5 + 60*b**13*x**6) + 822*a**5*b*d**3*x/(60*a**6*b**7 + 360
*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x
**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6*a**4*b**2*c*d**2/(60*a**6*b**7
+ 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b*
*11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**3*x**2*log(
a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b
**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*
a**4*b**2*d**3*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) - 36*a**3*b**3*c*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9
*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) + 1200*a**3*b**3*d**3*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
+ 360*a*b**12*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**3*x**3/(60*a**6*b**
7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*
b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 3*a**2*b**4*c**2*d/(60*a**
6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*
a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 90*a**2*b**4*c*d**2*x
**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*
x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b
**4*d**3*x**4*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*
x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b
```

13*x6) + 1350*a**2*b**4*d**3*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 18*a*b**5*c**2*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 120*a*b**5*c*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**3*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**3*x**5/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 10*b**6*c**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 45*b**6*c**2*d*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 90*b**6*c*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**3*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 137*a**6*d**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**5*b*d**3*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 625*a**5*b*d**3*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 6*a**4*b**2*c*d**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**4*b**2*d**3*x**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 1100*a**4*b**2*d**3*x**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 30*a**3*b**3*c*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**3*b**3*d**3*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 900*a**3*b**3*d**3*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**2*b**4*c**2*d/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 60*a**2*b**4*c*d**2*x**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**3*x**4*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**3*x**4/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 5*a*b**5* c**2*d*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 60*a*b**5*c*d**2*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 60*a*b**5*d**3*x**5*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 2*b**6*c**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 10*b**6*c**2*d*x**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 30*b**6*c*d**2*x**4/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) + 10*b**6*d**3*x**6/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5), Eq(n, -6)), (60*a**6*d**3*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 125*a**6*d**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**5*b*d**3*x*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 440*a**5*b*d**3*x/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 12*a**4*b**2*c*d**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 25*a**4*b**2*c*d**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 360*a**4*b**2*d**3*x**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 540*a**4*b**2*d**3*x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 48*a**3*b**3*c*d**2*x*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 88*a**3*b**3*c*d**2*x/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**3*x**3*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**3*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - a**2*b**4*c**2*d/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 72*a**2*b**4*c*d**2*x**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 108*a**2*b**4*c*d**2*x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 60*a**2*b**4*d**3*x**4*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 4*a*b**5*c**2*d*x/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 48*a*b**5*c*d**2*x**3*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 48*a*b**5*c*d**2*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 12*a*b**5*d**3*x**5/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - b**6*c**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 6*b**6*c**2*d*x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4)

+ 12*b**6*c*d**2*x**4*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 2*b**6*d**3*x**6/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4), Eq(n, -5)), (-60*a**6*d**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*d**3/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**5*b*d**3*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 270*a**5*b*d**3*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 36*a**4*b**2*c*d**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 66*a**4*b**2*c*d**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**3*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**3*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 108*a**3*b**3*c*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 162*a**3*b**3*c*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 60*a**3*b**3*d**3*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 3*a**2*b**4*c**2*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 108*a**2*b**4*c*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 108*a**2*b**4*c*d**2*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**2*b**4*d**3*x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 9*a*b**5*c**2*d*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 36*a*b**5*c*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 3*a*b**5*d**3*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - b**6*c**3/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 9*b**6*c**2*d*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 9*b**6*c*d**2*x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**3*x**6/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**3*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**3*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**3*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 72*a**4*b**2*c*d**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 108*a**4*b**2*c*d**2/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 60*a**4*b**2*d**3*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 144*a**3*b**3*c*d**2*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 144*a**3*b**3*c*d**2*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 20*a**3*b**3*d**3*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 12*a**2*b**4*c**2*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 18*a**2*b**4*c**2*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 72*a**2*b**4*c*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**3*x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 24*a*b**5*c**2*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 24*a*b**5*c**2*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**5*c*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*a*b**5*d**3*x**5/(4*a

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**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*b**6*c**3/(4*a**2*b**7 + 8*a*b**8*
x + 4*b**9*x**2) + 12*b**6*c**2*d*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8
*x + 4*b**9*x**2) + 6*b**6*c*d**2*x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + b**6*d**3*x**6/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)),
 (-60*a**6*d**3*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**3/(10*a*b
**7 + 10*b**8*x) - 60*a**5*b*d**3*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) -
120*a**4*b**2*c*d**2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 120*a**4*b**2*c
*d**2/(10*a*b**7 + 10*b**8*x) + 30*a**4*b**2*d**3*x**2/(10*a*b**7 + 10*b**8
*x) - 120*a**3*b**3*c*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 10*a**3
*b**3*d**3*x**3/(10*a*b**7 + 10*b**8*x) - 60*a**2*b**4*c**2*d*log(a/b + x)/
(10*a*b**7 + 10*b**8*x) - 60*a**2*b**4*c**2*d/(10*a*b**7 + 10*b**8*x) + 60*
a**2*b**4*c*d**2*x**2/(10*a*b**7 + 10*b**8*x) + 5*a**2*b**4*d**3*x**4/(10*a
*b**7 + 10*b**8*x) - 60*a*b**5*c**2*d*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x
) - 20*a*b**5*c*d**2*x**3/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**3*x**5/(10*
a*b**7 + 10*b**8*x) - 10*b**6*c**3/(10*a*b**7 + 10*b**8*x) + 30*b**6*c**2*d
*x**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d**2*x**4/(10*a*b**7 + 10*b**8*x)
 + 2*b**6*d**3*x**6/(10*a*b**7 + 10*b**8*x), Eq(n, -2)), (a**6*d**3*log(a/b
 + x)/b**7 - a**5*d**3*x/b**6 + 3*a**4*c*d**2*log(a/b + x)/b**5 + a**4*d**3
*x**2/(2*b**5) - 3*a**3*c*d**2*x/b**4 - a**3*d**3*x**3/(3*b**4) + 3*a**2*c*
*2*d*log(a/b + x)/b**3 + 3*a**2*c*d**2*x**2/(2*b**3) + a**2*d**3*x**4/(4*b*
*3) - 3*a*c**2*d*x/b**2 - a*c*d**2*x**3/b**2 - a*d**3*x**5/(5*b**2) + c**3*
log(a/b + x)/b + 3*c**2*d*x**2/(2*b) + 3*c*d**2*x**4/(4*b) + d**3*x**6/(6*b
), Eq(n, -1)), (720*a**7*d**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) - 720*a**6*b*d**3*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 72*a**5*b**2*c*d**2*n**2*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 936*a**5*b**2*c*d**2*n*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3024*a**5*b**2*c*d**2*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b**2*d**3*n
**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*
a**5*b**2*d**3*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) - 72*a**4*b**3*c*d**2*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) - 936*a**4*b**3*c*d**2*n**2*x*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) - 3024*a**4*b**3*c*d**2*n*x*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**3*n**3*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
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4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4
*b**3*d**3*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) - 240*a**4*b**3*d**3*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) + 6*a**3*b**4*c**2*d*n**4*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 132*a**3*b**4*c**2*d*n**3*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1074*a**3*b**4*c**2*d*n**2*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769
*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3828*a**3*b**4*c
**2*d*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*
a**3*b**4*c**2*d*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) + 36*a**3*b**4*c*d**2*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) + 504*a**3*b**4*c*d**2*n**3*x**2*(a + b*x)**n/(b**7*n**7
 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b
**7*n**2 + 13068*b**7*n + 5040*b**7) + 1980*a**3*b**4*c*d**2*n**2*x**2*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1512*a**3*b**4*c*
d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3
0*a**3*b**4*d**3*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**
7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
 5040*b**7) + 180*a**3*b**4*d**3*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**
7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
+ 13068*b**7*n + 5040*b**7) + 330*a**3*b**4*d**3*n**2*x**4*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**3*n*x**4*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769
*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c**2
*d*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 132
*a**2*b**5*c**2*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) - 1074*a**2*b**5*c**2*d*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) - 3828*a**2*b**5*c**2*d*n**2*x*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13
132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 5040*a**2*b**5*c**2*d*n*x*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 12*a**2*b**5*c*d**
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2*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1
92*a**2*b**5*c*d**2*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) - 996*a**2*b**5*c*d**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) - 1824*a**2*b**5*c*d**2*n**2*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1008*a**2*b**5*c*d**2
*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**
2*b**5*d**3*n**5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) - 60*a**2*b**5*d**3*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) - 210*a**2*b**5*d**3*n**3*x**5*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) - 300*a**2*b**5*d**3*n**2*x**5*(a + b
*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b*
*7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**3*
n*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6
*c**3*n**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27
*a*b**6*c**3*n**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 295*a*b**6*c**3*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1665*a*b**6*c**3*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 5104*a*b**6*c**3*n**2*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 8028*a*b**6*c**3*n*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*a*b**6*c**3*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3*a*b**6*c**2*d*n**6*x**2*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 69*a*b**6*c**2*d*n**5
*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 603*a*b
**6*c**2*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 2451*a*b**6*c**2*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
```

8*b**7*n + 5040*b**7) + 4434*a*b**6*c**2*d*n**2*x**2*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a*b**6*c**2*d*n*x**2*(a + b*x)
n/(b7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3*a*b**6*c*d**2*n**6*x
4*(a + b*x)n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 57*a*b**6
*c*d**2*n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 393*a*b**6*c*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 1203*a*b**6*c*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 1620*a*b**6*c*d**2*n**2*x**4*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
3 + 13132*b7*n**2 + 13068*b**7*n + 5040*b**7) + 756*a*b**6*c*d**2*n*x**
4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*d**3
*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 15
*a*b**6*d**3*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n*
*5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 504
0*b**7) + 85*a*b**6*d**3*n**4*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 225*a*b**6*d**3*n**3*x**6*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**3*n**2*x**6*(a + b*x)**n/(
b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
+ 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 120*a*b**6*d**3*n*x**6*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*c**3*n**6*x*
(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*b**7*c**3*
n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
4 + 6769*b7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*b*
*7*c**3*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 1665*b**7*c**3*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
5 + 1960*b7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 5104*b**7*c**3*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 8028*b**7*c**3*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13
068*b**7*n + 5040*b**7) + 5040*b**7*c**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**
7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2

+ 13068*b**7*n + 5040*b**7) + 3*b**7*c**2*d*n**6*x**3*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 75*b**7*c**2*d*n**5*x**3*(a + b*x)
n/(b7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 741*b**7*c**2*d*n**4*x
3*(a + b*x)n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3657*b**7
*c**2*d*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 9336*b**7*c**2*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 11388*b**7*c**2*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*d*x**3*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3*b**7*c*d**2*n**6*x**5*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 69*b**7*c*d**2*n**5*x**5
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 621*b**7*c*d
2*n4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960
*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
 2775*b**7*c*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b*
*7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
+ 5040*b**7) + 6432*b**7*c*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 7236*b**7*c*d**2*n*x**5*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3024*b**7*c*d**2*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**3*n**6*x**7*(a + b
*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b*
*7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**3*n**5*x
7*(a + b*x)n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*
d**3*n**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 735*b**7*d**3*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1624*b**7*d**3*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 1764*b**7*d**3*n*x**7*(a + b*x)**n/(b**7*n**7 + 28
*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n
2 + 13068*b7*n + 5040*b**7) + 720*b**7*d**3*x**7*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*

```
b**7*n**2 + 13068*b**7*n + 5040*b**7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(223) = 446$.

Time = 0.21 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.12

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$
$$= \frac{(bx+a)^{n+1}c^3}{b(n+1)} + \frac{3\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n c^2 d}{(n^3+6n^2+11n+6)b^3}$$
$$+ \frac{3\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+1\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$
$$+ \frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n^2-\right.}{}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2*d/((n^3 + 6*n^2 + 11*n + 6
)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11
*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d^2/((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624
*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 +
 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*
b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*
d^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n +
5040)*b^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2085 vs. $2(223) = 446$.

Time = 0.28 (sec) , antiderivative size = 2085, normalized size of antiderivative = 9.35

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d^2*
```

n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*n^5*
x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x^7 +
3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4 + 621*
(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 + 225*(b
*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*(b*x + a
)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b*x + a)^n
*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30*(b*x + a)
^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 210*(b*x + a)
^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 1764*(b*x + a)
^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*a*b^6*c^2*d*n
^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*a^2*b^5*c*d^2*
n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^
3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x + a)^n*a^2*b^5*d^
3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a)^n*b^7*d^3*x^7 +
(b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x - 6*(b*x + a)^n*a
^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2 + 36*(b*x + a)^n*a
^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^3*x^3 - 996*(b*x + a)^n
*a^2*b^5*c*d^2*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^3*n^3*x^3 + 1620*(b*x +
a)^n*a*b^6*c*d^2*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^3*n^2*x^4 + 7236*(b*x
+ a)^n*b^7*c*d^2*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^3*n*x^5 + 27*(b*x + a)^n
*a*b^6*c^3*n^5 + 295*(b*x + a)^n*b^7*c^3*n^4*x - 132*(b*x + a)^n*a^2*b^5*c^
2*d*n^4*x + 2451*(b*x + a)^n*a*b^6*c^2*d*n^3*x^2 + 504*(b*x + a)^n*a^3*b^4*
c*d^2*n^3*x^2 + 9336*(b*x + a)^n*b^7*c^2*d*n^2*x^3 - 1824*(b*x + a)^n*a^2*b
^5*c*d^2*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^3*n^2*x^3 + 756*(b*x + a)^n*a*
b^6*c*d^2*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^3*n*x^4 + 3024*(b*x + a)^n*b^7*
c*d^2*x^5 + 295*(b*x + a)^n*a*b^6*c^3*n^4 + 6*(b*x + a)^n*a^3*b^4*c^2*d*n^4
+ 1665*(b*x + a)^n*b^7*c^3*n^3*x - 1074*(b*x + a)^n*a^2*b^5*c^2*d*n^3*x -
72*(b*x + a)^n*a^4*b^3*c*d^2*n^3*x + 4434*(b*x + a)^n*a*b^6*c^2*d*n^2*x^2 +
1980*(b*x + a)^n*a^3*b^4*c*d^2*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^3*n^2*x
^2 + 11388*(b*x + a)^n*b^7*c^2*d*n*x^3 - 1008*(b*x + a)^n*a^2*b^5*c*d^2*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^3*n*x^3 + 1665*(b*x + a)^n*a*b^6*c^3*n^3 + 1
32*(b*x + a)^n*a^3*b^4*c^2*d*n^3 + 5104*(b*x + a)^n*b^7*c^3*n^2*x - 3828*(b
*x + a)^n*a^2*b^5*c^2*d*n^2*x - 936*(b*x + a)^n*a^4*b^3*c*d^2*n^2*x + 2520*
(b*x + a)^n*a*b^6*c^2*d*n*x^2 + 1512*(b*x + a)^n*a^3*b^4*c*d^2*n*x^2 + 360*
(b*x + a)^n*a^5*b^2*d^3*n*x^2 + 5040*(b*x + a)^n*b^7*c^2*d*x^3 + 5104*(b*x
+ a)^n*a*b^6*c^3*n^2 + 1074*(b*x + a)^n*a^3*b^4*c^2*d*n^2 + 72*(b*x + a)^n*
a^5*b^2*c*d^2*n^2 + 8028*(b*x + a)^n*b^7*c^3*n*x - 5040*(b*x + a)^n*a^2*b^5
*c^2*d*n*x - 3024*(b*x + a)^n*a^4*b^3*c*d^2*n*x - 720*(b*x + a)^n*a^6*b*d^3
*n*x + 8028*(b*x + a)^n*a*b^6*c^3*n + 3828*(b*x + a)^n*a^3*b^4*c^2*d*n + 93
6*(b*x + a)^n*a^5*b^2*c*d^2*n + 5040*(b*x + a)^n*b^7*c^3*x + 5040*(b*x + a)
^n*a*b^6*c^3 + 5040*(b*x + a)^n*a^3*b^4*c^2*d + 3024*(b*x + a)^n*a^5*b^2*c*
d^2 + 720*(b*x + a)^n*a^7*d^3)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b
^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)

Mupad [B] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 1144, normalized size of antiderivative = 5.13

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$
$$= \frac{(a+b\,x)^n\,(720\,a^7\,d^3+72\,a^5\,b^2\,c\,d^2\,n^2+936\,a^5\,b^2\,c\,d^2\,n+3024\,a^5\,b^2\,c\,d^2+6\,a^3\,b^4\,c^2\,d\,n^4+132\,a^3\,b^4\,c^2\,d\,n^3+}{b^7\,(n^7+28}$$
$$-\frac{x\,(a+b\,x)^n\,(720\,a^6\,b\,d^3\,n+72\,a^4\,b^3\,c\,d^2\,n^3+936\,a^4\,b^3\,c\,d^2\,n^2+3024\,a^4\,b^3\,c\,d^2\,n+6\,a^2\,b^5\,c^2\,d\,n^5+132\,a^2}{b^7\,(n^7+2}$$
$$+\frac{d^3\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$
$$+\frac{3\,d^2\,x^5\,(a+b\,x)^n\,(-2\,d\,a^2\,n+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,d\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,(-40\,a^4\,d^2\,n-4\,a^2\,b^2\,c\,d\,n^3-52\,a^2\,b^2\,c\,d\,n^2-168\,a^2\,b^2\,c\,d\,n+b^4\,c^2\,n^4+2}{b^4\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n}$$
$$+\frac{a\,d^3\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,a\,d^2\,n\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,(10\,d\,a^2+c\,b^2\,n^2+13\,c\,b^2\,n+42\,c\,b^2)}{b^3\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}$$
$$+\frac{3\,a\,d\,n\,x^2\,(n+1)\,(a+b\,x)^n\,(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+156\,a^2\,b^2\,c\,d\,n+504\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c^2}{b^5\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+}$$

```
[In] int((c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(720*a^7*d^3 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3024*a^5*
b^2*c*d^2 + 5104*a*b^6*c^3*n^2 + 1665*a*b^6*c^3*n^3 + 295*a*b^6*c^3*n^4 + 2
7*a*b^6*c^3*n^5 + a*b^6*c^3*n^6 + 8028*a*b^6*c^3*n + 3828*a^3*b^4*c^2*d*n +
 936*a^5*b^2*c*d^2*n + 1074*a^3*b^4*c^2*d*n^2 + 72*a^5*b^2*c*d^2*n^2 + 132*
a^3*b^4*c^2*d*n^3 + 6*a^3*b^4*c^2*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (x*(a + b*x)^n*(720*a^6*
b*d^3*n - 8028*b^7*c^3*n - 5104*b^7*c^3*n^2 - 1665*b^7*c^3*n^3 - 295*b^7*c^
3*n^4 - 27*b^7*c^3*n^5 - b^7*c^3*n^6 - 5040*b^7*c^3 + 5040*a^2*b^5*c^2*d*n
+ 3024*a^4*b^3*c*d^2*n + 3828*a^2*b^5*c^2*d*n^2 + 936*a^4*b^3*c*d^2*n^2 + 1
074*a^2*b^5*c^2*d*n^3 + 72*a^4*b^3*c*d^2*n^3 + 132*a^2*b^5*c^2*d*n^4 + 6*a^
2*b^5*c^2*d*n^5))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
 + 28*n^6 + n^7 + 5040)) + (d^3*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^
3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (3*d^2*x^5*(a + b*x)^n*(42*b^2*c + b^
2*c*n^2 - 2*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2
*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040
)) + (3*d*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(840*b^4*c^2 - 40*a^4*d^2*n + 638
*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 - 168*a^2*b^2*c
*d*n - 52*a^2*b^2*c*d*n^2 - 4*a^2*b^2*c*d*n^3))/(b^4*(13068*n + 13132*n^2 +
```

```
 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^3*n*x^6*(a +
b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132
*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (3*a*d^2*n*x
^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(10*a^2*d + 42*b^2*c + b^2*c*n^2 +
13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28
*n^6 + n^7 + 5040)) + (3*a*d*n*x^2*(n + 1)*(a + b*x)^n*(120*a^4*d^2 + 840*b
^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 5
04*a^2*b^2*c*d + 156*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))
```

3.362 $\int \frac{(a+bx)^n(c+dx^2)^3}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{(a+bx)^n(c+dx^2)^3}{x}\,dx = -\frac{ad(3b^4c^2+3a^2b^2cd+a^4d^2)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d(3b^4c^2+9a^2b^2cd+5a^4d^2)(a+bx)^{2+n}}{b^6(2+n)} - \frac{ad^2(9b^2c+10a^2d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{d^2(3b^2c+10a^2d)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^3(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^3(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -a*d*(a^4*d^2+3*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(1+n)/b^6/(1+n)+d*(5*a^4*d^2
+9*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(2+n)/b^6/(2+n)-a*d^2*(10*a^2*d+9*b^2*c)*
(b*x+a)^(3+n)/b^6/(3+n)+d^2*(10*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*
d^3*(b*x+a)^(5+n)/b^6/(5+n)+d^3*(b*x+a)^(6+n)/b^6/(6+n)-c^3*(b*x+a)^(1+n)*h
ypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {966, 1634, 67}

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x} dx = -\frac{ad^2(10a^2d+9b^2c)(a+bx)^{n+3}}{b^6(n+3)} + \frac{d^2(10a^2d+3b^2c)(a+bx)^{n+4}}{b^6(n+4)} - \frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)(a+bx)^{n+1}}{b^6(n+1)} + \frac{d(5a^4d^2+9a^2b^2cd+3b^4c^2)(a+bx)^{n+2}}{b^6(n+2)} - \frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)} + \frac{d^3(a+bx)^{n+6}}{b^6(n+6)} - \frac{c^3(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] -((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 + n
))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*(2
 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (d^
2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a + b*x
)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^3*(a
+ b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \frac{(a+bx)^n\left(b^6c^3(6+n)-a^5bd^3(6+n)x+b^2d\left(3b^4c^2-5a^4d^2\right)(6+n)x^2-10a^3b^3d^3(6+n)x^3+b^4d^2\left(3b^2c-10a^2d\right)(6+n)x^4-5ab^5d^3(6+n)x^5\right)}{x}\,dx}{b^6(6+n)}$$

$$= \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \left(-abd\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(6+n)(a+bx)^n + \frac{\left(6b^6c^3+b^6c^3n\right)(a+bx)^n}{x} + bd\left(3b^4c^2+9a^2b^2cd+5 \cdots\right.}{\cdots}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+n}}{b^6(2+n)} - \frac{ad^2\left(9b^2c+10a^2d\right)(a+bx)^{3+n}}{b^6(3+n)} + \frac{d^2\left(3b^2c+10a^2d\right)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^3(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + c^3\int \frac{(a+bx)^n}{x}\,dx$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+n}}{b^6(2+n)} - \frac{ad^2\left(9b^2c+10a^2d\right)(a+bx)^{3+n}}{b^6(3+n)} + \frac{d^2\left(3b^2c+10a^2d\right)(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^3(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^3(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = (a+bx)^{1+n}\left(-\frac{ad(3b^4c^2+3a^2b^2cd+a^4d^2)}{b^6(1+n)} + \frac{d(3b^4c^2+9a^2b^2cd+5a^4d^2)(a+bx)}{b^6(2+n)} - \frac{ad^2(9b^2c+10a^2d)(a+bx)^2}{b^6(3+n)} + \frac{d^2(3b^2c+10a^2d)(a+bx)^3}{b^6(4+n)} - \frac{5ad^3(a+bx)^4}{b^6(5+n)} + \frac{d^3(a+bx)^5}{b^6(6+n)} - \frac{c^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2))/(b^6*(1 +
n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x))/(b^6*(2 + n))
- (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^2)/(b^6*(3 + n)) + (d^2*(3*b^2*c +
10*a^2*d)*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^3*(a + b*x)^4)/(b^6*(5 + n))
+ (d^3*(a + b*x)^5)/(b^6*(6 + n)) - (c^3*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*x)/a])/(a + a*n))
```

Maple [F]

$$\int \frac{(bx+a)^n (d\,x^2+c)^3}{x}dx$$

```
[In] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(dx^2+c)^3(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. 2(226) = 452.

Time = 4.96 (sec) , antiderivative size = 5622, normalized size of antiderivative = 22.85

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3/x,x)
```

```
[Out] 3*c**2*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*
x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)),
(-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3
*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) +
b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b
*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + 3*c*d**2*Piecewise((a**n*x
**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 + 18*a**2*b
**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*x**3*lo
g(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), E
q(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) -
 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log(a/b + x)
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq
(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(2*a*b**4 + 2
*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*x**2/(2
*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*l
og(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b), Eq(n, -1)),
(-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
+ 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**n/(b**4*n**4
 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n*x**2*
(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4
) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3*(a + b*x)**
n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + b**4*n*
*3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b**4*n**4 + 1
```

```
0*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4*(a + b*x)**
n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True)) +
 d**3*Piecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**
3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*
a**4*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*
b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b*
*10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) + 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5
) + 900*a**2*b**3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4
*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5
*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b
**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*
a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 +
60*b**11*x**5), Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*
b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12
*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10
*x**4) - 240*a**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 +
 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360
*a**3*b**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*
x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 +
 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240
*a**2*b**3*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*
x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 +
 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*
a*b**4*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2
 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b
**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (6
0*a**5*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3) + 110*a**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x
**3) + 180*a**4*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 270*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**
8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**
6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(
a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15
```

*a*b**4*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
+ 3*b**5*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
, Eq(n, -4)), (-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x*
*2) - 90*a**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(
a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b
6 + 12*a*b7*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b
6 + 12*a*b7*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b*
*7*x + 6*b**8*x**2) - 5*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**
2) + 2*b**5*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60
*a**5*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**7*x
) + 60*a**4*b*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(1
2*a*b**6 + 12*b**7*x) + 10*a**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**
4*x**4/(12*a*b**6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n,
 -2)), (-a**5*log(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x
3/(3*b3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 +
21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
 + 720*b**6) - 60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) - 60*a**4*b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
4 + 735*b6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b*
*3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
6*n3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**
3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n**6
+ 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6
*n + 720*b**6) - 30*a**2*b**4*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
5 + 175*b6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) - 55*a**2*b**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
6*n4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a*
*2*b**4*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735
*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5*(a
 + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b*
*6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b
6*n5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*
a*b**5*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*
b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b*

```
*6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 225*b**6*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*
x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6), True)) - b**(n + 1)*c**3*n*(a/b + x)**(n + 1)*lerc
hphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b**(n + 1)*c**3*(
a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2
))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(dx^2+c)^3(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(dx^2+c)^3(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^2)^3}{x}\,dx = \int \frac{(d\,x^2+c)^3\,(a+b\,x)^n}{x}\,dx$$

```
[In] int(((c + d*x^2)^3*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)^3*(a + b*x)^n)/x, x)
```

3.363 $\int \frac{x^4(d+ex)^n}{a+cx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{x^4(d+ex)^n}{a+cx^2} dx = \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(-a)^{3/2}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] (-a*e^2+c*d^2)*(e*x+d)^(1+n)/c^2/e^3/(1+n)-2*d*(e*x+d)^(2+n)/c/e^3/(2+n)+(e
*x+d)^(3+n)/c/e^3/(3+n)+1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(-e*(-a)^(1/2)+d*c^
(1/2))-1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2
)/(e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

$= \{1643, 726, 70\}$

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \frac{(cd^2-ae^2)(d+ex)^{n+1}}{c^2e^3(n+1)} + \frac{(-a)^{3/2}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{(-a)^{3/2}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} - \frac{2d(d+ex)^{n+2}}{ce^3(n+2)} + \frac{(d+ex)^{n+3}}{ce^3(n+3)}$$

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((c*d^2 - a*e^2)*(d + e*x)^(1 + n))/(c^2*e^3*(1 + n)) - (2*d*(d + e*x)^(2 +
 n))/(c*e^3*(2 + n)) + (d + e*x)^(3 + n)/(c*e^3*(3 + n)) + ((-a)^(3/2)*(d +
 e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[
c]*d - Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((-a)^(3/2)
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(cd^2-ae^2)(d+ex)^n}{c^2e^2} - \frac{2d(d+ex)^{1+n}}{ce^2} + \frac{(d+ex)^{2+n}}{ce^2} + \frac{a^2(d+ex)^n}{c^2(a+cx^2)}\right) dx$$

$$= \frac{(cd^2 - ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c^2}$$

$$= \frac{(cd^2 - ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)} + \frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right)dx}{c^2}$$

$$= \frac{(cd^2 - ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2c^2} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2c^2}$$

$$= \frac{(cd^2 - ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$$
$$= \frac{(d+ex)^{1+n}\left(\frac{2(cd^2-ae^2)}{e^3(1+n)} - \frac{4cd(d+ex)}{e^3(2+n)} + \frac{2c(d+ex)^2}{e^3(3+n)} + \frac{(-a)^{3/2}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)} + \frac{\sqrt{-a}a\,\text{Hypergeomet}}{(\sqrt{c}}\right.}{2c^2}$$

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(c*d^2 - a*e^2))/(e^3*(1 + n)) - (4*c*d*(d + e*x))/(
e^3*(2 + n)) + (2*c*(d + e*x)^2)/(e^3*(3 + n)) + ((-a)^(3/2)*Hypergeometric
2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[
c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt[-a]*a*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*
(1 + n))))/(2*c^2)
```

Maple [F]

$$\int \frac{x^4 (ex+d)^n}{c\,x^2+a}dx$$

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a),x)
[Out] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((e*x + d)^n*x^4/(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$$

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a),x)
[Out] Integral(x**4*(d + e*x)**n/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^4\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2), x)
```

3.364 $\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{a(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] -d*(e*x+d)^(1+n)/c/e^2/(1+n)+(e*x+d)^(2+n)/c/e^2/(2+n)+1/2*a*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)
/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*a*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)/(1+n)/(e*(-a)^(1/2)+d*
c^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

$= \{1643, 845, 70\}$

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \frac{a(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{a(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d*(d + e*x)^(1 + n))/(c*e^2*(1 + n))) + (d + e*x)^(2 + n)/(c*e^2*(2 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d(d+ex)^n}{ce} + \frac{(d+ex)^{1+n}}{ce} - \frac{ax(d+ex)^n}{c\left(a+cx^2\right)}\right) dx$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int\left(-\frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}-\sqrt{c}x)} + \frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}+\sqrt{c}x)}\right)dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a\int\frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}dx}{2c^{3/2}} - \frac{a\int\frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}dx}{2c^{3/2}}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int\frac{x^3(d+ex)^n}{a+cx^2}dx = \frac{(d+ex)^{1+n}\left(-\frac{2\sqrt{c}(d-e(1+n)x)}{e^2(2+n)} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}(1+n)}$$

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*Sqrt[c]*(d - e*(1 + n)*x))/(e^2*(2 + n)) + (a*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/(Sqrt[c]*d - Sqrt[-a]*e) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(2*c^(3/2
)*(1 + n))
```

Maple [F]

$$\int\frac{x^3(ex+d)^n}{c\,x^2+a}dx$$

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((e*x + d)^n*x^3/(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$$

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a),x)
[Out] Integral(x**3*(d + e*x)**n/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^3\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2), x)
```

3.365 $\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \frac{(d+ex)^{1+n}}{ce(1+n)} + \frac{\sqrt{-a}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{\sqrt{-a}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] (e*x+d)^(1+n)/c/e/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*
c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(-e*(-a)^(1/2)+d*c^(1
/2))-1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/
2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1643, 726, 70}

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \frac{\sqrt{-a}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{\sqrt{-a}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{(d+ex)^{n+1}}{ce(n+1)}$$

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] (d + e*x)^(1 + n)/(c*e*(1 + n)) + (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(
Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(d+ex)^n}{c} - \frac{a(d+ex)^n}{c\left(a+cx^2\right)}\right) dx \\
&= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c} \\
&= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx}{c} \\
&= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2c} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2c} \\
&= \frac{(d+ex)^{1+n}}{ce(1+n)} + \frac{\sqrt{-a}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} \\
&\quad - \frac{\sqrt{-a}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \frac{(d+ex)^{1+n}\left(2(cd^2+ae^2)+e\left(\sqrt{-a}\sqrt{c}d-ae\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)-e\left(\sqrt{-a}\sqrt{\right.\right.}{2ce\left(cd^2+ae^2\right)(1+n)}$$

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(2*(c*d^2 + a*e^2) + e*(Sqrt[-a]*Sqrt[c]*d - a*e)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
- e*(Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*c*e*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]

$$\int \frac{x^2(ex+d)^n}{c\,x^2+a}dx$$

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^2/(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$$

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x)**n/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \frac{x^2\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2), x)
```

3.366 $\int \frac{x(d+ex)^n}{a+cx^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{x(d+ex)^n}{a+cx^2} dx = -\frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] -1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+
d*c^(1/2)))/(1+n)/c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/c^(1/2)
/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {845, 70}

$$\int \frac{x(d+ex)^n}{a+cx^2} dx = -\frac{(d+ex)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{(d+ex)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -1/2*((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) -
```

```
 ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx$$

$$= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{c}}+\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{c}}$$

$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = -\frac{(d+ex)^{1+n}\left(\left(\sqrt{c}d+\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(\sqrt{c}d-\sqrt{-a}e\right)\text{Hyper}\right.}{2\sqrt{c}\left(cd^2+ae^2\right)(1+n)}$$

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -1/2*((d + e*x)^(1 + n)*((Sqrt[c]*d + Sqrt[-a]*e)*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (Sqrt[c]*d - Sqrt
[-a]*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)]))/(Sqrt[c]*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]

$$\int \frac{x(ex+d)^n}{c\,x^2+a}dx$$

[In] int(x*(e*x+d)^n/(c*x^2+a),x)

[Out] int(x*(e*x+d)^n/(c*x^2+a),x)

Fricas [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")

[Out] integral((e*x + d)^n*x/(c*x^2 + a), x)

Sympy [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{x(d+ex)^n}{a+cx^2}\,dx$$

[In] integrate(x*(e*x+d)**n/(c*x**2+a),x)

[Out] Integral(x*(d + e*x)**n/(a + c*x**2), x)

Maxima [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)

Giac [F]

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \frac{x\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int((x*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x*(d + e*x)^n)/(a + c*x^2), x)
```

3.367 $\int \frac{(d+ex)^n}{a+cx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 167

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx=\frac{(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] 1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2)))/(1+n)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyp
ergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/(-a)^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {726, 70}

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx=\frac{(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[In] Int[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
```

```
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)\,dx\\
&= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{-a}}\\
&= \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(1+n)}$$

```
[In] Integrate[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1, 1
 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d + Sqr
t[-a]*e)))/(2*Sqrt[-a]*(1 + n))
```

Maple [F]

$$\int \frac{(ex+d)^n}{c\,x^2+a}dx$$

[In] int((e*x+d)^n/(c*x^2+a),x)

[Out] int((e*x+d)^n/(c*x^2+a),x)

Fricas [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n}{cx^2+a}\,dx$$

[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="fricas")

[Out] integral((e*x + d)^n/(c*x^2 + a), x)

Sympy [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(d+ex)^n}{a+cx^2}\,dx$$

[In] integrate((e*x+d)**n/(c*x**2+a),x)

[Out] Integral((d + e*x)**n/(a + c*x**2), x)

Maxima [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n}{cx^2+a}\,dx$$

[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x + d)^n/(c*x^2 + a), x)

Giac [F]

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n}{cx^2+a}\,dx$$

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \frac{(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int((d + e*x)^n/(a + c*x^2),x)
[Out] int((d + e*x)^n/(a + c*x^2), x)
```

3.368 $\int \frac{(d+ex)^n}{x(a+cx^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\int \frac{(d+ex)^n}{x(a+cx^2)} dx = \frac{\sqrt{c}(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} - \frac{(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{ex}{d}\right)}{ad(1+n)}$$

```
[Out] -(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],1+e*x/d)/a/d/(1+n)+1/2*(e*x+d)^(1+n
)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/
2)/a/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[
2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a/(1+n)/(e*(-a)^(1/2
)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {975, 67, 845, 70}

$$\int \frac{(d+ex)^n}{x(a+cx^2)}\,dx = \frac{\sqrt{c}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\sqrt{c}(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} - \frac{(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{ex}{d}+1\right)}{ad(n+1)}$$

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) +
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) -
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a*d*(1
 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(d+ex)^n}{ax} - \frac{cx(d+ex)^n}{a\,(a+cx^2)}\right) dx \\
&= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a} \\
&= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}-\sqrt{c}x)} + \frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}+\sqrt{c}x)}\right) dx}{a} \\
&= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2a} - \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2a} \\
&= \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} \\
&\quad + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} \\
&\quad - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.91

$$
\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \frac{(d+ex)^{1+n}\left(\left(cd^2+\sqrt{-a}\sqrt{c}de\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(cd^2-\sqrt{-a}\sqrt{c}de\right)\text{Hyp}\ldots\right.}{2ad\,(cd^2+\ldots}
$$

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*((c*d^2 + Sqrt[-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (c*d^2 - Sqrt[-a
]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)] - 2*(c*d^2 + a*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
 1 + (e*x)/d]))/(2*a*d*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)}dx$$

```
[In] int((e*x+d)^n/x/(c*x^2+a),x)
[Out] int((e*x+d)^n/x/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="fricas")
[Out] integral((e*x + d)^n/(c*x^3 + a*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx$$

```
[In] integrate((e*x+d)**n/x/(c*x**2+a),x)
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx=\int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx=\int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x)^n/(x*(a + c*x^2)),x)
[Out] int((d + e*x)^n/(x*(a + c*x^2)), x)
```

3.369 $\int \frac{(d+ex)^n}{x^2(a+cx^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)} dx = \frac{c(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{c(d+ex)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{e(d+ex)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{ad^2(1+n)}$$

```
[Out] e*(e*x+d)^(1+n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a/d^2/(1+n)+1/2*c*(e*x+d)
^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/
(-a)^(3/2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*c*(e*x+d)^(1+n)*hypergeom([1
, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(3/2)/(1+n)/(e*
(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{975, 67, 726, 70\}$

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \frac{c(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} - \frac{c(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{e(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{ex}{d}+1\right)}{ad^2(n+1)}$$

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 +
n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(d+ex)^n}{ax^2} - \frac{c(d+ex)^n}{a(a+cx^2)}\right) dx$$

$$= \frac{\int \frac{(d+ex)^n}{x^2} dx}{a} - \frac{c\int \frac{(d+ex)^n}{a+cx^2} dx}{a}$$

$$= \frac{e(d+ex)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)} - \frac{c\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)} + \frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right) dx}{a}$$

$$= \frac{e(d+ex)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)} - \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x} dx}{2(-a)^{3/2}} - \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x} dx}{2(-a)^{3/2}}$$

$$= \frac{c(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{c(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{e(d+ex)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)} dx = \frac{(d+ex)^{1+n}\left(-\frac{c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae} + \frac{c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae} + \frac{2e\,\text{Hypergeometri}}{}\right.}{2a(1+n)}$$

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*(-((c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[c]*d + a*e)) + (c*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sq
rt[-a]*Sqrt[c]*d - a*e) + (2*e*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)
/d])/d^2))/(2*a*(1 + n))
```

Maple [F]

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)}dx$$

[In] int((e*x+d)^n/x^2/(c*x^2+a),x)

[Out] int((e*x+d)^n/x^2/(c*x^2+a),x)

Fricas [F]

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="fricas")

[Out] integral((e*x + d)^n/(c*x^4 + a*x^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)}\,dx = \text{Timed out}$$

[In] integrate((e*x+d)**n/x**2/(c*x**2+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)

Giac [F]

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx = \int \frac{(d+e\,x)^n}{x^2\,(c\,x^2+a)}\,dx$$

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)),x)
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)), x)
```

3.370 $\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 332

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} + \frac{\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

```
[Out] (e*x+d)^(1+n)/c^2/e/(1+n)+1/2*a*(c*d*x+a*e)*(e*x+d)^(1+n)/c^2/(a*e^2+c*d^2)
/(c*x^2+a)-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(3*c*d^2*(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)-a*d*e*n*c^(
1/2))/c^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(3*c*d^2*
(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)+a*d*e*n*c^(1/2))/c^2/(a*e^2+c*d^2)/(1+n)/
(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

= {1663, 1643, 70}

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$= \frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(n+3)\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)}$$
$$-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{-a}ae^2(n+3)\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)}$$
$$+\frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2(a+cx^2)(ae^2+cd^2)}+\frac{(d+ex)^{n+1}}{c^2e(n+1)}$$

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (d + e*x)^(1 + n)/(c^2*e*(1 + n)) + (a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*
c^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((3*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e*n + S
qrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d - Sqrt[-a
]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((3*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e*n + Sqrt
[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d + Sqrt[-a]*e
)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
```

```
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2(cd^2+ae^2(1+n))}{c^2}+\frac{a^2denx}{c}-2a\left(d^2+\frac{ae^2}{c}\right)x^2\right)}{a+cx^2}\,dx}{2a(cd^2+ae^2)}$$

$$= \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\int \left(-\frac{2a(cd^2+ae^2)(d+ex)^n}{c^2} + \frac{\left(-\frac{a^3den}{c^{3/2}}+\sqrt{-a}\left(\frac{3a^2d^2}{c}+\frac{3a^3e^2}{c^2}+\frac{a^3e^2n}{c^2}\right)\right)(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)} + \frac{\left(\frac{a^3den}{c^{3/2}}+\sqrt{-a}\left(\frac{3a^2d^2}{c}+\frac{3a^3e^2}{c^2}+\frac{a^3e^2n}{c^2}\right)\right)(}{2a(\sqrt{-a}+\sqrt{c}x)}\right.}{2a(cd^2+ae^2)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\left(3\sqrt{-a}cd^2 - a\sqrt{c}den + \sqrt{-a}ae^2(3+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4c^2(cd^2+ae^2)} - \frac{\left(3\sqrt{-a}cd^2 + a\sqrt{c}den + \sqrt{-a}ae^2(3+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4c^2(cd^2+ae^2)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} + \frac{\left(3\sqrt{-a}cd^2 + a\sqrt{c}den + \sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(3\sqrt{-a}cd^2 - a\sqrt{c}den + \sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^2\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.24

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$= \frac{(d+ex)^{1+n}\left(\frac{4}{e+en} + \frac{2a(ae+cdx)}{(cd^2+ae^2)(a+cx^2)} + \frac{4\sqrt{-a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)} - \frac{4\sqrt{-a}\,\text{Hypergeometric2F1}\left(1,1+n,2\right.}{(\sqrt{c}d+\sqrt{-a}e)(1+n}\right.}{}$$

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*(4/(e + e*n) + (2*a*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a +
 c*x^2)) + (4*Sqrt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (4*Sqrt[
-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e
^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(Sq
rt[-a]*(c*d^2 + a*e^2)*(1 + n))))/(4*c^2)
```

Maple [F]

$$\int \frac{x^4 (ex+d)^n}{(c\,x^2+a)^2}dx$$

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{x^4 (d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^4/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 (d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x^4\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.371 $\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 297

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(\sqrt{-a}den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)} - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}$$

```
[Out] 1/2*a*(-e*x+d)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(d*e*n*(-
a)^(1/2)+(-2*c*d^2-a*e^2*(2+n))/c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/
2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e
*(-a)^(1/2)+d*c^(1/2)))*(2*c*d^2+a*e^2*(2+n)+d*e*n*(-a)^(1/2)*c^(1/2))/c^(3
/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

$= \{1663, 845, 70\}$

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx =$$

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)}$$
$$+\frac{(d+ex)^{n+1}\left(\sqrt{-a}den-\frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)}$$
$$+\frac{a(d-ex)(d+ex)^{n+1}}{2c\,(a+cx^2)\,(ae^2+cd^2)}$$

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[
-a]*d*e*n - (2*c*d^2 + a*e^2*(2 + n))/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*
c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((2*c*d^2 + Sqrt[-a]*
Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^(3/2)*(Sqrt[c]
*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
```

```
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2den}{c}-\frac{a\left(2cd^2+ae^2(2+n)\right)x}{c}\right)}{a+cx^2}\,dx}{2a\,(cd^2+ae^2)} \\
&= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \left(\frac{\left(\frac{\sqrt{-a}a^2den}{c}+\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(\frac{\sqrt{-a}a^2den}{c}-\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2a\,(cd^2+ae^2)} \\
&= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4c^{3/2}\,(cd^2+ae^2)} - \frac{\left(\sqrt{-a}den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4c\,(cd^2+ae^2)} \\
&= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} + \frac{\left(\sqrt{-a}den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)\,(1+n)} \\
&\quad - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)\,(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.83

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = -\frac{(d+ex)^{1+n}\left(\frac{2a\sqrt{c}(-d+ex)}{a+cx^2} + \frac{\left(2cd^2-\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{(2cd^2+\sqrt{-a}\sqrt{c}den}{}\right.}{4c^{3/2}\,(cd^2+ae^2)}$$

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/4*((d + e*x)^(1 + n)*((2*a*Sqrt[c]*(-d + e*x))/(a + c*x^2) + ((2*c*d^2 -
 Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(
1 + n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt
[c]*d + Sqrt[-a]*e)*(1 + n))))/(c^(3/2)*(c*d^2 + a*e^2))
```

Maple [F]

$$\int \frac{x^{3} \left(e x +d \right)^{n}}{\left(c \,x^{2}+a \right)^{2}}d x$$

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx=\int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^3/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x^3\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.372 $\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 306

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} + \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(cd^2+\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

```
[Out] -1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+
a*e^2*(1+n)-d*e*n*(-a)^(1/2)*c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(-e*
(-a)^(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1+n)+d*e*n*(-a)^(1/2)*c^(1/2)
)/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

$= \{1663, 845, 70\}$

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$$

$$= \frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)} - \frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4\sqrt{-a}c(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)} - \frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((a*e + c*d*x)*(d + e*x)^(1 + n))/(c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((
c*d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4
*Sqrt[-a]*c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((c*d^2 + S
qrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*Sqrt[-a]
*c*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
```

```
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} - \frac{\int \frac{(d+ex)^n\left(-\frac{a(cd^2+ae^2(1+n))}{c}-adenx\right)}{a+cx^2}\,dx}{2a(cd^2+ae^2)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} - \frac{\int \left(\frac{\left(\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}a(cd^2+ae^2(1+n))}{c}\right)(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)}+\frac{\left(-\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}a(cd^2+ae^2(1+n))}{c}\right)(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right)dx}{2a(cd^2+ae^2)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} - \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4\sqrt{-a}c(cd^2+ae^2)} - \frac{\left(cd^2+\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4\sqrt{-a}c(cd^2+ae^2)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c(cd^2+ae^2)(a+cx^2)} + \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} - \frac{\left(cd^2+\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4\sqrt{-a}c\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.32

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(d+ex)^{1+n}\left(-\frac{2(ae+cdx)}{(cd^2+ae^2)(a+cx^2)}+\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}(\sqrt{c}d-\sqrt{-a}e)(1+n)}-\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}(\sqrt{c}d+\sqrt{-a}e)(1+n)}\right.}{}$$

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a + c*x^2)) + (2*H
ypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]
*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (2*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a]*(Sq
rt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n) + Sqrt[-a]*Sq
rt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c
]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e^2*(-1 + n) - S
qrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/((-a)^(3/2)*(c*d^2
 + a*e^2)*(1 + n))))/(4*c)
```

Maple [F]

$$\int \frac{x^{2} \left(e x +d \right)^{n}}{\left(c \,x^{2}+a \right)^{2}}d x$$

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((e*x + d)^n*x^2/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x^2\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.373 $\int \frac{x(d+ex)^n}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 279

$$\begin{aligned}
&\int \frac{x(d+ex)^n}{(a+cx^2)^2} dx \\
&= -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} \\
&\quad + \frac{e(\sqrt{c}d+\sqrt{-a}e)\, n(d+ex)^{1+n} \,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}\,(\sqrt{c}d-\sqrt{-a}e)\,(cd^2+ae^2)\,(1+n)} \\
&\quad + \frac{e(\sqrt{-a}\sqrt{c}d+ae)\, n(d+ex)^{1+n} \,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}\,(\sqrt{c}d+\sqrt{-a}e)\,(cd^2+ae^2)\,(1+n)}
\end{aligned}$$

```
[Out] -1/2*(-e*x+d)*(e*x+d)^(1+n)/(a*e^2+c*d^2)/(c*x^2+a)+1/4*e*n*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(e*(-a)^
(1/2)+d*c^(1/2))/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/c^(1/2)/(-e*(-a)^(1/2)+d*c^
(1/2))+1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(a*e+d*(-a)^(1/2)*c^(1/2))/a/(a*e^2+c*d^2)/(1+n)/c^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {837, 845, 70}

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{en\left(\sqrt{-a}e+\sqrt{c}d\right)(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)} + \frac{en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)} - \frac{(d-ex)(d+ex)^{n+1}}{2(a+cx^2)(ae^2+cd^2)}$$

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((d - e*x)*(d + e*x)^(1 + n))/((c*d^2 + a*e^2)*(a + c*x^2)) + (e*(Sqrt
[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*Sqrt[-a]*Sqrt[c]*(Sqrt[c]
*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(Sqrt[-a]*Sqrt[c]*d + a*e)*n
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(4*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*
e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\text{integral} = -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-acden+ace^2nx\right)}{a+cx^2}\,dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(-\sqrt{-a}acden-a^2\sqrt{c}e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(-\sqrt{-a}acden+a^2\sqrt{c}e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(e\left(\sqrt{-a}\sqrt{c}d-ae\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4a\sqrt{c}\left(cd^2+ae^2\right)} + \frac{\left(e\left(\sqrt{-a}d+\frac{ae}{\sqrt{c}}\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(\sqrt{c}d+\sqrt{-a}e\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)} + \frac{e\left(\sqrt{-a}\sqrt{c}d+ae\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{(d+ex)^{1+n}\left(-\frac{2ac(d-ex)}{a+cx^2} - \frac{\left(\sqrt{-a}cden-a\sqrt{c}e^2n\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\left(\sqrt{-a}cden+a\sqrt{c}e^2n\right)\text{Hypergeome}}{\left(\sqrt{c}d+\sqrt{-\right.}}\right.}{4ac\left(cd^2+ae^2\right)}$$

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*a*c*(d - e*x))/(a + c*x^2) - ((Sqrt[-a]*c*d*e*n - a
*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqr
t[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*c*d*
e*n + a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*c*(
c*d^2 + a*e^2))
```

Maple [F]

$$\int \frac{x(ex+d)^n}{(c\,x^2+a)^2}dx$$

```
[In] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((e*x + d)^n*x/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{x\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

```
[In] int((x*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.374 $\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 304

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)}$$

```
[Out] 1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*
e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^
(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-
a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used

$= \{755, 845, 70\}$

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = -\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)} + \frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)} + \frac{(d+ex)^{n+1}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

```
[In] Int[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + ((c*d^2 + a*e^
2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-cd^2-ae^2(1-n)+cdenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)} \\
&= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} \\
&\quad - \frac{\int \left(\frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)-a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)+a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2a\left(cd^2+ae^2\right)} \\
&= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{3/2}\left(cd^2+ae^2\right)} \\
&\quad + \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4(-a)^{3/2}\left(cd^2+ae^2\right)} \\
&= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} \\
&\quad - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)} \\
&\quad + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.83

$$
\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{2(ae+cdx)}{a+cx^2} + \frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\sqrt{c}den\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\left(-cd^2+ae^2(-1+n)+\sqrt{-a}\sqrt{\right.}}{}\right.}{4a\left(cd^2+ae^2\right)}
$$

```
[In] Integrate[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 +
n)) + ((-(c*d^2) + a*e^2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2
F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a
]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*(c*d^2 + a*e^2))
```

Maple [F]

$$\int \frac{(ex+d)^n}{(c\,x^2+a)^2}dx$$

[In] int((e*x+d)^n/(c*x^2+a)^2,x)

[Out] int((e*x+d)^n/(c*x^2+a)^2,x)

Fricas [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")

[Out] integral((e*x + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

[In] integrate((e*x+d)**n/(c*x**2+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")

[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)

Giac [F]

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

```
[In] int((d + e*x)^n/(a + c*x^2)^2,x)
[Out] int((d + e*x)^n/(a + c*x^2)^2, x)
```

3.375 $\int \frac{(d+ex)^n}{x(a+cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 489

$$\begin{aligned}
&\int \frac{(d+ex)^n}{x(a+cx^2)^2} dx \\
&= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} \\
&\quad + \frac{\sqrt{c}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} \\
&\quad + \frac{\sqrt{c}e\left(\sqrt{c}d+\sqrt{-a}e\right)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\
&\quad + \frac{\sqrt{c}(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} \\
&\quad - \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d+ae\right)n(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+n)} \\
&\quad - \frac{(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d(1+n)}
\end{aligned}$$

```
[Out] 1/2*c*(-e*x+d)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)-(e*x+d)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+e*x/d)/a^2/d/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n
],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(-e*(-
a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/
4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)
+d*c^(1/2)))*c^(1/2)*(e*(-a)^(1/2)+d*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n
```

```
)/(-e*(-a)^(1/2)+d*c^(1/2))-1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],
(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)*(a*e+d*(-a)^(1/2)*c^(1/2)
)/a^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {975, 67, 837, 845, 70}

$$\begin{aligned}
&\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx \\
&= -\frac{\sqrt{c}en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)} \\
&+\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} \\
&+\frac{\sqrt{c}(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} \\
&-\frac{(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{ex}{d}+1\right)}{a^2d(n+1)} \\
&+\frac{\sqrt{c}en\left(\sqrt{-a}e+\sqrt{c}d\right)(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)} \\
&+\frac{c(d-ex)(d+ex)^{n+1}}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}
\end{aligned}$$

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] (c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c
]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt
[c]*e*(Sqrt[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(Sq
rt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]*Sqr
t[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(c
*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(d+ex)^n}{a^2 x} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{cx(d+ex)^n}{a^2\left(a+cx^2\right)} \right) dx$$

$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx}{a}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} - \frac{c\int\left(-\frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}-\sqrt{c}x)}+\frac{(d+ex)^n}{2\sqrt{c}(\sqrt{-a}+\sqrt{c}x)}\right)dx}{a^2} + \frac{\int\frac{(d+ex)^n(-acden+ace^2nx)}{a+cx^2}dx}{2a^2(cd^2+ae^2)}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} + \frac{\sqrt{c}\int\frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}dx}{2a^2} - \frac{\sqrt{c}\int\frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}dx}{2a^2} + \frac{\int\left(\frac{(-\sqrt{-a}acden-a^2\sqrt{c}e^2n)(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)}+\frac{(-\sqrt{-a}acden+a^2\sqrt{c}e^2n)(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right)dx}{2a^2(cd^2+ae^2)}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(\sqrt{c}d-\sqrt{-a}e)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2(\sqrt{c}d+\sqrt{-a}e)(1+n)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} - \frac{\left(\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d-ae\right)n\right)\int\frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}dx}{4a^2(cd^2+ae^2)} - \frac{\left(\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d+ae\right)n\right)\int\frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}dx}{4a^2(cd^2+ae^2)}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a(cd^2+ae^2)(a+cx^2)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(\sqrt{c}d-\sqrt{-a}e)(1+n)} + \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d-ae\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4a^2(\sqrt{c}d-\sqrt{-a}e)(cd^2+ae^2)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a^2(\sqrt{c}d+\sqrt{-a}e)(1+n)} - \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d+ae\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2(\sqrt{c}d+\sqrt{-a}e)(cd^2+ae^2)(1+n)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \frac{(d+ex)^{1+n}\left(\frac{2ac(d-ex)}{(cd^2+ae^2)(a+cx^2)} - \frac{4\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d+ex}{d}\right)}{d+dn} + \frac{2\sqrt{c}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(\sqrt{c}d-\sqrt{-a}e)(1+n)} + \right.}{}$$

[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]

[Out] ((d + e*x)^(1 + n)*((2*a*c*(d - e*x))/((c*d^2 + a*e^2)*(a + c*x^2)) - (4*Hypergeometric2F1[1, 1 + n, 2 + n, (d + e*x)/d])/(d + d*n) + (2*Sqrt[c]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (2*Sqrt[c]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (Sqrt[c]*e*n*((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*e + (-a)^(3/2)*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (-(Sqrt[-a]*c*d^2) - 2*a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/((c*d^2 + a*e^2)^2*(1 + n))))/(4*a^2)

Maple [F]

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)^2}dx$$

[In] int((e*x+d)^n/x/(c*x^2+a)^2,x)

[Out] int((e*x+d)^n/x/(c*x^2+a)^2,x)

Fricas [F]

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2x}\,dx$$

[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="fricas")

[Out] integral((e*x + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)

Sympy [F]

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx$$

```
[In] integrate((e*x+d)**n/x/(c*x**2+a)**2,x)
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2 x}\,dx$$

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2 x}\,dx$$

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx = \int \frac{(d+e\,x)^n}{x\,(c\,x^2+a)^2}\,dx$$

```
[In] int((d + e*x)^n/(x*(a + c*x^2)^2),x)
[Out] int((d + e*x)^n/(x*(a + c*x^2)^2), x)
```

3.376 $\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 513

$$\begin{aligned}\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx &= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)}\\
&-\frac{c(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}\\
&-\frac{c\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}\\
&+\frac{c(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}\\
&+\frac{c\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}\\
&+\frac{e(d+ex)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d^2(1+n)}\end{aligned}$$

```
[Out] -1/2*c*(c*d*x+a*e)*(e*x+d)^(1+n)/a^2/(a*e^2+c*d^2)/(c*x^2+a)+e*(e*x+d)^(1+n
)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a^2/d^2/(1+n)-1/2*c*(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(
1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n]
,(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(1+n)/(e*(-a)^(1/2)+d
*c^(1/2))+1/4*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(
-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/
2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))-1/4*c*(e*x+d)^(1+n)*hyperge
om([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(
```

```
1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2
)+d*c^(1/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {975, 67, 755, 845, 70, 726}

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx = -\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)}+\frac{e(d+ex)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)}+\frac{c(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)}-\frac{c(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{c(d+ex)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] -1/2*(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
- (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n
)) - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*
Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a
]*e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) +
 (c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)
])/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d
+ e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1
 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
|| GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(d+ex)^n}{a^2x^2} - \frac{c(d+ex)^n}{a\,(a+cx^2)^2} - \frac{c(d+ex)^n}{a^2\,(a+cx^2)} \right) dx$$

$$= \frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx}{a}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} - \frac{c\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)} + \frac{\sqrt{-a}(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right)dx}{a^2} + \frac{c\int \frac{(d+ex)^n(-cd^2-ae^2(1-n)+cdenx)}{a+cx^2}\,dx}{2a^2(cd^2+ae^2)}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} + \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2(-a)^{5/2}} + \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2(-a)^{5/2}} + \frac{c\int \left(\frac{(\sqrt{-a}(-cd^2-ae^2(1-n))-a\sqrt{c}den)(d+ex)^n}{2a(\sqrt{-a}-\sqrt{c}x)} + \frac{(\sqrt{-a}(-cd^2-ae^2(1-n))+a\sqrt{c}den)(d+ex)^n}{2a(\sqrt{-a}+\sqrt{c}x)}\right)dx}{2a^2(cd^2+ae^2)}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}(\sqrt{c}d-\sqrt{-a}e)(1+n)} + \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}(\sqrt{c}d+\sqrt{-a}e)(1+n)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} + \frac{\left(c\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{5/2}(cd^2+ae^2)} + \frac{\left(c\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4(-a)^{5/2}(cd^2+ae^2)}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}(\sqrt{c}d-\sqrt{-a}e)(1+n)} - \frac{c\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{5/2}(\sqrt{c}d-\sqrt{-a}e)(cd^2+ae^2)(1+n)} + \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{5/2}(\sqrt{c}d+\sqrt{-a}e)(1+n)} + \frac{c\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4(-a)^{5/2}(\sqrt{c}d+\sqrt{-a}e)(cd^2+ae^2)(1+n)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx = \frac{1}{4}(d+ex)^{1+n}\left(-\frac{2c(ae+cdx)}{a^2(cd^2+ae^2)(a+cx^2)} + \frac{2c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(-a)^{5/2}\left(-\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{2c\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(-a)^{5/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} + \frac{ac\left(\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\sqrt{c}den\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} - \frac{\left(cd^2-ae^2(-1+n)-\sqrt{-a}\sqrt{c}den\right)\text{Hypergeometric2F}}{\sqrt{c}d+\sqrt{-a}e}\right.}{(-a)^{7/2}(cd^2+ae^2)(1+n)} + \frac{4e\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{ex}{d}\right)}{a^2d^2(1+n)}\right)$$

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*c*(a*e + c*d*x))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
+ (2*c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d -
Sqrt[-a]*e)])/((-a)^(5/2)*(-(Sqrt[c]*d) + Sqrt[-a]*e)*(1 + n)) + (2*c*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]
)/((-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*c*(((c*d^2 - a*e^2*(-1
 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 -
 a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))
/((-a)^(7/2)*(c*d^2 + a*e^2)*(1 + n)) + (4*e*Hypergeometric2F1[2, 1 + n, 2
+ n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))))/4
```

Maple [F]

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)^2}dx$$

```
[In] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
[Out] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2x^2}\,dx$$

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((e*x + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n}{(cx^2+a)^2x^2}\,dx$$

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{(d+ex)^n}{x^2 (a+cx^2)^2} dx = \int \frac{(ex+d)^n}{(cx^2+a)^2 x^2} dx$$

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^n}{x^2 (a+cx^2)^2} dx = \int \frac{(d+e\,x)^n}{x^2\,(c\,x^2+a)^2} dx$$

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)^2),x)
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)^2), x)
```

3.377 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 399

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx = -\frac{cd(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$
$$+\frac{c(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n)))\,(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)}$$
$$-\frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)}+\frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)}$$
$$+\frac{(a^2e^4(2+m+n)(3+m+n)(4+m+n)(5+m+n)+cd^2(1+m)(2+m)\left(cd^2(12+7m+m^2)+2ae^2\right.}{e^4g(1+m)(2+m+n)(3}$$

```
[Out] -c*d*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(1
+m)*(e*x+d)^(1+n)/e^4/g/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)+c*(c*d^2*(m^2+7*m+1
2)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(2+m)*(e*x+d)^(1+n)/e^3/g^2/(3
+m+n)/(4+m+n)/(5+m+n)-c^2*d*(4+m)*(g*x)^(3+m)*(e*x+d)^(1+n)/e^2/g^3/(4+m+n)
/(5+m+n)+c^2*(g*x)^(4+m)*(e*x+d)^(1+n)/e/g^4/(5+m+n)+(a^2*e^4*(2+m+n)*(3+m+
n)*(4+m+n)*(5+m+n)+c*d^2*(1+m)*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*
n+n^2+m*(9+2*n))))*(g*x)^(1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/
e^4/g/(1+m)/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)/((1+e*x/d)^n)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {966, 1637, 81, 68, 66}

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$$
$$=\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a^2}{m+1}+\frac{cd^2(m+2)(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12))}{e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)}\right)\text{Hypergeometric}}{g}$$
$$-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}\left(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12)\right)}{e^4g(m+n+2)(m+n+3)(m+n+4)(m+n+5)}$$
$$+\frac{c(gx)^{m+2}(d+ex)^{n+1}\left(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12)\right)}{e^3g^2(m+n+3)(m+n+4)(m+n+5)}$$
$$-\frac{c^2d(m+4)(gx)^{m+3}(d+ex)^{n+1}}{e^2g^3(m+n+4)(m+n+5)}+\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] -((c*d*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*
(9 + 2*n)))*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^4*g*(2 + m + n)*(3 + m + n)
*(4 + m + n)*(5 + m + n))) + (c*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2
 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e^3*g^2*(3 +
 m + n)*(4 + m + n)*(5 + m + n)) - (c^2*d*(4 + m)*(g*x)^(3 + m)*(d + e*x)^(
1 + n))/(e^2*g^3*(4 + m + n)*(5 + m + n)) + (c^2*(g*x)^(4 + m)*(d + e*x)^(1
 + n))/(e*g^4*(5 + m + n)) + ((a^2/(1 + m) + (c*d^2*(2 + m)*(c*d^2*(12 + 7*
m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n))))/(e^4*(2 + m + n)*
(3 + m + n)*(4 + m + n)*(5 + m + n)))*(g*x)^(1 + m)*(d + e*x)^n*Hypergeomet
ric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1637

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Dist[1/(d*b^q*(
m + n + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q +
 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)^
(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
x] && PolyQ[Px, x] && GtQ[Expon[Px, x], 2]
```

Rubi steps

integral

$$= \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2eg^4(5+m+n)+2aceg^4(5+m+n)x^2-c^2dg^4(4+m)x^3\right)dx}{eg^4(5+m+n)}$$

$$= -\frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2e^2g^7(4+m+n)(5+m+n)+cg^7(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2\right.}{e^2g^7(4+m+n)(5+m+n)}$$

$$= \frac{c(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n)))(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} - \frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2e^3g^9(3+m+n)(4+m+n)(5+m+n)-cdg^9(2+m)\left(cd^2(12+7m+m^2)+2a\right.\right.}{e^3g^9(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)} + \frac{c(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n)))(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} - \frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \left(a^2 + \frac{cd^2(1+m)(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)}{e^4(2+m+n)(3+m+n)(4+m+n)(5+m+n)}\right)\int (gx)^m(d+ex)^n\,dx$$

$$= -\frac{cd(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)} + \frac{c(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n)))(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} - \frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \left(\left(a^2 + \frac{cd^2(1+m)(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)}{e^4(2+m+n)(3+m+n)(4+m+n)(5+m+n)}\right)(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int (gx)^m\left(1+\frac{ex}{d}\right)^n dx$$

$$= -\frac{cd(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)} + \frac{c(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n)))(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} - \frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\left(a^2+\frac{cd^2(1+m)(2+m)\left(cd^2(12+7m+m^2)+2ae^2(20+m^2+9n+n^2+m(9+2n))\right)}{e^4(2+m+n)(3+m+n)(4+m+n)(5+m+n)}\right)(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}{}_2F_1(1+m}{g(1+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$$
$$=\frac{x(gx)^m(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\left(c^2d^4\,\text{Hypergeometric2F1}\left(1+m,-4-n,2+m,-\frac{ex}{d}\right)-4c^2d^4\,\text{Hypergeomet}\right.}{}$$

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c^2*d^4*Hypergeometric2F1[1 + m, -4 - n, 2 + m, -((
e*x)/d)] - 4*c^2*d^4*Hypergeometric2F1[1 + m, -3 - n, 2 + m, -((e*x)/d)] +
6*c^2*d^4*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] + 2*a*c*d^2*e
^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] - 4*c^2*d^4*Hypergeo
metric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] - 4*a*c*d^2*e^2*Hypergeometric2
F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + c^2*d^4*Hypergeometric2F1[1 + m, -n,
 2 + m, -((e*x)/d)] + 2*a*c*d^2*e^2*Hypergeometric2F1[1 + m, -n, 2 + m, -((
e*x)/d)] + a^2*e^4*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^4*(
1 + m)*(1 + (e*x)/d)^n)
```

Maple [F]

$$\int (gx)^m\,(ex+d)^n\,\left(c\,x^2+a\right)^2\,dx$$

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx=\int \left(cx^2+a\right)^2(ex+d)^n(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^n*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.32

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \frac{a^2 d^n g^m x^{m+1} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix} \middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)} + \frac{2acd^n g^m x^{m+3} \Gamma(m+3)\, {}_2F_1\left(\begin{matrix} -n, m+3 \\ m+4 \end{matrix} \middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)} + \frac{c^2 d^n g^m x^{m+5} \Gamma(m+5)\, {}_2F_1\left(\begin{matrix} -n, m+5 \\ m+6 \end{matrix} \middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+6)}$$

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**n*g**m*x**(m + 1)*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp
_polar(I*pi)/d)/gamma(m + 2) + 2*a*c*d**n*g**m*x**(m + 3)*gamma(m + 3)*hype
r((-n, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4) + c**2*d**n*g*
*m*x**(m + 5)*gamma(m + 5)*hyper((-n, m + 5), (m + 6,), e*x*exp_polar(I*pi)
/d)/gamma(m + 6)
```

Maxima [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \int \left(cx^2+a\right)^2 (ex+d)^n (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 dx = \int \left(cx^2+a\right)^2 (ex+d)^n (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 \, dx = \int (g\,x)^m \,\left(c\,x^2+a\right)^2 \,(d+e\,x)^n \, dx$$

```
[In] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n,x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n, x)
```

3.378 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 164

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=-\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)}+\frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)}+\frac{\left(cd^2(1+m)(2+m)+ae^2(2+m+n)(3+m+n)\right)(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\text{Hypergeometric2F1}}{e^2g(1+m)(2+m+n)(3+m+n)}$$

```
[Out] -c*d*(2+m)*(g*x)^(1+m)*(e*x+d)^(1+n)/e^2/g/(2+m+n)/(3+m+n)+c*(g*x)^(2+m)*(e
*x+d)^(1+n)/e/g^2/(3+m+n)+(c*d^2*(1+m)*(2+m)+a*e^2*(2+m+n)*(3+m+n))*(g*x)^(
1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/e^2/g/(1+m)/(2+m+n)/(3+m+n
)/((1+e*x/d)^n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {966, 81, 68, 66}

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$$
$$=\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a}{m+1}+\frac{cd^2(m+2)}{e^2(m+n+2)(m+n+3)}\right)\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{ex}{d}\right)}{g}-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{e^2g(m+n+2)(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] -((c*d*(2 + m)*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^2*g*(2 + m + n)*(3 + m +
 n))) + (c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + ((a/(1 +
```

```
m) + (c*d^2*(2 + m))/(e^2*(2 + m + n)*(3 + m + n)))*(g*x)^(1 + m)*(d + e*x)
^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(aeg^2(3+m+n) - cdg^2(2+m)x\right)\,dx}{eg^2(3+m+n)} \\ &= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} \\ &\quad + \left(a + \frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right)\int (gx)^m(d+ex)^n\,dx\end{aligned}$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(\left(a+\frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right)(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int (gx)^m\left(1+\frac{ex}{d}\right)^n dx$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\left(a+\frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right)(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{ex}{d}\right)}{g(1+m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.69

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right) dx = \frac{x(gx)^m(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\left(cd^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,-\frac{ex}{d}\right)-2cd^2\,\text{Hypergeomet}\right.}{e^2(1+m}$$

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c*d^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*
x)/d)] - 2*c*d^2*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + (c*d
^2 + a*e^2)*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^2*(1 + m)*
(1 + (e*x)/d)^n)
```

Maple [F]

$$\int (gx)^m\,(ex+d)^n\,\left(c\,x^2+a\right)dx$$

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\int \left(cx^2+a\right)(ex+d)^n(gx)^m\,dx$$

[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="fricas")

[Out] integral((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\frac{ad^ng^mx^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n,m+1\\m+2\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)}+\frac{cd^ng^mx^{m+3}\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}-n,m+3\\m+4\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)}$$

[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)

[Out] a*d**n*g**m*x**(m + 1)*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_polar(I*pi)/d)/gamma(m + 2) + c*d**n*g**m*x**(m + 3)*gamma(m + 3)*hyper((-n, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4)

Maxima [F]

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\int \left(cx^2+a\right)(ex+d)^n(gx)^m\,dx$$

[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)

Giac [F]

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\int \left(cx^2+a\right)(ex+d)^n(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="giac")
[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx=\int (g\,x)^m\,\left(c\,x^2+a\right)\,\left(d+e\,x\right)^n\,dx$$

```
[In] int((g*x)^m*(a + c*x^2)*(d + e*x)^n,x)
[Out] int((g*x)^m*(a + c*x^2)*(d + e*x)^n, x)
```

3.379 $\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)}$$

```
[Out] 1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a/g/(1+m)/((1+e*x/d)^n)+1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+m,
x*c^(1/2)/(-a)^(1/2),-e*x/d)/a/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {926, 140, 138}

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\operatorname{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)} + \frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\operatorname{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)}$$

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e
*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(2*a
*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx \\
&= -\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{-a}} \\
&= -\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{-a}}-\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{-a}} \\
&= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)} \\
&\quad +\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2), x]
```

Maple [F]

$$\int \frac{(gx)^m\,(ex+d)^n}{c\,x^2+a}dx$$

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \frac{(g\,x)^m\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2),x)
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2), x)
```

3.380 $\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,2,2+m,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,2,2+m,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$

```
[Out] 1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+
m,x*c^(1/2)/(-a)^(1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e
*x+d)^n*AppellF1(1+m,-n,2,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2))/a^2/g/(1+m)/((1
+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,2,-n,2+m,x*c^(1/2)/(-a)^(
1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {975, 140, 138, 926}

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$
$$= \frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$
$$+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$
$$+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$
$$+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4
*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m
, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 +
 (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((
e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}-cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}+cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{2a\left(-ac-c^2x^2\right)} \right) dx$$

$$= -\frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\sqrt{c}-cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\sqrt{c}+cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{-ac-c^2x^2}\,dx}{2a}$$

$$= -\frac{c\int \left(-\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}-\sqrt{c}x\right)} - \frac{\sqrt{-a}(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx}{2a} - \frac{\left(c(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\left(\sqrt{-a}\sqrt{c}-cx\right)^2}\,dx}{4a} - \frac{\left(c(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\left(\sqrt{-a}\sqrt{c}+cx\right)^2}\,dx}{4a}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4(-a)^{3/2}}$$

$$= \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$
$$+ \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$
$$+ \frac{\left((d+ex)^n \left(1+\frac{ex}{d}\right)^{-n}\right) \int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{3/2}}$$
$$+ \frac{\left((d+ex)^n \left(1+\frac{ex}{d}\right)^{-n}\right) \int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4(-a)^{3/2}}$$

$$= \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$
$$+ \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$
$$+ \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$
$$+ \frac{(gx)^{1+m}(d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)}$$

Mathematica [F]

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]
```

Maple [F]

$$\int \frac{(gx)^m\,(ex+d)^n}{(c\,x^2+a)^2}dx$$

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(gx)^m (d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n (gx)^m}{(cx^2+a)^2}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((e*x + d)^n*(g*x)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (d+ex)^n}{(a+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(gx)^m (d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n (gx)^m}{(cx^2+a)^2}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(gx)^m (d+ex)^n}{(a+cx^2)^2}\,dx = \int \frac{(ex+d)^n (gx)^m}{(cx^2+a)^2}\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx=\int \frac{{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^n}{{\left(c\,x^2+a\right)}^2} \,d x$$

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.381 $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2d(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{ad(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{d(a+bx^2)^{3+p}}{2b^3(3+p)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*d*(b*x^2+a)^(p+1)/b^3/(p+1)-a*d*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*d*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/7*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 272, 45, 372, 371}

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2d(a+bx^2)^{p+1}}{2b^3(p+1)}-\frac{ad(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{d(a+bx^2)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*d*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (d*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int x^5\left(a+bx^2\right)^p\,dx+e\int x^6\left(a+bx^2\right)^p\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)+\left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\\
&\quad+\frac{1}{2}d\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{a^2d(a+bx^2)^{1+p}}{2b^3(1+p)} - \frac{ad(a+bx^2)^{2+p}}{b^3(2+p)} + \frac{d(a+bx^2)^{3+p}}{2b^3(3+p)} + \frac{1}{7}ex^7(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = \frac{1}{14}(a+bx^2)^p\left(\frac{7d(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)} + 2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^5*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((7*d*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*e*x^7*Hypergeometric2F1[7/2
, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F]

$$\int x^5(ex+d)\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = \int (ex+d)\left(bx^2+a\right)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^6 + d*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.

Time = 12.80 (sec) , antiderivative size = 950, normalized size of antiderivative = 7.60

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = \frac{a^p e x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7} + d\begin{cases} \frac{a^p x^6}{6} \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} \\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{}{2b} \end{cases}$$

```
[In] integrate(x**5*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b
**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**
4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*
x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p,
 -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2
), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b)
)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
 + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Maxima [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\big(bx^2+a\big)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^6, x) + 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p
)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d/((p^3 + 6*p^2 + 11*p +
 6)*b^3)
```

Giac [F]

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\big(bx^2+a\big)^p x^5\,dx$$

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx=\int x^5\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^5*(a + b*x^2)^p*(d + e*x), x)
```

3.382 $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2e(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{ae(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{e(a+bx^2)^{3+p}}{2b^3(3+p)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*e*(b*x^2+a)^(p+1)/b^3/(p+1)-a*e*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/5*d*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 372, 371, 272, 45}

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^2e(a+bx^2)^{p+1}}{2b^3(p+1)}-\frac{ae(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{e(a+bx^2)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int x^4(a+bx^2)^p\,dx + e\int x^5(a+bx^2)^p\,dx \\
&= \frac{1}{2}e\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx \\
&= \frac{1}{5}dx^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) \\
&\quad + \frac{1}{2}e\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx, x, x^2\right)
\end{aligned}$$

$$= \frac{a^2e(a+bx^2)^{1+p}}{2b^3(1+p)} - \frac{ae(a+bx^2)^{2+p}}{b^3(2+p)} + \frac{e(a+bx^2)^{3+p}}{2b^3(3+p)} + \frac{1}{5}dx^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx = \frac{1}{10}(a+bx^2)^p\left(\frac{5e(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)} + 2dx^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^4*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((5*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeometric2F1[5/2
, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F]

$$\int x^4(ex+d)\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx = \int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^5 + d*x^4)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.

Time = 8.51 (sec) , antiderivative size = 950, normalized size of antiderivative = 7.60

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx = \frac{a^p d x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + e\left(\begin{cases} \frac{a^p x^6}{6} \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} \\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \end{cases}\right.$$

```
[In] integrate(x**4*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + e*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b
**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**
4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*
x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p,
 -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2
), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b)
)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
 + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Maxima [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)\left(a+bx^2\right)^p\,dx=\int x^4\left(b\,x^2+a\right)^p(d+e\,x)\,dx$$

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x), x)
```

3.383 $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*d*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*d*(b*x^2+a)^(2+p)/b^2/(2+p)+1/5*e*x^
5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 272, 45, 372, 371}

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ad\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{d\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*d*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b
^2*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2
)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int x^3\left(a+bx^2\right)^p\,dx+e\int x^4\left(a+bx^2\right)^p\,dx\\
&=\frac{1}{2}d\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)+\left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&=\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\\
&\quad+\frac{1}{2}d\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,x^2\right)\\
&=-\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{10}\left(a+bx^2\right)^p\left(-\frac{5d(a+bx^2)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2ex^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^3*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*e*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/10
```

Maple [F]

$$\int x^3(ex+d)\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^4 + d*x^3)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(82) = 164$.

Time = 6.90 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.64

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^pex^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+d\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**3*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2
) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x
**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
 sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/
(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2
*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**
2*p + 4*b**2), True))
```

Maxima [F]

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^4, x) + 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)
*(b*x^2 + a)^p*d/((p^2 + 3*p + 2)*b^2)
```

Giac [F]

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^3\,dx$$

[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^3, x)

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx=\int x^3\left(b\,x^2+a\right)^p(d+e\,x)\,dx$$

[In] int(x^3*(a + b*x^2)^p*(d + e*x),x)

[Out] int(x^3*(a + b*x^2)^p*(d + e*x), x)

3.384 $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ae\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*e*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*e*(b*x^2+a)^(2+p)/b^2/(2+p)+1/3*d*x^
3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 372, 371, 272, 45}

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=-\frac{ae\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (e*(a + b*x^2)^(2 + p))/(2*b
^2*(2 + p)) + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int x^2(a+bx^2)^p\,dx + e\int x^3(a+bx^2)^p\,dx\\
&= \frac{1}{2}e\text{Subst}\left(\int x(a+bx)^p\,dx, x, x^2\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{1}{3}dx^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\\
&\quad+\frac{1}{2}e\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx, x, x^2\right)\\
&= -\frac{ae(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{e(a+bx^2)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\frac{1}{6}\left(a+bx^2\right)^p\left(-\frac{3e(a+bx^2)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2dx^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^2*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((-3*e*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/6
```

Maple [F]

$$\int x^2(ex+d)\left(b\,x^2+a\right)^p\,dx$$

```
[In] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^3 + d*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(82) = 164.

Time = 4.56 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.64

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\frac{a^p d x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+e\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + e*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2
) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x
**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
 sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/
(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2
*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**
2*p + 4*b**2), True))
```

Maxima [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int(ex+d)\left(bx^2+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)\left(a+bx^2\right)^p\,dx=\int x^2\left(b\,x^2+a\right)^p(d+e\,x)\,dx$$

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x), x)
```

3.385 $\int x(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*d*(b*x^2+a)^(p+1)/b/(p+1)+1/3*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/
2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {778, 267, 372, 371}

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int x(a+bx^2)^p\,dx + e\int x^2(a+bx^2)^p\,dx\\ &= \frac{d(a+bx^2)^{1+p}}{2b(1+p)} + \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{d(a+bx^2)^{1+p}}{2b(1+p)} + \frac{1}{3}ex^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx = \frac{1}{6}(a+bx^2)^p\left(\frac{3d(a+bx^2)}{b(1+p)} + 2ex^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((3*d*(a + b*x^2))/(b*(1 + p)) + (2*e*x^3*Hypergeometric2F1[
3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/6
```

Maple [F]

$$\int x(ex+d)\left(b\,x^{2}+a\right)^{p}dx$$

```
[In] int(x*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x(d+ex)\left(a+bx^{2}\right)^{p}\,dx=\int(ex+d)\left(bx^{2}+a\right)^{p}x\,dx$$

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + d*x)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int x(d+ex)\left(a+bx^{2}\right)^{p}\,dx=\frac{a^{p}ex^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+d\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\ p\neq-1\\ \log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(e*x+d)*(b*x**2+a)**p,x)
[Out] a**p*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] e*integrate((b*x^2 + a)^p*x^2, x) + 1/2*(b*x^2 + a)^(p + 1)*d/(b*(p + 1))
```

Giac [F]

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx=\int x\left(b\,x^2+a\right)^p(d+e\,x)\,dx$$

```
[In] int(x*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x*(a + b*x^2)^p*(d + e*x), x)
```

3.386 $\int (d+ex)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*e*(b*x^2+a)^(p+1)/b/(p+1)+d*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*
x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {655, 252, 251}

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[In] Int[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(a+bx^2)^{1+p}}{2b(1+p)} + d\int \left(a+bx^2\right)^p\,dx \\
&= \frac{e(a+bx^2)^{1+p}}{2b(1+p)} + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx \\
&= \frac{e(a+bx^2)^{1+p}}{2b(1+p)} + dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int (d+ex)\left(a+bx^2\right)^p\,dx$$
$$= \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(bex^2\left(1+\frac{bx^2}{a}\right)^p + ae\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right) + 2bd(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{2b(1+p)}$$

```
[In] Integrate[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(b*e*x^2*(1 + (b*x^2)/a)^p + a*e*(-1 + (1 + (b*x^2)/a)^p) +
2*b*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int \left(e x +d \right) \left(b \,x^{2}+a \right)^{p}d x$$

```
[In] int((e*x+d)*(b*x^2+a)^p,x)
[Out] int((e*x+d)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x + d)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=a^p d x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+e\left(\begin{cases}\frac{a^px^2}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)*(b*x**2+a)**p,x)
[Out] a**p*d*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + e*Piecewise((
a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\int (ex+d)\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\frac{e\,(b\,x^2+a)^{p+1}}{2\,b\,(p+1)}+\frac{d\,x\,(b\,x^2+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

```
[In] int((a + b*x^2)^p*(d + e*x),x)
[Out] (e*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (d*x*(a + b*x^2)^p*hypergeom([1/2,
-p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.387 $\int \frac{(d+ex)(a+bx^2)^p}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{(d+ex)(a+bx^2)^p}{x}\,dx = ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d*(
b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {778, 272, 67, 252, 251}

$$\int \frac{(d+ex)(a+bx^2)^p}{x}\,dx = ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(a+bx^2)^p}{x}\,dx + e\int (a+bx^2)^p\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx$$

$$= ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int\frac{(d+ex)\,(a+bx^2)^p}{x}\,dx = ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)-\frac{d(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x,x]
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Maple [F]

$$\int\frac{(ex+d)\,(b\,x^2+a)^p}{x}dx$$

```
[In] int((e*x+d)*(b*x^2+a)^p/x,x)
[Out] int((e*x+d)*(b*x^2+a)^p/x,x)
```

Fricas [F]

$$\int\frac{(d+ex)\,(a+bx^2)^p}{x}\,dx=\int\frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((e*x + d)*(b*x^2 + a)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.70 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = a^p e x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p d x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x,x)
```

```
[Out] a**p*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*x**(2*
p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma
(1 - p))
```

Maxima [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = \int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x))/x,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x, x)
```

3.388 $\int \frac{(d+ex)(a+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2} dx = -\frac{d(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] -d*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*e
*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 372, 371, 272, 67}

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2} dx = -\frac{d(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{(a+bx^2)^p}{x^2}\,dx + e\int \frac{(a+bx^2)^p}{x}\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx\end{aligned}$$

$$= -\frac{d(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = -\frac{d(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^2,x]
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Maple [F]

$$\int \frac{(ex+d)(b\,x^2+a)^p}{x^2}dx$$

```
[In] int((e*x+d)*(b*x^2+a)^p/x^2,x)
[Out] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="fricas")
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^2}\,dx = -\frac{a^p d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x} - \frac{b^p e x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**2,x)
[Out] -a**p*d*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*e*x**(
2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gam
ma(1 - p))
```

Maxima [F]

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)}{x^2}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^2,x)
[Out] int(((a + b*x^2)^p*(d + e*x))/x^2, x)
```

3.389 $\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 92

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx = -\frac{e(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] -e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)+1/2*b
*d*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x^2/a)/a^2/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {778, 272, 67, 372, 371}

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx = \frac{bd(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(a+bx^2)^p}{x^3}\,dx + e\int \frac{(a+bx^2)^p}{x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx, x, x^2\right) + \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx$$

$$= -\frac{e(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2)^{1+p} {}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3}\,dx = \frac{1}{2}(a+bx^2)^p\left(-\frac{2e\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2)\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^2}{a}\right)}{a^2(1+p)}\right)$$

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^3,x]
[Out] ((a + b*x^2)^p*((-2*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1
 + (b*x^2)/a)^p) + (b*d*(a + b*x^2)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(a^2*(1 + p))))/2
```

Maple [F]

$$\int \frac{(ex+d)(b\,x^2+a)^p}{x^3}dx$$

```
[In] int((e*x+d)*(b*x^2+a)^p/x^3,x)
[Out] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)(a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="fricas")
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = -\frac{a^p e\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d x^{2p-2}\Gamma(1-p)\, {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{a e^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)}$$

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**3,x)
```

```
[Out] -a**p*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*x**(
2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**
2))/(2*gamma(2 - p))
```

Maxima [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = \int \frac{(ex+d)(bx^2+a)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx=\int \frac{\left(b\,x^2+a\right)^p\,(d+e\,x)}{x^3}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^3,x)
[Out] int(((a + b*x^2)^p*(d + e*x))/x^3, x)
```

3.390 $\int x^5(d+ex)^2 (a+bx^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int x^5(d+ex)^2 (a+bx^2)^p \, dx = \frac{a^2(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^4(1+p)} - \frac{a(2bd^2-3ae^2)(a+bx^2)^{2+p}}{2b^4(2+p)} + \frac{(bd^2-3ae^2)(a+bx^2)^{3+p}}{2b^4(3+p)} + \frac{e^2(a+bx^2)^{4+p}}{2b^4(4+p)} + \frac{2}{7}dex^7(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)-1/2*a*(-3*a*e^2+2*b*d^2)*(
b*x^2+a)^(2+p)/b^4/(2+p)+1/2*(-3*a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2
*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)+2/7*d*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],
[9/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1666, 457, 78, 12, 372, 371}

$$\int x^5(d+ex)^2 (a+bx^2)^p \, dx = \frac{a^2(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^4(p+1)} - \frac{a(2bd^2-3ae^2)(a+bx^2)^{p+2}}{2b^4(p+2)} + \frac{(bd^2-3ae^2)(a+bx^2)^{p+3}}{2b^4(p+3)} + \frac{e^2(a+bx^2)^{p+4}}{2b^4(p+4)} + \frac{2}{7}dex^7(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^
2)^(3 + p))/(2*b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (
2*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
```

```
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int 2dex^6(a+bx^2)^p\,dx+\int x^5(a+bx^2)^p\,(d^2+e^2x^2)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\,(d^2+e^2x)\,dx,x,x^2\right)+(2de)\int x^6(a+bx^2)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2(-bd^2+ae^2)\,(a+bx)^p}{b^3}+\frac{a(-2bd^2+3ae^2)\,(a+bx)^{1+p}}{b^3}+\frac{(bd^2-3ae^2)\,(a+bx)^{2+p}}{b^3}+\frac{e^2(a+bx)^{3+p}}{b^3}\right)dx,x,x^2\right)\\
&\quad+\left(2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{a^2(bd^2-ae^2)\,(a+bx^2)^{1+p}}{2b^4(1+p)}-\frac{a(2bd^2-3ae^2)\,(a+bx^2)^{2+p}}{2b^4(2+p)}\\
&\quad+\frac{(bd^2-3ae^2)\,(a+bx^2)^{3+p}}{2b^4(3+p)}+\frac{e^2(a+bx^2)^{4+p}}{2b^4(4+p)}+\frac{2}{7}dex^7(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09

$$
\begin{aligned}
&\int x^5(d+ex)^2\,(a+bx^2)^p\,dx\\
&= \frac{1}{14}(a+bx^2)^p\left(\frac{7d^2(a+bx^2)\,(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4)}{b^3(1+p)(2+p)(3+p)}\right.\\
&\quad+\frac{7e^2(a+bx^2)\,(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)\,x^4+b^3(6+11p+6p^2+p^3)\,x^6)}{b^4(1+p)(2+p)(3+p)(4+p)}\\
&\quad\left.+4dex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)
\end{aligned}
$$

```
[In] Integrate[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*
p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*e^2*(a + b*x^2)*(-6*a^3 +
 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p^2
+ p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (4*d*e*x^7*Hypergeomet
ric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F]

$$\int x^5 (ex+d)^2 \left(b\,x^2+a\right)^p dx$$

```
[In] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^5 (d+ex)^2 \left(a+bx^2\right)^p \,dx = \int (ex+d)^2 (bx^2+a)^p x^5 \,dx$$

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(163) = 326$.

Time = 17.85 (sec) , antiderivative size = 2883, normalized size of antiderivative = 15.34

$$\int x^5 (d+ex)^2 \left(a+bx^2\right)^p \,dx = \text{Too large to display}$$

```
[In] integrate(x**5*(e*x+d)**2*(b*x**2+a)**p,x)
[Out] 2*a**p*d*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**2
*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3
+ 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b*
*5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b
**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) +
2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
, Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**
2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*
x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**
2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b*
*4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqr
t(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a +
b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*
x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a
```

*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 1
2*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b*
*3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*
b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3
 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + e**2*Piecewise((a**p*x**8/
8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2
 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*b**
4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b
4 + 36*a2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*l
og(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*
b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36
*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x - s
qrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**
6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 +
 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2*b*
*5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b))/
(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3
*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**
4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4 +
8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*
a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x
4) - 12*a2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*
b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**
2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**
4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), Eq(
p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(
x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2)
+ 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**
2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 +
4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(x
 - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/(2*
b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/
(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3
*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*
b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20
*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a +
 b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b*
*4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**
4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4
*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x*
6(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p

```
+ 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 7
0*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b
**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*
x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*
p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70
*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

Maxima [F]

$$\int x^5(d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 (bx^2+a)^p x^5 \, dx$$

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^2*x^7 + 2*
d*e*x^6)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int x^5(d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 (bx^2+a)^p x^5 \, dx$$

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^2 \left(a+bx^2\right)^p \, dx = \int x^5 \left(b\,x^2+a\right)^p (d+e\,x)^2 \, dx$$

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.391 $\int x^4(d+ex)^2 (a+bx^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int x^4(d+ex)^2 (a+bx^2)^p \, dx = \frac{a^2de(a+bx^2)^{1+p}}{b^3(1+p)} + \frac{e^2x^5(a+bx^2)^{1+p}}{b(7+2p)} - \frac{2ade(a+bx^2)^{2+p}}{b^3(2+p)} + \frac{de(a+bx^2)^{3+p}}{b^3(3+p)} - \frac{(5ae^2-bd^2(7+2p))\, x^5(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}$$

```
[Out] a^2*d*e*(b*x^2+a)^(p+1)/b^3/(p+1)+e^2*x^5*(b*x^2+a)^(p+1)/b/(7+2*p)-2*a*d*e
*(b*x^2+a)^(2+p)/b^3/(2+p)+d*e*(b*x^2+a)^(3+p)/b^3/(3+p)-1/5*(5*a*e^2-b*d^2
*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1
+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\int x^4(d+ex)^2 (a+bx^2)^p \, dx = \frac{a^2de(a+bx^2)^{p+1}}{b^3(p+1)} - \frac{2ade(a+bx^2)^{p+2}}{b^3(p+2)} + \frac{de(a+bx^2)^{p+3}}{b^3(p+3)} + \frac{1}{5}x^5(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{5ae^2}{2bp+7b}\right) \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + \frac{e^2x^5(a+bx^2)^{p+1}}{b(2p+7)}$$

```
[In] Int[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*e*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^2*x^5*(a + b*x^2)^(1 + p))
/(b*(7 + 2*p)) - (2*a*d*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (d*e*(a + b*
x^2)^(3 + p))/(b^3*(3 + p)) + ((d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x
^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2dex^5\left(a+bx^2\right)^p\,dx+\int x^4\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{e^2x^5(a+bx^2)^{1+p}}{b(7+2p)}+(2de)\int x^5\left(a+bx^2\right)^p\,dx-\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\int x^4\left(a+bx^2\right)^p\,dx\\
&= \frac{e^2x^5(a+bx^2)^{1+p}}{b(7+2p)}+(de)\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)\\
&\quad-\left(\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{e^2x^5(a+bx^2)^{1+p}}{b(7+2p)}+\frac{1}{5}\left(d^2-\frac{5ae^2}{7b+2bp}\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\right.\\
&\quad\left.-\frac{bx^2}{a}\right)+(de)\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2de(a+bx^2)^{1+p}}{b^3(1+p)}+\frac{e^2x^5(a+bx^2)^{1+p}}{b(7+2p)}-\frac{2ade(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{de(a+bx^2)^{3+p}}{b^3(3+p)}\\
&\quad+\frac{1}{5}\left(d^2-\frac{5ae^2}{7b+2bp}\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx\\
&= \frac{1}{35}\left(a+bx^2\right)^p\left(\frac{35de(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)}\right.\\
&\quad+7d^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\\
&\quad\left.+5e^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)
\end{aligned}$$

```
[In] Integrate[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((35*d*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*d^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (5*e^2*x^7*Hypergeometri
c2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/35
```

Maple [F]

$$\int x^{4}(ex+d)^{2}\left(b\,x^{2}+a\right)^{p}dx$$

```
[In] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2(bx^2+a)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(151) = 302$.

Time = 16.94 (sec) , antiderivative size = 986, normalized size of antiderivative = 5.57

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\frac{a^p d^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^p e^2 x^7 {}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$
$$+2de\left(\begin{cases}\frac{a^px^6}{6}\\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2}\\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}\\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\end{cases}\right.$$

```
[In] integrate(x**4*(e*x+d)**2*(b*x**2+a)**p,x)
```

[Out] a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*
e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 2*d*e*Piec
ewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x
4*log(x - sqrt(-a/b))/(4*a2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**
2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(
p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log
(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log
(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x*
*2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/
b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**
2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*
(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2
*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p
+ 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 2
2*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3), True))

Maxima [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)

Giac [F]

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^4\,\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.392 $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=-\frac{a(bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{(bd^2-2ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+1/2*(-2*a*e^2+b*d^2)*(b*x^2
+a)^(2+p)/b^3/(2+p)+1/2*e^2*(b*x^2+a)^(3+p)/b^3/(3+p)+2/5*d*e*x^5*(b*x^2+a)
^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1666, 457, 78, 12, 372, 371}

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=-\frac{a(bd^2-ae^2)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{(bd^2-2ae^2)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + ((b*d^2 - 2*a*
e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^
3*(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2dex^4\left(a+bx^2\right)^p\,dx+\int x^3\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^4\left(a+bx^2\right)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a\left(-bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^2-2ae^2\right)(a+bx)^{1+p}}{b^2}\right.\right.\\
&\quad\left.\left.+\frac{e^2(a+bx)^{2+p}}{b^2}\right)\,dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= -\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}\\
&\quad+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\begin{aligned}
\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx &= \frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5d^2\left(a+bx^2\right)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}\right.\\
&\quad+\frac{5e^2\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}\\
&\quad\left.+4dex^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)
\end{aligned}$$

```
[In] Integrate[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
p)) + (5*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x
^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (4*d*e*x^5*Hypergeometric2F1[5/2, -p,
7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F]

$$\int x^3(ex+d)^2\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx = \int (ex+d)^2(bx^2+a)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(124) = 248.

Time = 9.55 (sec) , antiderivative size = 1294, normalized size of antiderivative = 8.68

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx = \text{Too large to display}$$

```
[In] integrate(x**3*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**2
*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b*
*3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2
*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*
log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(
-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)),
(-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2
+ 6*b**2*p + 4*b**2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a
**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**
2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/
(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b)
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*
```

```
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b
))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4
*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(
2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4
*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt
(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x*
*4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*
p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/
(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*
x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6
*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**
3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3),
True))
```

Maxima [F]

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^2/((p^2 + 3*p + 2)*
b^2) + integrate((e^2*x^5 + 2*d*e*x^4)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^3\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^2\,dx$$

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.393 $\int x^2(d+ex)^2 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int x^2(d+ex)^2 \left(a+bx^2\right)^p \, dx = -\frac{ade(a+bx^2)^{1+p}}{b^2(1+p)} + \frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + \frac{de(a+bx^2)^{2+p}}{b^2(2+p)} - \frac{(3ae^2-bd^2(5+2p))\, x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{3b(5+2p)}$$

```
[Out] -a*d*e*(b*x^2+a)^(p+1)/b^2/(p+1)+e^2*x^3*(b*x^2+a)^(p+1)/b/(5+2*p)+d*e*(b*x
^2+a)^(2+p)/b^2/(2+p)-1/3*(3*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hypergeom
([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\int x^2(d+ex)^2 \left(a+bx^2\right)^p \, dx = -\frac{ade(a+bx^2)^{p+1}}{b^2(p+1)} + \frac{de(a+bx^2)^{p+2}}{b^2(p+2)} + \frac{1}{3}x^3\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp+5b}\right) \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

```
[In] Int[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -((a*d*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (e^2*x^3*(a + b*x^2)^(1 + p)
)/(b*(5 + 2*p)) + (d*e*(a + b*x^2)^(2 + p))/(b^2*(2 + p)) + ((d^2 - (3*a*e^
2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^
2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int 2dex^3(a+bx^2)^p\,dx + \int x^2(a+bx^2)^p(d^2+e^2x^2)\,dx$$

$$= \frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + (2de)\int x^3(a+bx^2)^p\,dx - \left(-d^2+\frac{3ae^2}{5b+2bp}\right)\int x^2(a+bx^2)^p\,dx$$

$$= \frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + (de)\text{Subst}\left(\int x(a+bx)^p\,dx, x, x^2\right) - \left(\left(-d^2+\frac{3ae^2}{5b+2bp}\right)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p dx$$

$$= \frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + \frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right) + (de)\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx, x, x^2\right)$$

$$= -\frac{ade(a+bx^2)^{1+p}}{b^2(1+p)} + \frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + \frac{de(a+bx^2)^{2+p}}{b^2(2+p)} + \frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{15}\left(a+bx^2\right)^p\left(5d^2x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+\frac{3e\left(-\frac{5d(a+bx^2)(a-b(1+p)x^2)}{b^2}+e(2+3p+p^2)\,x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)}{(1+p)(2+p)}\right)$$

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((5*d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p + (3*e*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/b^2 + (e*(2
 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/((1 + p)*(2 + p))))/15
```

Maple [F]

$$\int x^2(ex+d)^2\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int(ex+d)^2(bx^2+a)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(126) = 252$.

Time = 8.84 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.63

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx = \frac{a^p d^2 x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{a^p e^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5} + 2de\left(\begin{cases}\frac{a^p x^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*
e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 2*d*e*Piec
ewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x*
*2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3
*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b)
)/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**
2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**
2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b
**2*p + 4*b**2), True))
```

Maxima [F]

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx = \int (ex+d)^2\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x^2\,\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.394 $\int x(d+ex)^2 (a+bx^2)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x(d+ex)^2 (a+bx^2)^p \, dx = \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2(1+p)} + \frac{e^2(a+bx^2)^{2+p}}{2b^2(2+p)} + \frac{2}{3}dex^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*e^2*(b*x^2+a)^(2+p)/b^2/(2
+p)+2/3*d*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a
)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1666, 455, 45, 12, 372, 371}

$$\int x(d+ex)^2 (a+bx^2)^p \, dx = \frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{e^2(a+bx^2)^{p+2}}{2b^2(p+2)} + \frac{2}{3}dex^3(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p
, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int 2dex^2\left(a+bx^2\right)^p\,dx+\int x\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(a+bx^2\right)^p\,dx\end{aligned}$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(bd^2-ae^2)(a+bx)^p}{b}+\frac{e^2(a+bx)^{1+p}}{b}\right)dx, x, x^2\right)$$
$$+\left(2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p dx$$
$$= \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{e^2(a+bx^2)^{2+p}}{2b^2(2+p)}$$
$$+\frac{2}{3}dex^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.63

$$\int x(d+ex)^2\left(a+bx^2\right)^p dx$$
$$= \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3b^2x^2\left(1+\frac{bx^2}{a}\right)^p(d^2(2+p)+e^2(1+p)x^2)-3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+3ab\left(e^2\right.\right.}{6b^2(1}$$

```
[In] Integrate[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2)
 - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 4*b^2*d*e*(2 + 3*p + p^2)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b^2*(1 + p)*(2 + p)*(1 + (b
*x^2)/a)^p)
```

Maple [F]

$$\int x(ex+d)^2\left(b\,x^2+a\right)^p dx$$

```
[In] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.61

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{2a^pdex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+d^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$
$$+e^2\left(\begin{cases}\frac{a^px^4}{4} & \text{for}\ b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\ p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for}\ p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d**2
*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p +
1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**2*Piecewise((a*
*p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*l
og(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) +
b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-
a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2
) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*
x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*
b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4
*b**2), True))
```

Maxima [F]

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^2/(b*(p + 1)) + integrate((e^2*x^3 + 2*d*e*x^2)*(
b*x^2 + a)^p, x)
```

Giac [F]

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx=\int (ex+d)^2\left(bx^2+a\right)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx=\int x\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.395 $\int (d+ex)^2 (a+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\begin{aligned}\int (d+ex)^2 (a+bx^2)^p \, dx &= \frac{de(2+p)(a+bx^2)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)(a+bx^2)^{1+p}}{b(3+2p)} \\ &\quad - \frac{(ae^2-bd^2(3+2p))\,x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}\end{aligned}$$

```
[Out] d*e*(2+p)*(b*x^2+a)^(p+1)/b/(2*p^2+5*p+3)+e*(e*x+d)*(b*x^2+a)^(p+1)/b/(3+2*
p)-(a*e^2-b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/
b/(3+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {757, 655, 252, 251}

$$\begin{aligned}\int (d+ex)^2 (a+bx^2)^p \, dx &= x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(d^2 - \frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) \\ &\quad + \frac{e(d+ex)(a+bx^2)^{p+1}}{b(2p+3)} + \frac{de(p+2)(a+bx^2)^{p+1}}{b(p+1)(2p+3)}\end{aligned}$$

```
[In] Int[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (d*e*(2 + p)*(a + b*x^2)^(1 + p))/(b*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)}+\frac{\int\left(-ae^2+bd^2(3+2p)+2bde(2+p)x\right)\left(a+bx^2\right)^p\,dx}{b(3+2p)}\\
&= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)}+\frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)}+\left(d^2-\frac{ae^2}{3b+2bp}\right)\int\left(a+bx^2\right)^p\,dx\\
&= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)}+\frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)}\\
&\quad+\left(\left(d^2-\frac{ae^2}{3b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\,dx
\end{aligned}
$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2 - \frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int (d+ex)^2\left(a+bx^2\right)^p\,dx = \frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3bd^2(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+e\left(3d\left(bx^2\left(1+\frac{bx^2}{a}\right)^p+a\left(-\right.\right.\right.\right.}{3b(1+p)}$$

```
[In] Integrate[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*e*
(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(3*b*(1 + p)*(
1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int (ex+d)^2\left(b\,x^2+a\right)^p dx$$

```
[In] int((e*x+d)^2*(b*x^2+a)^p,x)
[Out] int((e*x+d)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^2\left(a+bx^2\right)^p\,dx = \int (ex+d)^2\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = a^p d^2 x\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^2 x^3\, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + 2de \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e**2*
x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewise
((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int (ex+d)^2 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p \left(d+e\,x\right)^2 dx$$

```
[In] int((a + b*x^2)^p*(d + e*x)^2,x)
[Out] int((a + b*x^2)^p*(d + e*x)^2, x)
```

3.396 $\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx = \frac{e^2(a+bx^2)^{1+p}}{2b(1+p)} + 2dex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{d^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] 1/2*e^2*(b*x^2+a)^(p+1)/b/(p+1)+2*d*e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/
2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+
p],1+b*x^2/a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1666, 457, 81, 67, 12, 252, 251}

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx = -\frac{d^2(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)} + 2dex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e^2(a+bx^2)^{p+1}}{2b(p+1)}$$

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] (e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (2*d*e*x*(a + b*x^2)^p*Hypergeome
tric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int 2de\left(a+bx^2\right)^p\,dx+\int \frac{\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)+(2de)\int \left(a+bx^2\right)^p\,dx\\
&= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{2}d^2\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)\\
&\quad+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+2dex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\\
&\quad-\frac{d^2\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x}\,dx\\
&= \frac{1}{2}\left(a+bx^2\right)^p\left(4dex\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)\right.\\
&\quad\left.+\frac{\left(a+bx^2\right)\left(ae^2-bd^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)\right)}{ab(1+p)}\right)
\end{aligned}$$

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p + ((a + b*x^2)*(a*e^2 - b*d^2*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 + (b*x^2)/a]))/(a*b*(1 + p))))/2
```

Maple [F]

$$\int \frac{(ex+d)^{2} (b\,x^{2}+a)^{p}}{x}dx$$

[In] int((e*x+d)^2*(b*x^2+a)^p/x,x)

[Out] int((e*x+d)^2*(b*x^2+a)^p/x,x)

Fricas [F]

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x}\,dx=\int \frac{(ex+d)^{2}(bx^{2}+a)^{p}}{x}\,dx$$

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x, x)

Sympy [A] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x}\,dx=2a^pdex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pd^2x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}+e^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

[In] integrate((e*x+d)**2*(b*x**2+a)**p/x,x)

[Out] 2*a**p*d*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p)) + e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))

Maxima [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x} dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x} dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x, x)
```

3.397 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x^2} dx = -\frac{d^2(a+bx^2)^{1+p}}{ax} + \frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{de(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{a(1+p)}$$

```
[Out] -d^2*(b*x^2+a)^(p+1)/a/x+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2
, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-d*e*(b*x^2+a)^(p+1)*hypergeom([1, p
+1],[2+p],1+b*x^2/a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 778, 272, 67, 252, 251}

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x^2} dx = \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{a(p+1)}$$

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) -
 (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+bx^2)^{1+p}}{ax} - \frac{\int \frac{(-2ade-(ae^2+bd^2(1+2p))x)(a+bx^2)^p}{x}\,dx}{a} \\
&= -\frac{d^2(a+bx^2)^{1+p}}{ax} + (2de)\int \frac{(a+bx^2)^p}{x}\,dx + \frac{(ae^2+bd^2(1+2p))\int (a+bx^2)^p\,dx}{a} \\
&= -\frac{d^2(a+bx^2)^{1+p}}{ax} + (de)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) \\
&\quad + \frac{\left((ae^2+bd^2(1+2p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\,dx}{a} \\
&= -\frac{d^2(a+bx^2)^{1+p}}{ax} \\
&\quad + \frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a} \\
&\quad - \frac{de(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{a(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx =$$

$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(ad^2(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+ex\left(-ae(1+p)x\,\text{Hypergeo}\right.\right.}{a(1+}$$

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(a*d^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/
a)] + e*x*(-(a*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]) +
 d*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^2)/a])))/(a*(1 + p)*x*(1 + (b*x^2)/a)^p))
```

Maple [F]

$$\int \frac{(ex+d)^2 (b\,x^2+a)^p}{x^2}dx$$

[In] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)

[Out] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)

Fricas [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^2, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2}\,dx = -\frac{a^p d^2 {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d e x^{2p} \Gamma(-p)\, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{a e^{i\pi}}{bx^2}\right)}{\Gamma(1-p)}$$

[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**2,x)

[Out] -a**p*d**2*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*e*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/gamma(1 - p
)

Maxima [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^2} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^2} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^2} dx = \int \frac{(b x^2+a)^p (d+e x)^2}{x^2} dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^2,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.398 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{2de(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(p+1)/a/x^2-2*d*e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2]
,-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*(b*d^2*p+a*e^2)*(b*x^2+a)^(p+1)*hypergeom(
[1, p+1],[2+p],1+b*x^2/a)/a^2/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 778, 372, 371, 272, 67}

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = -\frac{(a+bx^2)^{p+1} (ae^2+bd^2p) \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^2(a+bx^2)^{p+1}}{2ax^2} - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d^2*(a + b*x^2)^(1 + p))/(a*x^2) - (2*d*e*(a + b*x^2)^p*Hypergeometri
c2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - ((a*e^2 + b*d^2*
p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{\int \frac{(-4ade-2(ae^2+bd^2p)x)(a+bx^2)^p}{x^2}\,dx}{2a}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} + (2de)\int \frac{(a+bx^2)^p}{x^2}\,dx + \frac{(ae^2+bd^2p)\int \frac{(a+bx^2)^p}{x}\,dx}{a}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} + \frac{(ae^2+bd^2p)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2a} + \left(2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{(ae^2+bd^2p)\,(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2\,(a+bx^2)^p}{x^3}\,dx$$

$$= \frac{1}{2}(a+bx^2)^p\left(-\frac{4de\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x} - \frac{(a+bx^2)\left(ae^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)-bd^2\,\text{Hypergeometric2F1}\left(2,1+p,2+p\right.\right.}{a^2(1+p)}\right.$$

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^p) - ((a + b*x^2)*(a*e^2*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a]))
/(a^2*(1 + p))))/2
```

Maple [F]

$$\int \frac{(ex+d)^2 (b\,x^2+a)^p}{x^3}dx$$

[In] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)

[Out] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)

Fricas [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)^2(bx^2+a)^p}{x^3}\,dx$$

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^3, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3}\,dx = -\frac{2a^p de_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x} - \frac{b^p d^2 x^{2p-2}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)} - \frac{b^p e^2 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**3,x)

[Out] -2*a**p*d*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d**2*x**(2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(2 - p)) - b**p*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))

Maxima [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^3} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = \int \frac{(ex+d)^2 (bx^2+a)^p}{x^3} dx$$

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (a+bx^2)^p}{x^3} dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x^3} dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^3,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.399 $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 247

$$\begin{aligned}\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx &= \frac{a^2d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}\\ &\quad-\frac{ad(2bd^2-9ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{d(bd^2-9ae^2)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{3de^2\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}\\ &\quad-\frac{e(7ae^2-3bd^2(9+2p))\,x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)}{7b(9+2p)}\end{aligned}$$

```
[Out] 1/2*a^2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)+e^3*x^7*(b*x^2+a)^(p+1
)/b/(9+2*p)-1/2*a*d*(-9*a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*d*(-9*
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+3/2*d*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)-
1/7*e*(7*a*e^2-3*b*d^2*(9+2*p))*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-
b*x^2/a)/b/(9+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1666, 457, 78, 470, 372, 371\}$

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{a^2d(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)} - \frac{ad(2bd^2-9ae^2)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)} + \frac{d(bd^2-9ae^2)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)} + \frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)} + \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2 - \frac{7ae^2}{2bp+9b}\right)\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right) + \frac{e^3x^7\left(a+bx^2\right)^{p+1}}{b(2p+9)}$$

```
[In] Int[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (e^3*x^7*(a
 + b*x^2)^(1 + p))/(b*(9 + 2*p)) - (a*d*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(2*b^4*(2 + p)) + (d*(b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(2*b^4*(3
 + p)) + (3*d*e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (e*(3*d^2 - (7*a*e
^2)/(9*b + 2*b*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x
^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^5\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^6\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx \\
&= \frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right) \\
&\quad+\left(e\left(3d^2-\frac{7ae^2}{9b+2bp}\right)\right)\int x^6\left(a+bx^2\right)^p\,dx \\
&= \frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)} \\
&\quad+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2d\left(-bd^2+3ae^2\right)(a+bx)^p}{b^3}+\frac{ad\left(-2bd^2+9ae^2\right)(a+bx)^{1+p}}{b^3}\right.\right. \\
&\qquad\left.\left.+\frac{\left(bd^3-9ade^2\right)(a+bx)^{2+p}}{b^3}+\frac{3de^2(a+bx)^{3+p}}{b^3}\right)\,dx,x,x^2\right) \\
&\quad+\left(e\left(3d^2-\frac{7ae^2}{9b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,dx
\end{aligned}
$$

$$= \frac{a^2d(bd^2-3ae^2)(a+bx^2)^{1+p}}{2b^4(1+p)} + \frac{e^3x^7(a+bx^2)^{1+p}}{b(9+2p)} - \frac{ad(2bd^2-9ae^2)(a+bx^2)^{2+p}}{2b^4(2+p)} + \frac{d(bd^2-9ae^2)(a+bx^2)^{3+p}}{2b^4(3+p)} + \frac{3de^2(a+bx^2)^{4+p}}{2b^4(4+p)} + \frac{1}{7}e\left(3d^2-\frac{7ae^2}{9b+2bp}\right)x^7(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{1}{126}\left(a+bx^2\right)^p\left(\frac{63d^3(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)x^4\right)}{b^3(1+p)(2+p)(3+p)} + \frac{189de^2(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)x^4+b^3(6+11p+6p^2+p^3)x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)} + 54d^2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right) + 14e^3x^9\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{2},-p,\frac{11}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((63*d^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (189*d*e^2*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (54*d^2*e*x^7*Hyp
ergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (14*e^3*x^9
*Hypergeometric2F1[9/2, -p, 11/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/126
```

Maple [F]

$$\int x^5(ex+d)^3\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3(bx^2+a)^px^5\,dx$$

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(218) = 436.

Time = 31.28 (sec) , antiderivative size = 2919, normalized size of antiderivative = 11.82

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] 3*a**p*d**2*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + a
**p*e**3*x**9*hyper((9/2, -p), (11/2,), b*x**2*exp_polar(I*pi)/a)/9 + d**3*
Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b
**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b*
*2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2
*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4),
 Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x
**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**
4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt
(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b
*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x
**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*
b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**
3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3
+ 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + 3*d*e**2*Piecewise((a**p*x*
*8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x
```

```
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*
b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**
3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**
2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 +
12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*
b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 +
 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x
- sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**
2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b
))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b
**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*
x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4
 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 +
 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
 4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*
x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*
x**4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*
b**6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2
 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4),
Eq(p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*l
og(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**
2) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*
x**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4
 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*lo
g(x - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/
(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)*
*p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a
**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 1
00*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 +
 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(
a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48
*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*
b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b
**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p
*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4
*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
+ 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(
2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4
*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b*
*4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

Maxima [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^5\,dx$$

[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")

[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6)*(b*x^2 + a)^p, x)

Giac [F]

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^5\,dx$$

[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^5, x)

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^5\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^3\,dx$$

[In] int(x^5*(a + b*x^2)^p*(d + e*x)^3,x)

[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^3, x)

3.400 $\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 249

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{a^2e(3bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} - \frac{3ae(2bd^2-ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{3e(bd^2-ae^2)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{e^3\left(a+bx^2\right)^{4+p}}{2b^4(4+p)} - \frac{d(15ae^2-bd^2(7+2p))\,x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}$$

```
[Out] 1/2*a^2*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(p+1)/b^4/(p+1)+3*d*e^2*x^5*(b*x^2+a)^
(p+1)/b/(7+2*p)-3/2*a*e*(-a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+3/2*e*(-
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*e^3*(b*x^2+a)^(4+p)/b^4/(4+p)-1/
5*d*(15*a*e^2-b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1666, 470, 372, 371, 457, 78\}$

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{a^2e(3bd^2-ae^2)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)} - \frac{3ae(2bd^2-ae^2)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)} + \frac{3e(bd^2-ae^2)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)} + \frac{e^3(a+bx^2)^{p+4}}{2b^4(p+4)} + \frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2 - \frac{15ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + \frac{3de^2x^5(a+bx^2)^{p+1}}{b(2p+7)}$$

```
[In] Int[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*d*e^2*x^
5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(2*b^
4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (d*(d^2 - (15*a*e^
2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^5\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx \\
&= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right) \\
&\quad+\left(d\left(d^2-\frac{15ae^2}{7b+2bp}\right)\right)\int x^4\left(a+bx^2\right)^p\,dx \\
&= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} \\
&\quad+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2e\left(-3bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{3ae\left(-2bd^2+ae^2\right)(a+bx)^{1+p}}{b^3}\right.\right. \\
&\quad\left.\left.+\frac{3\left(bd^2e-ae^3\right)(a+bx)^{2+p}}{b^3}+\frac{e^3(a+bx)^{3+p}}{b^3}\right)\,dx,x,x^2\right) \\
&\quad+\left(d\left(d^2-\frac{15ae^2}{7b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx
\end{aligned}$$

$$= \frac{a^2e(3bd^2-ae^2)(a+bx^2)^{1+p}}{2b^4(1+p)} + \frac{3de^2x^5(a+bx^2)^{1+p}}{b(7+2p)} - \frac{3ae(2bd^2-ae^2)(a+bx^2)^{2+p}}{2b^4(2+p)} + \frac{3e(bd^2-ae^2)(a+bx^2)^{3+p}}{2b^4(3+p)} + \frac{e^3(a+bx^2)^{4+p}}{2b^4(4+p)} + \frac{1}{5}d\left(d^2-\frac{15ae^2}{7b+2bp}\right)x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{1}{70}(a+bx^2)^p\left(\frac{105d^2e(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)} + \frac{35e^3(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)\,x^4+b^3(6+11p+6p^2+p^3)\,x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)} + 14d^3x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + 30de^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((105*d^2*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2
+ 3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (35*e^3*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyper
geometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (30*d*e^2*x^7
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F]

$$\int x^4(ex+d)^3\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^px^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(216) = 432$.

Time = 21.45 (sec) , antiderivative size = 2919, normalized size of antiderivative = 11.72

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*a**
p*d*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 3*d**2
*e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2
*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
+ 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**
4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a
**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**
4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x
**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*
b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + s
qrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*
p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
 a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p
**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p*
*3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + e**3*Piecewise((a**p*x**
8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x*
```

```
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*b
**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3
*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2
*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 1
2*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 +
36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x -
 sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x
**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2
 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2*
b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b)
)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b*
*3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4
+ 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 +
8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6
*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x
**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x
**4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b
**6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2
+ 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), E
q(p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*lo
g(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2
) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x
**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4
+ 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log
(x - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/(
2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**
p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a*
*3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 10
0*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 +
20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a
 + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*
b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b
**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b*
*4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*
x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*
p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2
*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*
p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**
4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

Maxima [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^4\,dx$$

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^4\left(b\,x^2+a\right)^p(d+e\,x)^3\,dx$$

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.401 $\int x^3(d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\int x^3(d+ex)^3 \left(a+bx^2\right)^p \, dx = -\frac{ad(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5(a+bx^2)^{1+p}}{b(7+2p)}+\frac{d(bd^2-6ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{3de^2(a+bx^2)^{3+p}}{2b^3(3+p)}-\frac{e(5ae^2-3bd^2(7+2p))\,x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)}{5b(7+2p)}$$

```
[Out] -1/2*a*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+e^3*x^5*(b*x^2+a)^(p+1)
/b/(7+2*p)+1/2*d*(-6*a*e^2+b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+3/2*d*e^2*(b*x^
2+a)^(3+p)/b^3/(3+p)-1/5*e*(5*a*e^2-3*b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hyperg
eom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1666, 457, 78, 470, 372, 371\}$

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx = -\frac{ad(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)} + \frac{d(bd^2-6ae^2)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)} + \frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)} + \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2 - \frac{5ae^2}{2bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + \frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

```
[In] Int[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^3*x^5*(
a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p
))/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*(3*
d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p,
 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \int x^3(a+bx^2)^p\left(d^3+3de^2x^2\right)\,dx + \int x^4(a+bx^2)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$= \frac{e^3x^5(a+bx^2)^{1+p}}{b(7+2p)} + \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^3+3de^2x\right)\,dx, x, x^2\right) + \left(e\left(3d^2-\frac{5ae^2}{7b+2bp}\right)\right)\int x^4(a+bx^2)^p\,dx$$

$$= \frac{e^3x^5(a+bx^2)^{1+p}}{b(7+2p)} + \frac{1}{2}\text{Subst}\left(\int\left(\frac{ad(-bd^2+3ae^2)(a+bx)^p}{b^2} + \frac{(bd^3-6ade^2)(a+bx)^{1+p}}{b^2} + \frac{3de^2(a+bx)^{2+p}}{b^2}\right)dx, x, x^2\right) + \left(e\left(3d^2-\frac{5ae^2}{7b+2bp}\right)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx$$

$$= -\frac{ad(bd^2-3ae^2)(a+bx^2)^{1+p}}{2b^3(1+p)} + \frac{e^3x^5(a+bx^2)^{1+p}}{b(7+2p)} + \frac{d(bd^2-6ae^2)(a+bx^2)^{2+p}}{2b^3(2+p)} + \frac{3de^2(a+bx^2)^{3+p}}{2b^3(3+p)} + \frac{1}{5}e\left(3d^2-\frac{5ae^2}{7b+2bp}\right)x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{35d^3(a+bx^2)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{105de^2(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+42d^2ex^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+10e^3x^7\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d^3*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
 p)) + (105*d*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p
^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (42*d^2*e*x^5*Hypergeometric2F1[5
/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (10*e^3*x^7*Hypergeometric2
F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F]

$$\int x^3(ex+d)^3\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int(ex+d)^3\left(bx^2+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 1329, normalized size of antiderivative = 6.42

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx = \text{Too large to display}$$

```
[In] integrate(x**3*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] 3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a
**p*e**3*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**3*P
iecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3
*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b
**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*lo
g(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a
/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-
a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*
x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2
*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 +
6*b**2*p + 4*b**2), True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2
*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a
**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**
2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-
a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a
/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))
/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sq
rt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) +
x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b
**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
, True))
```

Maxima [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^3/((p^2 + 3*p + 2)*
b^2) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^3\left(b\,x^2+a\right)^p(d+e\,x)^3\,dx$$

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.402 $\int x^2(d+ex)^3 (a+bx^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int x^2(d+ex)^3 (a+bx^2)^p \, dx = -\frac{ae(3bd^2-ae^2)(a+bx^2)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)} + \frac{e(3bd^2-2ae^2)(a+bx^2)^{2+p}}{2b^3(2+p)}+\frac{e^3(a+bx^2)^{3+p}}{2b^3(3+p)} - \frac{d(9ae^2-bd^2(5+2p))x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{3b(5+2p)}$$

```
[Out] -1/2*a*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(p+1)/b^3/(p+1)+3*d*e^2*x^3*(b*x^2+a)^(
p+1)/b/(5+2*p)+1/2*e*(-2*a*e^2+3*b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e^3*(
b*x^2+a)^(3+p)/b^3/(3+p)-1/3*d*(9*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hype
rgeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1666, 470, 372, 371, 457, 78\}$

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx = -\frac{ae(3bd^2-ae^2)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)} + \frac{e(3bd^2-2ae^2)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)} + \frac{e^3(a+bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2 - \frac{9ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + \frac{3de^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

```
[In] Int[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (3*d*e^2*x
^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^
(2 + p))/(2*b^3*(2 + p)) + (e^3*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*(
d^2 - (9*a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p,
 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2(a+bx^2)^p\left(d^3+3de^2x^2\right)\,dx+\int x^3(a+bx^2)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= \frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right)\\
&\quad+\left(d\left(d^2-\frac{9ae^2}{5b+2bp}\right)\right)\int x^2(a+bx^2)^p\,dx\\
&= \frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ae(-3bd^2+ae^2)(a+bx)^p}{b^2}\right.\right.\\
&\quad\left.\left.+\frac{(3bd^2e-2ae^3)(a+bx)^{1+p}}{b^2}+\frac{e^3(a+bx)^{2+p}}{b^2}\right)dx,x,x^2\right)\\
&\quad+\left(d\left(d^2-\frac{9ae^2}{5b+2bp}\right)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p dx\\
&= -\frac{ae(3bd^2-ae^2)(a+bx^2)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{e(3bd^2-2ae^2)(a+bx^2)^{2+p}}{2b^3(2+p)}\\
&\quad+\frac{e^3(a+bx^2)^{3+p}}{2b^3(3+p)}+\frac{1}{3}d\left(d^2-\frac{9ae^2}{5b+2bp}\right)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\right.\\
&\quad\left.-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\frac{1}{30}\left(a+bx^2\right)^p\left(\frac{45d^2e(a+bx^2)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{15e^3(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)}+10d^3x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+18de^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((45*d^2*e*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2
 + p)) + (15*e^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^
2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (10*d^3*x^3*Hypergeometric2F1[3/2,
 -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (18*d*e^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/30
```

Maple [F]

$$\int x^2(ex+d)^3\left(b\,x^2+a\right)^p dx$$

```
[In] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int(ex+d)^3\left(bx^2+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(180) = 360.

Time = 11.37 (sec) , antiderivative size = 1329, normalized size of antiderivative = 6.33

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x**2*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 3*a**
p*d*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*d**2
*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*
b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 +
 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**
2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqr
t(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1))
, (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a
+ b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**
p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2), True)) + e**3*Piecewise((a**p*x**6/6, Eq(b, 0)), (2
*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a
**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**
2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-
a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a
/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))
/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sq
rt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) +
x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b
**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
, True))
```

Maxima [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3\left(bx^2+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^2\left(b\,x^2+a\right)^p(d+e\,x)^3\,dx$$

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.403 $\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 167

$$\begin{aligned}\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx &= \frac{d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{3de^2(a+bx^2)^{2+p}}{2b^2(2+p)}\\ &\quad-\frac{e(ae^2-bd^2(5+2p))\,x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{b(5+2p)}\end{aligned}$$

```
[Out] 1/2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/(p+1)+e^3*x^3*(b*x^2+a)^(p+1)/b/
(5+2*p)+3/2*d*e^2*(b*x^2+a)^(2+p)/b^2/(2+p)-e*(a*e^2-b*d^2*(5+2*p))*x^3*(b*
x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1666, 455, 45, 470, 372, 371}

$$\begin{aligned}\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx &= \frac{d(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2(a+bx^2)^{p+2}}{2b^2(p+2)}\\ &\quad+ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2\right.\\ &\quad\left.-\frac{ae^2}{2bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\\ &\quad+\frac{e^3x^3(a+bx^2)^{p+1}}{b(2p+5)}\end{aligned}$$

```
[In] Int[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b
*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)
) + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^2\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int (a+bx)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)\\
&\quad+\left(3e\left(d^2-\frac{ae^2}{5b+2bp}\right)\right)\int x^2\left(a+bx^2\right)^p\,dx\\
&= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(bd^3-3ade^2\right)(a+bx)^p}{b}\right.\right.\\
&\qquad\left.\left.+\frac{3de^2(a+bx)^{1+p}}{b}\right)\,dx,x,x^2\right)\\
&\quad+\left(3e\left(d^2-\frac{ae^2}{5b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}\\
&\quad+e\left(d^2-\frac{ae^2}{5b+2bp}\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx$$
$$=\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(5d\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p\left(d^2(2+p)+3e^2(1+p)x^2\right)-3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+ab\left(3\right.\right.\right.}{}$$

```
[In] Integrate[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(5*d*(b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + 3*e^2*(1 + p)
*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(3*e^2*p*x^2*(1 + (b*x^2)/
a)^p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p))) + 10*b^2*d^2*e*(2 + 3*p + p^2
)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 2*b^2*e^3*(2 + 3*p +
p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(10*b^2*(1 + p)*(2
 + p)*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int x(ex+d)^3 \left(b\,x^2+a\right)^p dx$$

```
[In] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
[Out] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p x \, dx$$

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.63

$$\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx = a^p d^2 e x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d^3 \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + 3de^2 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\ b=0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\ p=-2 \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\ p=-1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x*(e*x+d)**3*(b*x**2+a)**p,x)
[Out] a**p*d**2*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + a**p*
e**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**3*Piece
wise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne
```

```
(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + 3*d*e**2*Piecewise((a**p
*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*
x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/
b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2)
- a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x*
*2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b
**2), True))
```

Maxima [F]

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3(bx^2+a)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^3/(b*(p + 1)) + integrate((e^3*x^4 + 3*d*e^2*x^3
+ 3*d^2*e*x^2)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int (ex+d)^3(bx^2+a)^p x\,dx$$

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x\left(b\,x^2+a\right)^p\,(d+e\,x)^3\,dx$$

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.404 $\int (d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 176

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = -\frac{e\left(ae^2-3bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)(2+p)}+\frac{3de^2x\left(a+bx^2\right)^{1+p}}{b(3+2p)}+\frac{e^3x^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{d\left(3ae^2-bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}$$

```
[Out] -1/2*e*(a*e^2-3*b*d^2*(2+p))*(b*x^2+a)^(p+1)/b^2/(p+1)/(2+p)+3*d*e^2*x*(b*x
^2+a)^(p+1)/b/(3+2*p)+1/2*e^3*x^2*(b*x^2+a)^(p+1)/b/(2+p)-d*(3*a*e^2-b*d^2*
(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*
x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {757, 794, 252, 251}

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = -\frac{e\left(a+bx^2\right)^{p+1}\left((2p+3)\left(ae^2-bd^2(2p+5)\right)-2bde(p+1)(p+3)x\right)}{2b^2(p+2)\left(2p^2+5p+3\right)} + dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

```
[In] Int[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (e*((3 + 2*p)*(a*e^2 -
b*d^2*(5 + 2*p)) - 2*b*d*e*(1 + p)*(3 + p)*x)*(a + b*x^2)^(1 + p))/(2*b^2*(
2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2
)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e(d+ex)^2\,(a+bx^2)^{1+p}}{2b(2+p)}\\ &\quad+\frac{\int (d+ex)\,(-2(ae^2-bd^2(2+p))+2bde(3+p)x)\,(a+bx^2)^p\,dx}{2b(2+p)}\end{aligned}$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x)\left(a+bx^2\right)^{1+p}}{2b^2(2+p)\left(3+5p+2p^2\right)} + \left(d\left(d^2-\frac{3ae^2}{3b+2bp}\right)\right)\int \left(a+bx^2\right)^p\,dx$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x)\left(a+bx^2\right)^{1+p}}{2b^2(2+p)\left(3+5p+2p^2\right)} + \left(d\left(d^2-\frac{3ae^2}{3b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x)\left(a+bx^2\right)^{1+p}}{2b^2(2+p)\left(3+5p+2p^2\right)} + d\left(d^2-\frac{3ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int (d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(2b^2d^3\left(2+3p+p^2\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+e\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p\right.\right.}{}$$

```
[In] Integrate[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(2*b^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*x^2)/a)] + e*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p)*
x^2) - a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ 3*d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*d*e*(2 + 3*p + p^2)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)*(1 +
 (b*x^2)/a)^p)
```

Maple [F]

$$\int (ex+d)^3 \left(b\,x^2+a\right)^p dx$$

```
[In] int((e*x+d)^3*(b*x^2+a)^p,x)
[Out] int((e*x+d)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(148) = 296$.

Time = 5.71 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.48

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = a^p d^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$
$$+ 3d^2 e \left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$
$$+ e^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p,x)
[Out] a**p*d**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*e**
2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piecew
```

```
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**3*Piecewise((a**p*x**4
/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log(x +
sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*
log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/b))/(
2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a*l
og(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**
p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p*
*2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2
*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2),
 True))
```

Maxima [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int (ex+d)^3 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^3 \,dx$$

```
[In] int((a + b*x^2)^p*(d + e*x)^3,x)
[Out] int((a + b*x^2)^p*(d + e*x)^3, x)
```

3.405 $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx = \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)} - \frac{e(ae^2-3bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)} - \frac{d^3(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] 3/2*d*e^2*(b*x^2+a)^(p+1)/b/(p+1)+e^3*x*(b*x^2+a)^(p+1)/b/(3+2*p)-e*(a*e^2-
3*b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p
)/((1+b*x^2/a)^p)-1/2*d^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/
a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{1666, 457, 81, 67, 396, 252, 251\}$

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} dx = -\frac{d^3(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left(1, p+1, p+2, \frac{bx^2}{a}+1\right)}{2a(p+1)} + ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3d^2 - \frac{ae^2}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^2}{a}\right) + \frac{3de^2(a+bx^2)^{p+1}}{2b(p+1)} + \frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] (3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(
b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a+bx^2)^p\,(d^3+3de^2x^2)}{x}\,dx+\int (a+bx^2)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= \frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)}+\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p\,(d^3+3de^2x)}{x}\,dx,x,x^2\right)\\
&\quad+\left(e\left(3d^2-\frac{ae^2}{3b+2bp}\right)\right)\int (a+bx^2)^p\,dx\\
&= \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)}+\frac{1}{2}d^3\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)\\
&\quad+\left(e\left(3d^2-\frac{ae^2}{3b+2bp}\right)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)}\\
&\quad+e\left(3d^2-\frac{ae^2}{3b+2bp}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\\
&\quad-\frac{d^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} \, dx$$
$$= \frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(18abd^2e(1+p)x \text{ Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - 3bd^3(a+bx^2)\left(1+\frac{bx^2}{a}\right)\right.}{}$$

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x,x]
[Out] ((a + b*x^2)^p*(18*a*b*d^2*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] - 3*b*d^3*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*x^2)/a] + a*e^2*(9*d*(a + b*x^2)*(1 + (b*x^2)/a)^p + 2*b*e
*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(6*a*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int \frac{(ex+d)^3 (b\,x^2+a)^p}{x}dx$$

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x,x)
[Out] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} \, dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x} \, dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} dx = 3a^p d^2 e x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} - \frac{b^p d^3 x^{2p} \Gamma(-p) {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)} + 3de^2 \left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x,x)
```

```
[Out] 3*a**p*d**2*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e
**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - b**p*d**3*x
**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*
gamma(1 - p)) + 3*d*e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
+ b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), Tru
e))
```

Maxima [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x} dx = \int \frac{(ex+d)^3 (bx^2+a)^p}{x} dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x, x)
```

3.406 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(d+ex)^3(a+bx^2)^p}{x^2} dx = \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{d(3ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] 1/2*e^3*(b*x^2+a)^(p+1)/b/(p+1)-d^3*(b*x^2+a)^(p+1)/a/x+d*(3*a*e^2+b*d^2*(1
+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-
3/2*d^2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{1821, 1666, 457, 81, 67, 12, 252, 251\}$

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx$$
$$= -\frac{d^3(a+bx^2)^{p+1}}{ax} + \frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)} + \frac{e^3(a+bx^2)^{p+1}}{2b(p+1)}$$

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] (e^3*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d^3*(a + b*x^2)^(1 + p))/(a*x) +
 (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (3*d^2*e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^3(a+bx^2)^{1+p}}{ax} - \frac{\int \frac{(a+bx^2)^p(-3ad^2e-d(3ae^2+bd^2(1+2p))x-ae^3x^2)}{x}\,dx}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{\int d(3ae^2+bd^2(1+2p))\,(a+bx^2)^p\,dx}{a} - \frac{\int \frac{(a+bx^2)^p(-3ad^2e-ae^3x^2)}{x}\,dx}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{ax} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(-3ad^2e-ae^3x)}{x}\,dx, x, x^2\right)}{2a} \\
&\quad + \frac{(d(3ae^2+bd^2(1+2p)))\int (a+bx^2)^p\,dx}{a}
\end{aligned}
$$

$$= \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{1}{2}(3d^2e)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \frac{\left(d(3ae^2+bd^2(1+2p))\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p dx}{a}$$

$$= \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{d(3ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx = \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(-2abd^3(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+ex\Big(6abde(1+p)x\,\text{Hype}\right.}{}$$

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*a*b*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^
2)/a)] + e*x*(6*a*b*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + (a + b*x^2)*(1 + (b*x^2)/a)^p*(a*e^2 - 3*b*d^2*Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*x^2)/a]))))/(2*a*b*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int \frac{(ex+d)^3\,(b\,x^2+a)^p}{x^2}dx$$

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^2}\,dx=\int \frac{(ex+d)^3(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^2}\,dx=-\frac{a^p d^3\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}+3a^pde^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{3b^pd^2ex^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}+e^3\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**3*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + 3*a**p*d
*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - 3*b**p*d**2*e*
x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2
*gamma(1 - p)) + e**3*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b
*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^2}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x^2}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^2,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.407 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx = -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{3d^2e(a+bx^2)^{1+p}}{ax} + \frac{e(ae^2+3bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{d(3ae^2+bd^2p)\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] -1/2*d^3*(b*x^2+a)^(p+1)/a/x^2-3*d^2*e*(b*x^2+a)^(p+1)/a/x+e*(a*e^2+3*b*d^2
*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^
p)-1/2*d*(b*d^2*p+3*a*e^2)*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2
/a)/a^2/(p+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1821, 778, 272, 67, 252, 251\}$

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^3}\,dx$$
$$= -\frac{d(a+bx^2)^{p+1}\,(3ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^3(a+bx^2)^{p+1}}{2ax^2} + \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+3bd^2(2p+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{p+1}}{ax}$$

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d^3*(a + b*x^2)^(1 + p))/(a*x^2) - (3*d^2*e*(a + b*x^2)^(1 + p))/(a*x
) + (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*(3*a*e^2 + b*d^2*p)*(a +
b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*(1
 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{\int \frac{(a+bx^2)^p(-6ad^2e-2d(3ae^2+bd^2p)x-2ae^3x^2)}{x^2}\,dx}{2a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{3d^2e(a+bx^2)^{1+p}}{ax} + \frac{\int \frac{(2ad(3ae^2+bd^2p)+2ae(ae^2+3bd^2(1+2p))x)(a+bx^2)^p}{x}\,dx}{2a^2} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{3d^2e(a+bx^2)^{1+p}}{ax} + \frac{(d(3ae^2+bd^2p))\int \frac{(a+bx^2)^p}{x}\,dx}{a} \\
&\quad + \frac{(e(ae^2+3bd^2(1+2p)))\int (a+bx^2)^p\,dx}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{3d^2e(a+bx^2)^{1+p}}{ax} + \frac{(d(3ae^2+bd^2p))\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2a} \\
&\quad + \frac{\left(e(ae^2+3bd^2(1+2p))\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{2ax^2} - \frac{3d^2e(a+bx^2)^{1+p}}{ax} \\
&\quad + \frac{e(ae^2+3bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a} \\
&\quad - \frac{d(3ae^2+bd^2p)\,(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} \, dx =$$
$$-\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(6a^2d^2e(1+p) \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)+x\left(-2a^2e^3(1+p)x \text{Hy}\right.\right.}{}$$

[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]

[Out] -1/2*((a + b*x^2)^p*(6*a^2*d^2*e*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)] + x*(-2*a^2*e^3*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*(3*a*e^2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a]))))/(a^2*(1 + p)*x*(1 + (b*x^2)/a)^p)

Maple [F]

$$\int \frac{(ex+d)^3 (b\,x^2+a)^p}{x^3}dx$$

[In] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)

[Out] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)

Fricas [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} \, dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^3} \, dx$$

[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="fricas")

[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^3, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3}\,dx = -\frac{3a^p d^2 e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x} + a^p e^3 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) - \frac{b^p d^3 x^{2p-2}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\ 2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)} - \frac{3b^p d e^2 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)
```

```
[Out] -3*a**p*d**2*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p
*e**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**3*x**(
2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**
2))/(2*gamma(2 - p)) - 3*b**p*d*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1
- p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^3 (a+bx^2)^p}{x^3}\,dx = \int \frac{(ex+d)^3(bx^2+a)^p}{x^3}\,dx$$

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^3}\,dx = \int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x^3}\,dx$$

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^3,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.408 $\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \frac{(bd^2-ae^2)\,(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{d^4(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^3\,(bd^2+ae^2)\,(1+p)}$$

```
[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(p+1)/b^2/e^3/(p+1)+1/2*(b*x^2+a)^(2+p)/b^2/e/
(2+p)+1/5*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,7/2,e^2*x^2/d^2,-b*x^2/a)/d/((1
+b*x^2/a)^p)-1/2*d^4*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)
/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {973, 525, 524, 457, 90, 70}

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} + \frac{(bd^2-ae^2)\,(a+bx^2)^{p+1}}{2b^2e^3(p+1)} + \frac{(a+bx^2)^{p+2}}{2b^2e(p+2)} - \frac{d^4(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p+1)\,(ae^2+bd^2)}$$

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*e^3*(1 + p)) + (a + b*x^2)^(2
+ p)/(2*b^2*e*(2 + p)) + (x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*
x^2)/a), (e^2*x^2)/d^2])/(5*d*(1 + (b*x^2)/a)^p) - (d^4*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
e^3*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{x^4(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^5(a+bx^2)^p}{d^2-e^2x^2}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{x^2(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\text{Subst}\left(\int \left(\frac{(-bd^2+ae^2)(a+bx)^p}{be^4} - \frac{(a+bx)^{1+p}}{be^2} + \frac{d^4(a+bx)^p}{e^4(d^2-e^2x)}\right)dx, x, x^2\right)$$

$$= \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{d^4\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2e^3}$$

$$= \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{d^4(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^3(bd^2+ae^2)(1+p)}$$

Mathematica [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$$

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x),x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x), x]
```

Maple [F]

$$\int \frac{x^4(b\,x^2+a)^p}{ex+d}dx$$

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x), x)
```

3.409 $\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)(1+p)}$$

```
[Out] -1/2*d*(b*x^2+a)^(p+1)/b/e^2/(p+1)-1/5*e*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,
7/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/a)^p)+1/2*d^3*(b*x^2+a)^(p+1)*hyper
geom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {973, 457, 81, 70, 525, 524}

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = -\frac{ex^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},-p,1,\frac{7}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{p+1}\operatorname{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)} - \frac{d(a+bx^2)^{p+1}}{2be^2(p+1)}$$

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d*(a + b*x^2)^(1 + p))/(b*e^2*(1 + p)) - (e*x^5*(a + b*x^2)^p*AppellF
1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1 + (b*x^2)/a)^p)
+ (d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x
^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{x^3(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^4(a+bx^2)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2e^2}$$

$$= -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)(1+p)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.60

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(-\frac{3d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + \frac{e\left(1+\frac{bx^2}{a}\right)^{-p}\left(6bd^2(1+p)\right.}{}\right.}{6e}$$

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*((-3*d^3*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(
d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*(6*b*d^2*(1 + p)*x*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + e*(-3*d*(b*x^2*(1 + (b*x^2)/
a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + 2*b*e*(1 + p)*x^3*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])))/(b*(1 + p)*(1 + (b*x^2)/a)^p)))/(6*e^4)
```

Maple [F]

$$\int \frac{x^3(b\,x^2+a)^p}{ex+d}dx$$

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^3/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^3}{ex+d}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx = \int \frac{x^3\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x), x)
```

3.410 $\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^{1+p}}{2be(1+p)} + \frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e(bd^2+ae^2)(1+p)}$$

```
[Out] 1/2*(b*x^2+a)^(p+1)/b/e/(p+1)+1/3*x^3*(b*x^2+a)^p*AppellF1(3/2,1,-p,5/2,e^2
*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(p+1)*hypergeom([1,
p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {973, 525, 524, 457, 81, 70}

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \frac{x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)} + \frac{(a+bx^2)^{p+1}}{2be(p+1)}$$

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e*(1 + p)) + (x^3*(a + b*x^2)^p*AppellF1[3/2, -p,
1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d*(1 + (b*x^2)/a)^p) - (d^2*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*e*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{x^2(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^3(a+bx^2)^p}{d^2-e^2x^2}\,dx\\
&= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&= \frac{(a+bx^2)^{1+p}}{2be(1+p)} + \frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}\\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2e}\\
&= \frac{(a+bx^2)^{1+p}}{2be(1+p)} + \frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}\\
&\quad - \frac{d^2(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e\left(bd^2+ae^2\right)(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(ae^2p+be^2px^2-ae^2p\left(1+\frac{bx^2}{a}\right)^{-p}+bd^2(1+p)\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-\right.\right.}{2be^3p(}$$

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(a*e^2*p + b*e^2*p*x^2 - (a*e^2*p)/(1 + (b*x^2)/a)^p + (b*d^
2*(1 + p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*
(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*b*d*e*p*(1 + p)*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*b*e^3*p*(1 + p))
```

Maple [F]

$$\int \frac{x^2(b\,x^2+a)^p}{ex+d}dx$$

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^2/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx=\int \frac{(bx^2+a)^p x^2}{ex+d}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx=\int \frac{x^2\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x), x)
```

3.411 $\int \frac{x(a+bx^2)^p}{d+ex} dx$

Optimal result

Integrand size = 18, antiderivative size = 173

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = -\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e} + \frac{d(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\,(bd^2+ae^2)\,(1+p)}$$

```
[Out] -x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/((1+b*x^2/a)^p
)+x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/((1+b*x^2/a)^p)+1/2*d
*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e
^2+b*d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = -\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{d(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)} + \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e}$$

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(e*(1 + (b*x^2)/a)^p)) + (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)
*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (a+bx^2)^p\,dx}{e} - \frac{d\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e} \\
&= -\frac{d\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e} + \frac{\left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{e} \\
&= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - d\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx - \frac{d^2\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e} \\
&= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right) \\
&\quad - \frac{\left(d^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e} \\
&= -\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} \\
&\quad + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} \\
&\quad + \frac{d(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \frac{(a+bx^2)^p\left(-\frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}+2ex\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hyper}$$

$$2e^2$$

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(-((d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(
d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*e*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^2)
```

Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{ex+d}dx$$

```
[In] int(x*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{x(a+bx^2)^p}{d+ex}\,dx$$

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d),x)
[Out] Integral(x*(a + b*x**2)**p/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

Giac [F]

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p x}{ex+d}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx = \int \frac{x\,(b\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int((x*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x), x)
```

3.412 $\int \frac{(a+bx^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\,(bd^2+ae^2)\,(1+p)}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)
-1/2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2)
)/(a*e^2+b*d^2)/(p+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {771, 441, 440, 455, 70}

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)}$$

```
[In] Int[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right) dx \\
&= d\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx + e\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx \\
&= \frac{1}{2}e\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx, x, x^2\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx
\end{aligned}$$

$$= \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)(1+p)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\,\mathrm{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2ep}$$

```
[In] Integrate[(a + b*x^2)^p/(d + e*x),x]
[Out] ((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*
x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{ex+d}dx$$

```
[In] int((b*x^2+a)^p/(e*x+d),x)
[Out] int((b*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(a+bx^2)^p}{d+ex}\,dx$$

```
[In] integrate((b*x**2+a)**p/(e*x+d),x)
[Out] Integral((a + b*x**2)**p/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(bx^2+a)^p}{ex+d}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx = \int \frac{(b\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int((a + b*x^2)^p/(d + e*x),x)
[Out] int((a + b*x^2)^p/(d + e*x), x)
```

3.413 $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 176

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = &-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}\\ &+\frac{e^2(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d\left(bd^2+ae^2\right)(1+p)}\\ &-\frac{(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad(1+p)}\end{aligned}$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/
a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2
+b*d^2))/d/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p]
,1+b*x^2/a)/a/d/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {973, 457, 88, 67, 70, 441, 440}

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = -\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad(p+1)}$$

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^2*(1 + (b*x^2)/a)^p)) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1,
1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d*(b*d^2 + a*e^2)*(1 +
 p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/
a])/(2*a*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{(a+bx^2)^p}{x\,(d^2-e^2x^2)}\,dx - e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x\,(d^2-e^2x)}\,dx, x, x^2\right) - \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}\\
&\quad + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d} + \frac{e^2\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2d}
\end{aligned}$$

$$= -\frac{ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d\left(bd^2+ae^2\right)(1+p)} - \frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad(1+p)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \frac{(a+bx^2)^p \left(-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right) + (1$$
$$2dp$$

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)),x]
[Out] ((a + b*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p
, -(a/(b*x^2))]/(1 + a/(b*x^2))^p))/(2*d*p)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)}dx$$

```
[In] int((b*x^2+a)^p/x/(e*x+d),x)
[Out] int((b*x^2+a)^p/x/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(a+bx^2)^p}{x\,(d+ex)}\,dx$$

```
[In] integrate((b*x**2+a)**p/x/(e*x+d),x)
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*x^2)^p/(x*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x*(d + e*x)), x)
```

3.414 $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2(bd^2+ae^2)(1+p)} + \frac{e(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^2(1+p)}$$

```
[Out] -(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d^2,-b*x^2/a)/d/x/((1+b*x^2/a)^
p)-1/2*e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*
d^2))/d^2/(a*e^2+b*d^2)/(p+1)+1/2*e*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p
],1+b*x^2/a)/a/d^2/(p+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{973, 525, 524, 457, 88, 67, 70\}$

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = -\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)\left(ae^2+bd^2\right)} + \frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^2(p+1)}$$

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*x*(1 + (b*x^2)/a)^p)) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 +
p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(2*a*d^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{(a+bx^2)^p}{x^2(d^2-e^2x^2)}\,dx - e\int \frac{(a+bx^2)^p}{x(d^2-e^2x^2)}\,dx \\
&= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x(d^2-e^2x)}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2(d^2-e^2x^2)}\,dx \\
&= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} \\
&\quad - \frac{e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e^3\,\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2d^2} \\
&= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} \\
&\quad - \frac{e^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2(bd^2+ae^2)(1+p)} \\
&\quad + \frac{e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \frac{(a+bx^2)^p\left(\frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}-\frac{2d\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2}}{x}\right.}{2d^2}$$

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*((e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1/2,
 -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -
p, 1 - p, -(a/(b*x^2))])/(p*(1 + a/(b*x^2))^p)))/(2*d^2)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)}dx$$

```
[In] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)), x)
```

3.415 $\int \frac{(a+bx^2)^p}{x^3(d+ex)} dx$

Optimal result

Integrand size = 20, antiderivative size = 213

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)} dx = -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a^2d^3(1+p)}$$

```
[Out] -1/2*(b*x^2+a)^(p+1)/a/d/x^2+e*(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d
^2,-b*x^2/a)/d^2/x/((1+b*x^2/a)^p)+1/2*e^4*(b*x^2+a)^(p+1)*hypergeom([1, p+
1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)-1/2*(b*d^2*p+
a*e^2)*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a^2/d^3/(p+1)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used

= {973, 457, 105, 162, 67, 70, 525, 524}

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx =$$

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)} + \frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}}{2adx^2}$$

```
[In] Int[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(a + b*x^2)^(1 + p)/(a*d*x^2) + (e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1,
 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1 + (b*x^2)/a)^p) + (e^4*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*d^3*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\text{integral} = d\int \frac{(a+bx^2)^p}{x^3\,(d^2-e^2x^2)}\,dx - e\int \frac{(a+bx^2)^p}{x^2\,(d^2-e^2x^2)}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x^2(d^2-e^2x)}\,dx,x,x^2\right) - \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2(d^2-e^2x^2)}\,dx$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(-ae^2-bd^2p+be^2px)}{x(d^2-e^2x)}\,dx,x,x^2\right)}{2ad}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx,x,x^2\right)}{2d^3} + \frac{(ae^2+bd^2p)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2ad^3}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2d^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$$

$$= \frac{(a+bx^2)^p\left(-\frac{e^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + \frac{2de\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeon}}{}\right.}{}$$

```
[In] Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-((e^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)
/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d
 + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*d*e*Hypergeometric2F
1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) + ((d^2*Hypergeometri
c2F1[1 - p, -p, 2 - p, -(a/(b*x^2))])/((-1 + p)*x^2) + (e^2*Hypergeometric2
F1[-p, -p, 1 - p, -(a/(b*x^2))])/p)/(1 + a/(b*x^2))^p))/(2*d^3)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^3\,(ex+d)}dx$$

[In] int((b*x^2+a)^p/x^3/(e*x+d),x)

[Out] int((b*x^2+a)^p/x^3/(e*x+d),x)

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p/(e*x^4 + d*x^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \text{Timed out}$$

[In] integrate((b*x**2+a)**p/x**3/(e*x+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)x^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = \int \frac{(b\,x^2+a)^p}{x^3\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x^2)^p/(x^3*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^3*(d + e*x)), x)
```

3.416 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 392

$$
\begin{aligned}
&\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx \\
&= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} \\
&\quad - \frac{2d^2\left(2ae^2+bd^2(2+p)\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(bd^2+ae^2\right)} \\
&\quad - \frac{\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4(6+7p+2p^2)\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\right.}{be^4\left(bd^2+ae^2\right)(3+2p)} \\
&\quad + \frac{d^3\left(2ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{e^3\left(bd^2+ae^2\right)^2(1+p)}
\end{aligned}
$$

```
[Out] -d*(4+3*p)*(b*x^2+a)^(p+1)/b/e^3/(p+1)/(3+2*p)-d^4*(b*x^2+a)^(p+1)/e^3/(a*e
^2+b*d^2)/(e*x+d)+(e*x+d)*(b*x^2+a)^(p+1)/b/e^3/(3+2*p)-2*d^2*(2*a*e^2+b*d^
2*(2+p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e
^2+b*d^2)/((1+b*x^2/a)^p)-(a^2*e^4-2*a*b*d^2*e^2*(4+3*p)-2*b^2*d^4*(2*p^2+7
*p+6))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/e^4/(a*e^2+b*d^2
)/(3+2*p)/((1+b*x^2/a)^p)+d^3*(2*a*e^2+b*d^2*(2+p))*(b*x^2+a)^(p+1)*hyperge
om([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = -\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4(2p^2+7p+6))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.}{be^4(2p+3)\,(ae^2+bd^2)}$$
$$-\frac{2d^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(p+2))\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\,(ae^2+bd^2)}$$
$$-\frac{d^4(a+bx^2)^{p+1}}{e^3(d+ex)\,(ae^2+bd^2)}$$
$$+\frac{d^3(a+bx^2)^{p+1}\,(2ae^2+bd^2(p+2))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^3(p+1)\,(ae^2+bd^2)^2}$$
$$-\frac{d(3p+4)\,(a+bx^2)^{p+1}}{be^3(p+1)(2p+3)}+\frac{(d+ex)\,(a+bx^2)^{p+1}}{be^3(2p+3)}$$

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d*(4 + 3*p)*(a + b*x^2)^(1 + p))/(b*e^3*(1 + p)*(3 + 2*p))) - (d^4*(a +
b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*(a + b*x^2)^(1
 + p))/(b*e^3*(3 + 2*p)) - (2*d^2*(2*a*e^2 + b*d^2*(2 + p))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^
2)*(1 + (b*x^2)/a)^p) - ((a^2*e^4 - 2*a*b*d^2*e^2*(4 + 3*p) - 2*b^2*d^4*(6
+ 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b*e^4*(b*d^2 + a*e^2)*(3 + 2*p)*(1 + (b*x^2)/a)^p) + (d^3*(2*a*e^2 + b
*d^2*(2 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(e^3*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

integral

$$= -\frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(a+bx^2)^p\left(\frac{ad^3}{e^2}-\frac{d^2(ae^2+2bd^2(1+p))x}{e^3}+d\left(a+\frac{bd^2}{e^2}\right)x^2-\frac{(bd^2+ae^2)x^3}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{(a+bx^2)^p\left(ade(ae^2+2bd^2(2+p))+(a^2e^4-4b^2d^4(1+p)^2)x+2bde(bd^2+ae^2)(4+3p)x^2\right)}{d+ex}\,dx}{be^3(bd^2+ae^2)(3+2p)}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{(2abde^3(1+p)(ae^2+2bd^2(2+p))+2be^2(1+p)(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4(6+7p+2p^2))x)(a+bx^2)^p}{d+ex}\,dx}{2b^2e^5(bd^2+ae^2)(1+p)(3+2p)}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{(2d^3(2ae^2+bd^2(2+p)))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e^4(bd^2+ae^2)} - \frac{(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4(6+7p+2p^2))\int (a+bx^2)^p\,dx}{be^4(bd^2+ae^2)(3+2p)}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(2d^3\left(2ae^2+bd^2(2+p)\right)\right)\int\left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e^4\left(bd^2+ae^2\right)} - \frac{\left(\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4\left(6+7p+2p^2\right)\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p dx}{be^4\left(bd^2+ae^2\right)(3+2p)}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4\left(6+7p+2p^2\right)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4\left(bd^2+ae^2\right)(3+2p)} - \frac{\left(2d^4\left(2ae^2+bd^2(2+p)\right)\right)\int\frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}dx}{e^4\left(bd^2+ae^2\right)} - \frac{\left(2d^3\left(2ae^2+bd^2(2+p)\right)\right)\int\frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}dx}{e^3\left(bd^2+ae^2\right)}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4\left(6+7p+2p^2\right)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4\left(bd^2+ae^2\right)(3+2p)} - \frac{\left(d^3\left(2ae^2+bd^2(2+p)\right)\right)\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}dx,x,x^2\right)}{e^3\left(bd^2+ae^2\right)} - \frac{\left(2d^4\left(2ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}dx}{e^4\left(bd^2+ae^2\right)}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{2d^2\left(2ae^2+bd^2(2+p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(bd^2+ae^2\right)} - \frac{\left(a^2e^4-2abd^2e^2(4+3p)-2b^2d^4\left(6+7p+2p^2\right)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4\left(bd^2+ae^2\right)(3+2p)} + \frac{d^3\left(2ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2\left(a+bx^2\right)}{bd^2+ae^2}\right)}{e^3\left(bd^2+ae^2\right)^2(1+p)}$$

Mathematica [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$$

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2, x]
```

Maple [F]

$$\int \frac{x^4(b\,x^2+a)^p}{(ex+d)^2}dx$$

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.417 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 321

$$\begin{aligned}\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx &= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} \\ &+ \frac{d(3ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(bd^2+ae^2)} \\ &- \frac{d(2ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)} \\ &- \frac{d^2(3ae^2+bd^2(3+2p))\,(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)^2(1+p)}\end{aligned}$$

```
[Out] 1/2*(b*x^2+a)^(p+1)/b/e^2/(p+1)+d^3*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)/(e*x+
d)+d*(3*a*e^2+b*d^2*(3+2*p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^
2,-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/a)^p)-d*(2*a*e^2+b*d^2*(3+2*p))*x*(
b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/
a)^p)-1/2*d^2*(3*a*e^2+b*d^2*(3+2*p))*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2
+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+3))\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\,(ae^2+bd^2)}$$
$$- \frac{d^2(a+bx^2)^{p+1}(3ae^2+bd^2(2p+3))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)\,(ae^2+bd^2)^2}$$
$$- \frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^3\,(ae^2+bd^2)}$$
$$+ \frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)\,(ae^2+bd^2)} + \frac{(a+bx^2)^{p+1}}{2be^2(p+1)}$$

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^2*(1 + p)) + (d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d
^2 + a*e^2)*(d + e*x)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*App
ellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2 + a*e^2)*(
1 + (b*x^2)/a)^p) - (d*(2*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)
^p) - (d^2*(3*a*e^2 + b*d^2*(3 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^
2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
```

```
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(a+bx^2)^p\left(-\frac{ad^2}{e}+d\left(a+\frac{2bd^2(1+p)}{e^2}\right)x-\frac{(bd^2+ae^2)x^2}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(-2abd^2e(1+p)+2bd(1+p)(2ae^2+bd^2(3+2p))x)(a+bx^2)^p}{d+ex}\,dx}{2be^2(bd^2+ae^2)(1+p)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{(d(2ae^2+bd^2(3+2p)))\int (a+bx^2)^p\,dx}{e^3(bd^2+ae^2)} + \frac{(d^2(3ae^2+bd^2(3+2p)))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} + \frac{(d^2(3ae^2+bd^2(3+2p)))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e^3(bd^2+ae^2)} - \frac{\left(d(2ae^2+bd^2(3+2p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{d(2ae^2+bd^2(3+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)} + \frac{(d^3(3ae^2+bd^2(3+2p)))\int\frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e^3(bd^2+ae^2)} + \frac{(d^2(3ae^2+bd^2(3+2p)))\int\frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{e^2(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{d(2ae^2+bd^2(3+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)} + \frac{(d^2(3ae^2+bd^2(3+2p)))\,\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2e^2(bd^2+ae^2)} + \frac{\left(d^3(3ae^2+bd^2(3+2p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} + \frac{d(3ae^2+bd^2(3+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(bd^2+ae^2)} - \frac{d(2ae^2+bd^2(3+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)} - \frac{d^2(3ae^2+bd^2(3+2p))(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.07

$$\int\frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{(a+bx^2)^p\left(-\frac{2d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{3d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\right.}{}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((-2*d^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (
3*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(S
qrt[-(a/b)] + x))/(d + e*x))^p) + e*((e*(a + b*x^2 - a/(1 + (b*x^2)/a)^p))/
(b + b*p) - (4*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p)))/(2*e^4)
```

Maple [F]

$$\int \frac{x^{3}\left(b\,x^{2}+a\right)^{p}}{\left(ex+d\right)^{2}}dx$$

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx=\int \frac{\left(bx^{2}+a\right)^{p}x^{3}}{\left(ex+d\right)^{2}}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^2}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^3\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.418 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 281

$$\begin{aligned}
&\int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx \\
&= -\frac{d^2(a+bx^2)^{1+p}}{e(bd^2+ae^2)(d+ex)} \\
&\quad - \frac{2(ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2(bd^2+ae^2)} \\
&\quad + \frac{(ae^2+2bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)} \\
&\quad + \frac{d(ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{e(bd^2+ae^2)^2(1+p)}
\end{aligned}$$

```
[Out] -d^2*(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)/(e*x+d)-2*(a*e^2+b*d^2*(p+1))*x*(b*x^2
+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x
^2/a)^p)+(a*e^2+2*b*d^2*(p+1))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x
^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+d*(a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+
1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)^2/
(p+1)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1665, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= -\frac{2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(p+1))\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\,(ae^2+bd^2)}$$
$$+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a+\frac{2bd^2(p+1)}{e^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{ae^2+bd^2}$$
$$+\frac{d(a+bx^2)^{p+1}(ae^2+bd^2(p+1))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e(p+1)\,(ae^2+bd^2)^2}$$
$$-\frac{d^2(a+bx^2)^{p+1}}{e(d+ex)\,(ae^2+bd^2)}$$

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (2*(a*e^2 + b*
d^2*(1 + p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e^2*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + ((a + (2*b*d^2*(1 + p))
/e^2)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((b*d^
2 + a*e^2)*(1 + (b*x^2)/a)^p) + (d*(a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 +
 p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/
(e*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{\int \frac{\left(ad-\frac{\left(ae^2+2bd^2(1+p)\right)x}{e}\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{bd^2+ae^2}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e^2\left(bd^2+ae^2\right)}\\
&\quad+\frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)\int \left(a+bx^2\right)^p\,dx}{bd^2+ae^2}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e^2\left(bd^2+ae^2\right)}\\
&\quad+\frac{\left(\left(a+\frac{2bd^2(1+p)}{e^2}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{bd^2+ae^2}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)}+\frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2}\\
&\quad-\frac{\left(2d^2\left(ae^2+bd^2(1+p)\right)\right)\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{e^2\left(bd^2+ae^2\right)}-\frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)}\\
&\quad+\frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2}\\
&\quad-\frac{\left(d\left(ae^2+bd^2(1+p)\right)\right)\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{e\left(bd^2+ae^2\right)}\\
&\quad-\frac{\left(2d^2\left(ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e^2\left(bd^2+ae^2\right)}
\end{aligned}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e(bd^2+ae^2)(d+ex)} - \frac{2(ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2(bd^2+ae^2)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2} + \frac{d(ae^2+bd^2(1+p))(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{e(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{(a+bx^2)^p\left(\frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}\right.}{d+e}\right.}{}\right.}{e^3}$$

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^p))/e^3
```

Maple [F]

$$\int \frac{x^2(b\,x^2+a)^p}{(ex+d)^2}dx$$

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x^2\,\left(b\,x^2+a\right)^p}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.419 $\int \frac{x(a+bx^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 273

$$\begin{aligned}\int \frac{x(a+bx^2)^p}{(d+ex)^2} dx &= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} \\ &\quad + \frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\,(bd^2+ae^2)} \\ &\quad - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e\,(bd^2+ae^2)} \\ &\quad - \frac{(ae^2+bd^2(1+2p))\,(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\,(bd^2+ae^2)^2\,(1+p)}\end{aligned}$$

```
[Out] d*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(e*x+d)+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/e/(a*e^2+b*d^2)/((1+b*x^2/a)
^p)-b*d*(1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/(a*e^2+
b*d^2)/((1+b*x^2/a)^p)-1/2*(a*e^2+b*d^2*(1+2*p))*(b*x^2+a)^(p+1)*hypergeom(
[1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {849, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$$
$$= \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\,(ae^2+bd^2)}$$
$$-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e\,(ae^2+bd^2)}$$
$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2(2p+1))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)^2}$$
$$+\frac{d(a+bx^2)^{p+1}}{(d+ex)\,(ae^2+bd^2)}$$

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + ((a*e^2 + b*d^2*(1 +
2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d*e*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)*(1 + (b*
x^2)/a)^p) - ((a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^
2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(-ae+bd(1+2p)x)(a+bx^2)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{(bd(1+2p))\int (a+bx^2)^p\,dx}{e(bd^2+ae^2)} + \frac{(ae^2+bd^2(1+2p))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e(bd^2+ae^2)}$$

$$= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} + \frac{(ae^2+bd^2(1+2p))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e(bd^2+ae^2)} - \frac{\left(bd(1+2p)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{e(bd^2+ae^2)}$$

$$= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)} + \frac{(ae^2+bd^2(1+2p))\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{bd^2+ae^2} + \frac{(d(ae^2+bd^2(1+2p)))\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e(bd^2+ae^2)}$$

$$= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)} + \frac{(ae^2+bd^2(1+2p))\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2(bd^2+ae^2)} + \frac{\left(d(ae^2+bd^2(1+2p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e(bd^2+ae^2)}$$

$$= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} + \frac{(ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de(bd^2+ae^2)} - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)} - \frac{(ae^2+bd^2(1+2p))(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-2dp\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)\right.}{2e^2p(-1+2p)(d+ex)}$$

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*Appel
lF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)
]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^2}dx$$

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$$

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Integral(x*(a + b*x**2)**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{x\,(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.420 $\int \frac{(a+bx^2)^p}{(d+ex)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 244

$$\int \frac{(a+bx^2)^p}{(d+ex)^2} dx = \frac{e^2x(a+bx^2)^{1+p}}{(bd^2+ae^2)(d^2-e^2x^2)} - \frac{2bpx(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{bd^2+ae^2} + \frac{b(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{bde(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}$$

```
[Out] e^2*x*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)-2*b*p*x*(b*x^2+a)^p*Appe
llF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+b*(1+
2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/(a*e^2+b*d^2)/((1+b*
x^2/a)^p)-b*d*e*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e
^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {771, 441, 440, 455, 70, 525, 524}

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4} - \frac{bde(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)\left(ae^2+bd^2\right)^2}$$

```
[In] Int[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
^2*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(
(b*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (b*x^2)/a)^p) - (b*d*e*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right) dx \\
&= d^2 \int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2} dx - (2de) \int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2} dx + e^2 \int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2} dx \\
&= -\left((de)\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2} dx, x, x^2\right)\right) \\
&\quad + \left(d^2(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^2} dx \\
&\quad + \left(e^2(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-d^2+e^2x^2)^2} dx
\end{aligned}
$$

$$= \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{3d^4} - \frac{bde(a+bx^2)^{1+p}\ {}_2F_1\left(2, 1+p; 2+p; \frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{e(-1+2p)(d+ex)}$$

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^2,x]
[Out] ((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(ex+d)^2}dx$$

```
[In] int((b*x^2+a)^p/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$$

```
[In] integrate((b*x**2+a)**p/(e*x+d)**2,x)
[Out] Integral((a + b*x**2)**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \frac{(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*x^2)^p/(d + e*x)^2,x)
[Out] int((a + b*x^2)^p/(d + e*x)^2, x)
```

3.421 $\int \frac{(a+bx^2)^p}{x(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 368

$$
\begin{aligned}
\int \frac{(a+bx^2)^p}{x(d+ex)^2} dx = &-\frac{ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} \\
&-\frac{ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} \\
&-\frac{e^3x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5} \\
&+\frac{e^2(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2(bd^2+ae^2)(1+p)} \\
&-\frac{(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} \\
&+\frac{be^2(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}
\end{aligned}
$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/
a)^p)-e*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b
*x^2/a)^p)-1/3*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2
/a)/d^5/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)*hype
rgeom([1, p+1],[2+p],1+b*x^2/a)/a/d^2/(p+1)+b*e^2*(b*x^2+a)^(p+1)*hypergeom
([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {975, 272, 67, 771, 441, 440, 455, 70, 525, 524}

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = -\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5} - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)} + \frac{be^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^2(p+1)}$$

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^3*(1 + (b*x^2)/a)^p)) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)
^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^5*(1 + (b*x
^2)/a)^p) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1
 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(a+bx^2)^p}{d^2x} - \frac{e(a+bx^2)^p}{d(d+ex)^2} - \frac{e(a+bx^2)^p}{d^2(d+ex)}\right) dx \\
&= \frac{\int \frac{(a+bx^2)^p}{x}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right) dx}{d^2} \\
&\quad - \frac{e\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right) dx}{d} \\
&= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} - \frac{e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d} \\
&\quad - (de)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx + \left(2e^2\right)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx \\
&\quad - \frac{e^2\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{d^2} - \frac{e^3\int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\,dx}{d}
\end{aligned}$$

$$= -\frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} + e^2\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx, x, x^2\right)$$

$$- \frac{e^2\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx, x, x^2\right)}{2d^2} - \frac{\left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{d}$$

$$- \left(de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx$$

$$- \frac{\left(e^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-d^2+e^2x^2)^2}\,dx}{d}$$

$$= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}$$

$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}$$

$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5}$$

$$+ \frac{e^2(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2(bd^2+ae^2)(1+p)}$$

$$- \frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)}$$

$$+ \frac{be^2(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$$

$$= \frac{(a+bx^2)^p\left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{2d^2}$$

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
```

```
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (-(
AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-
(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b
)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1
 + a/(b*x^2))^p)/p))/(2*d^2)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)^2}dx$$

```
[In] int((b*x^2+a)^p/x/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx=\int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx=\int \frac{(a+bx^2)^p}{x\,(d+ex)^2}\,dx$$

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**2,x)
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx=\int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^2),x)
[Out] int((a + b*x^2)^p/(x*(d + e*x)^2), x)
```

3.422 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 421

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx &= \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}\\ &+\frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}\\ &+\frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}\\ &-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^2x}\\ &-\frac{e^3(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^3(bd^2+ae^2)(1+p)}\\ &+\frac{e(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{ad^3(1+p)}\\ &-\frac{be^3(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}\end{aligned}$$

```
[Out] 2*e^2*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x
^2/a)^p)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/
((1+b*x^2/a)^p)+1/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-
b*x^2/a)/d^6/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/
a)/d^2/x/((1+b*x^2/a)^p)-e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(
b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)+e*(b*x^2+a)^(p+1)*hypergeom
```

```
([1, p+1],[2+p],1+b*x^2/a)/a/d^3/(p+1)-b*e^3*(b*x^2+a)^(p+1)*hypergeom([2,
p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(p+1)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {975, 372, 371, 272, 67, 771, 441, 440, 455, 70, 525, 524}

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \frac{e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6} + \frac{2e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{ad^3(p+1)} - \frac{be^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^2x} - \frac{e^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^3(p+1)(ae^2+bd^2)}$$

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d^4*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x
^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (
b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(d^2*x*(1 + (b*x^2)/a)^p) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1
, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)*(1
 + p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x
^2)/a])/(a*d^3*(1 + p)) - (b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d*(b*d^2 + a*e^2)^2*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+bx^2)^p}{d^2x^2} - \frac{2e(a+bx^2)^p}{d^3x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^2} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)} \right) dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^2} - \frac{(2e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^3} + \frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^2}$$

$$= -\frac{e\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{d^3} + \frac{(2e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)\,dx}{d^3}$$

$$+ \frac{e^2\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right)\,dx}{d^2}$$

$$+ \frac{\left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx}{d^2}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x}$$

$$+ \frac{e(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)}$$

$$+ e^2\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx + \frac{(2e^2)\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d^2}$$

$$+ \frac{(2e^3)\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{d^3} - \frac{(2e^3)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx}{d} + \frac{e^4\int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\,dx}{d^2}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x}$$

$$+ \frac{e(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)} + \frac{e^3\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx, x, x^2\right)}{d^3}$$

$$- \frac{e^3\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx, x, x^2\right)}{d} + \left(e^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx$$

$$+ \frac{\left(2e^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{d^2}$$

$$+ \frac{\left(e^4(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-d^2+e^2x^2)^2}\,dx}{d^2}$$

$$= \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6} - \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} - \frac{e^3(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^3(bd^2+ae^2)(1+p)} + \frac{e(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)} - \frac{be^3(a+bx^2)^{1+p}{}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}$$

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$$

$$= \frac{(a+bx^2)^p\left(\frac{de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}\right.}{d+e}\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (e*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) - (d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))
])/(p*(1 + a/(b*x^2))^p)))/d^3
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)^2}dx$$

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^2x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)^2}\,dx$$

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.423 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 449

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}$$
$$+ \frac{d(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\right.}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\right.}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(}{bd}\right.}{2e^3(bd^2+ae^2)^3(1+p)}$$

```
[Out] 1/2*(b*x^2+a)^(p+1)/b/e^3/(p+1)-1/2*d^4*(b*x^2+a)^(p+1)/e^3/(a*e^2+b*d^2)/(
e*x+d)^2+d^3*(4*a*e^2+b*d^2*(3+p))*(b*x^2+a)^(p+1)/e^3/(a*e^2+b*d^2)^2/(e*x
+d)+d*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/
a)^p)-d*(3*a^2*e^4+2*a*b*d^2*e^2*(5+4*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)
^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-
1/2*d^2*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*(b*x^2+a)^(
p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2
)^3/(p+1)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$$

$$= \frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4(2p^2+7p+6))\operatorname{AppellF1}\big(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}}{e^4\left(ae^2+bd^2\right)^2}$$

$$-\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4(2p^2+7p+6))\,\text{Hypergeometric2F1}\big(\frac{1}{2},-p,\frac{3}{2}}{e^4\left(ae^2+bd^2\right)^2}$$

$$-\frac{d^2(a+bx^2)^{p+1}(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4(2p^2+7p+6))\,\text{Hypergeometric2F1}\big(1,p+1,p+2,\frac{e^2(bx}{bd^2}}{2e^3(p+1)\left(ae^2+bd^2\right)^3}$$

$$-\frac{d^4(a+bx^2)^{p+1}}{2e^3(d+ex)^2\left(ae^2+bd^2\right)}+\frac{d^3(a+bx^2)^{p+1}\left(4ae^2+bd^2(p+3)\right)}{e^3(d+ex)\left(ae^2+bd^2\right)^2}+\frac{(a+bx^2)^{p+1}}{2be^3(p+1)}$$

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^3*(1 + p)) - (d^4*(a + b*x^2)^(1 + p))/(2*e^3*(b
*d^2 + a*e^2)*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1
+ p))/(e^3*(b*d^2 + a*e^2)^2*(d + e*x)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4
+ 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/
2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p)
- (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^4*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2
*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(a+bx^2)^p\left(\frac{2ad^3}{e^2}-\frac{2d^2\left(ae^2+bd^2(1+p)\right)x}{e^3}+2d\left(a+\frac{bd^2}{e^2}\right)x^2-2\left(\frac{bd^2}{e}+ae\right)x^3\right)}{(d+ex)^2}\,dx}{2(bd^2+ae^2)}$$

$$= -\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{\int \frac{(a+bx^2)^p\left(2ad^2\left(3a+\frac{bd^2(2+p)}{e^2}\right)-\frac{2d\left(2a^2e^4+8abd^2e^2(1+p)+b^2d^4\left(5+7p+2p^2\right)\right)x}{e^3}+\frac{2\left(bd^2+ae^2\right)^2x^2}{e^2}\right)}{d+ex}\,dx}{2(bd^2+ae^2)^2}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{\int \frac{\left(4abd^2(1+p)\left(3ae^2+bd^2(2+p)\right)-\frac{4bd(1+p)\left(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4\left(6+7p+2p^2\right)\right)x}{e}\right)(a+bx^2)^p}{d+ex}\,dx}{4be^2(bd^2+ae^2)^2(1+p)}$$

$$=\frac{(a+bx^2)^{1+p}}{2be^3(1+p)}-\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2}+\frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}+\frac{(d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2)))\int\frac{(a+bx^2)^p}{d+ex}\,dx}{e^4(bd^2+ae^2)^2}-\frac{(d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2)))\int(a+bx^2)^p\,dx}{e^4(bd^2+ae^2)^2}$$

$$=\frac{(a+bx^2)^{1+p}}{2be^3(1+p)}-\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2}+\frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}+\frac{(d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2)))\int\left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e^4(bd^2+ae^2)^2}-\frac{\left(d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p dx}{e^4(bd^2+ae^2)^2}$$

$$=\frac{(a+bx^2)^{1+p}}{2be^3(1+p)}-\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2}+\frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}-\frac{d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^4(bd^2+ae^2)^2}+\frac{(d^3(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2)))\int\frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e^4(bd^2+ae^2)^2}+\frac{(d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2)))\int\frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{e^3(bd^2+ae^2)^2}$$

$$=\frac{(a+bx^2)^{1+p}}{2be^3(1+p)}-\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2}+\frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}-\frac{d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^4(bd^2+ae^2)^2}+\frac{(d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2)))\,\mathrm{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2e^3(bd^2+ae^2)^2}+\frac{\left(d^3(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e^4(bd^2+ae^2)^2}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}$$
$$+ \frac{d(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\ldots\right)}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d(3a^2e^4+2abd^2e^2(5+4p)+b^2d^4(6+7p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^4(bd^2+ae^2)^2}$$
$$- \frac{d^2(6a^2e^4+3abd^2e^2(4+3p)+b^2d^4(6+7p+2p^2))(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^3(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.03

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p\left(\frac{ae^2}{b+bp}+\frac{e^2x^2}{1+p}-\frac{8d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\mathrm{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^4\left(\frac{e\left(-\sqrt{\ldots}\right.}{d\ldots}\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((a*e^2)/(b + b*p) + (e^2*x^2)/(1 + p) - (8*d^3*AppellF1[1 -
 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)
/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(
a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d^4*AppellF1[2 - 2*p, -p, -p, 3 - 2*
p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 +
p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))
^p*(d + e*x)^2) + (6*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/
(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (6*d*e*x*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^5)
```

Maple [F]

$$\int \frac{x^4 (b\,x^2+a)^p}{(ex+d)^3}dx$$

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^4}{(ex+d)^3}\,dx$$

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^4\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.424 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 416

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3} dx = \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)}$$
$$- \frac{(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x}{d^2}\right.}{e^3(bd^2+ae^2)^2}$$
$$+ \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\right.}{e^3(bd^2+ae^2)^2}$$
$$+ \frac{d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+}{bd^2+}\right.}{2e^2(bd^2+ae^2)^3(1+p)}$$

```
[Out] 1/2*d^3*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)/(e*x+d)^2-d^2*(3*a*e^2+b*d^2*(2+p
))*(b*x^2+a)^(p+1)/e^2/(a*e^2+b*d^2)^2/(e*x+d)-(3*a^2*e^4+a*b*d^2*e^2*(6+7*
p)+b^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-
b*x^2/a)/e^3/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+(a^2*e^4+a*b*d^2*e^2*(5+6*p)+b
^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3
/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d
^4*(2*p^2+5*p+3))*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a
*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^3/(p+1)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1665, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = -\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4(2p^2+5p+3)\right)\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right.}{e^3\left(ae^2+bd^2\right)^2}$$
$$+\frac{d(a+bx^2)^{p+1}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4(2p^2+5p+3)\right)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+}{bd^2+ae}\right.}{2e^2(p+1)\left(ae^2+bd^2\right)^3}$$
$$+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(6p+5)+b^2d^4(2p^2+5p+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{}\right.}{e^3\left(ae^2+bd^2\right)^2}$$
$$-\frac{d^2(a+bx^2)^{p+1}\left(3ae^2+bd^2(p+2)\right)}{e^2(d+ex)\left(ae^2+bd^2\right)^2}+\frac{d^3(a+bx^2)^{p+1}}{2e^2(d+ex)^2\left(ae^2+bd^2\right)}$$

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) - (d^2*(3*a*e
^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)^2*(d + e*x))
- ((3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*(a + b
*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2
 + a*e^2)^2*(1 + (b*x^2)/a)^p) + ((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^
4*(3 + 5*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^2)/a)])/(e^3*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (d*(3*a^2*e^4 + a*b*d^
2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2
+ a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(a+bx^2)^p\left(-\frac{2ad^2}{e}+2d\left(a+\frac{bd^2(1+p)}{e^2}\right)x-2\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}\,dx}{2(bd^2+ae^2)} \\
&= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{\int \frac{\left(-\frac{2ad\left(2ae^2+bd^2(1+p)\right)}{e}+\frac{2\left(a^2e^4+abd^2e^2(5+6p)+b^2d^4\left(3+5p+2p^2\right)\right)x}{e^2}\right)(a+bx^2)^p}{d+ex}\,dx}{2(bd^2+ae^2)^2} \\
&= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))\int (a+bx^2)^p\,dx}{e^3(bd^2+ae^2)^2} \\
&\quad - \frac{(d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e^3(bd^2+ae^2)^2} \\
&= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} \\
&\quad - \frac{(d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e^3(bd^2+ae^2)^2} \\
&\quad + \frac{\left((a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx}{e^3(bd^2+ae^2)^2} \\
&= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)^2} \\
&\quad - \frac{(d^2(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)))\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e^3(bd^2+ae^2)^2} \\
&\quad - \frac{(d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)))\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{e^2(bd^2+ae^2)^2}
\end{aligned}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)}$$
$$+ \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)^2}$$
$$- \frac{(d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)))\,\mathrm{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2e^2(bd^2+ae^2)^2}$$
$$- \frac{\left(d^2(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e^3(bd^2+ae^2)^2}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)}$$
$$- \frac{(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(bd^2+ae^2)^2}$$
$$+ \frac{(a^2e^4+abd^2e^2(5+6p)+b^2d^4(3+5p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(bd^2+ae^2)^2}$$
$$+ \frac{d(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2))(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e^2(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.05

$$\int\frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p\left(\frac{6d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\mathrm{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((6*d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d
^3*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) - (3*d*AppellF1[-2*p, -p
, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*
```

```
x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
e*x))^p) + (2*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/(2*e^4)
```

Maple [F]

$$\int \frac{x^{3} \left(b\,x^{2}+a \right)^{p}}{\left(e x +d \right)^{3}}d x$$

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^3}{(ex+d)^3}\,dx$$

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^3\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.425 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 396

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)}$$
$$+ \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2(bd^2+ae^2)^2}$$
$$- \frac{bd(1+2p)(2ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)^2}$$
$$- \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e(bd^2+ae^2)^3(1+p)}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)/(e*x+d)^2+d*(2*a*e^2+b*d^2*(p+1))*
(b*x^2+a)^(p+1)/e/(a*e^2+b*d^2)^2/(e*x+d)+(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*
d^4*(2*p^2+3*p+1))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a
)/d/e^2/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-b*d*(1+2*p)*(2*a*e^2+b*d^2*(p+1))*x
*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^2/(a*e^2+b*d^2)^2/((1+b*
x^2/a)^p)-1/2*(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*d^4*(2*p^2+3*p+1))*(b*x^2+a)
^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2
)^3/(p+1)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 849, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a^2e^4+abd^2e^2(5p+2)+b^2d^4(2p^2+3p+1))\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2(ae^2+bd^2)^2}$$
$$-\frac{(a+bx^2)^{p+1}(a^2e^4+abd^2e^2(5p+2)+b^2d^4(2p^2+3p+1))\operatorname{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)^3}$$
$$-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(p+1))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e^2(ae^2+bd^2)^2}$$
$$+\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{e(d+ex)(ae^2+bd^2)^2}-\frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -1/2*(d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x)^2) + (d*(2*a*e^
2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)^2*(d + e*x)) + (
(a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*x*(a + b*x^2)
^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*e^2*(b*d^2 +
a*e^2)^2*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - ((a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1
+ 3*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{\left(2ad-\frac{2\left(ae^2+bd^2(1+p)\right)x}{e}\right)(a+bx^2)^p}{(d+ex)^2}\,dx}{2(bd^2+ae^2)}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} + \frac{\int \frac{\left(2a(ae^2+bd^2p)-\frac{2bd(1+2p)\left(2ae^2+bd^2(1+p)\right)x}{e}\right)(a+bx^2)^p}{d+ex}\,dx}{2(bd^2+ae^2)^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} - \frac{(bd(1+2p)(2ae^2+bd^2(1+p)))\int (a+bx^2)^p\,dx}{e^2(bd^2+ae^2)^2} + \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e^2(bd^2+ae^2)^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} + \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e^2(bd^2+ae^2)^2} - \frac{\left(bd(1+2p)(2ae^2+bd^2(1+p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{e^2(bd^2+ae^2)^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} - \frac{bd(1+2p)(2ae^2+bd^2(1+p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)^2} + \frac{(d(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2)))\int\frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e^2(bd^2+ae^2)^2} + \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\int\frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{e(bd^2+ae^2)^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} - \frac{bd(1+2p)(2ae^2+bd^2(1+p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)^2} + \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))\,\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2e(bd^2+ae^2)^2} + \frac{\left(d(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e^2(bd^2+ae^2)^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)} + \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2(bd^2+ae^2)^2} - \frac{bd(1+2p)(2ae^2+bd^2(1+p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^2(bd^2+ae^2)^2} - \frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2e(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.73

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-\frac{4d\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^2\,\text{AppellF1}\left(2-2\right.}{}\right.}{2e^3}$$

[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]

[Out] ((a + b*x^2)^p*((-4*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*(d + e*x)) + (d^2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*(d + e*x)^2) + AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/p))/(2*e^3*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)

Maple [F]

$$\int \frac{x^2(b\,x^2+a)^p}{(ex+d)^3}dx$$

[In] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)

[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)

Fricas [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x^2}{(ex+d)^3}\,dx$$

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{x^2\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.426 $\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 336

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} - \frac{bp(3ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e(bd^2+ae^2)^2} + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)^2} + \frac{bdp(3ae^2+bd^2(1+2p))(a+bx^2)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

```
[Out] 1/2*d*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(e*x+d)^2-(b*d^2*p+a*e^2)*(b*x^2+a)^(p+
1)/(a*e^2+b*d^2)^2/(e*x+d)-b*p*(3*a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*Appell
F1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+b*(
1+2*p)*(b*d^2*p+a*e^2)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/
(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*b*d*p*(3*a*e^2+b*d^2*(1+2*p))*(b*x^2+a)
^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^
3/(p+1)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {849, 858, 252, 251, 771, 441, 440, 455, 70}

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= -\frac{bpx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e\,(ae^2+bd^2)^2}$$
$$+\frac{b(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{e\,(ae^2+bd^2)^2}$$
$$+\frac{bdp(a+bx^2)^{p+1}(3ae^2+bd^2(2p+1))\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\,(ae^2+bd^2)^3}$$
$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)\,(ae^2+bd^2)^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2\,(ae^2+bd^2)}$$

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p
)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^
2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b
*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(
b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(-2ae+2bdpx)(a+bx^2)^p}{(d+ex)^2}\,dx}{2(bd^2+ae^2)} \\
&= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{\int \frac{(2abde(1-p)+2b(1+2p)(ae^2+bd^2p)x)(a+bx^2)^p}{d+ex}\,dx}{2(bd^2+ae^2)^2} \\
&= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{(b(1+2p)(ae^2+bd^2p))\int (a+bx^2)^p\,dx}{e(bd^2+ae^2)^2} \\
&\quad - \frac{(bdp(3ae^2+bd^2(1+2p)))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e(bd^2+ae^2)^2} \\
&= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} \\
&\quad - \frac{(bdp(3ae^2+bd^2(1+2p)))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e(bd^2+ae^2)^2} \\
&\quad + \frac{\left(b(1+2p)(ae^2+bd^2p)(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx}{e(bd^2+ae^2)^2} \\
&= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} \\
&\quad + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)^2} \\
&\quad - \frac{(bdp(3ae^2+bd^2(1+2p)))\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{(bd^2+ae^2)^2} \\
&\quad - \frac{(bd^2p(3ae^2+bd^2(1+2p)))\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e(bd^2+ae^2)^2}
\end{aligned}
$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)^2} - \frac{(bdp(3ae^2+bd^2(1+2p)))\,\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2(bd^2+ae^2)^2} - \frac{\left(bd^2p(3ae^2+bd^2(1+2p))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{e(bd^2+ae^2)^2}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} - \frac{bp(3ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e(bd^2+ae^2)^2} + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)^2} + \frac{bdp(3ae^2+bd^2(1+2p))(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.68

$$\int\frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(2(-1+p)(d+ex)\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-}}{d+e}\right.\right.}{2e^2(-1+p)(-1+2p)(d+}$$

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*(2*(-1 + p)*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
- Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + d*(1 - 2*p)*
AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)]))/(2*e^2*(-1 + p)*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x
))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F]

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^3}dx$$

[In] int(x*(b*x^2+a)^p/(e*x+d)^3,x)

[Out] int(x*(b*x^2+a)^p/(e*x+d)^3,x)

Fricas [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

[In] integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)

Giac [F]

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx = \int \frac{(bx^2+a)^p x}{(ex+d)^3} dx$$

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx = \int \frac{x (b x^2+a)^p}{(d+e x)^3} dx$$

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.427 $\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 322

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx &= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2}+\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\&\quad+\frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}\\&\quad+\frac{be(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4(bd^2+ae^2)^3(1+p)}\\&\quad-\frac{3b^2d^2e(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}\end{aligned}$$

```
[Out] -1/4*d^2*e*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+x*(b*x^2+a)^p*App
ellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/a)^p)+e^2*x^3*(b*x^2
+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+1/4*b
*e*(2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^2*(b*x^
2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(p+1)-3/2*b^2*d^2*e*(b*x^2+a)^(p+1)*hyp
ergeom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(p+1)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {771, 441, 440, 455, 70, 525, 524, 457, 79}

$$\begin{aligned}\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx\\ =\ &\frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}\\ &+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\ &-\frac{3b^2d^2e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)^3}\\ &+\frac{be(a+bx^2)^{p+1}\left(2ae^2+bd^2(p+1)\right)\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4(p+1)\left(ae^2+bd^2\right)^3}\\ &-\frac{d^2e(a+bx^2)^{p+1}}{4\left(d^2-e^2x^2\right)^2\left(ae^2+bd^2\right)}\end{aligned}$$

```
[In] Int[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -1/4*(d^2*e*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(
a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(
1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x
^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2*(1
+ p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^
2))/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3}-\frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{e^3x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\right)dx\\
&= d^3\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx-\left(3d^2e\right)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad+\left(3de^2\right)\int \frac{x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx+e^3\int \frac{x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\,dx\\
&= -\left(\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^3}\,dx,x,x^2\right)\right)\\
&\quad+\frac{1}{2}e^3\text{Subst}\left(\int \frac{x(a+bx)^p}{(-d^2+e^2x)^3}\,dx,x,x^2\right)\\
&\quad+\left(d^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad+\left(3de^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx\\
&= -\frac{d^2e(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}+\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}\\
&\quad+\frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}\\
&\quad-\frac{3b^2d^2e(a+bx^2)^{1+p}\,{}_2F_1\left(3,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2\left(bd^2+ae^2\right)^3(1+p)}\\
&\quad+\frac{\left(e(2ae^2+bd^2(1+p))\right)\text{Subst}\left(\int \frac{(a+bx)^p}{(-d^2+e^2x)^2}\,dx,x,x^2\right)}{4\left(bd^2+ae^2\right)}
\end{aligned}
$$

$$= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3}$$
$$+ \frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{3}{2}; -p, 3; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+ \frac{be(2ae^2+bd^2(1+p))(a+bx^2)^{1+p} {}_2F_1\left(2, 1+p; 2+p; \frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4(bd^2+ae^2)^3(1+p)}$$
$$- \frac{3b^2d^2e(a+bx^2)^{1+p} {}_2F_1\left(3, 1+p; 2+p; \frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$$
$$= \frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p \operatorname{AppellF1}\left(2-2p, -p, -p, 3-2p, \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2e(-1+p)(d+ex)^2}$$

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{(ex+d)^3}dx$$

```
[In] int((b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx=\int \frac{(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*x^2)^p/(d + e*x)^3,x)
```

```
[Out] int((a + b*x^2)^p/(d + e*x)^3, x)
```

3.428 $\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 700

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{de^2(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}$$
$$+ \frac{e^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)}$$
$$- \frac{(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}$$
$$+ \frac{be^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}$$
$$- \frac{be^2(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d(bd^2+ae^2)^3(1+p)}$$
$$+ \frac{3b^2de^2(a+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

```
[Out] 1/4*d*e^2*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2-e*x*(b*x^2+a)^p*Ap
pellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x^2+a)
^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x
^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-1/3
*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^6/((1+b*
x^2/a)^p)-e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d
^6/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^2*(b*
x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(p+1)-1/2*(b*x^2+a)^(p+1)*hypergeom
([1, p+1],[2+p],1+b*x^2/a)/a/d^3/(p+1)+b*e^2*(b*x^2+a)^(p+1)*hypergeom([2,
p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(p+1)-1/4*b*e^2*(
```

```
2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^2*(b*x^2+a)
/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^3/(p+1)+3/2*b^2*d*e^2*(b*x^2+a)^(p+1)*hyper
geom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(p+1)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {975, 272, 67, 771, 441, 440, 455, 70, 525, 524, 457, 79}

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$
$$= -\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}$$
$$-\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}$$
$$-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$+\frac{3b^2de^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)^3}$$
$$-\frac{(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^3(p+1)}$$
$$-\frac{be^2(a+bx^2)^{p+1}\left(2ae^2+bd^2(p+1)\right)\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d(p+1)\left(ae^2+bd^2\right)^3}$$
$$+\frac{be^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)\left(ae^2+bd^2\right)^2}$$
$$+\frac{de^2(a+bx^2)^{p+1}}{4\left(d^2-e^2x^2\right)^2\left(ae^2+bd^2\right)}$$
$$+\frac{e^2(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)\left(ae^2+bd^2\right)}$$

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (e*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1
 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a
), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/
2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e^3
*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(3*d^6*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^6*(1 + (b*x^2)/a)^p) + (e^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (b*e^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
])/(d*(b*d^2 + a*e^2)^2*(1 + p)) - (b*e^2*(2*a*e^2 + b*d^2*(1 + p))*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(4*d*(b*d^2 + a*e^2)^3*(1 + p)) + (3*b^2*d*e^2*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(a+bx^2)^p}{d^3x} - \frac{e(a+bx^2)^p}{d(d+ex)^3} - \frac{e(a+bx^2)^p}{d^2(d+ex)^2} - \frac{e(a+bx^2)^p}{d^3(d+ex)}\right) dx \\
&= \frac{\int \frac{(a+bx^2)^p}{x}\,dx}{d^3} - \frac{e\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^3} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^3} - \frac{e\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right) dx}{d^3} \\
&\quad - \frac{e\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right) dx}{d^2} \\
&\quad - \frac{e\int \left(\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3} - \frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{e^3x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\right) dx}{d} \\
&= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)} - e\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx \\
&\quad - \frac{e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d^2} - (d^2e)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx - \frac{e^2\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{d^3} \\
&\quad + \frac{(2e^2)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx}{d} + (3de^2)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx \\
&\quad - (3e^3)\int \frac{x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx - \frac{e^3\int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\,dx}{d^2} - \frac{e^4\int \frac{x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\,dx}{d}
\end{aligned}$$

$$=-\frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}-\frac{e^2\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2d^3}$$

$$+\frac{e^2\text{Subst}\left(\int\frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx,x,x^2\right)}{d}+\frac{1}{2}\left(3de^2\right)\text{Subst}\left(\int\frac{(a+bx)^p}{(d^2-e^2x)^3}\,dx,x,x^2\right)$$

$$-\frac{e^4\text{Subst}\left(\int\frac{x(a+bx)^p}{(-d^2+e^2x)^3}\,dx,x,x^2\right)}{2d}-\left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx$$

$$-\frac{\left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{d^2}$$

$$-\left(d^2e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx$$

$$-\left(3e^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx$$

$$-\frac{\left(e^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-d^2+e^2x^2)^2}\,dx}{d^2}$$

$$= \frac{de^2(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}$$
$$+ \frac{e^2(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)}$$
$$- \frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}$$
$$+ \frac{be^2(a+bx^2)^{1+p}\,{}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}$$
$$+ \frac{3b^2de^2(a+bx^2)^{1+p}\,{}_2F_1\left(3,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$
$$- \frac{(e^2(2ae^2+bd^2(1+p)))\,\text{Subst}\left(\int\frac{(a+bx)^p}{(-d^2+e^2x)^2}\,dx,x,x^2\right)}{4d(bd^2+ae^2)}$$

$$= \frac{de^2(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}$$
$$- \frac{e^3x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}$$
$$+ \frac{e^2(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^3(bd^2+ae^2)(1+p)}$$
$$- \frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}$$
$$+ \frac{be^2(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d(bd^2+ae^2)^2(1+p)}$$
$$- \frac{be^2(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d(bd^2+ae^2)^3(1+p)}$$
$$+ \frac{3b^2de^2(a+bx^2)^{1+p}\ {}_2F_1\left(3,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$
$$= \frac{(a+bx^2)^p\left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e}{-}\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
```

```
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*
((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (-(AppellF1[-2*p, -p, -
p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]
/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^
p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1 + a/(b*x^2))^p)/p))
/(2*d^3)
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)^3}dx$$

[In] int((b*x^2+a)^p/x/(e*x+d)^3,x)

[Out] int((b*x^2+a)^p/x/(e*x+d)^3,x)

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)

Sympy [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(a+bx^2)^p}{x\,(d+ex)^3}\,dx$$

[In] integrate((b*x**2+a)**p/x/(e*x+d)**3,x)

[Out] Integral((a + b*x**2)**p/(x*(d + e*x)**3), x)

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^3),x)
[Out] int((a + b*x^2)^p/(x*(d + e*x)^3), x)
```

3.429 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 754

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$$
$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2}$$
$$+\frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{2e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7}$$
$$+\frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7}$$
$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^3x}$$
$$-\frac{3e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^4(bd^2+ae^2)(1+p)}$$
$$+\frac{3e(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2ad^4(1+p)}$$
$$-\frac{2be^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^2(bd^2+ae^2)^2(1+p)}$$
$$+\frac{be^3(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(2,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d^2(bd^2+ae^2)^3(1+p)}$$
$$-\frac{3b^2e^3(a+bx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(3,1+p,2+p,\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

```
[Out] -1/4*e^3*(b*x^2+a)^(p+1)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+3*e^2*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+2*e^2*x*(b
*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+e
^2*x*(b*x^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/
a)^p)+2/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d
```

```
^7/((1+b*x^2/a)^p)+e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b
*x^2/a)/d^7/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a
)/d^3/x/((1+b*x^2/a)^p)-3/2*e^3*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^4/(a*e^2+b*d^2)/(p+1)+3/2*e*(b*x^2+a)^(p+1)*hy
pergeom([1, p+1],[2+p],1+b*x^2/a)/a/d^4/(p+1)-2*b*e^3*(b*x^2+a)^(p+1)*hyper
geom([2, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^2/(p+1)+
1/4*b*e^3*(2*a*e^2+b*d^2*(p+1))*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^3/(p+1)-3/2*b^2*e^3*(b*x^2+a)^
(p+1)*hypergeom([3, p+1],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3
/(p+1)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules

used = {975, 372, 371, 272, 67, 771, 441, 440, 455, 70, 525, 524, 457, 79}

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$$
$$= \frac{2e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7}$$
$$+\frac{e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,3,\frac{5}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7}$$
$$+\frac{3e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{2e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$+\frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}$$
$$-\frac{3b^2e^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)^3}$$
$$+\frac{3e(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2ad^4(p+1)}$$
$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{d^3x}$$
$$+\frac{be^3(a+bx^2)^{p+1}\left(2ae^2+bd^2(p+1)\right)\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d^2(p+1)\left(ae^2+bd^2\right)^3}$$
$$-\frac{2be^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)\left(ae^2+bd^2\right)^2}$$
$$-\frac{e^3(a+bx^2)^{p+1}}{4\left(d^2-e^2x^2\right)^2\left(ae^2+bd^2\right)}$$
$$-\frac{3e^3(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^4(p+1)\left(ae^2+bd^2\right)}$$

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] -1/4*(e^3*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (3*e^2
*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
^5*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -(
(b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*
AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a
```

```
)^p) + (2*e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^
2*x^2)/d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1[3/
2, -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)/a)^p) - ((a
+ b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(d^3*x*(1 + (b*x
^2)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (
e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(
a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*
d^4*(1 + p)) - (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 + p)) + (
b*e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
+ p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(4*d^2*(b*d^2 + a*e^2)^3*(1
 + p)) - (3*b^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p,
(e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(a+bx^2)^p}{d^3x^2}-\frac{3e(a+bx^2)^p}{d^4x}+\frac{e^2(a+bx^2)^p}{d^2(d+ex)^3}+\frac{2e^2(a+bx^2)^p}{d^3(d+ex)^2}+\frac{3e^2(a+bx^2)^p}{d^4(d+ex)}\right)dx\\
&= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^3}-\frac{(3e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^4}+\frac{(3e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^4}\\
&\quad+\frac{(2e^2)\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^3}+\frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx}{d^2}\\
&= -\frac{(3e)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2d^4}+\frac{(3e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^4}\\
&\quad+\frac{(2e^2)\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2}-\frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2}+\frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right)dx}{d^3}\\
&\quad+\frac{e^2\int \left(\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3}-\frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}+\frac{e^3x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\right)dx}{d^2}\\
&\quad+\frac{\left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} \\
&+ \frac{3e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} + \frac{(3e^2)\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d^3} \\
&+ \frac{(2e^2)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx}{d} + (de^2)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx \\
&- (3e^3)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx + \frac{(3e^3)\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx}{d^4} - \frac{(4e^3)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx}{d^2} \\
&+ \frac{(2e^4)\int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\,dx}{d^3} + \frac{(3e^4)\int \frac{x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx}{d} + \frac{e^5\int \frac{x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\,dx}{d^2} \\
&= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} \\
&+ \frac{3e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} \\
&- \frac{1}{2}(3e^3)\,\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^3}\,dx,x,x^2\right) + \frac{(3e^3)\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2d^4} \\
&- \frac{(2e^3)\,\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx,x,x^2\right)}{d^2} + \frac{e^5\text{Subst}\left(\int \frac{x(a+bx)^p}{(-d^2+e^2x)^3}\,dx,x,x^2\right)}{2d^2} \\
&+ \frac{\left(3e^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx}{d^3} \\
&+ \frac{\left(2e^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx}{d} \\
&+ \left(de^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx \\
&+ \frac{\left(2e^4(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(-d^2+e^2x^2)^2}\,dx}{d^3} \\
&+ \frac{\left(3e^4(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} \\
&+ \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
&+ \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
&+ \frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
&+ \frac{2e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7} \\
&+ \frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7} \\
&- \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} \\
&- \frac{3e^3(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^4(bd^2+ae^2)(1+p)} \\
&+ \frac{3e(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} \\
&- \frac{2be^3(a+bx^2)^{1+p}{}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^2(bd^2+ae^2)^2(1+p)} \\
&- \frac{3b^2e^3(a+bx^2)^{1+p}{}_2F_1\left(3,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)} \\
&+ \frac{(e^3(2ae^2+bd^2(1+p)))\,\mathrm{Subst}\left(\int\frac{(a+bx)^p}{(-d^2+e^2x)^2}\,dx,x,x^2\right)}{4d^2(bd^2+ae^2)}
\end{aligned}
$$

$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7} + \frac{e^4x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7} - \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} - \frac{3e^3(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^4(bd^2+ae^2)(1+p)} + \frac{3e(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} - \frac{2be^3(a+bx^2)^{1+p}{}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{d^2(bd^2+ae^2)^2(1+p)} + \frac{be^3(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}{}_2F_1\left(2,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{4d^2(bd^2+ae^2)^3(1+p)} - \frac{3b^2e^3(a+bx^2)^{1+p}{}_2F_1\left(3,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)^3(1+p)}$$

Mathematica [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \frac{(a+bx^2)^p\left(\frac{4de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{d^2e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d
^2*e*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d
+ Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (3*e*AppellF1[-2*p,
-p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d +
e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d
+ e*x))^p) - (2*d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (
b*x^2)/a)^p) - (3*e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))])/(p*(1 +
 a/(b*x^2))^p)))/(2*d^4)
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{x^{2}\left(ex+d\right)^{3}}d x$$

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx=\int \frac{(bx^2+a)^p}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(bx^2+a)^p}{(ex+d)^3x^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)^3}\,dx$$

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^3),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.430 $\int (gx)^m(d+ex)^3 \left(a+cx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 276

$$\int (gx)^m(d+ex)^3 \left(a+cx^2\right)^p \, dx = \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} - \frac{d(3ae^2(1+m)-cd^2(3+m+2p))(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},\right.}{cg(1+m)(3+m+2p)} - \frac{e(ae^2(2+m)-3cd^2(4+m+2p))(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},\right.}{cg^2(2+m)(4+m+2p)}$$

```
[Out] 3*d*e^2*(g*x)^(1+m)*(c*x^2+a)^(p+1)/c/g/(3+m+2*p)+e^3*(g*x)^(2+m)*(c*x^2+a)
^(p+1)/c/g^2/(4+m+2*p)-d*(3*a*e^2*(1+m)-c*d^2*(3+m+2*p))*(g*x)^(1+m)*(c*x^2
+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/g/(1+m)/(3+m+2*p)/(
(1+c*x^2/a)^p)-e*(a*e^2*(2+m)-3*c*d^2*(4+m+2*p))*(g*x)^(2+m)*(c*x^2+a)^p*hy
pergeom([-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/c/g^2/(2+m)/(4+m+2*p)/((1+c*x^2/a
)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{1823, 822, 372, 371\}$

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx$$
$$= \frac{e(gx)^{m+2} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(\frac{3d^2}{m+2}-\frac{ae^2}{c(m+2p+4)}\right) \text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},-\frac{cx^2}{a}\right)}{g^2}$$
$$+ \frac{d(gx)^{m+1} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(\frac{d^2}{m+1}-\frac{3ae^2}{c(m+2p+3)}\right) \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g}$$
$$+ \frac{3de^2(gx)^{m+1} \left(a+cx^2\right)^{p+1}}{cg(m+2p+3)} + \frac{e^3(gx)^{m+2} \left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

```
[In] Int[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (3*d*e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + (e^3*(g*x
)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) + (d*(d^2/(1 + m) - (3
*a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1
 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (e*((3*d^2)/
(2 + m) - (a*e^2)/(c*(4 + m + 2*p)))*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeome
tric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
```

```
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(a+cx^2\right)^p\left(cd^3(4+m+2p)-e\left(ae^2(2+m)-3cd^2(4+m+2p)\right)x+3cde^2(4+m+2p)x^2\right)dx}{c(4+m+2p)}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(-cd(4+m+2p)\left(3ae^2(1+m)-cd^2(3+m+2p)\right)-ce(3+m+2p)\left(ae^2(2+m)-3c\right.\right.}{c^2(3+m+2p)(4+m+2p)}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)\right)\int (gx)^m\left(a+cx^2\right)^p dx + \frac{\left(e\left(3d^2-\frac{ae^2(2+m)}{c(4+m+2p)}\right)\right)\int (gx)^{1+m}\left(a+cx^2\right)^p dx}{g}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p dx + \frac{\left(e\left(3d^2-\frac{ae^2(2+m)}{c(4+m+2p)}\right)\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\frac{cx^2}{a}\right)^p dx}{g}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e\left(3d^2-\frac{ae^2(2+m)}{c(4+m+2p)}\right)(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};-\frac{cx^2}{a}\right)}{g^2(2+m)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.66

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$$
$$=x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{1+m}+ex\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{2+m}+ex\left(\frac{3d\,\text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{cx^2}{a}\right)}{3+m}+\frac{ex\,\text{Hypergeometric2F1}\left(\frac{4+m}{2},-p,\frac{6+m}{2},-\frac{cx^2}{a}\right)}{4+m}\right)\right)\right)$$

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p, (4 +
m)/2, -((c*x^2)/a)])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2, -p, (
5 + m)/2, -((c*x^2)/a)])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2, -p, (6
 + m)/2, -((c*x^2)/a)])/(4 + m)))))/(1 + (c*x^2)/a)^p
```

Maple [F]

$$\int (gx)^m\,(ex+d)^3\,(c\,x^2+a)^p\,dx$$

```
[In] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx=\int (ex+d)^3(cx^2+a)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + a)^p*(g*x)^m, x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 113.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.83

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \frac{a^p d^3 g^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3a^p d^2 e g^m x^{m+2} \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3a^p d e^2 g^m x^{m+3} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{a^p e^3 g^m x^{m+4} \Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**3*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + 3*a**p*d**2*e*g**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I
*pi)/a)/(2*gamma(m/2 + 2)) + 3*a**p*d*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2)
*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/
2 + 5/2)) + a**p*e**3*g**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-p, m/2 + 2), (
m/2 + 3,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3))
```

Maxima [F]

$$\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (ex+d)^3 \left(cx^2+a\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (ex+d)^3(cx^2+a)^p(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^3 \left(a+cx^2\right)^p \, dx = \int (g\,x)^m \left(c\,x^2+a\right)^p \left(d+e\,x\right)^3 dx$$

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3,x)
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3, x)
```

3.431 $\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}$$
$$-\frac{(ae^2(1+m)-cd^2(3+m+2p))\,(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\right.}{cg(1+m)(3+m+2p)}$$
$$+\frac{2de(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{g^2(2+m)}$$

```
[Out] e^2*(g*x)^(1+m)*(c*x^2+a)^(p+1)/c/g/(3+m+2*p)-(a*e^2*(1+m)-c*d^2*(3+m+2*p))
*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/
g/(1+m)/(3+m+2*p)/((1+c*x^2/a)^p)+2*d*e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([
-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/g^2/(2+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1823, 822, 372, 371}

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx$$
$$=\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{ae^2}{c(m+2p+3)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g}$$
$$+\frac{2de(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},-\frac{cx^2}{a}\right)}{g^2(m+2)}$$
$$+\frac{e^2(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}$$

```
[In] Int[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + ((d^2/(1 + m)
 - (a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1
[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (2*d*e*(g
*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*
x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} \\
&\quad+\frac{\int (gx)^m\left(-ae^2(1+m)+cd^2(3+m+2p)+2cde(3+m+2p)x\right)\left(a+cx^2\right)^p\,dx}{c(3+m+2p)} \\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{(2de)\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g} \\
&\quad+\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)\int (gx)^m\left(a+cx^2\right)^p\,dx \\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(2de\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\frac{cx^2}{a}\right)^p\,dx}{g} \\
&\quad+\left(\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx \\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} \\
&\quad+\frac{\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)} \\
&\quad+\frac{2de(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};-\frac{cx^2}{a}\right)}{g^2(2+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int (gx)^m(d+ex)^2 \left(a+cx^2\right)^p \, dx = \frac{x(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d^2(6+5m+m^2) \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)+e(1+m)\right.}{(1+m}$$

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)/2,
-p, (3 + m)/2, -((c*x^2)/a)] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1[(
2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)] + e*(2 + m)*x*Hypergeometric2F1[(3 +
 m)/2, -p, (5 + m)/2, -((c*x^2)/a)])))/((1 + m)*(2 + m)*(3 + m)*(1 + (c*x^2
)/a)^p)
```

Maple [F]

$$\int (gx)^m (ex+d)^2 \left(c\,x^2+a\right)^p dx$$

```
[In] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)^2 \left(a+cx^2\right)^p \, dx = \int (ex+d)^2(cx^2+a)^p(gx)^m \, dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.82

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{a^pd^2g^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^pdeg^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{a^pe^2g^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**2*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*d*e*g**m*x**(m +
 2)*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/
a)/gamma(m/2 + 2) + a**p*e**2*g**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-p, m
/2 + 3/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\int (ex+d)^2\left(cx^2+a\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\int (ex+d)^2\left(cx^2+a\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2,x)
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2, x)
```

3.432 $\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{2},-p,\frac{4+m}{2},-\frac{cx^2}{a}\right)}{g^2(2+m)}$$

```
[Out] d*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g
/(1+m)/((1+c*x^2/a)^p)+e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([-p, 1+1/2*m],[2
+1/2*m],-c*x^2/a)/g^2/(2+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {822, 372, 371}

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g(m+1)} + \frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{2},-p,\frac{m+4}{2},-\frac{cx^2}{a}\right)}{g^2(m+2)}$$

```
[In] Int[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^2)
^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*
(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d\int (gx)^m\left(a+cx^2\right)^p\,dx+\frac{e\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}\\
&= \left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx\\
&\quad+\frac{\left(e\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\frac{cx^2}{a}\right)^p\,dx}{g}\\
&= \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}\\
&\quad+\frac{e(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};-\frac{cx^2}{a}\right)}{g^2(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d(2+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)+e(1+m)x\,\text{Hyperg}\right.}{(1+m)(2+m)}$$

```
[In] Integrate[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3 + m
)/2, -((c*x^2)/a)] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2
, -((c*x^2)/a)]))/((1 + m)*(2 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F]

$$\int (gx)^m(ex+d)\left(c\,x^2+a\right)^p dx$$

```
[In] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \int (ex+d)\left(cx^2+a\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = \frac{a^p d g^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{a^p e g^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

```
[In] integrate((g*x)**m*(e*x+d)*(c*x**2+a)**p,x)
```

```
[Out] a**p*d*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,)
, c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*e*g**m*x**(m + 2)*g
amma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)/(2
*gamma(m/2 + 2))
```

Maxima [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx=\int (ex+d)\left(cx^2+a\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx=\int (ex+d)\left(cx^2+a\right)^p(gx)^m\,dx$$

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx=\int {\left(g\,x\right)}^m\,{\left(c\,x^2+a\right)}^p\,\left(d+e\,x\right)\,dx$$

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x),x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x), x)
```

3.433 $\int (gx)^m (a+cx^2)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int (gx)^m (a+cx^2)^p \, dx = \frac{(gx)^{1+m} (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{cx^2}{a}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g/(
1+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {372, 371}

$$\int (gx)^m (a+cx^2)^p \, dx = \frac{(gx)^{m+1} (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{cx^2}{a}\right)}{g(m+1)}$$

```
[In] Int[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int(gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx\\ &= \frac{(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int(gx)^m\left(a+cx^2\right)^p\,dx = \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},-\frac{cx^2}{a}\right)}{1+m}$$

```
[In] Integrate[(g*x)^m*(a + c*x^2)^p,x]
[Out] (x*(g*x)^m*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(c*x^2)/a)])/((1 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F]

$$\int(gx)^m\left(c\,x^2+a\right)^p\,dx$$

```
[In] int((g*x)^m*(c*x^2+a)^p,x)
[Out] int((g*x)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (gx)^m \left(a+cx^2\right)^p \, dx = \int \left(cx^2+a\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (gx)^m \left(a+cx^2\right)^p \, dx = \frac{a^p g^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix} \middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

```
[In] integrate((g*x)**m*(c*x**2+a)**p,x)
[Out] a**p*g**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), 
c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int (gx)^m \left(a+cx^2\right)^p \, dx = \int \left(cx^2+a\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m \left(a+cx^2\right)^p \, dx = \int \left(cx^2+a\right)^p (gx)^m \, dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(a+cx^2\right)^p \, dx = \int (g\,x)^m \left(c\,x^2+a\right)^p \,dx$$

```
[In] int((g*x)^m*(a + c*x^2)^p,x)
[Out] int((g*x)^m*(a + c*x^2)^p, x)
```

3.434 $\int \frac{(gx)^m(a+cx^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex}\,dx = \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)} - \frac{ex^2(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{2+m}{2},-p,1,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(2+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d/(1+m)/((1+c*x^2/a)^p)-e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,1,-p,
2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^2/(2+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {973, 525, 524}

$$\int \frac{(gx)^m (a+cx^2)^p}{d+ex}\,dx = \frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{m+1}{2},-p,1,\frac{m+3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)} - \frac{ex^2(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{m+2}{2},-p,1,\frac{m+4}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+2)}$$

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^2*(g*x)^m*(a + c*x^2
)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(dx^{-m}(gx)^m\right)\int\frac{x^m(a+cx^2)^p}{d^2-e^2x^2}\,dx-\left(ex^{-m}(gx)^m\right)\int\frac{x^{1+m}(a+cx^2)^p}{d^2-e^2x^2}\,dx\\
&=\left(dx^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&\quad-\left(ex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^{1+m}\left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&=\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)}\\
&\quad-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{2+m}{2};-p,1;\frac{4+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(2+m)}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx$$

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x), x]
```

Maple [F]

$$\int \frac{(gx)^m\,(c\,x^2+a)^p}{ex+d}dx$$

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+a)^p(gx)^m}{ex+d}\,dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+a)^p(gx)^m}{ex+d}\,dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

Giac [F]

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+a)^p(gx)^m}{ex+d}\,dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \int \frac{(g\,x)^m\,(c\,x^2+a)^p}{d+e\,x}\,dx$$

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x),x)
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x), x)
```

3.435 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^2}\,dx = \frac{x(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{2},-p,2,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)} - \frac{2ex^2(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{2+m}{2},-p,2,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(2+m)} + \frac{e^2x^3(gx)^m(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{3+m}{2},-p,2,\frac{5+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(3+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,2,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^2/(1+m)/((1+c*x^2/a)^p)-2*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,2
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^3/(2+m)/((1+c*x^2/a)^p)+e^2*x^3*(g*x)^m
*(c*x^2+a)^p*AppellF1(3/2+1/2*m,2,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(3
+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {976, 525, 524}

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} \, dx = \frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2},-p,2,\frac{m+3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)} + \frac{e^2x^3(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+3}{2},-p,2,\frac{m+5}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(m+3)} - \frac{2ex^2(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+2}{2},-p,2,\frac{m+4}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+2)}$$

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF1[
(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*(3 + m)*(1
+ (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 976

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
```

```
d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-m}(gx)^m\right)\int \left(\frac{d^2x^m(a+cx^2)^p}{(d^2-e^2x^2)^2}-\frac{2dex^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^2}+\frac{e^2x^{2+m}(a+cx^2)^p}{(-d^2+e^2x^2)^2}\right)dx\\
&= \left(d^2x^{-m}(gx)^m\right)\int \frac{x^m(a+cx^2)^p}{(d^2-e^2x^2)^2}\,dx-\left(2dex^{-m}(gx)^m\right)\int \frac{x^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^2}\,dx\\
&\quad+\left(e^2x^{-m}(gx)^m\right)\int \frac{x^{2+m}(a+cx^2)^p}{(-d^2+e^2x^2)^2}\,dx\\
&= \left(d^2x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx\\
&\quad-\left(2dex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^{1+m}\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx\\
&\quad+\left(e^2x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^{2+m}\left(1+\frac{cx^2}{a}\right)^p}{(-d^2+e^2x^2)^2}\,dx\\
&= \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,2;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)}\\
&\quad-\frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{2+m}{2};-p,2;\frac{4+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(2+m)}\\
&\quad+\frac{e^2x^3(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{3+m}{2};-p,2;\frac{5+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(3+m)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx=\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx$$

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x]
```

Maple [F]

$$\int \frac{(gx)^m (c\,x^2 + a)^p}{(ex + d)^2} dx$$

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(gx)^m (a + cx^2)^p}{(d + ex)^2} dx = \int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^2} dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a + cx^2)^p}{(d + ex)^2} dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(gx)^m (a + cx^2)^p}{(d + ex)^2} dx = \int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^2} dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^2} dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} dx = \int \frac{(g\,x)^m \left(c\,x^2+a\right)^p}{(d+e\,x)^2} dx$$

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x)
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x)
```

3.436 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3}\,dx = \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m}{2},-p,3,\frac{3+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)} - \frac{3ex^2(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{2+m}{2},-p,3,\frac{4+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(2+m)} + \frac{3e^2x^3(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{3+m}{2},-p,3,\frac{5+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5(3+m)} - \frac{e^3x^4(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{4+m}{2},-p,3,\frac{6+m}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(4+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,3,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^3/(1+m)/((1+c*x^2/a)^p)-3*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,3
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(2+m)/((1+c*x^2/a)^p)+3*e^2*x^3*(g*x)
^m*(c*x^2+a)^p*AppellF1(3/2+1/2*m,3,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^5/
(3+m)/((1+c*x^2/a)^p)-e^3*x^4*(g*x)^m*(c*x^2+a)^p*AppellF1(2+1/2*m,3,-p,3+1
/2*m,e^2*x^2/d^2,-c*x^2/a)/d^6/(4+m)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {976, 525, 524}

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$
$$= -\frac{e^3x^4(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+4}{2},-p,3,\frac{m+6}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(m+4)}$$
$$+\frac{3e^2x^3(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+3}{2},-p,3,\frac{m+5}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5(m+3)}$$
$$-\frac{3ex^2(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+2}{2},-p,3,\frac{m+4}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(m+2)}$$
$$+\frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2},-p,3,\frac{m+3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+1)}$$

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF
1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^5*(3 + m)*(
1 + (c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a + c*x^2)^p*AppellF1[(4 + m)/2, -p,
3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 976

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-m}(gx)^m\right)\int\left(\frac{d^3x^m(a+cx^2)^p}{(d^2-e^2x^2)^3}-\frac{3d^2ex^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^3}+\frac{3de^2x^{2+m}(a+cx^2)^p}{(d^2-e^2x^2)^3}+\frac{e^3x^{3+m}(a+cx^2)^p}{(-d^2+e^2x^2)^3}\right)dx\\
&= \left(d^3x^{-m}(gx)^m\right)\int\frac{x^m(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx-\left(3d^2ex^{-m}(gx)^m\right)\int\frac{x^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad+\left(3de^2x^{-m}(gx)^m\right)\int\frac{x^{2+m}(a+cx^2)^p}{(d^2-e^2x^2)^3}\,dx+\left(e^3x^{-m}(gx)^m\right)\int\frac{x^{3+m}(a+cx^2)^p}{(-d^2+e^2x^2)^3}\,dx\\
&= \left(d^3x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad-\left(3d^2ex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^{1+m}\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad+\left(3de^2x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^{2+m}\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^3}\,dx\\
&\quad+\left(e^3x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^{3+m}\left(1+\frac{cx^2}{a}\right)^p}{(-d^2+e^2x^2)^3}\,dx\\
&= \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,3;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)}\\
&\quad-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{2+m}{2};-p,3;\frac{4+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(2+m)}\\
&\quad+\frac{3e^2x^3(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{3+m}{2};-p,3;\frac{5+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5(3+m)}\\
&\quad-\frac{e^3x^4(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{4+m}{2};-p,3;\frac{6+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(4+m)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3}\,dx = \int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3}\,dx$$

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]
```

Maple [F]

$$\int \frac{(gx)^m (c\,x^2+a)^p}{(ex+d)^3}dx$$

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3}\,dx = \int \frac{(cx^2+a)^p(gx)^m}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} \, dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} \, dx = \int \frac{(cx^2+a)^p (gx)^m}{(ex+d)^3} \, dx$$

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} \, dx = \int \frac{(g\,x)^m\,(c\,x^2+a)^p}{(d+e\,x)^3} \, dx$$

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x)
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x)
```

3.437 $\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 345

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e}$$
$$-\frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{192c^3d^3e^4}$$
$$+\frac{(cd^2-ae^2)\,(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(5*a^3*e^6+9*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+35*c^3*d^6
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)+1/24*(a/c/d-7*d/e^2)*x^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e-1/192*(105*c^3*d^6-25*a*c^2*d^4*e^2-17*a^2*c*d^2*e^4-15*a^
3*e^6-2*c*d*e*(-5*a^2*e^4-6*a*c*d^2*e^2+35*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 846, 793, 635, 212}

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{(cd^2-ae^2)\,(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6)\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}$$
$$-\frac{(-15a^3e^6-2cdex(-5a^2e^4-6acd^2e^2+35c^2d^4)-17a^2cd^2e^4-25ac^2d^4e^2+105c^3d^6)\,\sqrt{x\,(ae^2+cd^2)+}}{192c^3d^3e^4}$$
$$+\frac{1}{24}x^2\left(\frac{a}{cd}-\frac{7d}{e^2}\right)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}+\frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4e}$$

```
[In] Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^3(ae+cdx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$

$$= \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} + \frac{\int \frac{x^2\left(-3acd^2e-\frac{1}{2}cd(7cd^2-ae^2)x\right)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4cde}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2} + \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} + \frac{\int \frac{x\left(acd^2e(7cd^2-ae^2)+\frac{1}{4}cd(35c^2d^4-6acd^2e^2-5a^2e^4)x\right)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{12c^2d^2e^2}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2} + \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} - \frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)x)\sqrt{ade+(cd^2+}}{192c^3d^3e^4} + \frac{((cd^2-ae^2)(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6))\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128c^3d^3e^4}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}+\frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e}$$
$$-\frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)x)\sqrt{ade+(cd^2}}{192c^3d^3e^4}$$
$$+\frac{((cd^2-ae^2)(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6))\,\text{Subst}\left(\int\frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cd}}\right.}{64c^3d^3e^4}$$
$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}+\frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e}$$
$$-\frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)x)\sqrt{ade+(cd^2}}{192c^3d^3e^4}$$
$$+\frac{(cd^2-ae^2)(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.88

$$\int\frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{c}\sqrt{d}\sqrt{e}(-15a^3e^6+a^2cde^4(-17d+10ex)+ac^2d^2e^2(-25d^2+12dex-8e^2x^2)\right.}{192}$$

```
[In] Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a^
2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^2)
 + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 +
5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*S
qrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(315) = 630.

Time = 0.68 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.77

method	result
default	$\frac{x\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{4cde}-\frac{5\left(e^{2}a+c\,d^{2}\right)\left(\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{3cde}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(\frac{\left(2cdex+e^{2}a+c\,d^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{4cde}+\cdots\right.}{\cdots}\right.}{8cde}$

[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/e*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-5/8*(a*e^2+c*d^2)/c/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+d^2/e^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d^3/e^4*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.97

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \left[-\frac{3\left(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e\right.}{}\right.$$
$$-\frac{3\left(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+ac}\right.}{}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^
4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*
e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d
^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2
*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*e^4*x^
3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
- 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d^4*e
^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
4*d^4*e^5)]
```

Sympy [F]

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^3\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.88

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,x\left(\frac{6\,x}{e}-\frac{7\,c^3d^4e^2-ac^2d^2e^4}{c^3d^3e^4}\right)+\frac{35\,c^3d^5e-6\,ac^2d^3e^3-5\,a^2cde^5}{c^3d^3e^4}\right)x\right.$$
$$-\frac{(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+}\right.\right.\right.}{128\sqrt{cde}c^3d^3e^4}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*x*(6*x/e - (7*c^3*d
^4*e^2 - a*c^2*d^2*e^4)/(c^3*d^3*e^4)) + (35*c^3*d^5*e - 6*a*c^2*d^3*e^3 -
5*a^2*c*d*e^5)/(c^3*d^3*e^4))*x - (105*c^3*d^6 - 25*a*c^2*d^4*e^2 - 17*a^2*
c*d^2*e^4 - 15*a^3*e^6)/(c^3*d^3*e^4)) - 1/128*(35*c^4*d^8 - 20*a*c^3*d^6*e
^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*log(abs(-c*d^2 - a*e^
2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.438 $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 251

$$\begin{aligned}\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx &= \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} \\ &+ \frac{((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} \\ &- \frac{(cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\,\mathrm{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}\end{aligned}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e
*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/e+1/24*((-3*a*e^2+5*c*d^2)*(a*e^2+3*c*d^2)-2*c*d*e*(-a*e^2+5*c*d^2)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{865, 846, 793, 635, 212\}$

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$$
$$= -\frac{(cd^2-ae^2)(a^2e^4+2acd^2e^2+5c^2d^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}} + \frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24c^2d^2e^3} + \frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}$$

```
[In] Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
```

```
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{x^2(ae+cdx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{\int \frac{x\left(-2acd^2e-\frac{1}{2}cd(5cd^2-ae^2)x\right)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3cde}$$

$$= \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{\left((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} - \frac{\left((cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16c^2d^2e^3}$$

$$= \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{\left((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} - \frac{\left((cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^2d^2e^3}$$

$$= \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{\left((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} - \frac{(cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 10.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4+2acde^2(-2d+ex)+c^2d^2(15d^2-10dex+8e^2x^2))-\frac{3\sqrt{cd}\sqrt{cd^2\cdots}}{\cdots}\right.}{24c^{5/2}d^{5/2}e^{7/2}}$$

```
[In] Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*c
*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d^(5/2
)*e^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(225) = 450.

Time = 0.66 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.04

method	result
default	$\frac{\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{3cde}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cde\cdots}{\sqrt{cde}}\cdots\right)}{8cde\sqrt{cde}}\right.}{2cde}\cdots}{e}$

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e
*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-
d/e^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))
+d^2/e^3*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*l
n((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.14

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\left[-\frac{3\,(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade}\right.}{}\right.$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c
^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6 - 3*a
*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*
e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2
 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\,x\left(\frac{4\,x}{e}-\frac{5\,c^2d^3e-acde^3}{c^2d^2e^3}\right)+\frac{15\,c^2d^4-4\,acd^2e^2-3\,a^2e^4}{c^2d^2e^3}\right)$$
$$+\frac{(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ac}\right.\right.\right.}{16\,\sqrt{cde}c^2d^2e^3}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x*(4*x/e - (5*c^2*d^3*e
 - a*c*d*e^3)/(c^2*d^2*e^3)) + (15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4)/(c^
2*d^2*e^3)) + 1/16*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*
log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.439 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 207

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{(cd^2-ae^2)(3cd^2+ae^2)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e/(e*x+d)+1/8*(-a*e^2+c*d^2
)*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/4*(a/
c/d+3*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {808, 678, 635, 212}

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{(cd^2-ae^2)(ae^2+3cd^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}} + \frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}$$

```
[In] Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] -1/4*((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2
 - a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d
^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}\\&\quad+\frac{1}{4}\left(-\frac{3d}{e}-\frac{ae}{cd}\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx\end{aligned}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{\left(\left(\frac{3d}{e}+\frac{ae}{cd}\right)\left(2cd^2e-e(cd^2+ae^2)\right)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8e^2}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{\left(\left(\frac{3d}{e}+\frac{ae}{cd}\right)\left(2cd^2e-e(cd^2+ae^2)\right)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4e^2}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{(cd^2-ae^2)\,(3cd^2+ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(ae^2+cd(-3d+2ex))+\frac{(-6c^2d^4+4acd^2e^2+2a^2e^4)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\left(\sqrt{d-\frac{ae^2}{cd}}-\sqrt{d+ex}\right)}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4c^{3/2}d^{3/2}e^{5/2}}$$

```
[In] Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
+ 2*e*x)) + ((-6*c^2*d^4 + 4*a*c*d^2*e^2 + 2*a^2*e^4)*ArcTanh[(Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]
)/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.41

method	result
default	$\frac{\frac{(2cdex+e^2a+c\,d^2)\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}}{4cde}+\frac{(4ac\,d^2e^2-(e^2a+c\,d^2)^2)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}\right)}{8cde\sqrt{cde}}}{e}-\frac{d\left(\sqrt{cde}\right.}{}$

[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.02

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\left[-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade+(cd^2+ae\right.}{}\right.$$
$$-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(2\,c^2d^2e^2x-3\,c^2}{8\,c^2d^2e^3}$$

[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fricas")

[Out] [-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]

Sympy [F]

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
[Out] Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.79

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{e}-\frac{3\,cd^2-ae^2}{cde^2}\right)-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}cde^2}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="g
iac")
[Out] 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/e - (3*c*d^2 - a*e^2)/
(c*d*e^2)) - 1/8*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*log(abs(-c*d^2 - a*e
^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx = \int \frac{x\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{d + e\,x}\,dx$$

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.440 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} dx$

Optimal result

Integrand size = 37, antiderivative size = 131

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {678, 635, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} dx = \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e} - \frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2e^2}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{e^2}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}+\frac{(-cd^2+ae^2)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqrt
[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*S
qrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{cde\left(x+\frac{d}{e}\right)^2+(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}+\frac{\left(e^2a-c\,d^2\right)\ln\left(\frac{\frac{e^2a}{2}-\frac{c\,d^2}{2}+cde\left(x+\frac{d}{e}\right)}{\sqrt{cde}}+\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}}{e}$	131

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/
2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^
2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$$
$$=\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{c\right.}{4\,cde^2}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(c
*d*e^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{d + ex}\, dx = \int \frac{\sqrt{(d + ex) (ae + cdx)}}{d + ex}\, dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{d + ex}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{d + ex}\, dx = \frac{(cd^2 - ae^2) \log \left(\left| -cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right) \right| \right)}{2\sqrt{cde}e} + \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}}{e}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) + sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{d + e\,x}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

3.441 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 168

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx = \frac{\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-arctanh(1/2*(2*a*d*e+(a*e
^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))*a^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 857, 635, 212, 738}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx = \frac{\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{ae+cdx}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx\\ &= (cd)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx+(ae)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx\end{aligned}$$

$$= (2cd)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$
$$-(2ae)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$
$$= \frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{d}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 476 vs. $2(168) = 336$.

Time = 1.41 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx = -\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\left(\left(-\sqrt{c}d+\sqrt{cd^2-ae^2}\right)\sqrt{-2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2-ae^2}}\arctan\left(\frac{\sqrt{-2cd^2+ae^2}}{\sqrt{a}\sqrt{c}\sqrt{d}\sqrt{}}\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-((-(Sqrt[c]*d) + Sqrt[c*d^2 - a*e^2])
*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2*c
*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]
*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]) + (Sq
rt[c]*d + Sqrt[c*d^2 - a*e^2])*Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d
^2 - a*e^2]]*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2
]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c
*d)] - Sqrt[d + e*x]))] + 2*Sqrt[a]*Sqrt[c]*d*e*ArcTanh[(Sqrt[e]*Sqrt[a*e +
 c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]))/(Sq
rt[a]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(136) = 272$.

Time = 0.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}+\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{2\sqrt{cde}}-\frac{ade\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+}}{x}\right.}{\sqrt{ade}}}{d}$

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/d*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-1/d*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 947, normalized size of antiderivative = 5.64

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \left[\frac{1}{2}\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\left(2\,cde^2x + cd^2e + ae^3\right)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{\frac{cd}{e}} + 8\left(c^2d^3e + acde^3\right)x\right) + \frac{1}{2}\sqrt{\frac{ae}{d}}\log\left(\frac{8\,a^2d^2e^2 + (c^2d^4 + 6\,acd^2e^2 + a^2e^4)x^2 - 4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2\,ad^2e + (cd^3 + ac\ldots}{x^2}\right.\right.$$

$$-\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2\,cdex + cd^2 + ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2 + acd^2e + (c^2d^3 + acde^2)x)}\right)$$

$$+\frac{1}{2}\sqrt{\frac{ae}{d}}\log\left(\frac{8\,a^2d^2e^2 + (c^2d^4 + 6\,acd^2e^2 + a^2e^4)x^2 - 4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2\,ad^2e + (cd^3 + ac\ldots}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\left(2\,cde^2x + cd^2e + ae^3\right)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{\frac{cd}{e}} + 8\left(c^2d^3e + acde^3\right)x\right),$$

$$-\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2\,cdex + cd^2 + ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2 + acd^2e + (c^2d^3 + acde^2)x)}\right) + \sqrt{-\frac{ae}{d}}\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2\,ade + (cd^2 + ae^2)x)\sqrt{-\frac{ae}{d}}}{2\,(acde^2x^2 + a^2de^2 + (acd^2e + a^2e^3)x)}\right)\right]$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
+ 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^
2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
```

```
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*d/e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*
e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d
)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x))]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x\,(d + ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="g
iac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x(d + ex)}\, dx = \int \frac{\sqrt{c\, d\, e\, x^2 + (c\, d^2 + a\, e^2)\ x + a\, d\, e}}{x\ (d + e\, x)}\, dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.442 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{dx} - \frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {863, 820, 738, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx = -\frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{ae+cdx}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{dx}-\frac{(-2acd^2e+ae(cd^2+ae^2))\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2ade}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{dx}\\
&\quad+\frac{(-2acd^2e+ae(cd^2+ae^2))\,\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{ade}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{dx}-\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 841 vs. $2(137) = 274$.

Time = 9.25 (sec) , antiderivative size = 841, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx$$
$$= \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(-\frac{\sqrt{ae+cdx}\left(a^2e^4\sqrt{d+ex}+2acde^2\left(2d\sqrt{d-\frac{ae^2}{cd}}+2e\sqrt{d-\frac{ae^2}{cd}}x-4d\sqrt{d+ex}-3ex\sqrt{d+ex}\right)+c^2\left(d^2e^2x^2\left(-4\sqrt{d-\frac{ae}{cc}}\right.\right.\right.}{a^2de^4x-2acd^2e^2x\left(4d+3ex-2\sqrt{d-\frac{ae^2}{cd}}\sqrt{d+ex}\right)+c^2d^3x\left(8d^2+8dex+e^2x^2\right.}\right.}{}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-((Sqrt[a*e + c*d*x]*(a^2*e^4*Sqrt[d + e*
x] + 2*a*c*d*e^2*(2*d*Sqrt[d - (a*e^2)/(c*d)] + 2*e*Sqrt[d - (a*e^2)/(c*d)]
*x - 4*d*Sqrt[d + e*x] - 3*e*x*Sqrt[d + e*x]) + c^2*(d^2*e^2*x^2*(-4*Sqrt[d
 - (a*e^2)/(c*d)] + Sqrt[d + e*x]) + d^3*e*x*(-12*Sqrt[d - (a*e^2)/(c*d)] +
 8*Sqrt[d + e*x]) + d^4*(-8*Sqrt[d - (a*e^2)/(c*d)] + 8*Sqrt[d + e*x]))))/(
a^2*d*e^4*x - 2*a*c*d^2*e^2*x*(4*d + 3*e*x - 2*Sqrt[d - (a*e^2)/(c*d)]*Sqrt
[d + e*x]) + c^2*d^3*x*(8*d^2 + 8*d*e*x + e^2*x^2 - 8*d*Sqrt[d - (a*e^2)/(c
*d)]*Sqrt[d + e*x] - 4*e*Sqrt[d - (a*e^2)/(c*d)]*x*Sqrt[d + e*x]))) + (Sqrt
[c*d^2 - a*e^2]*(c*d^2 - a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 - a*e^2])*ArcTan[(Sqr
t[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(S
qrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))])
/(Sqrt[a]*d^(3/2)*Sqrt[e]*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 -
a*e^2]]) + (Sqrt[c*d^2 - a*e^2]*(-(c*d^2) + a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 -
a*e^2])*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)]
- Sqrt[d + e*x]))])/(Sqrt[a]*d^(3/2)*Sqrt[e]*Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt
[c]*d*Sqrt[c*d^2 - a*e^2]])))/Sqrt[(a*e + c*d*x)*(d + e*x)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(117) = 234$.

Time = 0.72 (sec) , antiderivative size = 708, normalized size of antiderivative = 5.17

method	result
default	$-\frac{\left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{adex}+\frac{\left(e^{2}a+c\,d^{2}\right)\left(\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}+\frac{\left(e^{2}a+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}e^{2}a+\frac{1}{2}c\,d^{2}+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}\right)}{2\sqrt{cde}}\right.}{2ade}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a
/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2
*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e
^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-e/d^2*((a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/
(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/
(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2
)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx$$
$$=\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade+(cd^2-ae^2)\sqrt{ade}x\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{cdex^2+ade+(c}}{x}\right.}{4\,ad^2ex}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade-(cd^2-ae^2)\sqrt{-ade}x\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)x)\sqrt{}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right.}{2\,ad^2ex}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^2
)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^
```

```
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2*(2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sqrt(-a
*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
+ a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^2\,(d + ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^2}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"maxima")
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \frac{(cd^2 - ae^2)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}d} - \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)cd^2 + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ae^2 + 2\sqrt{cde}}{\left(ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2\right)d}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] (c*d^2 - a*e^2)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))*c*d^2 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))*a*e^2 + 2*sqrt(c*d*e)*a*d*e)/((a*d*e - (sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.443 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 202

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x}+\frac{(cd^2-ae^2)\,(cd^2+3ae^2)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}$$

```
[Out] 1/8*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5
/2)/e^(3/2)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2-1/4*(c/a/e-3*
e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 848, 820, 738, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = \frac{(cd^2-ae^2)\,(3ae^2+cd^2)\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^2) - ((c/(a*e) - (3*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2
)*(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)
*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-3ae^2)+acde^2x}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2ade}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} - \frac{\left(\frac{c^2d^2}{a}+2ce^2-\frac{3ae^4}{d^2}\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8e}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} + \frac{\left(\frac{c^2d^2}{a}+2ce^2-\frac{3ae^4}{d^2}\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4e}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} + \frac{(cd^2-ae^2)\,(cd^2+3ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-cd^2x+ae(-2d+3ex))}{x^2}+\frac{(c^2d^4+2acd^2e^2-3a^2e^4)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e*
(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sqrt
[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(Sqrt[a*e + c*d*x]
*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. 2(176) = 352.

Time = 0.76 (sec) , antiderivative size = 1353, normalized size of antiderivative = 6.70

method	result	size
default	Expression too large to display	1353

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^
2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d
^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/
2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*
x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*
d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e^2/d^3*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/x))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2
*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*
d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/
4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*
d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-e^2/d^3
*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e
^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*
(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx$$
$$=\left[-\frac{(c^2d^4+2\,acd^2e^2-3\,a^2e^4)\sqrt{ade}x^2\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)}{x^2}\right.}{16\,a^2d^3e^2x^2}\right.$$
$$-\frac{(c^2d^4+2\,acd^2e^2-3\,a^2e^4)\sqrt{-ade}x^2\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right)+2\,(2\,a^2d^2e^2+}{8\,a^2d^3e^2x^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^
2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3
*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((c^2*d
^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^
2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^2*d^3*e^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x^3\,(d+ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(ex+d)x^3}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"maxima")
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(176) = 352$.

Time = 0.31 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx$$
$$=-\frac{(c^2d^4+2\,acd^2e^2-3\,a^2e^4)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{4\,\sqrt{-ade}ad^2e}$$
$$+\frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ac^2d^5e+10\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2cd^3e^3-}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"giac")
[Out] -1/4*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a*d^2*e)
+ 1/4*((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*
d^5*e + 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^
2*c*d^3*e^3 + 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^3*d*e^5 + 8*sqrt(c*d*e)*a^3*d^2*e^4 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^3*c^2*d^4 + 2*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c*d^2*e^2 - 3*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*e^4 + 8*sqrt(c*d*e)*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e)/((a*d*e - (sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*a*d^2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e}}{x^3\;(d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.444 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx &= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2}\\ &\quad+\frac{(3cd^2-5ae^2)\,(cd^2+3ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^3e^2x}\\ &\quad-\frac{(cd^2-ae^2)\,(c^2d^4+2acd^2e^2+5a^2e^4)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}\end{aligned}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/d/x^3-1/12*(c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2+1/2
4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^2/d^3/e^2/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 848, 820, 738, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx = \frac{(3cd^2-5ae^2)(3ae^2+cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24a^2d^3e^2x} - \frac{(cd^2-ae^2)(5a^2e^4+2acd^2e^2+c^2d^4)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12x^2} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3dx^3}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^3) - ((c/(a*e) - (5*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 -
5*a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24
*a^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
```

```
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{ae+cdx}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-5ae^2)+2acde^2x}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3ade} \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2} \\
&\quad + \frac{\int \frac{-\frac{1}{4}ae(3cd^2-5ae^2)(cd^2+3ae^2)-\frac{1}{2}acde^2(cd^2-5ae^2)x}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{6a^2d^2e^2} \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2} \\
&\quad + \frac{(3cd^2-5ae^2)\,(cd^2+3ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^3e^2x} \\
&\quad + \frac{((cd^2-ae^2)\,(c^2d^4+2acd^2e^2+5a^2e^4))\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16a^2d^3e^2}
\end{aligned}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{12x^2} + \frac{(3cd^2 - 5ae^2)\,(cd^2 + 3ae^2)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{24a^2d^3e^2x} - \frac{((cd^2 - ae^2)\,(c^2d^4 + 2acd^2e^2 + 5a^2e^4))\,\text{Subst}\left(\int \frac{1}{4ade - x^2}\,dx, x, \frac{2ade - (-cd^2 - ae^2)\,x}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\right)}{8a^2d^3e^2}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{12x^2} + \frac{(3cd^2 - 5ae^2)\,(cd^2 + 3ae^2)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{24a^2d^3e^2x} - \frac{(cd^2 - ae^2)\,(c^2d^4 + 2acd^2e^2 + 5a^2e^4)\tanh^{-1}\left(\frac{2ade + (cd^2 + ae^2)\,x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx$$

$$= \frac{\sqrt{(ae + cdx)(d + ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^2d^4x^2 - 2acd^2ex(d - 2ex) + a^2e^2(-8d^2 + 10dex - 15e^2x^2))}{x^3} - \frac{3(c^3d^6 + ac^2d^4e^2 + 3a^2cd^2e^4 - 5a^3e^6)\,\text{ar}}{\sqrt{ae + cdx}\sqrt{d + e}}\right.}{24a^{5/2}d^{7/2}e^{5/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 - 2
*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3 -
 (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. 2(256) = 512.

Time = 1.16 (sec) , antiderivative size = 2059, normalized size of antiderivative = 7.20

method	result	size
default	Expression too large to display	2059

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d^
2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^
2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*l
n((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c
*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d
^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^
(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^2/d^3*(-1/a/d/e/x*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)
^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/
2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)))-e^3/d^4*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2
*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(
a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))
-e/d^2*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c
*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+
c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln(
(1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d
*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2
+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e^3/d^4*((c*d*e*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))
/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 1.23 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx$$
$$= \left[-\frac{3\,(c^3d^6 + ac^2d^4e^2 + 3\,a^2cd^2e^4 - 5\,a^3e^6)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2 + (c^2d^4 + 6\,acd^2e^2 + a^2e^4)x^2 + 4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(}{x^2}\right.}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c^2*d
^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e
^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3), 1/48
*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d
*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(ex+d)x^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(256) = 512$.

Time = 0.33 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx = \frac{(c^3d^6+ac^2d^4e^2+3\,a^2cd^2e^4-5\,a^3e^6)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{8\sqrt{-ade}a^2d^3e^2} - \frac{3\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2c^3d^8e^2+51\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^3c^2d}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/8*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*arctan(-(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(
-a*d*e)*a^2*d^3*e^2) - 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 + 51*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 + 105*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 + 33*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 16*sqrt(c*d*e)*a^4*c
*d^5*e^5 + 48*sqrt(c*d*e)*a^5*d^3*e^7 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e + 72*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 + 24*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 - 40*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 + 48*sqrt(
c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*
c^2*d^6*e^2 + 144*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
```

```
2*x + a*e^2*x + a*d*e))^5*c^3*d^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c*d^2*e^4 + 15*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*e^6)/((a*d*e - (sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^3*a^2*d^3*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^4\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.445 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 389

$$\begin{aligned}
&\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24x^3} \\
&\quad + \frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96a^2d^3e^2x^2} \\
&\quad - \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{192a^3d^4e^3x} \\
&\quad + \frac{(cd^2-ae^2)\,(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6)\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}
\end{aligned}$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(35*a^3*e^6+15*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^6
)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/4*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/d/x^4-1/24*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/x^3+1/96*(-35*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/192*(-105*a^3*e^6+25*a^2*c*d^2
*e^4+17*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a
^3/d^4/e^3/x
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 848, 820, 738, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$$
$$= \frac{(-35a^2e^4+6acd^2e^2+5c^2d^4)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96a^2d^3e^2x^2} - \frac{(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192a^3d^4e^3x} + \frac{(cd^2-ae^2)(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6)\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24x^3}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^4) - ((c/(a*e) - (7*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4
+ 6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(96*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 -
 105*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3
*x) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35
*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{ae+cdx}{x^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4}-\frac{\int \frac{-\frac{1}{2}ae(cd^2-7ae^2)+3acde^2x}{x^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4ade} \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24x^3} \\
&\quad+\frac{\int \frac{-\frac{1}{4}ae(5c^2d^4+6acd^2e^2-35a^2e^4)-acde^2(cd^2-7ae^2)x}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{12a^2d^2e^2} \\
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24x^3} \\
&\quad+\frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96a^2d^3e^2x^2} \\
&\quad-\frac{\int \frac{-\frac{1}{8}ae(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)-\frac{1}{4}acde^2(5c^2d^4+6acd^2e^2-35a^2e^4)x}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{24a^3d^3e^3}
\end{aligned}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24x^3} + \frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96a^2d^3e^2x^2} - \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192a^3d^4e^3x} - \frac{((cd^2-ae^2)(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6))\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{128a^3d^4e^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24x^3} + \frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96a^2d^3e^2x^2} - \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192a^3d^4e^3x} + \frac{((cd^2-ae^2)(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6))\,\text{Subst}\left(\int\frac{1}{4ade-x^2}dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cd}}\right.}{64a^3d^4e^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24x^3} + \frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96a^2d^3e^2x^2} - \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192a^3d^4e^3x} + \frac{(cd^2-ae^2)(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 10.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.70

$$\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-15c^3d^6x^3+ac^2d^4ex^2(10d-17ex)+a^2cd^2e^2x(-8d^2+12dex-25e^2x^2)+a^3e^3(-48d^3+56d^2ex-70de^2x}{x^4}\right.}{192a^{7/2}d^{9/2}e^{7/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3 +
 a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 25*
e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/x^
4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3470 vs. $2(355) = 710$.

Time = 1.12 (sec) , antiderivative size = 3471, normalized size of antiderivative = 8.92

method	result	size
default	Expression too large to display	3471

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/8*(a*e^2+c*d^
2)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2
+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(
a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*
(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d
^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4
*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*
((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*
d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))-1/4*c/a*(-1/2/a/d/e/x^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d
*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/
d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/x))))+e^4/d^5*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*
d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e^2/d^3*(
-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/
d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a
/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2
```

```
*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e
^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x
)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*
e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/x)))-e^3/d^4*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2)))-e/d^2*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1
/2*(a*e^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(
a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*
e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2
*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))
+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2
*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))-e^4/d^5*((c*d*e*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*e^2*a-1/2*c*d
^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(c*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 8.55 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$$

$$= \left[-\frac{3\left(5c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+20a^3cd^2e^6-35a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8a^2d^2e^2+(c^2d^4+6acd^2e^2+a^2e^4)x^2-4\sqrt{cde}\cdots}{\cdots}\right)}{\cdots}\right.$$

$$-\frac{3\left(5c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+20a^3cd^2e^6-35a^4e^8\right)\sqrt{-ade}\,x^4\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2ade+(cd\cdots}{2(acd^2e^2x^2+a^2d^2e^2+(acd^3e+\cdots}\right)}{\cdots}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*
e^6 - 35*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*
(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d^3*e^5 -
 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a^4*d^2*e
^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4), -1/384*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2
+ 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-a*d*e)*x^4*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*
x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d
^3*e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a
^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{x^5(d + ex)} dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^5} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"maxima")
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1483 vs. $2(355) = 710$.

Time = 0.34 (sec) , antiderivative size = 1483, normalized size of antiderivative = 3.81

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{x^5(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"giac")
[Out] -1/64*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 -
 35*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) + 1/192*(15*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 + 396*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5 +
 1170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^2
*d^7*e^7 + 1212*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^6*c*d^5*e^9 + 279*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^7*d^3*e^11 + 128*sqrt(c*d*e)*a^5*c^2*d^8*e^6 + 256*sqrt(c*d*e)*
a^6*c*d^6*e^8 + 384*sqrt(c*d*e)*a^7*d^4*e^10 + 73*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 980*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*e^4 + 2238*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^2*d^6*
e^6 + 292*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a
^5*c*d^4*e^8 - 511*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))^3*a^6*d^2*e^10 + 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^2*a^3*c^3*d^9*e^3 + 1792*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 + 2432*
sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2
*a^5*c*d^5*e^7 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^5*a*c^4*d^9*e - 44*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^5*a^2*c^3*d^7*e^3 - 66*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
```

```
*x + a*e^2*x + a*d*e))^5*a^3*c^2*d^5*e^5 - 220*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*c*d^3*e^7 + 385*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^5*d*e^9 + 768*sqrt(c*d*e)*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^3*c^2*d^6*e
^4 + 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*c^4
*d^8 + 12*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a
*c^3*d^6*e^2 + 18*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^7*a^2*c^2*d^4*e^4 + 60*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))^7*a^3*c*d^2*e^6 - 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^7*a^4*e^8)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^2)^4*a^3*d^4*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^5\,\left(d+e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.446 $\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 449

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8)(cd^2+ae^2+2cdex)\sqrt{\ldots}}{512c^4d^4e^5}$$
$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{6e}$$
$$-\frac{(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde(21c^2d^4-6acd^2e^2-7a^2e^4)x)(ade+(cd^2+ae^2)x+\ldots}{960c^3d^3e^4}$$
$$-\frac{(cd^2-ae^2)^3(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}$$

```
[Out] 1/20*(a/c/d-3*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*x^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e-1/960*(105*c^3*d^6-21*a*c^2*d^4*e^2
-33*a^2*c*d^2*e^4-35*a^3*e^6-6*c*d*e*(-7*a^2*e^4-6*a*c*d^2*e^2+21*c^2*d^4)*
x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/1024*(-a*e^2+c*d^2
)^3*(7*a^3*e^6+15*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+21*c^3*d^6)*arctanh(1/2*(2
*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/512*(-7*a^4*e^8-8*a^3*c*d^2*e^6-6*a^
2*c^2*d^4*e^4+21*c^4*d^8)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 846, 793, 626, 635, 212}

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx =$$

$$-\frac{(7a^3e^6 + 15a^2cd^2e^4 + 21ac^2d^4e^2 + 21c^3d^6)(cd^2 - ae^2)^3 \operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}$$

$$-\frac{(-35a^3e^6 - 6cdex(-7a^2e^4 - 6acd^2e^2 + 21c^2d^4) - 33a^2cd^2e^4 - 21ac^2d^4e^2 + 105c^3d^6)\,(x(ae^2 + cd^2) + ade +}{960c^3d^3e^4}$$

$$+\frac{(-7a^4e^8 - 8a^3cd^2e^6 - 6a^2c^2d^4e^4 + 21c^4d^8)(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512c^4d^4e^5}$$

$$+\frac{1}{20}x^2\left(\frac{a}{cd} - \frac{3d}{e^2}\right)\left(x(ae^2 + cd^2) + ade + cdex^2\right)^{3/2} + \frac{x^3(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{6e}$$

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx \\
&= \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e} \\
&\quad+\frac{\int x^2\left(-3acd^2e-\frac{3}{2}cd\left(3cd^2-ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{6cde} \\
&= \frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e} \\
&\quad+\frac{\int x\left(3acd^2e\left(3cd^2-ae^2\right)+\frac{3}{4}cd\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{30c^2d^2e^2}
\end{aligned}$$

$$=\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$
$$-\frac{\left(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\left(ade+\left(cd^2+\right.\right.}{960c^3d^3e^4}$$
$$+\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\int\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{128c^3d^3e^4}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^5}$$
$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$
$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$
$$-\frac{\left(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\left(ade+\left(cd^2+\right.\right.}{960c^3d^3e^4}$$
$$-\frac{\left(\left(cd^2-ae^2\right)^3\left(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6\right)\right)\int\frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{1024c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^5}$$
$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$
$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$
$$-\frac{\left(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\left(ade+\left(cd^2+\right.\right.}{960c^3d^3e^4}$$
$$-\frac{\left(\left(cd^2-ae^2\right)^3\left(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6\right)\right)\text{Subst}\left(\int\frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+}}\right.}{512c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^5}$$
$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$
$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$
$$-\frac{\left(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\left(ade+\left(cd^2+\right.\right.}{960c^3d^3e^4}$$
$$-\frac{\left(cd^2-ae^2\right)^3\left(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.86

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^5e^{10}+5a^4cde^8(11d+1\right.}{}$$

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5*
a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e^2
*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3) +
a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*d^2*e
^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*(c*d^2 - a*e^2)^3*(21*c^3*d^6
 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(Sqrt[c]*Sqrt[d
]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d +
e*x])))/(7680*c^(9/2)*d^(9/2)*e^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. 2(415) = 830.

Time = 0.64 (sec) , antiderivative size = 1426, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1426

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/6*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-7/12*(a*e^2+c*d^2)
/c/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)
/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e*x+a*e^
2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^
2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2))))+d^2/e^3*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4
*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*
```

```
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d/e^2*(1
/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/d/e*(1
/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/
16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d^3/e^4*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/
c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d
/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.33

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{15\,(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c^2d\right.$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")

[Out] [-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3
*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e -
525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^
3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9
*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^
3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*
c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8
- 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^
4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e - 525*a*c^5*
d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^3*e^9 - 10
5*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9*c^6*d^8*e
^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^3 - 33*a*c
```

```
^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*c^6*d^10*e
^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8 - 35*a^4*c
^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.14

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \frac{1}{7680}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8\left(10\,cdx + \frac{c^6d^7}{}\right.\right.\right.\right.\right. + \frac{(21\,c^6d^{12} - 42\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 + 4\,a^3c^3d^6e^6 + 3\,a^4c^2d^4e^8 + 6\,a^5cd^2e^{10} - 7\,a^6e^{12})\log\left(\left|-cd^2 - ae^2 -\right.\right.}{1024\sqrt{cde}c^4d^4e^5}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c*d*x +
(c^6*d^7*e^4 + 13*a*c^5*d^5*e^6)/(c^5*d^5*e^5))*x - (9*c^6*d^8*e^3 - 14*a*c
```

```
^5*d^6*e^5 - 3*a^2*c^4*d^4*e^7)/(c^5*d^5*e^5))*x + (21*c^6*d^9*e^2 - 33*a*c
^5*d^7*e^4 + 3*a^2*c^4*d^5*e^6 - 7*a^3*c^3*d^3*e^8)/(c^5*d^5*e^5))*x - (105
*c^6*d^10*e - 168*a*c^5*d^8*e^3 + 18*a^2*c^4*d^6*e^5 + 16*a^3*c^3*d^4*e^7 -
 35*a^4*c^2*d^2*e^9)/(c^5*d^5*e^5))*x + (315*c^6*d^11 - 525*a*c^5*d^9*e^2 +
 78*a^2*c^4*d^7*e^4 + 54*a^3*c^3*d^5*e^6 + 55*a^4*c^2*d^3*e^8 - 105*a^5*c*d
*e^10)/(c^5*d^5*e^5)) + 1/1024*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^
4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^
6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx=\int \frac{x^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.447 $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 352

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = -\frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{(cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}}$$

```
[Out] 1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/240*(35*c^2*d^4-12*a*c*
d^2*e^2-15*a^2*e^4-6*c*d*e*(-3*a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/c^2/d^2/e^3+1/256*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-1/128*(-a*e^2+c
*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 846, 793, 626, 635, 212}

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}dx = \frac{(3a^2e^4+6acd^2e^2+7c^2d^4)(cd^2-ae^2)^3 \operatorname{arctanh}\left(\frac{ae^2+cd^2+2c}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd}}\right.}{256c^{7/2}d^{7/2}e^{9/2}}$$
$$+\frac{(-15a^2e^4-6cdex(7cd^2-3ae^2)-12acd^2e^2+35c^2d^4)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{240c^2d^2e^3}$$
$$-\frac{(3a^2e^4+6acd^2e^2+7c^2d^4)(cd^2-ae^2)(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128c^3d^3e^4}$$
$$+\frac{x^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5e}$$

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -1/128*((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*e^4)
 + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4
 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*
(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(256*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int x^2(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5e} + \frac{\int x\left(-2acd^2e-\frac{1}{2}cd(7cd^2-3ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx}{5cde}$$

$$= \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)\,x)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{240c^2d^2e^3} - \frac{((cd^2-ae^2)\,(7c^2d^4+6acd^2e^2+3a^2e^4))\int\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx}{32c^2d^2e^3}$$

$$= -\frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{\left((cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\right)\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{256c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{\left((cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\right)\text{Subst}\left(\int\frac{1}{4cde-x^2}dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{(cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.86

$$\int\frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(45a^4e^8-30a^3cde^6(d+ex)-6a\right.}{}$$

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(45*a^4*e^8 - 30*a^
3*c*d*e^6*(d + e*x) - 6*a^2*c^2*d^2*e^4*(6*d^2 - 3*d*e*x - 4*e^2*x^2) + 2*a
*c^3*d^3*e^2*(95*d^3 - 61*d^2*e*x + 48*d*e^2*x^2 + 264*e^3*x^3) + c^4*d^4*(
-105*d^4 + 70*d^3*e*x - 56*d^2*e^2*x^2 + 48*d*e^3*x^3 + 384*e^4*x^4)) + (15
```

```
*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sq
rt[d + e*x])))/(1920*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(322) = 644$.

Time = 0.64 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.26

method	result
default	$\frac{\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{5cde}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+e^2a+c \ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}}{e}$

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVER
BOSE)

[Out] 1/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)
/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d/e^2*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1
/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2)))+d^2/e^3*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/
2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d
^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.40

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{15\,(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8\right.$$

$$-\frac{15\,(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10})\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)}}{2\,(c^2d^2e^2x^2+acd^2e}\right.}{}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2
*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*
c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*
c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)
*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^
2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -
1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d
^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(384*c^5*d^5
*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5*e^5 - 30*a^3*
c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*x^3 - 8*
(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2 + 2*(35*c^5*d^8*
e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^2*e^8)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.15

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \frac{1}{1920}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(6\left(8\,cdx + \frac{c^5d^6e^3 + \cdots}{c^4\cdots}\right.\right.\right.\right.$$
$$- \frac{(7\,c^5d^{10} - 15\,ac^4d^8e^2 + 6\,a^2c^3d^6e^4 + 2\,a^3c^2d^4e^6 + 3\,a^4cd^2e^8 - 3\,a^5e^{10})\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x \cdots\right.\right.\right.}{256\sqrt{cde}c^3d^3e^4}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*c*d*x + (c^5
*d^6*e^3 + 11*a*c^4*d^4*e^5)/(c^4*d^4*e^4))*x - (7*c^5*d^7*e^2 - 12*a*c^4*d
^5*e^4 - 3*a^2*c^3*d^3*e^6)/(c^4*d^4*e^4))*x + (35*c^5*d^8*e - 61*a*c^4*d^6
*e^3 + 9*a^2*c^3*d^4*e^5 - 15*a^3*c^2*d^2*e^7)/(c^4*d^4*e^4))*x - (105*c^5*
d^9 - 190*a*c^4*d^7*e^2 + 36*a^2*c^3*d^5*e^4 + 30*a^3*c^2*d^3*e^6 - 45*a^4*
c*d*e^8)/(c^4*d^4*e^4)) - 1/256*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*
d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*log(abs(-c*d^2
- a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int \frac{x^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.448 $\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 295

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)\,(5cd^2+3ae^2)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2e^3}$$
$$-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{4cde(d+ex)}$$
$$-\frac{(cd^2-ae^2)^3\,(5cd^2+3ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}$$

```
[Out] -1/24*(3*a/c/d+5*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e/(e*x+d)-1/128*(-a*e^2+c*d^2)^3*(3*a*
e^2+5*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/64*(-a*e^2
+c*d^2)*(3*a*e^2+5*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used

$= \{808, 678, 626, 635, 212\}$

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = -\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}} + \frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3} + \frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{4cde(d+ex)} - \frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\text{integral} = \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{4cde(d + ex)} + \frac{1}{8}\left(-\frac{5d}{e} - \frac{3ae}{cd}\right)\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx$$

$$= -\frac{1}{24}\left(\frac{3a}{cd} + \frac{5d}{e^2}\right)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2} + \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{4cde(d + ex)} + \frac{\left(\left(\frac{5d}{e} + \frac{3ae}{cd}\right)(2cd^2e - e(cd^2 + ae^2))\right)\int \sqrt{ade + (cd^2 + ae^2)x + cdex^2}\,dx}{16e^2}$$

$$= \frac{(cd^2 - ae^2)(5cd^2 + 3ae^2)(cd^2 + ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{64c^2d^2e^3} - \frac{1}{24}\left(\frac{3a}{cd} + \frac{5d}{e^2}\right)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2} + \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{4cde(d + ex)} - \frac{\left((cd^2 - ae^2)^3(5cd^2 + 3ae^2)\right)\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\,dx}{128c^2d^2e^3}$$

$$= \frac{(cd^2 - ae^2)(5cd^2 + 3ae^2)(cd^2 + ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{64c^2d^2e^3} - \frac{1}{24}\left(\frac{3a}{cd} + \frac{5d}{e^2}\right)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2} + \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{4cde(d + ex)} - \frac{\left((cd^2 - ae^2)^3(5cd^2 + 3ae^2)\right)\text{Subst}\left(\int \frac{1}{4cde - x^2}\,dx, x, \frac{cd^2 + ae^2 + 2cdex}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{64c^2d^2e^3}$$

$$= \frac{(cd^2 - ae^2)(5cd^2 + 3ae^2)(cd^2 + ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{64c^2d^2e^3} - \frac{1}{24}\left(\frac{3a}{cd} + \frac{5d}{e^2}\right)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2} + \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{4cde(d + ex)} - \frac{(cd^2 - ae^2)^3(5cd^2 + 3ae^2)\tanh^{-1}\left(\frac{cd^2 + ae^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.80

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \frac{\sqrt{(ae + cdx)(d + ex)}\Big(\sqrt{c}\sqrt{d}\sqrt{e}(-9a^3e^6 + 3a^2cde^4(3d + 2ex) + a$$

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^2
*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2) +
c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*(c*d^2 - a*e
^2)^3*(5*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*
Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(5/2)*d^(5/
2)*e^(7/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.64

method	result
default	$\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(e^2\right.\right.}{}\right.}{16cde}$ … $/e$

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)
/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d/e^2*(1/3*(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^
2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/
c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.29

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \left[-\frac{3\,(5\,c^4d^8 - 12\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 - 3\,a^4e^8)\sqrt{cde}}{\right.$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*
e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*
d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^4
*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7
+ 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e
^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15
*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7 + 8*(c^4*
d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^
2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4
)]
```

Sympy [A] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 1093, normalized size of antiderivative = 3.71

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] a*e*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e/3
 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))*Pi
ecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e
*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/
(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d*
*2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) +
(x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6 + c*d**2
/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d*e*x**2 +
```

```
x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e**2 + c*d**
2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 + c*d**2)**2
, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True)) + c*d*Piecewise(((-a
*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c
) - (a*e**2 + c*d**2)*(-2*a*(a*e**2/8 + c*d**2/8)/(3*c) - (3*a*e**2/2 + 3*c
*d**2/2)*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*
e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sq
rt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a
*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d
*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**
2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*
(x**3/4 + x**2*(a*e**2/8 + c*d**2/8)/(3*c*d*e) + x*(a*d*e/4 - (a*e**2/8 + c
*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c*d*e) + (-2*a*(a*e**2/8 +
 c*d**2/8)/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(a*d*e/4 - (a*e**2/8 + c*d**2/
8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c*d*e))/(c*d*e)), Ne(c*d*e, 0)),
 (2*(a**2*d**2*e**2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 - 2*a*d*e*(a*d*e
 + x*(a*e**2 + c*d**2))**(5/2)/5 + (a*d*e + x*(a*e**2 + c*d**2))**(7/2)/7)/
(a*e**2 + c*d**2)**3, Ne(a*e**2 + c*d**2, 0)), (x**3*sqrt(a*d*e)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \frac{1}{192} \sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(6\,cdx + \frac{c^4d^5e^2 + 9\,ac^3d^3}{c^3d^3e^3}\right.\right.\right.$$
$$+ \frac{(5\,c^4d^8 - 12\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 - 3\,a^4e^8)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x}\right.\right.\right.}{128\sqrt{cde}c^2d^2e^3}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] 1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c*d*x + (c^4*d^5
*e^2 + 9*a*c^3*d^3*e^4)/(c^3*d^3*e^3))*x - (5*c^4*d^6*e - 10*a*c^3*d^4*e^3
- 3*a^2*c^2*d^2*e^5)/(c^3*d^3*e^3))*x + (15*c^4*d^7 - 31*a*c^3*d^5*e^2 + 9*
a^2*c^2*d^3*e^4 - 9*a^3*c*d*e^6)/(c^3*d^3*e^3)) + 1/128*(5*c^4*d^8 - 12*a*c
^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*log(abs(-c*d^
2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \int \frac{x\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{d + e\,x}\,dx$$

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.449 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 201

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e}+\frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}$$

```
[Out] 1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/16*(-a*e^2+c*d^2)^3*arctanh
(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+1/8*(a/c/d-d/e^2)*(2*c*d*e*x+a*e
^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {678, 626, 635, 212}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3e} \\ &\quad - \frac{(2cd^2e-e(cd^2+ae^2))\int\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx}{2e^2} \\ &= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2} \\ &\quad + \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3e} + \frac{(cd^2-ae^2)^3\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16cde^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} \\
&\quad+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} \\
&\quad+\frac{\left(cd^2-ae^2\right)^3 \operatorname{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8cde^2} \\
&= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} \\
&\quad+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} \\
&\quad+\frac{\left(cd^2-ae^2\right)^3 \tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4+2acde^2(4d+7ex)+c^2d^2(\cdots\right.}{24c^{3/2}d^{3/2}e^{5\cdots}}
$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c*
d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2 -
 a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x
])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{\left(cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(e^2a-c\,d^2\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+e^2a-c\,d^2\right)\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{4cde}-\frac{\left(e^2a-c\,d^2\right)^2\ln\left(\frac{\frac{e^2a}{2}-\frac{c\,d^2}{2}+\cdots}{\sqrt{c\cdots}}\right.}{\cdots}\right.}{2}}{e}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1
/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/
(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.65

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2+\right.$$

$$-\frac{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(8\,c^3d}{48\,c^2d^2e^3}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3*d^3*
e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 +
7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e
^3)]
```

Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.74

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = ae\left(\begin{cases}\left(\frac{x}{2}+\frac{\frac{ae^2}{4}+\frac{cd^2}{4}}{cde}\right)\sqrt{ade+cdex^2+x(ae^2+cd^2)}+\left(\frac{ade}{2}-\frac{\left(\frac{ae^2}{4}\ldots\right.}{}\right. \\ \frac{2(ade+x(ae^2+cd^2))^{\frac{3}{2}}}{3(ae^2+cd^2)} \\ x\sqrt{ade}\end{cases}\right) + cd\left(\begin{cases}\left(-\frac{a\left(\frac{ae^2}{6}+\frac{cd^2}{6}\right)}{2c}-\frac{(ae^2+cd^2)\left(\frac{ade}{3}-\frac{\left(\frac{ae^2}{6}+\frac{cd^2}{6}\right)\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)}{2cde}\right)}{2cde}\right)\left(\begin{cases}\frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\right)}{\sqrt{cde}} \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}}\end{cases}\right. \\ \frac{2\left(-\frac{ade\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5}\right)}{(ae^2+cd^2)^2} \\ \frac{x^2\sqrt{ade}}{2}\end{cases}\right.$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] a*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x**
2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d**
2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sq
rt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e*
*2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x -
 (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e)
)**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/(3*(a
*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True)) + c*d*Pie
cewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e/3 - (a*e
**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))*Piecewise
((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 +
 x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*
e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*
c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + (x**2/3
 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*
a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e
**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e**2 + c*d**2))**(3
/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 + c*d**2)**2, Ne(a*
e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.14

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \frac{1}{24}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\,cdx + \frac{c^3d^4e + 7\,ac^2d^2e^3}{c^2d^2e^2}\right)x - \frac{(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{16\sqrt{cde}cde^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e +
 7*a*c^2*d^2*e^3)/(c^2*d^2*e^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^2*c
*d*e^4)/(c^2*d^2*e^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
- a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{d + e\,x} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

3.450 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 251

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}$$
$$-\frac{\left(c^2d^4-6acd^2e^2-3a^2e^4\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}}$$
$$-a^{3/2}\sqrt{d}e^{3/2}\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)$$

```
[Out] -1/8*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)
/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c
^(1/2)/d^(1/2)-a^(3/2)*e^(3/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*d^(1/2)+1/4*(2*c
*d*e*x+5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 828, 857, 635, 212, 738}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=$$
$$-a^{3/2}\sqrt{d}e^{3/2}\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\operatorname{arctanh}\left(\frac{\ldots}{2\sqrt{c}\ldots}\right.}{8\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx$$

$$= \frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} - \frac{\int \frac{-4a^2cd^2e^3+\frac{1}{2}cd(c^2d^4-6acd^2e^2-3a^2e^4)x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4cde}$$

$$= \frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} + (a^2de^2)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx - \frac{(c^2d^4-6acd^2e^2-3a^2e^4)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8e}$$

$$= \frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} - (2a^2de^2)\,\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(c^2d^4-6acd^2e^2-3a^2e^4)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4e}$$

$$= \frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} - \frac{(c^2d^4-6acd^2e^2-3a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}} - a^{3/2}\sqrt{d}e^{3/2}\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(5ae^2+cd(d+2\right.}{4\sqrt{}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(5*a*e^2 + c*d*(d + 2*e*x)) + (-(c^2*d^4) + 6*a*c*d^2*e^2 +
3*a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*
x])] - 8*a^(3/2)*Sqrt[c]*d*e^3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqr
t[d]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(213) = 426$.

Time = 0.64 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.42

method	result
default	$\frac{\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{3}+\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cde}{\sqrt{cde}}\right.}{8cde\sqrt{cde}}\right.}{2}}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/d*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*
c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a
*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2
+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-1/d*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d
/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*
ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 2.60 (sec) , antiderivative size = 1327, normalized size of antiderivative = 5.29

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c^2*
d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4
*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - (
c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(8*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + (c^2
*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^
2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x
+ c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*
d*e^2)]
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{x\,(d+ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e)^{3/2}}{x\ (d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

3.451 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 240

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx = -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x} + \frac{\sqrt{c}\sqrt{d}(cd^2+3ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}(3cd^2+ae^2)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{d}}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-1/2*(
a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)*e^(1/2)/d^(1/2)-(-c*d*x+
a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

$= \{863, 826, 857, 635, 212, 738\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{c}\sqrt{d}(3ae^2+cd^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}(ae^2+3cd^2)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{d}} - \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(ae-cdx)}{x}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx \\
&= -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}-\frac{1}{2}\int \frac{-ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x} \\
&\quad +\frac{1}{2}\left(ae\left(3cd^2+ae^2\right)\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&\quad +\frac{1}{2}\left(cd\left(cd^2+3ae^2\right)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x} \\
&\quad -\left(ae\left(3cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)\,x}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\right) \\
&\quad +\left(cd\left(cd^2+3ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\right)
\end{aligned}$$

$$= -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x} + \frac{\sqrt{c}\sqrt{d}(cd^2+3ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}(3cd^2+ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx =$$

$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{d}\sqrt{e}(ae-cdx)\sqrt{ae+cdx}\sqrt{d+ex}-\sqrt{c}d(cd^2+3ae^2)\,x\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)+\sqrt{ae}\right.}{\sqrt{d}\sqrt{e}x\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*(a*e - c*d*x)*Sqrt[a*e
+ c*d*x]*Sqrt[d + e*x] - Sqrt[c]*d*(c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] + Sqrt[a]*e*(3*c*d^2 + a*e
^2)*x*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
)/(Sqrt[d]*Sqrt[e]*x*Sqrt[(a*e + c*d*x)*(d + e*x)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(202) = 404.

Time = 0.72 (sec) , antiderivative size = 1300, normalized size of antiderivative = 5.42

method	result	size
default	Expression too large to display	1300

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a
/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2
*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
```

```
1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/
c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c
*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e
)^(1/2))))-e/d^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*
d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+
a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^
2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e/d^2*(1/3*(c*d*e*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e
^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-
c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*
(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.33 (sec) , antiderivative size = 1221, normalized size of antiderivative = 5.09

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c
*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e +
(c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*(c*d^2
+ 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e +
 (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2
*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c
```

```
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (c*d^2 + 3*a
*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d^2 + 3*a*e^2)*sqrt(-c*d
/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^
2)*x)) - (3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2
*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{x^2\,(d + ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^2}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.41

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)} dx = \sqrt{cdex^2 + cd^2x + ae^2x + ade}cd + \frac{(3acd^2e + a^2e^3)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}} - \frac{(c^2d^3 + 3acde^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}} - \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)acd^2e + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2e^3 + 2\sqrt{cde}}{ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d + (3*a*c*d^2*e + a^2*e^3)*a
rctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-
a*d*e))/sqrt(-a*d*e) - 1/2*(c^2*d^3 + 3*a*c*d*e^2)*log(abs(-c*d^2 - a*e^2 -
 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)))/sqrt(c*d*e) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))*a*c*d^2*e + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^2*e^3 + 2*sqrt(c*d*e)*a^2*d*e^2)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^2\,(d + e\,x)} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

3.452 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)} dx$

Optimal result

Integrand size = 40, antiderivative size = 256

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)} dx = -\frac{(2ade+(5cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^2} + c^{3/2}d^{3/2}\sqrt{e}\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(3c^2d^4+6acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\quad}{2\sqrt{a}\sqrt{d}\sqrt{\quad}}\right)}{8\sqrt{a}d^{3/2}\sqrt{e}}$$

```
[Out] -1/8*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)
/a^(1/2)/e^(1/2)+c^(3/2)*d^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/4*(2*a
*d*e+(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

= {863, 824, 857, 635, 212, 738}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx =$$
$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{a}d^{3/2}\sqrt{e}}$$
$$+c^{3/2}d^{3/2}\sqrt{e}\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(x(ae^2+5cd^2)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+}}{4dx^2}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
 - ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2
)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)])/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
```

```
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx \\
&= -\frac{(2ade+(5cd^2+ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^2} \\
&\quad -\frac{\int \frac{-\frac{1}{2}ae(3c^2d^4+6acd^2e^2-a^2e^4)-4ac^2d^3e^2x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4ade} \\
&= -\frac{(2ade+(5cd^2+ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^2} \\
&\quad +\left(c^2d^2e\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&\quad +\frac{(3c^2d^4+6acd^2e^2-a^2e^4)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8d}
\end{aligned}$$

$$= -\frac{(2ade + (5cd^2 + ae^2)\,x)\,\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{4dx^2} + (2c^2d^2e)\,\text{Subst}\left(\int \frac{1}{4cde - x^2}\,dx, x, \frac{cd^2 + ae^2 + 2cdex}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\right) - \frac{(3c^2d^4 + 6acd^2e^2 - a^2e^4)\,\text{Subst}\left(\int \frac{1}{4ade - x^2}\,dx, x, \frac{2ade - (-cd^2 - ae^2)x}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{4d}$$

$$= -\frac{(2ade + (5cd^2 + ae^2)\,x)\,\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{4dx^2} + c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{cd^2 + ae^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\right) - \frac{(3c^2d^4 + 6acd^2e^2 - a^2e^4)\tanh^{-1}\left(\frac{2ade + (cd^2 + ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{8\sqrt{a}d^{3/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.95

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx = -\frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(5cd^2x + ae(2d + ex)) - 8\sqrt{a}c^{3/2}d^3ex^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d}}{\sqrt{e}\sqrt{ae+}}\right.\right.}{4\sqrt{a}d^{3/2}\sqrt{e}x^2\sqrt{(ae + cdx)(d + ex)}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*(5*c*d^2*x + a*e*(2*d + e*x)) - 8*Sqrt[a]*c^(3/2)*d^3*e
*x^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] +
 (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*x^2*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^2*Sqrt[(a
*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. 2(218) = 436.

Time = 0.77 (sec) , antiderivative size = 2438, normalized size of antiderivative = 9.52

method	result	size
default	Expression too large to display	2438

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^2/d^3*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d

```
^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-e^2/d^3*(1/3*(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^
2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/
c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.43 (sec) , antiderivative size = 1375, normalized size of antiderivative = 5.37

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{x^3(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d^4
+ 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*d^3*e*x^
2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*
e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), 1/8*(4*sqrt(c
*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d^4 + 6*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*
d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a
^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1
/8*(8*sqrt(-c*d*e)*a*c*d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*
e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2
```

```
 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)x^3}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(218) = 436.

Time = 0.44 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.45

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=-\frac{c^2d^2e\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{\sqrt{cde}}+\frac{\left(3\,c^2d^4+6\,acd^2e^2-a^2e^4\right)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}d}-\frac{3\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ac^2d^5e-2\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2cd^3e^3-\cdots}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -c^2*d^2*e*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/4*(3*c^2*d^4 + 6*a*c
*d^2*e^2 - a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - 1/4*(3*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*d^5*e - 2*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^3 - (sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^5 + 8*sqrt(c*d*e)*a
^2*c*d^4*e^2 - 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^3*c^2*d^4 - 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))^3*a*c*d^2*e^2 - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^3*a^2*e^4 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^2*a*c*d^3*e - 8*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*d*e^3)/((a*d*e - (sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^3\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

3.453 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 211

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3} + \frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+1/16*(-a*e^2+c*d^2)^3*ar
ctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-1/8*(c/a/e-e/d^2)*(2*a*d*
e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{863, 820, 734, 738, 212\}$

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \frac{(cd^2 - ae^2)^3 \operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}} - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8x^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3dx^3}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/8*((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/x^2 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
```

```
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx$$

$$= -\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3dx^3} - \frac{(-2acd^2e+ae(cd^2+ae^2))\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{2ade}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8x^2} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3dx^3} - \frac{(cd^2-ae^2)^3\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16ad^2e}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8x^2} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3dx^3} + \frac{(cd^2-ae^2)^3\,\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8ad^2e}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8x^2} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3dx^3} + \frac{(cd^2-ae^2)^3\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx=\frac{\left(-cd^2+ae^2\right)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(3c^2d^4x^2+2acd^2ex(7d+4ex)\right.}{\left(cd^2-ae^2\right)^3x}\right.}{24a^{3/2}d^{5/2}e^{3/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[
e]*(3*c^2*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x
- 3*e^2*x^2)))/((c*d^2 - a*e^2)^3*x^3) - (3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/
(24*a^(3/2)*d^(5/2)*e^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4329 vs. 2(185) = 370.

Time = 1.01 (sec) , antiderivative size = 4330, normalized size of antiderivative = 20.52

method	result	size
default	Expression too large to display	4330

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/6*(a*e^2+c*d^
2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^
2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^
2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*
d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e
^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e
^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1
/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/
```

2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*
d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2
)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+2/3*c/a*(-
1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/e*(
1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e
*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a
*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c
*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2
)))))+e^2/d^3*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^
2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^
2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*
d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e
^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e
^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2))))-e^3/d^4*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1
/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*
d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2
)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-e/d^2*(-1/2
/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e^2+c*d^2)/a/d/e*
(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/e
(1/3(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d
*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a

```
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*
d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2
+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^
2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*
d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*
a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^3/d^4*(1/3*(c*d*e*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+
e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2
-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.96 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.64

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \left[-\frac{3\,(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+(c^2c\right.}{}\right.$$

$$-\frac{3\,(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\sqrt{-ade}x^3\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right) + 2\,(8}{48\,a^2d^3e}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c^2*d
^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^
4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3), -1/48
*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
```

```
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*
e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(185) = 370$.

Time = 0.34 (sec) , antiderivative size = 1005, normalized size of antiderivative = 4.76

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx =$$

$$-\frac{(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{8\,\sqrt{-ade}ad^2e}$$

$$+\frac{3\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^3d^8e^2 - 9\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3c^2d^6e^4}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt
(-a*d*e)*a*d^2*e) + 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))*a^2*c^3*d^8*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 - 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 - 16*sqrt(c*d*e)*a^4*c*d^5*e
^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^
3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
3*a^2*c^2*d^5*e^3 - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))^3*a^3*c*d^3*e^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^3*a^4*d*e^7 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 48*sqrt(c*d*e)*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*d^2*e^6 - 3*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 - 39*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2
 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c*
d^2*e^4 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5
*a^3*e^6 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^4*a*c^2*d^5*e - 96*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^4*a^2*c*d^3*e^3)/((a*d*e - (sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^3*a*d^2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^4\,\left(d+e\,x\right)} \,d x$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

3.454 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 295

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx=\frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{64a^2d^3e^2x^2}$$
$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24x^3}$$
$$-\frac{\left(cd^2-ae^2\right)^3\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^4-1/24*(3*c/a/e-5*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3-1/128*(-a*e^2+c*d^2)^3*(5*a*e^2+
3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+1/64*(-a*e^2+c
*d^2)*(5*a*e^2+3*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

$= \{863, 848, 820, 734, 738, 212\}$

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = -\frac{\left(5ae^2 + 3cd^2\right)\left(cd^2 - ae^2\right)^3 \operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}} + \frac{\left(5ae^2 + 3cd^2\right)\left(cd^2 - ae^2\right)\left(x(ae^2 + cd^2) + 2ade\right)\sqrt{x\left(ae^2 + cd^2\right) + ade + cdex^2}}{64a^2d^3e^2x^2} - \frac{\left(x(ae^2 + cd^2) + ade + cdex^2\right)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae} - \frac{5e}{d^2}\right)\left(x(ae^2 + cd^2) + ade + cdex^2\right)^{3/2}}{24x^3}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx \\
&= -\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\int \frac{\left(-\frac{1}{2}ae(3cd^2-5ae^2)+acde^2x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx}{4ade} \\
&= -\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{24x^3} \\
&\quad - \frac{\left(\frac{3c^2d^2}{a}+2ce^2-\frac{5ae^4}{d^2}\right)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{16e} \\
&= \frac{(cd^2-ae^2)\,(3cd^2+5ae^2)\,(2ade+(cd^2+ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64a^2d^3e^2x^2} \\
&\quad - \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{24x^3} \\
&\quad + \frac{\left((cd^2-ae^2)^3\,(3cd^2+5ae^2)\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128a^2d^3e^2}
\end{aligned}$$

$$= \frac{(cd^2-ae^2)(3cd^2+5ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64a^2d^3e^2x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24x^3} - \frac{\left((cd^2-ae^2)^3(3cd^2+5ae^2)\right)\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64a^2d^3e^2}$$

$$= \frac{(cd^2-ae^2)(3cd^2+5ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64a^2d^3e^2x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24x^3} - \frac{(cd^2-ae^2)^3(3cd^2+5ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.84

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-9c^3d^6x^3+3ac^2d^4ex^2(2d+3ex)+a^2cd^2e^2x(7\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*d^6*x^3
+ 3*a*c^2*d^4*e*x^2*(2*d + 3*e*x) + a^2*c*d^2*e^2*x*(72*d^2 + 20*d*e*x - 31
*e^2*x^2) + a^3*e^3*(48*d^3 + 8*d^2*e*x - 10*d*e^2*x^2 + 15*e^3*x^3)))/x^4)
 - (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/
(192*a^(5/2)*d^(7/2)*e^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7420 vs. 2(265) = 530.

Time = 1.06 (sec) , antiderivative size = 7421, normalized size of antiderivative = 25.16

method	result	size
default	Expression too large to display	7421

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 4.94 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.39

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \left[-\frac{3(3c^4d^8 - 4ac^3d^6e^2 - 6a^2c^2d^4e^4 + 12a^3cd^2e^6 - 5a^4e^8)\sqrt{ade}x^4}{} \right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*
e^6 - 5*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(
48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 - 15
*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^6)*
x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*d^4*e^3*x^4), 1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a
^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
2*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 -
 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^
6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^5} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(265) = 530$.

Time = 0.37 (sec) , antiderivative size = 1618, normalized size of antiderivative = 5.48

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"giac")
[Out] 1/64*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 -
5*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/192*(9*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 12*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5 - 402
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^2*d^7*
e^7 - 348*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6
*c*d^5*e^9 - 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^7*d^3*e^11 - 128*sqrt(c*d*e)*a^6*c*d^6*e^8 - 33*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 - 724*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*e^4 - 185
4*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^2*d
^6*e^6 - 900*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
3*a^5*c*d^4*e^8 - 73*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^3*a^6*d^2*e^10 - 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 - 1024*sqrt(c*d*e)*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 - 384*s
qrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*
a^6*d^3*e^9 - 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^5*a*c^4*d^9*e - 596*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^5*a^2*c^3*d^7*e^3 - 1086*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^5*a^3*c^2*d^5*e^5 - 132*(sqrt(c*d*e)*x - sqrt(c*d*e*
```

```
x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*c*d^3*e^7 + 55*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^5*d*e^9 - 384*sqrt(c*d*e)*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^2*c^3*d^8*e^
2 - 1536*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^4*a^3*c^2*d^6*e^4 - 1536*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^4*c*d^4*e^6 + 9*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*c^4*d^8 - 12*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a*c^3*d^6*e^2 - 18*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^2*c^2*d^4*e^4 + 36*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^3*c*d^2*e^6
- 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^4*e^
8 - 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^6*a^2*c^2*d^5*e^3)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^2)^4*a^2*d^3*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^5\,(d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

3.455 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 395

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx &= -\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^3d^4e^3x^2} \\ &\quad -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{40x^4} \\ &\quad +\frac{\left(15c^2d^4+12acd^2e^2-35a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{240a^2d^3e^2x^3} \\ &\quad +\frac{\left(cd^2-ae^2\right)^3\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}}\end{aligned}$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5-1/40*(3*c/a/e-7*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4+1/240*(-35*a^2*e^4+12*a*c*d^2*e^
2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^3+1/256
*(-a*e^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+
(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*d^2*e^
2+3*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/x^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)} dx = \frac{(-35a^2e^4 + 12acd^2e^2 + 15c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)^3 \operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}} - \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128a^3d^4e^3x^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae} - \frac{7e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{40x^4}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] -1/128*((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^3*d^4*e^
3*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/
(a*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4)
+ ((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6
*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)
*d^(9/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx$$

$$= -\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{5dx^5} - \frac{\int \frac{\left(-\frac{1}{2}ae(3cd^2-7ae^2)+2acde^2x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5}\,dx}{5ade}$$

$$= -\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{40x^4} + \frac{\int \frac{\left(-\frac{1}{4}ae(15c^2d^4+12acd^2e^2-35a^2e^4)-\frac{1}{2}acde^2(3cd^2-7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx}{20a^2d^2e^2}$$

$$= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{((cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4))\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{32a^2d^3e^2}$$

$$= -\frac{(cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^3d^4e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} - \frac{\left((cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{256a^3d^4e^3}$$

$$= -\frac{(cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^3d^4e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{\left((cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^3d^4e^3}$$

$$= -\frac{(cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^3d^4e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{(cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.80

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(45c^4d^8x^4-30ac^3d^6ex^3(d+ex)+6a^2c^2d^4e^2x^2}{}\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(45*c^4*d^8*x^4
- 30*a*c^3*d^6*e*x^3*(d + e*x) + 6*a^2*c^2*d^4*e^2*x^2*(4*d^2 + 3*d*e*x - 6
*e^2*x^2) + 2*a^3*c*d^2*e^3*x*(264*d^3 + 48*d^2*e*x - 61*d*e^2*x^2 + 95*e^3
*x^3) + a^4*e^4*(384*d^4 + 48*d^3*e*x - 56*d^2*e^2*x^2 + 70*d*e^3*x^3 - 105
*e^4*x^4)))/x^5) + (15*(c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2
*e^4)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10574 vs. 2(361) = 722.

Time = 1.65 (sec) , antiderivative size = 10575, normalized size of antiderivative = 26.77

method	result	size
default	Expression too large to display	10575

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 14.93 (sec) , antiderivative size = 872, normalized size of antiderivative = 2.21

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = \left[-\frac{15\,(3\,c^5d^{10}-3\,ac^4d^8e^2-2\,a^2c^3d^6e^4-6\,a^3c^2d^4e^6+15\,a^4cd^2e^8}{}\right.$$

$$-\frac{15\,(3\,c^5d^{10}-3\,ac^4d^8e^2-2\,a^2c^3d^6e^4-6\,a^3c^2d^4e^6+15\,a^4cd^2e^8-7\,a^5e^{10})\sqrt{-ade}x^5\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2}}{2\,(acd^2e^2x^2}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*
d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2
 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e +
a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*
e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a
^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3
 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^
4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(a^4*d^5*e^4*x^5), -1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^
6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(-a*d*e)*x^5
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) + 2*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*e^3 - 36*
a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a^2*c^3*d^
8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3 + 8*(3*a
^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^4*c*d^6*e
^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*
e^4*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^6}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. 2(361) = 722.

Time = 0.40 (sec) , antiderivative size = 2352, normalized size of antiderivative = 5.95

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/128*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^
6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) + 1/
1920*(45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*
c^5*d^14*e^4 - 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^5*c^4*d^12*e^6 - 3870*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))*a^6*c^3*d^10*e^8 - 7770*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 - 3615*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 105*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 - 768*sqrt(c*d*e
)*a^7*c^2*d^9*e^9 - 1280*sqrt(c*d*e)*a^8*c*d^7*e^11 - 210*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^5*d^13*e^3 - 7470*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^4*d^11*e^
5 - 34420*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a
^5*c^3*d^9*e^7 - 41820*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))^3*a^6*c^2*d^7*e^9 - 12570*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a^7*c*d^5*e^11 - 790*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*d^3*e^13 - 7680*sqrt(c*d*e)*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*c^3*d^10*e^6 -
23040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^2*a^6*c^2*d^8*e^8 - 16640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c*d^6*e^10 - 3840*sqrt(c*d*e)*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^8*d^4*e^12 - 384*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c^5*d^12
*e^2 - 9600*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5
*a^3*c^4*d^10*e^4 - 37120*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^5*a^4*c^3*d^8*e^6 - 30720*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^5*a^5*c^2*d^6*e^8 - 1920*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^6*c*d^4*e^10 + 896*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*d^2*e^12 - 3840*sqrt(
c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^3*
c^4*d^11*e^3 - 26880*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))^4*a^4*c^3*d^9*e^5 - 53760*sqrt(c*d*e)*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^5*c^2*d^7*e^7 - 28160*sqrt
```

```
(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^6
*c*d^5*e^9 + 210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^7*a*c^5*d^11*e - 210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^7*a^2*c^4*d^9*e^3 - 5260*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^7*a^3*c^3*d^7*e^5 - 420*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^4*c^2*d^5*e^7 + 1050*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^5*c*d^3*e^9 - 490*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^6*d*e^11 - 1152
0*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^6*a^3*c^3*d^8*e^4 - 19200*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^6*a^4*c^2*d^6*e^6 - 45*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*c^5*d^10 + 45*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a*c^4*d^8*e^2 + 30*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^2*c^3*d^6*e^4 + 90*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^3*c^2*d^4*e^6
- 225*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^4*c
*d^2*e^8 + 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)^9*a^5*e^10)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^2)^5*a^3*d^4*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^6\,(d + e\,x)} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

3.456 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 498

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx = \frac{(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8)(2ade+(cd^2+ae^2)x)\sqrt{}}{512a^4d^5e^4x^2}$$
$$-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{20x^5}$$
$$+\frac{(7c^2d^4+6acd^2e^2-21a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{160a^2d^3e^2x^4}$$
$$-\frac{(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{960a^3d^4e^3x^3}$$
$$-\frac{(cd^2-ae^2)^3(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}$$

```
[Out] -1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6-1/20*(c/a/e-3*e/d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5+1/160*(-21*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/960*(-1
05*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^3-1/1024*(-a*e^2+c*d^2)^3*(21*a^3*e^6+2
1*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d
^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(
9/2)/d^(11/2)/e^(9/2)+1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*d^6*e^2+
7*c^4*d^8)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^4/d^5/e^4/x^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 848, 820, 734, 738, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx = \frac{(-21a^2e^4+6acd^2e^2+7c^2d^4)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{160a^2d^3e^2x^4} + \frac{(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8)(x(ae^2+cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512a^4d^5e^4x^2} - \frac{(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{960a^3d^4e^3x^3} - \frac{(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6)(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{20x^5}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^7}\,dx$$

$$= -\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{3}{2}ae(cd^2-3ae^2)+3acde^2x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^6}\,dx}{6ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5} + \frac{\int \frac{\left(-\frac{3}{4}ae\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)-3acde^2\left(cd^2-3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5}\,dx}{30a^2d^2e^2}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5} + \frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4} - \frac{\int \frac{\left(-\frac{3}{8}ae\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)-\frac{3}{4}acde^2\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}\,dx}{120a^3d^3e^3}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5} + \frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4} - \frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3} - \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx}{128a^3d^4e^3}$$

$$= \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512a^4d^5e^4x^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5} + \frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4} - \frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3} + \frac{\left(\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\right)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{1024a^4d^5e^4}$$

$$= \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512a^4d^5e^4x^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5} + \frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4} - \frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3} - \frac{\left(\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-\left(-cd^2-ae^2 \cdots\right.}{\sqrt{ade+\left(cd^2+ae^2\right)x+ \cdots}}\right.}{512a^4d^5e^4}$$

$$= \frac{(7c^4d^8 + 8ac^3d^6e^2 + 6a^2c^2d^4e^4 - 21a^4e^8)(2ade + (cd^2 + ae^2)x)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{512a^4d^5e^4x^2}$$
$$- \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{20x^5}$$
$$+ \frac{(7c^2d^4 + 6acd^2e^2 - 21a^2e^4)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{160a^2d^3e^2x^4}$$
$$- \frac{(35c^3d^6 + 33ac^2d^4e^2 + 21a^2cd^2e^4 - 105a^3e^6)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{960a^3d^4e^3x^3}$$
$$- \frac{(cd^2 - ae^2)^3(7c^3d^6 + 15ac^2d^4e^2 + 21a^2cd^2e^4 + 21a^3e^6)\tanh^{-1}\left(\frac{2ade + (cd^2 + ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.81

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \frac{\sqrt{(ae + cdx)(d + ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-105c^5d^{10}x^5 + 5ac^4d^8ex^4(14d + 11ex) - 2a^2c^3\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*x
^5 + 5*a*c^4*d^8*e*x^4*(14*d + 11*e*x) - 2*a^2*c^3*d^6*e^2*x^3*(28*d^2 + 16
*d*e*x - 27*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(8*d^3 + 4*d^2*e*x - 6*d*e^2*x
^2 + 13*e^3*x^3) + a^4*c*d^2*e^4*x*(1664*d^4 + 224*d^3*e*x - 264*d^2*e^2*x^
2 + 336*d*e^3*x^3 - 525*e^4*x^4) + a^5*e^5*(1280*d^5 + 128*d^4*e*x - 144*d^
3*e^2*x^2 + 168*d^2*e^3*x^3 - 210*d*e^4*x^4 + 315*e^5*x^5)))/x^6) - (15*(c*
d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^
6)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(S
qrt[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*a^(9/2)*d^(11/2)*e^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 16882 vs. 2(460) = 920.

Time = 1.93 (sec) , antiderivative size = 16883, normalized size of antiderivative = 33.90

method	result	size
default	Expression too large to display	16883

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 37.47 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.15

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^7(d+ex)}\,dx = \left[-\frac{15\,(7\,c^6d^{12} - 6\,ac^5d^{10}e^2 - 3\,a^2c^4d^8e^4 - 4\,a^3c^3d^6e^6 - 15\,a^4c^2d^4e^8}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^
3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(a*d*
e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 - (105*a*
c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7 +
 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^3*
c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*x^
4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*d^
3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^2
+ 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^5*d^6*e^5*x^6), 1/15360*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 -
 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*
e^10 - 21*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^
2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(1280*a^6*d^6*e^6 - (105*
a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7
 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^
3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*
x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*
d^3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^
2 + 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a^5*d^6*e^5*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)} dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^7} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3251 vs. $2(460) = 920$.

Time = 0.48 (sec) , antiderivative size = 3251, normalized size of antiderivative = 6.53

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"giac")
[Out] 1/512*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^
6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*arctan(-(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a
*d*e)*a^4*d^5*e^4) - 1/7680*(105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 90*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 - 15405*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 - 46140*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*e^11 - 46
305*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*c^2*d
^9*e^13 - 14730*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^10*c*d^7*e^15 - 315*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^11*d^5*e^17 - 3072*sqrt(c*d*e)*a^8*c^3*d^12*e^10 - 6144*sqrt(
```

c*d*e)*a^9*c^2*d^10*e^12 - 5120*sqrt(c*d*e)*a^10*c*d^8*e^14 - 595*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^6*d^16*e^4 - 3
0210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^
5*d^14*e^6 - 199425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^3*a^6*c^4*d^12*e^8 - 419500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 - 305925*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12 - 65010*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^9*c*d^6*e^14 - 3335
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^10*d^4*e
^16 - 30720*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^2*a^6*c^4*d^13*e^7 - 135168*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c^3*d^11*e^9 - 193536*sqrt(c*d*
e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^8*c^2*
d^9*e^11 - 92160*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^2*a^9*c*d^7*e^13 - 15360*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^10*d^5*e^15 - 1686*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*c^6*d^15*e^3 - 5341
2*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*c^5*d
^13*e^5 - 332370*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^5*a^5*c^4*d^11*e^7 - 581400*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^5*a^6*c^3*d^9*e^9 - 279450*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*c^2*d^7*e^11 - 10116*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^8*c*d^5*e^13 + 5058*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^9*d^3*e^15 -
15360*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^4*a^4*c^5*d^14*e^4 - 153600*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^5*c^4*d^12*e^6 - 506880*sqrt(c*d*e)*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^6*c^3*d^10*e
^8 - 552960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^4*a^7*c^2*d^8*e^10 - 184320*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^8*c*d^6*e^12 + 1386*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^2*c^6*d^14*e^2 - 1188*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^3*c^5*d^1
2*e^4 - 86610*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^7*a^4*c^4*d^10*e^6 - 135960*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^7*a^5*c^3*d^8*e^8 - 2970*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^7*a^6*c^2*d^6*e^10 + 8316*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^7*c*d^4*e^12 - 4158*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^8*d^2*e^14 - 97280*
sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6
*a^4*c^4*d^11*e^5 - 337920*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^6*a^5*c^3*d^9*e^7 - 261120*sqrt(c*d*e)*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^6*c^2*d^7*e^9 - 595
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a*c^6*d^13
e + 510(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^

```
2*c^5*d^11*e^3 + 255*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^9*a^3*c^4*d^9*e^5 + 340*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^9*a^4*c^3*d^7*e^7 + 1275*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^9*a^5*c^2*d^5*e^9 - 3570*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^6*c*d^3*e^11 + 1785*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^7*d*e^13 - 30720*
sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8
*a^4*c^3*d^8*e^6 + 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))^11*c^6*d^12 - 90*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^
2*x + a*d*e))^11*a*c^5*d^10*e^2 - 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^11*a^2*c^4*d^8*e^4 - 60*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^3*c^3*d^6*e^6 - 225*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^4*c^2*d^4*e^8 + 630*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^5*c*d^2*e^1
0 - 315*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^
6*e^12)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^2)^6*a^4*d^5*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7(d+ex)}\,dx=\int \frac{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}{x^7\,(d+e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

3.457 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 574

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = -\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16384c^5d^5e^6}$$
$$+\frac{\left(cd^2-ae^2\right)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2048c^4d^4e^5}$$
$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}$$
$$-\frac{\left(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde\left(33c^2d^4-10acd^2e^2-15a^2e^4\right)x\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{4480c^3d^3e^4}$$
$$+\frac{3\left(cd^2-ae^2\right)^5\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*
d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^
4/e^5+1/112*(5*a/c/d-11*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+
1/8*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-1/4480*(231*c^3*d^6-15*a*
c^2*d^4*e^2-95*a^2*c*d^2*e^4-105*a^3*e^6-10*c*d*e*(-15*a^2*e^4-10*a*c*d^2*e
^2+33*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e^4+3/327
68*(-a*e^2+c*d^2)^5*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^
6)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(11/2)/e^(13/2)-3/16384*(-a*e^2+c*d
^2)^3*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^6)*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 846, 793, 626, 635, 212}

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}dx = \frac{3(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6)(cd^2-ae^2)^5 \arctan}{32768c^{11/2}d^{11/2}e^{13/2}}$$
$$-\frac{(-105a^3e^6-10cdex(-15a^2e^4-10acd^2e^2+33c^2d^4)-95a^2cd^2e^4-15ac^2d^4e^2+231c^3d^6)(x(ae^2+cd^2)+}{4480c^3d^3e^4}$$
$$-\frac{3(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6)(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cd}}{16384c^5d^5e^6}$$
$$+\frac{(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6)(cd^2-ae^2)(ae^2+cd^2+2cdex)(x(ae^2+cd^2)+ade+cdex^2)^3}{2048c^4d^4e^5}$$
$$+\frac{1}{112}x^2\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}+\frac{x^3(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{8e}$$

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int x^3(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{8e} + \frac{\int x^2\left(-3acd^2e-\frac{1}{2}cd(11cd^2-5ae^2)\,x\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}\,dx}{8cde}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}$$

$$+\frac{\int x\left(acd^2e(11cd^2-5ae^2)+\frac{3}{4}cd(33c^2d^4-10acd^2e^2-15a^2e^4)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{56c^2d^2e^2}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}$$

$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x)\left(ade+(\right.}{4480c^3d^3e^4}$$

$$+\frac{\left((cd^2-ae^2)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\right)\int\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx}{256c^3d^3e^4}$$

$$=\frac{(cd^2-ae^2)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{2048c^4d^4e^5}$$

$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}$$

$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}$$

$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x)\left(ade+(\right.}{4480c^3d^3e^4}$$

$$-\frac{\left(3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\right)\int\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{4096c^4d^4e^5}$$

$$=$$

$$-\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+(cd^2+a}}{16384c^5d^5e^6}$$

$$+\frac{(cd^2-ae^2)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)\right.}{2048c^4d^4e^5}$$

$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}$$

$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}$$

$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x)\left(ade+(\right.}{4480c^3d^3e^4}$$

$$+\frac{\left(3(cd^2-ae^2)^5\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\right)\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{32768c^5d^5e^6}$$

$$= -\frac{3(cd^2-ae^2)^3(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae}{16384c^5d^5e^6}$$
$$+\frac{(cd^2-ae^2)(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x}{2048c^4d^4e^5}$$
$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}$$
$$+\frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8e}$$
$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x)(ade+(cd}{4480c^3d^3e^4}$$
$$+\frac{\left(3(cd^2-ae^2)^5(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)\right)\text{Subst}\left(\int\frac{1}{4cde-x^2}dx,x,\frac{cd^2+ae^2+2c}{\sqrt{ade+(cd^2+ae^2)}}\right.}{16384c^5d^5e^6}$$

$$= -\frac{3(cd^2-ae^2)^3(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae}{16384c^5d^5e^6}$$
$$+\frac{(cd^2-ae^2)(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x}{2048c^4d^4e^5}$$
$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}$$
$$+\frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8e}$$
$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)x)(ade+(cd}{4480c^3d^3e^4}$$
$$+\frac{3(cd^2-ae^2)^5(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.96

$$\int\frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(1575a^7e^{14}-525a^6cde^{12}(7d+2e\right.}{}$$

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(1575*a^7*e^14 - 52
5*a^6*c*d*e^12*(7*d + 2*e*x) + 35*a^5*c^2*d^2*e^10*(29*d^2 + 68*d*e*x + 24*
e^2*x^2) + 5*a^4*c^3*d^3*e^8*(185*d^3 - 110*d^2*e*x - 376*d*e^2*x^2 - 144*e
^3*x^3) + 5*a^3*c^4*d^4*e^6*(265*d^4 - 120*d^3*e*x + 80*d^2*e^2*x^2 + 320*d
*e^3*x^3 + 128*e^4*x^4) + a^2*c^5*d^5*e^4*(-11193*d^5 + 7034*d^4*e*x - 5488
```

```
*d^3*e^2*x^2 + 4640*d^2*e^3*x^3 + 137600*d*e^4*x^4 + 103680*e^5*x^5) + a*c^
6*d^6*e^2*(11445*d^6 - 7476*d^5*e*x + 5928*d^4*e^2*x^2 - 5056*d^3*e^3*x^3 +
 4480*d^2*e^4*x^4 + 212480*d*e^5*x^5 + 168960*e^6*x^6) + c^7*d^7*(-3465*d^7
 + 2310*d^6*e*x - 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e^4*x^4 +
1280*d^2*e^5*x^5 + 87040*d*e^6*x^6 + 71680*e^7*x^7)) + (105*(c*d^2 - a*e^2)
^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[
(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[a*e + c
*d*x]*Sqrt[d + e*x])))/(573440*c^(11/2)*d^(11/2)*e^(13/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. 2(536) = 1072.

Time = 0.66 (sec) , antiderivative size = 1895, normalized size of antiderivative = 3.30

method	result	size
default	Expression too large to display	1895

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/8*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-9/16*(a*e^2+c*d^2)
/c/d/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)
/c/d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*
d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+
1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
c*d*e)^(1/2)))))-1/8*a/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(
2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/
d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))+d^2/e^3*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/
d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^
2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(
a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d/e^2*(1/7*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/d/e*(1/12*(2*c*d*e*x+
a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*
```

```
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))-d^3/e^4
*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*
(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*
(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln(
(1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c
*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 1524, normalized size of antiderivative = 2.66

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*
d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*
e^3)*x) + 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3
 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 10
15*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*
c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^
8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 10
75*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*
c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*
e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7
- 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2
*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3
*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d
^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7), -1/
1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 - 40*
a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*d^4
*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/
(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(71680*c^8
*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3 - 11193*a^2*c^6*d^11*
```

```
e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 1015*a^5*c^3*d^5*e^11 -
3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*c^8*d^9*e^7 + 33*a*c^7
*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^8 + 81*a^2*c^6*d^6*e^1
0)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5
*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*c^7*d^10*e^6 + 290*a^2
*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8
*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 +
 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 -
3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4
*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Timed out}$$

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.37

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{1}{573440}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,c^2\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{3\,(33\,c^8d^{16}-120\,ac^7d^{14}e^2+140\,a^2c^6d^{12}e^4-40\,a^3c^5d^{10}e^6-10\,a^4c^4d^8e^8-8\,a^5c^3d^6e^{10}-20\,a^6c^2d^4e^{12}+40\,a^7}{32768\,\sqrt{cde}c^5d^5e^6}$$

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/573440*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(4*(14
*c^2*d^2*e*x + (17*c^9*d^10*e^7 + 33*a*c^8*d^8*e^9)/(c^7*d^7*e^7))*x + (c^9
*d^11*e^6 + 166*a*c^8*d^9*e^8 + 81*a^2*c^7*d^7*e^10)/(c^7*d^7*e^7))*x - (11
*c^9*d^12*e^5 - 35*a*c^8*d^10*e^7 - 1075*a^2*c^7*d^8*e^9 - 5*a^3*c^6*d^6*e^
11)/(c^7*d^7*e^7))*x + (99*c^9*d^13*e^4 - 316*a*c^8*d^11*e^6 + 290*a^2*c^7*
d^9*e^8 + 100*a^3*c^6*d^7*e^10 - 45*a^4*c^5*d^5*e^12)/(c^7*d^7*e^7))*x - (2
31*c^9*d^14*e^3 - 741*a*c^8*d^12*e^5 + 686*a^2*c^7*d^10*e^7 - 50*a^3*c^6*d^
8*e^9 + 235*a^4*c^5*d^6*e^11 - 105*a^5*c^4*d^4*e^13)/(c^7*d^7*e^7))*x + (11
55*c^9*d^15*e^2 - 3738*a*c^8*d^13*e^4 + 3517*a^2*c^7*d^11*e^6 - 300*a^3*c^6
*d^9*e^8 - 275*a^4*c^5*d^7*e^10 + 1190*a^5*c^4*d^5*e^12 - 525*a^6*c^3*d^3*e
^14)/(c^7*d^7*e^7))*x - (3465*c^9*d^16*e - 11445*a*c^8*d^14*e^3 + 11193*a^2
*c^7*d^12*e^5 - 1325*a^3*c^6*d^10*e^7 - 925*a^4*c^5*d^8*e^9 - 1015*a^5*c^4*
d^6*e^11 + 3675*a^6*c^3*d^4*e^13 - 1575*a^7*c^2*d^2*e^15)/(c^7*d^7*e^7)) -
3/32768*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 - 40*a^3*c
^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*d^4*e^12
 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e
)*c^5*d^5*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.458 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 452

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)^3\,(9c^2d^4+10acd^2e^2+5a^2e^4)\,(cd^2+ae^2+2cdex)\,\sqrt{c}}{1024c^4d^4e^5}$$
$$-\frac{(cd^2-ae^2)\,(9c^2d^4+10acd^2e^2+5a^2e^4)\,(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{384c^3d^3e^4}$$
$$+\frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{7e}$$
$$+\frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)\,x)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{840c^2d^2e^3}$$
$$-\frac{(cd^2-ae^2)^5\,(9c^2d^4+10acd^2e^2+5a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}}$$

```
[Out] -1/384*(-a*e^2+c*d^2)*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4+1/7*x^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e+1/840*(63*c^2*d^4-20*a*c*d^2*e^2-35*a^2*
e^4-10*c*d*e*(-5*a*e^2+9*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
c^2/d^2/e^3-1/2048*(-a*e^2+c*d^2)^5*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*ar
ctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/1024*(-a*e^2+c*d^2)^3*(5
*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 846, 793, 626, 635, 212}

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = -\frac{(5a^2e^4+10acd^2e^2+9c^2d^4)(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}} + \frac{(-35a^2e^4-10cdex(9cd^2-5ae^2)-20acd^2e^2+63c^2d^4)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{840c^2d^2e^3} + \frac{(5a^2e^4+10acd^2e^2+9c^2d^4)(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{1024c^4d^4e^5} - \frac{(5a^2e^4+10acd^2e^2+9c^2d^4)(cd^2-ae^2)(ae^2+cd^2+2cdex)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{384c^3d^3e^4} + \frac{x^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7e}$$

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int x^2(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{7e} + \frac{\int x\left(-2acd^2e-\frac{1}{2}cd(9cd^2-5ae^2)\,x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx}{7cde}$$

$$= \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3} - \frac{((cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4))\int (ade+(cd^2+ae^2)x+cdex^2)^{3/2}\,dx}{48c^2d^2e^3}$$

$$= -\frac{(cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3} + \frac{\left((cd^2-ae^2)^3(9c^2d^4+10acd^2e^2+5a^2e^4)\right)\int \sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx}{256c^3d^3e^4}$$

$$= \frac{(cd^2-ae^2)^3(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1024c^4d^4e^5} - \frac{(cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3} - \frac{\left((cd^2-ae^2)^5(9c^2d^4+10acd^2e^2+5a^2e^4)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2048c^4d^4e^5}$$

$$= \frac{(cd^2-ae^2)^3(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1024c^4d^4e^5} - \frac{(cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3} - \frac{\left((cd^2-ae^2)^5(9c^2d^4+10acd^2e^2+5a^2e^4)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^4d^4e^5}$$

$$= \frac{(cd^2-ae^2)^3(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1024c^4d^4e^5} - \frac{(cd^2-ae^2)(9c^2d^4+10acd^2e^2+5a^2e^4)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{384c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4-20acd^2e^2-35a^2e^4-10cde(9cd^2-5ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{840c^2d^2e^3} - \frac{(cd^2-ae^2)^5(9c^2d^4+10acd^2e^2+5a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.06

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-525a^6e^{12}+350a^5cde^{10}\right.}{}$$

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
[Out] ((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e
]*(-525*a^6*e^12 + 350*a^5*c*d*e^10*(4*d + e*x) - 35*a^4*c^2*d^2*e^8*(15*d^
2 + 26*d*e*x + 8*e^2*x^2) - 60*a^3*c^3*d^3*e^6*(10*d^3 - 5*d^2*e*x - 12*d*e
^2*x^2 - 4*e^3*x^3) + a^2*c^4*d^4*e^4*(3689*d^4 - 2332*d^3*e*x + 1824*d^2*e
^2*x^2 + 33520*d*e^3*x^3 + 23680*e^4*x^4) + 2*a*c^5*d^5*e^2*(-1680*d^5 + 10
99*d^4*e*x - 872*d^3*e^2*x^2 + 744*d^2*e^3*x^3 + 24320*d*e^4*x^4 + 18560*e^
5*x^5) + 3*c^6*d^6*(315*d^6 - 210*d^5*e*x + 168*d^4*e^2*x^2 - 144*d^3*e^3*x
^3 + 128*d^2*e^4*x^4 + 6400*d*e^5*x^5 + 5120*e^6*x^6)))/((c*d^2 - a*e^2)^5*
(a*e + c*d*x)*(d + e*x)) - (105*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*Ar
cTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e +
 c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*c^(9/2)*d^(9/2)*e^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(418) = 836.

Time = 0.66 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.39

method	result	size
default	Expression too large to display	1080

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2
)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2)))))-d/e^2*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d
*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*l
n((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(c*d*e)^(1/2))))+d^2/e^3*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*
d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*
(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*
d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 1272, normalized size of antiderivative = 2.81

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*
a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^1
2 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d
^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*d^7*e
^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*
a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d
*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 +
 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*
c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d
^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35
```

```
*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a
^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d
^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6), 1/2
15040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*a^3*c
^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^12 - 5
*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^6 + 945*c^7*d^13*e
- 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a^3*c^4*d^7*e^7 - 525*a^
4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d*e^13 + 1280*(15*c^7*d^8
*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 + 380*a*c^6*d^7*e^7 + 185
*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c
^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*
e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 -
 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3
*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Timed out}$$

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{d + ex}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.43

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{1}{107520}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\left(12\,c^2d^2e\right.\right.\right.\right.\right.\right.$$
$$+\frac{(9\,c^7d^{14}-35\,ac^6d^{12}e^2+45\,a^2c^5d^{10}e^4-15\,a^3c^4d^8e^6-5\,a^4c^3d^6e^8-9\,a^5c^2d^4e^{10}+15\,a^6cd^2e^{12}-5\,a^7e^{14})\log\left(\right.}{2048\,\sqrt{cde}c^4d^4e^5}$$

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/107520*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(12*c^
2*d^2*e*x + (15*c^8*d^9*e^6 + 29*a*c^7*d^7*e^8)/(c^6*d^6*e^6))*x + (3*c^8*d
^10*e^5 + 380*a*c^7*d^8*e^7 + 185*a^2*c^6*d^6*e^9)/(c^6*d^6*e^6))*x - (27*c
^8*d^11*e^4 - 93*a*c^7*d^9*e^6 - 2095*a^2*c^6*d^7*e^8 - 15*a^3*c^5*d^5*e^10
)/(c^6*d^6*e^6))*x + (63*c^8*d^12*e^3 - 218*a*c^7*d^10*e^5 + 228*a^2*c^6*d^
8*e^7 + 90*a^3*c^5*d^6*e^9 - 35*a^4*c^4*d^4*e^11)/(c^6*d^6*e^6))*x - (315*c
^8*d^13*e^2 - 1099*a*c^7*d^11*e^4 + 1166*a^2*c^6*d^9*e^6 - 150*a^3*c^5*d^7*
e^8 + 455*a^4*c^4*d^5*e^10 - 175*a^5*c^3*d^3*e^12)/(c^6*d^6*e^6))*x + (945*
c^8*d^14*e - 3360*a*c^7*d^12*e^3 + 3689*a^2*c^6*d^10*e^5 - 600*a^3*c^5*d^8*
e^7 - 525*a^4*c^4*d^6*e^9 + 1400*a^5*c^3*d^4*e^11 - 525*a^6*c^2*d^2*e^13)/(
c^6*d^6*e^6)) + 1/2048*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^
4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*
d^2*e^12 - 5*a^7*e^14)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^5
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \int \frac{x^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.459 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 381

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4} + \frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^2d^2e^3} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)} + \frac{\left(cd^2-ae^2\right)^5\left(7cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(5*a*e^2+7*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^3-1/60*(5*a/c/d+7*d/e^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)+1/1024*(-a*e^2+c*d^2)^5*(5*a*e^2+7*c*d^2)*arctanh(1/2*(2*c*d*e*x
+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/c^(7/2)/d^(7/2)/e^(9/2)-1/512*(-a*e^2+c*d^2)^3*(5*a*e^2+7*c*d^2)*(2*c*d
*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {808, 678, 626, 635, 212}

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}dx = \frac{(5ae^2+7cd^2)(cd^2-ae^2)^5 \operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}} - \frac{(5ae^2+7cd^2)(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512c^3d^3e^4} + \frac{(5ae^2+7cd^2)(cd^2-ae^2)(ae^2+cd^2+2cdex)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{192c^2d^2e^3} + \frac{(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{6cde(d+ex)} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}$$

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -1/512*((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*e^4) + ((c*d^2 - a*e^2
)*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^
2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)} \\
&\quad+\frac{1}{12}\left(-\frac{7d}{e}-\frac{5ae}{cd}\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx \\
&= -\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)} \\
&\quad+\frac{\left(\left(\frac{7d}{e}+\frac{5ae}{cd}\right)\left(cd^2-ae^2\right)\right)\int\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx}{24e} \\
&= \frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^2d^2e^3} \\
&\quad-\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)} \\
&\quad-\frac{\left(\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\right)\int\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx}{128c^2d^2e^3}
\end{aligned}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512c^3d^3e^4} + \frac{(cd^2-ae^2)(7cd^2+5ae^2)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192c^2d^2e^3} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{6cde(d+ex)} + \frac{\left((cd^2-ae^2)^5(7cd^2+5ae^2)\right)\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{1024c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512c^3d^3e^4} + \frac{(cd^2-ae^2)(7cd^2+5ae^2)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192c^2d^2e^3} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{6cde(d+ex)} + \frac{\left((cd^2-ae^2)^5(7cd^2+5ae^2)\right)\text{Subst}\left(\int\frac{1}{4cde-x^2}dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512c^3d^3e^4} + \frac{(cd^2-ae^2)(7cd^2+5ae^2)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192c^2d^2e^3} - \frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2} + \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{6cde(d+ex)} + \frac{(cd^2-ae^2)^5(7cd^2+5ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02

$$\int\frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}dx = \frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(75a^5e^{10}-5a^4cde^8(49d+10e\right.}{}$$

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e
]*(75*a^5*e^10 - 5*a^4*c*d*e^8*(49*d + 10*e*x) + 10*a^3*c^2*d^2*e^6*(15*d^2
 + 16*d*e*x + 4*e^2*x^2) - 6*a^2*c^3*d^3*e^4*(91*d^3 - 58*d^2*e*x - 564*d*e
^2*x^2 - 360*e^3*x^3) + a*c^4*d^4*e^2*(415*d^4 - 272*d^3*e*x + 216*d^2*e^2*
x^2 + 4448*d*e^3*x^3 + 3200*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x - 56*
```

```
d^3*e^2*x^2 + 48*d^2*e^3*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)))/((c*d^2 - a
*e^2)^5*(a*e + c*d*x)*(d + e*x)) + (15*(7*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*
(d + e*x)^(3/2))))/(7680*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.77

method	result
default	$\frac{\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2 \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2
)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2))))-d/e^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(
a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e
^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-
c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*
(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 1046, normalized size of antiderivative = 2.75

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \Bigg[-\frac{15\,(7\,c^6d^{12}-30\,ac^5d^{10}e^2+45\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6-15\,a^4c^2}{}$$

$$-\frac{15\,(7\,c^6d^{12}-30\,ac^5d^{10}e^2+45\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6-15\,a^4c^2d^4e^8+18\,a^5cd^2e^{10}-5\,a^6e^{12})\sqrt{-cde}\arctan\Big(}{}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3
*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e
+ 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4*c
^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^4
+ 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c
^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*x^
2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3*c
^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
- 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*
e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4
*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^
4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7
*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*
x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3
*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.38

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,c^2d^2ex+\frac{13}{}\right.\right.\right.\right.\right.$$
$$-\frac{\left(7\,c^6d^{12}-30\,ac^5d^{10}e^2+45\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6-15\,a^4c^2d^4e^8+18\,a^5cd^2e^{10}-5\,a^6e^{12}\right)\log\left(\left|-cd^2-ae^2\right.\right.}{1024\sqrt{cde}c^3d^3e^4}$$

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="g
iac")
[Out] 1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c^2*d^2*
e*x + (13*c^7*d^8*e^5 + 25*a*c^6*d^6*e^7)/(c^5*d^5*e^5))*x + (3*c^7*d^9*e^4
 + 278*a*c^6*d^7*e^6 + 135*a^2*c^5*d^5*e^8)/(c^5*d^5*e^5))*x - (7*c^7*d^10*
e^3 - 27*a*c^6*d^8*e^5 - 423*a^2*c^5*d^6*e^7 - 5*a^3*c^4*d^4*e^9)/(c^5*d^5*
e^5))*x + (35*c^7*d^11*e^2 - 136*a*c^6*d^9*e^4 + 174*a^2*c^5*d^7*e^6 + 80*a
^3*c^4*d^5*e^8 - 25*a^4*c^3*d^3*e^10)/(c^5*d^5*e^5))*x - (105*c^7*d^12*e -
```

```
415*a*c^6*d^10*e^3 + 546*a^2*c^5*d^8*e^5 - 150*a^3*c^4*d^6*e^7 + 245*a^4*c^
3*d^4*e^9 - 75*a^5*c^2*d^2*e^11)/(c^5*d^5*e^5)) - 1/1024*(7*c^6*d^12 - 30*a
*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^
8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e
)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \int \frac{x\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{d + e\,x}\,dx$$

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.460 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 274

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx=\frac{3(cd^2-ae^2)^3\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{128c^2d^2e^3}$$
$$+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(cd^2+ae^2+2cdex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$
$$+\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5e}-\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}$$

```
[Out] 1/16*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-3/256*(-a*e^2+c*d^2)^
5*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+3/128*(-a*e^2+c*d^2)^3*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {678, 626, 635, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = -\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}} + \frac{3(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128c^2d^2e^3} + \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5e} + \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(ae^2+cd^2+2cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{5e} \\
&\quad - \frac{(2cd^2e - e(cd^2 + ae^2)) \int (ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}\,dx}{2e^2} \\
&= \frac{1}{16}\left(\frac{a}{cd} - \frac{d}{e^2}\right)(cd^2 + ae^2 + 2cdex)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} \\
&\quad + \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{5e} \\
&\quad + \frac{\left(3(cd^2 - ae^2)^3\right) \int \sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}\,dx}{32cde^2} \\
&= \frac{3(cd^2 - ae^2)^3 (cd^2 + ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{128c^2d^2e^3} \\
&\quad + \frac{1}{16}\left(\frac{a}{cd} - \frac{d}{e^2}\right)(cd^2 + ae^2 + 2cdex)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} \\
&\quad + \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{5e} - \frac{\left(3(cd^2 - ae^2)^5\right) \int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{256c^2d^2e^3} \\
&= \frac{3(cd^2 - ae^2)^3 (cd^2 + ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{128c^2d^2e^3} \\
&\quad + \frac{1}{16}\left(\frac{a}{cd} - \frac{d}{e^2}\right)(cd^2 + ae^2 + 2cdex)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2} \\
&\quad + \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{5e} \\
&\quad - \frac{\left(3(cd^2 - ae^2)^5\right) \text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^2d^2e^3}
\end{aligned}$$

$$= \frac{3(cd^2 - ae^2)^3 (cd^2 + ae^2 + 2cdex) \sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{128c^2d^2e^3}$$
$$+ \frac{1}{16}\left(\frac{a}{cd} - \frac{d}{e^2}\right)(cd^2 + ae^2 + 2cdex)\left(ade + (cd^2 + ae^2) x + cdex^2\right)^{3/2}$$
$$+ \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{5e}$$
$$- \frac{3(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{d + ex} dx = \frac{((ae + cdx)(d + ex))^{3/2} \left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-15a^4e^8+10a^3cde^6(7d+ex)+2a^2c^2d^2e^4(64d^2}{}\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^4*e^8 + 1
0*a^3*c*d*e^6*(7*d + e*x) + 2*a^2*c^2*d^2*e^4*(64*d^2 + 233*d*e*x + 124*e^2
*x^2) + 2*a*c^3*d^3*e^2*(-35*d^3 + 23*d^2*e*x + 256*d*e^2*x^2 + 168*e^3*x^3
) + c^4*d^4*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4*
x^4)))/((a*e + c*d*x)*(d + e*x)) - (15*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3/2)*(d
 + e*x)^(3/2))))/(640*c^(5/2)*d^(5/2)*e^(7/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.19

method	result
default	$\frac{\left(cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(e^2a-c\,d^2\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+e^2a-c\,d^2\right)\left(cde\left(x+\frac{d}{e}\right)^2+\left(e^2a-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8cde}-\frac{3\left(e^2a-c\,d^2\right)^2\left(\frac{\left(2cde\left(x+\right.\right.}{}\right.}{}\right.}{e}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1
/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d
/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*
ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.08

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \left[\frac{15 (c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - c\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^
4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^
2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/12
80*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c^5*d^5*e^5*x^
4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 70*a^3*c^2*d^3*
e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*
d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 2
3*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.51

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{1}{640}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\,c^2d^2ex+\frac{11\,c^6d^7e^4+\cdots}{c^4d\cdots}\right.\right.\right.\right.$$
$$+\frac{3\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\cdots\right.\right.\right.}{256\sqrt{cde}c^2d^2e^3}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
[Out] 1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c^2*d^2*e*x +
 (11*c^6*d^7*e^4 + 21*a*c^5*d^5*e^6)/(c^4*d^4*e^4))*x + (c^6*d^8*e^3 + 64*a
*c^5*d^6*e^5 + 31*a^2*c^4*d^4*e^7)/(c^4*d^4*e^4))*x - (5*c^6*d^9*e^2 - 23*a
*c^5*d^7*e^4 - 233*a^2*c^4*d^5*e^6 - 5*a^3*c^3*d^3*e^8)/(c^4*d^4*e^4))*x +
(15*c^6*d^10*e - 70*a*c^5*d^8*e^3 + 128*a^2*c^4*d^6*e^5 + 70*a^3*c^3*d^4*e^
7 - 15*a^4*c^2*d^2*e^9)/(c^4*d^4*e^4)) + 3/256*(c^5*d^10 - 5*a*c^4*d^8*e^2
+ 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*log
(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

3.461 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 394

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx = -\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cde^2}$$
$$+\frac{\left(3cd^2+11ae^2+6cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e}$$
$$+\frac{\left(3c^4d^8-20ac^3d^6e^2+90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{5/2}}$$
$$-a^{5/2}d^{3/2}e^{5/2}\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)$$

```
[Out] 1/24*(6*c*d*e*x+11*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e
-a^(5/2)*d^(3/2)*e^(5/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/128*(-5*a^4*e^8+60*a
^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4-20*a*c^3*d^6*e^2+3*c^4*d^8)*arctanh(1/2*(2*
c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/64*(3*c^3*d^6-11*a*c^2*d^4*e^2-83*a^2*
c*d^2*e^4-5*a^3*e^6+2*c*d*e*(-5*a*e^2+c*d^2)*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 828, 857, 635, 212, 738}

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)}\,dx =$$

$$-a^{5/2}d^{3/2}e^{5/2}\text{arctanh}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\,(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(-5a^3e^6 - 83a^2cd^2e^4 - 11ac^2d^4e^2 + 2c}{}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] -1/64*((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d
*e*(c*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*
a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*
a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
```

```
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{x}\,dx$$

$$= \frac{(3cd^2+11ae^2+6cdex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{24e} - \frac{\int \frac{\left(-8a^2cd^2e^3+\frac{1}{2}cd(cd^2-5ae^2)(3cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx}{8cde}$$

$$= -\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde(cd^2-5ae^2)\,(3cd^2+ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)}}{64cde^2} + \frac{(3cd^2+11ae^2+6cdex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{24e} + \frac{\int \frac{32a^3c^2d^4e^5+\frac{1}{4}cd(3c^4d^8-20ac^3d^6e^2+90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8)x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{32c^2d^2e^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd^2 + ae^2)x)\sqrt{ade + (cd^2 + ae^2}}{64cde^2}$$
$$+ \frac{(3cd^2 + 11ae^2 + 6cdex)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{24e}$$
$$+ (a^3d^2e^3)\int \frac{1}{x\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}dx$$
$$+ \frac{(3c^4d^8 - 20ac^3d^6e^2 + 90a^2c^2d^4e^4 + 60a^3cd^2e^6 - 5a^4e^8)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{128cde^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd^2 + ae^2)x)\sqrt{ade + (cd^2 + ae^2}}{64cde^2}$$
$$+ \frac{(3cd^2 + 11ae^2 + 6cdex)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{24e}$$
$$- (2a^3d^2e^3)\,\text{Subst}\left(\int \frac{1}{4ade - x^2}dx, x, \frac{2ade - (-cd^2 - ae^2)x}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)$$
$$+ \frac{(3c^4d^8 - 20ac^3d^6e^2 + 90a^2c^2d^4e^4 + 60a^3cd^2e^6 - 5a^4e^8)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cd}}\right.}{64cde^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd^2 + ae^2)x)\sqrt{ade + (cd^2 + ae^2}}{64cde^2}$$
$$+ \frac{(3cd^2 + 11ae^2 + 6cdex)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{24e}$$
$$+ \frac{(3c^4d^8 - 20ac^3d^6e^2 + 90a^2c^2d^4e^4 + 60a^3cd^2e^6 - 5a^4e^8)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{5/2}}$$
$$- a^{5/2}d^{3/2}e^{5/2}\tanh^{-1}\left(\frac{2ade + (cd^2 + ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.87

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)}dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(15a^3e^6 + a^2cde\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(15*a^3*e^6 + a^2*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^2*
(57*d^2 + 244*d*e*x + 136*e^2*x^2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x
```

^2 + 48*d^3*e^3*x^3)) - 384*a^(5/2)*c^(3/2)*d^3*e^5*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(192*c^(3/2)*d^(3/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(352) = 704$.

Time = 0.60 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.53

method	result
default	$\frac{\frac{\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{5}+\frac{\left(e^2a+c\,d^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+e^2a+c\,d^2\right.}{}\right.}{}\right.}{}}{}$

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))-1/d*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 30.62 (sec) , antiderivative size = 1873, normalized size of antiderivative = 4.75

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/768*(384*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6
 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^
3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 +
 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^
4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3), 1/384*(192*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 6
0*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 -
9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(
9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^
4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
2*d^2*e^3), 1/768*(768*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/
(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8
 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sq
rt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c
*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e
+ 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^
3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c
^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3),
1/384*(384*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2
*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8 - 20*a*c^3*
d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*a
```

```
rctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3
)*x)) + 2*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^
2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(
3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d+ex)}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}{x\,(d+ex)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="g
iac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.462 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 352

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e}$$
$$-\frac{(3ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3x}$$
$$-\frac{\left(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6\right)\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16\sqrt{c}\sqrt{d}e^{3/2}}$$
$$-\frac{1}{2}a^{3/2}\sqrt{d}e^{3/2}\left(5cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)$$

```
[Out] -1/3*(-c*d*x+3*a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x-1/16*(-5*a^3*
e^6-45*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+c^3*d^6)*arctanh(1/2*(2*c*d*e*x+a*e^2
+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^
(3/2)/c^(1/2)/d^(1/2)-1/2*a^(3/2)*e^(3/2)*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*
a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))*d^(1/2)+1/8*(c^2*d^4+28*a*c*d^2*e^2+19*a^2*e^4+2*c*d*e*(7*a*e^
2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {863, 826, 828, 857, 635, 212, 738}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx =$$

$$-\frac{1}{2}a^{3/2}\sqrt{d}e^{3/2}(3ae^2+5cd^2)\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)+\frac{(19a^2e^4+2cdex(7ae^2+$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
```

```
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2}\,dx$$

$$=-\frac{(3ae-cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3x}-\frac{1}{2}\int\frac{\left(-ae(5cd^2+3ae^2)-cd(cd^2+7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e}-\frac{(3ae-cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3x}+\frac{\int\frac{4a^2cd^2e^3(5cd^2+3ae^2)-\frac{1}{2}cd\left(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6\right)x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8cde}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e}-\frac{(3ae-cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3x}+\frac{1}{2}\left(a^2de^2(5cd^2+3ae^2)\right)\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx-\frac{\left(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6\right)\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16e}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e}-\frac{(3ae-cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3x}-\left(a^2de^2(5cd^2+3ae^2)\right)\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)-\frac{\left(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6\right)\text{Subst}\left(\int\frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8e}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8e}-\frac{(3ae-cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3x}-\frac{\left(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16\sqrt{c}\sqrt{d}e^{3/2}}-\frac{1}{2}a^{3/2}\sqrt{d}e^{3/2}(5cd^2+3ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.88

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(3a^2e^3(-8d+11e\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(3*a^2*e^3*(-8*d + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) +
 c^2*d^2*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - 24*a^(3/2)*Sqrt[c]*d*e^3*(5*c*
d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[
d + e*x])] - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*
x*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(2
4*Sqrt[c]*Sqrt[d]*e^(3/2)*x*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. 2(308) = 616.

Time = 0.75 (sec) , antiderivative size = 2076, normalized size of antiderivative = 5.90

method	result	size
default	Expression too large to display	2076

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d^2)/a
/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2
*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d
/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2
*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*
e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+6
*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^
```

```
2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*
d*e)^(1/2)))))-e/d^2*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^
2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c
*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/x))))+e/d^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2
*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)
+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^
2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*
e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 10.89 (sec) , antiderivative size = 1717, normalized size of antiderivative = 4.88

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(
c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*
sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
- 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^3*x^3 -
 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e
 + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
```

```
a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*
e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^
5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(8*c^3*d^3*e^3*x^
3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^
5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(
-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*
e + a^2*d*e^3)*x)) - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a
^3*e^6)*sqrt(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^3 - 2
4*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e +
 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c*d*e^2*x), 1/48*(24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(-a*d*
e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) + 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^
6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^
2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c
^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a
^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{2}}}{(ex+d)x^2}\,dx$$

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")

[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x)

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.34

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,c^2d^2ex+\frac{7\,c^4d^5e^2+13\,ac^3d\cdots}{c^2d^2e^2}\cdots\right.\right.$$
$$+\frac{\left(5\,a^2cd^3e^2+3\,a^3de^4\right)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}$$
$$+\frac{\left(c^3d^6-15\,ac^2d^4e^2-45\,a^2cd^2e^4-5\,a^3e^6\right)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+a\cdots}\right.\right.\right.}{16\sqrt{cde}e}$$
$$-\frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2cd^3e^2+\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^3de^4+2\sqrt{\cdots}}{ade-\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)^2}$$

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")

[Out] 1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c^2*d^2*e*x + (7*c^4*d^5*e^2 + 13*a*c^3*d^3*e^4)/(c^2*d^2*e^2))*x + (3*c^4*d^6*e + 68*a*c^3*d^4*e^3 + 33*a^2*c^2*d^2*e^5)/(c^2*d^2*e^2)) + (5*a^2*c*d^3*e^2 + 3*a^3*d*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/sqrt(-a*d*e) + 1/16*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^2 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^4 + 2*sqrt(c*d*e)*a^3*d^2*e^3)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e)^{5/2}}{x^2\ (d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.463 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 339

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} - \frac{(ae-cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2x^2} + \frac{3\sqrt{c}\sqrt{d}(c^2d^4+10acd^2e^2+5a^2e^4)\,\mathrm{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{e}} - \frac{3\sqrt{a}\sqrt{e}(5c^2d^4+10acd^2e^2+a^2e^4)\,\mathrm{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{d}}$$

```
[Out] -1/2*(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2+3/8*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/
2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)
-3/8*(a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)
*e^(1/2)/d^(1/2)-3/4*(a*e*(a*e^2+3*c*d^2)-c*d*(3*a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 826, 857, 635, 212, 738}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{3\sqrt{c}\sqrt{d}(5a^2e^4+10acd^2e^2+c^2d^4)\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+c}}\right.}{8\sqrt{e}}$$
$$-\frac{3\sqrt{a}\sqrt{e}(a^2e^4+10acd^2e^2+5c^2d^4)\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{d}}$$
$$-\frac{(ae-cdx)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2x^2}$$
$$-\frac{3(ae(ae^2+3cd^2)-cdx(3ae^2+cd^2))\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4x}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{x^3}\,dx \\
&= -\frac{(ae-cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2x^2} \\
&\quad -\frac{3}{8}\int \frac{(-2ae(3cd^2+ae^2)-2cd(cd^2+3ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx \\
&= -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} \\
&\quad -\frac{(ae-cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2x^2} \\
&\quad +\frac{3}{16}\int \frac{2ae(5c^2d^4+10acd^2e^2+a^2e^4)+2cd(c^2d^4+10acd^2e^2+5a^2e^4)\,x}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx
\end{aligned}$$

$$= -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x} - \frac{(ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2x^2} + \frac{1}{8}\left(3ae\left(5c^2d^4+10acd^2e^2+a^2e^4\right)\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx + \frac{1}{8}\left(3cd\left(c^2d^4+10acd^2e^2+5a^2e^4\right)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$

$$= -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x} - \frac{(ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2x^2} - \frac{1}{4}\left(3ae\left(5c^2d^4+10acd^2e^2+a^2e^4\right)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) + \frac{1}{4}\left(3cd\left(c^2d^4+10acd^2e^2+5a^2e^4\right)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

$$= -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x} - \frac{(ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2x^2} + \frac{3\sqrt{c}\sqrt{d}(c^2d^4+10acd^2e^2+5a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{e}} - \frac{3\sqrt{a}\sqrt{e}(5c^2d^4+10acd^2e^2+a^2e^4)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.85

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(-9acdex(d-ex)+\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
+ e*x]*(-9*a*c*d*e*x*(d - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d +
 5*e*x)) - 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*Sqrt[c]*d*(
c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*x^2*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x
])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(4*Sqrt[d]*Sqrt[e]*x^2*Sqrt[(a*e + c*
d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3892 vs. 2(295) = 590.

Time = 0.81 (sec) , antiderivative size = 3893, normalized size of antiderivative = 11.48

method	result	size
default	Expression too large to display	3893

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+3/4*(a*e^2+c*d^
2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d
^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1
/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/
16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln
((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/
2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
)))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2
+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(
a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2
*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2)))))+5/2*c/a*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/
2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+
a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^
(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
```

```
d*e*x^2)^(1/2))/x)))))+e^2/d^3*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e
*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c
*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/x))))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)+5/2*(a*e^2+c*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*
e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+
c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d
*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e
*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/
d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))-e^2/d^3*(1/5*(c*d*e*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+e^2*a-c*d
^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^
2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*e^2*a-1/2*c*d^2+c*d*e
*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d
*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 4.42 (sec) , antiderivative size = 1569, normalized size of antiderivative = 4.63

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
 a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e/d)*
x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqr
t(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 - 2*a^2*d
*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*a*c*d^2*e
^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*
d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4
)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a
*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*c^2*d^2*e
*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1/16*(6*(5*c^2*d^4
 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3*(c^2*d^4 + 10*a*c*d^2*
e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*
d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*
a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/8*(3*(c^2*
d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^
2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c
*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2
+ a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2
+ (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{(ex+d)x^3}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(295) = 590$.

Time = 0.46 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.20

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \frac{1}{4}\left(2\,c^2d^2ex + \frac{5\,c^3d^4e + 9\,ac^2d^2e^3}{cde}\right)\sqrt{cdex^2 + cd^2x + ae^2x + ade}$$
$$+ \frac{3\left(5\,ac^2d^4e + 10\,a^2cd^2e^3 + a^3e^5\right)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}}$$
$$- \frac{3\left(c^3d^5 + 10\,ac^2d^3e^2 + 5\,a^2cde^4\right)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{8\sqrt{cde}}$$
$$- \frac{7\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^2d^5e^2 + 6\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3cd^3e^4 +}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"giac")
[Out] 1/4*(2*c^2*d^2*e*x + (5*c^3*d^4*e + 9*a*c^2*d^2*e^3)/(c*d*e))*sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e) + 3/4*(5*a*c^2*d^4*e + 10*a^2*c*d^2*e^3 + a^
```

```
3*e^5)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)/sqrt(-a*d*e))/sqrt(-a*d*e) - 3/8*(c^3*d^5 + 10*a*c^2*d^3*e^2 + 5*a^2*c*d*
e^4)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/4*(7*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c^2*d^5*e^2 + 6*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c*d^3*e^4 + 3*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*d*e^6 + 16*sqrt(c*
d*e)*a^3*c*d^4*e^3 + 8*sqrt(c*d*e)*a^4*d^2*e^5 - 9*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^2*d^4*e - 18*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c*d^2*e^3 - 5*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*e^5 - 24*sqrt(c*d*
e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c*d^
3*e^2 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))^2*a^3*d*e^4)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^3(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^3\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.464 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 371

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx = -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx} - \frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3} + \frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}\left(3cd^2+5ae^2\right)\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right) - \frac{\left(5c^3d^6+45ac^2d^4e^2+15a^2\right.}{}$$

```
[Out] -1/12*(4*a*d*e+3*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
/d/x^3-1/16*(-a^3*e^6+15*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+5*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)+1/2*c^(3/2)*d^(3/2)*(5*a*e^2+3*
c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/8*(5*c^2*d^4+12*a*c*d^2*e^2-a^2*
e^4-2*c*d*e*(a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {863, 824, 826, 857, 635, 212, 738}

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)} dx =$$
$$-\frac{(-a^2e^4 - 2cdex(ae^2 + 7cd^2) + 12acd^2e^2 + 5c^2d^4)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8dx}$$
$$-\frac{(-a^3e^6 + 15a^2cd^2e^4 + 45ac^2d^4e^2 + 5c^3d^6)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16\sqrt{a}d^{3/2}\sqrt{e}}$$
$$+\frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}(5ae^2+3cd^2)\operatorname{arctanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(3x(ae^2 + 3cd^2) + 4ade)}{}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/8*((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*x) - ((4*a*d*e + 3*(3*c*d^2
 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c
^(3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e
*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^4}\,dx$$

$$= -\frac{(4ade+3(3cd^2+ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{12dx^3} - \frac{\int \frac{\left(-\frac{1}{2}ae(5c^2d^4+12acd^2e^2-a^2e^4)-acde^2(7cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2}\,dx}{4ade}$$

$$= -\frac{(5c^2d^4+12acd^2e^2-a^2e^4-2cde(7cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8dx} - \frac{(4ade+3(3cd^2+ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{12dx^3} + \frac{\int \frac{\frac{1}{2}ae(5c^3d^6+45ac^2d^4e^2+15a^2cd^2e^4-a^3e^6)+4ac^2d^3e^2(3cd^2+5ae^2)x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8ade}$$

$$= -\frac{(5c^2d^4+12acd^2e^2-a^2e^4-2cde(7cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8dx} - \frac{(4ade+3(3cd^2+ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{12dx^3} + \frac{1}{2}\left(c^2d^2e\left(3cd^2+5ae^2\right)\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx + \frac{(5c^3d^6+45ac^2d^4e^2+15a^2cd^2e^4-a^3e^6)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16d}$$

$$= -\frac{(5c^2d^4+12acd^2e^2-a^2e^4-2cde(7cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8dx} - \frac{(4ade+3(3cd^2+ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{12dx^3} + \left(c^2d^2e\left(3cd^2+5ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(5c^3d^6+45ac^2d^4e^2+15a^2cd^2e^4-a^3e^6)\,\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8d}$$

$$= -\frac{(5c^2d^4+12acd^2e^2-a^2e^4-2cde(7cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8dx} - \frac{(4ade+3(3cd^2+ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{12dx^3} + \frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}\left(3cd^2+5ae^2\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) - \frac{(5c^3d^6+45ac^2}{}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.85

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)} dx =$$

$$-\frac{\sqrt{ae + cdx}\sqrt{d + ex}\Big(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(3c^2d^3x^2(11d - 8ex) + 2acd^2ex(13d + 34ex) + a^2e^2(8d^2 \cdots}{24 \cdots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/24*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]*(3*c^2*d^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d +
34*e*x) + a^2*e^2*(8*d^2 + 14*d*e*x + 3*e^2*x^2)) + 3*(5*c^3*d^6 + 45*a*c^2
*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*
x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] - 24*Sqrt[a]*c^(3/2)*d^3*e*(3*c*d^2 +
5*a*e^2)*x^3*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d +
e*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6849 vs. 2(327) = 654.

Time = 1.06 (sec) , antiderivative size = 6850, normalized size of antiderivative = 18.46

method	result	size
default	Expression too large to display	6850

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 5.78 (sec) , antiderivative size = 1741, normalized size of antiderivative = 4.69

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)
*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*d^4*e^2
*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^
2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sq
rt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d
^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4
- a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(24*a*c
^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3
*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e^
2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 12*(3*a*c^2*d^5
*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 2*(24*a*
c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^
3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e
^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a
*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 24*(3*a*c^2*d^
5*e + 5*a^2*c*d^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(24*a*c^2*d^4*e^2*x^3 -
8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(
13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(a*d^2*e*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(327) = 654$.

Time = 0.50 (sec) , antiderivative size = 1195, normalized size of antiderivative = 3.22

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\sqrt{cdex^2+cd^2x+ae^2x+ade}\,c^2d^2e$$
$$-\frac{\left(3\,c^3d^4e+5\,ac^2d^2e^3\right)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\,\sqrt{cde}}$$
$$+\frac{\left(5\,c^3d^6+45\,ac^2d^4e^2+15\,a^2cd^2e^4-a^3e^6\right)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{8\,\sqrt{-ade}d}$$
$$-\frac{15\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2c^3d^8e^2+39\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^3c^2d^6}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c^2*d^2*e - 1/2*(3*c^3*d^4*e +
5*a*c^2*d^2*e^3)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
```

```
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/8*(5*c^3*d^6 +
 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) -
 1/24*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2
*c^3*d^8*e^2 + 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^3*c^2*d^6*e^4 + 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^5*d^2*e^8 + 48*sqrt(c*d*e)*a^3*c^2*d^7*e^3 + 112*sqrt(c
*d*e)*a^4*c*d^5*e^5 - 40*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^3*a*c^3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 - 24*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 - 144*sqrt(c*d*e)*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c^2*d^6*e^2 - 240
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
2*a^3*c*d^4*e^4 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^5*c^3*d^6 + 153*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^5*a*c^2*d^4*e^2 + 99*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^5*a^2*c*d^2*e^4 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^5*a^3*e^6 + 144*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a*c^2*d^5*e + 288*sqrt(c*d*e)*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^2*c*d^3*e^3
 + 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^4*a^3*d*e^5)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))^2)^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^4\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.465 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 404

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx =$$
$$-\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+ae^2)x+cdex}}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24dx^4}$$
$$+c^{5/2}d^{5/2}e^{3/2}\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)+\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3c}{12}$$

```
[Out] -1/24*(6*a*d*e+(3*a*e^2+11*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/d/x^4+1/128*(-3*a^4*e^8+20*a^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-60*a*c^3*d^6*
e^2+5*c^4*d^8)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+c^(5/2)*
d^(5/2)*e^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-1/64*(2*a*d*e*(-a*e^2+5*c*d^2)*(3
*a*e^2+c*d^2)+(-3*a^3*e^6+11*a^2*c*d^2*e^4+83*a*c^2*d^4*e^2+5*c^3*d^6)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/x^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 824, 857, 635, 212, 738}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx =$$

$$-\frac{(x(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6)+2ade(5cd^2-ae^2)(3ae^2+cd^2))\sqrt{x(ae^2+cd^2)+ade+c}}{64ad^2ex^2}$$

$$+\frac{(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128a^{3/2}d^{5/2}e^{3/2}}$$

$$+c^{5/2}d^{5/2}e^{3/2}\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{(x(3ae^2+11cd^2)+6ade)(x(ae^2+cd^2)}{24dx^4}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] -1/64*((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2
*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2
*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*
e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{\left(6ade+(11cd^2+3ae^2)\,x\right)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{24dx^4} - \frac{\int \frac{\left(-\frac{1}{2}ae(5cd^2-ae^2)(cd^2+3ae^2)-8ac^2d^3e^2x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{8ade}$$

$$= -\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+a}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24dx^4}$$
$$+\frac{\int \frac{-\frac{1}{4}ae(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3cd^2e^6-3a^4e^8)+32a^2c^3d^5e^4x}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{32a^2d^2e^2}$$

$$= -\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+a}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24dx^4}$$
$$+\left(c^3d^3e^2\right)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$-\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3cd^2e^6-3a^4e^8)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128ad^2e}$$

$$= -\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+a}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24dx^4}$$
$$+\left(2c^3d^3e^2\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$
$$+\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3cd^2e^6-3a^4e^8)\,\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cd}}\right.}{64ad^2e}$$

$$= -\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+a}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24dx^4}$$
$$+c^{5/2}d^{5/2}e^{3/2}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)+\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2}{}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(-\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(15c^3d^6x^3+ac^2}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*(15*c^3*d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^2*
c*d^2*e^2*x*(136*d^2 + 244*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2
*e*x + 2*d*e^2*x^2 - 3*e^3*x^3))) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^
2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*x^4*ArcTanh[(Sqrt[d]*Sqrt[a*e
 + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 384*a^(3/2)*c^(5/2)*d^5*e^3*x
^4*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(
192*a^(3/2)*d^(5/2)*e^(3/2)*x^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11684 vs. $2(362) = 724$.

Time = 1.20 (sec) , antiderivative size = 11685, normalized size of antiderivative = 28.92

method	result	size
default	Expression too large to display	11685

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 16.14 (sec) , antiderivative size = 1917, normalized size of antiderivative = 4.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/768*(384*sqrt(c*d*e)*a^2*c^2*d^5*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4
 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
```

```
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*
(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a
^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(48*a^4*d^
4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*
e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 +
 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d^5*e^3*x^4*
arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^
3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^
2*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4
*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5
 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2
*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), 1/384*(192*sqrt(c*d*e)*a^2*c^2*d^5
*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*
a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e
^5 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d
^2*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/384*(384*sqrt(-c*d*e)*a^2*c^2
*d^5*e^3*x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4
+ 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(48*a^4*d^4*e
^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*e^7
)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 8*
(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a^2*d^3*e^2*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^5}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(362) = 724$.

Time = 0.68 (sec) , antiderivative size = 1750, normalized size of antiderivative = 4.33

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"giac")
[Out] -c^3*d^3*e^2*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/64*(5*c^4*d^8 - 60
*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*arctan(
-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e)
)/(sqrt(-a*d*e)*a*d^2*e) + 1/192*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 180*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5 + 114*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^2*d^7*e^7 + 60*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c*d^5*e^9 - 9*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*d^3*e^11 - 512*
sqrt(c*d*e)*a^5*c^2*d^8*e^6 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
```

```
2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*e^4 + 1374*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^2*d^6*e^6 + 548*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c*d^4*e^8 + 33
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*d^2*e^
10 + 2048*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^2*a^4*c^2*d^7*e^5 + 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 + 73*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*e + 276*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c^3*d^7*e^3 + 990*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*c^2*d^5*e^5
 + 676*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*
c*d^3*e^7 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)^5*a^5*d*e^9 - 1152*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))^4*a^3*c^2*d^6*e^4 + 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^4*c*d^4*e^6 + 384*sqrt(c*d*e
)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^5*d^2*e
^8 + 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*c^4
*d^8 + 588*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*
a*c^3*d^6*e^2 + 882*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^7*a^2*c^2*d^4*e^4 + 60*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^7*a^3*c*d^2*e^6 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^7*a^4*e^8 + 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a*c^3*d^7*e + 2304*sqrt(c*d*e)*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^2*c^2*d^5*e^
3 + 1152*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^6*a^3*c*d^3*e^5)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^2)^4*a*d^2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x^5\,(d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.466 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 289

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx = \frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}$$
$$-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{16x^4}$$
$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5dx^5}-\frac{3\left(cd^2-ae^2\right)^5\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

```
[Out] -1/16*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/x^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^5-3/256*(-a*
e^2+c*d^2)^5*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+3/128*(-a*
e^2+c*d^2)^3*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^2/d^3/e^2/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{863, 820, 734, 738, 212\}$

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx = -\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}} + \frac{3(cd^2-ae^2)^3\,(x(ae^2+cd^2)+2ade)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{128a^2d^3e^2x^2} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5dx^5} - \frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(x(ae^2+cd^2)+2ade)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{16x^4}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^6}\,dx$$

$$= -\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5dx^5} - \frac{\left(-2acd^2e+ae(cd^2+ae^2)\right)\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^5}\,dx}{2ade}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+(cd^2+ae^2)x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{16x^4} - \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5dx^5} - \frac{\left(3(cd^2-ae^2)^3\right)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{32ad^2e}$$

$$= \frac{3(cd^2-ae^2)^3\left(2ade+(cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^2d^3e^2x^2} - \frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+(cd^2+ae^2)x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{16x^4} - \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5dx^5} + \frac{\left(3(cd^2-ae^2)^5\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{256a^2d^3e^2}$$

$$= \frac{3(cd^2-ae^2)^3\left(2ade+(cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^2d^3e^2x^2} - \frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+(cd^2+ae^2)x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{16x^4} - \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5dx^5} - \frac{\left(3(cd^2-ae^2)^5\right)\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^2d^3e^2}$$

$$= \frac{3(cd^2-ae^2)^3\left(2ade+(cd^2+ae^2)x\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^2d^3e^2x^2} - \frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+(cd^2+ae^2)x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{16x^4} - \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5dx^5} - \frac{3(cd^2-ae^2)^5\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94

$$\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx = \frac{(-cd^2+ae^2)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)^3\left(15a^4e^4-\frac{15d^4}{\cdots}\right.}{\cdots}\right.}{64\cdots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[a]*Sqrt[d]*S
qrt[e]*(d + e*x)^3*(15*a^4*e^4 - (15*d^4*(a*e + c*d*x)^4)/(d + e*x)^4 + (70
*a*d^3*e*(a*e + c*d*x)^3)/(d + e*x)^3 + (128*a^2*d^2*e^2*(a*e + c*d*x)^2)/(
d + e*x)^2 - (70*a^3*d*e^3*(a*e + c*d*x))/(d + e*x)))/((-(c*d^2) + a*e^2)^5
*x^5*(a*e + c*d*x))) + (15*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqr
t[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(640*a^(5/2)*
d^(7/2)*e^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19538 vs. 2(259) = 518.

Time = 1.57 (sec) , antiderivative size = 19539, normalized size of antiderivative = 67.61

method	result	size
default	Expression too large to display	19539

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 13.96 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.02

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d + ex)} dx = \left[\frac{15\,(c^5d^{10} - 5\,ac^4d^8e^2 + 10\,a^2c^3d^6e^4 - 10\,a^3c^2d^4e^6 + 5\,a^4cd^2e^8 - a^5}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2 + (
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2) - 4*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3
- 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3
*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*
(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^
6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^
3*d^4*e^3*x^5), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4
 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-a*d*e)*x^5*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) - 2*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3 - 128*a^3*c^
2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3*d^8*e^2 + 2
33*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*(31*a^3*c^2*
d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^6*e^4 + 11*a
^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^
5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^6}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(259) = 518$.

Time = 0.40 (sec) , antiderivative size = 2449, normalized size of antiderivative = 8.47

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"giac")
[Out] 3/128*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6
 + 5*a^4*c*d^2*e^8 - a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/640*
(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^5*d
^14*e^4 - 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^5*c^4*d^12*e^6 + 150*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^6*c^3*d^10*e^8 + 1130*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 + 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 15*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 + 256*sqrt(c*d*e)*a^7*c^2
*d^9*e^9 - 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^3*a^3*c^5*d^13*e^3 + 350*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
```

*x + a*d*e))^3*a^4*c^4*d^11*e^5 + 5700*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^3*d^9*e^7 + 7100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 + 2210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d^5*e^11 + 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*d^3*e^13 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^6*c^2*d^8*e^8 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c*d^6*e^10 + 128*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c^5*d^12*e^2 + 3200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*c^4*d^10*e^4 + 12800*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*c^3*d^8*e^6 + 12800*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^5*c^2*d^6*e^8 + 3200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^6*c*d^4*e^10 + 128*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*d^2*e^12 + 6400*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^4*c^3*d^9*e^5 + 14080*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^5*c^2*d^7*e^7 + 6400*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^6*c*d^5*e^9 + 1280*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^7*d^3*e^11 + 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a*c^5*d^11*e + 2210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^2*c^4*d^9*e^3 + 7100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^3*c^3*d^7*e^5 + 5700*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^4*c^2*d^5*e^7 + 350*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^5*c*d^3*e^9 - 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^6*d*e^11 + 1280*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^2*c^4*d^10*e^2 + 6400*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^3*c^3*d^8*e^4 + 11520*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^4*c^2*d^6*e^6 + 6400*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^5*c*d^4*e^8 - 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*c^5*d^10 + 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a*c^4*d^8*e^2 + 1130*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^2*c^3*d^6*e^4 + 150*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^3*c^2*d^4*e^6 - 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^4*c*d^2*e^8 + 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^5*e^10 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^2*c^3*d^7*e^3 + 3840*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^3*c^2*d^5*e^5)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^5*a^2*d^3*e^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^6\,\left(d+e\,x\right)} \,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.467 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 386

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx = -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512a^3d^4e^3x^2} + \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} + \frac{(cd^2-ae^2)^5(5cd^2+7ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(7*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)/d/x^6-1/60*(5*c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)/x^5+1/1024*(-a*e^2+c*d^2)^5*(7*a*e^2+5*c*d^2)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/512*(-a*e^2+c*d^2)^3*(7*a*e^2+5*c*d^2)*(2
*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3
/x^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{(7ae^2+5cd^2)(cd^2-ae^2)^5 \operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}$$
$$-\frac{(7ae^2+5cd^2)(cd^2-ae^2)^3(x(ae^2+cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512a^3d^4e^3x^2}$$
$$+\frac{(7ae^2+5cd^2)(cd^2-ae^2)(x(ae^2+cd^2)+2ade)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{192a^2d^3e^2x^4}$$
$$-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6dx^6}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{60x^5}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] -1/512*((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^3*d^4*e^3*x^2) + ((c*d^2 -
 a*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2
+ 7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(1024*a^(7/2)*d^(9/2)*e^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^7}\,dx \\
&= -\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{1}{2}ae(5cd^2-7ae^2)+acde^2x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^6}\,dx}{6ade} \\
&= -\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{60x^5} \\
&\quad - \frac{\left(\frac{5c^2d^2}{a}+2ce^2-\frac{7ae^4}{d^2}\right)\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^5}\,dx}{24e}
\end{aligned}$$

$$= \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} + \frac{\left((cd^2-ae^2)^3(5cd^2+7ae^2)\right)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx}{128a^2d^3e^2}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512a^3d^4e^3x^2} + \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} - \frac{\left((cd^2-ae^2)^5(5cd^2+7ae^2)\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{1024a^3d^4e^3}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512a^3d^4e^3x^2} + \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} + \frac{\left((cd^2-ae^2)^5(5cd^2+7ae^2)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512a^3d^4e^3}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512a^3d^4e^3x^2} + \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{60x^5} + \frac{(cd^2-ae^2)^5(5cd^2+7ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.05

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)} dx = \frac{(-cd^2 + ae^2)^5 ((ae + cdx)(d + ex))^{3/2} \left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(75c^5d^{10}x^5 - 5ac^4d^8ex^4(10d\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sqr
t[e]*(75*c^5*d^10*x^5 - 5*a*c^4*d^8*e*x^4*(10*d + 49*e*x) + 10*a^2*c^3*d^6*
e^2*x^3*(4*d^2 + 16*d*e*x + 15*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(360*d^3 +
564*d^2*e*x + 58*d*e^2*x^2 - 91*e^3*x^3) + a^4*c*d^2*e^4*x*(3200*d^4 + 4448
*d^3*e*x + 216*d^2*e^2*x^2 - 272*d*e^3*x^3 + 415*e^4*x^4) + a^5*e^5*(1280*d
^5 + 1664*d^4*e*x + 48*d^3*e^2*x^2 - 56*d^2*e^3*x^3 + 70*d*e^4*x^4 - 105*e^
5*x^5)))/((c*d^2 - a*e^2)^5*x^6*(a*e + c*d*x)*(d + e*x)) - (15*(5*c*d^2 + 7
*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])
])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7680*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32290 vs. 2(352) = 704.

Time = 1.97 (sec) , antiderivative size = 32291, normalized size of antiderivative = 83.66

method	result	size
default	Expression too large to display	32291

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 41.44 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)} dx = \left[-\frac{15\,(5\,c^6d^{12} - 18\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 + 20\,a^3c^3d^6e^6 - 45\,a^4c^2d^4}{} \right.$$

$$-\frac{15\,(5\,c^6d^{12} - 18\,ac^5d^{10}e^2 + 15\,a^2c^4d^8e^4 + 20\,a^3c^3d^6e^6 - 45\,a^4c^2d^4e^8 + 30\,a^5cd^2e^{10} - 7\,a^6e^{12})\sqrt{-ade}x^6 \arctan}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3
*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(a*
d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 + (75*a
*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546*a^4*c^2*d^5*e
^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*d^10*e^2 - 80*
a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 35*a^6*d^2*e^10
)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c*d^5*e^7 - 7*a
^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6 + 3*a^6*d^4*e
^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6), -1/15360*(15*(5*c^6*d^12 - 18*a*c
^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8
+ 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(
a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(1280*a^6*d
^6*e^6 + (75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546
*a^4*c^2*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*
d^10*e^2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 3
5*a^6*d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c
*d^5*e^7 - 7*a^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6
 + 3*a^6*d^4*e^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)x^7}\,dx$$

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")

[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. 2(352) = 704.

Time = 0.51 (sec) , antiderivative size = 3388, normalized size of antiderivative = 8.78

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \text{Too large to display}$$

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")

[Out] -1/512*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) + 1/7680*(75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 270*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 + 225*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 + 15660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*e^11 + 14685*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*c^2*d^9*e^13 + 450*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^10*c*d^7*e^15 - 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*d^5*e^17 + 3072*sqrt(c*d*e)*a^9*c^2*d^10*e^12 - 425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^6*d^16*e^4 + 1530*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^5*d^14*e^6 + 75525*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^4*d^12*e^8 + 203100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 + 142065*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12 + 28170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^9*c*d^6*e^14 + 595*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^10*d^4*e^16 + 46080*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c^3*d^11*e^9 + 73728*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*

e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^8*c^2*d^9*e^11 + 30720*sqrt(c*d*e)*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^9*c*d^7*e
^13 + 990*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a
^3*c^6*d^15*e^3 + 42516*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^5*a^4*c^5*d^13*e^5 + 279450*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^5*a^5*c^4*d^11*e^7 + 526200*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^6*c^3*d^9*e^9 + 313650*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*c^2*d^7*e^11 +
 52020*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^8*
c*d^5*e^13 + 1686*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^5*a^9*d^3*e^15 + 76800*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))^4*a^5*c^4*d^12*e^6 + 307200*sqrt(c*d*e)*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^6*c^3*d^10*e^8 +
368640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))^4*a^7*c^2*d^8*e^10 + 122880*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^8*c*d^6*e^12 + 15360*sqrt(c*d*e)*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^9*d^4*e^14 + 9
90*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^2*c^6*
d^14*e^2 + 39444*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^7*a^3*c^5*d^12*e^4 + 233370*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^7*a^4*c^4*d^10*e^6 + 372600*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^5*c^3*d^8*e^8 + 160050*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^6*c^2*d^6*e^10 + 5940*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^7*c*d^4*e^
12 - 1386*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a
^8*d^2*e^14 + 15360*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^6*a^3*c^5*d^13*e^3 + 143360*sqrt(c*d*e)*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^4*c^4*d^11*e^5 + 460800*s
qrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*
a^5*c^3*d^9*e^7 + 430080*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^6*a^6*c^2*d^7*e^9 + 128000*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^7*c*d^5*e^11 - 425*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a*c^6*d^13*e
 + 1530*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^2*
c^5*d^11*e^3 + 44805*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^9*a^3*c^4*d^9*e^5 + 64860*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))^9*a^4*c^3*d^7*e^7 + 3825*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^5*c^2*d^5*e^9 - 2550*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^6*c*d^3*e^11 + 595*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^7*d*e^13 + 61440
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
8*a^3*c^4*d^10*e^4 + 184320*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))^8*a^4*c^3*d^8*e^6 + 138240*sqrt(c*d*e)*(sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^5*c^2*d^6*e^8 + 75
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*c^6*d^12

```
- 270*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a*c^
5*d^10*e^2 + 225*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^11*a^2*c^4*d^8*e^4 + 300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^11*a^3*c^3*d^6*e^6 - 675*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^11*a^4*c^2*d^4*e^8 + 450*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^5*c*d^2*e^10 - 105*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^6*e^12 + 15360*sqrt
(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^10*a^
3*c^3*d^7*e^5)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^2)^6*a^3*d^4*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^7\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.468 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 500

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx = \frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{a}}{1024a^4d^5e^4x^2}$$

$$-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}$$

$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}$$

$$+\frac{\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}$$

$$-\frac{\left(cd^2-ae^2\right)^5\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}}$$

```
[Out] -1/384*(-a*e^2+c*d^2)*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+
c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^4-1/7*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^7-1/84*(5*c/a/e-9*e/d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6+1/840*(-63*a^2*e^4+20*a*c*d^2*e^2+35*c^2*d
^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/e^2/x^5-1/2048*(-a*e^2+
c*d^2)^5*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c
*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a
^(9/2)/d^(11/2)/e^(9/2)+1/1024*(-a*e^2+c*d^2)^3*(9*a^2*e^4+10*a*c*d^2*e^2+5
*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/a^4/d^5/e^4/x^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{(-63a^2e^4+20acd^2e^2+35c^2d^4)\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{840a^2d^3e^2x^5} - \frac{(9a^2e^4+10acd^2e^2+5c^2d^4)\,(cd^2-ae^2)^5\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}} + \frac{(9a^2e^4+10acd^2e^2+5c^2d^4)\,(cd^2-ae^2)^3\,(x(ae^2+cd^2)+2ade)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{1024a^4d^5e^4x^2} - \frac{(9a^2e^4+10acd^2e^2+5c^2d^4)\,(cd^2-ae^2)\,(x(ae^2+cd^2)+2ade)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{384a^3d^4e^3x^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7dx^7} - \frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{84x^6}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e
^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*
a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7)
 - (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c
^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
2048*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^8}\,dx$$

$$= -\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(-\frac{1}{2}ae(5cd^2-9ae^2)+2acde^2x\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^7}\,dx}{7ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}+\frac{\int\frac{\left(-\frac{1}{4}ae\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)-\frac{1}{2}acde^2\left(5cd^2-9ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6}dx}{42a^2d^2e^2}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}+\frac{\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}+\frac{\left(\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}dx}{48a^2d^3e^2}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}+\frac{\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}-\frac{\left(\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}dx}{256a^3d^4e^3}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}+\frac{\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}+\frac{\left(\left(cd^2-ae^2\right)^5\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\right)\int\frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{2048a^4d^5e^4}$$

$$= \frac{(cd^2 - ae^2)^3 (5c^2d^4 + 10acd^2e^2 + 9a^2e^4) (2ade + (cd^2 + ae^2) x) \sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{1024a^4d^5e^4x^2} - \frac{(cd^2 - ae^2) (5c^2d^4 + 10acd^2e^2 + 9a^2e^4) (2ade + (cd^2 + ae^2) x) (ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{384a^3d^4e^3x^4} - \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{7dx^7} - \frac{\left(\frac{5c}{ae} - \frac{9e}{d^2}\right) (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{84x^6} + \frac{(35c^2d^4 + 20acd^2e^2 - 63a^2e^4) (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{840a^2d^3e^2x^5} - \frac{\left((cd^2 - ae^2)^5 (5c^2d^4 + 10acd^2e^2 + 9a^2e^4)\right) \text{Subst}\left(\int \frac{1}{4ade - x^2} \, dx, x, \frac{2ade - (-cd^2 - ae^2) x}{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}\right)}{1024a^4d^5e^4}$$

$$= \frac{(cd^2 - ae^2)^3 (5c^2d^4 + 10acd^2e^2 + 9a^2e^4) (2ade + (cd^2 + ae^2) x) \sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{1024a^4d^5e^4x^2} - \frac{(cd^2 - ae^2) (5c^2d^4 + 10acd^2e^2 + 9a^2e^4) (2ade + (cd^2 + ae^2) x) (ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{384a^3d^4e^3x^4} - \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{7dx^7} - \frac{\left(\frac{5c}{ae} - \frac{9e}{d^2}\right) (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{84x^6} + \frac{(35c^2d^4 + 20acd^2e^2 - 63a^2e^4) (ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{840a^2d^3e^2x^5} - \frac{(cd^2 - ae^2)^5 (5c^2d^4 + 10acd^2e^2 + 9a^2e^4) \tanh^{-1}\left(\frac{2ade + (cd^2 + ae^2) x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.99

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^8(d + ex)} \, dx = \frac{(-cd^2 + ae^2)^5 ((ae + cdx)(d + ex))^{3/2} \left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-525c^6d^{12}x^6 + 350ac^5d^1}{} \right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sqr
t[e]*(-525*c^6*d^12*x^6 + 350*a*c^5*d^10*e*x^5*(d + 4*e*x) - 35*a^2*c^4*d^8
*e^2*x^4*(8*d^2 + 26*d*e*x + 15*e^2*x^2) + 60*a^3*c^3*d^6*e^3*x^3*(4*d^3 +
12*d^2*e*x + 5*d*e^2*x^2 - 10*e^3*x^3) + a^4*c^2*d^4*e^4*x^2*(23680*d^4 + 3
3520*d^3*e*x + 1824*d^2*e^2*x^2 - 2332*d*e^3*x^3 + 3689*e^4*x^4) + 2*a^5*c*
d^2*e^5*x*(18560*d^5 + 24320*d^4*e*x + 744*d^3*e^2*x^2 - 872*d^2*e^3*x^3 +
1099*d*e^4*x^4 - 1680*e^5*x^5) + 3*a^6*e^6*(5120*d^6 + 6400*d^5*e*x + 128*d
^4*e^2*x^2 - 144*d^3*e^3*x^3 + 168*d^2*e^4*x^4 - 210*d*e^5*x^5 + 315*e^6*x^
6)))/((c*d^2 - a*e^2)^5*x^7*(a*e + c*d*x)*(d + e*x)) + (105*(5*c^2*d^4 + 10
*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x]])/(Sqrt[a]*Sqrt
```

```
[e]*Sqrt[d + e*x]])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*a^(9/2
)*d^(11/2)*e^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45105 vs. 2(462) = 924.

Time = 3.19 (sec) , antiderivative size = 45106, normalized size of antiderivative = 90.21

method	result	size
default	Expression too large to display	45106

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 78.30 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.60

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{x^8(d + ex)}\, dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^
3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^1
2 - 9*a^7*e^14)*sqrt(a*d*e)*x^7*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1
5360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^3*c^4*
d^9*e^5 + 600*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^3*e^11
- 945*a^7*d*e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*e^4 + 15
0*a^4*c^3*d^8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 315*a^7*d^
2*e^12)*x^5 - 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a^5*c^2*d^7
*e^7 + 218*a^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3*d^10*e^4 +
 2095*a^5*c^2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3 + 128*(185*
a^5*c^2*d^9*e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 1280*(29*a^6*c*d
^8*e^6 + 15*a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a
^5*d^6*e^5*x^7), 1/215040*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*
d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 3
5*a^6*c*d^2*e^12 - 9*a^7*e^14)*sqrt(-a*d*e)*x^7*arctan(1/2*sqrt(c*d*e*x^2 +
```

```
 a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c
*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(15360*a^7*d^7
*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^3*c^4*d^9*e^5 + 60
0*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^3*e^11 - 945*a^7*d*
e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*e^4 + 150*a^4*c^3*d^
8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 315*a^7*d^2*e^12)*x^5
- 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a^5*c^2*d^7*e^7 + 218*a
^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3*d^10*e^4 + 2095*a^5*c^
2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3 + 128*(185*a^5*c^2*d^9*
e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 1280*(29*a^6*c*d^8*e^6 + 15*
a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*d^6*e^5*x
^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)x^8}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4452 vs. $2(462) = 924$.

Time = 0.78 (sec) , antiderivative size = 4452, normalized size of antiderivative = 8.90

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"giac")
```

[Out] 1/1024*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a^7*e^14)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^4*d^5*e^4) - 1/107520*(525*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^7*d^20*e^6 - 1575*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^6*d^18*e^8 + 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^5*d^16*e^10 + 215565*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*c^4*d^14*e^12 + 431655*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^10*c^3*d^12*e^14 + 210315*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^2*d^10*e^16 + 3675*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^12*c*d^8*e^18 - 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^13*d^6*e^20 + 30720*sqrt(c*d*e)*a^10*c^3*d^13*e^13 + 43008*sqrt(c*d*e)*a^11*c^2*d^11*e^15 - 3500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^7*d^19*e^5 + 10500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^6*d^17*e^7 + 1068900*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^5*d^15*e^9 + 4655700*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^4*d^13*e^11 + 6225660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^9*c^3*d^11*e^13 + 2827020*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^10*c^2*d^9*e^15 + 405580*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^11*c*d^7*e^17 + 6300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^12*d^5*e^19 + 645120*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^8*c^4*d^14*e^10 + 1935360*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^9*c^3*d^12*e^12 + 1634304*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^10*c^2*d^10*e^14 + 430080*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^11*c*d^8*e^16 + 9905*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^4*c^7*d^18*e^4 + 615405*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^5*c^6*d^16*e^6 + 5823909*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^6*c^5*d^14*e^8 + 17499825*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*c^4*d^12*e^10 + 19168275*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^8*c^3*d^10*e^12 + 7652295*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^9*c^2*d^8*e^14 + 929495*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^10*c*d^6*e^16 + 25179*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^11*d^4*e^18 + 1075200*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^6*c^5*d^15*e^7 + 6666240*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^7*c^4*d^13*e^9 + 13332480*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^8*c^3*d^11*e^11 + 9719808*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^9*c^2*d^9*e^13 + 2365440*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^

2*x + a*e^2*x + a*d*e))^4*a^10*c*d^7*e^15 + 215040*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^11*d^5*e^17 + 15360*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^3*c^7*d^17
e^3 + 752640(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^7*a^4*c^6*d^15*e^5 + 6730752*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^7*a^5*c^5*d^13*e^7 + 18170880*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^6*c^4*d^11*e^9 + 16665600*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^7*c^3*d^9*e^11 + 462
3360*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^8*c^
2*d^7*e^13 + 107520*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^7*a^9*c*d^5*e^15 - 27648*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^7*a^10*d^3*e^17 + 215040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^4*c^6*d^16*e^4 + 2938880*sqr
t(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^
5*c^5*d^14*e^6 + 14192640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^6*a^6*c^4*d^12*e^8 + 23654400*sqrt(c*d*e)*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^7*c^3*d^10*e^10 +
 14120960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^6*a^8*c^2*d^8*e^12 + 2795520*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^9*c*d^6*e^14 - 9905*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^2*c^7*d^16*e^2 + 29715*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^3*c^6*d^1
4*e^4 + 1745499*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))^9*a^4*c^5*d^12*e^6 + 5294415*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^9*a^5*c^4*d^10*e^8 + 3625965*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^6*c^3*d^8*e^10 + 89145*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^7*c^2*d^6*e^12 - 69335
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^8*c*d^4*
e^14 + 17829*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
9*a^9*d^2*e^16 + 1576960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^8*a^4*c^5*d^13*e^5 + 8171520*sqrt(c*d*e)*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^5*c^4*d^11*e^7 + 12
257280*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))^8*a^6*c^3*d^9*e^9 + 5232640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^7*c^2*d^7*e^11 + 3500*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a*c^7*d^15*e - 10500*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^2*c^6*d^13*e^
3 + 6300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a
^3*c^5*d^11*e^5 + 290220*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^11*a^4*c^4*d^9*e^7 + 10500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^11*a^5*c^3*d^7*e^9 - 31500*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^6*c^2*d^5*e^11 + 24500*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^7*c*d^3*e^13 -
6300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^8*d
*e^15 + 1075200*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e

```
^2*x + a*d*e))^10*a^4*c^4*d^10*e^6 + 1505280*sqrt(c*d*e)*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^10*a^5*c^3*d^8*e^8 - 525*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*c^7*d^14 + 1575*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a*c^6*d^12
*e^2 - 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13
*a^2*c^5*d^10*e^4 - 525*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^13*a^3*c^4*d^8*e^6 - 1575*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))^13*a^4*c^3*d^6*e^8 + 4725*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^5*c^2*d^4*e^10 - 3675*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^6*c*d^2*e^12 + 945*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^7*e^14)/
((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^
7*a^4*d^5*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^8\,\left(d + e\,x\right)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.469 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 628

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx =$$

$$-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{16384a^5d^6e^5x^2}$$

$$+\frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{2048a^4d^5e^4x^4}$$

$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7}$$

$$+\frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}$$

$$-\frac{\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5}$$

$$+\frac{3\left(cd^2-ae^2\right)^5\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*
d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^4/
d^5/e^4/x^4-1/8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^8-1/112*(5*c/a/
e-11*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7+1/448*(-33*a^2*e^4+
10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/
e^2/x^6-1/4480*(-231*a^3*e^6+15*a^2*c*d^2*e^4+95*a*c^2*d^4*e^2+105*c^3*d^6)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^3/d^4/e^3/x^5+3/32768*(-a*e^2+c*
d^2)^5*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*arctanh(1/
2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/a^(11/2)/d^(13/2)/e^(11/2)-3/16384*(-a*e^2+c*d^2)^3*(33*a
```

```
^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*(2*a*d*e+(a*e^2+c*d^2)
*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^5/d^6/e^5/x^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{(-33a^2e^4+10acd^2e^2+15c^2d^4)\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{448a^2d^3e^2x^6}$$
$$-\frac{(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3d^6)\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4480a^3d^4e^3x^5}$$
$$+\frac{3(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6)\,(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}}$$
$$-\frac{3(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6)\,(cd^2-ae^2)^3\,(x(ae^2+cd^2)+2ade)\,\sqrt{x\,(ae^2+cd^2)+ade+cd}}{16384a^5d^6e^5x^2}$$
$$+\frac{(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6)\,(cd^2-ae^2)\,(x(ae^2+cd^2)+2ade)\,(x(ae^2+cd^2)+ade+cdex^2)}{2048a^4d^5e^4x^4}$$
$$-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{112x^7}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-11ae^2\right)+3acde^2x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^8}\,dx}{8ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7} + \frac{\int \frac{\left(-\frac{3}{4}ae\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)-acde^2\left(5cd^2-11ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7}\,dx}{56a^2d^2e^2}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7} + \frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6} - \frac{\int \frac{\left(-\frac{3}{8}ae\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)-\frac{3}{4}acde^2\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6}\,dx}{336a^3d^3e^3}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7} + \frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6} - \frac{\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5} - \frac{\left(\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}\,dx}{256a^3d^4e^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{2048a^4d^5e^4x^4} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7} + \frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6} - \frac{\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5} + \frac{\left(3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx}{4096a^4d^5e^4}$$

$$= -\frac{3(cd^2-ae^2)^3(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2-}{16384a^5d^6e^5x^2} + \frac{(cd^2-ae^2)(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae}{2048a^4d^5e^4x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{112x^7} + \frac{(15c^2d^4+10acd^2e^2-33a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{448a^2d^3e^2x^6} - \frac{(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4480a^3d^4e^3x^5} - \frac{\left(3(cd^2-ae^2)^5(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)\right)\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{32768a^5d^6e^5}$$

$$= -\frac{3(cd^2-ae^2)^3(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2-}{16384a^5d^6e^5x^2} + \frac{(cd^2-ae^2)(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae}{2048a^4d^5e^4x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{112x^7} + \frac{(15c^2d^4+10acd^2e^2-33a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{448a^2d^3e^2x^6} - \frac{(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4480a^3d^4e^3x^5} + \frac{\left(3(cd^2-ae^2)^5(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)\right)\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2}{\sqrt{ade+(cd^2+ae}}\right.}{16384a^5d^6e^5}$$

$$= -\frac{3(cd^2-ae^2)^3(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2-}{16384a^5d^6e^5x^2} + \frac{(cd^2-ae^2)(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae}{2048a^4d^5e^4x^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{112x^7} + \frac{(15c^2d^4+10acd^2e^2-33a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{448a^2d^3e^2x^6} - \frac{(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4480a^3d^4e^3x^5} + \frac{3(cd^2-ae^2)^5(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.91

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)} dx = \frac{\sqrt{(ae + cdx)(d + ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(1575c^7d^{14}x^7 - 525ac^6d^{12}ex^6(2d+7ex) + 35a^2c^5d}{}\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(1575*c^7*d^14*x
^7 - 525*a*c^6*d^12*e*x^6*(2*d + 7*e*x) + 35*a^2*c^5*d^10*e^2*x^5*(24*d^2 +
 68*d*e*x + 29*e^2*x^2) - 5*a^3*c^4*d^8*e^3*x^4*(144*d^3 + 376*d^2*e*x + 11
0*d*e^2*x^2 - 185*e^3*x^3) + 5*a^4*c^3*d^6*e^4*x^3*(128*d^4 + 320*d^3*e*x +
 80*d^2*e^2*x^2 - 120*d*e^3*x^3 + 265*e^4*x^4) + a^5*c^2*d^4*e^5*x^2*(10368
0*d^5 + 137600*d^4*e*x + 4640*d^3*e^2*x^2 - 5488*d^2*e^3*x^3 + 7034*d*e^4*x
^4 - 11193*e^5*x^5) + a^6*c*d^2*e^6*x*(168960*d^6 + 212480*d^5*e*x + 4480*d
^4*e^2*x^2 - 5056*d^3*e^3*x^3 + 5928*d^2*e^4*x^4 - 7476*d*e^5*x^5 + 11445*e
^6*x^6) + a^7*e^7*(71680*d^7 + 87040*d^6*e*x + 1280*d^5*e^2*x^2 - 1408*d^4*
e^3*x^3 + 1584*d^3*e^4*x^4 - 1848*d^2*e^5*x^5 + 2310*d*e^6*x^6 - 3465*e^7*x
^7)))/x^8) + (105*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2
*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[
e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(573440*a^(11/2)*d^
(13/2)*e^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70735 vs. 2(586) = 1172.

Time = 3.29 (sec) , antiderivative size = 70736, normalized size of antiderivative = 112.64

method	result	size
default	Expression too large to display	70736

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 176.45 (sec) , antiderivative size = 1550, normalized size of antiderivative = 2.47

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^9(d + ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"fricas")
[Out] [1/2293760*(105*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*
a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d
^4*e^12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*sqrt(a*d*e)*x^8*log((8*a^2*d^2*
e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e
 + a^2*d*e^3)*x)/x^2) - 4*(71680*a^8*d^8*e^8 + (1575*a*c^7*d^15*e - 3675*a^
2*c^6*d^13*e^3 + 1015*a^3*c^5*d^11*e^5 + 925*a^4*c^4*d^9*e^7 + 1325*a^5*c^3
*d^7*e^9 - 11193*a^6*c^2*d^5*e^11 + 11445*a^7*c*d^3*e^13 - 3465*a^8*d*e^15)
*x^7 - 2*(525*a^2*c^6*d^14*e^2 - 1190*a^3*c^5*d^12*e^4 + 275*a^4*c^4*d^10*e
^6 + 300*a^5*c^3*d^8*e^8 - 3517*a^6*c^2*d^6*e^10 + 3738*a^7*c*d^4*e^12 - 11
55*a^8*d^2*e^14)*x^6 + 8*(105*a^3*c^5*d^13*e^3 - 235*a^4*c^4*d^11*e^5 + 50*
a^5*c^3*d^9*e^7 - 686*a^6*c^2*d^7*e^9 + 741*a^7*c*d^5*e^11 - 231*a^8*d^3*e^
13)*x^5 - 16*(45*a^4*c^4*d^12*e^4 - 100*a^5*c^3*d^10*e^6 - 290*a^6*c^2*d^8*
e^8 + 316*a^7*c*d^6*e^10 - 99*a^8*d^4*e^12)*x^4 + 128*(5*a^5*c^3*d^11*e^5 +
 1075*a^6*c^2*d^9*e^7 + 35*a^7*c*d^7*e^9 - 11*a^8*d^5*e^11)*x^3 + 1280*(81*
a^6*c^2*d^10*e^6 + 166*a^7*c*d^8*e^8 + a^8*d^6*e^10)*x^2 + 5120*(33*a^7*c*d
^9*e^7 + 17*a^8*d^7*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a
^6*d^7*e^6*x^8), -1/1146880*(105*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*
c^6*d^12*e^4 + 8*a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^1
0 - 140*a^6*c^2*d^4*e^12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*sqrt(-a*d*e)*x
^8*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2
*d*e^3)*x)) + 2*(71680*a^8*d^8*e^8 + (1575*a*c^7*d^15*e - 3675*a^2*c^6*d^13
*e^3 + 1015*a^3*c^5*d^11*e^5 + 925*a^4*c^4*d^9*e^7 + 1325*a^5*c^3*d^7*e^9 -
 11193*a^6*c^2*d^5*e^11 + 11445*a^7*c*d^3*e^13 - 3465*a^8*d*e^15)*x^7 - 2*(
525*a^2*c^6*d^14*e^2 - 1190*a^3*c^5*d^12*e^4 + 275*a^4*c^4*d^10*e^6 + 300*a
^5*c^3*d^8*e^8 - 3517*a^6*c^2*d^6*e^10 + 3738*a^7*c*d^4*e^12 - 1155*a^8*d^2
*e^14)*x^6 + 8*(105*a^3*c^5*d^13*e^3 - 235*a^4*c^4*d^11*e^5 + 50*a^5*c^3*d^
9*e^7 - 686*a^6*c^2*d^7*e^9 + 741*a^7*c*d^5*e^11 - 231*a^8*d^3*e^13)*x^5 -
16*(45*a^4*c^4*d^12*e^4 - 100*a^5*c^3*d^10*e^6 - 290*a^6*c^2*d^8*e^8 + 316*
a^7*c*d^6*e^10 - 99*a^8*d^4*e^12)*x^4 + 128*(5*a^5*c^3*d^11*e^5 + 1075*a^6*
c^2*d^9*e^7 + 35*a^7*c*d^7*e^9 - 11*a^8*d^5*e^11)*x^3 + 1280*(81*a^6*c^2*d^
10*e^6 + 166*a^7*c*d^8*e^8 + a^8*d^6*e^10)*x^2 + 5120*(33*a^7*c*d^9*e^7 + 1
7*a^8*d^7*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^6*d^7*e^6
*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^9}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5681 vs. 2(586) = 1172.

Time = 1.30 (sec) , antiderivative size = 5681, normalized size of antiderivative = 9.05

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"giac")
[Out] -3/16384*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^5
*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^12
 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^5*d^6*e^5) +
1/573440*(1575*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)*a^7*c^8*d^23*e^7 - 4200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))*a^8*c^7*d^21*e^9 + 2100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^9*c^6*d^19*e^11 + 1147720*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^10*c^5*d^17*e^13 + 3441690*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^4*d^15*e^15 +
 3444840*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^12
*c^3*d^13*e^17 + 1132180*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
```

x + a*d*e))*a^13*c^2*d^11*e^19 + 12600*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^14*c*d^9*e^21 - 3465*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^15*d^7*e^23 + 163840*sqrt(c*d*e)*a^11*c^4*d^16*e^14 + 327680*sqrt(c*d*e)*a^12*c^3*d^14*e^16 + 229376*sqrt(c*d*e)*a^13*c^2*d^12*e^18 - 12075*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^8*d^22*e^6 + 32200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^7*d^20*e^8 + 5718300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^6*d^18*e^10 + 34399960*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^9*c^5*d^16*e^12 + 71098510*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^10*c^4*d^14*e^14 + 59605560*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^11*c^3*d^12*e^16 + 19609660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^12*c^2*d^10*e^18 + 2197160*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^13*c*d^8*e^20 + 26565*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^14*d^6*e^22 + 3440640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^9*c^5*d^17*e^11 + 14745600*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^10*c^4*d^15*e^13 + 21463040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^11*c^3*d^13*e^15 + 11927552*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^12*c^2*d^11*e^17 + 2293760*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^13*c*d^9*e^19 + 40215*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^5*c^8*d^21*e^5 + 3333400*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^6*c^7*d^19*e^7 + 41341300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^7*c^6*d^17*e^9 + 175494088*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^8*c^5*d^15*e^11 + 301656250*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^9*c^4*d^13*e^13 + 219161320*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^10*c^3*d^11*e^15 + 63849940*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^11*c^2*d^9*e^17 + 6056120*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^12*c*d^7*e^19 + 140903*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^13*d^5*e^21 + 5734400*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^7*c^6*d^18*e^8 + 48168960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^8*c^5*d^16*e^10 + 141066240*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^9*c^4*d^14*e^12 + 169738240*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^10*c^3*d^12*e^14 + 85557248*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^11*c^2*d^10*e^16 + 16056320*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^12*c*d^8*e^18 + 1146880*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^13*d^6*e^20 + 88045*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^4*c^8*d^20*e^4 + 5117320*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a* d*e))^7*a^5*c^7*d^18*e^6 + 62321980*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^6*c^6*d^16*e^8 + 248341016*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^7*c^5*d^14*e^10 + 382679710*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^8*c^4*d^12*e^12 + 230156920*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^9*c^3*d^10*e^14 + 45435740*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^10*c^2*d^8*e^16 + 704360*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^11*c*d^6*e^18 - 193699*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^7*a^12*d^4*e^20 + 1146880*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^5*c^7*d^19*e^5 + 20643840*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^6*c^6*d^17*e^7 + 136478720*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^7*c^5*d^15*e^9 + 344064000*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^8*c^4*d^13*e^11 + 358973440*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^9*c^3*d^11*e^13 + 150011904*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^10*c^2*d^9*e^15 + 21790720*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^6*a^11*c*d^7*e^17 - 75795*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^3*c^8*d^19*e^3 + 202120*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^4*c^7*d^17*e^5 + 21034300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^5*c^6*d^15*e^7 + 104522264*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^6*c^5*d^13*e^9 + 149863070*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^7*c^4*d^11*e^11 + 61565560*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^8*c^3*d^9*e^13 + 707420*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^9*c^2*d^7*e^15 - 606360*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^10*c*d^5*e^17 + 166749*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^9*a^11*d^3*e^19 + 13762560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^5*c^6*d^16*e^6 + 105512960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^6*c^5*d^14*e^8 + 260341760*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^7*c^4*d^12*e^10 + 231669760*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^8*c^3*d^10*e^12 + 65372160*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^8*a^9*c^2*d^8*e^14 + 40215*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^2*c^8*d^18*e^2 - 107240*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^3*c^7*d^16*e^4 + 53620*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^4*c^6*d^14*e^6 + 10113992*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^5*c^5*d^12*e^8 + 13789370*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^6*c^4*d^10*e^10 + 107240*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^7*c^3*d^8*e^12 - 375340*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^11*a^8*c^2*d^

```
6*e^14 + 321720*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))^11*a^9*c*d^4*e^16 - 88473*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^11*a^10*d^2*e^18 + 19496960*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^10*a^5*c^5*d^13*e^7 + 57344000*s
qrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^10
*a^6*c^4*d^11*e^9 + 37847040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^10*a^7*c^3*d^9*e^11 - 12075*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a*c^8*d^17*e + 32200*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^2*c^7*d^15*e^3 -
 16100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^3
*c^6*d^13*e^5 - 6440*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^13*a^4*c^5*d^11*e^7 - 8050*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^13*a^5*c^4*d^9*e^9 - 32200*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^6*c^3*d^7*e^11 + 112700*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^7*c^2*d^5*e^13 - 9
6600*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^13*a^8*c
*d^3*e^15 + 26565*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^13*a^9*d*e^17 + 2293760*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^12*a^5*c^4*d^10*e^8 + 1575*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*c^8*d^16 - 4200*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*a*c^7*d^14*e^2 + 2100*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*a^2*c^6*d^12
*e^4 + 840*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15
*a^3*c^5*d^10*e^6 + 1050*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^15*a^4*c^4*d^8*e^8 + 4200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^15*a^5*c^3*d^6*e^10 - 14700*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*a^6*c^2*d^4*e^12 + 12600*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*a^7*c*d^2*e^14 -
3465*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^15*a^8*e
^16)/((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)^2)^8*a^5*d^6*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e)^{5/2}}{x^9\;(d + e\,x)}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.470 $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\begin{aligned}\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{3(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3} \\ &\quad -\frac{2d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^3\,(cd^2-ae^2)\,(d+ex)}+\frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cde^3} \\ &\quad +\frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}\end{aligned}$$

```
[Out] 3/8*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c
^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(
5/2)/e^(7/2)-3/4*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
2/d^2/e^3-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/
(e*x+d)+1/2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{863, 832, 793, 635, 212\}$

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{3(a^2e^4+2acd^2e^2+5c^2d^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}$$
$$- \frac{((5cd^2-3ae^2)\,(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^3\,(cd^2-ae^2)}$$
$$- \frac{2dx^2(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{e\,(cd^2-ae^2)^2\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*
c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
```

```
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^3(ae+cdx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{2\int \frac{x\left(2acd^2e(cd^2-ae^2)+\frac{1}{2}cd(cd^2-ae^2)(5cd^2-ae^2)x\right)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cde\,(cd^2-ae^2)^2}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2-3ae^2)(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3\,(cd^2-ae^2)} + \frac{(3(5c^2d^4+2acd^2e^2+a^2e^4))\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8c^2d^2e^3}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2-3ae^2)(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3\,(cd^2-ae^2)} + \frac{(3(5c^2d^4+2acd^2e^2+a^2e^4))\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^2d^2e^3}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3\,(cd^2-ae^2)} + \frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(3a^3e^5(d+ex)+a^2cde^3(4d^2+5dex+e^2x^2)+c^3d^4x(-15d^2-5dex+2e^2x^2)-ac^2d^2e(15d^3+\ldots}{4c^{5/2}d^{5/2}e^{7/2}\,(cd^2-ae^2\ldots}$$

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e*
x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(15*
d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2
- a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*c^(5/2)*d^(5/2)*e^
(7/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(243) = 486.

Time = 0.69 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.90

method	result
default	$\frac{\frac{x\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{2cde}-\frac{3\left(e^2a+c\,d^2\right)\left(\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{cde}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{2cde\sqrt{cde}}\ldots\right.}{4cde}\ldots}{e}$

```
[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/2*x/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c*d^2)/
c/d/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/
d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
```

```
*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-1/2*a/c*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c
*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+d^2/e^3
*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(c*d*e)^(1/2)-d/e^2*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+2*d^3/e^4/(a*e^
2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.80

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+\right.}{}\right.$$
$$-\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex}}{}\right.}{}$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*
d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c^2
*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2
*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a
^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 -
a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^7 - 3*a*
c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 -
a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*
x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6*e - 4*a*c^2
*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d
^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)
*x)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{x^3}{\sqrt{(d+ex)(ae+cdx)}(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx\\&=\frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2x}{cde^2}-\frac{7cd^2e^5+3ae^7}{c^2d^2e^8}\right)\\&\quad-\frac{2d^3}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}\\&\quad-\frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\log\left(\left|cd^2+ae^2+2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\sqrt{cde}c^2d^2e^3}\end{aligned}$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/(c*d*e^2) - (7*c*d^2*e
^5 + 3*a*e^7)/(c^2*d^2*e^8)) - 2*d^3/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^3) - 3/8*(5*c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.471 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 195

$$\begin{aligned}\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^2\,(cd^2-ae^2)\,(d+ex)}\\ &\quad-\frac{(3cd^2+ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}\end{aligned}$$

```
[Out] -1/2*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1652, 806, 635, 212}

$$\begin{aligned}\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{(ae^2+3cd^2)\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}\\ &\quad+\frac{2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)\,(cd^2-ae^2)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{cde^2}\end{aligned}$$

```
[In] Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1652

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{cde^2} + \frac{\int \frac{-\frac{1}{2}de(cd^2+ae^2)-\frac{1}{2}e^2(3cd^2+ae^2)x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cde^3}$$

$$
\begin{aligned}
&= \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{e^2\,(cd^2 - ae^2)\,(d + ex)} \\
&\quad - \frac{1}{2}\left(\frac{a}{cd} + \frac{3d}{e^2}\right)\int \frac{1}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx \\
&= \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{e^2\,(cd^2 - ae^2)\,(d + ex)} \\
&\quad - \left(\frac{a}{cd} + \frac{3d}{e^2}\right)\text{Subst}\left(\int \frac{1}{4cde - x^2}\,dx, x, \frac{cd^2 + ae^2 + 2cdex}{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\right) \\
&= \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{e^2\,(cd^2 - ae^2)\,(d + ex)} \\
&\quad - \frac{(3cd^2 + ae^2)\tanh^{-1}\left(\frac{cd^2 + ae^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$
\int \frac{x^2}{(d + ex)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx
$$

$$
= \frac{\sqrt{c}\sqrt{d}\sqrt{e}(-a^2e^3(d + ex) + c^2d^3x(3d + ex) + acde(3d^2 - e^2x^2)) - (3c^2d^4 - 2acd^2e^2 - a^2e^4)\sqrt{ae + cdx}\sqrt{\cdots}}{c^{3/2}d^{3/2}e^{5/2}\,(cd^2 - ae^2)\,\sqrt{(ae + cdx)(d + ex)}}
$$

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a*
c*d*e*(3*d^2 - e^2*x^2)) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*Sqrt[a*e +
 c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*
Sqrt[d + e*x])])/(c^(3/2)*d^(3/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{cde}-\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}\right)}{2cde\sqrt{cde}}}{e}-\frac{d\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+(e^2a\cdots}\right.}{e^2\sqrt{cde}}$

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/
e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-2*d^2/e^3/(a
*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.01

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{(3\,c^2d^5-2\,acd^3e^2-a^2de^4+(3\,c^2d^4e-2\,acd^2e^3-a^2e^5)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4\right.}{}\right.$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e - a*
c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x),
 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 + (c^2*d^
3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^5
*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,d^2}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^2} + \frac{(3\,cd^2+ae^2)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\,\sqrt{cde}cde^2} + \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}}{cde^2}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
[Out] 2*d^2/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + s
qrt(c*d*e)*d)*e^2) + 1/2*(3*c*d^2 + a*e^2)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c
*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt
(c*d*e)*c*d*e^2) + sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/(c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$=\int \frac{x^2}{(d+e\,x)\ \sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\ x+a\,d\,e}}\,dx$$

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.472 $\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 139

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e\,(cd^2-ae^2)\,(d+ex)} + \frac{\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)-2*d*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {806, 635, 212}

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}} - \frac{2d\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)\,(cd^2-ae^2)}$$

```
[In] Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)}+\frac{\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{e}\\
&= -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)}+\frac{2\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{e}\\
&= -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)}+\frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(-\frac{d^{3/2}\sqrt{e}(ae+cdx)}{cd^2-ae^2}+\frac{\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}}\right)}{\sqrt{d}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((d^(3/2)*Sqrt[e]*(a*e + c*d*x))/(c*d^2 - a*e^2)) + (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])])/Sqrt[c]))/(Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}e^2a+\frac{1}{2}c\,d^2+cdex}{\sqrt{cde}}+\sqrt{ade+(e^2a+c\,d^2)x+cde\,x^2}\right)}{e\sqrt{cde}}+\frac{2d\sqrt{cde\left(x+\frac{d}{e}\right)^2+(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}}{e^2(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}$	131

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.19

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,a\right.}{2\,(c^2d^4e^2-acd^2e^4+(c}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e+(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2}}{2\,(c^2d^2e^2x^2+acd^2}\right.}{c^2d^4e^2-acd^2e^4+(c^2d^3e^3-acde^5)x}$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^
4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*
```

```
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)
*x)]
```

Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{1,[0,0,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]%%%
}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.473 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 52

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {664}

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)\,(cd^2-ae^2)}$$

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2(ae+cdx)}{(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{(e^2a-c\,d^2)(ex+d)}$	50
gosper	$-\frac{2(cdx+ae)}{(e^2a-c\,d^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	51
default	$-\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^2+(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}}{e(e^2a-c\,d^2)\left(x+\frac{d}{e}\right)}$	65

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBO
SE)
[Out] -2/(a*e^2-c*d^2)/(e*x+d)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{cd^3-ade^2+(cd^2e-ae^3)x}$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e -
 a*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,0,1]%%%},[2]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,0,
%%%{-1
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=-\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] -(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e*
x))
```

3.474 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 143

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

```
[Out] -arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-2*e*(c*d*x+a*e)/d/(-a*
e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {865, 836, 12, 738, 212}

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}} - \frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int -\frac{ae\left(cd^2-ae^2\right)^2}{2x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{ade\left(cd^2-ae^2\right)^2}$$

$$=-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{\int\frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{d}$$

$$=-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}-\frac{2\text{Subst}\left(\int\frac{1}{4ade-x^2}dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{d}$$

$$=-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}-\frac{\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int\frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx=\frac{2\left(-\frac{\sqrt{d}e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(\frac{2ade+\left(e^{2}a+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}{x}\right)}{d\sqrt{ade}}+\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{d\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$	136

```
[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.17

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{ade}\log\left(\frac{8a^2d^2e^2+(c^2d^4+6acd^2e^2+a^2\ldots}{\ldots}\right)}{2(acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x)}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-ade}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2\ldots}}{2(acd^2e^2x^2+a^2\ldots}\right)}{acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x}$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^2*e -
a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^
2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 -
a^2*d^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,1,5]%%%},[2,2]%%%}+%%%{%%%{-2,[1,3,3]%%%},[2,1]%%%}+
%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{1}{x\,(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.475 $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\ &\quad - \frac{(cd^2-3ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{ad^2e\,(cd^2-ae^2)\,x} \\ &\quad + \frac{(cd^2+3ae^2)\,\text{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}\end{aligned}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-2*e
*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(-3
*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/(-a*e^2+c*d^2
)/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{865, 836, 820, 738, 212\}$

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{(3ae^2+cd^2)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}$$
$$-\frac{(cd^2-3ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{ad^2ex\,(cd^2-ae^2)}$$
$$-\frac{2e(ae+cdx)}{dx\,(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
```

```
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(cd^2-3ae^2\right)\left(cd^2-ae^2\right)+acde^2\left(cd^2-ae^2\right)x}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{ade\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{ad^2e\left(cd^2-ae^2\right)x} - \frac{1}{2}\left(\frac{c}{ae}+\frac{3e}{d^2}\right)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{ad^2e\left(cd^2-ae^2\right)x} - \left(-\frac{c}{ae}-\frac{3e}{d^2}\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-\left(-cd^2-ae^2\right)x}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{ad^2e\left(cd^2-ae^2\right)x} + \frac{\left(cd^2+3ae^2\right)\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(-c^2d^3x(d+ex)+a^2e^3(d+3ex)-acde(d^2-3e^2x^2))+\left(c^2d^4+2acd^2e^2-3a^2e^4\right)x\sqrt{ae+cdx}\sqrt{}}{a^{3/2}d^{5/2}e^{3/2}\left(cd^2-ae^2\right)x\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a*
c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{adex}+\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{x}\right)}{2ade\sqrt{ade}}}{d}+\frac{e\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}}{}\right.}{d^2\sqrt{a}}$

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)/a
/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d
^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-2*e/d^2/(
a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.97 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{\sqrt{ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{}}{}\right.}{4\,((a^2cd^5e^3}\right.$$

$$-\frac{\sqrt{-ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ad}{2\,(acd^2e^2x^2+a^2d^2e^2+(a}\right.}{2\,((a^2cd^5e^3-a^3d^3e^5)x^2+(a}$$

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5 +
 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(
a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*
e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a
^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*
d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(a*c*
d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*e^2
- a^3*d^4*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,0,1]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,
0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.476 $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 329

$$
\begin{aligned}
\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx
&= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} \\
&\quad -\frac{\left(cd^2-5ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2} \\
&\quad +\frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4a^2d^3e^2\left(cd^2-ae^2\right)x} \\
&\quad -\frac{3(c^2d^4+2acd^2e^2+5a^2e^4)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}
\end{aligned}
$$

```
[Out] -3/8*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x
)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/
d^(7/2)/e^(5/2)-2*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-1/2*(-5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a/d^2/e/(-a*e^2+c*d^2)/x^2+1/4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)/x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {865, 836, 848, 820, 738, 212}

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{(3cd^2-5ae^2)\,(3ae^2+cd^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4a^2d^3e^2x\,(cd^2-ae^2)} - \frac{3(5a^2e^4+2acd^2e^2+c^2d^4)\,\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}} - \frac{(cd^2-5ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2\,(cd^2-ae^2)} - \frac{2e(ae+cdx)}{dx^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae + cdx}{x^3 \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae + cdx)}{d\left(cd^2 - ae^2\right)x^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(cd^2 - 5ae^2\right)\left(cd^2 - ae^2\right) + 2acde^2\left(cd^2 - ae^2\right)x}{x^3\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}\,dx}{ade\left(cd^2 - ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2} + \frac{\int \frac{-\frac{1}{4}ae\left(3cd^2-5ae^2\right)\left(cd^2-ae^2\right)\left(cd^2+3ae^2\right)-\frac{1}{2}acde^2\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)x}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{a^2d^2e^2\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2} + \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^2d^3e^2\left(cd^2-ae^2\right)x} + \frac{\left(3\left(c^2d^4+2acd^2e^2+5a^2e^4\right)\right)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8a^2d^3e^2}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2} + \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^2d^3e^2\left(cd^2-ae^2\right)x} - \frac{\left(3\left(c^2d^4+2acd^2e^2+5a^2e^4\right)\right)\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-\left(-cd^2-ae^2\right)x}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4a^2d^3e^2}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2ad^2e\left(cd^2-ae^2\right)x^2} + \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^2d^3e^2\left(cd^2-ae^2\right)x} - \frac{3\left(c^2d^4+2acd^2e^2+5a^2e^4\right)\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^3d^5x^2(d+ex)+a^3e^4(2d^2-5dex-15e^2x^2)+ac^2d^3ex(d^2+5dex+4e^2x^2)-a^2cde^2(2d^3-4\cdots}{4a^{5/2}d^{7/2}e^{5/2}\,(cd^2-ae\cdots}$$

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e*
x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2*(
2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(
7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.66

method	result
default	$\frac{-\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{2ade\,x^2}-\frac{3\left(e^2a+c\,d^2\right)\left(-\frac{\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}{adex}+\frac{\left(e^2a+c\,d^2\right)\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+\left(e^2a+c\,d^2\right)x\cdots}}{x}\right)}{2ade\sqrt{ade}}\cdots\right)}{4ade}\cdots}{d}$

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c*d^
2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d
^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*
e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-e
^2/d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c
*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*e^2/
d^3/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 2.25 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \left[\frac{3\left((c^3d^6e+ac^2d^4e^3+3a^2cd^2e^5-5a^3e^7)x^3+(c^3d^7+ac^2d^5e^2+3a^2cd^3e^4-5a^3de^6)x^2\right)\sqrt{ade}\log\left(\frac{8a^2d^2e^2\cdots}{}\right.}{}\right.$$

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 + (
c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)*l
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e)
+ 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 -
(3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2
*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2
), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(-a*d*e)
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c
*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2
*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c*d^6*e^4 - a^4
*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{1}{x^3\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%%}+
%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.477 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 515

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x^2(ade(cd^2-ae^2)\left(7c^2d^4-12acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(105c^4d^8-190ac^3d^6e^2+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-2cde(35c^3d^6-61ac^2d^4e^2+9a^2cd^2e^4-15a^3e}{12c^3d^3e^4\left(cd^2-ae^2\right)^3} + \frac{5\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}$$

```
[Out] -2/3*d*x^4*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/8*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-2/3*x^2*(a*d*e*(-a*e^2+
c*d^2)*(-3*a^2*e^4-12*a*c*d^2*e^2+7*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2
*c*d^2*e^4-11*a*c^2*d^4*e^2+7*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/12*(105*c^4*d^8-190*a*c^3*d^6*e^2+36*a^2*
c^2*d^4*e^4+30*a^3*c*d^2*e^6-45*a^4*e^8-2*c*d*e*(-15*a^3*e^6+9*a^2*c*d^2*e^
4-61*a*c^2*d^4*e^2+35*c^3*d^6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^3/d^3/e^4/(-a*e^2+c*d^2)^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 793, 635, 212}

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{5(3a^2e^4+6acd^2e^2+7c^2d^4)\operatorname{arctanh}\Big(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+a}}}{8c^{7/2}d^{7/2}e^{9/2}}$$
$$-\frac{2x^2(ade(cd^2-ae^2)(-3a^2e^4-12acd^2e^2+7c^2d^4)+x(cd^2-ae^2)(-3a^3e^6-a^2cd^2e^4-11ac^2d^4e^2+7c^3d^6)}{3cde^2(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$
$$-\frac{(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex(-15a^3e^6+9a^2cd^2e^4-61ac^2d^4e^2+35c^3d^6)-190ac^3d^6e^2+}{12c^3d^3e^4(cd^2-ae^2)^3}$$
$$-\frac{2dx^4(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e(cd^2-ae^2)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx \\
&= -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{2\int \frac{x^3\left(4acd^2e\left(cd^2-ae^2\right)+\frac{1}{2}cd\left(7cd^2-3ae^2\right)\left(cd^2-ae^2\right)x\right)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cde\left(cd^2-ae^2\right)^2} \\
&= -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad -\frac{2x^2(ade(cd^2-ae^2)\left(7c^2d^4-12acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3a\right.}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{4\int \frac{x\left(acd^2e\left(cd^2-ae^2\right)\left(7c^2d^4-12acd^2e^2-3a^2e^4\right)+\frac{1}{4}cd\left(cd^2-ae^2\right)\left(35c^3d^6-61ac^2d^4e^2+9a^2cd^2e^4-15a^3e^6\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3c^2d^2e^2\left(cd^2-ae^2\right)^4}
\end{aligned}$$

$$= -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x^2(ade(cd^2-ae^2)\,(7c^2d^4-12acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$-\frac{(105c^4d^8-190ac^3d^6e^2+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-2cde(35c^3d^6-61ac^2d^4e^2+9a^2cd}{12c^3d^3e^4\,(cd^2-ae^2)^3}$$
$$+\frac{(5(7c^2d^4+6acd^2e^2+3a^2e^4))\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8c^3d^3e^4}$$
$$= -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x^2(ade(cd^2-ae^2)\,(7c^2d^4-12acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$-\frac{(105c^4d^8-190ac^3d^6e^2+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-2cde(35c^3d^6-61ac^2d^4e^2+9a^2cd}{12c^3d^3e^4\,(cd^2-ae^2)^3}$$
$$+\frac{(5(7c^2d^4+6acd^2e^2+3a^2e^4))\,\text{Subst}\left(\int\frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^3d^3e^4}$$
$$= -\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x^2(ade(cd^2-ae^2)\,(7c^2d^4-12acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(7c^3d^6-11ac^2d^4e^2-a^2cd^2e^4-3}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$-\frac{(105c^4d^8-190ac^3d^6e^2+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-2cde(35c^3d^6-61ac^2d^4e^2+9a^2cd}{12c^3d^3e^4\,(cd^2-ae^2)^3}$$
$$+\frac{5(7c^2d^4+6acd^2e^2+3a^2e^4)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.75

$$\int\frac{x^5}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\,(-45a^5e^9(d+ex)^2+15a^4cde^7(2d-ex)(d+ex)^2+6a^3c^2d^2e}{}$$

```
[In] Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*a^5*e^9*(d + e*x)^2 + 15*a^4
*c*d*e^7*(2*d - e*x)*(d + e*x)^2 + 6*a^3*c^2*d^2*e^5*(d + e*x)^2*(6*d^2 + 2
*d*e*x + e^2*x^2) + c^5*d^8*x*(105*d^3 + 140*d^2*e*x + 21*d*e^2*x^2 - 6*e^3
*x^3) - 2*a^2*c^3*d^4*e^3*(95*d^4 + 111*d^3*e*x - 6*d^2*e^2*x^2 - 9*d*e^3*x
^3 + 9*e^4*x^4) + a*c^4*d^6*e*(105*d^4 - 50*d^3*e*x - 237*d^2*e^2*x^2 - 48*
```

```
d*e^3*x^3 + 18*e^4*x^4)))/(c*d^2 - a*e^2)^3) + 15*(7*c^2*d^4 + 6*a*c*d^2*e^
2 + 3*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(12*c^(7/2)*d^(7/2)*e^(9/2)*(
(a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1922 vs. 2(485) = 970.

Time = 0.87 (sec) , antiderivative size = 1923, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	1923

```
[In] int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/2*x^3/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^2
)/c/d/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2
)/c/d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)
/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/
e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d/e
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)))-3/2*a/c*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e
^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*
d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(
c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+2*d^4
/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+d^2/e^3*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2))-d/e^2*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*
e^2+c*d^2)/c/d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e
^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*
d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(
c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*
(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*
d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)))-d^3/e^4*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

```
-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-d^5/e^6*(-2/3/(a*e^2-c*d^2)/(x+d
/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3
*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(485) = 970$.

Time = 4.28 (sec) , antiderivative size = 2120, normalized size of antiderivative = 4.12

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4
*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^
5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*
a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5
 + 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x
^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6
 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*e^4 + 36*a
^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6*d^9*e^4 -
3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3*(7*c^6*d^10
*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^9 - 5*a^4*c^2
*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a^2*c^4*d^7*e^6
 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12*e - 50*a*c^5*d
^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4*c^2*d^4*e^9 - 9
0*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^7*d^
12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^4*d^6*e^12 + (c^8*
d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^5*d^5*e^13)*x^3 + (
2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^10 + a^3*c^5*d^6*e^12 -
 a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7*d^11*e^7 - 3*a^2*c^6*d^9*e^9
 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x), -1/24*(15*(7*a*c^5*d^12*e -
 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4*c^2*d^6*e^7 + 3*a^5*c*d^4*
e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^5*d^9*e^4 + 6*a^2*c^4*d^7*e
^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*a^5*c*d*e^12)*x^3 + (14*c^6
*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^
4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x^2 + (7*c^6*d^13 - a*c^5*d^
```

```
11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^
5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(105*a*c^5*d^11*e^2
 - 190*a^2*c^4*d^9*e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c
*d^3*e^10 - 6*(c^6*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*
d^3*e^10)*x^4 + 3*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 +
8*a^3*c^3*d^4*e^9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5
*d^9*e^4 + 12*a^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 +
 (105*c^6*d^12*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6
*e^7 + 45*a^4*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^
10 - a^4*c^4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^1
1 - a^3*c^5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*
d^8*e^10 + a^3*c^5*d^6*e^12 - a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7
*d^11*e^7 - 3*a^2*c^6*d^9*e^9 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x)
]
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
[Out] integrate(x^5/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.478 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 438

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{2x(ade(cd^2-ae^2)\left(5c^2d^4-10acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2e^3\left(cd^2-ae^2\right)^3} - \frac{(5cd^2+3ae^2)\,\text{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}}$$

```
[Out] -2/3*d*x^3*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*c
*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)-2/3*x*(a*d*e*(-a*e^2+c*d^2)*(-3*a^2*e^4-1
0*a*c*d^2*e^2+5*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2*c*d^2*e^4-9*a*c^2*d
^4*e^2+5*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/3*(-9*a^3*e^6+9*a^2*c*d^2*e^4-31*a*c^2*d^4*e^2+15*c^3*d^6)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3/(-a*e^2+c*d^2)^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 832, 654, 635, 212}

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2x(ade(cd^2-ae^2)(-3a^2e^4-10acd^2e^2+5c^2d^4)+x(cd^2-ae^2)(-3a^3e^6-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6))}{3cde^2(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e^3(cd^2-ae^2)^3} - \frac{(3ae^2+5cd^2)\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}} - \frac{2dx^3(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e(cd^2-ae^2)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^4(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x^2\left(3acd^2e(cd^2-ae^2)+\frac{1}{2}cd\left(5cd^2-3ae^2\right)\left(cd^2-ae^2\right)x\right)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cde\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x(ade(cd^2-ae^2)\left(5c^2d^4-10acd^2e^2-3a^2e^4\right)+\left(cd^2-ae^2\right)\left(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3e}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{4\int \frac{\frac{1}{2}acd^2e\left(cd^2-ae^2\right)\left(5c^2d^4-10acd^2e^2-3a^2e^4\right)+\frac{1}{4}cd\left(cd^2-ae^2\right)\left(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6\right)x}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3c^2d^2e^2\left(cd^2-ae^2\right)^4}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x(ade(cd^2-ae^2)\,(5c^2d^4-10acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3}{3cde^2\,(cd^2-ae^2)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$+\frac{(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e^3\,(cd^2-ae^2)^3}$$
$$-\frac{(5cd^2+3ae^2)\int\frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2c^2d^2e^3}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x(ade(cd^2-ae^2)\,(5c^2d^4-10acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3}{3cde^2\,(cd^2-ae^2)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$+\frac{(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e^3\,(cd^2-ae^2)^3}$$
$$-\frac{(5cd^2+3ae^2)\,\text{Subst}\left(\int\frac{1}{4cde-x^2}\,dx,x,\frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^2d^2e^3}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$
$$-\frac{2x(ade(cd^2-ae^2)\,(5c^2d^4-10acd^2e^2-3a^2e^4)+(cd^2-ae^2)\,(5c^3d^6-9ac^2d^4e^2-a^2cd^2e^4-3a^3}{3cde^2\,(cd^2-ae^2)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$
$$+\frac{(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e^3\,(cd^2-ae^2)^3}$$
$$-\frac{(5cd^2+3ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.68

$$\int\frac{x^4}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\,(-9a^4e^7(d+ex)^2+3a^3cde^5(3d-ex)(d+ex)^2+c^4d^7x(15d^2+}{}$$

```
[In] Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*a^4*e^7*(d + e*x)^2 + 3*a^3*c*d
*e^5*(3*d - e*x)*(d + e*x)^2 + c^4*d^7*x*(15*d^2 + 20*d*e*x + 3*e^2*x^2) +
a*c^3*d^5*e*(15*d^3 - 11*d^2*e*x - 39*d*e^2*x^2 - 9*e^3*x^3) + a^2*c^2*d^3*
e^3*(-31*d^3 - 33*d^2*e*x + 9*d*e^2*x^2 + 9*e^3*x^3)))/(c*d^2 - a*e^2)^3 -
3*(5*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*
```

```
Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(3*c^(5/2)*d^(5/2)*e^(
7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. 2(408) = 816.

Time = 0.86 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.54

method	result	size
default	Expression too large to display	1112

```
[In] int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/c/
d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d
/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d/e/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c
*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)))+d^2/e^3*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2
)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-d/e^2*(-x/c/d/
e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*(-1/c/d/e
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+d^4/e^5*(-2/3/(a*e^2-c*d^2)/(x+
d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^
3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(408) = 816.

Time = 1.67 (sec) , antiderivative size = 1782, normalized size of antiderivative = 4.07

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c
*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^
5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4
*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*
d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*
d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(15*a
*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3
*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3
+ (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^
8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^
6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9
 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10
- a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7
*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^1
0*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x), 1/
6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c*d^4
*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^5*e^
6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4*d^8
*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*d^11
 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*d^3*
e^8 - 6*a^5*d*e^10)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
 a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*a*c^4*d^9*e^2 - 31*a^2*c
^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4
*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*
a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2
 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*
e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*
c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (
c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3
 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11
 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e
^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x)]
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
[Out] integrate(x^4/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.479 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 297

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{2(ade(cd^2-3ae^2)(3cd^2+ae^2)+(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6)x)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\operatorname{arctanh}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

```
[Out] -2/3*d*x^2*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3
/2)/e^(5/2)-2/3*(a*d*e*(-3*a*e^2+c*d^2)*(a*e^2+3*c*d^2)+(-3*a^3*e^6-a^2*c*d
^2*e^4-7*a*c^2*d^4*e^2+3*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 791, 635, 212}

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(x(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6)+ade(cd^2-3ae^2)(ae^2+3cd^2))}{3cde^2(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}} - \frac{2dx^2(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

```
[In] Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\text{integral} = \int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\int \frac{x\left(2acd^2e\left(cd^2-ae^2\right)+\frac{3}{2}cd\left(cd^2-ae^2\right)^2x\right)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cde\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2(ade(cd^2-3ae^2)\left(3cd^2+ae^2\right)+\left(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cde^2}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2(ade(cd^2-3ae^2)\left(3cd^2+ae^2\right)+\left(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6\right)x)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{cde^2}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2(ade(cd^2-3ae^2)\,(3cd^2+ae^2)+(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6)\,x)}{3cde^2\,(cd^2-ae^2)^3\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)(-3a^3e^5(d+ex)^2+c^3d^6x(3d+4ex)-a^2cd^3e^3(8d+9ex)}{(cd^2-ae^2)^3}\right.}{3c^{3/2}d^{3/2}e}$$

[In] Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]

[Out] (2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*a^3*e^5*(d + e*x)^2 + c^3*d^6*x*(3*d + 4*e*x) - a^2*c*d^3*e^3*(8*d + 9*e*x) + a*c^2*d^4*e*(3*d^2 - 4*d*e*x - 9*e^2*x^2)))/(c*d^2 - a*e^2)^3) + 3*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]))/(3*c^(3/2)*d^(3/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(273) = 546.

Time = 0.69 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.15

method	result
default	$\frac{-\frac{x}{cde\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(-\frac{1}{cde\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(2cdex+e^{2}a+c\,d^{2}\right)}{cde\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}}}\right)}{2cde}}{e}$

[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*e^2*a+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+2*d^2/e^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e

```
*x^2)^(1/2)-d/e^2*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+
c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-d^3/e^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d
*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*
e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(273) = 546$.

Time = 2.06 (sec) , antiderivative size = 1466, normalized size of antiderivative = 4.94

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*e^2 - 8
*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e^4 - 3*a
^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e^5 - 6*a^
3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^5*d^10*e^
4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (c^6*d^9*e^5
 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3 + (2*c^6*d^1
0*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10 - a^4*c^2*d^
2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e^7 + 5*a^3*c^3
*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3
 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c
^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*
d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c
^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*a
*c^3*d^7*e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c
^3*d^5*e^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^
2*d^4*e^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(a*c^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^1
0 + (c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*
x^3 + (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e
```

```
^10 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7
*e^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^3}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
[Out] integrate(x^3/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.480 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 126

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8ae(2ade+\left(cd^2+ae^2\right)x)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/3*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
a*e*(2*a*d*e+(a*e^2+c*d^2)*x)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {868, 12, 650}

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae(x(ae^2+cd^2)+2ade)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\int -\frac{2ade^2\left(cd^2-ae^2\right)x}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3de\left(cd^2-ae^2\right)^2}\\
&= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{(4ae)\int \frac{x}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3\left(cd^2-ae^2\right)}\\
&= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&\quad -\frac{8ae\left(2ade+\left(cd^2+ae^2\right)x\right)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2(ae+cdx)^3\left(d^2-\frac{6ade(d+ex)}{ae+cdx}-\frac{3a^2e^2(d+ex)^2}{(ae+cdx)^2}\right)}{3\left(cd^2-ae^2\right)^3\left((ae+cdx)(d+ex)\right)^{3/2}}$$

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(d^2 - (6*a*d*e*(d + e*x))/(a*e + c*d*x) - (3*a^2*e^2*(d
 + e*x)^2)/(a*e + c*d*x)^2))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))
^(3/2))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

method	result
gosper	$\frac{2(cdx+ae)(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2)}{3(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
trager	$\frac{2(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(ex+d)^2(e^2a-c\,d^2)(cdx+ae)}$
default	$\frac{-\frac{1}{cde\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\left(e^2a+c\,d^2\right)\left(2cdex+e^2a+c\,d^2\right)}{cde\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}}{e}-\frac{2d(2cdex+e^2a+c\,d^2)}{e^2\left(4ac\,d^2e^2-(e^2a+c\,d^2)^2\right)\sqrt{ade+(e^2a+}}$

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*x
+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*
d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(118) = 236.

Time = 1.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.44

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8\,a^2d^2e^2-(c^2d^4-6\,acd^2e^2-3\,a^2e^4)x^2+4\,(acd^3\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8)x^3+(2\,c^4d^8e-5\right.}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d^
3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*
a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c
^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 -
a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 1071, normalized size of antiderivative = 8.50

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{4\,c\,d^3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\,e^3+3\,x\,a\,c^2\,d^4\,e^4-c^3\,d^7\,e-x\,c^3\,d^6\,e^2\right)}$$
$$-\frac{2\,d^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,a^2\,d^2\,e^5+6\,a^2\,d\,e^6\,x+3\,a^2\,e^7\,x^2-6\,a\,c\,d^4\,e^3-12\,a\,c\,d^3\,e^4\,x-6\,a\,c\,d^2\,e^5\,x^2+3\,c^2\,d^6\,e+6\,c^2\,d^5\,e^2\,x+3\,c^2\,d^4\,e^3\,x^2}$$
$$-\frac{4\,a\,d\,e^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\,e^3+3\,x\,a\,c^2\,d^4\,e^4-c^3\,d^7\,e-x\,c^3\,d^6\,e^2\right)}$$
$$+\frac{2\,c^4\,d^7\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{22\,a^3\,c\,d^2\,e^5}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$-\frac{28\,a^2\,c^2\,d^4\,e^3}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{2\,a\,c^3\,d^6\,e}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{10\,a^2\,c^2\,d^3\,e^4\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{2\,a^3\,c\,d\,e^6\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$-\frac{22\,a\,c^3\,d^5\,e^2\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3
*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*
a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*x +
 c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2*d^5
*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^3*e^4
*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2
)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*
d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) + (2*c^4*
d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*
d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) + (22*a^3*c
*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a
*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) - (28*
a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
```

```
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*
(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*
c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2
)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^
4*c*d*e^9)) - (22*a*c^3*d^5*e^2*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^
2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e
^7 + a^4*c*d*e^9))
```

3.481 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 138

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2(cd^2+3ae^2)\left(cd^2+ae^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*d/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/3
*(3*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {806, 627}

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(3ae^2+cd^2)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad -\frac{\left(cd^2+3ae^2\right)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3e\left(cd^2-ae^2\right)} \\
&= -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{2\left(cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(c^2d^3x(3d+2ex)+a^2e^3(2d+3ex)+2acde\left(3d^2+5dex+\right.\right.}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*
d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.08

method	result
gosper	$-\frac{2(cdx+ae)(6acd\,e^{3}x^{2}+2c^{2}d^{3}e\,x^{2}+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+2d\,e^{3}a^{2}+6ac\,d^{3}e)}{3(a^{3}e^{6}-3a^{2}c\,d^{2}e^{4}+3a\,c^{2}d^{4}e^{2}-c^{3}d^{6})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$
trager	$-\frac{2(6acd\,e^{3}x^{2}+2c^{2}d^{3}e\,x^{2}+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+2d\,e^{3}a^{2}+6ac\,d^{3}e)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ex+d)^{2}(e^{2}a-c\,d^{2})(cdx+ae)}$
default	$\frac{4cdex+2e^{2}a+2c\,d^{2}}{e\left(4ac\,d^{2}e^{2}-(e^{2}a+c\,d^{2})^{2}\right)\sqrt{ade+(e^{2}a+c\,d^{2})x+cde\,x^{2}}}-\frac{d\left(-\frac{2}{3\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2\right.}{3\left(e^{2}a-c\,d^{2}\right)^{3}\sqrt{}}\right.}{e^{2}}$

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e^
2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d
^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(130) = 260$.

Time = 1.87 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.28

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\,(6\,acd^3e}{3\,(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3}$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*d
^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^
4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4
*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2
 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d
*e^8)*x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
[Out] integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.62

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c\,d}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,x-}$$

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 12*a
*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^9*x -
3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 - 9*a^2*c
^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*a^2*c^2*
d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c*d^3*e^6*
x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^2 + 3*a^3*
c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

3.482 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 121

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8cd(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*c*d*
(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {672, 627}

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd(ae^2+cd^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3\left(cd^2-ae^2\right)\left(d+ex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad+\frac{(4cd)\int\frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3\left(cd^2-ae^2\right)} \\ &= \frac{2}{3\left(cd^2-ae^2\right)\left(d+ex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad-\frac{8cd(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int\frac{1}{\left(d+ex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2(3d^2+12dex+8e^2x^2)}{3\left(cd^2-ae^2\right)^3\left(d+ex\right)\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
[Out] (2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-8c^{2}d^{2}e^{2}x^{2}-4acd\,e^{3}x-12c^{2}d^{3}ex+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)}{3(a^{3}e^{6}-3a^{2}c\,d^{2}e^{4}+3a\,c^{2}d^{4}e^{2}-c^{3}d^{6})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	138
default	$\frac{-\frac{2}{3\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+e^{2}a-c\,d^{2}\right)}{3\left(e^{2}a-c\,d^{2}\right)^{3}\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(e^{2}a-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}}{e}$	146
trager	$-\frac{2\left(-8c^{2}d^{2}e^{2}x^{2}-4acd\,e^{3}x-12c^{2}d^{3}ex+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ex+d)^{2}(e^{2}a-c\,d^{2})(cdx+ae)}$	146

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-6
*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(113) = 226$.

Time = 2.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8\,c^{2}d^{2}e^{2}x^{2}+3\,c^{2}d^{4}+6\,acd^{2}e^{2}-a^{2}e^{4}+4\,(3\,c^{2}d^{3}e\right.}{3\left(ac^{3}d^{8}e-3\,a^{2}c^{2}d^{6}e^{3}+3\,a^{3}cd^{4}e^{5}-a^{4}d^{2}e^{7}+(c^{4}d^{7}e^{2}-3\,ac^{3}d^{5}e^{4}+3\,a^{2}c^{2}d^{3}e^{6}-a^{3}cde^{8})x^{3}+(2\,c^{4}d^{8}e-5\,a\right.}$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d^
3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*
e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*
c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3
*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*
c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+\right.}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\right.}$$

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] (2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*c
^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e +
 c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

3.483 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde(3cd^2-ae^2)\left(cd^2+3ae^2\right)x)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
-arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+2/3*(3*c^3*d^6+a*c^2*d
^4*e^2+7*a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*x)/
a/d^2/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{865, 836, 12, 738, 212\}$

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}} + \frac{2(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex(3cd^2-ae^2)\left(3ae^2+cd^2\right)+3c^3d^6)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
```

```
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{3}{2}ae\left(cd^2-ae^2\right)^2+2acde^2\left(cd^2-ae^2\right)x}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3ade\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde\left(3cd^2-ae^2\right)\left(cd^2+3ae^2\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{4\int \frac{3ae\left(cd^2-ae^2\right)^4}{4x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3a^2d^2e^2\left(cd^2-ae^2\right)^4}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde\left(3cd^2-ae^2\right)\left(cd^2+3ae^2\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{ad^2e}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde\left(3cd^2-ae^2\right)\left(cd^2+3ae^2\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\text{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-\left(-cd^2-ae^2\right)x}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{ad^2e}$$

$$
= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde(3cd^2-ae^2)\left(cd^2+3ae^2\right)x)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.80

$$
\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-3c^3d^5(d+ex)^2+a^3e^6(4d+3ex)-ac^2d^3e^3x(9d+8ex)\right.}{\left(-cd^2+ae^2\right)^3}\right.}{3a^{3/2}d^{5/2}}
$$

```
[In] Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*c^3*d^5*(d + e*x)^2 + a^3*e^
6*(4*d + 3*e*x) - a*c^2*d^3*e^3*x*(9*d + 8*e*x) + a^2*c*d*e^4*(-9*d^2 - 4*d
*e*x + 3*e^2*x^2)))/(-(c*d^2) + a*e^2)^3 - 3*(a*e + c*d*x)^(3/2)*(d + e*x)^
(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
)/(3*a^(3/2)*d^(5/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{1}{ade\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\left(e^2a+c\,d^2\right)\left(2cdex+e^2a+c\,d^2\right)}{ade\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\sqrt{ade+\left(e^2a+c\,d^2\right)x+cde\,x^2}}-\frac{\ln\left(\frac{2ade+\left(e^2a+c\,d^2\right)x+2\sqrt{ade}\sqrt{ade+\left(e^2a+c\,d^2\right.}}{x}\right.}{ade\sqrt{ade}}}{d}$

```
[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-1/d*(-2/3/(a*e^2-c*d
^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-
c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(247) = 494$.

Time = 5.46 (sec) , antiderivative size = 1476, normalized size of antiderivative = 5.45

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*c^3*d
^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c^2*d^4*e
^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 + 4*a^3*c*
d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3
*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^9 + (a^2*c^
4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*e^10)*x^3 +
(2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 + a^5*c*d^5*e^9
 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 - 3*a^4*c^2*d^8
*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d^8*e - 3*a^2*c^
2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4
+ 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3
*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2
- 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-a*d*e)*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) + 2*(3*a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3
+ 8*a^2*c^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d
^5*e^4 + 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*
d^5*e^9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c
*d^4*e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^
7 + a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^
4 - 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x}\,dx$$

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x\,(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.484 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 394

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde\left(3c^2d^4+10acd^2e^2-5a^2e^4\right)x\right)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x} + \frac{\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)+1/2*(5*a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2
)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+9*a^2*c*d^2*e^4-5*a^3*e^6+c*d*e*(-5*a^2*e^4+
10*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/x/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-1/3*(-15*a^3*e^6+31*a^2*c*d^2*e^4-9*a*c^2*d^4*e^2+9*c^
3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3
/x
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {865, 836, 820, 738, 212}

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{(5ae^2+3cd^2)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}} + \frac{2(-5a^3e^6+cdex(-5a^2e^4+10acd^2e^2+3c^2d^4)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{3ad^2ex\,(cd^2-ae^2)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}} - \frac{(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3a^2d^3e^2x\,(cd^2-ae^2)^3} - \frac{2e(ae+cdx)}{3dx\,(cd^2-ae^2)\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
```

```
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ae(3cd^2-5ae^2)(cd^2-ae^2)+3acde^2(cd^2-ae^2)x}{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx}{3ade\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde(3c^2d^4+10acd^2e^2-5a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{4\int \frac{\frac{1}{4}ae(cd^2-ae^2)(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6)+\frac{1}{2}acde^2(cd^2-ae^2)(3c^2d^4+10acd^2e^2-5a^2e^4)x}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3a^2d^2e^2\left(cd^2-ae^2\right)^4}$$

$$
\begin{aligned}
&= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad + \frac{2(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde(3c^2d^4+10acd^2e^2-5a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad - \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x} \\
&\quad - \frac{\left(3cd^2+5ae^2\right)\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2a^2d^3e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad + \frac{2(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde(3c^2d^4+10acd^2e^2-5a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad - \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x} \\
&\quad + \frac{\left(3cd^2+5ae^2\right)\operatorname{Subst}\left(\int \frac{1}{4ade-x^2}\,dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^2d^3e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad + \frac{2(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde(3c^2d^4+10acd^2e^2-5a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad - \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x} \\
&\quad + \frac{\left(3cd^2+5ae^2\right)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.77

$$
\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-9c^4d^7x(d+ex)^2-3ac^3d^5e(d-3ex)(d+ex)^2+a^4e^7(3\right.}{}
$$

```
[In] Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] (-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*c^4*d^7*x*(d + e*x)^2 - 3*a*c
^3*d^5*e*(d - 3*e*x)*(d + e*x)^2 + a^4*e^7*(3*d^2 + 20*d*e*x + 15*e^2*x^2)
+ a^2*c^2*d^3*e^3*(9*d^3 + 9*d^2*e*x - 33*d*e^2*x^2 - 31*e^3*x^3) - a^3*c*d
*e^5*(9*d^3 + 39*d^2*e*x + 11*d*e^2*x^2 - 15*e^3*x^3)))/((-(c*d^2) + a*e^2)
```

```
^3*x)) + 3*(3*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[
(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(3*a^(5/2)*d^
(7/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{1}{adex\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cdex^{2}}}-\frac{3\left(e^{2}a+c\,d^{2}\right)\left(\frac{1}{ade\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cdex^{2}}}-\frac{\left(e^{2}a+c\,d^{2}\right)\left(2cdex+e^{2}a+c\,d^{2}\right)}{ade\left(4ac\,d^{2}e^{2}-\left(e^{2}a+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(e^{2}a+c\,d^{2}\right)x+cdex^{2}}}-\cdots\right.}{2ade}}{d}$

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a
/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*
e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))-e/d^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^
2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2*(-2
/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*
c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(364) = 728.

Time = 11.47 (sec) , antiderivative size = 1812, normalized size of antiderivative = 4.60

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2
*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2*c
^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3*c^
5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a^5*d
*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 + 12*a
^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4
+ 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x
)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5
*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a
^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5
*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*
d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^
7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c
^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3
*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*
a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^
8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3
*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c
^4*d^8*e^3 - 16*a^2*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*
a^5*e^11)*x^3 + (3*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4
*c*d^3*e^8 - 10*a^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a
^3*c^2*d^6*e^5 + 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(-a*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5*d^3*e
^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a^4*c*d
^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5*e^6 +
 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*d^8*e^
3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^7 + 3*
a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c^3*d^1
0*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3*c^4*d
^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*a^7*d^
5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^8 - a^
7*d^6*e^10)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)

[Out] Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")

[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
x)

Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")

[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)

[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)

3.485 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 522

$$\begin{aligned}
&\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \\
&-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{2(3c^3d^6+ac^2d^4e^2+11a^2cd^2e^4-7a^3e^6+cde(3c^2d^4+12acd^2e^2-7a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{\left(15c^3d^6-9ac^2d^4e^2+61a^2cd^2e^4-35a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6a^2d^3e^2\left(cd^2-ae^2\right)^3x^2} \\
&+\frac{\left(45c^4d^8-30ac^3d^6e^2-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3d^4e^3\left(cd^2-ae^2\right)^3x} \\
&-\frac{5(3c^2d^4+6acd^2e^2+7a^2e^4)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}
\end{aligned}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-5/8*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*
d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^
(7/2)/d^(9/2)/e^(7/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+11*a^2*c*d^2*e^4-7*a^3*e
^6+c*d*e*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/
x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6*(-35*a^3*e^6+61*a^2*c*d^2*e
^4-9*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/
d^3/e^2/(-a*e^2+c*d^2)^3/x^2+1/12*(-105*a^4*e^8+190*a^3*c*d^2*e^6-36*a^2*c^
2*d^4*e^4-30*a*c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 836, 848, 820, 738, 212}

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{5(7a^2e^4+6acd^2e^2+3c^2d^4)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}} + \frac{2(-7a^3e^6+cdex(-7a^2e^4+12acd^2e^2+3c^2d^4)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{3ad^2ex^2(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6a^2d^3e^2x^2(cd^2-ae^2)^3} + \frac{(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6e^2+45c^4d^8)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12a^3d^4e^3x(cd^2-ae^2)^3} - \frac{2e(ae+cdx)}{3dx^2(cd^2-ae^2)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])]])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae + cdx}{x^3 \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae + cdx)}{3d\left(cd^2 - ae^2\right)x^2\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(3cd^2 - 7ae^2\right)\left(cd^2 - ae^2\right) + 4acde^2\left(cd^2 - ae^2\right)x}{x^3\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}\,dx}{3ade\left(cd^2 - ae^2\right)^2}$$

$$= -\frac{2e(ae + cdx)}{3d\left(cd^2 - ae^2\right)x^2\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6 + ac^2d^4e^2 + 11a^2cd^2e^4 - 7a^3e^6 + cde\left(3c^2d^4 + 12acd^2e^2 - 7a^2e^4\right)x\right)}{3ad^2e\left(cd^2 - ae^2\right)^3x^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}} + \frac{4\int \frac{\frac{1}{4}ae\left(cd^2 - ae^2\right)\left(15c^3d^6 - 9ac^2d^4e^2 + 61a^2cd^2e^4 - 35a^3e^6\right) + acde^2\left(cd^2 - ae^2\right)\left(3c^2d^4 + 12acd^2e^2 - 7a^2e^4\right)x}{x^3\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}\,dx}{3a^2d^2e^2\left(cd^2 - ae^2\right)^4}$$

$$= -\frac{2e(ae + cdx)}{3d\left(cd^2 - ae^2\right)x^2\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6 + ac^2d^4e^2 + 11a^2cd^2e^4 - 7a^3e^6 + cde\left(3c^2d^4 + 12acd^2e^2 - 7a^2e^4\right)x\right)}{3ad^2e\left(cd^2 - ae^2\right)^3x^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}} - \frac{\left(15c^3d^6 - 9ac^2d^4e^2 + 61a^2cd^2e^4 - 35a^3e^6\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{6a^2d^3e^2\left(cd^2 - ae^2\right)^3x^2} - \frac{2\int \frac{\frac{1}{8}ae\left(cd^2 - ae^2\right)\left(45c^4d^8 - 30ac^3d^6e^2 - 36a^2c^2d^4e^4 + 190a^3cd^2e^6 - 105a^4e^8\right) + \frac{1}{4}acde^2\left(cd^2 - ae^2\right)\left(15c^3d^6 - 9ac^2d^4e^2 + 61a^2cd^2e^4 - 3}{x^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}}{3a^3d^3e^3\left(cd^2 - ae^2\right)^4}$$

$$= -\frac{2e(ae + cdx)}{3d\left(cd^2 - ae^2\right)x^2\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6 + ac^2d^4e^2 + 11a^2cd^2e^4 - 7a^3e^6 + cde\left(3c^2d^4 + 12acd^2e^2 - 7a^2e^4\right)x\right)}{3ad^2e\left(cd^2 - ae^2\right)^3x^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}} - \frac{\left(15c^3d^6 - 9ac^2d^4e^2 + 61a^2cd^2e^4 - 35a^3e^6\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{6a^2d^3e^2\left(cd^2 - ae^2\right)^3x^2} + \frac{\left(45c^4d^8 - 30ac^3d^6e^2 - 36a^2c^2d^4e^4 + 190a^3cd^2e^6 - 105a^4e^8\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{12a^3d^4e^3\left(cd^2 - ae^2\right)^3x} + \frac{\left(5\left(3c^2d^4 + 6acd^2e^2 + 7a^2e^4\right)\right)\int \frac{1}{x\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}\,dx}{8a^3d^4e^3}$$

$$= -\frac{2e(ae+cdx)}{3d(cd^2-ae^2)x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+11a^2cd^2e^4-7a^3e^6+cde(3c^2d^4+12acd^2e^2-7a^2e^4)x)}{3ad^2e(cd^2-ae^2)^3x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(15c^3d^6-9ac^2d^4e^2+61a^2cd^2e^4-35a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{6a^2d^3e^2(cd^2-ae^2)^3x^2} + \frac{(45c^4d^8-30ac^3d^6e^2-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^3(cd^2-ae^2)^3x} - \frac{(5(3c^2d^4+6acd^2e^2+7a^2e^4))\,\text{Subst}\left(\int\frac{1}{4ade-x^2}\,dx,x,\frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^3d^4e^3}$$

$$= -\frac{2e(ae+cdx)}{3d(cd^2-ae^2)x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+11a^2cd^2e^4-7a^3e^6+cde(3c^2d^4+12acd^2e^2-7a^2e^4)x)}{3ad^2e(cd^2-ae^2)^3x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(15c^3d^6-9ac^2d^4e^2+61a^2cd^2e^4-35a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{6a^2d^3e^2(cd^2-ae^2)^3x^2} + \frac{(45c^4d^8-30ac^3d^6e^2-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^3(cd^2-ae^2)^3x} - \frac{5(3c^2d^4+6acd^2e^2+7a^2e^4)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.75

$$\int\frac{1}{x^3(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(-45c^5d^9x^2(d+ex)^2-15ac^4d^7ex(d-2ex)(d+ex)^2+6a^2c}{}$$

```
[In] Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]

[Out] ((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*c^5*d^9*x^2*(d + e*x)^2 - 15*a
*c^4*d^7*e*x*(d - 2*e*x)*(d + e*x)^2 + 6*a^2*c^3*d^5*e^2*(d + e*x)^2*(d^2 +
 2*d*e*x + 6*e^2*x^2) + a^5*e^8*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 105*
e^3*x^3) - 2*a^3*c^2*d^3*e^4*(9*d^4 - 9*d^3*e*x - 6*d^2*e^2*x^2 + 111*d*e^3
*x^3 + 95*e^4*x^4) + a^4*c*d*e^6*(18*d^4 - 48*d^3*e*x - 237*d^2*e^2*x^2 - 5
0*d*e^3*x^3 + 105*e^4*x^4)))/((-(c*d^2) + a*e^2)^3*x^2) - 15*(3*c^2*d^4 + 6
*a*c*d^2*e^2 + 7*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt
[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(12*a^(7/2)*d^(9/2
)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(488) = 976$.

Time = 0.77 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.59

method	result	size
default	Expression too large to display	1354

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^
2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d
^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/
d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+
2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e
^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e
^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e
^2/d^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-e/d^2*(-1/a/d/e/x
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2
+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a
*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-e^2/d^
3*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d^2)/(c*d*e*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(488) = 976$.

Time = 28.52 (sec) , antiderivative size = 2162, normalized size of antiderivative = 4.14

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith m="fricas")

[Out] [1/48*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^10 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(6*a^3*c^3*d^10*e^3 - 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3*d^12*e^6 - 3*a^6*c^2*d^10*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (a^5*c^3*d^13*e^5 - 3*a^6*c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2), 1/24*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^10 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(6*a^3*c^3*d^10*e^3 - 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3*d^12*e^6 - 3*a^6*c^2*d^1

```
0*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (a^5*c^3*d^13*e^5 - 3*a^6*
c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
x)
```

Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\ (d+e\,x)\ \left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\ x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.486 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 664

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)x)}{3ad^2e\left(cd^2-ae^2\right)^3x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x^3} + \frac{\left(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3d^4e^3\left(cd^2-ae^2\right)^3x^2} - \frac{\left(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24a^4d^5e^4\left(cd^2-ae^2\right)^3x} + \frac{5\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\operatorname{arctanh}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+5/16*(21*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+1
3*a^2*c*d^2*e^4-9*a^3*e^6+c*d*e*(-9*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*x)/a/
d^2/e/(-a*e^2+c*d^2)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/3*(-21
*a^3*e^6+33*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+7*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3/x^3+1/12*(-105*a^4*e^8+168*a
^3*c*d^2*e^6-18*a^2*c^2*d^4*e^4-16*a*c^3*d^6*e^2+35*c^4*d^8)*(a*d*e+(a*e^2+
```

```
c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x^2-1/24*(-315*a^5*e
^10+525*a^4*c*d^2*e^8-78*a^3*c^2*d^4*e^6-54*a^2*c^3*d^6*e^4-55*a*c^4*d^8*e^
2+105*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/(-a*e^2
+c*d^2)^3/x
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 836, 848, 820, 738, 212}

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}dx = \frac{2(-9a^3e^6+cdex(-9a^2e^4+14acd^2e^2+3c^2d^4)+13a^2c}{3ad^2ex^3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ac}}$$
$$-\frac{(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}$$
$$+\frac{5(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6)\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}$$
$$+\frac{(-105a^4e^8+168a^3cd^2e^6-18a^2c^2d^4e^4-16ac^3d^6e^2+35c^4d^8)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12a^3d^4e^3x^2\left(cd^2-ae^2\right)^3}$$
$$-\frac{(-315a^5e^{10}+525a^4cd^2e^8-78a^3c^2d^4e^6-54a^2c^3d^6e^4-55ac^4d^8e^2+105c^5d^{10})\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{24a^4d^5e^4x\left(cd^2-ae^2\right)^3}$$
$$-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \frac{ae+cdx}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{3}{2}ae\left(cd^2-3ae^2\right)\left(cd^2-ae^2\right)+5acde^2\left(cd^2-ae^2\right)x}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3ade\left(cd^2-ae^2\right)^2}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{4\int \frac{\frac{3}{4}ae\left(cd^2-ae^2\right)\left(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6\right)+\frac{3}{2}acde^2\left(cd^2-ae^2\right)\left(3c^2d^4+14acd^2e^2-9a^2e^4\right)x}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3a^2d^2e^2\left(cd^2-ae^2\right)^4}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)\,x)}{3ad^2e\left(cd^2-ae^2\right)^3x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e^2\left(cd^2-ae^2\right)^3x^3} - \frac{4\int \frac{\frac{3}{8}ae\left(cd^2-ae^2\right)\left(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8\right)+\frac{3}{2}acde^2\left(cd^2-ae^2\right)\left(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-\cdots\right.}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\cdots}{9a^3d^3e^3\left(cd^2-ae^2\right)^4}$$

$$= -\frac{2e(ae+cdx)}{3d(cd^2-ae^2)x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)x)}{3ad^2e(cd^2-ae^2)^3x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3a^2d^3e^2(cd^2-ae^2)^3x^3} + \frac{(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^3(cd^2-ae^2)^3x^2} + \frac{2\int \frac{\frac{3}{16}ae(cd^2-ae^2)(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10})+\frac{3}{8}acde^2(cd^2-ae^2)(35c^4d^8-16ac^3d}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}}{9a^4d^4e^4(cd^2-ae^2)^4}$$

$$= -\frac{2e(ae+cdx)}{3d(cd^2-ae^2)x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)x)}{3ad^2e(cd^2-ae^2)^3x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3a^2d^3e^2(cd^2-ae^2)^3x^3} + \frac{(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^3(cd^2-ae^2)^3x^2} - \frac{(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10})\sqrt{ade+(cd^2+ae^2)}}{24a^4d^5e^4(cd^2-ae^2)^3x} - \frac{(5(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6))\int \frac{1}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{16a^4d^5e^4}$$

$$= -\frac{2e(ae+cdx)}{3d(cd^2-ae^2)x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)x)}{3ad^2e(cd^2-ae^2)^3x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3a^2d^3e^2(cd^2-ae^2)^3x^3} + \frac{(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^3(cd^2-ae^2)^3x^2} - \frac{(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10})\sqrt{ade+(cd^2+ae^2)}}{24a^4d^5e^4(cd^2-ae^2)^3x} + \frac{(5(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6))\,\mathrm{Subst}\left(\int \frac{1}{4ade-x^2}dx, x, \frac{2ade-(-cd^2-ae^2)x}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^4d^5e^4}$$

$$= -\frac{2e(ae+cdx)}{3d\,(cd^2-ae^2)\,x^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+14acd^2e^2-9a^2e^4)\,x)}{3ad^2e\,(cd^2-ae^2)^3\,x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{(7c^3d^6-3ac^2d^4e^2+33a^2cd^2e^4-21a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3a^2d^3e^2\,(cd^2-ae^2)^3\,x^3} + \frac{(35c^4d^8-16ac^3d^6e^2-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12a^3d^4e^3\,(cd^2-ae^2)^3\,x^2} - \frac{(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10})\sqrt{ade+(cd^2+ae^2}}{24a^4d^5e^4\,(cd^2-ae^2)^3\,x} + \frac{5(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4(d+ex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(-105c^6d^{11}x^3(d+ex)^2-5ac^5d^9ex^2(7d-11ex)(d+ex)^2+a^2c^4d^7e^2}{}$$

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^6*d^11*x^3*(d + e*x)^2 - 5*a*c^5*d^9*e*x^
2*(7*d - 11*e*x)*(d + e*x)^2 + a^2*c^4*d^7*e^2*x*(d + e*x)^2*(14*d^2 + 23*d
*e*x + 54*e^2*x^2) - 2*a^3*c^3*d^5*e^3*(d + e*x)^2*(4*d^3 + 4*d^2*e*x - 9*d
*e^2*x^2 - 39*e^3*x^3) + a^6*e^9*(8*d^4 - 18*d^3*e*x + 63*d^2*e^2*x^2 + 420
*d*e^3*x^3 + 315*e^4*x^4) + a^4*c^2*d^3*e^5*(24*d^5 - 12*d^4*e*x + 62*d^3*e
^2*x^2 + 3*d^2*e^3*x^3 - 636*d*e^4*x^4 - 525*e^5*x^5) - a^5*c*d*e^7*(24*d^5
 - 40*d^4*e*x + 135*d^3*e^2*x^2 + 651*d^2*e^3*x^3 + 105*d*e^4*x^4 - 315*e^5
*x^5)))/((c*d^2 - a*e^2)^3*x^3*(d + e*x)) + 15*(7*c^3*d^6 + 15*a*c^2*d^4*e^
2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[
(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(24*a^(9/2)*d
^(11/2)*e^(9/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2408 vs. 2(626) = 1252.

Time = 1.14 (sec) , antiderivative size = 2409, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	2409

```
[In] int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/6*(a*e^2+c*d^
2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^
2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^
2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*
c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x
)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+
c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)
/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*
d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)))+e^2/d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x
))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-e^3/d^4*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)
*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/x))-e/d^2*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-5/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x
```

```
))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)
*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/x)))+e^3/d^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+e^2*a-c*d
^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(626) = 1252.

Time = 64.20 (sec) , antiderivative size = 2526, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/96*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c^5*d^9*e^6 - 4*a^3*c
^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12 - 21*a^6*c*d*e^14)*x
^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^10*e^5 - 11*a^3*c^4*d
^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*a^7*e^15)*x^5 + (7*c
^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a^3*c^4*d^9*e^6 - 23*
a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^12 - 42*a^7*d*e^14)*
x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*d^10*e^5 - 4*a^4*c^3
*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*a^7*d^2*e^13)*x^3)*s
qrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^4*c^3*d^11*e^4 - 2
4*a^5*c^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*c^6*d^12*e^3
 - 55*a^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e^9 + 525*a^5*
c^2*d^4*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2 - 75*a^2*c^5*d
^11*e^4 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^5*c^2*d^5*e^10
+ 105*a^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^14*e + 15*a^2*c^5
*d^12*e^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3*a^5*c^2*d^6*e^9
+ 651*a^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^5*d^13*e^2 - 51*a^
3*c^4*d^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8 + 135*a^6*c*d^5*e^1
0 - 63*a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 12*a^4*c^3*d^10*e^5 - 6*
a^5*c^2*d^8*e^7 + 20*a^6*c*d^6*e^9 - 9*a^7*d^4*e^11)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/((a^5*c^4*d^13*e^7 - 3*a^6*c^3*d^11*e^9 + 3*a^7*c
^2*d^9*e^11 - a^8*c*d^7*e^13)*x^6 + (2*a^5*c^4*d^14*e^6 - 5*a^6*c^3*d^12*e^
8 + 3*a^7*c^2*d^10*e^10 + a^8*c*d^8*e^12 - a^9*d^6*e^14)*x^5 + (a^5*c^4*d^1
```

```
5*e^5 - a^6*c^3*d^13*e^7 - 3*a^7*c^2*d^11*e^9 + 5*a^8*c*d^9*e^11 - 2*a^9*d^
7*e^13)*x^4 + (a^6*c^3*d^14*e^6 - 3*a^7*c^2*d^12*e^8 + 3*a^8*c*d^10*e^10 -
a^9*d^8*e^12)*x^3), -1/48*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c
^5*d^9*e^6 - 4*a^3*c^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12
- 21*a^6*c*d*e^14)*x^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^1
0*e^5 - 11*a^3*c^4*d^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*
a^7*e^15)*x^5 + (7*c^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a
^3*c^4*d^9*e^6 - 23*a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^
12 - 42*a^7*d*e^14)*x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*
d^10*e^5 - 4*a^4*c^3*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*
a^7*d^2*e^13)*x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a
^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(8*a^4*c^3*d^11*e^4 - 24*a^5*c
^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*c^6*d^12*e^3 - 55*a
^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e^9 + 525*a^5*c^2*d^4
*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2 - 75*a^2*c^5*d^11*e^4
 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^5*c^2*d^5*e^10 + 105*a
^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^14*e + 15*a^2*c^5*d^12*e
^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3*a^5*c^2*d^6*e^9 + 651*a
^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^5*d^13*e^2 - 51*a^3*c^4*d
^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8 + 135*a^6*c*d^5*e^10 - 63*
a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 12*a^4*c^3*d^10*e^5 - 6*a^5*c^2
*d^8*e^7 + 20*a^6*c*d^6*e^9 - 9*a^7*d^4*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/((a^5*c^4*d^13*e^7 - 3*a^6*c^3*d^11*e^9 + 3*a^7*c^2*d^9*
e^11 - a^8*c*d^7*e^13)*x^6 + (2*a^5*c^4*d^14*e^6 - 5*a^6*c^3*d^12*e^8 + 3*a
^7*c^2*d^10*e^10 + a^8*c*d^8*e^12 - a^9*d^6*e^14)*x^5 + (a^5*c^4*d^15*e^5 -
 a^6*c^3*d^13*e^7 - 3*a^7*c^2*d^11*e^9 + 5*a^8*c*d^9*e^11 - 2*a^9*d^7*e^13)
*x^4 + (a^6*c^3*d^14*e^6 - 3*a^7*c^2*d^12*e^8 + 3*a^8*c*d^10*e^10 - a^9*d^8
*e^12)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

```
[In] integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4), x)

Giac [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^4\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

[In] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)

[Out] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)

3.487 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 259

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8(ade(cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(c^3d^6+a^2cd^2e^4-2a^3e^6\right)x)}{15e\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{8(c^2d^4+10acd^2e^2+5a^2e^4)\left(cd^2+ae^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/5*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/15
*(a*d*e*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)+(-2*a^3*e^6+a^2*c*d^2*e^4+c^3*d^6)*x
)/e/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/15*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {868, 791, 627}

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{8(5a^2e^4+10acd^2e^2+c^2d^4)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{8(x(-2a^3e^6+a^2cd^2e^4+c^3d^6)+ade(cd^2-ae^2)\left(3ae^2+cd^2\right))}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}} + \frac{2x^2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\text{integral} = \frac{2x^2}{5\left(cd^2-ae^2\right)\left(d+ex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x\left(-2ade^2\left(cd^2-ae^2\right)+2cd^2e\left(cd^2-ae^2\right)x\right)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx}{5de\left(cd^2-ae^2\right)^2}$$

$$
\begin{aligned}
&= \frac{2x^2}{5\left(cd^2-ae^2\right)\left(d+ex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad - \frac{8(ade(cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(c^3d^6+a^2cd^2e^4-2a^3e^6\right)x)}{15e\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad - \frac{\left(4(c^2d^4+10acd^2e^2+5a^2e^4)\right)\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{15e\left(cd^2-ae^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x^2}{5\left(cd^2-ae^2\right)\left(d+ex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad - \frac{8(ade(cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(c^3d^6+a^2cd^2e^4-2a^3e^6\right)x)}{15e\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad + \frac{8(c^2d^4+10acd^2e^2+5a^2e^4)\left(cd^2+ae^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

$$
\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(c^4d^6x^2(15d^2+20dex+8e^2x^2)+a^4e^6(8d^2+20dex+15e
$$

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e^3
*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3) +
 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*x^3
+ 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(
3/2))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.41

method	result
gosper	$-\frac{2(cdx+ae)(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180}{15(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10}$
trager	$-\frac{2(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a^3c\,d^2e^6}{15(a^4e^8-4a^3c\,d^2e^6+}$
default	$\frac{-\frac{1}{3cde\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\frac{4}{3}cdex+\frac{2}{3}e^2a+\frac{2}{3}c\,d^2}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}+\frac{16cde\left(2cdex+e\right.}{3\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)^2\sqrt{a}}\right.}{2cde}}{e}$

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e^
2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20*c
^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^2+1
80*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+220*
a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a^2*c^
2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+
5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(247) = 494$.

Time = 16.88 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.17

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{}{15\left(a^2c^5d^{13}e^2-5\,a^3c^4d^{11}e^4+10\,a^4c^3d^9e^6-10\,a^5c^2d^7e^8+\right.}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(40*a^2*c^2*d^6*e^2 + 80*a^3*c*d^4*e^4 + 8*a^4*d^2*e^6 + 8*(c^4*d^6*e^
2 + 10*a*c^3*d^4*e^4 + 5*a^2*c^2*d^2*e^6)*x^4 + 4*(5*c^4*d^7*e + 53*a*c^3*d
^5*e^3 + 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7)*x^3 + 3*(5*c^4*d^8 + 60*a*c^3
*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 + 5*a^4*e^8)*x^2 + 4*(15*
a*c^3*d^7*e + 55*a^2*c^2*d^5*e^3 + 53*a^3*c*d^3*e^5 + 5*a^4*d*e^7)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e
^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e
^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^
6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a
*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^1
```

```
0 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12
*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*
c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2
- 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d
^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^
5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 +
 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
[Out] integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 3099, normalized size of antiderivative = 11.97

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

```
[Out] (((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^2
)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e*(
2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^
2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 + 2*a*
d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2))/(c*d*e)
 - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 + 4*c^2*d^4 +
 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*d*(a*e^2 + c*d^2
)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 + c*d^2)*((4*c^4*d^4
*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^
3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d
^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/
(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4
*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*
e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^4*d^4*e^
3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^3*d
^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3*(c
^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d^2)
```

*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36*a^2*c*d^2*e^3))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*a^3*c^2*d^3*e^6)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(12*a^3*e^5 - 36*a^2*c*d^2*e^3)*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) + (a*((a*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36*a^2*c*d^2*e^3))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (4*a^3*c*d^2*e^5*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e*x)^2) - (2*d^2*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^3*(5*a^3*e^7 - 5*c^3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5))

3.488 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 341

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8\left(2ade\left(cd^2+2ae^2\right)+\left(2c^2d^4+acd^2e^2+3a^2e^4\right)x\right)}{35e\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} + \frac{16\left(3c^2d^4+14acd^2e^2+7a^2e^4\right)\left(cd^2+ae^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{128cd\left(3c^2d^4+14acd^2e^2+7a^2e^4\right)\left(cd^2+ae^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/7*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-8/35
*(2*a*d*e*(2*a*e^2+c*d^2)+(3*a^2*e^4+a*c*d^2*e^2+2*c^2*d^4)*x)/e/(-a*e^2+c*
d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+16/105*(7*a^2*e^4+14*a*c*d^2
*e^2+3*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-128/105*c*d*(7*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*(2
*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {868, 791, 628, 627}

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = -\frac{128cd(7a^2e^4+14acd^2e^2+3c^2d^4)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} + \frac{16(7a^2e^4+14acd^2e^2+3c^2d^4)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}} - \frac{8(x(3a^2e^4+acd^2e^2+2c^2d^4)+2ade(2ae^2+cd^2))}{35e\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}} + \frac{2x^2}{7(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}$$

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
```

```
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} \\
&\quad+\frac{2\int \frac{x\left(-2ade^2\left(cd^2-ae^2\right)+4cd^2e\left(cd^2-ae^2\right)x\right)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx}{7de\left(cd^2-ae^2\right)^2} \\
&= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} \\
&\quad-\frac{8(2ade(cd^2+2ae^2)+\left(2c^2d^4+acd^2e^2+3a^2e^4\right)x)}{35e\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} \\
&\quad-\frac{(8(3c^2d^4+14acd^2e^2+7a^2e^4))\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx}{35e\left(cd^2-ae^2\right)^3} \\
&= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} \\
&\quad-\frac{8(2ade(cd^2+2ae^2)+\left(2c^2d^4+acd^2e^2+3a^2e^4\right)x)}{35e\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} \\
&\quad+\frac{16(3c^2d^4+14acd^2e^2+7a^2e^4)\left(cd^2+ae^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad+\frac{(64cd(3c^2d^4+14acd^2e^2+7a^2e^4))\int \frac{1}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{105\left(cd^2-ae^2\right)^5}
\end{aligned}$$

$$= \frac{2x^2}{7(cd^2-ae^2)(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} - \frac{8(2ade(cd^2+2ae^2)+(2c^2d^4+acd^2e^2+3a^2e^4)x)}{35e(cd^2-ae^2)^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} + \frac{16(3c^2d^4+14acd^2e^2+7a^2e^4)(cd^2+ae^2+2cdex)}{105e(cd^2-ae^2)^5(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} - \frac{128cd(3c^2d^4+14acd^2e^2+7a^2e^4)(cd^2+ae^2+2cdex)}{105(cd^2-ae^2)^7\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.29

$$\int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}\,dx = -\frac{2\sqrt{(ae+cdx)(d+ex)}(-15d^2e^4(ae+cdx)^6+84cd^3e^3(ae+cdx)^5(d+ex)+42ade^5(ae+cdx)^5(d+ex)-$$

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-15*d^2*e^4*(a*e + c*d*x)^6 + 84*c*d^3*e
^3*(a*e + c*d*x)^5*(d + e*x) + 42*a*d*e^5*(a*e + c*d*x)^5*(d + e*x) - 210*c
^2*d^4*e^2*(a*e + c*d*x)^4*(d + e*x)^2 - 280*a*c*d^2*e^4*(a*e + c*d*x)^4*(d
 + e*x)^2 - 35*a^2*e^6*(a*e + c*d*x)^4*(d + e*x)^2 + 420*c^3*d^5*e*(a*e + c
*d*x)^3*(d + e*x)^3 + 1260*a*c^2*d^3*e^3*(a*e + c*d*x)^3*(d + e*x)^3 + 420*
a^2*c*d*e^5*(a*e + c*d*x)^3*(d + e*x)^3 + 105*c^4*d^6*(a*e + c*d*x)^2*(d +
e*x)^4 + 840*a*c^3*d^4*e^2*(a*e + c*d*x)^2*(d + e*x)^4 + 630*a^2*c^2*d^2*e^
4*(a*e + c*d*x)^2*(d + e*x)^4 - 70*a*c^4*d^5*e*(a*e + c*d*x)*(d + e*x)^5 -
140*a^2*c^3*d^3*e^3*(a*e + c*d*x)*(d + e*x)^5 + 21*a^2*c^4*d^4*e^2*(d + e*x
)^6))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(325) = 650.

Time = 0.66 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.94

method	result
gosper	$-\frac{2(cdx+ae)\left(-896a^2c^4d^4e^8x^6-1792a\,c^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232a\,c^5d^7e^5x^5-1344c^6d^9e^3x\right.}{}$
trager	$-\frac{2\left(-896a^2c^4d^4e^8x^6-1792a\,c^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232a\,c^5d^7e^5x^5-1344c^6d^9e^3x^5-1680a\right.}{}$
default	$\frac{-\frac{1}{5cde\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}-\frac{\left(e^2a+c\,d^2\right)\left(\frac{\frac{4}{5}cdex+\frac{2}{5}e^2a+\frac{2}{5}c\,d^2}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right)\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}+\frac{16cde\left(\frac{\frac{4}{3}cdex}{\left(4ac\,d^2e^2-\left(e^2a+c\,d^2\right)^2\right.}\right.}{}\right.}{2cde}}{e}$

[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x^6-384*c^6*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*e^7*x^5-7232*a*c^5*d^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*d^2*e^10*x^4-11200*a^3*c^3*d^4*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^10*e^2*x^4-280*a^5*c*d*e^11*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*e^7*x^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^11*e*x^3+35*a^6*e^12*x^2-910*a^5*c*d^2*e^10*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^3*c^3*d^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^10*e^2*x^2-105*c^6*d^12*x^2+28*a^6*d*e^11*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120*a^3*c^3*d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^11*e*x+8*a^6*d^2*e^10-224*a^5*c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d^10*e^2)/(a^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3*d^6*e^8+35*a^3*c^4*d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(7/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(325) = 650$.

Time = 115.68 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.52

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}\,dx = \text{Too large to display}$$

[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm="fricas")

```
[Out] -2/105*(56*a^2*c^4*d^10*e^2 + 1120*a^3*c^3*d^8*e^4 + 1680*a^4*c^2*d^6*e^6 +
 224*a^5*c*d^4*e^8 - 8*a^6*d^2*e^10 + 128*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e^6
 + 7*a^2*c^4*d^4*e^8)*x^6 + 64*(21*c^6*d^9*e^3 + 113*a*c^5*d^7*e^5 + 119*a^
2*c^4*d^5*e^7 + 35*a^3*c^3*d^3*e^9)*x^5 + 80*(21*c^6*d^10*e^2 + 140*a*c^5*d
^8*e^4 + 254*a^2*c^4*d^6*e^6 + 140*a^3*c^3*d^4*e^8 + 21*a^4*c^2*d^2*e^10)*x
^4 + 40*(21*c^6*d^11*e + 203*a*c^5*d^9*e^3 + 602*a^2*c^4*d^7*e^5 + 542*a^3*
c^3*d^5*e^7 + 161*a^4*c^2*d^3*e^9 + 7*a^5*c*d*e^11)*x^3 + 5*(21*c^6*d^12 +
518*a*c^5*d^10*e^2 + 2639*a^2*c^4*d^8*e^4 + 4004*a^3*c^3*d^6*e^6 + 1859*a^4
*c^2*d^4*e^8 + 182*a^5*c*d^2*e^10 - 7*a^6*e^12)*x^2 + 4*(35*a*c^5*d^11*e +
749*a^2*c^4*d^9*e^3 + 2030*a^3*c^3*d^7*e^5 + 1610*a^4*c^2*d^5*e^7 + 191*a^5
*c*d^3*e^9 - 7*a^6*d*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(
a^3*c^7*d^18*e^3 - 7*a^4*c^6*d^16*e^5 + 21*a^5*c^5*d^14*e^7 - 35*a^6*c^4*d^
12*e^9 + 35*a^7*c^3*d^10*e^11 - 21*a^8*c^2*d^8*e^13 + 7*a^9*c*d^6*e^15 - a^
10*d^4*e^17 + (c^10*d^17*e^4 - 7*a*c^9*d^15*e^6 + 21*a^2*c^8*d^13*e^8 - 35*
a^3*c^7*d^11*e^10 + 35*a^4*c^6*d^9*e^12 - 21*a^5*c^5*d^7*e^14 + 7*a^6*c^4*d
^5*e^16 - a^7*c^3*d^3*e^18)*x^7 + (4*c^10*d^18*e^3 - 25*a*c^9*d^16*e^5 + 63
*a^2*c^8*d^14*e^7 - 77*a^3*c^7*d^12*e^9 + 35*a^4*c^6*d^10*e^11 + 21*a^5*c^5
*d^8*e^13 - 35*a^6*c^4*d^6*e^15 + 17*a^7*c^3*d^4*e^17 - 3*a^8*c^2*d^2*e^19)
*x^6 + 3*(2*c^10*d^19*e^2 - 10*a*c^9*d^17*e^4 + 15*a^2*c^8*d^15*e^6 + 7*a^3
*c^7*d^13*e^8 - 49*a^4*c^6*d^11*e^10 + 63*a^5*c^5*d^9*e^12 - 35*a^6*c^4*d^7
*e^14 + 5*a^7*c^3*d^5*e^16 + 3*a^8*c^2*d^3*e^18 - a^9*c*d*e^20)*x^5 + (4*c^
10*d^20*e - 10*a*c^9*d^18*e^3 - 30*a^2*c^8*d^16*e^5 + 155*a^3*c^7*d^14*e^7
- 245*a^4*c^6*d^12*e^9 + 147*a^5*c^5*d^10*e^11 + 35*a^6*c^4*d^8*e^13 - 95*a
^7*c^3*d^6*e^15 + 45*a^8*c^2*d^4*e^17 - 5*a^9*c*d^2*e^19 - a^10*e^21)*x^4 +
 (c^10*d^21 + 5*a*c^9*d^19*e^2 - 45*a^2*c^8*d^17*e^4 + 95*a^3*c^7*d^15*e^6
- 35*a^4*c^6*d^13*e^8 - 147*a^5*c^5*d^11*e^10 + 245*a^6*c^4*d^9*e^12 - 155*
a^7*c^3*d^7*e^14 + 30*a^8*c^2*d^5*e^16 + 10*a^9*c*d^3*e^18 - 4*a^10*d*e^20)
*x^3 + 3*(a*c^9*d^20*e - 3*a^2*c^8*d^18*e^3 - 5*a^3*c^7*d^16*e^5 + 35*a^4*c
^6*d^14*e^7 - 63*a^5*c^5*d^12*e^9 + 49*a^6*c^4*d^10*e^11 - 7*a^7*c^3*d^8*e^
13 - 15*a^8*c^2*d^6*e^15 + 10*a^9*c*d^4*e^17 - 2*a^10*d^2*e^19)*x^2 + (3*a^
2*c^8*d^19*e^2 - 17*a^3*c^7*d^17*e^4 + 35*a^4*c^6*d^15*e^6 - 21*a^5*c^5*d^1
3*e^8 - 35*a^6*c^4*d^11*e^10 + 77*a^7*c^3*d^9*e^12 - 63*a^8*c^2*d^7*e^14 +
25*a^9*c*d^5*e^16 - 4*a^10*d^3*e^18)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume?`
for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{7}{2}}(ex+d)}\,dx$$

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(7/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 11469, normalized size of antiderivative = 33.63

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)),x)
```

```
[Out] ((6*c^3*d^5 + 36*a*c^2*d^3*e^2 - 10*a^2*c*d*e^4)/(105*(a*e^2 - c*d^2)^6) -
x*((16*c^2*d^2*e)/(105*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(
105*(a*e^2 - c*d^2)^6)) + (8*a*c^2*d^3*e^2)/(105*(a*e^2 - c*d^2)^6))/(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (x*((a*((64*c^5*d^5*e^4*(a*e^2 +
c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*(
(64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c
```

^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6 *(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a *e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4* e^3*(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(60*c^4*d^7 - 204*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c* d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^3*d^3*e^2*(a*e^ 2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6 *(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((64* c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^ 3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^4*d ^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^ 3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*e^3* (a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2* a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(60*c^4*d^7 - 20 4*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*d^ 2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))/(x*(a*e^2 + c*d^2) + a*d *e + c*d*e*x^2)^(1/2) + (x*((a*((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6)))/ c + (36*c^4*d^7*e - 76*a*c^3*d^5*e^3 - 36*a^2*c^2*d^3*e^5 + 12*a^3*c*d*e^7) /(105*e*(a*e^2 - c*d^2)^6) + ((a*e^2 + c*d^2)*((8*a*c^3*d^4*e^3)/(105*(a*e^ 2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6))*(a*e^2 + c*d^2))/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^4 + 14*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6)))/(c*d*e)) + (30*a^4*e^8 - 22*c^4*d^8 + 20*a*c^3*d^6*e^2 - 132*a^3*c*d^2*e^6 + 72*a^2*c^2*d^4*e^4)/(105*e*(a*e^2 - c*d^2)^6) + (a*((8*a*c^3*d^4*e^3)/(105*(a*e^2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6))*(a*e^2 + c*d^2))/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^ 4 + 14*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6)))/c)/(x*(a*e^2 + c*d^2) + a*d* e + c*d*e*x^2)^(3/2) - (((d*((e*(2*a*e^4 - 2*c*d^2*e^2))/(7*(a*e^2 - c*d^2) ^4*(5*a*e^3 - 5*c*d^2*e)) - (4*c*d^2*e^3)/(7*(a*e^2 - c*d^2)^4*(5*a*e^3 - 5 *c*d^2*e))))/e + (e*(2*a*d*e^3 + 2*c*d^3*e))/(7*(a*e^2 - c*d^2)^4*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((e*(10*a*e^3 - 14*c*d^2*e))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^2*e^2)/(7*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c *d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*((a*((a*((4*c^5*d^5*e^4 *(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2* c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^ 5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d ^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a *c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4 *d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (32*a^4*c^2*d^3*e^8)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(16*a^4*e^7 - 64*a^3*c*d^2*e^5)*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((a*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d

^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*(
(a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d
^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 -
c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))
)/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*
e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*
e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (
2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4
*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a
*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(
16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5
*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (16*a^4*c*d^2*e^7*(a*e^2 + c*d^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e + c*d*x)^3*(d + e*x)^3) -
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((16*c^5*d^5*e^4*(a*e
^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e
^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2
)*((16*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e
^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*
c^4*d^4*e^3*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(
a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(484*c^4*d^7 + 1228*
a*c^3*d^5*e^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*c^3*d^3*e^2*(a*e^2 +
 c*d^2)*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((16*c^5*d^
5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*c^4*d^4*e^3*(
c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(a*e^2 + c*d^2
)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +

a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(484*c^4*d^7 + 1228*a*c^3*d^5*e ^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*d ^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a *e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((a*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^ 5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)* ((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c ^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^ 5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^ 2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4* (a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a* c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c^3* d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5 *e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(2*c ^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2* a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)* ((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c ^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^ 5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^ 2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4* (a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a* c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6*e^5*(a* e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d *e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5 *e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^ 2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2* a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a* e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32 *c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^ 6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a *e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e ^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2 *a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c ^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3* d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4*e^8 - 3 *c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e

^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c
^2*d^2*e*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4*c*d^2*e^8
 - 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^2*d^2*e*(a*e^2 + c*d^2)*(7*
a^4*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 -
 c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((a*(((a*e^2 +
 c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(10
5*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e)
- (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(1
05*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*
d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6
e^5(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*
(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^
2 + c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/
(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*
e) - (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 -
 c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)
/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c
^5*d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e
^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)
^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 +
c*d^2)*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4
*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2
 + c*d^2)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(
105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c
d(a*e^2 + c*d^2)*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4
*c*d^2*e^8 - 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e
*x)^2) - (2*d^2*e^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*
x)^4*(7*a^4*e^9 + 7*c^4*d^8*e - 28*a*c^3*d^6*e^3 - 28*a^3*c*d^2*e^7 + 42*a^
2*c^2*d^4*e^5)) + (8*c*d*e*(5*a*e^2 + c*d^2)*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(105*(a*e^2 - c*d^2)^6*(d + e*x))

3.489 $\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

```
[Out] 6/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)-4/5
5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*
(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)
/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {929, 285, 327, 224}

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{6}{55}\sqrt{x+1}\sqrt{x^2-x+1}x+\frac{2}{11}\sqrt{x+1}\sqrt{x^2-x+1}x^4$$

```
[In] Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (6*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/11 - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 +
x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{11\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\left(6\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{55\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.67 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$

$$= \frac{2\left(x\sqrt{1+x}(3-3x+3x^2+5x^3-5x^4+5x^5) + \sqrt{-\frac{6i}{3i+\sqrt{3}}}\left(3i+\sqrt{3}\right)(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{3}+}{\left(3i+\sqrt{\right.}\right.}}\right.}{55\sqrt{1-x+x^2}}$$

```
[In] Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/(
3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqrt[
3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3])*x
)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^4\sqrt{x^3+1}}{11}+\frac{6x\sqrt{x^3+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x(5x^3+3)\sqrt{1+x}\sqrt{x^2-x+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{55\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(5x^7+3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{i\cdots}\right.}{55(x^3+1)}$

```
[In] int(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/11*x^4*(x^3+1)^(1/2)
+6/55*x*(x^3+1)^(1/2)-12/55*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(
1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{55}\left(5\,x^4+3\,x\right)\sqrt{x^2-x+1}\sqrt{x+1} - \frac{12}{55}\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/55*(5*x^4 + 3*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 12/55*weierstrassPInvers
e(0, -4, x)
```

Sympy [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

```
[In] integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

```
[In] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.490 $\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

```
[Out] 2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {927}

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

```
[In] Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\text{integral}=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

```
[In] Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{9}$	18
default	$\frac{2\left(x^3+1\right)\sqrt{1+x}\,\sqrt{x^2-x+1}}{9}$	23
risch	$\frac{2\left(x^3+1\right)\sqrt{1+x}\,\sqrt{x^2-x+1}}{9}$	23
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^3\sqrt{x^3+1}}{9}+\frac{2\sqrt{x^3+1}}{9}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	53

```
[In] int(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

```
[In] integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(17) = 34.

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\begin{aligned}\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx&=\frac{2}{315}\left((5\,(7\,x-23)(x+1)+258)(x+1)-213\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\\&\quad+\frac{2}{105}\left(3\,(5\,x-12)(x+1)+71\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\end{aligned}$$

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] 2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sq
rt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x
+ 1)
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2\,(x^3+1)\,\sqrt{x+1}\,\sqrt{x^2-x+1}}{9}$$

```
[In] int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] (2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

3.491 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 294

$$\begin{aligned}&\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx\\&=\frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}\\&-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}\\&+\frac{2\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}\end{aligned}$$

```
[Out] 2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+6/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^
(1/2))+2/7*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-3/7*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2
))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {823, 285, 309, 224, 1891}

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$
$$= \frac{2\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$
$$+\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}$$

```
[In] Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (6*Sqrt[1 + x]*Sqrt[1 - x + x^2])
/(7*(1 + Sqrt[3] + x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1
- x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*(1 + x^3)) + (2*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1
 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}} \\
&= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}} \\
&= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}} \\
&\quad +\frac{\left(3\left(-1+\sqrt{3}\right)\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}}
\end{aligned}
$$

$$= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

$$+ \frac{2\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$

$$= \frac{\sqrt{1+x}\left(4x^2\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}(1-x+x^2) - 3\sqrt{2}\left(-3i+\sqrt{3}\right)\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\right.\right.}{14\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-}}$$

```
[In] Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3*
Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt
[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] + 3*Sq
rt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I
 + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(14*Sqrt
[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.73

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right)}{7\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x^2\sqrt{1+x}\,\sqrt{x^2-x+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right)\right)}{7\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(3i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+2x^5+9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\cdots}\right)}{7x^3+7}$

```
[In] int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/7*x^2*(x^3+1)^(1/2)+
6/7*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*
I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{7}\sqrt{x^2-x+1}\sqrt{x+1}x^2-\frac{6}{7}\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 - 6/7*weierstrassZeta(0, -4, weierstr
assPInverse(0, -4, x))
```

Sympy [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

```
[In] integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Giac [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

```
[In] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.492 $\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

```
[Out] 2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^
(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {727, 201, 224}

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{2}{5}x\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x
)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}\\
&= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x^3}}\\
&= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2}\\
&\quad+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$$
$$=\frac{2x\sqrt{1+x}(1-x+x^2)+\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{5\sqrt{1-x+x^2}}$$

```
[In] Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3]
)*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]
)/Sqrt[(-I)/(3*I + Sqrt[3])])/(5*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\sqrt{x^2-x+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{5\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+\cdots}{-3+\cdots}}\right.}{5(x^3+1)}$

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)+6/
5*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x + \frac{6}{5}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x + 6/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Giac [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

```
[In] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.493 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} - \frac{2\sqrt{1+x}\sqrt{1-x+x^2}\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2
-x+1)^(1/2)/(x^3+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 52, 65, 213}

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} dx = \frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1}\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcT
anh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 15.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \frac{2}{3}\left(\sqrt{1+x}\sqrt{1-x+x^2}-\operatorname{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
[Out] (2*(Sqrt[1 + x]*Sqrt[1 - x + x^2] - ArcTanh[Sqrt[1 + x]*Sqrt[1 - x + x^2]])
)/3
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(-\sqrt{x^3+1}+\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x^3+1}}$	43
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\,\left(\frac{2\sqrt{x^3+1}}{3}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	51

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x
^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) +
 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}\,dx$$

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

3.494 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}+\frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)} + \frac{\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

```
[Out] -(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+
3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^
(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 283, 309, 224, 1891}

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \frac{\sqrt{2}3^{3/4}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)} - \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x} + \frac{3\sqrt{x^2-x+1}\sqrt{x+1}}{x+\sqrt{3}+1}$$

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + x]*Sqrt[1 - x + x^2])/(1
 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*(1 + x^3)) + (Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{\sqrt{1+x^3}}{x^2}\,dx}{\sqrt{1+x^3}}\\
&= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}\\
&= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}\\
&\quad+\frac{\left(3\left(-1+\sqrt{3}\right)\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}
\end{aligned}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)} + \frac{\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.30 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+\frac{2i(1+x)}{-3i+\sqrt{3}}}\sqrt{1-\frac{2i(1+x)}{3i+\sqrt{3}}}\left(-\frac{\left(-3i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1+x}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}} + \frac{\left(-i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1+x}\cdots}{}\right)}{2\sqrt{2}\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{3-3(1+x)+(1+x)^2}}$$

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*Sq
rt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)
*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(
1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt
[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]],
(3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(2
*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right)}{\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right)\right)}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(3i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}\cdots}{-3\cdots}}\cdots\right)}{2x(x^3+1\cdots}$

[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{3\,x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")

[Out] -(3*x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x + 1)*sqrt(x + 1))/x

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}\,dx$$

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2,x)
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2, x)
```

3.495 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

```
[Out] -1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+1/2*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {929, 283, 224}

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} dx = \frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 +
x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^3}\,dx}{\sqrt{1+x^3}}\\
&= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x^3}}\\
&= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2}\\
&\quad+\frac{3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{\sqrt{1+x}\left(-\frac{2(1-x+x^2)}{x^2}-\frac{3i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{4\sqrt{1-x+x^2}}$$

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (
2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqr
t[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[1 -
x + x^2])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{2\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}x^2-9\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{4(x^3+1)x^2}$

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2)
+3/2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{3\,x^2\mathrm{weierstrassPInverse}(0,-4,x)-\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(3*x^2*weierstrassPInverse(0, -4, x) - sqrt(x^2 - x + 1)*sqrt(x + 1))/x
^2
```

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3,x)
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3, x)
```

3.496 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{935\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

```
[Out] 54/935*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/187*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)
+2/17*x^4*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-36/935*3^(3/4)*(1+x)^(3/2)*El
lipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{929, 285, 327, 224\}$

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = -\frac{36\;3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{935\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)} + \frac{54}{935}\sqrt{x+1}\sqrt{x^2-x+1}x + \frac{18}{187}\sqrt{x+1}\sqrt{x^2-x+1}x^4 + \frac{2}{17}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)x^4$$

```
[In] Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (54*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/935 + (18*x^4*Sqrt[1 + x]*Sqrt[1 - x +
 x^2])/187 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/17 - (36*3^(3/
4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(935*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
```

```
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3(1+x^3)^{3/2}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{17\sqrt{1+x^3}}$$
$$= \frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{187\sqrt{1+x^3}}$$
$$= \frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) - \frac{\left(54\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{935\sqrt{1+x^3}}$$
$$= \frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) - \frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{935\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.17

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2\left(x\sqrt{1+x}(27-27x+27x^2+100x^3-100x^4+100x^5+55x^6-55x^7+55x^8) - \frac{9i\sqrt{6}(1+x)\sqrt{\frac{3i+\cdots}{(\cdots}}\cdots}{\cdots}\right)}{935\sqrt{1-x+x^2}}$$

```
[In] Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

[Out] (2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x^
6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I +
 Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt
[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + S
qrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + S
qrt[3])]))/(935*Sqrt[1 - x + x^2])

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2x\left(55x^{6}+100x^{3}+27\right)\sqrt{1+x}\,\sqrt{x^{2}-x+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)}}{935\sqrt{x^{3}+1}\,\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{(1+x)(x^{2}-x+1)}\left(\frac{2x^{7}\sqrt{x^{3}+1}}{17}+\frac{40x^{4}\sqrt{x^{3}+1}}{187}+\frac{54x\sqrt{x^{3}+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\right.}{935\sqrt{x^{3}+1}}\right.}{x^{3}+1}$
default	$\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\left(55x^{10}+155x^{7}+27i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-81\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.}{935(x^{3}+1)}$

[In] int(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/935*x*(55*x^6+100*x^3+27)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-108/935*(3/2-1/2*I*
3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^
(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{935}\left(55\,x^7+100\,x^4+27\,x\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{108}{935}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")

[Out] 2/935*(55*x^7 + 100*x^4 + 27*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 108/935*wei
erstrassPInverse(0, -4, x)

Sympy [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
[Out] Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

```
[In] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.497 $\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2}{15}(1+x)^{5/2} \left(1-x+x^2\right)^{5/2}$$

```
[Out] 2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {927}

$$\int x^2(1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2}{15}(x+1)^{5/2} \left(x^2-x+1\right)^{5/2}$$

```
[In] Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{15}(1+x)^{5/2} \left(1-x+x^2\right)^{5/2}$$

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

```
[In] Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(1+x)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}{15}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(x^6+2x^3+1\right)}{15}$	28
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(x^6+2x^3+1\right)}{15}$	28
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^6\sqrt{x^3+1}}{15}+\frac{4x^3\sqrt{x^3+1}}{15}+\frac{2\sqrt{x^3+1}}{15}\right)}{x^3+1}$	72

```
[In] int(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^2(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
[Out] Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(17) = 34.

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 7.52

$$\begin{aligned}&\int x^2(1+x)^{3/2}\left(1-x\right.\\&\left.+x^2\right)^{3/2}dx=\frac{2}{45045}\left(\left(\left(7\left(3\left(11\left(13\,x-80\right)(x+1)+3165\right)(x+1)-16442\right)(x+1)+121227\right)(x+1)-8018\right.\\&+\frac{2}{45045}\left(\left(5\left(7\left(9\left(11\,x-57\right)(x+1)+1601\right)(x+1)-15837\right)(x+1)+65172\right)(x+1)-34077\right)\sqrt{(x+1)^2-}\\&+\frac{2}{315}\left(\left(5\left(7\,x-23\right)(x+1)+258\right)(x+1)-213\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\\&+\frac{2}{105}\left(3\left(5\,x-12\right)(x+1)+71\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\end{aligned}$$

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] 2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) +
121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
 + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1) +
 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*(7*x
 - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/
105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}\left(x^2+2\,x+1\right)}{15}$$

```
[In] int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] (2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15
```

3.498 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 325

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}+\frac{18\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

```
[Out] 18/91*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/13*x^2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1
)^(1/2)+54/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+18/91*3^(3/4)*(1+x)
^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+
1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)-27/91*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {823, 285, 309, 224, 1891}

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{18\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{91\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)} + \frac{18}{91}\sqrt{x+1}\sqrt{x^2-x+1}x^2 + \frac{54\sqrt{x+1}\sqrt{x^2-x+1}}{91\left(x+\sqrt{3}+1\right)} + \frac{2}{13}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)x^2$$

```
[In] Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/91 + (54*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/(91*(1 + Sqrt[3] + x)) + (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3)
)/13 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + S
qrt[3] + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x
^3)) + (18*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}\\
&= \frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{13\sqrt{1+x^3}}\\
&= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)\\
&\quad+\frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{x}{\sqrt{1+x^3}}\,dx}{91\sqrt{1+x^3}}
\end{aligned}$$

$$= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{91\sqrt{1+x^3}} + \frac{\left(27\left(-1+\sqrt{3}\right)\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{91\sqrt{1+x^3}}$$

$$= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)} + \frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)} + \frac{18\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\sqrt{1+x}\left(4x^2(1-x+x^2)(16+7x^3) - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-\left(\right.\right.}{\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}}\right.}{182\sqrt{1-x+x^2}}$$

```
[In] Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2x^2(7x^3+16)\sqrt{1+x}\,\sqrt{x^2-x+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\right.\right.}{91\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{32x^2\sqrt{x^3+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{91\sqrt{x^3+1}}\right.}{x^3+1}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(14x^8+27i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+46x^5-162\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.}{91}$

```
[In] int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/91*x^2*(7*x^3+16)*(1+x)^(1/2)*(x^2-x+1)^(1/2)+54/91*(3/2-1/2*I*3^(1/2))*(
(1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(
(-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2)))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.12

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{91}\left(7\,x^5+16\,x^2\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{54}{91}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/91*(7*x^5 + 16*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 54/91*weierstrassZeta
(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
[Out] Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Giac [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

```
[In] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.499 $\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{18}{55} x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11} x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

```
[Out] 18/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1
/2)+18/55*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/
2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {727, 201, 224}

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)} + \frac{18}{55}x\sqrt{x^2-x+1}\sqrt{x+1}+\frac{2}{11}x\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}$$

```
[In] Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2
]*(1 + x^3))/11 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x +
x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}\\
&= \frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{11\sqrt{1+x^3}}\\
&= \frac{18}{55}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)\\
&\quad+\frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{55\sqrt{1+x^3}}
\end{aligned}
$$

$$= \frac{18}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2x\sqrt{1+x}(1-x+x^2)(14+5x^3) + \frac{9i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{55\sqrt{1-x+x^2}}$$

```
[In] Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I)
/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
[Out] int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{2}{55}\left(5\,x^4+14\,x\right)\sqrt{x^2-x+1}\sqrt{x+1} + \frac{54}{55}\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] 2/55*(5*x^4 + 14*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 54/55*weierstrassPInver
se(0, -4, x)
```

Sympy [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int (x+1)^{\frac{3}{2}} \left(x^2-x+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int \left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Giac [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int \left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int (x+1)^{3/2} \left(x^2-x+1\right)^{3/2} dx$$

```
[In] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.500 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/9*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 52, 65, 213}

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}+\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}+\frac{2}{9}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)$$

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{\left(1+x^3\right)^{3/2}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\mathrm{Subst}\left(\int\frac{(1+x)^{3/2}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}
\end{aligned}$$

$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x^3}}$$

$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) + \frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3) - \frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x}\,dx = \frac{2}{9}\left(\sqrt{1+x}\sqrt{1-x+x^2}(4+x^3) - 3\text{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
[Out] (2*(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(4 + x^3) - 3*ArcTanh[Sqrt[1 + x]*Sqrt[1
 - x + x^2]]))/9
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(-x^3\sqrt{x^3+1}+3\,\text{arctanh}\left(\sqrt{x^3+1}\right)-4\sqrt{x^3+1}\right)}{9\sqrt{x^3+1}}$	57
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\,\left(\frac{2x^3\sqrt{x^3+1}}{9}+\frac{8\sqrt{x^3+1}}{9}-\frac{2\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{x^3+1}$	70

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] -2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/2
))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\frac{2}{9}\left(x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
[Out] 2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}{x}\,dx$$

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

3.501 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}
\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx &= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} \\
&+ \frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x} \\
&- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)} \\
&+ \frac{9\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}
\end{aligned}$$

```
[Out] 9/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)-(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+2
7/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+9/7*3^(3/4)*(1+x)^(3/2)*Ellip
ticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-27/14*
3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x
^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(
x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {929, 283, 285, 309, 224, 1891}

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx = \frac{9\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),\right.}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}$$
$$+\frac{9}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{27\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}{x}$$

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (9*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (27*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)/(7*(1 + Sqrt[3] + x)) - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/x - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(14*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3)) + (9*
Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^2}\,dx}{\sqrt{1+x^3}}\\
&= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x} + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{2\sqrt{1+x^3}}\\
&= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x}\\
&\quad + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{14\sqrt{1+x^3}}
\end{aligned}
$$

$$= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{14\sqrt{1+x^3}} + \frac{\left(27\left(-1+\sqrt{3}\right)\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{14\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{x} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)} + \frac{9\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{1+x}\left(\frac{4(1-x+x^2)(-7+2x^3)}{x} - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+}{3i+}}\right.\right.\right.}{}\right)}{28\sqrt{1-x+x^2}}$$

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(2x^3-7)}{7x}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{}}{}\right.\right.}{7\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}}{-\frac{3}{2}}}\right.\right.}{7\sqrt{x^3+1}}\right.}{x^3+1}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+4x^6-162\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{14x(x}$

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(2*x^3-7)/x+27/7*(3/2-1/2*I*3^(1/2))*((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2
-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3
/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
)*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.12

$$\int \frac{(1+x)^{3/2}\,(1-x+x^2)^{3/2}}{x^2}\,dx=\frac{(2\,x^3-7)\sqrt{x^2-x+1}\sqrt{x+1}-27\,x\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPIn}}{7\,x}$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*((2*x^3 - 7)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 27*x*weierstrassZeta(0, -4
, weierstrassPInverse(0, -4, x)))/x
```

Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2,x)
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2, x)
```

3.502 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2}-\frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{2x^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{10\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

```
[Out] 9/10*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)-1/2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/
x^2+9/10*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2
)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {929, 283, 201, 224}

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right.\right.}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1}-\frac{\sqrt{x^2-x+1}(x^3+1)\sqrt{x+1}}{2x^2}$$

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (9*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/10 - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
+ x^3))/(2*x^2) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(10*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{\left(1+x^3\right)^{3/2}}{x^3}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{2x^2}+\frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\sqrt{1+x^3}\,dx}{4\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{2x^2} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{20\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)}{2x^2} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{10\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}(1+x^3)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{1+x}\left(\frac{2(1-x+x^2)(-5+4x^3)}{x^2} - \frac{27i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\right.\right.}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right.}{20\sqrt{1-x+x^2}}$$

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))
/(20*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(4x^3-5)}{10x^2}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{10\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{10\sqrt{x^3+1}}-\frac{\sqrt{x}}{2}\right.}{x^3+1}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-81\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{20(x^3+1)x^2}$

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(4*x^3-5)/x^2+27/10*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*El
lipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I
*3^(1/2)))^(1/2))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=\frac{27\,x^2\text{weierstrassPInverse}(0,-4,x)+(4\,x^3-5)\sqrt{x^2-x+1}\sqrt{x+1}}{10\,x^2}$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*(27*x^2*weierstrassPInverse(0, -4, x) + (4*x^3 - 5)*sqrt(x^2 - x + 1)*
sqrt(x + 1))/x^2
```

Sympy [F]

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^3} \, dx = \int \frac{(x+1)^{\frac{3}{2}} \left(x^2-x+1\right)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^3} \, dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^3} \, dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^3} \, dx = \int \frac{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}}{x^3} \, dx$$

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3,x)
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3, x)
```

3.503 $\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2/5*x*(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-4/15*EllipticF((1+x-3^(1/2))/(1+x
+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {929, 327, 224}

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2x(x^3+1)}{5\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[In] Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*x*(1 + x^3))/(5*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (4*Sqrt[2 + Sqrt[3]]*Sq
rt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$
$$= \frac{6x\sqrt{1+x}(1-x+x^2) - \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{15\sqrt{1-x+x^2}}$$

```
[In] Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqr
t[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcS
inh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\sqrt{x^2-x+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{5\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x}{-3+i\sqrt{}}}\right.}{5(x^3+1)}$

```
[In] int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)-4/
5*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.18

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x - \frac{4}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 4/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.504 $\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

```
[In] Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

```
[In] Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
elliptic	$\frac{2\sqrt{(1+x)(x^2-x+1)}\,\sqrt{x^3+1}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	39

```
[In] int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\,(x^3+1)}{3\,\sqrt{x^2-x+1}\sqrt{x+1}}$$

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{(x+1)^2-3\,x}\sqrt{x+1}$$

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\,\sqrt{x^3+1}}{3}$$

```
[In] int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] (2*(x^3 + 1)^(1/2))/3
```

3.505 $\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 253

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2(1+x^3)}{\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/3*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4)*
EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/
2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {823, 309, 224, 1891}

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{2(x^3+1)}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

```
[In] Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*(1 + x^3))/(Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (2*Sqrt[2]*Sqrt[1 + x]*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2(1+x^3)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&\quad + \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.74 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}\left(3i+\sqrt{\right.}}{}\right.}{6\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

[In] Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]

[Out] ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\right)}{\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\left(iE\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}-iF\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i}{i\sqrt{3}+}}\right.\right.}{2x^3+2}$

[In] int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/

```
(-3/2-1/2*I*3^(1/2)))^(1/2)))*((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.506 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {727, 224}

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[In] Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$

$$= \frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

[Out] (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\left(3-i\sqrt{3}\right)\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)}{x^{3}+1}$	137
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^{2}-x+1)}}{\sqrt{x^{3}+1}\,\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	145

[In] int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] (3-I*3^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))/(x^3+1)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = 2\,\text{weierstrassPInverse}(0,-4,x)$$

[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")

[Out] 2*weierstrassPInverse(0, -4, x)

Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

[In] integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)

[Out] Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.507 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2\sqrt{1+x^3}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 272, 65, 213}

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2\sqrt{x^3+1}\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= -\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2}{3}\text{arctanh}\left(\sqrt{1+x}\sqrt{3-3(1+x)+(1+x)^2}\right)$$

```
[In] Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (-2*ArcTanh[Sqrt[1 + x]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]])/3
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\sqrt{x^{3}+1}\right) \sqrt{1+x}\, \sqrt{x^{2}-x+1}}{3 \sqrt{x^{3}+1}}$	33
elliptic	$-\frac{2 \sqrt{\left(1+x\right) \left(x^{2}-x+1\right)}\, \operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3 \sqrt{1+x}\, \sqrt{x^{2}-x+1}}$	40

```
[In] int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{1}{3}\,\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\,\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) - 1)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.508 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&\quad +\frac{\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

```
[Out] (-x^3-1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x
^2-x+1)^(1/2)+1/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1
/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/
2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 331, 309, 224, 1891}

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} \\ &\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} \\ &\quad -\frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{x^3+1}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}\end{aligned}$$

```
[In] Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((1 + x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]*(1
+ Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 - x + x^2]) + (Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

$$= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i-\dots\right.}{\dots}\right)}{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I +
 Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I
+ Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1
 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*
I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3
*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right)}{\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\cdots\right)\right)}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x-6\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+\cdots}{-3+\cdots}}\cdots\right)}{2x(x^3+1)\cdots}$

```
[In] int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+(3/
2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2))
)^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/
2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2
-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x +
1)*sqrt(x + 1))/x
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

```
[In] integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.509 $\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{-1-x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 1/2*(-x^3-1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-1/6*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {929, 331, 224}

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$
$$= -\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$
$$-\frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -1/2*(1 + x^3)/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (Sqrt[2 + Sqrt[3]]*Sqr
t[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt
[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 +
Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$
$$= \frac{-\frac{6\sqrt{1+x}(1-x+x^2)}{x^2} - \frac{i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{12\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt
[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{2\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2}{-3+i}}\right.}{4(x^3+1)x^2}$

[In] int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2) -1/2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x^2\mathrm{weierstrassPInverse}(0,-4,x)+\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")

[Out] -1/2*(x^2*weierstrassPInverse(0, -4, x) + sqrt(x^2 - x + 1)*sqrt(x + 1))/x^2

Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

[In] integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

```
[In] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.510 $\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] -2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+4/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)
),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {929, 294, 224}

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} - \frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (-2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{x^3}{(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{-\frac{6x}{\sqrt{1+x}}+\frac{2i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3}{3}\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{9\sqrt{1-x+x^2}}$$

[In] Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]

[Out] ((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{3\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{2x}{3\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+}{-3+}}\right.}{3(x^3+1)}$

[In] int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-2/3*x/(x^3+1)^(1/2)+4/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = -\frac{2 \left(\sqrt{x^2-x+1}\sqrt{x+1}x - 2 \left(x^3+1\right) \text{weierstrassPInverse}(0,-4,x)\right)}{3 \left(x^3+1\right)}$$

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 2*(x^3 + 1)*weierstrassPInverse(0,
-4, x))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{x^3}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{x^3}{(x+1)^{\frac{3}{2}} (x^2-x+1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{x^3}{(x^2-x+1)^{\frac{3}{2}} (x+1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x+1)^{3/2}(x^2-x+1)^{3/2}}\,dx$$

```
[In] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.511 $\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[Out] -2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx = -\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[In] Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	18
risch	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	18
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3(x^3+1)}$	25
elliptic	$-\frac{2\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{x^3+1}}$	39

```
[In] int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{x^2}{(x+1)^{\frac{3}{2}} (x^2-x+1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = -\frac{2}{3 \sqrt{x^2-x+1}\sqrt{x+1}}$$

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [F]

$$\int \frac{x^2}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{x^2}{(x^2-x+1)^{\frac{3}{2}} (x+1)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = -\frac{2}{3 \sqrt{x+1} \sqrt{x^2-x+1}}$$

```
[In] int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] -2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

3.512 $\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 282

$$
\begin{aligned}
\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} \\
&- \frac{2(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&+ \frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&- \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}
$$

```
[Out] 2/3*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(
x^2-x+1)^(1/2)-2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(
1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)+1/3*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {823, 296, 309, 224, 1891}

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2(x^3+1)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

```
[In] Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*(1 + x^3))/(3*Sqrt[1 + x]*(1
 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) + (Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sq
rt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 - x + x^2]) - (2*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{x}{(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

$$= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} - \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.43

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\right.}{}\right)}{18\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/(
3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[
(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*
I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3
])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqr
t[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/
Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F$
risch	$\frac{2x^2}{3\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)F\left(\sqrt{\right.\right.}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2}{-3+i}}\right.}{3(x^3+1)}$

[In] int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/3/(x^3+1)^(1/2)*x^2-2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x^2+(x^3+1)\text{weierstrassZeta}(0,-4,\text{weierstrassPIn}\right.}{3\left(x^3+1\right)}$$

[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")

[Out] 2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 + (x^3 + 1)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^3 + 1)

Sympy [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

```
[In] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.513 $\int \frac{1}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{1}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2))
,I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {727, 205, 224}

$$\int \frac{1}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 - x + x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1+x)^{3/2}\,(1-x+x^2)^{3/2}}\,dx = \sqrt{3-3(1+x)+(1+x)^2}\left(-\frac{2}{9\sqrt{1+x}}+\frac{2(1+x)^{3/2}}{9\,(3-3(1+x)+(1+x)^2)}\right)+\frac{i\sqrt{\ }}{-}$$

```
[In] Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9*
(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 - I
*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSi
nh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])
/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2}{-3+i}}\right.}{3(x^3+1)}$

```
[In] int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/3*x/(x^3+1)^(1/2)+2/
3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x+(x^3+1)\text{weierstrassPInverse}(0,-4,x)\right)}{3\left(x^3+1\right)}$$

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] 2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x + (x^3 + 1)*weierstrassPInverse(0, -4,
 x))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

```
[In] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.514 $\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2\sqrt{1+x^3}\,\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1
+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 53, 65, 213}

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2\sqrt{x^3+1}\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{1+x}-(1+x)^2\sqrt{\frac{1-x+x^2}{(1+x)^2}}\operatorname{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)\right)}{3(1+x)\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
[Out] (2*(Sqrt[1 + x] - (1 + x)^2*Sqrt[(1 - x + x^2)/(1 + x)^2]*ArcTanh[1/((1 + x
)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])]))/(3*(1 + x)*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-1\right)}{3(x^3+1)}$	43
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\,\left(\frac{2}{3\sqrt{x^3+1}}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	51
risch	$\frac{2}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{(1+x)(x^2-x+1)}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	58

```
[In] int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(
x^3+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(x^3+1)\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)-(x^3+1)\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)-2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqrt
(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(x+1\right)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)

[Out] Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x}\,dx$$

[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)

Giac [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x}\,dx$$

[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")

[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(x+1\right)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

[In] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)

[Out] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)

3.515 $\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 316

$$\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x^3)}{3\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} - \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{5\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2/3/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-5/3*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)
+5/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+5/9*EllipticF((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-5/6*3
^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {929, 296, 331, 309, 224, 1891}

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{5\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} - \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{2}{3x\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{5(x^3+1)}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5(x^3+1)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

```
[In] Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (5*(1 + x^3))/(3*x*Sqrt[1 + x]*Sqrt
[1 - x + x^2]) + (5*(1 + x^3))/(3*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x
+ x^2]) - (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (5*S
qrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x^2(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{5\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{3+5x^3}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\sqrt{\right.}}{}\right.}{36\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -1/3*(3 + 5*x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12*
Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqr
t[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sq
rt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3
*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]
+ (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sq
rt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{3\sqrt{x^3+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\cdots\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$-\frac{5x^3+3}{3x\sqrt{1+x}\,\sqrt{x^2-x+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right)}{3\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(5i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+15\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\cdots\right)}{6(x^3+1\cdots}$

[In] int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)-2/3/(x^3+1)^(1/2)*x^2+5/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(5\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+5\,(x^4+x)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{3\,(x^4+x)}$$

[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")

[Out] -1/3*((5*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^4 + x)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^4 + x)

Sympy [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(x+1\right)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(x+1\right)}^{3/2}\,{\left(x^2-x+1\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.516 $\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2/3/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-7/6*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(
1/2)-7/18*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 296, 331, 224}

$$\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = -\frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{7(x^3+1)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[In] Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (7*(1 + x^3))/(6*x^2*Sqrt[1 + x]*
Sqrt[1 - x + x^2]) - (7*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{1}{x^3(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{12\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{-\frac{6(3+7x^3)}{x^2\sqrt{1+x}} - \frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right), \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{36\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
+ Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{3\sqrt{x^3+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{x^3+1}}\right)}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{7x^3+3}{6x^2\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(x^2-x+1)}}{6\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(7i\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^2-21\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\cdots\right.}{12(x^3+1)x^2}$

```
[In] int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2)
-2/3*x/(x^3+1)^(1/2)-7/6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2
))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(7\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+7\,(x^5+x^2)\text{weierstrassPInverse}(0,-4,x)}{6\,(x^5+x^2)}$$

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*((7*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^5 + x^2)*weierstrass
PInverse(0, -4, x))/(x^5 + x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,{(x+1)}^{3/2}\,{(x^2-x+1)}^{3/2}}\,dx$$

```
[In] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.517 $\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 4/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2
)+4/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^
(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {929, 294, 205, 224}

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\,(x^3+1)}$$

```
[In] Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (4*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
+ x^2]*(1 + x^3)) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{x^3}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{6x(-1+2x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

```
[In] Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}+\frac{4x}{27\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{2\left(i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-3F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\cdots}\right.}{\cdots}$

```
[In] int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-2/9*x/(x^3+1)^(3/2)+4
/27*x/(x^3+1)^(1/2)+4/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2
))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{2\left((2x^4-x)\sqrt{x^2-x+1}\sqrt{x+1}+2(x^6+2x^3+1)\text{weierstrassPInverse}(\right.}{27(x^6+2x^3+1)}$$

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*((2*x^4 - x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 2*(x^6 + 2*x^3 + 1)*weier
strassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x^3}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x^3}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

```
[In] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.518 $\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}} \, dx = -\frac{2}{9(1+x)^{3/2}(1-x+x^2)^{3/2}}$$

```
[Out] -2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}} \, dx = -\frac{2}{9(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

```
[In] Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\text{integral} = -\frac{2}{9(1+x)^{3/2}(1-x+x^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=-\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

```
[In] Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{9(1+x)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}$	18
default	$-\frac{2}{9(x^{3}+1)\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	25
elliptic	$-\frac{2\sqrt{(1+x)(x^{2}-x+1)}}{9\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,(x^{3}+1)^{\frac{3}{2}}}$	39

```
[In] int(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=-\frac{2\,\sqrt{x^2-x+1}\sqrt{x+1}}{9\,(x^6+2\,x^3+1)}$$

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] -2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^2}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}}$$

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] -2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [F]

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^2}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.57

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{18\sqrt{x+1}\left(x^2-x+1\right)^{5/2}-18\,x\sqrt{x+1}\left(x^2-x+1\right)^{5/2}}{(x+1)\,\left(81\,x\left(x^2-x+1\right)^4-162\left(x^2-x+1\right)^4+81\left(x^2-x+1\right)^5\right)}$$

```
[In] int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] (18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(5
/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x + 1
)^5))
```

3.519 $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 318

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{10(1+x^3)}{27\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} - \frac{10\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 10/27*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-10/27*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)-10/81*Ellipt
icF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)+5/27*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {823, 296, 309, 224, 1891}

$$\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = -\frac{10\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{10x^2}{27\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2x^2}{9\sqrt{x+1}\sqrt{x^2-x+1}\,(x^3+1)} - \frac{10(x^3+1)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

```
[In] Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (10*x^2)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x^2)/(9*Sqrt[1 + x]*Sqrt[1
 - x + x^2]*(1 + x^3)) - (10*(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*S
qrt[1 - x + x^2]) + (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(9*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^
2]) - (10*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{x}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}+\frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}-\frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}\\
&\quad-\frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{\left(5\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{10(1+x^3)}{27\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} + \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} - \frac{10\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x)^{5/2}\,(1-x+x^2)^{5/2}}\,dx = \frac{2x^2(8+5x^3)}{27(1+x)^{3/2}\,(1-x+x^2)^{3/2}} - \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i\cdots\right.}{}\right.}{162\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3/
2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1
 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-
3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(
-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt
[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(
3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{9(x^3+1)^{\frac{3}{2}}}+\frac{10x^2}{27\sqrt{x^3+1}}-\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\right.}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{5i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}+15F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i}{}}}{}$

```
[In] int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x^2/(x^3+1)^(3/2)+
10/27/(x^3+1)^(1/2)*x^2-10/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)
))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3
^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*Ell
ipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.19

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left((5\,x^5+8\,x^2)\sqrt{x^2-x+1}\sqrt{x+1}+5\,(x^6+2\,x^3+1)\mathrm{weierstrassZeta}(0,\right.}{27\,(x^6+2\,x^3+1)}$$

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*((5*x^5 + 8*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^6 + 2*x^3 + 1)*w
eierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

```
[In] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.520 $\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}+\frac{14\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 14/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)+14/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {727, 205, 224}

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{14\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{14x}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\,(x^3+1)}$$

```
[In] Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (14*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) + (14*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x +
 x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^3}\int \frac{1}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{6x(10+7x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-}}{\sqrt{}}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}+\frac{14x}{27\sqrt{x^3+1}}+\frac{14\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{7i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-21F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{i}}{}$

```
[In] int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x/(x^3+1)^(3/2)+14
/27*x/(x^3+1)^(1/2)+14/27*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(
1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/
2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{2\left((7x^4+10x)\sqrt{x^2-x+1}\sqrt{x+1}+7(x^6+2x^3+1)\text{weierstrassPInver}\right.}{27(x^6+2x^3+1)}$$

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] 2/27*((7*x^4 + 10*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^6 + 2*x^3 + 1)*we
ierstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{5/2} (1-x+x^2)^{5/2}} \, dx = \int \frac{1}{(x+1)^{5/2} (x^2-x+1)^{5/2}} \, dx$$

```
[In] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.521 $\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx=\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}-\frac{2\sqrt{1+x^3}\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 53, 65, 213}

$$\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx=-\frac{2\sqrt{x^3+1}\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\,(x^3+1)}$$

```
[In] Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x(1+x)^{5/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}+\frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\sqrt{1+x^3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(1+x)^{5/2}\,(1-x+x^2)^{5/2}}\,dx = \frac{\frac{2(4+3x^3)}{3(1+x)^{3/2}(1-x+x^2)} - 2(1+x)\sqrt{\frac{1-x+x^2}{(1+x)^2}}\text{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)}{3\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((2*(4 + 3*x^3))/(3*(1 + x)^(3/2)*(1 - x + x^2)) - 2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + x)^2]*ArcTanh[1/((1 + x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])])/(
3*Sqrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2}{9(x^3+1)^{\frac{3}{2}}}+\frac{2}{3\sqrt{x^3+1}}-\frac{2\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	60
default	$-\frac{2\left(3\,\text{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}\,x^3-3x^3+3\,\text{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-4\right)}{9(x^3+1)\sqrt{x^2-x+1}\,\sqrt{1+x}}$	69

```
[In] int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(2/9/(x^3+1)^(3/2)+2/3/
(x^3+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\,(3\,x^3+4)\sqrt{x^2-x+1}\sqrt{x+1}-3\,(x^6+2\,x^3+1)\log\left(\sqrt{x^2-x+1}\sqrt{x\right.}{9\,(x^6+2\,x^3+1}$$

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] 1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log(
sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x +
 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x\,(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x\,(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

```
[In] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.522 $\int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 349

$$\int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{55(1+x^3)}{27\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} - \frac{55\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{55\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 22/27/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)-55/27*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/27*(x^3+1)/(1+x+3^(1/2))/
(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3
^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(
x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-55/54*3^(1/4)*EllipticE((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {929, 296, 331, 309, 224, 1891}

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{55\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$
$$-\frac{55\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$
$$+\frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{55(x^3+1)}{27x\sqrt{x+1}\sqrt{x^2-x+1}}$$
$$+\frac{55(x^3+1)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2}{9x\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}$$

```
[In] Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 22/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x*Sqrt[1 + x]*Sqrt[1 - x + x
^2]*(1 + x^3)) - (55*(1 + x^3))/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (55*
(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (55*Sqrt[
2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (55*Sqrt[2]*Sqrt[1 +
x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int \frac{1}{x^2(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(11\sqrt{1+x^3}\right)\int \frac{1}{x^2(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(55\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(55\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{54\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(55\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{54\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(55\left(-1+\sqrt{3}\right)\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{54\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{55(1+x^3)}{27\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{55\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} + \frac{55\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{27+88x^3+55x^6}{27x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} + \frac{55(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3\right.}{}\right.}{324\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

```
[In] Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -1/27*(27 + 88*x^3 + 55*x^6)/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(1
 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*S
qrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])
]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSi
nh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3
])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(
1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt
[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sqrt[3])]*S
qrt[1 - x + x^2])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.69

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{9\left(x^3+1\right)^{\frac{3}{2}}}-\frac{28x^2}{27\sqrt{x^3+1}}+\frac{55\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)E\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{55i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^4\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}+165F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^4\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{i}}{}$

```
[In] int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)-2/9
*x^2/(x^3+1)^(3/2)-28/27/(x^3+1)^(1/2)*x^2+55/27*(3/2-1/2*I*3^(1/2))*((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2
-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3
/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx =$$

$$-\frac{\left(55\,x^6+88\,x^3+27\right)\sqrt{x^2-x+1}\sqrt{x+1}+55\left(x^7+2\,x^4+x\right)\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,}{27\left(x^7+2\,x^4+x\right)}$$

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*((55*x^6 + 88*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 55*(x^7 + 2*x
^4 + x)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^7 + 2*x^4
 + x)
```

Sympy [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

```
[In] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.523 $\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}} dx = \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}(1+x^3)} - \frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{91\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right), -7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 26/27/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-91/54*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-91/162*EllipticF((1+x-
3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{929, 296, 331, 224\}$

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{91\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}} + \frac{26}{27x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{91(x^3+1)}{54x^2\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2}{9x^2\sqrt{x+1}\sqrt{x^2-x+1}\,(x^3+1)}$$

```
[In] Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 26/(27*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x^2*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) - (91*(1 + x^3))/(54*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])
- (91*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5
4*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^3}\int\frac{1}{x^3(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}+\frac{\left(13\sqrt{1+x^3}\right)\int\frac{1}{x^3(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}+\frac{\left(91\sqrt{1+x^3}\right)\int\frac{1}{x^3\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}\\
&\quad-\frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{\left(91\sqrt{1+x^3}\right)\int\frac{1}{\sqrt{1+x^3}}\,dx}{108\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}\\
&\quad-\frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}}\\
&\quad-\frac{91\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int\frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{-\frac{6\left(27+130x^3+91x^6\right)}{x^2(1+x)^{3/2}}-\frac{91i(1+x)\left(1-x+x^2\right)\sqrt{6+\frac{36i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6i}{\left(3i+\sqrt{3}\right)(1+x)}}\text{EllipticI}}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{324\left(1-x+x^2\right)^{3/2}}$$

```
[In] Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
+ x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I + S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 +
 x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(324*(1
 - x + x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}-\frac{32x}{27\sqrt{x^3+1}}-\frac{91\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{54\sqrt{x^3+1}}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$\frac{91i\sqrt{3}\,F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^5\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-273F\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^5\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{i}}{}$

```
[In] int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(x^2-x+1))^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2)
-2/9*x/(x^3+1)^(3/2)-32/27*x/(x^3+1)^(1/2)-91/54*(3/2-1/2*I*3^(1/2))*((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellipt
icF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{\left(91\,x^6+130\,x^3+27\right)\sqrt{x^2-x+1}\sqrt{x+1}+91\,\left(x^8+2\,x^5+x^2\right)\mathrm{weierstrassPInverse}(0,-4,x)}{54\,\left(x^8+2\,x^5+x^2\right)}$$

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/54*((91*x^6 + 130*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 91*(x^8 + 2*
x^5 + x^2)*weierstrassPInverse(0, -4, x))/(x^8 + 2*x^5 + x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(1/(x**3*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(x+1\right)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

```
[In] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.524 $\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = -\frac{21}{736(1-x)^2} - \frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{6023 \arctan\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{11\log(1-x)}{2304} - \frac{11\log(3+5x+4x^2)}{4608}$$

```
[Out] -21/736/(1-x)^2-97/4416/(1-x)+1/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+11/2304
*ln(1-x)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(1/2))
*23^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {836, 814, 648, 632, 210, 642}

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{6023 \arctan\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{44x+39}{276(1-x)^2 (4x^2+5x+3)} - \frac{11\log(4x^2+5x+3)}{4608} - \frac{97}{4416(1-x)} - \frac{21}{736(1-x)^2} + \frac{11\log(1-x)}{2304}$$

```
[In] Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{39+44x}{276(1-x)^2(3+5x+4x^2)}+\frac{1}{276}\int\frac{57+132x}{(-1+x)^3(3+5x+4x^2)}\,dx\\
&= \frac{39+44x}{276(1-x)^2(3+5x+4x^2)}\\
&\quad+\frac{1}{276}\int\left(\frac{63}{4(-1+x)^3}-\frac{97}{16(-1+x)^2}+\frac{253}{192(-1+x)}+\frac{2379-1012x}{192(3+5x+4x^2)}\right)dx\\
&= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2(3+5x+4x^2)}+\frac{11\log(1-x)}{2304}+\frac{\int\frac{2379-1012x}{3+5x+4x^2}\,dx}{52992}\\
&= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2(3+5x+4x^2)}\\
&\quad+\frac{11\log(1-x)}{2304}-\frac{11\int\frac{5+8x}{3+5x+4x^2}\,dx}{4608}+\frac{6023\int\frac{1}{3+5x+4x^2}\,dx}{105984}\\
&= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2(3+5x+4x^2)}+\frac{11\log(1-x)}{2304}\\
&\quad-\frac{11\log(3+5x+4x^2)}{4608}-\frac{6023\text{Subst}\left(\int\frac{1}{-23-x^2}\,dx,x,5+8x\right)}{52992}\\
&= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2(3+5x+4x^2)}\\
&\quad+\frac{6023\tan^{-1}\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}}+\frac{11\log(1-x)}{2304}-\frac{11\log(3+5x+4x^2)}{4608}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int\frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{-\frac{25392}{(-1+x)^2}+\frac{59248}{-1+x}+\frac{184(975+2204x)}{3+5x+4x^2}+36138\sqrt{23}\arctan\left(\frac{5+8x}{\sqrt{23}}\right)+34914\log(1-x)-17457\log(3+5x+4x^2)}{7312896}$$

```
[In] Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^2
) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*Lo
g[3 + 5*x + 4*x^2])/7312896
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result
default	$-\frac{1}{288(-1+x)^{2}}+\frac{7}{864(-1+x)}+\frac{11\ln(-1+x)}{2304}-\frac{-\frac{2204x}{23}-\frac{975}{23}}{6912\left(x^{2}+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{11\ln\left(4x^{2}+5x+3\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}$
risch	$\frac{\frac{97}{1104}x^{3}-\frac{407}{4416}x^{2}-\frac{5}{184}x-\frac{15}{1472}}{(-1+x)^{2}\left(4x^{2}+5x+3\right)}-\frac{11\ln\left(64x^{2}+80x+48\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}+\frac{11\ln(-1+x)}{2304}$

```
[In] int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/288/(-1+x)^2+7/864/(-1+x)+11/2304*ln(-1+x)-1/6912*(-2204/23*x-975/23)/(x
^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(
1/2))*23^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$$
$$=\frac{214176\,x^3+12046\,\sqrt{23}(4\,x^4-3\,x^3-3\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right)-224664\,x^2-5819\,(4\,x^4-}{2437632\,(4\,x^4-3}$$

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arct
an(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
+ 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x - 1
) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx=\frac{388x^3-407x^2-120x-45}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{11\log\left(x-1\right)}{2304}-\frac{11\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{6023\sqrt{23}\,\mathrm{atan}\left(\frac{8\sqrt{23}x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

```
[In] integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] (388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 + 60
23*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x}{(-1+x)^3\,(3+5x+4x^2)^2}\,dx = \frac{6023}{1218816}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right) + \frac{388\,x^3-407\,x^2-120\,x-45}{4416\,(4\,x^4-3\,x^3-3\,x^2-x+3)} - \frac{11}{4608}\,\log\left(4\,x^2+5\,x+3\right) + \frac{11}{2304}\,\log\left(x-1\right)$$

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{x}{(-1+x)^3\,(3+5x+4x^2)^2}\,dx = \frac{6023}{1218816}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right) + \frac{388\,x^3-407\,x^2-120\,x-45}{4416\,(4\,x^2+5\,x+3)(x-1)^2} - \frac{11}{4608}\,\log\left(4\,x^2+5\,x+3\right) + \frac{11}{2304}\,\log\left(|x-1|\right)$$

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 + 5*
x + 3) + 11/2304*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{11 \ln(x-1)}{2304} + \frac{-\frac{97 x^3}{4416} + \frac{407 x^2}{17664} + \frac{5 x}{736} + \frac{15}{5888}}{-x^4 + \frac{3 x^3}{4} + \frac{3 x^2}{4} + \frac{x}{4} - \frac{3}{4}} - \ln\left(x + \frac{5}{8} - \frac{\sqrt{23}\, 1\mathrm{i}}{8}\right) \left(\frac{11}{4608} + \frac{\sqrt{23}\, 6023\mathrm{i}}{2437632}\right) + \ln\left(x + \frac{5}{8} + \frac{\sqrt{23}\, 1\mathrm{i}}{8}\right) \left(-\frac{11}{4608} + \frac{\sqrt{23}\, 6023\mathrm{i}}{2437632}\right)$$

```
[In] int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

```
[Out] (11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/58
88)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 + 5
/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8)*(
(23^(1/2)*6023i)/2437632 - 11/4608)
```

3.525 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 490

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3} - \frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$
$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e-\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e+\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2/3*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(e*x+d)^(3/2)/c^3/e^3-2/5*(b*e+2*c*d)*
(e*x+d)^(5/2)/c^2/e^3+2/7*(e*x+d)^(7/2)/c/e^3-2*b*(-2*a*c+b^2)*(e*x+d)^(1/2
)/c^4+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*(b^3*c*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(5*a^2*b*
c^2*e-2*a^2*c^3*d-5*a*b^3*c*e+4*a*b^2*c^2*d+b^5*e-b^4*c*d)/(-4*a*c+b^2)^(1/
2))/c^(9/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*c*d-2*a*
b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e
-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 11.99 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$= \frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\text{arctanh}\left(\frac{\sqrt{\ }}{\sqrt{2cd\cdots}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{2}\left(\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\text{arctanh}\left(\frac{\ }{\sqrt{2cd\cdots}}\right)}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(d+ex)^{3/2}\left(ce(bd-ae)+b^2e^2+c^2d^2\right)}{3c^3e^3}$$
$$-\frac{2(d+ex)^{5/2}(be+2cd)}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3}$$

```
[In] Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int\left(-\frac{(b^3-2abc)e}{c^4}+\frac{(c^2d^2+b^2e^2+ce(bd-ae))x^2}{c^3e^2}-\frac{(2cd+be)x^4}{c^2e^2}+\frac{x^6}{ce^2}+\frac{b(b^2-2ac)(cd^2-bde+ae^2)-(b^3cd-2abc^2d}{c^4e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)}{e^2}\right.}\right.\right.}{e}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3} - \frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3} + \frac{2\text{Subst}\left(\int \frac{b(b^2-2ac)(cd^2-bde+ae^2)+(-b^3cd+2abc^2d+b^4e-3ab^2ce+a^2c^2e)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^4e^2}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3} - \frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$

$$- \frac{\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e-\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{-\frac{\sqrt{b^2-4ac}}{2e}} \right.}{c^4e^2}$$

$$- \frac{\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e+\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{b^2-4ac}}{2e}} \right.}{c^4e^2}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3} - \frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$

$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e-\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{}{\sqrt{2cd}} \right.}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+ \frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e+\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{}{\sqrt{2cd}} \right.}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.28

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{2\sqrt{d+ex}(-105b^3e^3-7c^2e(d+ex)(-2bd+5ae+3bex)+c^3(8d^3-4d^2ex+3de^2x^2+15e^3x^3)+35bce^2(}{105c^4e^3}$$

$$+ \frac{\left(ib^5e-b^3c\left(\sqrt{-b^2+4ac}d+5iae\right)+abc^2\left(2\sqrt{-b^2+4ac}d+5iae\right)+ab^2c\left(4icd-3\sqrt{-b^2+4ac}e\right)+b^4\left(-\right.\right.}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\right.}}$$

$$+ \frac{\left(-ib^5e+abc^2\left(2\sqrt{-b^2+4ac}d-5iae\right)+b^3c\left(-\sqrt{-b^2+4ac}d+5iae\right)+ab^2c\left(-4icd-3\sqrt{-b^2+4ac}e\right)-\right.}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+\right.}}$$

```
[In] Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*
x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e
 + b*(d + e*x))))/(105*c^4*e^3) + ((I*b^5*e - b^3*c*(Sqrt[-b^2 + 4*a*c]*d +
 (5*I)*a*e) + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e) + a*b^2*c*((4*I)
*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*((-I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a^
2*c^2*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2
 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)*b^5*e + a*b
*c^2*(2*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e) + b^3*c*(-(Sqrt[-b^2 + 4*a*c]*d)
+ (5*I)*a*e) + a*b^2*c*((-4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*(I*c*d +
 Sqrt[-b^2 + 4*a*c]*e) + a^2*c^2*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e
]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c]
)*e])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,e^{3}\left(\left(\left(a^{2}c^{2}-3a\,b^{2}c+b^{4}\right)e+2ab\,c^{2}d-b^{3}cd\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}-5e\left(b\left(a^{2}c^{2}-a\,b^{2}c+\frac{1}{5}b \cdots\right.\right.\right.}{\cdots}$
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex+d}\cdots\right.}{c^{4}}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex+d}\cdots\right.}{c^{4}}$
risch	$\frac{2\left(15c^{3}e^{3}x^{3}-21c^{2}e^{3}b\,x^{2}+3c^{3}d\,e^{2}x^{2}-35a\,c^{2}e^{3}x+35b^{2}c\,e^{3}x-7b\,c^{2}d\,e^{2}x-4c^{3}d^{2}ex+210c\,e^{3}ba-35a\,c^{2}d\,e^{2}-105b^{3}e^{3}+35b^{2}c \cdots\right.}{105e^{3}c^{4}}$

```
[In] int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2)*(-2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*e^3*(((
a^2*c^2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*c-1/4*b^2))^(1/2)-
5*e*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e-2/5*d*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c))*ar
ctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2))+((((a^2*c^2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*c-1/4*
b^2))^(1/2)+5*e*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e-2/5*d*(a^2*c^2-2*a*b^2*c+1/2
*b^4)*c))*2^(1/2)*e^3*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a
*c-1/4*b^2))^(1/2))*c)^(1/2))+4*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-
4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/14*c^3*x^3-1/6*(3/5*b*x+a)*x*c^2+b
*(1/6*b*x+a)*c-1/2*b^3)*e^3-1/6*d*(-3/35*c^2*x^2+(1/5*b*x+a)*c-b^2)*c*e^2+1
/15*d^2*(-2/7*c*x+b)*c^2*e+4/105*c^3*d^3)*(e*x+d)^(1/2))*((-b*e+2*c*d+(-4*e
^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2
))*c)^(1/2)/e^3/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5507 vs. $2(436) = 872$.

Time = 0.99 (sec) , antiderivative size = 5507, normalized size of antiderivative = 11.24

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{\sqrt{ex+d}x^4}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(436)=872$.

Time = 0.38 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.45

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e)*c^2*
e^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c
^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2
)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^4*
c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b
*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d +
2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2
*c^8*d*e^24 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^2
6)*c^8*e^24 + (2*c^8*d*e^24 - b*c^7*e^25)^2))/(c^8*e^24)))/((sqrt(b^2 - 4*a
*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^
2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c -
 6*a*b^3*c^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c
^3)*e)*c^2*e^2 + 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2
- 2*a*b^2*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 -
 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e)
 + (2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c^3 - 14*a*b^3*c^4
 + 12*a^2*b*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(
-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x +
d)/sqrt(-(2*c^8*d*e^24 - b*c^7*e^25 - sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25
+ a*c^7*e^26)*c^8*e^24 + (2*c^8*d*e^24 - b*c^7*e^25)^2))/(c^8*e^24)))/((sqr
t(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*
c^6*e^2)*c^2*abs(e)) + 2/105*(15*(e*x + d)^(7/2)*c^6*e^18 - 42*(e*x + d)^(5
/2)*c^6*d*e^18 + 35*(e*x + d)^(3/2)*c^6*d^2*e^18 - 21*(e*x + d)^(5/2)*b*c^5
*e^19 + 35*(e*x + d)^(3/2)*b*c^5*d*e^19 + 35*(e*x + d)^(3/2)*b^2*c^4*e^20 -
 35*(e*x + d)^(3/2)*a*c^5*e^20 - 105*sqrt(e*x + d)*b^3*c^3*e^21 + 210*sqrt(
e*x + d)*a*b*c^4*e^21)/(c^7*e^21)
```

Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 13879, normalized size of antiderivative = 28.32

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^2
*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d
*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d
*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^
7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^
(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d -
 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e
- 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2
) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d
*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2
*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*
d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^1
0*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7
*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a
^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^
2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e
^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a
*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^
11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6
*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b
^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^
9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c -
 b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1
/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*
```

c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*

b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a
^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8
*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c
^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) +
15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^
2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^
4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e
^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a
*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*
d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*
c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4
*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4
*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*
c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^
2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(1
6*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7*c^2*e
^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*d^3*e^
2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a^4*b^4
*c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*
e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e
^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7
+ (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/
2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63
*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c
- b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^
4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*
e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)
^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5
*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(
-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e -
b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6
*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1
/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5
*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2
*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^

2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e

+ 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^

8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7*c^2*e^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*d^3*e^2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a^4*b^4*c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*

```
e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3
)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^
2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2
*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4
 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 2
0*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^
4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4
*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^1
0*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7
*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2)))*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^
11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d -
63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a
*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c
*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) -
 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 +
b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((8*d)/(5*c*e^3) + (2*(b*e^4 - 2*c*d*e
^3))/(5*c^2*e^6))*(d + e*x)^(5/2) - (d + e*x)^(1/2)*((8*d^3)/(c*e^3) - (((8
*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e
^4))/(c*e^3) + ((b*e^4 - 2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e
^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e
^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3)) + (2*(d + e*x)^(7/2))/(7*c*e^3
)
```

3.526 $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2} + \frac{\left(b^3-3abc-\sqrt{b^2-4ac}(b^2-ac)\right)\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}} - \frac{\left(b^3-3abc+\sqrt{b^2-4ac}(b^2-ac)\right)\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}}$$

```
[Out] -2/3*(b*e+c*d)*(e*x+d)^(3/2)/c^2/e^2+2/5*(e*x+d)^(5/2)/c/e^2+2*(-a*c+b^2)*(
e*x+d)^(1/2)/c^3+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2)))^(1/2))*(b^3-3*a*b*c-(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*arct
anh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3-3*a*b*c+(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 4.71 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{\sqrt{2}\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\sqrt{2}\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(d+ex)^{3/2}(be+cd)}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2}$$

```
[In] Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) - (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e
+ 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d
- b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2} - \frac{2\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)+(-b^2cd+ac^2d+b^3e-2abce)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{c^3e^2}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2} + \frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx, x, \sqrt{d\right.}{c^3e^2} + \frac{\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx, x, \sqrt{d+\right.}{c^3e^2}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2}$$
$$- \frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$- \frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.43

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex}\left(15b^2e^2+c^2\left(-2d^2+dex+3e^2x^2\right)-5ce(3ae+b(d+ex))\right)}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd+\sqrt{b\right.\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(\right.}}}{}$$

```
[In] Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) -
5*c*e*(3*a*e + b*(d + e*x))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b^2
 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d +
Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e
 + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e
) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]
*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4
*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]))/(
15*c^(7/2))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.55

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(\left(-\frac{1}{2}b^3+abc\right)e-\frac{cd\left(ac-b^2\right)}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(a^2c^2-2a\,b^2c+\frac{1}{2}b^4\right)e+\frac{3db\left(ac-\frac{b^2}{3}\right)c}{2}\right)\right)\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{\cdots}\right.}\cdots}{\cdots}$
risch	$-\frac{2\left(-3c^2x^2e^2+5bc\,e^2x-c^2dex+15ac\,e^2-15b^2e^2+5bcde+2c^2d^2\right)\sqrt{ex+d}}{15e^2c^3}+\frac{-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cde+\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cde+\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4a\,b^2c\,e^2+3ab\,c^2de+b^4e^2-b^3cde+\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c
)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((((-1/2*b^3+a*
b*c)*e-1/2*c*d*(a*c-b^2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((a^2*c^2-2*a*b^2*
c+1/2*b^4)*e+3/2*d*b*(a*c-1/3*b^2)*c))*2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4
*b^2))^(1/2))*c)^(1/2)*e^2*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4
*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1
/2))*c)^(1/2)*((((1/2*b^3-a*b*c)*e+1/2*c*d*(a*c-b^2))*(-4*e^2*(a*c-1/4*b^2)
)^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e+3/2*d*b*(a*c-1/3*b^2)*c))*2^(1/2)*
e^2*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2)
)*c)^(1/2))+((-1/5*c^2*x^2+(1/3*b*x+a)*c-b^2)*e^2+1/3*d*(-1/5*c*x+b)*c*e+2/
15*c^2*d^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))
^(1/2))*c)^(1/2)*(e*x+d)^(1/2)))/c^3/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4245 vs. $2(273) = 546$.

Time = 0.95 (sec) , antiderivative size = 4245, normalized size of antiderivative = 13.02

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3
*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4
*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9
*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c
^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*
c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 +
 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (b^5*c^7 -
 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*
b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e
 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3
 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*
a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9
*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^
7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (
b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^
5*c^5)*e - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8
*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9
*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c -
```

```
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 +
 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9
*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))
 + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e^2*sqrt(((b^
6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*
c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*
a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)
*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*
c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 +
 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 2
9*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)
*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b
^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^
15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10
*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 -
 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5
 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/
(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^
3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(
2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt((
(b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*
d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^
3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^1
5)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c
^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8
+ 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4
*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 -
 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 +
```

```
 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5
*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^
2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 +
 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt
(e*x + d)) + 4*(3*c^2*e^2*x^2 - 2*c^2*d^2 - 5*b*c*d*e + 15*(b^2 - a*c)*e^2
+ (c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e^2)
```

Sympy [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

```
[In] integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
[Out] Integral(x**3*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^3}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(273) = 546.

Time = 0.38 (sec) , antiderivative size = 1074, normalized size of antiderivative = 3.29

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2
 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e)*c^2*e^2 - 2*((b^2*c^3
- a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e
+ (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqr
t(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^
4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*
```

```
c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1
/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12
- b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c^
6*e^12)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(
b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*e)*c^2*e^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3
*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*
a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) +
(2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e
^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sq
rt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^12
 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12
 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c^6*e^12)))/((sqrt(b^2 - 4*a*c)*c^6*d^2
 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) +
 2/15*(3*(e*x + d)^(5/2)*c^4*e^8 - 5*(e*x + d)^(3/2)*c^4*d*e^8 - 5*(e*x + d
)^(3/2)*b*c^3*e^9 + 15*sqrt(e*x + d)*b^2*c^2*e^10 - 15*sqrt(e*x + d)*a*c^3*
e^10)/(c^5*e^10)
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 11143, normalized size of antiderivative = 34.18

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e
^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*
d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5
*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b
^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*
c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^
(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)
*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2
*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c
^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^
4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*
c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 +
```

20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e
^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d
*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^
5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a
^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3
*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*
c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5
*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*
b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x
)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d -
33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e
+ a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a
^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c
- b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9
+ b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^
8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)
^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*
e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e +
10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^
4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3
*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*
c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2
- 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^
2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^
2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e
+ 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c
^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*
e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6
*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c
- b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b
^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7
*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3

- 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c
^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3
*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b
^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4
- 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4
- 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4
*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4
*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a
^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*
d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2
*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*
e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d
^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 -
b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5
*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d +
e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*
d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3
*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 2
8*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*
c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b
*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b
^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c
^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e
 + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a
*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^
3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1
/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*
a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b
^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*
e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c -
 b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5
*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c
*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5
*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2

)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*e - 8*
a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 3
8*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c
^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*
c^8)))^(1/2)*2i - ((2*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(3*c^2*e^4))*(d
+ e*x)^(3/2) + atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 -
5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3
- 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a
^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^
3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-
(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b
^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8
*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^
9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8
*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*
c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d
- 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-
(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a
^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a
^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 +
b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 1
4*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c
^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^
8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3
*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d
- 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(
-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*
a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c
^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^
2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 +
(8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2)
+ b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3
*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c
^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3
d(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2
- 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*

a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*

e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e
^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8
*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b
^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^
3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*
d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16
*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 -
4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c
 - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b
^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7
*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x
)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4
- 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a
^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*
d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*
c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*
b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^
7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*((8*a^4*c^
5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d -
 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*2i + (d + e*x)^(1/2)*((6*d^2)/(c*e^2) - (2*(a*e^4 + c*d^2*e^
2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^
4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)) + (2*(d + e*x)^(5/2))/(5*c*e^2)

3.527 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^
2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$= \frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}$$

```
[In] Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b(cd^2-bde+ae^2)-(bcd-b^2e+ace)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx,x,\sqrt{d+ex}\right)}{e}\\
&= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{2\text{Subst}\left(\int \frac{b(cd^2-bde+ae^2)+(-bcd+b^2e-ace)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^2e^2}\\
&= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}\\
&\quad-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^2e^2}\\
&\quad-\frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^2e^2}
\end{aligned}$$

$$
= -\frac{2b\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3ce}
+ \frac{\sqrt{2}\left(bcd - b^2e + ace - \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}
+ \frac{\sqrt{2}\left(bcd - b^2e + ace + \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}
$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.19

$$
\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx
$$

$$
= \frac{\frac{2\sqrt{c}\sqrt{d+ex}(-3be+c(d+ex))}{e} + \frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{3c^{5/2}}
$$

```
[In] Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/e + (3*Sqrt[2]*(-(b^3*e)
+ b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) -
a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d +
(b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a
*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e])
)/(3*c^(5/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{2(-xce+3be-cd)\sqrt{ex+d}}{3e\,c^2}-\frac{8\left(-\frac{\left(-3c\,e^2ba+2a\,c^2de+b^3e^2-b^2cde+\sqrt{-e^2\left(4ac-b^2\right)}\,ace-\sqrt{-e^2\left(4ac-b^2\right)}\,b^2e+\sqrt{-e^2\left(4ac-b\right.}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac\right.}\right.}}\right.}{}$
pseudoelliptic	$-\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c\,e}\left(\left(\left(ac-b^2\right)e+bcd\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+\left(-3abc+b^3\right)e^2+d\left(2a\,c^2-b^2c\right)e\right)\operatorname{arctanh}}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}\right)}{c^2}+\frac{8e\left(\frac{\left(-3c\,e^2ba+2a\,c^2de+b^3e^2-b^2cde-\sqrt{-e^2\left(4ac-b^2\right)}\,ace+\sqrt{-e^2\left(4ac-b^2\right)}\,b^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,b\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}}{}$
default	$\frac{-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}\right)}{c^2}+\frac{8e\left(\frac{\left(-3c\,e^2ba+2a\,c^2de+b^3e^2-b^2cde-\sqrt{-e^2\left(4ac-b^2\right)}\,ace+\sqrt{-e^2\left(4ac-b^2\right)}\,b^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,b\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}}{}$

```
[In] int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-c*e*x+3*b*e-c*d)*(e*x+d)^(1/2)/e/c^2-8/c*(-1/8*(-3*c*e^2*b*a+2*a*c^2
*d*e+b^3*e^2-b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a*c-b^2))^(1
/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(3*c*e^2*b*a-2*
a*c^2*d*e-b^3*e^2+b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a*c-b^2
))^(1/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. 2(270) = 540.

Time = 0.50 (sec) , antiderivative size = 2966, normalized size of antiderivative = 9.39

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*b^4*c
^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e
- (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a
^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*e*sqrt(((
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b
^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 +
8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^
5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*e)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^
6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)
*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c
```

```
^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 -
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c -
 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*s
qrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*
a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*
c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*
a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) + 4*(c*e*x + c*d - 3*b*e)*sqrt(e
*x + d))/(c^2*e)
```

Sympy [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

```
[In] integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
[Out] Integral(x**2*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^2}{cx^2+bx+a}\,dx$$

[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(270) = 540$.

Time = 0.37 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.84

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2-2\left(\sqrt{b^2-4\,ac}bc^3d^2\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2+2\left(\sqrt{b^2-4\,ac}bc^3d^2\right.\right.}{}$$

$$+\frac{2\left((ex+d)^{\frac{3}{2}}c^2e^2-3\sqrt{ex+d}bce^3\right)}{3\,c^3e^3}$$

[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")

[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*

```
a*b*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e)
 + (2*(b^2*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2
- 3*a*b^2*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arcta
n(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d
^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(
c^4*e^4)))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt
(b^2 - 4*a*c)*a*c^4*e^2)*c^2*abs(e)) + 2/3*((e*x + d)^(3/2)*c^2*e^2 - 3*sqr
t(e*x + d)*b*c*e^3)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 8171, normalized size of antiderivative = 25.86

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 -
 b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/
c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^
(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^
7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c
^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
+ 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4
- 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 -
 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 -
4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3
)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*
e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b
^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*
c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
 + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^
```

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2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^
3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c
^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^
3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-
(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^
2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*
a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b
^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*
d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3
*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(
1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18
*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2
*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*
a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b
^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2)
)/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d -
18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*
e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2
 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^
4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*
e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a
^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a
*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c
- b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a
```

c - b^2)^3)^(1/2))/(2(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e* x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^

```
(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*((b^4*e*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d
 - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*
a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((4*d)/(c*e) + (2*(b*e^2 - 2*
c*d*e))/(c^2*e^2))*(d + e*x)^(1/2)
```

3.528 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}}{c} + \frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2*(e*x+d)^(1/2)/c+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(
1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {838, 840, 1180, 214}

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\sqrt{2}\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\sqrt{2}\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{2\sqrt{d+ex}}{c}$$

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{d+ex}}{c} + \frac{\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2\sqrt{d+ex}}{c} + \frac{2\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2\sqrt{d+ex}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{c\sqrt{b^2-4ac}} + \frac{\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{c\sqrt{b^2-4ac}}$$

$$= \frac{2\sqrt{d+ex}}{c} + \frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.19

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{d+ex} - \frac{\left(-ibcd-c\sqrt{-b^2+4ac}d+ib^2e-2iace+b\sqrt{-b^2+4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} - \frac{\left(ibcd-c\sqrt{-b^2+4ac}d-ib^2e+2iace+b\sqrt{-b^2+4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{c^{3/2}}$$

```
[In] Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*e
- (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*S
qrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 + 4*a
*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/
2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}+\frac{\left(2ac\,e^2-b^2e^2+bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-2a\cdots\right.}{\cdots}}{c}$
derivativedivides	$\frac{2\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^2+b^2e^2-bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\cdots}}\right.}{c\sqrt{-e^2(4ac-b^2)}\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\cdots$
default	$\frac{2\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^2+b^2e^2-bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\cdots}}\right.}{c\sqrt{-e^2(4ac-b^2)}\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\cdots$
risch	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde+\sqrt{-e^2(4ac-b^2)}\,be-\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{c\sqrt{-e^2(4ac-b^2)}\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\cdots$

```
[In] int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*
e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-2*a*c*e^2+b^2*e^2-b*c*d*e+(-e
^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(241) = 482$.

Time = 0.40 (sec) , antiderivative size = 1721, normalized size of antiderivative = 6.00

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c
- 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sq
```

```
rt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^
2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*
d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d
 - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a
*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d
*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c -
2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) +
 sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c - 4*
a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^
3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d -
(a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (
b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c
^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) + 4*s
qrt(e*x + d))/c
```

Sympy [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

```
[In] integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x}{cx^2+bx+a}\,dx$$

[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(241) = 482$.

Time = 0.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.64

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{ex+d}}{c}$$

$$+ \frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^2c-4\,ac^2)d-(b^3-4\,abc)e)c^2e^2-2\,(\sqrt{b^2-4\,ac}c^3d^2-\sqrt{b^2-4}\right.}{}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^2c-4\,ac^2)d-(b^3-4\,abc)e)c^2e^2+2\,(\sqrt{b^2-4\,ac}c^3d^2-\sqrt{b^2-4}\right.}{}$$

[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")

```
[Out] 2*sqrt(e*x + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*c^4*d^2*e - (3
*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2
*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^
2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(
b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 + 2*(sqr
t(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*
c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (
2*b*c^4*d^2*e - (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x
 + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 +
(2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)
*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 5664, normalized size of antiderivative = 19.74

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4
*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d +
e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d
 + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b
^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b
^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b
^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*
a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a
^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 -
2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e
- b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c -
 b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*
c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e
^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(
4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1
/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^
2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d
*e^3))/c - (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)
^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*
e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(
1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*
```

c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c ^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4* c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^ 3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b ^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^ 4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2 *e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x) ^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7 *a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^ 3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6 *a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1 /2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c ^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c ^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c *e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2* d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a ^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2* c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e ^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c* e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3 *e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^ 2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(- (4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 1 2*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e* x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a *b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b ^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^ 2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*a^2*c^3*e ^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4* a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4* a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16 *a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2 *c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i

3.529 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2))*2^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

$= \{713, 1144, 214\}$

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\text{integral} = (2e)\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)\right)$$

$$+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)$$

$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} + \frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}}$$

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*
d + (b - I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a
*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-
b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(Sqrt[c]*Sq
rt[-b^2 + 4*a*c])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{c\sqrt{}}{\sqrt{\left(-be+2cd\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)}}\right.}{\sqrt{-e^2(4ac-b^2)}}$
derivativedivides	$8ec\left(\frac{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)\sqrt{2}\,\mathrm{arct}}{8c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-b\right.}}\right.$
default	$8ec\left(\frac{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)\sqrt{2}\,\mathrm{arct}}{8c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-b\right.}}\right.$

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+
d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(158) = 316$.

Time = 0.52 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.61

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c-4}{b^2c-4}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c-4}{b^2c-4}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^
3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*
```

```
e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log
(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
 + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*
sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e
^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*s
qrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2
/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) - 1/2*sqrt(2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2
*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))
*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*
c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
[Out] Integral(sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}}{cx^2+bx+a}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(158) = 316$.

Time = 0.32 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,a\right.}}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c|}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-\right.}}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)}$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 - (
4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e + sqrt((2*c
*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2*d^2
 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e)) -
1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 - (
4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e - sqrt((2*c
*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2*d^2
 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4)+\frac{\sqrt{d+e\,x}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b}{}\right.}{16\,c^2\,d^2\,e^3-16}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4)-\frac{\sqrt{d+e\,x}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b}{}\right.}{16\,c^2\,d^2\,e^3-16\,b}\right.$$

```
[In] int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)

[Out] - 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
+ 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32
*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^
2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^
3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16
*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d
- 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d
 + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*c^2*d*e^3
) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 6
4*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d
+ 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3
)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3
 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*c*d*e^4))*((e*
(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.530 $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 275

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=-\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}+\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{2}\sqrt{c}\left(bd-\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e+d
*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1
/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {911, 1301, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx = \frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

```
[In] Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{de}{a(d-x^2)}+\frac{e(cd^2-bde+ae^2-cdx^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx,x,\sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{a}-\frac{(2d)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx,x,\sqrt{d+ex}\right)}{a}\\
&= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}\\
&\quad+\frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+ex}\right)}{a\sqrt{b^2-4ac}}\\
&\quad-\frac{\left(c\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+ex}\right)}{a\sqrt{b^2-4ac}}
\end{aligned}$$

$$=-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}+\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{2}\sqrt{c}\left(bd-\sqrt{b^2-4ac}d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.97

$$\int\frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=-\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}\text{a}}{a}$$

```
[In] Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] -(((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 -
4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b*d)
 + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b
+ Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

method	result
derivativedivides	$2e^2\left(\frac{4c\left(\frac{\left(-2e^2a+bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(2e^2a-bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{(}}\right.}{a\,e^2}\right.$
default	$2e^2\left(\frac{4c\left(\frac{\left(-2e^2a+bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(2e^2a-bde-\sqrt{-e^2\left(4ac-b^2\right)}\,d\right)\sqrt{2}}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{(}}\right.}{a\,e^2}\right.$
pseudoelliptic	$-\frac{2\left(\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\,c\left(e^2a-\frac{bde}{2}-\frac{\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}}\right.\right.}{\sqrt{}}$

[In] int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)

[Out] 2*e^2*(4/a/e^2*c*(1/8*(-2*e^2*a+b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2*e^2*a-b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d^(1/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. 2(225) = 450.

Time = 0.54 (sec) , antiderivative size = 2446, normalized size of antiderivative = 8.89

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^
2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log
(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) + 2*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, 1/2*(sqrt(2)
*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
```

```
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + 4*sqrt(-d
)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
[Out] Integral(sqrt(d + e*x)/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(225) = 450.

Time = 0.32 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx = \frac{2\,d\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2-2\,\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,ac}\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2+2\,\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,ac}\right.\right.}{}$$

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 - 2*(sqrt(b^2 - 4*a*c)
*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e +
2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2
- 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e + s
qrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((
sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*
c)*a^3*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2
- 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqr
t(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e + 2*a^3*b*e^3 -
(a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)
)*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^
2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a
*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*ab
s(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 10894, normalized size of antiderivative = 39.61

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b
*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a
^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*
a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*
c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e
^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^
8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 3
84*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b
^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)
^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576
*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d
*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e
^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*
c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*
a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^
3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10
) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 +
 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i - (((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*
((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*
c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d
^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9
) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*
a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d +
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e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 -

64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)

^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c -
b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(
2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b
^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d
^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e
^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96
*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*
a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e
^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b
^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^
3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^
2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(
1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))
^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 6
4*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*
c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c
^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*
d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)
*1i)/((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d
*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4
*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*
c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d
 + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^1
0 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384
*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3
*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(
1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a
^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e
^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9
 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c
- b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^
2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*
e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10)
+ (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 6
4*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d +
8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^

3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (2*d^(1/2)*atanh((640*c^4*d^(5/2)*e^10*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) + (576*c^5*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (64*b^3*c^2*d^(3/2)*e^11*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (192*b^2*c^3*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) - (64*b^4*c^2*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (192*b^3*c^3*d^(7/2)*e^9*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3* d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) - (128*b^2*c^4*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (64*a*c^3*d^(1/2)*e^12*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (384*b*c^3*d^(3/2)*e^11*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (896*b*c^4*d^(7/2)*e^9*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11)))/a

3.531 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)} dx$

Optimal result

Integrand size = 25, antiderivative size = 368

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)} dx = -\frac{\sqrt{d+ex}}{ax} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}} + \frac{2(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2*(-a*e+b*d)*arctanh((e*x+d)^(1/
2)/d^(1/2))/a^2/d^(1/2)-(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d
-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d-a*b*e-(-a*e+b*d)
*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{2(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}-\frac{\sqrt{d+ex}}{ax}$$

```
[In] Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{de^2}{a(d-x^2)^2}-\frac{e(-bd+ae)}{a^2(d-x^2)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2}\\
&\quad+\frac{(2de)\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex}\right)}{a}\\
&\quad+\frac{(2(bd-ae))\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a^2}
\end{aligned}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(c\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{a^2\sqrt{b^2-4ac}} + \frac{\left(c\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)}{a^2\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{-\frac{a\sqrt{d+ex}}{x} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\sqrt{\cdots}\right.}{\sqrt{b^2-4ac}\sqrt{-2c\cdots}}}{a^2}$$

```
[In] Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^2 -
4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2
*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a*c*d +
 b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 -
4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*ArcTanh[S
qrt[d + e*x]/Sqrt[d]])/Sqrt[d])/a^2
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.01

method	result
derivativedivides	$2e^3 \left(\frac{-\frac{a\sqrt{ex+d}}{2x}-\frac{(ae-2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{a^2e^3} + \frac{4c\Big(-\frac{\big(-ab\,e^2-2acde+b^2de-\sqrt{-e^2(4ac-b^2)}\,ae+\sqrt{-e^2(4ac-b^2)}\,bd\big)\sqrt{2}\,a \ldots}{8\sqrt{-e^2(4ac-b^2)}\sqrt{\big(-be+2cd+\sqrt{-e^2 \ldots}}} \ldots}{\ldots} \right.$
default	$2e^3 \left(\frac{-\frac{a\sqrt{ex+d}}{2x}-\frac{(ae-2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{a^2e^3} + \frac{4c\Big(-\frac{\big(-ab\,e^2-2acde+b^2de-\sqrt{-e^2(4ac-b^2)}\,ae+\sqrt{-e^2(4ac-b^2)}\,bd\big)\sqrt{2}\,a \ldots}{8\sqrt{-e^2(4ac-b^2)}\sqrt{\big(-be+2cd+\sqrt{-e^2 \ldots}}} \ldots}{\ldots} \right.$
risch	$-\frac{\sqrt{ex+d}}{ax}-\frac{e\Big(-\frac{(-ae+2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{8c\Big(-\frac{\big(ab\,e^2+2acde-b^2de+\sqrt{-e^2(4ac-b^2)}\,ae-\sqrt{-e^2(4ac-b^2)}\,bd\big)\sqrt{2}\,\operatorname{arct}\ldots}{8\sqrt{-e^2(4ac-b^2)}\sqrt{\big(-be+2cd+\sqrt{-e^2(4 \ldots}}}\ldots}{\ldots}\ldots}{\ldots}$
pseudoelliptic	$\frac{2\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\sqrt{2}\,\left(\frac{\left(\sqrt{d}\,ea-b\,d^{\frac{3}{2}}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}+\frac{b\sqrt{d}\,ea}{2}\right)e\right)xc\operatorname{arctanh}\Big(\frac{\ldots}{\sqrt{(-b \ldots}}\ldots}{\ldots}$

```
[In] int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^3*(1/a^2/e^3*(-1/2*a*(e*x+d)^(1/2)/x-1/2*(a*e-2*b*d)/d^(1/2)*arctanh((e
*x+d)^(1/2)/d^(1/2)))+4/a^2/e^3*c*(-1/8*(-a*b*e^2-2*a*c*d*e+b^2*d*e-(-e^2*(
4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)
^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(a*b*e^
2+2*a*c*d*e-b^2*d*e-(-e^2*(4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b
*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2427 vs. $2(298) = 596$.

Time = 6.03 (sec) , antiderivative size = 4860, normalized size of antiderivative = 13.21

$$\int \frac{\sqrt{d+ex}}{x^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2
- 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b
^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 -
 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b
^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2
 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x
*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 -
 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*
c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8
*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e +
(a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(
a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a
*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^
```

2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - (2*b*d - a*e)*sqrt(d)*x*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*sqrt(e*x + d)*a*d)/(a^2*d*x), 1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^

```
2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a
^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b
^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a
^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4
*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*
a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*
b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2
*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*
c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b
^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)
*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a
*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d
 - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4
*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4
*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5
 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt
(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^
5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c +
 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9
*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*
c^3)*e)*sqrt(e*x + d)) - 2*(2*b*d - a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sq
rt(-d)/d) - 2*sqrt(e*x + d)*a*d)/(a^2*d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Integral(sqrt(d + e*x)/(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(298) = 596$.

Time = 0.34 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = -\frac{(2\,bd-ae)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2-2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\cdots\right.}{\cdots}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2+2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\cdots\right.}{\cdots}$$

$$-\frac{\sqrt{ex+d}}{ax}$$

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
[Out] -(2*b*d - a*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/4*(sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a
^2*c)*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^2*d*e + s
qrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt(-4*c^2*d
+ 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-
(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2
*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2
- 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/4*(sqrt(
-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4
*a^2*c)*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^2*d*e +
 sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e
```

```
)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt
(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c +
(2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^
2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - sqrt(e*x
 + d)/(a*x)
```

Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 19887, normalized size of antiderivative = 54.04

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e
 - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8
*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^
4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b
^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d
^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^
11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*(
(8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e
^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*
a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108
*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*
a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 -
(((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c
^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4
*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^
8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*
c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 +
8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^
6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a
^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)) + ((a*
e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*
c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d
*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*(
(8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^
3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e
^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*
e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3
```

*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4
*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x
)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^
5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2
*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^
5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3
*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^
8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9
+ 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 -
64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*
b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 +
4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c
^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d
*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))
))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)))/((16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +
 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*
b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e - 2
*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4
*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^
4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c
^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e
^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 -
 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*((8*(
d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11
- 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*
b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4
*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*
b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 - (((8
*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d
^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2
*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 +
24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*
e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^
6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^
5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d
^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)) + ((a*e - 2*b
d)((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2
*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 -
 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*((8*(16*
a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12
+ 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27
*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 -

8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(a^2*d^(1/2)) - atan(((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3 *d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12 *a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 -

20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(a^3*c^5*e^13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x)^(1/2)/(a*x) - atan(((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4

*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5
*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*
c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*
e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10
 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11
 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5
*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5
*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d +
e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*
e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*
(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(
64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2
*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9
 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d
 + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*
d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2
*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4
*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4
*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a
^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4
*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^
3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^
5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-
(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a
*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8
 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^
3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3
*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*
d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3
*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((8*(16*a^5*b*
c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a
^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b
^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^
5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*
d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^
10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^
11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a

^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b
^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d
+ e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d
^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e
^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6
*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*
c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a
^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c
^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40
*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b
^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*
b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*
c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*
(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2
e(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4
*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16
*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c -
b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a
^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^
2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a
^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3
)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((8*(16*a^5*
b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20
*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2
*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*
b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^
2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*
e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*
e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6
*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5
*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(
d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*
b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*
c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/

2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5
*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d
*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b
^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^
4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e +
a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*
(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b
*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 +
40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3
*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^
4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*
b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*
d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4
*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 1
8*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8
*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*
c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12
*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)
^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((8*(16*a^5*b
*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*
a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*
b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b
^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2
*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e
^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e
^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*
a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*
b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d
+ e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b
^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c
d(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*
d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*
e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^
6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4
*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e +
a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(

a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(a^3*c^5*e^13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i

3.532 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 531

$$\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx = -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{e(bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}-\frac{2(b^2d-acd-abe)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-(b}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-(b}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

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[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(3/2)-e*(-a*e+b*d)*arctanh((e*x
+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-2*(-a*b*e-a*c*d+b^2*d)*arctanh((e*x+d)^(1/2)
/d^(1/2))/a^3/d^(1/2)-1/2*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*x+d)^(1/2)/a/d/x+(-a
*e+b*d)*(e*x+d)^(1/2)/a^2/d/x+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d-a*c*(-2*a*e+d*(-4*a
*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d+e*(-4*a*c+b^2)^(1
/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh
(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1
```

```
/2)*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))+a*c*(2*a*e+d*(-4*a*c+b^2)
^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {911, 1301, 205, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(-abe-acd+b^2d)}{a^3\sqrt{d}}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-e\ldots}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

$$-\frac{\sqrt{2}\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\ldots}{\sqrt{2cd\ldots}}\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$-\frac{e(bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}+\frac{\sqrt{d+ex}(bd-ae)}{a^2dx}$$

$$-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}$$

```
[In] Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*Sqrt[d + e*x]/(a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*S
qrt[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/
2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^
2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[
2]*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a
*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b
^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3
*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{de^3}{a(d-x^2)^3} + \frac{e^2(-bd+ae)}{a^2(d-x^2)^2} + \frac{e(-b^2d+acd+abe)}{a^3(d-x^2)} + \frac{e((b^2-ac)(cd^2-bde+ae^2)-c(b^2d-acd-abe)x^2)}{a^3(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right) dx, x,$$

$$= \frac{2\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)-c(b^2d-acd-abe)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{(2de^2)\,\text{Subst}\left(\int \frac{1}{(d-x^2)^3} dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2e(bd-ae))\text{Subst}\left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex}\right)}{a^2} - \frac{(2(b^2d-acd-abe))\,\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex}\right)}{a^3}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2} + \frac{(bd-ae)\sqrt{d+ex}}{a^2dx} - \frac{2(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} - \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex}\right)}{2a} - \frac{(e(bd-ae))\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex}\right)}{a^2d} + \frac{\left(c\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{}}\right.}{a^3\sqrt{b^2-4ac}} - \frac{\left(c\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\right)\text{Subst}\left(\int \frac{}{-}\right.}{a^3\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4adx} + \frac{(bd-ae)\sqrt{d+ex}}{a^2dx} - \frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}} - \frac{2(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{2}\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex}\right)}{4ad}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4adx} + \frac{(bd-ae)\sqrt{d+ex}}{a^2dx} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}$$
$$- \frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}} - \frac{2(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$
$$+ \frac{\sqrt{2}\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\Bigg(}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$- \frac{\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\Bigg(}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{\frac{a\sqrt{d+ex}(4bdx-a(2d+ex))}{dx^2} + \frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{}$$

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((a*Sqrt[d + e*x]*(4*b*d*x - a*(2*d + e*x)))/(d*x^2) + (4*Sqrt[2]*Sqrt[c]*(
-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d)
+ a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2*(Sqr
t[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d +
 Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d +
 (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^
2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*ArcTanh[S
qrt[d + e*x]/Sqrt[d]])/d^(3/2))/(4*a^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\sqrt{ex+d}\,(aex-4bdx+2ad)}{4d\,a^2x^2}-\frac{e\left(-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea\sqrt{d}}-\frac{32dc\left(-\frac{\left(-2a^2c\,e^2+a\,b^2e^2+3abcde-\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{a^3e^4}+\frac{4c\left(-\frac{\left(-2a^2c\right.}{}\right.}{}\right.$
default	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{a^3e^4}+\frac{4c\left(-\frac{\left(-2a^2c\right.}{}\right.}{}\right.$
pseudoelliptic	$-\frac{-8\left(\frac{\left(-a\,d^{\frac{3}{2}}be-d^{\frac{5}{2}}\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}-\frac{3b\,d^{\frac{5}{2}}\left(ac-\frac{b^2}{3}\right)}{2}\right)e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\right.}\right.}}{}$

[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)

```
[Out] -1/4*(e*x+d)^(1/2)*(a*e*x-4*b*d*x+2*a*d)/d/a^2/x^2-1/4/a^2/d*e*(-1/e*(a^2*e
^2+4*a*b*d*e+8*a*c*d^2-8*b^2*d^2)/a/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))-
32*d/a/e*c*(-1/8*(-2*a^2*c*e^2+a*b^2*e^2+3*a*b*c*d*e-b^3*d*e+(-e^2*(4*a*c-b
^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(1/2)*b^
2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))+1/8*(2*a^2*c*e^2-a*b^2*e^2-3*a*b*c*d*e+b^3*d*e+(-e^2*(4*a*
c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(1/2)
*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3710 vs. $2(450) = 900$.

Time = 104.00 (sec) , antiderivative size = 7425, normalized size of antiderivative = 13.98

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Integral(sqrt(d + e*x)/(x**3*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^3}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(450) = 900$.

Time = 0.36 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx =$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}\left(\left(b^4-5\,ab^2c+4\,a^2c^2\right)d-\left(ab^3-4\,a^2bc\right)e\right)a^2e^2-2\,\left(\left(ab^2c-a^2c^2\right)\sqrt{\cdots}\right.\right.}{\cdots}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}\left(\left(b^4-5\,ab^2c+4\,a^2c^2\right)d-\left(ab^3-4\,a^2bc\right)e\right)a^2e^2+2\,\left(\left(ab^2c-a^2c^2\right)\sqrt{\cdots}\right.\right.}{\cdots}$$

$$+\frac{\left(8\,b^2d^2-8\,acd^2-4\,abde-a^2e^2\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,a^3\sqrt{-d}d}$$

$$+\frac{4\,(ex+d)^{\frac{3}{2}}bde-4\,\sqrt{ex+d}bd^2e-(ex+d)^{\frac{3}{2}}ae^2-\sqrt{ex+d}ade^2}{4\,a^2de^2x^2}$$

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c
 - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - s
qrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^3*c*d -
 a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a
^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4
*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (
a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^
2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 -
4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e)
- (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*
e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)
*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(
-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3
*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^
2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 4*
```

a*b*d*e - a^2*e^2)*arctan(sqrt(e*x + d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/4*(4
*(e*x + d)^(3/2)*b*d*e - 4*sqrt(e*x + d)*b*d^2*e - (e*x + d)^(3/2)*a*e^2 -
sqrt(e*x + d)*a*d*e^2)/(a^2*d*e^2*x^2)

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 33838, normalized size of antiderivative = 63.73

$$\int \frac{\sqrt{d+ex}}{x^3\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

[In] int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)

[Out] atan(((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^
10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*
e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*
d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*
b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^1
1*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*
d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b
^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*
a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*
a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 3
8*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c
^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^
2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*
c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 5
12*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1
152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^
10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3
*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b
^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 451
2*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 +
896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*
c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*
b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(

4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)
+ (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*
e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 12
8*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 +
960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^
10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d
^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*
c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a
^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12
+ 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d
^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e
^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e
^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-
(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25
*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(
1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^
6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 70
4*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10
- 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*
e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6
*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^
6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^
5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*
d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2
*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a
^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6
*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3
*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*
c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^1
0*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8
*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2
*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^
3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e
+ a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b
^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^1

2*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^1
0*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 6
4*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e
^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) -
 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(
8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e
^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8
 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^
4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*
c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^
7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 +
568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2
*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^1
2 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d
 + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*
d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^
4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*
b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*
e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^
9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^
5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c
^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*
b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 140
8*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^
11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^
5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*
b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3
*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((
b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4
*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 +
 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2
*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*
e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e
^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*
d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*

c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6* c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b

^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33 *a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (((a*e^2 + 4*b*d*e)*(d + e*x)^(1/2))/(4*a^2) + ((a*e^2 - 4*b*d*e)*(d + e*x)^(3/2))/(4*a^2*d))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*

e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^
8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c
^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20
*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a
^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d
^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5
*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4
*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^
4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a
^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 +
1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3
*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3
*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a
^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*
c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))
*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*
b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e
 + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^
4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14
- 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d
^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^
5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c
^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b
^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a
^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b
^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3
*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)
^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e -
 a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2
)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3
)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c -
 b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((1
28*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8
*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a
^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 2
4*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*
e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*
e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a
^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38
*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10

*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a

^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e -

4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^

4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2 *e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + (atan((((((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6* b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)) - (((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*

d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6
*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^
4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a
^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3
)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^
9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8
+ 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e
^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*
d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3
*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*
b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^
8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a
^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)))/((7*a^
5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e
^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^1
1 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9
- 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 +
9*a^4*b*c^7*d^2*e^13)/(a^8*d^2) + (((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 +
7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^
7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*
d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*
c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4
*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 67
2*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8
+ 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*
d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*
b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 13
2*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b
^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^
2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^1
0 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e
^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d
^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b
^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11
*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a
^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a
^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 89
6*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^1
0 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3
)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)
) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*
e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 1
28*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8
+ 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d
^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*

c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a
^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10
- 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9
+ 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12)
)/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^
(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2))
- ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8
+ 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2
*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512
*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10
- 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*
e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e
^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9
- 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 -
8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + (((((4*a^9*c^5*e^
14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*
c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d
^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^
5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9
*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a
^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 +
1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*
e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*
c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a
^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*
b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b
^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^
8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^
9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4
*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c
^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b
^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^
2) + ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a
^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a
^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 +
64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3
*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*
d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b
*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 +
512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 -
1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e
^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^
3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*
b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 45
12*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12

```
+ 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4
*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*
e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^
14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b
^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^
2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 7
52*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^1
0 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2
*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9
 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/
(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1
/2))))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(4*a^3*(d^3)^(1/2)
)
```

3.533 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 650

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5}$$
$$-\frac{2b(b^2-2ac)\,(d+ex)^{3/2}}{3c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))\,(d+ex)^{5/2}}{5c^3e^3}$$
$$-\frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$
$$+\frac{\sqrt{2}\Big((bcd-b^2e+ace)\,(b^2cd-2ac^2d-b^3e+3abce)+\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-6a}{\sqrt{b^2-4ac}}}{c^{11/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{2}\Big((bcd-b^2e+ace)\,(b^2cd-2ac^2d-b^3e+3abce)-\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-6a}{\sqrt{b^2-4ac}}}{c^{11/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2/3*b*(-2*a*c+b^2)*(e*x+d)^(3/2)/c^4+2/5*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*
(e*x+d)^(5/2)/c^3/e^3-2/7*(b*e+2*c*d)*(e*x+d)^(7/2)/c^2/e^3+2/9*(e*x+d)^(9/
2)/c/e^3-2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*(e*x+d)^(1/2)
/c^5+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2))*2^(1/2)*((a*c*e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(2
*b^5*c*d*e-10*a*b^3*c^2*d*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*e^2+4*
c*d^2)-b^4*c*(-6*a*e^2+c*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))
/c^(11/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e
*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((a*c*e-b^2*e+b
*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(-2*b^5*c*d*e+10*a*b^3*c^2*d*e-10
```

```
*a^2*b*c^3*d*e+b^6*e^2-a*b^2*c^2*(-9*a*e^2+4*c*d^2)+b^4*c*(-6*a*e^2+c*d^2)+
2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(11/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}+(ace+b\cdots\right.}{c^{11/2}\sqrt{2cd-e(b-\cdots}}$$

$$+\frac{\sqrt{2}\left((ace+b^2(-e)+bcd)(3abce-2ac^2d+b^3(-e)+b^2cd)-\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2\cdots}{\sqrt{b^2-4ac}}\right.}{c^{11/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$-\frac{2\sqrt{d+ex}(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd)}{c^5}-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}$$

$$+\frac{2(d+ex)^{5/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{5c^3e^3}-\frac{2(d+ex)^{7/2}(be+2cd)}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

```
[In] Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{e(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)}{c^5}-\frac{b(b^2-2ac)ex^2}{c^4}+\frac{(c^2d^2+b^2e^2+ce(bd-ae))x^4}{c^3e^2}-\frac{(2cd+be)x^6}{c^2e^2}+\frac{x^8}{ce^2}\right.\right.}{e}$$

$$= -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5}-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}$$
$$+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{5/2}}{5c^3e^3}-\frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$
$$+\frac{2\text{Subst}\left(\int \frac{(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)(cd^2-bde+ae^2)-(bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+\right.}{c^5e^2}$$

$$= -\frac{2(b^3cd - 2abc^2d - b^4e + 3ab^2ce - a^2c^2e)\sqrt{d+ex}}{c^5} - \frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{5/2}}{5c^3e^3} - \frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3} + \frac{2(d+ex)^{9/2}}{9ce^3} - \frac{\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce) - \frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4}{\sqrt{b^2-4ac}}\right.}{c^5e^2} - \frac{\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce) + \frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4}{\sqrt{b^2-4ac}}\right.}{c^5e^2}$$

$$= -\frac{2(b^3cd - 2abc^2d - b^4e + 3ab^2ce - a^2c^2e)\sqrt{d+ex}}{c^5} - \frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4} + \frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{5/2}}{5c^3e^3} - \frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3} + \frac{2(d+ex)^{9/2}}{9ce^3} + \frac{\sqrt{2}\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce) + \frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)}{\sqrt{b^2-4ac}}\right.}{c^{11/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce) - \frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)}{\sqrt{b^2-4ac}}\right.}{c^{11/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.39

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}(315b^4e^4 - 105b^2ce^3(4bd+9ae+bex) - 9c^3e(d+ex)^2(-2bd+7ae+5bex)}{31} - \frac{\left(ib^6e^2 + b^5e\left(-2icd+\sqrt{-b^2+4ac}e\right) + ib^4c\left(cd^2+2i\sqrt{-b^2+4ac}de-6ae^2\right) + ab^2c^2\left(-4icd^2+6\sqrt{-b^2+4a}\right.\right.}{} - \frac{\left(-ib^6e^2 + b^5e\left(2icd+\sqrt{-b^2+4ac}e\right) + b^4c\left(-icd^2-2\sqrt{-b^2+4ac}de+6iae^2\right) + ab^2c^2\left(4icd^2+6\sqrt{-b^2+4a}\right.\right.}{}$$

```
[In] Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c
^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d
```

```
*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*
(4*d + e*x))))/(315*c^5*e^3) - ((I*b^6*e^2 + b^5*e*((-2*I)*c*d + Sqrt[-b^2
+ 4*a*c]*e) + I*b^4*c*(c*d^2 + (2*I)*Sqrt[-b^2 + 4*a*c]*d*e - 6*a*e^2) + a*
b^2*c^2*((-4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e + (9*I)*a*e^2) + a*b*c^2*(
3*a*Sqrt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^
3*c*(-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (10*I)*a*e))
 - (2*I)*a^2*c^3*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]]
)/(c^(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])
*e]) - (((-I)*b^6*e^2 + b^5*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + b^4*c*((
-I)*c*d^2 - 2*Sqrt[-b^2 + 4*a*c]*d*e + (6*I)*a*e^2) + a*b^2*c^2*((4*I)*c*d^
2 + 6*Sqrt[-b^2 + 4*a*c]*d*e - (9*I)*a*e^2) + a*b*c^2*(3*a*Sqrt[-b^2 + 4*a*
c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*(-4*a*Sqrt[-b^2
+ 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (10*I)*a*e)) + (2*I)*a^2*c^3*(-(
c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(11/2)*Sqrt[-1/2*b
^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$2\left(\frac{3\left(\left(ac-b^2\right)e+bcd\right)\left(\left(abc-\frac{1}{3}b^3\right)e-\frac{2d\left(ac-\frac{b^2}{2}\right)c}{3}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(-\frac{9}{2}a^2b^2c^2+3a\,b^4c+a^3c^3-\frac{1}{2}b^6\right)e^2+5db\left(a^2c^2-a\right.\right.\right.$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

```
[In] int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((3/2*((a*c-b^2)*e+b*c*d)*((a*b*c-1/3*b^3)*e-2/3*d*(a*c-1/2*b^2)*c)*(-4*e
^2*(a*c-1/4*b^2))^(1/2)+e*((-9/2*a^2*b^2*c^2+3*a*b^4*c+a^3*c^3-1/2*b^6)*e^2
+5*d*b*(a^2*c^2-a*b^2*c+1/5*b^4)*c*e-d^2*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c^2))*
2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*e^3*arctanh(c*(e
*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(2
^(1/2)*e^3*(-3/2*((a*c-b^2)*e+b*c*d)*((a*b*c-1/3*b^3)*e-2/3*d*(a*c-1/2*b^2)
*c)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((-9/2*a^2*b^2*c^2+3*a*b^4*c+a^3*c^3-1/2
*b^6)*e^2+5*d*b*(a^2*c^2-a*b^2*c+1/5*b^4)*c*e-d^2*(a^2*c^2-2*a*b^2*c+1/2*b^
4)*c^2))*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^
(1/2))*c)^(1/2))+(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*
```

```
b^2))^(1/2))*c)^(1/2)*((1/9*c^4*x^4-1/5*(5/7*b*x+a)*x^2*c^3+(2/3*a*b*x+1/5*
b^2*x^2+a^2)*c^2+(-1/3*b^3*x-3*b^2*a)*c+b^4)*e^4+8/3*d*(5/84*c^3*x^3-3/20*(
4/7*b*x+a)*x*c^2+b*(3/20*b*x+a)*c-1/2*b^3)*c*e^3-1/5*d^2*(-1/21*c^2*x^2+(1/
7*b*x+a)*c-b^2)*c^2*e^2+2/35*d^3*(-2/9*c*x+b)*c^3*e+8/315*c^4*d^4)*(e*x+d)^
(1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/(-4*e^2*(a*c-1/
4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c
*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/e^3/c^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14340 vs. $2(592) = 1184$.

Time = 17.37 (sec) , antiderivative size = 14340, normalized size of antiderivative = 22.06

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx = \int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a} \, dx$$

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(592) = 1184.

Time = 0.40 (sec) , antiderivative size = 1612, normalized size of antiderivative = 2.48

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c^2 - 6*a*b^3*
c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^6*c - 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^
4)*d*e + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 - 12*a^3*b*c^3)*e^2)*c^2*e^2 - 2
*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 +
a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt
(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2 - 4*a*
c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2
*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3*e - (5*b^5*c^4 - 24*a*b^3*c^5 + 22
*a^2*b*c^6)*d^2*e^2 + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3*
c^6)*d*e^3 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^4)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x
+ d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*e
^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))/(c^10*e^30))
)/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 - 4*
a*c)*a*c^7*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^6*c - 7*a*b^4*c^
2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 - 1
2*a^3*b*c^3)*e^2)*c^2*e^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3
- (2*b^4*c^3 - 5*a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 -
2*a*b^3*c^3 - a^2*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c
^3 + a^3*c^4)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*
a*c)*c)*e)*abs(c)*abs(e) + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3*e - (
5*b^5*c^4 - 24*a*b^3*c^5 + 22*a^2*b*c^6)*d^2*e^2 + 2*(2*b^6*c^3 - 11*a*b^4*
c^4 + 14*a^2*b^2*c^5 - 2*a^3*c^6)*d*e^3 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^
3*c^4 - 2*a^3*b*c^5)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))
*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 - sqrt(
-4*(c^10*d^2*e^30 - b*c^9*d*e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 -
 b*c^9*e^31)^2))/(c^10*e^30)))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a
*c)*b*c^7*d*e + sqrt(b^2 - 4*a*c)*a*c^7*e^2)*c^2*abs(e)) + 2/315*(35*(e*x +
 d)^(9/2)*c^8*e^24 - 90*(e*x + d)^(7/2)*c^8*d*e^24 + 63*(e*x + d)^(5/2)*c^8
*d^2*e^24 - 45*(e*x + d)^(7/2)*b*c^7*e^25 + 63*(e*x + d)^(5/2)*b*c^7*d*e^25
 + 63*(e*x + d)^(5/2)*b^2*c^6*e^26 - 63*(e*x + d)^(5/2)*a*c^7*e^26 - 105*(e
*x + d)^(3/2)*b^3*c^5*e^27 + 210*(e*x + d)^(3/2)*a*b*c^6*e^27 - 315*sqrt(e*
x + d)*b^3*c^5*d*e^27 + 630*sqrt(e*x + d)*a*b*c^6*d*e^27 + 315*sqrt(e*x + d
)*b^4*c^4*e^28 - 945*sqrt(e*x + d)*a*b^2*c^5*e^28 + 315*sqrt(e*x + d)*a^2*c
^6*e^28)/(c^9*e^27)
```

Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 31485, normalized size of antiderivative = 48.44

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

[In] int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)

[Out] (d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3) + ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 - 2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 +

363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6 *e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*

b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^
6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^
3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*
c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^
2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4
*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^
4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5
 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c
^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c
^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 8
8*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3
*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^
4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*
b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5
*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^
5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4
*a^7*b^3*c*e^8 + 3*a^8*b*c^2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*
b^7*d^2*e^6 - 2*a^6*c^5*d^5*e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 -
 4*a^4*b^4*c^3*d^5*e^3 + 6*a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 1
6*a^5*b^3*c^3*d^4*e^4 + 8*a^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*
a^6*b^3*c^2*d^2*e^6 + 6*a^6*b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5
*d^6*e^2 + 2*a^5*b^5*c*d^2*e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6)
)/c^9 + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7
*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^
7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4
- 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2
*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^1
0*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b
*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^
3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e
^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2
)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^1
2*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^

3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a

^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4 *b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 -

8*a*b^2*c^12)))^(1/2)))*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e

^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^

2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4 *a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a ^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c ^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12* a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c ^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3 *c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9 *c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^ 3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8 *d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d ^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3 *b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4 *a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e ^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2* b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4* a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b* c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a *c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7 *c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^ 4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d *e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^ 2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1 /2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2* (-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4*a^7*b^3*c*e^8 + 3*a^8*b*c^ 2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*b^7*d^2*e^6 - 2*a^6*c^5*d^5 *e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 - 4*a^4*b^4*c^3*d^5*e^3 + 6* a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 16*a^5*b^3*c^3*d^4*e^4 + 8*a ^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*a^6*b^3*c^2*d^2*e^6 + 6*a^6 *b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5*d^6*e^2 + 2*a^5*b^5*c*d^2* e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6))/c^9 + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9* d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13* a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a *c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2* c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b

^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c
^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^
2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2
*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 -
 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5
d^3(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d
^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 -
 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 67
5*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
5*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^
2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)
^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (((8*(4*a^4
*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c
^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*
b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e
^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^
9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c
^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*
a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^
4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*
b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^
9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*
c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*
d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2
*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^
3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30
*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c

^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13
*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e -
52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c
^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 -
a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*
e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e
+ 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 3
06*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^
2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a
^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6
 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^
4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c
*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4
*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c
^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*
e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 1
76*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a
^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*
c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66
*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c
^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2
*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b
^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b
*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3
*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^
2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(

1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)))*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i - ((8*d)/(7*c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(7*c^2*e^6))*(d + e*x)^(7/2) + (d + e*x)^(5/2)*((12*d^2)/(5*c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(5*c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(5*c*e^3)) - (d + e*x)^(3/2)*((8*d^3)/(3*c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c*e^3) + ((b*e^4 - 2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(3*c*e^3)) + (2*(d + e*x)^(9/2))/(9*c*e^3)

3.534 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}$$

$$+\frac{2(b^2-ac)\,(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$

$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)-\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)+}{\sqrt{b^2-4ac}}}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)+\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)+}{\sqrt{b^2-4ac}}}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2/3*(-a*c+b^2)*(e*x+d)^(3/2)/c^3-2/5*(b*e+c*d)*(e*x+d)^(5/2)/c^2/e^2+2/7*(e
*x+d)^(7/2)/c/e^2+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(e*x+d)^(1/2)/c^4+arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a
*e^2+c*d^2)+(-2*b^4*c*d*e+8*a*b^2*c^2*d*e-4*a^2*c^3*d*e+b^5*e^2+b^3*c*(-5*a
*e^2+c*d^2)-a*b*c^2*(-5*a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e
^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a*e^2+c*d^2)+(2*b^4*c*d*e-8*a*b^2*c^2*d*e
+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3*c*d^2))/(
-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 12.67 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(-\frac{4a^2c^3de-b^3c(cd^2-5ae^2)-8ab^2c^2de+abc^2(3cd^2-5ae^2)+b^5(-e^2)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2\right.}}$$

$$+\frac{\sqrt{2}\left(\frac{4a^2c^3de-b^3c(cd^2-5ae^2)-8ab^2c^2de+abc^2(3cd^2-5ae^2)+b^5(-e^2)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc^2de+ac^2(cd^2-ae^2)+\right.}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}+\frac{2\sqrt{d+ex}(2abce-ac^2d+b^3(-e)+b^2cd)}{c^4}$$
$$-\frac{2(d+ex)^{5/2}(be+cd)}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$

```
[In] Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{e(b^2cd-ac^2d-b^3e+2abce)}{c^4}+\frac{(b^2-ac)ex^2}{c^3}-\frac{(cd+be)x^4}{c^2e}+\frac{x^6}{ce}-\frac{(b^2cd-ac^2d-b^3e+2abce)(cd^2-bde+ae^2)+(2b\cdots}{c^4e\left(\frac{cd^2-bde\cdots}{e^2}\right.}\cdots\right.\right.}{e}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4} + \frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3} - \frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2} + \frac{2(d+ex)^{7/2}}{7ce^2} - \frac{2\text{Subst}\left(\int \frac{(b^2cd-ac^2d-b^3e+2abce)(cd^2-bde+ae^2)+(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2))x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{\cdots}\right.}{c^4e^2}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4} + \frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3} - \frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2} + \frac{2(d+ex)^{7/2}}{7ce^2} - \frac{\left(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)-\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-}{\sqrt{b^2-4ac}}\right.}{c^4e^2} - \frac{\left(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)+\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-}{\sqrt{b^2-4ac}}\right.}{c^4e^2}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4} + \frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3} - \frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2} + \frac{2(d+ex)^{7/2}}{7ce^2} + \frac{\sqrt{2}\left(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)-\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(}{\sqrt{b^2-4ac}}\right.}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)+\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(}{\sqrt{b^2-4ac}}\right.}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.30

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = -\frac{2\sqrt{d+ex}(105b^3e^3+3c^3(2d-5ex)(d+ex)^2-35bce^2(4bd+6ae+bex)+7c^2e(3b(d+ex)^2+5ae(4d+ex)}{105c^4e^2} + \frac{\left(ib^5e^2+b^4e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ib^3c\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-5ae\right)\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2-cd(\sqrt{\right.\right.}{c^9} + \frac{\left(-ib^5e^2+b^4e\left(2icd+\sqrt{-b^2+4ac}e\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2+cd\left(-\sqrt{-b^2+4ac}d+4iae\right)\right)+abc^2\left(3icd^2+e\right.\right.}{}$$

```
[In] Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*e
^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)))
)/(105*c^4*e^2) + ((I*b^5*e^2 + b^4*e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) +
 I*b^3*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sqrt[-
b^2 + 4*a*c]*e^2 - c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)) + a*b*c^2*((-3*I
)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 +
 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*S
qrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)
*b^5*e^2 + b^4*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a*c^2*(a*Sqrt[-b^2 +
4*a*c]*e^2 + c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + a*b*c^2*((3*I)*c*
d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*((-I)*c*d^2 + e*(-2*S
qrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*
d*(Sqrt[-b^2 + 4*a*c]*d - (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2
*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$-\frac{\left(\left(\left(a^2c^2-3a\,b^2c+b^4\right)e^2+2d\left(2ba\,c^2-b^3c\right)e-c^2d^2\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-5\left(b\left(a^2c^2-a\,b^2c+\frac{1}{5}b^4\right)e^2-\frac{4d\left(a^2c^2-2a\,b^2\right.}{5}\right.\right.}{}$
risch	$\frac{2\left(15c^3e^3x^3-21c^2e^3b\,x^2+24c^3d\,e^2x^2-35a\,c^2e^3x+35b^2c\,e^3x-42b\,c^2d\,e^2x+3c^3d^2ex+210c\,e^3ba-140a\,c^2d\,e^2-105b^3e^3+14\right.}{105e^2c^4}$
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+d}-a\,c^2d\,e^2\sqrt{ex+d}-b^3e^3\sqrt{ex+d}+b^2c\right.}{c^4}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+d}-a\,c^2d\,e^2\sqrt{ex+d}-b^3e^3\sqrt{ex+d}+b^2c\right.}{c^4}$

```
[In] int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-(((a^2*c^2-3*a
*b^2*c+b^4)*e^2+2*d*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b^2))*(-4*e^2*(a*c-1/4
*b^2))^(1/2)-5*(b*(a^2*c^2-a*b^2*c+1/5*b^4)*e^2-4/5*d*(a^2*c^2-2*a*b^2*c+1/
2*b^4)*c*e-3/5*d^2*b*(a*c-1/3*b^2)*c^2)*e)*2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c
-1/4*b^2))^(1/2))*c)^(1/2)*e^2*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d
+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((((a^2*c^2-3*a*b^2*c+b^4)*e^2+2*d
*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b^2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+5*(b*(
a^2*c^2-a*b^2*c+1/5*b^4)*e^2-4/5*d*(a^2*c^2-2*a*b^2*c+1/2*b^4)*c*e-3/5*d^2*
b*(a*c-1/3*b^2)*c^2)*e)*2^(1/2)*e^2*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*
c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+4*(-4*e^2*(a*c-1/4*b^2))^(1/2)*
((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*((1/14*c^3
*x^3-1/6*(3/5*b*x+a)*x*c^2+b*(1/6*b*x+a)*c-1/2*b^3)*e^3-2/3*d*(-6/35*c^2*x^
2+(3/10*b*x+a)*c-b^2)*c*e^2-1/10*d^2*(-1/7*c*x+b)*c^2*e-1/35*c^3*d^3))*((-b
*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/e^2/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. $2(527) = 1054$.

Time = 7.34 (sec) , antiderivative size = 11459, normalized size of antiderivative = 19.72

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(527) = 1054$.

Time = 0.40 (sec) , antiderivative size = 1397, normalized size of antiderivative = 2.40

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c^2 - 5*a*b^2*c
^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e + (b^6 - 7*
```

```
a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 - 2*((b^2*c^4 - a*c^5)*s
qrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b
^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*
b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)
*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e - (5*b^4*c^4 - 19*a*b^2*
c^5 + 8*a^2*c^6)*d^2*e^2 + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^3
- (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqr
t(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^8*d*e^16
- b*c^7*e^17 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16
+ (2*c^8*d*e^16 - b*c^7*e^17)^2))/(c^8*e^16)))/((sqrt(b^2 - 4*a*c)*c^7*d^2
- sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2*abs(e)) -
1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c^2 - 5*a*b^2*c
^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e + (b^6 - 7*
a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 + 2*((b^2*c^4 - a*c^5)*s
qrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b
^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*
b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e - (5*b^4*c^4 - 19*a*b^2*
c^5 + 8*a^2*c^6)*d^2*e^2 + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^3
- (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^8*d*e^16
- b*c^7*e^17 - sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16
+ (2*c^8*d*e^16 - b*c^7*e^17)^2))/(c^8*e^16)))/((sqrt(b^2 - 4*a*c)*c^7*d^2
- sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2*abs(e)) +
2/105*(15*(e*x + d)^(7/2)*c^6*e^12 - 21*(e*x + d)^(5/2)*c^6*d*e^12 - 21*(e*
x + d)^(5/2)*b*c^5*e^13 + 35*(e*x + d)^(3/2)*b^2*c^4*e^14 - 35*(e*x + d)^(3
/2)*a*c^5*e^14 + 105*sqrt(e*x + d)*b^2*c^4*d*e^14 - 105*sqrt(e*x + d)*a*c^5
*d*e^14 - 105*sqrt(e*x + d)*b^3*c^3*e^15 + 210*sqrt(e*x + d)*a*b*c^4*e^15)/
(c^7*e^14)
```

Mupad [B] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 25497, normalized size of antiderivative = 43.88

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e
^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^
2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12
*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11
*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*
a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*
```

a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3
*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3
d^3(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^
4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*
b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c
^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*
b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c
^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^
3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^1
1 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*
c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*
a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3
+ 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^
2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e
^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b
^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^
6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*
c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^
5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^
2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3
+ a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^

6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)

^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4 *e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e ^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e ^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a *b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^ 5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e ^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a* b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^ 2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e ^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d ^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4* c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1 /2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^ 8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2) ^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5 *d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(- (4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^ 4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2* e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)) /(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5 *d^6*e^2 - a^5*c^4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3* b^3*c^3*d^5*e^3 + 6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^ 3*c^2*d^3*e^5 - 12*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d* e^7 - 4*a^3*b^5*c*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a ^5*b*c^3*d^3*e^5))/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^ 5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3 *e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b ^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3* b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^ 3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2) ^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10* c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3* (-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^ 7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4* a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^ 5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d* e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^ 2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d* e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^

2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7
*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3
 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 +
 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d
^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13
*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e
 - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*
e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 -
 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*
a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c
^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2
 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c
^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*
b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
e^3(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33
*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2
*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c
^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*
d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2
) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*
a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e
^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e
^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a
^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b
*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*
e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5)
)/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c
^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 1
38*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(

1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e
^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3
*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 -
 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*
a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*
a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^1
1*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10
*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33
*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^
3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b
^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a
*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*
c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a
*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((6*d)/(5*c*e^2)
 + (2*(b*e^3 - 2*c*d*e^2))/(5*c^2*e^4))*(d + e*x)^(5/2) + atan(((((8*(4*a^3
*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^
6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c
^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^
3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^
3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(

4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^

5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*
e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)
*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2
*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3
*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 1
5*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e
 + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a
^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^
5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^
2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 -
 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*
a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4
*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c
^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 12
9*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7
*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d
*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e
+ 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16
*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6
- 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*
e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*
d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^
6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c
^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2
+ 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*
a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^
3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*
a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 3
8*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*

c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*
a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a
^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*
b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*
b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
+ b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a
^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5*d^6*e^2 - a^5*c^
4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3*b^3*c^3*d^5*e^3 +
6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^3*c^2*d^3*e^5 - 1
2*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d*e^7 - 4*a^3*b^5*c
*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a^5*b*c^3*d^3*e^5)
)/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d
*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*
d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 -
12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^
11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
0*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 3
3*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c
^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*
b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*
a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3
*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*
a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2
*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^
7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
+ 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*
b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6

*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2

```
*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c
^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*
d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2
)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*
(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2
) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*
e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*
b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15
*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e
+ 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^
3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2
*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^
(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 +
 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*
e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^
5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^
4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*
b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5
*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^
3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2
*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^
3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8
*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*
d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e
```

```
*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^11*e^3 - 8*a^4*c^7
*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
+ 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b
^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 +
84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 12
6*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a
^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a
^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i + (d + e*x)^(3/2)*((2*d^2)/(c*e^2)
- (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(3*c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e
^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(3*c*e^2)) - (d + e*x)^(1/
2)*((2*d^3)/(c*e^2) - (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^4))*
(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c*e^2) + ((b*e^3 - 2*c*d*e^2)*((6*d^2)/(c*e
^2) - (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b
*e^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)))/(c*e^2)) + (2*
(d + e*x)^(7/2))/(7*c*e^2)
```

3.535 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 441

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$

$$+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)+\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)-\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*(a*c*e-b^2*e+b*c*d)*(e*x+d
)^(1/2)/c^3+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))*2^(1/2)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*a*b
*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(
1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((-b*e+c
*d)*(2*a*c*e-b^2*e+b*c*d)+(-2*b^3*c*d*e+6*a*b*c^2*d*e+b^4*e^2+b^2*c*(-4*a*e
^2+c*d^2)-2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {911, 1301, 1180, 214}

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)+\frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

$$+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)-\frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{2cd-e\left(\sqrt{b^2}\right.}}\right.}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$-\frac{2\sqrt{d+ex}\left(ace+b^2(-e)+bcd\right)}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}$$

```
[In] Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{e(bcd-b^2e+ace)}{c^3}-\frac{bex^2}{c^2}+\frac{x^4}{c}+\frac{(bcd-b^2e+ace)(cd^2-bde+ae^2)-(cd-be)(bcd-b^2e+2ace)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{\right.}{e}$$

$$= -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce} + \frac{2\text{Subst}\left(\int \frac{(bcd-b^2e+ace)(cd^2-bde+ae^2)-(cd-be)(bcd-b^2e+2ace)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{c^3e^2}$$

$$= -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce} - \frac{\left((cd-be)(bcd-b^2e+2ace)-\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b^2-4ac}}{2e}-2\right.}{c^3e^2} - \frac{\left((cd-be)(bcd-b^2e+2ace)+\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{-\frac{\sqrt{b^2-4ac}}{2e}-\right.}{c^3e^2}$$

$$= -\frac{2(bcd - b^2e + ace)\sqrt{d+ex}}{c^3} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$

$$+ \frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right) + \frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-(b}}\right.}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+ \frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right) - \frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-(b}}\right.}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\frac{2\sqrt{c}\sqrt{d+ex}\left(15b^2e^2+3c^2(d+ex)^2-5ce(4bd+3ae+bex)\right)}{e} - \frac{15\sqrt{2}\left(-b^4e^2+b^3e\left(2cd+\sqrt{b^2-4ac}e\right)+bc\left(-2a\sqrt{b^2-4ac}e^2+\right.\right.}{}}{}$$

```
[In] Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d +
3*a*e + b*e*x)))/e - (15*Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 - 4*
a*c]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6*a*
e)) - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 +
e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b
- Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sqrt[b^2
- 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^2*c*(c
*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2
 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e]))/(15*c^(7/2))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,e\left(\left(\left(\left(ac-\frac{b^{2}}{2}\right)e+\frac{bcd}{2}\right)(be-cd)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}+e\left(\left(a^{2}c^{2}-2a\,b^{2}c+\frac{1}{2}b^{4}\right)e^{2}+\cdots\right.\right.\right.\right.}{\cdots}$
risch	$-\frac{2\left(-3c^{2}x^{2}e^{2}+5bc\,e^{2}x-6c^{2}dex+15ac\,e^{2}-15b^{2}e^{2}+20bcde-3c^{2}d^{2}\right)\sqrt{ex+d}}{15e\,c^{3}}+\frac{-\frac{\left(2a^{2}c^{2}e^{3}-4a\,b^{2}c\,e^{3}+6ab\,c^{2}d\,e^{2}-2a\,c^{3}d^{2}\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^{2}}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ac\,e^{2}\sqrt{ex+d}-b^{2}e^{2}\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^{3}}+8e\left(-\frac{\left(2a^{2}c^{2}e^{3}-4a\,b^{2}c\,e^{3}+6ab\,c^{2}d\,e^{2}-2a\,c^{3}d^{2}e+b^{4}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^{2}}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ac\,e^{2}\sqrt{ex+d}-b^{2}e^{2}\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^{3}}+8e\left(-\frac{\left(2a^{2}c^{2}e^{3}-4a\,b^{2}c\,e^{3}+6ab\,c^{2}d\,e^{2}-2a\,c^{3}d^{2}e+b^{4}\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-4*e^2*(a*c-1/4*b^2))^(1/2)*(2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))
^(1/2))*c)^(1/2)*e*(((a*c-1/2*b^2)*e+1/2*b*c*d)*(b*e-c*d)*(-4*e^2*(a*c-1/4*
b^2))^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e^2+d*(3*a*b*c^2-b^3*c)*e-d^2*(a
*c-1/2*b^2)*c^2))*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c
-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(
1/2)*(2^(1/2)*e*(-((a*c-1/2*b^2)*e+1/2*b*c*d)*(b*e-c*d)*(-4*e^2*(a*c-1/4*b^
2))^(1/2)+e*((a^2*c^2-2*a*b^2*c+1/2*b^4)*e^2+d*(3*a*b*c^2-b^3*c)*e-d^2*(a*c
-1/2*b^2)*c^2))*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4
*b^2))^(1/2))*c)^(1/2))+(-4*e^2*(a*c-1/4*b^2))^(1/2)*((-1/5*c^2*x^2+(1/3*b*
x+a)*c-b^2)*e^2+4/3*d*(-3/10*c*x+b)*c*e-1/5*c^2*d^2)*((b*e-2*c*d+(-4*e^2*(a
*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)))/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b
^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/e/
c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8530 vs. $2(391) = 782$.

Time = 3.55 (sec) , antiderivative size = 8530, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(391) = 782$.

Time = 0.37 (sec) , antiderivative size = 1195, normalized size of antiderivative = 2.71

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c^
3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2
```

```
*b*c^2)*e^2)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b
^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a
^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c
)*e)*abs(c)*abs(e) + (2*(b^2*c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a*b*c^5
)*d^2*e^2 + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2 - 4*a*
b^3*c^3 + 2*a^2*b*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7 + sqrt(-
4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6 - b*c^5*e^
7)^2))/(c^6*e^6)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*
e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c
^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2*e^2 + 2*(sqr
t(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a
*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*
c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e^2 + 2*(2*b^4*c^3 - 7*
a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^4)*s
qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*
x + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7 - sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7
+ a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6 - b*c^5*e^7)^2))/(c^6*e^6)))/((sqrt(b^2
 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e
^2)*c^2*abs(e)) + 2/15*(3*(e*x + d)^(5/2)*c^4*e^4 - 5*(e*x + d)^(3/2)*b*c^3
*e^5 - 15*sqrt(e*x + d)*b*c^3*d*e^5 + 15*sqrt(e*x + d)*b^2*c^2*e^6 - 15*sqr
t(e*x + d)*a*c^3*e^6)/(c^5*e^5)
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 19465, normalized size of antiderivative = 44.14

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e
^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^
3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*
(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e
- 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^
3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2
```

+ 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3
*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^
3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*
b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 -
3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b
^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^
3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x
)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6
+ 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*
e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^
2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*
a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^
3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2
+ 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c
^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^
5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 9
9*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*
a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5
*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3
+ 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d
+ e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*
b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^
3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2
*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2

*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/
c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^
5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*
d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)
+ (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^
3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 -
4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*
a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5
*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e
^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 -
b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24
*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3
- 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2
- 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3
*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5
*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^
4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3
))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*
c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*
a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60
*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*

d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a

^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c
^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^
6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^
4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 6
3*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 -
60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^
4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c
^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 2
0*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2
*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*
e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*
b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2
*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^
3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^
3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*
d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)
^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c
*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d
^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^
4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*
c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 -
2*a^3*b^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4
*a^4*c^3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^
4*e^4 + 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3
*b*c^3*d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e
^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b
^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2
*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a
*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3
- 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*

b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + atan((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b

^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^

3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)

- 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2

)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 - 2*a^3*b^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4*a^4*c^3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^4*e^4 + 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3*b*c^3*d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + (d + e*x)^(1/2)*((2*d^2)/(c*e) - (2*(a*e^3 - b*d*e^2 + c*d^2*e))/(c^2*e^2) + (((4*d)/(c*e) + (2*(b*e^2 - 2*c*d*e))/(c^2*e^2))*(b*e^2 - 2*c*d*e))/(c*e)) - ((4*d)/(3*c*e) + (2*(b*e^2 - 2*c*d*e))/(3*c^2*e^2))*(d + e*x)^(3/2) + (2*(d + e*x)^(5/2))/(5*c*e)

3.536 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 453

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} \, dx = \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c}$$

$$+ \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)+bc\left(cd^2+e\left(2\sqrt{b^2-4ac}\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd-\sqrt{b^2-4ac}e\right)+bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right)-c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c+2*(-b*e+c*d)*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^
2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e
+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/
2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {838, 840, 1180, 214}

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2}\right.}}$$

$$-\frac{\sqrt{2}\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$+\frac{2\sqrt{d+ex}(cd-be)}{c^2}+\frac{2(d+ex)^{3/2}}{3c}$$

```
[In] Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}(-ae+(cd-be)x)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{-ae(2cd-be)+(c^2d^2+b^2e^2-ce(2bd+ae))x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{2\text{Subst}\left(\int \frac{-ae^2(2cd-be)-d(c^2d^2+b^2e^2-ce(2bd+ae))+(c^2d^2+b^2e^2-ce(2bd+ae))x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)+bc\left(cd^2+e\left(2\sqrt{b}\right.\right.\right.}{c^2\sqrt{b^2-4ac}} + \frac{\left(b^3e^2-b^2e\left(2cd-\sqrt{b^2-4ac}e\right)+bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right)-c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b}\right.\right.\right.}{c^2\sqrt{b^2-4ac}}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c}$$

$$+ \frac{\sqrt{2}\left(b^3e^2 - b^2e\left(2cd + \sqrt{b^2-4ac}e\right) + c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d - 4ae\right)\right) + bc\left(cd^2 + e\left(2\sqrt{\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd - \left(b - \sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\sqrt{2}\left(b^3e^2 - b^2e\left(2cd - \sqrt{b^2-4ac}e\right) + bc\left(cd^2 - e\left(2\sqrt{b^2-4ac}d + 3ae\right)\right) - c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{\right.\right.\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd - \left(b + \sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.09

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{d+ex}(4cd-3be+cex) + \frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-3ae\right)\right)+\right.}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-}}}{}$$

```
[In] Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x]*(4*c*d - 3*b*e + c*e*x) + (3*(I*b^3*e^2 + b^2*e*((
-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a
*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*
d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e -
 I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqr
t[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^2 + 4*
a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e)) + c*(-
(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)))*ArcTa
n[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*
e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/
(3*c^(5/2))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,\left(\left(\left((ac-b^{2})e^{2}+2bcde-c^{2}d^{2}\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}+(-3abc+b^{3})e^{3}+2d(2a\,c^{2}-b^{2}c\right.\right.}{}$
risch	$-\frac{2(-xce+3be-4cd)\sqrt{ex+d}}{3c^{2}}-\frac{8\left(-\frac{\left(-3c\,e^{3}ba+4a\,c^{2}d\,e^{2}+b^{3}e^{3}-2b^{2}cd\,e^{2}+b\,c^{2}d^{2}e+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right.}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^{2}}+\frac{\left(-3c\,e^{3}ba+4a\,c^{2}d\,e^{2}+b^{3}e^{3}-2b^{2}cd\,e^{2}+b\,c^{2}d^{2}e-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}+\sqrt{-e^{2}\left(\right.}\right.}{c\sqrt{-e^{2}\left(4a\right.}}}$
default	$-\frac{2\left(-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^{2}}+\frac{\left(-3c\,e^{3}ba+4a\,c^{2}d\,e^{2}+b^{3}e^{3}-2b^{2}cd\,e^{2}+b\,c^{2}d^{2}e-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}+\sqrt{-e^{2}\left(\right.}\right.}{c\sqrt{-e^{2}\left(4a\right.}}}$

```
[In] int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)

[Out] -(-2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(((a*c-b^2)*e
^2+2*b*c*d*e-c^2*d^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(-3*a*b*c+b^3)*e^3+2*d*(
2*a*c^2-b^2*c)*e^2+b*c^2*d^2*e)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((((a*c-b^2)*e^2+2*b*c*d*e-c^2*d^
2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(3*a*b*c-b^3)*e^3+2*d*(-2*a*c^2+b^2*c)*e^2-
b*c^2*d^2*e)*2^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*
c-1/4*b^2))^(1/2))*c)^(1/2))+2*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4
*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*((-1/3*c*x+b)*e-4/3*c*d))
*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/(-4*e^2*(a*c-1/4*b^2)
)^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4
*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5572 vs. 2(392) = 784.

Time = 1.65 (sec) , antiderivative size = 5572, normalized size of antiderivative = 12.30

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(392) = 784.

Time = 0.36 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.18

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\left((ex+d)^{\frac{3}{2}}c^2 + 3\sqrt{ex+d}c^2d - 3\sqrt{ex+d}bce\right)}{3\,c^3}$$

$$+ \frac{\left(\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}c\right)e}\left((b^2c^2 - 4\,ac^3)d^2 - 2\,(b^3c - 4\,abc^2)de + (b^4 - 5\,ab^2c + 4\,a^2c^2)e^2\right)c^2e^2 - \cdots\right.}{\cdots}$$

$$- \frac{\left(\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}c\right)e}\left((b^2c^2 - 4\,ac^3)d^2 - 2\,(b^3c - 4\,abc^2)de + (b^4 - 5\,ab^2c + 4\,a^2c^2)e^2\right)c^2e^2 + \cdots\right.}{\cdots}$$

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2/3*((e*x + d)^(3/2)*c^2 + 3*sqrt(e*x + d)*c^2*d - 3*sqrt(e*x + d)*b*c*e)/c
^3 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c^2 - 4*a*
c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c
^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - s
qrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*s
qrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*c^5*d^
3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^3 - (b^
4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))
*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d
^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2
- 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^
2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^
2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c
^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a
*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) +
 (2*b*c^5*d^3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)
*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4
*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d - b*c^3*e - sq
rt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))
/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a
*c)*a*c^4*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 13841, normalized size of antiderivative = 30.55

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1/
2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*
c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5
*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*
c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3
- 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2
+ 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54
*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d
*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(
-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4
*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 -
 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c
^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b
^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b
*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e
^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 3
6*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5
 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*
b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x
```

)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2 *c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a* c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^

4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*

c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*
a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^
6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 -
 b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*
b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c
^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b
^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*
c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4
*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*
(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*
e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*
d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2
*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^
2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^
3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^
2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^
(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5
*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^
2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5
*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 2
0*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3
*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b
^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6
*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24
*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^
2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2

```
) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*
d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*
c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^
4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4)
)/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a
^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 -
b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a
^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b
^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e +
24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b
^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i
```

3.537 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 322

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx=\frac{2e\sqrt{d+ex}}{c}-\frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2*e*(e*x+d)^(1/2)/c-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*
e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))
-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {717, 840, 1180, 214}

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx =$$

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{2e\sqrt{d+ex}}{c}$$

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\text{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{c\sqrt{b^2-4ac}} - \frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex}+\frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}d+iae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\cdots}{c^{3/2}}$$

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*c]
)*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*
b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2*d^2
 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 + 4*a*
c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I
*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[
-b^2 + 4*a*c])*e]))/c^(3/2)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{e\left(2\sqrt{ex+d}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,be-2\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right.}}\right.}{\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right.}{c}$
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\right.}}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
default	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\right.}}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
risch	$\frac{2e\sqrt{ex+d}}{c}-8e\left(\frac{\left(-2ac\,e^{2}+b^{2}e^{2}-2bcde+2c^{2}d^{2}+\sqrt{-e^{2}(4ac-b^{2})}\,be-2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e}}{\sqrt{\left(be-2cd+\sqrt{\right.}}}\right.}{8c\sqrt{-e^{2}(4ac-b^{2})}\,\sqrt{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] e/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2+(-e^2*(4*a*c-b^
2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-2*a*c*e^2+b^2*e
^2-2*b*c*d*e+2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2))^(1
/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(272) = 544$.

Time = 1.11 (sec) , antiderivative size = 2770, normalized size of antiderivative = 8.60

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*
a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3
+ 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt(2)*c*sqrt((2*
c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 +
 (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2
- 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2
- 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6
*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b
^2 - a^2*c)*e^5)*sqrt(e*x + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(
b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4
- 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
```

```
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d
^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4
)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^
2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*
x + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - 4*sqrt(e*x +
d)*e)/c
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Integral((d + e*x)**(3/2)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(272) = 544$.

Time = 0.33 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{ex+d}e}{c}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-\left(b^3-4\,abc\right)e^2)c^2e^2-2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-4}\right.\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-\left(b^3-4\,abc\right)e^2)c^2e^2+2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-4}\right.\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(e*x + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)
*(2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 - 2*(sqrt(b^2 - 4*
a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^
3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*
d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)
*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*
sqrt(e*x + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2
)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2
- 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/4*(sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d*e - (b^3
- 4*a*b*c)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c
)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*
c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sq
rt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2*d - b*
c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2
))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4
*a*c)*a*c^3*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 8334, normalized size of antiderivative = 25.88

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c
^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d
+ e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*
c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^
2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*
e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^
2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 -
 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a
^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2
*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
```

```
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^
3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 -
 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4
*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*
e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^
6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*
a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*
(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d +
 e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4
*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b
*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) - (16*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5
*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*
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d^2*e^6))/c))*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^5 - a*b^2*c^

2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4
))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d
e^2))/c)(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d
^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^
5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 +
b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*
e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a
*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 1
2*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4
*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(2*
c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*b*c^2*d^4*e^4 +
 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d^2*e^6))/c))*(-
(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b
^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a

```
*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.538 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 340

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = -\frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b\left(cd^2+ae^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b\left(cd^2+ae^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a-arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(-b*(a*e^2+c*d^2)+a*
e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*(a*e^2+c*d
^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-
4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {911, 1301, 212, 1180, 214}

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx =$$

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b(ae^2+cd^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

```
[In] Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{d^2e}{a(d-x^2)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)\,dx, x, \sqrt{d+ex}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a}-\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}\\
&= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}\\
&\quad+\frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x\right.}{a\sqrt{b^2-4ac}}\\
&\quad+\frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x,\right.}{a\sqrt{b^2-4ac}}
\end{aligned}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d - 4ae\right) - b(cd^2+ae^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d + 4ae\right) + b(cd^2+ae^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx = -\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d+4iae\right)-ib(cd^2+ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} + \frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d-4i\right.\right.}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{}}}{a}$$

```
[In] Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] -(((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I
)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-
2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2
*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^
2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]]
)/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
+ 2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.09

method	result
derivativedivides	$2e^2\left(\frac{4c\left(\frac{\left(ab\,e^3-4acd\,e^2+bc\,d^2e+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-\cdots\right.}{\cdots}\right)}{a\,e^2}\cdots\right.$
default	$2e^2\left(\frac{4c\left(\frac{\left(ab\,e^3-4acd\,e^2+bc\,d^2e+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-\cdots\right.}{\cdots}\right)}{a\,e^2}\cdots\right.$
pseudoelliptic	$\frac{-\sqrt{2}\left(\left(e^2a-c\,d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-ab\,e^3+4acd\,e^2-bc\,d^2e\right)\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex\cdots}}{\sqrt{\left(-be+2cd+\sqrt{\cdots}\right.}}\right.}{\cdots}$

```
[In] int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^2*(4/a/e^2*c*(1/8*(a*b*e^3-4*a*c*d*e^2+b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1/
2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1
/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-a*b*e^3+4*a*c*d*e
^2-b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)
/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)))-d^(3/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. $2(286) = 572$.

Time = 4.77 (sec) , antiderivative size = 5167, normalized size of antiderivative = 15.20

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{x(a+bx+cx^2)}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
[Out] Integral((d + e*x)**(3/2)/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(286) = 572$.

Time = 0.33 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,d^2\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2-2\,\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-4\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2+2\,\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-4\right.\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*
(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e
^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*
e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*
c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4
 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-
4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(
b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c
)*a^3*c*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2
 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2)*a^2*e^2 + 2
*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 -
4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)
*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4 - (a^2*b^2*c +
 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arct
an(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*
b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^
2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*ab
s(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 20897, normalized size of antiderivative = 61.46

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(
512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5
*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4
*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^
8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9
 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11
 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3
*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8
 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2
*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*
b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*
b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d
^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b
^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*
a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5
*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14
 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^1
1 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a
^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^
4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^
6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 +
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256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e
^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*
c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d
*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e -
3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*
e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*(
(b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^
2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^
10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 38
4*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2
*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*
a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*
x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576
*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d
^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c
*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3
*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*
e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4
*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9
- 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*
a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a
^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d +
e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4
*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14
+ 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*
b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*
b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c -

b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^

3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d^8*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128*b^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2*d^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e +

6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i + atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(

4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^
2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2
*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e
 + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*
d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*
e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e
^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) +
384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a
^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 12
8*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d +
e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 5
76*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2
*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4
*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d
^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*
d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c
- 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c
^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^
9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 3
2*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280
*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d
+ e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c
^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^1
4 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*
a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^
2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 -
 b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 +
a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*
c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d
*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e +
 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*
e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^
2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -

3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^

3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16
*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^
4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^
2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^
9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 9
6*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 -
 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 3
2*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*
e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64
*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*
b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*
c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c
^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3
*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^
2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*
a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2
)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 +
32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^
4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e
^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 -
160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 +
128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 -
 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d
^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^1
1 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 38
4*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 +
 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^
3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*
d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b
^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d
^8*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128
*b^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2
*d^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2
i - (2*atanh((64*a^3*c*e^16*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10
 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*
c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*
e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d
^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 102
4*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*
c^4*d^9*e^9)/a) + (576*c^5*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^

10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c*d^4*e^14) + (2304*c^4*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (128*b^2*c^4*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) + (320*b^3*c^3*d^7*e^9*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) - (192*b^4*c^2*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) + (1920*a*c^3*d^4*e^12*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (3328*b*c^3*d^5*e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (192*b^3*c*d^3*e^13*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b

^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2
+ (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5
*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/
a) + (640*b^2*c^3*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e^8 +
2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^2*c^
3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*d^7*
e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^2*d^
8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*d^3*
e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c*d^4
*e^14) + (384*b^3*c^2*d^5*e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e
^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^
2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*
d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^
2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*
d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c
*d^4*e^14) + (256*a^2*c^2*d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d
^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 1
92*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c
^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^
2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^
2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (
1536*b*c^4*d^9*e^9)/a) + (640*b^2*c^2*d^4*e^12*(d^3)^(1/2)*(d + e*x)^(1/2))
/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*
d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a
+ 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)
/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*
d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*
d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1536*b*c^4*d^7*e^9*(d^3)^(1/2)*(d + e
*x)^(1/2))/(576*c^5*d^10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 15
36*b*c^4*d^9*e^9 + 1920*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c
^3*d^8*e^10 + 384*b^3*c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^
3*d^9*e^9)/a - (192*b^4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3
*c*d^5*e^13 - 256*a^3*b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^
2*d^5*e^13 + 384*a^2*b^2*c*d^4*e^14) - (256*a^2*b*c*d*e^15*(d^3)^(1/2)*(d +
e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 -
3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5
*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c
^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (
192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 -
256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1024*a*b*c^2*d^3*e^13*(d^
3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3
*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^
14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a
+ (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9
*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*

c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) + (384*a*b^2*c*d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a))*(d^3)^(1/2))/a

3.539 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 403

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{2}\sqrt{c}\left(b^2d^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2a\left(cd^2+e\left(\sqrt{b^2-4ac}d-ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(b^2d^2-bd\left(\sqrt{b^2-4ac}d+2ae\right)-2a\left(cd^2-e\left(\sqrt{b^2-4ac}d+ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*(-2*a*e+b*d)*arctanh((e*x+d)^(
1/2)/d^(1/2))*d^(1/2)/a^2-d*(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+
d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d^2+b
*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b^2)^(1/2))))
/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1
/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c
^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2-e*(a*e+d*(-4*a*
c+b^2)^(1/2))))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = -\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{\sqrt{2}\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{2\sqrt{d}(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{d\sqrt{d+ex}}{ax}$$

```
[In] Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(Sqr
t[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[
b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{d^2e^2}{a(d-x^2)^2}-\frac{de(-bd+2ae)}{a^2(d-x^2)}+\frac{e\left(-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2\right)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{(2d^2e)\,\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex}\right)}{a} + \frac{(2d(bd-2ae))\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a^2}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{2\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$+ \frac{\left(c\left(b^2d^2-2acd^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2ae\left(\sqrt{b^2-4ac}d-ae\right)\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2c\right.}{a^2\sqrt{b^2-4ac}}$$

$$- \frac{\left(c\left(b^2d^2-2acd^2+2ae\left(\sqrt{b^2-4ac}d+ae\right)-bd\left(\sqrt{b^2-4ac}d+2ae\right)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd\right.}{a^2\sqrt{b^2-4ac}}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}$$

$$- \frac{\sqrt{2}\sqrt{c}\left(b^2d^2-2acd^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2ae\left(\sqrt{b^2-4ac}d-ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e}}{\sqrt{2cd-\left(b-\sqrt{b^2-\right.}}\right.}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+ \frac{\sqrt{2}\sqrt{c}\left(b^2d^2-2acd^2+2ae\left(\sqrt{b^2-4ac}d+ae\right)-bd\left(\sqrt{b^2-4ac}d+2ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e}}{\sqrt{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \frac{-\frac{ad\sqrt{d+ex}}{x} + \frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}d+ae\right)\right)\right)\arctan\left(\frac{}{\sqrt{-2c}}\right.}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{}$$

```
[In] Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[-b^2
 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d
 + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt
[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*
a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d - (2*I)
*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(S
qrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sqrt[d]*(2
*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.01

method	result
derivativedivides	$2e^3 \left(\frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2ac\,d^2e+b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac\right.}}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}\right.$
default	$2e^3 \left(\frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2ac\,d^2e+b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac\right.}}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}\right.$
risch	$-\frac{d\sqrt{ex+d}}{ax}-\frac{e\left(\frac{\sqrt{d}\,(3ae-2bd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea}+\frac{8c\left(-\frac{\left(-2a^2e^3+2abd\,e^2+2ac\,d^2e-b^2d^2e+2\sqrt{-e^2\left(4ac-b^2\right)}\,ade-\sqrt{-e^2\left(4ac\right.}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd\right.}}\right.}{}\right.}{}$
pseudoelliptic	$-\frac{2\left(\left(\left(-d^{\frac{3}{2}}ea+\frac{b\,d^{\frac{5}{2}}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-ac+\frac{b^2}{2}\right)d^{\frac{5}{2}}+ae\left(\sqrt{d}\,ea-b\,d^{\frac{3}{2}}\right)\right)\right)\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\right.}{}$

[In] int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)

```
[Out] 2*e^3*(4/a^2/e^3*c*(-1/8*(2*a^2*e^3-2*a*b*d*e^2-2*a*c*d^2*e+b^2*d^2*e-2*(-e
^2*(4*a*c-b^2))^(1/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c
*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8
*(-2*a^2*e^3+2*a*b*d*e^2+2*a*c*d^2*e-b^2*d^2*e-2*(-e^2*(4*a*c-b^2))^(1/2)*a
*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d/a^2/e^3*(1/2*a*(e*x+d)^(1/
2)/x+1/2*(3*a*e-2*b*d)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4324 vs. $2(344) = 688$.

Time = 24.31 (sec) , antiderivative size = 8653, normalized size of antiderivative = 21.47

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(344) = 688.

Time = 0.36 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.21

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = -\frac{\sqrt{ex+d}d}{ax} - \frac{(2bd^2-3ade)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}$$

$$+ \frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}c\right)e}((b^3-4abc)d^2-2(ab^2-4a^2c)de)e^2-2\left(\sqrt{b^2-4ac}bcd^3+2\sqrt{b^2-4a\right.\right.}{}$$

$$- \frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}c\right)e}((b^3-4abc)d^2-2(ab^2-4a^2c)de)e^2+2\left(\sqrt{b^2-4ac}bcd^3+2\sqrt{b^2-4a\right.\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(e*x + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(e*x + d)/sqrt(-d))
/(a^2*sqrt(-d)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b
^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c
*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a
*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*
e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*
e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(
-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2
*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^
2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2
 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 + 2*(sq
rt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)
*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c + sq
rt(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (
b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^2*c*
d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d
- a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*
a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 29890, normalized size of antiderivative = 74.17

$$\int \frac{(d+ex)^{3/2}}{x^2 (a+bx+cx^2)} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8
*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b
^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2
*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*
a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3
*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*
d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 1
00*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11
*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 +
 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e
^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*
d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9
+ 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252
*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(
1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c
^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d
^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d
^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^
3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b
^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b
*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*
(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4
*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*
c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*
d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*
x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a
^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b
*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3
*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^
2) + (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a
^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8
*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d
^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^
3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e
^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))
/a^4 + (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4
```

*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^2))/((16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d

^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c
^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^
3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c
^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*
d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d
^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e
^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4
*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^
3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*
e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*
e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2) + (d^(1/2)*((8*(d + e*x)^(1/2
)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^
4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10
- 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^1
2 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15
 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a
^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 + (d^(1/2)*((8*(56*a^4*c^6*
d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e
^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^
6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2
*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^
4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c
^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*
e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14
))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^1
2 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*
b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b
^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4
*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234
*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 +
16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^
4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 +
8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*
a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^
6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (
4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*
e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 5
6*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*
b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(
2*a^2))*(3*a*e - 2*b*d))/(2*a^2)))*(3*a*e - 2*b*d)*1i)/a^2 - atan(((((((8*(
80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*
c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c
^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d
^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*
b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a

*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^
3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2
*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*
c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*
e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^
4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2
+ 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*
b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2
*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c
^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184
*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*
b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^
4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*
a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11
- 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 -
84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*
e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4
*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*
e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*
b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(
56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^
3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3
*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*
a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12
- 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14
- 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3
*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3
*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a
*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 2
4*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*
b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*
e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)
^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(
1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5
*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^
10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*

e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 +

111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^
11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d
^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 +
 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*
a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 -
8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^
4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 +
 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b
^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*
a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^
8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*
d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c
^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*
e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13)
)/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)
^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*
e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e -
2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b
*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(
2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^4*d*e^
11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2
*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 3
2*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^
7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3
*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d
^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^
2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a
^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b
*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 -
a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^
2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3
*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^
7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^

10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3

*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*

d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - atan((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e

^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b

^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c
^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*
d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 +
4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*
b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1
/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3
*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e
^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^
9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*
e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*
d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^
2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3
- a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*
c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a
^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^1
1 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10
- 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e
^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*
d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2
*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6
*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10
+ 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*
e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 -
8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d
*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8
*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^
8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*
c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^
4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4
*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^
5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3
e^13))/a^4)((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b
^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*
c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^
2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^

3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36
*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^
4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e
^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^
10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 -
28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 -
8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b
^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2
- 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10
+ 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112
*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*
d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*
b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e -
18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 +
88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*
d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d
^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5
*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c
^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5
*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^
2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 +
24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a
*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2
*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^
6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7
*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*
d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c
^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*
b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5
*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^
7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^
14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)
^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2

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*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e
 + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^
3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a
^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12
 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a
^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28
*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a
*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c
^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*
c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^
3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^
4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*
e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11
 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2
)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2
) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 +
 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*
d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*
e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 5
6*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*
b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4
*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 +
 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*
a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^
2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^
2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)
^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5
*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2
*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2
*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*
c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5
*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6
*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*
a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)
```

^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d*(d + e*x)^(1/2))/(a*x)

3.540 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 607

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}$$
$$-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}$$
$$-\frac{2(b^2d^2-2abde-a(cd^2-ae^2))\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}d-2ae\right)+a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)-ab\left(3cd^2+e\left(2\sqrt{b^2-\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)-e*(-2*a*e+b*d)*arctanh((e
*x+d)^(1/2)/d^(1/2))/a^2/d^(1/2)-2*(b^2*d^2-2*a*b*d*e-a*(-a*e^2+c*d^2))*arc
tanh((e*x+d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/2*d*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*
x+d)^(1/2)/a/x+(-2*a*e+b*d)*(e*x+d)^(1/2)/a^2/x+arctanh(2^(1/2)*c^(1/2)*(e*
x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d^2
+b^2*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+a*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*
e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2+e*(-a*e+2*d*(-4*a*c+b^2)^(1/2))))/a^3
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*
```

```
c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/
2)*(b^3*d^2-b^2*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*(a*e^2*(-4*a*c+b^2)^(1/2)-
c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2-e*(a*e+2*d*(-4*a*c+b^2)^(1/2
))))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$\int \frac{(d+ex)^{3/2}}{x^3 (a+bx+cx^2)} \, dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(-2abde-a(cd^2-ae^2)+b^2d^2)}{a^3\sqrt{d}}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)+b^2d\left(d\sqrt{b^2-4ac}\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

$$-\frac{\sqrt{2}\sqrt{c}\left(-ab\left(3cd^2-e\left(2d\sqrt{b^2-4ac}+ae\right)\right)-a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2d\left(d\sqrt{b^2-4ac}\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$-\frac{e(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}+\frac{\sqrt{d+ex}(bd-2ae)}{a^2x}$$

$$-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}$$

```
[In] Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(d*Sqrt[d + e*x])/(a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*
e)*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqr
t[d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2
*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
(a^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*
a*e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*
b*(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^
2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d
 + a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{d^2e^3}{a(d-x^2)^3}+\frac{de^2(-bd+2ae)}{a^2(d-x^2)^2}+\frac{e\left(-b^2d^2+2abde+a\left(cd^2-ae^2\right)\right)}{a^3(d-x^2)}+\frac{e\left(\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d^2-2a\right.\right.}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2\right.}\right.\right.}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^3}$$
$$- \frac{\left(2d^2e^2\right)\text{Subst}\left(\int \frac{1}{(d-x^2)^3}\,dx, x, \sqrt{d+ex}\right)}{a}$$
$$- \frac{(2de(bd-2ae))\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex}\right)}{a^2}$$
$$- \frac{\left(2\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a^3}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{2\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$
$$- \frac{\left(3de^2\right)\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex}\right)}{2a}-\frac{(e(bd-2ae))\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a^2}$$
$$- \frac{\left(c\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}d-2ae\right)+a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)-ab\left(3cd^2+e\left(2\right.\right.\right.\right.}{a^3\sqrt{b^2-4ac}}$$
$$+ \frac{\left(c\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{\right.\right.\right.\right.}{a^3\sqrt{b^2-4ac}}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4ax} + \frac{(bd-2ae)\sqrt{d+ex}}{a^2x} - \frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{2(b^2d^2-2abde-a(cd^2-ae^2))\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{2}\sqrt{c}\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}d-2ae\right)+a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)-ab\left(3cd^2\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}e^2\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{(3e^2)\operatorname{Subst}\left(\int\frac{1}{d-x^2}\,dx,x,\sqrt{d+ex}\right)}{4a}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4ax} + \frac{(bd-2ae)\sqrt{d+ex}}{a^2x} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}} - \frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{2(b^2d^2-2abde-a(cd^2-ae^2))\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{2}\sqrt{c}\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}d-2ae\right)+a\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)-ab\left(3cd^2\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}e^2\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.92

$$\int\frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{\frac{a\sqrt{d+ex}(-2ad+4bdx-5aex)}{x^2} + \frac{4\sqrt{2}\sqrt{c}\left(ib^3d^2-b^2d\left(\sqrt{-b^2+4ac}d+2iae\right)+ab\left(-3icd^2+e\left(2\sqrt{-b^2+4ac}d+iae\right)\right.\right.}{\sqrt{-b^2+4ac}\sqrt{-}}}{}$$

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((a*Sqrt[d + e*x]*(-2*a*d + 4*b*d*x - 5*a*e*x))/x^2 + (4*Sqrt[2]*Sqrt[c]*(I
*b^3*d^2 - b^2*d*(Sqrt[-b^2 + 4*a*c]*d + (2*I)*a*e) + a*b*((-3*I)*c*d^2 + e
*(2*Sqrt[-b^2 + 4*a*c]*d + I*a*e)) + a*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(
Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c
*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(I*b^3*d^2 + b^2*d
*(Sqrt[-b^2 + 4*a*c]*d - (2*I)*a*e) + a*(a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(-(
Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + I*a*b*(-3*c*d^2 + e*((2*I)*Sqrt[-b^2
+ 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*
e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt
[-b^2 + 4*a*c])*e]) + ((-8*b^2*d^2 + 12*a*b*d*e + a*(8*c*d^2 - 3*a*e^2))*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/(4*a^3)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{-8\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x^2c\left(\frac{\left(-a\,d^{\frac{3}{2}}be+\frac{\left(-ac+b^2\right)d^{\frac{5}{2}}}{2}+\frac{e^2a^2\sqrt{d}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}\right.\right.}{}$
risch	$-\frac{\sqrt{ex+d}\,(5aex-4bdx+2ad)}{4a^2x^2}-\frac{e\left(-\frac{\left(-3e^2a^2+12abde+8c\,d^2a-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{32c\left(-\frac{\left(a^2b\,e^3+4a^2cd\,e^2-2a\,b^2\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^4\left(-\frac{\frac{\frac{ae(5ae-4bd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{1}{2}ab\,d^2e-\frac{3}{8}d\,a^2e^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(3e^2a^2-12abde-8c\,d^2a+8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{a^3e^4}+\frac{4c\left(-\right.}{}\right.$
default	$2e^4\left(-\frac{\frac{\frac{ae(5ae-4bd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{1}{2}ab\,d^2e-\frac{3}{8}d\,a^2e^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(3e^2a^2-12abde-8c\,d^2a+8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{a^3e^4}+\frac{4c\left(-\right.}{}\right.$

```
[In] int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-4*e^2*(a*c-1/4*b^2))^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/d^(1/2)*(-8*2
^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*x^2*c*(1/2*(-a*d^
```

```
(3/2)*b*e+1/2*(-a*c+b^2)*d^(5/2)+1/2*e^2*a^2*d^(1/2))*(-4*e^2*(a*c-1/4*b^2)
)^(1/2)+e*(a*e*(a*c-1/2*b^2)*d^(3/2)+1/4*b*((-3*a*c+b^2)*d^(5/2)+e^2*a^2*d^
(1/2))))*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2)
)^(1/2))*c)^(1/2))+(-8*2^(1/2)*(1/2*(a*d^(3/2)*b*e+1/2*d^(5/2)*(a*c-b^2)-1/
2*e^2*a^2*d^(1/2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*e*(a*c-1/2*b^2)*d^(3/2
)+1/4*b*((-3*a*c+b^2)*d^(5/2)+e^2*a^2*d^(1/2))))*x^2*c*arctan(c*(e*x+d)^(1/
2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(-4*e^2*(a*c
-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(3/2*(e
^2*a^2+4*(-b*d*e-2/3*c*d^2)*a+8/3*b^2*d^2)*x^2*arctanh((e*x+d)^(1/2)/d^(1/2
))+((-2*b*x+a)*d^(3/2)+5/2*a*d^(1/2)*e*x)*a*(e*x+d)^(1/2)))*((-b*e+2*c*d+(-
4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/x^2/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7205 vs. $2(523) = 1046$.

Time = 213.71 (sec) , antiderivative size = 14417, normalized size of antiderivative = 23.75

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^3}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(523) = 1046.

Time = 0.38 (sec) , antiderivative size = 1129, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^{3/2}}{x^3 (a+bx+cx^2)} dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(e*x + d)/s
qrt(-d))/(a^3*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c
)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a^2*
b^2 - 4*a^3*c)*e^2)*a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 -
 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2
- a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c
)*c)*e)*abs(a)*abs(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3*e +
(a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d*e^3)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x
 + d)/sqrt(-(2*a^3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2
)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2
 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*a
bs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*
b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^
2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3
 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2
 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*ab
s(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3*e + (a^2*b^4 + a^3*b^
2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d*e^3)*sqrt(-4*c^2*d + 2*(
b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^
3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*
c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a
*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(4*(
e*x + d)^(3/2)*b*d*e - 4*sqrt(e*x + d)*b*d^2*e - 5*(e*x + d)^(3/2)*a*e^2 +
3*sqrt(e*x + d)*a*d*e^2)/(a^2*e^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 44649, normalized size of antiderivative = 73.56

$$\int \frac{(d+ex)^{3/2}}{x^3 (a+bx+cx^2)} dx = \text{Too large to display}$$

```
[In] int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] (((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*(
d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan(((((((1
92*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*
c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*
c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b
^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 14
08*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576
*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 +
8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5
*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*
c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*
e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2
*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^
2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10
 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 1
28*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9
- 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^
3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*
e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e
^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^
2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3
*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d +
 e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^
13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 -
128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^
8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d
^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*
c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a
^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10
 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4
*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4
*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3
 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e
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*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2

- 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^

3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7

5*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b

^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 409
6*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^1
0 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c
^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^
4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))
*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*
d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e
 - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*
a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))
)^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 -
114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^
8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*
b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840
*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8
 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*
d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b
^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3
497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*
e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b
^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 +
3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 +
 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14
)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^
3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a
*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*
d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a
^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8
 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*
b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344
*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9
- 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*
d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5
*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 -

436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896
*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*
a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e
 + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*
a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12
*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*
d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^
5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*
b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^1
0*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8
*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a
^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 -
a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a
*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2
)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536
*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1
792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2
a^8))((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^
2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7
*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*
e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b
^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^1
2 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 217
6*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 11
52*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10

- 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5

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*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^
6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*
d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^
4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c
^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*
d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e -
24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e +
 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^5*e
^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 1
2*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24
*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c
*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^
2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + atan(((((
((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^
10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b
^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^
9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 +
 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 -
576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3
 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*
a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a
^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d
^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*
d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e
^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8
+ 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e
^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 -
 a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3
*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*
d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^
```

2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c
d^3(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27
*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*
a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((
d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2
*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10
 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5
*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^
5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b
^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 782
4*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e
^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*
d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*
c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*
d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*
c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60
*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 +
 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^
8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 1
5*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*
c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*
c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*
b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 217
6*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^1
0 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7
*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5
*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 +
3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*
d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^
7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024
*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 19
71*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b
^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^
2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*

d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a ^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c ^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b ^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a ^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b ^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944 *a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^1 1 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^ 5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a ^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a* b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6 *b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^ 3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3 *(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4 *c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a *b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b ^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^ 3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d* e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10 *b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2 *e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d ^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c ^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9* b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a *c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a* c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d ^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e *(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^ 4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2* (-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3* a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^ 3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c ^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512* a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896* a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960 *a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^ 5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c *e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2 *b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^

(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3 *a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d

^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2 *b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^

5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (atan(((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^

6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 + ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 +

52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*
a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 -
2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^
5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b
^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1
110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 4
48*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 78
0*a^6*b*c^6*d^3*e^13))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14
+ (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672
*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*
b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a
^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1
088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e
^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^
7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5
*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^
13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*
b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^
12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6
*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^
8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(8
76*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b
^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^
3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^
7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a
^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 +
4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3
*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b
^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^
8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a
^8) + (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 +
384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*
a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 1
60*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^
10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9
- 288*a^10*b^3*c^3*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 -
8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a
^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a
^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*
a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2
- 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(
8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3
*d^(1/2)))/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12
+ 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^

5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 - ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 + ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4* c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) + (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 +

```
1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8
 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9)
)/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a
^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/
2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*
a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2))))*(3*a^2*e^2
 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(4*a^3*d^(1/2))
```

3.541 $\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

```
[Out] 2*c*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-
f*x/e)/(1+m)/((1+f*x/e)^n)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^
(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b+(-4*a*c+b^2)^(1/2)),-f*x/e)
/(1+m)/((1+f*x/e)^n)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {925, 140, 138}

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*
x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1
+ m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b
+ Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$

$$= \frac{(2c)\int \frac{x^m(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^m(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

Mathematica [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$$

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2), x]
```

Maple [F]

$$\int \frac{x^m(fx+e)^n}{c\,x^2+bx+a}dx$$

```
[In] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^n x^m}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^m}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^m}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x^m\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

```
[In] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.542 $\int \frac{x^3(e+fx)^n}{a+bx+cx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 290

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2} dx = -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

```
[Out] -(b*f+c*e)*(f*x+e)^(1+n)/c^2/f^2/(1+n)+(f*x+e)^(2+n)/c/f^2/(2+n)+(f*x+e)^(1
+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*
(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)/(2*c*e-f*(b-(-4*a*c+b
^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+
(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)
/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{1642, 70\}$

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$$
$$= \frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$-\frac{(bf+ce)(e+fx)^{n+1}}{c^2f^2(n+1)}+\frac{(e+fx)^{n+2}}{cf^2(n+2)}$$

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(c
*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))
 + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2
- 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(-ce-bf)(e+fx)^n}{c^2f}+\frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\right.$$
$$\left.+\frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}+\frac{(e+fx)^{1+n}}{cf}\right)dx$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \left(\frac{b^2}{c^2} - \frac{a}{c} + \frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx - \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c}$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.10

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \frac{(e+fx)^{1+n}\left(\frac{\left(b^3-3abc-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f} + \frac{-\left(\left(b\left(b+\sqrt{b^2-4ac}\right)f-2c\right.\right.}{}\right.}{c^2\sqrt{b^2}}$$

[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]

[Out] ((e + f*x)^(1 + n)*(((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f) + (-((b*(b + Sqrt[b^2 - 4*a*c])*f - 2*c*(Sqrt[b^2 - 4*a*c]*e + 2*a*f))*(b*f*(2 + n) + c*(e - f*(1 + n)*x))) + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*f^2*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(f^2*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*f)*(2 + n))))/(c^2*Sqrt[b^2 - 4*a*c]*(1 + n))

Maple [F]

$$\int \frac{x^3 (fx+e)^n}{c\,x^2+bx+a}dx$$

```
[In] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")

[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{x^3\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

[In] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2),x)

[Out] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2), x)

3.543 $\int \frac{x^2(e+fx)^n}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 237

$$\begin{aligned}
&\int \frac{x^2(e+fx)^n}{a+bx+cx^2} \, dx \\
&= \frac{(e+fx)^{1+n}}{cf(1+n)} \\
&\quad + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\
&\quad + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}
\end{aligned}$$

```
[Out] (f*x+e)^(1+n)/c/f/(1+n)+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/
(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(1+n
)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2
*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1642, 70}

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$$
$$=\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$+\frac{(e+fx)^{n+1}}{cf(n+1)}$$

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e -
 (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n
)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])
/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral}=\int\left(\frac{(e+fx)^n}{c}+\frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int\frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int\frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{c}$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.22

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = -\frac{2(e+fx)^{1+n}\left(2\sqrt{b^2-4ac}(ce^2+f(-be+af))+f\left(-b^2e+2ace+b\sqrt{b^2-4ac}e+abf-a\sqrt{b^2-4ac}f\right.\right.}{\sqrt{b^2}}$$

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(e + f*x)^(1 + n)*(2*Sqrt[b^2 - 4*a*c]*(c*e^2 + f*(-(b*e) + a*f)) + f*(
-(b^2*e) + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2
 - 4*a*c])*f)] + f*(b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e - a*b*f - a*Sqr
t[b^2 - 4*a*c]*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e
 - (b + Sqrt[b^2 - 4*a*c])*f)]))/(Sqrt[b^2 - 4*a*c]*f*(2*c*e + (-b + Sqrt[b
^2 - 4*a*c])*f)*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Maple [F]

$$\int \frac{x^2(fx+e)^n}{c\,x^2+bx+a}dx$$

```
[In] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$$

```
[In] integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral(x**2*(e + f*x)**n/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx=\int \frac{x^2\,{\left(e+f\,x\right)}^n}{c\,x^2+b\,x+a}\,dx$$

```
[In] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.544 $\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2
)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)
^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

= {844, 70}

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)} - \frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[In] Int[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b -
 Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right) dx$$
$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2\right.}{2ce-\left(b+\sqrt{b^2-4ac}\right)}\right.}{1+n}$$

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
[Out] ((e + f*x)^(1 + n)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*
e - (b + Sqrt[b^2 - 4*a*c])*f)))/(1 + n)
```

Maple [F]

$$\int \frac{x(fx+e)^n}{c\,x^2+bx+a}dx$$

```
[In] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$$

```
[In] integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral(x*(e + f*x)**n/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{(fx+e)^n x}{cx^2+bx+a}\,dx$$

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \frac{x\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

```
[In] int((x*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.545 $\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = -\frac{2c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{2c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

```
[Out] -2*c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c
+b^2)^(1/2))))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+2*
c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^
2)^(1/2))))/(1+n)/(-4*a*c+b^2)^(1/2)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{725, 70\}$

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = \frac{2c(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} - \frac{2c(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[In] Int[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/
(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sqrt
[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2
 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx \\
&= \frac{(2c)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} \\
&= -\frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\
&\quad + \frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \frac{2c(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}(1+n)}$$

```
[In] Integrate[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x)
)/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)]/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f
)) + Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)]/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(Sqrt[b^2 - 4*a*c]*
(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{c\,x^2+bx+a}dx$$

```
[In] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n}{cx^2+bx+a} \, dx$$

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(e+fx)^n}{a+bx+cx^2} \, dx$$

```
[In] integrate((f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral((e + f*x)**n/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n}{cx^2+bx+a} \, dx$$

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(fx+e)^n}{cx^2+bx+a} \, dx$$

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx = \int \frac{(e+f\,x)^n}{c\,x^2+b\,x+a} \, dx$$

```
[In] int((e + f*x)^n/(a + b*x + c*x^2),x)
[Out] int((e + f*x)^n/(a + b*x + c*x^2), x)
```

3.546 $\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ae(1+n)}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a/e/(1+n)+c*(f*x+e)^(1+n)*
hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(1+b
/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+c*(f*x+e)^(1+
n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(
1-b/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {974, 67, 844, 70}

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx$$
$$=\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ae(n+1)}$$

```
[In] Int[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 +
n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(e+fx)^n}{ax}+\frac{(-b-cx)(e+fx)^n}{a\,(a+bx+cx^2)}\right)dx \\
&= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a}+\frac{\int \frac{(-b-cx)(e+fx)^n}{a+bx+cx^2}\,dx}{a} \\
&= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} \\
&\quad+\frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{a} \\
&= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} \\
&\quad-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{a} \\
&= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\
&\quad+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} \\
&\quad-\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx = \frac{(e+fx)^{1+n}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n\right.}{2ce-\left(b+\sqrt{b^2-4ac}\right)}\right.}{a(1+n)}$$

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[
1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c
*e - (b + Sqrt[b^2 - 4*a*c])*f) - Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f
*x)/e]/e))/(a*(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{x\,(c\,x^2+bx+a)}dx$$

```
[In] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^3 + b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx$$

```
[In] integrate((f*x+e)**n/x/(c*x**2+b*x+a),x)
[Out] Integral((e + f*x)**n/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx = \int \frac{(e+f\,x)^n}{x\,(c\,x^2+b\,x+a)}\,dx$$

```
[In] int((e + f*x)^n/(x*(a + b*x + c*x^2)),x)
[Out] int((e + f*x)^n/(x*(a + b*x + c*x^2)), x)
```

3.547 $\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 296

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx &= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} \\ &\quad -\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} \\ &\quad +\frac{b(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{a^2e(1+n)} \\ &\quad +\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ae^2(1+n)}\end{aligned}$$

```
[Out] b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a^2/e/(1+n)+f*(f*x+e)^(1+
n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1+n)-c*(f*x+e)^(1+n)*hypergeom(
[1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-c*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/
2)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {974, 67, 844, 70}

$$\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$$
$$= -\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$+\frac{b(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{a^2e(n+1)}$$
$$+\frac{f(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/
(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*
c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(e+fx)^n}{ax^2}-\frac{b(e+fx)^n}{a^2x}+\frac{(b^2-ac+bcx)(e+fx)^n}{a^2(a+bx+cx^2)}\right)dx\\
&= \frac{\int \frac{(b^2-ac+bcx)(e+fx)^n}{a+bx+cx^2}\,dx}{a^2}+\frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a}-\frac{b\int \frac{(e+fx)^n}{x}\,dx}{a^2}\\
&= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}\\
&\quad+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}\\
&\quad+\frac{\int \left(\frac{\left(bc+\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(bc-\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{a^2}\\
&= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}\\
&\quad+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a^2}+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)} + \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.83

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f} - \frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+n \right.}{2ce-\left(b+\sqrt{b^2-4ac}\right.}\right.}{a^2(1+n)}$$

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)
])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[b^2
- 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b +
Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f) + (b*Hypergeome
tric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/e + (a*f*Hypergeometric2F1[2, 1 + n,
 2 + n, 1 + (f*x)/e])/e^2))/(a^2*(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{x^2\left(c\,x^2+bx+a\right)}dx$$

```
[In] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx=\int \frac{(e+f\,x)^n}{x^2\,\left(c\,x^2+b\,x+a\right)}\,dx$$

```
[In] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)), x)
```

3.548 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{d^2(7e^2f^2+16defg+8d^2g^2)\,x}{e^2} - \frac{d(2e^2f^2+7defg+4d^2g^2)\,x^2}{e} - \frac{1}{3}(ef+dg)(ef+7dg)x^3 - \frac{1}{2}eg(ef+2dg)x^4 - \frac{1}{5}e^2g^2x^5 - \frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-d*(4*d^2*g^2+7*d*e*f*g+2*e^2*f^
2)*x^2/e-1/3*(d*g+e*f)*(7*d*g+e*f)*x^3-1/2*e*g*(2*d*g+e*f)*x^4-1/5*e^2*g^2*
x^5-8*d^3*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3} - \frac{dx^2(4d^2g^2+7defg+2e^2f^2)}{e} - \frac{d^2x(8d^2g^2+16defg+7e^2f^2)}{e^2} - \frac{1}{2}egx^4(2dg+ef) - \frac{1}{3}x^3(dg+ef)(7dg+ef) - \frac{1}{5}e^2g^2x^5$$

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^3(f+gx)^2}{d-ex}\,dx \\
&= \int \left(-\frac{d^2(7e^2f^2+16defg+8d^2g^2)}{e^2} - \frac{2d(2e^2f^2+7defg+4d^2g^2)\,x}{e} \right. \\
&\qquad \left. + (-ef-7dg)(ef+dg)x^2 - 2eg(ef+2dg)x^3 - e^2g^2x^4 - \frac{8d^3(ef+dg)^2}{e^2(-d+ex)}\right)dx \\
&= -\frac{d^2(7e^2f^2+16defg+8d^2g^2)\,x}{e^2} - \frac{d(2e^2f^2+7defg+4d^2g^2)\,x^2}{e} \\
&\quad -\frac{1}{3}(ef+dg)(ef+7dg)x^3 - \frac{1}{2}eg(ef+2dg)x^4 - \frac{1}{5}e^2g^2x^5 - \frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{x(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2(3f^2+3fgx+g^2x^2)+10de^3x(6f^2+8fgx+3g^2x^2)+e^4x^2(10}{30e^2} - \frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}$$

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/30*(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g
*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2
+ 15*f*g*x + 6*g^2*x^2)))/e^2 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.10

method	result
norman	$\left(-\frac{7}{3}d^2g^2-\frac{8}{3}defg-\frac{1}{3}e^2f^2\right)x^3-\frac{e^2g^2x^5}{5}-\frac{d(4d^2g^2+7defg+2e^2f^2)x^2}{e}-\frac{d^2(8d^2g^2+16defg+7e^2f^2)x}{e^2}-eg$
default	$-\frac{\frac{1}{5}g^2e^4x^5+d\,e^3g^2x^4+\frac{1}{2}e^4fg\,x^4+\frac{7}{3}d^2e^2g^2x^3+\frac{8}{3}d\,e^3fg\,x^3+\frac{1}{3}e^4f^2x^3+4d^3e\,g^2x^2+7d^2e^2fg\,x^2+2d\,e^3f^2x^2+8d^4g^2x+16d^3efg}{e^2}$
risch	$-\frac{e^2g^2x^5}{5}-ed\,g^2x^4-\frac{e^2fg\,x^4}{2}-\frac{7d^2g^2x^3}{3}-\frac{8edfg\,x^3}{3}-\frac{e^2f^2x^3}{3}-\frac{4d^3g^2x^2}{e}-7d^2fg\,x^2-2ed\,f^2x^2-$
parallelrisch	$-\frac{6g^2e^5x^5+30x^4d\,e^4g^2+15x^4e^5fg+70x^3d^2e^3g^2+80x^3d\,e^4fg+10x^3e^5f^2+120x^2d^3e^2g^2+210x^2d^2e^3fg+60x^2d\,e^4f^2+240\ln(e}{30e^3}$

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
[Out] (-7/3*d^2*g^2-8/3*d*e*f*g-1/3*e^2*f^2)*x^3-1/5*e^2*g^2*x^5-d*(4*d^2*g^2+7*d
*e*f*g+2*e^2*f^2)*x^2/e-d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*e*g*
(2*d*g+e*f)*x^4-8*d^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{6\,e^5g^2x^5+15\,(e^5fg+2\,de^4g^2)x^4+10\,(e^5f^2+8\,de^4fg+7\,d^2e^3g^2)x^3+30\,(2\,de^4f^2+7\,d^2e^3fg+4\,d^3e^2g}{30\,e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
[Out] -1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*e
^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*f*g + 4*d^3*e^2*g
^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3*e^
2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{8d^3(dg+ef)^2\log(-d+ex)}{e^3}-\frac{e^2g^2x^5}{5} - x^4\left(deg^2+\frac{e^2fg}{2}\right)-x^3\cdot\left(\frac{7d^2g^2}{3}+\frac{8defg}{3}+\frac{e^2f^2}{3}\right)-x^2 \cdot\left(\frac{4d^3g^2}{e}+7d^2fg+2def^2\right)-x\left(\frac{8d^4g^2}{e^2}+\frac{16d^3fg}{e}+7d^2f^2\right)$$

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g*
*2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**2*
(4*d**3*g**2/e + 7*d**2*f*g + 2*d*e*f**2) - x*(8*d**4*g**2/e**2 + 16*d**3*f
*g/e + 7*d**2*f**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{6\,e^4g^2x^5+15\,(e^4fg+2\,de^3g^2)x^4+10\,(e^4f^2+8\,de^3fg+7\,d^2e^2g^2)x^3+30\,(2\,de^3f^2+7\,d^2e^2fg+4\,d^3eg^2)}{30\,e^2} - \frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*e
^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*f*g + 4*d^3*e*g^2
)*x^2 + 30*(7*d^2*e^2*f^2 + 16*d^3*e*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^2*f
^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{8\,(d^3e^2f^2+2\,d^4efg+d^5g^2)\log(|ex-d|)}{e^3} - \frac{6\,e^7g^2x^5+15\,e^7fgx^4+30\,de^6g^2x^4+10\,e^7f^2x^3+80\,de^6fgx^3+70\,d^2e^5g^2x^3+60\,de^6f^2x^2+210\,d^2e^5fgx^2}{30\,e^5}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(abs(e*x - d))/e^3 - 1/30*(6*e^
7*g^2*x^5 + 15*e^7*f*g*x^4 + 30*d*e^6*g^2*x^4 + 10*e^7*f^2*x^3 + 80*d*e^6*f
*g*x^3 + 70*d^2*e^5*g^2*x^3 + 60*d*e^6*f^2*x^2 + 210*d^2*e^5*f*g*x^2 + 120*
d^3*e^4*g^2*x^2 + 210*d^2*e^5*f^2*x + 480*d^3*e^4*f*g*x + 240*d^4*e^3*g^2*x
)/e^5
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$$
$$= -x^2\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{2\,e}+\frac{d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e}+\frac{d\,\left(e\,g\,\left(3\,d\,g+2\,e\,f\right)+d\,e\,g^2\right)}{e}\right)}{2\,e}\right) - x^3\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{3\,e}+\frac{d\,\left(e\,g\,\left(3\,d\,g+2\,e\,f\right)+d\,e\,g^2\right)}{3\,e}\right) - x^4\left(\frac{e\,g\,\left(3\,d\,g+2\,e\,f\right)}{4}+\frac{d\,e\,g^2}{4}\right) - x\left(\frac{d\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{e}+\frac{d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e}+\frac{d\,\left(e\,g\,\left(3\,d\,g+2\,e\,f\right)+d\,e\,g^2\right)}{e}\right)}{e}\right)}{e}+\frac{d^2\,f\,\left(2\,d\,g+3\,e\,f\right)}{e}\right) - \frac{\ln\left(e\,x-d\right)\,\left(8\,d^5\,g^2+16\,d^4\,e\,f\,g+8\,d^3\,e^2\,f^2\right)}{e^3} - \frac{e^2\,g^2\,x^5}{5}$$

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/(2*e) + (d*((e^3*f^2 + 3*d^2*e
*g^2 + 6*d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/(2*e)) - x^
3*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/(3*e) + (d*(e*g*(3*d*g + 2*e*f) +
d*e*g^2))/(3*e)) - x^4*((e*g*(3*d*g + 2*e*f))/4 + (d*e*g^2)/4) - x*((d*((d^
3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e + (d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2
*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/e))/e + (d^2*f*(2*d*g + 3
*e*f))/e) - (log(e*x - d)*(8*d^5*g^2 + 8*d^3*e^2*f^2 + 16*d^4*e*f*g))/e^3 -
 (e^2*g^2*x^5)/5
```

3.549 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{d(ef+2dg)(3ef+2dg)x}{e^2} - \frac{(e^2f^2+6defg+4d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+3dg)x^3 - \frac{1}{4}eg^2x^4 - \frac{4d^2(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -d*(2*d*g+e*f)*(2*d*g+3*e*f)*x/e^2-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x^2/e-
1/3*g*(3*d*g+2*e*f)*x^3-1/4*e*g^2*x^4-4*d^2*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3} - \frac{x^2(4d^2g^2+6defg+e^2f^2)}{2e} - \frac{dx(2dg+ef)(2dg+3ef)}{e^2} - \frac{1}{3}gx^3(3dg+2ef) - \frac{1}{4}eg^2x^4$$

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex)^2(f+gx)^2}{d-ex}\,dx\\ &= \int \left(\frac{d(-3ef-2dg)(ef+2dg)}{e^2} - \frac{(e^2f^2+6defg+4d^2g^2)\,x}{e} - g(2ef+3dg)x^2 - eg^2x^3 - \frac{4d^2(ef+dg)^2}{e^2(-d+ex)}\right)dx\\ &= -\frac{d(ef+2dg)(3ef+2dg)x}{e^2} - \frac{(e^2f^2+6defg+4d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+3dg)x^3 - \frac{1}{4}eg^2x^4 - \frac{4d^2(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{ex(48d^3g^2+24d^2eg(4f+gx)+12de^2(3f^2+3fgx+g^2x^2)+e^3x(6f^2+8fgx+3g^2x^2))+48d^2(ef+}{12e^3}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/12*(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*x
 + g^2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2*L
og[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
norman	$-\frac{e\,g^2x^4}{4}-\frac{g(3dg+2ef)x^3}{3}-\frac{(4d^2g^2+6defg+e^2f^2)x^2}{2e}-\frac{d(4d^2g^2+8defg+3e^2f^2)x}{e^2}-\frac{4d^2(d^2g^2+2defg+e^2f^2)\ln(-ex+}{e^3}$
risch	$-\frac{e\,g^2x^4}{4}-x^3d\,g^2-\frac{2e\,x^3fg}{3}-\frac{2x^2g^2d^2}{e}-3x^2fgd-\frac{e\,x^2f^2}{2}-\frac{4d^3g^2x}{e^2}-\frac{8d^2fgx}{e}-3d\,f^2x-\frac{4d^4\ln(-ex+}{e^3}$
default	$-\frac{\frac{g^2e^3x^4}{4}+\frac{((2dg+ef)e^2g+eg(deg+e^2f))x^3}{3}+\frac{((2dg+ef)(deg+e^2f)+eg(2d^2g+3def))x^2}{2}+(2dg+ef)(2d^2g+3def)x}{e^2}-\frac{4d^2(d^2g^2+2}{}$
parallelrisch	$-\frac{3g^2e^4x^4+12x^3d\,e^3g^2+8x^3e^4fg+24x^2d^2e^2g^2+36x^2d\,e^3fg+6x^2e^4f^2+48\ln(ex-d)d^4g^2+96\ln(ex-d)d^3efg+48\ln(ex-d)d^2e^2}{12e^3}$

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*e*g^2*x^4-1/3*g*(3*d*g+2*e*f)*x^3-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x^
2/e-d*(4*d^2*g^2+8*d*e*f*g+3*e^2*f^2)/e^2*x-4*d^2*(d^2*g^2+2*d*e*f*g+e^2*f^
2)/e^3*ln(-e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{3\,e^4g^2x^4+4\,(2\,e^4fg+3\,de^3g^2)x^3+6\,(e^4f^2+6\,de^3fg+4\,d^2e^2g^2)x^2+12\,(3\,de^3f^2+8\,d^2e^2fg+4\,d^3eg^2)x}{12\,e^3}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^4*g^2*x^4 + 4*(2*e^4*f*g + 3*d*e^3*g^2)*x^3 + 6*(e^4*f^2 + 6*d*e
^3*f*g + 4*d^2*e^2*g^2)*x^2 + 12*(3*d*e^3*f^2 + 8*d^2*e^2*f*g + 4*d^3*e*g^2
)*x + 48*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{4d^2(dg+ef)^2\log(-d+ex)}{e^3}-\frac{eg^2x^4}{4}-x^3\left(dg^2+\frac{2efg}{3}\right) - x^2\cdot\left(\frac{2d^2g^2}{e}+3dfg+\frac{ef^2}{2}\right)-x\left(\frac{4d^3g^2}{e^2}+\frac{8d^2fg}{e}+3df^2\right)$$

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
2*e*f*g/3) - x**2*(2*d**2*g**2/e + 3*d*f*g + e*f**2/2) - x*(4*d**3*g**2/e**
2 + 8*d**2*f*g/e + 3*d*f**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{3\,e^3g^2x^4+4\,(2\,e^3fg+3\,de^2g^2)x^3+6\,(e^3f^2+6\,de^2fg+4\,d^2eg^2)x^2+12\,(3\,de^2f^2+8\,d^2efg+4\,d^3g^2)x}{12\,e^2} - \frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^3*g^2*x^4 + 4*(2*e^3*f*g + 3*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 6*d*e
^2*f*g + 4*d^2*e*g^2)*x^2 + 12*(3*d*e^2*f^2 + 8*d^2*e*f*g + 4*d^3*g^2)*x)/e
^2 - 4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)\log(|ex-d|)}{e^3} - \frac{3\,e^5g^2x^4+8\,e^5fgx^3+12\,de^4g^2x^3+6\,e^5f^2x^2+36\,de^4fgx^2+24\,d^2e^3g^2x^2+36\,de^4f^2x+96\,d^2e^3fgx+4}{12\,e^4}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(abs(e*x - d))/e^3 - 1/12*(3*e^
5*g^2*x^4 + 8*e^5*f*g*x^3 + 12*d*e^4*g^2*x^3 + 6*e^5*f^2*x^2 + 36*d*e^4*f*g
*x^2 + 24*d^2*e^3*g^2*x^2 + 36*d*e^4*f^2*x + 96*d^2*e^3*f*g*x + 48*d^3*e^2*
g^2*x)/e^4
```

Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx = -x^3\left(\frac{2\,g\,(d\,g+e\,f)}{3}+\frac{d\,g^2}{3}\right) - x^2\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{2\,e}+\frac{d\,(2\,g\,(d\,g+e\,f)+d\,g^2)}{2\,e}\right) - x\left(\frac{d\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e}+\frac{d\,(2\,g\,(d\,g+e\,f)+d\,g^2)}{e}\right)}{e} + \frac{2\,d\,f\,(d\,g+e\,f)}{e}\right) - \frac{\ln(e\,x-d)\,\left(4\,d^4\,g^2+8\,d^3\,e\,f\,g+4\,d^2\,e^2\,f^2\right)}{e^3} - \frac{e\,g^2\,x^4}{4}$$

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^3*((2*g*(d*g + e*f))/3 + (d*g^2)/3) - x^2*((d^2*g^2 + e^2*f^2 + 4*d*e*f
*g)/(2*e) + (d*(2*g*(d*g + e*f) + d*g^2))/(2*e)) - x*((d*((d^2*g^2 + e^2*f^
2 + 4*d*e*f*g)/e + (d*(2*g*(d*g + e*f) + d*g^2))/e))/e + (2*d*f*(d*g + e*f)
)/e) - (log(e*x - d)*(4*d^4*g^2 + 4*d^2*e^2*f^2 + 8*d^3*e*f*g))/e^3 - (e*g^
2*x^4)/4
```

3.550 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2} dx = -\frac{2dg(ef+dg)x}{e^2} - \frac{d(f+gx)^2}{e} - \frac{(f+gx)^3}{3g} - \frac{2d(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -2*d*g*(d*g+e*f)*x/e^2-d*(g*x+f)^2/e-1/3*(g*x+f)^3/g-2*d*(d*g+e*f)^2*ln(-e*
x+d)/e^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 78}

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2} dx = -\frac{2d(dg+ef)^2\log(d-ex)}{e^3} - \frac{2dgx(dg+ef)}{e^2} - \frac{d(f+gx)^2}{e} - \frac{(f+gx)^3}{3g}$$

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex)(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{2dg(ef+dg)}{e^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)}-\frac{2dg(f+gx)}{e}-(f+gx)^2\right)dx\\ &= -\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{ex(6d^2g^2+3deg(4f+gx)+e^2(3f^2+3fgx+g^2x^2))+6d(ef+dg)^2\log(d-ex)}{3e^3}$$

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/3*(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2
)) + 6*d*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result
norman	$-\frac{g^2x^3}{3}-\frac{(2d^2g^2+4defg+e^2f^2)x}{e^2}-\frac{g(dg+ef)x^2}{e}-\frac{2d(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{e^3}$
default	$-\frac{\frac{1}{3}g^2x^3e^2+de\,g^2x^2+e^2fg\,x^2+2d^2g^2x+4defgx+e^2f^2x}{e^2}-\frac{2d(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{e^3}$
risch	$-\frac{g^2x^3}{3}-\frac{d\,g^2x^2}{e}-fg\,x^2-\frac{2d^2g^2x}{e^2}-\frac{4dfgx}{e}-f^2x-\frac{2d^3\ln(-ex+d)g^2}{e^3}-\frac{4d^2\ln(-ex+d)fg}{e^2}-\frac{2d\ln(-ex+d)}{e}$
parallelrisch	$-\frac{g^2x^3e^3+3x^2d\,e^2g^2+3x^2e^3fg+6\ln(ex-d)d^3g^2+12\ln(ex-d)d^2efg+6\ln(ex-d)d\,e^2f^2+6x\,d^2e\,g^2+12xd\,e^2fg+3x\,e^3f^2}{3e^3}$

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*g^2*x^3-(2*d^2*g^2+4*d*e*f*g+e^2*f^2)/e^2*x-1/e*g*(d*g+e*f)*x^2-2*d*(d
^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{e^3g^2x^3+3\,(e^3fg+de^2g^2)x^2+3\,(e^3f^2+4\,de^2fg+2\,d^2eg^2)x+6\,(de^2f^2+2\,d^2efg+d^3g^2)\log{(ex-d)}}{3\,e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g
+ 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2d(dg+ef)^2\log{(-d+ex)}}{e^3}-\frac{g^2x^3}{3}-x^2\left(\frac{dg^2}{e}+fg\right)-x\left(\frac{2d^2g^2}{e^2}+\frac{4dfg}{e}+f^2\right)$$

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2/e + f*g
) - x*(2*d**2*g**2/e**2 + 4*d*f*g/e + f**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{e^2g^2x^3+3\,(e^2fg+deg^2)x^2+3\,(e^2f^2+4\,defg+2\,d^2g^2)x}{3\,e^2} - \frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2*
d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)\log{(|ex-d|)}}{e^3} - \frac{e^3g^2x^3+3\,e^3fgx^2+3\,de^2g^2x^2+3\,e^3f^2x+12\,de^2fgx+6\,d^2eg^2x}{3\,e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(abs(e*x - d))/e^3 - 1/3*(e^3*g^2
*x^3 + 3*e^3*f*g*x^2 + 3*d*e^2*g^2*x^2 + 3*e^3*f^2*x + 12*d*e^2*f*g*x + 6*d
^2*e*g^2*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = -x^2\left(\frac{d\,g^2+2\,e\,f\,g}{2\,e}+\frac{d\,g^2}{2\,e}\right) - x\left(\frac{e\,f^2+2\,d\,g\,f}{e}+\frac{d\,\left(\frac{d\,g^2+2\,e\,f\,g}{e}+\frac{d\,g^2}{e}\right)}{e}\right) - \frac{g^2\,x^3}{3} - \frac{\ln{(e\,x-d)}\,\left(2\,d^3\,g^2+4\,d^2\,e\,f\,g+2\,d\,e^2\,f^2\right)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d*g^2 + 2*e*f*g)/(2*e) + (d*g^2)/(2*e)) - x*((e*f^2 + 2*d*f*g)/e +
(d*((d*g^2 + 2*e*f*g)/e + (d*g^2)/e))/e) - (g^2*x^3)/3 - (log(e*x - d)*(2*d
^3*g^2 + 2*d*e^2*f^2 + 4*d^2*e*f*g))/e^3
```

3.551 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx = -\frac{g(ef+dg)x}{e^2} - \frac{(f+gx)^2}{2e} - \frac{(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -g*(d*g+e*f)*x/e^2-1/2*(g*x+f)^2/e-(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {813, 45}

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx = -\frac{(dg+ef)^2\log(d-ex)}{e^3} - \frac{gx(dg+ef)}{e^2} - \frac{(f+gx)^2}{2e}$$

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{g(ef+dg)}{e^2}+\frac{(ef+dg)^2}{e^2(d-ex)}-\frac{g(f+gx)}{e}\right)dx\\ &= -\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{egx(4ef+2dg+egx)+2(ef+dg)^2\log(d-ex)}{2e^3}$$

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
[Out] -1/2*(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{g\left(\frac{1}{2}eg\,x^2+dgx+2efx\right)}{e^2}+\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln\left(-ex+d\right)}{e^3}$	59
norman	$-\frac{g^2x^2}{2e}-\frac{g(dg+2ef)x}{e^2}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln\left(-ex+d\right)}{e^3}$	61
risch	$-\frac{g^2x^2}{2e}-\frac{g^2dx}{e^2}-\frac{2gfx}{e}-\frac{\ln\left(-ex+d\right)d^2g^2}{e^3}-\frac{2\ln\left(-ex+d\right)dfg}{e^2}-\frac{\ln\left(-ex+d\right)f^2}{e}$	79
parallelrisch	$-\frac{g^2x^2e^2+2\ln\left(ex-d\right)d^2g^2+4\ln\left(ex-d\right)defg+2\ln\left(ex-d\right)e^2f^2+2xde\,g^2+4x\,e^2fg}{2e^3}$	79

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
[Out] -g/e^2*(1/2*e*g*x^2+d*g*x+2*e*f*x)+(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3*ln(-e*x
+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{e^2g^2x^2+2\,(2\,e^2fg+deg^2)x+2\,(e^2f^2+2\,defg+d^2g^2)\log{(ex-d)}}{2\,e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
[Out] -1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d^
2*g^2)*log(e*x - d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -x\left(\frac{dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{g^2x^2}{2e}-\frac{(dg+ef)^2\log{(-d+ex)}}{e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
[Out] -x*(d*g**2/e**2 + 2*f*g/e) - g**2*x**2/(2*e) - (d*g + e*f)**2*log(-d + e*x)
/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{eg^2x^2+2\,(2\,efg+dg^2)x}{2\,e^2}-\frac{(e^2f^2+2\,defg+d^2g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
[Out] -1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g
^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{eg^2x^2+4\,efgx+2\,dg^2x}{2\,e^2} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(e*g^2*x^2 + 4*e*f*g*x + 2*d*g^2*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g
^2)*log(abs(e*x - d))/e^3
```

Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx = -x\left(\frac{d\,g^2}{e^2}+\frac{2\,f\,g}{e}\right) - \frac{\ln(e\,x-d)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{e^3} - \frac{g^2\,x^2}{2\,e}$$

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2),x)
```

```
[Out] - x*((d*g^2)/e^2 + (2*f*g)/e) - (log(e*x - d)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*
g))/e^3 - (g^2*x^2)/(2*e)
```

3.552 $\int \frac{(f+gx)^2}{d^2-e^2x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{(f+gx)^2}{d^2-e^2x^2} dx = -\frac{g^2x}{e^2} - \frac{(ef+dg)^2\log(d-ex)}{2de^3} + \frac{(ef-dg)^2\log(d+ex)}{2de^3}$$

```
[Out] -g^2*x/e^2-1/2*(d*g+e*f)^2*ln(-e*x+d)/d/e^3+1/2*(-d*g+e*f)^2*ln(e*x+d)/d/e^
3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {716, 647, 31}

$$\int \frac{(f+gx)^2}{d^2-e^2x^2} dx = -\frac{(dg+ef)^2\log(d-ex)}{2de^3} + \frac{(ef-dg)^2\log(d+ex)}{2de^3} - \frac{g^2x}{e^2}$$

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
```

```
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{g^2}{e^2}+\frac{e^2f^2+d^2g^2+2e^2fgx}{e^2\left(d^2-e^2x^2\right)}\right)\,dx\\
&= -\frac{g^2x}{e^2}+\frac{\int \frac{e^2f^2+d^2g^2+2e^2fgx}{d^2-e^2x^2}\,dx}{e^2}\\
&= -\frac{g^2x}{e^2}-\frac{(ef-dg)^2\int \frac{1}{-de-e^2x}\,dx}{2de}+\frac{(ef+dg)^2\int \frac{1}{de-e^2x}\,dx}{2de}\\
&= -\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx=\frac{\left(e^2f^2+d^2g^2\right)\operatorname{arctanh}\left(\frac{ex}{d}\right)-deg\left(gx+f\log\left(d^2-e^2x^2\right)\right)}{de^3}$$

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
[Out] ((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2]))
/(d*e^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

method	result	size
norman	$-\frac{g^{2} x}{e^{2}}+\frac{\left(d^{2} g^{2}-2 d e f g+e^{2} f^{2}\right) \ln \left(e x+d\right)}{2 e^{3} d}-\frac{\left(d^{2} g^{2}+2 d e f g+e^{2} f^{2}\right) \ln \left(-e x+d\right)}{2 d\, e^{3}}$	82
default	$-\frac{g^{2} x}{e^{2}}+\frac{\left(-d^{2} g^{2}-2 d e f g-e^{2} f^{2}\right) \ln \left(-e x+d\right)}{2 d\, e^{3}}+\frac{\left(d^{2} g^{2}-2 d e f g+e^{2} f^{2}\right) \ln \left(e x+d\right)}{2 e^{3} d}$	84
parallelrisch	$-\frac{\ln \left(e x-d\right) d^{2} g^{2}+2 \ln \left(e x-d\right) d e f g+\ln \left(e x-d\right) e^{2} f^{2}-\ln \left(e x+d\right) d^{2} g^{2}+2 \ln \left(e x+d\right) d e f g-\ln \left(e x+d\right) e^{2} f^{2}+2 x d e\, g^{2}}{2 d\, e^{3}}$	102
risch	$-\frac{g^{2} x}{e^{2}}+\frac{d \ln \left(-e x-d\right) g^{2}}{2 e^{3}}-\frac{\ln \left(-e x-d\right) f g}{e^{2}}+\frac{\ln \left(-e x-d\right) f^{2}}{2 e d}-\frac{d \ln \left(e x-d\right) g^{2}}{2 e^{3}}-\frac{\ln \left(e x-d\right) f g}{e^{2}}-\frac{\ln \left(e x-d\right) f^{2}}{2 e d}$	116

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
[Out] -g^2*x/e^2+1/2/e^3*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d*ln(e*x+d)-1/2*(d^2*g^2+2*d
*e*f*g+e^2*f^2)/d/e^3*ln(-e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{2\,deg^2x-(e^2f^2-2\,defg+d^2g^2)\log\left(ex+d\right)+(e^2f^2+2\,defg+d^2g^2)\log\left(ex-d\right)}{2\,de^3}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
[Out] -1/2*(2*d*e*g^2*x - (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d) + (e^2*f^2
 + 2*d*e*f*g + d^2*g^2)*log(e*x - d))/(d*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2}+\frac{(dg-ef)^2\log\left(x+\frac{2d^2fg+\frac{d(dg-ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3} - \frac{(dg+ef)^2\log\left(x+\frac{2d^2fg-\frac{d(dg+ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
[Out] -g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d*
*2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d*(
d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2} + \frac{(e^2f^2-2\,defg+d^2g^2)\log{(ex+d)}}{2\,de^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log{(ex-d)}}{2\,de^3}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d*e^3) - 1/2
*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = -\frac{g^2x}{e^2} + \frac{(e^2f^2-2\,defg+d^2g^2)\log{(|ex+d|)}}{2\,de^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log{(|ex-d|)}}{2\,de^3}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d*e^3)
- 1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = \frac{\ln{(d+e\,x)}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3} - \frac{g^2\,x}{e^2} - \frac{\ln{(d-e\,x)}\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3}$$

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2),x)
```

```
[Out] (log(d + e*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(2*d*e^3) - (g^2*x)/e^2 - (l
og(d - e*x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(2*d*e^3)
```

3.553 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 86

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}$$

```
[Out] -1/2*(-d*g+e*f)^2/d/e^3/(e*x+d)-1/4*(d*g+e*f)^2*ln(-e*x+d)/d^2/e^3+1/4*(-d*
g+e*f)*(3*d*g+e*f)*ln(e*x+d)/d^2/e^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = \frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/2*(e*f - d*g)^2/(d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*
e^3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^2}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{2de^2(d+ex)^2}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)}\right)dx \\
&= -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = \frac{-(ef+dg)^2(d+ex)\log(d-ex)+(ef-dg)(2d(-ef+dg)+(ef+3dg)(d+ex)\log(d+ex))}{4d^2e^3(d+ex)}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
[Out] (-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
+ (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result
default	$\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}+\frac{(-3d^2g^2+2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}-\frac{d^2g^2-2defg+e^2f^2}{2e^3d(ex+d)}$
norman	$\frac{-d^2g^2+2defg-e^2f^2}{2d\,e^3(ex+d)}-\frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{4d^2e^3}-\frac{(3d^2g^2-2defg-e^2f^2)\ln(ex+d)}{4d^2e^3}$
risch	$-\frac{d\,g^2}{2e^3(ex+d)}+\frac{fg}{e^2(ex+d)}-\frac{f^2}{2ed(ex+d)}-\frac{\ln(ex-d)g^2}{4e^3}-\frac{\ln(ex-d)fg}{2d\,e^2}-\frac{\ln(ex-d)f^2}{4d^2e}-\frac{3\ln(-ex-d)g^2}{4e^3}+\frac{\ln(-ex-d)}{2d\,e^2}$
parallelrisch	$-\frac{\ln(ex-d)x\,d^2e\,g^2+2\ln(ex-d)xd\,e^2fg+\ln(ex-d)x\,e^3f^2+3\ln(ex+d)x\,d^2e\,g^2-2\ln(ex+d)xd\,e^2fg-\ln(ex+d)x\,e^3f^2+\ln(ex-d)}{4d^2e^3(ex}$

[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)

[Out] 1/4*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/d^2/e^3*ln(-e*x+d)+1/4*(-3*d^2*g^2+2*d*e*f*g+e^2*f^2)/d^2/e^3*ln(e*x+d)-1/2/e^3*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d/(e*x+d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(80) = 160.

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = -\frac{2\,de^2f^2-4\,d^2efg+2\,d^3g^2-(de^2f^2+2\,d^2efg-3\,d^3g^2+(e^3f^2+2\,de^2fg-3\,d^2eg^2)x)\log(ex+d)+(d}{4\,(d^2e^4x+d^3e^3)}$$

[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")

[Out] -1/4*(2*d*e^2*f^2 - 4*d^2*e*f*g + 2*d^3*g^2 - (d*e^2*f^2 + 2*d^2*e*f*g - 3*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x + d) + (d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x - d))/(d^2*e^4*x + d^3*e^3)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(75) = 150.

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = -\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x} - \frac{(dg-ef)(3dg+ef)\log\left(x+\frac{-2d^3g^2+d(dg-ef)(3dg+ef)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3} - \frac{(dg+ef)^2\log\left(x+\frac{-2d^3g^2+d(dg+ef)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e*
f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d**2
*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(x +
 (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**2*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = -\frac{e^2f^2-2\,defg+d^2g^2}{2\,(de^4x+d^2e^3)}+\frac{(e^2f^2+2\,defg-3\,d^2g^2)\log\left(ex+d\right)}{4\,d^2e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log\left(ex-d\right)}{4\,d^2e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(d*e^4*x + d^2*e^3) + 1/4*(e^2*f^2 + 2
*d*e*f*g - 3*d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 + 2*d*e*f*g + d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx = \frac{(e^2f^2+2\,defg-3\,d^2g^2)\log\left(|ex+d|\right)}{4\,d^2e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log\left(|ex-d|\right)}{4\,d^2e^3} - \frac{de^2f^2-2\,d^2efg+d^3g^2}{2\,(ex+d)d^2e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(e^2*f^2 + 2*d*e*f*g - 3*d^2*g^2)*log(abs(e*x + d))/(d^2*e^3) - 1/4*(e^
2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^2*e^3) - 1/2*(d*e^2*f^2 -
 2*d^2*e*f*g + d^3*g^2)/((e*x + d)*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx = \frac{\ln(d+e\,x)\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3} - \frac{\ln(d-e\,x)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3} - \frac{d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,(d+e\,x)}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d^2*e^3) - (log(d - e*x
)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^3) - (d^2*g^2 + e^2*f^2 - 2*d*e
*f*g)/(2*d*e^3*(d + e*x))
```

3.554 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

```
[Out] -1/4*(-d*g+e*f)^2/d/e^3/(e*x+d)^2-1/4*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)+1/4*(d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(dg+ef)^2}{4d^3e^3}-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/4*(e*f - d*g)^2/(d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2
*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}\,dx \\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^3} + \frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^2} + \frac{(ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2} - \frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)} + \frac{(ef+dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2} \\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2} - \frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)} + \frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx = \frac{\frac{2d(-ef+dg)\left(2d^2g+e^2fx+de(2f+3gx)\right)}{(d+ex)^2} - (ef+dg)^2\log(d-ex) + (ef+dg)^2\log(d+ex)}{8d^3e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
[Out] ((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2 -
 (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

method	result
norman	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{(ex+d)^2}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(-ex+d)}{8e^3d^3}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}$
default	$\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln(-ex+d)}{8e^3d^3}-\frac{-3d^2g^2+2defg+e^2f^2}{4d^2e^3(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{4e^3d(ex+d)^2}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}$
risch	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{(ex+d)^2}-\frac{\ln(-ex+d)g^2}{8e^3d}-\frac{\ln(-ex+d)fg}{4e^2d^2}-\frac{\ln(-ex+d)f^2}{8e\,d^3}+\frac{\ln(ex+d)g^2}{8e^3d}+\frac{\ln(ex+d)fg}{4e^2d^2}$
parallelrisch	$-\frac{4d^2e^2f^2-4d^4g^2-\ln(ex+d)d^2e^2f^2+\ln(ex-d)x^2e^4f^2-\ln(ex+d)x^2e^4f^2+\ln(ex-d)d^2e^2f^2-6x\,d^3e\,g^2+2xd\,e^3f^2+2\ln(ex-d)}{}$

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(d^2*g^2-e^2*f^2)/d/e^3+1/4*(3*d^2*g^2-2*d*e*f*g-e^2*f^2)/d^2/e^2*x)/(
e*x+d)^2-1/8*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(-e*x+d)+1/8*(d^2*g^2+2*
d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.11

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx =$$
$$-\frac{4\,d^2e^2f^2-4\,d^4g^2+2\,(de^3f^2+2\,d^2e^2fg-3\,d^3eg^2)x-(d^2e^2f^2+2\,d^3efg+d^4g^2+(e^4f^2+2\,de^3fg+d}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g - 3*d^3*e*g^
2)*x - (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*
e^2*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) +
(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2
)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5
*x^2 + 2*d^4*e^4*x + d^5*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(75) = 150$.

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.13

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx = -\frac{-2d^3g^2+2de^2f^2+x(-3d^2eg^2+2de^2fg+e^3f^2)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 + 2*d*e**2*f*g + e**3*f*
*2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*log(
-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3
) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f*
*2)) + x)/(8*d**3*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx = -\frac{2\,de^2f^2-2\,d^3g^2+(e^3f^2+2\,de^2fg-3\,d^2eg^2)x}{4\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex+d)}{8\,d^3e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{8\,d^3e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 2*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d
^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d
^3*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = -\frac{(e^2f^2+2\,defg+d^2g^2)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{8\,d^3e^3} - \frac{\frac{e^5f^2}{ex+d}+\frac{de^5f^2}{(ex+d)^2}+\frac{2\,de^4fg}{ex+d}-\frac{2\,d^2e^4fg}{(ex+d)^2}-\frac{3\,d^2e^3g^2}{ex+d}+\frac{d^3e^3g^2}{(ex+d)^2}}{4\,d^2e^6}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(-2*d/(e*x + d) + 1))/(d^3*e^3)
 - 1/4*(e^5*f^2/(e*x + d) + d*e^5*f^2/(e*x + d)^2 + 2*d*e^4*f*g/(e*x + d) -
 2*d^2*e^4*f*g/(e*x + d)^2 - 3*d^2*e^3*g^2/(e*x + d) + d^3*e^3*g^2/(e*x + d
)^2)/(d^2*e^6)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx = \frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{4\,d^3\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2)/(2*d*e^3) - (x*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d
^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) + (atanh((e*x)/d)*(d*g + e*f)^2)/(4*d^3*
e^3)
```

3.555 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx=-\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] -1/6*(-d*g+e*f)^2/d/e^3/(e*x+d)^3-1/8*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)^2-1/8*(d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(d*g+e*f)^2*arctanh(e*x/d)/d^4/e^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx=\frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(dg+ef)^2}{8d^4e^3}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/6*(e*f - d*g)^2/(d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2
*e^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*Ar
cTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}\,dx\\ &= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^4}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^3}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef+dg)^2}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx\\ &= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}\\ &= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx = \frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3}+\frac{6d^2\left(-e^2f^2-2defg+3d^2g^2\right)}{(d+ex)^2}-\frac{6d(ef+dg)^2}{d+ex}-3(ef+dg)^2\log(d-ex)+3(ef+dg)^2\log(d+ex)}{48d^4e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] ((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d^
2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log[d
 - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

method	result
norman	$\frac{-\frac{\left(d^2g^2-2defg-5e^2f^2\right)x^3}{12d^4}-\frac{\left(d^2g^2+2defg-7e^2f^2\right)x}{8d^2e^2}-\frac{\left(3d^2g^2-2defg-9e^2f^2\right)x^2}{8d^3e}}{(ex+d)^3}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(-ex+d)}{16e^3d^4}+\frac{\left(d^2g^2+2\right.}{}$
default	$\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln(-ex+d)}{16e^3d^4}-\frac{-3d^2g^2+2defg+e^2f^2}{8d^2e^3(ex+d)^2}-\frac{d^2g^2-2defg+e^2f^2}{6e^3d(ex+d)^3}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{16e^3d^4}-$
risch	$\frac{-\frac{\left(d^2g^2+2defg+e^2f^2\right)x^2}{8d^3e}+\frac{\left(d^2g^2-6defg-3e^2f^2\right)x}{8d^2e^2}+\frac{d^2g^2-2defg-5e^2f^2}{12de^3}}{(ex+d)^3}-\frac{\ln(-ex+d)g^2}{16e^3d^2}-\frac{\ln(-ex+d)fg}{8e^2d^3}-\frac{\ln(-ex+d)f^2}{16e\,d^4}+$
parallelrisch	$-\frac{-6\ln(ex+d)d^4efg+3\ln(ex-d)x^3d^2e^3g^2-18\ln(ex+d)x\,d^3e^2fg+18\ln(ex-d)x\,d^3e^2fg+18\ln(ex-d)x^2d^2e^3fg-18\ln(ex+d)x}{}$

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12*(d^2*g^2-2*d*e*f*g-5*e^2*f^2)/d^4*x^3-1/8*(d^2*g^2+2*d*e*f*g-7*e^2*f
^2)/d^2/e^2*x-1/8*(3*d^2*g^2-2*d*e*f*g-9*e^2*f^2)/d^3/e*x^2)/(e*x+d)^3-1/16
*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^4*ln(-e*x+d)+1/16*(d^2*g^2+2*d*e*f*g+e^2
*f^2)/e^3/d^4*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx = -\frac{20\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2+6\,(de^4f^2+2\,d^2e^3fg+d^3e^2g^2)x^2+6\,(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2)x-3\,(}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/48*(20*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 + 6*(d*e^4*f^2 + 2*d^2*e^3*
f*g + d^3*e^2*g^2)*x^2 + 6*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x -
3*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g
^2)*x^3 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2
+ 2*d^3*e^2*f*g + d^4*e*g^2)*x)*log(e*x + d) + 3*(d^3*e^2*f^2 + 2*d^4*e*f*g
 + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g^2)*x^3 + 3*(d*e^4*f^2 + 2*d
^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2 + 2*d^3*e^2*f*g + d^4*e*g^2)
*x)*log(e*x - d))/(d^4*e^6*x^3 + 3*d^5*e^5*x^2 + 3*d^6*e^4*x + d^7*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(99) = 198$.

Time = 0.59 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.19

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = -\frac{-2d^4g^2+4d^3efg+10d^2e^2f^2+x^2\cdot(3d^2e^2g^2+6de^3fg+3e^4f^2)+x(-3d^3eg^2+18d^2e^2fg+9de^3f^2)}{24d^6e^3+72d^5e^4x+72d^4e^5x^2+24d^3e^6x^3} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
+ 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*d*
e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3*e*
*6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g +
 e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(
d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = -\frac{10\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2+3\,(e^4f^2+2\,de^3fg+d^2e^2g^2)x^2+3\,(3\,de^3f^2+6\,d^2e^2fg-d^3eg^2)x}{24\,(d^3e^6x^3+3\,d^4e^5x^2+3\,d^5e^4x+d^6e^3)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex+d)}{16\,d^4e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(ex-d)}{16\,d^4e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/24*(10*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 + 3*(e^4*f^2 + 2*d*e^3*f*g
+ d^2*e^2*g^2)*x^2 + 3*(3*d*e^3*f^2 + 6*d^2*e^2*f*g - d^3*e*g^2)*x)/(d^3*e^
6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3) + 1/16*(e^2*f^2 + 2*d*e*f*g
+ d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*lo
g(e*x - d)/(d^4*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.65

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex+d|)}{16\,d^4e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log(|ex-d|)}{16\,d^4e^3} - \frac{10\,d^3e^2f^2+4\,d^4efg-2\,d^5g^2+3\,(de^4f^2+2\,d^2e^3fg+d^3e^2g^2)x^2+3\,(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2)x}{24\,(ex+d)^3d^4e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*(e^
2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) - 1/24*(10*d^3*e^2
*f^2 + 4*d^4*e*f*g - 2*d^5*g^2 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2
)*x^2 + 3*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x)/((e*x + d)^3*d^4*e
^3)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{(d+ex)^3\,(d^2-e^2x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)^2}{8\,d^4\,e^3} - \frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2}{12\,d\,e^3}+\frac{x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{8\,d^3\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^3),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(8*d^4*e^3) - ((5*e^2*f^2 - d^2*g^2 + 2*d*e*
f*g)/(12*d*e^3) + (x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(8*d^2*e^2) + (x^2*
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 +
3*d^2*e*x)
```

3.556 $\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2\text{arctanh}\left(\frac{ex}{d}\right)}{16d^5e^3}$$

```
[Out] -1/8*(-d*g+e*f)^2/d/e^3/(e*x+d)^4-1/12*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+
d)^3-1/16*(d*g+e*f)^2/d^3/e^3/(e*x+d)^2-1/16*(d*g+e*f)^2/d^4/e^3/(e*x+d)+1/
16*(d*g+e*f)^2*arctanh(e*x/d)/d^5/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = \frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+ef)^2}{16d^5e^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/8*(e*f - d*g)^2/(d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^
2*e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2
/(16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}\,dx \\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^5}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^4}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^3}+\frac{(ef+dg)^2}{16d^4e^2(d+ex)^2}+\frac{(ef+dg)^2}{16d^4e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{16d^4e^2} \\
&= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{8d^3(e^2f^2+2defg-3d^2g^2)}{(d+ex)^3}+\frac{6d^2(ef+dg)^2}{(d+ex)^2}+\frac{6d(ef+dg)^2}{d+ex}+3(ef+dg)^2\log(d-ex)-3(ef+dg)^2\log}{96d^5e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/96*((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g - 3
*d^2*g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f + d*
g)^2)/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d + e*
x])/(d^5*e^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.43

method	result
norman	$\frac{-\frac{(3d^2g^2-26defg-61e^2f^2)x^3}{48d^4}-\frac{(d^2g^2-2defg-7e^2f^2)x^2}{4e\,d^3}+\frac{e^2(dfg+2e\,f^2)x^4}{6d^5}-\frac{(d^2g^2+2defg-15e^2f^2)x}{16d^2e^2}}{(ex+d)^4}-\frac{(d^2g^2+2defg+e^2f^2)\ln}{32e^3d^5}$
default	$\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{32e^3d^5}-\frac{-3d^2g^2+2defg+e^2f^2}{12d^2e^3(ex+d)^3}-\frac{d^2g^2-2defg+e^2f^2}{8e^3d(ex+d)^4}+\frac{(d^2g^2+2defg+e^2f^2)\ln(ex+d)}{32e^3d^5}-$
risch	$\frac{-\frac{(d^2g^2+2defg+e^2f^2)x^3}{16d^4}-\frac{(d^2g^2+2defg+e^2f^2)x^2}{4d^3e}-\frac{(3d^2g^2+38defg+19e^2f^2)x}{48d^2e^2}-\frac{f(dg+2ef)}{6e^2d}}{(ex+d)^4}-\frac{\ln(-ex+d)g^2}{32e^3d^3}-\frac{\ln(-ex+d)fg}{16e^2d^4}$
parallelrisch	$-\frac{-12\ln(ex+d)x\,d^3e^3f^2+6\ln(ex-d)d^5efg-6\ln(ex+d)d^5efg+3\ln(ex-d)x^4d^2e^4g^2-3\ln(ex+d)x^4d^2e^4g^2+12\ln(ex-d)x^3d^3}{}$

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/48*(3*d^2*g^2-26*d*e*f*g-61*e^2*f^2)/d^4*x^3-1/4/e*(d^2*g^2-2*d*e*f*g-7
*e^2*f^2)/d^3*x^2+1/6*e^2*(d*f*g+2*e*f^2)/d^5*x^4-1/16*(d^2*g^2+2*d*e*f*g-1
5*e^2*f^2)/d^2/e^2*x)/(e*x+d)^4-1/32*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^5*ln
(-e*x+d)+1/32*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^5*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(129) = 258.

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.68

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{32\,d^4e^2f^2+16\,d^5efg+6\,(de^5f^2+2\,d^2e^4fg+d^3e^3g^2)x^3+24\,(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2)x^2+2\,(19\,d^3e}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e
^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*d^
3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + 2*d^5*e*f*g
+ d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^
2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2
)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*log(e*x + d) + 3*(d^
4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x
^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d
^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)
*x)*log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 + 4*d^8*e^4*
x + d^9*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(122) = 244.

Time = 0.68 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.03

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = -\frac{8d^4fg+16d^3ef^2+x^3\cdot(3d^2e^2g^2+6de^3fg+3e^4f^2)+x^2\cdot(12d^3eg^2+24d^2e^2fg+12de^3f^2)+x(3d^4g^2+}{48d^8e^2+192d^7e^3x+288d^6e^4x^2+192d^5e^5x^3+48d^4e^6x^4} - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{32d^5e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{32d^5e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

```
[Out] -(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e
**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(
3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*
e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*
g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) +
 x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*
d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.70

$$\int \frac{(f+gx)^2}{(d+ex)^4 (d^2-e^2x^2)}\,dx = -\frac{16\,d^3ef^2+8\,d^4fg+3\,(e^4f^2+2\,de^3fg+d^2e^2g^2)x^3+12\,(de^3f^2+2\,d^2e^2fg+d^3eg^2)x^2+(19\,d^2e^2f^2+3}{48\,(d^4e^6x^4+4\,d^5e^5x^3+6\,d^6e^4x^2+4\,d^7e^3x+d^8e^2)} + \frac{(e^2f^2+2\,defg+d^2g^2)\log{(ex+d)}}{32\,d^5e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log{(ex-d)}}{32\,d^5e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*x
^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 38*
d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 + 4*
d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d
^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^2}{(d+ex)^4 (d^2-e^2x^2)}\,dx = \frac{(e^2f^2+2\,defg+d^2g^2)\log{(|ex+d|)}}{32\,d^5e^3} - \frac{(e^2f^2+2\,defg+d^2g^2)\log{(|ex-d|)}}{32\,d^5e^3} - \frac{16\,d^4e^2f^2+8\,d^5efg+3\,(de^5f^2+2\,d^2e^4fg+d^3e^3g^2)x^3+12\,(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2)x^2+(19\,d^3e^3}{48\,(ex+d)^4d^5e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^5*e^3) - 1/32*(e^
2*f^2 + 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^5*e^3) - 1/48*(16*d^4*e^2
*f^2 + 8*d^5*e*f*g + 3*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 12*(
d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + (19*d^3*e^3*f^2 + 38*d^4*e
^2*f*g + 3*d^5*e*g^2)*x)/((e*x + d)^4*d^5*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$$
$$=\frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)^2}{16\,d^5\,e^3}-\frac{\frac{x^3\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{16\,d^4}+\frac{2\,e\,f^2+d\,g\,f}{6\,d\,e^2}+\frac{x\,\left(3\,d^2\,g^2+38\,d\,e\,f\,g+19\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^3\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^4),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(16*d^5*e^3) - ((x^3*(d^2*g^2 + e^2*f^2 + 2*
d*e*f*g))/(16*d^4) + (2*e*f^2 + d*f*g)/(6*d*e^2) + (x*(3*d^2*g^2 + 19*e^2*f
^2 + 38*d*e*f*g))/(48*d^2*e^2) + (x^2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d
^3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

3.557 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 218

$$\begin{aligned}\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2} \\ &\quad+\frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e} \\ &\quad+\frac{1}{3}d\left(7e^2f^2+46defg+49d^2g^2\right)x^3 \\ &\quad+\frac{1}{4}e\left(e^2f^2+14defg+23d^2g^2\right)x^4+\frac{1}{5}e^2g(2ef+7dg)x^5+\frac{1}{6}e^3g^2x^6 \\ &\quad+\frac{32d^5(ef+dg)^2}{e^3(d-ex)}+\frac{16d^4(ef+dg)(5ef+9dg)\log(d-ex)}{e^3}\end{aligned}$$

```
[Out] d^3*(112*d^2*g^2+160*d*e*f*g+49*e^2*f^2)*x/e^2+1/2*d^2*(80*d^2*g^2+98*d*e*f
*g+23*e^2*f^2)*x^2/e+1/3*d*(49*d^2*g^2+46*d*e*f*g+7*e^2*f^2)*x^3+1/4*e*(23*
d^2*g^2+14*d*e*f*g+e^2*f^2)*x^4+1/5*e^2*g*(7*d*g+2*e*f)*x^5+1/6*e^3*g^2*x^6
+32*d^5*(d*g+e*f)^2/e^3/(-e*x+d)+16*d^4*(d*g+e*f)*(9*d*g+5*e*f)*ln(-e*x+d)/
e^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{862, 90\}$

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{32d^5(dg+ef)^2}{e^3(d-ex)} + \frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3} + \frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right) + \frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right) + \frac{d^2x^2(80d^2g^2+98defg+23e^2f^2)}{2e} + \frac{d^3x(112d^2g^2+160defg+49e^2f^2)}{e^2} + \frac{1}{5}e^2gx^5(7dg+2ef) + \frac{1}{6}e^3g^2x^6$$

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}\,dx$$

$$= \int \left(\frac{d^3(49e^2f^2+160defg+112d^2g^2)}{e^2} + \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x}{e} + d\left(7e^2f^2+46defg+49d^2g^2\right)x^2 + e\left(e^2f^2+14defg+23d^2g^2\right)x^3 + e^2g(2ef+7dg)x^4 + e^3g^2x^5 + \frac{16d^4(-5ef-9dg)(ef+dg)}{e^2(d-ex)} + \frac{32d^5(ef+dg)^2}{e^2(-d+ex)^2}\right)dx$$

$$
\begin{aligned}
&= \frac{d^3(49e^2f^2 + 160defg + 112d^2g^2)\,x}{e^2} + \frac{d^2(23e^2f^2 + 98defg + 80d^2g^2)\,x^2}{2e} \\
&\quad + \frac{1}{3}d\left(7e^2f^2 + 46defg + 49d^2g^2\right)x^3 + \frac{1}{4}e\left(e^2f^2 + 14defg + 23d^2g^2\right)x^4 + \frac{1}{5}e^2g(2ef \\
&\quad + 7dg)x^5 + \frac{1}{6}e^3g^2x^6 + \frac{32d^5(ef + dg)^2}{e^3(d - ex)} + \frac{16d^4(ef + dg)(5ef + 9dg)\log(d - ex)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx &= \frac{d^3(49e^2f^2 + 160defg + 112d^2g^2)\,x}{e^2} \\
&\quad + \frac{d^2(23e^2f^2 + 98defg + 80d^2g^2)\,x^2}{2e} \\
&\quad + \frac{1}{3}d\left(7e^2f^2 + 46defg + 49d^2g^2\right)x^3 \\
&\quad + \frac{1}{4}e\left(e^2f^2 + 14defg + 23d^2g^2\right)x^4 + \frac{1}{5}e^2g(2ef + 7dg)x^5 + \frac{1}{6}e^3g^2x^6 \\
&\quad - \frac{32d^5(ef + dg)^2}{e^3(-d + ex)} + \frac{16d^4(5e^2f^2 + 14defg + 9d^2g^2)\log(d - ex)}{e^3}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*x))
+ (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{1}{6}g^2e^5x^6+\frac{7}{5}x^5de^4g^2+\frac{2}{5}x^5e^5fg+\frac{23}{4}x^4d^2e^3g^2+\frac{7}{2}x^4de^4fg+\frac{1}{4}x^4e^5f^2+\frac{49}{3}x^3d^3e^2g^2+\frac{46}{3}x^3d^2e^3fg+\frac{7}{3}x^3de^4f^2+40x^2d^4eg^2+4}{e^2}$
risch	$\frac{e^3g^2x^6}{6}+\frac{7e^2x^5dg^2}{5}+\frac{2e^3x^5fg}{5}+\frac{23ex^4d^2g^2}{4}+\frac{7e^2x^4dfg}{2}+\frac{e^3x^4f^2}{4}+\frac{49x^3d^3g^2}{3}+\frac{46ex^3d^2fg}{3}+\frac{7e^2x^3df^2}{3}+$
norman	$\frac{\left(-\frac{287}{3}g^2d^5-\frac{434}{3}fgd^4e-\frac{140}{3}f^2d^3e^2\right)x^3+\left(-\frac{67}{12}g^2d^2e^3-\frac{7}{2}fgde^4-\frac{1}{4}f^2e^5\right)x^6+\left(-\frac{224}{15}g^2d^3e^2-\frac{224}{15}fgd^2e^3-\frac{7}{3}f^2de^4\right)x^5+\left(-\frac{137}{4}\right.}{-e^2}$
parallelrisch	$\frac{-4860d^5e^2f^2+8640\ln(ex-d)xd^6eg^2+4800\ln(ex-d)xd^4e^3f^2-13440\ln(ex-d)d^6efg-8640d^7g^2+2250x^2d^3e^4f^2+1420x^3d^4e}{}$

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/6*g^2*e^5*x^6+7/5*x^5*d*e^4*g^2+2/5*x^5*e^5*f*g+23/4*x^4*d^2*e^3*g
^2+7/2*x^4*d*e^4*f*g+1/4*x^4*e^5*f^2+49/3*x^3*d^3*e^2*g^2+46/3*x^3*d^2*e^3*
f*g+7/3*x^3*d*e^4*f^2+40*x^2*d^4*e*g^2+49*x^2*d^3*e^2*f*g+23/2*x^2*d^2*e^3*
f^2+112*g^2*d^5*x+160*f*g*d^4*e*x+49*f^2*d^3*e^2*x)+32*d^5*(d^2*g^2+2*d*e*f
*g+e^2*f^2)/e^3/(-e*x+d)+16*d^4/e^3*(9*d^2*g^2+14*d*e*f*g+5*e^2*f^2)*ln(-e*
x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{10\,e^7g^2x^7 - 1920\,d^5e^2f^2 - 3840\,d^6efg - 1920\,d^7g^2 + 2\,(12\,e^7fg + 37\,de^6g^2)x^6 + 3\,(5\,e^7f^2 + 62\,de^6fg + 87\,d}{}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 + 2
*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e^5
*g^2)*x^5 + 5*(25*d*e^6*f^2 + 142*d^2*e^5*f*g + 127*d^3*e^4*g^2)*x^4 + 10*(
55*d^2*e^5*f^2 + 202*d^3*e^4*f*g + 142*d^4*e^3*g^2)*x^3 + 90*(25*d^3*e^4*f^
2 + 74*d^4*e^3*f*g + 48*d^5*e^2*g^2)*x^2 - 60*(49*d^4*e^3*f^2 + 160*d^5*e^2
*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g^2 - (
5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e
^3)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{16d^4(dg+ef)\,(9dg+5ef)\log(-d+ex)}{e^3} + \frac{e^3g^2x^6}{6} + x^5 \cdot \left(\frac{7de^2g^2}{5} + \frac{2e^3fg}{5}\right) + x^4 \cdot \left(\frac{23d^2eg^2}{4} + \frac{7de^2fg}{2} + \frac{e^3f^2}{4}\right) + x^3 \cdot \left(\frac{49d^3g^2}{3} + \frac{46d^2efg}{3} + \frac{7de^2f^2}{3}\right) + x^2 \cdot \left(\frac{40d^4g^2}{e} + 49d^3fg + \frac{23d^2ef^2}{2}\right) + x\left(\frac{112d^5g^2}{e^2} + \frac{160d^4fg}{e} + 49d^3f^2\right) + \frac{-32d^7g^2 - 64d^6efg - 32d^5e^2f^2}{-de^3 + e^4x}$$

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6 +
 x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**2*
f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2*f*
*2/3) + x**2*(40*d**4*g**2/e + 49*d**3*f*g + 23*d**2*e*f**2/2) + x*(112*d**
5*g**2/e**2 + 160*d**4*f*g/e + 49*d**3*f**2) + (-32*d**7*g**2 - 64*d**6*e*f
*g - 32*d**5*e**2*f**2)/(-d*e**3 + e**4*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{32\,(d^5e^2f^2+2\,d^6efg+d^7g^2)}{e^4x-de^3} + \frac{10\,e^5g^2x^6+12\,(2\,e^5fg+7\,de^4g^2)x^5+15\,(e^5f^2+14\,de^4fg+23\,d^2e^3g^2)x^4+20\,(7\,de^4f^2+46\,d^2e^3fg+\cdots}{60\,e^2} + \frac{16\,(5\,d^4e^2f^2+14\,d^5efg+9\,d^6g^2)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g^
2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 23*
d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^3 +
 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e^2*f
^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g
 + 9*d^6*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{16\,(5\,d^4e^2f^2+14\,d^5efg+9\,d^6g^2)\log(|ex-d|)}{e^3} - \frac{32\,(d^5e^2f^2+2\,d^6efg+d^7g^2)}{(ex-d)e^3} + \frac{10\,e^{15}g^2x^6+24\,e^{15}fgx^5+84\,de^{14}g^2x^5+15\,e^{15}f^2x^4+210\,de^{14}fgx^4+345\,d^2e^{13}g^2x^4+140\,de^{14}f^2x^3+\cdots}{\cdots}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g + 9*d^6*g^2)*log(abs(e*x - d))/e^3 - 32*(d
^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/((e*x - d)*e^3) + 1/60*(10*e^15*g^2*x^6
 + 24*e^15*f*g*x^5 + 84*d*e^14*g^2*x^5 + 15*e^15*f^2*x^4 + 210*d*e^14*f*g*x
^4 + 345*d^2*e^13*g^2*x^4 + 140*d*e^14*f^2*x^3 + 920*d^2*e^13*f*g*x^3 + 980
*d^3*e^12*g^2*x^3 + 690*d^2*e^13*f^2*x^2 + 2940*d^3*e^12*f*g*x^2 + 2400*d^4
*e^11*g^2*x^2 + 2940*d^3*e^12*f^2*x + 9600*d^4*e^11*f*g*x + 6720*d^5*e^10*g
^2*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.72

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$

$$= x^5\left(\frac{e^2\,g\,(5\,d\,g+2\,e\,f)}{5}+\frac{2\,d\,e^2\,g^2}{5}\right)+x^3\left(\frac{5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)}{3}+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{3\,e}-\frac{d^2\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{3\,e^2}\right)$$

$$+x^4\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{4\,e^2}-\frac{d^2\,e\,g^2}{4}+\frac{d\,(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2)}{2\,e}\right)$$

$$+x^2\left(\frac{5\,d^2\,(d^2\,g^2+4\,d\,e\,f\,g+2\,e^2\,f^2)}{2\,e}-\frac{d^2\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{2\,e^2}\right.$$

$$+\frac{d\left(5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{e}-\frac{d^2\,(e^2\,g\,(5\,d}{}\right.}{e}$$

$$+x\left(\frac{d^5\,g^2+10\,d^4\,e\,f\,g+10\,d^3\,e^2\,f^2}{e^2}\right.$$

$$d^2\left(5\,d\,(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2)+\frac{2\,d\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2+\frac{2\,d\,\left(e^2\,g\,(5\,d\,g+2\,e\,f)+2\,d\,e^2\,g^2\right)}{e}\right)}{e}-\frac{d^2\,(e^2\,g\,(5}{}\right.$$

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^2,x)

[Out] x^5*((e^2*g*(5*d*g + 2*e*f))/5 + (2*d*e^2*g^2)/5) + x^3*((5*d*(2*d^2*g^2 +
e^2*f^2 + 4*d*e*f*g))/3 + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e
^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(3*e) - (d
^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/(3*e^2)) + x^4*((e^5*f^2 + 10*d^2
*e^3*g^2 + 10*d*e^4*f*g)/(4*e^2) - (d^2*e*g^2)/4 + (d*(e^2*g*(5*d*g + 2*e*f
) + 2*d*e^2*g^2))/(2*e)) + x^2*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/(
2*e) - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2
*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(2*e^2) + (d*(5*d*(2*d^2*g^2
+ e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^
2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e
^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e) + x*((d^5*g^2 + 10*d^3*e^2*f^
2 + 10*d^4*e*f*g)/e^2 - (d^2*(5*d*(2*d^2*g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*
((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5
*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2
*g^2))/e^2))/e^2 + (2*d*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/e - (d^2
*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(
5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e^2 + (2*d*(5*d*(2*d^2*g^2 + e^2*f^2 + 4
*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2
 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g +
 2*e*f) + 2*d*e^2*g^2))/e^2))/e))/e) + (log(e*x - d)*(144*d^6*g^2 + 80*d^4*
e^2*f^2 + 224*d^5*e*f*g))/e^3 + (32*(d^7*g^2 + d^5*e^2*f^2 + 2*d^6*e*f*g))/
(e*(d*e^2 - e^3*x)) + (e^3*g^2*x^6)/6
```

3.558 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 177

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3 + \frac{1}{2}eg(ef+3dg)x^4 + \frac{1}{5}e^2g^2x^5 + \frac{16d^4(ef+dg)^2}{e^3(d-ex)} + \frac{32d^3(ef+dg)(ef+2dg)\log(d-ex)}{e^3}$$

```
[Out] d^2*(48*d^2*g^2+64*d*e*f*g+17*e^2*f^2)*x/e^2+d*(16*d^2*g^2+17*d*e*f*g+3*e^2
*f^2)*x^2/e+1/3*(17*d^2*g^2+12*d*e*f*g+e^2*f^2)*x^3+1/2*e*g*(3*d*g+e*f)*x^4
+1/5*e^2*g^2*x^5+16*d^4*(d*g+e*f)^2/e^3/(-e*x+d)+32*d^3*(d*g+e*f)*(2*d*g+e*
f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{16d^4(dg+ef)^2}{e^3(d-ex)} + \frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3} + \frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right) + \frac{dx^2(16d^2g^2+17defg+3e^2f^2)}{e} + \frac{d^2x(48d^2g^2+64defg+17e^2f^2)}{e^2} + \frac{1}{2}egx^4(3dg+ef) + \frac{1}{5}e^2g^2x^5$$

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}\,dx\\
&= \int \left(\frac{d^2(17e^2f^2+64defg+48d^2g^2)}{e^2}+\frac{2d(3e^2f^2+17defg+16d^2g^2)\,x}{e}\right.\\
&\qquad +\left(e^2f^2+12defg+17d^2g^2\right)x^2+2eg(ef+3dg)x^3+e^2g^2x^4\\
&\qquad \left.+\frac{32d^3(-ef-2dg)(ef+dg)}{e^2(d-ex)}+\frac{16d^4(ef+dg)^2}{e^2(-d+ex)^2}\right)dx\\
&= \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2}+\frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e}\\
&\quad +\frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3+\frac{1}{2}eg(ef+3dg)x^4+\frac{1}{5}e^2g^2x^5\\
&\quad +\frac{16d^4(ef+dg)^2}{e^3(d-ex)}+\frac{32d^3(ef+dg)(ef+2dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3 + \frac{1}{2}eg(ef+3dg)x^4 + \frac{1}{5}e^2g^2x^5 - \frac{16d^4(ef+dg)^2}{e^3(-d+ex)} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)\log(d-ex)}{e^3}$$

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3*(-
d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.23

method	result
default	$\frac{\frac{1}{5}g^2e^4x^5+\frac{3}{2}de^3g^2x^4+\frac{1}{2}e^4fgx^4+\frac{17}{3}d^2e^2g^2x^3+4de^3fgx^3+\frac{1}{3}e^4f^2x^3+16d^3eg^2x^2+17d^2e^2fgx^2+3de^3f^2x^2+48d^4g^2x+64d^3e}{e^2}$
risch	$\frac{e^2g^2x^5}{5}+\frac{3edg^2x^4}{2}+\frac{e^2fgx^4}{2}+\frac{17d^2g^2x^3}{3}+4edfgx^3+\frac{e^2f^2x^3}{3}+\frac{16d^3g^2x^2}{e}+17d^2fgx^2+3edf^2x^2+$
norman	$\frac{\left(-\frac{127}{3}d^4g^2-60fged^3-\frac{50}{3}d^2e^2f^2\right)x^3+\left(-\frac{82}{15}d^2g^2e^2-4dfge^3-\frac{1}{3}f^2e^4\right)x^5+\left(-\frac{29}{2}g^2ed^3-\frac{33}{2}e^2fgd^2-3e^3f^2d\right)x^4+\frac{d^2\left(32g^2d^5+4\right.}{}}{-e^2x^2+d^2}$
parallelrisch	$\frac{6g^2e^6x^6+39x^5de^5g^2+15x^5e^6fg+125x^4d^2e^4g^2+105de^5fgx^4+10e^6f^2x^4+310d^3e^3g^2x^3+390d^2e^4fgx^3+80de^5f^2x^3+1920\ln}{}$

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/5*g^2*e^4*x^5+3/2*d*e^3*g^2*x^4+1/2*e^4*f*g*x^4+17/3*d^2*e^2*g^2*x
^3+4*d*e^3*f*g*x^3+1/3*e^4*f^2*x^3+16*d^3*e*g^2*x^2+17*d^2*e^2*f*g*x^2+3*d*
e^3*f^2*x^2+48*d^4*g^2*x+64*d^3*e*f*g*x+17*d^2*e^2*f^2*x)+32*d^3/e^3*(2*d^2
*g^2+3*d*e*f*g+e^2*f^2)*ln(-e*x+d)+16*d^4*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(
-e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{6\,e^6g^2x^6 - 480\,d^4e^2f^2 - 960\,d^5efg - 480\,d^6g^2 + 3\,(5\,e^6fg + 13\,de^5g^2)x^5 + 5\,(2\,e^6f^2 + 21\,de^5fg + 25\,d^2e^4g^2}{}$$

[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] 1/30*(6*e^6*g^2*x^6 - 480*d^4*e^2*f^2 - 960*d^5*e*f*g - 480*d^6*g^2 + 3*(5*e^6*f*g + 13*d*e^5*g^2)*x^5 + 5*(2*e^6*f^2 + 21*d*e^5*f*g + 25*d^2*e^4*g^2)*x^4 + 10*(8*d*e^5*f^2 + 39*d^2*e^4*f*g + 31*d^3*e^3*g^2)*x^3 + 30*(14*d^2*e^4*f^2 + 47*d^3*e^3*f*g + 32*d^4*e^2*g^2)*x^2 - 30*(17*d^3*e^3*f^2 + 64*d^4*e^2*f*g + 48*d^5*e*g^2)*x - 960*(d^4*e^2*f^2 + 3*d^5*e*f*g + 2*d^6*g^2 - (d^3*e^3*f^2 + 3*d^4*e^2*f*g + 2*d^5*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{32d^3(dg+ef)\,(2dg+ef)\log(-d+ex)}{e^3} + \frac{e^2g^2x^5}{5} + x^4 \cdot \left(\frac{3deg^2}{2} + \frac{e^2fg}{2}\right) + x^3 \cdot \left(\frac{17d^2g^2}{3} + 4defg + \frac{e^2f^2}{3}\right) + x^2 \cdot \left(\frac{16d^3g^2}{e} + 17d^2fg + 3def^2\right) + x\left(\frac{48d^4g^2}{e^2} + \frac{64d^3fg}{e} + 17d^2f^2\right) + \frac{-16d^6g^2 - 32d^5efg - 16d^4e^2f^2}{-de^3 + e^4x}$$

[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)

[Out] 32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 + x**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f**2/3) + x**2*(16*d**3*g**2/e + 17*d**2*f*g + 3*d*e*f**2) + x*(48*d**4*g**2/e**2 + 64*d**3*f*g/e + 17*d**2*f**2) + (-16*d**6*g**2 - 32*d**5*e*f*g - 16*d**4*e**2*f**2)/(-d*e**3 + e**4*x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{16\,(d^4e^2f^2+2\,d^5efg+d^6g^2)}{e^4x-de^3} + \frac{6\,e^4g^2x^5+15\,(e^4fg+3\,de^3g^2)x^4+10\,(e^4f^2+12\,de^3fg+17\,d^2e^2g^2)x^3+30\,(3\,de^3f^2+17\,d^2e^2fg+16\,\ldots}{30\,e^2} + \frac{32\,(d^3e^2f^2+3\,d^4efg+2\,d^5g^2)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -16*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/(e^4*x - d*e^3) + 1/30*(6*e^4*g^2
*x^5 + 15*(e^4*f*g + 3*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 12*d*e^3*f*g + 17*d^2
*e^2*g^2)*x^3 + 30*(3*d*e^3*f^2 + 17*d^2*e^2*f*g + 16*d^3*e*g^2)*x^2 + 30*(
17*d^2*e^2*f^2 + 64*d^3*e*f*g + 48*d^4*g^2)*x)/e^2 + 32*(d^3*e^2*f^2 + 3*d^
4*e*f*g + 2*d^5*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{32\,(d^3e^2f^2+3\,d^4efg+2\,d^5g^2)\log(|ex-d|)}{e^3} - \frac{16\,(d^4e^2f^2+2\,d^5efg+d^6g^2)}{(ex-d)e^3} + \frac{6\,e^{12}g^2x^5+15\,e^{12}fgx^4+45\,de^{11}g^2x^4+10\,e^{12}f^2x^3+120\,de^{11}fgx^3+170\,d^2e^{10}g^2x^3+90\,de^{11}f^2x^2+510\ldots}{30\,e^{10}}$$

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 32*(d^3*e^2*f^2 + 3*d^4*e*f*g + 2*d^5*g^2)*log(abs(e*x - d))/e^3 - 16*(d^4*
e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/((e*x - d)*e^3) + 1/30*(6*e^12*g^2*x^5 + 1
5*e^12*f*g*x^4 + 45*d*e^11*g^2*x^4 + 10*e^12*f^2*x^3 + 120*d*e^11*f*g*x^3 +
 170*d^2*e^10*g^2*x^3 + 90*d*e^11*f^2*x^2 + 510*d^2*e^10*f*g*x^2 + 480*d^3*
e^9*g^2*x^2 + 510*d^2*e^10*f^2*x + 1920*d^3*e^9*f*g*x + 1440*d^4*e^8*g^2*x)
/e^10
```

Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.19

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$= x^2\left(\frac{2\,d\,(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2)}{e}-\frac{d^2\,(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2)}{2\,e^2}+\frac{d\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2)}{e}\right)}{e}\right)$$
$$+x^4\left(\frac{e\,g\,(2\,d\,g+e\,f)}{2}+\frac{d\,e\,g^2}{2}\right)+x\left(\frac{d^4\,g^2+8\,d^3\,e\,f\,g+6\,d^2\,e^2\,f^2}{e^2}-\frac{d^2\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2)}{e}\right)}{e^2}+\frac{2\,d\left(\frac{4\,d\,(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2)}{e}-\frac{d^2\,(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2)}{e^2}+\frac{2\,d\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,\left(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2\right)}{e}\right)}{e}\right)}{e}\right)$$
$$+x^3\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{3\,e^2}-\frac{d^2\,g^2}{3}+\frac{2\,d\,(2\,e\,g\,(2\,d\,g+e\,f)+2\,d\,e\,g^2)}{3\,e}\right)$$
$$+\frac{\ln(e\,x-d)\,\left(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2\right)}{e^3}$$
$$+\frac{16\,(d^6\,g^2+2\,d^5\,e\,f\,g+d^4\,e^2\,f^2)}{e\,\left(d\,e^2-e^3\,x\right)}+\frac{e^2\,g^2\,x^5}{5}$$

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((2*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) +
2*d*e*g^2))/(2*e^2) + (d*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2
*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e) + x^4*((e*g*(2*d*g +
e*f))/2 + (d*e*g^2)/2) + x*((d^4*g^2 + 6*d^2*e^2*f^2 + 8*d^3*e*f*g)/e^2 - (
d^2*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2
*d*g + e*f) + 2*d*e*g^2))/e))/e^2 + (2*d*((4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f
```

```
*g))/e - (d^2*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e^2 + (2*d*((e^4*f^2 + 6*d
^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e
*g^2))/e))/e))/e) + x^3*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(3*e^2) -
(d^2*g^2)/3 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/(3*e)) + (log(e*x - d
)*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g))/e^3 + (16*(d^6*g^2 + d^4*e^
2*f^2 + 2*d^5*e*f*g))/(e*(d*e^2 - e^3*x)) + (e^2*g^2*x^5)/5
```

3.559 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4+\frac{8d^3(ef+dg)^2}{e^3(d-ex)}+\frac{4d^2(ef+dg)(3ef+7dg)\log(d-ex)}{e^3}$$

```
[Out] d*(20*d^2*g^2+24*d*e*f*g+5*e^2*f^2)*x/e^2+1/2*(12*d^2*g^2+10*d*e*f*g+e^2*f^
2)*x^2/e+1/3*g*(5*d*g+2*e*f)*x^3+1/4*e*g^2*x^4+8*d^3*(d*g+e*f)^2/e^3/(-e*x+
d)+4*d^2*(d*g+e*f)*(7*d*g+3*e*f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{x^2(12d^2g^2+10defg+e^2f^2)}{2e}+\frac{dx(20d^2g^2+24defg+5e^2f^2)}{e^2}+\frac{1}{3}gx^3(5dg+2ef)+\frac{1}{4}eg^2x^4$$

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3
*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d
- e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2}\,dx \\
&= \int \left(\frac{d(5e^2f^2+24defg+20d^2g^2)}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x}{e}+g(2ef+5dg)x^2 \right.\\
&\qquad \left. +eg^2x^3+\frac{4d^2(-3ef-7dg)(ef+dg)}{e^2(d-ex)}+\frac{8d^3(ef+dg)^2}{e^2(-d+ex)^2}\right)dx \\
&= \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3 \\
&\quad +\frac{1}{4}eg^2x^4+\frac{8d^3(ef+dg)^2}{e^3(d-ex)}+\frac{4d^2(ef+dg)(3ef+7dg)\log(d-ex)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx &= \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e} \\
&\quad +\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4-\frac{8d^3(ef+dg)^2}{e^3(-d+ex)} \\
&\quad +\frac{4d^2(3e^2f^2+10defg+7d^2g^2)\log(d-ex)}{e^3}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g + 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d^3 *(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g ^2)*Log[d - e*x])/e^3

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{4}g^2e^3x^4+\frac{5}{3}x^3d\,e^2g^2+\frac{2}{3}x^3e^3fg+6x^2d^2e\,g^2+5x^2d\,e^2fg+\frac{1}{2}x^2e^3f^2+20d^3g^2x+24d^2efgx+5d\,e^2f^2x}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2\right.}{e^3}$
risch	$\frac{e\,g^2x^4}{4}+\frac{5x^3d\,g^2}{3}+\frac{2e\,x^3fg}{3}+\frac{6x^2g^2d^2}{e}+5x^2fgd+\frac{e\,x^2f^2}{2}+\frac{20d^3g^2x}{e^2}+\frac{24d^2fgx}{e}+5d\,f^2x+\frac{28d^4\ln(-ex+}{e^3}$
norman	$\frac{\left(-\frac{55}{3}d^3g^2-\frac{70}{3}d^2efg-5d\,e^2f^2\right)x^3+\left(-\frac{23}{4}d^2g^2e-5dfg\,e^2-\frac{1}{2}f^2e^3\right)x^4+\frac{d^3\left(28d^2g^2+40defg+13e^2f^2\right)x}{e^2}+\frac{d^2\left(28d^4g^2+42fge\,d^3+17d^2e^2\right.}{2e^3}}{-e^2x^2+d^2}$
parallelrisch	$\frac{3g^2e^5x^5+17x^4d\,e^4g^2+8x^4e^5fg+52x^3d^2e^3g^2+52x^3d\,e^4fg+6x^3e^5f^2+336\ln(ex-d)x\,d^4e\,g^2+480\ln(ex-d)x\,d^3e^2fg+144\ln(ex-}{}$

[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/e^2*(1/4*g^2*e^3*x^4+5/3*x^3*d*e^2*g^2+2/3*x^3*e^3*f*g+6*x^2*d^2*e*g^2+5* x^2*d*e^2*f*g+1/2*x^2*e^3*f^2+20*d^3*g^2*x+24*d^2*e*f*g*x+5*d*e^2*f^2*x)+4* d^2/e^3*(7*d^2*g^2+10*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)+8*d^3*(d^2*g^2+2*d*e*f* g+e^2*f^2)/e^3/(-e*x+d)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$=\frac{3\,e^5g^2x^5-96\,d^3e^2f^2-192\,d^4efg-96\,d^5g^2+(8\,e^5fg+17\,de^4g^2)x^4+2\,(3\,e^5f^2+26\,de^4fg+26\,d^2e^3g^2)x^3}{}$$

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] 1/12*(3*e^5*g^2*x^5 - 96*d^3*e^2*f^2 - 192*d^4*e*f*g - 96*d^5*g^2 + (8*e^5* f*g + 17*d*e^4*g^2)*x^4 + 2*(3*e^5*f^2 + 26*d*e^4*f*g + 26*d^2*e^3*g^2)*x^3 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3*f^ 2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g + 7 *d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*log(e*x - d))/ (e^4*x - d*e^3)

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4d^2(dg+ef)\,(7dg+3ef)\log{(-d+ex)}}{e^3} + \frac{eg^2x^4}{4} + x^3 \cdot \left(\frac{5dg^2}{3} + \frac{2efg}{3}\right) + x^2 \cdot \left(\frac{6d^2g^2}{e} + 5dfg + \frac{ef^2}{2}\right) + x\left(\frac{20d^3g^2}{e^2} + \frac{24d^2fg}{e} + 5df^2\right) + \frac{-8d^5g^2 - 16d^4efg - 8d^3e^2f^2}{-de^3 + e^4x}$$

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x**
3*(5*d*g**2/3 + 2*e*f*g/3) + x**2*(6*d**2*g**2/e + 5*d*f*g + e*f**2/2) + x*
(20*d**3*g**2/e**2 + 24*d**2*f*g/e + 5*d*f**2) + (-8*d**5*g**2 - 16*d**4*e*
f*g - 8*d**3*e**2*f**2)/(-d*e**3 + e**4*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{8\,(d^3e^2f^2 + 2\,d^4efg + d^5g^2)}{e^4x - de^3} + \frac{3\,e^3g^2x^4 + 4\,(2\,e^3fg + 5\,de^2g^2)x^3 + 6\,(e^3f^2 + 10\,de^2fg + 12\,d^2eg^2)x^2 + 12\,(5\,de^2f^2 + 24\,d^2efg + 20\,d^3g}{12\,e^2} + \frac{4\,(3\,d^2e^2f^2 + 10\,d^3efg + 7\,d^4g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/(e^4*x - d*e^3) + 1/12*(3*e^3*g^2*
x^4 + 4*(2*e^3*f*g + 5*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 10*d*e^2*f*g + 12*d^2*
e*g^2)*x^2 + 12*(5*d*e^2*f^2 + 24*d^2*e*f*g + 20*d^3*g^2)*x)/e^2 + 4*(3*d^2
*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{4\left(3\,d^2e^2f^2+10\,d^3efg+7\,d^4g^2\right)\log\left(|ex-d|\right)}{e^3}-\frac{8\left(d^3e^2f^2+2\,d^4efg+d^5g^2\right)}{(ex-d)e^3} + \frac{3\,e^9g^2x^4+8\,e^9fgx^3+20\,de^8g^2x^3+6\,e^9f^2x^2+60\,de^8fgx^2+72\,d^2e^7g^2x^2+60\,de^8f^2x+288\,d^2e^7fgx+2}{12\,e^8}$$

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 4*(3*d^2*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(abs(e*x - d))/e^3 - 8*(d^3
*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/((e*x - d)*e^3) + 1/12*(3*e^9*g^2*x^4 + 8
*e^9*f*g*x^3 + 20*d*e^8*g^2*x^3 + 6*e^9*f^2*x^2 + 60*d*e^8*f*g*x^2 + 72*d^2
*e^7*g^2*x^2 + 60*d*e^8*f^2*x + 288*d^2*e^7*f*g*x + 240*d^3*e^6*g^2*x)/e^8
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.16

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = x\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{e^2}-\frac{d^2\,\left(g\,\left(3\,d\,g+2\,e\,f\right)+2\,d\,g^2\right)}{e^2} + \frac{2\,d\,\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^2}-\frac{d^2\,g^2}{e}+\frac{2\,d\,\left(g\,\left(3\,d\,g+2\,e\,f\right)+2\,d\,g^2\right)}{e}\right)}{e}\right) + x^2\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{2\,e^2}-\frac{d^2\,g^2}{2\,e} + \frac{d\,\left(g\,\left(3\,d\,g+2\,e\,f\right)+2\,d\,g^2\right)}{e}\right)+x^3\left(\frac{g\,\left(3\,d\,g+2\,e\,f\right)}{3}+\frac{2\,d\,g^2}{3}\right) + \frac{\ln\left(e\,x-d\right)\,\left(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2\right)}{e^3} + \frac{8\,\left(d^5\,g^2+2\,d^4\,e\,f\,g+d^3\,e^2\,f^2\right)}{e\,\left(d\,e^2-e^3\,x\right)}+\frac{e\,g^2\,x^4}{4}$$

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e^2 - (d^2*(g*(3*d*g + 2*e*f) + 2*
d*g^2))/e^2 + (2*d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/e^2 - (d^2*g^2)/e
 + (2*d*(g*(3*d*g + 2*e*f) + 2*d*g^2))/e))/e) + x^2*((e^3*f^2 + 3*d^2*e*g^2
 + 6*d*e^2*f*g)/(2*e^2) - (d^2*g^2)/(2*e) + (d*(g*(3*d*g + 2*e*f) + 2*d*g^2
```

```
))/e) + x^3*((g*(3*d*g + 2*e*f))/3 + (2*d*g^2)/3) + (log(e*x - d)*(28*d^4*g
^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g))/e^3 + (8*(d^5*g^2 + d^3*e^2*f^2 + 2*d^
4*e*f*g))/(e*(d*e^2 - e^3*x)) + (e*g^2*x^4)/4
```

3.560 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2} + \frac{g(ef+2dg)x^2}{e} + \frac{g^2x^3}{3} + \frac{4d^2(ef+dg)^2}{e^3(d-ex)} + \frac{4d(ef+dg)(ef+3dg)\log(d-ex)}{e^3}$$

```
[Out] (8*d^2*g^2+8*d*e*f*g+e^2*f^2)*x/e^2+g*(2*d*g+e*f)*x^2/e+1/3*g^2*x^3+4*d^2*(
d*g+e*f)^2/e^3/(-e*x+d)+4*d*(d*g+e*f)*(3*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{4d^2(dg+ef)^2}{e^3(d-ex)} + \frac{x(8d^2g^2+8defg+e^2f^2)}{e^2} + \frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3} + \frac{gx^2(2dg+ef)}{e} + \frac{g^2x^3}{3}$$

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}\,dx \\
&= \int \left(\frac{e^2f^2+8defg+8d^2g^2}{e^2}+\frac{2g(ef+2dg)x}{e}+g^2x^2+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)}+\frac{4d^2(ef+dg)^2}{e^2(-d+ex)^2}\right)dx \\
&= \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}+\frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg)(ef+3dg)\log(d-ex)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$
\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}-\frac{4d^2(ef+dg)^2}{e^3(-d+ex)}+\frac{4d(e^2f^2+4defg+3d^2g^2)\log(d-ex)}{e^3}
$$

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f*g
 + 3*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{1}{3}g^2x^3e^2+2de\,g^2x^2+e^2fg\,x^2+8d^2g^2x+8defgx+e^2f^2x}{e^2}+\frac{4d(3d^2g^2+4defg+e^2f^2)\ln(-ex+d)}{e^3}+\frac{4d^2(d^2g^2+2defg+e^2f^2)}{e^3(-ex+d)}$
risch	$\frac{g^2x^3}{3}+\frac{2d\,g^2x^2}{e}+fg\,x^2+\frac{8d^2g^2x}{e^2}+\frac{8dfgx}{e}+f^2x+\frac{12d^3\ln(-ex+d)g^2}{e^3}+\frac{16d^2\ln(-ex+d)fg}{e^2}+\frac{4d\ln(-ex+d)f^2}{e}$
norman	$\frac{(-\frac{23}{3}d^2g^2-8defg-e^2f^2)x^3+\frac{d^2(6d^3g^2+9d^2efg+4d\,e^2f^2)}{e^3}+\frac{d^2(12d^2g^2+16defg+5e^2f^2)x}{e^2}-\frac{e^2g^2x^5}{3}-eg(2dg+ef)x^4}{-e^2x^2+d^2}+\frac{4d(3d^2g^2+$
parallelrisch	$\frac{g^2e^4x^4+5x^3d\,e^3g^2+3x^3e^4fg+36\ln(ex-d)x\,d^3e\,g^2+48\ln(ex-d)x\,d^2e^2fg+12\ln(ex-d)xd\,e^3f^2+18x^2d^2e^2g^2+21x^2d\,e^3fg+3x^2}{3e^3(ex-d)}$

[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/e^2*(1/3*g^2*x^3*e^2+2*d*e*g^2*x^2+e^2*f*g*x^2+8*d^2*g^2*x+8*d*e*f*g*x+e^2*f^2*x)+4*d/e^3*(3*d^2*g^2+4*d*e*f*g+e^2*f^2)*ln(-e*x+d)+4*d^2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.93

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$=\frac{e^4g^2x^4-12\,d^2e^2f^2-24\,d^3efg-12\,d^4g^2+(3\,e^4fg+5\,de^3g^2)x^3+3\,(e^4f^2+7\,de^3fg+6\,d^2e^2g^2)x^2-3\,(de^3}{3\,(e^4x-}$$

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] 1/3*(e^4*g^2*x^4 - 12*d^2*e^2*f^2 - 24*d^3*e*f*g - 12*d^4*g^2 + (3*e^4*f*g + 5*d*e^3*g^2)*x^3 + 3*(e^4*f^2 + 7*d*e^3*f*g + 6*d^2*e^2*g^2)*x^2 - 3*(d*e^3*f^2 + 8*d^2*e^2*f*g + 8*d^3*e*g^2)*x - 12*(d^2*e^2*f^2 + 4*d^3*e*f*g + 3*d^4*g^2 - (d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4d(dg+ef)\,(3dg+ef)\log{(-d+ex)}}{e^3}+\frac{g^2x^3}{3}+x^2\cdot\left(\frac{2dg^2}{e}+fg\right) + x\left(\frac{8d^2g^2}{e^2}+\frac{8dfg}{e}+f^2\right)+\frac{-4d^4g^2-8d^3efg-4d^2e^2f^2}{-de^3+e^4x}$$

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 + x**2*(2*d*
g**2/e + f*g) + x*(8*d**2*g**2/e**2 + 8*d*f*g/e + f**2) + (-4*d**4*g**2 - 8
*d**3*e*f*g - 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)}{e^4x-de^3} + \frac{e^2g^2x^3+3\,(e^2fg+2\,deg^2)x^2+3\,(e^2f^2+8\,defg+8\,d^2g^2)x}{3\,e^2} + \frac{4\,(de^2f^2+4\,d^2efg+3\,d^3g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)/(e^4*x - d*e^3) + 1/3*(e^2*g^2*x^3
 + 3*(e^2*f*g + 2*d*e*g^2)*x^2 + 3*(e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2
 + 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{4\,(de^2f^2+4\,d^2efg+3\,d^3g^2)\log{(|ex-d|)}}{e^3}-\frac{4\,(d^2e^2f^2+2\,d^3efg+d^4g^2)}{(ex-d)e^3} + \frac{e^6g^2x^3+3\,e^6fgx^2+6\,de^5g^2x^2+3\,e^6f^2x+24\,de^5fgx+24\,d^2e^4g^2x}{3\,e^6}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(abs(e*x - d))/e^3 - 4*(d^2*e^2*
f^2 + 2*d^3*e*f*g + d^4*g^2)/((e*x - d)*e^3) + 1/3*(e^6*g^2*x^3 + 3*e^6*f*g
*x^2 + 6*d*e^5*g^2*x^2 + 3*e^6*f^2*x + 24*d*e^5*f*g*x + 24*d^2*e^4*g^2*x)/e
^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x^2\left(\frac{g\,(d\,g+e\,f)}{e}+\frac{d\,g^2}{e}\right)+x\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e^2}+\frac{2\,d\,\left(\frac{2\,g\,(d\,g+e\,f)}{e}+\frac{2\,d\,g^2}{e}\right)}{e}-\frac{d^2\,g^2}{e^2}\right)+\frac{g^2\,x^3}{3}+\frac{4\,(d^4\,g^2+2\,d^3\,e\,f\,g+d^2\,e^2\,f^2)}{e\,(d\,e^2-e^3\,x)}+\frac{\ln(e\,x-d)\,(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((g*(d*g + e*f))/e + (d*g^2)/e) + x*((d^2*g^2 + e^2*f^2 + 4*d*e*f*g)/e^
2 + (2*d*((2*g*(d*g + e*f))/e + (2*d*g^2)/e))/e - (d^2*g^2)/e^2) + (g^2*x^3
)/3 + (4*(d^4*g^2 + d^2*e^2*f^2 + 2*d^3*e*f*g))/(e*(d*e^2 - e^3*x)) + (log(
e*x - d)*(12*d^3*g^2 + 4*d*e^2*f^2 + 16*d^2*e*f*g))/e^3
```

3.561 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2} dx = \frac{g(2ef+3dg)x}{e^2} + \frac{g^2x^2}{2e} + \frac{2d(ef+dg)^2}{e^3(d-ex)} + \frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}$$

```
[Out] g*(3*d*g+2*e*f)*x/e^2+1/2*g^2*x^2/e+2*d*(d*g+e*f)^2/e^3/(-e*x+d)+(d*g+e*f)*
(5*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 78}

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2} dx = \frac{2d(dg+ef)^2}{e^3(d-ex)} + \frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3} + \frac{gx(3dg+2ef)}{e^2} + \frac{g^2x^2}{2e}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{g(2ef+3dg)}{e^2}+\frac{g^2x}{e}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)}+\frac{2d(ef+dg)^2}{e^2(-d+ex)^2}\right)dx\\ &= \frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{2eg(2ef+3dg)x+e^2g^2x^2+\frac{4d(ef+dg)^2}{d-ex}+2(e^2f^2+6defg+5d^2g^2)\log(d-ex)}{2e^3}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
[Out] (2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2*
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

method	result
default	$\frac{g\left(\frac{1}{2}eg\,x^{2}+3dgx+2efx\right)}{e^{2}}+\frac{\left(5d^{2}g^{2}+6defg+e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}+\frac{2d\left(d^{2}g^{2}+2defg+e^{2}f^{2}\right)}{e^{3}\left(-ex+d\right)}$
risch	$\frac{g^{2}x^{2}}{2e}+\frac{3g^{2}dx}{e^{2}}+\frac{2gfx}{e}+\frac{5\ln\left(-ex+d\right)d^{2}g^{2}}{e^{3}}+\frac{6\ln\left(-ex+d\right)dfg}{e^{2}}+\frac{\ln\left(-ex+d\right)f^{2}}{e}+\frac{2d^{3}g^{2}}{e^{3}\left(-ex+d\right)}+\frac{4d^{2}fg}{e^{2}\left(-ex+d\right)}+$
norman	$\frac{\frac{d\left(5d^{2}g^{2}+6defg+2e^{2}f^{2}\right)x}{e^{2}}+\frac{d^{2}\left(5d^{2}g^{2}+8defg+4e^{2}f^{2}\right)}{2e^{3}}-\frac{e\,g^{2}x^{4}}{2}-g\left(3dg+2ef\right)x^{3}}{-e^{2}x^{2}+d^{2}}+\frac{\left(5d^{2}g^{2}+6defg+e^{2}f^{2}\right)\ln\left(-ex+d\right)}{e^{3}}$
parallelrisch	$\frac{g^{2}x^{3}e^{3}+10\ln\left(ex-d\right)x\,d^{2}e\,g^{2}+12\ln\left(ex-d\right)xd\,e^{2}fg+2\ln\left(ex-d\right)x\,e^{3}f^{2}+5x^{2}d\,e^{2}g^{2}+4x^{2}e^{3}fg-10\ln\left(ex-d\right)d^{3}g^{2}-12\ln\left(ex-d\right)d^{2}}{2e^{3}\left(ex-d\right)}$

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] g/e^2*(1/2*e*g*x^2+3*d*g*x+2*e*f*x)+1/e^3*(5*d^2*g^2+6*d*e*f*g+e^2*f^2)*ln(
-e*x+d)+2*d*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{e^3g^2x^3 - 4\,de^2f^2 - 8\,d^2efg - 4\,d^3g^2 + (4\,e^3fg + 5\,de^2g^2)x^2 - 2\,(2\,de^2fg + 3\,d^2eg^2)x - 2\,(de^2f^2 + 6\,d^2ef}{2\,(e^4x - de^3)}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(e^3*g^2*x^3 - 4*d*e^2*f^2 - 8*d^2*e*f*g - 4*d^3*g^2 + (4*e^3*f*g + 5*d
*e^2*g^2)*x^2 - 2*(2*d*e^2*f*g + 3*d^2*e*g^2)*x - 2*(d*e^2*f^2 + 6*d^2*e*f*
g + 5*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)*log(e*x - d))/(e^4
*x - d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = x\left(\frac{3dg^2}{e^2}+\frac{2fg}{e}\right)+\frac{-2d^3g^2-4d^2efg-2de^2f^2}{-de^3+e^4x} + \frac{g^2x^2}{2e}+\frac{(dg+ef)\,(5dg+ef)\log\left(-d+ex\right)}{e^3}$$

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] x*(3*d*g**2/e**2 + 2*f*g/e) + (-2*d**3*g**2 - 4*d**2*e*f*g - 2*d*e**2*f**2)
/(-d*e**3 + e**4*x) + g**2*x**2/(2*e) + (d*g + e*f)*(5*d*g + e*f)*log(-d +
e*x)/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)}{e^4x-de^3} + \frac{eg^2x^2+2\,(2\,efg+3\,dg^2)x}{2\,e^2} + \frac{(e^2f^2+6\,defg+5\,d^2g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/(e^4*x - d*e^3) + 1/2*(e*g^2*x^2 + 2
*(2*e*f*g + 3*d*g^2)*x)/e^2 + (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(e*x - d
)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2+6\,defg+5\,d^2g^2)\log{(|ex-d|)}}{e^3} + \frac{e^3g^2x^2+4\,e^3fgx+6\,de^2g^2x}{2\,e^4} - \frac{2\,(de^2f^2+2\,d^2efg+d^3g^2)}{(ex-d)e^3}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(abs(e*x - d))/e^3 + 1/2*(e^3*g^2*x^2
+ 4*e^3*f*g*x + 6*d*e^2*g^2*x)/e^4 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/
((e*x - d)*e^3)
```

Mupad [B] (verification not implemented)

Time = 12.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = x\left(\frac{d\,g^2+2\,e\,f\,g}{e^2}+\frac{2\,d\,g^2}{e^2}\right) + \frac{\ln(e\,x-d)\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{e^3} + \frac{g^2\,x^2}{2\,e}+\frac{2\,\left(d^3\,g^2+2\,d^2\,e\,f\,g+d\,e^2\,f^2\right)}{e\,\left(d\,e^2-e^3\,x\right)}$$

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d*g^2 + 2*e*f*g)/e^2 + (2*d*g^2)/e^2) + (log(e*x - d)*(5*d^2*g^2 + e^2*
f^2 + 6*d*e*f*g))/e^3 + (g^2*x^2)/(2*e) + (2*(d^3*g^2 + d*e^2*f^2 + 2*d^2*e
*f*g))/(e*(d*e^2 - e^3*x))
```

3.562 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 50

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}$$

```
[Out] g^2*x/e^2+(d*g+e*f)^2/e^3/(-e*x+d)+2*g*(d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 45}

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{g^2}{e^2}+\frac{(ef+dg)^2}{e^2(-d+ex)^2}+\frac{2g(ef+dg)}{e^2(-d+ex)}\right)dx\\ &= \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx=\frac{eg^2x+\frac{(ef+dg)^2}{d-ex}+2g(ef+dg)\log(d-ex)}{e^3}$$

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
[Out] (e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{g^2x}{e^2}+\frac{2g(dg+ef)\ln(-ex+d)}{e^3}+\frac{d^2g^2+2defg+e^2f^2}{e^3(-ex+d)}$	63
risch	$\frac{g^2x}{e^2}+\frac{2g^2\ln(-ex+d)d}{e^3}+\frac{2g\ln(-ex+d)f}{e^2}+\frac{d^2g^2}{e^3(-ex+d)}+\frac{2dfg}{e^2(-ex+d)}+\frac{f^2}{e(-ex+d)}$	89
norman	$\frac{\frac{d\left(d^2g^2+2defg+e^2f^2\right)}{e^3}+\frac{\left(2d^2g^2+2defg+e^2f^2\right)x}{e^2}-g^2x^3}{-e^2x^2+d^2}+\frac{2g(dg+ef)\ln(-ex+d)}{e^3}$	99
parallelrisch	$\frac{2\ln(ex-d)xde\,g^2+2\ln(ex-d)x\,e^2fg+g^2x^2e^2-2\ln(ex-d)d^2g^2-2\ln(ex-d)defg-2d^2g^2-2defg-e^2f^2}{e^3(ex-d)}$	109

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
[Out] g^2*x/e^2+2*g*(d*g+e*f)*ln(-e*x+d)/e^3+(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*
x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$$
$$= \frac{e^2g^2x^2 - deg^2x - e^2f^2 - 2\,defg - d^2g^2 - 2\,(defg + d^2g^2 - (e^2fg + deg^2)x)\log{(ex-d)}}{e^4x - de^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] (e^2*g^2*x^2 - d*e*g^2*x - e^2*f^2 - 2*d*e*f*g - d^2*g^2 - 2*(d*e*f*g + d^2
*g^2 - (e^2*f*g + d*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-d^2g^2 - 2defg - e^2f^2}{-de^3 + e^4x} + \frac{g^2x}{e^2} + \frac{2g(dg+ef)\log{(-d+ex)}}{e^3}$$

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g
*(d*g + e*f)*log(-d + e*x)/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{g^2x}{e^2} - \frac{e^2f^2 + 2\,defg + d^2g^2}{e^4x - de^3} + \frac{2\,(efg + dg^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] g^2*x/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(e^4*x - d*e^3) + 2*(e*f*g + d*
g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{g^2x}{e^2} + \frac{2\,(efg+dg^2)\log\left(|ex-d|\right)}{e^3} - \frac{e^2f^2+2\,defg+d^2g^2}{(ex-d)e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] g^2*x/e^2 + 2*(e*f*g + d*g^2)*log(abs(e*x - d))/e^3 - (e^2*f^2 + 2*d*e*f*g
+ d^2*g^2)/((e*x - d)*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{e\,\left(d\,e^2-e^3\,x\right)} + \frac{g^2\,x}{e^2} + \frac{\ln\left(e\,x-d\right)\,\left(2\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(e*(d*e^2 - e^3*x)) + (g^2*x)/e^2 + (log(e*
x - d)*(2*d*g^2 + 2*e*f*g))/e^3
```

3.563 $\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(ef+dg)^2}{2de^3(d-ex)} - \frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3} + \frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}$$

```
[Out] 1/2*(d*g+e*f)^2/d/e^3/(-e*x+d)-1/4*(-3*d*g+e*f)*(d*g+e*f)*ln(-e*x+d)/d^2/e^
3+1/4*(-d*g+e*f)^2*ln(e*x+d)/d^2/e^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {813, 90}

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(ef-dg)^2\log(d+ex)}{4d^2e^3} - \frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3} + \frac{(dg+ef)^2}{2de^3(d-ex)}$$

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)}\right)dx\\ &= \frac{(ef+dg)^2}{2de^3(d-ex)}-\frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{2d(ef+dg)^2+(-e^2f^2+2defg+3d^2g^2)(d-ex)\log(d-ex)+(ef-dg)^2(d-ex)\log(d+ex)}{4d^2e^3(d-ex)}$$

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
[Out] (2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d -
 e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

method	result
default	$\frac{d^2g^2+2defg+e^2f^2}{2d\,e^3(-ex+d)}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}$
norman	$\frac{-\frac{-d^2g^2-2defg-e^2f^2}{2e^3}+\frac{(d^2g^2+2defg+e^2f^2)x}{2d\,e^2}}{-e^2x^2+d^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}$
risch	$\frac{d\,g^2}{2e^3(-ex+d)}+\frac{fg}{e^2(-ex+d)}+\frac{f^2}{2de(-ex+d)}+\frac{\ln(-ex-d)g^2}{4e^3}-\frac{\ln(-ex-d)fg}{2d\,e^2}+\frac{\ln(-ex-d)f^2}{4d^2e}+\frac{3\ln(ex-d)g^2}{4e^3}+\frac{\ln(e}{2}$
parallelrisch	$\frac{3\ln(ex-d)x\,d^2e\,g^2+2\ln(ex-d)xd\,e^2fg-\ln(ex-d)x\,e^3f^2+\ln(ex+d)x\,d^2e\,g^2-2\ln(ex+d)xd\,e^2fg+\ln(ex+d)x\,e^3f^2-3\ln(ex-d)d}{4d^2e^3(ex-}$

[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/d/e^3/(-e*x+d)+1/4*(3*d^2*g^2+2*d*e*f*g-e^2
*f^2)/d^2/e^3*ln(-e*x+d)+1/4*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3*ln(e*x+d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,de^2f^2+4\,d^2efg+2\,d^3g^2+(de^2f^2-2\,d^2efg+d^3g^2-(e^3f^2-2\,de^2fg+d^2eg^2)x)\log(ex+d)-(de^2}{4\,(d^2e^4x-d^3e^3)}$$

[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] -1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d^
3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2 -
2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x - d^3*e^3)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-d^2g^2-2defg-e^2f^2}{-2d^2e^3+2de^4x}+\frac{(dg-ef)^2\log\left(x+\frac{2d^3g^2-d(dg-ef)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}+\frac{(dg+ef)(3dg-ef)\log\left(x+\frac{2d^3g^2-d(dg+ef)(3dg-ef)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g - e
*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f*g
 - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3*g*
*2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))
/(4*d**2*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{e^2f^2+2\,defg+d^2g^2}{2\,(de^4x-d^2e^3)} + \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex+d)}{4\,d^2e^3} - \frac{(e^2f^2-2\,defg-3\,d^2g^2)\log(ex-d)}{4\,d^2e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 - 2
*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3*d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex+d|)}{4\,d^2e^3} - \frac{(e^2f^2-2\,defg-3\,d^2g^2)\log(|ex-d|)}{4\,d^2e^3} - \frac{de^2f^2+2\,d^2efg+d^3g^2}{2\,(ex-d)d^2e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^2*e^3) - 1/4*(e^2*
f^2 - 2*d*e*f*g - 3*d^2*g^2)*log(abs(e*x - d))/(d^2*e^3) - 1/2*(d*e^2*f^2 +
 2*d^2*e*f*g + d^3*g^2)/((e*x - d)*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,\left(d-e\,x\right)}+\frac{\ln\left(d+e\,x\right)\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}+\frac{\ln\left(d-e\,x\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(2*d*e^3*(d - e*x)) + (log(d + e*x)*(d^2*g^
2 + e^2*f^2 - 2*d*e*f*g))/(4*d^2*e^3) + (log(d - e*x)*(3*d^2*g^2 - e^2*f^2
+ 2*d*e*f*g))/(4*d^2*e^3)
```

3.564 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{(d^2g+e^2fx)\,(f+gx)}{2d^2e^2\,(d^2-e^2x^2)} + \frac{(ef-dg)(ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

```
[Out] 1/2*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)+1/2*(-d*g+e*f)*(d*g+e*f)
*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {737, 214}

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx = \frac{\text{arctanh}\left(\frac{ex}{d}\right)(ef-dg)(dg+ef)}{2d^3e^3} + \frac{(f+gx)\,(d^2g+e^2fx)}{2d^2e^2\,(d^2-e^2x^2)}$$

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d^2g+e^2fx)(f+gx)}{2d^2e^2(d^2-e^2x^2)} - \frac{1}{2}\left(-\frac{f^2}{d^2}+\frac{g^2}{e^2}\right)\int \frac{1}{d^2-e^2x^2}\,dx \\ &= \frac{(d^2g+e^2fx)(f+gx)}{2d^2e^2(d^2-e^2x^2)} + \frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{-2d^2fg-e^2f^2x-d^2g^2x}{2d^2e^2(-d^2+e^2x^2)} - \frac{(-e^2f^2+d^2g^2)\,\text{arctanh}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
[Out] (-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^2
*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

method	result
norman	$\frac{\frac{fg}{e^2}+\frac{(d^2g^2+e^2f^2)x}{2d^2e^2}}{-e^2x^2+d^2}+\frac{(d^2g^2-e^2f^2)\ln(-ex+d)}{4e^3d^3}-\frac{(d^2g^2-e^2f^2)\ln(ex+d)}{4e^3d^3}$
risch	$\frac{\frac{fg}{e^2}+\frac{(d^2g^2+e^2f^2)x}{2d^2e^2}}{-e^2x^2+d^2}+\frac{\ln(ex-d)g^2}{4e^3d}-\frac{\ln(ex-d)f^2}{4e\,d^3}-\frac{\ln(-ex-d)g^2}{4e^3d}+\frac{\ln(-ex-d)f^2}{4e\,d^3}$
default	$\frac{(d^2g^2-e^2f^2)\ln(-ex+d)}{4e^3d^3}+\frac{d^2g^2+2defg+e^2f^2}{4d^2e^3(-ex+d)}+\frac{(-d^2g^2+e^2f^2)\ln(ex+d)}{4e^3d^3}-\frac{d^2g^2-2defg+e^2f^2}{4d^2e^3(ex+d)}$
parallelrisch	$\frac{\ln(ex-d)x^2d^2e^2g^2-\ln(ex-d)x^2e^4f^2-\ln(ex+d)x^2d^2e^2g^2+\ln(ex+d)x^2e^4f^2-\ln(ex-d)d^4g^2+\ln(ex-d)d^2e^2f^2+\ln(ex+d)d^4g^2-}{4d^3e^3(e^2x^2-d^2)}$

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
[Out] (f*g/e^2+1/2*(d^2*g^2+e^2*f^2)/d^2/e^2*x)/(-e^2*x^2+d^2)+1/4/e^3*(d^2*g^2-e
^2*f^2)/d^3*ln(-e*x+d)-1/4/e^3*(d^2*g^2-e^2*f^2)/d^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.09

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = -\frac{4\,d^3efg+2\,(de^3f^2+d^3eg^2)x+(d^2e^2f^2-d^4g^2-(e^4f^2-d^2e^2g^2)x^2)\log\left(ex+d\right)-(d^2e^2f^2-d^4g^2-(}{4\,(d^3e^5x^2-d^5e^3)}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*e*f*g + 2*(d*e^3*f^2 + d^3*e*g^2)*x + (d^2*e^2*f^2 - d^4*g^2 -
(e^4*f^2 - d^2*e^2*g^2)*x^2)*log(e*x + d) - (d^2*e^2*f^2 - d^4*g^2 - (e^4*f
^2 - d^2*e^2*g^2)*x^2)*log(e*x - d))/(d^3*e^5*x^2 - d^5*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(66) = 132$.

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.11

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx = \frac{-2d^2fg+x(-d^2g^2-e^2f^2)}{-2d^4e^2+2d^2e^4x^2} + \frac{(dg-ef)\,(dg+ef)\log\left(-\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)}+x\right)}{4d^3e^3} - \frac{(dg-ef)\,(dg+ef)\log\left(\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)}+x\right)}{4d^3e^3}$$

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**2*f*g + x*(-d**2*g**2 - e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**2
) + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2 -
e**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*f)*
(d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{2\,d^2fg+(e^2f^2+d^2g^2)x}{2\,(d^2e^4x^2-d^4e^2)}+\frac{(e^2f^2-d^2g^2)\log{(ex+d)}}{4\,d^3e^3} - \frac{(e^2f^2-d^2g^2)\log{(ex-d)}}{4\,d^3e^3}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*g + (e^2*f^2 + d^2*g^2)*x)/(d^2*e^4*x^2 - d^4*e^2) + 1/4*(e^2
*f^2 - d^2*g^2)*log(e*x + d)/(d^3*e^3) - 1/4*(e^2*f^2 - d^2*g^2)*log(e*x -
d)/(d^3*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = -\frac{e^2f^2x+d^2g^2x+2\,d^2fg}{2\,(e^2x^2-d^2)d^2e^2}+\frac{(e^3f^2-d^2eg^2)\log{(|ex+d|)}}{4\,d^3e^4} - \frac{(e^3f^2-d^2eg^2)\log{(|ex-d|)}}{4\,d^3e^4}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^2*f^2*x + d^2*g^2*x + 2*d^2*f*g)/((e^2*x^2 - d^2)*d^2*e^2) + 1/4*(e
^3*f^2 - d^2*e*g^2)*log(abs(e*x + d))/(d^3*e^4) - 1/4*(e^3*f^2 - d^2*e*g^2)
*log(abs(e*x - d))/(d^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \frac{\frac{f\,g}{e^2}+\frac{x\,(d^2\,g^2+e^2\,f^2)}{2\,d^2\,e^2}}{d^2-e^2\,x^2} - \frac{2\,\mathrm{atanh}\left(\frac{4\,e\,x\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d\,(d^2\,g^2-e^2\,f^2)}\right)\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d^3\,e^3}$$

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^2,x)
```

```
[Out] ((f*g)/e^2 + (x*(d^2*g^2 + e^2*f^2))/(2*d^2*e^2))/(d^2 - e^2*x^2) - (2*atan
h((4*e*x*((d^2*g^2)/4 - (e^2*f^2)/4))/(d*(d^2*g^2 - e^2*f^2)))*((d^2*g^2)/4
 - (e^2*f^2)/4))/(d^3*e^3)
```

3.565 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 121

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2} dx = \frac{(ef+dg)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)} + \frac{(3ef-dg)(ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] 1/8*(d*g+e*f)^2/d^3/e^3/(-e*x+d)-1/8*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^2+1/4*(d^
2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)+1/8*(-d*g+3*e*f)*(d*g+e*f)*arctanh(e*x/d)/d^
4/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2} dx = \frac{\text{arctanh}\left(\frac{ex}{d}\right)(3ef-dg)(dg+ef)}{8d^4e^3} + \frac{(dg+ef)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{8d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2}+\frac{(3ef-dg)(ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{((3ef-dg)(ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2} \\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{(3ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{2d(ef+dg)^2}{d-ex}-\frac{2d^2(ef-dg)^2}{(d+ex)^2}+\frac{4d\left(-e^2f^2+d^2g^2\right)}{d+ex}+\left(-3e^2f^2-2defg+d^2g^2\right)\log(d-ex)+\left(3e^2f^2+2defg-d^2g^2\right)}{16d^4e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*(
-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log[d
 - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

method	result
default	$\frac{\left(d^{2} g^{2}-2 d e f g-3 e^{2} f^{2}\right) \ln \left(-e x+d\right)}{16 e^{3} d^{4}}+\frac{d^{2} g^{2}+2 d e f g+e^{2} f^{2}}{8 e^{3} d^{3} \left(-e x+d\right)}-\frac{-d^{2} g^{2}+e^{2} f^{2}}{4 e^{3} d^{3} \left(e x+d\right)}+\frac{\left(-d^{2} g^{2}+2 d e f g+3 e^{2} f^{2}\right) \ln \left(e x+d\right)}{16 e^{3} d^{4}}-\frac{d^{2} g^{2}-}{8 d^{2}}$
norman	$\frac{-\frac{-d^{2} g^{2}-2 d e f g+e^{2} f^{2}}{4 d e^{3}}-\frac{\left(-3 d^{2} g^{2}-2 d e f g-3 e^{2} f^{2}\right) x}{8 e^{2} d^{2}}+\frac{\left(-d^{2} g^{2}+2 d e f g+3 e^{2} f^{2}\right) x^{2}}{8 d^{3} e}}{\left(-e x+d\right) \left(e x+d\right)^{2}}+\frac{\left(d^{2} g^{2}-2 d e f g-3 e^{2} f^{2}\right) \ln \left(-e x+d\right)}{16 e^{3} d^{4}}-\frac{\left(d^{2} g^{2}-\right.}{}$
risch	$\frac{-\frac{\left(d^{2} g^{2}-2 d e f g-3 e^{2} f^{2}\right) x^{2}}{8 e d^{3}}+\frac{\left(3 d^{2} g^{2}+2 d e f g+3 e^{2} f^{2}\right) x}{8 d^{2} e^{2}}+\frac{d^{2} g^{2}+2 d e f g-e^{2} f^{2}}{4 d e^{3}}}{\left(e x+d\right) \left(-e^{2} x^{2}+d^{2}\right)}+\frac{\ln \left(e x-d\right) g^{2}}{16 e^{3} d^{2}}-\frac{\ln \left(e x-d\right) f g}{8 e^{2} d^{3}}-\frac{3 \ln \left(e x-d\right) f^{2}}{16 e d^{4}}-$
parallelrisch	$\frac{-2 \ln \left(e x+d\right) d^{4} e f g+\ln \left(e x-d\right) x^{3} d^{2} e^{3} g^{2}-2 \ln \left(e x+d\right) x d^{3} e^{2} f g+2 \ln \left(e x-d\right) x d^{3} e^{2} f g-2 \ln \left(e x-d\right) x^{2} d^{2} e^{3} f g+2 \ln \left(e x+d\right) x^{2} d^{2} e^{3} f}{}$

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/e^3*(d^2*g^2-2*d*e*f*g-3*e^2*f^2)/d^4*ln(-e*x+d)+1/8*(d^2*g^2+2*d*e*f*
g+e^2*f^2)/e^3/d^3/(-e*x+d)-1/4*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)+1/16*(-d
^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*g+e^2*f^
2)/d^2/e^3/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(114) = 228$.

Time = 0.35 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.45

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{4\,d^3e^2f^2 - 8\,d^4efg - 4\,d^5g^2 - 2\left(3\,de^4f^2 + 2\,d^2e^3fg - d^3e^2g^2\right)x^2 - 2\left(3\,d^2e^3f^2 + 2\,d^3e^2fg + 3\,d^4eg^2\right)x - (}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
- (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*
f^2 + 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) + (3*d^3*e^2*f^2 + 2*d^4*e
*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + d^5*e^5*x^2 - d^6*e^4*x - d^7*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(105) = 210.

Time = 0.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.31

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{-2d^4g^2-4d^3efg+2d^2e^2f^2+x^2(d^2e^2g^2-2de^3fg-3e^4f^2)+x(-3d^3eg^2-2d^2e^2fg-3de^3f^2)}{-8d^6e^3-8d^5e^4x+8d^4e^5x^2+8d^3e^6x^3} + \frac{(dg-3ef)\,(dg+ef)\log\left(-\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)}+x\right)}{16d^4e^3} - \frac{(dg-3ef)\,(dg+ef)\log\left(\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)}+x\right)}{16d^4e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*
d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*
f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2
 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f
)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{2\,d^2e^2f^2-4\,d^3efg-2\,d^4g^2-(3\,e^4f^2+2\,de^3fg-d^2e^2g^2)x^2-(3\,de^3f^2+2\,d^2e^2fg+3\,d^3eg^2)x}{8\left(d^3e^6x^3+d^4e^5x^2-d^5e^4x-d^6e^3\right)} + \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log\left(ex+d\right)}{16\,d^4e^3} - \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log\left(ex-d\right)}{16\,d^4e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.66

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log\left(|ex+d|\right)}{16\,d^4e^3} - \frac{(3\,e^2f^2+2\,defg-d^2g^2)\log\left(|ex-d|\right)}{16\,d^4e^3} + \frac{2\,d^3e^2f^2-4\,d^4efg-2\,d^5g^2-(3\,de^4f^2+2\,d^2e^3fg-d^3e^2g^2)x^2-(3\,d^2e^3f^2+2\,d^3e^2fg+3\,d^4eg^2)x}{8\,(ex+d)^2(ex-d)d^4e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*(
3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) + 1/8*(2*d^3*e
^2*f^2 - 4*d^4*e*f*g - 2*d^5*g^2 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g
^2)*x^2 - (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x)/((e*x + d)^2*(e*
x - d)*d^4*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.64

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3+d^2\,e\,x-d\,e^2\,x^2-e^3\,x^3} + \frac{\mathrm{atanh}\left(\frac{e\,x\,(d\,g+e\,f)\,(d\,g-3\,e\,f)}{d\,(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2)}\right)\,(d\,g+e\,f)\,(d\,g-3\,e\,f)}{8\,d^4\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 + 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 - e^3*x^3 - d*e^2*x^2 + d^2*e*x) + (atanh((e*x*(d*g + e*f)*(d*g - 3*e*
f))/(d*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g)))*(d*g + e*f)*(d*g - 3*e*f))/(8*d^
4*e^3)
```

3.566 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{f(ef+dg)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

```
[Out] 1/16*(d*g+e*f)^2/d^4/e^3/(-e*x+d)-1/12*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^3+1/8*(
d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^2-1/16*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*x
+d)+1/4*f*(d*g+e*f)*arctanh(e*x/d)/d^5/e^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{f\operatorname{arctanh}\left(\frac{ex}{d}\right)(dg+ef)}{4d^5e^2} + \frac{(dg+ef)^2}{16d^4e^3(d-ex)} - \frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^4} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3} + \frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^2} + \frac{f(ef+dg)}{4d^4e\,(d^2-e^2x^2)}\right) dx \\
&= \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{(f(ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{4d^4e} \\
&= \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{f(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{2d(2d^5g^2+3e^5f^2x^3+d^3e^2f(-4f+gx)+3de^4fx^2(2f+gx)+2d^4eg(f+2gx)+d^2e^3fx(f+6gx))+3ef}{24d^5e^3(d-ex)(d+e}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(2
*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f +
 d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*(d
+ e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11

method	result
norman	$\frac{\frac{dfg-3ef^2}{8de^2}-\frac{\left(-d^2g^2-defg-e^2f^2\right)x^3}{3d^4}+\frac{f(dg+ef)x^2}{2d^3}-\frac{e\left(-4d^2g^2-defg-e^2f^2\right)x^4}{24d^5}}{(ex+d)^3(-ex+d)}-\frac{f(dg+ef)\ln(-ex+d)}{8d^5e^2}+\frac{f(dg+ef)\ln(ex+d)}{8d^5e^2}$
risch	$\frac{\frac{ef(dg+ef)x^3}{4d^4}+\frac{f(dg+ef)x^2}{2d^3}+\frac{\left(4d^2g^2+defg+e^2f^2\right)x}{12d^2e^2}+\frac{d^2g^2+defg-2e^2f^2}{6de^3}}{(ex+d)^2(-e^2x^2+d^2)}-\frac{f\ln(-ex+d)g}{8d^4e^2}-\frac{\ln(-ex+d)f^2}{8ed^5}+\frac{f\ln(ex+d)g}{8d^4e^2}+$
default	$\frac{d^2g^2+2defg+e^2f^2}{16e^3d^4(-ex+d)}-\frac{f(dg+ef)\ln(-ex+d)}{8d^5e^2}-\frac{-d^2g^2+e^2f^2}{8e^3d^3(ex+d)^2}-\frac{-d^2g^2+2defg+3e^2f^2}{16e^3d^4(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{12d^2e^3(ex+d)^3}+\frac{f(dg+e}{8}$
parallelrisch	$-\frac{3\ln(ex-d)x^4e^5f^2-3\ln(ex+d)x^4e^5f^2-3\ln(ex-d)d^5fg-3\ln(ex-d)d^4ef^2+3\ln(ex+d)d^5fg+3\ln(ex+d)d^4ef^2+18f^2d^3e^2x+}{}$

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(d*f*g-3*e*f^2)/d/e^2-1/3*(-d^2*g^2-d*e*f*g-e^2*f^2)/d^4*x^3+1/2/d^3*f
*(d*g+e*f)*x^2-1/24*e*(-4*d^2*g^2-d*e*f*g-e^2*f^2)/d^5*x^4)/(e*x+d)^3/(-e*x
+d)-1/8*f*(d*g+e*f)/d^5/e^2*ln(-e*x+d)+1/8*f*(d*g+e*f)/d^5/e^2*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(137) = 274$.

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.31

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^2}\, dx = \frac{8\, d^4e^2f^2 - 4\, d^5efg - 4\, d^6g^2 - 6\, (de^5f^2 + d^2e^4fg)x^3 - 12\, (d^2e^4f^2 + d^3e^3fg)x^2 - 2\, (d^3e^3f^2 + d^4e^2fg + 4\, d^5eg^2)x - \dots}{\dots}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(8*d^4*e^2*f^2 - 4*d^5*e*f*g - 4*d^6*g^2 - 6*(d*e^5*f^2 + d^2*e^4*f*g)
*x^3 - 12*(d^2*e^4*f^2 + d^3*e^3*f*g)*x^2 - 2*(d^3*e^3*f^2 + d^4*e^2*f*g +
4*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2
*(d*e^5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x +
 d) + 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2*(d*e^5*f^2
 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x - d))/(d^5*e
^7*x^4 + 2*d^6*e^6*x^3 - 2*d^8*e^4*x - d^9*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.65

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^2}\, dx = \frac{-2d^5g^2 - 2d^4efg + 4d^3e^2f^2 + x^3(-3de^4fg - 3e^5f^2) + x^2(-6d^2e^3fg - 6de^4f^2) + x(-4d^4eg^2 - d^3e^2fg - \dots)}{-12d^8e^3 - 24d^7e^4x + 24d^5e^6x^3 + 12d^4e^7x^4} - \frac{f(dg+ef)\log\left(-\frac{df(dg+ef)}{e(dfg+ef^2)} + x\right)}{8d^5e^2} + \frac{f(dg+ef)\log\left(\frac{df(dg+ef)}{e(dfg+ef^2)} + x\right)}{8d^5e^2}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**5*g**2 - 2*d**4*e*f*g + 4*d**3*e**2*f**2 + x**3*(-3*d*e**4*f*g - 3*e
**5*f**2) + x**2*(-6*d**2*e**3*f*g - 6*d*e**4*f**2) + x*(-4*d**4*e*g**2 - d
**3*e**2*f*g - d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e
**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(d*f*
g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e*(d*f
*g + e*f**2)) + x)/(8*d**5*e**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = \frac{4\,d^3e^2f^2-2\,d^4efg-2\,d^5g^2-3\,(e^5f^2+de^4fg)x^3-6\,(de^4f^2+d^2e^3fg)x^2-(d^2e^3f^2+d^3e^2fg+4\,d^4eg^2)x}{12\,(d^4e^7x^4+2\,d^5e^6x^3-2\,d^7e^4x-d^8e^3)} + \frac{(ef^2+dfg)\log(ex+d)}{8\,d^5e^2} - \frac{(ef^2+dfg)\log(ex-d)}{8\,d^5e^2}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*d^3*e^2*f^2 - 2*d^4*e*f*g - 2*d^5*g^2 - 3*(e^5*f^2 + d*e^4*f*g)*x^3
 - 6*(d*e^4*f^2 + d^2*e^3*f*g)*x^2 - (d^2*e^3*f^2 + d^3*e^2*f*g + 4*d^4*e*g
^2)*x)/(d^4*e^7*x^4 + 2*d^5*e^6*x^3 - 2*d^7*e^4*x - d^8*e^3) + 1/8*(e*f^2 +
 d*f*g)*log(e*x + d)/(d^5*e^2) - 1/8*(e*f^2 + d*f*g)*log(e*x - d)/(d^5*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2}\,dx = -\frac{(ef^2+dfg)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{8\,d^5e^2} + \frac{e^2f^2+2\,defg+d^2g^2}{32\,d^5e^3\left(\frac{2\,d}{ex+d}-1\right)} - \frac{\frac{9\,d^2e^5f^2}{ex+d}+\frac{6\,d^3e^5f^2}{(ex+d)^2}+\frac{4\,d^4e^5f^2}{(ex+d)^3}+\frac{6\,d^3e^4fg}{ex+d}-\frac{8\,d^5e^4fg}{(ex+d)^3}-\frac{3\,d^4e^3g^2}{ex+d}-\frac{6\,d^5e^3g^2}{(ex+d)^2}+\frac{4\,d^6e^3g^2}{(ex+d)^3}}{48\,d^6e^6}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(e*f^2 + d*f*g)*log(abs(-2*d/(e*x + d) + 1))/(d^5*e^2) + 1/32*(e^2*f^2
 + 2*d*e*f*g + d^2*g^2)/(d^5*e^3*(2*d/(e*x + d) - 1)) - 1/48*(9*d^2*e^5*f^2
/(e*x + d) + 6*d^3*e^5*f^2/(e*x + d)^2 + 4*d^4*e^5*f^2/(e*x + d)^3 + 6*d^3*
e^4*f*g/(e*x + d) - 8*d^5*e^4*f*g/(e*x + d)^3 - 3*d^4*e^3*g^2/(e*x + d) - 6
*d^5*e^3*g^2/(e*x + d)^2 + 4*d^6*e^3*g^2/(e*x + d)^3)/(d^6*e^6)
```

Mupad [B] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^2} \, dx = \frac{\frac{d^2 g^2+d e f g-2 e^2 f^2}{6 d e^3}+\frac{f x^2 (d g+e f)}{2 d^3}+\frac{x (4 d^2 g^2+d e f g+e^2 f^2)}{12 d^2 e^2}+\frac{e f x^3 (d g+e f)}{4 d^4}}{d^4+2 d^3 e x-2 d e^3 x^3-e^4 x^4} + \frac{f \operatorname{atanh}\left(\frac{e x}{d}\right) (d g+e f)}{4 d^5 e^2}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + d*e*f*g)/(6*d*e^3) + (f*x^2*(d*g + e*f))/(2*d^3) +
(x*(4*d^2*g^2 + e^2*f^2 + d*e*f*g))/(12*d^2*e^2) + (e*f*x^3*(d*g + e*f))/(4
*d^4))/(d^4 - e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x) + (f*atanh((e*x)/d)*(d*g +
 e*f))/(4*d^5*e^2)
```

3.567 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 178

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx = \frac{(ef+dg)^2}{32d^5e^3(d-ex)} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3} - \frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2} - \frac{f(ef+dg)}{8d^5e^2(d+ex)} + \frac{(ef+dg)(5ef+dg)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{32d^6e^3}$$

```
[Out] 1/32*(d*g+e*f)^2/d^5/e^3/(-e*x+d)-1/16*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^4+1/12*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^3-1/32*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^2-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)+1/32*(d*g+e*f)*(d*g+5*e*f)*arctanh(e
*x/d)/d^6/e^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(dg+ef)(dg+5ef)}{32d^6e^3} + \frac{(dg+ef)^2}{32d^5e^3(d-ex)} - \frac{f(dg+ef)}{8d^5e^2(d+ex)} - \frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^5}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^4}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^3}\right. \\
&\qquad \left.+\frac{f(ef+dg)}{8d^5e(d+ex)^2}+\frac{(ef+dg)(5ef+dg)}{32d^5e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= \frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2} \\
&\quad -\frac{f(ef+dg)}{8d^5e^2(d+ex)}+\frac{((ef+dg)(5ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{32d^5e^2} \\
&= \frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3} \\
&\quad -\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-\frac{f(ef+dg)}{8d^5e^2(d+ex)}+\frac{(ef+dg)(5ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{\frac{6d(ef+dg)^2}{d-ex}-\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{16d^3\left(-e^2f^2+d^2g^2\right)}{(d+ex)^3}+\frac{6d^2\left(-3e^2f^2-2defg+d^2g^2\right)}{(d+ex)^2}-\frac{24def(ef+dg)}{d+ex}-3(5e^2f^2+6defg+d^2g^2)}{192d^6e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d
^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d
^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*
d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*Log[d
 + e*x])/(192*d^6*e^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-d^2g^2-6defg-5e^2f^2\right)\ln(-ex+d)}{64e^3d^6}+\frac{d^2g^2+2defg+e^2f^2}{32e^3d^5(-ex+d)}+\frac{\left(d^2g^2+6defg+5e^2f^2\right)\ln(ex+d)}{64e^3d^6}-\frac{-d^2g^2+e^2f^2}{12e^3d^3(ex+d)^3}-$
norman	$\frac{\frac{\left(25d^2g^2+54defg-19e^2f^2\right)x^3}{96d^4}-\frac{\left(3d^2g^2-14defg-33e^2f^2\right)x^2}{32e\,d^3}+\frac{3e\left(3d^2g^2+2defg-9e^2f^2\right)x^4}{32d^5}+\frac{e^2\left(d^2g^2-4e^2f^2\right)x^5}{12d^6}-\cdots}{(ex+d)^4(-ex+d)}$
risch	$\frac{\frac{e\left(d^2g^2+6defg+5e^2f^2\right)x^4}{32d^5}+\frac{3\left(d^2g^2+6defg+5e^2f^2\right)x^3}{32d^4}+\frac{7\left(d^2g^2+6defg+5e^2f^2\right)x^2}{96d^3e}+\frac{\left(7d^2g^2-6defg-5e^2f^2\right)x}{32d^2e^2}+\frac{d^2g^2-4e^2f^2}{12d\,e^3}}{(ex+d)^3\left(-e^2x^2+d^2\right)}-$
parallelrisch	$-\frac{-15\ln(ex+d)x^5e^7f^2+15\ln(ex-d)x^5e^7f^2+162x\,d^4e^3f^2+54\ln(ex+d)x\,d^5e^2fg+3\ln(ex-d)x^5d^2e^5g^2-3\ln(ex+d)x^5d^2e^5g^2+\cdots}{}$

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-d^2*g^2-6*d*e*f*g-5*e^2*f^2)/e^3/d^6*ln(-e*x+d)+1/32*(d^2*g^2+2*d*e*
f*g+e^2*f^2)/e^3/d^5/(-e*x+d)+1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^6*ln
(e*x+d)-1/12*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)^3-1/32*(-d^2*g^2+2*d*e*f*g+
3*e^2*f^2)/e^3/d^4/(e*x+d)^2-1/16*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+
d)^4-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(167) = 334$.

Time = 0.35 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.64

$$\int \frac{(f+gx)^2}{(d+ex)^3 (d^2-e^2x^2)^2}\,dx$$
$$= \frac{64\,d^5e^2f^2 - 16\,d^7g^2 - 6\,(5\,de^6f^2 + 6\,d^2e^5fg + d^3e^4g^2)x^4 - 18\,(5\,d^2e^5f^2 + 6\,d^3e^4fg + d^4e^3g^2)x^3 - 14\,(5\,d^3}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(64*d^5*e^2*f^2 - 16*d^7*g^2 - 6*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e
^4*g^2)*x^4 - 18*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 14*(5*
d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 6*(5*d^4*e^3*f^2 + 6*d^5*e
^2*f*g - 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7*g^2 - (5*e^7
*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^
3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 + 2*(5
*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*
e^2*f*g + d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7
*g^2 - (5*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2
*e^5*f*g + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^
2)*x^3 + 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3
*f^2 + 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + 3*d^7*e^7
*x^4 + 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 - 3*d^10*e^4*x - d^11*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(162) = 324$.

Time = 0.80 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.11

$$\int \frac{(f+gx)^2}{(d+ex)^3 (d^2-e^2x^2)^2}\,dx$$
$$= \frac{-8d^6g^2 + 32d^4e^2f^2 + x^4(-3d^2e^4g^2 - 18de^5fg - 15e^6f^2) + x^3(-9d^3e^3g^2 - 54d^2e^4fg - 45de^5f^2) + x^2(-7}{-96d^{10}e^3 - 288d^9e^4x - 192d^8e^5x^2 + 192d^7e^6x^3 + 28}$$
$$- \frac{(dg+ef)\,(dg+5ef)\log\left(-\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)} + x\right)}{64d^6e^3}$$
$$+ \frac{(dg+ef)\,(dg+5ef)\log\left(\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)} + x\right)}{64d^6e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-8*d**6*g**2 + 32*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2 - 18*d*e**5*f*g
 - 15*e**6*f**2) + x**3*(-9*d**3*e**3*g**2 - 54*d**2*e**4*f*g - 45*d*e**5*f
**2) + x**2*(-7*d**4*e**2*g**2 - 42*d**3*e**3*f*g - 35*d**2*e**4*f**2) + x*
(-21*d**5*e*g**2 + 18*d**4*e**2*f*g + 15*d**3*e**3*f**2))/(-96*d**10*e**3 -
 288*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*e**7*
x**4 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e*f)*(d
*g + 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3) +
 (d*g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2*g**2 +
6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.67

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{32\,d^4e^2f^2-8\,d^6g^2-3\,(5\,e^6f^2+6\,de^5fg+d^2e^4g^2)x^4-9\,(5\,de^5f^2+6\,d^2e^4fg+d^3e^3g^2)x^3-7\,(5\,d^2e^4f^2+6}{96\,(d^5e^8x^5+3\,d^6e^7x^4+2\,d^7e^6x^3-2\,d^8e^5x^2-3\,d^9e^4x-}$$
$$+\frac{(5\,e^2f^2+6\,defg+d^2g^2)\log{(ex+d)}}{64\,d^6e^3}-\frac{(5\,e^2f^2+6\,defg+d^2g^2)\log{(ex-d)}}{64\,d^6e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d^2*e^4*g^2
)*x^4 - 9*(5*d*e^5*f^2 + 6*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 - 7*(5*d^2*e^4*f^
2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d^3*e^3*f^2 + 6*d^4*e^2*f*g - 7
*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6*e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5*x^2
 - 3*d^9*e^4*x - d^10*e^3) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x
 + d)/(d^6*e^3) - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^6*
e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{(5\,e^2f^2+6\,defg+d^2g^2)\log{(|ex+d|)}}{64\,d^6e^3}-\frac{(5\,e^2f^2+6\,defg+d^2g^2)\log{(|ex-d|)}}{64\,d^6e^3}$$
$$+\frac{32\,d^5e^2f^2-8\,d^7g^2-3\,(5\,de^6f^2+6\,d^2e^5fg+d^3e^4g^2)x^4-9\,(5\,d^2e^5f^2+6\,d^3e^4fg+d^4e^3g^2)x^3-7\,(5\,d^3e^4}{96\,(ex+d)^4(ex-d)d^6e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^6*e^3) - 1/64*(
5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^6*e^3) + 1/96*(32*d^5
*e^2*f^2 - 8*d^7*g^2 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e^4*g^2)*x^4 -
9*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 7*(5*d^3*e^4*f^2 + 6*
d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*e^2*f*g - 7*d^6*e
*g^2)*x)/((e*x + d)^4*(e*x - d)*d^6*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$$
$$= \frac{\frac{d^2\,g^2-4\,e^2\,f^2}{12\,d\,e^3}+\frac{3\,x^3\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^4}+\frac{e\,x^4\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^5}-\frac{x\,\left(-7\,d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^2\,e^2}+\frac{7\,x^2\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\right.}{96\,d^3\,e}}{d^5+3\,d^4\,e\,x+2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3-3\,d\,e^4\,x^4-e^5\,x^5}$$
$$+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{d\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{32\,d^6\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^3),x)
```

```
[Out] ((d^2*g^2 - 4*e^2*f^2)/(12*d*e^3) + (3*x^3*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g
))/(32*d^4) + (e*x^4*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g))/(32*d^5) - (x*(5*e^
2*f^2 - 7*d^2*g^2 + 6*d*e*f*g))/(32*d^2*e^2) + (7*x^2*(d^2*g^2 + 5*e^2*f^2
+ 6*d*e*f*g))/(96*d^3*e))/(d^5 - e^5*x^5 - 3*d*e^4*x^4 + 2*d^3*e^2*x^2 - 2*
d^2*e^3*x^3 + 3*d^4*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 5*e*f))/(d*(d^2*g
^2 + 5*e^2*f^2 + 6*d*e*f*g)))*(d*g + e*f)*(d*g + 5*e*f))/(32*d^6*e^3)
```

3.568 $\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3}-\frac{f(ef+dg)}{16d^5e^2(d+ex)^2}-\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d+ex)}+\frac{(ef+dg)(3ef+dg)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{32d^7e^3}$$

```
[Out] 1/64*(d*g+e*f)^2/d^6/e^3/(-e*x+d)-1/20*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^5+1/16*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^4-1/48*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^3-1/16*f*(d*g+e*f)/d^5/e^2/(e*x+d)^2-1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^
3/(e*x+d)+1/32*(d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/d^7/e^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(dg+ef)(dg+3ef)}{32d^7e^3}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{64d^6e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^6}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^5}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^4}\right. \\
&\qquad \left.+\frac{f(ef+dg)}{8d^5e(d+ex)^3}+\frac{(ef+dg)(5ef+dg)}{64d^6e^2(d+ex)^2}+\frac{(ef+dg)(3ef+dg)}{32d^6e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} \\
&\quad -\frac{f(ef+dg)}{16d^5e^2(d+ex)^2}-\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d+ex)}+\frac{((ef+dg)(3ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{32d^6e^2} \\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} \\
&\quad -\frac{f(ef+dg)}{16d^5e^2(d+ex)^2}-\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d+ex)}+\frac{(ef+dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{15d(ef+dg)^2}{d-ex}-\frac{48d^5(ef-dg)^2}{(d+ex)^5}+\frac{60d^4\left(-e^2f^2+d^2g^2\right)}{(d+ex)^4}+\frac{20d^3\left(-3e^2f^2-2defg+d^2g^2\right)}{(d+ex)^3}-\frac{60d^2ef(ef+dg)}{(d+ex)^2}-\frac{15d\left(5e^2f^2+6defg+d^2g^2\right)}{d+ex}-\cdots}{960d^7e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60*
d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5*e^
2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g
^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/(960*
d^7*e^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(-d^2g^2-4defg-3e^2f^2\right)\ln(-ex+d)}{64e^3d^7}+\frac{d^2g^2+2defg+e^2f^2}{64e^3d^6(-ex+d)}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)\ln(ex+d)}{64e^3d^7}-\frac{d^2g^2+6defg+5e^2f^2}{64e^3d^6(ex+d)}-\cdots$
norman	$\frac{\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}-\frac{\left(d^2g^2-4defg-13e^2f^2\right)x^2}{8e\,d^3}+\frac{e\left(7d^2g^2+4defg-27e^2f^2\right)x^4}{24d^5}+\frac{e^2\left(79d^2g^2-68defg-531e^2f^2\right)x^5}{480d^6}+\frac{e^3\left(d^2g^2-2defg\cdots\right.}{30d\cdots}}{(ex+d)^5(-ex+d)}$
risch	$\frac{\frac{e^2\left(d^2g^2+4defg+3e^2f^2\right)x^5}{32d^6}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)e\,x^4}{8d^5}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^2}{24d^3e}+\frac{\left(49d^2g^2-188defg-141e\cdots\right.}{480e^2d^2}}{(ex+d)^4(-e^2x^2+d^2)}$
parallelrisch	$-\frac{-300\ln(ex-d)x^2d^5e^3fg-64x^6d\,e^7fg-136x^5d^2e^6fg+160x^4d^3e^5fg+640x^3d^4e^4fg-180\ln(ex-d)x\,d^5e^3f^2+60\ln(ex+d)x\,d^7\cdots}{\cdots}$

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-d^2*g^2-4*d*e*f*g-3*e^2*f^2)/e^3/d^7*ln(-e*x+d)+1/64*(d^2*g^2+2*d*e*
f*g+e^2*f^2)/e^3/d^6/(-e*x+d)+1/64/e^3*(d^2*g^2+4*d*e*f*g+3*e^2*f^2)/d^7*ln
(e*x+d)-1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^6/(e*x+d)-1/16*(-d^2*g^2+e
^2*f^2)/e^3/d^3/(e*x+d)^4-1/48*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4/(e*x+
d)^3-1/20*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+d)^5-1/16*f*(d*g+e*f)/d^
5/e^2/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(197) = 394$.

Time = 0.34 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.30

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx = \frac{288\,d^6e^2f^2+64\,d^7efg-32\,d^8g^2-30\,(3\,de^7f^2+4\,d^2e^6fg+d^3e^5g^2)x^5-120\,(3\,d^2e^6f^2+4\,d^3e^5fg+d^4e^4g}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/960*(288*d^6*e^2*f^2 + 64*d^7*e*f*g - 32*d^8*g^2 - 30*(3*d*e^7*f^2 + 4*d^
2*e^6*f*g + d^3*e^5*g^2)*x^5 - 120*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4
*g^2)*x^4 - 160*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 40*(3*d
^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 2*(141*d^5*e^3*f^2 + 188*d^
6*e^2*f*g - 49*d^7*e*g^2)*x - 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g + d^8*g^2 - (
3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 + 4*d^2*e^6*f*g
 + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4 +
 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d^5*e^3*f^2 + 4
*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x + d) + 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g
 + d^8*g^2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 +
 4*d^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*
e^4*g^2)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d
^5*e^3*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 4*d
^8*e^8*x^5 + 5*d^9*e^7*x^4 - 5*d^11*e^5*x^2 - 4*d^12*e^4*x - d^13*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(192) = 384$.

Time = 0.93 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.03

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx = \frac{-16d^7g^2+32d^6efg+144d^5e^2f^2+x^5(-15d^2e^5g^2-60de^6fg-45e^7f^2)+x^4(-60d^3e^4g^2-240d^2e^5fg-}{-480d^{12}e^3-1920d^{11}e^4x} - \frac{(dg+ef)(dg+3ef)\log\left(-\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3} + \frac{(dg+ef)(dg+3ef)\log\left(\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-16*d**7*g**2 + 32*d**6*e*f*g + 144*d**5*e**2*f**2 + x**5*(-15*d**2*e**5*g
**2 - 60*d*e**6*f*g - 45*e**7*f**2) + x**4*(-60*d**3*e**4*g**2 - 240*d**2*e
**5*f*g - 180*d*e**6*f**2) + x**3*(-80*d**4*e**3*g**2 - 320*d**3*e**4*f*g -
 240*d**2*e**5*f**2) + x**2*(-20*d**5*e**2*g**2 - 80*d**4*e**3*f*g - 60*d**
3*e**4*f**2) + x*(-49*d**6*e*g**2 + 188*d**5*e**2*f*g + 141*d**4*e**3*f**2)
)/(-480*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*d**8*e
**7*x**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(d*g + 3
*e*f)*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f
**2)) + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g + e*f)*(d*
g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**7*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.63

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx$$
$$= \frac{144\,d^5e^2f^2+32\,d^6efg-16\,d^7g^2-15\,(3\,e^7f^2+4\,de^6fg+d^2e^5g^2)x^5-60\,(3\,de^6f^2+4\,d^2e^5fg+d^3e^4g^2)x^4 \cdots}{480\,(d^6e^9x^6+4\,d^7e^8x^5+5 \cdots}$$
$$+\frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(ex+d)}{64\,d^7e^3}-\frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(ex-d)}{64\,d^7e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(144*d^5*e^2*f^2 + 32*d^6*e*f*g - 16*d^7*g^2 - 15*(3*e^7*f^2 + 4*d*e^
6*f*g + d^2*e^5*g^2)*x^5 - 60*(3*d*e^6*f^2 + 4*d^2*e^5*f*g + d^3*e^4*g^2)*x
^4 - 80*(3*d^2*e^5*f^2 + 4*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 20*(3*d^3*e^4*f
^2 + 4*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + (141*d^4*e^3*f^2 + 188*d^5*e^2*f*g
- 49*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 4*d^7*e^8*x^5 + 5*d^8*e^7*x^4 - 5*d^10*e^
5*x^2 - 4*d^11*e^4*x - d^12*e^3) + 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^7*e^3) - 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^7*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.47

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2}\,dx$$
$$= \frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(|ex+d|)}{64\,d^7e^3}-\frac{(3\,e^2f^2+4\,defg+d^2g^2)\log(|ex-d|)}{64\,d^7e^3}$$
$$+\frac{144\,d^6e^2f^2+32\,d^7efg-16\,d^8g^2-15\,(3\,de^7f^2+4\,d^2e^6fg+d^3e^5g^2)x^5-60\,(3\,d^2e^6f^2+4\,d^3e^5fg+d^4e^4g \cdots}{480 \cdots}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^7*e^3) - 1/64*(
3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^7*e^3) + 1/480*(144*d
^6*e^2*f^2 + 32*d^7*e*f*g - 16*d^8*g^2 - 15*(3*d*e^7*f^2 + 4*d^2*e^6*f*g +
d^3*e^5*g^2)*x^5 - 60*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4 - 8
0*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 20*(3*d^4*e^4*f^2 + 4
*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + (141*d^5*e^3*f^2 + 188*d^6*e^2*f*g - 49*d
^7*e*g^2)*x)/((e*x + d)^5*(e*x - d)*d^7*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.50

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \frac{\frac{x^3\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{6\,d^4}-\frac{-d^2\,g^2+2\,d\,e\,f\,g+9\,e^2\,f^2}{30\,d\,e^3}+\frac{e\,x^4\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^5}-\frac{x\,\left(-49\,d^2\,g^2+188\,d\,e\,f\,g+141\,e^2\,f^2\right)}{480\,d^2\,e^2}+\frac{x^2\,\left(d^2\right.}{}}{d^6+4\,d^5\,e\,x+5\,d^4\,e^2\,x^2-5\,d^2\,e^4\,x^4-4\,d\,e^5\,x^5-e^6\,x^6} + \frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g+3\,e\,f\right)}{32\,d^7\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^4),x)
```

```
[Out] ((x^3*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(6*d^4) - (9*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g)/(30*d*e^3) + (e*x^4*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(8*d^5) -
(x*(141*e^2*f^2 - 49*d^2*g^2 + 188*d*e*f*g))/(480*d^2*e^2) + (x^2*(d^2*g^2
+ 3*e^2*f^2 + 4*d*e*f*g))/(24*d^3*e) + (e^2*x^5*(d^2*g^2 + 3*e^2*f^2 + 4*d*
e*f*g))/(32*d^6))/(d^6 - e^6*x^6 - 4*d*e^5*x^5 + 5*d^4*e^2*x^2 - 5*d^2*e^4*
x^4 + 4*d^5*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 3*e*f))/(d*(d^2*g^2 + 3*e
^2*f^2 + 4*d*e*f*g)))*(d*g + e*f)*(d*g + 3*e*f))/(32*d^7*e^3)
```

3.569 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 179

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(ef+2dg)(ef+12dg)x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} - \frac{32d^3(ef+dg)(ef+2dg)}{e^3(d-ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}$$

```
[Out] -d*(56*d^2*g^2+48*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*(2*d*g+e*f)*(12*d*g+e*f)*x^2
/e-1/3*g*(7*d*g+2*e*f)*x^3-1/4*e*g^2*x^4+8*d^4*(d*g+e*f)^2/e^3/(-e*x+d)^2-3
2*d^3*(d*g+e*f)*(2*d*g+e*f)/e^3/(-e*x+d)-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2
*f^2)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{862, 90\}$

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{8d^4(dg+ef)^2}{e^3(d-ex)^2} - \frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)} - \frac{dx(56d^2g^2+48defg+7e^2f^2)}{e^2} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{1}{3}gx^3(7dg+2ef) - \frac{x^2(2dg+ef)(12dg+ef)}{2e} - \frac{1}{4}eg^2x^4$$

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}\,dx \\
&= \int \left(-\frac{d(7e^2f^2+48defg+56d^2g^2)}{e^2} + \frac{(-ef-12dg)(ef+2dg)x}{e} - g(2ef+7dg)x^2 - eg^2x^3 + \frac{32d^3(-ef-2dg)(ef+dg)}{e^2(d-ex)^2} - \frac{16d^4(ef+dg)^2}{e^2(-d+ex)^3} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)}{e^2(-d+ex)}\right) dx \\
&= -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(ef+2dg)(ef+12dg)x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} - \frac{32d^3(ef+dg)(ef+2dg)}{e^3(d-ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(e^2f^2+14defg+24d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)}{e^3(-d+ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}$$

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*g
 + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*
d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2
*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[
d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\frac{1}{4}g^2e^3x^4+\frac{7}{3}x^3de^2g^2+\frac{2}{3}x^3e^3fg+12x^2d^2eg^2+7x^2de^2fg+\frac{1}{2}x^2e^3f^2+56d^3g^2x+48d^2efgx+7de^2f^2x}{e^2} - \frac{8d^2\left(13d^2g^2+14defg\right.}{e}$
risch	$-\frac{eg^2x^4}{4}-\frac{7x^3dg^2}{3}-\frac{2ex^3fg}{3}-\frac{12x^2g^2d^2}{e}-7x^2fgd-\frac{ex^2f^2}{2}-\frac{56d^3g^2x}{e^2}-\frac{48d^2fgx}{e}-7df^2x+\frac{(64g^2d^5+}{}$
norman	$\frac{\left(\frac{521}{3}g^2d^5+\frac{574}{3}fgd^4e+46f^2d^3e^2\right)x^3+\left(-\frac{23}{2}g^2d^2e^3-7fgde^4-\frac{1}{2}f^2e^5\right)x^6+\left(-\frac{154}{3}g^2d^3e^2-\frac{140}{3}fgd^2e^3-7f^2de^4\right)x^5-\frac{d^4\left(319g^2d^4e\right.}{}}{(-e^2x^2+}$
parallelrisch	$-\frac{1344\ln(ex-d)d^5efg+1248\ln(ex-d)x^2d^4e^2g^2+288\ln(ex-d)x^2d^2e^4f^2-2496\ln(ex-d)xd^5eg^2-576\ln(ex-d)xd^3e^3f^2+6e^6f}{}$

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/4*g^2*e^3*x^4+7/3*x^3*d*e^2*g^2+2/3*x^3*e^3*f*g+12*x^2*d^2*e*g^2+
7*x^2*d*e^2*f*g+1/2*x^2*e^3*f^2+56*d^3*g^2*x+48*d^2*e*f*g*x+7*d*e^2*f^2*x)-
8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3-32*d^3/e^3*(2*d^2*g^
2+3*d*e*f*g+e^2*f^2)/(-e*x+d)+8*d^4*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d
)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{3\,e^6g^2x^6 + 288\,d^4e^2f^2 + 960\,d^5efg + 672\,d^6g^2 + 2\,(4\,e^6fg + 11\,de^5g^2)x^5 + (6\,e^6f^2 + 68\,de^5fg + 91\,d^2e^4}{}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^6*g^2*x^6 + 288*d^4*e^2*f^2 + 960*d^5*e*f*g + 672*d^6*g^2 + 2*(4
*e^6*f*g + 11*d*e^5*g^2)*x^5 + (6*e^6*f^2 + 68*d*e^5*f*g + 91*d^2*e^4*g^2)*
x^4 + 4*(18*d*e^5*f^2 + 104*d^2*e^4*f*g + 103*d^3*e^3*g^2)*x^3 - 6*(27*d^2*
e^4*f^2 + 178*d^3*e^3*f*g + 200*d^4*e^2*g^2)*x^2 - 12*(25*d^3*e^3*f^2 + 48*
d^4*e^2*f*g + 8*d^5*e*g^2)*x + 96*(3*d^4*e^2*f^2 + 14*d^5*e*f*g + 13*d^6*g^
2 + (3*d^2*e^4*f^2 + 14*d^3*e^3*f*g + 13*d^4*e^2*g^2)*x^2 - 2*(3*d^3*e^3*f^
2 + 14*d^4*e^2*f*g + 13*d^5*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x +
d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.22

$$\begin{aligned}
&\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx \\
&= -\frac{8d^2\cdot(13d^2g^2+14defg+3e^2f^2)\log(-d+ex)}{e^3} - \frac{eg^2x^4}{4} - x^3\cdot\left(\frac{7dg^2}{3}+\frac{2efg}{3}\right) \\
&\quad - x^2\cdot\left(\frac{12d^2g^2}{e}+7dfg+\frac{ef^2}{2}\right) - x\left(\frac{56d^3g^2}{e^2}+\frac{48d^2fg}{e}+7df^2\right) \\
&\quad - \frac{56d^6g^2+80d^5efg+24d^4e^2f^2+x(-64d^5eg^2-96d^4e^2fg-32d^3e^3f^2)}{d^2e^3-2de^4x+e^5x^2}
\end{aligned}$$

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g*
*2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) - x**2*(12*d**2*g**2/e + 7*d*f*g
+ e*f**2/2) - x*(56*d**3*g**2/e**2 + 48*d**2*f*g/e + 7*d*f**2) - (56*d**6*g
**2 + 80*d**5*e*f*g + 24*d**4*e**2*f**2 + x*(-64*d**5*e*g**2 - 96*d**4*e**2
*f*g - 32*d**3*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= -\frac{8\,(3\,d^4e^2f^2+10\,d^5efg+7\,d^6g^2-4\,(d^3e^3f^2+3\,d^4e^2fg+2\,d^5eg^2)x)}{e^5x^2-2\,de^4x+d^2e^3}$$
$$-\frac{3\,e^3g^2x^4+4\,(2\,e^3fg+7\,de^2g^2)x^3+6\,(e^3f^2+14\,de^2fg+24\,d^2eg^2)x^2+12\,(7\,de^2f^2+48\,d^2efg+56\,d^3g^2}{12\,e^2}$$
$$-\frac{8\,(3\,d^2e^2f^2+14\,d^3efg+13\,d^4g^2)\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -8*(3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f
*g + 2*d^5*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/12*(3*e^3*g^2*x^4
+ 4*(2*e^3*f*g + 7*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 14*d*e^2*f*g + 24*d^2*e*g^
2)*x^2 + 12*(7*d*e^2*f^2 + 48*d^2*e*f*g + 56*d^3*g^2)*x)/e^2 - 8*(3*d^2*e^2
*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{8\,(3\,d^2e^2f^2+14\,d^3efg+13\,d^4g^2)\log{(|ex-d|)}}{e^3}$$
$$-\frac{8\,(3\,d^4e^2f^2+10\,d^5efg+7\,d^6g^2-4\,(d^3e^3f^2+3\,d^4e^2fg+2\,d^5eg^2)x)}{(ex-d)^2e^3}$$
$$-\frac{3\,e^{13}g^2x^4+8\,e^{13}fgx^3+28\,de^{12}g^2x^3+6\,e^{13}f^2x^2+84\,de^{12}fgx^2+144\,d^2e^{11}g^2x^2+84\,de^{12}f^2x+576\,d^2e^{11}}{12\,e^{12}}$$

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -8*(3*d^2*e^2*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(abs(e*x - d))/e^3 - 8*(3
*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f*g +
2*d^5*e*g^2)*x)/((e*x - d)^2*e^3) - 1/12*(3*e^13*g^2*x^4 + 8*e^13*f*g*x^3 +
 28*d*e^12*g^2*x^3 + 6*e^13*f^2*x^2 + 84*d*e^12*f*g*x^2 + 144*d^2*e^11*g^2*
x^2 + 84*d*e^12*f^2*x + 576*d^2*e^11*f*g*x + 672*d^3*e^10*g^2*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

$$= \frac{x\,(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2)-\frac{8\,(7\,d^6\,g^2+10\,d^5\,e\,f\,g+3\,d^4\,e^2\,f^2)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x^2\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{2\,e^3}-\frac{3\,d^2\,g^2}{2\,e}+\frac{3\,d\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{2\,e}\right) - x\left(\frac{d^3\,g^2}{e^2}-\frac{3\,d^2\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{e^2}+\frac{4\,d\,(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2)}{e^2}+\frac{3\,d\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^3}-\frac{3\,d^2\,g^2}{e}+\frac{3\,d\,(2\,g\,(2\,d\,g+e\,f)+3\,d\,g^2)}{e}\right)}{e}\right) - x^3\left(\frac{2\,g\,(2\,d\,g+e\,f)}{3}+d\,g^2\right) - \frac{\ln(e\,x-d)\,(104\,d^4\,g^2+112\,d^3\,e\,f\,g+24\,d^2\,e^2\,f^2)}{e^3} - \frac{e\,g^2\,x^4}{4}$$

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g) - (8*(7*d^6*g^2 + 3*d^4*e^2
*f^2 + 10*d^5*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x^2*((e^4*f^2 +
6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(2*e^3) - (3*d^2*g^2)/(2*e) + (3*d*(2*g*(2*d*g
 + e*f) + 3*d*g^2))/(2*e)) - x*((d^3*g^2)/e^2 - (3*d^2*(2*g*(2*d*g + e*f) +
 3*d*g^2))/e^2 + (4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e^2 + (3*d*((e^4*f^2
 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^3 - (3*d^2*g^2)/e + (3*d*(2*g*(2*d*g + e*
f) + 3*d*g^2))/e))/e) - x^3*((2*g*(2*d*g + e*f))/3 + d*g^2) - (log(e*x - d)
*(104*d^4*g^2 + 24*d^2*e^2*f^2 + 112*d^3*e*f*g))/e^3 - (e*g^2*x^4)/4
```

3.570 $\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx=-\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d^2(ef+dg)(3ef+7dg)}{e^3(d-ex)}-\frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}$$

```
[Out] -(18*d^2*g^2+12*d*e*f*g+e^2*f^2)*x/e^2-g*(3*d*g+e*f)*x^2/e-1/3*g^2*x^3+4*d^
3*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d^2*(d*g+e*f)*(7*d*g+3*e*f)/e^3/(-e*x+d)-2*d
*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx=\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{x(18d^2g^2+12defg+e^2f^2)}{e^2}-\frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3}-\frac{gx^2(3dg+ef)}{e}-\frac{g^2x^3}{3}$$

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3}\,dx \\
&= \int \left(\frac{-e^2f^2-12defg-18d^2g^2}{e^2}-\frac{2g(ef+3dg)x}{e}-g^2x^2 \right.\\
&\quad +\frac{4d^2(-3ef-7dg)(ef+dg)}{e^2(d-ex)^2}-\frac{8d^3(ef+dg)^2}{e^2(-d+ex)^3}\\
&\quad \left. -\frac{2d(3e^2f^2+18defg+19d^2g^2)}{e^2(-d+ex)}\right)dx \\
&= -\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}\\
&\quad -\frac{4d^2(ef+dg)(3ef+7dg)}{e^3(d-ex)}-\frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2} - \frac{g(ef+3dg)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(ef+dg)^2}{e^3(d-ex)^2} + \frac{4d^2(3e^2f^2+10defg+7d^2g^2)}{e^3(-d+ex)} - \frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}$$

[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]

[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e - (g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^2)*Log[d - e*x])/e^3

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\frac{1}{3}g^2x^3e^2+3de\,g^2x^2+e^2fg\,x^2+18d^2g^2x+12defgx+e^2f^2x}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\ln(-ex+d)}{e^3}-\frac{4d^2\left(7d^2g^2+10def\right.}{e^3(-ex+}$
risch	$-\frac{g^2x^3}{3}-\frac{3d\,g^2x^2}{e}-fg\,x^2-\frac{18d^2g^2x}{e^2}-\frac{12dfgx}{e}-f^2x-\frac{\left(-28d^4g^2-40fge\,d^3-12d^2e^2f^2\right)x+\frac{8d^3\left(3d^2g^2+4defg+e^2\right.}{e}}{e^2(-ex+d)^2}$
norman	$\frac{\left(\frac{191}{3}d^4g^2+64fge\,d^3+14d^2e^2f^2\right)x^3+\left(-\frac{52}{3}d^2g^2e^2-12dfg\,e^3-f^2e^4\right)x^5+\frac{d^2\left(41g^2e\,d^3+51e^2fg\,d^2+16e^3f^2d\right)x^2}{e^2}-\frac{d^4\left(30g^2e\,d^3+34e^2fg\right.}{e^4}}{\left(-e^2x^2+d^2\right)^2}$
parallelrisch	$-\frac{g^2e^5x^5+7x^4d\,e^4g^2+3x^4e^5fg+114\ln(ex-d)x^2d^3e^2g^2+108\ln(ex-d)x^2d^2e^3fg+18\ln(ex-d)x^2d\,e^4f^2+37x^3d^2e^3g^2+30x^3d\,e^4}{}$

[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)

[Out] -1/e^2*(1/3*g^2*x^3*e^2+3*d*e*g^2*x^2+e^2*f*g*x^2+18*d^2*g^2*x+12*d*e*f*g*x+e^2*f^2*x)-2*d*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3-4*d^2/e^3*(7*d^2*g^2+10*d*e*f*g+3*e^2*f^2)/(-e*x+d)+4*d^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{e^5g^2x^5+24\,d^3e^2f^2+96\,d^4efg+72\,d^5g^2+(3\,e^5fg+7\,de^4g^2)x^4+(3\,e^5f^2+30\,de^4fg+37\,d^2e^3g^2)x^3-$$

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(e^5*g^2*x^5 + 24*d^3*e^2*f^2 + 96*d^4*e*f*g + 72*d^5*g^2 + (3*e^5*f*g
 + 7*d*e^4*g^2)*x^4 + (3*e^5*f^2 + 30*d*e^4*f*g + 37*d^2*e^3*g^2)*x^3 - 3*(
2*d*e^4*f^2 + 23*d^2*e^3*f*g + 33*d^3*e^2*g^2)*x^2 - 3*(11*d^2*e^3*f^2 + 28
*d^3*e^2*f*g + 10*d^4*e*g^2)*x + 6*(3*d^3*e^2*f^2 + 18*d^4*e*f*g + 19*d^5*g
^2 + (3*d*e^4*f^2 + 18*d^2*e^3*f*g + 19*d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2
 + 18*d^3*e^2*f*g + 19*d^4*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d
^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx &= -\frac{2d(19d^2g^2+18defg+3e^2f^2)\log(-d+ex)}{e^3}-\frac{g^2x^3}{3} \\ &\quad -x^2\cdot\left(\frac{3dg^2}{e}+fg\right)-x\left(\frac{18d^2g^2}{e^2}+\frac{12dfg}{e}+f^2\right) \\ &\quad -\frac{24d^5g^2+32d^4efg+8d^3e^2f^2+x(-28d^4eg^2-40d^3e^2fg-12d^2e^3f^2)}{d^2e^3-2de^4x+e^5x^2}\end{aligned}$$

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x*
*3/3 - x**2*(3*d*g**2/e + f*g) - x*(18*d**2*g**2/e**2 + 12*d*f*g/e + f**2)
- (24*d**5*g**2 + 32*d**4*e*f*g + 8*d**3*e**2*f**2 + x*(-28*d**4*e*g**2 - 4
0*d**3*e**2*f*g - 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{4\,(2\,d^3e^2f^2+8\,d^4efg+6\,d^5g^2-(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2)x)}{e^5x^2-2\,de^4x+d^2e^3} - \frac{e^2g^2x^3+3\,(e^2fg+3\,deg^2)x^2+3\,(e^2f^2+12\,defg+18\,d^2g^2)x}{3\,e^2} - \frac{2\,(3\,de^2f^2+18\,d^2efg+19\,d^3g^2)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f
*g + 7*d^4*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/3*(e^2*g^2*x^3 + 3
*(e^2*f*g + 3*d*e*g^2)*x^2 + 3*(e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2 -
 2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2\,(3\,de^2f^2+18\,d^2efg+19\,d^3g^2)\log(|ex-d|)}{e^3} - \frac{4\,(2\,d^3e^2f^2+8\,d^4efg+6\,d^5g^2-(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2)x)}{(ex-d)^2e^3} - \frac{e^9g^2x^3+3\,e^9fgx^2+9\,de^8g^2x^2+3\,e^9f^2x+36\,de^8fgx+54\,d^2e^7g^2x}{3\,e^9}$$

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(abs(e*x - d))/e^3 - 4*(2*d
^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*
d^4*e*g^2)*x)/((e*x - d)^2*e^3) - 1/3*(e^9*g^2*x^3 + 3*e^9*f*g*x^2 + 9*d*e^
8*g^2*x^2 + 3*e^9*f^2*x + 36*d*e^8*f*g*x + 54*d^2*e^7*g^2*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{x\,(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2)-\frac{8\,(3\,d^5\,g^2+4\,d^4\,e\,f\,g+d^3\,e^2\,f^2)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^3}+\frac{3\,d\left(\frac{g\,(3\,d\,g+2\,e\,f)}{e}+\frac{3\,d\,g^2}{e}\right)}{e}-\frac{3\,d^2\,g^2}{e^2}\right) - x^2\left(\frac{g\,(3\,d\,g+2\,e\,f)}{2\,e}+\frac{3\,d\,g^2}{2\,e}\right) - \frac{g^2\,x^3}{3} - \frac{\ln(e\,x-d)\,(38\,d^3\,g^2+36\,d^2\,e\,f\,g+6\,d\,e^2\,f^2)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(28*d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g) - (8*(3*d^5*g^2 + d^3*e^2*f
^2 + 4*d^4*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((e^3*f^2 + 3*d^2
*e*g^2 + 6*d*e^2*f*g)/e^3 + (3*d*((g*(3*d*g + 2*e*f))/e + (3*d*g^2)/e))/e -
 (3*d^2*g^2)/e^2) - x^2*((g*(3*d*g + 2*e*f))/(2*e) + (3*d*g^2)/(2*e)) - (g^
2*x^3)/3 - (log(e*x - d)*(38*d^3*g^2 + 6*d*e^2*f^2 + 36*d^2*e*f*g))/e^3
```

3.571 $\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 118

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2} - \frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)} - \frac{(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{e^3}$$

```
[Out] -g*(5*d*g+2*e*f)*x/e^2-1/2*g^2*x^2/e+2*d^2*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d*(
d*g+e*f)*(3*d*g+e*f)/e^3/(-e*x+d)-(13*d^2*g^2+10*d*e*f*g+e^2*f^2)*ln(-e*x+d
)/e^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{(13d^2g^2+10defg+e^2f^2)\log(d-ex)}{e^3} - \frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(-\frac{g(2ef+5dg)}{e^2}-\frac{g^2x}{e}+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)^2}-\frac{4d^2(ef+dg)^2}{e^2(-d+ex)^3}\right.\\ &\qquad\left.+\frac{-e^2f^2-10defg-13d^2g^2}{e^2(-d+ex)}\right)dx\\ &= -\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2}\\ &\quad-\frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)}-\frac{(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2eg(2ef+5dg)x+e^2g^2x^2-\frac{4d^2(ef+dg)^2}{(d-ex)^2}+\frac{8d(e^2f^2+4defg+3d^2g^2)}{d-ex}+2(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{2e^3}$$

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -1/2*(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e*
x)^2 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 10*
d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
default	$-\frac{g\left(\frac{1}{2}eg\,x^2+5dgx+2efx\right)}{e^2}+\frac{\left(-13d^2g^2-10defg-e^2f^2\right)\ln(-ex+d)}{e^3}-\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)}{e^3(-ex+d)}+\frac{2d^2\left(d^2g^2+2defg+e^2f^2\right.}{e^3(-ex+d)^2}$
risch	$-\frac{g^2x^2}{2e}-\frac{5g^2dx}{e^2}-\frac{2gfx}{e}+\frac{(12d^3g^2+16d^2efg+4d\,e^2f^2)x-\frac{2d^2\left(5d^2g^2+6defg+e^2f^2\right)}{e}}{e^2(-ex+d)^2}-\frac{13\ln(-ex+d)d^2g^2}{e^3}-\frac{10\ln(-ex}{e^2}$
norman	$\frac{(22d^3g^2+20d^2efg+4d\,e^2f^2)x^3-\frac{d^4\left(11d^2g^2e+12dfg\,e^2+2f^2e^3\right)}{e^4}-\frac{e^3g^2x^6}{2}+\frac{d^2\left(31d^2g^2e+40dfg\,e^2+12f^2e^3\right)x^2}{2e^2}-e^2g(5dg+2ef)x^5-\frac{d^4}{}}{(-e^2x^2+d^2)^2}$
parallelrisch	$-\frac{g^2e^4x^4+26\ln(ex-d)x^2d^2e^2g^2+20\ln(ex-d)x^2d\,e^3fg+2\ln(ex-d)x^2e^4f^2+8x^3d\,e^3g^2+4x^3e^4fg-52\ln(ex-d)x\,d^3e\,g^2-40\ln(e}{}$

[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)

[Out] -g/e^2*(1/2*e*g*x^2+5*d*g*x+2*e*f*x)+(-13*d^2*g^2-10*d*e*f*g-e^2*f^2)/e^3*l
n(-e*x+d)-4*d/e^3*(3*d^2*g^2+4*d*e*f*g+e^2*f^2)/(-e*x+d)+2*d^2*(d^2*g^2+2*d
*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(118) = 236.

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx =$$
$$-\frac{e^4g^2x^4+4\,d^2e^2f^2+24\,d^3efg+20\,d^4g^2+4\,(e^4fg+2\,de^3g^2)x^3-(8\,de^3fg+19\,d^2e^2g^2)x^2-2\,(4\,de^3f^2+}{}$$

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")

[Out] -1/2*(e^4*g^2*x^4 + 4*d^2*e^2*f^2 + 24*d^3*e*f*g + 20*d^4*g^2 + 4*(e^4*f*g
+ 2*d*e^3*g^2)*x^3 - (8*d*e^3*f*g + 19*d^2*e^2*g^2)*x^2 - 2*(4*d*e^3*f^2 +
14*d^2*e^2*f*g + 7*d^3*e*g^2)*x + 2*(d^2*e^2*f^2 + 10*d^3*e*f*g + 13*d^4*g^
2 + (e^4*f^2 + 10*d*e^3*f*g + 13*d^2*e^2*g^2)*x^2 - 2*(d*e^3*f^2 + 10*d^2*e
^2*f*g + 13*d^3*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= -x\left(\frac{5dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{10d^4g^2+12d^3efg+2d^2e^2f^2+x(-12d^3eg^2-16d^2e^2fg-4de^3f^2)}{d^2e^3-2de^4x+e^5x^2}$$
$$-\frac{g^2x^2}{2e}-\frac{(13d^2g^2+10defg+e^2f^2)\log(-d+ex)}{e^3}$$

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -x*(5*d*g**2/e**2 + 2*f*g/e) - (10*d**4*g**2 + 12*d**3*e*f*g + 2*d**2*e**2*
f**2 + x*(-12*d**3*e*g**2 - 16*d**2*e**2*f*g - 4*d*e**3*f**2))/(d**2*e**3 -
 2*d*e**4*x + e**5*x**2) - g**2*x**2/(2*e) - (13*d**2*g**2 + 10*d*e*f*g + e
**2*f**2)*log(-d + e*x)/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= -\frac{2\left(d^2e^2f^2+6\,d^3efg+5\,d^4g^2-2\left(de^3f^2+4\,d^2e^2fg+3\,d^3eg^2\right)x\right)}{e^5x^2-2\,de^4x+d^2e^3}$$
$$-\frac{eg^2x^2+2\left(2\,efg+5\,dg^2\right)x}{2\,e^2}-\frac{\left(e^2f^2+10\,defg+13\,d^2g^2\right)\log(ex-d)}{e^3}$$

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -2*(d^2*e^2*f^2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g +
3*d^3*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/2*(e*g^2*x^2 + 2*(2*e*f
*g + 5*d*g^2)*x)/e^2 - (e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(e^2f^2+10\,defg+13\,d^2g^2)\log(|ex-d|)}{e^3} - \frac{2\,(d^2e^2f^2+6\,d^3efg+5\,d^4g^2-2\,(de^3f^2+4\,d^2e^2fg+3\,d^3eg^2)x)}{(ex-d)^2e^3} - \frac{e^5g^2x^2+4\,e^5fgx+10\,de^4g^2x}{2\,e^6}$$

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -(e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(abs(e*x - d))/e^3 - 2*(d^2*e^2*f^2
 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)
/((e*x - d)^2*e^3) - 1/2*(e^5*g^2*x^2 + 4*e^5*f*g*x + 10*d*e^4*g^2*x)/e^6
```

Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{2\,(5\,d^4\,g^2+6\,d^3\,e\,f\,g+d^2\,e^2\,f^2)}{e}-x\,(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - x\left(\frac{2\,g\,(d\,g+e\,f)}{e^2}+\frac{3\,d\,g^2}{e^2}\right) - \frac{\ln(e\,x-d)\,\left(13\,d^2\,g^2+10\,d\,e\,f\,g+e^2\,f^2\right)}{e^3} - \frac{g^2\,x^2}{2\,e}$$

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((2*(5*d^4*g^2 + d^2*e^2*f^2 + 6*d^3*e*f*g))/e - x*(12*d^3*g^2 + 4*d*e^2*
f^2 + 16*d^2*e*f*g))/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((2*g*(d*g + e*f))
/e^2 + (3*d*g^2)/e^2) - (log(e*x - d)*(13*d^2*g^2 + e^2*f^2 + 10*d*e*f*g))/
e^3 - (g^2*x^2)/(2*e)
```

3.572 $\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 81

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)} - \frac{2g(ef+2dg)\log(d-ex)}{e^3}$$

```
[Out] -g^2*x/e^2+d*(d*g+e*f)^2/e^3/(-e*x+d)^2-(d*g+e*f)*(5*d*g+e*f)/e^3/(-e*x+d)-
2*g*(2*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 78}

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2} - \frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(-\frac{g^2}{e^2}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)^3}-\frac{2g(ef+2dg)}{e^2(-d+ex)}\right)dx\\ &= -\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{-4d^3g^2+4d^2eg(-f+gx)+2de^2gx(3f+gx)+e^3x(f^2-g^2x^2)-2g(ef+2dg)(d-ex)^2\log(d-ex)}{e^3(d-ex)^2}$$

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2 -
 g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{g^{2} x}{e^{2}}-\frac{\left(-5 d^{2} g^{2}-6 d e f g-e^{2} f^{2}\right) x+\frac{4 d^{2} g (d g+e f)}{e}}{e^{2} (-e x+d)^{2}}-\frac{4 g^{2} \ln (-e x+d) d}{e^{3}}-\frac{2 g \ln (-e x+d) f}{e^{2}}$
default	$-\frac{g^{2} x}{e^{2}}-\frac{2 g (2 d g+e f) \ln (-e x+d)}{e^{3}}+\frac{-5 d^{2} g^{2}-6 d e f g-e^{2} f^{2}}{e^{3} (-e x+d)}+\frac{d\left(d^{2} g^{2}+2 d e f g+e^{2} f^{2}\right)}{e^{3} (-e x+d)^{2}}$
norman	$\frac{\left(7 d^{2} g^{2}+6 d e f g+e^{2} f^{2}\right) x^{3}-\frac{d^{4}\left(4 d e g^{2}+4 e^{2} f g\right)}{e^{4}}-e^{2} g^{2} x^{5}+\frac{2 d\left(3 d^{2} g^{2} e+4 d f g e^{2}+f^{2} e^{3}\right) x^{2}}{e^{2}}-\frac{d^{2}\left(4 d^{2} g^{2}+2 d e f g-e^{2} f^{2}\right) x}{e^{2}}}{\left(-e^{2} x^{2}+d^{2}\right)^{2}}-\frac{2 g (2 d g+e f \cdots}{\cdots}$
parallelrisch	$-\frac{4 \ln (e x-d) x^{2} d e^{2} g^{2}+2 \ln (e x-d) x^{2} e^{3} f g+g^{2} x^{3} e^{3}-8 \ln (e x-d) x d^{2} e g^{2}-4 \ln (e x-d) x d e^{2} f g+4 \ln (e x-d) d^{3} g^{2}+2 \ln (e x-d) d^{2} e \cdots}{e^{3} (e x-d)^{2}}$

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -g^2*x/e^2-((-5*d^2*g^2-6*d*e*f*g-e^2*f^2)*x+4*d^2*g*(d*g+e*f)/e)/e^2/(-e*x
+d)^2-4*g^2*ln(-e*x+d)/e^3*d-2*g*ln(-e*x+d)/e^2*f
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{e^3g^2x^3-2\,de^2g^2x^2+4\,d^2efg+4\,d^3g^2-(e^3f^2+6\,de^2fg+4\,d^2eg^2)x+2\,(d^2efg+2\,d^3g^2+(e^3fg+2\,\cdots}{e^5x^2-2\,de^4x+d^2e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*
e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*g^
2)*x^2 - 2*(d*e^2*f*g + 2*d^2*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x
+ d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{4d^3g^2+4d^2efg+x(-5d^2eg^2-6de^2fg-e^3f^2)}{d^2e^3-2de^4x+e^5x^2} - \frac{g^2x}{e^2} - \frac{2g(2dg+ef)\log(-d+ex)}{e^3}$$

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(4*d**3*g**2 + 4*d**2*e*f*g + x*(-5*d**2*e*g**2 - 6*d*e**2*f*g - e**3*f**2
))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*l
og(-d + e*x)/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2x}{e^2} - \frac{4\,d^2efg + 4\,d^3g^2 - (e^3f^2 + 6\,de^2fg + 5\,d^2eg^2)x}{e^5x^2 - 2\,de^4x + d^2e^3} - \frac{2\,(efg + 2\,dg^2)\log\left(ex - d\right)}{e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^
2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/e^
3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2x}{e^2} - \frac{2\,(efg + 2\,dg^2)\log\left(|ex - d|\right)}{e^3} - \frac{4\,d^2efg + 4\,d^3g^2 - (e^3f^2 + 6\,de^2fg + 5\,d^2eg^2)x}{(ex-d)^2e^3}$$

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -g^2*x/e^2 - 2*(e*f*g + 2*d*g^2)*log(abs(e*x - d))/e^3 - (4*d^2*e*f*g + 4*d
^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)/((e*x - d)^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{4\,(d^3\,g^2+e\,f\,d^2\,g)}{e}-x\,(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2} - \frac{g^2\,x}{e^2} - \frac{\ln(e\,x-d)\,(4\,d\,g^2+2\,e\,f\,g)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((4*(d^3*g^2 + d^2*e*f*g))/e - x*(5*d^2*g^2 + e^2*f^2 + 6*d*e*f*g))/(d^2*
e^2 + e^4*x^2 - 2*d*e^3*x) - (g^2*x)/e^2 - (log(e*x - d)*(4*d*g^2 + 2*e*f*g
))/e^3
```

3.573 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(ef+dg)^2}{2e^3(d-ex)^2} - \frac{2g(ef+dg)}{e^3(d-ex)} - \frac{g^2\log(d-ex)}{e^3}$$

```
[Out] 1/2*(d*g+e*f)^2/e^3/(-e*x+d)^2-2*g*(d*g+e*f)/e^3/(-e*x+d)-g^2*ln(-e*x+d)/e^
3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 45}

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2g(dg+ef)}{e^3(d-ex)} + \frac{(dg+ef)^2}{2e^3(d-ex)^2} - \frac{g^2\log(d-ex)}{e^3}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^3}\,dx \\ &= \int \left(\frac{(ef+dg)^2}{e^2(d-ex)^3} - \frac{2g(ef+dg)}{e^2(d-ex)^2} + \frac{g^2}{e^2(d-ex)}\right)dx \\ &= \frac{(ef+dg)^2}{2e^3(d-ex)^2} - \frac{2g(ef+dg)}{e^3(d-ex)} - \frac{g^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{(ef+dg)(-3dg+e(f+4gx))}{(d-ex)^2} - 2g^2\log(d-ex)}{2e^3}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
[Out] (((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/(
2*e^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{\frac{2g(dg+ef)x}{e^2}-\frac{3d^2g^2+2defg-e^2f^2}{2e^3}}{(-ex+d)^2}-\frac{g^2\ln(-ex+d)}{e^3}$	69
default	$-\frac{g^2\ln(-ex+d)}{e^3}-\frac{2g(dg+ef)}{e^3(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{2e^3(-ex+d)^2}$	74
parallelrisch	$-\frac{2\ln(ex-d)x^2e^2g^2-4\ln(ex-d)xde\,g^2+2\ln(ex-d)d^2g^2-4xde\,g^2-4x\,e^2fg+3d^2g^2+2defg-e^2f^2}{2e^3(ex-d)^2}$	105
norman	$\frac{(2d\,g^2+2fge)x^3-\frac{d^2(3d^2g^2e+2dfg\,e^2-f^2e^3)}{2e^4}+\frac{(5d^2g^2e+6dfg\,e^2+f^2e^3)x^2}{2e^2}-\frac{d(d^2g^2-e^2f^2)x}{e^2}}{(-e^2x^2+d^2)^2}-\frac{g^2\ln(-ex+d)}{e^3}$	139

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
[Out] (2*g*(d*g+e*f)*x/e^2-1/2*(3*d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3)/(-e*x+d)^2-g^2*
ln(-e*x+d)/e^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$=\frac{e^2f^2-2\,defg-3\,d^2g^2+4\,(e^2fg+deg^2)x-2\,(e^2g^2x^2-2\,deg^2x+d^2g^2)\log{(ex-d)}}{2\,(e^5x^2-2\,de^4x+d^2e^3)}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^2
*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx=-\frac{3d^2g^2+2defg-e^2f^2+x(-4deg^2-4e^2fg)}{2d^2e^3-4de^4x+2e^5x^2}-\frac{g^2\log{(-d+ex)}}{e^3}$$

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
[Out] -(3*d**2*g**2 + 2*d*e*f*g - e**2*f**2 + x*(-4*d*e*g**2 - 4*e**2*f*g))/(2*d*
*2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx=\frac{e^2f^2-2\,defg-3\,d^2g^2+4\,(e^2fg+deg^2)x}{2\,(e^5x^2-2\,de^4x+d^2e^3)}-\frac{g^2\log{(ex-d)}}{e^3}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x)/(e^5*x^2 -
2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{g^2\log(|ex-d|)}{e^3}+\frac{4\,(efg+dg^2)x+\frac{e^2f^2-2\,defg-3\,d^2g^2}{e}}{2\,(ex-d)^2e^2}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -g^2*log(abs(e*x - d))/e^3 + 1/2*(4*(e*f*g + d*g^2)*x + (e^2*f^2 - 2*d*e*f*
g - 3*d^2*g^2)/e)/((e*x - d)^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{\frac{3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{2\,e^3}-\frac{2\,g\,x\,(d\,g+e\,f)}{e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}-\frac{g^2\,\ln(e\,x-d)}{e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(2*e^3) - (2*g*x*(d*g + e*f))/e^2)/(d^
2 + e^2*x^2 - 2*d*e*x) - (g^2*log(e*x - d))/e^3
```

3.574 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 88

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

```
[Out] 1/4*(d*g+e*f)^2/d/e^3/(-e*x+d)^2+1/4*(-3*d*g+e*f)*(d*g+e*f)/d^2/e^3/(-e*x+d
)+1/4*(-d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(ef-dg)^2}{4d^3e^3}+\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^3}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d^2-e^2x^2)}\right)dx \\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2} \\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$
\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{-\frac{2d(ef+dg)(2d^2g+e^2fx-de(2f+3gx))}{(d-ex)^2}-(ef-dg)^2\log(d-ex)+(ef-dg)^2\log(d+ex)}{8d^3e^3}
$$

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
[Out] ((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 - (
e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

method	result
default	$\frac{-3d^2g^2-2defg+e^2f^2}{4e^3d^2(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{4d\,e^3(-ex+d)^2}+\frac{(-d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{8e^3d^3}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{8e^3d^3}$
risch	$\frac{\frac{(3d^2g^2+2defg-e^2f^2)x}{4d^2e^2}-\frac{d^2g^2-e^2f^2}{2d\,e^3}}{(-ex+d)^2}-\frac{\ln(-ex+d)g^2}{8e^3d}+\frac{\ln(-ex+d)fg}{4e^2d^2}-\frac{\ln(-ex+d)f^2}{8e\,d^3}+\frac{\ln(ex+d)g^2}{8e^3d}-\frac{\ln(ex+d)fg}{4e^2d^2}+$
norman	$\frac{\frac{d(-d^2g^2e+f^2e^3)}{2e^4}-\frac{(d^2g^2-2defg-3e^2f^2)x}{4e^2}+\frac{(3d^2g^2+2defg-e^2f^2)x^3}{4d^2}-\frac{(-de\,g^2-e^2fg)x^2}{e^2}}{(-e^2x^2+d^2)^2}-\frac{(d^2g^2-2defg+e^2f^2)\ln(-ex+d)}{8e^3d^3}+$
parallelrisch	$-\frac{-4d^2e^2f^2+4d^4g^2-\ln(ex+d)d^2e^2f^2+\ln(ex-d)x^2e^4f^2-\ln(ex+d)x^2e^4f^2+\ln(ex-d)d^2e^2f^2-6x\,d^3e\,g^2+2xd\,e^3f^2-2\ln(ex-d)}{}$

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3*d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2/(-e*x+d)-1/4*(-d^2*g^2-2*d*e*f*g
-e^2*f^2)/d/e^3/(-e*x+d)^2+1/8*(-d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3/d^3*ln(-e*x
+d)+1/8*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(84) = 168.

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.08

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$=\frac{4\,d^2e^2f^2-4\,d^4g^2-2\,(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2)x+(d^2e^2f^2-2\,d^3efg+d^4g^2+(e^4f^2-2\,de^3fg+d^2e^2g}{}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2
)*x + (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e
^2*g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) - (
d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)
*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5*
x^2 - 2*d^4*e^4*x + d^5*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(75) = 150.

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{2d^3g^2-2de^2f^2+x(-3d^2eg^2-2de^2fg+e^3f^2)}{4d^4e^3-8d^3e^4x+4d^2e^5x^2} - \frac{(dg-ef)^2\log\left(-\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}{8d^3e^3} + \frac{(dg-ef)^2\log\left(\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}{8d^3e^3}$$

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**3*g**2 - 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 - 2*d*e**2*f*g + e**3*f**
2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log(-
d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
 + (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**
2)) + x)/(8*d**3*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{2\,de^2f^2-2\,d^3g^2-(e^3f^2-2\,de^2fg-3\,d^2eg^2)x}{4\,(d^2e^5x^2-2\,d^3e^4x+d^4e^3)} + \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex+d)}{8\,d^3e^3} - \frac{(e^2f^2-2\,defg+d^2g^2)\log(ex-d)}{8\,d^3e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*e^2*f^2 - 2*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d^
2*e^5*x^2 - 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*lo
g(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^
3*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex+d|)}{8\,d^3e^3} - \frac{(e^2f^2-2\,defg+d^2g^2)\log(|ex-d|)}{8\,d^3e^3} + \frac{2\,d^2e^2f^2-2\,d^4g^2-(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2)x}{4\,(ex-d)^2d^3e^3}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x + d))/(d^3*e^3) - 1/8*(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(e*x - d))/(d^3*e^3) + 1/4*(2*d^2*e^2*f^2
 - 2*d^4*g^2 - (d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2)*x)/((e*x - d)^2*d^
3*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,(d\,g-e\,f)^2}{4\,d^3\,e^3} - \frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2)}{4\,d^2\,e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}$$

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (atanh((e*x)/d)*(d*g - e*f)^2)/(4*d^3*e^3) - ((d^2*g^2 - e^2*f^2)/(2*d*e^3)
 - (x*(3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^2))/(d^2 + e^2*x^2 - 2*d*
e*x)
```

3.575 $\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(ef-dg)(3ef+dg)\text{arctanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] 1/8*(d*g+e*f)^2/d^2/e^3/(-e*x+d)^2+1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-
1/8*(-d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(-d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/
d^4/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {813, 90, 214}

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{\text{arctanh}\left(\frac{ex}{d}\right)(dg+3ef)(ef-dg)}{8d^4e^3} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(dg+ef)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)^3} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^2} + \frac{(ef-dg)(3ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right) dx$$
$$= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{((ef-dg)(3ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}$$
$$= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(ef-dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$
$$= \frac{\frac{2d^2(ef+dg)^2}{(d-ex)^2} + \frac{4de^2f^2-4d^3g^2}{d-ex} - \frac{2d(ef-dg)^2}{d+ex} + \left(-3e^2f^2+2defg+d^2g^2\right)\log(d-ex) + \left(3e^2f^2-2defg-d^2g^2\right)\log}{16d^4e^3}$$

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
(2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d -
e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.50

method	result
default	$\frac{-d^2g^2+e^2f^2}{4d^3e^3(-ex+d)} - \frac{-d^2g^2-2defg-e^2f^2}{8e^3d^2(-ex+d)^2} + \frac{(d^2g^2+2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4} + \frac{(-d^2g^2-2defg+3e^2f^2)\ln(ex+d)}{16e^3d^4} - \frac{d^2}{}$
norman	$\frac{\frac{-d^2g^2e+2dfg\,e^2+f^2e^3}{4e^4} + \frac{(d^2g^2+2defg-3e^2f^2)x^3}{8d^3} + \frac{g^2x^2}{2e} + \frac{(d^2g^2+2defg+5e^2f^2)x}{8d\,e^2}}{(-e^2x^2+d^2)^2} + \frac{(d^2g^2+2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4} - (d^2$
risch	$\frac{\frac{(d^2g^2+2defg-3e^2f^2)x^2}{8d^3e} + \frac{(3d^2g^2-2defg+3e^2f^2)x}{8e^2d^2} - \frac{d^2g^2-2defg-e^2f^2}{4d\,e^3}}{(-ex+d)(-e^2x^2+d^2)} - \frac{\ln(-ex-d)g^2}{16e^3d^2} - \frac{\ln(-ex-d)fg}{8e^2d^3} + \frac{3\ln(-ex-d)f^2}{16e\,d^4}$
parallelrisch	$\frac{-2\ln(ex+d)d^4efg+\ln(ex-d)x^3d^2e^3g^2+2\ln(ex+d)x\,d^3e^2fg-2\ln(ex-d)x\,d^3e^2fg-2\ln(ex-d)x^2d^2e^3fg+2\ln(ex+d)x^2d^2e^3f}{}$

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-1/8*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^
3/d^2/(-e*x+d)^2+1/16/e^3*(d^2*g^2+2*d*e*f*g-3*e^2*f^2)/d^4*ln(-e*x+d)+1/16
*(-d^2*g^2-2*d*e*f*g+3*e^2*f^2)/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*g+e^
2*f^2)/e^3/d^3/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$
$$= \frac{4\,d^3e^2f^2 + 8\,d^4efg - 4\,d^5g^2 - 2\,(3\,de^4f^2 - 2\,d^2e^3fg - d^3e^2g^2)x^2 + 2\,(3\,d^2e^3f^2 - 2\,d^3e^2fg + 3\,d^4eg^2)x + (}{}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 - 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 + 2*(3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
+ (3*d^3*e^2*f^2 - 2*d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*
f^2 - 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) - (3*d^3*e^2*f^2 - 2*d^4*e
*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 - d^5*e^5*x^2 - d^6*e^4*x + d^7*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(105) = 210.

Time = 0.58 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = -\frac{2d^4g^2-4d^3efg-2d^2e^2f^2+x^2(-d^2e^2g^2-2de^3fg+3e^4f^2)+x(-3d^3eg^2+2d^2e^2fg-3de^3f^2)}{8d^6e^3-8d^5e^4x-8d^4e^5x^2+8d^3e^6x^3} + \frac{(dg-ef)\,(dg+3ef)\log\left(-\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}{16d^4e^3} - \frac{(dg-ef)\,(dg+3ef)\log\left(\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}{16d^4e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**4*g**2 - 4*d**3*e*f*g - 2*d**2*e**2*f**2 + x**2*(-d**2*e**2*g**2 - 2
*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 2*d**2*e**2*f*g - 3*d*e**3
*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2
 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f
)*log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = \frac{2\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2-(3\,e^4f^2-2\,de^3fg-d^2e^2g^2)x^2+(3\,de^3f^2-2\,d^2e^2fg+3\,d^3eg^2)x}{8\,(d^3e^6x^3-d^4e^5x^2-d^5e^4x+d^6e^3)} + \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log{(ex+d)}}{16\,d^4e^3} - \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log{(ex-d)}}{16\,d^4e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 - 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 + (3*d*e^3*f^2 - 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 - d^4*e^5*x^2 - d^5*e^4*x + d^6*e^3) + 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log\left(|ex+d|\right)}{16\,d^4e^3} - \frac{(3\,e^2f^2-2\,defg-d^2g^2)\log\left(|ex-d|\right)}{16\,d^4e^3} + \frac{2\,d^3e^2f^2+4\,d^4efg-2\,d^5g^2-(3\,de^4f^2-2\,d^2e^3fg-d^3e^2g^2)x^2+(3\,d^2e^3f^2-2\,d^3e^2fg+3\,d^4eg^2)x}{8\,(ex+d)(ex-d)^2d^4e^3}$$

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^4*e^3) - 1/16*(
3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^4*e^3) + 1/8*(2*d^3*e
^2*f^2 + 4*d^4*e*f*g - 2*d^5*g^2 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g
^2)*x^2 + (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x)/((e*x + d)*(e*x
- d)^2*d^4*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2-2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3-d^2\,e\,x-d\,e^2\,x^2+e^3\,x^3} - \frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}\right)\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{8\,d^4\,e^3}$$

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((e^2*f^2 - d^2*g^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 - 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 + e^3*x^3 - d*e^2*x^2 - d^2*e*x) - (atanh((e*x*(d*g - e*f)*(d*g + 3*e*
f))/(d*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)))*(d*g - e*f)*(d*g + 3*e*f))/(8*d^
4*e^3)
```

3.576 $\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2}+\frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} + \frac{(3e^2f^2-d^2g^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

```
[Out] 1/4*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)^2+1/8*(2*d^2*f*g+(-d^2*g
^2+3*e^2*f^2)*x)/d^4/e^2/(-e^2*x^2+d^2)+1/8*(-d^2*g^2+3*e^2*f^2)*arctanh(e*
x/d)/d^5/e^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {753, 653, 214}

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)(3e^2f^2-d^2g^2)}{8d^5e^3}+\frac{(f+gx)(d^2g+e^2fx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{x(3e^2f^2-d^2g^2)+2d^2fg}{8d^4e^2(d^2-e^2x^2)}$$

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} - \frac{\int \frac{-3e^2f^2+d^2g^2-2e^2fgx}{(d^2-e^2x^2)^2}\,dx}{4d^2e^2} \\
&= \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} - \frac{\left(-\frac{3e^2f^2}{d^2}+g^2\right)\int \frac{1}{d^2-e^2x^2}\,dx}{8d^2e^2} \\
&= \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} + \frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$
\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{-3de^5f^2x^3+d^5eg(4f+gx)+d^3e^3x(5f^2+g^2x^2)+(3e^2f^2-d^2g^2)(d^2-e^2x^2)^2\operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^5e^3(d^2-e^2x^2)^2}
$$

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3*
e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 - e^
2*x^2)^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

method	result
norman	$\frac{\frac{fg}{2e^2}+\frac{\left(d^2g^2-3e^2f^2\right)x^3}{8d^4}+\frac{\left(d^2g^2+5e^2f^2\right)x}{8d^2e^2}}{\left(-e^2x^2+d^2\right)^2}+\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(-ex+d\right)}{16d^5e^3}-\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(ex+d\right)}{16d^5e^3}$
risch	$\frac{\frac{fg}{2e^2}+\frac{\left(d^2g^2-3e^2f^2\right)x^3}{8d^4}+\frac{\left(d^2g^2+5e^2f^2\right)x}{8d^2e^2}}{\left(-e^2x^2+d^2\right)^2}-\frac{\ln\left(-ex-d\right)g^2}{16d^3e^3}+\frac{3\ln\left(-ex-d\right)f^2}{16d^5e}+\frac{\ln\left(ex-d\right)g^2}{16d^3e^3}-\frac{3\ln\left(ex-d\right)f^2}{16d^5e}$
default	$\frac{\left(d^2g^2-3e^2f^2\right)\ln\left(-ex+d\right)}{16d^5e^3}-\frac{-d^2g^2-2defg-e^2f^2}{16e^3d^3\left(-ex+d\right)^2}+\frac{-d^2g^2+2defg+3e^2f^2}{16e^3d^4\left(-ex+d\right)}+\frac{\left(-d^2g^2+3e^2f^2\right)\ln\left(ex+d\right)}{16e^3d^5}-\frac{-d^2g^2-2def}{16e^3d^4(e}$
parallelrisch	$\ln\left(ex-d\right)x^4d^2e^5g^2-3\ln\left(ex-d\right)x^4e^7f^2-\ln\left(ex+d\right)x^4d^2e^5g^2+3\ln\left(ex+d\right)x^4e^7f^2-2\ln\left(ex-d\right)x^2d^4e^3g^2+6\ln\left(ex-d\right)x^2d^2e^5f^2+2\,\mathrm{l}$

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*f*g/e^2+1/8/d^4*(d^2*g^2-3*e^2*f^2)*x^3+1/8*(d^2*g^2+5*e^2*f^2)/d^2/e^
2*x)/(-e^2*x^2+d^2)^2+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3*ln(-e*x+d)-1/16*(d^2
*g^2-3*e^2*f^2)/d^5/e^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.98

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$
$$=\frac{8\,d^5efg-2\,(3\,de^5f^2-d^3e^3g^2)x^3+2\,(5\,d^3e^3f^2+d^5eg^2)x+(3\,d^4e^2f^2-d^6g^2+(3\,e^6f^2-d^2e^4g^2)x^4-2\,(3\,d}{16\,(d^5e^7x^4-}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 +
d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2
*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log(e*x + d) - (3*d^4*e^2*f^2 - d^6*g^2
 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log
(e*x - d))/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{-4d^4fg + x^3(-d^2e^2g^2+3e^4f^2) + x(-d^4g^2-5d^2e^2f^2)}{8d^8e^2-16d^6e^4x^2+8d^4e^6x^4} + \frac{(d^2g^2-3e^2f^2)\log\left(-\frac{d}{e}+x\right)}{16d^5e^3} - \frac{(d^2g^2-3e^2f^2)\log\left(\frac{d}{e}+x\right)}{16d^5e^3}$$

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**4*f*g + x**3*(-d**2*e**2*g**2 + 3*e**4*f**2) + x*(-d**4*g**2 - 5*d*
*2*e**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d**2
*g**2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*f**
2)*log(d/e + x)/(16*d**5*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{4\,d^4fg - (3\,e^4f^2-d^2e^2g^2)x^3 + (5\,d^2e^2f^2+d^4g^2)x}{8\,(d^4e^6x^4-2\,d^6e^4x^2+d^8e^2)} + \frac{(3\,e^2f^2-d^2g^2)\log\left(ex+d\right)}{16\,d^5e^3} - \frac{(3\,e^2f^2-d^2g^2)\log\left(ex-d\right)}{16\,d^5e^3}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)*
x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*log
(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = -\frac{3\,e^4f^2x^3-d^2e^2g^2x^3-5\,d^2e^2f^2x-d^4g^2x-4\,d^4fg}{8\,(e^2x^2-d^2)^2d^4e^2} + \frac{(3\,e^3f^2-d^2eg^2)\log\left(|ex+d|\right)}{16\,d^5e^4} - \frac{(3\,e^3f^2-d^2eg^2)\log\left(|ex-d|\right)}{16\,d^5e^4}$$

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*e^4*f^2*x^3 - d^2*e^2*g^2*x^3 - 5*d^2*e^2*f^2*x - d^4*g^2*x - 4*d^4
*f*g)/((e^2*x^2 - d^2)^2*d^4*e^2) + 1/16*(3*e^3*f^2 - d^2*e*g^2)*log(abs(e*
x + d))/(d^5*e^4) - 1/16*(3*e^3*f^2 - d^2*e*g^2)*log(abs(e*x - d))/(d^5*e^4
)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{\frac{x^3\,\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^4}+\frac{f\,g}{2\,e^2}+\frac{x\,\left(d^2\,g^2+5\,e^2\,f^2\right)}{8\,d^2\,e^2}}{d^4-2\,d^2\,e^2\,x^2+e^4\,x^4}-\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^5\,e^3}$$

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((x^3*(d^2*g^2 - 3*e^2*f^2))/(8*d^4) + (f*g)/(2*e^2) + (x*(d^2*g^2 + 5*e^2*
f^2))/(8*d^2*e^2))/(d^4 + e^4*x^4 - 2*d^2*e^2*x^2) - (atanh((e*x)/d)*(d^2*g
^2 - 3*e^2*f^2))/(8*d^5*e^3)
```

3.577 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} -\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)} +\frac{(5e^2f^2+2defg-d^2g^2)\,\text{arctanh}\left(\frac{ex}{d}\right)}{16d^6e^3}$$

```
[Out] 1/32*(d*g+e*f)^2/d^4/e^3/(-e*x+d)^2+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/24*(
-d*g+e*f)^2/d^3/e^3/(e*x+d)^3-1/32*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^3/(e*x+d)^2
+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)+1/16*(-d^2*g^2+2*d*e*f*g+5*e^2*f^
2)*arctanh(e*x/d)/d^6/e^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \frac{\text{arctanh}\left(\frac{ex}{d}\right)\left(-d^2g^2+2defg+5e^2f^2\right)}{16d^6e^3} +\frac{f(dg+ef)}{8d^5e^2(d-ex)}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2} +\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}$$

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^3}+\frac{f(ef+dg)}{8d^5e(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^4}+\frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^3}\right. \\
&\qquad \left.+\frac{3e^2f^2-d^2g^2}{16d^5e^2(d+ex)^2}+\frac{-5e^2f^2-2defg+d^2g^2}{16d^5e^2\left(-d^2+e^2x^2\right)}\right)dx \\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} \\
&\quad -\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}-\frac{\left(5e^2f^2+2defg-d^2g^2\right)\int\frac{1}{-d^2+e^2x^2}\,dx}{16d^5e^2} \\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} \\
&\quad -\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{\left(5e^2f^2+2defg-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$$
$$= \frac{\frac{3d^2(ef+dg)^2}{(d-ex)^2}+\frac{12def(ef+dg)}{d-ex}-\frac{4d^3(ef-dg)^2}{(d+ex)^3}+\frac{3d^2\left(-3e^2f^2+2defg+d^2g^2\right)}{(d+ex)^2}+\frac{6d\left(-3e^2f^2+d^2g^2\right)}{d+ex}+3\left(-5e^2f^2-2defg+d^2g\right.}{96d^6e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4*
d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 - 2*d
*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d
+ e*x])/(96*d^6*e^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.30

method	result
default	$-\frac{-d^2g^2-2defg-e^2f^2}{32e^3d^4(-ex+d)^2}+\frac{\left(d^2g^2-2defg-5e^2f^2\right)\ln(-ex+d)}{32d^6e^3}+\frac{f(dg+ef)}{8d^5e^2(-ex+d)}-\frac{-d^2g^2+3e^2f^2}{16e^3d^5(ex+d)}-\frac{-d^2g^2-2defg+3e^2}{32e^3d^4(ex+d)^2}$
norman	$\frac{\frac{\left(11d^2g^2+26defg-31e^2f^2\right)x^3}{48d^4}+\frac{\left(d^2g^2+14defg+3e^2f^2\right)x^2}{16e\,d^3}-\frac{e\left(d^2g^2+22defg+7e^2f^2\right)x^4}{48d^5}-\frac{e^2\left(d^2g^2+4defg-2e^2f^2\right)x^5}{12d^6}+\frac{\left(d^2g^2-2defg+\right.}{16d^2e^2}}{(ex+d)^3(-ex+d)^2}$
risch	$\frac{\frac{\left(d^2g^2-2defg-5e^2f^2\right)e\,x^4}{16d^5}+\frac{\left(d^2g^2-2defg-5e^2f^2\right)x^3}{16d^4}-\frac{5\left(d^2g^2-2defg-5e^2f^2\right)x^2}{48d^3e}+\frac{\left(7d^2g^2+10defg+25e^2f^2\right)x}{48d^2e^2}+\frac{d^2g^2+4defg-2e^2f^2}{12d\,e^3}}{(ex+d)\left(-e^2x^2+d^2\right)^2}$
parallelrisch	$\frac{15\ln(ex+d)x^5e^7f^2-15\ln(ex-d)x^5e^7f^2+66x\,d^4e^3f^2+6\ln(ex+d)x\,d^5e^2fg+3\ln(ex-d)x^5d^2e^5g^2-3\ln(ex+d)x^5d^2e^5g^2+3\ln(}{}$

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^4/(-e*x+d)^2+1/32*(d^2*g^2-2*d*e*f
*g-5*e^2*f^2)/d^6/e^3*ln(-e*x+d)+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/16*(-d^
2*g^2+3*e^2*f^2)/e^3/d^5/(e*x+d)-1/32*(-d^2*g^2-2*d*e*f*g+3*e^2*f^2)/e^3/d^
4/(e*x+d)^2+1/32*(-d^2*g^2+2*d*e*f*g+5*e^2*f^2)/e^3/d^6*ln(e*x+d)-1/24*(d^2
*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(178) = 356.

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.52

$$\int \frac{(f+gx)^2}{(d+ex)\,(d^2-e^2x^2)^3}\,dx = -\frac{16\,d^5e^2f^2 - 32\,d^6efg - 8\,d^7g^2 + 6\,(5\,de^6f^2 + 2\,d^2e^5fg - d^3e^4g^2)x^4 + 6\,(5\,d^2e^5f^2 + 2\,d^3e^4fg - d^4e^3g^2)x^3}{}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 + 2*d^2*e
^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)
*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*e^3
*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d
^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2
*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^
2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f
^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 2*d^6*
e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^
2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d
^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*
d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + d^
7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)^2}{(d+ex)\,(d^2-e^2x^2)^3}\,dx = -\frac{-4d^6g^2 - 16d^5efg + 8d^4e^2f^2 + x^4(-3d^2e^4g^2 + 6de^5fg + 15e^6f^2) + x^3(-3d^3e^3g^2 + 6d^2e^4fg + 15de^5f^2)}{48d^{10}e^3 + 48d^9e^4x - 96d^8e^5x^2 - 96d^7e^6x^3 + } + \frac{(d^2g^2 - 2defg - 5e^2f^2)\log\left(-\frac{d}{e}+x\right)}{32d^6e^3} - \frac{(d^2g^2 - 2defg - 5e^2f^2)\log\left(\frac{d}{e}+x\right)}{32d^6e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**6*g**2 - 16*d**5*e*f*g + 8*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2
 + 6*d*e**5*f*g + 15*e**6*f**2) + x**3*(-3*d**3*e**3*g**2 + 6*d**2*e**4*f*g
 + 15*d*e**5*f**2) + x**2*(5*d**4*e**2*g**2 - 10*d**3*e**3*f*g - 25*d**2*e*
*4*f**2) + x*(-7*d**5*e*g**2 - 10*d**4*e**2*f*g - 25*d**3*e**3*f**2))/(48*d
**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**
6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*lo
g(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e
+ x)/(32*d**6*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.64

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = -\frac{8\,d^4e^2f^2-16\,d^5efg-4\,d^6g^2+3\,(5\,e^6f^2+2\,de^5fg-d^2e^4g^2)x^4+3\,(5\,de^5f^2+2\,d^2e^4fg-d^3e^3g^2)x^3-}{48\,(d^5e^8x^5+d^6e^7x^4-2\,d^7e^6x^3-2\,d^8e^5x^2+} + \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(ex+d)}{32\,d^6e^3} - \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(ex-d)}{32\,d^6e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(8*d^4*e^2*f^2 - 16*d^5*e*f*g - 4*d^6*g^2 + 3*(5*e^6*f^2 + 2*d*e^5*f*
g - d^2*e^4*g^2)*x^4 + 3*(5*d*e^5*f^2 + 2*d^2*e^4*f*g - d^3*e^3*g^2)*x^3 -
5*(5*d^2*e^4*f^2 + 2*d^3*e^3*f*g - d^4*e^2*g^2)*x^2 - (25*d^3*e^3*f^2 + 10*
d^4*e^2*f*g + 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + d^6*e^7*x^4 - 2*d^7*e^6*x^3 -
2*d^8*e^5*x^2 + d^9*e^4*x + d^10*e^3) + 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g
^2)*log(e*x + d)/(d^6*e^3) - 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e*x
 - d)/(d^6*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3}\,dx = \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(|ex+d|)}{32\,d^6e^3} - \frac{(5\,e^2f^2+2\,defg-d^2g^2)\log(|ex-d|)}{32\,d^6e^3} - \frac{8\,d^5e^2f^2-16\,d^6efg-4\,d^7g^2+3\,(5\,de^6f^2+2\,d^2e^5fg-d^3e^4g^2)x^4+3\,(5\,d^2e^5f^2+2\,d^3e^4fg-d^4e^3g^2)x^3}{48\,(ex+d)^3(ex-d)^2d^6e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x + d))/(d^6*e^3) - 1/32*(
5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(abs(e*x - d))/(d^6*e^3) - 1/48*(8*d^5*
e^2*f^2 - 16*d^6*e*f*g - 4*d^7*g^2 + 3*(5*d*e^6*f^2 + 2*d^2*e^5*f*g - d^3*e
^4*g^2)*x^4 + 3*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 5*(5*d^
3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (25*d^4*e^3*f^2 + 10*d^5*e^2
*f*g + 7*d^6*e*g^2)*x)/((e*x + d)^3*(e*x - d)^2*d^6*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$$
$$= \frac{\frac{d^2\,g^2+4\,d\,e\,f\,g-2\,e^2\,f^2}{12\,d\,e^3}-\frac{x^3\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^5}+\frac{x\,\left(7\,d^2\,g^2+10\,d\,e\,f\,g+25\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2\right.}{}}{d^5+d^4\,e\,x-2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3+d\,e^4\,x^4+e^5\,x^5}$$
$$+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^6\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + 4*d*e*f*g)/(12*d*e^3) - (x^3*(5*e^2*f^2 - d^2*g^2 +
 2*d*e*f*g))/(16*d^4) - (e*x^4*(5*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(16*d^5)
+ (x*(7*d^2*g^2 + 25*e^2*f^2 + 10*d*e*f*g))/(48*d^2*e^2) + (5*x^2*(5*e^2*f^
2 - d^2*g^2 + 2*d*e*f*g))/(48*d^3*e))/(d^5 + e^5*x^5 + d*e^4*x^4 - 2*d^3*e^
2*x^2 - 2*d^2*e^3*x^3 + d^4*e*x) + (atanh((e*x)/d)*(5*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g))/(16*d^6*e^3)
```

3.578 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 235

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx = {} & \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2}+\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} \\ & -\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} \\ & -\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}-\frac{5e^2f^2+2defg-d^2g^2}{32d^6e^3(d+ex)} \\ & +\frac{(15e^2f^2+10defg-d^2g^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{64d^7e^3}\end{aligned}$$

```
[Out] 1/64*(d*g+e*f)^2/d^5/e^3/(-e*x+d)^2+1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^3/(-e*
x+d)-1/32*(-d*g+e*f)^2/d^3/e^3/(e*x+d)^4-1/48*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^
3/(e*x+d)^3+1/32*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)^2+1/32*(d^2*g^2-2*d*e*
f*g-5*e^2*f^2)/d^6/e^3/(e*x+d)+1/64*(-d^2*g^2+10*d*e*f*g+15*e^2*f^2)*arctan
h(e*x/d)/d^7/e^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{862, 90, 214\}$

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)\left(-d^2g^2+10defg+15e^2f^2\right)}{64d^7e^3} + \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)} + \frac{(dg+ef)^2}{64d^5e^3(d-ex)^2} - \frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)} - \frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}$$

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^3} + \frac{(ef+dg)(5ef+dg)}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^5} + \frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^4} + \frac{3e^2f^2-d^2g^2}{16d^5e^2(d+ex)^3} + \frac{5e^2f^2+2defg-d^2g^2}{32d^6e^2(d+ex)^2} + \frac{-15e^2f^2-10defg+d^2g^2}{64d^6e^2\left(-d^2+e^2x^2\right)}\right)dx$$

$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} - \frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2} - \frac{5e^2f^2+2defg-d^2g^2}{32d^6e^3(d+ex)} - \frac{(15e^2f^2+10defg-d^2g^2)\int \frac{1}{-d^2+e^2x^2}\,dx}{64d^6e^2}$$

$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} - \frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2} - \frac{5e^2f^2+2defg-d^2g^2}{32d^6e^3(d+ex)} + \frac{(15e^2f^2+10defg-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx = \frac{\frac{6d^2(ef+dg)^2}{(d-ex)^2} + \frac{6d(5e^2f^2+6defg+d^2g^2)}{d-ex} - \frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{8d^3(-3e^2f^2+2defg+d^2g^2)}{(d+ex)^3} + \frac{12d^2(-3e^2f^2+d^2g^2)}{(d+ex)^2} + \frac{12d(-5e^2f^2-2def\cdots}{d+ex}}{384d^7e^3}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)
)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2*d
*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*x)^
2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^2 -
10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*
Log[d + e*x])/(384*d^7*e^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.22

method	result
norman	$\frac{\frac{\left(31d^2g^2+74defg-81e^2f^2\right)x^3}{96d^4}+\frac{\left(d^2g^2+22defg+17e^2f^2\right)x^2}{32e\,d^3}+\frac{e\left(11d^2g^2-14defg-69e^2f^2\right)x^4}{96d^5}-\frac{e^2\left(29d^2g^2+94defg-51e^2f^2\right)x^5}{192d^6}-\frac{e^3\left(d^2\cdots\right.}{\cdots}}{(ex+d)^4(-ex+d)^2}$
default	$\frac{d^2g^2+6defg+5e^2f^2}{64e^3d^6(-ex+d)}-\frac{-d^2g^2-2defg-e^2f^2}{64e^3d^5(-ex+d)^2}+\frac{\left(d^2g^2-10defg-15e^2f^2\right)\ln(-ex+d)}{128d^7e^3}-\frac{-d^2g^2+3e^2f^2}{32e^3d^5(ex+d)^2}-\frac{-d^2g^2-2def\cdots}{48e^3d^4(ex\cdots}$
risch	$\frac{\frac{\left(d^2g^2-10defg-15e^2f^2\right)e^2x^5}{64d^6}+\frac{\left(d^2g^2-10defg-15e^2f^2\right)e\,x^4}{32d^5}-\frac{\left(d^2g^2-10defg-15e^2f^2\right)x^3}{96d^4}-\frac{5\left(d^2g^2-10defg-15e^2f^2\right)x^2}{96d^3e}+\frac{\left(35d^2g^2+3\cdots\right.}{1\cdots}}{(ex+d)^2\left(-e^2x^2+d^2\right)^2}$
parallelrisch	Expression too large to display

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/96*(31*d^2*g^2+74*d*e*f*g-81*e^2*f^2)/d^4*x^3+1/32/e*(d^2*g^2+22*d*e*f*g
+17*e^2*f^2)/d^3*x^2+1/96*e*(11*d^2*g^2-14*d*e*f*g-69*e^2*f^2)/d^5*x^4-1/19
2*e^2*(29*d^2*g^2+94*d*e*f*g-51*e^2*f^2)/d^6*x^5-1/12*e^3*(d^2*g^2+2*d*e*f*
g-3*e^2*f^2)/d^7*x^6+1/64*(d^2*g^2-10*d*e*f*g+49*e^2*f^2)/d^2/e^2*x)/(e*x+d
)^4/(-e*x+d)^2+1/128*(d^2*g^2-10*d*e*f*g-15*e^2*f^2)/d^7/e^3*ln(-e*x+d)-1/1
28*(d^2*g^2-10*d*e*f*g-15*e^2*f^2)/d^7/e^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(223) = 446$.

Time = 0.35 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.37

$$\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)^3}\,dx = -\frac{96\,d^6e^2f^2-64\,d^7efg-32\,d^8g^2+6\,(15\,de^7f^2+10\,d^2e^6fg-d^3e^5g^2)x^5+12\,(15\,d^2e^6f^2+10\,d^3e^5fg-d^4e$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*d
^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e
^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20*(1
5*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 + 34*
d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2
+ (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*
e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2
)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4
*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f
*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^
2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^
2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g
^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e
^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2
*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*x^5 - d^9*e^7*x
^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.58

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^3} \, dx = -\frac{-16d^7g^2 - 32d^6efg + 48d^5e^2f^2 + x^5(-3d^2e^5g^2 + 30de^6fg + 45e^7f^2) + x^4(-6d^3e^4g^2 + 60d^2e^5fg + 90d}{192d^{12}e^3 + 384d^{11}e^4x - 192d^1} + \frac{(d^2g^2 - 10defg - 15e^2f^2)\log\left(-\frac{d}{e}+x\right)}{128d^7e^3} - \frac{(d^2g^2 - 10defg - 15e^2f^2)\log\left(\frac{d}{e}+x\right)}{128d^7e^3}$$

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-16*d**7*g**2 - 32*d**6*e*f*g + 48*d**5*e**2*f**2 + x**5*(-3*d**2*e**5*g*
*2 + 30*d*e**6*f*g + 45*e**7*f**2) + x**4*(-6*d**3*e**4*g**2 + 60*d**2*e**5
*f*g + 90*d*e**6*f**2) + x**3*(2*d**4*e**3*g**2 - 20*d**3*e**4*f*g - 30*d**
2*e**5*f**2) + x**2*(10*d**5*e**2*g**2 - 100*d**4*e**3*f*g - 150*d**3*e**4*
f**2) + x*(-35*d**6*e*g**2 - 34*d**5*e**2*f*g - 51*d**4*e**3*f**2))/(192*d*
*12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 19
2*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 -
10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*
d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^3} \, dx = -\frac{48\,d^5e^2f^2 - 32\,d^6efg - 16\,d^7g^2 + 3\,(15\,e^7f^2 + 10\,de^6fg - d^2e^5g^2)x^5 + 6\,(15\,de^6f^2 + 10\,d^2e^5fg - d^3e^4g}{192\,(d^6e^9x^6 + 2\,d^7e^8x^5 - e} + \frac{(15\,e^2f^2 + 10\,defg - d^2g^2)\log(ex+d)}{128\,d^7e^3} - \frac{(15\,e^2f^2 + 10\,defg - d^2g^2)\log(ex-d)}{128\,d^7e^3}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*e
^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^2)
*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^3*e
^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e^2*f
*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d^9*e^
6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 + 10*d*
e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e*f*g -
d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.43

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx = -\frac{(15\,e^2f^2+10\,defg-d^2g^2)\log\left(\left|-\frac{2\,d}{ex+d}+1\right|\right)}{128\,d^7e^3} - \frac{11\,e^2f^2+14\,defg+3\,d^2g^2-\frac{8\,(3\,de^3f^2+4\,d^2e^2fg+d^3eg^2)}{(ex+d)e}}{256\,d^7e^3\left(\frac{2\,d}{ex+d}-1\right)^2} - \frac{\frac{15\,d^6e^{11}f^2}{ex+d}+\frac{9\,d^7e^{11}f^2}{(ex+d)^2}+\frac{6\,d^8e^{11}f^2}{(ex+d)^3}+\frac{3\,d^9e^{11}f^2}{(ex+d)^4}+\frac{6\,d^7e^{10}fg}{ex+d}-\frac{4\,d^9e^{10}fg}{(ex+d)^3}-\frac{6\,d^{10}e^{10}fg}{(ex+d)^4}-\frac{3\,d^8e^9g^2}{ex+d}-\frac{3\,d^9e^9g^2}{(ex+d)^2}-\frac{2\,d^{10}e^9g^2}{(ex+d)^3}+}{96\,d^{12}e^{12}}$$

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/128*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*log(abs(-2*d/(e*x + d) + 1))/(d^
7*e^3) - 1/256*(11*e^2*f^2 + 14*d*e*f*g + 3*d^2*g^2 - 8*(3*d*e^3*f^2 + 4*d^
2*e^2*f*g + d^3*e*g^2)/((e*x + d)*e))/(d^7*e^3*(2*d/(e*x + d) - 1)^2) - 1/9
6*(15*d^6*e^11*f^2/(e*x + d) + 9*d^7*e^11*f^2/(e*x + d)^2 + 6*d^8*e^11*f^2/
(e*x + d)^3 + 3*d^9*e^11*f^2/(e*x + d)^4 + 6*d^7*e^10*f*g/(e*x + d) - 4*d^9
*e^10*f*g/(e*x + d)^3 - 6*d^10*e^10*f*g/(e*x + d)^4 - 3*d^8*e^9*g^2/(e*x +
d) - 3*d^9*e^9*g^2/(e*x + d)^2 - 2*d^10*e^9*g^2/(e*x + d)^3 + 3*d^11*e^9*g^
2/(e*x + d)^4)/(d^12*e^12)
```

Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx = \frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2}{12\,d\,e^3}+\frac{x^3\,(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2)}{96\,d^4}-\frac{e\,x^4\,(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2)}{32\,d^5}+\frac{x\,(35\,d^2\,g^2+34\,d\,e\,f\,g+51\,e^2\,f^2)}{192\,d^2\,e^2}+\frac{5\,x^2\,(-}{}}{d^6+2\,d^5\,e\,x-d^4\,e^2\,x^2-4\,d^3\,e^3\,x^3-d^2\,e^4\,x^4+2\,d\,e^5\,x^5+e^6} + \frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2)}{64\,d^7\,e^3}$$

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)/(12*d*e^3) + (x^3*(15*e^2*f^2 - d^2*g^2
+ 10*d*e*f*g))/(96*d^4) - (e*x^4*(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(32*d
^5) + (x*(35*d^2*g^2 + 51*e^2*f^2 + 34*d*e*f*g))/(192*d^2*e^2) + (5*x^2*(15
*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(96*d^3*e) - (e^2*x^5*(15*e^2*f^2 - d^2*g
^2 + 10*d*e*f*g))/(64*d^6))/(d^6 + e^6*x^6 + 2*d*e^5*x^5 - d^4*e^2*x^2 - 4*
d^3*e^3*x^3 - d^2*e^4*x^4 + 2*d^5*e*x) + (atanh((e*x)/d)*(15*e^2*f^2 - d^2*
g^2 + 10*d*e*f*g))/(64*d^7*e^3)
```

3.579 $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 269

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^5(d+ex)^3}{5de^6(d^2-e^2x^2)^{5/2}}+\frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6(d^2-e^2x^2)^{3/2}}+\frac{(ef+dg)^3(2e^2f^2-21defg+127d^2g^2)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}}+\frac{g^4(5ef+3dg)\sqrt{d^2-e^2x^2}}{e^6}+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{g^3(20e^2f^2+30defg+13d^2g^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/5*(d*g+e*f)^5*(e*x+d)^3/d/e^6/(-e^2*x^2+d^2)^(5/2)+1/15*(-23*d*g+2*e*f)*(
d*g+e*f)^4*(e*x+d)^2/d^2/e^6/(-e^2*x^2+d^2)^(3/2)-1/2*g^3*(13*d^2*g^2+30*d*
e*f*g+20*e^2*f^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(d*g+e*f)^3*(12
7*d^2*g^2-21*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^3/e^6/(-e^2*x^2+d^2)^(1/2)+g^4*(3
*d*g+5*e*f)*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*g^5*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1649, 1829, 655, 223, 209}

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(13d^2g^2+30defg+20e^2f^2)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}(3dg+5ef)}{e^6}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6(d^2-e^2x^2)^{3/2}}+\frac{(d+ex)^3(dg+ef)^5}{5de^6(d^2-e^2x^2)^{5/2}}+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{(d+ex)(dg+ef)^3(127d^2g^2-21defg+2e^2f^2)}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^5f^5-15de^4f^4g-30d^2e^3f^3g^2-30d^3e^2f^2g^3-15d^4efg^4-3d^5g^5}{e^5}+\frac{5dg^2\left(10e^3f^3+10de^2f^2g+5d^2efg^2+d^3g^3\right)x}{e^4}+\frac{5dg^3\left(10e^2f^2+5defg+d^2g\cdots\right.}{e^3}\cdots\right.}{\left(d^2-e^2x^2\right)^{5/2}}\cdots}{5d}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^5f^5-15de^4f^4g+70d^2e^3f^3g^2+170d^3e^2f^2g^3+135d^4efg^4+37d^5g^5}{e^5}+\frac{15d^2g^3\left(10e^2f^2+10defg+3d^2g^2\right)x}{e^4}+\frac{15d^2g^4(5ef+2dg)x^2}{e^3}+\cdots\right.}{\left(d^2-e^2x^2\right)^{3/2}}\cdots}{15d^2}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d^2g^2\right)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{15d^3g^3\left(10e^2f^2+15defg+6d^2g^2\right)}{e^5}+\frac{15d^3g^4(5ef+3dg)x}{e^4}+\frac{15d^3g^5x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d^2g^2\right)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}} + \frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-\frac{15d^3g^3\left(20e^2f^2+30defg+13d^2g^2\right)}{e^3}-\frac{30d^3g^4(5ef+3dg)x}{e^2}}{\sqrt{d^2-e^2x^2}}\,dx}{30d^3e^2}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d^2g^2\right)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}} + \frac{g^4(5ef+3dg)\sqrt{d^2-e^2x^2}}{e^6} + \frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5} - \frac{\left(g^3\left(20e^2f^2+30defg+13d^2g^2\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^5}$$

$$
\begin{aligned}
&= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}}+\frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad +\frac{(ef+dg)^3\left(2e^2f^2-21defg+127d^2g^2\right)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}} \\
&\quad +\frac{g^4(5ef+3dg)\sqrt{d^2-e^2x^2}}{e^6}+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5} \\
&\quad -\frac{\left(g^3\left(20e^2f^2+30defg+13d^2g^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^5} \\
&= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}}+\frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad +\frac{(ef+dg)^3\left(2e^2f^2-21defg+127d^2g^2\right)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}}+\frac{g^4(5ef+3dg)\sqrt{d^2-e^2x^2}}{e^6} \\
&\quad +\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{g^3\left(20e^2f^2+30defg+13d^2g^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.12

$$
\int \frac{(d+ex)^3(f+gx)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(304d^7g^5+4e^7f^5x^2+3d^6eg^4(240f-239gx)-6de^6f^4x(2f+5gx)+2d^2e^5f^3\left(7f^2+45fgx+70g^2x^2\right)+d^5\right.}{}
$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(304*d^7*g^5 + 4*e^7*f^5*x^2 + 3*d^6*e*g^4*(240*f - 2
39*g*x) - 6*d*e^6*f^4*x*(2*f + 5*g*x) + 2*d^2*e^5*f^3*(7*f^2 + 45*f*g*x + 7
0*g^2*x^2) + d^5*e^2*g^3*(440*f^2 - 1710*f*g*x + 479*g^2*x^2) + 5*d^4*e^3*g
^2*(8*f^3 - 204*f^2*g*x + 234*f*g^2*x^2 - 9*g^3*x^3) - 5*d^3*e^4*g*(6*f^4 +
 24*f^3*g*x - 128*f^2*g^2*x^2 + 30*f*g^3*x^3 + 3*g^4*x^4)))/(d^3*(d - e*x)^
3) + 30*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(e*x)/(Sqrt[d^2]
- Sqrt[d^2 - e^2*x^2])])/(30*e^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(247) = 494.

Time = 1.14 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.25

method	result
risch	$\frac{g^4(egx+6dg+10ef)\sqrt{-e^2x^2+d^2}}{2e^6} - \frac{\frac{13d^2g^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{20e^2f^2g^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{30def\,g^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \cdots}{}$
default	Expression too large to display

```
[In] int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g^4*(e*g*x+6*d*g+10*e*f)/e^6*(-e^2*x^2+d^2)^(1/2)-1/2/e^5*(13*d^2*g^5/(
e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+20*e^2*f^2*g^3/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+30*d*e*f*g^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+20*g^2*(d^3*g^3+3*d^2*e*f*g^2+3*d
*e^2*f^2*g+e^3*f^3)/e^2/d/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+10*g
*(d^4*g^4+4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2+4*d*e^3*f^3*g+e^4*f^4)/e^2*(1/3/d
/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)
^2*e^2-2*d*e*(x-d/e))^(1/2))+(2*d^5*g^5+10*d^4*e*f*g^4+20*d^3*e^2*f^2*g^3+2
0*d^2*e^3*f^3*g^2+10*d*e^4*f^4*g+2*e^5*f^5)/e^3*(1/5/d/e/(x-d/e)^3*(-(x-d/e
)^2*e^2-2*d*e*(x-d/e))^(1/2)-2/5*e/d*(1/3/d/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d
*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(247) = 494$.

Time = 0.33 (sec) , antiderivative size = 807, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{14\,d^3e^5f^5 - 30\,d^4e^4f^4g + 40\,d^5e^3f^3g^2 + 440\,d^6e^2f^2g^3 + 720\,d^7efg^4 + 304\,d^8g^5 - 2\,(7\,e^8f^5 - 15\,de^7f^4g + 2\cdots}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(14*d^3*e^5*f^5 - 30*d^4*e^4*f^4*g + 40*d^5*e^3*f^3*g^2 + 440*d^6*e^2
*f^2*g^3 + 720*d^7*e*f*g^4 + 304*d^8*g^5 - 2*(7*e^8*f^5 - 15*d*e^7*f^4*g +
20*d^2*e^6*f^3*g^2 + 220*d^3*e^5*f^2*g^3 + 360*d^4*e^4*f*g^4 + 152*d^5*e^3*
g^5)*x^3 + 6*(7*d*e^7*f^5 - 15*d^2*e^6*f^4*g + 20*d^3*e^5*f^3*g^2 + 220*d^4
*e^4*f^2*g^3 + 360*d^5*e^3*f*g^4 + 152*d^6*e^2*g^5)*x^2 - 6*(7*d^2*e^6*f^5
- 15*d^3*e^5*f^4*g + 20*d^4*e^4*f^3*g^2 + 220*d^5*e^3*f^2*g^3 + 360*d^6*e^2
*f*g^4 + 152*d^7*e*g^5)*x + 30*(20*d^6*e^2*f^2*g^3 + 30*d^7*e*f*g^4 + 13*d^
8*g^5 - (20*d^3*e^5*f^2*g^3 + 30*d^4*e^4*f*g^4 + 13*d^5*e^3*g^5)*x^3 + 3*(2
```

```
0*d^4*e^4*f^2*g^3 + 30*d^5*e^3*f*g^4 + 13*d^6*e^2*g^5)*x^2 - 3*(20*d^5*e^3*
f^2*g^3 + 30*d^6*e^2*f*g^4 + 13*d^7*e*g^5)*x)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*d^3*e^4*g^5*x^4 - 14*d^2*e^5*f^5 + 30*d^3*e^4*f^4*g - 40
*d^4*e^3*f^3*g^2 - 440*d^5*e^2*f^2*g^3 - 720*d^6*e*f*g^4 - 304*d^7*g^5 + 15
*(10*d^3*e^4*f*g^4 + 3*d^4*e^3*g^5)*x^3 - (4*e^7*f^5 - 30*d*e^6*f^4*g + 140
*d^2*e^5*f^3*g^2 + 640*d^3*e^4*f^2*g^3 + 1170*d^4*e^3*f*g^4 + 479*d^5*e^2*g
^5)*x^2 + 3*(4*d*e^6*f^5 - 30*d^2*e^5*f^4*g + 40*d^3*e^4*f^3*g^2 + 340*d^4*
e^3*f^2*g^3 + 570*d^5*e^2*f*g^4 + 239*d^6*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))/(
d^3*e^9*x^3 - 3*d^4*e^8*x^2 + 3*d^5*e^7*x - d^6*e^6)
```

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^5}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3*(f + g*x)**5/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(247) = 494$.

Time = 0.29 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.96

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] -1/2*e*g^5*x^7/(-e^2*x^2 + d^2)^(5/2) + 7/30*d^2*e*g^5*x*(15*x^4/((-e^2*x^2
 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2
*x^2 + d^2)^(5/2)*e^6)) - 7/6*d^2*g^5*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2)
 - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 1/5*d*f^5*x/(-e^2*x^2 + d^2)^(5/
2) + 3/5*d^2*f^5/((-e^2*x^2 + d^2)^(5/2)*e) + d^3*f^4*g/((-e^2*x^2 + d^2)^(
5/2)*e^2) + 4/15*f^5*x/((-e^2*x^2 + d^2)^(3/2)*d) + 14/15*d^4*g^5*x/((-e^2*
x^2 + d^2)^(3/2)*e^5) + 1/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5
)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 8/15*f^5*x/(sqrt(-e^2*x^2 +
 d^2)*d^3) - 49/30*d^2*g^5*x/(sqrt(-e^2*x^2 + d^2)*e^5) - (5*e^3*f*g^4 + 3*
d*e^2*g^5)*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/2*d^2*g^5*arcsin(e^2*x/(d*s
qrt(e^2)))/(sqrt(e^2)*e^5) - 1/3*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e
*g^5)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)
*e^4))/e^2 + 6*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*
```

```
e^4) + (10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*x^4/(
(-e^2*x^2 + d^2)^(5/2)*e^2) + 5/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^
2*g^3 + d^3*f*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8*(5*e^3*f*g^4 + 3*d*
e^2*g^5)*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6) - 4/3*(10*e^3*f^3*g^2 + 30*d*
e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^5 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f
^2*g^3 + d^3*f*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*(3*d*e^2*f^5 +
 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*(5*
e^3*f*g^4 + 3*d*e^2*g^5)*d^6/((-e^2*x^2 + d^2)^(5/2)*e^8) + 8/15*(10*e^3*f^
3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*d^4/((-e^2*x^2 + d^2)^
(5/2)*e^6) - 2/15*(e^3*f^5 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2
*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 4/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*
g^4 + 3*d^2*e*g^5)*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^6) + 1/2*(e^3*f^4*g + 6*
d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
 - 1/15*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)
^(3/2)*d^2*e^2) - 7/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x/(s
qrt(-e^2*x^2 + d^2)*e^6) + (e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 +
 d^3*f*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/15*(3*d*e^2*f^5 + 15*d^2*e
*f^4*g + 10*d^3*f^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (10*e^3*f^2*g^3
 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^6
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(247) = 494$.

Time = 0.32 (sec) , antiderivative size = 969, normalized size of antiderivative = 3.60

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{g^5x}{e^5}+\frac{2\left(5\,e^{11}fg^4+3\,de^{10}g^5\right)}{e^{16}}\right) - \frac{\left(20\,e^2f^2g^3+30\,defg^4+13\,d^2g^5\right)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^5|e|} + \frac{2\left(7\,e^5f^5-15\,de^4f^4g+20\,d^2e^3f^3g^2+220\,d^3e^2f^2g^3+285\,d^4efg^4+107\,d^5g^5-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^3f^5}{x}+\cdots\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-e^2*x^2 + d^2)*(g^5*x/e^5 + 2*(5*e^11*f*g^4 + 3*d*e^10*g^5)/e^16)
 - 1/2*(20*e^2*f^2*g^3 + 30*d*e*f*g^4 + 13*d^2*g^5)*arcsin(e*x/d)*sgn(d)*sg
n(e)/(e^5*abs(e)) + 2/15*(7*e^5*f^5 - 15*d*e^4*f^4*g + 20*d^2*e^3*f^3*g^2 +
 220*d^3*e^2*f^2*g^3 + 285*d^4*e*f*g^4 + 107*d^5*g^5 - 20*(d*e + sqrt(-e^2*
```

```
x^2 + d^2)*abs(e))*e^3*f^5/x + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e^2
*f^4*g/x - 100*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*f^3*g^2/x - 950*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*f^2*g^3/x - 1200*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))*d^4*f*g^4/(e*x) - 445*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d
^5*g^5/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e*f^5/x^2 - 75*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f^4*g/x^2 + 200*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^2*d^2*f^3*g^2/(e*x^2) + 1450*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^2*d^3*f^2*g^3/(e^2*x^2) + 1800*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d
^4*f*g^4/(e^3*x^2) + 665*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*g^5/(e^4
*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^5/(e*x^3) + 75*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^3*d*f^4*g/(e^2*x^3) - 750*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3*d^3*f^2*g^3/(e^4*x^3) - 1050*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3*d^4*f*g^4/(e^5*x^3) - 405*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^5
*g^5/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^5/(e^3*x^4) + 1
50*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*f^2*g^3/(e^6*x^4) + 225*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*f*g^4/(e^7*x^4) + 90*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^4*d^5*g^5/(e^8*x^4))/(d^3*e^5*((d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(f+g\,x)^5\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.580 $\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 215

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{(ef+dg)^4(d+ex)^3}{5de^5(d^2-e^2x^2)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5(d^2-e^2x^2)^{3/2}}+\frac{2(ef+dg)^2(e^2f^2-8defg+36d^2g^2)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}-\frac{g^3(4ef+3dg)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/5*(d*g+e*f)^4*(e*x+d)^3/d/e^5/(-e^2*x^2+d^2)^(5/2)+2/15*(-9*d*g+e*f)*(d*g
+e*f)^3*(e*x+d)^2/d^2/e^5/(-e^2*x^2+d^2)^(3/2)-g^3*(3*d*g+4*e*f)*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(d*g+e*f)^2*(36*d^2*g^2-8*d*e*f*g+e^2*f^2)*
(e*x+d)/d^3/e^5/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1649, 655, 223, 209}

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx=-\frac{g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(3dg+4ef)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5(d^2-e^2x^2)^{3/2}}+\frac{(d+ex)^3(dg+ef)^4}{5de^5(d^2-e^2x^2)^{5/2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}+\frac{2(d+ex)(dg+ef)^2(36d^2g^2-8defg+e^2f^2)}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^4f^4-12de^3f^3g-18d^2e^2f^2g^2-12d^3efg^3-3d^4g^4}{e^4}+\frac{5dg^2\left(6e^2f^2+4defg+d^2g^2\right)x}{e^3}+\frac{5dg^3(4ef+dg)x^2}{e^2}+\frac{5dg^4x^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^4f^4-12de^3f^3g+42d^2e^2f^2g^2+68d^3efg^3+27d^4g^4}{e^4}+\frac{30d^2g^3(2ef+dg)x}{e^3}+\frac{15d^2g^4x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$=\frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(ef+dg)^2\left(e^2f^2-8defg+36d^2g^2\right)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}-\frac{\int\frac{\frac{15d^3g^3(4ef+3dg)}{e^4}+\frac{15d^3g^4x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$=\frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(ef+dg)^2\left(e^2f^2-8defg+36d^2g^2\right)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}-\frac{\left(g^3(4ef+3dg)\right)\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$=\frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(ef+dg)^2\left(e^2f^2-8defg+36d^2g^2\right)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}-\frac{\left(g^3(4ef+3dg)\right)\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$=\frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(ef+dg)^2\left(e^2f^2-8defg+36d^2g^2\right)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}-\frac{g^3(4ef+3dg)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int\frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(72d^6g^4+2e^6f^4x^2+d^5eg^3(88f-171gx)-6de^5f^3x(f+2gx)+3d}{}$$
$$+\frac{g^3(4ef+3dg)\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{e^4\sqrt{-e^2}}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(72*d^6*g^4 + 2*e^6*f^4*x^2 + d^5*e*g^3*(88*f - 171*g*
x) - 6*d*e^5*f^3*x*(f + 2*g*x) + 3*d^4*e^2*g^2*(4*f^2 - 68*f*g*x + 39*g^2*x
^2) + d^2*e^4*f^2*(7*f^2 + 36*f*g*x + 42*g^2*x^2) - d^3*e^3*g*(12*f^3 + 36*
f^2*g*x - 128*f*g^2*x^2 + 15*g^3*x^3)))/(15*d^3*e^5*(d - e*x)^3) + (g^3*(4*
e*f + 3*d*g)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(e^4*Sqrt[-e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(199) = 398$.

Time = 0.90 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.24

method	result
risch	$\frac{g^4\sqrt{-e^2x^2+d^2}}{e^5} - \frac{\frac{(3dg+4ef)g^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}} + \frac{6g^2\left(d^2g^2+2defg+e^2f^2\right)\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{e^3d\left(x-\frac{d}{e}\right)} + \frac{4g\left(d^3g^3+3d^2efg^2+3de^2f^2g+\right.}{}}{}$
default	$d^3f^4\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right) + e^3g^4\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{}\right.$

```
[In] int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] g^4*(-e^2*x^2+d^2)^(1/2)/e^5-1/e^3*((3*d*g+4*e*f)*g^3/e/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+6*g^2/e^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/d
/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+4/e^3*g*(d^3*g^3+3*d^2*e*f*g^
2+3*d*e^2*f^2*g+e^3*f^3)*(1/3/d/e/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^
(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2))+1/e^4*(d^4*g^4+
4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2+4*d*e^3*f^3*g+e^4*f^4)*(1/5/d/e/(x-d/e)^3*(
-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-2/5*e/d*(1/3/d/e/(x-d/e)^2*(-(x-d/e)^2*
e^2-2*d*e*(x-d/e))^(1/2)-1/3/d^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/
2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(199) = 398$.

Time = 0.40 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.90

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{7\,d^3e^4f^4-12\,d^4e^3f^3g+12\,d^5e^2f^2g^2+88\,d^6efg^3+72\,d^7g^4-(7\,e^7f^4-12\,de^6f^3g+12\,d^2e^5f^2g^2+88\,d^3e^4f}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^4*f^4 - 12*d^4*e^3*f^3*g + 12*d^5*e^2*f^2*g^2 + 88*d^6*e*f*g
^3 + 72*d^7*g^4 - (7*e^7*f^4 - 12*d*e^6*f^3*g + 12*d^2*e^5*f^2*g^2 + 88*d^3
*e^4*f*g^3 + 72*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 - 12*d^2*e^5*f^3*g + 12*d
^3*e^4*f^2*g^2 + 88*d^4*e^3*f*g^3 + 72*d^5*e^2*g^4)*x^2 - 3*(7*d^2*e^5*f^4
- 12*d^3*e^4*f^3*g + 12*d^4*e^3*f^2*g^2 + 88*d^5*e^2*f*g^3 + 72*d^6*e*g^4)*
x + 30*(4*d^6*e*f*g^3 + 3*d^7*g^4 - (4*d^3*e^4*f*g^3 + 3*d^4*e^3*g^4)*x^3 +
 3*(4*d^4*e^3*f*g^3 + 3*d^5*e^2*g^4)*x^2 - 3*(4*d^5*e^2*f*g^3 + 3*d^6*e*g^4
)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^3*g^4*x^3 - 7*d^
2*e^4*f^4 + 12*d^3*e^3*f^3*g - 12*d^4*e^2*f^2*g^2 - 88*d^5*e*f*g^3 - 72*d^6
*g^4 - (2*e^6*f^4 - 12*d*e^5*f^3*g + 42*d^2*e^4*f^2*g^2 + 128*d^3*e^3*f*g^3
 + 117*d^4*e^2*g^4)*x^2 + 3*(2*d*e^5*f^4 - 12*d^2*e^4*f^3*g + 12*d^3*e^3*f^
2*g^2 + 68*d^4*e^2*f*g^3 + 57*d^5*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*
x^3 - 3*d^4*e^7*x^2 + 3*d^5*e^6*x - d^6*e^5)
```

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(199) = 398$.

Time = 0.29 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.53

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*e
) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 + d^
2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)
^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/((-e^
2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 4/15*
f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*d^3) -
1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d
```

```
^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d
^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 18*d*e^2*f^
2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(2*e
^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e
*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^4 + 4*d
^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(2*e^3*f^2
*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(
e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*d^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*x/((-e^2*x^2 + d^2)
^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g
^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3
*f^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(4*e^3*f*g^3 + 3*d*e^2*
g^4)*x/(sqrt(-e^2*x^2 + d^2)*e^6) + 1/5*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 1
2*d^2*e*f*g^3 + d^3*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^4
+ 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (4*e^3*
f*g^3 + 3*d*e^2*g^4)*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(199) = 398$.

Time = 0.31 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.52

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}\,g^4}{e^5} - \frac{(4\,efg^3+3\,dg^4)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^4|e|} + \frac{2\left(7\,e^4f^4-12\,de^3f^3g+12\,d^2e^2f^2g^2+88\,d^3efg^3+57\,d^4g^4-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)e^2f^4}{x}+\frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)de\cdots}{x}\right.}{\cdots}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] sqrt(-e^2*x^2 + d^2)*g^4/e^5 - (4*e*f*g^3 + 3*d*g^4)*arcsin(e*x/d)*sgn(d)*s
gn(e)/(e^4*abs(e)) + 2/15*(7*e^4*f^4 - 12*d*e^3*f^3*g + 12*d^2*e^2*f^2*g^2
+ 88*d^3*e*f*g^3 + 57*d^4*g^4 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^2*
f^4/x + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*e*f^3*g/x - 60*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))*d^2*f^2*g^2/x - 380*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))*d^3*f*g^3/(e*x) - 240*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*g^4/(e^
2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*f^4/x^2 - 60*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*d*f^3*g/(e*x^2) + 120*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^2*d^2*f^2*g^2/(e^2*x^2) + 580*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*
d^3*f*g^3/(e^3*x^2) + 360*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*g^4/(e^
4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^4/(e^2*x^3) + 60*(d*e +
```

```
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*f^3*g/(e^3*x^3) - 300*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^3*d^3*f*g^3/(e^5*x^3) - 210*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3*d^4*g^4/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^4/(e
^4*x^4) + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^3*f*g^3/(e^7*x^4) + 45
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^4*g^4/(e^8*x^4))/(d^3*e^4*((d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

```
[In] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.581 $\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 183

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^3(d+ex)^3}{5de^4(d^2-e^2x^2)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(ef+dg)(2e^2f^2-11defg+32d^2g^2)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}} - \frac{g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*(d*g+e*f)^3*(e*x+d)^3/d/e^4/(-e^2*x^2+d^2)^(5/2)+1/15*(-13*d*g+2*e*f)*(
d*g+e*f)^2*(e*x+d)^2/d^2/e^4/(-e^2*x^2+d^2)^(3/2)-g^3*arctan(e*x/(-e^2*x^2+
d^2)^(1/2))/e^4+1/15*(d*g+e*f)*(32*d^2*g^2-11*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^
3/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1649, 792, 223, 209}

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{g^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}} + \frac{(d+ex)(dg+ef)(32d^2g^2-11defg+2e^2f^2)}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
```

```
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^3f^3-9de^2f^2g-9d^2efg^2-3d^3g^3}{e^3}+\frac{5dg^2(3ef+dg)x}{e^2}+\frac{5dg^3x^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad + \frac{\int \frac{(d+ex)\left(\frac{2e^3f^3-9de^2f^2g+21d^2efg^2+17d^3g^3}{e^3}+\frac{15d^2g^3x}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad + \frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g^2\right)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}} - \frac{g^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}
\end{aligned}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4(d^2-e^2x^2)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(ef+dg)(2e^2f^2-11defg+32d^2g^2)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}} - \frac{g^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4(d^2-e^2x^2)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(ef+dg)(2e^2f^2-11defg+32d^2g^2)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}} - \frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\frac{(ef+dg)\sqrt{d^2-e^2x^2}(22d^4g^2+2e^4f^2x^2-de^3fx(6f+11gx)-d^3eg(16f+51gx)+d^2e^2(7f^2+33fgx+32g^2x^2))}{d^3(d-ex)^3} + 3}{15e^4}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (((e*f + d*g)*Sqrt[d^2 - e^2*x^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2 - d*e^3*f*x*(
6*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33*f*g*x + 32*g^
2*x^2)))/(d^3*(d - e*x)^3) + 30*g^3*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^
2*x^2])])/(15*e^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(169) = 338$.

Time = 0.65 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.76

method	result
default	$d^3 f^3 \left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3g^3\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{e^2\sqrt{}}{}}{}\right.$

```
[In] int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*f^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(
3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+e^3*g^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(
5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1
/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+(3*d*e^
2*g^3+3*e^3*f*g^2)*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e
^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+1/5*(3*d^3*f^2*g+3*d^
2*e*f^3)/e^2/(-e^2*x^2+d^2)^(5/2)+(3*d^2*e*g^3+9*d*e^2*f*g^2+3*e^3*f^2*g)*(
1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2
)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+
d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+(3*d^3*f*g^2+9*d^2*e*f^2*g+3*
d*e^2*f^3)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2
+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2
)^(1/2))))+(d^3*g^3+9*d^2*e*f*g^2+9*d*e^2*f^2*g+e^3*f^3)*(1/3*x^2/e^2/(-e^2
*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(169) = 338$.

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{7\,d^3e^3f^3-9\,d^4e^2f^2g+6\,d^5efg^2+22\,d^6g^3-(7\,e^6f^3-9\,de^5f^2g+6\,d^2e^4fg^2+22\,d^3e^3g^3)x^3+3\,(7\,de^5f^3-}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^3*f^3 - 9*d^4*e^2*f^2*g + 6*d^5*e*f*g^2 + 22*d^6*g^3 - (7*e^
6*f^3 - 9*d*e^5*f^2*g + 6*d^2*e^4*f*g^2 + 22*d^3*e^3*g^3)*x^3 + 3*(7*d*e^5*
f^3 - 9*d^2*e^4*f^2*g + 6*d^3*e^3*f*g^2 + 22*d^4*e^2*g^3)*x^2 - 3*(7*d^2*e^
4*f^3 - 9*d^3*e^3*f^2*g + 6*d^4*e^2*f*g^2 + 22*d^5*e*g^3)*x - 30*(d^3*e^3*g
^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*arctan(-(d - sqrt(-e^
2*x^2 + d^2))/(e*x)) + (7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g^2 + 2
2*d^5*g^3 + (2*e^5*f^3 - 9*d*e^4*f^2*g + 21*d^2*e^3*f*g^2 + 32*d^3*e^2*g^3)
*x^2 - 3*(2*d*e^4*f^3 - 9*d^2*e^3*f^2*g + 6*d^3*e^2*f*g^2 + 17*d^4*e*g^3)*x
)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^3 - 3*d^4*e^6*x^2 + 3*d^5*e^5*x - d^6*e^
4)
```

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(169) = 338$.

Time = 0.28 (sec) , antiderivative size = 903, normalized size of antiderivative = 4.93

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}\,e^3g^3x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}\,eg^3x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{df^3x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,d^2f^3}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{3\,d^3f^2g}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{4\,f^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{4\,d^2g^3x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{8\,f^3x}{15\,\sqrt{-e^2x^2+d^2}d^3} - \frac{7\,g^3x}{15\,\sqrt{-e^2x^2+d^2}e^3} + \frac{3\,(e^3fg^2+de^2g^3)x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{g^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}e^3} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4\,(e^3fg^2+de^2g^3)d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{(e^3f^3+9\,de^2f^2g+9\,d^2efg^2+d^3g^3)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{9\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{3\,(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{8\,(e^3fg^2+de^2g^3)d^4}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^6} - \frac{2\,(e^3f^3+9\,de^2f^2g+9\,d^2efg^2+d^3g^3)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} + \frac{3\,(e^3f^2g+3\,de^2fg^2+d^2eg^3)x}{5\,\sqrt{-e^2x^2+d^2}d^2e^4} - \frac{2\,(de^2f^3+3\,d^2ef^2g+d^3fg^2)x}{5\,\sqrt{-e^2x^2+d^2}d^4e^2}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*g^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2
 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*g^3*x*(3*x
^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 1/5
*d*f^3*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^3/((-e^2*x^2 + d^2)^(5/2)*e) +
3/5*d^3*f^2*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f^3*x/((-e^2*x^2 + d^2)^(
3/2)*d) + 4/15*d^2*g^3*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 8/15*f^3*x/(sqrt(-e
^2*x^2 + d^2)*d^3) - 7/15*g^3*x/(sqrt(-e^2*x^2 + d^2)*e^3) + 3*(e^3*f*g^2 +
 d*e^2*g^3)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - g^3*arcsin(e^2*x/(d*sqrt(e^2
)))/(sqrt(e^2)*e^3) + 3/2*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x^3/((-e^
2*x^2 + d^2)^(5/2)*e^2) - 4*(e^3*f*g^2 + d*e^2*g^3)*d^2*x^2/((-e^2*x^2 + d^
2)^(5/2)*e^4) + 1/3*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3
)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3
*f*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e^2*g^3)*d^4/((
-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2
```

```
+ d^3*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/10*(e^3*f^2*g + 3*d*e^2*f*g
^2 + d^2*e*g^3)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^3 + 3*d^2*e*f
^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) + 3/5*(e^3*f^2*g + 3*d
*e^2*f*g^2 + d^2*e*g^3)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^3 +
 3*d^2*e*f^2*g + d^3*f*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(169) = 338.

Time = 0.31 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{g^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^3|e|} + \frac{2\left(7\,e^3f^3-9\,de^2f^2g+6\,d^2efg^2+22\,d^3g^3-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)ef^3}{x}+\frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)df^2g}{x}-\frac{30\left(de+\sqrt{-e^2x^2+\cdots}\right.}{ex}\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -g^3*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^3*abs(e)) + 2/15*(7*e^3*f^3 - 9*d*e^2*f
^2*g + 6*d^2*e*f*g^2 + 22*d^3*g^3 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*
e*f^3/x + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f^2*g/x - 30*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))*d^2*f*g^2/(e*x) - 95*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))*d^3*g^3/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*f^3/(e*x^
2) - 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f^2*g/(e^2*x^2) + 60*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*f*g^2/(e^3*x^2) + 145*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^2*d^3*g^3/(e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*f^3/(e^3*x^3) + 45*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*f^2*g/(e^
4*x^3) - 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^3*g^3/(e^6*x^3) + 15*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^3/(e^5*x^4) + 15*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^4*d^3*g^3/(e^8*x^4))/(d^3*e^3*((d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{{(f+g\,x)}^3\,{(d+e\,x)}^3}{{(d^2-e^2\,x^2)}^{7/2}}\,dx$$

```
[In] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.582 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 145

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{(ef+dg)^2(d+ex)^3}{5de^3(d^2-e^2x^2)^{5/2}}+\frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3(d^2-e^2x^2)^{3/2}}+\frac{(2e^2f^2-6defg+7d^2g^2)(d+ex)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)^2*(e*x+d)^3/d/e^3/(-e^2*x^2+d^2)^(5/2)+2/15*(-4*d*g+e*f)*(d*g
+e*f)*(e*x+d)^2/d^2/e^3/(-e^2*x^2+d^2)^(3/2)+1/15*(7*d^2*g^2-6*d*e*f*g+2*e^
2*f^2)*(e*x+d)/d^3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1649, 803, 651}

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3(d^2-e^2x^2)^{3/2}}+\frac{(d+ex)(7d^2g^2-6defg+2e^2f^2)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-2f^2+\frac{6dfg}{e}+\frac{3d^2g^2}{e^2}+\frac{5dg^2x}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} \\
&\quad + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e^2} \\
&= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)(d+ex)}{15d^3e^3\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(2d^4g^2+2e^4f^2x^2-6d^3eg(f+gx)-6de^3fx(f+gx)+d^2e^2(7f^2+}{15d^3e^3(d-ex)^3}$$

[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]

[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*d*e^3*f*x*(f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3*(d - e*x)^3)

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

method	result
trager	$\frac{(7x^2d^2e^2g^2-6x^2d\,e^3fg+2x^2e^4f^2-6x\,d^3e\,g^2+18x\,d^2e^2fg-6xd\,e^3f^2+2d^4g^2-6fge\,d^3+7d^2e^2f^2)\sqrt{-e^2x^2+d^2}}{15d^3e^3(-ex+d)^3}$
gosper	$\frac{(-ex+d)(ex+d)^4(7x^2d^2e^2g^2-6x^2d\,e^3fg+2x^2e^4f^2-6x\,d^3e\,g^2+18x\,d^2e^2fg-6xd\,e^3f^2+2d^4g^2-6fge\,d^3+7d^2e^2f^2)}{15d^3e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$d^3f^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+g^2e^3\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{e^2}\right.$

[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(7*d^2*e^2*g^2*x^2-6*d*e^3*f*g*x^2+2*e^4*f^2*x^2-6*d^3*e*g^2*x+18*d^2*e^2*f*g*x-6*d*e^3*f^2*x+2*d^4*g^2-6*d^3*e*f*g+7*d^2*e^2*f^2)/d^3/e^3/(-e*x+d)^3*(-e^2*x^2+d^2)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(133) = 266.

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{7\,d^3e^2f^2-6\,d^4efg+2\,d^5g^2-(7\,e^5f^2-6\,de^4fg+2\,d^2e^3g^2)x^3+3\,(7\,de^4f^2-6\,d^2e^3fg+2\,d^3e^2g^2)x^2-3\,(7}{1}$$

[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15*(7*d^3*e^2*f^2 - 6*d^4*e*f*g + 2*d^5*g^2 - (7*e^5*f^2 - 6*d*e^4*f*g + 2*d^2*e^3*g^2)*x^3 + 3*(7*d*e^4*f^2 - 6*d^2*e^3*f*g + 2*d^3*e^2*g^2)*x^2 - 3*(7*d^2*e^3*f^2 - 6*d^3*e^2*f*g + 2*d^4*e*g^2)*x + (7*d^2*e^2*f^2 - 6*d^3*e*f*g + 2*d^4*g^2 + (2*e^4*f^2 - 6*d*e^3*f*g + 7*d^2*e^2*g^2)*x^2 - 6*(d*e^3*f^2 - 3*d^2*e^2*f*g + d^3*e*g^2)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 - 3*d^4*e^5*x^2 + 3*d^5*e^4*x - d^6*e^3)

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(133) = 266.

Time = 0.19 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.02

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{eg^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{4\,d^2g^2x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{df^2x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3\,d^2f^2}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{2\,d^3fg}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{8\,d^4g^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{4\,f^2x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{8\,f^2x}{15\,\sqrt{-e^2x^2+d^2}d^3} + \frac{(2\,e^3fg+3\,de^2g^2)x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{(e^3f^2+6\,de^2fg+3\,d^2eg^2)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{3\,(2\,e^3fg+3\,de^2g^2)d^2x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2\,(e^3f^2+6\,de^2fg+3\,d^2eg^2)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{(2\,e^3fg+3\,de^2g^2)x}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e^4} - \frac{(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} + \frac{(2\,e^3fg+3\,de^2g^2)x}{5\,\sqrt{-e^2x^2+d^2}d^2e^4} - \frac{2\,(3\,de^2f^2+6\,d^2efg+d^3g^2)x}{15\,\sqrt{-e^2x^2+d^2}d^4e^2}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*g^2*x^4/(-e^2*x^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2/((-e^2*x^2 + d^2)^(5/2)*
e) + 1/5*d*f^2*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^2/((-e^2*x^2 + d^2)^(5/
2)*e) + 2/5*d^3*f*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/15*d^4*g^2/((-e^2*x^2
+ d^2)^(5/2)*e^3) + 4/15*f^2*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^2*x/(sqr
t(-e^2*x^2 + d^2)*d^3) + 1/2*(2*e^3*f*g + 3*d*e^2*g^2)*x^3/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 1/3*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^2) - 3/10*(2*e^3*f*g + 3*d*e^2*g^2)*d^2*x/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 2/15*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*d^2/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 1/10*(2*e^3*f*g + 3*d*e^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
- 1/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*
e^2) + 1/5*(2*e^3*f*g + 3*d*e^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/1
5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(133) = 266.

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.55

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(7\,e^2f^2-6\,defg+2\,d^2g^2-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f^2}{x}+\frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dfg}{ex}-\frac{10\left(\right.}{}\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(7*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))*f^2/x + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f*g/(e*x) - 10*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*g^2/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^2*f^2/(e^2*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f
*g/(e^3*x^2) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*g^2/(e^4*x^2) -
 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^2/(e^4*x^3) + 30*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3*d*f*g/(e^5*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^4*f^2/(e^6*x^4))/(d^3*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x
) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4\,g^2-6\,d^3\,e\,f\,g-6\,d^3\,e\,g^2\,x+7\,d^2\,e^2\,f^2+18\,d^2\,e^2\,f\,g\,x+7\,d^2\,e^2\right.}{15\,d^3\,e^3\,\left(d-e\,x\right)^3}$$

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4*g^2 + 7*d^2*e^2*f^2 + 2*e^4*f^2*x^2 - 6*d^3*e
*f*g + 7*d^2*e^2*g^2*x^2 - 6*d*e^3*f^2*x - 6*d^3*e*g^2*x + 18*d^2*e^2*f*g*x
 - 6*d*e^3*f*g*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

3.583 $\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)(d+ex)^3}{5de^2(d^2-e^2x^2)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} + \frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)*(e*x+d)^3/d/e^2/(-e^2*x^2+d^2)^(5/2)+2/15*(-3*d*g+2*e*f)*(e*x
+d)/d/e^2/(-e^2*x^2+d^2)^(3/2)+1/15*(-3*d*g+2*e*f)*x/d^3/e/(-e^2*x^2+d^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {803, 667, 197}

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}} + \frac{2(d+ex)(2ef-3dg)}{15de^2(d^2-e^2x^2)^{3/2}} + \frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[In] Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\left(-5ef+3(ef+dg)\right)\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de} \\
&= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\left(-5ef+3(ef+dg)\right)\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de} \\
&= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-3d^3g+2e^3fx^2-3de^2x(2f+gx)+d^2e(7f+9gx)\right)}{15d^3e^2(d-ex)^3}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^3*g + 2*e^3*f*x^2 - 3*d*e^2*x*(2*f + g*x) + d^2*
e*(7*f + 9*g*x)))/(15*d^3*e^2*(d - e*x)^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

method	result
trager	$-\frac{\left(3d\,e^2g\,x^2-2e^3f\,x^2-9d^2egx+6d\,e^2fx+3d^3g-7d^2ef\right)\sqrt{-e^2x^2+d^2}}{15d^3\left(-ex+d\right)^3e^2}$
gosper	$-\frac{(-ex+d)(ex+d)^4\left(3d\,e^2g\,x^2-2e^3f\,x^2-9d^2egx+6d\,e^2fx+3d^3g-7d^2ef\right)}{15d^3e^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
default	$d^3f\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3g\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\cdots\right.}{\cdots}\right.$

[In] int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/15*(3*d*e^2*g*x^2-2*e^3*f*x^2-9*d^2*e*g*x+6*d*e^2*f*x+3*d^3*g-7*d^2*e*f)/d^3/(-e*x+d)^3/e^2*(-e^2*x^2+d^2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx =$$
$$-\frac{7\,d^3ef-3\,d^4g-(7\,e^4f-3\,de^3g)x^3+3\,(7\,de^3f-3\,d^2e^2g)x^2-3\,(7\,d^2e^2f-3\,d^3eg)x+(7\,d^2ef-3\,d^3g+\cdots}{15\,(d^3e^5x^3-3\,d^4e^4x^2+3\,d^5e^3x-d^6e^2)}$$

[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15*(7*d^3*e*f - 3*d^4*g - (7*e^4*f - 3*d*e^3*g)*x^3 + 3*(7*d*e^3*f - 3*d^2*e^2*g)*x^2 - 3*(7*d^2*e^2*f - 3*d^3*e*g)*x + (7*d^2*e*f - 3*d^3*g + (2*e^3*f - 3*d*e^2*g)*x^2 - 3*(2*d*e^2*f - 3*d^2*e*g)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^5*x^3 - 3*d^4*e^4*x^2 + 3*d^5*e^3*x - d^6*e^2)

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3\,(f+gx)}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.19

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = {} & \frac{egx^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dfx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{3\,d^2gx}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}e} \\ & + \frac{3\,d^2f}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^3g}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{4\,fx}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{gx}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}e} \\ & + \frac{8\,fx}{15\,\sqrt{-e^2x^2+d^2}d^3} + \frac{gx}{5\,\sqrt{-e^2x^2+d^2}d^2e} + \frac{(e^3f+3\,de^2g)x^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{3\,(de^2f+d^2eg)x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e^2} \\ & - \frac{2\,(e^3f+3\,de^2g)d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e^4} - \frac{(de^2f+d^2eg)x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} - \frac{2\,(de^2f+d^2eg)x}{5\,\sqrt{-e^2x^2+d^2}d^4e^2}\end{aligned}$$

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] 1/2*e*g*x^3/(-e^2*x^2 + d^2)^(5/2) + 1/5*d*f*x/(-e^2*x^2 + d^2)^(5/2) - 3/1
0*d^2*g*x/((-e^2*x^2 + d^2)^(5/2)*e) + 3/5*d^2*f/((-e^2*x^2 + d^2)^(5/2)*e)
 + 1/5*d^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f*x/((-e^2*x^2 + d^2)^(3/2
)*d) + 1/10*g*x/((-e^2*x^2 + d^2)^(3/2)*e) + 8/15*f*x/(sqrt(-e^2*x^2 + d^2)
*d^3) + 1/5*g*x/(sqrt(-e^2*x^2 + d^2)*d^2*e) + 1/3*(e^3*f + 3*d*e^2*g)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) + 3/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 2/15*(e^3*f + 3*d*e^2*g)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/
5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 2/5*(d*e^2*f + d
^2*e*g)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(105) = 210.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.36

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(7\,ef-3\,dg-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f}{ex}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dg}{e^2x}+\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right.}{e^3x^2}\right.}{15\,d^3e}$$

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(7*e*f - 3*d*g - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*f/(e*x) + 15*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*g/(e^2*x) + 40*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^2*f/(e^3*x^2) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*g/(
e^4*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f/(e^5*x^3) + 15*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*g/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^4*f/(e^7*x^4))/(d^3*e*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2
*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,g\,d^3-9\,g\,d^2\,e\,x-7\,f\,d^2\,e+3\,g\,d\,e^2\,x^2+6\,f\,d\,e^2\,x-2\,f\,e^3\,x^2\right)}{15\,d^3\,e^2\,{\left(d-e\,x\right)}^3}$$

```
[In] int(((f + g*x)*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^3*g - 2*e^3*f*x^2 - 7*d^2*e*f + 6*d*e^2*f*x -
9*d^2*e*g*x + 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)
```

3.584 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {669, 673, 665}

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
trager	$\frac{\left(2e^{2}x^{2}-6dex+7d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}\left(-ex+d\right)^{3}e}$
gosper	$\frac{\left(-ex+d\right)\left(ex+d\right)^{4}\left(2e^{2}x^{2}-6dex+7d^{2}\right)}{15d^{3}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
default	$d^{3}\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)+e^{3}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)+3d\,e^{2}$

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/(-e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-(2\,e^2x^2-6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e)}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{2\,x}{15\sqrt{-e^2x^2+d^2}d^3}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 40*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/
(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 7)/(d^3*((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

3.585 $\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 242

$$\begin{aligned}\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} \\ &\quad - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}} \\ &\quad + \frac{g^3\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef+dg)^3\sqrt{e^2f^2-d^2g^2}}\end{aligned}$$

```
[Out] 4/5*d*(e*x+d)/(d*g+e*f)/(-e^2*x^2+d^2)^(5/2)+1/15*(-5*d*(-d*g+e*f)+e*(11*d*
g+e*f)*x)/d/(d*g+e*f)^2/(-e^2*x^2+d^2)^(3/2)+g^3*arctan((e^2*f*x+d^2*g)/(-d
^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(d*g+e*f)^3/(-d^2*g^2+e^2*f^2)^
(1/2)+1/15*(15*d^3*g^2+e*(22*d^2*g^2+9*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^
3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1661, 837, 12, 739, 210}

$$\begin{aligned}\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{g^3\arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}} \\ &\quad - \frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2} + \frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)} \\ &\quad + \frac{15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}\end{aligned}$$

```
[In] Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{\frac{d^3e^2(ef+5dg)}{ef+dg}-\frac{d^2e^3(5ef-11dg)x}{ef+dg}}{(f+gx)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad-\frac{\int \frac{-\frac{d^3e^4(ef-dg)\left(2e^2f^2+7defg+15d^2g^2\right)}{ef+dg}-\frac{2d^3e^5g(ef-dg)(ef+11dg)x}{ef+dg}}{(f+gx)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4\left(e^2f^2-d^2g^2\right)}\\
&= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}}+\frac{\int \frac{15d^6e^6g^3(ef-dg)^2}{(ef+dg)(f+gx)\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6\left(e^2f^2-d^2g^2\right)^2}\\
&= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}}+\frac{g^3\int \frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}\,dx}{(ef+dg)^3}\\
&= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}}\\
&\quad-\frac{g^3\text{Subst}\left(\int \frac{1}{-e^2f^2+d^2g^2-x^2}\,dx,x,\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}}\right)}{(ef+dg)^3}\\
&= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}\\
&\quad+\frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}}+\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef+dg)^3\sqrt{e^2f^2-d^2g^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\frac{\left(-e^2f^2+d^2g^2\right)(d+ex)\left(32d^4g^2+2e^4f^2x^2+3d^3eg(8f-17gx)+3de^3fx(-2f+3gx)+d^2e^2\left(7f^2-27fgx+22g\right.\right.}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}}}{15(-ef+dg)(ef+dg)}$$

```
[In] Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g*
(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x + 2
2*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2 -
d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x
^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. 2(223) = 446.

Time = 0.52 (sec) , antiderivative size = 1667, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	1667

```
[In] int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/g^4*(1/5/(d^2*g^2-e^2*f^2)*g^
2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-e^2*f*g/(d
^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g
^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^
(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x
+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)
^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2
-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^2*g^2-e^2*f^2)*g^2*(1/
3/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)
/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e
^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^
2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*
g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(d^2*g^2-e^2*f^2)*g^2/(
-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-2*e^2*f*g/(d^
2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e
^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-
1/(d^2*g^2-e^2*f^2)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^
```

```
2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-e^2*(x+f/g)^2+2*
e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g)))))+e/g^3*(e^2*f^2*(1
/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x
/d^4/(-e^2*x^2+d^2)^(1/2)))+g^2*e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2
/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2
)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*d^2*g^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/
2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
-3*d*e*f*g*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2
)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+1/5*(3*d*e*g^2-e^2*f*g)/e^2/(-e^2*
x^2+d^2)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(223) = 446$.

Time = 0.37 (sec) , antiderivative size = 1767, normalized size of antiderivative = 7.30

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g
^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3
*e^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d
^3*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 + 15*(d^3*e^3*g^3*x
^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(-e^2*f^2 + d^2*g^2)*
log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sq
rt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x +
f)) - 3*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2
*f*g^3 - 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f
^2*g^2 - 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*
e^4*f^2*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^
2*e^4*f^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(
-e^2*x^2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9
*e^2*f^2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g +
 2*d^5*e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3
 + 3*(d^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3
 - 3*d^8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g +
2*d^7*e^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x), 1
/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g^3
 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3*e
^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d^3
*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 - 30*(d^3*e^3*g^3*x^3
 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(e^2*f^2 - d^2*g^2)*arc
```

```
tan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - 3
*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2*f*g^3
- 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f^2*g^2
- 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*e^4*f^2
*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^2*e^4*f
^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(-e^2*x^
2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9*e^2*f^
2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g + 2*d^5*
e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3 + 3*(d
^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3 - 3*d^
8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g + 2*d^7*e
^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d+ex)^3}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}(f+gx)}\,dx$$

```
[In] integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(223) = 446$.

Time = 0.31 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.65

$$\int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\,eg^3\arctan\left(\frac{dg+\frac{\left(de+\sqrt{-e^2x^2+d^2}|e|\right)f}{ex}}{\sqrt{e^2f^2-d^2g^2}}\right)}{\left(e^3f^3|e|+3\,de^2f^2g|e|+3\,d^2efg^2|e|+d^3g^3|e|\right)\sqrt{e^2f^2-d^2g^2}} + \frac{2\left(7\,e^3f^2+24\,de^2fg+32\,d^2eg^2-\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)ef^2}{x}-\frac{75\left(de+\sqrt{-e^2x^2+d^2}|e|\right)dfg}{x}-\frac{115\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2g^2}{ex}\right.}{}$$

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*e*g^3*arctan((d*g + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*f/(e*x))/sqrt(e^
2*f^2 - d^2*g^2))/((e^3*f^3*abs(e) + 3*d*e^2*f^2*g*abs(e) + 3*d^2*e*f*g^2*a
bs(e) + d^3*g^3*abs(e))*sqrt(e^2*f^2 - d^2*g^2)) + 2/15*(7*e^3*f^2 + 24*d*e
^2*f*g + 32*d^2*e*g^2 - 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e*f^2/x - 75
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d*f*g/x - 115*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))*d^2*g^2/(e*x) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*f^2/(e
*x^2) + 135*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d*f*g/(e^2*x^2) + 185*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2*g^2/(e^3*x^2) - 30*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^3*f^2/(e^3*x^3) - 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)^3*d*f*g/(e^4*x^3) - 135*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*g^2/(e^
5*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*f^2/(e^5*x^4) + 45*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^4*d*f*g/(e^6*x^4) + 45*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^4*d^2*g^2/(e^7*x^4))/((d^3*e^3*f^3*abs(e) + 3*d^4*e^2*f^2*g*a
bs(e) + 3*d^5*e*f*g^2*abs(e) + d^6*g^3*abs(e))*((d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))/(e^2*x) - 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^3}{(f+g\,x)\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)), x)
```

3.586 $\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 311

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\left(45d^3g^2+e\left(2e^2f^2+14defg+57d^2g^2\right)x\right)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(ef-dg)(ef+dg)^4(f+gx)}+\frac{eg^3(4ef-3dg)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef-dg)(ef+dg)^4\sqrt{e^2f^2-d^2g^2}}$$

```
[Out] 4/5*d*e*(e*x+d)/(d*g+e*f)^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(5*d*(-3*d*g+e*f)-e
*(21*d*g+e*f)*x)/d/(d*g+e*f)^3/(-e^2*x^2+d^2)^(3/2)+e*g^3*(-3*d*g+4*e*f)*ar
ctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(-d*g+e
*f)/(d*g+e*f)^4/(-d^2*g^2+e^2*f^2)^(1/2)+1/15*e*(45*d^3*g^2+e*(57*d^2*g^2+1
4*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^4/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+
d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g*x+f)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

= {1661, 821, 739, 210}

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{eg^3(4ef-3dg)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4} - \frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d\,(d^2-e^2x^2)^{3/2}\,(dg+ef)^3} + \frac{4de(d+ex)}{5\,(d^2-e^2x^2)^{5/2}\,(dg+ef)^2} + \frac{e(45d^3g^2+ex(57d^2g^2+14defg+2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}$$

```
[In] Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} + \frac{\int \frac{\frac{d^3e^2(e^2f^2+10defg+5d^2g^2)}{(ef+dg)^2} - \frac{d^2e^3(ef-5dg)(5ef+3dg)x}{(ef+dg)^2} + \frac{16d^3e^4g^2x^2}{(ef+dg)^2}}{(f+gx)^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\frac{d^3e^4(2e^3f^3+12de^2f^2g+45d^2efg^2+15d^3g^3)}{(ef+dg)^3} + \frac{d^3e^5g(4e^2f^2+69defg+45d^2g^2)x}{(ef+dg)^3} + \frac{2d^3e^6g^2(ef+21dg)x^2}{(ef+dg)^3}}{(f+gx)^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3(d^2-e^2x^2)^{3/2}} + \frac{e(45d^3g^2+e(2e^2f^2+14defg+57d^2g^2)x)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{\frac{15d^6e^6g^3(4ef+dg)}{(ef+dg)^4} + \frac{45d^6e^7g^4x}{(ef+dg)^4}}{(f+gx)^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3(d^2-e^2x^2)^{3/2}} + \frac{e(45d^3g^2+e(2e^2f^2+14defg+57d^2g^2)x)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(ef-dg)(ef+dg)^4(f+gx)} + \frac{(eg^3(4ef-3dg))\int \frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}\,dx}{(ef-dg)(ef+dg)^4}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3(d^2-e^2x^2)^{3/2}} + \frac{e(45d^3g^2+e(2e^2f^2+14defg+57d^2g^2)x)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(ef-dg)(ef+dg)^4(f+gx)} - \frac{(eg^3(4ef-3dg))\,\text{Subst}\left(\int \frac{1}{-e^2f^2+d^2g^2-x^2}\,dx, x, \frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}}\right)}{(ef-dg)(ef+dg)^4}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2(d^2-e^2x^2)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3(d^2-e^2x^2)^{3/2}} + \frac{e(45d^3g^2+e(2e^2f^2+14defg+57d^2g^2)x)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(ef-dg)(ef+dg)^4(f+gx)} + \frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{(ef-dg)(ef+dg)^4\sqrt{e^2f^2-d^2g^2}}$$

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(e^2f^2-d^2g^2)(d+ex)(15d^6g^4+2e^6f^3x^2(f+gx)-9d^5eg^3(8f+13gx)+6de^5f^2x(-f^2+fgx+2g^2x^2)+d^4e}{d^3(d-e}$$

```
[In] Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*d
^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*e^
2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*g*x
 + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2*x^2
 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*e*g^3
*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2
*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3288 vs. 2(291) = 582.

Time = 0.52 (sec) , antiderivative size = 3289, normalized size of antiderivative = 10.58

method	result	size
default	Expression too large to display	3289

```
[In] int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/g^3*(1/5/e*g/(-e^2*x^2+d^2)^(5/2)+3*d*g*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)
+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))-2
*e*f*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2
)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*e/g^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)*(1/
5/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)
/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e
^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^
2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*
f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/
3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*
f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^
2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+
f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x
+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)
^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2
-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2
```

```
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(
d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^
2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^
2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^
2-e^2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)
*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1
/2)*(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g)
))))+1/g^5*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)*(-1/(d^2*g^2-e^2*f
^2)*g^2/(x+f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5
/2)-7*e^2*f*g/(d^2*g^2-e^2*f^2)*(1/5/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+
2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2
/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/
(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4
*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(
-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/
g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^
4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2
)*g^2/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*
g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^
2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g
^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(
x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^
(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(d^2*g^2-e^2*f^2)*g^2/(-e^2*(x+f/g)^2+2*e
^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2
*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*
(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2
)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(
x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d
^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g)))))+6*e^2/(d^2*g^2-e^2*f^2)*g^2*(2/5*(-
2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2
*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*
(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^
2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d
^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2
/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-e^2*(x+f/g)^2+2*e^2*f/g*(x+f/g)+(d^2*g
^2-e^2*f^2)/g^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1623 vs. 2(290) = 580.

Time = 0.75 (sec) , antiderivative size = 3305, normalized size of antiderivative = 10.63

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*e^4*f^5*g^2 - 99*d^6*e^3*f
^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 - 15*d^9*f*g^6 - (7*e^9*f^6*
g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^3*e^6*f^3*g^4 - 23*d^4*e^5
*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4 - (7*e^9*f^7 + 6*d*e^8*f^
6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 + 274*d^4*e^5*f^3*g^4 + 141*
d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*g^7)*x^3 + 3*(7*d*e^8*f^7
+ 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*e^5*f^4*g^3 + 76*d^5*e^4*f
^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 + 15*d^8*e*g^7)*x^2 - 15*(4*
d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^2*g^4 - 3*d^4*e^4*f*g^5)*x
^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*d^5*e^3*f*g^5)*x^3 + 3*(4*
d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f*g^5)*x^2 - (12*d^5*e^3*f^
3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log
((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(
-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f))
 - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5*g^2 - 328*d^5*e^4*f^
4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117*d^8*e*f*g^6 + 15*d^9
*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e^4*f^5*g^2 - 99*d^5*e
^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 15*d^8*f*g^6 + (2*e^8*
f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3*e^5*f^3*g^4 - 43*d^4
*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*d*e^7*f^6*g + 5*d^2*e
^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4 + 141*d^5*e^3*f^2*g^
5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6*f^6*g + 51*d^3*e^5*f
^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 164*d^6*e^2*f^2*g^5 - 1
17*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9 + 3*d^7*e^6*f^8*g + d
^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^4 + d^11*e^2*f^4*g^5
+ 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g + 3*d^4*e^9*f^7*g^2 + d
^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5 + d^8*e^5*f^3*g^6 +
3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^9 - 8*d^5*e^8*f^7*g^2
 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5*f^4*g^5 - 8*d^10*e^3
*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*d^5*e^8*f^8*g - 2*d^6
*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 + 2*d^10*e^3*f^3*g^6 -
 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f^9 + 8*d^6*e^7*f^8*g
- 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3*f^4*g^5 + 8*d^11*e^2
*f^3*g^6 - d^13*f*g^8)*x), 1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*
e^4*f^5*g^2 - 99*d^6*e^3*f^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 -
```

```
15*d^9*f*g^6 - (7*e^9*f^6*g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^
3*e^6*f^3*g^4 - 23*d^4*e^5*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4
 - (7*e^9*f^7 + 6*d*e^8*f^6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 +
274*d^4*e^5*f^3*g^4 + 141*d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*
g^7)*x^3 + 3*(7*d*e^8*f^7 + 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*
e^5*f^4*g^3 + 76*d^5*e^4*f^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 +
15*d^8*e*g^7)*x^2 + 30*(4*d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^
2*g^4 - 3*d^4*e^4*f*g^5)*x^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*
d^5*e^3*f*g^5)*x^3 + 3*(4*d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f
*g^5)*x^2 - (12*d^5*e^3*f^3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sq
rt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e
^2*f^2 - d^2*g^2)*x)) - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5
*g^2 - 328*d^5*e^4*f^4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117
*d^8*e*f*g^6 + 15*d^9*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e
^4*f^5*g^2 - 99*d^5*e^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 1
5*d^8*f*g^6 + (2*e^8*f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3
*e^5*f^3*g^4 - 43*d^4*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*
d*e^7*f^6*g + 5*d^2*e^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4
 + 141*d^5*e^3*f^2*g^5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6
*f^6*g + 51*d^3*e^5*f^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 16
4*d^6*e^2*f^2*g^5 - 117*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9
+ 3*d^7*e^6*f^8*g + d^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^
4 + d^11*e^2*f^4*g^5 + 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g +
3*d^4*e^9*f^7*g^2 + d^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5
 + d^8*e^5*f^3*g^6 + 3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^
9 - 8*d^5*e^8*f^7*g^2 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5
*f^4*g^5 - 8*d^10*e^3*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*
d^5*e^8*f^8*g - 2*d^6*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 +
 2*d^10*e^3*f^3*g^6 - 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f
^9 + 8*d^6*e^7*f^8*g - 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3
*f^4*g^5 + 8*d^11*e^2*f^3*g^6 - d^13*f*g^8)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)^2\,(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}\,(f+gx)^2}\,dx$$

```
[In] integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^2 (d^2-e^2x^2)^{7/2}} dx = \int \frac{(d+e\,x)^3}{(f+g\,x)^2\,(d^2-e^2\,x^2)^{7/2}} \,dx$$

```
[In] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.587 $\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 398

$$\begin{aligned}\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}}\\
&-\frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}}\\
&+\frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2}+\frac{3eg^4(3ef-2dg)\sqrt{d^2-e^2x^2}}{2(ef-dg)^2(ef+dg)^5(f+gx)}\\
&+\frac{e^2g^3\left(20e^2f^2-30defg+13d^2g^2\right)\arctan\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{2(ef-dg)^2(ef+dg)^5\sqrt{e^2f^2-d^2g^2}}\end{aligned}$$

```
[Out] 4/5*d*e^2*(e*x+d)/(d*g+e*f)^3/(-e^2*x^2+d^2)^(5/2)-1/15*e^2*(5*d*(-5*d*g+e*
f)-e*(31*d*g+e*f)*x)/d/(d*g+e*f)^4/(-e^2*x^2+d^2)^(3/2)+1/2*e^2*g^3*(13*d^2
*g^2-30*d*e*f*g+20*e^2*f^2)*arctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2))/(-d*g+e*f)^2/(d*g+e*f)^5/(-d^2*g^2+e^2*f^2)^(1/2)+1/
15*e^2*(90*d^3*g^2+e*(107*d^2*g^2+19*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^5/
(-e^2*x^2+d^2)^(1/2)+1/2*g^4*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g
*x+f)^2+3/2*e*g^4*(-2*d*g+3*e*f)*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)^2/(d*g+e*f
)^5/(g*x+f)
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1661, 1665, 821, 739, 210}

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}} dx = \frac{e^2g^3(13d^2g^2-30defg+20e^2f^2) \arctan\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}} + \frac{3eg^4\sqrt{d^2-e^2x^2}(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4} - \frac{e^2(5d(ef-5dg)-ex(31dg+ef))}{15d\,(d^2-e^2x^2)^{3/2}\,(dg+ef)^4} + \frac{4de^2(d+ex)}{5\,(d^2-e^2x^2)^{5/2}\,(dg+ef)^3} + \frac{e^2(90d^3g^2+ex(107d^2g^2+19defg+2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^5}$$

```
[In] Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^3f^3+15de^2f^2g+15d^2efg^2+5d^3g^3\right)}{(ef+dg)^3}-\frac{d^2e^3\left(5e^3f^3-33de^2f^2g-45d^2efg^2-15d^3g^3\right)x}{(ef+dg)^3}+\frac{4d^3e^4g^2(12ef+5dg)x^2}{(ef+dg)^3}+\frac{16d^3e^5g^3x^3}{(ef+dg)^3}}{(f+gx)^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{d^3e^4\left(2e^4f^4+17de^3f^3g+90d^2e^2f^2g^2+60d^3efg^3+15d^4g^4\right)}{(ef+dg)^4}+\frac{3d^3e^5g\left(2e^2f^2+45defg+15d^2g^2\right)x}{(ef+dg)^3}+\frac{3d^3e^6g^2\left(2e^2f^2+57defg+25d^2g^2\right)x^2}{(ef+dg)^4}+2d^3e\cdots}{(f+gx)^3(d^2-e^2x^2)^{3/2}}}{15d^4e^4}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{\frac{15d^6e^6g^3\left(10e^2f^2+5defg+d^2g^2\right)}{(ef+dg)^5}+\frac{45d^6e^7g^4(5ef+dg)x}{(ef+dg)^5}+\frac{90d^6e^8g^5x^2}{(ef+dg)^5}}{(f+gx)^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2} + \frac{\int \frac{\frac{30d^6e^7g^3\left(10e^2f^2-5defg-3d^2g^2\right)}{(ef+dg)^4}+\frac{15d^6e^8g^4(11ef-13dg)x}{(ef+dg)^4}}{(f+gx)^2\sqrt{d^2-e^2x^2}}\,dx}{30d^6e^6\left(e^2f^2-d^2g^2\right)}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2} + \frac{3eg^4(3ef-2dg)\sqrt{d^2-e^2x^2}}{2(ef-dg)^2(ef+dg)^5(f+gx)} + \frac{\left(e^2g^3\left(20e^2f^2-30defg+13d^2g^2\right)\right)\int \frac{1}{(f+gx)\sqrt{d^2-e^2x^2}}\,dx}{2(ef-dg)^2(ef+dg)^5}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2} + \frac{3eg^4(3ef-2dg)\sqrt{d^2-e^2x^2}}{2(ef-dg)^2(ef+dg)^5(f+gx)} - \frac{\left(e^2g^3\left(20e^2f^2-30defg+13d^2g^2\right)\right)\text{Subst}\left(\int \frac{1}{-e^2f^2+d^2g^2-x^2}\,dx, x, \frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}}\right)}{2(ef-dg)^2(ef+dg)^5}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+19defg+107d^2g^2\right)x\right)}{15d^3(ef+dg)^5\sqrt{d^2-e^2x^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(ef-dg)(ef+dg)^4(f+gx)^2} + \frac{3eg^4(3ef-2dg)\sqrt{d^2-e^2x^2}}{2(ef-dg)^2(ef+dg)^5(f+gx)} + \frac{e^2g^3\left(20e^2f^2-30defg+13d^2g^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{2(ef-dg)^2(ef+dg)^5\sqrt{e^2f^2-d^2g^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.99 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(\frac{6e^2(ef+dg)^2}{d(d-ex)^3}+\frac{2e^2(ef+dg)(2ef+17dg)}{d^2(d-ex)^2}+\frac{2e^2(2e^2f^2+19defg+107d^2g^2)}{d^3(d-ex)}+\frac{15}{(ef-}\right.}{}$$

```
[In] Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f +
 d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*g
+ 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f + g*
x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)*e^2
*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f + d*g
)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2]))/(e
^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*(f +
g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6395 vs. 2(370) = 740.

Time = 0.48 (sec) , antiderivative size = 6396, normalized size of antiderivative = 16.07

method	result	size
default	Expression too large to display	6396

```
[In] int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2651 vs. 2(369) = 738.

Time = 2.54 (sec) , antiderivative size = 5361, normalized size of antiderivative = 13.47

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(f+gx)^3}\,dx$$

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1401 vs. $2(369) = 738$.

Time = 0.43 (sec) , antiderivative size = 1401, normalized size of antiderivative = 3.52

$$\int \frac{(d+ex)^3}{(f+gx)^3 (d^2-e^2x^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] -(20*e^5*f^2*g^3 - 30*d*e^4*f*g^4 + 13*d^2*e^3*g^5)*arctan((d*g + (d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))*f/(e*x))/sqrt(e^2*f^2 - d^2*g^2))/((e^7*f^7*abs(
e) + 3*d*e^6*f^6*g*abs(e) + d^2*e^5*f^5*g^2*abs(e) - 5*d^3*e^4*f^4*g^3*abs(
e) - 5*d^4*e^3*f^3*g^4*abs(e) + d^5*e^2*f^2*g^5*abs(e) + 3*d^6*e*f*g^6*abs(
e) + d^7*g^7*abs(e))*sqrt(e^2*f^2 - d^2*g^2)) - (10*d*e^5*f^4*g^4 - 6*d^2*e
^4*f^3*g^5 - d^3*e^3*f^2*g^6 + 29*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e
^2*f^3*g^5/x - 18*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^3*e*f^2*g^6/x - 2*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^4*f*g^7/x + 10*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^2*d*e*f^4*g^4/x^2 - 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^
2*f^3*g^5/x^2 + 19*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^3*f^2*g^6/(e*x^2
```

```
) - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^4*f*g^7/(e^2*x^2) - 2*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^5*g^8/(e^3*x^2) + 11*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^3*d^2*f^3*g^5/(e^2*x^3) - 6*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*d^3*f^2*g^6/(e^3*x^3) - 2*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^4*
f*g^7/(e^4*x^3))/((e^7*f^9*abs(e) + 3*d*e^6*f^8*g*abs(e) + d^2*e^5*f^7*g^2*
abs(e) - 5*d^3*e^4*f^6*g^3*abs(e) - 5*d^4*e^3*f^5*g^4*abs(e) + d^5*e^2*f^4*
g^5*abs(e) + 3*d^6*e*f^3*g^6*abs(e) + d^7*f^2*g^7*abs(e))*(e*f + 2*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))*d*g/(e^2*x) + (d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^2*f/(e^3*x^2))^2) + 2/15*(7*e^5*f^2 + 44*d*e^4*f*g + 127*d^2*e^3*g^2 - 2
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*e^3*f^2/x - 145*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))*d*e^2*f*g/x - 485*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2*e*
g^2/x + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*e*f^2/x^2 + 245*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*d*f*g/x^2 + 745*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^2*d^2*g^2/(e*x^2) - 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*f^2/(e*x^
3) - 195*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d*f*g/(e^2*x^3) - 525*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2*g^2/(e^3*x^3) + 15*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^4*f^2/(e^3*x^4) + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*
d*f*g/(e^4*x^4) + 150*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^2*g^2/(e^5*x^
4))/((d^3*e^5*f^5*abs(e) + 5*d^4*e^4*f^4*g*abs(e) + 10*d^5*e^3*f^3*g^2*abs(
e) + 10*d^6*e^2*f^2*g^3*abs(e) + 5*d^7*e*f*g^4*abs(e) + d^8*g^5*abs(e))*((d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(f+gx)^3\,(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(d+e\,x)^3}{(f+g\,x)^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

```
[In] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.588 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}} + \frac{2c\sqrt{d+ex}}{e^2g} - \frac{2(cf^2+ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*g^2+c*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/g^(3/2)/(-d
*g+e*f)^(3/2)-2*(a*e^2+c*d^2)/e^2/(-d*g+e*f)/(e*x+d)^(1/2)+2*c*(e*x+d)^(1/2
)/e^2/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {912, 1275, 211}

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2(ag^2+cf^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}} - \frac{2(ae^2+cd^2)}{e^2\sqrt{d+ex}(ef-dg)} + \frac{2c\sqrt{d+ex}}{e^2g}$$

```
[In] Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cd^2+ae^2}{e^2}-\frac{2cdx^2}{e^2}+\frac{cx^4}{e^2}}{x^2\left(\frac{ef-dg}{e}+\frac{gx^2}{e}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$=\frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cd^2+ae^2}{e(ef-dg)x^2}-\frac{e(cf^2+ag^2)}{g(-ef+dg)(-ef+dg-gx^2)}\right)\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$=-\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}+\frac{(2(cf^2+ag^2))\,\text{Subst}\left(\int \frac{1}{-ef+dg-gx^2}\,dx,x,\sqrt{d+ex}\right)}{g(ef-dg)}$$

$$=-\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2(cf^2+ag^2)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2(cd^2g+ae^2g-cef(d+ex)+cdg(d+ex))}{e^2g(ef-dg)\sqrt{d+ex}} - \frac{2(cf^2+ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2*g + a*e^2*g - c*e*f*(d + e*x) + c*d*g*(d + e*x)))/(e^2*g*(e*f -
d*g)*Sqrt[d + e*x]) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqr
t[e*f - d*g]])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^2\left(a\,g^2+c\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}+\frac{2\left(e^2a+c\,d^2\right)}{(dg-ef)\sqrt{ex+d}}}{e^2}$	111
derivativedivides	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^2\left(a\,g^2+c\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}-\frac{2\left(-e^2a-c\,d^2\right)}{(dg-ef)\sqrt{ex+d}}}{e^2}$	114
default	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2e^2\left(a\,g^2+c\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)g\sqrt{(dg-ef)g}}-\frac{2\left(-e^2a-c\,d^2\right)}{(dg-ef)\sqrt{ex+d}}}{e^2}$	114
risch	$\frac{2c\sqrt{ex+d}}{e^2g}-\frac{2\left(\frac{e^2\left(a\,g^2+c\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(dg-ef)\sqrt{(dg-ef)g}}-\frac{\left(e^2a+c\,d^2\right)g}{(dg-ef)\sqrt{ex+d}}\right)}{g\,e^2}$	117

```
[In] int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(c/g*(e*x+d)^(1/2)-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)*
arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+(a*e^2+c*d^2)/(d*g-e*f)/(e*x+d
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(98) = 196.

Time = 0.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.46

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \left[\frac{(cde^2f^2+ade^2g^2+(ce^3f^2+ae^3g^2)x)\sqrt{-efg+dg^2}\log\left(\frac{egx-ef+2\,dg-2\,\sqrt{-efg+dg^2}}{gx+f}\right.}{de^4f^2g^2-2\,d^2e^3f}\right.$$

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] [((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*sqrt(-e*f*g + d*g
^2)*log((e*g*x - e*f + 2*d*g - 2*sqrt(-e*f*g + d*g^2)*sqrt(e*x + d))/(g*x +
 f)) + 2*(c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g
^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4
*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d
^2*e^3*g^4)*x), 2*((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*
sqrt(e*f*g - d*g^2)*arctan(sqrt(e*f*g - d*g^2)*sqrt(e*x + d)/(e*g*x + d*g))
 + (c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g^3 + (
c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4*f^2*g
^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d^2*e^3
*g^4)*x)]
```

Sympy [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{d+ex}}{eg}+\frac{e\left(ag^2+cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}(dg-ef)}+\frac{ae^2+cd^2}{e\sqrt{d+ex}(dg-ef)}\right)}{e} & \text{for } e\neq 0 \\ \frac{-\frac{cfx}{g^2}+\frac{cx^2}{2g}+\frac{\left(ag^2+cf^2\right)\left(\begin{cases}\frac{x}{f} & \text{for } g=0\\ \frac{\log(f+gx)}{g} & \text{otherwise}\end{cases}\right)}{g^2}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

```
[Out] Piecewise((2*(c*sqrt(d + e*x)/(e*g) + e*(a*g**2 + c*f**2)*atan(sqrt(d + e*x
)/sqrt(-(d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + (a*e**2 +
 c*d**2)/(e*sqrt(d + e*x)*(d*g - e*f)))/e, Ne(e, 0)), ((-c*f*x/g**2 + c*x**
2/(2*g) + (a*g**2 + c*f**2)*Piecewise((x/f, Eq(g, 0)), (log(f + g*x)/g, Tru
e))/g**2)/d**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2\,(cf^2+ag^2)\arctan\left(\frac{\sqrt{ex+d}g}{\sqrt{efg-dg^2}}\right)}{(efg-dg^2)^{\frac{3}{2}}} - \frac{2\,(cd^2+ae^2)}{(e^3f-de^2g)\sqrt{ex+d}} + \frac{2\sqrt{ex+d}c}{e^2g}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/(e*f*g - d*g
^2)^(3/2) - 2*(c*d^2 + a*e^2)/((e^3*f - d*e^2*g)*sqrt(e*x + d)) + 2*sqrt(e*
x + d)*c/(e^2*g)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,c\,\sqrt{d+e\,x}}{e^2\,g} + \frac{2\,(c\,g\,d^2+a\,g\,e^2)}{e^2\,g\,(d\,g-e\,f)\,\sqrt{d+e\,x}} + \frac{\mathrm{atan}\left(\frac{d\,g^{3/2}\,\sqrt{d+e\,x}\,1\mathrm{i}-e\,f\,\sqrt{g}\,\sqrt{d+e\,x}\,1\mathrm{i}}{(d\,g-e\,f)^{3/2}}\right)\,(c\,f^2+a\,g^2)\,2\mathrm{i}}{g^{3/2}\,(d\,g-e\,f)^{3/2}}$$

```
[In] int((a + c*x^2)/((f + g*x)*(d + e*x)^(3/2)),x)
```

```
[Out] (atan((d*g^(3/2)*(d + e*x)^(1/2)*1i - e*f*g^(1/2)*(d + e*x)^(1/2)*1i)/(d*g
- e*f)^(3/2))*(a*g^2 + c*f^2)*2i)/(g^(3/2)*(d*g - e*f)^(3/2)) + (2*c*(d + e
*x)^(1/2))/(e^2*g) + (2*(a*e^2*g + c*d^2*g))/(e^2*g*(d*g - e*f)*(d + e*x)^(
1/2))
```

3.589 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx = & -\frac{2(ef-dg)^3\,(cf^2+ag^2)\,\sqrt{f+gx}}{g^6} \\ & +\frac{2(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))\,(f+gx)^{3/2}}{3g^6} \\ & -\frac{2(ef-dg)\,(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))\,(f+gx)^{5/2}}{5g^6} \\ & +\frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))\,(f+gx)^{7/2}}{7g^6} \\ & -\frac{2ce^2(5ef-3dg)(f+gx)^{9/2}}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}\end{aligned}$$

```
[Out] 2/3*(-d*g+e*f)^2*(3*a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-2/5*(-d*g
+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6+2/7*
e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*c*e
^2*(-3*d*g+5*e*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e
*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {912, 1167}

$$\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2e(f+gx)^{7/2}\left(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2)\right)}{7g^6} - \frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2)\right)}{5g^6} - \frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^3}{g^6} + \frac{2(f+gx)^{3/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{3g^6} - \frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

```
[In] Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3 \left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^3(cf^2+ag^2)}{g^5}+\frac{(ef-dg)^2(3aeg^2+cf(5ef-2dg))x^2}{g^5}+\frac{(ef-dg)(-3ae^2g^2-c(10e^2f^2-8defg+d^2g^2))x^4}{g^5}+\right.}{g}$$

$$= -\frac{2(ef-dg)^3\,(cf^2+ag^2)\,\sqrt{f+gx}}{g^6} + \frac{2(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))\,(f+gx)^{3/2}}{3g^6} - \frac{2(ef-dg)\,(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))\,(f+gx)^{5/2}}{5g^6} + \frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))\,(f+gx)^{7/2}}{7g^6} - \frac{2ce^2(5ef-3dg)(f+gx)^{9/2}}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(99ag^2(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-6fg^2}{}$$

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(99*a*g^2*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^
2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2
+ 5*g^3*x^3)) + c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e*g^
2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4 - 6
4*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f^5 -
128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5))
))/(3465*g^6)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2c e^3 (gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fce^3\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a g^2+c f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-6\right.}{}$
default	$\frac{2c e^3 (gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fce^3\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a g^2+c f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-6\right.}{}$
pseudoelliptic	$\frac{2\left(\left(\frac{x^3\left(\frac{7c x^2}{11}+a\right)e^3}{7}+\frac{3x^2\left(\frac{5c x^2}{9}+a\right)d e^2}{5}+d^2x\left(\frac{3c x^2}{7}+a\right)e+d^3\left(\frac{c x^2}{5}+a\right)\right)g^5-2\left(\left(\frac{5}{99}c x^4+\frac{3}{35}a x^2\right)e^3+\frac{2\left(\frac{10c x^2}{21}+a\right)x}{5}\right.\right.}{}$
gosper	$\frac{2\sqrt{gx+f}\,\left(315c e^3x^5g^5+1155cd e^2g^5x^4-350c e^3f g^4x^4+495a e^3g^5x^3+1485c d^2e g^5x^3-1320cd e^2f g^4x^3+400c e^3f^2g^3x^3\right.}{}$
trager	$\frac{2\sqrt{gx+f}\,\left(315c e^3x^5g^5+1155cd e^2g^5x^4-350c e^3f g^4x^4+495a e^3g^5x^3+1485c d^2e g^5x^3-1320cd e^2f g^4x^3+400c e^3f^2g^3x^3\right.}{}$
risch	$\frac{2\sqrt{gx+f}\,\left(315c e^3x^5g^5+1155cd e^2g^5x^4-350c e^3f g^4x^4+495a e^3g^5x^3+1485c d^2e g^5x^3-1320cd e^2f g^4x^3+400c e^3f^2g^3x^3\right.}{}$

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c-2*f*c*e^3)*(g*x+f)^
(9/2)+1/7*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g*x+f)
^(7/2)+1/5*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2+c*f^2)
)*(g*x+f)^(5/2)+1/3*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^2))*(g*x
+f)^(3/2)+(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\left(315\,ce^3g^5x^5-1280\,ce^3f^5+4224\,cde^2f^4g-6930\,ad^2efg^4+3465\,ad^3g^5-1584\,(3\,cd^2e+ae^3)f^3g^2+18\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d^
2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d^3
 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80*c*
e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(160*
```

```
c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 - 231*(
c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2 + 346
5*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d*e^2)*f
*g^4)*x)*sqrt(g*x + f)/g^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(241) = 482.

Time = 1.00 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{11}{2}}}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+cd^3g^3-9cd^2efg^2+18cde^2\right.}{5g^5}\right.}{} \\ \frac{ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{\sqrt{f}} \end{cases}$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*e**3*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(3*c*d*
e**2*g - 5*c*e**3*f)/(9*g**5) + (f + g*x)**(7/2)*(a*e**3*g**2 + 3*c*d**2*e*
g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(7*g**5) + (f + g*x)**(5/2)*(3*a*d
*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d*e**
2*f**2*g - 10*c*e**3*f**3)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d**2*e*g**4 - 6
*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*f**2*g
**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(3*g**5) + sqrt(f + g*x)*(a*d**3*
g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 - a*e**3*f**3*g**2 + c*d**3
*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*f**4*g - c*e**3*f**5)/g**5)/
g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x
**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3)/3)/sqrt(f
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3-385\,(5\,ce^3f-3\,cde^2g)(gx+f)^{\frac{9}{2}}+495\,(10\,ce^3f^2-12\,cde^2fg+(3\,cd^2e+ae^3)g^2)(g\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f
)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*
x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)
*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 12*
c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a
*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2
)*f^2*g^3)*sqrt(g*x + f))/g^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\left(3465\sqrt{gx+f}ad^3 + \frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ad^2e}{g} + \frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd^3}{g^2} + \frac{693\left(3(gx+f)^{\frac{5}{2}}\right.}{}\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5*(g*x
 + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
 f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(
g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e
^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x
 + f)*f^5)*c*e^3/g^5)/g
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}} dx$$
$$= \frac{(f+g x)^{7/2} (6 c d^2 e g^2 - 24 c d e^2 f g + 20 c e^3 f^2 + 2 a e^3 g^2)}{7 g^6}$$
$$+ \frac{2 \sqrt{f+g x} (c f^2 + a g^2) (d g - e f)^3}{g^6} + \frac{2 c e^3 (f+g x)^{11/2}}{11 g^6}$$
$$+ \frac{2 (f+g x)^{3/2} (d g - e f)^2 (5 c e f^2 - 2 c d f g + 3 a e g^2)}{3 g^6}$$
$$+ \frac{2 (f+g x)^{5/2} (d g - e f) (c d^2 g^2 - 8 c d e f g + 10 c e^2 f^2 + 3 a e^2 g^2)}{5 g^6}$$
$$+ \frac{2 c e^2 (f+g x)^{9/2} (3 d g - 5 e f)}{9 g^6}$$

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g
^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*
e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*(f +
g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

3.590 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\begin{aligned}\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx &= \frac{2(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}{g^5} \\ &\quad - \frac{4(ef-dg)(aeg^2+cf(2ef-dg))(f+gx)^{3/2}}{3g^5} \\ &\quad + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))(f+gx)^{5/2}}{5g^5} \\ &\quad - \frac{4ce(2ef-dg)(f+gx)^{7/2}}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}\end{aligned}$$

```
[Out] -4/3*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+f)^(3/2)/g^5+2/5*(a*e^2*g^2
+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(5/2)/g^5-4/7*c*e*(-d*g+2*e*f)*(g
*x+f)^(7/2)/g^5+2/9*c*e^2*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g
*x+f)^(1/2)/g^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used

$= \{912, 1167\}$

$$\int \frac{(d+ex)^2\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2(f+gx)^{5/2}\left(ae^2g^2+c(d^2g^2-6defg+6e^2f^2)\right)}{5g^5} + \frac{2\sqrt{f+gx}(ag^2+cf^2)\,(ef-dg)^2}{g^5} - \frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{3g^5} - \frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

```
[In] Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int\left(\frac{(-ef+dg)^2(cf^2+ag^2)}{g^4}+\frac{2(ef-dg)(-aeg^2-cf(2ef-dg))x^2}{g^4}+\frac{(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))x^4}{g^4}-\frac{2ce(2ef-dg}{g^4}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^2\,(cf^2+ag^2)\,\sqrt{f+gx}}{g^5} - \frac{4(ef-dg)\,(aeg^2+cf(2ef-dg))\,(f+gx)^{3/2}}{3g^5} + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))\,(f+gx)^{5/2}}{5g^5} - \frac{4ce(2ef-dg)(f+gx)^{7/2}}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(21ag^2(15d^2g^2+10deg(-2f+gx)+e^2(8f^2-4fgx+3g^2x^2))+c(21d^2g^2(8f^2-4fgx+3g^2x^2}{315}$$

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(21*a*g^2*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(8*f^2
 - 4*f*g*x + 3*g^2*x^2)) + c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18
*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 - 64*
f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4))))/(315*g^5)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{x^2\left(\frac{5cx^2}{9}+a\right)e^2}{5}+\frac{2xd\left(\frac{3cx^2}{7}+a\right)e}{3}+d^2\left(\frac{cx^2}{5}+a\right)\right)g^4-\frac{4\left(\left(\frac{2}{21}cx^3+\frac{1}{5}ax\right)e^2+d\left(\frac{9cx^2}{35}+a\right)e+\frac{cd^2x}{5}\right)fg^3}{3}+\frac{8f^2}{}\right.\right.}{g^5}$
derivativedivides	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2ce^2f\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4(dg-ef)ecf+e^2\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+2(c\right.}{}}{g^5}$
default	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2ce^2f\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4(dg-ef)ecf+e^2\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+2(c\right.}{}}{g^5}$
gosper	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+63cd^2g^4x^2-108cdefg^3x^2+48ce^2f^2g^2x^2+210adeg^4x-}{}$
trager	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+63cd^2g^4x^2-108cdefg^3x^2+48ce^2f^2g^2x^2+210adeg^4x-}{}$
risch	$\frac{2\sqrt{gx+f}\,(35ce^2x^4g^4+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+63cd^2g^4x^2-108cdefg^3x^2+48ce^2f^2g^2x^2+210adeg^4x-}{}$

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 2*(g*x+f)^(1/2)*((1/5*x^2*(5/9*c*x^2+a)*e^2+2/3*x*d*(3/7*c*x^2+a)*e+d^2*(1/5*c*x^2+a))*g^4-4/3*((2/21*c*x^3+1/5*a*x)*e^2+d*(9/35*c*x^2+a)*e+1/5*c*d^2*x)*f*g^3+8/15*f^2*((2/7*c*x^2+a)*e^2+6/7*c*d*e*x+c*d^2)*g^2-32/35*e*c*f^3*(2/9*e*x+d)*g+128/315*c*e^2*f^4)/g^5

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\,(35\,ce^2g^4x^4+128\,ce^2f^4-288\,cdef^3g-420\,adefg^3+315\,ad^2g^4+168\,(cd^2+ae^2)f^2g^2-10\,(4\,ce^2fg^3-9$$

[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^3 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*d*e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g^4)*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2 + a*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{9}{2}}}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2cdeg-4ce^2f\right)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(2adeg^3-2ae^2fg^2-2cd^2fg^2+6cdef^2g-4ce^2f^3\right)}{3g^4}+\frac{\sqrt{f-}}{}\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{\sqrt{f}} \end{cases}$$

[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)

[Out] Piecewise((2*(c*e**2*(f + g*x)**(9/2)/(9*g**4) + (f + g*x)**(7/2)*(2*c*d*e*g - 4*c*e**2*f)/(7*g**4) + (f + g*x)**(5/2)*(a*e**2*g**2 + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**4) + (f + g*x)**(3/2)*(2*a*d*e*g**3 - 2*a*e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/(3*g**4) + sqrt(f + g*x)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g**2 + c*d**2*f**2*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/g**4)/g, Ne(g, 0)), ((a*d**2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)/sqrt(f), True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2 - 90\,(2\,ce^2f - cdeg)(gx+f)^{\frac{7}{2}} + 63\,(6\,ce^2f^2 - 6\,cdefg + (cd^2+ae^2)g^2)(gx+f)^{\frac{5}{2}} - 2\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
+ 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) - 21
0*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x +
f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 + (c*
d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,\sqrt{gx+f}ad^2 + \frac{210\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ade}{g} + \frac{21\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}f^2\right)cd^2}{g^2} + \frac{21\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(g\right.}{}\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*
x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{5/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{5\,g^5} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^2}{g^5} + \frac{4\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2)}{3\,g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{9/2}}{9\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{7/2}\,(d\,g-2\,e\,f)}{7\,g^5}$$

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f +
g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e^2*
(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5)
```

3.591 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = -\frac{2(ef-dg)(cf^2+ag^2)\sqrt{f+gx}}{g^4} + \frac{2(aeg^2+cf(3ef-2dg))(f+gx)^{3/2}}{3g^4} - \frac{2c(3ef-dg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[Out] 2/3*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(3/2)/g^4-2/5*c*(-d*g+3*e*f)*(g*x+
f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(
1/2)/g^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {786}

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)}{g^4} + \frac{2(f+gx)^{3/2}(aeg^2+cf(3ef-2dg))}{3g^4} - \frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[In] Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}} + \frac{\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^3} \right. \\ &\qquad \left. + \frac{c(-3ef+dg)(f+gx)^{3/2}}{g^3} + \frac{ce(f+gx)^{5/2}}{g^3}\right) dx \\ &= -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4} + \frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4} \\ &\quad - \frac{2c(3ef-dg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\left(35ag^2(-2ef+3dg+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)}{105g^4}$$

```
[In] Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f*
g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))/(
105*g^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{x^2\left(\frac{5ex}{7}+d\right)c}{5}+a\left(\frac{ex}{3}+d\right)\right)g^3-\frac{2f\left(\frac{2\left(\frac{9ex}{14}+d\right)xc}{5}+ae\right)g^2}{3}+\frac{8c\left(\frac{3ex}{7}+d\right)f^2g}{15}-\frac{16cef^3}{35}\right)}{g^4}$
gosper	$\frac{2\sqrt{gx+f}\,(15ce x^3g^3+21cd g^3x^2-18cef g^2x^2+35ae g^3x-28cdf g^2x+24ce f^2gx+105ad g^3-70aef g^2+56cd f^2g-48ce f^3}{105g^4}$
trager	$\frac{2\sqrt{gx+f}\,(15ce x^3g^3+21cd g^3x^2-18cef g^2x^2+35ae g^3x-28cdf g^2x+24ce f^2gx+105ad g^3-70aef g^2+56cd f^2g-48ce f^3}{105g^4}$
risch	$\frac{2\sqrt{gx+f}\,(15ce x^3g^3+21cd g^3x^2-18cef g^2x^2+35ae g^3x-28cdf g^2x+24ce f^2gx+105ad g^3-70aef g^2+56cd f^2g-48ce f^3}{105g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(ag^2+cf^2\right)\sqrt{gx+f}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(ag^2+cf^2\right)\sqrt{gx+f}}{g^4}$

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*((1/5*x^2*(5/7*e*x+d)*c+a*(1/3*e*x+d))*g^3-2/3*f*(2/5*(9/14
*e*x+d)*x*c+a*e)*g^2+8/15*c*(3/7*e*x+d)*f^2*g-16/35*c*e*f^3)/g^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(15\,ceg^3x^3-48\,cef^3+56\,cdf^2g-70\,aefg^2+105\,adg^3-3\left(6\,cefg^2-7\,cdg^3\right)x^2+\left(24\,cef^2g-28\,cdfg^2\right.\right.}{105\,g^4}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d*
g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35*a
*e*g^3)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{7}{2}}}{7g^3}+\frac{(f+gx)^{\frac{5}{2}}(cdg-3cef)}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}\left(aeg^2-2cdfg+3cef^2\right)}{3g^3}+\frac{\sqrt{f+gx}\left(adg^3-aefg^2+cdf^2g-cef^3\right)}{g^3}\right)}{g} & \text{for } g\neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*e*(f + g*x)**(7/2)/(7*g**3) + (f + g*x)**(5/2)*(c*d*g - 3*c
*e*f)/(5*g**3) + (f + g*x)**(3/2)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/(3*g*
*3) + sqrt(f + g*x)*(a*d*g**3 - a*e*f*g**2 + c*d*f**2*g - c*e*f**3)/g**3)/g
, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce-21\,(3\,cef-cdg)(gx+f)^{\frac{5}{2}}+35\,(3\,cef^2-2\,cdfg+aeg^2)(gx+f)^{\frac{3}{2}}-105\,(cef^3-cdf^2}{105\,g^4}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*(
3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\left(105\sqrt{gx+f}ad + \frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ae}{g} + \frac{7\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd}{g^2} + \frac{3\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}\right.}{}\right.}{105g}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e
/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*
d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{3/2}\,(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2)}{3\,g^4} + \frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4} + \frac{2\,c\,(f+g\,x)^{5/2}\,(d\,g-3\,e\,f)}{5\,g^4} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)}{g^4}$$

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f +
 g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(f
+ g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

3.592 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx=\frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] -4/3*c*f*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2+c*f^2)*(g*x+f)^
(1/2)/g^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {711}

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(ag^2+cf^2)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

```
[In] Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{cf^2+ag^2}{g^2\sqrt{f+gx}} - \frac{2cf\sqrt{f+gx}}{g^2} + \frac{c(f+gx)^{3/2}}{g^2}\right) dx \\ &= \frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3} - \frac{4cf(f+gx)^{3/2}}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}} dx = \frac{2\sqrt{f+gx}(15ag^2+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

```
[In] Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
[Out] (2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{2\left(3\left(c\,x^{2}+5a\right)g^{2}-4cfxg+8c\,f^{2}\right)\sqrt{gx+f}}{15g^{3}}$	40
gosper	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
trager	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
risch	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52

```
[In] int((c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(3*(c*x^2+5*a)*g^2-4*c*f*x*g+8*c*f^2)*(g*x+f)^(1/2)/g^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\,(3\,cg^2x^2-4\,cfgx+8\,cf^2+15\,ag^2)\sqrt{gx+f}}{15\,g^3}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2a\sqrt{f+gx}+\frac{2c\left(f^2\sqrt{f+gx}-\frac{2f(f+gx)^{\frac{3}{2}}}{3}+\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Piecewise(((2*a*sqrt(f + g*x) + 2*c*(f**2*sqrt(f + g*x) - 2*f*(f + g*x)**(3
/2)/3 + (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15\,g}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15\,g}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+g\,x}\,\left(3\,c\,(f+g\,x)^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,(f+g\,x)\right)}{15\,g^3}$$

```
[In] int((a + c*x^2)/(f + g*x)^(1/2),x)
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*x
)))/(15*g^3)
```

3.593 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=-\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)-2*c*(d*g+e*f)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {912, 1167, 214}

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=-\frac{2(ae^2+cd^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[In] Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=\frac{2c\sqrt{f+gx}(-2ef-3dg+egx)}{3e^2g^2}+\frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) + (2*(c*d^2 + a*e^
2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*Sqrt[-(e*f)
 + d*g])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{2c(-egx+3dg+2ef)\sqrt{gx+f}}{3g^2e^2}+\frac{2(e^2a+c\,d^2)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}$	82
pseudoelliptic	$\frac{-\frac{2c\sqrt{gx+f}\,(-egx+3dg+2ef)}{3}+\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{g^2e^2}$	83
derivativedivides	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	96
default	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	96

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*c*(-e*g*x+3*d*g+2*e*f)*(g*x+f)^(1/2)/g^2/e^2+2*(a*e^2+c*d^2)/e^2/((d*g
-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.86

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$=\left[\frac{3\,(cd^2+ae^2)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\,(2\,ce^3f^2+cde^2fg-3\,cd^2eg^2-(ce^3fg\right.}{3\,(e^4fg^2-de^3g^3)}$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^2*
f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^
2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt
(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2*f*g
- 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 -
d*e^3*g^3)]
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx=\begin{cases}\frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3eg}+\frac{\sqrt{f+gx}(-cdg-cef)}{e^2g}+\frac{g\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^3\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g\neq 0\\ \frac{-\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{\left(ae^2+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{\sqrt{f}} & \text{otherwise}\end{cases}$$

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*(f + g*x)**(3/2)/(3*e*g) + sqrt(f + g*x)*(-c*d*g - c*e*f)/(
e**2*g) + g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**3
*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x**2/(2*e) + (a*e**2
 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2)/sqrt(f)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^2} + \frac{2\left((gx+f)^{\frac{3}{2}}ce^2g^4-3\sqrt{gx+f}ce^2fg^4-3\sqrt{gx+f}cdeg^5\right)}{3\,e^3g^6}$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f
 + d*e*g)*e^2) + 2/3*((g*x + f)^(3/2)*c*e^2*g^4 - 3*sqrt(g*x + f)*c*e^2*f*g
^4 - 3*sqrt(g*x + f)*c*d*e*g^5)/(e^3*g^6)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}} - \sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right) + \frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(5
/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^4)
 + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.594 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{(ae^2g+cd(4ef-3dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

```
[Out] (a*e^2*g+c*d*(-3*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+c*d^2/e^2)*(g*x+f)^(1
/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {912, 1171, 396, 214}

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{(ae^2g+cd(4ef-3dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}} - \frac{\sqrt{f+gx}(ae^2+cd^2)}{e^2(d+ex)(ef-dg)} + \frac{2c\sqrt{f+gx}}{e^2g}$$

```
[In] Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((c*d^2 + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d
*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cf^2}{g^2}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} - \frac{\left(a+\frac{cd(4ef-3dg)}{e^2g}\right)\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}(-ae^2g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)} + \frac{(ae^2g+cd(4ef-3dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(-(a*e^2*g) + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x))))/(
e^2*g*(e*f - d*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTan[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

method	result
risch	$\frac{2c\sqrt{gx+f}}{e^2 g}-\frac{-\frac{g\left(e^2 a+c\,d^2\right)\sqrt{gx+f}}{(dg-ef)(e(gx+f)+dg-ef)}-\frac{\left(a\,e^2 g-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}}{e^2}$
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2 a+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2 g-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2 a+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2 g-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
pseudoelliptic	$\frac{g(ex+d)\left(a\,e^2 g-3c\,d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{(dg-ef)e}\,\sqrt{gx+f}\,\left((-2cfx+ag)e^2-2cd(-gx+f)e+3c\,d^2 g\right)}{\sqrt{(dg-ef)e}\,g\,e^2(dg-ef)(ex+d)}$

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g-1/e^2*(-g*(a*e^2+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2)/(e*
(g*x+f)+d*g-e*f)-(a*e^2*g-3*c*d^2*g+4*c*d*e*f)/(d*g-e*f)/((d*g-e*f)*e)^(1/2
)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.42

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \left[-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{}}{ex+d}\right)}{2\,(de^5f^2g-2\,d^2e^4fg^2+d^3e^3g^3}\right.$$
$$-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)}{de^5f^2g-2\,d^2e^4fg^2+d^3e^3g^3+(e^6}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2
*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e
^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 +
 a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*
d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3
 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (3*c*d^
3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt(-e^2*f
 + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d
*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4
*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^
2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

Sympy [F]

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{(4\,cdef-3\,cd^2g+ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f-de^2g)\sqrt{-e^2f+deg}} - \frac{\sqrt{gx+f}cd^2g+\sqrt{gx+f}ae^2g}{(e^3f-de^2g)((gx+f)e-ef+dg)} + \frac{2\sqrt{gx+f}c}{e^2g}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -(4*c*d*e*f - 3*c*d^2*g + a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e
*g))/((e^3*f - d*e^2*g)*sqrt(-e^2*f + d*e*g)) - (sqrt(g*x + f)*c*d^2*g + sq
rt(g*x + f)*a*e^2*g)/((e^3*f - d*e^2*g)*((g*x + f)*e - e*f + d*g)) + 2*sqrt
(g*x + f)*c/(e^2*g)
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)} + \frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g + 4
*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^2*
g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/2))
/(e^2*g)
```

3.595 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

```
[Out] -1/4*(3*a*e^2*g^2+c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+c*d^2/e^2)*(g*x+f
)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*(-5*d*g+8*e*f))*(g*x+f)^(1/
2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {912, 1171, 393, 214}

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{(3ae^2g^2+c(3d^2g^2-8defg+8e^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}} - \frac{\sqrt{f+gx}(ae^2+cd^2)}{2e^2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}(3ae^2g+cd(8ef-5dg))}{4e^2(d+ex)(ef-dg)^2}$$

```
[In] Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*((c*d^2 + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d*g)*(d + e*x)^2) + ((3*a*
e^2*g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x))
```

```
 - ((3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*
Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cf^2}{g^2}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$
\begin{aligned}
&= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} \\
&\quad + \frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\,\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{4e^2g(ef-dg)^2} \\
&= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} \\
&\quad - \frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$
\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\frac{\sqrt{e}\sqrt{f+gx}(ae^2(-2ef+5dg+3egx)+cd(-3d^2g+8e^2fx+de(6f-5gx)))}{(ef-dg)^2(d+ex)^2} + \frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{5/2}}}{4e^{5/2}}
$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((Sqrt[e]*Sqrt[f + g*x]*(a*e^2*(-2*e*f + 5*d*g + 3*e*g*x) + c*d*(-3*d^2*g +
 8*e^2*f*x + d*e*(6*f - 5*g*x))))/((e*f - d*g)^2*(d + e*x)^2) + ((3*a*e^2*g
^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/
Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{3\left(\left(a\,g^2+\frac{8c\,f^2}{3}\right)e^2-\frac{8cdefg}{3}+c\,d^2g^2\right)(ex+d)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{4}+\frac{5\left(-\frac{2a\left(-\frac{3gx}{2}+f\right)e^3}{5}+d\left(\frac{8cfx}{5}+ag\right)e^2+\frac{6c\left(-\frac{5gx}{6}+f\right)d^2e}{5}-\right.}{4}}{\sqrt{(dg-ef)e}\,(dg-ef)^2(ex+d)^2e^2}$
derivativedivides	$\frac{\frac{g\left(3a\,e^2g-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+3c\,d^2g^2-8cdefg+8c\,e^2f^2)\arctan\left(\frac{e\sqrt{gx}}{\sqrt{(dg-}}\right.}{4(d^2g^2-2defg+e^2f^2)e^2\sqrt{(dg-ef)e}}$
default	$\frac{\frac{g\left(3a\,e^2g-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+3c\,d^2g^2-8cdefg+8c\,e^2f^2)\arctan\left(\frac{e\sqrt{gx}}{\sqrt{(dg-}}\right.}{4(d^2g^2-2defg+e^2f^2)e^2\sqrt{(dg-ef)e}}$

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/4/((d*g-e*f)*e)^(1/2)/(d*g-e*f)^2*(3/5*((a*g^2+8/3*c*f^2)*e^2-8/3*c*d*e*f
*g+c*d^2*g^2)*(e*x+d)^2*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+(-2/5*a
*(-3/2*g*x+f)*e^3+d*(8/5*c*f*x+a*g)*e^2+6/5*c*(-5/6*g*x+f)*d^2*e-3/5*c*d^3*
g)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2))/(e*x+d)^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(158) = 316$.

Time = 0.30 (sec) , antiderivative size = 896, normalized size of antiderivative = 5.03

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(8\,cd^2e^2f^2 - 8\,cd^3efg + 3\,(cd^4 + ad^2e^2)g^2 + (8\,ce^4f^2 - 8\,cde^3fg + 3\,(cd^2e^2 + ae^4)g^2)x^2 + 2\,(8\,cde^3f^2 -}{8\,(d^2e^6f^3 -}\right.$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*e
^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2
- 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((
e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(
2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e -
5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3
*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*
g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*
c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(-e^2*f +
 d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*(3*c*
d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e - 5*a*d^2
*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3*e^2 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.60

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{(8\,ce^2f^2-8\,cdefg+3\,cd^2g^2+3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^2-2\,de^3fg+d^2e^2g^2)\sqrt{-e^2f+deg}} + \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\,\sqrt{gx+f}cde^2f^2g-5\,(gx+f)^{\frac{3}{2}}cd^2eg^2+3\,(gx+f)^{\frac{3}{2}}ae^3g^2+11\,\sqrt{gx+f}cd^2efg^2}{4\,(e^4f^2-2\,de^3fg+d^2e^2g^2)((gx+f)e-ef+dg}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c*e^2*f^2 - 8*c*d*e*f*g + 3*c*d^2*g^2 + 3*a*e^2*g^2)*arctan(sqrt(g*x
 + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)*sqrt(-
e^2*f + d*e*g)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 8*sqrt(g*x + f)*c*d*
e^2*f^2*g - 5*(g*x + f)^(3/2)*c*d^2*e*g^2 + 3*(g*x + f)^(3/2)*a*e^3*g^2 + 1
1*sqrt(g*x + f)*c*d^2*e*f*g^2 - 5*sqrt(g*x + f)*a*e^3*f*g^2 - 3*sqrt(g*x +
f)*c*d^3*g^3 + 5*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f^2 - 2*d*e^3*f*g + d^2*e
^2*g^2)*((g*x + f)*e - e*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 12.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$=\frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*(d
*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 +
e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(
3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g - e
*f)^(5/2))
```

3.596 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3(cf^2+ag^2)}{g^6\sqrt{f+gx}} \\ &+ \frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))\sqrt{f+gx}}{g^6} \\ &- \frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{3/2}}{3g^6} \\ &+ \frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{5/2}}{5g^6} \\ &- \frac{2ce^2(5ef-3dg)(f+gx)^{7/2}}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}\end{aligned}$$

```
[Out] -2/3*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3/2
)/g^6+2/5*e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g
^6-2/7*c*e^2*(-3*d*g+5*e*f)*(g*x+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2
*(-d*g+e*f)^3*(a*g^2+c*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(3*a*e*g^2+c*f
*(-2*d*g+5*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {912, 1275}

$$\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2e(f+gx)^{5/2}(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{5g^6} - \frac{2(f+gx)^{3/2}(ef-dg)(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{3g^6} + \frac{2(ag^2+cf^2)(ef-dg)^3}{g^6\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{g^6} - \frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

```
[In] Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{g^5} + \frac{(-ef+dg)^3(cf^2+ag^2)}{g^5x^2} + \frac{(ef-dg)(-3ae^2g^2-c(10e^2f^2-8defg+d^2g^2))x^2}{g^5} + \frac{e}{}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^3(cf^2+ag^2)}{g^6\sqrt{f+gx}} + \frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))\sqrt{f+gx}}{g^6}$$
$$- \frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{3/2}}{3g^6}$$
$$+ \frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{5/2}}{5g^6}$$
$$- \frac{2ce^2(5ef-3dg)(f+gx)^{7/2}}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(63ag^2(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g(-8f^2-4fgx+g^2x^2)+e^3(16f^3+}{}$$

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(63*a*g^2*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4
*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + c*(
105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8*f^2*g*
x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2
*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2
*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5))))/(315*g^6*Sqrt[f + g*x]
)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{x^{3}\left(\frac{5c\,x^{2}}{9}+a\right)e^{3}}{5}-x^{2}d\left(\frac{3c\,x^{2}}{7}+a\right)e^{2}-3\left(\frac{c\,x^{2}}{5}+a\right)x\,d^{2}e+d^{3}\left(-\frac{c\,x^{2}}{3}+a\right)\right)g^{5}-6f\left(-\frac{\left(\frac{25c\,x^{2}}{63}+a\right)x^{2}e^{3}}{15}-\frac{2x\left(\frac{6c\,x^{2}}{35}\right.}{3}\right.\right.\right.}{}$
risch	$\frac{2\left(35c\,e^{3}x^{4}g^{4}+135cd\,e^{2}g^{4}x^{3}-85c\,e^{3}f\,g^{3}x^{3}+63a\,e^{3}g^{4}x^{2}+189c\,d^{2}e\,g^{4}x^{2}-351cd\,e^{2}f\,g^{3}x^{2}+165c\,e^{3}f^{2}g^{2}x^{2}+315ad\,e^{2}g^{4}x\right.}{}$
gosper	$-\frac{2\left(-35c\,e^{3}x^{5}g^{5}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+216cd\,e^{2}f\,g^{4}x^{3}-80c\,e^{3}f^{2}g^{3}x^{3}-315ad\,e^{2}g\right.}{}$
trager	$-\frac{2\left(-35c\,e^{3}x^{5}g^{5}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+216cd\,e^{2}f\,g^{4}x^{3}-80c\,e^{3}f^{2}g^{3}x^{3}-315ad\,e^{2}g\right.}{}$
derivativedivides	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}+\frac{6cd\,e^{2}g(gx+f)^{\frac{7}{2}}}{7}-\frac{10c\,e^{3}f(gx+f)^{\frac{7}{2}}}{7}+\frac{2a\,e^{3}g^{2}(gx+f)^{\frac{5}{2}}}{5}+\frac{6c\,d^{2}e\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}+4c\,e^{3}f^{2}(gx+}{}$
default	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}+\frac{6cd\,e^{2}g(gx+f)^{\frac{7}{2}}}{7}-\frac{10c\,e^{3}f(gx+f)^{\frac{7}{2}}}{7}+\frac{2a\,e^{3}g^{2}(gx+f)^{\frac{5}{2}}}{5}+\frac{6c\,d^{2}e\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}+4c\,e^{3}f^{2}(gx+}{}$

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((-1/5*x^3*(5/9*c*x^2+a)*e^3-x^2*d*(3/7*c*x^2+a)*e^2-3*(1/5*c*x^2+a)*x*d
^2*e+d^3*(-1/3*c*x^2+a))*g^5-6*f*(-1/15*(25/63*c*x^2+a)*x^2*e^3-2/3*x*(6/35
*c*x^2+a)*d*e^2+d^2*(-1/5*c*x^2+a)*e-2/9*c*d^3*x)*g^4+8*f^2*((-2/63*c*x^3-1
/5*a*x)*e^3+d*(-6/35*c*x^2+a)*e^2-3/5*c*d^2*e*x+1/3*c*d^3)*g^3-16/5*e*((-10
/63*c*x^2+a)*e^2-12/7*c*d*e*x+3*c*d^2)*f^3*g^2+384/35*e^2*c*(-5/27*e*x+d)*f
^4*g-256/63*c*e^3*f^5)/(g*x+f)^(1/2)/g^6
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(35\,ce^3g^5x^5+1280\,ce^3f^5-3456\,cde^2f^4g+1890\,ad^2efg^4-315\,ad^3g^5+1008\,(\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2*
e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 + 3
*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*f^2
*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3*f^3
*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^3 + 3
*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*d^2*e*
g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^4)*x)*s
qrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{9}{2}}}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(3ade^2g^3\right.}{}\right.}{} \\ \frac{ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e**3*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(3*c*d*e*
*2*g - 5*c*e**3*f)/(7*g**5) + (f + g*x)**(5/2)*(a*e**3*g**2 + 3*c*d**2*e*g*
*2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d*e
**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d*e**2*
f**2*g - 10*c*e**3*f**3)/(3*g**5) + sqrt(f + g*x)*(3*a*d**2*e*g**4 - 6*a*d*
e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*f**2*g**2 -
 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/g**5 - (a*g**2 + c*f**2)*(d*g - e*f)**
3/(g**5*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*
d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*
e**2 + c*d**3)/3)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\,(5\,ce^3f-3\,cde^2g)(gx+f)^{\frac{7}{2}}+63\,(10\,ce^3f^2-12\,cde^2fg+(3\,cd^2e+ae^3)g^2)(gx+f)^{\frac{5}{2}}-105}{}\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^(
7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x +
f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*f*g
^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c*d*e
^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d
*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a*d^
2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2)*f
^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(218) = 436$.

Time = 0.28 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^3 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(ce^3f^5 - 3\,cde^2f^4g + 3\,cd^2ef^3g^2 + ae^3f^3g^2 - cd^3f^2g^3 - 3\,ade^2f^2g^3 + 3\,ad^2ef}{\sqrt{gx+f}g^6}$$
$$+ \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^3g^{48} - 225\,(gx+f)^{\frac{7}{2}}ce^3fg^{48} + 630\,(gx+f)^{\frac{5}{2}}ce^3f^2g^{48} - 1050\,(gx+f)^{\frac{3}{2}}ce^3f^3g^{48} + 1575\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*c*d^2*e*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3*
f^2*g^3 - 3*a*d*e^2*f^2*g^3 + 3*a*d^2*e*f*g^4 - a*d^3*g^5)/(sqrt(g*x + f)*g
^6) + 2/315*(35*(g*x + f)^(9/2)*c*e^3*g^48 - 225*(g*x + f)^(7/2)*c*e^3*f*g^
48 + 630*(g*x + f)^(5/2)*c*e^3*f^2*g^48 - 1050*(g*x + f)^(3/2)*c*e^3*f^3*g^
48 + 1575*sqrt(g*x + f)*c*e^3*f^4*g^48 + 135*(g*x + f)^(7/2)*c*d*e^2*g^49 -
 756*(g*x + f)^(5/2)*c*d*e^2*f*g^49 + 1890*(g*x + f)^(3/2)*c*d*e^2*f^2*g^49
 - 3780*sqrt(g*x + f)*c*d*e^2*f^3*g^49 + 189*(g*x + f)^(5/2)*c*d^2*e*g^50 +
 63*(g*x + f)^(5/2)*a*e^3*g^50 - 945*(g*x + f)^(3/2)*c*d^2*e*f*g^50 - 315*(
g*x + f)^(3/2)*a*e^3*f*g^50 + 2835*sqrt(g*x + f)*c*d^2*e*f^2*g^50 + 945*sqr
t(g*x + f)*a*e^3*f^2*g^50 + 105*(g*x + f)^(3/2)*c*d^3*g^51 + 315*(g*x + f)^
(3/2)*a*d*e^2*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 - 1890*sqrt(g*x + f)*a*
d*e^2*f*g^51 + 945*sqrt(g*x + f)*a*d^2*e*g^52)/g^54
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2 - 24\,c\,d\,e^2\,f\,g + 20\,c\,e^3\,f^2 + 2\,a\,e^3\,g^2)}{5\,g^6}$$
$$- \frac{2\,c\,d^3\,f^2\,g^3 + 2\,a\,d^3\,g^5 - 6\,c\,d^2\,e\,f^3\,g^2 - 6\,a\,d^2\,e\,f\,g^4 + 6\,c\,d\,e^2\,f^4\,g + 6\,a\,d\,e^2\,f^2\,g^3 - 2\,c\,e^3\,f^5 - 2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}$$
$$+ \frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2 - 2\,c\,d\,f\,g + 3\,a\,e\,g^2)}{g^6}$$
$$+ \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,\left(c\,d^2\,g^2 - 8\,c\,d\,e\,f\,g + 10\,c\,e^2\,f^2 + 3\,a\,e^2\,g^2\right)}{3\,g^6}$$
$$+ \frac{2\,c\,e^2\,(f+g\,x)^{7/2}\,(3\,d\,g - 5\,e\,f)}{7\,g^6}$$

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g
^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f^3*
g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f + g*
x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(f + g
*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g
))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

3.597 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2(ef-dg)^2\,(cf^2+ag^2)}{g^5\sqrt{f+gx}} - \frac{4(ef-dg)\,(aeg^2+cf(2ef-dg))\,\sqrt{f+gx}}{g^5} + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))\,(f+gx)^{3/2}}{3g^5} - \frac{4ce(2ef-dg)(f+gx)^{5/2}}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

```
[Out] 2/3*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3/2)/g^5-4/5*c*e*(
-d*g+2*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a
*g^2+c*f^2)/g^5/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+
f)^(1/2)/g^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {912, 1275}

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(f+gx)^{3/2}\,(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(ag^2+cf^2)\,(ef-dg)^2}{g^5\sqrt{f+gx}} - \frac{4\sqrt{f+gx}(ef-dg)\,(aeg^2+cf(2ef-dg))}{g^5} - \frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

```
[In] Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{2(ef-dg)(-aeg^2-cf(2ef-dg))}{g^4}+\frac{(-ef+dg)^2(cf^2+ag^2)}{g^4x^2}+\frac{(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))x^2}{g^4}-\frac{2ce(2ef-dg)x}{g^4}\right.\right.}{g}$$

$$= -\frac{2(ef-dg)^2(cf^2+ag^2)}{g^5\sqrt{f+gx}}-\frac{4(ef-dg)(aeg^2+cf(2ef-dg))\sqrt{f+gx}}{g^5}+\frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))(f+gx)^{3/2}}{3g^5}-\frac{4ce(2ef-dg)(f+gx)^{5/2}}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{-70ag^2(3d^2g^2-6deg(2f+gx)+e^2(8f^2+4fgx-g^2x^2))+2c(35d^2g^2(-8f^2-\cdots}{\cdots}$$

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-70*a*g^2*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x - g^2*x^
2)) + 2*c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f
^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*
x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{x^{2}\left(\frac{3c\,x^{2}}{7}+a\right)e^{2}}{3}-2\left(\frac{c\,x^{2}}{5}+a\right)xde+d^{2}\left(-\frac{c\,x^{2}}{3}+a\right)\right)g^{4}-4\left(-\frac{x\left(\frac{6c\,x^{2}}{35}+a\right)e^{2}}{3}+d\left(-\frac{c\,x^{2}}{5}+a\right)e-\frac{c\,d^{2}x}{3}\right)f\,g^{3}+\frac{8f^{2}\cdots}{\cdots}\cdots}{\sqrt{gx+f}\,g^{5}}$
risch	$\frac{2(15c\,e^{2}x^{3}g^{3}+42cde\,g^{3}x^{2}-39c\,e^{2}f\,g^{2}x^{2}+35a\,e^{2}g^{3}x+35c\,d^{2}g^{3}x-126cdef\,g^{2}x+87c\,e^{2}f^{2}gx+210ade\,g^{3}-175a\,e^{2}f\,g^{2}-1\cdots}{105g^{5}}$
gosper	$-\frac{2(-15c\,e^{2}x^{4}g^{4}-42cde\,g^{4}x^{3}+24c\,e^{2}f\,g^{3}x^{3}-35a\,e^{2}g^{4}x^{2}-35c\,d^{2}g^{4}x^{2}+84cdef\,g^{3}x^{2}-48c\,e^{2}f^{2}g^{2}x^{2}-210ade\,g^{4}x+140a\cdots}{\cdots}$
trager	$-\frac{2(-15c\,e^{2}x^{4}g^{4}-42cde\,g^{4}x^{3}+24c\,e^{2}f\,g^{3}x^{3}-35a\,e^{2}g^{4}x^{2}-35c\,d^{2}g^{4}x^{2}+84cdef\,g^{3}x^{2}-48c\,e^{2}f^{2}g^{2}x^{2}-210ade\,g^{4}x+140a\cdots}{\cdots}$
derivativedivides	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^{2}f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}+\frac{2c\,d^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c\,e^{2}f^{2}(gx+f)^{\frac{3}{2}}-\cdots}{\cdots}$
default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^{2}f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}+\frac{2c\,d^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c\,e^{2}f^{2}(gx+f)^{\frac{3}{2}}-\cdots}{\cdots}$

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(g*x+f)^(1/2)*((-1/3*x^2*(3/7*c*x^2+a)*e^2-2*(1/5*c*x^2+a)*x*d*e+d^2*(-1
/3*c*x^2+a))*g^4-4*(-1/3*x*(6/35*c*x^2+a)*e^2+d*(-1/5*c*x^2+a)*e-1/3*c*d^2*
x)*f*g^3+8/3*f^2*((-6/35*c*x^2+a)*e^2-6/5*c*d*e*x+c*d^2)*g^2-32/5*e*(-2/7*e
*x+d)*c*f^3*g+128/35*c*e^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(15\,ce^2g^4x^4 - 384\,ce^2f^4 + 672\,cdef^3g + 420\,adefg^3 - 105\,ad^2g^4 - 280\,(cd^2 + a$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^3
 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d*e
*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)*x^
2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 + a*e
^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{(f+gx)^{\frac{5}{2}}\cdot\left(2cdeg-4ce^2f\right)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{3g^4} + \frac{\sqrt{f+gx}\left(2adeg^3-2ae^2fg^2\right.}{}\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e**2*(f + g*x)**(7/2)/(7*g**4) + (f + g*x)**(5/2)*(2*c*d*e*
g - 4*c*e**2*f)/(5*g**4) + (f + g*x)**(3/2)*(a*e**2*g**2 + c*d**2*g**2 - 6*
c*d*e*f*g + 6*c*e**2*f**2)/(3*g**4) + sqrt(f + g*x)*(2*a*d*e*g**3 - 2*a*e**
2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/g**4 - (a*g**2
 + c*f**2)*(d*g - e*f)**2/(g**4*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**2*x +
a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)/f**(3
/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15(gx+f)^{\frac{7}{2}}ce^2-42(2ce^2f-cdeg)(gx+f)^{\frac{5}{2}}+35(6ce^2f^2-6cdefg+(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}-210(2ce^2f\cdots}{g^4}\cdots\right.}{10\cdots}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2)
 + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) - 2
10*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqrt(g
*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2(ce^2f^4-2cdef^3g+cd^2f^2g^2+ae^2f^2g^2-2adefg^3+ad^2g^4)}{\sqrt{gx+f}\,g^5} + \frac{2\left(15(gx+f)^{\frac{7}{2}}ce^2g^{30}-84(gx+f)^{\frac{5}{2}}ce^2fg^{30}+210(gx+f)^{\frac{3}{2}}ce^2f^2g^{30}-420\sqrt{gx+f}ce^2f^3g^{30}+42(gx+\cdots\right.}{\cdots}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*e^2*f^4 - 2*c*d*e*f^3*g + c*d^2*f^2*g^2 + a*e^2*f^2*g^2 - 2*a*d*e*f*g
^3 + a*d^2*g^4)/(sqrt(g*x + f)*g^5) + 2/105*(15*(g*x + f)^(7/2)*c*e^2*g^30
- 84*(g*x + f)^(5/2)*c*e^2*f*g^30 + 210*(g*x + f)^(3/2)*c*e^2*f^2*g^30 - 42
0*sqrt(g*x + f)*c*e^2*f^3*g^30 + 42*(g*x + f)^(5/2)*c*d*e*g^31 - 210*(g*x +
 f)^(3/2)*c*d*e*f*g^31 + 630*sqrt(g*x + f)*c*d*e*f^2*g^31 + 35*(g*x + f)^(3
/2)*c*d^2*g^32 + 35*(g*x + f)^(3/2)*a*e^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*
g^32 - 210*sqrt(g*x + f)*a*e^2*f*g^32 + 210*sqrt(g*x + f)*a*d*e*g^33)/g^35
```

Mupad [B] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{3\,g^5} - \frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}} + \frac{4\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{5/2}\,(d\,g-2\,e\,f)}{5\,g^5}$$

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 -
 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/2)*
(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(7/2)
)/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

3.598 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}} dx = \frac{2(ef-dg)(cf^2+ag^2)}{g^4\sqrt{f+gx}} + \frac{2(aeg^2+cf(3ef-2dg))\sqrt{f+gx}}{g^4} - \frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[Out] -2/3*c*(-d*g+3*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(-d*g+e*f
)*(a*g^2+c*f^2)/g^4/(g*x+f)^(1/2)+2*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(1
/2)/g^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {786}

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}} dx = \frac{2(ag^2+cf^2)(ef-dg)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(aeg^2+cf(3ef-2dg))}{g^4} - \frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[In] Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3(f+gx)^{3/2}} + \frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}} + \frac{c(-3ef+dg)\sqrt{f+gx}}{g^3} + \frac{ce(f+gx)^{3/2}}{g^3} \right) dx$$

$$= \frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}} + \frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4} - \frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{30ag^2(2ef-dg+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(16f^3+8f^2gx-2fg\right.}{15g^4\sqrt{f+gx}}$$

```
[In] Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6
*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{((6e\,x^3+10d\,x^2)c-30a(-ex+d))g^3+60((-\frac{1}{5}e\,x^2-\frac{2}{3}dx)c+ae)f\,g^2-80c\,f^2(-\frac{3ex}{5}+d)g+96ce\,f^3}{15\sqrt{gx+f}\,g^4}$
gosper	$-\frac{2(-3ce\,x^3g^3-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x+20cdf\,g^2x-24ce\,f^2gx+15ad\,g^3-30aef\,g^2+40cd\,f^2g-48ce\,f^3)}{15\sqrt{gx+f}\,g^4}$
trager	$-\frac{2(-3ce\,x^3g^3-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x+20cdf\,g^2x-24ce\,f^2gx+15ad\,g^3-30aef\,g^2+40cd\,f^2g-48ce\,f^3)}{15\sqrt{gx+f}\,g^4}$
risch	$\frac{2(3ce\,x^2g^2+5cdx\,g^2-9cefgx+15ae\,g^2-25cdfg+33ce\,f^2)\sqrt{gx+f}}{15g^4}-\frac{2(ad\,g^3-aef\,g^2+cd\,f^2g-ce\,f^3)}{g^4\sqrt{gx+f}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae\,g^2\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6ce\,f^2\sqrt{gx+f}-\frac{2(ad\,g^3-aef\,g^2+cd\,f^2g-ce\,f^3)}{\sqrt{gx+f}}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae\,g^2\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6ce\,f^2\sqrt{gx+f}-\frac{2(ad\,g^3-aef\,g^2+cd\,f^2g-ce\,f^3)}{\sqrt{gx+f}}}{g^4}$

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(((6*e*x^3+10*d*x^2)*c-30*a*(-e*x+d))*g^3+60*((-1/5*e*x^2-2/3*d*x)*c+a
*e)*f*g^2-80*c*f^2*(-3/5*e*x+d)*g+96*c*e*f^3)/(g*x+f)^(1/2)/g^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(3\,ceg^3x^3+48\,cef^3-40\,cdf^2g+30\,aefg^2-15\,adg^3-(6\,cefg^2-5\,cdg^3)x^2+}{15\,(g^5x+fg^4)}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^3
 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*g^
3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{5}{2}}}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}(cdg-3cef)}{3g^3}+\frac{\sqrt{f+gx}\left(aeg^2-2cdfg+3cef^2\right)}{g^3}-\frac{\left(ag^2+cf^2\right)(dg-ef)}{g^3\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e*(f + g*x)**(5/2)/(5*g**3) + (f + g*x)**(3/2)*(c*d*g - 3*c
*e*f)/(3*g**3) + sqrt(f + g*x)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/g**3 - (
a*g**2 + c*f**2)*(d*g - e*f)/(g**3*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d*x +
a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\,(3\,cef-cdg)(gx+f)^{\frac{3}{2}}+15\,(3\,cef^2-2\,cdfg+aeg^2)\sqrt{gx+f}}{g^3}+\frac{15\,(cef^3-cdf^2g+aefg^2-adg^3}{\sqrt{gx+f}g^3}\right.}{15\,g}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3*
c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(cef^3-cdf^2g+aefg^2-adg^3)}{\sqrt{gx+f}g^4} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}ceg^{16}-15\,(gx+f)^{\frac{3}{2}}cefg^{16}+45\,\sqrt{gx+f}cef^2g^{16}+5\,(gx+f)^{\frac{3}{2}}cdg^{17}-30\,\sqrt{gx+f}cdfg^{17}+\right.}{15\,g^{20}}$$

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*e*f^3 - c*d*f^2*g + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^4) + 2/15*(3
*(g*x + f)^(5/2)*c*e*g^16 - 15*(g*x + f)^(3/2)*c*e*f*g^16 + 45*sqrt(g*x + f
)*c*e*f^2*g^16 + 5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 +
 15*sqrt(g*x + f)*a*e*g^18)/g^20
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{f+g\,x}\,(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2)}{g^4} - \frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\,\sqrt{f+g\,x}} + \frac{2\,c\,e\,(f+g\,x)^{5/2}}{5\,g^4} + \frac{2\,c\,(f+g\,x)^{3/2}\,(d\,g-3\,e\,f)}{3\,g^4}$$

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2*
c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f + g*
x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

3.599 $\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2+c*f^2)/g^3/(g*x+f)^(1/2)-4*c*f*(g*x+f)^(1/
2)/g^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {711}

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(ag^2+cf^2)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

```
[In] Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(\frac{cf^2+ag^2}{g^2(f+gx)^{3/2}}-\frac{2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\&=-\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int\frac{a+cx^2}{(f+gx)^{3/2}}\,dx=\frac{2(-3ag^2+c(-8f^2-4fgx+g^2x^2))}{3g^3\sqrt{f+gx}}$$

```
[In] Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
[Out] (2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^2-3a\right)g^2}{3}-\frac{8cfxg}{3}-\frac{16c\,f^2}{3}}{\sqrt{gx+f}\,g^3}$	39
gosper	$-\frac{2\left(-c\,x^2g^2+4cfxg+3a\,g^2+8c\,f^2\right)}{3\sqrt{gx+f}\,g^3}$	41
trager	$-\frac{2\left(-c\,x^2g^2+4cfxg+3a\,g^2+8c\,f^2\right)}{3\sqrt{gx+f}\,g^3}$	41
risch	$-\frac{2c(-gx+5f)\sqrt{gx+f}}{3g^3}-\frac{2\left(a\,g^2+c\,f^2\right)}{g^3\sqrt{gx+f}}$	46
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{3}{2}}}{3}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^2+c\,f^2\right)}{\sqrt{gx+f}}}{g^3}$	48
default	$\frac{\frac{2c(gx+f)^{\frac{3}{2}}}{3}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^2+c\,f^2\right)}{\sqrt{gx+f}}}{g^3}$	48

```
[In] int((c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*((c*x^2-3*a)*g^2-4*c*f*x*g-8*c*f^2)/(g*x+f)^(1/2)/g^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\,(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2)\sqrt{gx+f}}{3\,(g^4x+fg^3)}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g^
3)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{2cf\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{\frac{3}{2}}}{3g^2}-\frac{ag^2+cf^2}{g^2\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{cx^3}{3}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(-2*c*f*sqrt(f + g*x)/g**2 + c*(f + g*x)**(3/2)/(3*g**2) - (a*
g**2 + c*f**2)/(g**2*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*x + c*x**3/3)/f**(3/
2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-6\sqrt{gx+f}cf}{g^2}-\frac{3\,(cf^2+ag^2)}{\sqrt{gx+f}g^2}\right)}{3\,g}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sqr
t(g*x + f)*g^2))/g
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{2\,(cf^2+ag^2)}{\sqrt{gx+f}\,g^3} + \frac{2\left((gx+f)^{\frac{3}{2}}cg^6 - 6\sqrt{gx+f}\,cfg^6\right)}{3\,g^9}$$

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sqr
t(g*x + f)*c*f*g^6)/g^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{6\,a\,g^2 - 2\,c\,(f+g\,x)^2 + 6\,c\,f^2 + 12\,c\,f\,(f+g\,x)}{3\,g^3\,\sqrt{f+g\,x}}$$

```
[In] int((a + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] -(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)^
(1/2))
```

3.600 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-
d*g+e*f)^(3/2)+2*(a*g^2+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(g*x+f)^(1/
2)/e/g^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {912, 1275, 214}

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2(ae^2+cd^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2+cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

```
[In] Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{g}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)\,dx, x, \sqrt{f+gx}\right)}{g}\\
&= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx, x, \sqrt{f+gx}\right)}{e(ef-dg)}\\
&= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2(aeg^2-cdg(f+gx)+cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}} - \frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(a*e*g^2 - c*d*g*(f + g*x) + c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f) + d*g)*
Sqrt[f + g*x]) - (2*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e
*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
pseudoelliptic	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
risch	$\frac{2c\sqrt{gx+f}}{e\,g^2}-\frac{2\left(\frac{g^2\left(e^2a+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}+\frac{\left(a\,g^2+c\,f^2\right)e}{(dg-ef)\sqrt{gx+f}}\right)}{g^2e}$	116

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(c/e*(g*x+f)^(1/2)-g^2*(a*e^2+c*d^2)/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*
arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))-(a*g^2+c*f^2)/(d*g-e*f)/(g*x+f)
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(98) = 196$.

Time = 0.28 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.39

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \left[-\frac{((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\sqrt{g}}{ex+d}\right.}{e^4f^3g^2-2\,de^3f^2g}\right.$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log(
(e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*
(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c
*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2
- 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2*f + d
*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f
^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e^3*f^2*g
 - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*
f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{f+gx}}{eg}-\frac{ag^2+cf^2}{g\sqrt{f+gx}(dg-ef)}-\frac{g(ae^2+cd^2)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}(dg-ef)}\right)}{g} & \text{for } g\neq 0 \\ \frac{-\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{(ae^2+cd^2)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*sqrt(f + g*x)/(e*g) - (a*g**2 + c*f**2)/(g*sqrt(f + g*x)*(d
*g - e*f)) - g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e
**2*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x**2/
(2*e) + (a*e**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True)
)/e**2)/f**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^2f-deg)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2+ag^2)}{(efg^2-dg^3)\sqrt{gx+f}} + \frac{2\sqrt{gx+f}c}{eg^2}$$

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^2*f - d*
e*g)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e*f*g^2 - d*g^3)*sqrt(g*x
+ f)) + 2*sqrt(g*x + f)*c/(e*g^2)
```

Mupad [B] (verification not implemented)

Time = 12.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{2\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)\,(e^2\,f-d\,e\,g)}{\sqrt{e}\,(2\,c\,d^2+2\,a\,e^2)\,(d\,g-e\,f)^{3/2}}\right)\,(c\,d^2+a\,e^2)}{e^{3/2}\,(d\,g-e\,f)^{3/2}} + \frac{2\,c\sqrt{f+g\,x}}{e\,g^2} - \frac{2\,(c\,e\,f^2+a\,e\,g^2)}{e\,g^2\,\sqrt{f+g\,x}\,(d\,g-e\,f)}$$

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*e
^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(3/
2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f + g
*x)^(1/2)*(d*g - e*f))
```

3.601 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}} dx = -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(4ef-dg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

```
[Out] (3*a*e^2*g+c*d*(-d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2+c*f^2)/g/(-d*g+e*f)^2/(g*x+f)^(1/2)-(a*
e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {912, 1273, 464, 214}

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}} dx = \frac{(3ae^2g+cd(4ef-dg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}} - \frac{\sqrt{f+gx}(ae^2+cd^2)}{e(d+ex)(ef-dg)^2} - \frac{2(ag^2+cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

```
[In] Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(ae^2g^2-c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx,x,\sqrt{f+gx}\right)}{e^2(ef-dg)^2}$$

$$= -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(3ae^2g+cd(4ef-dg))\operatorname{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{eg(ef-dg)^2}$$

$$= -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(4ef-dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))-aeg(2dg+e(f+3gx))}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}} + \frac{(-3ae^2g+cd(-4ef+dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) - a*e*g*(2*d*g + e*(f + 3
*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((-3*a*e^2*g + c*d*(-
4*e*f + d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*
(-(e*f) + d*g)^(5/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(e^2a+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
default	$\frac{-\frac{2g\left(\frac{g\left(e^2a+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
pseudoelliptic	$-\frac{2\left(\frac{3\sqrt{gx+f}\left(a\,e^2g-\frac{1}{3}c\,d^2g+\frac{4}{3}cdef\right)(ex+d)g\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2}+\sqrt{(dg-ef)e}\left(\left(\frac{3}{2}a\,g^2x+c\,f^2x+\frac{1}{2}afg\right)e^2+d(a\,g^2+c\,f^2\right.\right.}{\sqrt{(dg-ef)e}\sqrt{gx+f}\,g(ex+d)(dg-ef)^2e}$

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f
)+1/2*(3*a*e^2*g-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 906, normalized size of antiderivative = 6.29

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \left[\frac{(4\,cd^2ef^2g-(cd^3-3\,ade^2)fg^2+(4\,cde^2fg^2-(cd^2e-3\,ae^3)g^3)x^2+(4\,cde^2f\ldots}{\ldots}\right.$$

$$-\frac{(4\,cd^2ef^2g-(cd^3-3\,ade^2)fg^2+(4\,cde^2fg^2-(cd^2e-3\,ae^3)g^3)x^2+(4\,cde^2f^2g+3\,(cd^2e+ae^3)fg^2-(cd^3\ldots}{de^5f^4g-3\,d^2e^4f^3g^2+3\ldots}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c*
d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2 -
(c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g +
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a*d^
2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*
g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^
3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g
^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 -
 (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^
2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d
*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*
d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*c*d*e
^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^3)*g^3)*x)*sqrt
(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f
*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
+ (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.58

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{(4\,cdef - cd^2g + 3\,ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f^2 - 2\,de^2fg + d^2eg^2)\sqrt{-e^2f+deg}} - \frac{2\,(gx+f)ce^2f^2 - 2\,ce^2f^3 + 2\,cdef^2g + (gx+f)cd^2g^2 + 3\,(gx+f)ae^2g^2 - 2\,ae^2fg^2 + 2\,adeg^3}{(e^3f^2g - 2\,de^2fg^2 + d^2eg^3)\left((gx+f)^{\frac{3}{2}}e - \sqrt{gx+f}ef + \sqrt{gx+f}dg\right)}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*c*d*e*f - c*d^2*g + 3*a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e
*g))/((e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*sqrt(-e^2*f + d*e*g)) - (2*(g*x +
 f)*c*e^2*f^2 - 2*c*e^2*f^3 + 2*c*d*e*f^2*g + (g*x + f)*c*d^2*g^2 + 3*(g*x
+ f)*a*e^2*g^2 - 2*a*e^2*f*g^2 + 2*a*d*e*g^3)/((e^3*f^2*g - 2*d*e^2*f*g^2 +
 d^2*e*g^3)*((g*x + f)^(3/2)*e - sqrt(g*x + f)*e*f + sqrt(g*x + f)*d*g))
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.30

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{(f+g\,x)\,\left(c\,d^2\,g^2+2\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{e\,{(d\,g-e\,f)}^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,{(f+g\,x)}^{3/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,{(d\,g-e\,f)}^{5/2}}\right)\,\left(-c\,g\,d^2+4\,c\,f\,d\,e+3\,a\,g\,e^2\right)}{e^{3/2}\,{(d\,g-e\,f)}^{5/2}}$$

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] - ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f
+ g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))
/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/2)*(
d*g - e*f)^(5/2))
```

3.602 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

```
[Out] -1/4*(15*a*e^2*g^2+c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^2+c*f^2)/(-d*g+e*
f)^3/(g*x+f)^(1/2)-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)^2
+1/4*(7*a*e^2*g+c*d*(-d*g+8*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {912, 1273, 467, 464, 214}

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = -\frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}} - \frac{\sqrt{f+gx}(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2} + \frac{\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{4e(d+ex)(ef-dg)^3} + \frac{2(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)^3}$$

```
[In] Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
```

```
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)(cf^2+ag^2)}{g^5}+\frac{e\left(3ae^2g^2-c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2e^2(ef-dg)^2}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\text{Subst}\left(\int \frac{\frac{8e^2(cf^2+ag^2)}{g^4}+\frac{e(7ae^2g+cd(8ef-dg))x^2}{g^3(ef-dg)}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{4e^2(ef-dg)^2}$$

$$= \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{4eg(ef-dg)^3}$$

$$= \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\frac{\sqrt{e}\left(c\left(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e\left(14f^2+5fgx-g^2x^2\right)\right)+ae\left(8d^2g^2+deg(9f+25gx)+e^2(-2\cdots\right.\right.}{(ef-dg)^3(d+ex)^2\sqrt{f+gx}}\cdots}{4e^{3/2}}$$

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) + d
^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) + a*e*(8*d^2*g^2 + d*e*g*(9*f + 25*g*x)
+ e^2*(-2*f^2 + 5*f*g*x + 15*g^2*x^2))))/((e*f - d*g)^3*(d + e*x)^2*Sqrt[f
+ g*x]) - ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*ArcTan[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(7/2))/(4*e^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg+c\,d^3g^2+7c\,d^2efg-8cd\,e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\cdots\right.}{(dg-ef)^3}$
default	$-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg+c\,d^3g^2+7c\,d^2efg-8cd\,e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\cdots\right.}{(dg-ef)^3}$
pseudoelliptic	$-\frac{2\left(\frac{15\sqrt{gx+f}\left(\left(a\,g^2+\frac{8c\,f^2}{15}\right)e^2+\frac{8cdefg}{15}-\frac{c\,d^2g^2}{15}\right)(ex+d)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{8}+\left(\left(\frac{15a\,g^2x^2}{8}+\frac{5afgx}{8}-\frac{f^2\left(-4c\,x^2+a\right)}{4}\right)e^3\cdots\right.\right.}{\sqrt{(dg-ef)e}\,\sqrt{gx+f}\,(dg-\cdots}$

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*g^2+c*f^2)/(d*g-e*f)^3/(g*x+f)^(1/2)-2/(d*g-e*f)^3*(((7/8*a*e^2*g^2-1
/8*c*d^2*g^2+c*d*e*f*g)*(g*x+f)^(3/2)+1/8*g*(9*a*d*e^2*g^2-9*a*e^3*f*g+c*d^
3*g^2+7*c*d^2*e*f*g-8*c*d*e^2*f^2)/e*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^2+1
/8*(15*a*e^2*g^2-c*d^2*g^2+8*c*d*e*f*g+8*c*e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*a
rctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(192) = 384$.

Time = 0.34 (sec) , antiderivative size = 1539, normalized size of antiderivative = 7.19

$$\int \frac{a + cx^2}{(d + ex)^3 (f + gx)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
(8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e
^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*
a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a
*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g
*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a
*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*e^4)*f^
2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 - 5*a*e^5
)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (19*c*d^2*e
^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e + 25*a*d^2
*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3
*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6
*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*
f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3
*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*
f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 8*c*d^
3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2
 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(
c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*
f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^
2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*
d^3*e^2 - 11*a*d*e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
3*(3*c*d^2*e^3 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c
*d*e^4*f^3 - (19*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g
^2 - (c*d^4*e + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e
^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4
*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x
^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d
^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*
e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.72

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(8\,ce^2f^2+8\,cdefg-cd^2g^2+15\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)\sqrt{-e^2f+deg}}$$
$$+\frac{2\,(cf^2+ag^2)}{(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\sqrt{gx+f}}$$
$$+\frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\,\sqrt{gx+f}cde^2f^2g-(gx+f)^{\frac{3}{2}}cd^2eg^2+7\,(gx+f)^{\frac{3}{2}}ae^3g^2+7\,\sqrt{gx+f}cd^2efg^2-9\,\sqrt{}}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)((gx+f)e-ef+d}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
[Out] 1/4*(8*c*e^2*f^2 + 8*c*d*e*f*g - c*d^2*g^2 + 15*a*e^2*g^2)*arctan(sqrt(g*x
+ f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2 -
d^3*e*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e^3*f^3 - 3*d*e^2*f^
2*g + 3*d^2*e*f*g^2 - d^3*g^3)*sqrt(g*x + f)) + 1/4*(8*(g*x + f)^(3/2)*c*d*
e^2*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g - (g*x + f)^(3/2)*c*d^2*e*g^2 + 7*(
g*x + f)^(3/2)*a*e^3*g^2 + 7*sqrt(g*x + f)*c*d^2*e*f*g^2 - 9*sqrt(g*x + f)*
a*e^3*f*g^2 + sqrt(g*x + f)*c*d^3*g^3 + 9*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*
f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2 - d^3*e*g^3)*((g*x + f)*e - e*f + d*g
)^2)
```

Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.45

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2\right.}{4\,e^{3/2}\,\left(d\,g-e\,f\right)^{7/2}}$$

$$-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,\left(d\,g-e\,f\right)^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g+16\,c\,e^2\,f^2+25\,a\,e^2\,g^2\right)}{4\,e\,\left(d\,g-e\,f\right)^2}}{e^2\,\left(f+g\,x\right)^{5/2}-\left(f+g\,x\right)^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 +
 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g -
e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f*g))
/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 +
8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(3/2)*(
2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))
```

3.603 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

```
[Out] 1/4*(8*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2))*arctanh(g^(1/2)*(e*x+d)
^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)^(3/2)*(g*x+f)^(
1/2)/e^2/g-1/4*c*(5*d*g+3*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {966, 81, 65, 223, 212}

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}} - \frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

```
[In] Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2) + (c*(d + e*
x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*
g + 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(
4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}ce(3ef+5dg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}\\ &= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}\\ &\quad+\frac{1}{8}\left(8a+\frac{c(3e^2f^2+2defg+3d^2g^2)}{e^2g^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} \\
&+ \frac{\left(8a+\frac{c(3e^2f^2+2defg+3d^2g^2)}{e^2g^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{4e} \\
&= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} \\
&+ \frac{\left(8a+\frac{c(3e^2f^2+2defg+3d^2g^2)}{e^2g^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{4e} \\
&= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} \\
&+ \frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx &= \frac{c\sqrt{d+ex}\sqrt{f+gx}(-3ef-3dg+2egx)}{4e^2g^2} \\
&+ \frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}
\end{aligned}
$$

```
[In] Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (c*Sqrt[d + e*x]*Sqrt[f + g*x]*(-3*e*f - 3*d*g + 2*e*g*x))/(4*e^2*g^2) + ((
8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[
f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5/2)*g^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(121) = 242.

Time = 0.42 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.08

method	result
default	$\frac{\left(8\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)a\,e^2g^2+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg}{2\sqrt{eg}}\right.\right.}{}$

```
[In] int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.29

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,cdefg+(3\,cd^2+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\,egx+ef+dg)\right.}{16\,e^3g^3}\right.$$
$$-\frac{(3\,ce^2f^2+2\,cdefg+(3\,cd^2+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-2\,(2\,ce^2g^2x-3\,ce}{8\,e^3g^3}$$

[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] [1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]

Sympy [F]

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

[In] integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)

[Out] Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg+3\,cd^2g^2+8\,ae^2g^2)\log\left(\left|-\sqrt{eg}\sqrt{ex+\cdots}\right.\right.}{\sqrt{eg}e^2g^2}\cdots\right.}{4\,|e|}$$

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3*
g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2)/(e^8*g^3)) - (3*c*e^2*f^2 + 2*c*d*e*f*g
+ 3*c*d^2*g^2 + 8*a*e^2*g^2)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^2))*e/abs(e)
```

Mupad [B] (verification not implemented)

Time = 32.61 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.87

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}} - \frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\,\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^3\,\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^3}+\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^7\,\left(\frac{3\,c\,d^2\,g^2}{2}+c\,d\cdots\right.}{e^2\,g^3\,\left(\sqrt{f+g\,x}-\sqrt{\cdots}\right.}\cdots}{\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^8}{{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^8}+\frac{e^4}{g^4}-\frac{4\,e\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^6}{g\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^6}-\frac{4\,e^3\,\left(\sqrt{\cdots}\right.}{g^3\,\left(\sqrt{f\cdots}\right.}\cdots}$$

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*a*
atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/
2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 + (3*c
*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*
x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))
/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^7*((3*c
*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/2) - f^(
1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2
)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(1/2)*f^(1/
2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/
2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/2) - f^(1/2)
)^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g*x)^(1/2) - f
^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f + g*x)^(1/2) -
f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f + g*x)^(1/2) -
 f^(1/2))^4))
```

3.604 $\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}} dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}} dx = \sqrt{-1+x}x\sqrt{1+x}$$

```
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {392}

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}} dx = \sqrt{x-1}x\sqrt{x+1}$$

```
[In] Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rule 392

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2)), x] /; FreeQ[{a1, b1, a2, b2, c, d, n
, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d - b1*b2*
c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\text{integral} = \sqrt{-1+x}x\sqrt{1+x}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx=\sqrt{-1+x}x\sqrt{1+x}$$

```
[In] Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
default	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
risch	$x\sqrt{-1+x}\,\sqrt{1+x}$	13

```
[In] int((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx=\sqrt{x+1}\sqrt{x-1}x$$

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \text{Timed out}$$

```
[In] integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x^2-1}x$$

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x-1}x$$

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \frac{(x^2+x)\sqrt{x-1}}{\sqrt{x+1}}$$

```
[In] int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
[Out] ((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

3.605 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 411

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx=\frac{e\sqrt{d+ex}\sqrt{f+gx}}{c}+\frac{\sqrt{e}(ef+3dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$
$$+\frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}-\sqrt{-a}(cd^2f-ae(ef+2dg))\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$
$$+\frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] (3*d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/c/
g^(1/2)+e*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2
)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(-(c*d^2*
f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/c^(1/2))/a/c/(-e*
(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)
^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)^(1/2))*((c*d^2*f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/
c^(1/2))/a/c/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {918, 81, 65, 223, 212, 6857, 95, 214}

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}-\sqrt{-a}(cd^2f-ae(2dg+ef))\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} + \frac{\left(\sqrt{-a}(cd^2f-ae(2dg+ef))+\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{ac\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}} + \frac{\sqrt{e}(3dg+ef)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c}$$

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 918

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[g/c, Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
- 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c, Int[Simp[c*d*f^2 - 2*a*e*f*g - a
*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*((f + g*x)
^(n - 2)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2
+ a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cd^2f-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c} + \frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$
\begin{aligned}
&= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} \\
&+ \frac{\int \left(\frac{-\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))}{2a(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} \right) dx}{c} \\
&+ \frac{(e(ef+3dg)) \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}} \, dx}{2c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}} \, dx, x, \sqrt{d+ex}\right)}{c} \\
&+ \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}} + \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} \, dx}{2ac} \\
&+ \frac{\left(\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}} + \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} \, dx}{2ac}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}} \, dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} \\
&+ \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}} + \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}f+\sqrt{-a}g)x^2} \, dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ac} \\
&+ \frac{\left(\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}} + \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{c}f+\sqrt{-a}g)x^2} \, dx, x, \frac{\sqrt{d+}}{\sqrt{f+}}\right.}{ac}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} \\
&+ \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}} - \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\
&+ \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}} + \sqrt{-a}(cd^2f-ae(ef+2dg))\right) \tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{ac\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\sqrt{c}e\sqrt{d+ex}\sqrt{f+gx} + \frac{(i\sqrt{c}d+\sqrt{a}e)\sqrt{cd^2+ae^2}(\sqrt{c}f-i\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}}\right.}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}}}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + ((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*
d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f +
 g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d
 + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]
*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f + I*S
qrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I
*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((S
qrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (Sqrt[c]*Sqrt[e]*(e*
f + 3*d*g)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g
])/c^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2384 vs. 2(331) = 662.

Time = 0.45 (sec) , antiderivative size = 2385, normalized size of antiderivative = 5.80

method	result	size
default	Expression too large to display	2385

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+c*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g
+e*f)/(e*g)^(1/2))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/
2)*(-a*c)^(1/2)*c*d*e*g+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)
^(1/2)*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(
1/2)*c*e^2*f-2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln
((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)
^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*(e*g)^(1/2)*d*e*g-(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*
g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((
```

g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*(e*g)^(1/2)*e^2*f+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(e*g)^(1/2)*a*e^2*g-(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(e*g)^(1/2)*c*d^2*g-2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(e*g)^(1/2)*c*d*e*f+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(e*g)^(1/2)*c^2*d^2*f+2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*a*c*(e*g)^(1/2)*d*e*g+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*a*c*(e*g)^(1/2)*e^2*f+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*(e*g)^(1/2)*a*e^2*g-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*(e*g)^(1/2)*c*d^2*g-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*(e*g)^(1/2)*c*d*e*f-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*c^2*d^2*f+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*(-a*c)^(1/2)*(e*g)^(1/2)*c*e)/((g*x+f)*(e*x+d))^(1/2)/(-a*c)^(1/2)/c^2/(e*g)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}}{c\,x^2+a}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

3.606 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 342

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{2\sqrt{e}\sqrt{g}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] 2*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)*g^(1/2)/c+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d*f-a*e*g-(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(
1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctan
h((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2))*(c*d*f-a*e*g+(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{920, 65, 223, 212, 6857, 95, 214\}$

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\left(-\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}} + \frac{2\sqrt{e}\sqrt{g}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[e*(g/c), Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)^
(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c}+\frac{(eg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}\\
&= \frac{\int \left(\frac{-a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}\\
&\quad+\frac{(2g)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{c}\\
&= \frac{(2g)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c}\\
&\quad-\frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c}\\
&\quad-\frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c}
\end{aligned}
$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}c} - \frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\frac{\sqrt{cd^2+ae^2}(i\sqrt{c}f+\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}} + \frac{\sqrt{cd^2+ae^2}(-i\sqrt{c}f+\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}}}{c}$$

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] ((Sqrt[c*d^2 + a*e^2]*(I*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]
*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f -
I*Sqrt[a]*g))]) + (Sqrt[c*d^2 + a*e^2]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt
[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt
[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + 2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/c
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(262) = 524$.

Time = 0.45 (sec) , antiderivative size = 1499, normalized size of antiderivative = 4.38

method	result	size
default	Expression too large to display	1499

```
[In] int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*((e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1
/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e*g-
(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((
2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-
a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*f-(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*
f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e
*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)
))*(-a*c)^(1/2)*d*g-(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-
c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-a*c)^(1/2)*e*f-(e
*g)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2
)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*e*g+(e*g)^(1/2)*ln((-2*(-a*c)^(1/2)*e*
g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^
(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/
(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/
2)*c*d*f-(e*g)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*d*g-(e*g)^(1/2)*
ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*
c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*e*f+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x
+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
f)/c)^(1/2)*(-a*c)^(1/2)*e*g)/((g*x+f)*(e*x+d))^(1/2)/(-a*c)^(1/2)/c/(e*g)^
(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)/(((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{gx+f}}{cx^2+a}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Hanged}$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
[Out] \text{Hanged}
```

3.607 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}} - \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(g*(-a)^(1/2)+f*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {924, 95, 214}

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}} - \frac{\sqrt{\sqrt{-a}g+\sqrt{c}f}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

```
[In] Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

integral

$$= \int \left(\frac{\sqrt{-a}f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}f+\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$= \frac{1}{2}\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx + \frac{1}{2}\left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$= \left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right) + \left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)$$

$$= \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}} - \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}}$$

Mathematica [A] (verified)

Time = 10.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}}-\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}d+\sqrt{-a}e}}}{\sqrt{-a}\sqrt{c}}$$

```
[In] Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[-(Sqr
t[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(176) = 352$.

Time = 0.40 (sec) , antiderivative size = 1387, normalized size of antiderivative = 5.78

method	result	size
default	Expression too large to display	1387

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*(ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+
2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+
d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*
(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*c*e^2*f-ln((-2*
(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a
*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2
)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*e^2*g+ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*
e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)
*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1
/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c^2*d^2*f-l
n((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c
)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c*d^2*g-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g
*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*
```

```
x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c
)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*a*c*e^
2*f-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*a*e^2*g-ln((2*(-a*c)^(1/2)*e*g*x+c
*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*
((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(
-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c^
2*d^2*f-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*c*d^2*g)/((g*x+f)*(e*x+d))^(1/
2)/(-e*(-a*c)^(1/2)+c*d)/(-a*c)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
a*e*g-c*d*f)/c)^(1/2)/(e*(-a*c)^(1/2)+c*d)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f-a*e*g+c*d*f)/c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(176) = 352$.

Time = 6.81 (sec) , antiderivative size = 1921, normalized size of antiderivative = 8.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a
^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e +
a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^
2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a
*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c
*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*
f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^
2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*
c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*
```

```
e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d
^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) - 1/4*sqrt(-(c*d*f
 + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(
e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-
(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^
2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^
3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*s
qrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a
*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g + (a*
c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f +
a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*
f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f +
(c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Hanged}$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.608 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 351

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] -2*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{922, 37, 6857, 95, 214\}$

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(ae^2+cd^2)\,\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\,(ae^2+cd^2)\,\sqrt{\sqrt{-a}g+\sqrt{c}f}} - \frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\int \left(\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} + \frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cd^2+ae^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cd^2+ae^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\,\sqrt{d+ex}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (I*Sqrt[-((Sqrt[c]*d
 + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqr
t[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*S
qrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]) + (
I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt
[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[
c*d^2 + a*e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. 2(279) = 558.

Time = 0.46 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5816 vs. $2(279) = 558$.

Time = 35.16 (sec) , antiderivative size = 5816, normalized size of antiderivative = 16.57

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.609 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 613

$$\begin{aligned}\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx &= -\frac{2e\sqrt{f+gx}}{3\,(cd^2+ae^2)\,(d+ex)^{3/2}} \\ &+ \frac{4eg\sqrt{f+gx}}{3\,(cd^2+ae^2)\,(ef-dg)\sqrt{d+ex}} \\ &+ \frac{e\big(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\big)\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)\,(ef-dg)\sqrt{d+ex}} \\ &- \frac{e\big(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\big)\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)\,(ef-dg)\sqrt{d+ex}} \\ &+ \frac{\sqrt{c}\big(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\big)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\,\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\ &+ \frac{\sqrt{c}\big(\sqrt{-a}cdf+\sqrt{-a}aeg+a\sqrt{c}(ef-dg)\big)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\,\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\,\sqrt{\sqrt{c}f+\sqrt{-a}g}}\end{aligned}$$

```
[Out] -2/3*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(3/2)+4/3*e*g*(g*x+f)^(1/2)/(a*e
^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)+e*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(
1/2))*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(
1/2))/(e*x+d)^(1/2)-e*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(g*x+f)^(
1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*
c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*(-a)^
(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f*(-a)^(1/2)+a*
```

```
e*g*(-a)^(1/2)+a*(-d*g+e*f)*c^(1/2))/a/(a*e^2+c*d^2)/(e*(-a)^(1/2)+d*c^(1/2
))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {922, 47, 37, 6857, 98, 95, 214}

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \frac{\sqrt{c}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(ae^2+cd^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$
$$+\frac{\sqrt{c}\left(a\sqrt{c}(ef-dg)+\sqrt{-a}cdf+\sqrt{-a}aeg\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{a\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}(ae^2+cd^2)\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$
$$+\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ae^2+cd^2)(ef-dg)}$$
$$-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ae^2+cd^2)(ef-dg)}$$
$$+\frac{4eg\sqrt{f+gx}}{3\sqrt{d+ex}(ae^2+cd^2)(ef-dg)}-\frac{2e\sqrt{f+gx}}{3(d+ex)^{3/2}(ae^2+cd^2)}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
x])]])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\text{integral} = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(a+cx^2)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} + \frac{\int \left(\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}+\sqrt{c}x)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx}{cd^2+ae^2} - \frac{(2eg)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}(cd^2+ae^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{e\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{\left(\sqrt{c}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)} - \frac{\left(\sqrt{c}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{e\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{\left(\sqrt{c}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)} - \frac{\left(\sqrt{c}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)}$$

$$= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} - \frac{e\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(ef-dg)\sqrt{d+ex}} + \frac{\sqrt{c}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{c}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}dx = \frac{2\sqrt{f+gx}(ae^4(f+gx)+cde(-6d^2g+6e^2fx+de(7f-5gx)))}{3(cd^2+ae^2)^2(-ef+dg)(d+ex)^{3/2}} - \frac{i\sqrt{c}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)^2\sqrt{cd^2+ae^2}} + \frac{i\sqrt{c}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)^2\sqrt{cd^2+ae^2}}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[f + g*x]*(a*e^4*(f + g*x) + c*d*e*(-6*d^2*g + 6*e^2*f*x + d*e*(7*f
- 5*g*x))))/(3*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)*(d + e*x)^(3/2)) - (I*Sqrt[
c]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqr
t[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*
f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*Sq
rt[c*d^2 + a*e^2]) + (I*Sqrt[c]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[
c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(
Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[c*d^2 + a*e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 14860 vs. $2(501) = 1002$.

Time = 0.46 (sec) , antiderivative size = 14861, normalized size of antiderivative = 24.24

method	result	size
default	Expression too large to display	14861

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10812 vs. $2(501) = 1002$.

Time = 160.90 (sec) , antiderivative size = 10812, normalized size of antiderivative = 17.64

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx=\int \frac{\sqrt{f+gx}}{\left(a+cx^2\right)(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,(d+e\,x)^{5/2}}\,dx$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

3.610 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 337

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d^2-a*e^2-2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e
*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^
(1/2))^(1/2))*(c*d^2-a*e^2+2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(e*(-a)^(
1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {924, 65, 223, 212, 6857, 95, 214}

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}} + \frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}+\frac{cd^2-ae^2+2cdex}{c\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\right)dx\\
&= \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c}+\frac{e^2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}\\
&= \frac{\int \left(\frac{-2a\sqrt{c}de+\sqrt{-a}(cd^2-ae^2)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}+\frac{2a\sqrt{c}de+\sqrt{-a}(cd^2-ae^2)}{2a(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{c}\\
&= \frac{(2e)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c}\\
&\quad-\frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c}\\
&\quad-\frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c}
\end{aligned}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}c} - \frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\frac{\left(i\sqrt{c}d+\sqrt{a}e\right)\sqrt{cd^2+ae^2}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} + \frac{\left(-i\sqrt{c}d+\sqrt{a}e\right)\sqrt{cd^2+ae^2}\arctan\left(\cdots\right.}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{\cdots}\right.\right.}}}{c}$$

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[(Sqrt[c*d^2 + a*e^2]
*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f -
I*Sqrt[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt
[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt
[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. 2(257) = 514.

Time = 0.46 (sec) , antiderivative size = 2336, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	2336

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2
*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d)
)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^
2*e^2*g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^
(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2)*(-((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*
g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(
1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(
c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f+a*e*g-c*d*f)/c)^(1/2)-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1
/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*d*e*
g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*
(-a*c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f^2*(e*g)^(1/2
)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-2*ln((2*(-a*c)
^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*
d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2
*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*
d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*
((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(
-a*c)^(1/2)))*a^2*e^2*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/
c)^(1/2)*(e*g)^(1/2)+ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*
(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)-ln((-
2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1
/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)-2*ln((-2*(-a*c)^(1/2)*e*g*x+c*d
*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(
```

```
(g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-
a*c)^(1/2)))*a*d*e*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^
(1/2)*(e*g)^(1/2)*(-a*c)^(1/2)+ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d)
)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c^
2*d^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(
1/2)-2*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d
*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c*d*e*f^2*(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*(-a*c)^(1/2)+2*ln(1/2
*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^2
*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)+2*ln(1/2*(2*e*g
*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2*((
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2))/((g*x+f)*(e*x+d))^(1/
2)/(g*(-a*c)^(1/2)+c*f)/(c*f-g*(-a*c)^(1/2))/(e*g)^(1/2)/(-a*c)^(1/2)/(-((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)/(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,(c\,x^2+a)}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

3.611 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+cx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(e*(-a)^(1/2)+d*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {924, 95, 214}

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

integral

$$=\int \left(\frac{\sqrt{-a}d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}d+\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$=\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}+\frac{e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$=\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left(\frac{ad}{(-a)^{3/2}}+\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)$$

$$=\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.75 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = -\frac{1}{4}(ef-dg)\text{RootSum}\Big[ce^4f^2+ae^4g^2+4ce^3f^2g\#1^2 - 8cde^2fg^2\#1^2-4ae^3g^3\#1^2+6ce^2f^2g^2\#1^4-16cdefg^3\#1^4+16cd^2g^4\#1^4+6ae^2g^4\#1^4 + 4cef^2g^3\#1^6-8cdfg^4\#1^6-4aeg^5\#1^6+cf^2g^4\#1^8 + ag^6\#1^8\&, \frac{-e^3\log(f+gx)+2e^3\log\left(\sqrt{d-\frac{ef}{g}}-\sqrt{d+ex}+\sqrt{f+gx}\#1\right)-e^2g\log(f+gx)\#1^2+2e^2g}{ce^3f^2\#1-2cde^2fg\#1-}$$

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -1/4*((e*f - d*g)*RootSum[c*e^4*f^2 + a*e^4*g^2 + 4*c*e^3*f^2*g*#1^2 - 8*c*
d*e^2*f*g^2*#1^2 - 4*a*e^3*g^3*#1^2 + 6*c*e^2*f^2*g^2*#1^4 - 16*c*d*e*f*g^3
*#1^4 + 16*c*d^2*g^4*#1^4 + 6*a*e^2*g^4*#1^4 + 4*c*e*f^2*g^3*#1^6 - 8*c*d*f
*g^4*#1^6 - 4*a*e*g^5*#1^6 + c*f^2*g^4*#1^8 + a*g^6*#1^8 & , (-(e^3*Log[f +
 g*x]) + 2*e^3*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1] -
e^2*g*Log[f + g*x]*#1^2 + 2*e^2*g*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + S
qrt[f + g*x]*#1]*#1^2 + e*g^2*Log[f + g*x]*#1^4 - 2*e*g^2*Log[Sqrt[d - (e*f
)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + g^3*Log[f + g*x]*#1^6 - 2*g
^3*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^6)/(c*e^3*f
^2*#1 - 2*c*d*e^2*f*g*#1 - a*e^3*g^2*#1 + 3*c*e^2*f^2*g*#1^3 - 8*c*d*e*f*g^
2*#1^3 + 8*c*d^2*g^3*#1^3 + 3*a*e^2*g^3*#1^3 + 3*c*e*f^2*g^2*#1^5 - 6*c*d*f
*g^3*#1^5 - 3*a*e*g^4*#1^5 + c*f^2*g^3*#1^7 + a*g^5*#1^7) & ])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. 2(176) = 352.

Time = 0.46 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.76

method	result	size
default	Expression too large to display	1383

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*((-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*
g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^
(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e*
g^2+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)
```

```
*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*e*f^2+(-((-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2
*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d)
)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*
c*d*g^2+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-
a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d*f^2+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e
*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*
(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/
2)))*a*e*g^2+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*
c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)
*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c*e*f^2-(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x
+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x
+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)
^(1/2)))*a*c*d*g^2-(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-
a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c^2*d*f^2)/((g*x+f)*(e*x+d))^(1
/2)/(c*f-g*(-a*c)^(1/2))/(-a*c)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
a*e*g-c*d*f)/c)^(1/2)/(g*(-a*c)^(1/2)+c*f)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f-a*e*g+c*d*f)/c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(176) = 352$.

Time = 8.91 (sec) , antiderivative size = 1913, normalized size of antiderivative = 7.97

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a
^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g +
a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2
*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^
```

```
2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^
2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^
2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*
c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2
*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^
2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*
f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*
g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f
*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^
2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e
*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) - 1/4*sqrt(-(c
*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2
)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*lo
g(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3)*sq
rt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*
g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*
c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2
 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2
*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3
)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 +
 a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(
-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4
)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g
 - (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f
^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)
/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2
*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*
f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] \text{Hanged}
```

3.612 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1
/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {926, 95, 214}

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx\\
&= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}-\frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx$$
$$= \frac{\sqrt[4]{-1}\left(-\frac{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f-i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f-i\sqrt{a}g}}+\frac{\sqrt{i\sqrt{c}d+\sqrt{a}e}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f+i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f+i\sqrt{a}g}}\right)}{\sqrt{a}\sqrt{cd^2+ae^2}}$$

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((-1)^(1/4)*(-((Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*ArcTan[((-1)^(1/4)*Sqrt[c*
d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*
f - I*Sqrt[a]*g]*Sqrt[d + e*x])])/Sqrt[Sqrt[c]*f - I*Sqrt[a]*g]) + (Sqrt[I*
Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[((-1)^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x
])/(Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]*Sqrt[d + e*
x])])/Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]))/(Sqrt[a]*Sqrt[c*d^2 + a*e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. 2(170) = 340.

Time = 0.45 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.15

method	result	size
default	Expression too large to display	1415

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^2*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*
g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g
*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*
c)^(1/2)))*a*c*d^2*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)
^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-((-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x
+c*e*f*x+2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*((g*x+
```

```
f)*(e*x+d))^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^
(1/2)))*c^2*d^2*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-
(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*(((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+
c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+
f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^
(1/2)))*a*c*d^2*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/
2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(((-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c
*e*f*x+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(
1/2)))*c^2*d^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
))*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*
d*f)/c)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)/(-
a*c)^(1/2)/(c*f-g*(-a*c)^(1/2))/(g*(-a*c)^(1/2)+c*f)/(-e*(-a*c)^(1/2)+c*d)/
(e*(-a*c)^(1/2)+c*d)/((g*x+f)*(e*x+d))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4325 vs. $2(170) = 340$.

Time = 15.59 (sec) , antiderivative size = 4325, normalized size of antiderivative = 18.80

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3
*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*
c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c
*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^
2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g +
 d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e +
 a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)
*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2
*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d
*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*
(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^
```

4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) - 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*

```
f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f
^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)
/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((
e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*
f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a
^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2
 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*
f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqr
t(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 +
a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)
*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3
+ a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f
^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 +
 a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x
)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{1}{(a+cx^2)\,\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.613 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

```
[Out] -e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1
/2)+e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)
)^(3/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)
+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a
)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{926, 98, 95, 214\}$

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\sqrt{\sqrt{-a}g+\sqrt{c}f}} - \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}$$

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right) dx$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} - \frac{\sqrt{c}\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\left(\sqrt{-a}\sqrt{c}d-ae\right)} - \frac{\sqrt{c}\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\left(\sqrt{-a}\sqrt{c}d+ae\right)}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}d-ae} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}d+ae}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^2\sqrt{f+gx}}{(cd^2+ae^2)\,(-ef+dg)\sqrt{d+ex}} + \frac{i\sqrt{c}\left(\sqrt{c}d+i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} - \frac{i\sqrt{c}\left(\sqrt{c}d-i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}$$

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]) + (I*S
qrt[c]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x
])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e
*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqr
t[c]*f - I*Sqrt[a]*g))]) - (I*Sqrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcTan[(S
qrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c
]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.

Time = 0.47 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.01

method	result	size
default	Expression too large to display	10977

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11846 vs. $2(270) = 540$.

Time = 56.03 (sec) , antiderivative size = 11846, normalized size of antiderivative = 33.46

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.614 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 625

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx &= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} \\ &- \frac{2\sqrt{e}(ef-dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)} \\ &- \frac{\sqrt{e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)} \\ &+ \frac{\sqrt{e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)} \\ &+ \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} \\ &- \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}\end{aligned}$$

```
[Out] -2*(-d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/
(a*g^2+c*f^2)/g^(1/2)-arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*
(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2
)/c^(1/2)/g^(1/2)+arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*(c*d
*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^
(1/2)/g^(1/2)+2*(-d*g+e*f)*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctan
h((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2
)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(-e*(-a)^(1
/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(-g*(-a)^(1/2)+f*c^(1
```

```
/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/
2))*(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {922, 49, 65, 223, 212, 6857, 132, 12, 95, 214}

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx=&-\frac{2\sqrt{e}(ef-dg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(ag^2+cf^2)}\\&-\frac{\sqrt{e}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(ag^2+cf^2)}\\&+\frac{\sqrt{e}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(ag^2+cf^2)}\\&+\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(ag^2+cf^2)}\\&-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}\,(ag^2+cf^2)}\\&+\frac{2\sqrt{d+ex}(ef-dg)}{\sqrt{f+gx}\,(ag^2+cf^2)}\end{aligned}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(a+cx^2)}\,dx}{cf^2+ag^2}-\frac{(g(ef-dg))\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}\,dx}{cf^2+ag^2}\\
&= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}}\\
&\quad+\frac{\int \left(\frac{\left(-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{f+gx}}+\frac{\left(a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{f+gx}}\right)dx}{cf^2+ag^2}\\
&\quad-\frac{(e(ef-dg))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{cf^2+ag^2}\\
&= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}}-\frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{cf^2+ag^2}\\
&\quad-\frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{\sqrt{d+ex}}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(cf^2+ag^2\right)}\\
&\quad-\frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{\sqrt{d+ex}}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(cf^2+ag^2\right)}
\end{aligned}
$$

$$
= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{cf^2+ag^2}
- \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{d-\frac{\sqrt{-a}e}{\sqrt{c}}}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)}
- \frac{\left(e\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}
- \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{d+\frac{\sqrt{-a}e}{\sqrt{c}}}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)}
+ \frac{\left(e\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}
$$

$$
= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)}
- \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}
- \frac{\left(\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)}
+ \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}
- \frac{\left(\left(d+\frac{\sqrt{-a}e}{\sqrt{c}}\right)\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)}
$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)}$$
$$- \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}$$
$$- \frac{\left(\left(d-\frac{\sqrt{-a}e}{\sqrt{c}}\right)\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\mathrm{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cf^2+ag^2)}$$
$$+ \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\,(cf^2+ag^2)}$$
$$- \frac{\left(\left(d+\frac{\sqrt{-a}e}{\sqrt{c}}\right)\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cf^2+ag^2)}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)}$$
$$- \frac{\sqrt{e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)}$$
$$+ \frac{\sqrt{e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\,(cf^2+ag^2)}$$
$$+ \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)}$$
$$- \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.15 (sec) , antiderivative size = 1049, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \frac{(ef-dg)\left(2\sqrt{d+ex}-\frac{1}{2}\sqrt{f+gx}\mathrm{RootSum}\left[ce^4f^2+ae^4g^2+4ce^3f^2g\#1^2-8\right.\right.}{}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] ((e*f - d*g)*(2*Sqrt[d + e*x] - (Sqrt[f + g*x]*RootSum[c*e^4*f^2 + a*e^4*g^
2 + 4*c*e^3*f^2*g*#1^2 - 8*c*d*e^2*f*g^2*#1^2 - 4*a*e^3*g^3*#1^2 + 6*c*e^2*
```

```
f^2*g^2*#1^4 - 16*c*d*e*f*g^3*#1^4 + 16*c*d^2*g^4*#1^4 + 6*a*e^2*g^4*#1^4 +
 4*c*e*f^2*g^3*#1^6 - 8*c*d*f*g^4*#1^6 - 4*a*e*g^5*#1^6 + c*f^2*g^4*#1^8 +
a*g^6*#1^8 & , (-(c*d*e^3*f*Log[f + g*x]) - a*e^4*g*Log[f + g*x] + 2*c*d*e^
3*f*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1] + 2*a*e^4*g*L
og[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1] - c*d*e^2*f*g*Log[
f + g*x]*#1^2 + 2*c*d^2*e*g^2*Log[f + g*x]*#1^2 + a*e^3*g^2*Log[f + g*x]*#1
^2 + 2*c*d*e^2*f*g*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x]*#1
]*#1^2 - 4*c*d^2*e*g^2*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g*x
]*#1]*#1^2 - 2*a*e^3*g^2*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x] + Sqrt[f + g
*x]*#1]*#1^2 + c*d*e*f*g^2*Log[f + g*x]*#1^4 - 2*c*d^2*g^3*Log[f + g*x]*#1^
4 - a*e^2*g^3*Log[f + g*x]*#1^4 - 2*c*d*e*f*g^2*Log[Sqrt[d - (e*f)/g] - Sqr
t[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + 4*c*d^2*g^3*Log[Sqrt[d - (e*f)/g] - S
qrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + 2*a*e^2*g^3*Log[Sqrt[d - (e*f)/g] -
 Sqrt[d + e*x] + Sqrt[f + g*x]*#1]*#1^4 + c*d*f*g^3*Log[f + g*x]*#1^6 + a*e
*g^4*Log[f + g*x]*#1^6 - 2*c*d*f*g^3*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x]
+ Sqrt[f + g*x]*#1]*#1^6 - 2*a*e*g^4*Log[Sqrt[d - (e*f)/g] - Sqrt[d + e*x]
+ Sqrt[f + g*x]*#1]*#1^6)/(c*e^3*f^2*#1 - 2*c*d*e^2*f*g*#1 - a*e^3*g^2*#1 +
 3*c*e^2*f^2*g*#1^3 - 8*c*d*e*f*g^2*#1^3 + 8*c*d^2*g^3*#1^3 + 3*a*e^2*g^3*#
1^3 + 3*c*e*f^2*g^2*#1^5 - 6*c*d*f*g^3*#1^5 - 3*a*e*g^4*#1^5 + c*f^2*g^3*#1
^7 + a*g^5*#1^7) & ])/2))/((c*f^2 + a*g^2)*Sqrt[f + g*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8263 vs. $2(497) = 994$.

Time = 0.48 (sec) , antiderivative size = 8264, normalized size of antiderivative = 13.22

method	result	size
default	Expression too large to display	8264

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.615 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 351

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

```
[Out] -2*g*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{922, 37, 6857, 95, 214\}$

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(ag^2+cf^2)}$$
$$-\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}\,(ag^2+cf^2)} - \frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\int \left(\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,(cf^2+ag^2)}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}f+\sqrt{-a}g)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\,(cf^2+ag^2)}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{cf^2+ag^2}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f-i\sqrt{a}g\right)\sqrt{cf^2+ag^2}}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (I*Sqrt[-((Sqrt[c]*d
 + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqr
t[d + e*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*S
qrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[c*f^2 + a*g^2]) - (
I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt
[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[
c*f^2 + a*g^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.

Time = 0.46 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5844 vs. $2(279) = 558$.

Time = 46.42 (sec) , antiderivative size = 5844, normalized size of antiderivative = 16.65

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.616 $\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)} dx = \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}$$

```
[Out] g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/
2)-g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(
1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-
e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))
^(3/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+
f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)
^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {926, 98, 95, 214}

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}} + \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}$$

```
[In] Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\right) dx$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{\sqrt{c}\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\left(\sqrt{-a}\sqrt{c}f-ag\right)} - \frac{\sqrt{c}\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\left(\sqrt{-a}\sqrt{c}f+ag\right)}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}f-ag} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}f+ag}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{2g^2\sqrt{d+ex}}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}} - \frac{i\sqrt{c}\left(\sqrt{c}f-i\sqrt{a}g\right)^2\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}(cf^2+ag^2)^{3/2}} + \frac{i\sqrt{c}\left(\sqrt{c}f+i\sqrt{a}g\right)^2\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}(cf^2+ag^2)^{3/2}}$$

```
[In] Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (I*Sqrt
[c]*(Sqrt[c]*f - I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/
(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[f + g*x]
)])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*(
c*f^2 + a*g^2)^(3/2)) + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[(Sqrt
[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e
)*(Sqrt[c]*f + I*Sqrt[a]*g))]*(c*f^2 + a*g^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.

Time = 0.46 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.01

method	result	size
default	Expression too large to display	10977

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12028 vs. $2(270) = 540$.

Time = 90.51 (sec) , antiderivative size = 12028, normalized size of antiderivative = 33.98

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\left(c\,x^2+a\right)\,\sqrt{d+e\,x}}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

3.617 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 549

$$\begin{aligned}\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx =\\ &-\frac{e}{\sqrt{-a}\,(\sqrt{c}d-\sqrt{-a}e)\,(ef-dg)\sqrt{d+ex}\sqrt{f+gx}}\\ &+\frac{e}{\sqrt{-a}\,(\sqrt{c}d+\sqrt{-a}e)\,(ef-dg)\sqrt{d+ex}\sqrt{f+gx}}\\ &+\frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\,(\sqrt{c}d-\sqrt{-a}e)\,(\sqrt{c}f-\sqrt{-a}g)\,(ef-dg)^2\sqrt{f+gx}}\\ &+\frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\,(\sqrt{c}d+\sqrt{-a}e)\,(\sqrt{c}f+\sqrt{-a}g)\,(ef-dg)^2\sqrt{f+gx}}\\ &+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,(\sqrt{c}d-\sqrt{-a}e)^{3/2}\,(\sqrt{c}f-\sqrt{-a}g)^{3/2}}\\ &-\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,(\sqrt{c}d+\sqrt{-a}e)^{3/2}\,(\sqrt{c}f+\sqrt{-a}g)^{3/2}}\end{aligned}$$

```
[Out] c*arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(
-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*
(-a)^(1/2)+f*c^(1/2))^(3/2)-c*arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2)
)^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1
/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)-e/(-d*g+e*f)/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+e/(-d*g+e*f)/(-a)^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2
)-(d*g+e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2)+(d*g+
```

```
e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {926, 106, 157, 12, 95, 214}

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}} - \frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)} + \frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)} + \frac{g\sqrt{d+ex}\left(2aeg-\sqrt{-a}\sqrt{c}(dg+ef)\right)}{a\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)^2} + \frac{g\sqrt{d+ex}\left(\sqrt{-a}\sqrt{c}(dg+ef)+2aeg\right)}{a\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2}$$

```
[In] Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*
x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(Sqrt[c
]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x]) + (
c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sq
rt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*(Sqrt[c
]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\right) dx$$

$$= -\frac{\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} - \frac{\int \frac{\frac{1}{2}\left(2\sqrt{-a}eg+\sqrt{c}(ef-dg)\right)+\sqrt{c}egx}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)} + \frac{\int \frac{\frac{1}{2}\left(2\sqrt{-a}eg-\sqrt{c}(ef-dg)\right)-\sqrt{c}egx}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{2\int -\frac{c(ef-dg)^2}{4(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2} + \frac{2\int -\frac{c(ef-dg)^2}{4(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} - \frac{c\int \frac{1}{(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)} - \frac{c\int \frac{1}{(\sqrt{-a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\left(\sqrt{-a}cdf+(-a)^{3/2}eg+a\sqrt{c}(ef+dg)\right)}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)} - \frac{c\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}cdf+(-a)^{3/2}eg+a\sqrt{c}(ef+dg)}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg-\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{g\left(2\sqrt{-a}eg+\sqrt{c}(ef+dg)\right)\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)^2\sqrt{f+gx}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(\sqrt{-a}cdf+(-a)^{3/2}eg+a\sqrt{c}(ef+dg)\right)} - \frac{c\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = -\frac{2(c(d^3g^3+d^2eg^3x+e^3f^2(f+gx))+ae^2g^2(dg+e(f+2gx)))}{(cd^2+ae^2)\,(ef-dg)^2\,(cf^2+ag^2)\,\sqrt{d+ex}\sqrt{f+gx}} - \frac{ic\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f-i\sqrt{a}g\right)^2} + \frac{ic\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f+i\sqrt{a}g\right)^2}$$

```
[In] Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*(c*(d^3*g^3 + d^2*e*g^3*x + e^3*f^2*(f + g*x)) + a*e^2*g^2*(d*g + e*(f
+ 2*g*x))))/((c*d^2 + a*e^2)*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[d + e*x]*Sq
rt[f + g*x]) - (I*c*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]
*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqr
t[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d -
I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)^2) + (I*c*Sqrt[-
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqr
t[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[c*d^2 +
a*e^2]*(Sqrt[c]*f + I*Sqrt[a]*g)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 30647 vs. 2(433) = 866.

Time = 0.46 (sec) , antiderivative size = 30648, normalized size of antiderivative = 55.83

method	result	size
default	Expression too large to display	30648

```
[In] int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)} \, dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)} \, dx = \int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)} \, dx = \int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)} \, dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\,(c\,x^2+a)\,\,(d+e\,x)^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.618 $\int \frac{\sqrt{x}}{\sqrt{1+x}(1+x^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = -\frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{1+x}}\right)$$

```
[Out] -1/2*(1-I)^(3/2)*arctanh((1-I)^(1/2)*x^(1/2)/(1+x)^(1/2))-1/2*(1+I)^(3/2)*a
rctanh((1+I)^(1/2)*x^(1/2)/(1+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {924, 95, 214}

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = -\frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[In] Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -1/2*((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]]) - ((1 + I)^
(3/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{1+x}}+\frac{1}{2\sqrt{x}(i+x)\sqrt{1+x}}\right) dx\\ &= -\left(\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\sqrt{1+x}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}(i+x)\sqrt{1+x}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)+\text{Subst}\left(\int \frac{1}{i+(1-i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)\\ &= -\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{1+x}}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = -\text{RootSum}\left[16+32\#1+16\#1^2+\#1^4\&,\frac{\log\left(-2x+2\sqrt{x}\sqrt{1+x}+\#1\right)\#1^2}{8+8\#1+\#1^3}\&\right]$$

```
[In] Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x]*Sqrt[1 + x]
+ #1]*#1^2)/(8 + 8*#1 + #1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(45) = 90$.

Time = 0.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.69

method	result
default	$\frac{\sqrt{\frac{(1+x)x}{\left(\sqrt{2}-1+x\right)^{2}}}\left(\sqrt{2}-1+x\right)\left(\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\sqrt{\frac{\left(3\sqrt{2}-4\right)x\left(4+3\sqrt{2}\right)(1+x)}{\left(\sqrt{2}-1+x\right)^{2}}}\left(3+2\sqrt{2}\right)\left(\sqrt{2}+1-x\right)\left(3\sqrt{2}-4\right)\left(\sqrt{2}-1+x\right)}{4(1+x)x}\right)\sqrt{1+}$

```
[In] int(x^(1/2)/(x^2+1)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^(1/2)/(1+x)^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2+
2*2^(1/2))^(1/2)*arctan(1/4*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2)-4)*x*(4+3*2^(1
/2))*(1+x)/(2^(1/2)-1+x)^2)^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)
*(2^(1/2)-1+x)/(1+x)/x)*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*ar
ctan(1/4*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2)-4)*x*(4+3*2^(1/2))*(1+x)/(2^(1/2)
-1+x)^2)^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x)/(1+x
)/x)*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)/(1
+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)/
(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx &= \frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left((i+1)\sqrt{2}\sqrt{i-1}+2\sqrt{x+1}\sqrt{x}-2x-2i\right)\\&\quad-\frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left(-(i+1)\sqrt{2}\sqrt{i-1}+2\sqrt{x+1}\sqrt{x}-2x-2i\right)\\&\quad+\frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left(-(i-1)\sqrt{2}\sqrt{-i-1}+2\sqrt{x+1}\sqrt{x}-2x+2i\right)\\&\quad-\frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left((i-1)\sqrt{2}\sqrt{-i-1}+2\sqrt{x+1}\sqrt{x}-2x+2i\right)\end{aligned}$$

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(I - 1)*log((I + 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt(x + 1)*sqr
t(x) - 2*x - 2*I) - 1/4*sqrt(2)*sqrt(I - 1)*log(-(I + 1)*sqrt(2)*sqrt(I - 1
) + 2*sqrt(x + 1)*sqrt(x) - 2*x - 2*I) + 1/4*sqrt(2)*sqrt(-I - 1)*log(-(I -
 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt(x + 1)*sqrt(x) - 2*x + 2*I) - 1/4*sqrt(2)
*sqrt(-I - 1)*log((I - 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt(x + 1)*sqrt(x) - 2*
x + 2*I)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{\sqrt{x}}{\sqrt{x+1}\,(x^2+1)}\,dx$$

```
[In] integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
[Out] Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \frac{\sqrt{x}}{(x^2+1)\sqrt{x+1}}\,dx$$

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(37) = 74$.

Time = 0.78 (sec) , antiderivative size = 375, normalized size of antiderivative = 5.77

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}-\frac{1}{x+1}+1\right) + \frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}-\frac{1}{x+1}+1\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\right)}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\right)}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right) + \frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right)$$

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(2*(1/2)^(3/4)*((1/2)
^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) + 1/
4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(-2*(1/2)^(3/4)*((1/2)^
(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) - 1/8
*(sqrt(2*sqrt(2) + 2) - sqrt(2*sqrt(2) - 2))*log((1/2)^(1/4)*sqrt(sqrt(2) +
 2)*sqrt(-1/(x + 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) + 1/8*(sqrt(2*sqrt(2)
 + 2) - sqrt(2*sqrt(2) - 2))*log(-(1/2)^(1/4)*sqrt(sqrt(2) + 2)*sqrt(-1/(x
+ 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) - 1/4*sqrt(2*sqrt(2) + 2)*arctan(2*(
1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2) + 2) + 2)/sqrt(-sqrt(2) + 2)) - 1/4*sq
rt(2*sqrt(2) + 2)*arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2) + 2) - 2)
/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log((1/2)^(1/4)*sqrt(sqrt(2)
 + 2) + sqrt(1/2) + 1) + 1/8*sqrt(2*sqrt(2) - 2)*log(-(1/2)^(1/4)*sqrt(sqrt
(2) + 2) + sqrt(1/2) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 1610, normalized size of antiderivative = 24.77

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \text{Too large to display}$$

[In] int(x^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)

[Out] - atan(((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454
158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((53150
2202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/
2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16
 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280))*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (13555990528*x)/((
x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(1/2) - 1
))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i - (((- 2^(1
/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x)/((x + 1)
^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x +
1)^(1/2) - 1) + ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/
16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (12079595520*x)/((x + 1)^(1/2) - 1
)^2 - 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2
)) + 9529458688) - (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i)/((((- 2^(1/2)/16 - 1/16)^(1
/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))/((x + 1)^(1/2) - 1
) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((1127428915
20*x^(1/2))/((x + 1)^(1/2) - 1) - ((531502202880*x)/((x + 1)^(1/2) - 1)^2 -
 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*(
(- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) - (12079595520*x)/
((x + 1)^(1/2) - 1)^2 + 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2)) + (13555990528*x)/((x + 1)^(1/2) - 1)^2 + 9529458688)
+ (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) -
(2^(1/2)/16 - 1/16)^(1/2)) + (((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 -
1/16)^(1/2))*((13555990528*x)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2)
)/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x
 + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/
16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)
) + (12079595520*x)/((x + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 -
1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2
))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2)) + (7549747200*x)/((x + 1)^(1/2) - 1)^2 + 503316480))*((- 2^(1/2)/16

- 1/16)^(1/2)*2i - (2^(1/2)/16 - 1/16)^(1/2)*2i) - atan(((x^(1/2)*(- 2^(1/2
)/16 - 1/16)^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2)*678
4i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(3/2)*6784i)/((x + 1
)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2)
- 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2) - 1) + (x^
(1/2)*(2^(1/2)/16 - 1/16)^2*(- 2^(1/2)/16 - 1/16)^(1/2)*134400i)/((x + 1)^(
1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^(1/2)/16 + 1/16)^2*134400
i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)*(- 2^(1/2)/16 - 1/16)
^(1/2)*20352i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^
(1/2)/16 + 1/16)*20352i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
*(- 2^(1/2)/16 - 1/16)^(3/2)*268800i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/
2)/16 - 1/16)^(3/2)*(2^(1/2)/16 + 1/16)*268800i)/((x + 1)^(1/2) - 1))/(4544
(2^(1/2)/16 - 1/16)^(1/2)(- 2^(1/2)/16 - 1/16)^(1/2) + 65280*(2^(1/2)/16
- 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2) + 65280*(2^(1/2)/16 - 1/16)^(3/2)
(- 2^(1/2)/16 - 1/16)^(1/2) + 345600(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/
16 - 1/16)^(5/2) + 1152000*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^
(3/2) + 345600*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + x/((
x + 1)^(1/2) - 1)^2 + (6464*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/1
6)^(1/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(
1/2)/16 - 1/16)^(3/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)
^(3/2)*(- 2^(1/2)/16 - 1/16)^(1/2))/((x + 1)^(1/2) - 1)^2 - (760320*x*(2^(1
/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2))/((x + 1)^(1/2) - 1)^2 - (
2534400*x*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^(3/2))/((x + 1)^(
1/2) - 1)^2 - (760320*x*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/
2))/((x + 1)^(1/2) - 1)^2 + 1))*((- 2^(1/2)/16 - 1/16)^(1/2)*2i + (2^(1/2)/
16 - 1/16)^(1/2)*2i)

3.619 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4} \, dx = \frac{(f+g)^2(1+x)^4}{5(1-x^2)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15(1-x^2)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2 \arcsin(x)$$

```
[Out] 1/5*(f+g)^2*(1+x)^4/(-x^2+1)^(5/2)+1/15*(f-9*g)*(f+g)*(1+x)^3/(-x^2+1)^(3/2
)-g^2*arcsin(x)+2*g^2*(1+x)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {867, 1649, 803, 667, 222}

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4} \, dx = -g^2 \arcsin(x) + \frac{(x+1)^4(f+g)^2}{5(1-x^2)^{5/2}} + \frac{(x+1)^3(f-9g)(f+g)}{15(1-x^2)^{3/2}} + \frac{2g^2(x+1)}{\sqrt{1-x^2}}$$

```
[In] Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+x)^4(f+gx)^2}{(1-x^2)^{7/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\,(1-x^2)^{5/2}} - \frac{1}{5}\int \frac{(1+x)^3\left(-f^2+8fg+4g^2+5g^2x\right)}{(1-x^2)^{5/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\,(1-x^2)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\,(1-x^2)^{3/2}} + g^2\int \frac{(1+x)^2}{(1-x^2)^{3/2}}\,dx
\end{aligned}$$

$$= \frac{(f+g)^2(1+x)^4}{5(1-x^2)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15(1-x^2)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2 \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{(f+g)^2(1+x)^4}{5(1-x^2)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15(1-x^2)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2 \sin^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \frac{\sqrt{1-x^2}(f^2(-4-3x+x^2)-2fg(-1+3x+4x^2)-3g^2(8-19x+13x^2))}{15(-1+x)^3} + 2g^2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
[Out] (Sqrt[1 - x^2]*(f^2*(-4 - 3*x + x^2) - 2*f*g*(-1 + 3*x + 4*x^2) - 3*g^2*(8
- 19*x + 13*x^2)))/(15*(-1 + x)^3) + 2*g^2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{(1+x)(f^2x^2-8x^2fg-39g^2x^2-3f^2x-6fgx+57g^2x-4f^2+2fg-24g^2)}{15(-1+x)^2\sqrt{-x^2+1}} - g^2\arcsin(x)$
trager	$\frac{(f^2x^2-8x^2fg-39g^2x^2-3f^2x-6fgx+57g^2x-4f^2+2fg-24g^2)\sqrt{-x^2+1}}{15(-1+x)^3} + g^2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2\right.\right.$
default	$g^2\left(\frac{\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}}{(-1+x)^2} + \sqrt{-\left(-1+x\right)^2+2-2x} - \arcsin(x)\right) + \frac{2g(f+g)\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}}{3(-1+x)^3} + (f^2 +$

```
[In] int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x,method=_RETURNVERBOSE)
[Out] -1/15*(1+x)*(f^2*x^2-8*f*g*x^2-39*g^2*x^2-3*f^2*x-6*f*g*x+57*g^2*x-4*f^2+2*
f*g-24*g^2)/(-1+x)^2/(-x^2+1)^(1/2)-g^2*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.41

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$$
$$= \frac{2\,(2\,f^2-fg+12\,g^2)x^3-6\,(2\,f^2-fg+12\,g^2)x^2-4\,f^2+2\,fg-24\,g^2+6\,(2\,f^2-fg+12\,g^2)x+30\,(g^2}{}$$

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2 +
 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 + 3*
g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^2 -
 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(x^3
- 3*x^2 + 3*x - 1)
```

Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \int \frac{\sqrt{-(x-1)\,(x+1)}\,(f+gx)^2}{(x-1)^4}\,dx$$

```
[In] integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

Maxima [F]

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \int \frac{(gx+f)^2\sqrt{-x^2+1}}{(x-1)^4}\,dx$$

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.32

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = -g^2\arcsin(x) + \frac{2\left(4f^2-2fg+24g^2+\frac{5f^2\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{10fg\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{105g^2\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{25f^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{10fg(\cdots}{}\right.}{}$$

[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")

[Out] -g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)/x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f^2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*(sqrt(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(sqrt(-x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) - 1)/x + 1)^5

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

$$\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx = \sqrt{1-x^2}\left(\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{x-1}-\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{\frac{2\,f^2}{5}+\frac{4\,f\,g}{5}+\frac{2\,g^2}{5}}{(x-1)^3}+\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}+\frac{4\,g^2}{15}}{x-1}-\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}+\frac{4\,g^2}{15}}{(x-1)^2}\right) - g^2\,\mathrm{asin}(x) - \frac{\sqrt{1-x^2}\,(4\,g^2+2\,f\,g)}{x-1}$$

[In] int(((f + g*x)^2*(1 - x^2)^(1/2))/(x - 1)^4,x)

[Out] (1 - x^2)^(1/2)*((2*f*g + f^2/3 + (5*g^2)/3)/(x - 1) - (2*f*g + f^2/3 + (5*g^2)/3)/(x - 1)^2) - (1 - x^2)^(1/2)*(((4*f*g)/5 + (2*f^2)/5 + (2*g^2)/5)/(x - 1)^3 + ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1) - ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1)^2) - g^2*asin(x) - ((1 - x^2)^(1/2)*(2*f*g + 4*g^2))/(x - 1)

3.620 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)} dx$

Optimal result

Integrand size = 30, antiderivative size = 107

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)} dx = -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\arcsin(ax)}{d^2} + \frac{(ac-d)^2\arctan\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {867, 1668, 858, 222, 739, 210}

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)} dx = \frac{(ac-d)^2\arctan\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{\arcsin(ax)(ac-2d)}{d^2}$$

```
[In] Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$$

$$
\begin{aligned}
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2} + \frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} - \frac{(ac-d)^2\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx, x, \frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} + \frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$
\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \frac{-d\sqrt{1-a^2x^2} + (-2ac+4d)\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right) - \frac{2(ac-d)\sqrt{a^2c^2-d^2}\arctan\left(\frac{\sqrt{a^2c^2}}{c+dx-c\sqrt{}}\right.}{ac+d}}{d^2}
$$

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] (-(d*Sqrt[1 - a^2*x^2]) + (-2*a*c + 4*d)*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^
2])] - (2*(a*c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(Sqrt[a^2*c^2 - d^2]*x)/(c +
 d*x - c*Sqrt[1 - a^2*x^2])])/(a*c + d))/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}}-\frac{\frac{a(ac-2d)\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}}-\frac{\left(-a^2c^2+2acd-d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{-a^2\left(x+\frac{c}{d}\right)^2+\frac{2a^2c(}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{a^2c^2-d^2}{d^2}}}}{d}$
default	$\frac{-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{1}{a}\right)^2}-3a\left(\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3}-a\left(-\frac{\left(-2a^2\left(x-\frac{1}{a}\right)-2a\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a^2}+\frac{\arctan\left(\frac{\sqrt{}}{\sqrt{-a^2\left(x-\frac{1}{}\right.}}\right.}{2\sqrt{a}}\right.\right.}{a(ac+d)}$

[In] int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*x^2-1)/(-a^2*x^2+1)^(1/2)-1/d*(a*(a*c-2*d)/d/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-(-a^2*c^2+2*a*c*d-d^2)/d^2/(-(a^2*c^2-d^2)/
d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2
)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d)
))

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.97

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}\right.}{dx+c}\right.}{d^2}\right.$$

[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")

[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x

```
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F]

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}{(c+dx)\,(ax-1)^2}\,dx$$

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

Maxima [F]

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = \int \frac{(-a^2x^2+1)^{\frac{3}{2}}}{(ax-1)^2(dx+c)}\,dx$$

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(99) = 198$.

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.94

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx =$$

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\operatorname{sgn}\left(\frac{1}{ax-1}\right)\operatorname{sgn}(a)}{ad} - \frac{2\left(ac\operatorname{sgn}\left(\frac{1}{ax-1}\right)\operatorname{sgn}(a) - 2\,d\operatorname{sgn}\left(\frac{1}{ax-1}\right)\operatorname{sgn}(a)\right)\arctan\left(\sqrt{-\frac{2}{ax}}\right.}{ad^2}\right.$$

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*s
gn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x -
 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(a*x
 - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x - 1)
- 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*
a*d^2))*abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx = -\frac{\sqrt{1-a^2\,x^2}}{d} - \frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d} - \frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

```
[In] int((1 - a^2*x^2)^(3/2)/((a*x - 1)^2*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

3.621 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\arcsin(ax)}{d^2}+\frac{(ac-d)^2\arctan\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 222, 739, 210}

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \frac{(ac-d)^2\arctan\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\arcsin(ax)(ac-2d)}{d^2}$$

```
[In] Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} + \frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{-d\sqrt{1-a^2x^2}+(-2ac+4d)\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)-\frac{2(ac-d)\sqrt{a^2c^2-d^2}\arctan\left(\frac{\sqrt{a^2c^2-d^2}x}{c+dx-c\sqrt{1-a^2x^2}}\right)}{ac+d}}{d^2}$$

```
[In] Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-(d*Sqrt[1 - a^2*x^2]) + (-2*a*c + 4*d)*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^
2])] - (2*(a*c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(Sqrt[a^2*c^2 - d^2]*x)/(c +
 d*x - c*Sqrt[1 - a^2*x^2])])/(a*c + d))/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.

Time = 0.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

method	result
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}} - \frac{\frac{a(ac-2d)\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}} - \frac{\left(-a^2c^2+2acd-d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{-a^2\left(x+\frac{c}{d}\right)^2+\frac{2a^2}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{-\frac{a^2c^2-d^2}{d^2}}}}{d}$
default	$-\frac{a\left(-\frac{2d\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{ac\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{d\sqrt{-a^2x^2+1}}{a}\right)}{d^2} - \frac{\left(a^2c^2-2acd+d^2\right)\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\sqrt{-}\right.}{}\right.}{d^3\sqrt{-\frac{a^2c^2}{d^2}}}$

```
[In] int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*x^2-1)/(-a^2*x^2+1)^(1/2)-1/d*(a*(a*c-2*d)/d/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-(-a^2*c^2+2*a*c*d-d^2)/d^2/(-(a^2*c^2-d^2)/
d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2
)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.97

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$$
$$= \left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac}{d^2}\right.$$

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F]

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \int \frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

```
[In] integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a*c>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{(a^2c-2\,ad)\arcsin{(ax)}\,\mathrm{sgn}(a)}{d^2|a|} - \frac{\sqrt{-a^2x^2+1}}{d} - \frac{2\,(a^3c^2-2\,a^2cd+ad^2)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}d^2|a|}$$

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d - 2
*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a)*
c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2\,x^2}}{d} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d} - \frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

```
[In] int((a*x + 1)^2/((1 - a^2*x^2)^(1/2)*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

3.622 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 851

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-798d^3e}{3465c^2eg^4}$$
$$+\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$
$$-\frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3))(f+gx)^{3/2}\sqrt{a+cx^2}}{3465cg^4}$$
$$+\frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$
$$+\frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$
$$+\frac{4\sqrt{-a}(3a^2e^2g^4(26ef+231dg)-c^2f^2(64e^3f^3-264de^2f^2g+396d^2efg^2-231d^3g^3)-9acg^2(6e^3f^3-33}{3465c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{}}$$
$$-\frac{4\sqrt{-a}(cf^2+ag^2)(75a^2e^3g^4-3aceg^2(2e^2f^2-33defg+165d^2g^2)-c^2f(64e^3f^3-264de^2f^2g+396d^2ef}{3465c^{5/2}g^5\sqrt{f+gx}\sqrt{a+}}$$

```
[Out] -2/3465*(2*a*e^2*g^2*(-231*d*g+74*e*f)-c*(-567*d^3*g^3+1107*d^2*e*f*g^2-843
*d*e^2*f^2*g+233*e^3*f^3))*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^4+2/693*e*(18*
a*e^2*g^2-c*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+a)^(1/
2)/c/g^4+2/99*e^2*(-3*d*g+e*f)*(g*x+f)^(7/2)*(c*x^2+a)^(1/2)/g^4-2/3465*(15
```

```
0*a^2*e^4*g^4-6*a*c*e^2*g^2*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+c^2*(315*d^4
*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+187*e^4*f^4))*(g*
x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+a)
^(1/2)/e+4/3465*(3*a^2*e^2*g^4*(231*d*g+26*e*f)-c^2*f^2*(-231*d^3*g^3+396*d
^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)-9*a*c*g^2*(77*d^3*g^3+88*d^2*e*f*g^2
-33*d*e^2*f^2*g+6*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*
(1+c*x^2/a)^(1/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2)-4/3465*(a*g^2+c*f^2)*(75*a^2*e^3*g^4-3*a*c*e*g^2*(165*d
^2*g^2-33*d*e*f*g+2*e^2*f^2)-c^2*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*
f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2
*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(5/2)/g^5/(g*x+f)^(1/2)/(c*x^
2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {933, 1668, 858, 733, 435, 430}

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}$$

$$+\frac{4\sqrt{-a}(3a^2e^2(26ef+231dg)g^4-9ac(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3)\,g^2-c^2f^2(64e^3f^3-264d}{3465c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}$$

$$-\frac{4\sqrt{-a}(cf^2+ag^2)\,(75a^2e^3g^4-3ace(2e^2f^2-33degf+165d^2g^2)\,g^2-c^2f(64e^3f^3-264de^2gf^2+396d^2}{3465c^{5/2}g^5\sqrt{f+gx}\sqrt{cx}}$$

$$+\frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{99g^4}$$

$$+\frac{2e(18ae^2g^2-c(29e^2f^2-96degf+81d^2g^2))\,(f+gx)^{5/2}\sqrt{cx^2+a}}{693cg^4}$$

$$-\frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3))\,(f+gx)^{3/2}\sqrt{cx^2+a}}{3465cg^4}$$

$$-\frac{2(150a^2e^4g^4-6ace^2(2e^2f^2-33degf+165d^2g^2)\,g^2+c^2(187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d}{3465c^2eg^4}$$

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2)
 + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*
g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d
 + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f -
 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g
^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(
693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4)
 + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 26
4*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*
d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^
3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*
f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3
465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{\int \frac{(d+ex)^3(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4} + \frac{2\int \frac{-\frac{1}{2}acg^2(7e^4f^4-21de^3f^3g-27d^3efg^3+9d^4g^4)-\frac{1}{2}cg(3aeg^2(7e^3f^3-21de^2f^2g-27d^2efg^2+3d^3g^3)+2c(e^4f^5-3de^3f^4g+9d^4fg}{}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4} + \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4} + \frac{4\int \frac{-\frac{3}{4}acg^6(30ae^4f^2g^2-c(32e^4f^4-111de^3f^3g+135d^2e^2f^2g^2+63d^3efg^3-21d^4g^4))-\frac{1}{4}cg^5(180a^2e^4fg^4-aceg^2(107e^3f^3-519de}{}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$
$$- \frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3))\,(f+gx)^{3/2}\sqrt{a+c}}{3465cg^4}$$
$$+ \frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))\,(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$
$$+ \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$
$$+ \frac{8\int \frac{-\frac{3}{8}ac^2g^9\left(2ae^3fg^2(ef+231dg)+c\left(73e^4f^4-288de^3f^3g+432d^2e^2f^2g^2-882d^3efg^3+105d^4g^4\right)\right)-\frac{3}{4}c^2g^8\left(a^2e^3g^4(76ef+231dg)-1\right.}{}}{}$$

$$=$$
$$- \frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2 -}{3465c^2eg^4}$$
$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$
$$- \frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3))\,(f+gx)^{3/2}\sqrt{a+c}}{3465cg^4}$$
$$+ \frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))\,(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$
$$+ \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$
$$+ \frac{16\int \frac{\frac{3}{8}ac^2eg^{11}\left(75a^2e^3g^4-9aceg^2\left(e^2f^2+66defg+55d^2g^2\right)-c^2f\left(16e^3f^3-66de^2f^2g+99d^2efg^2-924d^3g^3\right)\right)-\frac{3}{8}c^3eg^{10}\left(3a^2e^2g^4(26e\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{10395c^4eg^{14}}$$

$$=$$
$$- \frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2 -}{3465c^2eg^4}$$
$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$
$$- \frac{2(2ae^2g^2(74ef-231dg)-c(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3))\,(f+gx)^{3/2}\sqrt{a+c}}{3465cg^4}$$
$$+ \frac{2e(18ae^2g^2-c(29e^2f^2-96defg+81d^2g^2))\,(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$
$$+ \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$
$$+ \frac{(2(cf^2+ag^2)\,(75a^2e^3g^4-3aceg^2(2e^2f^2-33defg+165d^2g^2)-c^2f(64e^3f^3-264de^2f^2g+396d}{3465c^2g^5}$$
$$- \frac{(2(3a^2e^2g^4(26ef+231dg)-c^2f^2(64e^3f^3-264de^2f^2g+396d^2efg^2-231d^3g^3)-9acg^2(6e^3f^3 -}{3465cg^5}$$

$$= -\frac{2(150a^2e^4g^4 - 6ace^2g^2(2e^2f^2 - 33defg + 165d^2g^2) + c^2(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2}{3465c^2eg^4}$$

$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$

$$- \frac{2(2ae^2g^2(74ef - 231dg) - c(233e^3f^3 - 843de^2f^2g + 1107d^2efg^2 - 567d^3g^3))\,(f+gx)^{3/2}\sqrt{a+}}{3465cg^4}$$

$$+ \frac{2e(18ae^2g^2 - c(29e^2f^2 - 96defg + 81d^2g^2))\,(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$

$$+ \frac{2e^2(ef - 3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$

$$- \frac{\Big(4a(3a^2e^2g^4(26ef + 231dg) - c^2f^2(64e^3f^3 - 264de^2f^2g + 396d^2efg^2 - 231d^3g^3) - 9acg^2(6e^3}{3465\sqrt{-a}c^{3/2}g}$$

$$+ \frac{\Big(4a(cf^2 + ag^2)\,(75a^2e^3g^4 - 3aceg^2(2e^2f^2 - 33defg + 165d^2g^2) - c^2f(64e^3f^3 - 264de^2f^2g + 3}{3465\sqrt{-a}c^{5/2}g^5\sqrt{}}$$

$$= -\frac{2(150a^2e^4g^4 - 6ace^2g^2(2e^2f^2 - 33defg + 165d^2g^2) + c^2(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2}{3465c^2eg^4}$$

$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e}$$

$$- \frac{2(2ae^2g^2(74ef - 231dg) - c(233e^3f^3 - 843de^2f^2g + 1107d^2efg^2 - 567d^3g^3))\,(f+gx)^{3/2}\sqrt{a+}}{3465cg^4}$$

$$+ \frac{2e(18ae^2g^2 - c(29e^2f^2 - 96defg + 81d^2g^2))\,(f+gx)^{5/2}\sqrt{a+cx^2}}{693cg^4}$$

$$+ \frac{2e^2(ef - 3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$

$$+ \frac{4\sqrt{-a}(3a^2e^2g^4(26ef + 231dg) - c^2f^2(64e^3f^3 - 264de^2f^2g + 396d^2efg^2 - 231d^3g^3) - 9acg^2(6e}{3465c^{3/2}g^5\sqrt{\frac{\sqrt{}}{\sqrt{c}}}}$$

$$- \frac{4\sqrt{-a}(cf^2 + ag^2)\,(75a^2e^3g^4 - 3aceg^2(2e^2f^2 - 33defg + 165d^2g^2) - c^2f(64e^3f^3 - 264de^2f^2g -}{3465c^{5/2}g^5\sqrt{f+gx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.02 (sec) , antiderivative size = 1045, normalized size of antiderivative = 1.23

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(-150a^2e^3g^4+2aceg^2\left(495d^2g^2+33deg(4f+7gx)+e^2\left(-23f^2+16fgx+45g^2x^2\right)\right)+c^2\left(231d^3g^3(f+3gx)+99d^2eg^2\left(-4f^2+3\right.\right.\right.}{c^2g^4}\right.}{}$$

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(-150*a^2*e^3*g^4 + 2*a*c*e*g^2*(495*d^2*g^2
 + 33*d*e*g*(4*f + 7*g*x) + e^2*(-23*f^2 + 16*f*g*x + 45*g^2*x^2)) + c^2*(2
31*d^3*g^3*(f + 3*g*x) + 99*d^2*e*g^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + 33*
d*e^2*g*(8*f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3) + e^3*(-64*f^4 + 48*
f^3*g*x - 40*f^2*g^2*x^2 + 35*f*g^3*x^3 + 315*g^4*x^4))))/(c^2*g^4) - (4*(f
 + g*x)*((g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a^2*e^2*g^4*(26*e*f + 231
*d*g) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3
) + c^2*f^2*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*f*g^2 + 231*d^3*g^3)
)*(a + c*x^2))/(f + g*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(3*a^2*e
^2*g^4*(26*e*f + 231*d*g) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*
e*f*g^2 + 77*d^3*g^3) + c^2*f^2*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*
f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*((-7
5*I)*a^2*e^3*g^4 - 3*a^(3/2)*Sqrt[c]*e^2*g^3*(e*f + 231*d*g) + (3*I)*a*c*e*
g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + I*c^2*f*(64*e^3*f^3 - 264*d*e^
2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 3*Sqrt[a]*c^(3/2)*g*(16*e^3*f^3
- 66*d*e^2*f^2*g + 99*d^2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt
[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Ellip
ticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
- I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^6*Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]])))/(3465*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1823 vs. 2(761) = 1522.

Time = 2.82 (sec) , antiderivative size = 1824, normalized size of antiderivative = 2.14

method	result	size
elliptic	Expression too large to display	1824
risch	Expression too large to display	2571
default	Expression too large to display	6457

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/11*e^3*x^4*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c/g*x^3*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3
*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c/g*x^2*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5
*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e^
3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*
e^3)/g*f)/g*f)/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d^2*e*g+3*a
*d*e^2*f+c*d^3*f-2/3*(3*c*d*e^2*g+1/11*f*c*e^3)/c/g*f*a-5/7*(2/11*a*e^3*g+3
*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c*a-4/5*(3*a*e^2
*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c*a-6/
7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)
/g*f)/g*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^3*f-2/5*(3*a*e^2*g*
d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c*a-6/7*(
2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/g*
f)/c/g*f*a-1/3*(3*a*d^2*e*g+3*a*d*e^2*f+c*d^3*f-2/3*(3*c*d*e^2*g+1/11*f*c*e
^3)/c/g*f*a-5/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11
*f*c*e^3)/g*f)/c*a-4/5*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3
*c*d*e^2*g+1/11*f*c*e^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*
(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/g*f)/g*f)/c*a)*(f/g-(-a*c)^(1/2)/c)*((x+f/g
)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x
+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(a*d^3*g+3*a*d^2*e*f-4/7*(2/11*a*e^3
*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/c/g*f*a-3/5*
(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-7/9*(3*c*d*e^2*g+1/11*f*c*e^3
)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2*g+1/11*f*c*e
^3)/g*f)/g*f)/c*a-2/3*(3*a*d^2*e*g+3*a*d*e^2*f+c*d^3*f-2/3*(3*c*d*e^2*g+1/1
1*f*c*e^3)/c/g*f*a-5/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9*(3*c*d*e^2
*g+1/11*f*c*e^3)/g*f)/c*a-4/5*(3*a*e^2*g*d+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f
-7/9*(3*c*d*e^2*g+1/11*f*c*e^3)/c*a-6/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2
*f-8/9*(3*c*d*e^2*g+1/11*f*c*e^3)/g*f)/g*f)/g*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^
```

```
2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c
)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 765, normalized size of antiderivative = 0.90

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{2\left(2\left(64\,c^3e^3f^6-264\,c^3de^2f^5g+6\left(66\,c^3d^2e+17\,ac^2e^3\right)f^4g^2-33\left(7\,c^3d^3+15\,ac^2de^2\right)f^3g^3+3\left(363\,ac^2d\right.\right.\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/10395*(2*(64*c^3*e^3*f^6 - 264*c^3*d*e^2*f^5*g + 6*(66*c^3*d^2*e + 17*a*
c^2*e^3)*f^4*g^2 - 33*(7*c^3*d^3 + 15*a*c^2*d*e^2)*f^3*g^3 + 3*(363*a*c^2*d
^2*e - 17*a^2*c*e^3)*f^2*g^4 - 99*(21*a*c^2*d^3 - 11*a^2*c*d*e^2)*f*g^5 + 4
5*(33*a^2*c*d^2*e - 5*a^3*e^3)*g^6)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^
2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)
 + 6*(64*c^3*e^3*f^5*g - 264*c^3*d*e^2*f^4*g^2 + 18*(22*c^3*d^2*e + 3*a*c^2
*e^3)*f^3*g^3 - 33*(7*c^3*d^3 + 9*a*c^2*d*e^2)*f^2*g^4 + 6*(132*a*c^2*d^2*e
 - 13*a^2*c*e^3)*f*g^5 + 693*(a*c^2*d^3 - a^2*c*d*e^2)*g^6)*sqrt(c*g)*weier
strassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g
^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(315*c^3*e^3*g^6*x^4 - 64*c^3*e^3*f^4*
g^2 + 264*c^3*d*e^2*f^3*g^3 - 2*(198*c^3*d^2*e + 23*a*c^2*e^3)*f^2*g^4 + 33
*(7*c^3*d^3 + 8*a*c^2*d*e^2)*f*g^5 + 30*(33*a*c^2*d^2*e - 5*a^2*c*e^3)*g^6
+ 35*(c^3*e^3*f*g^5 + 33*c^3*d*e^2*g^6)*x^3 - 5*(8*c^3*e^3*f^2*g^4 - 33*c^3
*d*e^2*f*g^5 - 9*(33*c^3*d^2*e + 2*a*c^2*e^3)*g^6)*x^2 + (48*c^3*e^3*f^3*g^
3 - 198*c^3*d*e^2*f^2*g^4 + (297*c^3*d^2*e + 32*a*c^2*e^3)*f*g^5 + 231*(3*c
^3*d^3 + 2*a*c^2*d*e^2)*g^6)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^3*g^6)
```

Sympy [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}(d+ex)^3\,\sqrt{f+gx}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^3\,\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^3\,\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{(d+e\,x)}^3\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3,x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3, x)
```

3.623 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 635

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e}$$
$$+\frac{4(7ae^2g^2-c(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3}$$
$$+\frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3}$$
$$+\frac{4\sqrt{-a}(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg+21d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{c}{}}}{315c^{3/2}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}(cf^2+ag^2)(3aeg^2(ef-10dg)+cf(8e^2f^2-24defg+21d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\Big(}{315c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 4/315*(7*a*e^2*g^2-c*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^
2+a)^(1/2)/c/g^3+2/63*e*(-3*d*g+e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/g^3-2/31
5*(6*a*e^2*g^2*(-10*d*g+e*f)-c*(-35*d^3*g^3+63*d^2*e*f*g^2-57*d*e^2*f^2*g+1
9*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/
2)*(c*x^2+a)^(1/2)/e+4/315*(21*a^2*e^2*g^4+3*a*c*g^2*(-21*d^2*g^2-16*d*e*f*
g+3*e^2*f^2)+c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*
```

```
c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2
))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/2)/g^4/(c*x^2+a)^(1/2)/(
(g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/315*(a*g^2+c*f^2)*(3*a*e*
g^2*(-10*d*g+e*f)+c*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))
^(1/2)/c^(3/2)/g^4/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {933, 1668, 858, 733, 435, 430}

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(21a^2e^2g^4+3acg^2(-21d^2g^2-16defg+3e^2f^2)+c^2f^2(21d^2g^2-24defg+8e^2f}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(3aeg^2(ef-10dg)+cf(21d^2g^2-24defg+8e^2f^2))\,\text{EllipticF}}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+\frac{4\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2-c(21d^2g^2-24defg+8e^2f^2)\right)}{315cg^3}$$

$$-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(6ae^2g^2(ef-10dg)-c(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3))}{315ceg^3}$$

$$+\frac{2e\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{63g^3}+\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e
*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*e*g^3) + (2*(d
+ e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*
f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*c*g^3
) + (2*e*(e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^3) + (4*Sqrt[
-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2
*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*
```

```
(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(315*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{\int \frac{(d+ex)^2\left(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{2\int \frac{-\frac{1}{2}acg^2\left(5e^3f^3-15de^2f^2g-21d^2efg^2+7d^3g^3\right)-cfg\left(ae^2g^2(5ef-36dg)+c\left(e^3f^3-3de^2f^2g+7d^3g^3\right)\right)x+\frac{1}{2}cg^2\left(4ae^2g^2(4ef+9dg\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{63ceg^4}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4\left(7ae^2g^2-c\left(8e^2f^2-24defg+21d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{4\int \frac{-\frac{1}{4}acg^5\left(42ae^3fg^2-c\left(23e^3f^3-69de^2f^2g+231d^2efg^2-35d^3g^3\right)\right)-\frac{1}{2}cg^4\left(21a^2e^3g^4+3aceg^2\left(5e^2f^2-36defg-21d^2g^2\right)-c^2f(11\right.}{\sqrt{f+gx}\sqrt{a+c}}}{315c^2eg^7}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4\left(7ae^2g^2-c\left(8e^2f^2-24defg+21d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{8\int \frac{-\frac{3}{2}ac^2eg^7\left(3aeg^2(3ef+5dg)-cf\left(e^2f^2-3defg+42d^2g^2\right)\right)-\frac{3}{4}c^2eg^6\left(21a^2e^2g^4+3acg^2\left(3e^2f^2-16defg-21d^2g^2\right)+c^2f^2\left(8e^2f^2-\right.\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{945c^3eg^9}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4(7ae^2g^2-c(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{(2(cf^2+ag^2)(3aeg^2(ef-10dg)+cf(8e^2f^2-24defg+21d^2g^2)))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{315cg^4} - \frac{(2(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg+21d^2g^2)))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{315cg^4}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4(7ae^2g^2-c(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} - \frac{\left(4a(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg+21d^2g^2))\sqrt{f+gx}\sqrt{\right.}{315\sqrt{-a}c^{3/2}g^4\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(4a(cf^2+ag^2)(3aeg^2(ef-10dg)+cf(8e^2f^2-24defg+21d^2g^2))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}}{315\sqrt{-a}c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4(7ae^2g^2-c(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^3} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{4\sqrt{-a}(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg+21d^2g^2))\sqrt{f+g}}{315c^{3/2}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}(cf^2+ag^2)(3aeg^2(ef-10dg)+cf(8e^2f^2-24defg+21d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F}{315c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.89 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.27

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(2aeg^2(4ef+30dg+7egx)+c(21d^2g^2(f+3gx)+6deg(-4f^2+3fgx+15g^2x^2)+e^2(8f^3-6f^2gx+5fg^2x^2+35g^3x^3)))}{cg^3} - \right.}{}$$

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(2
1*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*f^
3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21
*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x^2) -
 I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 -
 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2
))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt
[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqr
```

```
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*e^
2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^2 + 24*d*e*f
*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g + 21*d^2*g^2))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(315*Sqrt
[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(551) = 1102$.

Time = 1.40 (sec) , antiderivative size = 1142, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1142
risch	Expression too large to display	1677
default	Expression too large to display	4352

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9*e^2*x^3*(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2*c*d*e*g+1/9*c*e^2*f)/c/g*x^2*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*
g+1/9*c*e^2*f))/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*a*d*e*g+1/3*
a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1
/9*c*e^2*f))-5/7*a/c*(2*c*d*e*g+1/9*c*e^2*f))/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)+2*(a*d^2*f-2/5*f*a/c/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c
*d*e*g+1/9*c*e^2*f))-1/3*a/c*(2*a*d*e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*
e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*c*e^2*f))-5/7*a/c*(2*c*d*e*g
+1/9*c*e^2*f)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*(
(x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(
-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/
(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2))+2*(a*d^2*g+2*a*d*e*f-4/7*f*a/c/g*(2*c*d*e*g+1/9*c*e^2*f)-3/5*a/c*(2/9
*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*c*e^2*f))-2/3*f/g*(2*a*d*
e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c
*d*e*g+1/9*c*e^2*f))-5/7*a/c*(2*c*d*e*g+1/9*c*e^2*f)))*(f/g-(-a*c)^(1/2)/c)
*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2
)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x
^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(
1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^
(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
```

```
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.80

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{2\left(2\left(8\,c^2e^2f^5-24\,c^2def^4g-66\,acdef^2g^3-90\,a^2deg^5+3\left(7\,c^2d^2+5\,ace^2\right)f^3g^2+3\left(63\,acd^2-11\,a^2e^2\right)fg\right.\right.}{}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(2*(8*c^2*e^2*f^5 - 24*c^2*d*e*f^4*g - 66*a*c*d*e*f^2*g^3 - 90*a^2*d*
e*g^5 + 3*(7*c^2*d^2 + 5*a*c*e^2)*f^3*g^2 + 3*(63*a*c*d^2 - 11*a^2*e^2)*f*g
^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f
^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(8*c^2*e^2*f^4*g - 24*c^2*d
*e*f^3*g^2 - 48*a*c*d*e*f*g^4 + 3*(7*c^2*d^2 + 3*a*c*e^2)*f^2*g^3 - 21*(3*a
*c*d^2 - a^2*e^2)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*
(35*c^2*e^2*g^5*x^3 + 8*c^2*e^2*f^3*g^2 - 24*c^2*d*e*f^2*g^3 + 60*a*c*d*e*g
^5 + (21*c^2*d^2 + 8*a*c*e^2)*f*g^4 + 5*(c^2*e^2*f*g^4 + 18*c^2*d*e*g^5)*x^
2 - (6*c^2*e^2*f^2*g^3 - 18*c^2*d*e*f*g^4 - 7*(9*c^2*d^2 + 2*a*c*e^2)*g^5)*
x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)^2\,\sqrt{f+gx}\,dx$$

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2,x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2, x)
```

3.624 $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 434

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c} - \frac{4\sqrt{-a}(cf^2(4ef-7dg)+ag^2(8ef+21dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}(cf^2+ag^2)(5aeg^2+cf(4ef-7dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{\ }{\sqrt{-a}}\right)}{105c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/7*e*(c*x^2+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(5*a*e*g^2+c*f*(-7*d*g+4*e*f)-3
*c*g*(7*d*g+e*f)*x)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2-4/105*(c*f^2*(-7*d*
g+4*e*f)+a*g^2*(21*d*g+8*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/
2)*(1+c*x^2/a)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e*g^2+c*f*(-7*d*g+4*e*f))*E
llipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1
/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {847, 829, 858, 733, 435, 430}

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(5aeg^2+cf(4ef-7dg))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-}\right.}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ag^2(21dg+8ef)+cf^2(4ef-7dg))\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

$$-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{105cg^2}$$

$$+\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)
*Sqrt[a + c*x^2])/(105*c*g^2) + (2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c)
 - (4*Sqrt[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(c*f^2
+ a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^
3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2e\sqrt{f+gx}\left(a+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-aeg)+\frac{1}{2}c(ef+7dg)x\right)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx}{7c}$$

$$= -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c} + \frac{8\int \frac{-\frac{1}{4}acg(5aeg^2+cf(ef-28dg))+\frac{1}{4}c^2(cf^2(4ef-7dg)+ag^2(8ef+21dg))x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c} - \frac{(2(cf^2+ag^2)(5aeg^2+cf(4ef-7dg)))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{105cg^3} + \frac{(2(cf^2(4ef-7dg)+ag^2(8ef+21dg)))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{105g^3}$$

$$= -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c} + \frac{\left(4a(cf^2(4ef-7dg)+ag^2(8ef+21dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{}}{}\right.}{105\sqrt{-a}\sqrt{c}g^3\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(4a(cf^2+ag^2)(5aeg^2+cf(4ef-7dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,d\right.}{105\sqrt{-a}c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{f+gx}(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}(a+cx^2)^{3/2}}{7c} - \frac{4\sqrt{-a}(cf^2(4ef-7dg)+ag^2(8ef+21dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right.}{105\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}(cf^2+ag^2)(5aeg^2+cf(4ef-7dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}}\right.}{105c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.39 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.41

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(10aeg^2+7cdg(f+3gx)+ce(-4f^2+3fgx+15g^2x^2))}{cg^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(cf^2(4ef-7dg)+ag^2(8ef+21dg))(a+cx^2)+\right.}{}\right.}{}$$

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4*
f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*Sqrt
[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f + 21*d
*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 + I*c*f*(
4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(362) = 724$.

Time = 1.44 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e\,x^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7}+\frac{2\left(cdg+\frac{1}{7}cef\right)x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(\frac{2aeg}{7}+cdf-\frac{4f\left(cdg+\frac{1}{7}cef\right)}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+a}}{3cg}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e*x^2*(c*g*x^3
+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(c*d*g+1/7*c*e*f)/c/g*x*(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)+2/3*(2/7*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f))/c/g*(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d*f-2/5*a/c*f/g*(c*d*g+1/7*c*e*f)-1/3*a/c*(2/7
*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((
x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^
(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(a*d*g+3/7*a*e*f-3/5*a/c*(c*d*g+1/7*c*e*f)
-2/3*f/g*(2/7*a*e*g+c*d*f-4/5*f/g*(c*d*g+1/7*c*e*f)))*(f/g-(-a*c)^(1/2)/c)*
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c
)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.79

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{2\left(2\left(4c^2ef^4-7c^2df^3g+11acef^2g^2-63acdfg^3+15a^2eg^4\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8}{}\right.}{}$$

[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -2/315*(2*(4*c^2*e*f^4 - 7*c^2*d*f^3*g + 11*a*c*e*f^2*g^2 - 63*a*c*d*f*g^3 + 15*a^2*e*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2) , -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c^2*e*f^3*g - 7*c^2*d*f^2*g^2 + 8*a*c*e*f*g^3 + 21*a*c*d*g^4)*sqrt(c*g)*weierstrassZeta (4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstr assPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(15*c^2*e*g^4*x^2 - 4*c^2*e*f^2*g^2 + 7*c^2*d*f* g^3 + 10*a*c*e*g^4 + 3*(c^2*e*f*g^3 + 7*c^2*d*g^4)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^4)

Sympy [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)\sqrt{f+gx}\,dx$$

[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)

[Out] Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)

Maxima [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)

Giac [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x),x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x), x)
```

3.625 $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 362

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}$$
$$+\frac{4\sqrt{-a}(cf^2-3ag^2)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}f(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g-4/15*f*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+
4/15*(-3*a*g^2+c*f^2)*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x
^2/a)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)-4/15*f*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^
2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g^2/c^(1/2)/(g*
x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {749, 847, 858, 733, 435, 430}

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= -\frac{4\sqrt{-a}f\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(cf^2-3ag^2)\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}-\frac{4f\sqrt{a+cx^2}\sqrt{f+gx}}{15g}$$

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-4*f*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*g) + (2*(f + g*x)^(3/2)*Sqrt[a + c
*x^2])/(5*g) + (4*Sqrt[-a]*(c*f^2 - 3*a*g^2)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqr
t[-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*f*(c*f^2 + a*g^2)*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
 a*g)])/(15*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}+\frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{5g}\\
&= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}+\frac{4\int \frac{2acfg-\frac{1}{2}c(cf^2-3ag^2)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15cg}\\
&= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}\\
&\quad+\frac{1}{15}\left(2\left(3a-\frac{cf^2}{g^2}\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx\\
&\quad+\frac{1}{15}\left(2f\left(a+\frac{cf^2}{g^2}\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx
\end{aligned}
$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{\left(4a\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(4af\left(a+\frac{cf^2}{g^2}\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} - \frac{4\sqrt{-a}\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}f\left(a+\frac{cf^2}{g^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.44

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{\sqrt{f+gx}\left(\frac{2(f+3gx)(a+cx^2)}{g} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(cf^2-3ag^2)(a+cx^2)+\sqrt{c}\left(-ic^{3/2}f^3+\sqrt{a}cf^2g+3ia\sqrt{c}fg^2-3a^{3/2}g^3\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]*(c*f^2 - 3*a*g^2)*(a + c*x^2) + Sqrt[c]*((-I)*c^(3/2)*f^3 + S
qrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3)*Sqrt[(g*((I*Sqrt[a]
```

```
)/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]
*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[
f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*S
qrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f
 + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^3*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(290) = 580$.

Time = 0.93 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.66

method	result
risch	$\frac{2(3gx+f)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g}+\frac{2\left(\frac{8afg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2\left(3a\,g\right.}{}\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5}+\frac{2f\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{15g}+\frac{16fa\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F}{15\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*g*x+f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/15/g*(8*a*f*g*(f/g-(-a*c)^
(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*a*g^2-c*f^2)*(f/g
-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(
-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
```

```
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/
g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))*((g*x+f)*(c*x^2+a))^(1/2
)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.63

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{2\left(2\,(cf^3+9\,afg^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)+6\,(cf^2g-3\,ag^3)\sqrt{cg}\text{weier}\right.}{}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(2*(c*f^3 + 9*a*f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(c*
f^2*g - 3*a*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -
8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)
/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(3*c*g
^3*x + c*f*g^2)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^3)
```

Sympy [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

Maxima [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Giac [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2), x)
```

3.626 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 683

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}f(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e-2/3*(-3*d*g+e*f)*EllipticE(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/g/(c*x^2+a)^(1/2)/((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*(2*a*e^2*g-3*c*d*(-d*g+e
*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
```

```
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+2/3
*f*(-3*d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*
((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2
+a)^(1/2)-2*(a*e^2+c*d^2)*(-d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2)
)^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {933, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)} + \frac{2\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ef-3dg)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}} - \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(2ae^2g-3cd(ef-dg)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[(Sqrt
```

```
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqr
t[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2
]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c
]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && !LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \frac{a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \left(\frac{2ae^2g-3cd(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{c(ef-3dg)x}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3(cd^2+ae^2)(ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2} + \frac{((cd^2+ae^2)(ef-dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(2ag-\frac{3cd(ef-dg)}{e^2}\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3e^2 g} - \frac{(cf(ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2 g} + \frac{\left((cd^2+ae^2)(ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e^3\sqrt{a+cx^2}} + \frac{\left(2a\left(2ag-\frac{3cd(ef-dg)}{e^2}\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(cd^2+ae^2)(ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right.}{e^3\sqrt{a+cx^2}} + \frac{\left(2a\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}e^2g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(2a\sqrt{c}f(ef-3dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}e^2g\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}f(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx, x}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}f(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.08 (sec) , antiderivative size = 1216, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(f+gx)^{3/2}\left(2ce^2f\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-6cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+\frac{2ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2}-\frac{6cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2}+\frac{2ae^2fg^2\sqrt{-f-}}{(f+gx)^2}\right.}{}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
(2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*Sqrt[
c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x) - (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c
]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]
 + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[1 -
 f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*a*e^2*g^2*Sqrt[1 - f
/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]
*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(
3*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/
(f + g*x))^2)/g^2])
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3e}+\frac{2\left(\frac{a\,e^2g+c\,d^2g-cdef}{e^3}-\frac{ag}{3e}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{\cdots}{\frac{f}{g}\cdots}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/e*(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)+2*((a*e^2*g+c*d^2*g-c*d*e*f)/e^3-1/3/e*a*g)*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/
2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(-(d*g-e*f)/e^2*c
-2/3/e*c*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c
)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*
EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))-2*(a*d*e^
2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c
)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*
c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/
(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/
2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.627 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 650

$$\begin{aligned}
\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx &= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&+ \frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} \\
&- \frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&- \frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)-3*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)
*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+3*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(
```

```
1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-3*d*g+2*e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-
a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1
/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elliptic
Pi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),
2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))
/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {931, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$$

$$= -\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g-cd(2ef-3dg))\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)} + \frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}} - \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt
[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
```

```
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e
*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/
(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*
g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]
*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{c(2ef-3dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2e}$$

$$
\begin{aligned}
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{(c(2ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} \\
&\quad + \frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} \\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} \\
&\quad + \frac{\left((ae^2g-cd(2ef-3dg))\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2e^3\sqrt{a+cx^2}} \\
&\quad + \frac{\left(a\sqrt{c}(2ef-3dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} \\
&\quad - \frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&\quad - \frac{\left((ae^2g-cd(2ef-3dg))\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}\right.}{e^3\sqrt{a+cx^2}} \\
&\quad + \frac{\left(3a\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} \\
&\quad - \frac{\left(3a\sqrt{c}f\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^2\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

$$=-\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)}-\frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\left((ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx\right.}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)}-\frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.77 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(-\frac{e^2(a+cx^2)}{d+ex} - \frac{-3ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+3cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-3ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+3adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+6ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f-}{}\right.}{}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a
*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]*(2*e*f - 3
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
 + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*
(f + g*x))))/(e^3*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.38

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{e(ex+d)}+\frac{2\left(-\frac{c(2dg-ef)}{e^3}+\frac{cdg}{2e^3}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+}{\frac{f}{g}-\sqrt{}}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/e*(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(-c*(2*d*g-e*f)/e^3+1/2*c*d/e^3*g)*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/
2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+3*c/e^2*g*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/
2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-
a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/e^4*(a*e^2*g+3*c*d^2*g-2*c*d
*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2
)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g
)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c
)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2,x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.628 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1205

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)}$$
$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)^2-1/4*(a*e^2*g-c*d*(-3*d*g+2*e
*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-1/4*(
a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f
)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-3/2*g*EllipticF(1/2*(1-x*c^(1/
2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-
a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*(a*e^2*g-c*d*(-3*d*g+2*e*f
))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
```

```
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1
/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c
^(1/2)))^(1/2)/e^3/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-c
*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+
d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*(a*e^2*g-c*d*(-3*d
*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d
*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))
*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(a*
e^2+c*d^2)/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)
```

Rubi [A] (verified)

Time = 3.22 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules

used = {931, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$$

$$= \frac{\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)(ae^2g-cd(2ef-3dg))^2}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(ae^2g-cd(2ef-3dg))}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(ae^2g-cd(2ef-3dg))}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(ae^2g-cd(2ef-3dg))}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}(ae^2g-cd(2ef-3dg))}{4e(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$-\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x)^2) - ((a*e^2*g - c*d*(2*e
*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g
)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f -
d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
```

```
(3*Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4
*e^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]
*Sqrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^3*(c*d^2
 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqr
t[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]
/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt
[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))
^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^3*((Sqrt[c]
*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
```

```
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}+\frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e} \\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}+\frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{2c(ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4e} \\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}+\frac{(3cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^3} \\
&\quad +\frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} \\
&\quad +\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^3} \\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}-\frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} \\
&\quad -\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8e^3\left(cd^2+ae^2\right)(ef-dg)} \\
&\quad +\frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2e^3\sqrt{a+cx^2}} \\
&\quad +\frac{\left(3a\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{2\sqrt{-a}e^3\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{(ae^2g-cd(2ef-3dg))\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{8e^3(cd^2+ae^2)(ef-dg)} - \frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^3\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(cdg(ae^2g-cd(2ef-3dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e^3(cd^2+ae^2)(ef-dg)} + \frac{(cg(ae^2g-cd(2ef-3dg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e^2(cd^2+ae^2)(ef-dg)} - \frac{(ae^2g-cd(2ef-3dg))^2\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e^3(cd^2+ae^2)(ef-dg)} - \frac{\left(c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$
\begin{aligned}
= & -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}-\frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} \\
& -\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
& -\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}} \\
& +\frac{(c(ae^2g-cd(2ef-3dg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{8e^2\left(cd^2+ae^2\right)(ef-dg)} \\
& -\frac{(cf(ae^2g-cd(2ef-3dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8e^2\left(cd^2+ae^2\right)(ef-dg)} \\
& -\frac{\left((ae^2g-cd(2ef-3dg))^2\sqrt{1+\frac{cx^2}{a}}\right)\int\frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{8e^3\left(cd^2+ae^2\right)(ef-dg)\sqrt{a+cx^2}} \\
& +\frac{\left(a\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\sqrt{}\right.}{4\sqrt{-a}e^3\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)}$$

$$- \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^3\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{\left((ae^2g-cd(2ef-3dg))^2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\cdots}\right.}{4e^3\left(cd^2+ae^2\right)(ef-dg)\sqrt{a+cx^2}}$$

$$+ \frac{\left(a\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{4\sqrt{-a}e^2\left(cd^2+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$- \frac{\left(a\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx,x,\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\right.}{4\sqrt{-a}e^2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$-\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\left((ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx\right.}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2}-\frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)}$$
$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.36 (sec) , antiderivative size = 2526, normalized size of antiderivative = 2.10

$$\int\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\text{Result too large to show}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-2 + ((a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d +
 e*x))/((c*d^2 + a*e^2)*(-(e*f) + d*g))))/(4*e*(d + e*x)^2) + (-2*c^2*d*e^3
*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sq
rt[a]*g)/Sqrt[c]] + a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c^2*
```

d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d^3*e*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 10*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*c^2*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 5*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 3*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - a*c*d*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*d - I*Sqrt[a]*e)*g*(I*Sqrt[c]*f - Sqrt[a]*g)*(a*e^2*g + c*d*(4*e*f - 3*d*g) + (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (4*I)*a*c*e^4*f^2*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (4*I)*c^2*d^3*e*f*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (12*I)*a*c*d*e^3*f*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (3*I)*c^2*d^4*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (6*I)*a*c*d^2*e^2*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a^2*e^4*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (

```
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/(4*e^3*(c*d^2 + a*e^2)*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 1161, normalized size of antiderivative = 0.96

method	result	size
elliptic	Expression too large to display	1161
default	Expression too large to display	19181

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2/e*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/e/(a*d*e
^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2
*(c*g/e^3-1/8*c*g*(3*a*d*e^2*g-2*a*e^3*f+5*c*d^3*g-4*c*d^2*e*f)/e^3/(a*d*e^
2*g-a*e^3*f+c*d^3*g-c*d^2*e*f))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(
1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^
(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*Ell
ipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-
a*c)^(1/2)/c))^(1/2))-1/4*c/e^2*g*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-
a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(
-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)
^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)))+1/4*(a^2*e^4*g^2-6*a*c*d^2*e^2*g^2+12*a*c*d*e^3*f*g-4*a*c*e^4*f^2-3*c
^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-
(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/
(-f/g-(-a*c)^(1/2)/c))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^3}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.629 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 666

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} + \frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} + \frac{4\sqrt{-a}(21a^2e^3g^4-3aceg^2(10e^2f^2-39defg+63d^2g^2)-c^2f(64e^3f^3-216de^2f^2g+252d^2efg^2-105d}{315c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}(cf^2+ag^2)(9ae^2g^2(2ef-5dg)-c(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{}}{315c^{3/2}g^5\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 4/315*e*(7*a*e^2*g^2+c*(42*d^2*g^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(
c*x^2+a)^(1/2)/c/g^4-4/63*e^2*(-3*d*g+4*e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/
g^4-4/315*(9*a*e^2*g^2*(-5*d*g+2*e*f)+c*(-35*d^3*g^3+168*d^2*e*f*g^2-204*d*
e^2*f^2*g+76*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^4+2/9*(e*x+d)^3*(g
*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/315*(21*a^2*e^3*g^4-3*a*c*e*g^2*(63*d^2*g^2
```

```
-39*d*e*f*g+10*e^2*f^2)-c^2*f*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g
+64*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/
(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1
/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^
(1/2)-4/315*(a*g^2+c*f^2)*(9*a*e^2*g^2*(-5*d*g+2*e*f)-c*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1
+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^
5/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {935, 1668, 858, 733, 435, 430}

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(21a^2e^3g^4-3aceg^2(63d^2g^2-39defg+10e^2f^2)-c^2f(-105d^3g^3+252d^2efg^2-2}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(9ae^2g^2(2ef-5dg)-c(-105d^3g^3+252d^2efg^2-216de^2f^2g+6}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+\frac{4e\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c(42d^2g^2-111defg+64e^2f^2)\right)}{315cg^4}$$

$$-\frac{4\sqrt{a+cx^2}\sqrt{f+gx}(9ae^2g^2(2ef-5dg)+c(-35d^3g^3+168d^2efg^2-204de^2f^2g+76e^3f^3))}{315cg^4}$$

$$-\frac{4e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{63g^4}+\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}$$

```
[In] Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^
2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*g^4) + (2*(d
 + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) + (4*e*(7*a*e^2*g^2 + c*(64*
e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*
c*g^4) - (4*e^2*(4*e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^4) +
 (4*Sqrt[-a]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^
3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
```

```
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^5
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*S
qrt[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*
d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqr
t[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)
*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 935

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*
x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2\left(2a(3ef-4dg)+2(cdf-aeg)x+2c(4ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{2\int \frac{-acg^2\left(20e^3f^3-15de^2f^2g-21d^2efg^2+28d^3g^3\right)-cg\left(aeg^2\left(40e^2f^2-72defg+63d^2g^2\right)+c\left(8e^3f^4-6de^2f^3g-7d^3fg^3\right)\right)x+cg^2\left(ae^2g\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{63cg^5}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} + \frac{4e\left(7ae^2g^2+c\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{4\int \frac{\frac{1}{2}acg^5\left(21ae^3fg^2+c\left(92e^3f^3-258de^2f^2g+231d^2efg^2-140d^3g^3\right)\right)+\frac{1}{2}cg^4\left(21a^2e^3g^4+3aceg^2\left(2e^2f^2+9defg-63d^2g^2\right)+c^2f\left(88e^3\right.\right.}{\sqrt{f+gx}\sqrt{a+c}}}{315c^2g^8}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} + \frac{4e\left(7ae^2g^2+c\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{8\int \frac{\frac{3}{4}ac^2g^7\left(3ae^2g^2(ef+15dg)+c\left(16e^3f^3-54de^2f^2g+63d^2efg^2-105d^3g^3\right)\right)+\frac{3}{4}c^2g^6\left(21a^2e^3g^4-3aceg^2\left(10e^2f^2-39defg+63d^2g^2\right)\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{945c^3g^{10}}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g}$$
$$+\frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4}$$
$$-\frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4}$$
$$+\frac{(2(cf^2+ag^2)(9ae^2g^2(2ef-5dg)-c(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3)))\int\frac{1}{\sqrt{f+gx}\sqrt{}}}{315cg^5}$$
$$-\frac{(2(21a^2e^3g^4-3aceg^2(10e^2f^2-39defg+63d^2g^2)-c^2f(64e^3f^3-216de^2f^2g+252d^2efg^2-1}{315cg^5}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g}$$
$$+\frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4}$$
$$-\frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4}$$
$$-\frac{\left(4a(21a^2e^3g^4-3aceg^2(10e^2f^2-39defg+63d^2g^2)-c^2f(64e^3f^3-216de^2f^2g+252d^2efg^2-\right.}{315\sqrt{-a}c^{3/2}g^5\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{cg}}{\sqrt{-a}}}}\sqrt{a}}$$
$$+\frac{\left(4a(cf^2+ag^2)(9ae^2g^2(2ef-5dg)-c(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3))\sqrt{\frac{c(f+g}{cf-\frac{a\sqrt{}}{\sqrt{}}}}\right.}{315\sqrt{-a}c^{3/2}g^5\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^4} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} + \frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+cx^2}}{315cg^4} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} + \frac{4\sqrt{-a}(21a^2e^3g^4-3aceg^2(10e^2f^2-39defg+63d^2g^2)-c^2f(64e^3f^3-216de^2f^2g+252d^2efg^2-}{315c^{3/2}g^5\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx}} - \frac{4\sqrt{-a}(cf^2+ag^2)(9ae^2g^2(2ef-5dg)-c(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3))\sqrt{\frac{\sqrt{c}(f}{\sqrt{c}f+}}}{315c^{3/2}g^5\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.66 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(2ae^2g^2(-11ef+45dg+7egx)+c(105d^3g^3+63d^2eg^2(-4f+3gx)+27de^2g(8f^2-6fgx+5g^2x^2)+e^3(-64f^3+48f^2gx-40fg}{cg^4}\right.}{}$$

```
[In] Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e*g*x) +
c*(105*d^3*g^3 + 63*d^2*e*g^2*(-4*f + 3*g*x) + 27*d*e^2*g*(8*f^2 - 6*f*g*x
+ 5*g^2*x^2) + e^3*(-64*f^3 + 48*f^2*g*x - 40*f*g^2*x^2 + 35*g^3*x^3))))/(c
*g^4) + (4*(f + g*x)*((g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-21*a^2*e^3*g^
4 + 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(64*e^3*f^3
```

```
- 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*(a + c*x^2))/(f + g*x)^
2 + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*
e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*f^2*g -
 252*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I
*Sqrt[a]*g)*((-21*I)*a^(3/2)*e^3*g^3 + 9*a*Sqrt[c]*e^2*g^2*(2*e*f - 5*d*g)
+ (3*I)*Sqrt[a]*c*e*g*(16*e^2*f^2 - 54*d*e*f*g + 63*d^2*g^2) + c^(3/2)*(-64
*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g
*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*
g^6*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(315*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1156, normalized size of antiderivative = 1.74

method	result	size
elliptic	Expression too large to display	1156
risch	Expression too large to display	1937
default	Expression too large to display	5079

```
[In] int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9*e^3/g*x^3*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c/g*x^2*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/
g*c*f)/g*f)/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d*e^2+c*d^3-2/
3*e^3/g*f*a-5/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c*a-4/5*(2/9*a*e^3+3*c*d^2*e-6/7*
(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/g*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2
*(a*d^3-2/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/c/g*f*a
-1/3*(3*a*d*e^2+c*d^3-2/3*e^3/g*f*a-5/7*(3*c*d*e^2-8/9*e^3/g*c*f)/c*a-4/5*(
2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/g*f)/c*a)*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*a*d^2*e-4/7*(3*
c*d*e^2-8/9*e^3/g*c*f)/c/g*f*a-3/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*
e^3/g*c*f)/g*f)/c*a-2/3*(3*a*d*e^2+c*d^3-2/3*e^3/g*f*a-5/7*(3*c*d*e^2-8/9*e
^3/g*c*f)/c*a-4/5*(2/9*a*e^3+3*c*d^2*e-6/7*(3*c*d*e^2-8/9*e^3/g*c*f)/g*f)/g
*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*
c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(
1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Ell
```

```
ipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-
a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 578, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx =$$
$$-\frac{2\left(2\left(64\,c^2e^3f^5-216\,c^2de^2f^4g+6\left(42\,c^2d^2e+13\,ace^3\right)f^3g^2-3\left(35\,c^2d^3+93\,acde^2\right)f^2g^3+6\left(63\,acd^2e-\right.\right.\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/945*(2*(64*c^2*e^3*f^5 - 216*c^2*d*e^2*f^4*g + 6*(42*c^2*d^2*e + 13*a*c*
e^3)*f^3*g^2 - 3*(35*c^2*d^3 + 93*a*c*d*e^2)*f^2*g^3 + 6*(63*a*c*d^2*e - 2*
a^2*e^3)*f*g^4 - 45*(7*a*c*d^3 - 3*a^2*d*e^2)*g^5)*sqrt(c*g)*weierstrassPIn
verse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3
*(3*g*x + f)/g) + 6*(64*c^2*e^3*f^4*g - 216*c^2*d*e^2*f^3*g^2 + 6*(42*c^2*d
^2*e + 5*a*c*e^3)*f^2*g^3 - 3*(35*c^2*d^3 + 39*a*c*d*e^2)*f*g^4 + 21*(9*a*c
*d^2*e - a^2*e^3)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*
(35*c^2*e^3*g^5*x^3 - 64*c^2*e^3*f^3*g^2 + 216*c^2*d*e^2*f^2*g^3 - 2*(126*c
^2*d^2*e + 11*a*c*e^3)*f*g^4 + 15*(7*c^2*d^3 + 6*a*c*d*e^2)*g^5 - 5*(8*c^2*
e^3*f*g^4 - 27*c^2*d*e^2*g^5)*x^2 + (48*c^2*e^3*f^2*g^3 - 162*c^2*d*e^2*f*g
^4 + 7*(27*c^2*d^2*e + 2*a*c*e^3)*g^5)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c
^2*g^6)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)^3}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2), x)
```

3.630 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 508

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3}$$
$$+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}-\frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3}$$
$$+\frac{4\sqrt{-a}(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{105\sqrt{c}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{4\sqrt{-a}(cf^2+ag^2)(5ae^2g^2-c(24e^2f^2-56defg+35d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1}}{}\right.\right.}{105c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -4/35*e*(-2*d*g+3*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g^3+4/105*(5*a*e^2*g^2
+c*(10*d^2*g^2-34*d*e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^3+
2/7*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/105*(a*e*g^2*(-42*d*g+13*e*
f)+c*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)
*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^4/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e^2*g^2-c*(35*d
^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^4/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {935, 1668, 858, 733, 435, 430}

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$=\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(5ae^2g^2-c(35d^2g^2-56defg+24e^2f^2))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\ldots}}{\sqrt{\ldots}}\right.\right.}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(aeg^2(13ef-42dg)+cf(35d^2g^2-56defg+24e^2f^2))\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\ldots\right.}{105\sqrt{c}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$+\frac{4\sqrt{a+cx^2}\sqrt{f+gx}\left(e^2\left(\frac{5a}{c}+\frac{21f^2}{g^2}\right)+10d^2-\frac{34def}{g}\right)}{105g}$$
$$-\frac{4e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{35g^3}+\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}$$

```
[In] Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (4*(10*d^2 + e^2*((5*a)/c + (21*f^2)/g^2) - (34*d*e*f)/g)*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(105*g) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g)
 - (4*e*(3*e*f - 2*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*g^3) + (4*Sqrt
[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2
))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^4*
Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sq
rt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-
a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 935

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*
x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(2a(2ef-3dg)+2(cdf-aeg)x+2c(3ef-2dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{2\int \frac{-acg^2(9e^2f^2-16defg+15d^2g^2)+cg(aeg^2(ef-14dg)-cf(6e^2f^2-4defg-5d^2g^2))x-cg^2(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2}{\sqrt{f+gx}\sqrt{a+cx^2}}}{35cg^4}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{4\int \frac{\frac{1}{2}acg^4(5ae^2g^2-c(6e^2f^2-14defg+35d^2g^2))+\frac{1}{2}c^2g^3(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2))x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{105c^2g^6}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{(2(cf^2+ag^2)(5ae^2g^2-c(24e^2f^2-56defg+35d^2g^2)))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{105cg^4} - \frac{(2(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2)))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{105g^4}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{\left(4a(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+}}{}\right.}{105\sqrt{-a}\sqrt{c}g^4\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(4a(cf^2+ag^2)(5ae^2g^2-c(24e^2f^2-56defg+35d^2g^2))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}}\right.}{105\sqrt{-a}c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{4(5ae^2g^2 + c(21e^2f^2 - 34defg + 10d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} + \frac{4\sqrt{-a}(aeg^2(13ef-42dg) + cf(24e^2f^2 - 56defg + 35d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\sqrt{2}}\ldots\right.\right.}{105\sqrt{c}g^4\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}(cf^2+ag^2)(5ae^2g^2 - c(24e^2f^2 - 56defg + 35d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\ldots}}{\ldots}\right.\right.}{105c^{3/2}g^4\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.16 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(10ae^2g^2+c(35d^2g^2+14deg(-4f+3gx)+3e^2(8f^2-6fgx+5g^2x^2)))}{cg^3} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(aeg^2(13ef-42dg)+cf(24e^2f^2\ldots\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*(-4
*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))))/(c*g^3) - (4*(g^2*Sqrt
[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 -
 56*d*e*f*g + 35*d^2*g^2))*(a + c*x^2) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g
)*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
[a]*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)*Sqrt[a]*
Sqrt[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^2*g^2))*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)]))/(c*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a +
c*x^2])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{2e^2x^2\sqrt{cg x^3+cf x^2+agx+fa}}{7g}+\frac{2\left(2cde-\frac{6e^2cf}{7g}\right)x\sqrt{cg x^3+cf x^2+agx+fa}}{5cg}+\frac{2\left(\frac{2e^2a}{7}+c d^2-\frac{4\left(2cde-\frac{6e^2cf}{7g}\right)f}{5g}\right)\sqrt{cg x^3 \cdots}}{3cg}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^2/g*x^2*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2*c*d*e-6/7*e^2/g*c*f)/c/g*x*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2/3*(2/7*e^2*a+c*d^2-4/5*(2*c*d*e-6/7*e^2/g*c*f)/g*f
)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^2-2/5*(2*c*d*e-6/7*e^2/g*c*f
)/c/g*f*a-1/3*(2/7*e^2*a+c*d^2-4/5*(2*c*d*e-6/7*e^2/g*c*f)/g*f)/c*a)*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^
(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*a*d*e-4/7*e
^2/g*f*a-3/5*(2*c*d*e-6/7*e^2/g*c*f)/c*a-2/3*(2/7*e^2*a+c*d^2-4/5*(2*c*d*e-
6/7*e^2/g*c*f)/g*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c
)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a
*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left(2\left(24\,c^2e^2f^4-56\,c^2def^3g-84\,acdefg^3+(35\,c^2d^2+31\,ace^2)f^2g^2+15\,(7\,acd^2-a^2e^2)g^4\right)\sqrt{cg}\text{weierstras}\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(2*(24*c^2*e^2*f^4 - 56*c^2*d*e*f^3*g - 84*a*c*d*e*f*g^3 + (35*c^2*d^
2 + 31*a*c*e^2)*f^2*g^2 + 15*(7*a*c*d^2 - a^2*e^2)*g^4)*sqrt(c*g)*weierstra
ssPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, 1/3*(3*g*x + f)/g) + 6*(24*c^2*e^2*f^3*g - 56*c^2*d*e*f^2*g^2 - 42*a*c*d*
e*g^4 + (35*c^2*d^2 + 13*a*c*e^2)*f*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f
^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInver
se(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3
*g*x + f)/g)) + 3*(15*c^2*e^2*g^4*x^2 + 24*c^2*e^2*f^2*g^2 - 56*c^2*d*e*f*g
^3 + 5*(7*c^2*d^2 + 2*a*c*e^2)*g^4 - 6*(3*c^2*e^2*f*g^3 - 7*c^2*d*e*g^4)*x)
*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)^2}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}\,(ex+d)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2), x)
```

3.631 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} dx$

Optimal result

Integrand size = 26, antiderivative size = 364

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} dx = -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{4\sqrt{-a}(3aeg^2+cf(4ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{4\sqrt{-a}(4ef-5dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2/15*(-3*e*g*x-5*d*g+4*e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g^2-4/15*(3*a*e*
g^2+c*f*(-5*d*g+4*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2
),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c
*x^2/a)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)+4/15*(-5*d*g+4*e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/
g^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {829, 858, 733, 435, 430}

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3aeg^2+cf(4ef-5dg))\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

```
[In] Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) - (4*
Sqrt[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[
-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^
2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{4\int \frac{-\frac{1}{2}acg(ef-5dg)+\frac{1}{2}c(3aeg^2+cf(4ef-5dg))x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15cg^2}$$

$$= -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{(2(4ef-5dg)\left(cf^2+ag^2\right))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15g^3} + \frac{(2(3aeg^2+cf(4ef-5dg)))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{15g^3}$$

$$=-\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2}$$
$$+\frac{\left(4a(3aeg^2+cf(4ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}\sqrt{c}g^3\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$
$$-\frac{\left(4a(4ef-5dg)(cf^2+ag^2)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}\sqrt{c}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$=-\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2}$$
$$-\frac{4\sqrt{-a}(3aeg^2+cf(4ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{4\sqrt{-a}(4ef-5dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.32 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.50

$$\int\frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$=\frac{\sqrt{f+gx}\left(\frac{2(-4ef+5dg+3egx)(a+cx^2)}{g^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(3aeg^2+cf(4ef-5dg))(a+cx^2)-\sqrt{c}(i\sqrt{c}f-\sqrt{a}g)(3aeg^2+cf(4ef-5dg))\sqrt{\frac{g}{-}}\right.}{}\right.}{}$$

```
[In] Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x^2
) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqr
```

```
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*g + Sqr
t[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-
(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x))))/(15*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(298) = 596$.

Time = 1.08 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2ex\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5g}+\frac{2\left(cd-\frac{4cfe}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(ad-\frac{2fae}{5g}-\frac{a\left(cd-\frac{4cfe}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}}{\sqrt{cg}}\right.}{}$
risch	$\frac{2(3egx+5dg-4ef)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g^2}+\frac{2\left(\frac{10ad\,g^2\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e/g*x*(c*g*x^3
+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(c*d-4/5*c*f/g*e)/c/g*(c*g*x^3+c*f*x^2+a*g*x+
a*f)^(1/2)+2*(a*d-2/5*f*a/g*e-1/3*a/c*(c*d-4/5*c*f/g*e))*(f/g-(-a*c)^(1/2)/
c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1
/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
```

```
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2/5*a*e-2/3*f/g*(c*d-4/5*
c*f/g*e))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^
(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*El
lipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\left(2\left(4\,cef^3-5\,cdf^2g+6\,aefg^2-15\,adg^3\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right.}{}$$

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(2*(4*c*e*f^3 - 5*c*d*f^2*g + 6*a*e*f*g^2 - 15*a*d*g^3)*sqrt(c*g)*wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(
c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c*e*f^2*g - 5*c*d*f*g^2 + 3*a*e*g^3)*sqrt
(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g
^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^
3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e*g^3*x - 4*c*e*f*g^2
+ 5*c*d*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2), x)
```

3.632 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 322

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} + \frac{4\sqrt{-a}\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/3*f*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*
c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2
)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/3*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1
/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(
-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/g^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {749, 858, 733, 435, 430}

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= -\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

```
[In] Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) + (4*Sqrt[-a]*Sqrt[c]*f*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*g^2*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} + \frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} + \frac{1}{3}\left(2\left(a+\frac{cf^2}{g^2}\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx - \frac{(2cf)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} - \frac{\left(4a\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(4a\left(a+\frac{cf^2}{g^2}\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} + \frac{4\sqrt{-a}\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{4\sqrt{-a}\left(a+\frac{cf^2}{g^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.50 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(g^2(a+cx^2) - \frac{2\left(fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+\sqrt{c}f(-i\sqrt{c}f+\sqrt{a}g)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\ldots}}{\sqrt{\ldots}}\right.\right.\right.}{\ldots}\right)}{3g^3\sqrt{a+\ldots}}$$

```
[In] Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))
]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt
[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*
g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(256) = 512$.

Time = 0.98 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.81

method	result
risch	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}}{3g}+\frac{2\left(\frac{2ag\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}-\frac{2cf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}{3g}+\frac{4a\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{3\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}\cdots\right.}{\cdots}$
default	$-\frac{2\sqrt{c\,x^{2}+a}\,\sqrt{gx+f}\left(2\sqrt{-ac}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)a\,g^{3}+2\sqrt{-ac}\,\sqrt{-\cdots}\right.}{\cdots}$

```
[In] int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/3/g*(2*a*g*(f/g-(-a*c)^(1/2)/c)*((x+f
/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^
(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g
*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-2*c*f*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/
(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a
*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*El
lipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2))))*((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left(6\sqrt{cg}cfg\text{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3g}{}\right.\right.\right.}{}$$

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(6*sqrt(c*g)*c*f*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*
g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*sqrt(c*x^2
 + a)*sqrt(g*x + f)*c*g^2 + 2*(c*f^2 + 3*a*g^2)*sqrt(c*g)*weierstrassPInver
se(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3
*g*x + f)/g))/(c*g^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
[Out] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.633 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 473

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx
&= -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&\quad+\frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{f+gx}\sqrt{a+cx^2}} \\
&\quad-\frac{2(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)
/e/g/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(d*
g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f
*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*
c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
-2*(a*e^2+c*d^2)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {937, 947, 174, 552, 551, 858, 733, 435, 430}

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= -\frac{2\sqrt{\frac{cx^2}{a}+1}(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$
$$+\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(dg+ef)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$-\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])
/(e*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*Sqrt[c]*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*g*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e),
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f +
 Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 937

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Dist[(c*d^2 + a*e^2)/e^2, Int[1/((d + e*x)*Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sqrt
```

```
[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &
& NeQ[c*d^2 + a*e^2, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx - \frac{\int \frac{cd-cex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{eg} - \frac{(c(ef+dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2g} + \frac{\left(\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{a+cx^2}} + \frac{\left(2a\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}eg\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(2a\sqrt{c}(ef+dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^2g\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{2\left(a+\frac{cd^2}{e^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.77 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.32

$$\int\frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx =$$
$$-\frac{2\left(-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{-\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d
*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g
*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a
]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*
Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellip
ticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)]))/(e^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(386) = 772$.

Time = 0.78 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.76

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2cd\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cgx^3+cfx^2+agx+fa}}+\frac{2c\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{}{\frac{f}{g}-}}}{}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*c*d/e^2*(f/g-(-
```

```
a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/
g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(
c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*c/e*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-
a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
)+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*
c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(a*e^2+c*d^2)/e^3*(f/g-(-a*c)^
(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.634 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 694

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+\frac{(ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)-EllipticE(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/(-d*g+e*f)/(c*x^2+a)^(1/2
)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
```

```
(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/
2)))^(1/2)/e/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-d*g+2*e*f)*Elliptic
F(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^
2*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2)
,2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {939, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

$$= -\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}$$
$$+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g+cd(2ef-dg))\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)}$$
$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}$$
$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e\sqrt{a+cx^2}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt
[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[
-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (
```

```
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g
)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*(e*f - d*g)*Sqrt[
f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f
 + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((S
qrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2
*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*
f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 939

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
+ c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)}+\frac{\int \frac{-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(ef-dg)}\\
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)}+\frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{-ae^2g-cd(2ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2(ef-dg)}
\end{aligned}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2(ef-dg)} - \frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{\left(\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2(ef-dg)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}(2ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{\left(\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{(ef-dg)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e(ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(a\sqrt{c}f\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{\left(\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx, x, \sqrt{1 \ldots}\right.}{(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.29 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{a+cx^2}{d+ex} - \frac{ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} + ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - 2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx) + 2cdefg}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I
*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f
+ I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f
 + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d*
e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/
(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/S
qrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*d
^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*
e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((-(e*f) + d*g
)*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{(dg-ef)(ex+d)}+\frac{2\left(\frac{c}{e^2}-\frac{cdg}{2e^2(dg-ef)}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/(d*g-e*f)*(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(c/e^2-1/2*c*d/e^2*g/(d*g-e*f))*(f/g-
(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))
^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-c*g/(d*g-e*f)/e
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE((
(x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/
2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)
,((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g-c*d^2*g+2*c
*d*e*f)/e^3/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(
-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*El
lipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d
/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{(d+e\,x)}^2}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.635 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1241

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)}$$

$$+\frac{\sqrt{-a}\sqrt{c}(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}f(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}dg(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{(ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{4e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*
(d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(e*x+
d)+1/4*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)
*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/
2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(1/2*(1-
x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1
/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
```

```
c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*f*(3*a*e^2
*g+c*d*(d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-
2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1
/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(-d*g+
e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*d*g*(3*a*e^2*g+c*d*(d*g+2*e*f))*El
lipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/
2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(
g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2/(g*x+f)^(1/2)
/(c*x^2+a)^(1/2)-c*(d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*
2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)-1/4*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticPi
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^
(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 1241, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules

used = {939, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$

$$=\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(3age^2+cd(2ef+dg))}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(3age^2+cd(2ef+dg))}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(3age^2+cd(2ef+dg))}{4e^2\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)(3age^2}{4e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{f+gx}\sqrt{cx^2+a}(3age^2+cd(2ef+dg))}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e
*f - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*
Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)
*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a +
```

```
c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^2*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*S
qrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a +
 c*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
- (c*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1
+ (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e
^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
((a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*
e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^2*((Sqrt[c]*d)/Sqrt[-a]
+ e)*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 939

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
+ c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
```

```
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{-3ag+2cfx-cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-3ae^2g-cd(2ef+dg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef+dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(c(ef+dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2(ef-dg)} - \frac{(3ae^2g+cd(2ef+dg))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(2ef+dg))\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8e^2(cd^2+ae^2)(ef-dg)^2} + \frac{\left(c(ef+dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2e^2(ef-dg)\sqrt{a+cx^2}} - \frac{\left(a\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{2\sqrt{-a}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(3ae^2g+cd(2ef+dg))\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{8e^2(cd^2+ae^2)(ef-dg)^2} - \frac{\left(c(ef+dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^2(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{(cdg(3ae^2g+cd(2ef+dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e^2(cd^2+ae^2)(ef-dg)^2} - \frac{(cg(3ae^2g+cd(2ef+dg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e(cd^2+ae^2)(ef-dg)^2} + \frac{((ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg)))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e^2(cd^2+ae^2)(ef-dg)^2} - \frac{\left(c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)}$$

$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{(c(3ae^2g+cd(2ef+dg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{8e(cd^2+ae^2)(ef-dg)^2}$$

$$+ \frac{(cf(3ae^2g+cd(2ef+dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8e(cd^2+ae^2)(ef-dg)^2}$$

$$+ \frac{\left((ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg))\sqrt{1+\frac{cx^2}{a}}\right)\int\frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{8e^2(cd^2+ae^2)(ef-dg)^2\sqrt{a+cx^2}}$$

$$- \frac{\left(a\sqrt{c}dg(3ae^2g+cd(2ef+dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \sqrt{\ldots}\right.}{4\sqrt{-a}e^2(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$
\begin{aligned}
&= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} \\
&+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} \\
&+ \frac{\sqrt{-a}\sqrt{c}dg\left(3ae^2g+cd(2ef+dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}} \\
&- \frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} \\
&- \frac{\left(\left(ae^2g-cd(2ef-3dg)\right)\left(3ae^2g+cd(2ef+dg)\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}}{\sqrt{c}}\cdots}}\cdots\right.}{4e^2\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{a+cx^2}} \\
&- \frac{\left(a\sqrt{c}\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{4\sqrt{-a}e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} \\
&+ \frac{\left(a\sqrt{c}f\left(3ae^2g+cd(2ef+dg)\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx,x,\frac{\sqrt{1-\cdots}}{\sqrt{2}}\cdots\right.}{4\sqrt{-a}e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)}$$

$$+ \frac{\sqrt{-a}\sqrt{c}(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\sqrt{-a}\sqrt{c}f(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt{-a}\sqrt{c}dg(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\left((ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+\right.}\right.}{4e^2\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)}$$
$$+ \frac{\sqrt{-a}\sqrt{c}(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{\sqrt{-a}\sqrt{c}f(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}dg(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e^2(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{c(ef+dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{(ae^2g-cd(2ef-3dg))(3ae^2g+cd(2ef+dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{4e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.72 (sec) , antiderivative size = 2197, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (c^2*d^2*f^3 - 3*a*c*e^2*f^3 - (2*c^2*d*e*f^4)/g + (c^2*d^3*f^2*g)/e + a*c*
d*e*f^2*g + a*c*d^2*f*g^2 - 3*a^2*e^2*f*g^2 + (a*c*d^3*g^3)/e + 3*a^2*d*e*g
^3 - 2*c^2*d^2*f^2*(f + g*x) + 6*a*c*e^2*f^2*(f + g*x) + (4*c^2*d*e*f^3*(f
```

```
+ g*x))/g - (2*c^2*d^3*f*g*(f + g*x))/e - 6*a*c*d*e*f*g*(f + g*x) + c^2*d^2
*f*(f + g*x)^2 - 3*a*c*e^2*f*(f + g*x)^2 - (2*c^2*d*e*f^2*(f + g*x)^2)/g +
(c^2*d^3*g*(f + g*x)^2)/e + 3*a*c*d*e*g*(f + g*x)^2 - ((e*f - d*g)*(f + g*x
)*(a + c*x^2)*(a*e^2*(2*e*f - 5*d*g - 3*e*g*x) - c*d*(3*d^2*g + 2*e^2*f*x +
 d*e*g*x)))/(d + e*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d
*g)*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
 g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(-3*a*e^2*g
 - (6*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g
*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((4*I)*a*c*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)
), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
 ((4*I)*c^2*d^3*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]
*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[
a]*g)])/(e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) - ((4*I)*a*c*d*e*f*g*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f +
 I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]] + ((6*I)*a*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*c^2*d^4*g^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c
]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]) + ((3*I)*a^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(
3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4*(c*d^2
 + a*e^2)*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 1196, normalized size of antiderivative = 0.96

method	result	size
elliptic	Expression too large to display	1196
default	Expression too large to display	19170

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2/(d*g-e*f)*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d^2*g+2*c*d*e*f)/(
a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^
(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f-c*d^3*g+4*c*d^2*e*f)/(a*d*e^2*g-
a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((
x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^
(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(3*a*e^2*g+c*d^2*g+2*c*d*e*f)/(a*d*e
^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ell
ipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-
a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d^2*e^2*g^2-4*a*c*d*e^3*f*g
+4*a*c*e^4*f^2-c^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^
2*e*f)/e^3/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-
f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ell
ipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/
e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.636 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 531

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2}$$
$$+\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c}+\frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2}$$
$$+\frac{2\sqrt{-a}(ae^2g^2(19ef+189dg)-c(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsi\right.}{105c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt{-a}e(cf^2+ag^2)(25ae^2g^2-c(8e^2f^2-42defg+105d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{105c^{5/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/35*e^2*(11*d*g+e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^2-2/105*e*(25*a*e^2
*g^2+c*(-90*d^2*g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^
2/g^2+2/7*e*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c+2/105*(a*e^2*g^2*(189
*d*g+19*e*f)-c*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^2*f^2*g+8*e^3*f^3))*Elli
pticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/2)/g^3/(c
*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/105*e*(a*g
^2+c*f^2)*(25*a*e^2*g^2-c*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/2
*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2))
)^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2)/c^(5/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {956, 1668, 858, 733, 435, 430}

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(25ae^2g^2-c(105d^2g^2-42defg+8e^2f^2))\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ae^2g^2(189dg+19ef)-c(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3))\,E\left(\text{arc}\cdots\right.}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}} + \frac{2e\sqrt{a+cx^2}\sqrt{f+gx}\left(-e^2\left(\frac{25a}{c}+\frac{7f^2}{g^2}\right)+90d^2-\frac{12def}{g}\right)}{105c} + \frac{2e^2\sqrt{a+cx^2}(f+gx)^{3/2}(11dg+ef)}{35cg^2} + \frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(90*d^2 - e^2*((25*a)/c + (7*f^2)/g^2) - (12*d*e*f)/g)*Sqrt[f + g*x]*S
qrt[a + c*x^2])/(105*c) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
7*c) + (2*e^2*(e*f + 11*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*c*g^2) +
(2*Sqrt[-a]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g +
 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(105*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*e*(c*f^2 + a*g^2)*(25*a*e^2*g^2 - c*(8
*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(5/2)*g^3*Sqr
t[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+ae(4ef+dg)+\left(5ae^2g-cd(12ef+7dg)\right)x-ce(ef+11dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(35cd^3fg-ae\left(3e^2f^2+53defg+5d^2g^2\right)\right)+\frac{1}{2}cg\left(ae^2g^2(23ef+63dg)+c\left(2e^3f^3+22de^2f^2g-95d^2efg^2-35d^3g^3\right)\right)x+\frac{1}{2}ceg^2}{\sqrt{f+gx}\sqrt{a+cx^2}}}{35c^2g^3}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{4\int \frac{-\frac{1}{4}cg^4\left(105c^2d^3fg+25a^2e^3g^2-ace\left(2e^2f^2+147defg+105d^2g^2\right)\right)+\frac{1}{4}c^2g^3\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2f^2g+105d^2ef\right.\right.}{\sqrt{f+gx}\sqrt{a+cx^2}}}{105c^3g^5}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} + \frac{\left(e\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42defg+105d^2g^2\right)\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{105c^2g^3} - \frac{\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{105cg^3}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{\left(2a\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3\right)\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)}{105\sqrt{-a}c^{3/2}g^3\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2ae\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42defg+105d^2g^2\right)\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{105\sqrt{-a}c^{5/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2e(25ae^2g^2 + c(7e^2f^2 + 12defg - 90d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2}$$

$$+ \frac{2\sqrt{-a}(ae^2g^2(19ef+189dg) - c(8e^3f^3 - 42de^2f^2g + 105d^2efg^2 + 105d^3g^3))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E}{105c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$- \frac{2\sqrt{-a}e(cf^2+ag^2)(25ae^2g^2 - c(8e^2f^2 - 42defg + 105d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\right.\right.}{105c^{5/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.52 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(-25ae^3g^2+ce(105d^2g^2+21deg(f+3gx)+e^2(-4f^2+3fgx+15g^2x^2)))}{c^2g^2} - \frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ae^2g^2(19ef+189dg)-c(8e\right.}{}\right.}{}$$

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(-25*a*e^3*g^2 + c*e*(105*d^2*g^2 + 21*d*e*g
*(f + 3*g*x) + e^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2))))/(c^2*g^2) - (2*(g^2*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^
3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt[c]*
(I*a*Sqrt[c]*e^2*f*g^2*(19*e*f + 189*d*g) - a^(3/2)*e^2*g^3*(19*e*f + 189*d
*g) - I*c^(3/2)*f*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g
^3) + Sqrt[a]*c*g*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g
^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] - g*(Sqrt[c]*f + I*Sqrt[a]*g)*((105*I)*c^(3/2)*d^3*g^2 + 25*a^
(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(2*e*f - 63*d*g) + Sqrt[a]*c*e*(-8*e^
2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elli
```

```
pticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
 - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c^2*g^4*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.66

method	result
elliptic	$\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^3x^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7c}+\frac{2\left(3d\,e^2g+\frac{1}{7}e^3f\right)x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(3d^2eg+3d\,e^2f-\frac{4f\left(3d\,e^2g+\frac{1}{7}e^3f\right)}{5g}-\frac{5ga}{7c}\right.}{3cg}\right.$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^3/c*x^2*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(3*d*e^2*g+1/7*e^3*f)/c/g*x*(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)+2/3*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*e^3*f)
-5/7/c*g*a*e^3)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^3*f-2/5*a/c*f/g*
(3*d*e^2*g+1/7*e^3*f)-1/3/c*a*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*e
^3*f)-5/7/c*g*a*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(
-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x
+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2))+2*(d^3*g+3*d^2*e*f-4/7*a/c*f*e^3-3/5/c*a*(3*d*e^2*g+1/7*e^3*f)-
2/3*f/g*(3*d^2*e*g+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*e^3*f)-5/7/c*g*a*e^3))*
(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^
(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((
x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2
)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx =$$
$$-\frac{2\left((8\,c^2e^3f^4 - 42\,c^2de^2f^3g + (105\,c^2d^2e - 13\,ace^3)f^2g^2 - 42\,(5\,c^2d^3 - 6\,acde^2)fg^3 + 15\,(21\,acd^2e - 5\,a^2e\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*((8*c^2*e^3*f^4 - 42*c^2*d*e^2*f^3*g + (105*c^2*d^2*e - 13*a*c*e^3)*
f^2*g^2 - 42*(5*c^2*d^3 - 6*a*c*d*e^2)*f*g^3 + 15*(21*a*c*d^2*e - 5*a^2*e^3
)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(
c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c^2*e^3*f^3*g - 42*c^
2*d*e^2*f^2*g^2 + (105*c^2*d^2*e - 19*a*c*e^3)*f*g^3 + 21*(5*c^2*d^3 - 9*a*
c*d*e^2)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c
*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(15*c^2*e
^3*g^4*x^2 - 4*c^2*e^3*f^2*g^2 + 21*c^2*d*e^2*f*g^3 + 5*(21*c^2*d^2*e - 5*a
*c*e^3)*g^4 + 3*(c^2*e^3*f*g^3 + 21*c^2*d*e^2*g^4)*x)*sqrt(c*x^2 + a)*sqrt(
g*x + f))/(c^3*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^3}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2), x)
```

3.637 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 410

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c}$$
$$+\frac{2\sqrt{-a}(9ae^2g^2+c(2e^2f^2-10defg-15d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{-a}e(ef-5dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/15*e*(7*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/5*e*(e*x+d)*(g*x+f)^
(1/2)*(c*x^2+a)^(1/2)/c+2/15*(9*a*e^2*g^2+c*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f
^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(3/
2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/1
5*e*(-5*d*g+e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {956, 1668, 858, 733, 435, 430}

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(9ae^2g^2+c(-15d^2g^2-10defg+2e^2f^2))\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$-\frac{4\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)(ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{15cg}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(e*f + 7*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g) + (2*e*(d + e*x)
*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + (2*Sqrt[-a]*(9*a*e^2*g^2 + c*(2*e^2
*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(15*c^(3/2)*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]
*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*e*(e*f - 5*d*g)*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(15*c^(3/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

integral

$$= \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+ae(2ef+dg)+(3ae^2g-cd(8ef+5dg))x-ce(ef+7dg)x^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{5c}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}cg^2(15cd^2f-ae(7ef+10dg))+\frac{1}{2}cg(9ae^2g^2+c(2e^2f^2-10defg-15d^2g^2))x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15c^2g^2}$$

$$=\frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c}-\frac{1}{15}\left(-15d^2+e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right)-\frac{10def}{g}\right)\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx+\frac{(2e(ef-5dg)\left(cf^2+ag^2\right))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15cg^2}$$

$$=\frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c}-\frac{\left(2a\left(-15d^2+e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right)-\frac{10def}{g}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}+\frac{\left(4ae(ef-5dg)\left(cf^2+ag^2\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{15\sqrt{-a}c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=\frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c}-\frac{2\sqrt{-a}\left(15d^2-e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right)+\frac{10def}{g}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}-\frac{4\sqrt{-a}e(ef-5dg)\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.09 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e(a+cx^2)(10dg+e(f+3gx))}{cg} + \frac{(f+gx)\left(\frac{2g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(-2e^2f^2+10defg+15d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2} + \frac{2\sqrt{c}\left(-i\sqrt{c}f+\sqrt{a}g\right)\left(-9ae^2\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*e*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)))/(c*g) + ((f + g*
x)*((2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(-2*e^2*f^2 +
 10*d*e*f*g + 15*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (2*Sqrt[c]*((-I)*Sqrt
[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*g^2)
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[
f + g*x] + (2*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g - (9*I)*a
*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^3*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]])))/(15*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(338) = 676.

Time = 2.09 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5c}+\frac{2\left(2deg+\frac{1}{5}e^2f\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^2f-\frac{2e^2fa}{5c}-\frac{\left(2deg+\frac{1}{5}e^2f\right)a}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^2/c*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*d*e*g+1/5*e^2*f)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^2*f-2/5*e^2/c*f*a-1/3*(2*d*e*g+1/5*e^2*f)/c*a)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d^2*g+2*d*e*f-3/5*e^2/c*a*g-2/3*(2*d*e*g+1/5*e^2*f)/g*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\left(2\left(ce^2f^3-5\,cdef^2g-15\,adeg^3+3\left(5\,cd^2-2\,ae^2\right)fg^2\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,a\right.}{27\,cg^3}\right.\right.}{}$$

[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] 2/45*(2*(c*e^2*f^3 - 5*c*d*e*f^2*g - 15*a*d*e*g^3 + 3*(5*c*d^2 - 2*a*e^2)*f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c

```
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(2*c*e^2*f^2*g - 10*c*d*e
*f*g^2 - 3*(5*c*d^2 - 3*a*e^2)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 -
3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/
3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x
+ f)/g)) + 3*(3*c*e^2*g^3*x + c*e^2*f*g^2 + 10*c*d*e*g^3)*sqrt(c*x^2 + a)*s
qrt(g*x + f))/(c^2*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2\,\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^2}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2), x)
```

3.638 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 331

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}e(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c-2/3*(3*d*g+e*f)*EllipticE(1/2*(1-x*c^
(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))
*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g/c^(1/2)/(c*x^2+a)^(1/2)/((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*e*(a*g^2+c*f^2)*EllipticF(1
/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2
)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)/c^(3/2)/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {847, 858, 733, 435, 430}

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}}$$
$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3dg+ef)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*Sqrt[-a]*(e*f + 3*d*g)*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*e*(c*
f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*g*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-aeg)+\frac{1}{2}c(ef+3dg)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{(ef+3dg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g} - \frac{(e(cf^2+ag^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3cg}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{\left(2a(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(2ae(cf^2+ag^2)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}c^{3/2}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}e(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.52 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{f+gx}\left(e(a+cx^2)+\frac{(ef+3dg)(a+cx^2)}{f+gx}+\frac{ic\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef+3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{\cdots}}{\sqrt{\cdots}}}}{\sqrt{f+gx}}\cdots\right.\right.}{g^2}\cdots\right)}{3c\sqrt{a\cdots}}$$

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) + (
I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt
[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*Sqrt[
c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(265) = 530$.

Time = 1.29 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3c}+\frac{2\left(df-\frac{eag}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+}{-\frac{f}{g}-}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
risch	$\frac{2e\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3c}-\frac{\left(\frac{2aeg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{6cdf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)}{}\right.}{}$
default	Expression too large to display

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/c*e*(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)+2*(d*f-1/3/c*e*a*g)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d*g+1/3*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))
^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)
/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ceg^2-(cef^2-6\,cdfg+3\,aeg^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3}\right.\right.}{}$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e*g^2 - (c*e*f^2 - 6*c*d*f*g + 3*a*e
*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(c*e*f*g + 3*c*d*g^2)*sqr
t(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f
^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^2*g^2)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2), x)
```

3.639 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=-\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/c^(1/2)
/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {733, 435}

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\left(2a\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.38 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.16

$$\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2i\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}x\right)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\right.\right.\right.}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{g\left(i\sqrt{a}+\sqrt{c}x\right)}}\sqrt{a+cx^2}}$$

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqr
t[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(f
+ g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
```

```
+ I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c
*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(108) = 216$.

Time = 0.44 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.91

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,(cf-g\sqrt{-ac})\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)g-\sqrt{-a}\cdots\right.}{g(cg\,x^{3}+cf\,x^{2}+agx\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2f\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}+\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\cdots}{\frac{f}{g}-\sqrt{\cdots}}}\cdots}{\cdots}\right.}{\sqrt{gx+f}\,\sqrt{c\,x\cdots}}$

```
[In] int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-g*(-a*c)^(1/2))*(-(g*x+f)*c/(g*(-a*c)^
(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-a*c)^(1/2)*EllipticF((-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
))^(1/2))*g-(-a*c)^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),
(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*g+f*EllipticF((-(g*x+f)
*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))
^(1/2))*c-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/
2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2\left(2\sqrt{cg}f\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\sqrt{cg}g\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\right.\right.}{3\,cg}$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(c*g)*f*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*sqrt(c*g)*g*weierstras
sZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(
c*g^3), 1/3*(3*g*x + f)/g)))/(c*g)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2), x)
```

3.640 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 319

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d
*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1
/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c
*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(e+d*c^(1/
2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {958, 733, 430, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)} - \frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
 (2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
 (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*
((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\text{integral} = \frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}dx}{e\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{e\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.36 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2i\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticPi}\left(\frac{e\left(f-\frac{i\sqrt{a}g}{\sqrt{c}}\right)}{ef-dg},i\text{arcsin}\cdots\right.\right.}{e\sqrt{\frac{\sqrt{c}(f+gx)}{g(i\sqrt{a}+\sqrt{c}x)}}\sqrt{a+cx^2}}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqrt
[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sq
rt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Ellipti
cPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt[c]*
(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x)
)]*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.38

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{cx^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(fF\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)c-\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\cdots\right.\right.}{ec(cg\,x^3+cf\,x\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{2(dg-ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\cdots}{\cdots}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}$

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)-c*f))^(1/2)*(f*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)
,(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c-(-a*c)^(1/2)*Ellipti
cF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*g-EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g
*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
```

```
))^(1/2))*c*f+EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^
(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2)
)*(-a*c)^(1/2)*g)/e/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\;(d+e\,x)}\,dx$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.641 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 698

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{(ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)-EllipticE(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2)
)*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*d^2)/(c*x^2+a
)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1
-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(
```

```
1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2
*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {960, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\,(ae^2+cd^2)}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\,(ae^2+cd^2)}$$
$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\,(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g+cd(2ef-dg))\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ae^2+cd^2)}$$
$$-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\,(ae^2+cd^2)}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (Sqrt[-a
]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 +
```

```
a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[
a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/(
(Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*
d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 960

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1) -
 e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-2cdf-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{cgx}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(2ef-dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(cd^2+ae^2)} + \frac{(ae^2g+cd(2ef-dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)} + \frac{\left((ae^2g+cd(2ef-dg))\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2e(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}dg\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left((ae^2g+cd(2ef-dg))\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c\cdots}}{\sqrt{-\cdots}}}\cdots\right.}{e(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(cd^2+ae^2)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(a\sqrt{c}f\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\left((ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\right.}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{(ae^2g+cd(2ef-dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.59 (sec) , antiderivative size = 1330, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$$
$$= \frac{\sqrt{f+gx}\left(-\frac{e^2(a+cx^2)}{d+ex} - \frac{-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+g}{}\right.}{}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2
*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*
e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((
I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] +
e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a]
)/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]
*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[
f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d
*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*
d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a
*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((c*d^2*e + a*e^3
)*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.32

method	result
elliptic	$\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{\left(e^2a+c\,d^2\right)(ex+d)}+\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.}{\left(e^2a+c\,d^2\right)e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.$
default	Expression too large to display

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-e/(a*e^2+c*d^2)*(
c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+c*d*g/(a*e^2+c*d^2)/e*(f/g-(-a*c)^
(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+c*g/(a*e^2+c*d^2)*(f/g
-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(
-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/
g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g-c*d^2*g+2*c*d*e*
f)/e^2/(a*e^2+c*d^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-
f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ell
ipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/
e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{(d+e\,x)}^2}\,dx$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.642 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1246

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$$
$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)(d+ex)}$$
$$- \frac{\sqrt{-a}\sqrt{c}(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}f(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{\sqrt{-a}\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{(ae^2g+cd(6ef-5dg))(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{4e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^2-1/4*e*(a*e^2*g
+c*d*(-5*d*g+6*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*
f)/(e*x+d)-1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)
^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2
)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(c*x^2
+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
```

```
1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*
(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*
x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)
^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g+c*d*(-5*d*g+6*
e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c
^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(-d*g+e*f)/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+
f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)+1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*(a*e^2*g-c*d*(-3*d*g+2*e*f))
*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a
)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(
-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 3.34 (sec) , antiderivative size = 1246, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules

used = {960, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$$

$$= -\frac{(age^2+cd(6ef-5dg))\sqrt{f+gx}\sqrt{cx^2+ae}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\sqrt{f+gx}\sqrt{cx^2+ae}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$$-\frac{\sqrt{-a}\sqrt{c}(age^2+cd(6ef-5dg))\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f(age^2+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{cx^2+ae}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg(age^2+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+ae}}$$

$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{cx^2+ae}}$$

$$+\frac{(age^2+cd(6ef-5dg))\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+ae}}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)^2) - (e*(
a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a
*e^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f -
5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 +
a*e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
```

```
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*S
qrt[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f -
 5*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sq
rt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sq
rt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elliptic
F[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^
2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*
x^2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e*((Sqrt[c]*d)/Sqr
t[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 960

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
```

```
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1) -
 e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2}-\frac{\int \frac{-4cdf-aeg+2c(ef-2dg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4(cd^2+ae^2)}\\
&= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2}-\frac{\int \left(\frac{cg}{e\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{-ae^2g-cd(6ef-5dg)}{e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{2c(ef-3dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4(cd^2+ae^2)}\\
&= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2}-\frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e(cd^2+ae^2)}\\
&\quad-\frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(cd^2+ae^2)}\\
&\quad+\frac{(ae^2g+cd(6ef-5dg))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e(cd^2+ae^2)}\\
&= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2}-\frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)(d+ex)}\\
&\quad-\frac{(ae^2g+cd(6ef-5dg))\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8e(cd^2+ae^2)^2(ef-dg)}\\
&\quad-\frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2e(cd^2+ae^2)\sqrt{a+cx^2}}\\
&\quad-\frac{\left(a\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{2\sqrt{-a}e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{(ae^2g+cd(6ef-5dg))\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{8e\left(cd^2+ae^2\right)^2(ef-dg)} + \frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e\left(cd^2+ae^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(cg(ae^2g+cd(6ef-5dg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8\left(cd^2+ae^2\right)^2(ef-dg)} + \frac{(cdg(ae^2g+cd(6ef-5dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e\left(cd^2+ae^2\right)^2(ef-dg)} - \frac{((ae^2g+cd(6ef-5dg))(ae^2g-cd(2ef-3dg)))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8e\left(cd^2+ae^2\right)^2(ef-dg)} + \frac{\left(c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}\right.}{e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}$$

$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{(c(ae^2g+cd(6ef-5dg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)}$$

$$- \frac{(cf(ae^2g+cd(6ef-5dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)}$$

$$- \frac{\left((ae^2g+cd(6ef-5dg))(ae^2g-cd(2ef-3dg))\sqrt{1+\frac{cx^2}{a}}\right)\int\frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{8e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{a+cx^2}}$$

$$+ \frac{\left(a\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-}}{\sqrt{}}\right.}{4\sqrt{-a}e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\left((ae^2g+cd(6ef-5dg))\left(ae^2g-cd(2ef-3dg)\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-}}{\sqrt{}}}}\right.}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{a+cx^2}}$$

$$+\frac{\left(a\sqrt{c}(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{4\sqrt{-a}\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$-\frac{\left(a\sqrt{c}f(ae^2g+cd(6ef-5dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx,x,\frac{\sqrt{1-}}{}\right.}{4\sqrt{-a}\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\left(\left(ae^2g+cd(6ef-5dg)\right)\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-\right.}\right.}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{-a}\sqrt{c}(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2e(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}f(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg(ae^2g+cd(6ef-5dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4e(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{(ae^2g+cd(6ef-5dg))(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-}}}}{\sqrt{2}}\right.\right.}{4e\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.04 (sec) , antiderivative size = 2450, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (-11*c^2*d^2*e^2*f^3 + a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 5*c^2*d^3*e*f^2*
g + 5*a*c*d*e^3*f^2*g - 11*a*c*d^2*e^2*f*g^2 + a^2*e^4*f*g^2 + 5*a*c*d^3*e*
g^3 - a^2*d*e^3*g^3 + 22*c^2*d^2*e^2*f^2*(f + g*x) - 2*a*c*e^4*f^2*(f + g*x
```

```
) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 10*c^2*d^3*e*f*g*(f + g*x) + 2*a*c*d*e
^3*f*g*(f + g*x) - 11*c^2*d^2*e^2*f*(f + g*x)^2 + a*c*e^4*f*(f + g*x)^2 + (
6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 5*c^2*d^3*e*g*(f + g*x)^2 - a*c*d*e^3*g*(f
 + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*e^2)*(e*f
- d*g) + (a*e^2*g + c*d*(6*e*f - 5*d*g))*(d + e*x)))/(d + e*x)^2 + (Sqrt[c]
*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(6*e*f - 5*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + (e*(I*Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(a*e^2*g + (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g)
 + c*d*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt
[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I)
*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*S
qrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e
*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x
))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt
[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c
]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - ((12*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*
Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((12*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))
/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2
)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((10*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f +
 g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
```

```
I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4
*e*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 1224, normalized size of antiderivative = 0.98

method	result	size
elliptic	Expression too large to display	1224
default	Expression too large to display	20359

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2*e/(a*e^2+c*d^
2)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*e*(a*e^2*g-5*c*d^2*g+6*c
*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(3*a*d*e^2*g-2*a*e^3*f-3*c*d^3*g+4*c*d^2
*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)/e*(f/g-(-a*c)^(1/
2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)
^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-
f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(a*e^2*g-5*c*d^2*
g+6*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(a^2*e^4*g^2+10*a*c*d^2*e^2
*g^2-12*a*c*d*e^3*f*g+4*a*c*e^4*f^2-3*c^2*d^4*g^2+12*c^2*d^3*e*f*g-8*c^2*d^
2*e^2*f^2)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)/e^2*(f/g-(-a
*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c
*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^
(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^3}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.643 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 600

$$
\begin{aligned}
\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} \\
&- \frac{2\sqrt{-a}g(7ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{a\sqrt{c}x}{(-a)^{3/2}}}}{\sqrt{2}}\right)|\frac{2ag}{-\sqrt{-a}\sqrt{c}f+ag}\right)}{3\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&+ \frac{2\sqrt{-a}g(ae^2g^2+c(-2e^2f^2+6defg-3d^2g^2))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{a\sqrt{c}x}{(-a)^{3/2}}}}{\sqrt{2}}\right),\frac{2}{-\sqrt{-a}}\right.}{3c^{3/2}e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&- \frac{2(ef-dg)^2\sqrt{\frac{g(\sqrt{-a}-\sqrt{c}x)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{-\frac{g(\sqrt{-a}+\sqrt{c}x)}{\sqrt{c}f-\sqrt{-a}g}}\operatorname{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\sqrt{\frac{c}{cf+\sqrt{-a}\sqrt{c}g}}\sqrt{f+gx}\right),\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right.}{e^3\sqrt{\frac{c}{cf+\sqrt{-a}\sqrt{c}g}}\sqrt{a+cx^2}}
\end{aligned}
$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e-2/3*g*(-3*d*g+7*e*f)*EllipticE(1/
2*(1+a*x*c^(1/2)/(-a)^(3/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c
^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e^2/c^(1/2)/(c*x
^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*g*(a*e^2*g
^2+c*(-3*d^2*g^2+6*d*e*f*g-2*e^2*f^2))*EllipticF(1/2*(1+a*x*c^(1/2)/(-a)^(3
/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/
2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/
2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d*g+e*f)^2*EllipticPi((g*x+f)^(1/2
)*(c/(c*f+g*(-a)^(1/2)*c^(1/2)))^(1/2),e*(f+g*(-a)^(1/2)/c^(1/2))/(-d*g+e*f
),((g*(-a)^(1/2)+f*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(g*((-a)^(1/2
```

```
)-x*c^(1/2))/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)*(-g*((-a)^(1/2)+x*c^(1/2))/(-g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(c*x^2+a)^(1/2)/(c/(c*f+g*(-a)^(1/2)*c^(1
/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {972, 733, 430, 947, 174, 552, 551, 435, 757, 858}

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)(ef-dg)^3}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$-\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(ef-dg)^2}{\sqrt{c}e^3\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$-\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)(ef-dg)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$
$$-\frac{8\sqrt{-a}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$
$$+\frac{2\sqrt{-a}g(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}e\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$+\frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}}{3ce}$$

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) - (8*Sqrt[-a]*f*g*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*e*Sqrt[(Sqrt[c]*(f +
 g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*g*(e*f - d*
```

```
g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*g*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-
2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + (2*Sqrt[-a]*g*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*e*Sqrt[f +
 g*x]*Sqrt[a + c*x^2]) - (2*(e*f - d*g)^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a
] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqr
t[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a
 + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{g(ef-dg)\sqrt{f+gx}}{e^2\sqrt{a+cx^2}} + \frac{g(f+gx)^{3/2}}{e\sqrt{a+cx^2}}\right) dx$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(g(ef-dg)^2)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^3}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}(3cf^2-ag^2)+2cfgx}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e^3\sqrt{a+cx^2}}$$

$$+ \frac{\left(2ag(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}e^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$+ \frac{\left(2ag(ef-dg)^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$
\begin{aligned}
&= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} \\
&\quad - \frac{2\sqrt{-a}g(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&\quad - \frac{2\sqrt{-a}g(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&\quad + \frac{(4fg)\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3e} - \frac{(g(cf^2+ag^2))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce} \\
&\quad - \frac{\left(2(ef-dg)^3\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^3\sqrt{a+cx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} \\
&\quad - \frac{2\sqrt{-a}g(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&\quad - \frac{2\sqrt{-a}g(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&\quad - \frac{\left(2(ef-dg)^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}\,dx,x,\sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}\right.}{e^3\sqrt{f+gx}\sqrt{a+cx^2}} \\
&\quad + \frac{\left(8afg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}e\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} \\
&\quad - \frac{\left(2ag(cf^2+ag^2)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}c^{3/2}e\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} - \frac{8\sqrt{-a}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$- \frac{2\sqrt{-a}g(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$- \frac{2\sqrt{-a}g(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{2\sqrt{-a}g(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}e\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{2(ef-dg)^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.46 (sec) , antiderivative size = 1440, normalized size of antiderivative = 2.40

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce}$$

$$+ \frac{2(f+gx)^{3/2}\left(7ce^2f\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - 3cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} + \frac{7ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2} - \frac{3cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2} + \frac{7ae^2fg^2\sqrt{-f-\frac{i}{}}}{(f+gx)^2}\right.}{}$$

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (2*(f + g*x)^(3/2)*(7*c*e^2
*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] + (7*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*c*d
*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (7*a*e^2*f*g^2*Sqr
```

```
t[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x)^2 - (14*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])/(f + g*x) + (6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) +
(Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(7*e*f - 3*d*g)*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[
f + g*x] + (I*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(I*Sqrt[a]*e*g + Sqrt[c]*(6*e*f -
 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 -
 f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt
[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c*e^2*f^2*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sq
rt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt
[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((6*I)*c*d
*e*f*g*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f
/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f -
d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/
Sqrt[f + g*x] + ((3*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt
[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(3*c*e^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/(f + g*x))^2)/g^2])
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 948, normalized size of antiderivative = 1.58

method	result
elliptic	$\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2g^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3ec}+\frac{2\left(\frac{g\left(d^2g^2-3defg+3e^2f^2\right)}{e^3}-\frac{g^3a}{3ec}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-a}}{c}}}}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/e*g^2/c*(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)/e^3-1/3/e*g^
3/c*a)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)
^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/
2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(
-g^2/e^2*(d*g-3*e*f)-2/3/e*g^2*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c
)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*(
(-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x
+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)))-2*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)/e^4*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)^{5/2}}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.644 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 469

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$-\frac{2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*g*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/e/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2*g*(
-d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*
g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)-2*(-d*g+e*f)^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {972, 733, 430, 947, 174, 552, 551, 435}

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)} - \frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
 (2*Sqrt[-a]*g*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[
-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + S
qrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}}\right) dx$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e^2\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}e\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2ag(ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)^2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{e^2\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\left(2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right.}{e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.98

$$\int\frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}dx = \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(\frac{2i\sqrt{a}fg\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{\sqrt{c}e}-\frac{i\sqrt{a}dg^2\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}}{}\right.}{}$$

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g*
Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2
```

```
]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d*g^
2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqrt[(g*
(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + Sqrt[c]
*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(S
qrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I
*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*e*
Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (Sqrt[a]*f^2
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), Ar
cSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
 Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sqrt[1 + (c*x^2)
/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*
Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(I*
Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sq
rt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(e^2*(I*Sqrt[c]*
d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(382) = 764$.

Time = 1.37 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2g(dg-2ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2g^2\left(\frac{f}{g}-\frac{\sqrt{-}}{c}\right.}{}\right.}{}$
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)d\,g^2-\sqrt{-ac}\,F\left(\sqrt{-\frac{}{g}}\right.\right.}{}$

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*g*(d*g-2*e*f)/e
^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/
2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(
1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*g^2/e
```

```
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE((
(x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/
2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)
,((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(d^2*g^2-2*d*e*f*g
+e^2*f^2)/e^3*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x
-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a
*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi
(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)^{3/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

```
[In] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.645 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 457

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg}$$
$$+\frac{2\sqrt{-a}e(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt{-a}(ae^2g^2(7ef-15dg)-c(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}}{15c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -8/15*e^2*(-3*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2+2/5*e^2*(e*x+d)*
(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/15*e*(9*a*e^2*g^2-c*(45*d^2*g^2-30*d*e*
f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^
(1/2)/c^(3/2)/g^3/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)-2/15*(a*e^2*g^2*(-15*d*g+7*e*f)-c*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*
e^2*f^2*g+8*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {945, 1668, 858, 733, 435, 430}

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(9ae^2g^2-c(45d^2g^2-30defg+8e^2f^2))\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g^2(7ef-15dg)-c(-15d^3g^3+45d^2efg^2-30de^2f^2g+8e^3f^3))\,\text{Ellipti}}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{15cg^2}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-8*e^2*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g^2) + (2*e^2*(d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + (2*Sqrt[-a]*e*(9*a*e^2*g^2
 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(15*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(a*e^2*g^2*(7*e*f - 15*d*g)
- c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*Sqrt[(Sqrt[
c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/(15*c^(3/2)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\int \frac{-5cd^3g+ae^2(2ef+dg)+e(3ae^2g+cd(2ef-15dg))x+4ce^2(ef-3dg)x^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{5cg}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{2\int \frac{-\frac{1}{2}cg^2(15cd^3g-ae^2(2ef+15dg))+\frac{1}{2}ceg(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{15c^2g^3}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{(e(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2)))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}dx}{15cg^3} + \frac{(ae^2g^2(7ef-15dg)-c(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{15cg^3}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\left(2ae(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx\right.}{15\sqrt{-a}c^{3/2}g^3\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2a(ae^2g^2(7ef-15dg)-c(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Su}}{15\sqrt{-a}c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} + \frac{2\sqrt{-a}e(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2}{\sqrt{-a}}\right.}{15c^{3/2}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}(ae^2g^2(7ef-15dg)-c(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F}{15c^{3/2}g^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.72 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e^2(-4ef+15dg+3egx)(a+cx^2)}{cg^2}+\frac{2(f+gx)\left(\frac{eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(8e^2f^2-30defg+45d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2}+\frac{\sqrt{c}e(-i\sqrt{c}f+\sqrt{a}g)\left(-9\right.}{}\right.}{}\right.}{}$$

[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]

[Out] (Sqrt[f + g*x]*((2*e^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2))/(c*g^2) + (2*(f + g*x)*((e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^3*g^2 - I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*d*e*f*g - 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(15*Sqrt[a + c*x^2])

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^3x\sqrt{cgx^3+cfx^2+agx+fa}}{5cg}+\frac{2\left(3de^2-\frac{4fe^3}{5g}\right)\sqrt{cgx^3+cfx^2+agx+fa}}{3cg}+\frac{2\left(d^3-\frac{2fae^3}{5cg}-\frac{a\left(3de^2-\frac{4fe^3}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^3/c/g*x*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*d*e^2-4/5*f/g*e^3)/c/g*(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)+2*(d^3-2/5*f*a/c/g*e^3-1/3*a/c*(3*d*e^2-4/5*f/g*e^3))*(f
/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1
/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*d^2*e-3
/5*e^3/c*a-2/3*f/g*(3*d*e^2-4/5*f/g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*(
(x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ellip
ticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*
c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left((8\,ce^3f^3-30\,cde^2f^2g+3\,(15\,cd^2e-ae^3)fg^2-45\,(cd^3-ade^2)g^3)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,a}{3\,cg^2}\right.\right.}{}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*((8*c*e^3*f^3 - 30*c*d*e^2*f^2*g + 3*(15*c*d^2*e - a*e^3)*f*g^2 - 45*
(c*d^3 - a*d*e^2)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/
(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c*e^3
*f^2*g - 30*c*d*e^2*f*g^2 + 9*(5*c*d^2*e - a*e^3)*g^3)*sqrt(c*g)*weierstras
sZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(
c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e^3*g^3*x - 4*c*e^3*f*g^2 + 15*c*d*e^2
*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.646 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}} dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} + \frac{4\sqrt{-a}e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}((3cd^2-ae^2)g^2+2cef(ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+4/3*e*(-3*d*g+e*f)*EllipticE(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1
/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*((-a*e^2+3*c*d^2)*
g^2+2*c*e*f*(-3*d*g+e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1
/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)^(1/2
)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {945, 24, 858, 733, 435, 430}

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(g^2(3cd^2-ae^2)+2cef(ef-3dg)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2a}{\sqrt{-a}\sqrt{c}}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ef-3dg)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}} + \frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) + (4*Sqrt[-a]*e*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*((3*c*d^2 - a*e^2)*g^2 + 2*c*e*f*(e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3
/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-d(3cd^2-ae^2)g+e(ae^2g+cd(2ef-9dg))x+2ce^2(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3cg} \\
&= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-e^2(3cd^2-ae^2)g+2ce^3(ef-3dg)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce^2g} \\
&= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{(2e(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2} \\
&\quad + \frac{1}{3}\left(3d^2-\frac{ae^2}{c}+\frac{2ef(ef-3dg)}{g^2}\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx
\end{aligned}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\left(4ae(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}\sqrt{c}g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2a\left(3d^2-\frac{ae^2}{c}+\frac{2ef(ef-3dg)}{g^2}\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx, x, \frac{\sqrt{1-}}{\sqrt{2}}\right.}{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} + \frac{4\sqrt{-a}e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}\left(3d^2-\frac{ae^2}{c}+\frac{2ef(ef-3dg)}{g^2}\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.65 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(e^2g^2(a+cx^2)-\frac{2eg^2(ef-3dg)(a+cx^2)}{f+gx}-2ice\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+\right.}{}$$

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2))
/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
```

```
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(290) = 580$.

Time = 2.54 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^2-\frac{a\,e^2}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\cdots}{-\cdots}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
risch	$\frac{2e^2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3cg}-\frac{\left(\frac{2a\,e^2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}-\frac{6c\,d^2g\left(\frac{f}{g}-\frac{\sqrt{-\cdots}}{c}\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3*e^2/c/g*(c*g*x
^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^2-1/3*a*e^2/c)*(f/g-(-a*c)^(1/2)/c)*((x+f/
g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*d*e-2/3*e^2*f/g)*(f/g-(-a*c)^(1/
2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)
^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+
```

```
c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a
*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-
a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ce^2g^2+(2ce^2f^2-6cdefg+3(3cd^2-ae^2)g^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2}\right.\right.}{}$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e^2*g^2 + (2*c*e^2*f^2 - 6*c*d*e*f*g
 + 3*(3*c*d^2 - a*e^2)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(c*
e^2*f*g - 3*c*d*e*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^
2*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.647 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 288

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*e*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/g/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(-d
*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+
f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)
/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {858, 733, 435, 430}

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[a +
 c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
```

```
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{g}$$

$$= \frac{\left(2ae\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2a(-ef+dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.29 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.52

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left(-eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+i\sqrt{c}e\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{}}{}\right.\right.\right.}{cg^2\sqrt{}}$$

[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]

[Out] (-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(232) = 464.

Time = 1.22 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.81

method	result
default	$\frac{2\left(F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)ae\,g^2+F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)cdfg-\sqrt{-ac}\,F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)d\,g^2\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2d\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}+\frac{2e\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-}}{c}}}}{}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+}}$

[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 2*(EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)
/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f*g-(-
a*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*g^2+(-a*c)^(1/2)*EllipticF((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)
)^(1/2))*e*f*g-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2)
)*c*e*f^2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.62

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left(3\sqrt{cg}eg\,\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\right.\right.\right.}{3\,cg^2}$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*e*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + (e*f - 3*d*g)
*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
+ 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g))/(c*g^2)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.648 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {733, 430}

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\left(2a\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right), \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\sqrt{a+cx^2}}$$

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{2\left(cf-g\sqrt{-ac}\right)F\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{c\,x^{2}+a}\,\sqrt{gx+f}}{cg\left(cg\,x^{3}+cf\,x^{2}+agx+fa\right)}$	200
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^{2}+a\right)}\,\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,\sqrt{cg\,x^{3}+cf\,x^{2}+agx+fa}}$	242

[In] int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(c*f-g*(-a*c)^(1/2))*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-
(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/g/
(c*g*x^3+c*f*x^2+a*g*x+a*f)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\frac{2\,\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,\left(cf^{2}-3\,ag^{2}\right)}{3\,cg^{2}},-\frac{8\,\left(cf^{3}+9\,afg^{2}\right)}{27\,cg^{3}},\frac{3\,gx+f}{3\,g}\right)}{cg}$$

[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)/(c*g)

Sympy [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.649 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(
-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/
a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(e+d*c^(1/2)/(-a)
^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {947, 174, 552, 551}

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*
EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{cx^2}{a}}\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.36 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right), \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticPi}\left(\frac{\sqrt{c}(e\ldots}{e(\sqrt{c}f\ldots}\right.\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)\sqrt{a+cx^2}}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g))
, I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.41

method	result	s
default	$\frac{2(cf-g\sqrt{-ac})\Pi\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\frac{(g\sqrt{-ac}-cf)e}{c(dg-ef)},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\sqrt{cx^2+a}\sqrt{gx+f}}{c(dg-ef)(cg\,x^3+cf\,x^2+agx+fa)}$	2
elliptic	$\frac{2\sqrt{(gx+f)(cx^2+a)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\sqrt{cx^2+a}\,e\sqrt{cg\,x^3+cf\,x^2+agx+fa}\left(-\frac{f}{g}+\frac{d}{e}\right)}$	2

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*f-g*(-a*c)^(1/2))*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.650 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 746

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-e*Ellip
ticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*
c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+
c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)+e*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-
```

```
a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+
f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*
x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)
/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elli
pticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/
2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {954, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\,(ae^2+cd^2)\,(ef-dg)}$$
$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\,(ae^2+cd^2)\,(ef-dg)}$$
$$-\frac{\sqrt{-a}\sqrt{c}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\,(ae^2+cd^2)\,(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g-cd(2ef-3dg))\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ae^2+cd^2)\,(ef-dg)}$$
$$-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\,(ae^2+cd^2)\,(ef-dg)}$$

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]

[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
))) - (Sqrt[-a]*Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
```

```
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sq
rt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*e*f*Sqrt[(Sqrt[c]*(f + g*x)
)/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^
2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d
*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a]
+ e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[
c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)\,(ef-dg)(d+ex)} - \frac{\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\,(cd^2+ae^2)\,(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\int \left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right) dx}{2(cd^2+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} + \frac{(ceg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} - \frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} - \frac{(cef)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} - \frac{\left((ae^2g-cd(2ef-3dg))\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2(cd^2+ae^2)(ef-dg)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}dg\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{\left((ae^2g-cd(2ef-3dg))\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\cdots}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{a+cx^2}} + \frac{\left(a\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(cd^2+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} - \frac{\left(a\sqrt{c}ef\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{\left((ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,\right.}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.97 (sec) , antiderivative size = 1349, normalized size of antiderivative = 1.81

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(-\frac{2e^2(a+cx^2)}{d+ex}+\frac{2\left(-ce^2f^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+cdef^2g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}-ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+adeg^3\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+2ce^2f^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+\right.}{}\right.}{}$$

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*
e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c
*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g
*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2*d*g))*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(
Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] + I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[
(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)]))/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-(e*f) + d*g)*(f + g*
x))))/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{\left(a\,e^2gd-a\,e^3f+c\,d^3g-c\,d^2ef\right)(ex+d)}-\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\right.}{\left(a\,e^2gd-a\,e^3f+c\,d^3g-c\,d^2ef\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(e^2/(a*d*e^2*g-a*e
^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-c*d*g/(a*
d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c
)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*
c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*
EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2))-c*e*g/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^
(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g+3*c*d^2*g-2*c*d*e*
f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*
((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-
a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2\,\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.651 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1257

$$
\begin{aligned}
&\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx \\
&= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} \\
&+\frac{3\sqrt{-a}\sqrt{c}e(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} \\
&+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} \\
&-\frac{3\sqrt{-a}\sqrt{c}ef(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}} \\
&+\frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}} \\
&+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} \\
&-\frac{3(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\operatorname{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}
\end{aligned}
$$

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^2+3
/4*e^2*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*
d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/4*e*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(a*e^2+c*d^2)^
2/(-d*g+e*f)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(
1/2)+1/2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a
*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x
```

```
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f
)^(1/2)/(c*x^2+a)^(1/2)-3/4*e*f*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)
+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/
2)+3/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^
(1/2)*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a
*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*Ell
ipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1
/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/
2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f
)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-3/4*(a*e^2*g-c*d*(
-3*d*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*
e^2+c*d^2)^2/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^
(1/2)
```

Rubi [A] (verified)

Time = 3.28 (sec) , antiderivative size = 1257, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules

used = {954, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{3(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{cx^2+a}e^2}{4(cd^2+ae^2)^2(ef-dg)^2(d+ex)} - \frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2(cd^2+ae^2)(ef-dg)(d+ex)^2}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)e}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}$$
$$- \frac{3\sqrt{-a}\sqrt{c}f(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$- \frac{3(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}$$
$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d +
e*x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x
^2])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[-a]*Sqrt[c]*e*
(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]
*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]
*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[
```

```
Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g
)])/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*Sqrt
[-a]*Sqrt[c]*e*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2
 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt
[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/S
qrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)
^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sq
rt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(
2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)
*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*(a*e^2*g -
 c*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*
Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*
g)])/(4*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f +
 g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3ae^2g-4cd(ef-dg)+2ce(ef-2dg)x+ce^2gx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3\left(ae^2g-cd(2ef-3dg)\right)}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)} - \frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} - \frac{\left(3\left(ae^2g-cd(2ef-3dg)\right)\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \frac{\left(3\left(ae^2g-cd(2ef-3dg)\right)\right)\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2} - \frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{a+cx^2}} - \frac{\left(a\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{2\sqrt{-a}\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{(3(ae^2g-cd(2ef-3dg)))\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}}+\frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$
$$+ \frac{\left(c(ef-3dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{(3cdg(ae^2g-cd(2ef-3dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$
$$- \frac{(3ceg(ae^2g-cd(2ef-3dg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$
$$+ \frac{\left(3(ae^2g-cd(2ef-3dg))^2\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$
$$+ \frac{\left(c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx,x,\sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}$$

$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{(3ce(ae^2g-cd(2ef-3dg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$

$$+ \frac{(3cef(ae^2g-cd(2ef-3dg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2}$$

$$+ \frac{\left(3(ae^2g-cd(2ef-3dg))^2\sqrt{1+\frac{cx^2}{a}}\right)\int\frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{8\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{a+cx^2}}$$

$$- \frac{\left(3a\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\right.}{4\sqrt{-a}\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)^2(d+ex)}$$

$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$- \frac{\left(3(ae^2g-cd(2ef-3dg))^2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}dx,x,\sqrt{1-\right.}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{a+cx^2}}$$

$$- \frac{\left(3a\sqrt{c}e(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{4\sqrt{-a}(cd^2+ae^2)^2(ef-dg)^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$+ \frac{\left(3a\sqrt{c}ef(ae^2g-cd(2ef-3dg))\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx,x,\right.}{4\sqrt{-a}(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$=-\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}$$

$$+\frac{3\sqrt{-a}\sqrt{c}e(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{3\sqrt{-a}\sqrt{c}ef(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$-\frac{\left(3(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}dx\right.}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2(cd^2+ae^2)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)^2(ef-dg)^2(d+ex)}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}e(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{3\sqrt{-a}\sqrt{c}ef(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{3\sqrt{-a}\sqrt{c}dg(ae^2g-cd(2ef-3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$+ \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{3(ae^2g-cd(2ef-3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.75 (sec) , antiderivative size = 2491, normalized size of antiderivative = 1.98

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-15*c^2*d^2*e^2*f^3 - 3*a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 9*c^2*d^3*e*f^
2*g + 9*a*c*d*e^3*f^2*g - 15*a*c*d^2*e^2*f*g^2 - 3*a^2*e^4*f*g^2 + 9*a*c*d^
3*e*g^3 + 3*a^2*d*e^3*g^3 + 30*c^2*d^2*e^2*f^2*(f + g*x) + 6*a*c*e^4*f^2*(f
```

+ g*x) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 18*c^2*d^3*e*f*g*(f + g*x) - 6*a *c*d*e^3*f*g*(f + g*x) - 15*c^2*d^2*e^2*f*(f + g*x)^2 - 3*a*c*e^4*f*(f + g* x)^2 + (6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 9*c^2*d^3*e*g*(f + g*x)^2 + 3*a*c* d*e^3*g*(f + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a* e^2)*(e*f - d*g) - 3*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d + e*x)))/(d + e*x) ^2 + (3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*(a*e^2*g + c* d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh [Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g) /(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((Sqrt[c]*d - I*Sqrt[a]*e)*(3*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(e*f - 2*d*g) - Sqrt[a]*c*e*(2*e^2*f^2 - 6*d*e*f*g + d^2*g^2) - I*c^(3/2)*d*(4*e^2*f^2 - 9*d*e*f*g + 8*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*A rcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt [a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I)*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I* Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a] *g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqr t[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g* x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqr t[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqr t[c]] - ((20*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)] *Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticP i[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((4*I)*a*c*d*e^3*f*g*S qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I* Sqrt[a]*g)/Sqrt[c]] + ((15*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/ 2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSin h[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((6*I)*a*c *d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]) /Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((3*I)*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a]) /Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*

```
(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*
g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
- I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
)/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 1192, normalized size of antiderivative = 0.95

method	result	size
elliptic	Expression too large to display	1192
default	Expression too large to display	20366

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2*e^2/(a*d*e^2*g
-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+3/4
*e^2*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f+7*c*
d^3*g-4*c*d^2*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)^(1/2
)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^
(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-3/4*c*e*g*(a*e^2*g+3*c*d^2
*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)^(1/2)/c)*
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c
)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d^2*e^2*g^2+4*a*c
*d*e^3*f*g-4*a*c*e^4*f^2+15*c^2*d^4*g^2-20*c^2*d^3*e*f*g+8*c^2*d^2*e^2*f^2)
/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*
((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-
a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3} \,d x$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.652 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 387

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}} + \frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+2*g*EllipticE(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(-d*g+e*f)/(
a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2)-2*e*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/
2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*
x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {972, 759, 21, 733, 435, 947, 174, 552, 551}

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\left(ag^2+cf^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$
$$-\frac{2e\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)}$$
$$+\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (2*Sq
rt[-a]*Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e
*f - d*g)*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[a + c*x^2]) - (2*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[S
qrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a
]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right) dx$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}} + \frac{(2cg)\int \frac{-\frac{f}{2}-\frac{gx}{2}}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)\,(cf^2+ag^2)} + \frac{\left(e\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{(ef-dg)\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{(ef-dg)\,(cf^2+ag^2)} - \frac{\left(2e\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{(ef-dg)\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}} - \frac{\left(2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2a\sqrt{c}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(ef-dg)\,(cf^2+ag^2)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}} + \frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.30 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\sqrt{c}(ef-dg)E\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{}}{\sqrt{}}}}{\sqrt{f+gx}}\right.\right.\right.}{}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*ArcSi
nh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*
g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g))*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[a]*g)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(324) = 648$.

Time = 2.53 (sec) , antiderivative size = 929, normalized size of antiderivative = 2.40

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{2\left(cg\,x^2+ag\right)g}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{\left(x+\frac{f}{g}\right)\left(cg\,x^2+ag\right)}}+\frac{2cgf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,F\left(\sqrt{\frac{x+}{\frac{f}{g}-\sqrt{}}}\right.}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*(c*g*x^2+a*g)/(
a*g^2+c*f^2)*g/(d*g-e*f)/((x+f/g)*(c*g*x^2+a*g))^(1/2)+2/(a*g^2+c*f^2)*c*g*
f/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c
)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
)+2/(a*g^2+c*f^2)*c*g^2/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c
)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*(
(-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x
+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)
/c))^(1/2)))-2/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c
)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)
*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/
g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.653 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 818

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}$$
$$+ \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$+ \frac{2\sqrt{-a}\sqrt{c}eg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$
$$- \frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$
$$- \frac{2e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] 2/3*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(3/2)+8/3*c*f*g^2*
(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)^2/(g*x+f)^(1/2)+2*e*g^2*(c*x^2+a)^
```

```
(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)/(g*x+f)^(1/2)+8/3*c^(3/2)*f*g*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^
2)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*e*g
*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^
(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(1+c*x^2/a)^(1/2)/(
-d*g+e*f)^2/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)-2/3*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(1+c*x^2/a)^
(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(a*g^2+c*
f^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*e^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2))*(1+c*x^2/a)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/
2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules

used = {972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticPi}\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} + \frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)e}{(ef-dg)^2(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}} + \frac{2g^2\sqrt{cx^2+a}e}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}} + \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)(cf^2+ag^2)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}} - \frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)(cf^2+ag^2)\sqrt{f+gx}\sqrt{cx^2+a}} + \frac{8cfg^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}} + \frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) + (
8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])
+ (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]) +
 (8*Sqrt[-a]*c^(3/2)*f*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*e*g*Sqrt[f + g*x]*Sqrt
[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*
Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
qrt[-a]*Sqrt[c]*f - a*g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]*Sqr
t[a + c*x^2]) - (2*e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
```

```
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{e^2}{(ef-dg)^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right) dx$$

$$= \frac{e^2 \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2ceg)\int \frac{-\frac{f}{2}-\frac{gx}{2}}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2\left(cf^2+ag^2\right)} + \frac{(2cg)\int \frac{-\frac{3f}{2}+\frac{gx}{2}}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{3(ef-dg)\left(cf^2+ag^2\right)} + \frac{\left(e^2\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{(ef-dg)^2\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(4cg)\int \frac{\frac{1}{4}\left(3cf^2-ag^2\right)+cfgx}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3(ef-dg)\left(cf^2+ag^2\right)^2} - \frac{(ceg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{(ef-dg)^2\left(cf^2+ag^2\right)} - \frac{\left(2e^2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{(ef-dg)^2\sqrt{a+cx^2}}$$

$$=\frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}}+\frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}-\frac{\left(4c^2fg\right)\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3(ef-dg)\left(cf^2+ag^2\right)^2}+\frac{(cg)\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3(ef-dg)\left(cf^2+ag^2\right)}-\frac{\left(2e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}\,dx,x,\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{\left(2a\sqrt{c}eg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$=\frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}}+\frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}+\frac{2\sqrt{-a}\sqrt{c}eg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}-\frac{2e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}-\frac{\left(8ac^{3/2}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}+\frac{\left(2a\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx,x,\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$
= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}}
+ \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}
+ \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}
+ \frac{2\sqrt{-a}\sqrt{c}eg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}
- \frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}\sqrt{a+cx^2}}
- \frac{2e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+cx^2}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.37 (sec) , antiderivative size = 1917, normalized size of antiderivative = 2.34

$$
\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)\left(a+cx^2\right)\left(ag^2(4ef-dg+3egx)+cf(-\right.\right.}{}
$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4*
e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) - (f
 + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4*g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*a*c*d
*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*
```

```
a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*a*c
*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e^2*f^3*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*
x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (Sqrt[c]*f + I*Sqrt[a]*g)*(3*a^(3/2)*e^2*g^3 + (3*I)*a*Sqrt[c]*e
*g^2*(2*e*f - d*g) + Sqrt[a]*c*g*(2*e^2*f^2 + 2*d*e*f*g - d^2*g^2) + (3*I)*
c^(3/2)*f*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
 x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c^2*e^2*f^4*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt
[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (6*I)*a*c*e^2
*f^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3
*I)*a^2*e^2*g^4*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sq
rt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*
f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)])))/(3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(
f + g*x)^(3/2)*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 1079, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	1079
default	Expression too large to display	9415

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2/3/(a*g^2+c*f^2)
```

```
/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(x+f/g)^2+2/3*(c*g*x^2+a*g)/(a
*g^2+c*f^2)^2*g*(3*a*e*g^2-4*c*d*f*g+7*c*e*f^2)/(d*g-e*f)^2/((x+f/g)*(c*g*x
^2+a*g))^(1/2)+2*(-1/3*c*g/(a*g^2+c*f^2)/(d*g-e*f)-1/3*c*f*g*(3*a*e*g^2-4*c
*d*f*g+7*c*e*f^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g
)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x
+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-2/3*g^2*c*(3*a*e*g^2-4*c*d*f*g+7*c*e*f
^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(
1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^
(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-
f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f
/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c
))^(1/2)))+2*e/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/
e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-
f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{5/2}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.654 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\,\text{EllipticPi}\left(\frac{2e}{\sqrt{-c}d+e},\arcsin\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right),\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

```
[Out] -2*EllipticPi(1/2*(1-x*(-c)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*(-c)^(1/2)),2^(1/
2)*(g/(g+f*(-c)^(1/2)))^(1/2))*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/2)/
(e+d*(-c)^(1/2))/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {946, 174, 552, 551}

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\,\text{EllipticPi}\left(\frac{2e}{\sqrt{-c}d+e},\arcsin\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right),\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[-c]*(f + g*x))/(Sqrt[-c]*f + g)]*EllipticPi[(2*e)/(Sqrt[-c]*
d + e), ArcSin[Sqrt[1 - Sqrt[-c]*x]/Sqrt[2]], (2*g)/(Sqrt[-c]*f + g)])/((Sq
rt[-c]*d + e)*Sqrt[f + g*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{1-\sqrt{-c}x}\sqrt{1+\sqrt{-c}x}(d+ex)\sqrt{f+gx}}\,dx$$

$$= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}d+e-ex^2\right)\sqrt{f+\frac{g}{\sqrt{-c}}-\frac{gx^2}{\sqrt{-c}}}}\,dx, x, \sqrt{1-\sqrt{-c}x}\right)\right)$$

$$= -\frac{\left(2\sqrt{1+\frac{g\left(-1+\sqrt{-c}x\right)}{\sqrt{-c}f+g}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}d+e-ex^2\right)\sqrt{1-\frac{gx^2}{\sqrt{-c}\left(f+\frac{g}{\sqrt{-c}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c}x}\right)}{\sqrt{f+gx}}$$

$$= -\frac{2\sqrt{1-\frac{g\left(1-\sqrt{-c}x\right)}{\sqrt{-c}f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.37

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2i\sqrt{\frac{g\left(\frac{i}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{ig}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-ig}{\sqrt{c}f+ig}\right)-\text{EllipticPi}\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+ig)},i\text{arc}\right.\right.}{\sqrt{-f-\frac{ig}{\sqrt{c}}}(ef-dg)\sqrt{1+cx^2}}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f - d*
g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f + g*x]
], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e*f -
d*g)*Sqrt[1 + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(95) = 190.

Time = 1.81 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2(g+f\sqrt{-c})\Pi\left(\sqrt{\frac{(gx+f)\sqrt{-c}}{g+f\sqrt{-c}}},-\frac{(g+f\sqrt{-c})e}{\sqrt{-c}\,(dg-ef)},\sqrt{\frac{g+f\sqrt{-c}}{f\sqrt{-c}-g}}\right)\sqrt{-\frac{(x\sqrt{-c}-1)g}{g+f\sqrt{-c}}}\,\sqrt{-\frac{(x\sqrt{-c}+1)g}{f\sqrt{-c}-g}}\,\sqrt{\frac{(gx+f)\sqrt{-c}}{g+f\sqrt{-c}}}\,\sqrt{c\,x^2+1}\,\sqrt{gx+f}}{\sqrt{-c}\,(dg-ef)(cg\,x^3+cf\,x^2+gx+f)}$	215
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^2+1)}\left(\frac{f}{g}+\frac{1}{\sqrt{-c}}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x+\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}}\,\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}},\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^2+1}\,e\sqrt{cg\,x^3+cf\,x^2+gx+f}\,\left(-\frac{f}{g}+\frac{d}{e}\right)}$	240

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g+f*(-c)^(1/2))/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2
)))^(1/2),-(g+f*(-c)^(1/2))*e/(-c)^(1/2)/(d*g-e*f),((g+f*(-c)^(1/2))/(f*(-c
)^(1/2)-g))^(1/2))*(-(x*(-c)^(1/2)-1)*g/(g+f*(-c)^(1/2)))^(1/2)*(-(x*(-c)^(
1/2)+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/
2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{(d+ex)\,\sqrt{f+gx}\sqrt{cx^2+1}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+1}\,(d+e\,x)}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)), x)
```

3.655 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 454

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{\sqrt[4]{cf^2+ag^2}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2+ae^2)(f+gx)^2}{(cf^2+ag^2)(d+ex)^2}}{\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)^2}}\,\text{EllipticF}\left(2\right.}{\sqrt[4]{cd^2+ae^2}(ef-dg)\sqrt{a+cx^2}\sqrt{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2}{(cf^2}}}$$

```
[Out] -(a*g^2+c*f^2)^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2
)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arctan((a*e^2+c*d^2)^(
1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))*EllipticF(sin(2*arct
an((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/
2*(2+2*(a*e*g+c*d*f)/(a*e^2+c*d^2)^(1/2)/(a*g^2+c*f^2)^(1/2))^(1/2))*(1+(g*
x+f)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))*((-d*g+e*f)^2*(c*x^2+
a)/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)*((1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)
/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)/(1+(g*x+f)*(a*e^2
+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))^2)^(1/2)/(a*e^2+c*d^2)^(1/4)/(-d
*g+e*f)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a
*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {950, 1117}

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(\ldots\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx\ldots}{(d+ex)\ldots}}}$$

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^2
 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 +
a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)
*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2))/
(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^2]*El
lipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^2)^(1/
4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c*f^2 +
a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqrt[1 - (2
*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*
(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

Rule 950

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*((a + c*x^2)/(
(c*f^2 + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqrt[a + c*x^2])), Subst[Int[1/
Sqrt[1 - (2*c*d*f + 2*a*e*g)*(x^2/(c*f^2 + a*g^2)) + (c*d^2 + a*e^2)*(x^4/(
c*f^2 + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{(2cdf+2aeg)x^2}{cf^2+ag^2}+\frac{(cd^2+ae^2)x^4}{cf^2+ag^2}}}\,dx, x, \frac{\sqrt{f+gx}}{\sqrt{d+ex}}\right)}{(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2+ag^2}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2+ae^2)(f+gx)^2}{(cf^2+ag^2)(d+ex)^2}}{\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)^2}}F\Big(2}{\sqrt[4]{cd^2+ae^2}(ef-dg)\sqrt{a+cx^2}\sqrt{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.18 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.76

$$\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{2}(i\sqrt{a}+\sqrt{c}x)\sqrt{d+ex}\sqrt{\frac{d-\frac{i\sqrt{a}e}{\sqrt{c}}+\frac{i\sqrt{c}dx}{\sqrt{a}}+ex}{d+ex}}\sqrt{\frac{(i\sqrt{c}d+\sqrt{a}e)(f+gx)}{(i\sqrt{c}f+\sqrt{a}g)(d+ex)}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\right),-\frac{i\sqrt{}}{}\right.}{(\sqrt{c}d-i\sqrt{a}e)\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqrt
[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt[a
]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqr
t[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x
))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c])/(2*
e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[a] + S
qrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt[a + c*
x^2])
```

Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.88

method	result
default	$\frac{2(c e^2 f x^2-\sqrt{-ac}\, e^2 g x^2+2cdefx-2\sqrt{-ac}\, degx+c d^2 f-\sqrt{-ac}\, d^2 g) F\left(\sqrt{\frac{(e\sqrt{-ac}-cd)(gx+f)}{(g\sqrt{-ac}-cf)(ex+d)}}, \sqrt{\frac{(e\sqrt{-ac}+cd)(g\sqrt{-ac}-cf)}{(g\sqrt{-ac}+cf)(e\sqrt{-ac}-cd)}}\right)\sqrt{\frac{(dg-ef)}{(g\sqrt{-ac}}}}{\sqrt{-\frac{(gx+f)(ex+d)(-cx+\sqrt{-ac})(cx+\sqrt{-ac})}{c}}\,(-e\sqrt{-ac}+cd)(dg-ef)\sqrt{(gx}}$
elliptic	$\frac{2\sqrt{(gx+f)(ex+d)(c x^2+a)}\left(-\frac{f}{g}+\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{\left(\frac{d}{e}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{f}{g}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\left(x+\frac{d}{e}\right)^2\sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{\left(\frac{f}{g}+\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\, F\left(\sqrt{\frac{\left(\frac{d}{e}\right.}{\left(\frac{f}{g}\right.}}\right.}{\sqrt{gx+f}\,\sqrt{ex+d}\,\sqrt{c x^2+a}\left(\frac{d}{e}-\frac{\sqrt{-ac}}{c}\right)\left(-\frac{d}{e}+\frac{f}{g}\right)\sqrt{ceg\left(x+\frac{f}{g}\right)\left(x+\frac{d}{e}\right)\left(x-\frac{\sqrt{-ac}}{c}\right)\left(x+\right.}}$

[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(c*e^2*f*x^2-(-a*c)^(1/2)*e^2*g*x^2+2*c*d*e*f*x-2*(-a*c)^(1/2)*d*e*g*x+c*d^2*f-(-a*c)^(1/2)*d^2*g)*EllipticF(((e*(-a*c)^(1/2)-c*d)*(g*x+f)/(g*(-a*c)^(1/2)-c*f)/(e*x+d))^(1/2),((e*(-a*c)^(1/2)+c*d)*(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)/(e*(-a*c)^(1/2)-c*d))^(1/2))*((d*g-e*f)*(c*x+(-a*c)^(1/2))/(g*(-a*c)^(1/2)-c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-c*x+(-a*c)^(1/2))/(g*(-a*c)^(1/2)+c*f)/(e*x+d))^(1/2)*((e*(-a*c)^(1/2)-c*d)*(g*x+f)/(g*(-a*c)^(1/2)-c*f)/(e*x+d))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(-1/c*(g*x+f)*(e*x+d)*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2)))^(1/2)/(-e*(-a*c)^(1/2)+c*d)/(d*g-e*f)/((g*x+f)*(e*x+d)*(c*x^2+a))^(1/2)

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*e*f + c*d*g)*x^3 + a*d*f + (c*d*f + a*e*g)*x^2 + (a*e*f + a*d*g)*x), x)

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

3.656 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 52

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$

```
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1/
2)/(2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {533, 432, 430}

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt[
1 + x]*Sqrt[-1 + 2*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{-1+x^2}\int \frac{1}{\sqrt{-1+x^2}\sqrt{-1+2x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}}\\ &= \frac{\left(\sqrt{1-2x^2}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\\ &= \frac{\left(\sqrt{1-2x^2}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\\ &= \frac{\sqrt{1-2x^2}\sqrt{1-x^2}F\left(\sin^{-1}(x)|\,2\right)}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 34.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = -\frac{2(-1+x)^{3/2}\sqrt{\frac{1+x}{1-x}}\sqrt{\frac{1-2x^2}{(-1+x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+\sqrt{2}+\frac{1}{-1+x}}}{2^{3/4}}\right),4\left(-4+3\sqrt{2}\right)\right)}{\sqrt{3+2\sqrt{2}}\sqrt{1+x}\sqrt{-1+2x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Ellip
ticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2])]
)/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^2-1}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{2x^4-3x^2+1}$	58
elliptic	$\frac{\sqrt{(2x^2-1)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^2-1}\,\sqrt{2x^4-3x^2+1}}$	73

```
[In] int(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1+x)^(1/2)*(1+x)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-2
*x^2+1)^(1/2)*EllipticF(x,2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = F(\arcsin{(x)}\mid 2)$$

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] elliptic_f(arcsin(x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}\,dx$$

```
[In] integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

```
[In] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
[Out] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.657 $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{16(cdf-aeg)^2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} + \frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^3d^3e} + \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cd\sqrt{d+ex}}$$

```
[Out] -16/35*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/35*(-a*e*g+c*d*f)*(g*x+f)^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+2/7*(g*x+f)^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+16/35*g*(-a*e*g+c*
d*f)^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=-\frac{16\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2\,(2ae^2g-cd(3ef-dg))}{35c^4d^4e\sqrt{d+ex}}$$
$$+\frac{16g\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{35c^3d^3e}$$
$$+\frac{12(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}$$
$$+\frac{2(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cd\sqrt{d+ex}} \\
&\quad+\frac{\left(6(cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\right)\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{7cde^2} \\
&= \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cd\sqrt{d+ex}} \\
&\quad+\frac{\left(24(cdf-aeg)^2\right)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{35c^2d^2} \\
&= \frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^3d^3e} \\
&\quad+\frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cd\sqrt{d+ex}} \\
&\quad-\frac{\left(8(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{35c^3d^3e} \\
&= -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} \\
&\quad+\frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^3d^3e} \\
&\quad+\frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cd\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg(35f^2+14fgx+3g^2x^2)+c^3d^3(35f^3}{35c^4d^4\sqrt{d+ex}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
+ g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3
+ 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e}{35\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2+70a}{35c^4d^4\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$

```
[In] int((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/35/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-5*c^3*d^3*g^3*x^3+6*a*c^2
*d^2*e*g^3*x^2-21*c^3*d^3*f*g^2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^
2*x-35*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*e*f
^2*g-35*c^3*d^3*f^3)/c^4/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\,(5\,c^3d^3g^3x^3+35\,c^3d^3f^3-70\,ac^2d^2ef^2g+56\,a^2cde^2fg^2-16\,a^3e^3g^3+3\,(7\,c^3d^3fg^2-2\,ac^2d^2eg^3)x^2+(35}{35\,(c^4d^4ex+c^4d^5)}$$

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c*
d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2
+ (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{2\sqrt{cdx+ae}f^3}{cd} + \frac{2\,(c^2d^2x^2-acdex-2\,a^2e^2)f^2g}{\sqrt{cdx+ae}c^2d^2} \\ &\quad + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)fg^2}{5\sqrt{cdx+ae}c^3d^3} \\ &\quad + \frac{2\,(5\,c^4d^4x^4-ac^3d^3ex^3+2\,a^2c^2d^2e^2x^2-8\,a^3cde^3x-16\,a^4e^4)g^3}{35\sqrt{cdx+ae}c^4d^4}\end{aligned}$$

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^2
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*a
^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d
^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16*a^4
*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(245) = 490.

Time = 0.31 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{2e\left(\frac{35(c^3d^3f^3-3ac^2d^2ef^2g+3a^2cde^2fg^2-a^3e^3g^3)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^4d^4e} - \frac{35\sqrt{-cd^2e+ae^3}c^3d^3e^3f^3-35\sqrt{-cd^2e+ae^3}c^3d^4e^2f^2g-70\sqrt{-}}{}\right.}{}$$

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/35*e*(35*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*e
^3*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e) - (35*sqrt(-c*d
^2*e + a*e^3)*c^3*d^3*e^3*f^3 - 35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^2*f^2*g
 - 70*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^4*f^2*g + 21*sqrt(-c*d^2*e + a*e^3
)*c^3*d^5*e*f*g^2 + 28*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^3*f*g^2 + 56*sqrt
(-c*d^2*e + a*e^3)*a^2*c*d*e^5*f*g^2 - 5*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g^3
 - 6*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2*g^3 - 8*sqrt(-c*d^2*e + a*e^3)*a^
2*c*d^2*e^4*g^3 - 16*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g^3)/(c^4*d^4*e^4) + (3
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*f^2*g - 70*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d*e^5*f*g^2 + 35*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*a^2*e^6*g^3 + 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(5/2)*c*d*e^2*f*g^2 - 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3*g^
3 + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^3)/(c^4*d^4*e^7))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3)}{35\,c^4\,d^4\,e} - \frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e} + \right.}{x+\frac{d}{e}}$$

```
[In] int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))/(
35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*e*g
 - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e)
```

3.658 $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 200

$$\begin{aligned}\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^3d^3e\sqrt{d+ex}}\\ &\quad+\frac{8g(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e}\\ &\quad+\frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cd\sqrt{d+ex}}\end{aligned}$$

```
[Out] -8/15*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^3/d^3/e/(e*x+d)^(1/2)+2/5*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+8/15*g*(-a*e*g+c*d*f)*(e*x+d)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

= {884, 808, 662}

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{8\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)\,(2ae^2g-cd(3ef-dg))}{15c^3d^3e\sqrt{d+ex}} + \frac{8g\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
```

```
gerQ[n])
```

Rubi steps

$$\text{integral} = \frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cd\sqrt{d+ex}} + \frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5cde^2}$$

$$= \frac{8g(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cd\sqrt{d+ex}} - \frac{\left(4(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{15c^2d^2e}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} + \frac{8g(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cd\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2(15f^2+10fgx+3g^2x^2)\right)}{15c^3d^3\sqrt{d+ex}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x) +
 c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15c^2d^2f^2)}{15\sqrt{ex+d}\,c^3d^3}$	98
gosper	$\frac{2(cdx+ae)(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15c^2d^2f^2)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	116

```
[In] int((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2-4*a*c*d*e
*g^2*x+10*c^2*d^2*f*g*x+8*a^2*e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)/c^3/d^
3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-20\,acdefg+8\,a^2e^2g^2+2\,(5\,c^2d^2fg-2\,acdeg^2)x)\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{15\,(c^3d^3ex+c^3d^4)}$$

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2 +
 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}f^2}{cd} + \frac{4\,(c^2d^2x^2-acdex-2\,a^2e^2)fg}{3\sqrt{cdx+ae}c^2d^2} + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)g^2}{15\sqrt{cdx+ae}c^3d^3}$$

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*
a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\,e\left(\frac{15\,(c^2d^2f^2-2\,acdefg+a^2e^2g^2)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e} - \frac{15\sqrt{-cd^2e+ae^3}c^2d^2e^2f^2-10\sqrt{-cd^2e+ae^3}c^2d^3efg-20\sqrt{-cd^2e+ae^3}acde^3f \cdots}{c^3d^3 \cdots}\right.}{\cdots}$$

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/15*e*(15*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)/(c^3*d^3*e) - (15*sqrt(-c*d^2*e + a*e^3)*c^2*d^2*e^2*f^
2 - 10*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e*f*g - 20*sqrt(-c*d^2*e + a*e^3)*a*c
*d*e^3*f*g + 3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*g^2 + 4*sqrt(-c*d^2*e + a*e^3
)*a*c*d^2*e^2*g^2 + 8*sqrt(-c*d^2*e + a*e^3)*a^2*e^4*g^2)/(c^3*d^3*e^3) + (
10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^2*f*g - 10*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g^2 + 3*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2)*g^2)/(c^3*d^3*e^5))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(16\,a^2\,e^2\,g^2-40\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2)}{15\,c^3\,d^3\,e}+\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{5\,c\,d\,e}-\frac{4\,g\,x\,(2\,a\,e\,g-5\,c\,d\,f)\,\sqrt{d+e}}{15\,c^2\,d^2\,e}\right.}{x+\frac{d}{e}}$$

```
[In] int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d +
e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*c^2
*d^2*e)))/(x + d/e)
```

3.659 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$

Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e\sqrt{d+ex}} + \frac{2g\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde}$$

```
[Out] -2/3*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^2/d^2/e/(e*x+d)^(1/2)+2/3*g*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c/d/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2g\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cde} - \frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^2d^2e\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2g\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde} \\
&\quad+\frac{1}{3}\left(3f-\frac{dg}{e}-\frac{2aeg}{cd}\right)\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e\sqrt{d+ex}} \\
&\quad+\frac{2g\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.42

$$\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(3f+gx))}{3c^2d^2\sqrt{d+ex}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-cdgx+2aeg-3cdf)}{3\sqrt{ex+d}\,c^{2}d^{2}}$	49
gosper	$-\frac{2(cdx+ae)(-cdgx+2aeg-3cdf)\sqrt{ex+d}}{3c^{2}d^{2}\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$	67

```
[In] int((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*g*x+2*a*e*g-3*c*d*f)/c
^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx+3\,cdf-2\,aeg)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*
g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}f}{cd}+\frac{2\,(c^2d^2x^2-acdex-2\,a^2e^2)g}{3\sqrt{cdx+ae}c^2d^2}$$

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}(cdf-aeg)}{c^2d^2e}+\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}g}{c^2d^2e^3}-\frac{3\,\sqrt{-cd^2e+ae^3}cdef-\sqrt{-cd^2e+ae^3}cd^2g-2\,\sqrt{-cd^2e+ae^3}ae^2g}{c^2d^2e^2}\right)}{3\,|e|}$$

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f - a*e*g)/(c^2*d^2*e
) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*g/(c^2*d^2*e^3) - (3*sqrt(-c*
d^2*e + a*e^3)*c*d*e*f - sqrt(-c*d^2*e + a*e^3)*c*d^2*g - 2*sqrt(-c*d^2*e +
 a*e^3)*a*e^2*g)/(c^2*d^2*e^2))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=-\frac{\left(\frac{(4\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}}{3\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

```
[In] int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] -((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^(
1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

3.660 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}}{cd\sqrt{d+ex}}$$

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,cd}$	32
gosper	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	50

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cdex+cd^2}$$

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^
2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}}{cd}$$

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{cde} - \frac{\sqrt{-cd^2e+ae^3}}{cde}\right)}{|e|}$$

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
[Out] 2*e*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c*d*e) - sqrt(-c*d^2*e + a*e^
3)/(c*d*e))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

```
[In] int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
[Out] (2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(x
 + d/e))
```

3.661 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 80

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

```
[Out] 2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/
2)/(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {888, 211}

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

integral

$$= (2e^2)\,\text{Subst}\left(\int \frac{1}{-e\,(cd^2+ae^2)\,g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\right)$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}}$	77

```
[In] int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+(ef+dg)x}\right)}{cdfg-aeg^2},\ -\frac{2\,\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}\right)}{\sqrt{cdfg-aeg^2}}\right]$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e
*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x))/(
c*d*f*g - a*e*g^2), -2*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x))/sqrt(c*d*f*g - a*e*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)), x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}|e|} - \frac{2\,e\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}|e|}$$

[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")

[Out] 2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*abs(e)) - 2*e*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*abs(e))

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

[In] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)

[Out] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)

3.662 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 140

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(3/2)/g^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {886, 888, 211}

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}} + \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2(cdf-aeg)}\\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)}\\
&\quad + \frac{(cde^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{cdf-aeg}\\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdgx+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)(gx+f)\sqrt{(aeg-cdf)g}}$	158

```
[In] int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] ((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c*d*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(
a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(124) = 248.

Time = 0.31 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x+2\,\sqrt{cdex^2+ad}}{egx^2+df+(ef+dg)x}\right.}{2\,(c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2d}\right.$$
$$-\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}\right)-\sqrt{cde}}{c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2d^2ef^3g+a^2de^2g^4+}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*g
^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^
2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g -
 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f
*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a
*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x^2 + c*d
^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g - 2*a*c*d^2*e*
f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^
3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^2
*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(124) = 248$.

Time = 0.37 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=e^2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}cd}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)(cdf|e|-aeg|e|)}+\frac{cd\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}(cdf|e|-aeg|e|)e}\right)$$
$$-\frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}e}{\sqrt{cdfg-aeg^2}cdef^2|e|-\sqrt{cdfg-aeg^2}cd^2fg|e|-\sqrt{cdfg-aeg^2}ae^2fg|e|+\sqrt{cdfg-aeg^2}adeg^2|e|}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] e^2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d/((c*d*e^2*f - a*e^3*g + ((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f*abs(e) - a*e*g*abs(e))) + c*d*a
rctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
)/(sqrt(c*d*f*g - a*e*g^2)*(c*d*f*abs(e) - a*e*g*abs(e))*e)) - (c*d*e^2*f*a
rctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - c*d^2*e*g*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + sqrt(-c*d^2*e +
 a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*c*d*e*f^2*abs(e
) - sqrt(c*d*f*g - a*e*g^2)*c*d^2*f*g*abs(e) - sqrt(c*d*f*g - a*e*g^2)*a*e^
2*f*g*abs(e) + sqrt(c*d*f*g - a*e*g^2)*a*d*e*g^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.663 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 213

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)} + \frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}$$

```
[Out] 3/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)/g^(1/2)+1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+3/4*c*d*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used

$= \{886, 888, 211\}$

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}} + \frac{3cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^2} + \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{(3cd)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4(cdf-aeg)}\\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{(3c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8(cdf-aeg)^2}\\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{(3c^2d^2e^2)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4(cdf-aeg)^2}\\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$
\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx
$$
$$
=\frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)(-2aeg+cd(5f+3gx))+3c^2d^2\sqrt{ae+cdx}(f+gx)^2\arctan\left(\frac{\sqrt{g}\sqrt{ae+cd}}{\sqrt{cdf-aeg}}\right.\right.}{4\sqrt{g}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}
$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x)*(-2*a*e*g + c*d*(
5*f + 3*g*x)) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+6\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)^2(gx+f)^2\sqrt{(a}}$

```
[In] int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^2*d^2*f^2-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c
*d*g*x+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g-5*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^2/(g*x+f)^
2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(187) = 374.

Time = 0.35 (sec) , antiderivative size = 1283, normalized size of antiderivative = 6.02

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*
g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*g^3 - 3*
a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (2*c^3*d^3
*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2
*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^5)*x^2
+ (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*
g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)
*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g
```

```
^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g
^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (5*c^2*d
^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^
5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 +
 (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e
^4*g^6)*x^3 + (2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2
*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2
- 2*a^3*e^4)*f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4
 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*
a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^3}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(187) = 374$.

Time = 0.45 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.96

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{1}{4}\left(\frac{3\,c^2d^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg^2}e}+\frac{5\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^2f-5\,\sqrt{}}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+}\right.$$
$$-\frac{3\,c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-6\,c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+3\,c^2d^4eg}{4\left(\sqrt{cdfg-aeg^2}c^2d^2e^2f^4|e|-2\,\sqrt{cdfg-aeg^2}c^2d^3ef^3g|e|-2\,\sqrt{cdfg-aeg^2}acde^3f^3g|e|+\sqrt{cdfg-aeg}\right.}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/4*(3*c^2*d^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3
*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) + (5*sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c^3*d^3*e^2*f - 5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d
^2*e^3*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g)/((c^2*d^2
*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3*g^2*abs(e))*(c*d*e^2*f - a
*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^3 - 1/4*(3*c^2*d^2*e^
3*f^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 6*c^2*
d^3*e^2*f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) +
3*c^2*d^4*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f - 3*sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e*g - 2*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*f*g - a*e*g^2)*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^4*
abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f^3*g*abs(e) - 2*sqrt(c*d*f*g
- a*e*g^2)*a*c*d*e^3*f^3*g*abs(e) + sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*f^2*g^2
*abs(e) + 4*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^2*f^2*g^2*abs(e) + sqrt(c*d*f
*g - a*e*g^2)*a^2*e^4*f^2*g^2*abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d^3*e*
f*g^3*abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^3*f*g^3*abs(e) + sqrt(c*d*
f*g - a*e*g^2)*a^2*d^2*e^2*g^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.664 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 280

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}\\&\quad+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)}+\frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}\end{aligned}$$

```
[Out] 5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)/g^(1/2)+1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/12*c*d*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(
1/2)+5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(
g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{886, 888, 211\}$

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}} + \frac{5c^2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{8\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{5cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{12\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{(5cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{6(cdf-aeg)}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{(5c^2d^2)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8(cdf-aeg)^2}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{(5c^3d^3)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16(cdf-aeg)^3}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{(5c^3d^3e^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{8(cdf-aeg)^3}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{c^3d^3\sqrt{d+ex}\left(\frac{(ae+cdx)(8a^2e^2g^2-2acdeg(13f+5gx)+c^2d^2(33f^2+40fgx+15g^2x^2))}{c^3d^3(cdf-aeg)^3(f+gx)^3} + \frac{15\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{7/2}}\right)}{24\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (c^3*d^3*Sqrt[d + e*x]*(((a*e + c*d*x)*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(13*f +
 5*g*x) + c^2*d^2*(33*f^2 + 40*f*g*x + 15*g^2*x^2)))/(c^3*d^3*(c*d*f - a*e*
g)^3*(f + g*x)^3) + (15*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*(c*d*f - a*e*g)^(7/2))))/(24*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+45 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f^2\right.}{}$

```
[In] int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^3-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^
2*x^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-40*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-8*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+26*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*
a*c*d*e*f*g-33*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+
d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. 2(248) = 496.

Time = 0.67 (sec) , antiderivative size = 2027, normalized size of antiderivative = 7.24

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
```

+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x)]

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^4}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(248) = 496$.

Time = 0.66 (sec) , antiderivative size = 1460, normalized size of antiderivative = 5.21

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/24*(15*c^3*d^3*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d
*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e)
 + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g^2)
*e) + (33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2 - 66*sqrt
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g + 33*sqrt((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2 + 40*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*c^4*d^4*e^2*f*g - 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
```

```
/2)*a*c^3*d^3*e^3*g^2 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^
3*g^2)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*
e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*g)^3))*e^4 - 1/24*(15*c^3*d^3*e^4*f^3*arctan(sqrt(-c
*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 45*c^3*d^4*e^3*f^2*g*arcta
n(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 45*c^3*d^5*e^2*f*
g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c^3*d
^6*e*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 33*
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^2 - 40*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^2*f*g - 26*sqrt(-c*d^2*e
+ a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^4*f*g + 15*sqrt(-c*d^2*e + a*e^3)*
sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e*g^2 + 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*a*c*d^2*e^3*g^2 + 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a^2*e^5*g^2)/(sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^3*f^6*abs(e) - 3*sqr
t(c*d*f*g - a*e*g^2)*c^3*d^4*e^2*f^5*g*abs(e) - 3*sqrt(c*d*f*g - a*e*g^2)*a
*c^2*d^2*e^4*f^5*g*abs(e) + 3*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e*f^4*g^2*abs
(e) + 9*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^3*f^4*g^2*abs(e) + 3*sqrt(c*d*f
*g - a*e*g^2)*a^2*c*d*e^5*f^4*g^2*abs(e) - sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*
f^3*g^3*abs(e) - 9*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^2*f^3*g^3*abs(e) - 9
*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^4*f^3*g^3*abs(e) - sqrt(c*d*f*g - a*e*
g^2)*a^3*e^6*f^3*g^3*abs(e) + 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^5*e*f^2*g^4
*abs(e) + 9*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^3*e^3*f^2*g^4*abs(e) + 3*sqrt(c
*d*f*g - a*e*g^2)*a^3*d*e^5*f^2*g^4*abs(e) - 3*sqrt(c*d*f*g - a*e*g^2)*a^2*
c*d^4*e^2*f*g^5*abs(e) - 3*sqrt(c*d*f*g - a*e*g^2)*a^3*d^2*e^4*f*g^5*abs(e)
 + sqrt(c*d*f*g - a*e*g^2)*a^3*d^3*e^3*g^6*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.665 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 257

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{16g(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^4d^4e\sqrt{d+ex}}+\frac{16g^2(cdf-aeg)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^2d^2\sqrt{d+ex}}$$

```
[Out] -2*(g*x+f)^3*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/5
*g*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/5*g*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+16/5*g^2*(-a*e*g+c*d*f)*(e*x+d)^(1/
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{880, 884, 808, 662\}$

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{16g\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)\,(2ae^2g-cd(3ef-dg))}{5c^4d^4e\sqrt{d+ex}} + \frac{16g^2\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{5c^3d^3e} + \frac{12g(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
```

```
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{(6g)\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cd}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{12g(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{(24g(cdf-aeg))\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5c^2d^2}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&\quad+\frac{16g^2(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^3d^3e}\\
&\quad+\frac{12g(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^2d^2\sqrt{d+ex}}\\
&\quad-\frac{(8g(cdf-aeg)(2ae^2g-cd(3ef-dg)))\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5c^3d^3e}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&\quad-\frac{16g(cdf-aeg)(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^4d^4e\sqrt{d+ex}}\\
&\quad+\frac{16g^2(cdf-aeg)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^3d^3e}\\
&\quad+\frac{12g(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5c^2d^2\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(16a^3e^3g^3+8a^2cde^2g^2(-5f+gx)-2ac^2d^2eg(-15f^2+\dots}{5c^4d^4\sqrt{(ae+cdx\dots}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^2
*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x + 5*
f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.70

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2\dots}{5\sqrt{ex+d}\,(cdx+ae)c^4d^4}$
gosper	$\frac{2(cdx+ae)\,(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2+30a\,c^2d^2\dots}{5c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/5/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2*
e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+15
*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*g-5
*c^3*d^3*f^3)/(c*d*x+a*e)/c^4/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^3d^3g^3x^3-5\,c^3d^3f^3+30\,ac^2d^2ef^2g-40\,a^2cde^2fg^2+16\,a^3e^3g^3\dots}{\dots}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e^
2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (15*
```

```
c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*e +
(c^5*d^6 + a*c^4*d^4*e^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx &= -\frac{2\,f^3}{\sqrt{cdx+ae}cd}+\frac{6\,(cdx+2\,ae)f^2g}{\sqrt{cdx+ae}c^2d^2}\\ &\quad+\frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)fg^2}{\sqrt{cdx+ae}c^3d^3}+\frac{2\,(c^3d^3x^3-2\,ac^2d^2ex^2+8\,a^2cde^2x+16\,a^3e^3)g^3}{5\,\sqrt{cdx+ae}c^4d^4}\end{aligned}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e)
*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x + 16*
a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(235) = 470$.

Time = 0.31 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^3d^3e^2f^3-3\,ac^2d^2e^3f^2g+3\,a^2cde^4fg^2-a^3e^5g^3)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^4d^4|e|} + \frac{2\,(5\,c^3d^3e^3f^3+15\,c^3d^4e^2f^2g-30\,ac^2d^2e^4f^2g-5\,c^3d^5efg^2-20\,ac^2d^3e^3fg^2+40\,a^2cde^5fg^2+c^3d^6g^3+2\,a}{5\,\sqrt{-cd^2e+ae^3}c^4d^4e|e|} + \frac{2\left(15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^{18}d^{18}e^{24}f^2g-30\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^{17}d^{17}e^{25}fg^2+15\,\sqrt{(ex+}\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] -2*(c^3*d^3*e^2*f^3 - 3*a*c^2*d^2*e^3*f^2*g + 3*a^2*c*d*e^4*f*g^2 - a^3*e^5
*g^3)/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*abs(e)) + 2/5*(5*c^3
*d^3*e^3*f^3 + 15*c^3*d^4*e^2*f^2*g - 30*a*c^2*d^2*e^4*f^2*g - 5*c^3*d^5*e*
f*g^2 - 20*a*c^2*d^3*e^3*f*g^2 + 40*a^2*c*d*e^5*f*g^2 + c^3*d^6*g^3 + 2*a*c
^2*d^4*e^2*g^3 + 8*a^2*c*d^2*e^4*g^3 - 16*a^3*e^6*g^3)/(sqrt(-c*d^2*e + a*e
^3)*c^4*d^4*e*abs(e)) + 2/5*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^1
8*d^18*e^24*f^2*g - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^17*d^17*
e^25*f*g^2 + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^16*d^16*e^26*
g^3 + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^17*d^17*e^22*f*g^2 - 5*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^16*d^16*e^23*g^3 + ((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^16*d^16*e^20*g^3)/(c^20*d^20*e^24*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+60}{5\,c^5\,d^5\,e}\right.}{}$$

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e^
3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5*c
^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*e*g
 - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2*
e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2 + (x
*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

3.666 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 181

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{8g(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^3d^3e\sqrt{d+ex}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2e}$$

```
[Out] -2*(g*x+f)^2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
g*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/e/(e*x+d)^(1/2)+8/3*g^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c^2/d^2/e
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {880, 808, 662}

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^3d^3e\sqrt{d+ex}} + \frac{8g^2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e} - \frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{(4g)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cd}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{8g^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e}\\
&\quad-\frac{(4g(2ae^2g-cd(3ef-dg)))\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3c^2d^2e}
\end{aligned}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{8g(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^3d^3e\sqrt{d+ex}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2e}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(-8a^2e^2g^2-4acdeg(-3f+gx)+c^2d^2(-3f^2+6fgx+g}{3c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*f
^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-g^2x^2c^2d^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3c^2d^2f^2)}{3\sqrt{ex+d}\,(cdx+ae)c^3d^3}$	108
gosper	$-\frac{2(cdx+ae)(-g^2x^2c^2d^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3c^2d^2f^2)(ex+d)^{\frac{3}{2}}}{3c^3d^3(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	116

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^2*d^2*g^2*x^2+4*a*c*d*e*
g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)/(c*d*x+a*
e)/c^3/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^2d^2g^2x^2-3\,c^2d^2f^2+12\,acdefg-8\,a^2e^2g^2+2\,(3\,c^2d^2fg-2\,a}{3\,(c^4d^4ex^2+ac^3d^4e+(c^4d^5+}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*(
3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x
)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,f^2}{\sqrt{cdx+ae}cd} + \frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}c^2d^2} + \frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)g^2}{3\,\sqrt{cdx+ae}c^3d^3}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*c
^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a*e
)*c^3*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^2d^2e^2f^2-2\,acde^3fg+a^2e^4g^2)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3|e|} + \frac{2\,(3\,c^2d^2e^2f^2+6\,c^2d^3efg-12\,acde^3fg-c^2d^4g^2-4\,acd^2e^2g^2+8\,a^2e^4g^2)}{3\,\sqrt{-cd^2e+ae^3}c^3d^3|e|} + \frac{2\left(6\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^7d^7e^8fg-6\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^6d^6e^9g^2+((ex+d)cde-cd^2e+\dots\right.}{3\,c^9d^9e^8|e|}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] -2*(c^2*d^2*e^2*f^2 - 2*a*c*d*e^3*f*g + a^2*e^4*g^2)/(sqrt((e*x + d)*c*d*e
- c*d^2*e + a*e^3)*c^3*d^3*abs(e)) + 2/3*(3*c^2*d^2*e^2*f^2 + 6*c^2*d^3*e*f
*g - 12*a*c*d*e^3*f*g - c^2*d^4*g^2 - 4*a*c*d^2*e^2*g^2 + 8*a^2*e^4*g^2)/(s
qrt(-c*d^2*e + a*e^3)*c^3*d^3*abs(e)) + 2/3*(6*sqrt((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*c^7*d^7*e^8*f*g - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c
^6*d^6*e^9*g^2 + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*e^6*g^2)
/(c^9*d^9*e^8*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,(2\,a\,e\,g-3\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}\dots\right.}{\frac{a}{c}+x^2+\frac{x\,(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2)}{3\,c^4\,d^4\,e}}$$

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d + e
*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(3*c
^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e))
```

3.667 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 150

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\left(2ae^2g-cd(ef+dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}$$

```
[Out] -2*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-2*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c^2/d^2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {802, 662}

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)} - \frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad+\frac{\left(2\left(-\frac{1}{2}e\left(2cdef-\left(cd^2+ae^2\right)g\right)+\frac{3}{2}\left(cde^2f+\left(cd^2e-e\left(cd^2+ae^2\right)\right)g\right)\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd\left(2cd^2e-e\left(cd^2+ae^2\right)\right)} \\ &= -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad-\frac{2\left(2ae^2g-cd(ef+dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int\frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{d+ex}(2aeg+cd(-f+gx))}{c^2d^2\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdgx+2aeg-cdf)}{\sqrt{ex+d}\,(cdx+ae)c^{2}d^{2}}$	58
gosper	$\frac{2(cdx+ae)(cdgx+2aeg-cdf)(ex+d)^{\frac{3}{2}}}{c^{2}d^{2}(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	66

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c*d*g*x+2*a*e*g-c*d*f)/(c*d*x+
a*e)/c^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx-cdf+2\,aeg)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*s
qrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\,f}{\sqrt{cdx+ae}cd}+\frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}c^2d^2}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] -2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\,\sqrt{(ex+d)cde-cd^2e+ae^3}g}{c^2d^2|e|}-\frac{2\,(cde^2f-ae^3g)}{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2|e|}+\frac{2\,(cde^2f+cd^2eg-2\,ae^3g)}{\sqrt{-cd^2e+ae^3}c^2d^2|e|}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(c^2*d^2*abs(e)) - 2*(c*d*e^2*f
 - a*e^3*g)/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*abs(e)) + 2*(c
*d*e^2*f + c*d^2*e*g - 2*a*e^3*g)/(sqrt(-c*d^2*e + a*e^3)*c^2*d^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\left(\frac{(4\,a\,e\,g-2\,c\,d\,f)\,\sqrt{d+e\,x}}{c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\frac{a}{c}+x^2+\frac{x\,\left(c^3\,d^4+a\,c^2\,d^2\,e^2\right)}{c^3\,d^3\,e}}$$

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2),x)
```

```
[Out] ((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2
))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2 +
 (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

3.668 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral}=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{cd\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,(cdx+ae)cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	50

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
[Out] -2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x}$$

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2
 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2}{\sqrt{cdx+ae}cd}$$

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,e^2}{\sqrt{(ex+d)cde-cd^2e+ae^3}cd|e|} + \frac{2\,e^2}{\sqrt{-cd^2e+ae^3}cd|e|}$$

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
[Out] -2*e^2/(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*abs(e)) + 2*e^2/(sqrt(-
c*d^2*e + a*e^3)*c*d*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^2
*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

3.669 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 133

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(3/2)-2*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {882, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{g\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\
&\quad -\frac{(2e^2g)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{cdf-aeg}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{cdf-aeg}+\sqrt{g}\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g] + Sqrt[g]*Sqrt[a*e + c*d*x]*ArcTan[(
Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(3/2)*Sq
rt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(g\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}-\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)\sqrt{(aeg-cdf)g}}$	118

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*arctanh(g*(c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(c*d*x
+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 4.16

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2}{\cdots}\right.}{acd^2ef\cdots}\right.$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] [-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (
c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c
^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2
*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d*f - a*
e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*s
qrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*
d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d
*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(117) = 234$.

Time = 0.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -2e^2\left(\frac{g\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}(cdf|e|-aeg|e|)e}+\frac{1}{\sqrt{(ex+d)cde-cd^2e+ae^3}(cdf|e|-aeg|e|)}\right) + \frac{2\left(\sqrt{-cd^2e+ae^3}eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+\sqrt{cdfg-aeg^2}e^2\right)}{\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}cdf|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}aeg|e|}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] -2*e^2*(g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*(c*d*f*abs(e) - a*e*g*abs(e))*e) + 1/
(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f*abs(e) - a*e*g*abs(e)))) +
2*(sqrt(-c*d^2*e + a*e^3)*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e)) + sqrt(c*d*f*g - a*e*g^2)*e^2)/(sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*c*d*f*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a*e*g*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(f+g\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)), x)
```

3.670 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 202

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{3cd\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

```
[Out] -3*c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(5/2)-2*(e*x+d)^(1/2)/(-a*e*g
+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3*g*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {882, 886, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{3cd\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}} - \frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/(c*d*f - a*e*g)^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(3g)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{3g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{(3cdg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2(cdf-aeg)^2}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{3g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{(3cde^2g)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{(cdf-aeg)^2}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{3g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}(aeg+cd(2f+3gx))+3cd\sqrt{g}\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(a*e*g + c*d*(2*f + 3*g*x)) + 3*c*d*S
qrt[g]*Sqrt[a*e + c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f +
 g*x)))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cd\,g^2x+3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdfg-3\sqrt{(aeg-cdf)g}\,cdgx-\sqrt{}\right.}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)^2(gx+f)\sqrt{(aeg-cdf)g}}$

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*
(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*(c*d*x+a*e)^(1/2)*ar
ctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f*g-3*((a*e*g-c*d*f)
*g)^(1/2)*c*d*g*x-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/2)*c
*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 1067, normalized size of antiderivative = 5.28

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{3\,(c^2d^2egx^3+acd^2ef+(c^2d^2ef+(c^2d^3+acde^2)g)x^2}{2\,(ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+(c^3d^3ef^2g-}\right.$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e^
2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a*e
*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) -
(c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt
(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^
3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f
^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2
+ (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4 + a*c^2*d^
2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^2 - a^3*e^4
)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 +
 a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
```

```
*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x +
2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g +
a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^
3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
+ (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^
2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^2}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(182) = 364$.

Time = 0.50 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.73

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$

$$-\left(\frac{3\,cdg\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3g}}{\sqrt{cdfg-aeg^2e}}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg^2e}}+\frac{2\,c^2}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\left(\sqrt{(ex+\right.}}\right.$$

$$+\frac{3\sqrt{-cd^2e+ae^3}cde^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)-3\sqrt{-cd^2e+ae}}{\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^2d^2ef^3|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^2d^3f^2g|e|-2\sqrt{-cd^2e+ae^3}\sqrt{cdfg-}}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] -(3*c*d*g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3*g^2*
abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) + (2*c^2*d^2*e^2*f - 2*a*c*d*e^3*g + 3*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*
e^2*f*g*abs(e) + a^2*e^3*g^2*abs(e))*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*c*d*e^2*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3*g + ((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*g)))*e^3 + (3*sqrt(-c*d^2*e + a*e^3)*c*d*e^
2*f*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt
(-c*d^2*e + a*e^3)*c*d^2*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*
g - a*e*g^2)*e)) + 2*sqrt(c*d*f*g - a*e*g^2)*c*d*e^3*f - 3*sqrt(c*d*f*g - a
*e*g^2)*c*d^2*e^2*g + sqrt(c*d*f*g - a*e*g^2)*a*e^4*g)/(sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e*f^3*abs(e) - sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^2*d^3*f^2*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*a*c*d*e^2*f^2*g*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a*c*d^2*e*f*g^2*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*a^2*e^3*f*g^2*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
- a*e*g^2)*a^2*d*e^2*g^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

3.671 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 274

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{15cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{15c^2d^2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{7/2}}$$

```
[Out] -15/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(7/2)-2*(e*x+d)^(1/2)/
(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2*g*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2
)-15/4*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+
f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {882, 886, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{15c^2d^2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}} - \frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3} - \frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(5g)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{5g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{(15cdg)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4(cdf-aeg)^2}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{5g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{15cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(15c^2d^2g)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8(cdf-aeg)^3}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{5g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{15cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(15c^2d^2e^2g)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4(cdf-aeg)^3}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{5g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{15cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}(-2a^2e^2g^2+acdeg(9f+5gx)+c^2d^2(8f^2+25fgx+15g^2x^2))+15c^2d^2\sqrt{g}\sqrt{ae+c}\right.}{4(cdf-aeg)^{7/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(-2*a^2*e^2*g^2 + a*c*d*e*g*(9*f +
 5*g*x) + c^2*d^2*(8*f^2 + 25*f*g*x + 15*g^2*x^2)) + 15*c^2*d^2*Sqrt[g]*Sqr
t[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]]))/((c*d*f - a*e*g)^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2g^3x^2+30\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2fg^2x-15\sqrt{(aeg-\right.}{}$

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+30*arctanh(g
*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g^2
*x-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2+15*arctanh(g*(c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2*g-5*((a*e*g-c*d*f)
*g)^(1/2)*a*c*d*e*g^2*x-25*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*((a*e*g-
c*d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*((a*e*g
-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+f)^2/((a*e*g
-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(244) = 488.

Time = 0.49 (sec) , antiderivative size = 1863, normalized size of antiderivative = 6.80

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^
4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4
 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)
*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g +
 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d
)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 +
d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f
*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^
2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2
- 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4
+ (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e
 - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^
3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d
^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (
3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*
g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
(5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d^3*e*g^2*x^4 + a*c^
2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*
d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c
^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*f^2)*x)*sqrt(g/(c*d*f - a*e*g))*ar
ctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x
+ d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))
 + (15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 5*
(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e
^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g
^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g +
(c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2
*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*
g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4
*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a
^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4
```

```
*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*
f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*
e^5)*f^2*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^3}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(244) = 488$.

Time = 0.70 (sec) , antiderivative size = 1390, normalized size of antiderivative = 5.07

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] -1/4*(15*c^2*d^2*g*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(
e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g^
2)*e) + 8*c^2*d^2/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) +
 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt((e*x + d)*c*d*e - c*
```

```
d^2*e + a*e^3)) + (9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*e^2*f*
g - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3*g^2 + 7*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g^2)/((c^3*d^3*e^2*f^3*abs(e) -
3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs
(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^4
+ 1/4*(15*sqrt(-c*d^2*e + a*e^3)*c^2*d^2*e^3*f^2*g*arctan(sqrt(-c*d^2*e + a
*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 30*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e^
2*f*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*s
qrt(-c*d^2*e + a*e^3)*c^2*d^4*e*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e)) + 8*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^4*f^2 - 25*sqrt
(c*d*f*g - a*e*g^2)*c^2*d^3*e^3*f*g + 9*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^5*f
*g + 15*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e^2*g^2 - 5*sqrt(c*d*f*g - a*e*g^2)
*a*c*d^2*e^4*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*a^2*e^6*g^2)/(sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^2*f^5*abs(e) - 2*sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e*f^4*g*abs(e) - 3*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^3*f^4*g*abs(e) + sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*f^3*g^2*abs(e) + 6*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^2*f^3*g^2*abs(e) + 3*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^4*f^3*g^2*abs(e) - 3*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e*f^2*g^3*abs(e) - 6*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^3*f^2*g^3*abs(e) - sqrt
(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^5*f^2*g^3*abs(e) + 3*sqrt(
-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^3*e^2*f*g^4*abs(e) + 2*sq
rt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*d*e^4*f*g^4*abs(e) - sqrt(
-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*d^2*e^3*g^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

3.672 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 239

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{16g^2(2ae^2g-cd(3ef-dg))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^4d^4e\sqrt{d+ex}} + \frac{16g^3\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3e}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-4*
g*(g*x+f)^2*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1
6/3*g^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^4/d^4/e/(e*x+d)^(1/2)+16/3*g^3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used

$= \{880, 808, 662\}$

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(2ae^2g-cd(3ef-dg))}{3c^4d^4e\sqrt{d+ex}} + \frac{16g^3\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^3d^3e} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{(2g)\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx}{cd} \\
&= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} \\
&\quad - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{(8g^2)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{c^2d^2} \\
&= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad + \frac{16g^3\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^3d^3e} \\
&\quad - \frac{(8g^2(2ae^2g-cd(3ef-dg)))\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3c^3d^3e} \\
&= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad - \frac{16g^2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^4d^4e\sqrt{d+ex}} \\
&\quad + \frac{16g^3\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^3d^3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\,(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg(f^2-6}{3c^4d^4((ae+cdx)(d+}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c^
2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2*x
^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f}{3\sqrt{ex+d}\,(cdx+ae)^2c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f\,g^2+6a\,c^2}{3c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$

```
[In] int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^3*d^3*g^3*x^3+6*a*c^2*d^
2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x
+9*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2*g+
c^3*d^3*f^3)/(c*d*x+a*e)^2/c^4/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2\left(c^3d^3g^3x^3-c^3d^3f^3-6\,ac^2d^2ef^2g+24\,a^2cde^2fg^2-16\,a^3e^3g^3+3\right.}{3\left(c^6d^6ex^3+\right.}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*f
*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(3*
c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^5*e^
2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(3\,cdx+2\,ae)f^2g}{(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} + \frac{2\,(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2)fg^2}{(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}} + \frac{2\,(c^3d^3x^3-6\,ac^2d^2ex^2-24\,a^2cde^2x-16\,a^3e^3)g^3}{3\,(c^5d^5x+ac^4d^4e)\sqrt{cdx+ae}} - \frac{2\,f^3}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*d*e
^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) - 2/3*
f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(217) = 434.

Time = 0.34 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(c^3d^3e^3f^3-9\,c^3d^4e^2f^2g+6\,ac^2d^2e^4f^2g-9\,c^3d^5efg^2+36\,ac^2d^3e^3fg^2-24\,a^2cde^5fg^2+c^3d^6g^3+6\,ac^2d^4}{3\,\left(\sqrt{-cd^2e+ae^3}c^5d^6|e|-\sqrt{-cd^2e+ae^3}ac^4d^4e^2|e|\right)} - \frac{2\,(c^3d^3e^4f^3-3\,ac^2d^2e^5f^2g+3\,a^2cde^6fg^2-a^3e^7g^3+9\,((ex+d)cde-cd^2e+ae^3)c^2d^2e^2f^2g-18\,((ex+}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^4d^4|e|} + \frac{2\left(9\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^9d^9e^8fg^2-9\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^8d^8e^9g^3+((ex+d)cde-cd^2\right.}{3\,c^{12}d^{12}e^8|e|}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] -2/3*(c^3*d^3*e^3*f^3 - 9*c^3*d^4*e^2*f^2*g + 6*a*c^2*d^2*e^4*f^2*g - 9*c^3
*d^5*e*f*g^2 + 36*a*c^2*d^3*e^3*f*g^2 - 24*a^2*c*d*e^5*f*g^2 + c^3*d^6*g^3
+ 6*a*c^2*d^4*e^2*g^3 - 24*a^2*c*d^2*e^4*g^3 + 16*a^3*e^6*g^3)/(sqrt(-c*d^2
*e + a*e^3)*c^5*d^6*abs(e) - sqrt(-c*d^2*e + a*e^3)*a*c^4*d^4*e^2*abs(e)) -
```

```
2/3*(c^3*d^3*e^4*f^3 - 3*a*c^2*d^2*e^5*f^2*g + 3*a^2*c*d*e^6*f*g^2 - a^3*e
^7*g^3 + 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^2*f^2*g - 18*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^3*f*g^2 + 9*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*a^2*e^4*g^3)/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4
*abs(e)) + 2/3*(9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*e^8*f*g^2
 - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*g^3 + ((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*g^3)/(c^12*d^12*e^8*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\left(\frac{32\,a^3\,e^3\,g^3}{3}-16\,a^2\,c\,d\,e^2\,f\,g^2+4\,a\,c^2\,d^2\,e\,f^2\,g+\frac{2\,c^3\,d^3\,f^3}{3}\right)}{c^6\,d^6\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}+\frac{g^2\,x^2\,(}{}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}+\frac{x^2\,(c^6\,d^7+2\,a\,c^5\,d^5\,e^2)}{c^6\,d^6\,e}}$$

```
[In] int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3*
e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g^2
))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4*a*
e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*e)/(c
^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d^5*e^2
))/(c^6*d^6*e))
```

3.673 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 211

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\,(cd^2-ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{8g(2ae^2g-cd(ef+dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^3d^3\,(cd^2-ae^2)\,\sqrt{d+ex}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/
3*g*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-8/3*g*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^3/d^3/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {880, 802, 662}

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{3c^3d^3\sqrt{d+ex}\,(cd^2-ae^2)} - \frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\,(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
```

```
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(4g)\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}\\
&= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\
&\quad-\frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&\quad-\frac{\left(4g\left(2ae^2g-cd(ef+dg)\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3c^2d^2\left(cd^2-ae^2\right)}\\
&= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\
&\quad-\frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&\quad-\frac{8g\left(2ae^2g-cd(ef+dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3\left(cd^2-ae^2\right)\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2(f^2+6fgx-3g^2\right.}{3c^3d^3((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
+ 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-c^2d^2f^2)}{3\sqrt{ex+d}\,(cdx+ae)^2c^3d^3}$	108
gosper	$\frac{2(cdx+ae)(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-c^2d^2f^2)(ex+d)^{\frac{5}{2}}}{3c^3d^3(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	116

```
[In] int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2+12*a*c*d*e
*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)/(c*d*x+a*e)
^2/c^3/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(3\,c^2d^2g^2x^2-c^2d^2f^2-4\,acdefg+8\,a^2e^2g^2-6\,(c^2d^2fg-2\,acdeg}{3\,(c^5d^5ex^3+a^2c^3d^4e^2+(c^5d^6+2\,ac^4d^4e^2)x^2+(}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(c
^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2
)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{4\,(3\,cdx+2\,ae)fg}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} + \frac{2\,(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2)g^2}{3\,(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}} - \frac{2\,f^2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2\,(c^2d^2e^3f^2-6\,c^2d^3e^2fg+4\,acde^4fg-3\,c^2d^4eg^2+12\,acd^2e^3g^2-8\,a^2e^5g^2)}{3\,\left(\sqrt{-cd^2e+ae^3}c^4d^5|e|-\sqrt{-cd^2e+ae^3}ac^3d^3e^2|e|\right)} + \frac{2\,\sqrt{(ex+d)cde-cd^2e+ae^3}g^2}{c^3d^3|e|} - \frac{2\,(c^2d^2e^4f^2-2\,acde^5fg+a^2e^6g^2+6\,((ex+d)cde-cd^2e+ae^3)cde^2fg-6\,((ex+d)cde-cd^2e+ae^3)ae}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^3d^3|e|}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] -2/3*(c^2*d^2*e^3*f^2 - 6*c^2*d^3*e^2*f*g + 4*a*c*d*e^4*f*g - 3*c^2*d^4*e*g
^2 + 12*a*c*d^2*e^3*g^2 - 8*a^2*e^5*g^2)/(sqrt(-c*d^2*e + a*e^3)*c^4*d^5*ab
s(e) - sqrt(-c*d^2*e + a*e^3)*a*c^3*d^3*e^2*abs(e)) + 2*sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*g^2/(c^3*d^3*abs(e)) - 2/3*(c^2*d^2*e^4*f^2 - 2*a*c*d*
e^5*f*g + a^2*e^6*g^2 + 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^2*f*g -
 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3*g^2)/(((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*c^3*d^3*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,\left(-16\,a^2\,e^2\,g^2+8\right.}{3\,c^5\,d}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\,d^6+\right.}{3\,c^5}}$$

```
[In] int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2))
/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d*e
*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e
)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(3*c^
5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

3.674 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 154

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(2ae^2g+cd(ef-3dg))\sqrt{d+ex}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(e*x+d)^(5/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)+2/3*(2*a*e^2*g+c*d*(-3*d*g+e*f))*(e*x+d)^(1/2)/c^2/d^2/(-
a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {802, 662}

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(2ae^2g+cd(ef-3dg))}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{\left(2ae^2g+cd(ef-3dg)\right)\int\frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd\left(cd^2-ae^2\right)}\\ &= -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(2ae^2g+cd(ef-3dg)\right)\sqrt{d+ex}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int\frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3cdgx+2aeg+cdf)}{3\sqrt{ex+d}\,(cdx+ae)^{2}c^{2}d^{2}}$	58
gosper	$-\frac{2(cdx+ae)(3cdgx+2aeg+cdf)(ex+d)^{\frac{5}{2}}}{3c^{2}d^{2}(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	66

```
[In] int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c*d*g*x+2*a*e*g+c*d*f)/(c
*d*x+a*e)^2/c^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(3\,cdgx+cdf+2\,aeg)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+(c^4d^5+2\,ac^3d^3e^2)x^2+(2\,ac^3d^4e+a^2c^2d^2e^3)x\right)}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*e
*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3
*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(3\,cdx+2\,ae)g}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} - \frac{2\,f}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] -2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/
3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,(cde^3f-3\,cd^2e^2g+2\,ae^4g)}{3\,\left(\sqrt{-cd^2e+ae^3}c^3d^4|e|-\sqrt{-cd^2e+ae^3}ac^2d^2e^2|e|\right)} - \frac{2\,(cde^4f-ae^5g+3\,((ex+d)cde-cd^2e+ae^3)e^2g)}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2|e|}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] -2/3*(c*d*e^3*f - 3*c*d^2*e^2*g + 2*a*e^4*g)/(sqrt(-c*d^2*e + a*e^3)*c^3*d^
4*abs(e) - sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^2*abs(e)) - 2/3*(c*d*e^4*f -
a*e^5*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e^2*g)/(((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*c^2*d^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

```
[In] int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2),x)
```

```
[Out] -(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^
3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5 + 2
*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

3.675 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral}=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{3\sqrt{ex+d}\,(cdx+ae)^2 cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	50

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURNV
ERBOSE)
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{3\left(c^3d^3ex^3+a^2cd^2e^2+(c^3d^4+2\,ac^2d^2e^2)x^2+(2\,ac^2d^3e+a^2cde^3)x\right)}$$

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x
^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2
*c*d*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(42) = 84.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,e^3}{3\,\left(\sqrt{-cd^2e+ae^3}c^2d^3|e|-\sqrt{-cd^2e+ae^3}acde^2|e|\right)} - \frac{2\,e^4}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}cd|e|}$$

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
[Out] -2/3*e^3/(sqrt(-c*d^2*e + a*e^3)*c^2*d^3*abs(e) - sqrt(-c*d^2*e + a*e^3)*a*
c*d*e^2*abs(e)) - 2/3*e^4/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*ab
s(e))
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

```
[In] int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3*
d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x +
 2*a*c^2*d^2*e^2*x^2))
```

3.676 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 188

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+2
*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)+2*g*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {882, 888, 211}

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

integral

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{g\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cdf-aeg}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g^2\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{(cdf-aeg)^2}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} } + \frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(2e^2g^2\right)\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{(cdf-aeg)^2}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} } + \frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(4aeg-cd(f-3gx))+3g^{3/2}(ae\right.}{3(cdf-aeg)^{5/2}((ae+cdx)(d+e}$$

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(4*a*e*g - c*d*(f - 3*g*x)) + 3*g^(
3/2)*(a*e + c*d*x)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*
e*g]]))/(3*(c*d*f - a*e*g)^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cd\,g^2x+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)ae\,g^2\sqrt{cdx+ae}-3\sqrt{(aeg-cdf)g}\,cdgx-\right.}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)^2\sqrt{(aeg-cdf)g}}$

```
[In] int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2))*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f)*g)^(1/2)*c
*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(e*x+
d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(166) = 332$.

Time = 0.39 (sec) , antiderivative size = 1015, normalized size of antiderivative = 5.40

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \left[\frac{3\left(c^2d^2egx^3+a^2de^2g+(c^2d^3+2\,acde^2)gx^2+(2\right.}{3\left(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+(c^4d^4ef^2-2\,c\right.}\right.$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2
*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d
^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*
f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^
2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*
c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)
*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*
c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*d^2*e*g*x^3
 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*
c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*
c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e +
2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a
*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g
 + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(166) = 332.

Time = 0.44 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.63

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2}{3}\,e^3\left(\frac{3\,g^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2e}}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg}}\right.$$

$$-\frac{2\left(3\sqrt{-cd^2e+ae^3}cd^2eg^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\sqrt{-cd^2e}\right.}{3\left(\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}c^3d^4f^2|e|-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}ac^2d^2e^2f^2|e|-2\sqrt{-cd^2e+ae^3}\sqrt{cd}\right.}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] 2/3*e^3*(3*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3
*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (c*d*e^2*f - a*e^3*g - 3*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)*g)/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs
(e) + a^2*e^3*g^2*abs(e))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2))) - 2/3
*(3*sqrt(-c*d^2*e + a*e^3)*c*d^2*e*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*a*e^3*g^2*arctan(sqrt(-
c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + sqrt(c*d*f*g - a*e*g^2)*c
*d*e^3*f + 3*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e^2*g - 4*sqrt(c*d*f*g - a*e*g^2
)*a*e^4*g)/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*f^2*abs(
e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^2*f^2*abs(e
) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e*f*g*abs(e)
 + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^3*f*g*abs(e)
+ sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^2*g^2*abs(e) -
 sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^4*g^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{5/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)), x)
```

3.677 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$

Optimal result

Integrand size = 46, antiderivative size = 268

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} + \frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5cdg^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{7/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+5*c*d*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)+10/3*g*(e*x+d)^(1
/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1
/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {882, 886, 888, 211}

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{5cdg^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}} + \frac{5g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3(f+gx)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
```

```
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad -\frac{(5g)\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3(cdf-aeg)} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{\left(5g^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{(cdf-aeg)^2} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{5g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^3\sqrt{d+ex}(f+gx)}+\frac{\left(5cdg^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2(cdf-aeg)^3}
\end{aligned}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} + \frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{(5cde^2g^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{(cdf-aeg)^3}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} + \frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(3a^2e^2g^2+2acdeg(7f+10gx)\right.}{3(cdf-}$$

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] ((d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(7*f + 1
0*g*x) + c^2*d^2*(-2*f^2 + 10*f*g*x + 15*g^2*x^2)) + 15*c*d*g^(3/2)*(a*e +
c*d*x)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*
g]]))/(3*(c*d*f - a*e*g)^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}\,c^2d^2g^3x^2+15\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acde\,g^3x\sqrt{cdx+ae}+15\,\text{arctanh}\left(\right.\right.}{}$

```
[In] int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+15*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^3*x*(c*d*x+a*e)^(1/2)+15*arctanh(g*(
c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g^2*x
+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g^2*(c*d
*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a*e*g-c*d*f)*
g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-3*((a*e*g-c
*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g+2*((a*e*g
-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3/(
g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(240) = 480$.

Time = 0.63 (sec) , antiderivative size = 1907, normalized size of antiderivative = 7.12

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^
4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2
*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*
c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt
(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*
g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 + 14*
a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^4 - 3*
a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^
5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2
*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4
*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)
*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d
^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2
- a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a
^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^
3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2
*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x), 1/3*
(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^4 + 2*
```

```
a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2*d^3*e
 + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g)*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2
 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^
4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (
c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c
^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*
(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^
2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a
*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^
4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3
- (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e +
a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a
^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^2}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(240) = 480$.

Time = 0.64 (sec) , antiderivative size = 1404, normalized size of antiderivative = 5.24

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g^2/((c^3*d^3*e^2*f^3*ab
s(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*
g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g))
+ 15*c*d*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3
*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e)
- 2*(c^2*d^2*e^2*f - a*c*d*e^3*g - 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*
d*g)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^
4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)))*e^4 - 1/3*(15*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e^2*f*g^2*arctan(sqrt(-c
*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*
a*c*d*e^4*f*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - 15*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*e*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g
/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^3*g^3*a
rctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 2*sqrt(c*d*f*
g - a*e*g^2)*c^2*d^2*e^4*f^2 + 10*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^3*f*g -
 14*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^5*f*g - 15*sqrt(c*d*f*g - a*e*g^2)*c^2*
d^4*e^2*g^2 + 20*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^4*g^2 - 3*sqrt(c*d*f*g -
 a*e*g^2)*a^2*e^6*g^2)/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*
d^5*e*f^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^3
*e^3*f^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^6*f^
3*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^4*e^2
*f^3*g*abs(e) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^
2*e^4*f^3*g*abs(e) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3
*d^5*e*f^2*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^
3*c*d*e^5*f^2*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)
*a^2*c^2*d^4*e^2*f*g^3*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e
*g^2)*a^3*c*d^2*e^4*f*g^3*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a^4*e^6*f*g^3*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2
)*a^3*c*d^3*e^3*g^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)
*a^4*d*e^5*g^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{5/2}}{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
[Out] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

3.678 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 342

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2} + \frac{35cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^4\sqrt{d+ex}(f+gx)} + \frac{35c^2d^2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{9/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+35/4*c^2*d^2*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(9/2)+14/3*g*(
e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+35/6*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x
+f)^2/(e*x+d)^(1/2)+35/4*c*d*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-
a*e*g+c*d*f)^4/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {882, 886, 888, 211}

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{35c^2d^2g^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}} + \frac{35cdg^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4} + \frac{35g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3} + \frac{14g\sqrt{d+ex}}{3(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad -\frac{(7g)\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3(cdf-aeg)} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{\left(35g^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3(cdf-aeg)^2} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{35g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2}+\frac{\left(35cdg^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4(cdf-aeg)^3}
\end{aligned}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} } + \frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2} + \frac{35cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^4\sqrt{d+ex}(f+gx)} + \frac{\left(35c^2d^2g^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8(cdf-aeg)^4}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} } + \frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2} + \frac{35cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^4\sqrt{d+ex}(f+gx)} + \frac{\left(35c^2d^2e^2g^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\right)}{4(cdf-aeg)^4}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} } + \frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{35g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{6(cdf-aeg)^3\sqrt{d+ex}(f+gx)^2} + \frac{35cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^4\sqrt{d+ex}(f+gx)} + \frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4(cdf-aeg)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{c^2d^2\sqrt{d+ex}\left(\frac{-6a^3e^3g^3+3a^2cde^2g^2(13f+7gx)+2ac^2d^2eg(40f^2+119f}{c^2d^2(cdf-aeg)^4}\right.}{12}$$

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (c^2*d^2*Sqrt[d + e*x]*((-6*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(13*f + 7*g*x)
+ 2*a*c^2*d^2*e*g*(40*f^2 + 119*f*g*x + 70*g^2*x^2) + c^3*d^3*(-8*f^3 + 56*
```

```
f^2*g*x + 175*f*g^2*x^2 + 105*g^3*x^3))/(c^2*d^2*(c*d*f - a*e*g)^4*(a*e + c
*d*x)*(f + g*x)^2) + (105*g^(3/2)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*
e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(9/2)))/(12*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(304) = 608$.

Time = 0.54 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.93

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(105 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^4x^3\sqrt{cdx+ae}+105 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^2d^2e\,g^4x^2\sqrt{cdx+ae}+210 \operatorname{arct}\right.}{}$

```
[In] int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)

[Out] -1/12*((c*d*x+a*e)*(e*x+d))^(1/2)*(105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^3*d^3*g^4*x^3*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e
)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*g^4*x^2*(c*d*x+a*e)^(1/2)+210*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f*g^3*x^2*(c*d
*x+a*e)^(1/2)+210*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^
2*d^2*e*f*g^3*x*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^2*g^2*x*(c*d*x+a*e)^(1/2)-105*((a*e*g-c*d*f)*g)^(
1/2)*c^3*d^3*g^3*x^3+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*
d^2*e*g^3*x^2-175*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-21*((a*e*g-c*d*
f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-238*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2
*x-56*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+6*((a*e*g-c*d*f)*g)^(1/2)*a^3
*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e*g-c*d*f)*g)^
(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2
)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(304) = 608$.

Time = 1.31 (sec) , antiderivative size = 2935, normalized size of antiderivative = 8.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
+ (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*
a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^
2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c
^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f
+ 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3
 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*
d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*
g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*
e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*
g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e
^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c
^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*
c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2
*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f
^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c
^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*
a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*
g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^
6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*
g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^
4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2
*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*
a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d
*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x), 1/12*(105*(c^4*d^4
*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2 + (c^4*d^5 + 2*a*c^
3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*a*c^3*d^3*e^2)*f*g^
2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^2*d^3*e^2*g^3 + (c^
4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f*g^2)
*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2*g)*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + (105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3 + 80*a
*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*d^3*f*g
^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*g^2 + 3
*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4
*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4
*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^
5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*
e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5
```

```
*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^
5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*
c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*
e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3
*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^
3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 +
 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4
*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a
^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^
4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3
*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f
^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^3}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. 2(304) = 608.

Time = 1.14 (sec) , antiderivative size = 2347, normalized size of antiderivative = 6.86

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] 1/12*(105*c^2*d^2*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^4*d^4*e^3*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g*a
bs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) + a^4
*e^7*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - 8*(c^3*d^3*e^2*f - a*c^2*d^2*
e^3*g - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*g)/((c^4*d^4*e^3*f^4*
abs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs(e) -
4*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)) + 3*(13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*e^
2*f*g^2 - 13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3*g^3 + 11
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g^3)/((c^4*d^4*e^3*f^4*a
bs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs(e) - 4
*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^5 - 1/12*(105*sqrt(-c*d^2*e + a*e^
3)*c^3*d^4*e^3*f^2*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*
g^2)*e)) - 105*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^5*f^2*g^2*arctan(sqrt(-c*
d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 210*sqrt(-c*d^2*e + a*e^3)*
c^3*d^5*e^2*f*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*
e)) + 210*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^4*f*g^3*arctan(sqrt(-c*d^2*e +
 a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 105*sqrt(-c*d^2*e + a*e^3)*c^3*d^6
*e*g^4*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 105*s
qrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^3*g^4*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e)) + 8*sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^5*f^3 + 56*
sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^4*f^2*g - 80*sqrt(c*d*f*g - a*e*g^2)*a*c^
2*d^2*e^6*f^2*g - 175*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e^3*f*g^2 + 238*sqrt(
c*d*f*g - a*e*g^2)*a*c^2*d^3*e^5*f*g^2 - 39*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d
*e^7*f*g^2 + 105*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*e^2*g^3 - 140*sqrt(c*d*f*g
 - a*e*g^2)*a*c^2*d^4*e^4*g^3 + 21*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^6*g^
3 + 6*sqrt(c*d*f*g - a*e*g^2)*a^3*e^8*g^3)/(sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*c^5*d^6*e^2*f^6*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*a*c^4*d^4*e^4*f^6*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f
*g - a*e*g^2)*c^5*d^7*e*f^5*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*a*c^4*d^5*e^3*f^5*g*abs(e) + 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*a^2*c^3*d^3*e^5*f^5*g*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*c^5*d^8*f^4*g^2*abs(e) + 7*sqrt(-c*d^2*e + a*e^3)*sqrt(c
```

```
*d*f*g - a*e*g^2)*a*c^4*d^6*e^2*f^4*g^2*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*a^2*c^3*d^4*e^4*f^4*g^2*abs(e) - 6*sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c^2*d^2*e^6*f^4*g^2*abs(e) - 4*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^4*d^7*e*f^3*g^3*abs(e) - 8*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^3*d^5*e^3*f^3*g^3*abs(e) + 8*
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c^2*d^3*e^5*f^3*g^3*abs(
e) + 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^4*c*d*e^7*f^3*g^3*a
bs(e) + 6*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^3*d^6*e^2*f^
2*g^4*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c^2*d^4
*e^4*f^2*g^4*abs(e) - 7*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^4*
c*d^2*e^6*f^2*g^4*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a
^5*e^8*f^2*g^4*abs(e) - 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^
3*c^2*d^5*e^3*f*g^5*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a^4*c*d^3*e^5*f*g^5*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e
*g^2)*a^5*d*e^7*f*g^5*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a^4*c*d^4*e^4*g^6*abs(e) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2
)*a^5*d^2*e^6*g^6*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

3.679 $\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 336

$$\begin{aligned}&\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx\\&=-\frac{128(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3465c^5d^5e(d+ex)^{3/2}}\\&\quad+\frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}}\\&\quad+\frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}}\\&\quad+\frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}}\\&\quad+\frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}\end{aligned}$$

```
[Out] -128/3465*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/e/(e*x+d)^(3/2)+32/231*(-a*e*g+c*d*f)^2*(g*x
+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+16/99*(
-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*
x+d)^(3/2)+2/11*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+
d)^(3/2)+128/1155*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^4/d^4/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= -\frac{128(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)^3\,(2ae^2g-cd(5ef-3dg))}{3465c^5d^5e(d+ex)^{3/2}}$$
$$+\frac{128g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)^3}{1155c^4d^4e\sqrt{d+ex}}$$
$$+\frac{32(f+gx)^2\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)^2}{231c^3d^3(d+ex)^{3/2}}$$
$$+\frac{16(f+gx)^3\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}$$
$$+\frac{2(f+gx)^4\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{11cd(d+ex)^{3/2}}$$

```
[In] Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} \\
&\quad+\frac{(8(cdf-aeg))\int\frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{11cd} \\
&= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(16(cdf-aeg)^2\right)\int\frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{33c^2d^2} \\
&= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}} \\
&\quad+\frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(64(cdf-aeg)^3\right)\int\frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{231c^3d^3}
\end{aligned}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}}$$
$$+ \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}}$$
$$+ \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}}$$
$$+ \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$
$$+ \frac{\left(64(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{1155c^3d^3}$$
$$= \frac{128(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3465c^4d^4(d+ex)^{3/2}}$$
$$+ \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}}$$
$$+ \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}}$$
$$+ \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}}$$
$$+ \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.58

$$\int\frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2((ae+cdx)(d+ex))^{3/2}\left(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2(33f^2+22fgx+5g^2x^2)-8\right.}{3465}$$

```
[In] Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
1*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8*a
*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*d^4
*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4
)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.81

method	result
default	$\frac{2(cdx+ae)(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g}{}$
gosper	$\frac{2(cdx+ae)(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g}{}$

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^
4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4
*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376*a*
c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*
g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(
(c*d*x+a*e)*(e*x+d))^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4+}{}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*f
^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5*g
^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g^2
+ 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^3*g
+ 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^3*g^4
)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e^2*f^2
*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F]

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{(d+ex)(ae+cdx)}(f+gx)^4}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx\\&=\frac{2(cdx+ae)^{\frac{3}{2}}f^4}{3\,cd}+\frac{8(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}f^3g}{15\,c^2d^2}\\&\quad+\frac{4(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}f^2g^2}{35\,c^3d^3}\\&\quad+\frac{8(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}fg^3}{315\,c^4d^4}\\&\quad+\frac{2(315\,c^5d^5x^5+35\,ac^4d^4ex^4-40\,a^2c^3d^3e^2x^3+48\,a^3c^2d^2e^3x^2-64\,a^4cde^4x+128\,a^5e^5)\sqrt{cdx+ae}g^4}{3465\,c^5d^5}\end{aligned}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^
2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^3)
+ 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3
*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(315*c^
5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*d^2*e^
3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(306) = 612$.

Time = 0.32 (sec) , antiderivative size = 1107, normalized size of antiderivative = 3.29

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(1155*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*
e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2
+ 198*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3
)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
+ a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 44*f*g^3
*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e
^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^
3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a
*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/
e^2 + g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)
*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2
*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128
*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 15
40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e^2 - 924*f^3*g*((3*sqrt(-c
*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^
2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs
(e)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,g^4\,x^5}{11}+\frac{256\,a^5\,e^5\,g^4-1408\,a^4\,c\,d\,e^4\,f\,g^3+3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2-3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g+2310\,a\,c^4}{3465\,c^5\,d^5}\right.}{}$$

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*e
^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d*e
^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d^5*
f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*e*f^
3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*e^3*g
^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*
c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(
693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^(1/2)
```

3.680 $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{16(cdf-aeg)^2(2ae^2g-cd(5ef-3dg))(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{315c^4d^4e(d+ex)^{3/2}} + \frac{16g(cdf-aeg)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105c^3d^3e\sqrt{d+ex}} + \frac{4(cdf-aeg)(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{21c^2d^2(d+ex)^{3/2}} + \frac{2(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{9cd(d+ex)^{3/2}}$$

```
[Out] -16/315*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e/(e*x+d)^(3/2)+4/21*(-a*e*g+c*d*f)*(g*x+f)^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+2/9*(g*x+f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+16/105*g*(-a*e*g
+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= -\frac{16(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2(2ae^2g-cd(5ef-3dg))}{315c^4d^4e(d+ex)^{3/2}}$$
$$+\frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}$$
$$+\frac{4(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}$$
$$+\frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{9cd(d+ex)^{3/2}}$$

```
[In] Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(2\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int\frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{3cde^2} \\
&= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(8(cdf-aeg)^2\right)\int\frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{21c^2d^2} \\
&= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}} \\
&\quad+\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(8(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{105c^2d^2} \\
&= \frac{16(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{315c^3d^3(d+ex)^{3/2}} \\
&\quad+\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}} \\
&\quad+\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg(21f^2+18fgx+5g^2x^2)+c^3d^3(10\right.}{315c^4d^4(d+ex)^{3/2}}$$

```
[In] Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(3
*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(105*
f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3
/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2f\right.}{315c^4d^4\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2f\right.}{315c^4d^4\sqrt{ex+d}}$

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*
f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+
16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
((c*d*x+a*e)*(e*x+d))^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\left(35\,c^4d^4g^3x^4+105\,ac^3d^3ef^3-126\,a^2c^2d^2e^2f^2g+72\,a^3cde^3fg^2-16\,a^4e^4g^3+5\,(27\,c^4d^4fg^2+ac^3d^3eg^3)x^3\right.}{}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*g
 + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^3*
e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e^2*
g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2*f*g
^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^3}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx\\&=\frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd}+\frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}f^2g}{5\,c^2d^2}\\&\quad+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}fg^2}{35\,c^3d^3}\\&\quad+\frac{2\,(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}g^3}{315\,c^4d^4}\end{aligned}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3) + 2
/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3*c*
d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(245) = 490.

Time = 0.30 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.83

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{2\left(\frac{105\,f^3\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}+\frac{9\,fg^2\left(\frac{15\sqrt{-cd^2e+ae^3}c^3d^6-3\sqrt{-cd^2e+ae^3}ac^2d^4e^2-4\sqrt{-cd^2e+ae}}{c^3d^3e^2}\right.}{}\right.}{}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/315*(105*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^
2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2 +
9*f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^
2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^
3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - g^3*((35*sqrt(
-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt
(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6
 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2 - 63*f^
2*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2
 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^
2*d^2*e^2))*abs(e)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{2\,g^3\,x^4}{9}-\frac{32\,a^4\,e^4\,g^3-144\,a^3\,c\,d\,e^3\,f\,g^2+252\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4}+\frac{x\,(16\,a^3\,c\,d\,e^3\,g}{}\right.}{\sqrt{d+e\,x}}$$

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*f*g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c^2*d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*e*g + 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)

3.681 $\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 200

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{105c^3d^3e(d+ex)^{3/2}}\\ &\quad+\frac{8g(cdf-aeg)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}}\\ &\quad+\frac{2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\end{aligned}$$

```
[Out] -8/105*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(3/2)+2/7*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+8/35*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{884, 808, 662\}$

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=-\frac{8\big(x(ae^2+cd^2)+ade+cdex^2\big)^{3/2}\,(cdf-aeg)\,(2ae^2g-cd(5ef-3dg))}{105c^3d^3e(d+ex)^{3/2}}$$
$$+\frac{8g\big(x(ae^2+cd^2)+ade+cdex^2\big)^{3/2}\,(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}$$
$$+\frac{2(f+gx)^2\,\big(x(ae^2+cd^2)+ade+cdex^2\big)^{3/2}}{7cd(d+ex)^{3/2}}$$

```
[In] Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
```

```
gerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}} \\
&\quad+\frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int\frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{7cde^2} \\
&= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}} \\
&\quad-\frac{\left(4(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\right)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{35c^2d^2e} \\
&= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e(d+ex)^{3/2}} \\
&\quad+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}} \\
&\quad+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$
\int\frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx
=\frac{2((ae+cdx)(d+ex))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2\left(35f^2+42fgx+15g^2x^2\right)\right)}{105c^3d^3(d+ex)^{3/2}}
$$

```
[In] Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g*
x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2
))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2(cdx+ae)\left(15g^2x^2c^2d^2-12acde\,g^2x+42c^2d^2fgx+8a^2e^2g^2-28acdefg+35c^2d^2f^2\right)\sqrt{(cdx+ae)(ex+d)}}{105c^3d^3\sqrt{ex+d}}$	106
gosper	$\frac{2(cdx+ae)\left(15g^2x^2c^2d^2-12acde\,g^2x+42c^2d^2fgx+8a^2e^2g^2-28acdefg+35c^2d^2f^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{105c^3d^3\sqrt{ex+d}}$	116

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^3/d
^3/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\,(14\,c^3d^3fg+ac^2d^2eg^2)x^2+(35\,c^3d^3f^2+14\,ac\right.}{105\,(c^3d^3ex+c^3d^4)}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^3
*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 14*
a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^2}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}fg}{15\,c^2d^2}+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}g^2}{105\,c^3d^3}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(182) = 364.

Time = 0.30 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{35\,f^2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}+\frac{g^2\left(\frac{15\,\sqrt{-cd^2e+ae^3}c^3d^6-3\,\sqrt{-cd^2e+ae^3}ac^2d^4e^2-4\,\sqrt{-cd^2e+ae^3}a^2c}{c^3d^3e^2}\right.}{}\right.}{}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/105*(35*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2
)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2 + g
^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4
*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^
3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^
6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 14*f*g*((3*sqrt(-c*
d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2
*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(
e)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3}{7}+\frac{16\,a^3\,e^3\,g^2-56\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3}+\frac{x\,\left(-8\,a^2\,c\,d\,e^2\,g^2+28\,a\,c^2\,d^2\,e\,f\,g+70\right.}{105\,c^3\,d^3}\right.}{\sqrt{d+e\,x}}$$

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^3
*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^3*
d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*x^2
*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.682 $\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{2(2ae^2g-cd(5ef-3dg))\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{15c^2d^2e(d+ex)^{3/2}} + \frac{2g\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

```
[Out] -2/15*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^2/d^2/e/(e*x+d)^(3/2)+2/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d
/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5cde\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(2ae^2g-cd(5ef-3dg))}{15c^2d^2e(d+ex)^{3/2}}$$

```
[In] Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2g(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5cde\sqrt{d+ex}} \\ &\quad+\frac{1}{5}\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\int\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx \\ &= \frac{2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{15cd(d+ex)^{3/2}}+\frac{2g(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5cde\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int\frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}(-2aeg+cd(5f+3gx))}{15c^2d^2(d+ex)^{3/2}}$$

```
[In] Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*
d^2*(d + e*x)^(3/2))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{(cdx+ae)(ex+d)}}{15c^2d^2\sqrt{ex+d}}$	57
gosper	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15c^2d^2\sqrt{ex+d}}$	67

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)/
c^2/d^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\,(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+(5\,c^2d^2f+acdeg)x)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{15\,(c^2d^2ex+c^2d^3)}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
[Out] 2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*
x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2
),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd}+\frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}g}{15\,c^2d^2}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*e
^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.02

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(\frac{5\,f\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e^2}-\frac{g\left(\frac{3\,\sqrt{-cd^2e+ae^3}c^2d^4-\sqrt{-cd^2e+ae^3}acd^2e^2-2\,\sqrt{-cd^2e+ae^3}a^2e^4}{c^2d^2}+\frac{5\,\dots}{\dots}\right)\dots}{e^3}\right)}{15\,e}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] 2/15*(5*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c
*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^2 - g*((3
*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sq
rt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e
^2))*abs(e)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\,(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d)}{15\,c^2\,d^2}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/
2),x)
```

```
[Out] (((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2*
f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2))/(d + e*x)^(1/2)
```

3.683 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral}=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2((ae + cdx)(d + ex))^{3/2}}{3cd(d + ex)^{3/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3cd\sqrt{ex+d}}$	40
gosper	$\frac{2(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3cd\sqrt{ex+d}}$	50

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
[Out] 2/3*(c*d*x+a*e)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdx + ae)\sqrt{ex + d}}{3\,(cdex + cd^2)}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}}\,dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{\sqrt{d + ex}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\,(cdx + ae)^{\frac{3}{2}}}{3\,cd}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}}\,dx = \frac{2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd} + \frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{cde}\right)|e|}{3\,e^3}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
[Out] 2/3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d) +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e^3
```

Mupad [B] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{\left(\frac{2\,x}{3}+\frac{2\,a\,e}{3\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

```
[Out] (((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
/(d + e*x)^(1/2)
```

3.684 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 124

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(3/2)+2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {878, 888, 211}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),x
]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} \\
&\quad -\frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{e^2g} \\
&= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} \\
&\quad -\frac{\left(2e^2(cdf-aeg)\right)\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g} \\
&= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}-\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x] - Sqrt[c*d*f
- a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(3/2)
*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)aeg-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g\sqrt{(aeg-cdf)g}}$	143

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*a*e*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/
g/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}g\sqrt{-\frac{cdf-aeg}{g}}-(cdef-(cd^2+2\,ae^2)g)x}{egx^2+df+(ef+dg)x}\right)+}{egx+dg}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*
d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqr
t((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(108) = 216$.

Time = 0.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)}\,dx = \frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{g} + \frac{cdef\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-ae^2g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-\sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}}{\sqrt{cdfg-aeg^2}g} - \frac{(cde^2f-ae^3g)\,\mathrm{arc}}{\ldots}\right.}{e^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] 2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/g + (c*d*e*f*arctan(sqrt(-c*d^2*
e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - a*e^2*g*arctan(sqrt(-c*d^2*e +
a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2))/(sqrt(c*d*f*g - a*e*g^2)*g) - (c*d*e^2*f - a*e^3*g)*arctan(sqr
t((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c
*d*f*g - a*e*g^2)*e*g))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)), x)
```

3.685 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2} dx$

Optimal result

Integrand size = 46, antiderivative size = 132

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2} dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}(f+gx)} + \frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)-(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {876, 888, 211}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2} dx = \frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2)
,x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}(f+gx)}+\frac{(cd)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{(cde^2)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}(f+gx)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}}{f+gx}+\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^2),x]
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdgx-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g(gx+f)\sqrt{(aeg-cdf)g}}$	151

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
[Out] (-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g*x-arctanh(g*(c
*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2))*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/
g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(116) = 232$.

Time = 0.44 (sec) , antiderivative size = 562, normalized size of antiderivative = 4.26

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$$

$$=\left[-\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x-2\,\sqrt{cdex^2+ac}}{egx^2+df+(ef+dg)x}\right.}{2\,(cd^2f^2g^2-adefg^3+(cdefg^3-ae^2g^4)x^2+(cdej}\right.$$

$$-\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}\right)+\sqrt{cdex}}{cd^2f^2g^2-adefg^3+(cdefg^3-ae^2g^4)x^2+(cdef^2g^2-adeg^4+(cd^2-}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2
- a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4
+ (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)
*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
 a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*
g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4
)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^2}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^2}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(116) = 232$.

Time = 0.36 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx =$$

$$-\frac{\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,cde^2}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g} - \frac{cde\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{\sqrt{cdfg-aeg^2}\,g} + \frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)-cd^2eg\arctan\left(\frac{\sqrt{-\ldots}}{\sqrt{c\ldots}}\right.}{\sqrt{cdfg-aeg^2}\,efg-\sqrt{c\ldots}}\right.}{e^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] -(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^2/((c*d*e^2*f - a*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g) - c*d*e*arctan(sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2
)*g) + (c*d*e^2*f*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*
e)) - c*d^2*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g
^2)*e*f*g - sqrt(c*d*f*g - a*e*g^2)*d*g^2))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{{\left(f + g\,x\right)}^2\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)), x)
```

3.686 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 207

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx=-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}$$

```
[Out] 1/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^2/(e*x+d)^(1/2)+1/4*c*d*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {876, 886, 888, 211}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx=\frac{c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3)
,x]
```

```
[Out] -1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^2) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c
*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{2g\sqrt{d+ex}(f+gx)^2} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4g}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{(c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g(cdf-aeg)}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{(c^2d^2e^2)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4g(cdf-aeg)}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(2aeg+cd(-f+gx))+c^2d^2(f+gx)^2\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{4g^{3/2}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^3),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(2*a*e*g + c*d*(-f + g*x)) + c^2*d^2*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[
a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*g^(3/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+2\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-\sqrt{cdx}\right.}{4\sqrt{ex+d}\sqrt{cdx+ae}\,(aeg-cdf)g(gx+f)^2\sqrt{(aeg-cd}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^2*d^2*g^2*x^2+2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^2*d^2*f*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c
^2*d^2*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x-2*(c*d*x+a*e)^
(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)^2/((a*e*g
-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(181) = 362$.

Time = 0.40 (sec) , antiderivative size = 1056, normalized size of antiderivative = 5.10

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$$
$$= \Bigg[\frac{(c^2d^2eg^2x^3 + c^2d^3f^2 + (2\,c^2d^2efg + c^2d^3g^2)x^2 + (c^2d^2ef^2 + 2\,c^2d^3fg)x)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f}{}\right.}{8\,(c^2d^3f^4g^2 - 2\,acd^2ef^3g^3 + a^2de^2f^2g^4 + (c^2d^2ef^2g^4 - 2\,acde^2fg^5 + a^2e^3g^6)x^3 + (2\,c^2}$$
$$- \frac{(c^2d^2eg^2x^3 + c^2d^3f^2 + (2\,c^2d^2efg + c^2d^3g^2)x^2 + (c^2d^2ef^2 + 2\,c^2d^3fg)x)\sqrt{cdfg - aeg^2}\arctan\left(\frac{\sqrt{cdex^2+a}}{c}\right.}{4\,(c^2d^3f^4g^2 - 2\,acd^2ef^3g^3 + a^2de^2f^2g^4 + (c^2d^2ef^2g^4 - 2\,acde^2fg^5 + a^2e^3g^6)x^3 + (2\,c^2d^2ef^3g^3 + a^2de^2g^6}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g^2 +
2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*g^3 + a
^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3*g^6)*x^
3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^4
- 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2*d*e^2*f*g
^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^4)*x),
 -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*f^2*g - 3*a*c*
d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
```

```
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e
*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2
*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e
^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a
^2*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*
f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^3}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(181) = 362$.

Time = 0.49 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \frac{\left(\frac{c^2d^2e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(cdfg-aeg^2)^{\frac{3}{2}}} - \frac{c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-2\,c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+c^2d^4eg^2\arctan\left(\frac{\sqrt{-cd^2}}{\sqrt{cdfg}}\right.}{\sqrt{cdfg-aeg^2}cde^2f^3g-2\sqrt{cdfg-aeg^2}cd^2ef^2g^2-\sqrt{cdfg-aeg^2}}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/4*(c^2*d^2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e))/(c*d*f*g - a*e*g^2)^(3/2) - (c^2*d^2*e^3*f^2*arctan(sqrt(
-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 2*c^2*d^3*e^2*f*g*arctan
(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + c^2*d^4*e*g^2*arct
an(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2*e +
a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f - sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*c*d^2*e*g + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)
*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f^3*g - 2*sqrt(c*d*f*g - a*e*g^2
)*c*d^2*e*f^2*g^2 - sqrt(c*d*f*g - a*e*g^2)*a*e^3*f^2*g^2 + sqrt(c*d*f*g -
a*e*g^2)*c*d^3*f*g^3 + 2*sqrt(c*d*f*g - a*e*g^2)*a*d*e^2*f*g^3 - sqrt(c*d*f
*g - a*e*g^2)*a*d^2*e*g^4) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^3*d
^3*e^4*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^5*g - ((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^2*g)/((c*d*e^2*f - a*e^3*g +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2*(c*d*f*g - a*e*g^2)))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)), x)
```

3.687 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 277

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} + \frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}$$

```
[Out] 1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(5/2)-1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^3/(e*x+d)^(1/2)+1/12*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*
d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x
+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = \frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}} + \frac{c^2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^2} + \frac{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4)
,x]
```

```
[Out] -1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^3) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*
g)*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTa
n[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g
]*Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
```

```
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{(cd)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{6g}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{(c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g(cdf-aeg)}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)}+\frac{(c^3d^3)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16g(cdf-aeg)^2}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{(c^3d^3e^2)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{8g(cdf-aeg)^2}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$
$$=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(-8a^2e^2g^2-2acdeg(-7f+gx)+c^2d^2(-3f^2+8fgx+3g^2}{24g^{3/2}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^3}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^4),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(-8*a^2*e^2*g^2 - 2*a*c*d*e*g*(-7*f + g*x) + c^2*d^2*(-3*f^2 + 8*f*g*x
 + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/S
qrt[c*d*f - a*e*g]]))/(24*g^(3/2)*(c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*
(d + e*x)]*(f + g*x)^3)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\,\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f\,g^2x^2+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f^2}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^
2+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-8*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)*a^2*e^2*g^2-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*
e*f*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. 2(245) = 490.

Time = 0.80 (sec) , antiderivative size = 1732, normalized size of antiderivative = 6.25

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d
^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5
*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3
*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^
3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^
2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 +
 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g
^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^
2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*
d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e
^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c
^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2
 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^3*d^3*f^3*g - 17*a
*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^
3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d
*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3
*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4
 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(
a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^
7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3
*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^
3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e -
a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. 2(245) = 490.

Time = 0.66 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.45

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = \frac{\left(\frac{3\,c^3d^3e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2f^2g-2\,acdefg^2+a^2e^2g^3)\sqrt{cdfg-aeg^2}}-\frac{3\,c^3d^3e^4f^3\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-9\,c^3d^4e^3f^2g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+9\,c^3d^5e^2fg^2\arctan}{\sqrt{cdfg-aeg^2}c^2d^2e^3f^5g-3\sqrt{cdfg-aeg^2}c^2d^3e^2f^4g^2-2\sqrt{cdfg}}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*
d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 + a^2*e^2*g^3)*sqrt(
c*d*f*g - a*e*g^2)) - (3*c^3*d^3*e^4*f^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 9*c^3*d^4*e^3*f^2*g*arctan(sqrt(-c*d^2*e + a*e
^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 9*c^3*d^5*e^2*f*g^2*arctan(sqrt(-c*d^2
```

```
*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*c^3*d^6*e*g^3*arctan(sqrt(-c
*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^2 - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*c^2*d^3*e^2*f*g + 14*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*a*c*d*e^4*f*g + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^
2*d^4*e*g^2 + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^3*
g^2 - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^5*g^2)/(sqrt(c
*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^5*g - 3*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^2
*f^4*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^4*f^4*g^2 + 3*sqrt(c*d*f*g - a
*e*g^2)*c^2*d^4*e*f^3*g^3 + 6*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^3*f^3*g^3 +
 sqrt(c*d*f*g - a*e*g^2)*a^2*e^5*f^3*g^3 - sqrt(c*d*f*g - a*e*g^2)*c^2*d^5*
f^2*g^4 - 6*sqrt(c*d*f*g - a*e*g^2)*a*c*d^3*e^2*f^2*g^4 - 3*sqrt(c*d*f*g -
a*e*g^2)*a^2*d*e^4*f^2*g^4 + 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d^4*e*f*g^5 + 3*
sqrt(c*d*f*g - a*e*g^2)*a^2*d^2*e^3*f*g^5 - sqrt(c*d*f*g - a*e*g^2)*a^2*d^3
*e^2*g^6) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^6*f^2 - 6*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^7*f*g + 3*sqrt((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^8*g^2 - 8*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*c^4*d^4*e^4*f*g + 8*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a*c^3*d^3*e^5*g^2 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^
3*e^2*g^2)/((c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 + a^2*e^2*g^3)*(c*d*e^2*f - a*
e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^4\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)), x)
```

3.688 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 347

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx &= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\\ &\quad+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}\\ &\quad+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}\\ &\quad+\frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)}\\ &\quad+\frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}\end{aligned}$$

```
[Out] 5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)-1/4*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^4/(e*x+d)^(1/2)+1/24*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/96*c^
2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/
(e*x+d)^(1/2)+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*
g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {876, 886, 888, 211}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx=\frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)
,x]
```

```
[Out] -1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^4) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*
g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^
3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt
[d + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*
f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{(cd)\int\frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}\\
&\quad+\frac{(5c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{48g(cdf-aeg)}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}\\
&\quad+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{(5c^3d^3)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{64g(cdf-aeg)^2}\\
&= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}\\
&\quad+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)}\\
&\quad+\frac{(5c^4d^4)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128g(cdf-aeg)^3}
\end{aligned}
$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{(5c^4d^4e^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{64g(cdf-aeg)^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} + \frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$$

$$= \frac{c^4d^4\sqrt{(ae+cdx)(d+ex)}\Bigg(\frac{\sqrt{g}(48a^3e^3g^3+8a^2cde^2g^2(-17f+gx)-2ac^2d^2eg(-59f^2+18fgx+5g^2x^2)+c^3d^3(-15f^3+73f^2gx+55fg^2x^2}{c^4d^4(cdf-aeg)^3(f+gx)^4}}{192g^{3/2}\sqrt{d+ex}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^5),x]
```

```
[Out] (c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[g]*(48*a^3*e^3*g^3 + 8*a^2*c*
d*e^2*g^2*(-17*f + g*x) - 2*a*c^2*d^2*e*g*(-59*f^2 + 18*f*g*x + 5*g^2*x^2)
+ c^3*d^3*(-15*f^3 + 73*f^2*g*x + 55*f*g^2*x^2 + 15*g^3*x^3)))/(c^4*d^4*(c*
d*f - a*e*g)^3*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c
*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x])))/(192*g^(3/2)*Sq
rt[d + e*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(309) = 618.

Time = 0.54 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.98

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+60\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+90\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4f^2\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*
d^4*f^4+10*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-55
*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*c*d*e^2*
g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g^2*x*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*e^
3*g^3+136*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-118*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d
*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. 2(309) = 618.

Time = 1.35 (sec) , antiderivative size = 2610, normalized size of antiderivative = 7.52

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
```

```
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3 - 184*
a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*
x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*d^2*e^2*f*
g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2*d^3*e^2*f^
6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e*f^4*g^6 -
4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 +
 a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 16*
a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^7 + 2*(3
*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g
^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 6*a*c^3
*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^6 + 12*(a^2
*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^8
)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 8*a*c^3
*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c
^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7
)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - a*c^3*d^3*e
^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*(6*a^2*c^2*
d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^6)*x),
-1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c
*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c
*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*
x)) + (15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2
*g^3 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d
^3*e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^
2*e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2
*d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2
*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e
*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*
e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^
4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3
*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a
^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^
5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^
6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e
^5)*f^2*g^8)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^
5 - 8*a*c^3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 +
 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e
^5)*f^3*g^7)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 -
```

```
a*c^3*d^3*e^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*
(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^
4*g^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex+d}(gx+f)^5}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. $2(309) = 618$.

Time = 1.73 (sec) , antiderivative size = 1955, normalized size of antiderivative = 5.63

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/192*(15*c^4*d^4*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 + 3*a^2*c*d*
e^2*f*g^3 - a^3*e^3*g^4)*sqrt(c*d*f*g - a*e*g^2)) - (15*c^4*d^4*e^5*f^4*arc
tan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 60*c^4*d^5*e^4*
f^3*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 90*c^4
*d^6*e^3*f^2*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e
```

```
)) - 60*c^4*d^7*e^2*f*g^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e)) + 15*c^4*d^8*e*g^4*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*
g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^
3*e^4*f^3 - 73*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^3*f
^2*g + 118*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^5*f^2
*g + 55*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e^2*f*g^2 +
36*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^4*f*g^2 - 136
*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^6*f*g^2 - 15*sqrt
(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*e*g^3 - 10*sqrt(-c*d^2*e
 + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^3*g^3 - 8*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^5*g^3 + 48*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*a^3*e^7*g^3)/(sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^4*f^
7*g - 4*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^3*f^6*g^2 - 3*sqrt(c*d*f*g - a*e*
g^2)*a*c^2*d^2*e^5*f^6*g^2 + 6*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e^2*f^5*g^3
+ 12*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^4*f^5*g^3 + 3*sqrt(c*d*f*g - a*e*g
^2)*a^2*c*d*e^6*f^5*g^3 - 4*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*e*f^4*g^4 - 18*
sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^3*f^4*g^4 - 12*sqrt(c*d*f*g - a*e*g^2)*
a^2*c*d^2*e^5*f^4*g^4 - sqrt(c*d*f*g - a*e*g^2)*a^3*e^7*f^4*g^4 + sqrt(c*d*
f*g - a*e*g^2)*c^3*d^7*f^3*g^5 + 12*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^5*e^2*f
^3*g^5 + 18*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^3*e^4*f^3*g^5 + 4*sqrt(c*d*f*g
- a*e*g^2)*a^3*d*e^6*f^3*g^5 - 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^6*e*f^2*g^
6 - 12*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^4*e^3*f^2*g^6 - 6*sqrt(c*d*f*g - a*e
*g^2)*a^3*d^2*e^5*f^2*g^6 + 3*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^5*e^2*f*g^7 +
 4*sqrt(c*d*f*g - a*e*g^2)*a^3*d^3*e^4*f*g^7 - sqrt(c*d*f*g - a*e*g^2)*a^3*
d^4*e^3*g^8) - (15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^7*d^7*e^8*f^3
- 45*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^6*d^6*e^9*f^2*g + 45*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^5*d^5*e^10*f*g^2 - 15*sqrt((e*x +
d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^4*d^4*e^11*g^3 - 73*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*c^6*d^6*e^6*f^2*g + 146*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a*c^5*d^5*e^7*f*g^2 - 73*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a^2*c^4*d^4*e^8*g^3 - 55*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^5*
d^5*e^4*f*g^2 + 55*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^4*d^4*e^5*
g^3 - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^4*d^4*e^2*g^3)/((c^3*d
^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 + 3*a^2*c*d*e^2*f*g^3 - a^3*e^3*g^4)*(c*d*
e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^4))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(f + g\,x)^5\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)), x)
```

3.689 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 336

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{128(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^5d^5e(d+ex)^{5/2}} + \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

```
[Out] -128/15015*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/e/(e*x+d)^(5/2)+128/3003*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(3/2)+32/429*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*
x+d)^(5/2)+16/143*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)/c^2/d^2/(e*x+d)^(5/2)+2/13*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx =$$
$$-\frac{128(x(ae^2+cd^2)+ade+cdex^2)^{5/2}\,(cdf-aeg)^3\,(2ae^2g-cd(7ef-5dg))}{15015c^5d^5e(d+ex)^{5/2}}$$
$$+\frac{128g(x(ae^2+cd^2)+ade+cdex^2)^{5/2}\,(cdf-aeg)^3}{3003c^4d^4e(d+ex)^{3/2}}$$
$$+\frac{32(f+gx)^2\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}\,(cdf-aeg)^2}{429c^3d^3(d+ex)^{5/2}}$$
$$+\frac{16(f+gx)^3\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}\,(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}$$
$$+\frac{2(f+gx)^4\,(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{13cd(d+ex)^{5/2}}$$

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} \\
&\quad+\frac{(8(cdf-aeg))\int\frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{13cd} \\
&= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(48(cdf-aeg)^2\right)\int\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{143c^2d^2} \\
&= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} \\
&\quad+\frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(64(cdf-aeg)^3\right)\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{429c^3d^3}
\end{aligned}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} + \frac{\left(64(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx}{3003c^3d^3}$$

$$= \frac{128(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^4d^4(d+ex)^{5/2}} + \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.58

$$\int\frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f+\right.}{}$$

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
3*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) -
8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) + c^4
*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g
^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4}{}$
gosper	$\frac{2(cdx+ae)(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4d^4f^2g^2x^2-320a^3cd\,e}{}$

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15015*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(1155*c^4*d
^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*d^4*f*g^3*x^3+560*a^2*c^2*d^2*e
^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*c^4*d^4*f^2*g^2*x^2-320*a^3*c*d
*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720*a*c^3*d^3*e*f^2*g^2*x+8580*c^4
*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3*f*g^3+2288*a^2*c^2*d^2*e^2*f^2
*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4)/c^5/d^5
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.40

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(1155\,c^6d^6g^4x^6+3003\,a^2c^4d^4e^2f^4-3432\,a^3c^3d^3e^3f^3g\right.}{}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^3
*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a^6
*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*d^6
*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c^6*d
^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*c^3*d
^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*a^2*c^
4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g^4)*x^2
 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^3*d^3*e^
3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F]

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}(f+gx)^4}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**4/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.23

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f^4}{5\,cd}$$
$$+\frac{8\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}f^3g}{35\,c^2d^2}$$
$$+\frac{4\left(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4\right)\sqrt{cdx+ae}f^2g^2}{105\,c^3d^3}$$
$$+\frac{8\left(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5\right)\sqrt{cdx+ae}fg^3}{1155\,c^4d^4}$$
$$+\frac{2\left(1155\,c^6d^6x^6+1470\,ac^5d^5ex^5+35\,a^2c^4d^4e^2x^4-40\,a^3c^3d^3e^3x^3+48\,a^4c^2d^2e^4x^2-64\,a^5cde^5x+128\,a^6e^6\right)\sqrt{}}{15015\,c^5d^5}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g^2
/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*
e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x
+ a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*e*x^5 +
 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e^4*x^2 -
 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. 2(306) = 612.

Time = 0.38 (sec) , antiderivative size = 2535, normalized size of antiderivative = 7.54

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/45045*(15015*a*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3
)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/
e + 1716*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d
^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 257
4*a*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*
a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e +
a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - 858*c*d*f^2*
g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^
6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)
*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2
)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(
e)/e^2 - 572*a*f*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e
+ a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(
-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d
^4*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c
^4*d^4*e^7))*abs(e)/e + 52*c*d*f*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10
- 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c
^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e +
 a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4)
+ (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e
^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/
e^2 + 13*a*g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a
*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-
```

```
c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8
- 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6
 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e - 5*c*d*g^4*((693*sq
rt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 -
70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c
^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e
+ a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)/(c^6*d^6*e^5
) + (3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2
)*a^2*e^6 + 4095*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))/(c^6*d^6*e^11))*abs(e)/e^2 - 3003*
c*d*f^4*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2
*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))
/(c^2*d^2*e^2))*abs(e)/e^3 - 12012*a*f^3*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d
^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)
/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^3\,x^5\,\left(7\,a\,e\,g+26\,c\,d\,f\right)}{143}+2\right.}{}$$

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c*
d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^3*
e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(15015*c
^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3*d^3
*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/(1501
5*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a^2*c^4
*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^2*g^2))
/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 - 2*a^3*e
^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*d^2) + (2
*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*c*d)))/(d
+ e*x)^(1/2)
```

3.690 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^4d^4e(d+ex)^{5/2}} + \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

```
[Out] -16/1155*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(5/2)+16/231*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(3/2)+4/33*(-a*e*g+c
*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/
2)+2/11*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)}{1155c^4d^4e(d+ex)^{5/2}} + \frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(6(cdf-aeg)\right)\int\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{11cd} \\
&= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(8(cdf-aeg)^2\right)\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{33c^2d^2} \\
&= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} \\
&\quad+\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(8(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{231c^2d^2} \\
&= \frac{16(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^3d^3(d+ex)^{5/2}} \\
&\quad+\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} \\
&\quad+\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f+\right.}{}$$

[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2),x]

[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(11*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3*(231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d + e*x)^(5/2))

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2\left(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+1\right.}{1155\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)\left(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+16a^3e^3g^3-88a^2cd\,e^2\right.}{1155c^4d^4(ex+d)^{\frac{3}{2}}}$

[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/1155*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-105*c^3*d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^3*f*g^2*x^2-40*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-88*a^2*c*d*e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3)/c^4/d^4

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(105\,c^5d^5g^3x^5+231\,a^2c^3d^3e^2f^3-198\,a^3c^2d^2e^3f^2g+88\right.}{}$$

[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")

```
[Out] 2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^3
*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4*a
*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*c^3
*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*c^3*
d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99*a^2*
c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(f+gx)^3}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**3/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx&=\frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}f^3}{5\,cd}\\&+\frac{6\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}f^2g}{35\,c^2d^2}\\&+\frac{2\,(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4)\sqrt{cdx+ae}fg^2}{105\,c^3d^3}\\&+\frac{2\,(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5)\sqrt{cdx+ae}g^3}{1155\,c^4d^4}\end{aligned}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^2/(
c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*e^
2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x +
a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. $2(245) = 490$.

Time = 0.36 (sec) , antiderivative size = 1778, normalized size of antiderivative = 6.61

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(1155*a*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*
a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e
+ 99*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
 + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 99*a*f*
g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^
4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a
^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e
^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - 33*c*d*f*g^2*((35*sq
rt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*s
qrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*
e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 3
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2 - 11
*a*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3
*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e
^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*a
bs(e)/e + c*d*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e
+ a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqr
t(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e
^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*
e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e^2 - 231*c*d*f^3*(
(3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*
sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e +
```

```
a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2
*e^2))*abs(e)/e^3 - 693*a*f^2*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-
c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2)
+ (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^4\,(4\,a\,e\,g+11\,c\,d\,f)}{33}-\right.}{}$$

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c*
d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^3*
f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 - 12*
a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g))/(
1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^2*f^2
 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^3*f^3
+ 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d + e*x)^
(1/2)
```

3.691 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 200

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315c^3d^3e(d+ex)^{5/2}} + \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

```
[Out] -8/315*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(5/2)+8/63*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/e/(e*x+d)^(3/2)+2/9*(g*x+f)^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

= {884, 808, 662}

$$\int \frac{(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)}{315c^3d^3e(d+ex)^{5/2}} + \frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
```

```
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}} \\
&\quad+\frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{9cde^2} \\
&= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}} \\
&\quad-\frac{\left(4(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{63c^2d^2e} \\
&= -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315c^3d^3e(d+ex)^{5/2}} \\
&\quad+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} \\
&\quad+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+\right.}{315c^3d^3(d+ex)^{5/2}}$$

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g*
x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2
))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}(35g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+90c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-36acdefg+63c^{2}d^{2}f^{2})}{315\sqrt{ex+d}\,c^{3}d^{3}}$	108
gosper	$\frac{2(cdx+ae)(35g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+90c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-36acdefg+63c^{2}d^{2}f^{2})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}{315c^{3}d^{3}(ex+d)^{\frac{3}{2}}}$	116

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(35*c^2*d^2*g
^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a^2*e^2*g^2-36*a*c*d*e*f*g+63*c^
2*d^2*f^2)/c^3/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(35\,c^4d^4g^2x^4+63\,a^2c^2d^2e^2f^2-36\,a^3cde^3fg+8\,a^4e^4g^2}{}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g + 8
*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f
^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2
+ 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(f+gx)^2}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**2/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f^2}{5\,cd} + \frac{4\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}fg}{35\,c^2d^2} + \frac{2\left(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4\right)\sqrt{cdx+ae}g^2}{315\,c^3d^3}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a^2
*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(c^3*
d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(182) = 364.

Time = 0.33 (sec) , antiderivative size = 1145, normalized size of antiderivative = 5.72

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/315*(105*a*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*
e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e +
6*c*d*f*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*
c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*
e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 3*a*g^2*((15
*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 -
4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)/
(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - c*d*g^2*((35*sqrt(-c*d^2*e +
```

```
a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e
+ a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt
(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2
*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2 - 21*c*d*f^2*((3*
sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqr
t(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^
2))*abs(e)/e^3 - 42*a*f*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*
e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^3\,(5\,a\,e\,g+9\,c\,d\,f)}{63}+\frac{1\cdots}{}\right.}{}$$

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*f
))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(31
5*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d^3*
e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 - 2*a^
2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

3.692 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(2ae^2g-cd(7ef-5dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35c^2d^2e(d+ex)^{5/2}} + \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

```
[Out] -2/35*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/c^2/d^2/e/(e*x+d)^(5/2)+2/7*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d
/e/(e*x+d)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}} - \frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(2ae^2g-cd(7ef-5dg))}{35c^2d^2e(d+ex)^{5/2}}$$

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}} \\ &\quad +\frac{1}{7}\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx \\ &= \frac{2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35cd(d+ex)^{5/2}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}(-2aeg+cd(7f+5gx))}{35c^2d^2(d+ex)^{5/2}}$$

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}(-5cdgx+2aeg-7cdf)}{35\sqrt{ex+d}\,c^{2}d^{2}}$	59
gosper	$-\frac{2(cdx+ae)(-5cdgx+2aeg-7cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35c^{2}d^{2}(ex+d)^{\frac{3}{2}}}$	67

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/35*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-5*c*d*g*x+2
*a*e*g-7*c*d*f)/c^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(5\,c^3d^3gx^3+7\,a^2cde^2f-2\,a^3e^3g+\left(7\,c^3d^3f+8\,ac^2d^2eg\right)\right.}{35\,(}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a*
c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}f}{5\,cd}+\frac{2\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}g}{35\,c^2d^2}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*
(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*x
+ a*e)*g/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(113) = 226$.

Time = 0.30 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.06

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(\frac{35\,af\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e}\right.}{}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

```
[Out] 2/105*(35*a*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2
)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e + c*d
*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4
*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^
3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^
6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 7*c*d*f*((3*sqrt(-c
*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^
2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs
(e)/e^3 - 7*a*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)
*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 11.95 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\,\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\frac{4\,a}{}\right.}{\sqrt{d+e\,x}}$$

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*d
*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7 +
(2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.693 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral} = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2}{5\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	50

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
[Out] 2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(c^2d^2x^2 + 2\,acdex + a^2e^2)\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex+d}}{5\,(cdex + cd^2)}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}}{5\,cd}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(42) = 84.

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 5.29

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(\frac{5\,a\left(\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}}{cde}\right)|e|}{e}-\frac{cd\left(\frac{3\sqrt{-cd}}{}\right.}{}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="giac")
[Out] 2/15*(5*a*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c
*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)/e - c*d*((3
*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sq
rt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e
^2))*abs(e)/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \frac{\left(\frac{4\,a\,e\,x}{5} + \frac{2\,c\,d\,x^2}{5} + \frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{\sqrt{d + e\,x}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

```
[Out] (((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

3.694 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 179

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}+\frac{2(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+2*(-a*e*g+c*d*f
)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)-2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {878, 888, 211}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\frac{2(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{e^2g}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{(cdf-aeg)^2\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{g^2}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{(2e^2(cdf-aeg}{}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}} + \frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.74

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(4aeg+cd(-3f+gx))+3(c\right.}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c*
d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(3\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2e^2g^2-6\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acdefg+3\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2-\sqrt{\right.}}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2\sqrt{(aeg-cdf)g}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*a^2*e^2*g^2-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^
2*d^2*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x-4*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.28

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \left[-\frac{3\,(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+}{}\right.}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)/
g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^
2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^
2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt((c*d
*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\,(f+gx)}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(157) = 314$.

Time = 0.47 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.83

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)} dx = \frac{2\left(c^2d^2f^2|e| - 2\,acdefg|e| + a^2e^2g^2|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg - aeg^2}eg^2} - \frac{2\left(3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 6\,acde^3fg|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) + 3\,a^2e^4g^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) \cdots\right)}{3 \cdots} - \frac{2\left(3\sqrt{(ex+d)cde - cd^2e + ae^3}cde^{10}fg|e| - 3\sqrt{(ex+d)cde - cd^2e + ae^3}ae^{11}g^2|e| - ((ex+d)cde - cd^2 \cdots\right)}{3\,e^{12}g^3}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] 2*(c^2*d^2*f^2*abs(e) - 2*a*c*d*e*f*g*abs(e) + a^2*e^2*g^2*abs(e))*arctan(s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt
(c*d*f*g - a*e*g^2)*e*g^2) - 2/3*(3*c^2*d^2*e^2*f^2*abs(e)*arctan(sqrt(-c*d
^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 6*a*c*d*e^3*f*g*abs(e)*arcta
n(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 3*a^2*e^4*g^2*abs
(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-
c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e*f*abs(e) - sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*g*abs(e) + 4*sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*f*g - a*e*g^2)*a*e^2*g*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^3*g^2) - 2/
3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^10*f*g*abs(e) - 3*sqrt((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^11*g^2*abs(e) - ((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*e^8*g^2*abs(e))/(e^12*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)\,\left(d + e\,x\right)^{3/2}} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)), x)
```

3.695 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 178

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}(f+gx)} - \frac{3cd\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

```
[Out] -(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)-3*c*d*arct
an(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*
x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(5/2)+3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 878, 888, 211}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = -\frac{3cd\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^2\sqrt{d+ex}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{(3cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{2g}$$

$$= \frac{3cd\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{g^2\sqrt{d + ex}} - \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{g(d + ex)^{3/2}(f + gx)} - \frac{(3cd(cdf - aeg))\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g^2}$$

$$= \frac{3cd\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{g^2\sqrt{d + ex}} - \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{g(d + ex)^{3/2}(f + gx)} - \frac{(3cde^2(cdf - aeg))\,\mathrm{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g^2}$$

$$= \frac{3cd\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{g^2\sqrt{d + ex}} - \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{g(d + ex)^{3/2}(f + gx)} - \frac{3cd\sqrt{cdf - aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2}\,dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{g}\sqrt{ae + cdx}(-aeg + cd(3f + 2gx)) - 3cd\sqrt{c\ldots}\right.}{g^{5/2}\sqrt{(ae + cdx)(d + ex)}(f + \ldots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^2),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-(a*e*g) + c*d
*(3*f + 2*g*x)) - 3*c*d*Sqrt[c*d*f - a*e*g]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[
a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]
*(f + g*x))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.66

method	result
default	$\frac{\left(-3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acde\,g^2x+3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx-3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acdefg+3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf\ldots}}\right.\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2(gx+\ldots}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x,metho
d=_RETURNVERBOSE)
```

```
[Out] (-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^2*x+3*ar
ctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x-3*arctanh(
g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x
+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2+2*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)*c*d*g*x-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+3
*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/
2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.49

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2} dx = \left[\frac{3\left(cdegx^2 + cd^2f + (cdef + cd^2g)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f}{}\right.}{} \right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e*
g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(e
*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f + (c*d
*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f
 - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(
e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2} dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^2}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(158) = 316$.

Time = 0.42 (sec) , antiderivative size = 605, normalized size of antiderivative = 3.40

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\frac{3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\,c^2d^3efg|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}}{\sqrt{cdfg-aeg^2}}\right.}{}$$
$$+\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}cd|e|}{e^2g^2}-\frac{3\left(c^2d^2f|e|-acdeg|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^2}$$
$$+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2f|e|-\sqrt{(ex+d)cde-cd^2e+ae^3}acdeg|e|}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] (3*c^2*d^2*e^2*f^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e)) - 3*c^2*d^3*e*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e)) - 3*a*c*d*e^3*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 3*a*c*d^2*e^2*g^2*abs(e)*arctan(sqrt(-c
*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c*d*e*f*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f
*g - a*e*g^2)*c*d^2*g*abs(e) + sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*a*e^2*g*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^3*f*g^2 - sqrt(c*d*f*g - a*e*
g^2)*d*e^2*g^3) + 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*abs(e)/(e^2
*g^2) - 3*(c^2*d^2*f*abs(e) - a*c*d*e*g*abs(e))*arctan(sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)
*e*g^2) + (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*f*abs(e) - sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e*g*abs(e))/((c*d*e^2*f - a*e^3*g
+ ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^2} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^2\,\left(d + e\,x\right)^{3/2}} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)), x)
```

3.696 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 195

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=-\frac{3cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}$$

```
[Out] -1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^2+3/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(1/2)-3/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {876, 888, 211}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^3),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3cd)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx}{4g}$$

$$=-\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g^2}$$

$$=-\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3c^2d^2e^2)\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4g^2}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} + \frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}(2aeg+cd(3f+5gx))}{(f+gx)^2}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{4g^{5/2}\sqrt{d+ex}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/(
f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*
e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+6\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+3\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2f^2+5\right.}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^2(gx+f)^2\sqrt{(aeg-cdj}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^2*d^2*f^2+5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c*d*g*x+2*(c*d*
x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^2/((a*e*g-c*
d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(169) = 338$.

Time = 0.34 (sec) , antiderivative size = 840, normalized size of antiderivative = 4.31

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \left[-\frac{3\,(c^2d^2eg^2x^3 + c^2d^3f^2 + (2\,c^2d^2efg + c^2d^3g^2)x^2 + (c^2d^2ef^2 + 2\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*f*g^2
 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4 + (c*d
*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 - 2*a*e^2
)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*f^2*g^4)*
x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g
^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan
(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e
*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^2*d^2*f^2*g -
 a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*
f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c
*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*
e^2)*f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^3}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(169) = 338.

Time = 0.48 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.98

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \frac{3\,c^2d^2|e|\arctan\left(\frac{\sqrt{(ex+d)cde - cd^2e + ae^3}g}{\sqrt{cdfg - aeg^2e}}\right)}{4\sqrt{cdfg - aeg^2}eg^2} - \frac{3\,c^2d^2e^2f^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 6\,c^2d^3efg|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) + 3\,c^2d^4g^2|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - \cdots}{4\left(\sqrt{cdfg - aeg^2}e^3f^2g^2 - 2\sqrt{c\cdots}\right.} - \frac{3\sqrt{(ex+d)cde - cd^2e + ae^3}c^3d^3e^2f|e| - 3\sqrt{(ex+d)cde - cd^2e + ae^3}ac^2d^2e^3g|e| + 5\left((ex+d)cde - cd^2e\cdots\right.}{4\left(cde^2f - ae^3g + ((ex+d)cde - cd^2e + ae^3)g\right)^2g^2}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] 3/4*c^2*d^2*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^2) - 1/4*(3*c^2*d^2*e^2*
f^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 6
*c^2*d^3*e*f*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e)) + 3*c^2*d^4*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*
g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e*f
*abs(e) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*g*abs(e) -
 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*e^2*g*abs(e))/(sqrt(c*d
*f*g - a*e*g^2)*e^3*f^2*g^2 - 2*sqrt(c*d*f*g - a*e*g^2)*d*e^2*f*g^3 + sqrt(
c*d*f*g - a*e*g^2)*d^2*e*g^4) - 1/4*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*c^3*d^3*e^2*f*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*
d^2*e^3*g*abs(e) + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g*ab
s(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)), x)
```

3.697 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 265

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = -\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)} -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^3+1/8*
c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(3/2)-1/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{876, 886, 888, 211\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^4),x]
```

```
[Out] -1/4*(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*(
f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{(cd)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx}{2g}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\\
&\quad+\frac{\left(c^2d^2\right)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g^2}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}\\
&\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{\left(c^3d^3\right)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16g^2(cdf-aeg)}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}\\
&\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\\
&\quad+\frac{\left(c^3d^3e^2\right)\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{8g^2(cdf-aeg)}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}\\
&\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.76

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(8a^2e^2g^2-2acdeg(\right.}{24g^{5/2}(cdf-aeg)^3}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(f - 7*g*x) + c^2*d^2*(-3*f^2 - 8*f*g*x +
 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqr
t[c*d*f - a*e*g]]))/(24*g^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d
 + e*x)]*(f + g*x)^3)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+9\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f^2g\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2
-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+8*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)*a^2*e^2*g^2+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e
*f*g+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)
/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. 2(233) = 466.

Time = 0.44 (sec) , antiderivative size = 1434, normalized size of antiderivative = 5.41

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^
3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d
^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3
 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f
^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^
4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^
8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2
*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*
e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g
^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e
^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*
x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^
2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*
g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*
g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) +
 (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3 + 8*a^3*e^3*
g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f^2*g^2 - 11*a
*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^4 + a^2*d*e^2
*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^8)*x^4 + (3*c
^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^6 - (2*a*c
*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*e^2*f*g^7 + (
c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^6)*x^2 + (c^
2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g^4 -
 (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(233)=466$.

Time = 0.69 (sec) , antiderivative size = 1087, normalized size of antiderivative = 4.10

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{8\left(cdfg^2-aeg^3\right)\sqrt{cdfg-aeg^2e}}$$
$$-\frac{3c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)-9c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)+9c^3d^5efg^2|e|\arctan\left(\frac{\sqrt{-cd^2e+a}}{\sqrt{cdfg-ae}}\right.}{24\left(\sqrt{cdfg}\right.}$$
$$-\frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}c^5d^5e^4f^2|e|-6\sqrt{(ex+d)cde-cd^2e+ae^3}ac^4d^4e^5fg|e|+3\sqrt{(ex+d)cde-}}{24\left(cde\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] 1/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/((c*d*f*g^2 - a*e*g^3)*sqrt(c*d*f*g - a*e*g^2)*e) - 1
/24*(3*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e)) - 9*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*
```

```
g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 9*c^3*d^5*e*f*g^2*abs(e)*arctan(sqrt(-c*d^
2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*c^3*d^6*g^3*abs(e)*arctan(s
qrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) + 8*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) + 3*sqrt(-c*d^2*e + a*e^3)*sq
rt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2*abs(e) - 14*sqrt(-c*d^2*e + a*e^3)*sqrt(c
*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*abs(e) + 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c*d*e^4*f^4*g
^2 - 3*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e^3*f^3*g^3 - sqrt(c*d*f*g - a*e*g^2)*
a*e^5*f^3*g^3 + 3*sqrt(c*d*f*g - a*e*g^2)*c*d^3*e^2*f^2*g^4 + 3*sqrt(c*d*f*
g - a*e*g^2)*a*d*e^4*f^2*g^4 - sqrt(c*d*f*g - a*e*g^2)*c*d^4*e*f*g^5 - 3*sq
rt(c*d*f*g - a*e*g^2)*a*d^2*e^3*f*g^5 + sqrt(c*d*f*g - a*e*g^2)*a*d^3*e^2*g
^6) - 1/24*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2*abs(e
) - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g*abs(e) + 3*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2*abs(e) + 8*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^2*f*g*abs(e) - 8*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^3*g^2*abs(e) - 3*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^3*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3*(c*d*f*g^2 - a*e*g^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)), x)
```

3.698 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 335

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3} + \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4} + \frac{3c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^4+3/64
*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(5/2)-1/8*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^3/(e*x+d)^(1/2)+1/32*c^2*d^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)
+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^2/
(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{876, 886, 888, 211\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\frac{3c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}+\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^5),x]
```

```
[Out] -1/8*(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*(
f + g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2
*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x
)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f
+ g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^
(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
```

```
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{(3cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx}{8g}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\\
&\quad+\frac{\left(c^2d^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16g^2}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{\left(3c^3d^3\right)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{64g^2(cdf-aeg)}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\\
&\quad+\frac{\left(3c^4d^4\right)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128g^2(cdf-aeg)^2}\\
&= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad+\frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\\
&\quad+\frac{\left(3c^4d^4e^2\right)\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{64g^2(cdf-aeg)^2}
\end{aligned}$$

$$= -\frac{cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{8g^2\sqrt{d + ex}(f + gx)^3} + \frac{c^2d^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{32g^2(cdf - aeg)\sqrt{d + ex}(f + gx)^2} + \frac{3c^3d^3\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{64g^2(cdf - aeg)^2\sqrt{d + ex}(f + gx)} - \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{4g(d + ex)^{3/2}(f + gx)^4} + \frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{5/2}(cdf - aeg)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^5} dx = \frac{c^4d^4((ae + cdx)(d + ex))^{3/2}\left(\frac{\sqrt{g}(-16a^3e^3g^3 + 24a^2cde^2g^2(f - gx) - 2ac^2d^2eg(f^2}{c^4d^4(cdf - aeg}\right.}{64g^{5/2}(d}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^5),x]
```

```
[Out] (c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(-16*a^3*e^3*g^3 + 24*a^
2*c*d*e^2*g^2*(f - g*x) - 2*a*c^2*d^2*e*g*(f^2 - 22*f*g*x + g^2*x^2) + c^3*
d^3*(-3*f^3 - 11*f^2*g*x + 11*f*g^2*x^2 + 3*g^3*x^3)))/(c^4*d^4*(c*d*f - a*
e*g)^2*(a*e + c*d*x)*(f + g*x)^4) + (3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/S
qrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*(a*e + c*d*x)^(3/2))))/(64*g^(5
/2)*(d + e*x)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(297) = 594.

Time = 0.57 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+12\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+18\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/64*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f*g^3*x^3+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^3*g*x-3*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d
```

```
*f)*g)^(1/2)+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*d^4
*f^4+2*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-11*c^3
*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+24*a^2*c*d*e^2*g^3
*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-44*a*c^2*d^2*e*f*g^2*x*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+11*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+16*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*e^3*g
^3-24*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2+2*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+3*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g
*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(297) = 594$.

Time = 0.80 (sec) , antiderivative size = 2238, normalized size of antiderivative = 6.68

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^
4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d
*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11
*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*
c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^
3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
)/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*
d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^
3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*
d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 +
 (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*
d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^
3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g
^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e
^4)*f^3*g^7)*x^2 + (c^3*d^3*e*f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^
```

```
2*c*d^2*e^2 - a^3*e^4)*f^4*g^6)*x), -1/64*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f
^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4
*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*
e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*
g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 -
3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^
3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3
*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d
^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3
*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 +
(4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^
7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)
*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a
*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*
f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^7)*x^2 + (c^3*d^3*e*
f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(
4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^6
)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^5}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. 2(297) = 594.

Time = 1.21 (sec) , antiderivative size = 1785, normalized size of antiderivative = 5.33

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] 3/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*s
qrt(c*d*f*g - a*e*g^2)*e) - 1/64*(3*c^4*d^4*e^4*f^4*abs(e)*arctan(sqrt(-c*d
^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 12*c^4*d^5*e^3*f^3*g*abs(e)*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 18*c^4*d^6*e
^2*f^2*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*
e)) - 12*c^4*d^7*e*f*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e)) + 3*c^4*d^8*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*c^3*d^3*e^3*f^3*abs(e) + 11*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^
2)*c^3*d^4*e^2*f^2*g*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*a*c^2*d^2*e^4*f^2*g*abs(e) + 11*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*c^3*d^5*e*f*g^2*abs(e) - 44*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*a*c^2*d^3*e^3*f*g^2*abs(e) + 24*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*a^2*c*d*e^5*f*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*
g - a*e*g^2)*c^3*d^6*g^3*abs(e) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*a*c^2*d^4*e^2*g^3*abs(e) + 24*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
 a*e*g^2)*a^2*c*d^2*e^4*g^3*abs(e) - 16*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*a^3*e^6*g^3*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^5*f^6*g^
2 - 4*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^4*f^5*g^3 - 2*sqrt(c*d*f*g - a*e*g^
2)*a*c*d*e^6*f^5*g^3 + 6*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e^3*f^4*g^4 + 8*sq
rt(c*d*f*g - a*e*g^2)*a*c*d^2*e^5*f^4*g^4 + sqrt(c*d*f*g - a*e*g^2)*a^2*e^7
*f^4*g^4 - 4*sqrt(c*d*f*g - a*e*g^2)*c^2*d^5*e^2*f^3*g^5 - 12*sqrt(c*d*f*g
- a*e*g^2)*a*c*d^3*e^4*f^3*g^5 - 4*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^6*f^3*g^
5 + sqrt(c*d*f*g - a*e*g^2)*c^2*d^6*e*f^2*g^6 + 8*sqrt(c*d*f*g - a*e*g^2)*a
*c*d^4*e^3*f^2*g^6 + 6*sqrt(c*d*f*g - a*e*g^2)*a^2*d^2*e^5*f^2*g^6 - 2*sqrt
(c*d*f*g - a*e*g^2)*a*c*d^5*e^2*f*g^7 - 4*sqrt(c*d*f*g - a*e*g^2)*a^2*d^3*e
^4*f*g^7 + sqrt(c*d*f*g - a*e*g^2)*a^2*d^4*e^3*g^8) - 1/64*(3*sqrt((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*c^7*d^7*e^6*f^3*abs(e) - 9*sqrt((e*x + d)*c*d*e
- c*d^2*e + a*e^3)*a*c^6*d^6*e^7*f^2*g*abs(e) + 9*sqrt((e*x + d)*c*d*e - c*
d^2*e + a*e^3)*a^2*c^5*d^5*e^8*f*g^2*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*a^3*c^4*d^4*e^9*g^3*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*c^6*d^6*e^4*f^2*g*abs(e) - 22*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*a*c^5*d^5*e^5*f*g^2*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3
```

```
)^(3/2)*a^2*c^4*d^4*e^6*g^3*abs(e) - 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(5/2)*c^5*d^5*e^2*f*g^2*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5
/2)*a*c^4*d^4*e^3*g^3*abs(e) - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*
c^4*d^4*g^3*abs(e))/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*(c*d
*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^5\,\left(d + e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)), x)
```

3.699 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 405

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^5+3/12
8*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(7/2)-3/40*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^4/(e*x+d)^(1/2)+1/80*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/
2)+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^
2/(g*x+f)^2/(e*x+d)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g^2/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = \frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}} + \frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}(f+gx)^3(cdf-aeg)} - \frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4} - \frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^6),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])]])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(3cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx}{10g} \\
&= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} \\
&\quad +\frac{(3c^2d^2)\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{80g^2} \\
&= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
&\quad -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(c^3d^3)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{32g^2(cdf-aeg)} \\
&= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
&\quad +\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} \\
&\quad +\frac{(3c^4d^4)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128g^2(cdf-aeg)^2}
\end{aligned}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} + \frac{(3c^5d^5)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{256g^2(cdf-aeg)^3}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} + \frac{(3c^5d^5e^2)\,\mathrm{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{128g^2(cdf-aeg)^3}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4} + \frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} + \frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.75

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}dx = \frac{c^5d^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}(128a^4e^4g^4+16a^3cde^3g^3(-21f+11gx)+8a^2c^2d\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^6),x]
```

```
[Out] (c^5*d^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(128*a^4*e^4*g^4 + 16*a^
3*c*d*e^3*g^3*(-21*f + 11*g*x) + 8*a^2*c^2*d^2*e^2*g^2*(31*f^2 - 64*f*g*x +
 g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 - 233*f^2*g*x + 23*f*g^2*x^2 + 5*g^3*x^3
) + c^4*d^4*(-15*f^4 - 70*f^3*g*x + 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15*g^4
*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^3*(a*e + c*d*x)*(f + g*x)^5) + (15*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*(a
*e + c*d*x)^(3/2))))/(640*g^(5/2)*(d + e*x)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(361) = 722$.

Time = 0.58 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^{5}d^{5}g^{5}x^{5}+75\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^{5}d^{5}f\,g^{4}x^{4}+150\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^{5}d}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*a
rctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5-8*a^2*c^2*d
^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f*g
^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-128*c^4*d^4*f^2*g^2*x^2*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-176*a^3*c*d*e^3*g^4*x*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)+512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g
-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)-128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^4+336*(c*d*x+a*e
)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3-248*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5/g^2/(a*e*g-
c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(361) = 722$.

Time = 2.28 (sec) , antiderivative size = 3204, normalized size of antiderivative = 7.91

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="fricas")
```

[Out] [1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)*x^5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e^2)*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^4 + 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 + 5*(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)*f^7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c^4*d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*g^4 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x), -1/640*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(

```
c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3
*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e
^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^
12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e
^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2
*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)
*x^5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e
^2)*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^
3*e^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^
4 + 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2
)*f^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^
3*e^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 +
 5*(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)
*f^7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3
*e^2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c
^4*d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*
g^4 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^6}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2659 vs. $2(361) = 722$.

Time = 2.80 (sec) , antiderivative size = 2659, normalized size of antiderivative = 6.57

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^6} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] 3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^2 - 3*a*c^2*d^2*e*f^2*g^3 + 3*a^2*c
*d*e^2*f*g^4 - a^3*e^3*g^5)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*c^5*d^5*
e^5*f^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
 - 75*c^5*d^6*e^4*f^4*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*
g - a*e*g^2)*e)) + 150*c^5*d^7*e^3*f^3*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*
e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 150*c^5*d^8*e^2*f^2*g^3*abs(e)*arctan
(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 75*c^5*d^9*e*f*g^4
*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c
^5*d^10*g^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)
*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^4*e^4*f^4*ab
s(e) + 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^5*e^3*f^3*g*
abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^3*e^5*f^
3*g*abs(e) + 128*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^6*e^2
*f^2*g^2*abs(e) - 466*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*
d^4*e^4*f^2*g^2*abs(e) + 248*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)
*a^2*c^2*d^2*e^6*f^2*g^2*abs(e) - 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
a*e*g^2)*c^4*d^7*e*f*g^3*abs(e) - 46*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g -
a*e*g^2)*a*c^3*d^5*e^3*f*g^3*abs(e) + 512*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f
*g - a*e*g^2)*a^2*c^2*d^3*e^5*f*g^3*abs(e) - 336*sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*f*g - a*e*g^2)*a^3*c*d*e^7*f*g^3*abs(e) + 15*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^4*d^8*g^4*abs(e) + 10*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a*c^3*d^6*e^2*g^4*abs(e) + 8*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*a^2*c^2*d^4*e^4*g^4*abs(e) - 176*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*f*g - a*e*g^2)*a^3*c*d^2*e^6*g^4*abs(e) + 128*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*a^4*e^8*g^4*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c^
3*d^3*e^6*f^8*g^2 - 5*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^5*f^7*g^3 - 3*sqrt(
c*d*f*g - a*e*g^2)*a*c^2*d^2*e^7*f^7*g^3 + 10*sqrt(c*d*f*g - a*e*g^2)*c^3*d
^5*e^4*f^6*g^4 + 15*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^6*f^6*g^4 + 3*sqrt(
c*d*f*g - a*e*g^2)*a^2*c*d*e^8*f^6*g^4 - 10*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6
*e^3*f^5*g^5 - 30*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^5*f^5*g^5 - 15*sqrt(c
*d*f*g - a*e*g^2)*a^2*c*d^2*e^7*f^5*g^5 - sqrt(c*d*f*g - a*e*g^2)*a^3*e^9*f
^5*g^5 + 5*sqrt(c*d*f*g - a*e*g^2)*c^3*d^7*e^2*f^4*g^6 + 30*sqrt(c*d*f*g -
a*e*g^2)*a*c^2*d^5*e^4*f^4*g^6 + 30*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^3*e^6*f
```

```
^4*g^6 + 5*sqrt(c*d*f*g - a*e*g^2)*a^3*d*e^8*f^4*g^6 - sqrt(c*d*f*g - a*e*g
^2)*c^3*d^8*e*f^3*g^7 - 15*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^6*e^3*f^3*g^7 -
30*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^4*e^5*f^3*g^7 - 10*sqrt(c*d*f*g - a*e*g^
2)*a^3*d^2*e^7*f^3*g^7 + 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^7*e^2*f^2*g^8 +
15*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^5*e^4*f^2*g^8 + 10*sqrt(c*d*f*g - a*e*g^
2)*a^3*d^3*e^6*f^2*g^8 - 3*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^6*e^3*f*g^9 - 5*
sqrt(c*d*f*g - a*e*g^2)*a^3*d^4*e^5*f*g^9 + sqrt(c*d*f*g - a*e*g^2)*a^3*d^5
*e^4*g^10) - 1/640*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*e^8*
f^4*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*f^3*g
*abs(e) + 90*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^7*d^7*e^10*f^2*g
^2*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^6*d^6*e^11*f*g
^3*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^4*c^5*d^5*e^12*g^4
*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*f^3*g*ab
s(e) - 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^7*d^7*e^7*f^2*g^2*
abs(e) + 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^6*d^6*e^8*f*g^
3*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*c^5*d^5*e^9*g^4
*abs(e) - 128*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^7*d^7*e^4*f^2*g^2
*abs(e) + 256*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^6*d^6*e^5*f*g^3
*abs(e) - 128*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c^5*d^5*e^6*g^4
*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^6*d^6*e^2*f*g^3*ab
s(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c^5*d^5*e^3*g^4*abs(e
) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*c^5*d^5*g^4*abs(e))/((c^3*
d^3*f^3*g^2 - 3*a*c^2*d^2*e*f^2*g^3 + 3*a^2*c*d*e^2*f*g^4 - a^3*e^3*g^5)*(c
*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)),x)

[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)), x)
```

3.700 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 336

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{128(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{45045c^5d^5e(d+ex)^{7/2}} + \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} + \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} + \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

```
[Out] -128/45045*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/e/(e*x+d)^(7/2)+128/6435*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(5/2)+32/715*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*
x+d)^(7/2)+16/195*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)/c^2/d^2/(e*x+d)^(7/2)+2/15*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx =$$
$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)}{45045c^5d^5e(d+ex)^{7/2}}$$
$$+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3}{6435c^4d^4e(d+ex)^{5/2}}$$
$$+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{715c^3d^3(d+ex)^{7/2}}$$
$$+\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}$$
$$+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} \\
&\quad+\frac{(8(cdf-aeg))\int\frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{15cd} \\
&= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} \\
&\quad+\frac{\left(16(cdf-aeg)^2\right)\int\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{65c^2d^2} \\
&= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} \\
&\quad+\frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} \\
&\quad+\frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} \\
&\quad+\frac{\left(64(cdf-aeg)^3\right)\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{715c^3d^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} \\
&+ \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} \\
&+ \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} \\
&+ \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} \\
&+ \frac{\left(64(cdf-aeg)^3\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx}{6435c^3d^3} \\
&= \frac{128(cdf-aeg)^3\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{45045c^4d^4(d+ex)^{7/2}} \\
&+ \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} \\
&+ \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} \\
&+ \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} \\
&+ \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.61

$$
\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(128a^4e^4g^4-64a^3cde
$$

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3*
c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x + 21
*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 231*g
^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860*f*g
^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{3}\left(3003g^{4}x^{4}c^{4}d^{4}-1848a\,c^{3}d^{3}e\,g^{4}x^{3}+13860c^{4}d^{4}f\,g^{3}x^{3}+1008a^{2}c^{2}d^{2}e^{2}g^{4}x^{2}-7560a\,c^{3}d^{3}e f\,g^{3}x^{2}+24570c\cdots\right.}{\cdots}$
gosper	$\frac{2(cdx+ae)\left(3003g^{4}x^{4}c^{4}d^{4}-1848a\,c^{3}d^{3}e\,g^{4}x^{3}+13860c^{4}d^{4}f\,g^{3}x^{3}+1008a^{2}c^{2}d^{2}e^{2}g^{4}x^{2}-7560a\,c^{3}d^{3}ef\,g^{3}x^{2}+24570c^{4}d^{4}f^{2}g^{2}x^{2}-448a^{3}c\cdots\right.}{\cdots}$

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/45045*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(3003*c^4*d
^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^4*d^4*f*g^3*x^3+1008*a^2*c^2*d^
2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+24570*c^4*d^4*f^2*g^2*x^2-448*a^3*
c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-10920*a*c^3*d^3*e*f^2*g^2*x+2002
0*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*d*e^3*f*g^3+3120*a^2*c^2*d^2*e^
2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^4*f^4)/c^5/d^5
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(3003\,c^7d^7g^4x^7+6435\,a^3c^4d^4e^3f^4-5720\,a^4c^3d^3e^4f^3g+\cdots\right.}{\cdots}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^3
*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a^7
*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7*d^
7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(572*c
^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 + a^3*c
^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g + 8814*
a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e^4*g^4)
*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*a^3*c^4*
d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g^4)*x^2 +
 (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a^4*c^3*d^3
*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}f^4}{7\,cd}$$
$$+ \frac{8\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}f^3g}{63\,c^2d^2}$$
$$+ \frac{4\,(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5)\sqrt{cdx+ae}f^2g^2}{231\,c^3d^3}$$
$$+ \frac{8\,(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6)\sqrt{cdx}}{3003\,c^4d^4}$$
$$+ \frac{2\,(3003\,c^7d^7x^7+7161\,ac^6d^6ex^6+4473\,a^2c^5d^5e^2x^5+35\,a^3c^4d^4e^3x^4-40\,a^4c^3d^3e^4x^3+48\,a^5c^2d^2e^5x^2-64\,a}{45045\,c^5d^5}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2)
+ 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f^2*g
^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*
d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5
*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*c^7*d^7*
x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4*d^4*e^3*
x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*d*e^6*x +
128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4287 vs. $2(306) = 612$.

Time = 0.48 (sec) , antiderivative size = 4287, normalized size of antiderivative = 12.76

$$\int \frac{(f+gx)^4 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/45045*(15015*a^2*f^4*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e
^3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e
) + 429*c^2*d^2*f^4*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
 a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*
d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 34
32*a*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
 + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e + 2574*a^2*
f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^
2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^
3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 572*c^2*d^2*f^3*g*
((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^
2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3
*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*
e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e
^2 - 1716*a*c*d*f^2*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2
*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sq
rt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^
4*d^4*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))
/(c^4*d^4*e^7))*abs(e)/e - 572*a^2*f*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^
8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c
^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*
e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2
)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e
```

*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e) + 78*c^2*d^2*f^2*g^2*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e^2 + 104*a*c*d*f*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e + 13*a^2*g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e) - 20*c^2*d^2*f*g^3*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)/(c^6*d^6*e^5) + (3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))/(c^6*d^6*e^11))*abs(e)/e^2 - 10*a*c*d*g^4*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)/(c^6*d^6*e^5) + (3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))/(c^6*d^6*e^11))*abs(e)/e + c^2*d^2*g^4*((3003*sqrt(-c*d^2*e + a*e^3)*c^7*d^14 - 231*sqrt(-c*d^2*e + a*e^3)*a*c^6*d^12*e^2 - 252*sqrt(-c*d^2*e + a*e^3)*a^2

```
*c^5*d^10*e^4 - 280*sqrt(-c*d^2*e + a*e^3)*a^3*c^4*d^8*e^6 - 320*sqrt(-c*d^
2*e + a*e^3)*a^4*c^3*d^6*e^8 - 384*sqrt(-c*d^2*e + a*e^3)*a^5*c^2*d^4*e^10
- 512*sqrt(-c*d^2*e + a*e^3)*a^6*c*d^2*e^12 - 1024*sqrt(-c*d^2*e + a*e^3)*a
^7*e^14)/(c^7*d^7*e^6) + (15015*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^6*e^18 - 54054*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^5*e^15 + 96525*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^4*e^12 - 100100*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(9/2)*a^3*e^9 + 61425*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(11/2)*a^2*e^6 - 20790*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(13/2)*a*e^3
 + 3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(15/2))/(c^7*d^7*e^13))*abs(e)/
e^2 - 6006*a*c*d*f^4*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a
*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2 - 12012*a^2*f^3*g*((3*sqrt(-c*d^2*e
 + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e +
a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*
e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e)
/e
```

Mupad [B] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.56

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^5\,(71\,a^2\,e^2\,g^2+540\,a\,c\,d\,e}{715}\right.}{}$$

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
+ 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*c^
4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6240
*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 80*a^
4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3*g +
88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)/15 +
(2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 + 572*c^3
*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c*d) + (2
*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*f^3*g + 4
80*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*d^4) + (2*
a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f^3*g - 120*
a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^3)))/(d + e*
x)^(1/2)
```

3.701 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 269

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^4d^4e(d+ex)^{7/2}} + \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} + \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

```
[Out] -16/3003*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(7/2)+16/429*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(5/2)+12/143*(-a*e*g
+c*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(
7/2)+2/13*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/
2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)}{3003c^4d^4e(d+ex)^{7/2}} + \frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}} + \frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} \\
&\quad+\frac{(6(cdf-aeg))\int\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{13cd} \\
&= \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} \\
&\quad+\frac{\left(24(cdf-aeg)^2\right)\int\frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{143c^2d^2} \\
&= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} \\
&\quad+\frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} \\
&\quad+\frac{\left(8(cdf-aeg)^2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{429c^2d^2} \\
&= \frac{16(cdf-aeg)^2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^3d^3(d+ex)^{7/2}} \\
&\quad+\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} \\
&\quad+\frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} \\
&\quad+\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.55

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cde^2g$$

[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/2),x]

[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2*x^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(3003*c^4*d^4*Sqrt[d + e*x])

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3(-231g^3x^3c^3d^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2gx}{3003\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-231g^3x^3c^3d^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2gx+16a^3e^3g^3-104a^2cd}{3003c^4d^4(ex+d)^{\frac{5}{2}}}$

[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/3003*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-231*c^3*d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-104*a^2*c*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*f^3)/c^4/d^4

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(231\,c^6d^6g^3x^6+429\,a^3c^3d^3e^3f^3-286\,a^4c^2d^2e^4f^2g+10$$

[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")

```
[Out] 2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^4
*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 + 9*
a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53*a^
2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 1469*a
^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e*f^3
+ 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^2*e^4*
g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 52*a^4*c
^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}\,f^3}{7\,cd}$$
$$+\frac{2\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}\,f^2g}{21\,c^2d^2}$$
$$+\frac{2\,(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5)\sqrt{cdx+ae}\,fg^2}{231\,c^3d^3}$$
$$+\frac{2\,(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6)\sqrt{cdx}}{3003\,c^4d^4}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2)
+ 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f*g^2
```

```
/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*d^
4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5*x
 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. 2(245) = 490.

Time = 0.41 (sec) , antiderivative size = 3058, normalized size of antiderivative = 11.37

$$\int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/45045*(15015*a^2*f^3*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e
^3)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e
) + 429*c^2*d^2*f^3*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
 a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*
d^2*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 25
74*a*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
 + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e + 1287*a^2*
f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*
d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)
*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2
*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 429*c^2*d^2*f^2*g*((
35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2
- 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c
*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^
3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2
 - 858*a*c*d*f*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e +
a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c
*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4
*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4
```

```
*d^4*e^7))*abs(e)/e - 143*a^2*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*s
qrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*
e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^
4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e
^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(9/2))/(c^4*d^4*e^7))*abs(e) + 39*c^2*d^2*f*g^2*((315*sqrt(-c*d^2*e + a
*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e
 + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*
sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)
/(c^5*d^5*e^4) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 -
 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^
5*e^9))*abs(e)/e^2 + 26*a*c*d*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 3
5*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*
d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*
e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (
1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3
+ 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e -
 5*c^2*d^2*g^3*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a
*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(
-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e
^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)
*a^6*e^12)/(c^6*d^6*e^5) + (3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*
a^5*e^15 - 9009*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(11/2)*a*e^3 - 693*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))/(c^6*d^6*e^
11))*abs(e)/e^2 - 6006*a*c*d*f^3*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(
-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2)
 + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e^2 - 9009*a^2*f^2*g*((3*sq
rt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(
-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2)
)*abs(e)/e)/e
```

Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.41

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a\,c\,d\,e\right.}{429}\right.}{}$$

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d^3
*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*d^4
) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e^2*f
*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/13 + (
6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3 + 128
7*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003*c^3*d^
3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g +
13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

3.702 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}} dx$

Optimal result

Integrand size = 46, antiderivative size = 200

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}} dx = -\frac{8(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^3d^3e(d+ex)^{7/2}} + \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

```
[Out] -8/693*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(7/2)+8/99*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/e/(e*x+d)^(5/2)+2/11*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

= {884, 808, 662}

$$\int \frac{(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx =$$
$$-\frac{8(x(ae^2+cd^2)+ade+cdex^2)^{7/2}\,(cdf-aeg)\,(2ae^2g-cd(9ef-7dg))}{693c^3d^3e(d+ex)^{7/2}}$$
$$+\frac{8g(x(ae^2+cd^2)+ade+cdex^2)^{7/2}\,(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}$$
$$+\frac{2(f+gx)^2\,(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{11cd(d+ex)^{7/2}}$$

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
```

```
gerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} \\
&+\frac{(4(cdf-aeg))\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{11cd} \\
&= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} \\
&+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} \\
&+\frac{\left(4(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx}{99cd} \\
&= \frac{8(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^2d^2(d+ex)^{7/2}} \\
&+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} \\
&+\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$
\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^2g^2-4acdeg(11\right.}{693c^3d^3\sqrt{d+e}}
$$

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e
*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*d^
3*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{3}\left(63g^{2}x^{2}c^{2}d^{2}-28acde\,g^{2}x+154c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-44acdefg+99c^{2}d^{2}f^{2}\right)}{693\sqrt{ex+d}\,c^{3}d^{3}}$	108
gosper	$\frac{2(cdx+ae)\left(63g^{2}x^{2}c^{2}d^{2}-28acde\,g^{2}x+154c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-44acdefg+99c^{2}d^{2}f^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{693c^{3}d^{3}(ex+d)^{\frac{5}{2}}}$	116

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/693*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(63*c^2*d^2*g
^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*a^2*e^2*g^2-44*a*c*d*e*f*g+99*c
^2*d^2*f^2)/c^3/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(63\,c^5d^5g^2x^5+99\,a^3c^2d^2e^3f^2-44\,a^4cde^4fg+8\,a^5e^5g^2\right.}{}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g + 8
*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4*e*
f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3
*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.22

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f^2}{7\,cd} + \frac{4\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}fg}{63\,c^2d^2} + \frac{2\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}g^2}{693\,c^3d^3}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) +
2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3*a
^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^2/(c^
3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(182) = 364.

Time = 0.38 (sec) , antiderivative size = 2010, normalized size of antiderivative = 10.05

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(1155*a^2*f^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3
)*a*e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e)
+ 33*c^2*d^2*f^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*
e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2
*e + a*e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 132*a
*c*d*f*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c
^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e
^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*
a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
```

)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e + 33*a^2*g^2*((15 *sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)/ (c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42* ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d ^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e) - 22*c^2*d^2*f*g*((35*sqrt(-c*d^ 2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d ^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16 *sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e - c* d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2 - 22*a*c*d*g ^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6 *e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)* a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e *x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d ^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2) *a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e + c^2*d^2*g^2*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqr t(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e ^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)/(c^5*d^5*e^4) + (1155*((e*x + d)* c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2* e^6 - 1540*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d) *c*d*e - c*d^2*e + a*e^3)^(11/2))/(c^5*d^5*e^9))*abs(e)/e^2 - 462*a*c*d*f^2 *((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d ^2*e^2))*abs(e)/e^2 - 462*a^2*f*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt (-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e)/e

Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f\,g+1}{693\,c^3\,d^3}\right.}{}$$

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*c
^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^2
+ 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2*d^
2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(297*
c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*x^2*(
a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x)^(1/2
)
```

3.703 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 125

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(2ae^2g-cd(9ef-7dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63c^2d^2e(d+ex)^{7/2}} + \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

```
[Out] -2/63*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)/c^2/d^2/e/(e*x+d)^(7/2)+2/9*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}} - \frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(2ae^2g-cd(9ef-7dg))}{63c^2d^2e(d+ex)^{7/2}}$$

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2g(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{9cde(d+ex)^{5/2}} \\ &\quad + \frac{1}{9}\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx \\ &= \frac{2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{63cd(d+ex)^{7/2}}+\frac{2g(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{9cde(d+ex)^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(9f+7gx}{63c^2d^2\sqrt{d+ex}}$$

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{3}(-7cdgx+2aeg-9cdf)}{63\sqrt{ex+d}\,c^{2}d^{2}}$	59
gosper	$-\frac{2(cdx+ae)(-7cdgx+2aeg-9cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63c^{2}d^{2}(ex+d)^{\frac{5}{2}}}$	67

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/63*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-7*c*d*g*x+2
*a*e*g-9*c*d*f)/c^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+\left(9\,c^4d^4f+19\,ac^3d^3eg\right.\right.}{}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*a
*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2
*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2
),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f}{7\,cd} + \frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}\,g}{63\,c^2d^2}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*d^
2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. 2(113) = 226.

Time = 0.32 (sec) , antiderivative size = 1147, normalized size of antiderivative = 9.18

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

```
[Out] 2/315*(105*a^2*f*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*
e^2)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e) + 3*
c^2*d^2*f*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*
c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*
e^3)*a^3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 + 6*a*c*d*g*((
15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2
- 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6
)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 4
2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e - c^2*d^2*g*((35*sqrt(-c*d^2
*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^
2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*
sqrt(-c*d^2*e + a*e^3)*a^4*e^8)/(c^4*d^4*e^3) + (105*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)
```

```
*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(9/2))/(c^4*d^4*e^7))*abs(e)/e^2 - 42*a*c*d*f*
((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2
*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^
2*e^2))*abs(e)/e^2 - 21*a^2*g*((3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*
d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) +
(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*abs(e)/e)/e
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,(5\,a\,e\,g+$$

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a*
e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*a^
3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f))/
(63*c*d)))/(d + e*x)^(1/2)
```

3.704 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral} = \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{2((ae + cdx)(d + ex))^{7/2}}{7cd(d + ex)^{7/2}}$$

[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]

[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3}{7\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	50

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/7*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3/c/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}} dx = \frac{2\,(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3)\sqrt{cdex^2 + ade + (cd^2 + }}{7\,(cdex + cd^2)}$$

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{5}{2}}}{(d + ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3)\sqrt{cdx + ae}}{7\,cd}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 477, normalized size of antiderivative = 9.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}}\,dx = \frac{2\left(35\,a^2\left(\frac{\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2}{cd} + \frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{cde}\right)|e| + \right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
[Out] 2/105*(35*a^2*((sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2
)/(c*d) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c*d*e))*abs(e) + c^2*d
^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4
*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^
3*e^6)/(c^3*d^3*e^2) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^
```

```
6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(7/2))/(c^3*d^3*e^5))*abs(e)/e^2 - 14*a*c*d*((3*sqrt(-
c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d
^2*e + a*e^3)*a^2*e^4)/(c^2*d^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))/(c^2*d^2*e^2))*ab
s(e)/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+\frac{6\,a\,c\,d\,e\,x^2}{7}\right)}{\sqrt{d+e\,x}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d^
2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

3.705 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 236

$$\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2(cdf-aeg)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{2(cdf-aeg)^{5/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/
2)+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-2*(-a*e*g+c*
d*f)^(5/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)+2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{878, 888, 211\}$

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\, dx = -\frac{2(cdf-aeg)^{5/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} + \frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{g^3\sqrt{d+ex}} - \frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{3g^2(d+ex)^{3/2}} + \frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)),x]
```

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx}{e^2g}$$

$$= -\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} + \frac{(cdf-aeg)^2\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{g^2}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cdf-aeg)^3\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{g^3}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(2e^2(cdf-aeg)^3\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\right)}{g^3}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(23a^2e^2g^2+acdeg(-35f+11g\ldots\right.}{15g^{7/2}\sqrt{(\ldots}}$$

[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x)),x]

[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(23*a^2*e^2*g^2 + a*c*d*e*g*(-35*f + 11*g*x) + c^2*d^2*(15*f^2 - 5*f*g*x + 3*g^2*x^2)) - 15*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(208) = 416.

Time = 0.58 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^3e^3g^3-45\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2cde^2fg^2+45\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)ac^2d\ldots\right.}{\ldots}$

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,method=_RETURNVERBOSE)

[Out] -2/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a^3*e^3*g^3-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-11*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-23*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+35*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 587, normalized size of antiderivative = 2.49

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)}\,dx = \left[\frac{15\,(c^2d^3f^2 - 2\,acd^2efg + a^2de^2g^2 + (c^2d^2ef^2 - 2\,acde^2fg + a^2e^3\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x
^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)
/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 -
 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*g^3), 2
/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2
*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x +
d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (
3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c
^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(e*g^3*x + d*g^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(208) = 416.

Time = 0.64 (sec) , antiderivative size = 853, normalized size of antiderivative = 3.61

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)} dx =$$
$$-\frac{2 (c^3d^3f^3|e| - 3 ac^2d^2ef^2g|e| + 3 a^2cde^2fg^2|e| - a^3e^3g^3|e|) \arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg - aeg^2}eg^3}$$
$$+ \frac{2 \left(15 c^3d^3e^3f^3|e| \arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 45 ac^2d^2e^4f^2g|e| \arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) + 45 a^2cde^5fg^2|e| \arctan\left(\right.\right.}{}$$
$$+ \frac{2 \left(15 \sqrt{(ex + d)cde - cd^2e + ae^3}c^2d^2e^{28}f^2g^2|e| - 30 \sqrt{(ex + d)cde - cd^2e + ae^3}acde^{29}fg^3|e| + 15 \sqrt{(ex +\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] -2*(c^3*d^3*f^3*abs(e) - 3*a*c^2*d^2*e*f^2*g*abs(e) + 3*a^2*c*d*e^2*f*g^2*a
bs(e) - a^3*e^3*g^3*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*
g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) + 2/15*(15*c
^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e)) - 45*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e)) + 45*a^2*c*d*e^5*f*g^2*abs(e)*arctan(sqrt(-c*d^2*
e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*a^3*e^6*g^3*abs(e)*arctan(sq
rt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) - 5*sqrt(-c*d^2*e + a*e
^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) + 35*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2*abs(e) + 11*sqrt(-c*d^2*e + a*e^3)*sqrt(
```

```
c*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*abs(e) - 23*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*g^3) +
2/15*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^28*f^2*g^2*abs(e
) - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^29*f*g^3*abs(e) + 15
*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*e^30*g^4*abs(e) - 5*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^26*f*g^3*abs(e) + 5*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(3/2)*a*e^27*g^4*abs(e) + 3*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(5/2)*e^24*g^4*abs(e))/(e^30*g^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)), x)
```

3.706 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 235

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

```
[Out] 5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)-(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)+5*c*d*(-a*e*g+c*d*f)
^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^(1/2)/(e*x+d)^(1/2))/g^(7/2)-5*c*d*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{876, 878, 888, 211\}$

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2}\,dx = \frac{5cd(cdf - aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} - \frac{5cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf - aeg)}{g^3\sqrt{d+ex}} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^2),x]
```

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{(5cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx}{2g}$$

$$= \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} - \frac{(5cd(cdf-aeg))\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{2g^2}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{(5cd(cdf-aeg)^2)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g^3}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{(5cde^2(cdf-aeg)^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g^3}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-3a^2e^2g^2 + 2acdeg(10f+7gx\right.}{3g^{7/2}\sqrt{}}$$

[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x)^2),x]

[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-3*a^2*e^2*g^2 + 2*a*c*d*e*g*(10*f + 7*g*x) + c^2*d^2*(-15*f^2 - 10*f*g*x + 2*g^2*x^2)) + 15*c*d*(c*d*f - a*e*g)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(3*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(209) = 418.

Time = 0.60 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2cde^2g^3x - 30\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)ac^2d^2efg^2x + 15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right.\right.}{}$

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x,method=_RETURNVERBOSE)

[Out] -1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-14*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-20*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.86

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2}\,dx = \left[-\frac{15\,(c^2d^3f^2 - acd^2efg + (c^2d^2efg - acde^2g^2)x^2 + (c^2d^2ef^2 - ac\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^
2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-(c
*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c
*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(2*c^
2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(5*c^2*
d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(c^2*d^3*
f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d^2*e*f^2
- a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arc
tan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)) - (2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*
e^2*g^2 - 2*(5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^2} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(209) = 418.

Time = 0.60 (sec) , antiderivative size = 1025, normalized size of antiderivative = 4.36

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2} dx =$$
$$- \frac{15 c^3 d^3 e^3 f^3 |e| \arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 15 c^3 d^4 e^2 f^2 g |e| \arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right) - 30 ac^2 d^2 e^4 f^2 g |e| \arctan\left(\frac{\sqrt{-}}{\sqrt{c}}\right.}{}$$
$$+ \frac{5 (c^3 d^3 f^2 |e| - 2 ac^2 d^2 efg |e| + a^2 cde^2 g^2 |e|) \arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{\sqrt{cdfg - aeg^2}eg^3}$$
$$- \frac{\sqrt{(ex + d)cde - cd^2e + ae^3}c^3 d^3 f^2 |e| - 2 \sqrt{(ex + d)cde - cd^2e + ae^3}ac^2 d^2 efg |e| + \sqrt{(ex + d)cde - cd^2e}}{(cde^2 f - ae^3 g + ((ex + d)cde - cd^2e + ae^3)g)g^3}$$
$$- \frac{2 \left(6 \sqrt{(ex + d)cde - cd^2e + ae^3}c^2 d^2 e^{10} fg^3 |e| - 6 \sqrt{(ex + d)cde - cd^2e + ae^3}acde^{11} g^4 |e| - ((ex + d)cde\right.}{3 e^{12} g^6}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="giac")
[Out] -1/3*(15*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e)) - 15*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^
3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 30*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan(sq
rt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 30*a*c^2*d^3*e^3*f*g^
2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*
a^2*c*d*e^5*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*
e*g^2)*e)) - 15*a^2*c*d^2*e^4*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*c^2*d^2*e^2*f^2*abs(e) + 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*c^2*d^3*e*f*g*abs(e) + 20*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2
```

```
)*a*c*d*e^3*f*g*abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c
^2*d^4*g^2*abs(e) - 14*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d
^2*e^2*g^2*abs(e) - 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^
4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f*g^3 - sqrt(c*d*f*g - a*e*g^2)*
d*e^3*g^4) + 5*(c^3*d^3*f^2*abs(e) - 2*a*c^2*d^2*e*f*g*abs(e) + a^2*c*d*e^2
*g^2*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - (sqrt((e*x + d)*c*d*e - c
*d^2*e + a*e^3)*c^3*d^3*f^2*abs(e) - 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*a*c^2*d^2*e*f*g*abs(e) + sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c*
d*e^2*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*g)*g^3) - 2/3*(6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^10*f*
g^3*abs(e) - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^11*g^4*abs(e
) - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^8*g^4*abs(e))/(e^12*g^6
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^2}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^2\,\left(d + e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)), x)
```

3.707 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 246

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}-\frac{15c^2d^2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{7/2}}$$

```
[Out] -5/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)-
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^2-15/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(7/2)+15/4*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{876, 878, 888, 211\}$

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = -\frac{15c^2d^2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}} + \frac{15c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^3),x]
```

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
```

```
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} + \frac{(5cd)\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx}{4g}$$

$$= -\frac{5cd\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} + \frac{(15c^2d^2)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{8g^2}$$

$$= \frac{15c^2d^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} - \frac{(15c^2d^2(cdf-aeg))\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g^3}$$

$$= \frac{15c^2d^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} - \frac{(15c^2d^2e^2(cdf-aeg))\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4g^3}$$

$$= \frac{15c^2d^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{\left(ade + (cd^2 + ae^2)x + cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2} - \frac{15c^2d^2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{7/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2a^2e^2g^2 - acdeg(5f+9gx) + \right.}{4g^{7/2}\sqrt{(ae}}$$

[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x)^3),x]

[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a^2*e^2*g^2 - a*c*d*e*g*(5*f + 9*g*x) + c^2*d^2*(15*f^2 + 25*f*g*x + 8*g^2*x^2)) - 15*c^2*d^2*Sqrt[c*d*f - a*e*g]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(214) = 428.

Time = 0.56 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^2d^2e\,g^3x^2-15\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3f\,g^2x^2+30\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\right.}{}$

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2+9*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-25*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \left[\frac{15\,(c^2d^2eg^2x^3 + c^2d^3f^2 + (2\,c^2d^2efg + c^2d^3g^2)x^2 + (c^2d^2ef^2 + 2\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^
2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x^3 + d*
f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4*(15*(c^
2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*
d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*
sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (8*c
^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d
^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 +
2*d*f*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^3} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^3} dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(214) = 428$.

Time = 0.58 (sec) , antiderivative size = 1038, normalized size of antiderivative = 4.22

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^3} dx = \frac{2\sqrt{(ex + d)cde - cd^2e + ae^3}c^2d^2|e|}{e^2g^3}$$
$$+ \frac{15c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 30c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - 15ac^2d^2e^4f^2g|e|\arctan\left(\frac{\sqrt{-cd}}{\sqrt{cdf}}\right.}{}$$
$$- \frac{15(c^3d^3f|e| - ac^2d^2eg|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\sqrt{cdfg - aeg^2}eg^3}$$
$$+ \frac{7\sqrt{(ex + d)cde - cd^2e + ae^3}c^4d^4e^2f^2|e| - 14\sqrt{(ex + d)cde - cd^2e + ae^3}ac^3d^3e^3fg|e| + 7\sqrt{(ex + d)cde -}{4(cde^2f - ae^3g + ((ex}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="giac")
[Out] 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*abs(e)/(e^2*g^3) + 1/4*(1
5*c^3*d^3*e^3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*
e*g^2)*e)) - 30*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 15*a*c^2*d^2*e^4*f^2*g*abs(e)*arctan(sqrt(-c*d
^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*c^3*d^5*e*f*g^2*abs(e)*ar
ctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 30*a*c^2*d^3*e
^3*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - 15*a*c^2*d^4*e^2*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d
^2*e^2*f^2*abs(e) + 25*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d
^3*e*f*g*abs(e) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^
3*f*g*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*g^2
*abs(e) - 9*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^2*g^2*
```

```
abs(e) + 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*e^4*g^2*abs(e
))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f^2*g^3 - 2*sqrt(c*d*f*g - a*e*g^2)*d*e^3*f
*g^4 + sqrt(c*d*f*g - a*e*g^2)*d^2*e^2*g^5) - 15/4*(c^3*d^3*f*abs(e) - a*c^
2*d^2*e*g*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*
d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) + 1/4*(7*sqrt((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^2*f^2*abs(e) - 14*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*a*c^3*d^3*e^3*f*g*abs(e) + 7*sqrt((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*a^2*c^2*d^2*e^4*g^2*abs(e) + 9*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*c^3*d^3*f*g*abs(e) - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2
)*a*c^2*d^2*e*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*g)^2*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)), x)
```

3.708 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4} dx$

Optimal result

Integrand size = 46, antiderivative size = 253

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4} dx = -\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)} - \frac{5cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3} + \frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}$$

```
[Out] -5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^3+5/
8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(1/2)-5/8*c^2*d^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used

$= \{876, 888, 211\}$

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}} - \frac{5c^2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)} - \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^4),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{(5cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx}{6g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{\left(5c^2d^2\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx}{8g^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{\left(5c^3d^3\right)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{16g^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3} + \frac{\left(5c^3d^3e^2\right)\text{Subst}\left(\int\frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\right)}{8g^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.68

$$\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+33g\right.\right.}{(f+gx)^3}\right.}{24g^{7/2}\sqrt{d+ex}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5*
f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (15
*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d
*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+45\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+45\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^3+33*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g
^2*x^2+26*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x+40*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+8*(c*d*x+a*e)^(1/2)*((a*e
*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)
*a*c*d*e*f*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(221) = 442$.

Time = 0.82 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.51

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \left[-\frac{15\,(c^3d^3eg^3x^4+c^3d^4f^3+(3\,c^3d^3efg^2+c^3d^4g^3)x^3+3\,(c^3d^3ef^2\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d
*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
 + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(20*c^3
*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e*f^3*g^5
+ (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 + (c*d^2 - 3
*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a*e^2)*f^2*g
```

```
^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f^3*g^5)*x),
 -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g
 + (c*d^2 + a*e^2)*g*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^
2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2
+ 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d
*e*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
 (c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f
^3*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^4}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(221) = 442$.

Time = 0.71 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.72

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \frac{5\,c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{8\,\sqrt{cdfg-aeg^2}eg^3}$$

$$-\frac{15\,c^3d^3e^3f^3|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-45\,c^3d^4e^2f^2g|e|\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+45\,c^3d^5efg^2|e|\arctan\left(\frac{\sqrt{-cd^2}}{\sqrt{cdfg}}\right.}{}$$

$$-\frac{15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^5d^5e^4f^2|e|-30\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^4d^4e^5fg|e|+15\,\sqrt{(ex+d)c}}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] 5/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - 1/24*(15*c^3*d^3*e^
3*f^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) -
 45*c^3*d^4*e^2*f^2*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
- a*e*g^2)*e)) + 45*c^3*d^5*e*f*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/
(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c^3*d^6*g^3*abs(e)*arctan(sqrt(-c*d^2*e +
 a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*c^2*d^2*e^2*f^2*abs(e) + 40*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*c^2*d^3*e*f*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f
*g - a*e*g^2)*a*c*d*e^3*f*g*abs(e) - 33*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*c^2*d^4*g^2*abs(e) + 26*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*a*c*d^2*e^2*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*
e*g^2)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*e^4*f^3*g^3 - 3*sqrt(c*
d*f*g - a*e*g^2)*d*e^3*f^2*g^4 + 3*sqrt(c*d*f*g - a*e*g^2)*d^2*e^2*f*g^5 -
sqrt(c*d*f*g - a*e*g^2)*d^3*e*g^6) - 1/24*(15*sqrt((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*c^5*d^5*e^4*f^2*abs(e) - 30*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*a*c^4*d^4*e^5*f*g*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a
^2*c^3*d^3*e^6*g^2*abs(e) + 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^
4*d^4*e^2*f*g*abs(e) - 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d
^3*e^3*g^2*abs(e) + 33*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^3*g^
2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3*
g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)), x)
```

3.709 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 323

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4} + \frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}$$

```
[Out] -5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^3-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^4+5/
64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(3/2)-5/32*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^2/(e*x+d)^(1/2)+5/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^
(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{876, 886, 888, 211\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}} + \frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)(cdf-aeg)} - \frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2} - \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^5),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{(5cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx}{8g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{\left(5c^2d^2\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx}{16g^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{\left(5c^3d^3\right)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{64g^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{\left(5c^4d^4\right)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{128g^3(cdf-aeg)}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{\left(5c^4d^4e^2\right)\text{Subst}\left(\int\frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\right)}{64g^3(cdf-aeg)}$$

$$= -\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{5cd(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4} + \frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \frac{c^4d^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(48a^3e^3g^3-8a^2cde^2g^2(f-17gx)+2ac^2d^2eg(-5\ldots}{c^4d^4(cdf-\ldots}\right.}{192g^{7/2}\ldots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^5),x]
```

```
[Out] (c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(48*a^3*e^3*g^3 - 8*a^2*
c*d*e^2*g^2*(f - 17*g*x) + 2*a*c^2*d^2*e*g*(-5*f^2 - 18*f*g*x + 59*g^2*x^2)
 - c^3*d^3*(15*f^3 + 55*f^2*g*x + 73*f*g^2*x^2 - 15*g^3*x^3)))/(c^4*d^4*(c*
d*f - a*e*g)*(a*e + c*d*x)^2*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e +
c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(3/2)*(a*e + c*d*x)^(5/2))))
/(192*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(285) = 570.

Time = 0.55 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+60\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+90\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4f^2\ldots\right.}{\ldots}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
```

```
*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*
d^4*f^4-118*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+7
3*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-136*a^2*c*d*e
^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g^2*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+55*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3
*e^3*g^3+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2+10*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(
a*e*g-c*d*f)/g^3/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(285) = 570$.

Time = 0.55 (sec) , antiderivative size = 1862, normalized size of antiderivative = 5.76

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 56*a^3*c
*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 +
(73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*g^5)*x^2
+ (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*e^2*f*g^4
+ 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^4*g^6 + (c^2
*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^2*d^2*e*f^3*g
^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*c*d^2*e - 2*a^
2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g^9 + 2*(c^2*d^3
 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8)*x^3 + 2*(2*c^2
*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^6 -
2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g^4 + 4*a^2*d*e^2*f
^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e - a^2*e^3)*f^4*g^
6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c
^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*
d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)
```

```
*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*
e^2)*g*x)) + (15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*
f^2*g^3 - 56*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3
*d^3*e*g^5)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2
*d^2*e^2*g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*
c^2*d^2*e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*
e^2*f^4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 +
(4*c^2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2
*(a*c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*
f*g^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g
^8)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c
*d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e
 - a^2*e^3)*f^4*g^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^5}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. 2(285) = 570.

Time = 1.75 (sec) , antiderivative size = 1574, normalized size of antiderivative = 4.87

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="giac")
[Out] 5/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c*d*f*g^3 - a*e*g^4)*sqrt(c*d*f*g - a*e*g^2)*e) -
1/192*(15*c^4*d^4*e^4*f^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e)) - 60*c^4*d^5*e^3*f^3*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e
^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 90*c^4*d^6*e^2*f^2*g^2*abs(e)*arctan(s
qrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 60*c^4*d^7*e*f*g^3*a
bs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*c^4
*d^8*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^3*f^3*abs(e
) + 55*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^2*f^2*g*abs
(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^4*f^2*g
*abs(e) - 73*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e*f*g^2
*abs(e) + 36*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^3*f
*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^5*
f*g^2*abs(e) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*g^
3*abs(e) + 118*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^2
*g^3*abs(e) - 136*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*
e^4*g^3*abs(e) + 48*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*e^6*
g^3*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c*d*e^5*f^5*g^3 - 4*sqrt(c*d*f*g - a*e
*g^2)*c*d^2*e^4*f^4*g^4 - sqrt(c*d*f*g - a*e*g^2)*a*e^6*f^4*g^4 + 6*sqrt(c*
d*f*g - a*e*g^2)*c*d^3*e^3*f^3*g^5 + 4*sqrt(c*d*f*g - a*e*g^2)*a*d*e^5*f^3*
g^5 - 4*sqrt(c*d*f*g - a*e*g^2)*c*d^4*e^2*f^2*g^6 - 6*sqrt(c*d*f*g - a*e*g^
2)*a*d^2*e^4*f^2*g^6 + sqrt(c*d*f*g - a*e*g^2)*c*d^5*e*f*g^7 + 4*sqrt(c*d*f
*g - a*e*g^2)*a*d^3*e^3*f*g^7 - sqrt(c*d*f*g - a*e*g^2)*a*d^4*e^2*g^8) - 1/
192*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^7*d^7*e^6*f^3*abs(e) - 45
*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^6*d^6*e^7*f^2*g*abs(e) + 45*sq
rt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^5*d^5*e^8*f*g^2*abs(e) - 15*sqr
t((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^4*d^4*e^9*g^3*abs(e) + 55*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*e^4*f^2*g*abs(e) - 110*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^5*d^5*e^5*f*g^2*abs(e) + 55*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^4*d^4*e^6*g^3*abs(e) + 73*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^5*d^5*e^2*f*g^2*abs(e) - 73*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(5/2)*a*c^4*d^4*e^3*g^3*abs(e) - 15*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(7/2)*c^4*d^4*g^3*abs(e))/((c*d*f*g^3 - a*e*g^4)*(c*d*e
^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(f + g\,x)^5\,(d + e\,x)^{5/2}} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)), x)
```

3.710 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 393

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=-\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}$$

```
[Out] -1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^5+3/1
28*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(5/2)-1/16*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^3/(e*x+d)^(1/2)+1/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d
)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c
*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {876, 886, 888, 211}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}} + \frac{3c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^2} + \frac{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3} - \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^6),x]
```

```
[Out] -1/16*(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e
*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6
4*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
 + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d +
e*x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*
g*(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^
(7/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{(cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx}{2g}\\
&= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}\\
&\quad -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{\left(3c^2d^2\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx}{16g^2}\\
&= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}\\
&\quad -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{\left(c^3d^3\right)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{32g^3}\\
&= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\
&\quad -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}\\
&\quad +\frac{\left(3c^4d^4\right)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{128g^3(cdf-aeg)}
\end{aligned}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5} + \frac{(3c^5d^5)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{256g^3(cdf-aeg)^2}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5} + \frac{(3c^5d^5e^2)\operatorname{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{128g^3(cdf-aeg)^2}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5} + \frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.77

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}dx = \frac{c^5d^5((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(-128a^4e^4g^4+16a^3cde^3g^3(11f-21gx)-8a^2c^2d\right.}{}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^6),x]
```

```
[Out] (c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-128*a^4*e^4*g^4 + 16*a
^3*c*d*e^3*g^3*(11*f - 21*g*x) - 8*a^2*c^2*d^2*e^2*g^2*(f^2 - 64*f*g*x + 31
*g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 + 23*f^2*g*x - 233*f*g^2*x^2 + 5*g^3*x^3
) + c^4*d^4*(-15*f^4 - 70*f^3*g*x - 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15*g^4
*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^2*(a*e + c*d*x)^2*(f + g*x)^5) + (15*ArcTa
n[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*
(a*e + c*d*x)^(5/2))))/(640*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(349) = 698$.

Time = 0.57 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5g^5x^5+75\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5fg^4x^4+150\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5+248*a^2*c^
2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e
*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+128*c^4*d^4*f^2*g^2*x^
2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+336*a^3*c*d*e^3*g^4*x*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*x+a*e)^(1
/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^4-176*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3+8*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5/g^3/(a*e*g
-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(349) = 698$.

Time = 1.37 (sec) , antiderivative size = 2750, normalized size of antiderivative = 7.00

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^
6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^
3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2
+ (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f
^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*
c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
- 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 +
2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5
 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^
3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e
^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*
f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d
^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 +
 (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d
*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^3*g^9 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^
3*e*f^6*g^6 - a^3*d*e^3*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*
(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*
x^3 + 5*(c^3*d^3*e*f^7*g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2
*e^2)*f^6*g^6 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2
- a^3*e^4)*f^4*g^8)*x^2 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3
*d^4 - 3*a*c^2*d^2*e^2)*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 +
 (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^7)*x), -1/640*(15*(c^5*d^5*e*g^5*x^6 +
c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^
3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(
c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g
)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
+ a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*
e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*c*d*e^4*f*g^5 - 128*a^5
*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4
 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 -
 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*
g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3
*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c
^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e
*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*
x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*
f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a
```

```
^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4
- 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^9 + (3*a
^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^3*e*f^6*g^6 - a^3*d*e^3
*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*(a*c^2*d^3*e - a^2*c*d*
e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*x^3 + 5*(c^3*d^3*e*f^7*
g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^6 - 3*(2*a*
c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^8)*x^2
 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 + (15*a^2*c*d^2*e^2 - a^
3*e^4)*f^5*g^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^6}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2407 vs. 2(349) = 698.

Time = 2.83 (sec) , antiderivative size = 2407, normalized size of antiderivative = 6.12

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="giac")
```

[Out] 3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2*e^2*g^5)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*c^5*d^5*e^5*f^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 75*c^5*d^6*e^4*f^4*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 150*c^5*d^7*e^3*f^3*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 150*c^5*d^8*e^2*f^2*g^3*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 75*c^5*d^9*e*f*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*c^5*d^10*g^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^4*e^4*f^4*abs(e) + 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^5*e^3*f^3*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^3*e^5*f^3*g*abs(e) - 128*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^6*e^2*f^2*g^2*abs(e) + 46*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^4*e^4*f^2*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^2*e^6*f^2*g^2*abs(e) - 70*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^7*e*f*g^3*abs(e) + 466*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^5*e^3*f*g^3*abs(e) - 512*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^3*e^5*f*g^3*abs(e) + 176*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c*d*e^7*f*g^3*abs(e) + 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^4*d^8*g^4*abs(e) + 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^3*d^6*e^2*g^4*abs(e) - 248*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^2*d^4*e^4*g^4*abs(e) + 336*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c*d^2*e^6*g^4*abs(e) - 128*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^4*e^8*g^4*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^6*f^7*g^3 - 5*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^5*f^6*g^4 - 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d*e^7*f^6*g^4 + 10*sqrt(c*d*f*g - a*e*g^2)*c^2*d^4*e^4*f^5*g^5 + 10*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e^6*f^5*g^5 + sqrt(c*d*f*g - a*e*g^2)*a^2*e^8*f^5*g^5 - 10*sqrt(c*d*f*g - a*e*g^2)*c^2*d^5*e^3*f^4*g^6 - 20*sqrt(c*d*f*g - a*e*g^2)*a*c*d^3*e^5*f^4*g^6 - 5*sqrt(c*d*f*g - a*e*g^2)*a^2*d*e^7*f^4*g^6 + 5*sqrt(c*d*f*g - a*e*g^2)*c^2*d^6*e^2*f^3*g^7 + 20*sqrt(c*d*f*g - a*e*g^2)*a*c*d^4*e^4*f^3*g^7 + 10*sqrt(c*d*f*g - a*e*g^2)*a^2*d^2*e^6*f^3*g^7 - sqrt(c*d*f*g - a*e*g^2)*c^2*d^7*e*f^2*g^8 - 10*sqrt(c*d*f*g - a*e*g^2)*a*c*d^5*e^3*f^2*g^8 - 10*sqrt(c*d*f*g - a*e*g^2)*a^2*d^3*e^5*f^2*g^8 + 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d^6*e^2*f*g^9 + 5*sqrt(c*d*f*g - a*e*g^2)*a^2*d^4*e^4*f*g^9 - sqrt(c*d*f*g - a*e*g^2)*a^2*d^5*e^3*g^10) - 1/640*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*e^8*f^4*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*f^3*g*abs(e) + 90*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^7*d^7*e^10*f^2*g^2*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^6*d^6*e^11*f*g^3*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^4*c^5*d^5*e^12*g^4*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*f^3*g*abs(e) - 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^7*d^7*e^7*f^2*g^2*abs(e) + 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^6*d^6*e^8*f*g^3*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a

```
*e^3)^(3/2)*a^3*c^5*d^5*e^9*g^4*abs(e) + 128*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*c^7*d^7*e^4*f^2*g^2*abs(e) - 256*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*a*c^6*d^6*e^5*f*g^3*abs(e) + 128*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*a^2*c^5*d^5*e^6*g^4*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(7/2)*c^6*d^6*e^2*f*g^3*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(7/2)*a*c^5*d^5*e^3*g^4*abs(e) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
9/2)*c^5*d^5*g^4*abs(e))/((c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2*e^2*g^5)
*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^6\,\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)), x)
```

3.711 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 463

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx=-\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{5c^5d^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{5c^6d^6\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}$$

```
[Out] -1/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^5-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^6+5/
512*c^6*d^6*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(7/2)-1/32*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^4/(e*x+d)^(1/2)+1/192*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x
+d)^(1/2)+5/768*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g
+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)+5/512*c^5*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {876, 886, 888, 211}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = \frac{5c^6d^6\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}} + \frac{5c^5d^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{5c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{768g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192g^3\sqrt{d+ex}(f+gx)^3(cdf-aeg)} - \frac{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} - \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^7),x]
```

```
[Out] -1/32*(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e
*x]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1
92*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*
g^3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*
c^6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[
c*d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{(5cd)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx}{12g}\\
&= -\frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}\\
&\quad -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{(c^2d^2)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx}{8g^2}\\
&= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}-\frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}\\
&\quad -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{(c^3d^3)\int\frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{64g^3}\\
&= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}\\
&\quad -\frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}\\
&\quad +\frac{(5c^4d^4)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{384g^3(cdf-aeg)}
\end{aligned}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{(5c^5d^5)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{512g^3(cdf-aeg)^2}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^5d^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{(5c^6d^6)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{1024g^3(cdf-aeg)^3}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^5d^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{(5c^6d^6e^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{512g^3(cdf-aeg)^3}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \frac{5c^5d^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{5c^6d^6\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.80

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = \frac{c^6d^6((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(256a^5e^5g^5+640a^4cde^4g^4(-f+gx)+16a^3c^2d^2e^3\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^7),x]
```

```
[Out] (c^6*d^6*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(256*a^5*e^5*g^5 + 640*a
^4*c*d*e^4*g^4*(-f + g*x) + 16*a^3*c^2*d^2*e^3*g^3*(27*f^2 - 106*f*g*x + 27
*g^2*x^2) + 8*a^2*c^3*d^3*e^2*g^2*(-f^3 + 159*f^2*g*x - 159*f*g^2*x^2 + g^3
*x^3) - 2*a*c^4*d^4*e*g*(5*f^4 + 28*f^3*g*x - 594*f^2*g^2*x^2 + 28*f*g^3*x^
3 + 5*g^4*x^4) + c^5*d^5*(-15*f^5 - 85*f^4*g*x - 198*f^3*g^2*x^2 + 198*f^2*
g^3*x^3 + 85*f*g^4*x^4 + 15*g^5*x^5)))/(c^6*d^6*(c*d*f - a*e*g)^3*(a*e + c*
d*x)^2*(f + g*x)^6) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]])/((c*d*f - a*e*g)^(7/2)*(a*e + c*d*x)^(5/2))))/(1536*g^(7/2)*(d + e*
x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. 2(413) = 826.

Time = 0.58 (sec) , antiderivative size = 1251, normalized size of antiderivative = 2.70

method	result	size
default	Expression too large to display	1251

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/1536*((c*d*x+a*e)*(e*x+d))^(1/2)*(-1188*a*c^4*d^4*e*f^2*g^3*x^2*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*c^5*d^5*g^5*x^5*(c*d*x+a*e)^(1/2)*((a*e
*g-c*d*f)*g)^(1/2)+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*
c^6*d^6*f^5*g*x+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^
6*d^6*f^4*g^2*x^2+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*
c^6*d^6*f^2*g^4*x^4+300*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^6*d^6*f^3*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*c^6*d^6*f*g^5*x^5+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^6*d^6*f^6-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^5*e^5*g^5+15*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5+15*arctanh(g*(c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^6*d^6*g^6*x^6-432*a^3*c^2*d^2*e^3*g^5*
x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*a^4*c*d*e^4*g^5*x*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+10*a*c^4*d^4*e*g^5*x^4*(c*d*x+a*e)^(1/2)
```

```
*((a*e*g-c*d*f)*g)^(1/2)+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*
c*d*e^4*f*g^4-432*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c^2*d^2*e^3
*f^2*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^
2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^4*d^4*e*f^4*g-85*c^5*d^5
*f*g^4*x^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-198*c^5*d^5*f^2*g^3*x^
3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+198*c^5*d^5*f^3*g^2*x^2*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*c^5*d^5*f^4*g*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)-8*a^2*c^3*d^3*e^2*g^5*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2)+1272*a^2*c^3*d^3*e^2*f*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)+56*a*c^4*d^4*e*f*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)-1272*a^2*c^3*d^3*e^2*f^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2
)+56*a*c^4*d^4*e*f^3*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*a
^3*c^2*d^2*e^3*f*g^4*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^6/g^3/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c
*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1915 vs. $2(413) = 826$.

Time = 3.97 (sec) , antiderivative size = 3872, normalized size of antiderivative = 8.36

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="fricas")
```

```
[Out] [1/3072*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7
*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*
f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g
^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 +
6*c^6*d^7*f^5*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f +
2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) - 2*(15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*
d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^
5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5
*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*
e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4
*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a
*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^
6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*
g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^
4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
```

^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 - 4*a^3*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^4*g^14 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4*f*g^13 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^4*d*e^4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^10 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6 + 4*a^4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9*g^5 + 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f^10*g^4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x), -1/1536*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 + 6*c^6*d^7*f^5*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 - 4*a^3*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^4*g^14 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4*f*g^1

```
3 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*
d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 - (8*a^3
*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^4*d*e^
4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e - 2*a
^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^10 -
4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6 + 4*a^
4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3*d^4*e
 - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^
9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9*g^5 +
 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*a*c^3*
d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)
*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f^10*g^
4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6*(4*a*
c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**7,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^7}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3412 vs. 2(413) = 826.

Time = 7.62 (sec) , antiderivative size = 3412, normalized size of antiderivative = 7.37

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="giac")
```

```
[Out] 5/512*c^6*d^6*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3*a^2*c
*d*e^2*f*g^5 - a^3*e^3*g^6)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/1536*(15*c^6*d^6
*e^6*f^6*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)
) - 90*c^6*d^7*e^5*f^5*g*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f
*g - a*e*g^2)*e)) + 225*c^6*d^8*e^4*f^4*g^2*abs(e)*arctan(sqrt(-c*d^2*e + a
*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 300*c^6*d^9*e^3*f^3*g^3*abs(e)*arcta
n(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 225*c^6*d^10*e^2*
f^2*g^4*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))
 - 90*c^6*d^11*e*f*g^5*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e)) + 15*c^6*d^12*g^6*abs(e)*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*c^5*d^5*e^5*f^5*abs(e) + 85*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*c^5*d^6*e^4*f^4*g*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e
*g^2)*a*c^4*d^4*e^6*f^4*g*abs(e) - 198*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
- a*e*g^2)*c^5*d^7*e^3*f^3*g^2*abs(e) + 56*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*
f*g - a*e*g^2)*a*c^4*d^5*e^5*f^3*g^2*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt
(c*d*f*g - a*e*g^2)*a^2*c^3*d^3*e^7*f^3*g^2*abs(e) - 198*sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*c^5*d^8*e^2*f^2*g^3*abs(e) + 1188*sqrt(-c*d^2*
e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^4*d^6*e^4*f^2*g^3*abs(e) - 1272*sqrt
(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^3*d^4*e^6*f^2*g^3*abs(e) +
 432*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^3*c^2*d^2*e^8*f^2*g^3
*abs(e) + 85*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^5*d^9*e*f*g^4
*abs(e) + 56*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a*c^4*d^7*e^3*f
*g^4*abs(e) - 1272*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*c^3*d
^5*e^5*f*g^4*abs(e) + 1696*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a
^3*c^2*d^3*e^7*f*g^4*abs(e) - 640*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e
*g^2)*a^4*c*d*e^9*f*g^4*abs(e) - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a
*e*g^2)*c^5*d^10*g^5*abs(e) - 10*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*
g^2)*a*c^4*d^8*e^2*g^5*abs(e) - 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e
*g^2)*a^2*c^3*d^6*e^4*g^5*abs(e) + 432*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
- a*e*g^2)*a^3*c^2*d^4*e^6*g^5*abs(e) - 640*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*f*g - a*e*g^2)*a^4*c*d^2*e^8*g^5*abs(e) + 256*sqrt(-c*d^2*e + a*e^3)*sqrt(
c*d*f*g - a*e*g^2)*a^5*e^10*g^5*abs(e))/(sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^
```

7*f^9*g^3 - 6*sqrt(c*d*f*g - a*e*g^2)*c^3*d^4*e^6*f^8*g^4 - 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*e^8*f^8*g^4 + 15*sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*e^5*f^7*g^5 + 18*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^7*f^7*g^5 + 3*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d*e^9*f^7*g^5 - 20*sqrt(c*d*f*g - a*e*g^2)*c^3*d^6*e^4*f^6*g^6 - 45*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e^6*f^6*g^6 - 18*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^8*f^6*g^6 - sqrt(c*d*f*g - a*e*g^2)*a^3*e^10*f^6*g^6 + 15*sqrt(c*d*f*g - a*e*g^2)*c^3*d^7*e^3*f^5*g^7 + 60*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^5*e^5*f^5*g^7 + 45*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^3*e^7*f^5*g^7 + 6*sqrt(c*d*f*g - a*e*g^2)*a^3*d*e^9*f^5*g^7 - 6*sqrt(c*d*f*g - a*e*g^2)*c^3*d^8*e^2*f^4*g^8 - 45*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^6*e^4*f^4*g^8 - 60*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^4*e^6*f^4*g^8 - 15*sqrt(c*d*f*g - a*e*g^2)*a^3*d^2*e^8*f^4*g^8 + sqrt(c*d*f*g - a*e*g^2)*c^3*d^9*e*f^3*g^9 + 18*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^7*e^3*f^3*g^9 + 45*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^5*e^5*f^3*g^9 + 20*sqrt(c*d*f*g - a*e*g^2)*a^3*d^3*e^7*f^3*g^9 - 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^8*e^2*f^2*g^10 - 18*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^6*e^4*f^2*g^10 - 15*sqrt(c*d*f*g - a*e*g^2)*a^3*d^4*e^6*f^2*g^10 + 3*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^7*e^3*f*g^11 + 6*sqrt(c*d*f*g - a*e*g^2)*a^3*d^5*e^5*f*g^11 - sqrt(c*d*f*g - a*e*g^2)*a^3*d^6*e^4*g^12) - 1/1536*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^11*d^11*e^10*f^5*abs(e) - 75*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^10*d^10*e^11*f^4*g*abs(e) + 150*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^9*d^9*e^12*f^3*g^2*abs(e) - 150*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^8*d^8*e^13*f^2*g^3*abs(e) + 75*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^4*c^7*d^7*e^14*f*g^4*abs(e) - 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^5*c^6*d^6*e^15*g^5*abs(e) + 85*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^10*d^10*e^8*f^4*g*abs(e) - 340*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^9*d^9*e^9*f^3*g^2*abs(e) + 510*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^8*d^8*e^10*f^2*g^3*abs(e) - 340*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*c^7*d^7*e^11*f*g^4*abs(e) + 85*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*c^6*d^6*e^12*g^5*abs(e) + 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^9*d^9*e^6*f^3*g^2*abs(e) - 594*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^8*d^8*e^7*f^2*g^3*abs(e) + 594*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c^7*d^7*e^8*f*g^4*abs(e) - 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*c^6*d^6*e^9*g^5*abs(e) - 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^8*d^8*e^4*f^2*g^3*abs(e) + 396*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c^7*d^7*e^5*f*g^4*abs(e) - 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*c^6*d^6*e^6*g^5*abs(e) - 85*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*c^7*d^7*e^2*f*g^4*abs(e) + 85*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*c^6*d^6*e^3*g^5*abs(e) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*c^6*d^6*g^5*abs(e))/((c^3*d^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3*a^2*c*d*e^2*f*g^5 - a^3*e^3*g^6)*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^6)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^7\,\left(d + e\,x\right)^{5/2}} dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)), x)
```

3.712 $\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 313

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] 5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/12*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+5/8*(-a*e*g+c*d*f)^2*
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {884, 905, 65, 223, 212}

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$
$$+\frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}$$
$$+\frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}} \\
&\quad + \frac{\left(5(cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\right)\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{6cde^2} \\
&= \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2\sqrt{d+ex}} \\
&\quad + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}} \\
&\quad + \frac{\left(5(cdf-aeg)^2\right)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8c^2d^2} \\
&= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} \\
&\quad + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2\sqrt{d+ex}} \\
&\quad + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}} \\
&\quad + \frac{\left(5(cdf-aeg)^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16c^3d^3}
\end{aligned}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{16c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{8c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{8c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(15a^2e^2g^2-10acdeg(4f+gx)+\cdots\right.}{24c^{7/2}d^{7/2}\sqrt{(\cdots}}$$

[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]

[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*a^2*e^2*g^2 - 10*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)) + (15*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[g]))/(24*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{c\cdots}}{2\sqrt{cdg}}\right)\cdots\right.}{\cdots}$

[In] int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)+20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+80*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-66*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \Bigg[\frac{4\,(8\,c^3d^3g^3x^2+33\,c^3d^3f^2g-40\,ac^2d^2efg^2+15\,a^2cde^2g^3+2\,(13\,c^3d^3}{}$$

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*
a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g*x + c^4*d^5*g),
1/48*(2*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*a
^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^3
 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e
*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-
c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sq
rt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g*x + c^4*d^5*g)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(265) = 530.

Time = 0.64 (sec) , antiderivative size = 989, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{e\left(\frac{\left(\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\left(2\,(e^2f+(ex+d)e\right.\right.}{}\right.}{}$$

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] 1/24*e*((sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)
*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*abs(e)/(c*d*e^3*g) + 5*(c^4*d^4*e^2*f*a
bs(e) - a*c^3*d^3*e^3*g*abs(e))/(c^5*d^5*e^3*g)) + 15*(c^4*d^4*e^4*f^2*abs(
e) - 2*a*c^3*d^3*e^5*f*g*abs(e) + a^2*c^2*d^2*e^6*g^2*abs(e))/(c^5*d^5*e^3*
g)) - 15*(c^3*d^3*e^3*f^3*abs(e) - 3*a*c^2*d^2*e^4*f^2*g*abs(e) + 3*a^2*c*d
*e^5*f*g^2*abs(e) - a^3*e^6*g^3*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
 - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3))*g/(e^4*abs(g)) + (15*c^3*d^3*e
^4*f^3*g*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2))) - 45*a*c^2*d^2*e^5*f^2*g^2*abs(e)*log(abs(-sqrt(e^2*f - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 45*a^2*c*d*e^6*f*g^3*
abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3
*g^2))) - 15*a^3*e^7*g^4*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) +
sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 33*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e
^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(e) + 26*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(e) + 40*sqrt(-c
*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(e
```

```
) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^
4*g^2*abs(e) - 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c
*d*g)*a*c*d^2*e^2*g^2*abs(e) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^2*abs(e))/(sqrt(c*d*g)*c^3*d^3*e^5*abs(g)))
/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{5/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.713 $\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 244

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] 3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used

$= \{884, 905, 65, 223, 212\}$

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
```

```
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} \\
&\quad+\frac{\left(3\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4cde^2} \\
&= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} \\
&\quad+\frac{\left(3(cdf-aeg)^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8c^2d^2} \\
&= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} \\
&\quad+\frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{8c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} \\
&\quad+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} \\
&\quad+\frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{4c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}
\end{aligned}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{4c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(-3aeg+cd(5f+2gx)) + \frac{3(cdf-\dots}{\dots}\right.}{4c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(-3*a*e*g + c*d
*(5*f + 2*g*x)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[g]))/(4*c^(5/2)*d^
(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.30

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)ac\dots\right.}{8\sqrt{ex\dots}}$

```
[In] int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(g*x+f)^(1/2)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2
)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*
((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*
(c*d*x+a*e))^(1/2)*a*e*g+10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f
)/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x+5\,c^2d^2fg-3\,acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{}}{}\right.$$

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g*x
 + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c
^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2
*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g*x + c^3*
d^4*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

[In] integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)

[Out] Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(204) = 408.

Time = 0.52 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{e\left(\frac{\left(\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{2\left(e^2f+(ex+d)eg-\cdots\right)}{cde^2g}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")

[Out] 1/4*e*((sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*abs(e)/(c*d*e^2*g) + 3*(c^2*d^2*e^2*f*abs(e) - a*c*d*e^3*g*abs(e))/(c^3*d^3*e^2*g)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e) + a^2*e^4*g^2*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d

```
*g)*c^2*d^2))*g/(e^3*abs(g)) + (3*c^2*d^2*e^3*f^2*g*abs(e)*log(abs(-sqrt(e^
2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a*c*d*e^4*f
*g^2*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 +
a*e^3*g^2))) + 3*a^2*e^5*g^3*abs(e)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g
) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(e) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(e) + 3*sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(e))/(sqrt(c*d*g)*c^2
*d^2*e^4*abs(g)))/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.714 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 169

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] (-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {884, 905, 65, 223, 212}

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}} + \frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} \\ &\quad + \frac{(cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2cde^2} \\ &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} \\ &\quad + \frac{\left((cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{2cde^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\ &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} \\ &\quad + \frac{\left((cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{c^2d^2e^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\ &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} \\ &\quad + \frac{\left((cde^2f+cd^2eg-e(cd^2+ae^2)\,g)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{c^2d^2e^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\ &= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} \\ &\quad + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}(ae+cdx)\sqrt{f+gx}+(cdf-aeg)\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*(a*e + c*d*x)*Sqrt[f + g*x] + (c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf-\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,cd\sqrt{cdg}}$

[In] int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*f-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c/d/(c*d*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}cdg-(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{cdg}\log\left(-\frac{8\,c^2}{}\right.}{}\right.$$

[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g - (

```
c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^2*d^2*e*g*x + c^2*d^3*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(141) = 282.

Time = 0.43 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=$$
$$-\frac{e\left(\frac{g\left(\frac{\left(cdef|e|-ae^2g|e|\right)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\right)cdg}\right|\right)}{\sqrt{cdg}cd}-\frac{\sqrt{-cde^2fg+ae^3g^2+\left(e^2f+(ex+d)eg-deg\right)\cdots}}{cdeg}\cdots\right)}{e^2|g|}\cdots\right)}{\cdots}$$

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] -e*(g*((c*d*e*f*abs(e) - a*e^2*g*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*
abs(e)/(c*d*e*g))/(e^2*abs(g)) - (c*d*e^2*f*g*abs(e)*log(abs(-sqrt(e^2*f -
d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - a*e^3*g^2*abs(e)*lo
g(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) -
 sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*abs(e))/(sq
rt(c*d*g)*c*d*e^3*abs(g)))/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.715 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 105

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a
*e)^(1/2)*(e*x+d)^(1/2)/c^(1/2)/d^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {905, 65, 223, 212}

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{d+ex}\sqrt{ae+cdx}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/
(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)}{\sqrt{ex+d}\,\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}}$	102

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*ln(1/2*(2*c*d*g*x
+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))/(c
*d*g)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$

$$= \left[\frac{\sqrt{cdg}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdgx+cdf+aeg)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\,(c^2d^2efg+}{ex+d}\right.}{2\,cdg}\right.$$

$$\left.-\frac{\sqrt{-cdg}\arctan\left(\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2\,cdegx^2+cd^2f+adeg+(cdef+(2\,cd^2+ae^2)g)x}\right)}{cdg}\right]$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
+ a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d
*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(-c*d*
g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e
*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*
d^2 + a*e^2)*g)*x))/(c*d*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$
$$-\frac{2\,e^3\left(\frac{g\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}e|g|} - \frac{g\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae\cdots}\right.\right.}{\sqrt{cdg}e|g|}\right.}{|e|^2}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] -2*e^3*(g*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*
d*g)*e*abs(g)) - g*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)))/(sqrt(c*d*g)*e*abs(g)))/abs(e)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)), x)
```

3.716 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
```

```
0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(cdf - aeg)\sqrt{d + ex}\sqrt{f + gx}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d + ex}}{(f + gx)^{3/2}\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}\,dx = \frac{2\sqrt{(ae + cdx)(d + ex)}}{(cdf - aeg)\sqrt{d + ex}\sqrt{f + gx}}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,\sqrt{gx+f}\,(aeg-cdf)}$	45
gosper	$-\frac{2(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\,(aeg-cdf)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	63

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(55) = 110.

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx}}{cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2+}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(
c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^
2 + (c*d^2 - a*e^2)*f*g)*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.18

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$
$$-\frac{2\sqrt{cdg}e^2g}{cde^2fg|g|-cd^2eg^2|g|-\sqrt{-cd^2eg^2+ae^3g^2}\sqrt{e^2f-deg}\sqrt{cdg}|g|}$$
$$+\frac{4\sqrt{cdg}e^2g}{\left(cde^2fg-ae^3g^2+\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}-\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right.\right.}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] -2*sqrt(c*d*g)*e^2*g/(c*d*e^2*f*g*abs(g) - c*d^2*e*g^2*abs(g) - sqrt(-c*d^2
*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*abs(g)) + 4*sqrt(c*d*g)
*e^2*g/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g))^2)*abs(g))
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=-\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,(c\,d^2\,f-a\,d\,e\,g)}{a\,e^2\,g-c\,d\,e\,f}\right)\,(a\,e^2\,g-c\,d\,e\,f)}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f +
 g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*(a
*e^2*g - c*d*e*f))
```

3.717 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{4cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(e
*x+d)^(1/2)+4/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
2/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}}+\frac{(2cd)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3(cdf-aeg)}\\ &= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}}+\frac{4cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int\frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(-aeg+cd(3f+2gx))}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
- a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-2cdgx+aeg-3cdf)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}\,(aeg-cdf)^{2}}$	61
gosper	$-\frac{2(cdx+ae)(-2cdgx+aeg-3cdf)\sqrt{ex+d}}{3(gx+f)^{\frac{3}{2}}\,(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$	98

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-2*c*d*g*x+a*
e*g-3*c*d*f)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(113) = 226.

Time = 0.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{}{3\,(c^2d^3f^4-2\,acd^2ef^3g+a^2de^2f^2g^2+(c^2d^2ef^2g^2-2\,ac}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e^2
*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (2*c
^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a*c*d
^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c^2*d^
3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(113) = 226.

Time = 0.35 (sec) , antiderivative size = 676, normalized size of antiderivative = 5.24

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{8\left(cde^2fg-ae^3g^2+3\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}-\right.\right.}{3\left(cde^2fg-ae^3g^2+\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cd}\right.\right.}$$

$$-\frac{}{3\left(c^2d^2e^4f^3g|g|-6\,c^2d^3e^3f^2g^2|g|+3\,acde^5f^2g^2|g|+9\,c^2d^4e^2fg^3|g|-6\,acd^2e^4fg^3|g|-4\,c^2d^5eg^4|g|+3\,acd\right.}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] 8/3*(c*d*e^2*f*g - a*e^3*g^2 + 3*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(
c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g))^2)*sqrt(c*d*g)*c*d*e^4*g^2/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f +
(e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
 + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3*abs(g)) - 2/3*(2*sqrt(c*d*g)*c*d*e^4
*f*g - 3*sqrt(c*d*g)*c*d^2*e^3*g^2 + sqrt(c*d*g)*a*e^5*g^2 - 3*sqrt(-c*d^2*
e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c*d*e^2*g)/(c^2*d^2*e^4*f^3*g*abs(g)
 - 6*c^2*d^3*e^3*f^2*g^2*abs(g) + 3*a*c*d*e^5*f^2*g^2*abs(g) + 9*c^2*d^4*e^
2*f*g^3*abs(g) - 6*a*c*d^2*e^4*f*g^3*abs(g) - 4*c^2*d^5*e*g^4*abs(g) + 3*a*
c*d^3*e^3*g^4*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*c*d*e^2*f^2*abs(g) + 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2
*f - d*e*g)*sqrt(c*d*g)*c*d^2*e*f*g*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^3*f*g*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^3*g^2*abs(g) + sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*d*e^2*g^2*abs(g))
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{\left(\frac{(2\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,g\,(a\,e\,g-c\,d\,f)^2}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^2}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{d\,f\,\sqrt{f+g\,x}}{e\,g}+\frac{x\,\sqrt{f+g\,x}\,(d\,g+e\,f)}{e\,g}}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*d
*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x*(f
 + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

3.718 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(e
*x+d)^(1/2)+8/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)
^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)+16/15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{16c^2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} + \frac{8cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}}+\frac{(4cd)\int\frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5(cdf-aeg)}\\
&= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}}+\frac{8cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}\\
&\quad+\frac{(8c^2d^2)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{15(cdf-aeg)^2}\\
&= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}}+\frac{8cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}\\
&\quad+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d}{15(cdf-aeg)^3\sqrt{d+ex}(f+gx)^5}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x)
 + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d +
 e*x]*(f + g*x)^(5/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15c^2d^2f^2)}{15\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)^3}$	111
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15c^2d^2f^2)\sqrt{ex+d}}{15(gx+f)^{\frac{5}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	169

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/15/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g^
2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2*e^2*g^2-10*a*c*d*e*f*g+15*c^2*
d^2*f^2)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(174) = 348.

Time = 1.04 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{}{15\left(c^3d^4f^6-3\,ac^2d^3ef^5g+3\,a^2cd^2e^2f^4g^2-a^3de^3f^3g^3+(\right.}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2 +
 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
```

```
)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*a^2
*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e
^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4*g^2
- a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c^3*d^3*
e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(a*c^2*
d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x^2 + (
c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g - 3
*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^
3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(174) = 348$.

Time = 0.42 (sec) , antiderivative size = 1612, normalized size of antiderivative = 8.14

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")

[Out] 32/15*(c^2*d^2*e^4*f^2*g^2 - 2*a*c*d*e^5*f*g^3 + a^2*e^6*g^4 + 5*(sqrt(e^2*
f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c*d*e^2*f*g - 5*(sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e
*x + d)*e*g - d*e*g)*c*d*g))^2*a*e^3*g^2 + 10*(sqrt(e^2*f + (e*x + d)*e*g -
 d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g))^4)*sqrt(c*d*g)*c^2*d^2*e^6*g^3/((c*d*e^2*f*g - a*e^3*g^2
 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a
*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^5*abs(g)) - 2/15*(8*s
qrt(c*d*g)*c^2*d^2*e^6*f^2*g - 45*sqrt(c*d*g)*c^2*d^3*e^5*f*g^2 + 29*sqrt(c
*d*g)*a*c*d*e^7*f*g^2 + 40*sqrt(c*d*g)*c^2*d^4*e^4*g^3 - 35*sqrt(c*d*g)*a*c
*d^2*e^6*g^3 + 3*sqrt(c*d*g)*a^2*e^8*g^3 - 25*sqrt(-c*d^2*e*g^2 + a*e^3*g^2
)*sqrt(e^2*f - d*e*g)*c^2*d^2*e^4*f*g + 40*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*s
qrt(e^2*f - d*e*g)*c^2*d^3*e^3*g^2 - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt
(e^2*f - d*e*g)*a*c*d*e^5*g^2)/(c^3*d^3*e^6*f^5*g*abs(g) - 15*c^3*d^4*e^5*f
^4*g^2*abs(g) + 10*a*c^2*d^2*e^7*f^4*g^2*abs(g) + 55*c^3*d^5*e^4*f^3*g^3*ab
s(g) - 50*a*c^2*d^3*e^6*f^3*g^3*abs(g) + 5*a^2*c*d*e^8*f^3*g^3*abs(g) - 85*
c^3*d^6*e^3*f^2*g^4*abs(g) + 90*a*c^2*d^4*e^5*f^2*g^4*abs(g) - 15*a^2*c*d^2
*e^7*f^2*g^4*abs(g) + 60*c^3*d^7*e^2*f*g^5*abs(g) - 70*a*c^2*d^5*e^4*f*g^5*
abs(g) + 15*a^2*c*d^3*e^6*f*g^5*abs(g) - 16*c^3*d^8*e*g^6*abs(g) + 20*a*c^2
*d^6*e^3*g^6*abs(g) - 5*a^2*c*d^4*e^5*g^6*abs(g) - 5*sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^4*f^4*abs(g) + 30*sqrt(-
c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e^3*f^3*g*
abs(g) - 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*
a*c*d*e^5*f^3*g*abs(g) - 61*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e
*g)*sqrt(c*d*g)*c^2*d^4*e^2*f^2*g^2*abs(g) + 32*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^4*f^2*g^2*abs(g) - sqrt(-c*d^
2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^6*f^2*g^2*abs(g)
 + 52*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^
5*e*f*g^3*abs(g) - 34*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sq
rt(c*d*g)*a*c*d^3*e^3*f*g^3*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(
e^2*f - d*e*g)*sqrt(c*d*g)*a^2*d*e^5*f*g^3*abs(g) - 16*sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^6*g^4*abs(g) + 12*sqrt(-c*
d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^4*e^2*g^4*abs(
g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*d^2
*e^4*g^4*abs(g))
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\left(\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c\,d\,x\,\left(a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(d\,g+2\,e\,f\right)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(2\,d\,g+e\,f\right)}{e\,g^2}}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15*
e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g -
c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g - c*
d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2
) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*f))/(
e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

3.719 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} + \frac{32c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(7/2)/(e
*x+d)^(1/2)+12/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^2/(g*x+f)^(5/2)/(e*x+d)^(1/2)+16/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/35*c^3*d^3*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(
1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used

$= \{886, 874\}$

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{32c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} + \frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3} + \frac{12cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{(6cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{7(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} + \frac{(24c^2d^2)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{35(cdf-aeg)^2}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} + \frac{(16c^3d^3)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{35(cdf-aeg)^3}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} + \frac{32c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-5a^3e^3g^3+3a^2cde^2g^2(7f+2gx)}{35(cdf}$$

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f +
 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3 +
 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d + e
*x]*(f + g*x)^(7/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}\,(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e^2f\,g^2+35c}{35(gx+f)^{\frac{7}{2}}\,(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ad}}$

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/35/(e*x+d)^(1/2)/(g*x+f)^(7/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3*
g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x+28*a
*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*e^3*g^3-21*a^2*c*d*e^2*f*g^2+35
*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(235) = 470$.

Time = 2.75 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{}{35\,(c^4d^5f^8-4\,ac^3d^4ef^7g+6\,a^2c^2d^3e^2f^6g^2-4\,a^3cd^2e^3f^5}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*c
*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2 +
2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^
8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*f^5*g
^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a
^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4*c^4*d^
4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4 - 4*(a*c
^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^
4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^4*e*f^6*g^
2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 - 2*(4*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3
*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d^4*e*f^7*g +
3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 - 12*(a*c^3*d^4
*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g
^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*e*f^8 + 4*a^4*d*
e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c^3*d^4*e - 3*a^2*
c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^3 - (16*a^
3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. $2(235) = 470$.

Time = 0.55 (sec) , antiderivative size = 2971, normalized size of antiderivative = 11.13

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] 64/35*(c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5
- a^3*e^9*g^6 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d^2
*e^4*f^2*g^2 - 14*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c*d*e
^5*f*g^3 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e
^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*e^6*g^4
+ 21*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g +
 a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*e^2*f*g - 21*(sq
```

rt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*a*e^3*g^2 + 35*(sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g))^6)*sqrt(c*d*g)*c^3*d^3*e^8*g^4/((c*d*e^2
*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^7*abs
(g)) - 2/35*(16*sqrt(c*d*g)*c^3*d^3*e^8*f^3*g - 203*sqrt(c*d*g)*c^3*d^4*e^7
*f^2*g^2 + 155*sqrt(c*d*g)*a*c^2*d^2*e^9*f^2*g^2 + 462*sqrt(c*d*g)*c^3*d^5*
e^6*f*g^3 - 518*sqrt(c*d*g)*a*c^2*d^3*e^8*f*g^3 + 104*sqrt(c*d*g)*a^2*c*d*e
^10*f*g^3 - 280*sqrt(c*d*g)*c^3*d^6*e^5*g^4 + 378*sqrt(c*d*g)*a*c^2*d^4*e^7
*g^4 - 119*sqrt(c*d*g)*a^2*c*d^2*e^9*g^4 + 5*sqrt(c*d*g)*a^3*e^11*g^4 - 77*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c^3*d^3*e^6*f^2*g + 322*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c^3*d^4*e^5*f*g^2 - 168*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*a*c^2*d^2*e^7*f*g^2 - 28
0*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*c^3*d^5*e^4*g^3 + 238*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*a*c^2*d^3*e^6*g^3 - 35*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*a^2*c*d*e^8*g^3)/(c^4*d^4
*e^8*f^7*g*abs(g) - 28*c^4*d^5*e^7*f^6*g^2*abs(g) + 21*a*c^3*d^3*e^9*f^6*g^
2*abs(g) + 182*c^4*d^6*e^6*f^5*g^3*abs(g) - 196*a*c^3*d^4*e^8*f^5*g^3*abs(g
) + 35*a^2*c^2*d^2*e^10*f^5*g^3*abs(g) - 532*c^4*d^7*e^5*f^4*g^4*abs(g) + 6
86*a*c^3*d^5*e^7*f^4*g^4*abs(g) - 196*a^2*c^2*d^3*e^9*f^4*g^4*abs(g) + 7*a^
3*c*d*e^11*f^4*g^4*abs(g) + 833*c^4*d^8*e^4*f^3*g^5*abs(g) - 1204*a*c^3*d^6
*e^6*f^3*g^5*abs(g) + 434*a^2*c^2*d^4*e^8*f^3*g^5*abs(g) - 28*a^3*c*d^2*e^1
0*f^3*g^5*abs(g) - 728*c^4*d^9*e^3*f^2*g^6*abs(g) + 1141*a*c^3*d^7*e^5*f^2*
g^6*abs(g) - 476*a^2*c^2*d^5*e^7*f^2*g^6*abs(g) + 42*a^3*c*d^3*e^9*f^2*g^6*
abs(g) + 336*c^4*d^10*e^2*f*g^7*abs(g) - 560*a*c^3*d^8*e^4*f*g^7*abs(g) + 2
59*a^2*c^2*d^6*e^6*f*g^7*abs(g) - 28*a^3*c*d^4*e^8*f*g^7*abs(g) - 64*c^4*d^
11*e*g^8*abs(g) + 112*a*c^3*d^9*e^3*g^8*abs(g) - 56*a^2*c^2*d^7*e^5*g^8*abs
(g) + 7*a^3*c*d^5*e^7*g^8*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^
2*f - d*e*g)*sqrt(c*d*g)*c^3*d^3*e^6*f^6*abs(g) + 77*sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*e^5*f^5*g*abs(g) - 35*sqrt
(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^7*f^
5*g*abs(g) - 301*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*
d*g)*c^3*d^5*e^4*f^4*g^2*abs(g) + 217*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e
^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^6*f^4*g^2*abs(g) - 21*sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^8*f^4*g^2*abs(g)
 + 575*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^
6*e^3*f^3*g^3*abs(g) - 521*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*
g)*sqrt(c*d*g)*a*c^2*d^4*e^5*f^3*g^3*abs(g) + 87*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^7*f^3*g^3*abs(g) - sqrt(-c
*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^9*f^3*g^3*abs
(g) - 584*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^
3*d^7*e^2*f^2*g^4*abs(g) + 611*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*a*c^2*d^5*e^4*f^2*g^4*abs(g) - 135*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^3*e^6*f^2*g^4*abs(g) + 3*

```
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*d*e^8*f^
2*g^4*abs(g) + 304*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(
c*d*g)*c^3*d^8*e*f*g^5*abs(g) - 352*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2
*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^6*e^3*f*g^5*abs(g) + 93*sqrt(-c*d^2*e*g^2 +
 a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^4*e^5*f*g^5*abs(g) - 3*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*d^2*e^7*
f*g^5*abs(g) - 64*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c
*d*g)*c^3*d^9*g^6*abs(g) + 80*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d
*e*g)*sqrt(c*d*g)*a*c^2*d^7*e^2*g^6*abs(g) - 24*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^5*e^4*g^6*abs(g) + sqrt(-c*d^2*
e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*d^3*e^6*g^6*abs(g))
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,\left(10\,a^3\,e^3\,g^3-42\,a^2\,c\,d\,e^2\,f\,g^2+70\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,e\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^3\,d^3\,x^3\,\sqrt{d+e\,x}}{35\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c\cdots}{\cdots}\right.}{x^4\,\sqrt{f+g\,x}+\frac{d\,f^3\,\sqrt{f+g\,x}}{e\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,\left(d\,g+3\,e\,f\right)}{e\,g}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\cdots\right.}{e\,g^2}\cdots}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(35
*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*g -
 c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 14*
a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g - 7*c*d
*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (d
*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/(e*g) +
 (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(1/2)*(3*
d*g + e*f))/(e*g^3))
```

3.720 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 301

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3\sqrt{d+ex}}$$
$$+\frac{5g(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2\sqrt{d+ex}}$$
$$+\frac{15\sqrt{g}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(g*x+f)^(5/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/2*g*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+15/4*g*(-a*e*g+c*d*f)*(g*x+f)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {880, 884, 905, 65, 223, 212}

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{15\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$
$$+\frac{15g\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}$$
$$+\frac{5g(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{(5g)\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cd}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{(15g(cdf-aeg))\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4c^2d^2}
\end{aligned}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{(15g(cdf-aeg)^2)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8c^3d^3}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{\left(15g(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{8c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{\left(15g(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{4c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{\left(15g(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{4c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$
= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} + \frac{15\sqrt{g}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.61

$$
\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-15a^2e^2g^2-5acdeg(-5f+gx)+c^2d^2(\right.}{4c^{7/2}d^{7/2}}
$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-15*a^2*e^2*g^2 - 5*a*c*d*e*
g*(-5*f + g*x) + c^2*d^2*(-8*f^2 + 9*f*g*x + 2*g^2*x^2)) + 15*Sqrt[g]*(c*d*
f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqr
t[g]*Sqrt[a*e + c*d*x])]))/(4*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(255) = 510$.

Time = 0.55 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.12

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2cde^2g^3x-30\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)ac^2d^2efg^2x+15\ln\left(\right.\right.}{}$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
```

```
(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-30*ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*a^2*c*d*e^2*f*g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+4*c^2*d^2*
g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-10*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+18*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-16*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)
^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)/c^3/d^3/(e*x+d
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 971, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x^2-8\,c^2d^2f^2+25\,acdefg-15\,a^2e^2g^2+(9\,c^2d^2fg-5\,\ldots}{}\right.$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*g
^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2*e^
2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*e^3*
g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*f
*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*
x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c
^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(
g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*
c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g
 - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c^2*d^3*e*f^2 -
 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g
 + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e +
 a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan
(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*
```

```
c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*
x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(255) = 510.

Time = 0.63 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{e^2f+(ex+d)eg-deg}\Big((e^2f+(ex+d)eg-deg)\Big(\frac{2\,(e^2f+(ex+d)}{cde^4|g}}{4\sqrt{-cde^2fg+ae^3g^2+(}} - \frac{15\,(c^2d^2f^2g^2-2\,acdefg^3+a^2e^2g^4)\log\Big(\Big|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f}}{4\sqrt{cdg}c^3d^3|g|} + \frac{15\sqrt{-cd^2eg^2+ae^3g^2}c^2d^2e^2f^2g^2\log\Big(\Big|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\Big|\Big)-30\sqrt{-cd^2eg^2+ae^3g^2}}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(2
*(e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(c*d*e^4*abs(g)) + 5*(c^4*d^4*e^2*f*g^
2 - a*c^3*d^3*e^3*g^3)/(c^5*d^5*e^4*abs(g))) - 15*(c^4*d^4*e^4*f^2*g^2 - 2*
a*c^3*d^3*e^5*f*g^3 + a^2*c^2*d^2*e^6*g^4)/(c^5*d^5*e^4*abs(g)))/sqrt(-c*d*
e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g) - 15/4*(c^2*d^
2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*abs(g)) + 1/4*(15*sqrt(-c*d^2
*e*g^2 + a*e^3*g^2)*c^2*d^2*e^2*f^2*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c
*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 30*sqrt(-c*d^2*e*g^2 + a*e^3*g^2
)*a*c*d*e^3*f*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*
g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^4*g^4*log(abs(
-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 8*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g^2 + 9*sqrt(e^2*f - d*e*g)*sq
rt(c*d*g)*c^2*d^3*e*f*g^3 - 25*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*
g^3 - 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^4 - 5*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*a*c*d^2*e^2*g^4 + 15*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g
^4)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^3*e^2*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.721 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 227

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
3*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^
(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*g*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {880, 884, 905, 65, 223, 212}

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
```

```
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{(3g)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cd}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{(3g(cdf-aeg))\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2c^2d^2}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{\left(3g(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{\left(3g(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{\left(3g(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2\sqrt{d+ex}}\\
&\quad+\frac{3\sqrt{g}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-2cdf+3aeg+cdgx)+3\sqrt{g}(cdf-aeg)\right.}{c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-2*c*d*f + 3*a*e*g + c*d*g*x
) + 3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c^(5/2)*d^(5/2)*Sqrt[(a*e + c*
d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx+3\ln\left(\frac{2cdgx+aeg}{}\right.\right.}{}$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a^2*e^2*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g-2*((g*x+f)*(c*d*x+a*e))^
(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))
^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(c*d*g)^(1/2)/
c^2/d^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx-2\,cdf+3\,aeg)\sqrt{ex+d}\sqrt{}}{}\right.$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^
2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)
*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*
f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4
*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4
*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(
c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)*sqrt(e*x
+ d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^
2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c
^2*d^2*e^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(193) = 386.

Time = 0.52 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.32

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{(e^2f+(ex+d)eg-deg)g^2}{cde^2|g|}-\frac{3\,(c^2d^2e^2fg^2-acde^3g^3)}{c^3d^3e^2|g|}\right.}{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}$$
$$-\frac{3\,(cdfg^2-aeg^3)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)\right.\right.}{\sqrt{cdg}c^2d^2|g|}$$
$$+\frac{3\sqrt{-cd^2eg^2+ae^3g^2}cdefg^2\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\right|\right)-3\sqrt{-cd^2eg^2+ae^3g^2}ae^2g^3\log}{\sqrt{-cd}}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(c
*d*e^2*abs(g)) - 3*(c^2*d^2*e^2*f*g^2 - a*c*d*e^3*g^3)/(c^3*d^3*e^2*abs(g))
)/sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g) -
3*(c*d*f*g^2 - a*e*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c
*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d
*g)))/(sqrt(c*d*g)*c^2*d^2*abs(g)) + (3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c*d*
e*f*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^
3*g^2))) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*e^2*g^3*log(abs(-sqrt(e^2*f -
 d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 2*sqrt(e^2*f - d*e
*g)*sqrt(c*d*g)*c*d*e*f*g^2 + sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g^3 - 3
*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
*sqrt(c*d*g)*c^2*d^2*e*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{3/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.722 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 161

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*g^(1/2)*
(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {880, 905, 65, 223, 212}

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{g\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(g\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}-\sqrt{g}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x] - Sqrt[g]*Sqrt[a*e + c*d*x
]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c
^(3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-2\sqrt{}\right.}{\sqrt{cdg}\,(cdx+ae)\sqrt{(gx+f)(cdx+ae)}\,dc\sqrt{ex+d}}$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*(2
*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(
1/2))*a*e*g-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/(c*d
*x+a*e)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/c/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.53

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{\frac{g}{cd}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2ef}{}\right.}{}\right.$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2
*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*
e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c
*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^
3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*
x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2
)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(133) = 266.

Time = 0.47 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\left(cd^2eg^2\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg^2+ae^3g^2}\right|\right)-ae^3g^2\,\mathrm{l}\cdots\right.}{\sqrt{cdg}\cdots}$$
$$-\frac{2\,g^2\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}cd|g|}$$
$$+\frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}g^2}{(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)cd|g|}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2
 + a*e^3*g^2))) - a*e^3*g^2*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt
(-c*d^2*e*g^2 + a*e^3*g^2))) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g))/(sqrt(c*d*g)*c^2*d^3*e*abs(g) - sqrt(c*d*g)*a*c*d*e^3*a
bs(g)) - 2*g^2*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + s
qrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sq
rt(c*d*g)*c*d*abs(g)) + 2*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*g^2/((c*d*e^2*f*g
 - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*c*d*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,\,x+a\,d\,e)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.723 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
```

```
0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)}$	55
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(ex+d)^{\frac{3}{2}}}{(aeg-cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	63

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/(a*e*
g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}}{acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 +
a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)),
 x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e}}{(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)(} - \frac{2\sqrt{-cd^2eg^2+ae^3g^2}\sqrt{e^2f-deg}}{c^2d^3ef|g|-acde^3f|g|-acd^2e^2g|g|+a^2e^4g|g|}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*e*g^2/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g)*(c*d*e*f*abs(g) - a*e^2*g*abs(g))) - 2*sqrt
(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)/(c^2*d^3*e*f*abs(g) - a*c*d*
e^3*f*abs(g) - a*c*d^2*e^2*g*abs(g) + a^2*e^4*g*abs(g))
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{\left(\frac{2\,f\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)}{c\,d\,e}}$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/(
c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2
*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c*d^
2))/(c*d*e))
```

3.724 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 124

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-4*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {882, 874}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{(2g)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{4g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(aeg+cd(f+2gx))}{(cdf-aeg)^2\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+aeg+cdf)}{\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)(aeg-cdf)^2}$	70
gosper	$-\frac{2(cdx+ae)(2cdgx+aeg+cdf)(ex+d)^{\frac{3}{2}}}{\sqrt{gx+f}\,(a^2e^2g^2-2acdefg+c^2d^2f^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	97

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+a*e*g
+c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(112) = 224.

Time = 0.35 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.62

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{cdex^2+ade+}}{ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+(c^3d^3ef^2g-2\,ac^2d^2e^2fg^2+a^2cde^3g^3)x^3+(c^3d^3ef^3+(c^3d^4-ac^2d^2}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*
d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x
^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c
^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*
d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(112) = 224$.

Time = 0.38 (sec) , antiderivative size = 793, normalized size of antiderivative = 6.40

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=$$

$$-2\left(\frac{\sqrt{e^2f+(ex+d)eg-deg}cdg^2}{(c^2d^2e^2f^2|g|-2\,acde^3fg|g|+a^2e^4g^2|g|)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}+\frac{}{\Big(cde^2f}\right.$$

$$+\frac{}{\sqrt{e^2f-deg}\sqrt{cdg}c^3d^4f^2g|g|-\sqrt{e^2f-deg}\sqrt{cdg}ac^2d^2e^2f^2g|g|-2\sqrt{e^2f-deg}\sqrt{cdg}ac^2d^3efg^2|g|+2\sqrt{e^2f}}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g^2/((c^2*d^2*e^2*f^2*abs(g) -
2*a*c*d*e^3*f*g*abs(g) + a^2*e^4*g^2*abs(g))*sqrt(-c*d*e^2*f*g + a*e^3*g^2
+ (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) + 2*sqrt(c*d*g)*g^2/((c*d*e^2*f*g
 - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d
*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)*(c*d*f*ab
s(g) - a*e*g*abs(g))))*e^2 + 2*(sqrt(e^2*f - d*e*g)*c^2*d^2*e*f*g^2 - sqrt(
e^2*f - d*e*g)*c^2*d^3*g^3 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c*d
^2*g^2 - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*e^2*g^2)/(sqrt(e^2*f
- d*e*g)*sqrt(c*d*g)*c^3*d^4*f^2*g*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
*a*c^2*d^2*e^2*f^2*g*abs(g) - 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e
*f*g^2*abs(g) + 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^3*f*g^2*abs(g)
+ sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^2*g^3*abs(g) - sqrt(e^2*f - d
*e*g)*sqrt(c*d*g)*a^3*e^4*g^3*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^3*d
^3*e*f^3*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^3*d^4*f^2*g*abs(g) - 2*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^2*d^2*e^2*f^2*g*abs(g) + 2*sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*a*c^2*d^3*e*f*g^2*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*
a^2*c*d*e^3*f*g^2*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c*d^2*e^2*g^3
*abs(g))
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\left(\frac{4\,g\,x\,\sqrt{d+e\,x}}{e\,(a\,e\,g-c\,d\,f)^2}+\frac{(2\,a\,e\,g+2\,c\,d\,f)\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -(((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
+ e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1
/2)*(a*e^2 + c*d^2))/(c*d*e))
```

3.725 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 192

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} - \frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-8/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(
g*x+f)^(3/2)/(e*x+d)^(1/2)-16/3*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {882, 886, 874}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} - \frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{(4g)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{8g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} - \frac{(8cdg)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3(cdf-aeg)^2}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{8g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} - \frac{16cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2(3f^2+12fgx+8g^2x^2))}{3(cdf-aeg)^3\sqrt{(ae+cdx)(d+ex)}(f+gx)^{3/2}}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3*f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^(3/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3c^2d^2f^2)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)(aeg-cdf)^3}$	120
gosper	$-\frac{2(cdx+ae)(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3c^2d^2f^2)(ex+d)^{\frac{3}{2}}}{3(gx+f)^{\frac{3}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	168

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e^2*g^2-6*a*c*d*e*f*g-3*c^2*d^2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(170) = 340.

Time = 0.42 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.38

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{3\left(ac^3d^4ef^5-3\,a^2c^2d^3e^2f^4g+3\,a^3cd^2e^3f^3g^2-a^4de^4f^2g^3+\left(c^4d^4ef^3g^2-3\,ac^3d^3e^2f^2g^3+3\,a^2c^2d^2e^3fg^4-\right.\right.}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*(
3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3
*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d
^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^
4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*
d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3
 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*
d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d
^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 -
(2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c
^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^
2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2618 vs. 2(170) = 340.

Time = 1.84 (sec) , antiderivative size = 2618, normalized size of antiderivative = 13.64

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^2/((c^3*d^3*e^3*f^3*a
bs(g) - 3*a*c^2*d^2*e^4*f^2*g*abs(g) + 3*a^2*c*d*e^5*f*g^2*abs(g) - a^3*e^6
*g^3*abs(g))*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
)*c*d*g)) + 2*(5*sqrt(c*d*g)*c^3*d^3*e^4*f^2*g^4 - 10*sqrt(c*d*g)*a*c^2*d^2
*e^5*f*g^5 + 5*sqrt(c*d*g)*a^2*c*d*e^6*g^6 + 12*sqrt(c*d*g)*(sqrt(e^2*f + (
e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d^2*e^2*f*g^3 - 12*sqrt(c*d*g)*(sqrt
(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2
 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c*d*e^3*g^4 + 3*sqrt(c*d*g)*
(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^
3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*g^2)/((c^2*d^2*e*f^2*
abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a^2*e^3*g^2*abs(g))*(c*d*e^2*f*g - a*e^3*
g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g
+ a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3))*e^3 + 2/3*(3*s
qrt(e^2*f - d*e*g)*c^4*d^4*e^3*f^3*g^2 - 9*sqrt(e^2*f - d*e*g)*c^4*d^5*e^2*
f^2*g^3 + 21*sqrt(e^2*f - d*e*g)*c^4*d^6*e*f*g^4 - 24*sqrt(e^2*f - d*e*g)*a
*c^3*d^4*e^3*f*g^4 + 12*sqrt(e^2*f - d*e*g)*a^2*c^2*d^2*e^5*f*g^4 - 12*sqrt
(e^2*f - d*e*g)*c^4*d^7*g^5 + 15*sqrt(e^2*f - d*e*g)*a*c^3*d^5*e^2*g^5 - 3*
sqrt(e^2*f - d*e*g)*a^2*c^2*d^3*e^4*g^5 - 3*sqrt(e^2*f - d*e*g)*a^3*c*d*e^6
*g^5 - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^3*e^3*f^3*g + 18*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^4*e^2*f^2*g^2 - 6*sqrt(-c*
```

d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g^2 - 27*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^5*e*f*g^3 + 18*sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*sqrt(c*d*g)*a*c^2*d^3*e^3*f*g^3 - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*s
qrt(c*d*g)*a^2*c*d*e^5*f*g^3 + 12*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g
)*c^3*d^6*g^4 - 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^2*d^4*e^2*
g^4 + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^3*e^6*g^4)/(3*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*c^5*d^6*e^2*f^5*g*abs(g) - 3*sqrt(e^2*f - d*e*g)*sqrt
(c*d*g)*a*c^4*d^4*e^4*f^5*g*abs(g) - 7*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^5*
d^7*e*f^4*g^2*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^4*d^5*e^3*f^4*g^
2*abs(g) + 8*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^3*d^3*e^5*f^4*g^2*abs(g)
 + 4*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^5*d^8*f^3*g^3*abs(g) + 16*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*a*c^4*d^6*e^2*f^3*g^3*abs(g) - 14*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*a^2*c^3*d^4*e^4*f^3*g^3*abs(g) - 6*sqrt(e^2*f - d*e*g)*sqrt(c*
d*g)*a^3*c^2*d^2*e^6*f^3*g^3*abs(g) - 12*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*
c^4*d^7*e*f^2*g^4*abs(g) - 6*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^3*d^5*e^
3*f^2*g^4*abs(g) + 18*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^2*d^3*e^5*f^2*g
^4*abs(g) + 12*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^3*d^6*e^2*f*g^5*abs(g)
 - 8*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^2*d^4*e^4*f*g^5*abs(g) - 5*sqrt(
e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c*d^2*e^6*f*g^5*abs(g) + sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*a^5*e^8*f*g^5*abs(g) - 4*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c
^2*d^5*e^3*g^6*abs(g) + 5*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c*d^3*e^5*g^6
*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*d*e^7*g^6*abs(g) + sqrt(-c*d^
2*e*g^2 + a*e^3*g^2)*c^5*d^5*e^3*f^6*abs(g) - 6*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*c^5*d^6*e^2*f^5*g*abs(g) + 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^5*d^7*e*f
^4*g^2*abs(g) + 12*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^4*d^5*e^3*f^4*g^2*abs
(g) - 6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^3*d^3*e^5*f^4*g^2*abs(g) - 4*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^5*d^8*f^3*g^3*abs(g) - 24*sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*a*c^4*d^6*e^2*f^3*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*a^3*c^2*d^2*e^6*f^3*g^3*abs(g) + 12*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^4
*d^7*e*f^2*g^4*abs(g) + 18*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^3*d^5*e^3*f
^2*g^4*abs(g) - 12*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^2*d^3*e^5*f^2*g^4*a
bs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c*d*e^7*f^2*g^4*abs(g) - 12*sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^3*d^6*e^2*f*g^5*abs(g) + 6*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*a^4*c*d^2*e^6*f*g^5*abs(g) + 4*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*a^3*c^2*d^5*e^3*g^6*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c*d^
3*e^5*g^6*abs(g))

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\frac{8\,x\,(a\,e\,g+3\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^3}+\frac{\sqrt{d+e\,x}\,(-2\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2}{3\,c\,d\,e\,g\,(a\,e\,g-c\,d\,f)^3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a\,f\,\sqrt{f+g\,x}}{c\,g}+\frac{x\,\sqrt{f+g\,x}\,(c\,f\,d^2}{c\,d}}$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e
*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a*e
*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*f*(f
 + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g))/(c
*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*g))
```

3.726 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 262

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} - \frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} - \frac{32c^2d^2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-12/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/
(g*x+f)^(5/2)/(e*x+d)^(1/2)-16/5*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)-32/5*c^2*d^2*g*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used

$= \{882, 886, 874\}$

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} - \frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3} - \frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad -\frac{(6g)\int\frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdf-aeg} \\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad -\frac{12g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}}-\frac{(24cdg)\int\frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5(cdf-aeg)^2} \\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad -\frac{12g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}}-\frac{16cdg\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} \\
&\quad -\frac{(16c^2d^2g)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{5(cdf-aeg)^3} \\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\
&\quad -\frac{12g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}}-\frac{16cdg\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} \\
&\quad -\frac{32c^2d^2g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{d+ex}(a^3e^3g^3-a^2cde^2g^2(5f+2gx)+ac^2d^2eg(15f^2+20fgx+8g^2x^2)+c^3d^3(5f^3+30f^2gx+40fg^2}{5(cdf-aeg)^4\sqrt{(ae+cdx)(d+ex)}(f+gx)^{5/2}}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2*
e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*g^
2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f
 + g*x)^(5/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-5a^2cd\,e^2f}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(cdx+ae)(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-5a^2cd\,e^2f\,g^2+15a\,c}{5(gx+f)^{\frac{5}{2}}(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)(cde\,x^2+a\,e^2x+c\,d^2x+ade)}$

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/5/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(16*c^3*d^3*g^
3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2*x^2-2*a^2*c*d*e^2*g^3*x+20*a*c
^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*g^3-5*a^2*c*d*e^2*f*g^2+15*a*c^
2*d^2*e*f^2*g+5*c^3*d^3*f^3)/(c*d*x+a*e)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(232) = 464$.

Time = 1.40 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.05

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$

$$-\frac{}{5\,(ac^4d^5ef^7 - 4\,a^2c^3d^4e^2f^6g + 6\,a^3c^2d^3e^3f^5g^2 - 4\,a^4cd^2e^4f^4g^3 + a^5de^5f^3g^4 + (c^5d^5ef^4g^3 - 4\,ac^4d^4e^2f^3g^4 +}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*d
*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(1
5*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*f^7
- 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^4*f^4
*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g^4 + 6
*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7)*x^5 +
 (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2*a*c^4*d
^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f
^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4 + a^5*e^6
)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3*a*c^4*d^4*
e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2*(7*a^2*c^3*
d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5
)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5*e*f^7 + 3*a^5
*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^4*d^5*e + 2*a^2
*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4)*f^4*g^3 +
 (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c*d^2*e^4 - a^5*e^
6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4*d^4*e^2)*f^7 - (a
*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^
4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f^4*g^3 - (11*a^4*c*d^2*
e^4 - a^5*e^6)*f^3*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6137 vs. $2(232) = 464$.

Time = 24.65 (sec) , antiderivative size = 6137, normalized size of antiderivative = 23.42

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
[Out] -2/5*(5*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^3*d^3*g^2/((c^4*d^4*e^4*f^4*a
bs(g) - 4*a*c^3*d^3*e^5*f^3*g*abs(g) + 6*a^2*c^2*d^2*e^6*f^2*g^2*abs(g) - 4
*a^3*c*d*e^7*f*g^3*abs(g) + a^4*e^8*g^4*abs(g))*sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) + 2*(11*sqrt(c*d*g)*c^6*d^6*e^
8*f^4*g^6 - 44*sqrt(c*d*g)*a*c^5*d^5*e^9*f^3*g^7 + 66*sqrt(c*d*g)*a^2*c^4*d
^4*e^10*f^2*g^8 - 44*sqrt(c*d*g)*a^3*c^3*d^3*e^11*f*g^9 + 11*sqrt(c*d*g)*a^
4*c^2*d^2*e^12*g^10 + 50*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*s
qrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
)*c*d*g))^2*c^5*d^5*e^6*f^3*g^5 - 150*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g))^2*a*c^4*d^4*e^7*f^2*g^6 + 150*sqrt(c*d*g)*(sqrt(e^2*
f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*c^3*d^3*e^8*f*g^7 - 50*sqrt(c*d
*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g +
a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^3*c^2*d^2*e^9*g^8 +
 80*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*
d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c^4*d^4*e
^4*f^2*g^4 - 160*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*
g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)
)^4*a*c^3*d^3*e^5*f*g^5 + 80*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*
g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d
```

*e*g)*c*d*g))^4*a^2*c^2*d^2*e^6*g^6 + 30*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g))^6*c^3*d^3*e^2*f*g^3 - 30*sqrt(c*d*g)*(sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g))^6*a*c^2*d^2*e^3*g^4 + 5*sqrt(c*d*g)*(sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^8*c^2*d^2*g^2)/((c^3*d^3*e^2*f
^3*abs(g) - 3*a*c^2*d^2*e^3*f^2*g*abs(g) + 3*a^2*c*d*e^4*f*g^2*abs(g) - a^3
*e^5*g^3*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d
*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
- d*e*g)*c*d*g))^2)^5))*e^4 + 2/5*(5*sqrt(e^2*f - d*e*g)*c^6*d^6*e^5*f^5*g^
2 - 35*sqrt(e^2*f - d*e*g)*c^6*d^7*e^4*f^4*g^3 + 10*sqrt(e^2*f - d*e*g)*a*c
^5*d^5*e^6*f^4*g^3 + 150*sqrt(e^2*f - d*e*g)*c^6*d^8*e^3*f^3*g^4 - 160*sqrt
(e^2*f - d*e*g)*a*c^5*d^6*e^5*f^3*g^4 + 60*sqrt(e^2*f - d*e*g)*a^2*c^4*d^4*
e^7*f^3*g^4 - 305*sqrt(e^2*f - d*e*g)*c^6*d^9*e^2*f^2*g^5 + 465*sqrt(e^2*f
- d*e*g)*a*c^5*d^7*e^4*f^2*g^5 - 225*sqrt(e^2*f - d*e*g)*a^2*c^4*d^5*e^6*f^
2*g^5 + 15*sqrt(e^2*f - d*e*g)*a^3*c^3*d^3*e^8*f^2*g^5 + 260*sqrt(e^2*f - d
*e*g)*c^6*d^10*e*f*g^6 - 430*sqrt(e^2*f - d*e*g)*a*c^5*d^8*e^3*f*g^6 + 180*
sqrt(e^2*f - d*e*g)*a^2*c^4*d^6*e^5*f*g^6 + 30*sqrt(e^2*f - d*e*g)*a^3*c^3*
d^4*e^7*f*g^6 - 15*sqrt(e^2*f - d*e*g)*a^4*c^2*d^2*e^9*f*g^6 - 80*sqrt(e^2*
f - d*e*g)*c^6*d^11*g^7 + 140*sqrt(e^2*f - d*e*g)*a*c^5*d^9*e^2*g^7 - 65*sq
rt(e^2*f - d*e*g)*a^2*c^4*d^7*e^4*g^7 + 5*sqrt(e^2*f - d*e*g)*a^3*c^3*d^5*e
^6*g^7 - 10*sqrt(e^2*f - d*e*g)*a^4*c^2*d^3*e^8*g^7 + 5*sqrt(e^2*f - d*e*g)
*a^5*c*d*e^10*g^7 - 14*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^5*d^5*e
^5*f^5*g + 85*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^5*d^6*e^4*f^4*g^
2 - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^4*d^4*e^6*f^4*g^2 - 2
75*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^5*d^7*e^3*f^3*g^3 + 210*sqr
t(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^4*d^5*e^5*f^3*g^3 - 75*sqrt(-c*
d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*c^3*d^3*e^7*f^3*g^3 + 425*sqrt(-c*d^
2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^5*d^8*e^2*f^2*g^4 - 450*sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^4*d^6*e^4*f^2*g^4 + 135*sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*sqrt(c*d*g)*a^2*c^3*d^4*e^6*f^2*g^4 + 30*sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*sqrt(c*d*g)*a^3*c^2*d^2*e^8*f^2*g^4 - 300*sqrt(-c*d^2*e*g^2 + a*e^3
*g^2)*sqrt(c*d*g)*c^5*d^9*e*f*g^5 + 350*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt
(c*d*g)*a*c^4*d^7*e^3*f*g^5 - 75*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)
*a^2*c^3*d^5*e^5*f*g^5 - 40*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^3*
c^2*d^3*e^7*f*g^5 - 5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^4*c*d*e^
9*f*g^5 + 80*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^5*d^10*g^6 - 100*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^4*d^8*e^2*g^6 + 25*sqrt(-c*d
^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*c^3*d^6*e^4*g^6 + 10*sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*sqrt(c*d*g)*a^4*c*d^2*e^8*g^6 - sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*sqrt(c*d*g)*a^5*e^10*g^6)/(5*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^7*d^8*e^4
*f^8*g*abs(g) - 5*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^6*e^6*f^8*g*abs(g
) - 30*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^7*d^9*e^3*f^7*g^2*abs(g) + 20*sqrt
(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^7*e^5*f^7*g^2*abs(g) + 10*sqrt(e^2*f -

d*e*g)*sqrt(c*d*g)*a^2*c^5*d^5*e^7*f^7*g^2*abs(g) + 61*sqrt(e^2*f - d*e*g)*
sqrt(c*d*g)*c^7*d^10*e^2*f^6*g^3*abs(g) + 27*sqrt(e^2*f - d*e*g)*sqrt(c*d*g
)*a*c^6*d^8*e^4*f^6*g^3*abs(g) - 97*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5
*d^6*e^6*f^6*g^3*abs(g) + 9*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^4*e^8
*f^6*g^3*abs(g) - 52*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^7*d^11*e*f^5*g^4*abs
(g) - 158*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^9*e^3*f^5*g^4*abs(g) + 15
6*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*d^7*e^5*f^5*g^4*abs(g) + 90*sqrt(
e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^5*e^7*f^5*g^4*abs(g) - 36*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*a^4*c^3*d^3*e^9*f^5*g^4*abs(g) + 16*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*c^7*d^12*f^4*g^5*abs(g) + 180*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*
a*c^6*d^10*e^2*f^4*g^5*abs(g) + 35*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*
d^8*e^4*f^4*g^5*abs(g) - 295*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^6*e^
6*f^4*g^5*abs(g) + 35*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^4*e^8*f^4*g
^5*abs(g) + 29*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^2*e^10*f^4*g^5*abs
(g) - 64*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^11*e*f^3*g^6*abs(g) - 200*
sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*d^9*e^3*f^3*g^6*abs(g) + 220*sqrt(e
^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^7*e^5*f^3*g^6*abs(g) + 130*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*a^4*c^3*d^5*e^7*f^3*g^6*abs(g) - 80*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*a^5*c^2*d^3*e^9*f^3*g^6*abs(g) - 6*sqrt(e^2*f - d*e*g)*sqrt(c*
d*g)*a^6*c*d*e^11*f^3*g^6*abs(g) + 96*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c
^5*d^10*e^2*f^2*g^7*abs(g) + 40*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^8
*e^4*f^2*g^7*abs(g) - 205*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^6*e^6*f
^2*g^7*abs(g) + 45*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^4*e^8*f^2*g^7*
abs(g) + 25*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^6*c*d^2*e^10*f^2*g^7*abs(g) -
sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^7*e^12*f^2*g^7*abs(g) - 64*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*a^3*c^4*d^9*e^3*f*g^8*abs(g) + 60*sqrt(e^2*f - d*e*g)*sq
rt(c*d*g)*a^4*c^3*d^7*e^5*f*g^8*abs(g) + 34*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
*a^5*c^2*d^5*e^7*f*g^8*abs(g) - 32*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^6*c*d^
3*e^9*f*g^8*abs(g) + 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^7*d*e^11*f*g^8*abs
(g) + 16*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^8*e^4*g^9*abs(g) - 28*sq
rt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^6*e^6*g^9*abs(g) + 13*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*a^6*c*d^4*e^8*g^9*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*
g)*a^7*d^2*e^10*g^9*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^7*e^5*f^9
*abs(g) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^8*e^4*f^8*g*abs(g) + 6*sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^6*e^6*f^8*g*abs(g) + 55*sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*c^7*d^9*e^3*f^7*g^2*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*a*c^6*d^7*e^5*f^7*g^2*abs(g) - 29*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c
^5*d^5*e^7*f^7*g^2*abs(g) - 85*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^10*e^2*
f^6*g^3*abs(g) - 130*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^8*e^4*f^6*g^3*a
bs(g) + 95*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5*d^6*e^6*f^6*g^3*abs(g) +
36*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^4*e^8*f^6*g^3*abs(g) + 60*sqrt(
-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^11*e*f^5*g^4*abs(g) + 270*sqrt(-c*d^2*e*g^2
+ a*e^3*g^2)*a*c^6*d^9*e^3*f^5*g^4*abs(g) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*a^2*c^5*d^7*e^5*f^5*g^4*abs(g) - 180*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3
*c^4*d^5*e^7*f^5*g^4*abs(g) - 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^3*

e^9*f^5*g^4*abs(g) - 16*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^12*f^4*g^5*abs
(g) - 220*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^10*e^2*f^4*g^5*abs(g) - 23
5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5*d^8*e^4*f^4*g^5*abs(g) + 260*sqrt(
-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^6*e^6*f^4*g^5*abs(g) + 95*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*a^4*c^3*d^4*e^8*f^4*g^5*abs(g) - 10*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*a^5*c^2*d^2*e^10*f^4*g^5*abs(g) + 64*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*a*c^6*d^11*e*f^3*g^6*abs(g) + 280*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5
*d^9*e^3*f^3*g^6*abs(g) - 60*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^7*e^5
*f^3*g^6*abs(g) - 215*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^5*e^7*f^3*g^
6*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^5*c^2*d^3*e^9*f^3*g^6*abs(g)
 + 5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*c*d*e^11*f^3*g^6*abs(g) - 96*sqrt(-
c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5*d^10*e^2*f^2*g^7*abs(g) - 120*sqrt(-c*d^2*
e*g^2 + a*e^3*g^2)*a^3*c^4*d^8*e^4*f^2*g^7*abs(g) + 165*sqrt(-c*d^2*e*g^2 +
 a*e^3*g^2)*a^4*c^3*d^6*e^6*f^2*g^7*abs(g) + 30*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*a^5*c^2*d^4*e^8*f^2*g^7*abs(g) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*
c*d^2*e^10*f^2*g^7*abs(g) + 64*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^9*e
^3*f*g^8*abs(g) - 20*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^7*e^5*f*g^8*a
bs(g) - 50*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^5*c^2*d^5*e^7*f*g^8*abs(g) + 15
*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*c*d^3*e^9*f*g^8*abs(g) - 16*sqrt(-c*d^2
*e*g^2 + a*e^3*g^2)*a^4*c^3*d^8*e^4*g^9*abs(g) + 20*sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*a^5*c^2*d^6*e^6*g^9*abs(g) - 5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*c
*d^4*e^8*g^9*abs(g))

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,\left(a\,e\,g-c\,d\,f\right)^4}+\frac{\sqrt{d+e\,x}\,\left(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,a\,c^2\,d\cdots\right.}{c\,d\,e\,g^2\,\left(a\,e\,g-c\,d\,f\right)^4}\cdots\right.}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+2\,c\cdots\right.}{c\,d\,e\,g}\cdots}$$

[In] int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)

[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*c
^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + ((d
 + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g - 2*
a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(d +
e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d + e*
x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f + g*x)
^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*e^2*f*g

```
 + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + 2*
c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*a*d*e*g))
/(c*d*e*g^2))
```

3.727 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 289

$$\begin{aligned}\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\ &\quad -\frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{5g^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}} \\ &\quad +\frac{5g^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)-10/3*g*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+5*g^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2
)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{880, 884, 905, 65, 223, 212\}$

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
```

```
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(5g)\int\frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}\\
&= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\
&\quad-\frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(5g^2\right)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{c^2d^2}\\
&= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&\quad+\frac{5g^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}\\
&\quad+\frac{\left(5g^2(cdf-aeg)\right)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2c^3d^3}
\end{aligned}
$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^3d^3\sqrt{d+ex}} + \frac{\left(5g^2(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{2c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^3d^3\sqrt{d+ex}} + \frac{\left(5g^2(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^3d^3\sqrt{d+ex}} + \frac{\left(5g^2(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{c^4d^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^3d^3\sqrt{d+ex}} + \frac{5g^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{(d+ex)^{5/2}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(15a^2e^2g^2-10acdeg(f-2\right.}{3c}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

[Out] ((d + e*x)^(5/2)*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*a^2*e^2*g
^2 - 10*a*c*d*e*g*(f - 2*g*x) + c^2*d^2*(-2*f^2 - 14*f*g*x + 3*g^2*x^2)) +
15*g^(3/2)*(c*d*f - a*e*g)*(a*e + c*d*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(7/2)*d^(7/2)*((a*e + c*d*x
)*(d + e*x))^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(245) = 490$.

Time = 0.55 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^{3}d^{3}f\,g^{2}x^{2}+30\ln\left(\right.}{}$

[In] int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)

[Out] -1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*g^2
*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+
f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2-6*c^2
*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-40*(c*d*g)^(1/2)*((g
x+f)(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e
))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2
g^2+20((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+4*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*
x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)^2/c^3/d^3/
(e*x+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.89 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.65

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \left[\frac{4\,(3\,c^2d^2g^2x^2-2\,c^2d^2f^2-10\,acdefg+15\,a^2e^2g^2-2\,(7\,c^2d^2fg-\right.$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g
^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3
*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^
2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d
*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d
^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*
d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 +
(c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x), 1/6
*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g^2 -
2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3*g^2
+ (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*
g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d*e^3)
*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g
/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g
)*x)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 +
 (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(245) = 490$.

Time = 0.86 (sec) , antiderivative size = 1105, normalized size of antiderivative = 3.82

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{15\sqrt{-cd^2eg^2+ae^3g^2}c^2d^3efg^3\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg\cdots}\right|\right)}{\cdots} - \frac{\sqrt{e^2f+(ex+d)eg-deg}\left((e^2f+(ex+d)eg-deg)\left(\frac{3\,(c^5d^5e^2fg^5-ac^4d^4e^3g^6)\,(e^2f+(ex+d)eg-deg)}{c^6d^6e^4fg|g|-ac^5d^5e^5g^2|g|}-\frac{20\,(c^5d^5e^4f^2g^5-2\cdots}{c^6d^6e^4fg\cdots}\cdots\right)\right)}{3\,(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)\sqrt{-cde^2fg+ae^3g\cdots}} - \frac{5\,(cdfg^3-aeg^4)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)\cdots}\right|\right)}{\sqrt{cdg}c^3d^3|g|}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] 1/3*(15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^2*d^3*e*f*g^3*log(abs(-sqrt(e^2*f
- d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 15*sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*a*c*d*e^3*f*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) +
 sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c*d
```

```
^2*e^2*g^4*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a
*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^4*g^4*log(abs(-sqrt(e
^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g^2 + 14*sqrt(e^2*f - d*e*g)*sqrt(c*d
*g)*c^2*d^3*e*f*g^3 - 10*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g^3 +
3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^4 - 20*sqrt(e^2*f - d*e*g)*sqrt
(c*d*g)*a*c*d^2*e^2*g^4 + 15*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^4)/(
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^4*d^5*e*abs(g) - sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^3*d^3*e^3*abs(g)) - 1/3*sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(3*(c^5*d^5*e^2*f*g^5 -
 a*c^4*d^4*e^3*g^6)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^6*d^6*e^4*f*g*abs(g)
 - a*c^5*d^5*e^5*g^2*abs(g)) - 20*(c^5*d^5*e^4*f^2*g^5 - 2*a*c^4*d^4*e^5*f*
g^6 + a^2*c^3*d^3*e^6*g^7)/(c^6*d^6*e^4*f*g*abs(g) - a*c^5*d^5*e^5*g^2*abs(
g))) + 15*(c^5*d^5*e^6*f^3*g^5 - 3*a*c^4*d^4*e^7*f^2*g^6 + 3*a^2*c^3*d^3*e^
8*f*g^7 - a^3*c^2*d^2*e^9*g^8)/(c^6*d^6*e^4*f*g*abs(g) - a*c^5*d^5*e^5*g^2*
abs(g)))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)
*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) -
5*(c*d*f*g^3 - a*e*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c
*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d
*g)))/(sqrt(c*d*g)*c^3*d^3*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{\left(f+g\,x\right)^{5/2}\,\left(d+e\,x\right)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

3.728 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 219

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+2*g^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2)
)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-2*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {880, 905, 65, 223, 212}

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g^2\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{c^2d^2}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(g^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(2g^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(2g^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(3aeg+cd(f+4gx))-3g^{3/2}(ae+cdx)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{3c^{5/2}d^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(3*a*e*g + c*d*(f + 4*g*
x)) - 3*g^(3/2)*(a*e + c*d*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])
/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(5/2)*d^(5/2)*((a*e + c*d*x)*(d + e*x)
)^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{3\sqrt{cdg}\,(}$

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g^2
*x^2+6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^2-8*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a
*e))^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(c*d*g)
^(1/2)/(c*d*x+a*e)^2/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.45

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \left[-\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(4\,cdgx+cdf+3\,aeg)\sqrt{ex+d}\sqrt{}}{}\right.$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c
^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))*lo
g(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8
*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d
^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)
*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 + a^2*c^2
*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e
^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*
d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2
*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 +
 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(183) = 366$.

Time = 0.65 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.13

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{2\left(3\sqrt{-cd^2eg^2+ae^3g^2}cd^2g^3\log\left(\left|-\sqrt{e^2f-deg}\sqrt{cdg}+\sqrt{-cd^2eg\right.\right.\right.}}{}$$
$$+\frac{2\sqrt{e^2f+(ex+d)eg-deg}\left(\frac{4\,(c^3d^3e^2fg^4-ac^2d^2e^3g^5)(e^2f+(ex+d)eg-deg)}{c^4d^4e^2f|g|-ac^3d^3e^3g|g|}-\frac{3\,(c^3d^3e^4f^2g^4-2\,ac^2d^2e^5fg^5+a^2cde^6g^6)}{c^4d^4e^2f|g|-ac^3d^3e^3g|g|}\right)}{3\,(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}$$
$$-\frac{2\,g^3\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}c^2d^2|g|}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c*d^2*g^3*log(abs(-sqrt(e^2*f - d*e*g
)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*a*e^2*g^3*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e
*g^2 + a*e^3*g^2))) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*g^2 + 4*sqrt(
e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g^3 - 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*
e^2*g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^3*d^4*abs(g) - sqrt(
-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^2*d^2*e^2*abs(g)) + 2/3*sqrt(e^2*
f + (e*x + d)*e*g - d*e*g)*(4*(c^3*d^3*e^2*f*g^4 - a*c^2*d^2*e^3*g^5)*(e^2*
f + (e*x + d)*e*g - d*e*g)/(c^4*d^4*e^2*f*abs(g) - a*c^3*d^3*e^3*g*abs(g))
- 3*(c^3*d^3*e^4*f^2*g^4 - 2*a*c^2*d^2*e^5*f*g^5 + a^2*c*d*e^6*g^6)/(c^4*d^
4*e^2*f*abs(g) - a*c^3*d^3*e^3*g*abs(g)))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
(e*x + d)*e*g - d*e*g)*c*d*g)) - 2*g^3*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e
*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{5/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

3.729 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
```

```
0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d +
 e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2(gx+f)^{\frac{3}{2}}\sqrt{(cdx+ae)(ex+d)}}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)}$	55
gosper	$\frac{2(gx+f)^{\frac{3}{2}}(cdx+ae)(ex+d)^{\frac{5}{2}}}{3(aeg-cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	63

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/3/(e*x+d)^(1/2)*(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/(
a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx =$$

$$-\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{3\,(a^2cd^2e^2f-a^3de^3g+(c^3d^3ef-ac^2d^2e^2g)x^3+((c^3d^4+2\,ac^2d^2e^2)f-(ac^2d^3e+2\,a^2cde^3)g)x^2+((2\,ac^2}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(3
/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3 +
((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 + ((2
*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(55) = 110.

Time = 0.45 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.02

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(e^2f+(ex+d)eg-}{3\,(c^2d^2e^2f|g|-acde^3g|g|)(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-d}$$
$$-\frac{2\left(\sqrt{e^2f-deg}efg^2-\sqrt{e^2f-deg}dg^3\right)}{3\left(\sqrt{-cd^2eg^2+ae^3g^2}c^2d^3f|g|-\sqrt{-cd^2eg^2+ae^3g^2}acde^2f|g|-\sqrt{-cd^2eg^2+ae^3g^2}acd^2eg|g|+\sqrt{-cd^2eg^2+}\right.}$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] 2/3*(e^2*f + (e*x + d)*e*g - d*e*g)^(3/2)*c*d*e^2*g^4/((c^2*d^2*e^2*f*abs(g
) - a*c*d*e^3*g*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e
*g)*c*d*g)) - 2/3*(sqrt(e^2*f - d*e*g)*e*f*g^2 - sqrt(e^2*f - d*e*g)*d*g^3)
/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^2*d^3*f*abs(g) - sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*a*c*d*e^2*f*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c*d^2*e*g*abs
(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*e^3*g*abs(g))
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\left(\frac{2\,f\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}+\frac{x^2\,(c\,d^2+2\,a\,e^2)}{c\,d\,e}}$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] (((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2*
g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^
2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

3.730 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 128

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} + \frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3(cdf-aeg)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4/3*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {882, 874}

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad -\frac{(2g)\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3(cdf-aeg)} \\
&= -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}(-3aeg+cd(f-2gx))}{3(cdf-aeg)^2((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(-3*a*e*g + c*d*(f - 2*g*x)))/(3*(c*d*f -
 a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+3aeg-cdf)}{3\sqrt{ex+d}\,(cdx+ae)^{2}(aeg-cdf)^{2}}$	72
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(2cdgx+3aeg-cdf)(ex+d)^{\frac{5}{2}}}{3(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	99

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/3/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+3*a*
e*g-c*d*f)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(112) = 224.

Time = 0.44 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{}{3\,(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+(c^4d^4ef^2-2\,ac}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g +
 a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2
)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e
^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^
2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2
*e^3 + a^4*e^5)*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=$$
$$-\frac{2\left(\frac{2\,(e^2f+(ex+d)eg-deg)c^2d^2g^4}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3fg|g|+a^2cde^4g^2|g|}-\frac{3\,(c^2d^2e^2fg^4-acde^3g^5)}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3fg|g|+a^2cde^4g^2|g|}\right)\sqrt{e^2f+(ex+d)eg-deg}e^2}{3\,(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}}$$
$$-\frac{2\left(\sqrt{e^2f-deg}cdefg^2+2\sqrt{e^2f-deg}cd\cdots\right.}{3\left(\sqrt{-cd^2eg^2+ae^3g^2}c^3d^4f^2|g|-\sqrt{-cd^2eg^2+ae^3g^2}ac^2d^2e^2f^2|g|-2\sqrt{-cd^2eg^2+ae^3g^2}ac^2d^3efg|g|+2\sqrt{-\cdots}\right.}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] -2/3*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^4/(c^3*d^3*e^2*f^2*abs(g)
 - 2*a*c^2*d^2*e^3*f*g*abs(g) + a^2*c*d*e^4*g^2*abs(g)) - 3*(c^2*d^2*e^2*f*
g^4 - a*c*d*e^3*g^5)/(c^3*d^3*e^2*f^2*abs(g) - 2*a*c^2*d^2*e^3*f*g*abs(g) +
 a^2*c*d*e^4*g^2*abs(g)))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*e^2/((c*d*e^2
```

```
*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g
 + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 2/3*(sqrt(e^2*f -
d*e*g)*c*d*e*f*g^2 + 2*sqrt(e^2*f - d*e*g)*c*d^2*g^3 - 3*sqrt(e^2*f - d*e*g
)*a*e^2*g^3)/(sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^3*d^4*f^2*abs(g) - sqrt(-c*d
^2*e*g^2 + a*e^3*g^2)*a*c^2*d^2*e^2*f^2*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*a*c^2*d^3*e*f*g*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c*d*e^
3*f*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c*d^2*e^2*g^2*abs(g) - sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*e^4*g^2*abs(g))
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,(a\,e\,g-c\,d\,f)^2}-\frac{(2\,c\cdots}{3\cdots}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,(c\,d^2+\cdots}{c\,d\,e}}$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2))
/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/(3
*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/2))
/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g*x)^
(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*x*(f
+ g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.731 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 194

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}+\frac{8g\sqrt{d+ex}}{3(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{16g^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(
g*x+f)^(1/2)+8/3*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{882,\ 874\}$

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}$$
$$+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$
$$-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\text{integral} = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$-\frac{(4g)\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3(cdf-aeg)}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{8g\sqrt{d+ex}}{3(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{(8g^2)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3(cdf-aeg)^2}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{8g\sqrt{d+ex}}{3(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{16g^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53

$$\int\frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\,(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2(-f^2+}{3(cdf-aeg)^3((ae+cdx)(d+ex))^{3/2}\sqrt{f+}}$$

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^2
 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3
/2)*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-c^2d^2f^2)}{3\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)^2(aeg-cdf)^3}$	121
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-c^2d^2f^2)(ex+d)^{\frac{5}{2}}}{3\sqrt{gx+f}\,(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	169

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g^2
*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*e^2*g^2+6*a*c*d*e*f*g-c^2*d^2*f
^2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(170) = 340$.

Time = 0.53 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.44

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\cdots}{3\left(a^2c^3d^4e^2f^4-3\,a^3c^2d^3e^3f^3g+3\,a^4cd^2e^4f^2g^2-a^5de^5\cdots\right.}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(c
^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g
+ 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^
4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d
^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e
^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*
e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d
^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2
*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6
)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5
)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. $2(170) = 340$.

Time = 0.50 (sec) , antiderivative size = 1923, normalized size of antiderivative = 9.91

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] 2/3*(6*sqrt(c*d*g)*g^3/((c^2*d^2*e*f^2*abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a^
2*e^3*g^2*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g -
d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g))^2)) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(5*(c^5*d^5*e*f
^2*g^4*abs(g) - 2*a*c^4*d^4*e^2*f*g^5*abs(g) + a^2*c^3*d^3*e^3*g^6*abs(g))*
(e^2*f + (e*x + d)*e*g - d*e*g)/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^5*f^4*
g^3 + 10*a^2*c^4*d^4*e^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^4*c^2*d
^2*e^8*f*g^6 - a^5*c*d*e^9*g^7) - 6*(c^5*d^5*e^3*f^3*g^4*abs(g) - 3*a*c^4*d
^4*e^4*f^2*g^5*abs(g) + 3*a^2*c^3*d^3*e^5*f*g^6*abs(g) - a^3*c^2*d^2*e^6*g^
7*abs(g))/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^5*f^4*g^3 + 10*a^2*c^4*d^4*e
^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^4*c^2*d^2*e^8*f*g^6 - a^5*c*d
*e^9*g^7))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*
g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))
*e^3 - 2/3*(sqrt(e^2*f - d*e*g)*c^3*d^3*e^2*f^2*g^2 + 4*sqrt(e^2*f - d*e*g)
*c^3*d^4*e*f*g^3 - 6*sqrt(e^2*f - d*e*g)*a*c^2*d^2*e^3*f*g^3 - 5*sqrt(e^2*f
 - d*e*g)*c^3*d^5*g^4 + 6*sqrt(e^2*f - d*e*g)*a*c^2*d^3*e^2*g^4 - sqrt(-c*d
^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*g - sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(c*d*g)*c^2*d^3*e*f*g^2 + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sq
rt(c*d*g)*a*c*d*e^3*f*g^2 + 5*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^
2*d^4*g^3 - 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c*d^2*e^2*g^3 +
3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*e^4*g^3)/(sqrt(e^2*f - d*e
```

```
*g)*sqrt(c*d*g)*c^5*d^7*f^3*g*abs(g) - 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*
c^4*d^5*e^2*f^3*g*abs(g) + sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^3*d^3*e^4*
f^3*g*abs(g) - 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^4*d^6*e*f^2*g^2*abs(g)
 + 6*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^3*d^4*e^3*f^2*g^2*abs(g) - 3*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^2*d^2*e^5*f^2*g^2*abs(g) + 3*sqrt(e^2*f
- d*e*g)*sqrt(c*d*g)*a^2*c^3*d^5*e^2*f*g^3*abs(g) - 6*sqrt(e^2*f - d*e*g)*s
qrt(c*d*g)*a^3*c^2*d^3*e^4*f*g^3*abs(g) + 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
*a^4*c*d*e^6*f*g^3*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^2*d^4*e^3
*g^4*abs(g) + 2*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c*d^2*e^5*g^4*abs(g) -
sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*e^7*g^4*abs(g) + sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*c^5*d^6*e*f^4*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^4*d^4*e^
3*f^4*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^5*d^7*f^3*g*abs(g) - 2*sqrt
(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^4*d^5*e^2*f^3*g*abs(g) + 3*sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*a^2*c^3*d^3*e^4*f^3*g*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*a*c^4*d^6*e*f^2*g^2*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^2*d^
2*e^5*f^2*g^2*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^3*d^5*e^2*f*g
^3*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^2*d^3*e^4*f*g^3*abs(g) +
 sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c*d*e^6*f*g^3*abs(g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*a^3*c^2*d^4*e^3*g^4*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*
a^4*c*d^2*e^5*g^4*abs(g))
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^3}+\frac{\sqrt{d+e\,x}\,(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2)}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)^3}+\frac{8\,g\,x\,(3\,a\,e\,g+c\,d\,f)\,\sqrt{d+e}}{3\,c\,d\,e\,(a\,e\,g-c\,d\,f)^3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,(c\,d^2+2\,a\,e^2)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}}$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/2
))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*f^
2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g + c*
d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2) +
(a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(
c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.732 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 260

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{16g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} + \frac{32cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(3/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/3*c*d*g^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used

$= \{882, 886, 874\}$

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{32cdg^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}$$
$$+\frac{16g^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}$$
$$+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}$$
$$-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad -\frac{(2g)\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cdf-aeg} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{\left(8g^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{(cdf-aeg)^2} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{16g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}}+\frac{\left(16cdg^2\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3(cdf-aeg)^3} \\
&= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&\quad +\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&\quad +\frac{16g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}}+\frac{32cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(-a^3e^3g^3+3a^2cde^2g^2(3f+2gx)+3ac^2d\right.}{3(cdf-aeg)^4((\ldots}$$

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a*
c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x + 24
*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(
3/2)*(f + g*x)^(3/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e\ldots\right.}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)^2(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2f\,g^2-9a\ldots\right.}{3(gx+f)^{\frac{3}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\ldots}}$

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3*g
^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x-36*a
*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3*g^3-9*a^2*c*d*e^2*f*g^2-9*a*c^
2*d^2*e*f^2*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. 2(230) = 460.

Time = 1.03 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{\ldots}{3\left(a^2c^4d^5e^2f^6-4\,a^3c^3d^4e^3f^5g+6\,a^4c^2d^3e^4f^4g^2-4\,a^5\ldots\right.}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^2
*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*d^
3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 - 4*a
^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3
+ a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*
c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 +
(2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e -
 a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g
^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2
*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e
^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e
^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f
^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g -
2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*
c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d
^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a
^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e
^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f
^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6
*e^7)*f^2*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4668 vs. $2(230) = 460$.

Time = 3.08 (sec) , antiderivative size = 4668, normalized size of antiderivative = 17.95

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] -2/3*e^4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(8*(c^7*d^7*e^2*f^3*g^4*abs(g
) - 3*a*c^6*d^6*e^3*f^2*g^5*abs(g) + 3*a^2*c^5*d^5*e^4*f*g^6*abs(g) - a^3*c
^4*d^4*e^5*g^7*abs(g))*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^8*d^8*e^6*f^7*g^2
 - 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c^5*d^5*e^
9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2*g^7 + 7*a
^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9) - 9*(c^7*d^7*e^4*f^4*g^4*abs(g) -
 4*a*c^6*d^6*e^5*f^3*g^5*abs(g) + 6*a^2*c^5*d^5*e^6*f^2*g^6*abs(g) - 4*a^3*
c^4*d^4*e^7*f*g^7*abs(g) + a^4*c^3*d^3*e^8*g^8*abs(g))/(c^8*d^8*e^6*f^7*g^2
 - 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c^5*d^5*e^
9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2*g^7 + 7*a
^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9))/((c*d*e^2*f*g - a*e^3*g^2 - (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
 (e*x + d)*e*g - d*e*g)*c*d*g)) - 4*(4*sqrt(c*d*g)*c^3*d^3*e^4*f^2*g^5 - 8*
sqrt(c*d*g)*a*c^2*d^2*e^5*f*g^6 + 4*sqrt(c*d*g)*a^2*c*d*e^6*g^7 + 9*sqrt(c*
d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g +
 a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d^2*e^2*f*g^4 -
9*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*
e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c*d*e^3*g
^5 + 3*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*g^
3)/((c^3*d^3*e^2*f^3*abs(g) - 3*a*c^2*d^2*e^3*f^2*g*abs(g) + 3*a^2*c*d*e^4*
f*g^2*abs(g) - a^3*e^5*g^3*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3)) - 2/3*(sqrt(e^2*f - d*e*g)*c^5*d^
5*e^4*f^4*g^2 + 2*sqrt(e^2*f - d*e*g)*c^5*d^6*e^3*f^3*g^3 - 6*sqrt(e^2*f -
d*e*g)*a*c^4*d^4*e^5*f^3*g^3 - 24*sqrt(e^2*f - d*e*g)*c^5*d^7*e^2*f^2*g^4 +
 42*sqrt(e^2*f - d*e*g)*a*c^4*d^5*e^4*f^2*g^4 - 12*sqrt(e^2*f - d*e*g)*a^2*
c^3*d^3*e^6*f^2*g^4 + 56*sqrt(e^2*f - d*e*g)*c^5*d^8*e*f*g^5 - 120*sqrt(e^2
*f - d*e*g)*a*c^4*d^6*e^3*f*g^5 + 78*sqrt(e^2*f - d*e*g)*a^2*c^3*d^4*e^5*f*
g^5 - 18*sqrt(e^2*f - d*e*g)*a^3*c^2*d^2*e^7*f*g^5 - 32*sqrt(e^2*f - d*e*g)
*c^5*d^9*g^6 + 72*sqrt(e^2*f - d*e*g)*a*c^4*d^7*e^2*g^6 - 48*sqrt(e^2*f - d
*e*g)*a^2*c^3*d^5*e^4*g^6 + 6*sqrt(e^2*f - d*e*g)*a^3*c^2*d^3*e^6*g^6 + 3*s
qrt(e^2*f - d*e*g)*a^4*c*d*e^8*g^6 - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(
c*d*g)*c^4*d^4*e^4*f^4*g - 6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^4
```

*d^5*e^3*f^3*g^2 + 18*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^3*d^3*
e^5*f^3*g^2 + 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^4*d^6*e^2*f^2
*g^3 - 78*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^3*d^4*e^4*f^2*g^3
+ 12*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*c^2*d^2*e^6*f^2*g^3 - 7
2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*c^4*d^7*e*f*g^4 + 120*sqrt(-c*
d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*c^3*d^5*e^3*f*g^4 - 42*sqrt(-c*d^2*e*g
^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*c^2*d^3*e^5*f*g^4 + 6*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(c*d*g)*a^3*c*d*e^7*f*g^4 + 32*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
*sqrt(c*d*g)*c^4*d^8*g^5 - 56*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a*
c^3*d^6*e^2*g^5 + 24*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^2*c^2*d^4
*e^4*g^5 - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^3*c*d^2*e^6*g^5 -
 sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(c*d*g)*a^4*e^8*g^5)/(3*sqrt(e^2*f - d*
e*g)*sqrt(c*d*g)*c^7*d^9*e^2*f^6*g*abs(g) - 6*sqrt(e^2*f - d*e*g)*sqrt(c*d*
g)*a*c^6*d^7*e^4*f^6*g*abs(g) + 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*d
^5*e^6*f^6*g*abs(g) - 7*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^7*d^10*e*f^5*g^2*
abs(g) + 3*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^8*e^3*f^5*g^2*abs(g) + 1
5*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*d^6*e^5*f^5*g^2*abs(g) - 11*sqrt(
e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^4*e^7*f^5*g^2*abs(g) + 4*sqrt(e^2*f -
d*e*g)*sqrt(c*d*g)*c^7*d^11*f^4*g^3*abs(g) + 19*sqrt(e^2*f - d*e*g)*sqrt(c*
d*g)*a*c^6*d^9*e^2*f^4*g^3*abs(g) - 36*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*
c^5*d^7*e^4*f^4*g^3*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^5*e^
6*f^4*g^3*abs(g) + 14*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^3*e^8*f^4*g
^3*abs(g) - 16*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^6*d^10*e*f^3*g^4*abs(g)
- 6*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^5*d^8*e^3*f^3*g^4*abs(g) + 54*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^6*e^5*f^3*g^4*abs(g) - 26*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^4*e^7*f^3*g^4*abs(g) - 6*sqrt(e^2*f - d*e*g
)*sqrt(c*d*g)*a^5*c^2*d^2*e^9*f^3*g^4*abs(g) + 24*sqrt(e^2*f - d*e*g)*sqrt(
c*d*g)*a^2*c^5*d^9*e^2*f^2*g^5*abs(g) - 26*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*
a^3*c^4*d^7*e^4*f^2*g^5*abs(g) - 21*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3
*d^5*e^6*f^2*g^5*abs(g) + 24*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^3*e^
8*f^2*g^5*abs(g) - sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^6*c*d*e^10*f^2*g^5*abs
(g) - 16*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*c^4*d^8*e^3*f*g^6*abs(g) + 29*
sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^6*e^5*f*g^6*abs(g) - 9*sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^4*e^7*f*g^6*abs(g) - 5*sqrt(e^2*f - d*e*g)*
sqrt(c*d*g)*a^6*c*d^2*e^9*f*g^6*abs(g) + sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^
7*e^11*f*g^6*abs(g) + 4*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*c^3*d^7*e^4*g^7
*abs(g) - 9*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^5*c^2*d^5*e^6*g^7*abs(g) + 6*
sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^6*c*d^3*e^8*g^7*abs(g) - sqrt(e^2*f - d*e
*g)*sqrt(c*d*g)*a^7*d*e^10*g^7*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*
d^8*e^3*f^7*abs(g) - sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^6*e^5*f^7*abs(g
) - 6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^9*e^2*f^6*g*abs(g) + 5*sqrt(-c*d
^2*e*g^2 + a*e^3*g^2)*a*c^6*d^7*e^4*f^6*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*a^2*c^5*d^5*e^6*f^6*g*abs(g) + 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*
d^10*e*f^5*g^2*abs(g) + 9*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^8*e^3*f^5*
g^2*abs(g) - 24*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5*d^6*e^5*f^5*g^2*abs(

```
g) + 6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^4*e^7*f^5*g^2*abs(g) - 4*sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*c^7*d^11*f^4*g^3*abs(g) - 29*sqrt(-c*d^2*e*g^2
 + a*e^3*g^2)*a*c^6*d^9*e^2*f^4*g^3*abs(g) + 21*sqrt(-c*d^2*e*g^2 + a*e^3*g
^2)*a^2*c^5*d^7*e^4*f^4*g^3*abs(g) + 26*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*
c^4*d^5*e^6*f^4*g^3*abs(g) - 14*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^3*
e^8*f^4*g^3*abs(g) + 16*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a*c^6*d^10*e*f^3*g^4
*abs(g) + 26*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^2*c^5*d^8*e^3*f^3*g^4*abs(g)
- 54*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^6*e^5*f^3*g^4*abs(g) + sqrt(-
c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^4*e^7*f^3*g^4*abs(g) + 11*sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*a^5*c^2*d^2*e^9*f^3*g^4*abs(g) - 24*sqrt(-c*d^2*e*g^2 + a*
e^3*g^2)*a^2*c^5*d^9*e^2*f^2*g^5*abs(g) + 6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*
a^3*c^4*d^7*e^4*f^2*g^5*abs(g) + 36*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*
d^5*e^6*f^2*g^5*abs(g) - 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^5*c^2*d^3*e^8*
f^2*g^5*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*c*d*e^10*f^2*g^5*abs(
g) + 16*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^3*c^4*d^8*e^3*f*g^6*abs(g) - 19*sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^3*d^6*e^5*f*g^6*abs(g) - 3*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*a^5*c^2*d^4*e^7*f*g^6*abs(g) + 6*sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*a^6*c*d^2*e^9*f*g^6*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^4*c^
3*d^7*e^4*g^7*abs(g) + 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^5*c^2*d^5*e^6*g^7
*abs(g) - 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*a^6*c*d^3*e^8*g^7*abs(g))
```

Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g\,x^2\,\left(a\,e\,g+c\,d\,f\right)\,\sqrt{d+e}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right.}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,\left(g\,a^2\,e^3+2\,g\,a\,c\,d^2\right.}{c^2\,d^2\,e\,g}}$$

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*(
d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 + 2
*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*e*g
*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)
^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g))/(c*
d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*(a^2*e
^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (a*x*(f
+ g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*f*(f + g
*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g + c*d*e*f)
)/(c*d*e*g))
```

3.733 $\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 385

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] -5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(3/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/96*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+1/24*(a*e/c/d-f/
g)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/4*
(g*x+f)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-5/64*
(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/g/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}} - \frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{96c^2d^2g\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{24\sqrt{d+ex}}$$

```
[In] Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(5(cdf-aeg)^2)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{48cdg}$$

$$= -\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(5(cdf-aeg)^3)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{64c^2d^2g}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(5(cdf-aeg)^4)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128c^3d^3g}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g\sqrt{d+ex}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{128c^3d^3g\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{64c^4d^4g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{64c^4d^4g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.61

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(15a^3e^3g^3-5a\right.}{}$$

```
[In] Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(15*a
^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(73*f^2 + 36*
f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^2*x^2 + 48*g^3
*x^3)) - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqr
t[g]*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(192*c^(7/2)*d^(7/2)*g^(3/2)*
Sqrt[d + e*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(329) = 658$.

Time = 0.57 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}+15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-60\right.}{}$

```
[In] int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*
d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*
(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*a*c^2*d^2*e
*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-272*c^3*d^3*f*g^2*x^2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*
c^2*d^2*e*f*g^2*x-236*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f^2
*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3+110*((g*x+f)*
```

```
(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2-146*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e))^(1/2)/c^3/d^3/
(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 2.54 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.77

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \left[\frac{4\,(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g+73\,ac^3d^3ef^2g^2-55\,a^2c^2d^2e\right.$$

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2 -
 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g^2*x +
 c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d
^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g
^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g^2*x + c^4*d
^5*g^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
e*x + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6752 vs. 2(329) = 658.

Time = 2.15 (sec) , antiderivative size = 6752, normalized size of antiderivative = 17.54

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] 1/192*(48*f^2*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e
*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d
*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
```

)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d*g^3))*abs(e)^2/e^4 + g^2*((192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*e*f*abs(g)/g^2 - 192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^3*abs(g)/g + 8*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*e*f*abs(g)/g^2 - 24*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*d*abs(g)/g - 96*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(

c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*f*abs(g)/g^3 + 144*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^2*abs(g)/(e*g^2) + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*abs(g)/g)/(e^2*g) - (15*c^4*d^4*e^5*f^4*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a*c^3*d^3*e^6*f^3*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a^2*c^2*d^2*e^7*f^2*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a^3*c*d*e^8*f*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*a^4*e^9*g^5*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^3*e^3*f^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*e^2*f^2*g*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^5*e*f*g^2*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^3*f*g^2*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^5*f*g^2*abs(g) - 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^6*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^4*e^2*g^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e

^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^4*g^3*abs(g) + 15*sqrt(
-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^6*g^3*abs(g
))/(sqrt(c*d*g)*c^3*d^3*e^2*g^5))*abs(e)^2/e^4 - 16*f*g*((24*((c*d*e^2*f*g
- a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqr
t(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(
c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f
*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) +
sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sq
rt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*
e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3
*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e
^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7
- a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d
^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^
4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a
^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt
(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(
c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*
d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*
e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g
^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g -
d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g
+ a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d
)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2)
)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^
3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqr
t(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^
2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e
*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*
d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*lo
g(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) -
3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt
(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d
*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2
+ a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqr
t(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*a
bs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*
c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*s
qrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f

```
 - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^2*d
^2*g^4))*abs(e)^2/e^5)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{(f+g\,x)^{5/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

```
[In] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
[Out] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.734 $\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 313

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/12*(a*e/c/d-f/g)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{12\sqrt{d+ex}}$$

```
[In] Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] -1/8*((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^
(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((
c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
+ c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{6g}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)^2\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8cdg}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)^3\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16c^2d^2g}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{16c^2d^2g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{8c^3d^3g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{8c^3d^3g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.60

$$\int\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-3a^2e^2g^2+2ac\right.}{2}$$

```
[In] Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(-3*a
^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(3*f^2 + 14*f*g*x + 8*g^2*x^
2)) - (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]
*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(24*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt[
d + e*x])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.61

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{3}e^{3}g^{3}-9\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{}$

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^
3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/
2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*l
n(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+
a*e))^(1/2)+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+28*(c
*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)/(
e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.38 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.71

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2+3\,c^3d^3f^2g+8\,ac^2d^2efg^2-3\,a^2cde^2g^3+\right.$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2
*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 +
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
```

```
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2*c*d
*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2)]
```

Sympy [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
e*x + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3056 vs. $2(265) = 530$.

Time = 1.20 (sec) , antiderivative size = 3056, normalized size of antiderivative = 9.76

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

[Out] 1/24*(6*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d*g^3))*abs(e)^2/e^4 - g*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3

```
)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d
))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*
g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f -
 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*
d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(
g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2
))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*
g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(
-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3
*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqr
t(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2
*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sq
rt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^
2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)
*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^2*d^2*g^4))*abs(e)^2/e^5)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.735 $\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 241

$$\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -1/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)+1/4*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= -\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}} + \frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}} + \frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{4\sqrt{d+ex}}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*
x],x]
```

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{4g} \\
&= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}} \\
&\quad + \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}} \\
&\quad - \frac{(cdf-aeg)^2\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8cdg}
\end{aligned}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}}-\frac{\left((cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{8cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}}-\frac{\left((cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{4c^2d^2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}}-\frac{\left((cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{4c^2d^2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{f+gx}(aeg+cd(f+2gx))-(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[
d + e*x],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
*Sqrt[f + g*x]*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^2*ArcTanh[(Sqrt[
c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*c^(3/2)*d^(3/2)
*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{8\sqrt{}}$

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/8*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d
*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^2-
2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-4*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2
)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(e*x+d)^(1/2)/((
g*x+f)*(c*d*x+a*e))^(1/2)/c/d/g/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.29 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\left[\frac{4\,(2\,c^2d^2g^2x+c^2d^2fg+acdeg^2)\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}+(c^2d^3f^2-2\,acd^2efg+\right.$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*c*
d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)
*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g +
 a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x +
 c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
+ (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^2*x + c
^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f
^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a
^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a
*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^2*x + c^2*d^3*g^
2)]
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*
x + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(201) = 402$.

Time = 0.65 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.13

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\left(\frac{4\left(\frac{(cde^2fg-ae^3g^2)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{\cdots}\right.}{g^2}\cdots\right.}{\cdots}$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] 1/4*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*f*abs(g)/
g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*
x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*abs(g)/g
+ (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(
2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*
c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*
f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e
^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*
d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*
g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-sqrt(e^2*f
```

```
- d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d*e^4*f*g^2
*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^
3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sq
rt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f
- d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt
(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d*g^3))*abs
(e)^2/e^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.736 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 167

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(
1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/c^(1/2)/d^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {878, 905, 65, 223, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*x
]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

integral

$$= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{2g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{cdg\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}{g\sqrt{d+ex}} + \frac{(-cdf+aeg)\sqrt{(ae+cdx)(d+ex)}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f
 + g*x]),x]
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])/(g*Sqrt[d + e*x]) + ((-(c*d*f
) + a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*e + c*d
*x]*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf+2\sqrt{(}\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,g\sqrt{cdg}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/((
g*x+f)*(c*d*x+a*e))^(1/2)/g/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$=\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}cdg-(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{cdg}\log\left(-\frac{8\,c^2d^2}{}\right.}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g + (c*
d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2
*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*
d*e*g^2*x + c*d^2*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+d
)**(1/2),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}\sqrt{gx + f}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(g
*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(139) = 278$.

Time = 0.46 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx = \frac{\frac{\left(\frac{(cdef - ae^2g)\log\left(\left|-\sqrt{(ex+d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - acde^3g + \left((ex+d)cde - cd^2e + ae^3\right)cdg}\right|\right)}{\sqrt{cdg}g} + \frac{\sqrt{c^2d^2e^2f - acde^3g + \left((ex+d)cde - cd^2e + ae^3\right)cdg}\sqrt{}}{cdeg}}{cd}}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
[Out] (((c*d*e*f - a*e^2*g)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt
(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*c*d*g)))/(sqrt(c*d*g)*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
/(c*d*e*g))*abs(c)*abs(d)/(c*d) - (c^2*d^2*e^2*f*abs(c)*abs(d)*log(abs(-sqr
t(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - a*c
*d*e^3*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g))) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d
^2*e + a*e^3)*sqrt(c*d*g)*abs(c)*abs(d))/(sqrt(c*d*g)*c^2*d^2*e*g))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e}}{\sqrt{f + g\,x}\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)), x)
```

3.737 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 158

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*c^(1/2)*
d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {876, 905, 65, 223, 212}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{(cd)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{g}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(cd\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}dx,x,\sqrt{ae+cdx}\right)}{g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.84

$$\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}+\sqrt{c}\sqrt{d}\sqrt{f+gx}\text{arctanh}\left(\frac{\sqrt{c}\cdots}{\sqrt{g}\cdots}\right.\right.}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]) + Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt
[a*e + c*d*x])]))/(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf-2\sqrt{(gx+\cdots}\right.}{\sqrt{cdg}\sqrt{(gx+f)(cdx+ae)}\,g\sqrt{ex+d}\sqrt{gx+f}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] ((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*(2*c*d*g*x+a*e*g
```

```
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*f-2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((g*x+f)*(c*d*x+a*
e))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \left[\frac{(egx^2+df+(ef+dg)x)\sqrt{\frac{cd}{g}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g}{}\right.}{}\right.$$

$$-\frac{(egx^2+df+(ef+dg)x)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{-\frac{cd}{g}}g}{2\,cdegx^2+cd^2f+adeg+(cdef+(2\,cd^2+ae^2)g)x}\right)+2\sqrt{cdex^2+ade+(cd^2}}{eg^2x^2+dfg+(efg+dg^2)x}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g*x^2 +
d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g +
 (e*f*g + d*g^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(130) = 260$.

Time = 0.52 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=$$
$$-\frac{2\left(\frac{e^2|c||d|\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}\right|\right)}{\sqrt{cdg}g|e|}+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e^2|c}{\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e}}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] -2*(e^2*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt
(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*c*d*g)))/(sqrt(c*d*g)*g*abs(e)) + sqrt((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*e^2*abs(c)*abs(d)/(sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*c*d*g)*g*abs(e)) - (c^2*d^2*e^3*f*abs(c)*abs(d)*log(abs
(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)))
- c^2*d^3*e^2*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) +
 sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sq
rt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*e*abs(c)*abs(d))/(sqrt(c*d*g)*c^2*d^2*e*f*
g*abs(e) - sqrt(c*d*g)*c^2*d^3*g^2*abs(e)))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{3/2}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e}}{(f + g\,x)^{3/2}\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)), x)
```

3.738 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(5
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
```

```
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{3(cdf - aeg)(d + ex)^{3/2}(f + gx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \frac{2((ae + cdx)(d + ex))^{3/2}}{3(cdf - aeg)(d + ex)^{3/2}(f + gx)^{3/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(5/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f +
 g*x)^(3/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	53
gosper	$-\frac{2(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	63

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)/(e
*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(c}{3\,(cd^2f^3-adef^2g+(cdefg^2-ae^2g^3)x^3+(2\,cdef^2g-adeg^3+(cd}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 + (
2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 - 2*a
*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(55) = 110$.

Time = 0.51 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \frac{2\left(\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2e^4g|c||d|}{(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg)^{\frac{3}{2}}(cde^2fg|e|-ae^3g^2|e|)} + \frac{}{\sqrt{c^2d^2e^2f-c^2 \cdots}}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] 2/3*(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*g*abs(c)*abs(d)/
((c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^
(3/2)*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*abs(e))) + (sqrt(-c*d^2*e + a*e^3)*c*
d^2*e^2*abs(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a*e^4*abs(c)*abs(d))/(sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e*f^2*abs(e) - sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*c*d^2*f*g*abs(e) - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^2*f*g*abs(e)
 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*d*e*g^2*abs(e)))*abs(e)/e^2
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = -\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g} + \frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\,\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x} - \frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2 - 3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^(1
/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g^2
- 3*c*d*f*g))
```

3.739 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}} dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(5/2)+4/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}} dx = \frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(7
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{(2cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx}{5(cdf-aeg)}\\ &= \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}(-3aeg+cd(5f+2gx))}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d
*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)(-2cdgx+3aeg-5cdf)}{15(gx+f)^{\frac{5}{2}}\sqrt{ex+d}\,(aeg-cdf)^{2}}$	70
gosper	$-\frac{2(cdx+ae)(-2cdgx+3aeg-5cdf)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{15(gx+f)^{\frac{5}{2}}(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2})\sqrt{ex+d}}$	99

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/15*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*(
-2*c*d*g*x+3*a*e*g-5*c*d*f)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(113) = 226.

Time = 0.35 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.12

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{}{15\,(c^2d^3f^5-2\,acd^2ef^4g+a^2de^2f^3g^2+(c^2d^2ef^2g^3-2\,acde^2fg^4+a}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2*g^
3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d*e^2
*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^4)*x
^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^2
 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e^2*f^2*
g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(113) = 226$.

Time = 0.84 (sec) , antiderivative size = 762, normalized size of antiderivative = 5.91

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2\left(\frac{5\sqrt{-cd^2e}}{\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^2e^2f^4|e|-2\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^3ef^3g|e|-2\sqrt{c^2d^2e^2f-c^2d^3eg}ac}\right.}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] 2/15*((5*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*e^3*f*abs(c)*abs(d) - 5*sqrt(-c*d^2
*e + a*e^3)*a*c*d*e^5*f*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*e^
2*g*abs(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^4*g*abs(c)*abs(d) + 3*
sqrt(-c*d^2*e + a*e^3)*a^2*e^6*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d
^3*e*g)*c^2*d^2*e^2*f^4*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^
3*e*f^3*g*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f^3*g*abs(
e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4*f^2*g^2*abs(e) + 4*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^2*f^2*g^2*abs(e) + sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a^2*e^4*f^2*g^2*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a
*c*d^3*e*f*g^3*abs(e) - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d*e^3*f*g^3
*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^2*e^2*g^4*abs(e)) + (2*((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^6*g^3*abs(c)*abs(d)/(c^2*d^2*e^
4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e)) + 5*(c^5*
d^5*e^8*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e^9*g^3*abs(c)*abs(d))/(c^2*d^2*e^4
*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e)))*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*
c*d*e - c*d^2*e + a*e^3)*c*d*g)^(5/2))*abs(e)/e^2
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{\left(\frac{x\,(10\,c^2\,d^2\,f-2\,a\,c\,d\,e\,g)}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}-\frac{6\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}+\frac{4\,c^2\,d^2\,x^2}{15\,g\,(a\,e\,g-c\,d\,f)^2}\right)\sqrt{c\,d\,e\,x^2+}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{2\,f\,x\,\sqrt{f+g\,x}}{g}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] (((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2*
g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e*g
 - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2*f*x
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

3.740 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{16c^2d^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(7/2)+8/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(5/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\frac{16c^2d^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(9
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{(4cd)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}\\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}\\
&\quad+\frac{(8c^2d^2)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx}{35(cdf-aeg)^2}\\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}\\
&\quad+\frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\,(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2(35f^2+}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{7/2}}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*g
*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d +
 e*x)^(3/2)*(f + g*x)^(7/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)\left(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35c^2d^2f^2\right)}{105(gx+f)^{\frac{7}{2}}\sqrt{ex+d}\,(aeg-cdf)^3}$	119
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35c^2d^2f^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{105(gx+f)^{\frac{7}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)\sqrt{ex+d}}$	169

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/105*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*
(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*a^2*e^2*g^2-42*a*c*
d*e*f*g+35*c^2*d^2*f^2)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. 2(174) = 348.

Time = 0.55 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.78

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{}{105\,(c^3d^4f^7-3\,ac^2d^3ef^6g+3\,a^2cd^2e^2f^5g^2-a^3de^3f^4g^3+(c^3d^3ef^3g^4}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^3
*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14*a
*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
```

```
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g + 3*
a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^
2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e*f^4*g
^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e
- 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4 + 2*(3*
c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g
^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e^2 - a^3*e
^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 + 3*(c^3*d^4
- 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^3 + (9
*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*a^3*d*e^3*f^3
*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3
)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(174) = 348$.

Time = 0.79 (sec) , antiderivative size = 1426, normalized size of antiderivative = 7.20

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] 2/105*((35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^4*f^2*abs(c)*abs(d) - 35*sqrt(-
c*d^2*e + a*e^3)*a*c^2*d^2*e^6*f^2*abs(c)*abs(d) - 28*sqrt(-c*d^2*e + a*e^3
)*c^3*d^5*e^3*f*g*abs(c)*abs(d) - 14*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^5*f
*g*abs(c)*abs(d) + 42*sqrt(-c*d^2*e + a*e^3)*a^2*c*d*e^7*f*g*abs(c)*abs(d)
+ 8*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*e^2*g^2*abs(c)*abs(d) + 4*sqrt(-c*d^2*e
+ a*e^3)*a*c^2*d^4*e^4*g^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a^2*c*d
^2*e^6*g^2*abs(c)*abs(d) - 15*sqrt(-c*d^2*e + a*e^3)*a^3*e^8*g^2*abs(c)*abs
(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^3*e^3*f^6*abs(e) - 3*sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^4*e^2*f^5*g*abs(e) - 3*sqrt(c^2*d^2*e^2*f -
 c^2*d^3*e*g)*a*c^2*d^2*e^4*f^5*g*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g)*c^3*d^5*e*f^4*g^2*abs(e) + 9*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^
3*e^3*f^4*g^2*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d*e^5*f^4*
g^2*abs(e) - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^6*f^3*g^3*abs(e) - 9*s
qrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^4*e^2*f^3*g^3*abs(e) - 9*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^2*e^4*f^3*g^3*abs(e) - sqrt(c^2*d^2*e^2*f
- c^2*d^3*e*g)*a^3*e^6*f^3*g^3*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
*a*c^2*d^5*e*f^2*g^4*abs(e) + 9*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^3
*e^3*f^2*g^4*abs(e) + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d*e^5*f^2*g^4
*abs(e) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^4*e^2*f*g^5*abs(e) -
3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^2*e^4*f*g^5*abs(e) + sqrt(c^2*d^2
*e^2*f - c^2*d^3*e*g)*a^3*d^3*e^3*g^6*abs(e)) + ((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*(4*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^6*d^6*e^8*g^5*ab
s(c)*abs(d)/(c^3*d^3*e^6*f^3*g^3*abs(e) - 3*a*c^2*d^2*e^7*f^2*g^4*abs(e) +
3*a^2*c*d*e^8*f*g^5*abs(e) - a^3*e^9*g^6*abs(e)) + 7*(c^7*d^7*e^10*f*g^4*ab
s(c)*abs(d) - a*c^6*d^6*e^11*g^5*abs(c)*abs(d))/(c^3*d^3*e^6*f^3*g^3*abs(e)
 - 3*a*c^2*d^2*e^7*f^2*g^4*abs(e) + 3*a^2*c*d*e^8*f*g^5*abs(e) - a^3*e^9*g^
6*abs(e)))*((e*x + d)*c*d*e - c*d^2*e + a*e^3) + 35*(c^8*d^8*e^12*f^2*g^3*a
bs(c)*abs(d) - 2*a*c^7*d^7*e^13*f*g^4*abs(c)*abs(d) + a^2*c^6*d^6*e^14*g^5*
abs(c)*abs(d))/(c^3*d^3*e^6*f^3*g^3*abs(e) - 3*a*c^2*d^2*e^7*f^2*g^4*abs(e)
 + 3*a^2*c*d*e^8*f*g^5*abs(e) - a^3*e^9*g^6*abs(e)))/(c^2*d^2*e^2*f - a*c*d
*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(7/2))*abs(e)/e^2
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{}}{g^2}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^2
*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3*d
^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*f)^
3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g - 7
*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2)
 + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/2)*(d
+ e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.741 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{32c^3d^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{315(cdf-aeg)^4(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(9/2)+4/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(7/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(5/2)+32/315*c^3*d^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(3/2)/(g*x+f)^
(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{886, 874\}$

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{32c^3d^3\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4} + \frac{16c^2d^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3} + \frac{4cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2} + \frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(1
1/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx}{3(cdf-aeg)}$$

$$= \frac{2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{9(cdf - aeg)(d + ex)^{3/2}(f + gx)^{9/2}} + \frac{4cd(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{21(cdf - aeg)^2(d + ex)^{3/2}(f + gx)^{7/2}} + \frac{(8c^2d^2)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx}{21(cdf - aeg)^2}$$

$$= \frac{2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{9(cdf - aeg)(d + ex)^{3/2}(f + gx)^{9/2}} + \frac{4cd(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{21(cdf - aeg)^2(d + ex)^{3/2}(f + gx)^{7/2}} + \frac{16c^2d^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{105(cdf - aeg)^3(d + ex)^{3/2}(f + gx)^{5/2}} + \frac{(16c^3d^3)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx}{105(cdf - aeg)^3}$$

$$= \frac{2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{9(cdf - aeg)(d + ex)^{3/2}(f + gx)^{9/2}} + \frac{4cd(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{21(cdf - aeg)^2(d + ex)^{3/2}(f + gx)^{7/2}} + \frac{16c^2d^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{105(cdf - aeg)^3(d + ex)^{3/2}(f + gx)^{5/2}} + \frac{32c^3d^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{315(cdf - aeg)^4(d + ex)^{3/2}(f + gx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \frac{2((ae + cdx)(d + ex))^{3/2}\left(-35a^3e^3g^3 + 15a^2cde^2g^2(9f + 2gx) - 3ac^2 \cdots\right.}{315(cdf - aeg)^4 \cdots}$$

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(9
*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(10
5*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^4*(
d + e*x)^(3/2)*(f + g*x)^(9/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)\left(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3 \cdots\right.}{315(gx+f)^{\frac{9}{2}}\sqrt{ex+d}\,(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd\,e^2f \cdots\right.}{315(gx+f)^{\frac{9}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right)\cdots}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*
(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f*g^2*x^2-30*a^2*c*d
*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+35*a^3*e^3*g^3-135*a
^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(235) = 470$.

Time = 1.09 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.42

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{}{315\,(c^4d^5f^9 - 4\,ac^3d^4ef^8g + 6\,a^2c^2d^3e^2f^7g^2 - 4\,a^3cd^2e^3f^6g^3 + a^4de^4}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*g
 + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^3*
e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2*g^
3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f*g^2
 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*d^3*e^
2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e*f^4*g^
5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d*e^4*f*g
^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^4*d^5 - 2
0*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^6 +
 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 5*a^4*
e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 - 8*a
*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 + 2*(3*
a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^
2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^4*d^5 - 4*a*c^3
*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(3*a^
2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^
6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^4*d^5 - 2*a*c^3*d^
3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^3 + 4*(3*a^2*c
^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^5)*x^
2 + (c^4*d^4*e*f^9 + 5*a^4*d*e^4*f^4*g^5 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^
8*g - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^2 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^3 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+
d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2280 vs. $2(235) = 470$.

Time = 1.18 (sec) , antiderivative size = 2280, normalized size of antiderivative = 8.54

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

```
[Out] 2/315*((105*sqrt(-c*d^2*e + a*e^3)*c^4*d^5*e^5*f^3*abs(c)*abs(d) - 105*sqrt
(-c*d^2*e + a*e^3)*a*c^3*d^3*e^7*f^3*abs(c)*abs(d) - 126*sqrt(-c*d^2*e + a*
e^3)*c^4*d^6*e^4*f^2*g*abs(c)*abs(d) - 63*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^4*
e^6*f^2*g*abs(c)*abs(d) + 189*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^2*e^8*f^2*g*
abs(c)*abs(d) + 72*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e^3*f*g^2*abs(c)*abs(d) +
 36*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*e^5*f*g^2*abs(c)*abs(d) + 27*sqrt(-c*d
^2*e + a*e^3)*a^2*c^2*d^3*e^7*f*g^2*abs(c)*abs(d) - 135*sqrt(-c*d^2*e + a*e
^3)*a^3*c*d*e^9*f*g^2*abs(c)*abs(d) - 16*sqrt(-c*d^2*e + a*e^3)*c^4*d^8*e^2
*g^3*abs(c)*abs(d) - 8*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^4*g^3*abs(c)*abs(
d) - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^6*g^3*abs(c)*abs(d) - 5*sqrt(-c
```

*d^2*e + a*e^3)*a^3*c*d^2*e^8*g^3*abs(c)*abs(d) + 35*sqrt(-c*d^2*e + a*e^3)
*a^4*e^10*g^3*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^4*e^4
*f^8*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^5*e^3*f^7*g*abs(e)
- 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^3*e^5*f^7*g*abs(e) + 6*sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^6*e^2*f^6*g^2*abs(e) + 16*sqrt(c^2*d^2*e^
2*f - c^2*d^3*e*g)*a*c^3*d^4*e^4*f^6*g^2*abs(e) + 6*sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g)*a^2*c^2*d^2*e^6*f^6*g^2*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g)*c^4*d^7*e*f^5*g^3*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*
d^5*e^3*f^5*g^3*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^3*e
^5*f^5*g^3*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d*e^7*f^5*g^3
*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^8*f^4*g^4*abs(e) + 16*sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^6*e^2*f^4*g^4*abs(e) + 36*sqrt(c^2*d
^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^4*e^4*f^4*g^4*abs(e) + 16*sqrt(c^2*d^2*e^
2*f - c^2*d^3*e*g)*a^3*c*d^2*e^6*f^4*g^4*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*
d^3*e*g)*a^4*e^8*f^4*g^4*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3
*d^7*e*f^3*g^5*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^5*e^
3*f^3*g^5*abs(e) - 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^3*e^5*f^3*g
^5*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d*e^7*f^3*g^5*abs(e) +
6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^6*e^2*f^2*g^6*abs(e) + 16*sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^4*e^4*f^2*g^6*abs(e) + 6*sqrt(c^2*d^
2*e^2*f - c^2*d^3*e*g)*a^4*d^2*e^6*f^2*g^6*abs(e) - 4*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a^3*c*d^5*e^3*f*g^7*abs(e) - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g)*a^4*d^3*e^5*f*g^7*abs(e) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^4*e^4
*g^8*abs(e)) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*(2*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*(4*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^8*d^8*e^10
*g^7*abs(c)*abs(d)/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3*g^5*ab
s(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs(e) + a^
4*e^12*g^8*abs(e)) + 9*(c^9*d^9*e^12*f*g^6*abs(c)*abs(d) - a*c^8*d^8*e^13*g
^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3*g^5*abs
(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs(e) + a^4
*e^12*g^8*abs(e)))*((e*x + d)*c*d*e - c*d^2*e + a*e^3) + 63*(c^10*d^10*e^14
*f^2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9*e^15*f*g^6*abs(c)*abs(d) + a^2*c^8*d^8
*e^16*g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3*
g^5*abs(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs(e
) + a^4*e^12*g^8*abs(e))) + 105*(c^11*d^11*e^16*f^3*g^4*abs(c)*abs(d) - 3*a
*c^10*d^10*e^17*f^2*g^5*abs(c)*abs(d) + 3*a^2*c^9*d^9*e^18*f*g^6*abs(c)*abs
(d) - a^3*c^8*d^8*e^19*g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a
*c^3*d^3*e^9*f^3*g^5*abs(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d
*e^11*f*g^7*abs(e) + a^4*e^12*g^8*abs(e)))/(c^2*d^2*e^2*f - a*c*d*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(9/2))*abs(e)/e^2

Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{x\,\left(-10\,a^3\,c\,d\,e^3\,g^3+54\,a^2\,c^2\,d^2\,e^2\,f\,g^2-126\,a\,c^3\cdots\right.}{315\,g^4\,\left(a\,e\,g-c\,d\,f\right)^4}\cdots\right.}{x^4\,\sqrt{f+g\,x}\cdots}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e*
x)^(1/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a^
3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g^4
*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^2*d
^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32*c^4
*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21*c^2*
d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x^3*(a*
e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6
*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.742 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 382

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}$$
$$+\frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32cdg^2\sqrt{d+ex}}$$
$$-\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}$$
$$+\frac{3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 1/4*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+3
/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(5/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/32*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)*(
g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+3/64
*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2
/d^2/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$
$$+\frac{3\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{32cdg^2\sqrt{d+ex}}$$
$$-\frac{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{8g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{5/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}$$

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} - \frac{\left(3(cdf-aeg)\right)\int\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx}{8g}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{(cdf-aeg)^2\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{16g^2}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{(3(cdf-aeg)^3)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{64cdg^2}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{(3(cdf-aeg)^4)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{128c^2d^2g^2}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{\left(3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{128c^2d^2g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{\left(3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{64c^3d^3g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{\left(3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{64c^3d^3g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} + \frac{3(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-3a^3e^3g^3+a^2cde^2g^2(\cdots}{\cdots}\right.}{\cdots}$$

[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2),x]

[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(-3*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(11*f^2 + 44*f*g*x + 24*g^2*x^2) + c^3*d^3*(-3*f^3 + 2*f^2*g*x + 24*f*g^2*x^2 + 16*g^3*x^3)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(64*c^(5/2)*d^(5/2)*g^(5/2)*(d + e*x)^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(326) = 652.

Time = 0.54 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(32c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}+3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-12\ln\left(\frac{2cd\cdots}{\cdots}\right)\cdots\right.}{\cdots}$

[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/128*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(32*c^3*d^3*g^3*x^3*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-12*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+18*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-12*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4+48*a*c^2*d^2*e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+48*c^3*d^3*f*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*g^3*x+88*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f^2*g*x-6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3+22*((g*x+f)*(c*d*x+a*e

```
))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2+22*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*
d*g)^(1/2)*a*c^2*d^2*e*f^2*g-6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^
3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/g^2/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.64 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.77

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\left[\frac{4\left(16\,c^4d^4g^4x^3-3\,c^4d^4f^3g+11\,ac^3d^3ef^2g^2+11\,a^2c^2\right.}{}\right.$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*d^
3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e^2*
g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4
*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3
*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt
(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e
^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
 a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
 + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x + c^3*d^4*g
^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^
2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*
e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2
 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2
*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*
sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e
*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g^3*x + c^3*d^4*g^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*
x + d)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8807 vs. $2(326) = 652$.

Time = 2.74 (sec) , antiderivative size = 8807, normalized size of antiderivative = 23.05

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] 1/192*(48*a*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e
*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d
*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
```

)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d*g^3))*abs(e)^2/e^3 + c*d*g*((192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*e*f*abs(g)/g^2 - 192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^3*abs(g)/g + 8*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*e*f*abs(g)/g^2 - 24*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*d*abs(g)/g - 96*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)

/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*f*abs(g)/g^3 + 144*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^2*abs(g)/(e*g^2) + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*abs(g)/g)/(e^2*g) - (15*c^4*d^4*e^5*f^4*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a*c^3*d^3*e^6*f^3*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a^2*c^2*d^2*e^7*f^2*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a^3*c*d*e^8*f*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*a^4*e^9*g^5*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^3*e^3*f^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*e^2*f^2*g*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^5*e*f*g^2*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^3*f*g^2*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^5*f*g^2*abs(g) - 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^6*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^4*e^2*g^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a *e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^4*g^3*abs(g) + 15*sqr t(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^6*g^3*abs (g))/(sqrt(c*d*g)*c^3*d^3*e^2*g^5))*abs(e)^2/e^4 - 8*c*d*f*((24*((c*d*e^2*f *g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sq rt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g) *c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^ 2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))) /sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e *g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e ^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e *x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a* c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^ 5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g ^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^ 3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3 *e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + s qrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sq rt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c ^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5* f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e* g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d) *e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2* f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d ^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3 *e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(- sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3 *g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g) /(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt (c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g) *log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + s qrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g ^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2* sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f* g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g) *a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^

2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e
^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^
2*d^2*g^4))*abs(e)^2/e^5 - 8*a*g*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-s
qrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*
g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs
(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e
^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e
^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e
*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e
^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2)
- (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*
e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*
e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/
(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*
d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f
*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(
g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e
*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d
^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3
- a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + s
qrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sq
rt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f +
(e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^
2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d
)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f
*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e
^4*f^3*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e
*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g
^2))) + 3*a^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqr
t(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f
- d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*
g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*
abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a

```
*c*d^2*e^2*g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g
)*sqrt(c*d*g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^2*d^2*g^4))*abs(e)^2/e^4)/
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

3.743 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 310

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$
$$-\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}}$$
$$+\frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] 1/3*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+1
/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(5/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/4*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+1/8*(-a*e*g+c*d*f)^2*(g*x+f)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$
$$+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}$$
$$-\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}}$$
$$+\frac{(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)
^(3/2),x]
```

```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} \\
&\quad -\frac{(cdf-aeg)\int\frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{2g} \\
&= -\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} \\
&\quad +\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} \\
&\quad +\frac{(cdf-aeg)^2\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g^2}
\end{aligned}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{(cdf-aeg)^3\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16cdg^2}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{16cdg^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{8c^2d^2g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} + \frac{\left((cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{8c^2d^2g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$
\begin{aligned}
&= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}} \\
&\quad - \frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g^2\sqrt{d+ex}} \\
&\quad + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} \\
&\quad + \frac{(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.64

$$
\int \frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(3a^2e^2g^2+2acdeg(4f+7gx)}{ae+cdx}\right.}{24c^{3/2}d^{3/2}g^{5/2}}
$$

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
+ e*x)^(3/2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(3
*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2 + 2*f*g*x + 8*g^
2*x^2)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt
[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(24*c^(3/2)*
d^(3/2)*g^(5/2)*(d + e*x)^(3/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.63

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-9\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{}$

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+
c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g
^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*
```

```
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/
(c*d*g)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x
+a*e))^(1/2)-28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-4*(
c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*
e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)/
(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.01 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2-3\,c^3d^3f^2g+8\,ac^2d^2efg^2+3\,a^2cde^2g^3+\right.$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2
*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 -
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d
*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3)]
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\sqrt{f+gx}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
+ d)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3057 vs. 2(262) = 524.

Time = 1.21 (sec) , antiderivative size = 3057, normalized size of antiderivative = 9.86

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
[Out] 1/24*(6*a*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
 + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*f*a
bs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
 + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*abs
```

(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d*g^3))*abs(e)^2/e^3 - c*d*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g

```
)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g
^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*
d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(ab
s(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a
^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*
g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*s
qrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e
^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2
*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*
g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*
g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^2*d^2*g^4))*abs(e)^2/e^5)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

3.744 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 238

$$\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}} + \frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g/(e*x+d)^(3/2)+3
/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/c^(1/2)/d^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {878, 905, 65, 223, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx = \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$
$$-\frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}}$$
$$+\frac{\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g(d+ex)^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
```

```
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{\left(3(cdf-aeg)\right)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{4g}\\
&=-\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}}\\
&\quad+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}\\
&\quad+\frac{\left(3(cdf-aeg)^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8g^2}\\
&=-\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}}\\
&\quad+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}\\
&\quad+\frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{8g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&=-\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}}\\
&\quad+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}\\
&\quad+\frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{4cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}} + \frac{\left(3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{4cdg^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}} + \frac{3(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{f+gx}(5aeg+cd(-3f+\right.}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{(ae+cdx)(d}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
*Sqrt[f + g*x]*(5*a*e*g + c*d*(-3*f + 2*g*x)) + 3*(c*d*f - a*e*g)^2*ArcTanh
[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*
Sqrt[d]*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)ac\right.}{8\sqrt{ex}}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2
)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*
((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c*d*g*x+10*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f
)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.74

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x-3\,c^2d^2fg+5\,acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^3*x +
 c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2
*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f
*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2
*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^3*x + c*d^2*g^
3)]
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(1/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*sqrt(f + g*x)),
 x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sq
rt(g*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(198) = 396$.

Time = 0.55 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.98

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\frac{e\left(\frac{\left(\sqrt{c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg}\sqrt{(ex+d)cde-cd^2e+ae^3}\left(\frac{2\left((ex+d)cd\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="giac")
[Out] 1/4*e*((sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)*abs(e)/(c*d*e^2*g) - 3*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*abs(e
))/(c*d*e^2*g^3)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e) + a^
2*e^4*g^2*abs(e))*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt(c*d
```

```
*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*c*d*g)))/(sqrt(c*d*g)*g^2))*abs(c)*abs(d)/(c*d*e^3) + (3*c^3*d^3*e^3*f^2*
abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g))) - 6*a*c^2*d^2*e^4*f*g*abs(c)*abs(d)*abs(e)*log(
abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
)) + 3*a^2*c*d*e^5*g^2*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 3*sqrt(c^2*d^2*e^2*f -
 c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*abs(c)*abs(d)*abs(
e) + 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)
*c*d^2*g*abs(c)*abs(d)*abs(e) - 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*g)*a*e^2*g*abs(c)*abs(d)*abs(e))/(sqrt(c*d*g)*c^2*
d^2*e^4*g^2))/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}\sqrt{f + gx}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\sqrt{f + g\,x}\,\left(d + e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)), x)
```

3.745 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 222

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}-\frac{3\sqrt{c}\sqrt{d}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(1/2)-3*
(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+3*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{876, 878, 905, 65, 223, 212\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = -\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} + \frac{3cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
```

```
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}+\frac{(3cd)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{g}\\
&= \frac{3cd\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\\
&\quad -\frac{(3cd(cdf-aeg))\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g^2}\\
&= \frac{3cd\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\\
&\quad -\frac{\left(3cd(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{2g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= \frac{3cd\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\\
&\quad -\frac{\left(3(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}
$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} - \frac{\left(3(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{g^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} - \frac{3\sqrt{c}\sqrt{d}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2aeg+cd(3f+gx))-3\sqrt{c}\cdots\right.}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{\cdots}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a*e*g + c*d
*(3*f + g*x)) - 3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[f + g*x]*ArcTanh[(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(g^(5/2)*Sqrt[(
a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx+3\ln\left(\frac{2cdgx+aeg-\cdots}{\cdots}\right.\right.}{\cdots}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln(1/2
*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2))*a*c*d*e*f*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)
```

```
)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2)*c*d*g*x-4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e*
g+6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))^
(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/2)/(e*x+d)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.15 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.99

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx+3\,cdf-2\,aeg)\sqrt{ex+d}\sqrt{g}}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g
- a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt(c*
d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*
g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^
2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*g^2 + (e
*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*f^2 - a*d*
e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e
^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*
d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^
2 + d*g^3)*x)]
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(188) = 376.

Time = 0.59 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.52

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx=\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\left(\frac{((ex+d)cde-cd^2e+ae^3)|c||d|}{e^2g}+\frac{3\left(cde^2fg|c||d|-\right.}{e^2g^3}\right.}{\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}}+\frac{3\left(cdf|c||d|-aeg|c||d|\right)\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-c\right.\right.}{\sqrt{cdg}g^2}-\frac{3\sqrt{c^2d^2e^2f-c^2d^3eg}cdef|c||d|\log\left(\left|-\sqrt{-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-c^2d^3eg}\right|\right)-3\sqrt{c^2d^2e^2f-c^2d^3eg}}{}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*abs(c)*abs(d)/(e^2*g) + 3*(c*d*e^2*f*g*abs(c)*abs(d) - a*e^3*g^2*abs(c)*a
bs(d))/(e^2*g^3))/sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*c*d*g) + 3*(c*d*f*abs(c)*abs(d) - a*e*g*abs(c)*abs(d))*log(ab
s(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f
 - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*
g^2) - (3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e*f*abs(c)*abs(d)*log(abs(-
sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) -
3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^2*g*abs(c)*abs(d)*log(abs(-sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 3*sqrt(-
c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*abs(c)*abs(d) - sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*g)*c*d^2*g*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a
*e^2*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*e*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)), x)
```

3.746 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 214

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}} +\frac{2c^{3/2}d^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(3/2)+
2*c^(3/2)*d^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)
^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2
)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {876, 905, 65, 223, 212}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} -\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{(cd)\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{g}$$

$$= -\frac{2cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d + ex}\sqrt{f + gx}} - \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{(c^2d^2)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{g^2}$$

$$= -\frac{2cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d + ex}\sqrt{f + gx}} - \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{\left(c^2d^2\sqrt{ae + cdx}\sqrt{d + ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{g^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}$$

$$= -\frac{2cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d + ex}\sqrt{f + gx}} - \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{\left(2cd\sqrt{ae + cdx}\sqrt{d + ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae + cdx}\right)}{g^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}$$

$$= -\frac{2cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d + ex}\sqrt{f + gx}} - \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{\left(2cd\sqrt{ae + cdx}\sqrt{d + ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{g^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}$$

$$= -\frac{2cd\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d + ex}\sqrt{f + gx}} - \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d + ex)^{3/2}(f + gx)^{3/2}} + \frac{2c^{3/2}d^{3/2}\sqrt{ae + cdx}\sqrt{d + ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.71

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{5/2}}\,dx = \frac{2\sqrt{ae + cdx}\sqrt{d + ex}\left(-\sqrt{g}\sqrt{ae + cdx}(aeg + cd(3f + 4gx)) + 3c^{3/}\right.}{3g^{5/2}\sqrt{(ae + cdx)(d + ex)}(f + }$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(a*e*g + c*
d*(3*f + 4*g*x))) + 3*c^(3/2)*d^(3/2)*(f + g*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*g^(5/2)*Sqrt[(a*e + c*
d*x)*(d + e*x)]*(f + g*x)^(3/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx}{3\sqrt{cdg}\sqrt{(gx}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g^2*x^2+6*ln(1/2*
(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)
^(1/2))*c^2*d^2*f*g*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-8*((g*x+f)*(c*d*x+a*e))^
(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(c*d*g)^(1/2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.20

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(4\,cdgx+3\,cdf+aeg)\sqrt{ex+d}\sqrt{}}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f +
 a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d
*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d
```

```
*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*
d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*f*g^3 +
d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqrt(g*x + f)
 + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^
2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^
2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^3 + d*f^2*g^2
 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(178) = 356$.

Time = 0.72 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.21

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{5/2}}\,dx =$$

$$-\frac{2\,cd|c||d|\log\left(\left|-\sqrt{(ex+d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - acde^3g + ((ex+d)cde - cd^2e + ae^3)cdg}\right|\right.}{\sqrt{cdg}g^2}$$

$$+\frac{2\left(3\sqrt{c^2d^2e^2f - c^2d^3eg}cdef|c||d|\log\left(\left|-\sqrt{-cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - c^2d^3eg}\right|\right) - 3\sqrt{c^2d^2e^2f - c^2d^3eg}\right.}{3\,(\sqrt{}}$$

$$-\frac{2\sqrt{(ex+d)cde - cd^2e + ae^3}\left(\frac{4\left(c^3d^3e^2fg^2|c||d| - ac^2d^2e^3g^3|c||d|\right)\left((ex+d)cde - cd^2e + ae^3\right)}{cde^2fg^3 - ae^3g^4} + \frac{3\left(c^4d^4e^4f^2g|c||d| - 2\,ac^3d^3e^5fg^2|c||d| + \right.}{cde^2fg^3 - ae^3g^4}\right.}{3\left(c^2d^2e^2f - acde^3g + ((ex+d)cde - cd^2e + ae^3)cdg\right)^{\frac{3}{2}}}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] -2*c*d*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt(
c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)))/(sqrt(c*d*g)*g^2) + 2/3*(3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*
c*d*e*f*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^
2*d^2*e^2*f - c^2*d^3*e*g))) - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^2*g*
abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^
2*f - c^2*d^3*e*g))) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d*e*f*abs(c)*
abs(d) - 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c*d^2*g*abs(c)*abs(d) + sqrt(
-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*e^2*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*sqrt(c*d*g)*e*f*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c
*d*g)*d*g^3) - 2/3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(4*(c^3*d^3*e^2*
f*g^2*abs(c)*abs(d) - a*c^2*d^2*e^3*g^3*abs(c)*abs(d))*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)/(c*d*e^2*f*g^3 - a*e^3*g^4) + 3*(c^4*d^4*e^4*f^2*g*abs(c)*a
bs(d) - 2*a*c^3*d^3*e^5*f*g^2*abs(c)*abs(d) + a^2*c^2*d^2*e^6*g^3*abs(c)*ab
s(d))/(c*d*e^2*f*g^3 - a*e^3*g^4))/(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{5/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^{5/2}\,\left(d + e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)), x)
```

3.747 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
```

```
0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{5(cdf - aeg)(d + ex)^{5/2}(f + gx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{7/2}} dx = \frac{2((ae + cdx)(d + ex))^{5/2}}{5(cdf - aeg)(d + ex)^{5/2}(f + gx)^{5/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(7/2)),x]
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f +
 g*x)^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)}$	55
gosper	$-\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5(gx+f)^{\frac{5}{2}}(aeg-cdf)(ex+d)^{\frac{3}{2}}}$	63

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x,m
ethod=_RETURNVERBOSE)
[Out] -2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2)*(c*d*x+a*e)^2/
(a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(55) = 110.

Time = 0.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \frac{2\left(c^2d^2x^2+2\,acc\right.}{5\left(cd^2f^4-adef^3g+(cdefg^3-ae^2g^4)x^4+(3\,cdef^2g^2-adeg^4+\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*
g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*f*g
^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 + (c*
d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(55) = 110.

Time = 0.95 (sec) , antiderivative size = 446, normalized size of antiderivative = 7.08

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = -\frac{2\left(\sqrt{-cd^2e+ae^3}c^2d^4|c||d|-2\sqrt{-cd^2e+ae^3}acd^2e^2|c||d|+\cdots\right.}{5\left(\sqrt{c^2d^2e^2f-c^2d^3eg}cde^2f^3-2\sqrt{c^2d^2e^2f-c^2d^3eg}cd^2ef^2g-\sqrt{c^2d^2e^2f-c^2d^3eg}ae^3f^2g+\sqrt{c^2d^2e^2f-c^2d\cdots}\right.} + \frac{2\left(c^5d^5e^4fg^2|c||d|-ac^4d^4e^5g^3|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}}{5\left(c^2d^2e^4f^2g^2-2acde^5fg^3+a^2e^6g^4\right)\left(c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg\right)^{\frac{5}{2}}}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] -2/5*(sqrt(-c*d^2*e + a*e^3)*c^2*d^4*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^
3)*a*c*d^2*e^2*abs(c)*abs(d) + sqrt(-c*d^2*e + a*e^3)*a^2*e^4*abs(c)*abs(d)
)/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d*e^2*f^3 - 2*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*c*d^2*e*f^2*g - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^3*f^2*g +
 sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^3*f*g^2 + 2*sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g)*a*d*e^2*f*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*d^2*e*g^3) +
2/5*(c^5*d^5*e^4*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e^5*g^3*abs(c)*abs(d))*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)/((c^2*d^2*e^4*f^2*g^2 - 2*a*c*d*e^5*
f*g^3 + a^2*e^6*g^4)*(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*c*d*g)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = -\frac{\left(\frac{2a^2e^2}{5aeg^3-5cdfg^2}+\frac{2c^2d^2x^2}{5aeg^3-5cdfg^2}+\frac{4acdex}{5aeg^3-5cdfg^2}\right)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^2\sqrt{f+gx}\sqrt{d+ex}-\frac{\sqrt{f+gx}\,(5cdf^3-5aef^2g)\sqrt{d+ex}}{5aeg^3-5cdfg^2}+\frac{x\sqrt{f+gx}\,(10aefg^2-10cdf^2g)\sqrt{d+ex}}{5aeg^3-5cdfg^2}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -(((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*c
*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^
(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g^2)
+ (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5*a*e*
g^3 - 5*c*d*f*g^2))
```

3.748 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(7/2)+4/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{(2cd)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}$$

$$=\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx=\frac{2((ae+cdx)(d+ex))^{5/2}(-5aeg+cd(7f+2gx))}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(9/2)),x]

[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d
*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+5aeg-7cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35(gx+f)^{\frac{7}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2}\right)(ex+d)^{\frac{3}{2}}}$	99
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-2g\,x^{2}c^{2}d^{2}+3acdegx-7c^{2}d^{2}fx+5a^{2}e^{2}g-7acdef\right)(cdx+ae)}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}\left(aeg-cdf\right)^{2}}$	100

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(113) = 226.

Time = 0.50 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.08

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{}{35\,(c^2d^3f^6-2\,acd^2ef^5g+a^2de^2f^4g^2+(c^2d^2ef^2g^4-2\,acde^2fg^5+}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^
2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*a*c
*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 +
 a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 8*a*c
*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2*e*f^4
*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e
 - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2*g^4 + (
3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^3)*x^2 +
 (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g - (
8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 1001 vs. $2(113) = 226$.

Time = 0.74 (sec) , antiderivative size = 1001, normalized size of antiderivative = 7.76

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx =$$

$$-\frac{2\left(7\sqrt{-cd^2e+ae^3}c\right.}{35\left(\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^2e^3f^5 - 3\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^3e^2f^4g - 2\sqrt{c^2d^2e^2f-c^2d^3eg}acde^4f^4g + 3\sqrt{c^2d\right.}$$
$$+\frac{2\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\left(\frac{2\left(c^7d^7e^6fg^4|c||d|-ac^6d^6e^7g^5|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)}{c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2cde^8fg^5-a^3e^9g^6}+\frac{7\left(c^8d^8e^8f^2g^3|c||d|-2\,ac^7d^7e^9fg^4|c||d|\right.}{c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2c}\right.}{35\left(c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg\right)^{\frac{7}{2}}}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] -2/35*(7*sqrt(-c*d^2*e + a*e^3)*c^3*d^5*e*f*abs(c)*abs(d) - 14*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^3*e^3*f*abs(c)*abs(d) + 7*sqrt(-c*d^2*e + a*e^3)*a^2*c*d
*e^5*f*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g*abs(c)*abs(d) - s
qrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e
```

```
^3)*a^2*c*d^2*e^4*g*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g*abs(
c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^3*f^5 - 3*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*c^2*d^3*e^2*f^4*g - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*a*c*d*e^4*f^4*g + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4*e*f^3*g
^2 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^3*f^3*g^2 + sqrt(c^2*d^2
*e^2*f - c^2*d^3*e*g)*a^2*e^5*f^3*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c
^2*d^5*f^2*g^3 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^3*e^2*f^2*g^3 -
3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d*e^4*f^2*g^3 + 2*sqrt(c^2*d^2*e^2*
f - c^2*d^3*e*g)*a*c*d^4*e*f*g^4 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*
d^2*e^3*f*g^4 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^3*e^2*g^5) + 2/35*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*(2*(c^7*d^7*e^6*f*g^4*abs(c)*abs(d
) - a*c^6*d^6*e^7*g^5*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c
^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5 - a^3*e^
9*g^6) + 7*(c^8*d^8*e^8*f^2*g^3*abs(c)*abs(d) - 2*a*c^7*d^7*e^9*f*g^4*abs(c
)*abs(d) + a^2*c^6*d^6*e^10*g^5*abs(c)*abs(d))/(c^3*d^3*e^6*f^3*g^3 - 3*a*c
^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5 - a^3*e^9*g^6))/(c^2*d^2*e^2*f - a
*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.91

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^2\,e^2\,(5\,a\,e\,g-7\,c\,d\,f)}{35\,g^3\,{(a\,e\,g-c\,d\,f)}^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,{(a\,e\,g-c\,d\,f)}^2}+\frac{2\,c^2\,d^2\,x^2\,(a\,e\,g-7\,c\,d\,f)}{35\,g^3\,{(a\,e\,g-c\,d\,f)}^2}+\frac{4\,a\,c\,d\,e\,x\,(4\,a\,e\,g \cdots}{35\,g^3\,(a\,e\,g-c \cdots}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c*
d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f)^
2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a*c*
d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^2)
```

3.749 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{16c^2d^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(9/2)+8/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(7/2)+16/315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
```

```
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{(4cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx}{9(cdf-aeg)} \\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}} \\
&\quad+\frac{(8c^2d^2)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx}{63(cdf-aeg)^2} \\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}} \\
&\quad+\frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\,(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2(63}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{9/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2*
g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(9/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-20acde\,g^2x+36c^2d^2fgx+35a^2e^2g^2-90acdefg+63c^2d^2f^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}{315(gx+f)^{\frac{9}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(ex+d)^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^3c^3d^3-12a\,c^2d^2e\,g^2x^2+36c^3d^3fg\,x^2+15a^2cd\,e^2g^2x-54a\,c^2d^2efgx+63c^3d^3f^2x+35a^3e^3g^2-90a^2cd\,e^2fg}{315\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^3}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35*
a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(174) = 348.

Time = 1.03 (sec) , antiderivative size = 918, normalized size of antiderivative = 4.64

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{}{315\,(c^3d^4f^8-3\,ac^2d^3ef^7g+3\,a^2cd^2e^2f^6g^2-a^3de^3f^5g^3+(c^3d^3ef^3g}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 35
*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^2
```

```
- 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 - 72
*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*e*f^7
*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5 - 3*a
*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c^3*d^3*
e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2
*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^7)*x^5
+ 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^
4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*
e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 + (c^3*d^4 -
3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (3*a^2*c
*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3*d*e^3*f^3*g^5
 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*
f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d^3*e*f^8 - 5*a^3
*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(5*a*c^2*d^3*e - a
^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{11/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(174) = 348$.

Time = 1.11 (sec) , antiderivative size = 1760, normalized size of antiderivative = 8.89

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] -2/315*(63*sqrt(-c*d^2*e + a*e^3)*c^4*d^6*e^2*f^2*abs(c)*abs(d) - 126*sqrt(
-c*d^2*e + a*e^3)*a*c^3*d^4*e^4*f^2*abs(c)*abs(d) + 63*sqrt(-c*d^2*e + a*e^
3)*a^2*c^2*d^2*e^6*f^2*abs(c)*abs(d) - 36*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e*
f*g*abs(c)*abs(d) - 18*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*e^3*f*g*abs(c)*abs(
d) + 144*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^3*e^5*f*g*abs(c)*abs(d) - 90*sqrt
(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e^3)
*c^4*d^8*g^2*abs(c)*abs(d) + 4*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2*g^2*abs
(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4*g^2*abs(c)*abs(d) - 5
0*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6*g^2*abs(c)*abs(d) + 35*sqrt(-c*d^2*e
 + a*e^3)*a^4*e^8*g^2*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3
*d^3*e^4*f^7 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^4*e^3*f^6*g - 3*sq
rt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^2*e^5*f^6*g + 6*sqrt(c^2*d^2*e^2*f
- c^2*d^3*e*g)*c^3*d^5*e^2*f^5*g^2 + 12*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a
*c^2*d^3*e^4*f^5*g^2 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d*e^6*f^5*
g^2 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^6*e*f^4*g^3 - 18*sqrt(c^2*d
^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^4*e^3*f^4*g^3 - 12*sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g)*a^2*c*d^2*e^5*f^4*g^3 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*e^7
*f^4*g^3 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^7*f^3*g^4 + 12*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^5*e^2*f^3*g^4 + 18*sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g)*a^2*c*d^3*e^4*f^3*g^4 + 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*
d*e^6*f^3*g^4 - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^6*e*f^2*g^5 - 1
2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^4*e^3*f^2*g^5 - 6*sqrt(c^2*d^2*
e^2*f - c^2*d^3*e*g)*a^3*d^2*e^5*f^2*g^5 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e
*g)*a^2*c*d^5*e^2*f*g^6 + 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^3*e^4*f
*g^6 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^4*e^3*g^7) + 2/315*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*
(c^9*d^9*e^8*f*g^6*abs(c)*abs(d) - a*c^8*d^8*e^9*g^7*abs(c)*abs(d))*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e^8*f^4*g^4 - 4*a*c^3*d^3*e^9*f^3*g^5
 + 6*a^2*c^2*d^2*e^10*f^2*g^6 - 4*a^3*c*d*e^11*f*g^7 + a^4*e^12*g^8) + 9*(c
^10*d^10*e^10*f^2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9*e^11*f*g^6*abs(c)*abs(d)
+ a^2*c^8*d^8*e^12*g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4 - 4*a*c^3*d^3*e^
9*f^3*g^5 + 6*a^2*c^2*d^2*e^10*f^2*g^6 - 4*a^3*c*d*e^11*f*g^7 + a^4*e^12*g^
8)) + 63*(c^11*d^11*e^12*f^3*g^4*abs(c)*abs(d) - 3*a*c^10*d^10*e^13*f^2*g^5
*abs(c)*abs(d) + 3*a^2*c^9*d^9*e^14*f*g^6*abs(c)*abs(d) - a^3*c^8*d^8*e^15*
```

```
g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4 - 4*a*c^3*d^3*e^9*f^3*g^5 + 6*a^2*c
^2*d^2*e^10*f^2*g^6 - 4*a^3*c*d*e^11*f*g^7 + a^4*e^12*g^8))/(c^2*d^2*e^2*f
- a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.90

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,\left(a\,e\,g-c\,d\,f\right)^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d\cdots\right.}{315\,g^4\,\left(a\,e\,g-c\,d\,f\right)^3}\cdots\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\cdots}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2*
c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2*(
126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*(a*
e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3*d^3
*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25*a^2*
e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^3)))/(
x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.750 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155(cdf-aeg)^4(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(11/2)+4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(9/2)+16/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(7/2)+32/1155*c^3*d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(5/2)/(g*x+
f)^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{886, 874\}$

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\, dx = \frac{32c^3d^3\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4} + \frac{16c^2d^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3} + \frac{4cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^2} + \frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = \frac{2\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{(6cd) \int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\, dx}{11(cdf-aeg)}$$

$$
\begin{aligned}
&= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} \\
&\quad +\frac{\left(8c^2d^2\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{33(cdf-aeg)^2} \\
&= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} \\
&\quad +\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{\left(16c^3d^3\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{231(cdf-aeg)^3} \\
&= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} \\
&\quad +\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155(cdf-aeg)^4(d+ex)^{5/2}(f+gx)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)-\right.}{1155(cdf-a}
$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*(
11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3*(
231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*g)^
4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+40a\,c^2d^2e\,g^3x^2-88c^3d^3f\,g^2x^2-70a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-198c^3d^3f^2gx+105a^3e^3g^3-385a^2cd\,e^2\right.}{1155(gx+f)^{\frac{11}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right.}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(-16g^3x^4c^4d^4+24a\,c^3d^3e\,g^3x^3-88c^4d^4f\,g^2x^3-30a^2c^2d^2e^2g^3x^2+132a\,c^3d^3ef\,g^2x^2-198c^4d^4f^2g\,x^2+35a^3cd\,e\right.}{1155\sqrt{ex+d}\,(gx+}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3*
f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*x+
105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4*a^3
*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(
e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(235) = 470$.

Time = 1.44 (sec) , antiderivative size = 1420, normalized size of antiderivative = 5.32

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{13/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3*
f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a*c
^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c^3*
d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3*d^3
*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396*a^2*
c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*d^2*e^3*
f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7
+ 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)*x^7 + (6*
c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^6 -
 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c
*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 + 3*(5*c^4*d^4
*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^5 - 2
*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^
3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)*x^5 + 5*(4*c^4*
d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^
4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c^2*d^3*e^2 - 8
*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7)*x^4 + 5*(3*c
^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*
g^3 - 2*(8*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^4 + 12*(2*a^2*c^2*d^3*e^2
 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^6)*x^3 + 3*(
2*c^4*d^4*e*f^9*g + 5*a^4*d*e^4*f^4*g^6 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8
*g^2 - 4*(5*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^3 + 2*(15*a^2*c^2*d^3*e^
2 - 4*a^3*c*d*e^4)*f^6*g^4 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^5)*x^2 +
(c^4*d^4*e*f^10 + 6*a^4*d*e^4*f^5*g^5 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9
```

```
*g - 6*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^2 + 4*(9*a^2*c^2*d^3*e^2 - a
^3*c*d*e^4)*f^7*g^3 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(13/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2715 vs. $2(235) = 470$.

Time = 1.69 (sec) , antiderivative size = 2715, normalized size of antiderivative = 10.17

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*sqrt(-c*d^2*e + a*e^3)*c^5*d^7*e^3*f^3*abs(c)*abs(d) - 462*sqr
t(-c*d^2*e + a*e^3)*a*c^4*d^5*e^5*f^3*abs(c)*abs(d) + 231*sqrt(-c*d^2*e + a
*e^3)*a^2*c^3*d^3*e^7*f^3*abs(c)*abs(d) - 198*sqrt(-c*d^2*e + a*e^3)*c^5*d^
8*e^2*f^2*g*abs(c)*abs(d) - 99*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^6*e^4*f^2*g*a
bs(c)*abs(d) + 792*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^4*e^6*f^2*g*abs(c)*abs(
d) - 495*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^2*e^8*f^2*g*abs(c)*abs(d) + 88*sq
```

rt(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g^2*abs(c)*abs(d) + 44*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^7*e^3*f*g^2*abs(c)*abs(d) + 33*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^5*e^5*f*g^2*abs(c)*abs(d) - 550*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^3*e^7*f*g^2*abs(c)*abs(d) + 385*sqrt(-c*d^2*e + a*e^3)*a^4*c*d*e^9*f*g^2*abs(c)*abs(d) - 16*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^3*abs(c)*abs(d) - 8*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2*g^3*abs(c)*abs(d) - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4*g^3*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6*g^3*abs(c)*abs(d) + 140*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8*g^3*abs(c)*abs(d) - 105*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^3*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^4*e^5*f^9 - 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^5*e^4*f^8*g - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^3*e^6*f^8*g + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^6*e^3*f^7*g^2 + 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^4*e^5*f^7*g^2 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^2*e^7*f^7*g^2 - 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^7*e^2*f^6*g^3 - 40*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^5*e^4*f^6*g^3 - 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^3*e^6*f^6*g^3 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d*e^8*f^6*g^3 + 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^8*e*f^5*g^4 + 40*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^6*e^3*f^5*g^4 + 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^4*e^5*f^5*g^4 + 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^2*e^7*f^5*g^4 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*e^9*f^5*g^4 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^9*f^4*g^5 - 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^7*e^2*f^4*g^5 - 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^5*e^4*f^4*g^5 - 40*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^3*e^6*f^4*g^5 - 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d*e^8*f^4*g^5 + 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^8*e*f^3*g^6 + 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^6*e^3*f^3*g^6 + 40*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^4*e^5*f^3*g^6 + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^2*e^7*f^3*g^6 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^7*e^2*f^2*g^7 - 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^5*e^4*f^2*g^7 - 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^3*e^6*f^2*g^7 + 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^6*e^3*f*g^8 + 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^4*e^5*f*g^8 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^5*e^4*g^9) + 2/1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*(c^11*d^11*e^10*f*g^8*abs(c)*abs(d) - a*c^10*d^10*e^11*g^9*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^5*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^12*f^3*g^7 - 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^9 - a^5*e^15*g^10) + 11*(c^12*d^12*e^12*f^2*g^7*abs(c)*abs(d) - 2*a*c^11*d^11*e^13*f*g^8*abs(c)*abs(d) + a^2*c^10*d^10*e^14*g^9*abs(c)*abs(d))/(c^5*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^12*f^3*g^7 - 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^9 - a^5*e^15*g^10)) + 99*(c^13*d^13*e^14*f^3*g^6*abs(c)*abs(d) - 3*a*c^12*d^12*e^15*f^2*g^7*abs(c)*abs(d) + 3*a^2*c^11*d^11*e^16*f*g^8*abs(c)*abs(d) - a^3*c^10*d^10*e^17*g^9*abs(c)*abs(d))/(c^5*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^12*f^3*g^7 - 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d

```
*e^14*f*g^9 - a^5*e^15*g^10)) + 231*(c^14*d^14*e^16*f^4*g^5*abs(c)*abs(d) -
 4*a*c^13*d^13*e^17*f^3*g^6*abs(c)*abs(d) + 6*a^2*c^12*d^12*e^18*f^2*g^7*ab
s(c)*abs(d) - 4*a^3*c^11*d^11*e^19*f*g^8*abs(c)*abs(d) + a^4*c^10*d^10*e^20
*g^9*abs(c)*abs(d))/(c^5*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a
^2*c^3*d^3*e^12*f^3*g^7 - 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^
9 - a^5*e^15*g^10))/(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2
*e + a*e^3)*c*d*g)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.94

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{210\,a^5\,e^5\,g^3-770\,a^4\,c\,d\,e^4\,f\,g^2+990\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,(a\,e\,g-c\,d\,f)^4}-\frac{x^2\,(-10\,a^3\,c^2\,d^2\,e^3\,g^3+\cdots}{\cdots}\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\cdots}{\cdots}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^2
*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(1155
*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3 +
66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c*d*f
)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*(3*a^
2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*f)^4)
+ (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) + (4*a*c
*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g - 275*a^2*
c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2)*(d + e*
x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(
1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (
10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3)
```

3.751 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 448

$$\begin{aligned}\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = &-\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} \\ &-\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} \\ &+\frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} \\ &-\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} \\ &+\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} \\ &-\frac{3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\end{aligned}$$

```
[Out] -1/8*(-a*e*g+c*d*f)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g
^2/(e*x+d)^(3/2)+1/5*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
g/(e*x+d)^(5/2)-3/128*(-a*e*g+c*d*f)^5*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/64*(-a*e*g+c*d*f)^3*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^3/(e*x+d)^(1/2)+1/
16*(-a*e*g+c*d*f)^2*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
^3/(e*x+d)^(1/2)-3/128*(-a*e*g+c*d*f)^4*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/c^2/d^2/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^5\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{16g^3\sqrt{d+ex}} - \frac{(f+gx)^{5/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} \\
&\quad -\frac{(cdf-aeg)\int\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{2g} \\
&= -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} \\
&\quad +\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} \\
&\quad +\frac{\left(3(cdf-aeg)^2\right)\int\frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx}{16g^2} \\
&= \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} \\
&\quad -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} \\
&\quad +\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} \\
&\quad -\frac{(cdf-aeg)^3\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{32g^3} \\
&= -\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} \\
&\quad +\frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} \\
&\quad -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} \\
&\quad +\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} \\
&\quad -\frac{\left(3(cdf-aeg)^4\right)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{128cdg^3}
\end{aligned}
$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(3(cdf-aeg)^5)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{256c^2d^2g^3}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{256c^2d^2g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{128c^3d^3g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{128c^3d^3g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{3(cdf-aeg)^5\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.68

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-15a^4e^4g^4+10a^3cde^3\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
d + e*x)^(5/2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(-
15*a^4*e^4*g^4 + 10*a^3*c*d*e^3*g^3*(7*f + g*x) + 2*a^2*c^2*d^2*e^2*g^2*(64
```

```
*f^2 + 233*f*g*x + 124*g^2*x^2) + 2*a*c^3*d^3*e*g*(-35*f^3 + 23*f^2*g*x + 2
56*f*g^2*x^2 + 168*g^3*x^3) + c^4*d^4*(15*f^4 - 10*f^3*g*x + 8*f^2*g^2*x^2
+ 176*f*g^3*x^3 + 128*g^4*x^4)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^5*Ar
cTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e +
c*d*x)^(5/2)))/(640*c^(5/2)*d^(5/2)*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(384) = 768.

Time = 0.62 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.24

method	result	size
default	Expression too large to display	1005

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/1280*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(256*c^4*d^4*g^4*x^4*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+672*a*c^3*d^3*e*g^4*x^3*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)+352*c^4*d^4*f*g^3*x^3*((g*x+f)*(c*d*x+a*e))^(1/
2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*c*d*e^
4*f*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c
^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))
^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4*d^4*e*f^4*g-15*ln(1/2*(2*c*d*g*x
+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^
5*d^5*f^5+496*a^2*c^2*d^2*e^2*g^4*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2)+1024*a*c^3*d^3*e*f*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+1
6*c^4*d^4*f^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+20*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*c*d*e^3*g^4*x+932*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+92*((g*x+f)*(c*d*x+a*e))^(1/
2)*(c*d*g)^(1/2)*a*c^3*d^3*e*f^2*g^2*x-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2)*c^4*d^4*f^3*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^4*e
^4*g^4+140*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*c*d*e^3*f*g^3+256*
a^2*c^2*d^2*e^2*f^2*g^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-140*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^3*d^3*e*f^3*g+30*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/c^2/d^2/g^3/((g*x+f)*(c
*d*x+a*e))^(1/2)/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 3.55 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.97

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2
 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*
g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 +
 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2
- 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 1
0*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^
5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d
^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^4), 1/1280*(2*(
128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2 + 128*a^2*c
^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*g^5 + 16*(11
*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 + 64*a*c^4*d^
4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2 - 23*a*c^4*d
^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^5
*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 10*a^3*c^2*d^
3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^5*e*f^5 - 5*
a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d^2*e^4*f^2*g
^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^3*d^3*e*g^4*x + c^3*d^4*g^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e*
x + d)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18597 vs. 2(384) = 768.

Time = 5.44 (sec) , antiderivative size = 18597, normalized size of antiderivative = 41.51

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] 1/1920*(480*a^2*f*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2
+ (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2
+ (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d
```

*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^
2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (
3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e
^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d
*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(ab
s(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a
*c*d*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^
2*e*g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sq
rt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)
*c*d*g^3))*abs(e)^2/e^2 + 10*c^2*d^2*f*((192*((c*d*e^2*f*g - a*e^3*g^2)*log
(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g +
a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c
*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g))*d^2*e*f*abs(g)/g^2 - 192*((c*d*e^2*f*g - a*e^3*g^
2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2
*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + s
qrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(
e^2*f + (e*x + d)*e*g - d*e*g))*d^3*abs(g)/g + 8*(sqrt(-c*d*e^2*f*g + a*e^3
*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g -
d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g
)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^
6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 -
2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d
^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*
g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a
^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)
*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f
*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*
d^2*g))*e*f*abs(g)/g^2 - 24*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e
*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4
*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7))
+ 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6
+ 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*
e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f
^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*
a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f
+ (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*d*abs(g)/g -
96*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*
(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a

*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4
*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*
e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c
*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d
*g)*c*d))*d*f*abs(g)/g^3 + 144*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e
*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d
^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e
*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^
2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g -
d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^2*abs(g)/(e*g^2) + (sqrt(-c*d*e^2*f
*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x +
 d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e
*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*
f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (1
63*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12
+ 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c
^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 -
 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g
^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^
14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)
) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g
+ 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^
2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*
f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4
 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*abs(g)/g)/(e^2*g) - (15*c
^4*d^4*e^5*f^4*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*
d^2*e*g^2 + a*e^3*g^2))) - 12*a*c^3*d^3*e^6*f^3*g^2*abs(g)*log(abs(-sqrt(e^
2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a^2*c^2*d^2
*e^7*f^2*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*
e*g^2 + a*e^3*g^2))) - 12*a^3*c*d*e^8*f*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*a^4*e^9*g^5*abs(g)
*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))
) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d
^3*e^3*f^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*s
qrt(c*d*g)*c^3*d^4*e^2*f^2*g*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt
(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g*abs(g) + 8*sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^5*e*f*g^2*abs(g) - 4*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^3
*f*g^2*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c
*d*g)*a^2*c*d*e^5*f*g^2*abs(g) - 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2
*f - d*e*g)*sqrt(c*d*g)*c^3*d^6*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^4*e^2*g^3*abs(g) + 10*sqrt(-c*d^

2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^4*g^3*abs(
g) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*
e^6*g^3*abs(g))/(sqrt(c*d*g)*c^3*d^3*e^2*g^5))*abs(e)^2/e^4 + 20*a*c*d*g*((
192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e
*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*e*f*abs(g)/
g^2 - 192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g -
d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
 - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^3*abs(g
)/g + 8*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)
(4(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*
c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5
*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7
 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3
*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2
*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 +
4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*
e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*e*f*abs(g)/g^2 - 24*(sqrt(-c*d*e^
2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*
x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d
)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c
^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5
*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^
7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3
*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*
e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 -
 a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sq
rt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqr
t(c*d*g)*c^2*d^2*g))*d*abs(g)/g - 96*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5
*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^
5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*f*abs(g)/g^3 + 144*(sqrt(-c*d
*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*
(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(
c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^
2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(
abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g +
a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^2

*abs(g)/(e*g^2) + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*abs(g)/g)/(e^2*g) - (15*c^4*d^4*e^5*f^4*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a*c^3*d^3*e^6*f^3*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a^2*c^2*d^2*e^7*f^2*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a^3*c*d*e^8*f*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*a^4*e^9*g^5*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^3*e^3*f^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*e^2*f^2*g*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^5*e*f*g^2*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^3*f*g^2*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^5*f*g^2*abs(g) - 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^6*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^4*e^2*g^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^4*g^3*abs(g) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^6*g^3*abs(g))/(sqrt(c*d*g)*c^3*d^3*e^2*g^5))*abs(e)^2/e^3 - c^2*d^2*g*((1920*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^3*e*f*abs(g)/g^2 - 1920*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f

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+ (e*x + d)*e*g - d*e*g))*d^4*abs(g)/g + 240*(sqrt(-c*d*e^2*f*g + a*e^3*g^2
 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*
g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e
^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(
c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*
c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e
^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g -
3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c
*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log
(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g +
 a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*
g))*d*e*f*abs(g)/g^2 - 480*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*
x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*
d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7))
+ 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6
+ 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e
^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^
2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a
*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*d^2*abs(g)/g -
 1440*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*
g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g
- a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*
e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a
^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt
(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(
c*d*g)*c*d))*d^2*f*abs(g)/g^3 + 1920*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5
*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^
5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^3*abs(g)/(e*g^2) - 10*(sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e
*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6
*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^
14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^
8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9
*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f
^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^
6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d
^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*
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e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*e*f*abs(g)/g^2 + 40*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*d*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)*(8*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^4*g^4) - (41*c^8*d^8*e^8*f*g^19 - 40*c^8*d^9*e^7*g^20 - a*c^7*d^7*e^9*g^20)/(c^8*d^8*e^10*g^23)) + (513*c^8*d^8*e^10*f^2*g^19 - 1000*c^8*d^9*e^9*f*g^20 - 26*a*c^7*d^7*e^11*f*g^20 + 480*c^8*d^10*e^8*g^21 + 40*a*c^7*d^8*e^10*g^21 - 7*a^2*c^6*d^6*e^12*g^21)/(c^8*d^8*e^10*g^23)) - 5*(447*c^8*d^8*e^12*f^3*g^19 - 1304*c^8*d^9*e^11*f^2*g^20 - 37*a*c^7*d^7*e^13*f^2*g^20 + 1248*c^8*d^10*e^10*f*g^21 + 112*a*c^7*d^8*e^12*f*g^21 - 19*a^2*c^6*d^6*e^14*f*g^21 - 384*c^8*d^11*e^9*g^22 - 96*a*c^7*d^9*e^11*g^22 + 40*a^2*c^6*d^7*e^13*g^22 - 7*a^3*c^5*d^5*e^15*g^22)/(c^8*d^8*e^10*g^23)) + 15*(193*c^8*d^8*e^14*f^4*g^19 - 744*c^8*d^9*e^13*f^3*g^20 - 28*a*c^7*d^7*e^15*f^3*g^20 + 1056*c^8*d^10*e^12*f^2*g^21 + 120*a*c^7*d^8*e^14*f^2*g^21 - 18*a^2*c^6*d^6*e^16*f^2*g^21 - 640*c^8*d^11*e^11*f*g^22 - 192*a*c^7*d^9*e^13*f*g^22 + 72*a^2*c^6*d^7*e^15*f*g^22 - 12*a^3*c^5*d^5*e^17*f*g^22 + 128*c^8*d^12*e^10*g^23 + 128*a*c^7*d^10*e^12*g^23 - 96*a^2*c^6*d^8*e^14*g^23 + 40*a^3*c^5*d^6*e^16*g^23 - 7*a^4*c^4*d^4*e^18*g^23)/(c^8*d^8*e^10*g^23)) + 15*(63*c^5*d^5*e^6*f^5 - 280*c^5*d^6*e^5*f^4*g - 35*a*c^4*d^4*e^7*f^4*g + 480*c^5*d^7*e^4*f^3*g^2 + 160*a*c^4*d^5*e^6*f^3*g^2 - 10*a^2*c^3*d^3*e^8*f^3*g^2 - 384*c^5*d^8*e^3*f^2*g^3 - 288*a*c^4*d^6*e^5*f^2*g^3 + 48*a^2*c^3*d^4*e^7*f^2*g^3 - 6*a^3*c^2*d^2*e^9*f^2*g^3 + 128*c^5*d^9*e^2*f*g^4 + 2

56*a*c^4*d^7*e^4*f*g^4 - 96*a^2*c^3*d^5*e^6*f*g^4 + 32*a^3*c^2*d^3*e^8*f*g^
4 - 5*a^4*c*d*e^10*f*g^4 - 128*a*c^4*d^8*e^3*g^5 + 128*a^2*c^3*d^6*e^5*g^5
- 96*a^3*c^2*d^4*e^7*g^5 + 40*a^4*c*d^2*e^9*g^5 - 7*a^5*e^11*g^5)*log(abs(-
sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3
*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^4*d^4*g^3))*
abs(g)/g)/(e^3*g) - (105*c^5*d^5*e^6*f^5*g*abs(g)*log(abs(-sqrt(e^2*f - d*e
*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 75*a*c^4*d^4*e^7*f^4*g
^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*
e^3*g^2))) - 30*a^2*c^3*d^3*e^8*f^3*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)
*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 30*a^3*c^2*d^2*e^9*f^2*g^
4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e
^3*g^2))) - 75*a^4*c*d*e^10*f*g^5*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(
c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 105*a^5*e^11*g^6*abs(g)*log(abs
(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 105*
sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^4*d^4*e^4*
f^4*abs(g) + 70*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d
*g)*c^4*d^5*e^3*f^3*g*abs(g) - 40*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f
- d*e*g)*sqrt(c*d*g)*a*c^3*d^3*e^5*f^3*g*abs(g) + 56*sqrt(-c*d^2*e*g^2 + a
*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^4*d^6*e^2*f^2*g^2*abs(g) - 22*s
qrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^3*d^4*e^4
*f^2*g^2*abs(g) - 34*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqr
t(c*d*g)*a^2*c^2*d^2*e^6*f^2*g^2*abs(g) + 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)
*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^4*d^7*e*f*g^3*abs(g) - 16*sqrt(-c*d^2*e*
g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^3*d^5*e^3*f*g^3*abs(g)
- 22*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^
2*d^3*e^5*f*g^3*abs(g) - 40*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e
*g)*sqrt(c*d*g)*a^3*c*d*e^7*f*g^3*abs(g) - 384*sqrt(-c*d^2*e*g^2 + a*e^3*g^
2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^4*d^8*g^4*abs(g) + 48*sqrt(-c*d^2*e*g^
2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^3*d^6*e^2*g^4*abs(g) + 5
6*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c^2*d^
4*e^4*g^4*abs(g) + 70*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sq
rt(c*d*g)*a^3*c*d^2*e^6*g^4*abs(g) + 105*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqr
t(e^2*f - d*e*g)*sqrt(c*d*g)*a^4*e^8*g^4*abs(g))/(sqrt(c*d*g)*c^4*d^4*e^3*g
^6))*abs(e)^2/e^4 - 160*a*c*d*f*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g
^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f
*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x +
d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(
-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^
3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^
2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*
x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^
2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) -
(13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e

^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)*c^2*d^2*g^4))*abs(e)^2/e^4 - 80*a^2*g*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^

```
4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^
3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(
5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^
5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g
^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
 - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x
+ d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d
^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3
*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-
sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3
*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)
/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*
e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*
d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3
 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) +
sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(s
qrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(g)*log(abs(
-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a*c
^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-
c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(-sqrt(e^2*f
 - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3*e^7*g^4*ab
s(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g
^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^
2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)
*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e
^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e*g^2 + a*e^
3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sqrt(-c*d^2*e
*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(g) +
3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^
2*abs(g))/(sqrt(c*d*g)*c^2*d^2*g^4))*abs(e)^2/e^3)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

3.752 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 376

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -5/24*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
g^2/(e*x+d)^(3/2)+1/4*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/g/(e*x+d)^(5/2)-5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/32*(-a*e*g+c*d*f)^2*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)-5/64*(
-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g
^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {878, 884, 905, 65, 223, 212}

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} + \frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{32g^3\sqrt{d+ex}} - \frac{5(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}$$

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)
^(5/2),x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf-aeg))\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx}{8g}$$

$$= -\frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} + \frac{(5(cdf-aeg)^2)\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx}{16g^2}$$

$$= \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf-aeg)^3)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{64g^3}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf-aeg)^4)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{128cdg^3}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{128cdg^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{64c^2d^2g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{64c^2d^2g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{5(cdf-aeg)^4\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(15a^3e^3g^3+a^2cde^2g^2(73f+}{}\right.}{}$$

[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/2),x]

[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d^2*e*g*(-55*f^2 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x + 8*f*g^2*x^2 + 48*g^3*x^3)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(192*c^(3/2)*d^(3/2)*g^(7/2)*(d + e*x)^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(320) = 640.

Time = 0.59 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}+15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-60\ln\right.}{}$

[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-272*a*c^2*d^2*e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-16*c^3*d^3*f*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)-236*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f^2*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^3*e^3*g^3-146*((g*x+f)*

```
(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*c*d*e^2*f*g^2+110*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a*e))^(1/2)/g^3/(c
*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.73 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \left[\frac{4\left(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g-55\,ac^3d^3ef^2g^2+73\,a^2c^2d^2\right.}{}\right.$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2 +
 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*c^
3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^4*x +
 c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d
^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f
*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^
3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^4*x + c^2*d
^3*g^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
+ d)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6756 vs. $2(320) = 640$.

Time = 2.20 (sec) , antiderivative size = 6756, normalized size of antiderivative = 17.97

$$\int \frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] 1/192*(48*a^2*((4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e
*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d
*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
```

)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^
3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^
2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g
^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g)
+ sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/
(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))/g - (c^2*d^2*e^3*f^2*g*abs(g)*log(abs(-s
qrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 2*a*c*d
*e^4*f*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*
g^2 + a*e^3*g^2))) + a^2*e^5*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c
*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*s
qrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d*e*f*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3
*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c*d^2*g*abs(g) + sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*e^2*g*abs(g))/(sqrt(c*d*g)*c*d
*g^3))*abs(e)^2/e^2 + c^2*d^2*((192*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqr
t(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^
2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*
g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x +
d)*e*g - d*e*g))*d^2*e*f*abs(g)/g^2 - 192*((c*d*e^2*f*g - a*e^3*g^2)*log(ab
s(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*
e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*
e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (
e*x + d)*e*g - d*e*g))*d^3*abs(g)/g + 8*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2
*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^
2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d
^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d
^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^
7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c
^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^
6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(
-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^
3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*e
*f*abs(g)/g^2 - 24*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g
- d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e
*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*
f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*
c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*
d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7))
 + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*
c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4
*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x +
d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
 + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*d*abs(g)/g - 96*(sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f
+ 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*

g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*f*abs(g)/g^3 + 144*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d^2*abs(g)/(e*g^2) + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)*(6*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^3*g^3) - (25*c^6*d^6*e^5*f*g^11 - 24*c^6*d^7*e^4*g^12 - a*c^5*d^5*e^6*g^12)/(c^6*d^6*e^6*g^14)) + (163*c^6*d^6*e^7*f^2*g^11 - 312*c^6*d^7*e^6*f*g^12 - 14*a*c^5*d^5*e^8*f*g^12 + 144*c^6*d^8*e^5*g^13 + 24*a*c^5*d^6*e^7*g^13 - 5*a^2*c^4*d^4*e^9*g^13)/(c^6*d^6*e^6*g^14)) - 3*(93*c^6*d^6*e^9*f^3*g^11 - 264*c^6*d^7*e^8*f^2*g^12 - 15*a*c^5*d^5*e^10*f^2*g^12 + 240*c^6*d^8*e^7*f*g^13 + 48*a*c^5*d^6*e^9*f*g^13 - 9*a^2*c^4*d^4*e^11*f*g^13 - 64*c^6*d^9*e^6*g^14 - 48*a*c^5*d^7*e^8*g^14 + 24*a^2*c^4*d^5*e^10*g^14 - 5*a^3*c^3*d^3*e^12*g^14)/(c^6*d^6*e^6*g^14)) - 3*(35*c^4*d^4*e^5*f^4 - 120*c^4*d^5*e^4*f^3*g - 20*a*c^3*d^3*e^6*f^3*g + 144*c^4*d^6*e^3*f^2*g^2 + 72*a*c^3*d^4*e^5*f^2*g^2 - 6*a^2*c^2*d^2*e^7*f^2*g^2 - 64*c^4*d^7*e^2*f*g^3 - 96*a*c^3*d^5*e^4*f*g^3 + 24*a^2*c^2*d^3*e^6*f*g^3 - 4*a^3*c*d*e^8*f*g^3 + 64*a*c^3*d^6*e^3*g^4 - 48*a^2*c^2*d^4*e^5*g^4 + 24*a^3*c*d^2*e^7*g^4 - 5*a^4*e^9*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*g^2))*abs(g)/g)/(e^2*g) - (15*c^4*d^4*e^5*f^4*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a*c^3*d^3*e^6*f^3*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 6*a^2*c^2*d^2*e^7*f^2*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 12*a^3*c*d*e^8*f*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*a^4*e^9*g^5*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^3*e^3*f^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^4*e^2*f^2*g*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^2*e^4*f^2*g*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^5*e*f*g^2*abs(g) - 4*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^3*e^3*f*g^2*abs(g) - 7*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d*e^5*f*g^2*abs(g) - 48*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^3*d^6*g^3*abs(g) + 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c^2*d^4*e^2*g^3*abs(g) + 10*sqrt(-c*d^2*e*g^2 +

a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*c*d^2*e^4*g^3*abs(g) + 15*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^3*e^6*g^3*abs(g))/(sqrt(c*d*g)*c^3*d^3*e^2*g^5))*abs(e)^2/e^4 - 16*a*c*d*((24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d^2*abs(g)/g - (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(e^2*f + (e*x + d)*e*g - d*e*g)/(e^2*g^2) - (13*c^4*d^4*e^3*f*g^5 - 12*c^4*d^5*e^2*g^6 - a*c^3*d^3*e^4*g^6)/(c^4*d^4*e^3*g^7)) + 3*(11*c^4*d^4*e^5*f^2*g^5 - 20*c^4*d^5*e^4*f*g^6 - 2*a*c^3*d^3*e^6*f*g^6 + 8*c^4*d^6*e^3*g^7 + 4*a*c^3*d^4*e^5*g^7 - a^2*c^2*d^2*e^7*g^7)/(c^4*d^4*e^3*g^7)) + 3*(5*c^3*d^3*e^4*f^3 - 12*c^3*d^4*e^3*f^2*g - 3*a*c^2*d^2*e^5*f^2*g + 8*c^3*d^5*e^2*f*g^2 + 8*a*c^2*d^3*e^4*f*g^2 - a^2*c*d*e^6*f*g^2 - 8*a*c^2*d^4*e^3*g^3 + 4*a^2*c*d^2*e^5*g^3 - a^3*e^7*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*g))*abs(g)/g - 6*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*f*abs(g)/g^3 + 12*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c*d))*d*abs(g)/(e*g^2))/g - (3*c^3*d^3*e^4*f^3*g*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a*c^2*d^2*e^5*f^2*g^2*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) - 3*a^2*c*d*e^6*f*g^3*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*a^3*e^7*g^4*abs(g)*log(abs(-sqrt(e^2*f - d*e*g)*sqrt(c*d*g) + sqrt(-c*d^2*e*g^2 + a*e^3*g^2))) + 3*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(g) - 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(g) - 8*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*c^2*d^4*g^2*abs(g) + 2*sqrt(-c*d^2*e*g^2 + a*e^3*g^2)*sqrt

```
(e^2*f - d*e*g)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(g) + 3*sqrt(-c*d^2*e*g^2 +
a*e^3*g^2)*sqrt(e^2*f - d*e*g)*sqrt(c*d*g)*a^2*e^4*g^2*abs(g))/(sqrt(c*d*g)
*c^2*d^2*g^4))*abs(e)^2/e^4)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

3.753 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 304

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}}+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -5/12*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/
g^2/(e*x+d)^(3/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*(g*x+f)^(1/2)
/g/(e*x+d)^(5/2)-5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(
1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/c^(1/2)
/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/8*(-a*e*g+c*d*f)^2*(g*x+
f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {878, 905, 65, 223, 212}

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}} + \frac{5\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8g^3\sqrt{d+ex}} - \frac{5\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*
(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
 + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} \\
&\quad - \frac{\left(5(cdf-aeg)\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{6g} \\
&= -\frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}} \\
&\quad + \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} \\
&\quad + \frac{\left(5(cdf-aeg)^2\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{8g^2}
\end{aligned}
$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} - \frac{(5(cdf-aeg)^3)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16g^3}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{16g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{8cdg^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} - \frac{\left(5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{8cdg^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} - \frac{5(cdf-aeg)^3\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.62

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\sqrt{f+gx}(33a^2e^2g^2+2acdeg(-20f+13gx)+c^2d^2(15f\right.}{(ae+cdx)^2}}{24g^{7/2}(d+ex)^{5/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*Sqrt[f + g*x]*(33*a^2*e^2*g^2 +
2*a*c*d*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))/(a
*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)^(5/2))))/(2
4*g^(7/2)*(d + e*x)^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.64

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/48*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+
c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g
^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-
```

```
15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2
))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*
d*x+a*e))^(1/2)+52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-
20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x+66*((g*x+f)*(c*d
*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*
d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2
*f^2)/(e*x+d)^(1/2)/g^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.03 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \left[\frac{4\,(8\,c^3d^3g^3x^2 + 15\,c^3d^3f^2g - 40\,ac^2d^2efg^2 + 33\,a^2cde^2g^3 - 2\,(5\,c^3d\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*
a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^2*g^4), 1/
48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*a^2
*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^3*d^4*f^3 -
 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*
d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt
(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*
c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^4*x + c*d^2*g^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sq
rt(g*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(256) = 512$.

Time = 0.69 (sec) , antiderivative size = 1098, normalized size of antiderivative = 3.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] 1/24*e*((sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*abs(e)/(c*d*e^3*g) -
 5*(c*d*e^2*f*g^3*abs(e) - a*e^3*g^4*abs(e))/(c*d*e^3*g^5)) + 15*(c^2*d^2*e
^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e))/(c*d*e^3
*g^5)) + 15*(c^3*d^3*e^3*f^3*abs(e) - 3*a*c^2*d^2*e^4*f^2*g*abs(e) + 3*a^2*
c*d*e^5*f*g^2*abs(e) - a^3*e^6*g^3*abs(e))*log(abs(-sqrt((e*x + d)*c*d*e -
c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3))*abs(c)*abs(d)/(c*d*e
^4) - (15*c^4*d^4*e^4*f^3*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e
```

```
^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 45*a*c^3*d^3*e^5*f^
2*g*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt
(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 45*a^2*c^2*d^2*e^6*f*g^2*abs(c)*abs(d)*ab
s(e)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g))) - 15*a^3*c*d*e^7*g^3*abs(c)*abs(d)*abs(e)*log(abs(-sqrt(-c*d^2*
e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^2*e^2*f^2
*abs(c)*abs(d)*abs(e) + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e
+ a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*abs(e) - 40*sqrt(c^2*d^2*e
^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(c)
*abs(d)*abs(e) + 8*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*g)*c^2*d^4*g^2*abs(c)*abs(d)*abs(e) - 26*sqrt(c^2*d^2*e^2*f - c^2
*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(c)*abs(d)*
abs(e) + 33*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(-c*d^2*e + a*e^3)*sqrt(c
*d*g)*a^2*e^4*g^2*abs(c)*abs(d)*abs(e))/(sqrt(c*d*g)*c^2*d^2*e^5*g^3))/abs(
e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}\sqrt{f + gx}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\sqrt{f + g\,x}\,\left(d + e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)), x)
```

3.754 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 294

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(1/2)+5/
2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g^2/(e*x+d)^(3/
2)+15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/
(g*x+f)^(1/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/4*c*d*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {876, 878, 905, 65, 223, 212}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}dx = \frac{15\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$
$$-\frac{15cd\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^3\sqrt{d+ex}}$$
$$+\frac{5cd\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{(5cd)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{g}\\ &= \frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}\\ &\quad -\frac{(15cd(cdf-aeg))\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{4g^2}\end{aligned}$$

$$=-\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{(15cd(cdf-aeg)^2)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx}{8g^3}$$

$$=-\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{\left(15cd(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{8g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$=-\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{\left(15(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}dx,x,\sqrt{ae+cdx}\right)}{4g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$=-\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{\left(15(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{4g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.68

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-8a^2e^2g^2+acdeg(25f+9gx)\right.}{4g^{7/2}\sqrt{}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-8*a^2*e^2*g^2
 + a*c*d*e*g*(25*f + 9*g*x) + c^2*d^2*(-15*f^2 - 5*f*g*x + 2*g^2*x^2)) + 15
*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*
(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(248) = 496.

Time = 0.57 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.13

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2cd\,e^2g^3x-30\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2ef\,g^2x+15\ln\left(\right.\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
```

```
(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2-3
0*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+4*c^2*d^2*
g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)+18*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*c^2*d^2*f*g*x-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 915, normalized size of antiderivative = 3.11

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = \left[\frac{4\,(2\,c^2d^2g^2x^2-15\,c^2d^2f^2+25\,acdefg-8\,a^2e^2g^2-(5\,c^2d^2fg-9\,a\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g
^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g
+ a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2
 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*
d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*
f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*
f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*
x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*c^2*d^2*g^2
*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 9
*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e^2*f*g^2 + (c^
2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*d^2*e*f^3 + a^2
*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e - a^2*e^3)*f*g^2)
*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^
4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(248) = 496$.

Time = 0.77 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.04

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\left(\left((ex+d)cde-cd^2e+ae^3\right)\left(\frac{2\,((ex+d)c\cdots}{\cdots}\right.\right.}{4\sqrt{c^2d^2e^2f-acd\cdots}}$$
$$-\frac{15\left(c^2d^2f^2|c||d|-2\,acdefg|c||d|+a^2e^2g^2|c||d|\right)\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-a\cdots}\right.\right.}{4\sqrt{cdg}g^3}$$
$$+\frac{15\sqrt{c^2d^2e^2f-c^2d^3eg}c^2d^2e^2f^2|c||d|\log\left(\left|-\sqrt{-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-c^2d^3eg}\right|\right)-30\sqrt{c^2d^2e^2f-c\cdots}}{\cdots}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*abs(c)*abs(d)/(e^4*g) - 5*(c*d
*e^2*f*g^3*abs(c)*abs(d) - a*e^3*g^4*abs(c)*abs(d))/(e^4*g^5)) - 15*(c^2*d^
2*e^4*f^2*g^2*abs(c)*abs(d) - 2*a*c*d*e^5*f*g^3*abs(c)*abs(d) + a^2*e^6*g^4
```

```
*abs(c)*abs(d))/(e^4*g^5))/sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*c*d*g) - 15/4*(c^2*d^2*f^2*abs(c)*abs(d) - 2*a*c*d*e
*f*g*abs(c)*abs(d) + a^2*e^2*g^2*abs(c)*abs(d))*log(abs(-sqrt((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3) + 1/4*(15*sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*log(abs(-sqrt(-c*
d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 30*sqrt(
c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d
^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(c
^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*e^4*g^2*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*
e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(-c*d
^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d) - 5*sqrt(-c*d^2*e +
 a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d) - 25*sqrt(-c*d^2*e + a*e^3)
*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*g)*c^2*d^4*g^2*abs(c)*abs(d) + 9*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d
^2*e^2*g^2*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a^2*e^4*g^2
*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*e^2*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)), x)
```

3.755 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 284

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}-\frac{5c^{3/2}d^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(3/2)-
10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(1/2)-5*c^(3/2)*d^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*c^2*d^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{876, 878, 905, 65, 223, 212\}$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = -\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} + \frac{5c^2d^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
```

```
b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{3g}\\
&= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}\\
&\quad -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}+\frac{\left(5c^2d^2\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{g^2}\\
&= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}\\
&\quad -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}-\frac{\left(5c^2d^2(cdf-aeg)\right)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2g^3}
\end{aligned}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} - \frac{\left(5c^2d^2(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} - \frac{\left(5cd(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}dx,x,\sqrt{ae+cdx}\right)}{g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} - \frac{\left(5cd(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{cd}}dx,x,\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} - \frac{5c^{3/2}d^{3/2}(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.66

$$\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(-2a^2e^2g^2-2acdeg(5f+7gx)+c^2d^2(15f^2+20fgx+3g\ldots}{(ae+cdx)^2(f+gx)^{3/2}}\right.}{3g^{7/2}(d+ex)^{5/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-2*a^2*e^2*g^2 - 2*a*c*d*e*g*(5
*f + 7*g*x) + c^2*d^2*(15*f^2 + 20*f*g*x + 3*g^2*x^2)))/((a*e + c*d*x)^2*(f
 + g*x)^(3/2)) - (15*c^(3/2)*d^(3/2)*(c*d*f - a*e*g)*ArcTanh[(Sqrt[c]*Sqrt[
d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(3*g^
(7/2)*(d + e*x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(240) = 480$.

Time = 0.57 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.21

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2e\,g^3x^2-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3f\,g^2x^2+30\ln\left(\right.\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*g^2*
x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+
2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x
+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/
2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+6*c^2*d^
2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)-28*(c*d*g)^(1/2)*((g*x+
f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+40*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^
(1/2)*c^2*d^2*f*g*x-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2
-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/((g*x+
f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 973, normalized size of antiderivative = 3.43

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \left[\frac{4\,(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-10\,acdefg-2\,a^2e^2g^2+2\,(10\,c^2d^2fg}{}\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g
^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g
 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g
^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^
2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f
^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c
*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3
*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2
*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*
f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(-c*d/g)*ar
ctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2
*g^3 + 2*d*f*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. 2(240) = 480.

Time = 1.01 (sec) , antiderivative size = 1178, normalized size of antiderivative = 4.15

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] -1/3*(15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*lo
g(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g))) - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*log
(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g
))) - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^3*f*g*abs(c)*abs(d)*log(
abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
)) + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^2*e^2*g^2*abs(c)*abs(d)*log
(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g
))) + 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d) -
 20*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^3*e*f*g*abs(c)*abs(d) - 10*sqr
t(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d*e^3*f*g*abs(c)*abs(d) + 3*sqrt(-c*d^2
*e + a*e^3)*sqrt(c*d*g)*c^2*d^4*g^2*abs(c)*abs(d) + 14*sqrt(-c*d^2*e + a*e^
3)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(c)*abs(d) - 2*sqrt(-c*d^2*e + a*e^3)*sqr
t(c*d*g)*a^2*e^4*g^2*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt
(c*d*g)*e^2*f*g^3 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*d*e*g^4)
+ 1/3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*(3*(c^4*d^4*e^2*f*g^4*abs(c)*abs(d) - a*c^3*d^3*e^3*g^5*abs(c)*abs(
d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^2*d^2*e^4*f*g^5 - a*c*d*e^5*g^6)
 + 20*(c^5*d^5*e^4*f^2*g^3*abs(c)*abs(d) - 2*a*c^4*d^4*e^5*f*g^4*abs(c)*abs
(d) + a^2*c^3*d^3*e^6*g^5*abs(c)*abs(d))/(c^2*d^2*e^4*f*g^5 - a*c*d*e^5*g^6
)) + 15*(c^6*d^6*e^6*f^3*g^2*abs(c)*abs(d) - 3*a*c^5*d^5*e^7*f^2*g^3*abs(c)
*abs(d) + 3*a^2*c^4*d^4*e^8*f*g^4*abs(c)*abs(d) - a^3*c^3*d^3*e^9*g^5*abs(c
)*abs(d))/(c^2*d^2*e^4*f*g^5 - a*c*d*e^5*g^6))/(c^2*d^2*e^2*f - a*c*d*e^3*g
 + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(3/2) + 5*(c^2*d^2*f*abs(c)*a
bs(d) - a*c*d*e*g*abs(c)*abs(d))*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e -
c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{5/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)), x)
```

3.756 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 274

$$\begin{aligned}\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx &= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} \\ &\quad-\frac{2cd(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}} \\ &\quad+\frac{2c^{5/2}d^{5/2}\sqrt{ae+cdx}\sqrt{d+ex}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\end{aligned}$$

```
[Out] -2/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(3/2)-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(
5/2)+2*c^(5/2)*d^(5/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used

$= \{876, 905, 65, 223, 212\}$

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}\,\mathrm{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}} - \frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])]])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] && !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(cd)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx}{g}\\
&= -\frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}\\
&\quad-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(c^2d^2)\int\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{g^2}\\
&= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}\\
&\quad-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(c^3d^3)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{g^3}\\
&= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}\\
&\quad-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{\left(c^3d^3\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}\\
&\quad-\frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}\\
&\quad+\frac{\left(2c^2d^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx,x,\sqrt{ae+cdx}\right)}{g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}} + \frac{\left(2c^2d^2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{g^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}} + \frac{2c^{5/2}d^{5/2}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}(3a^2e^2g^2+acdeg(5f+11gx)\right.}{15g^{7/2}\sqrt{(ae}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(3*a^2*e^2*
g^2 + a*c*d*e*g*(5*f + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2)))
 + 15*c^(5/2)*d^(5/2)*(f + g*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]
*(f + g*x)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(230) = 460.

Time = 0.58 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3g^3x^3+45\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3f\right.}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+
f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*g^3*x^3+45*ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2
*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(
1/2))*c^3*d^3*f^3-46*c^2*d^2*g^2*x^2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)-22*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-70*(c*d*g)^(
1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*e))^(1/2
)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*
c*d*e*f*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.41

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \left[-\frac{4\,(23\,c^2d^2g^2x^2+15\,c^2d^2f^2+5\,acdefg+3\,a^2e^2g^2+(35\,c^2d^2fg\right.$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] [-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*
g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f
^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f
*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*
x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)
*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x), -1/15*(2
*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*g^2 + (35
*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f^3 + (3*c
^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f*g^2)*x^2
 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(
2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e
```

```
*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^
2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(230) = 460$.

Time = 1.59 (sec) , antiderivative size = 1224, normalized size of antiderivative = 4.47

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] -2*c^2*d^2*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*s
qrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3) + 2/15*(15*sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*
d*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) - 30*sqrt(c^2*d^2*e^2*f - c^2*d^
3*e*g)*c^2*d^3*e*f*g*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
```

```
*g) + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3
*e*g)*c^2*d^4*g^2*abs(c)*abs(d)*log(abs(-sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)
 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g))) + 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d
*g)*c^2*d^2*e^2*f^2*abs(c)*abs(d) - 35*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c
^2*d^3*e*f*g*abs(c)*abs(d) + 5*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d*e^3
*f*g*abs(c)*abs(d) + 23*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*c^2*d^4*g^2*abs(
c)*abs(d) - 11*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a*c*d^2*e^2*g^2*abs(c)*ab
s(d) + 3*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*g)*a^2*e^4*g^2*abs(c)*abs(d))/(sqr
t(c^2*d^2*e^2*f - c^2*d^3*e*g)*sqrt(c*d*g)*e^2*f^2*g^3 - 2*sqrt(c^2*d^2*e^2
*f - c^2*d^3*e*g)*sqrt(c*d*g)*d*e*f*g^4 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)
*sqrt(c*d*g)*d^2*g^5) - 2/15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*(23*(c^6*d^6*e^4*f^2*g^4*abs(c)*abs(d) - 2*a
*c^5*d^5*e^5*f*g^5*abs(c)*abs(d) + a^2*c^4*d^4*e^6*g^6*abs(c)*abs(d))*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)/(c^2*d^2*e^4*f^2*g^5 - 2*a*c*d*e^5*f*g^6 + a
^2*e^6*g^7) + 35*(c^7*d^7*e^6*f^3*g^3*abs(c)*abs(d) - 3*a*c^6*d^6*e^7*f^2*g
^4*abs(c)*abs(d) + 3*a^2*c^5*d^5*e^8*f*g^5*abs(c)*abs(d) - a^3*c^4*d^4*e^9*
g^6*abs(c)*abs(d))/(c^2*d^2*e^4*f^2*g^5 - 2*a*c*d*e^5*f*g^6 + a^2*e^6*g^7))
 + 15*(c^8*d^8*e^8*f^4*g^2*abs(c)*abs(d) - 4*a*c^7*d^7*e^9*f^3*g^3*abs(c)*a
bs(d) + 6*a^2*c^6*d^6*e^10*f^2*g^4*abs(c)*abs(d) - 4*a^3*c^5*d^5*e^11*f*g^5
*abs(c)*abs(d) + a^4*c^4*d^4*e^12*g^6*abs(c)*abs(d))/(c^2*d^2*e^4*f^2*g^5 -
 2*a*c*d*e^5*f*g^6 + a^2*e^6*g^7))/(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^{7/2}\,\left(d + e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)), x)
```

3.757 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}} dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}} dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {874}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}} dx = \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
```

```
0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{7/2}}{7(cdf - aeg)(d + ex)^{7/2}(f + gx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{9/2}} dx = \frac{2((ae + cdx)(d + ex))^{7/2}}{7(cdf - aeg)(d + ex)^{7/2}(f + gx)^{7/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f +
 g*x)^(7/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7(gx+f)^{\frac{7}{2}}(aeg-cdf)(ex+d)^{\frac{5}{2}}}$	63
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(c^2d^2x^2+2acdex+e^2a^2\right)(cdx+ae)}{7\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)}$	78

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/7/(g*x+f)^(7/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(5/2)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(55) = 110$.

Time = 0.44 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.75

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \frac{}{7\left(cd^2f^5-adef^4g+\left(cdefg^4-ae^2g^5\right)x^5+\left(4\,cdef^2g^3-adeg^5+(c\right.\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5 -
 a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*g^5
 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 + (2*c
*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3*c*d^
2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 - a*e^2
)*f^4*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(55) = 110.

Time = 0.78 (sec) , antiderivative size = 602, normalized size of antiderivative = 9.56

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \frac{2}{7\left(\sqrt{c^2d^2e^2f-c^2d^3eg}cde^3f^4-3\sqrt{c^2d^2e^2f-c^2d^3eg}cd^2e^2f^3g-\sqrt{}\right.}$$
$$+\frac{2\left(c^8d^8e^6f^2g^3|c||d|-2\,ac^7d^7e^7fg^4|c||d|+a^2c^6d^6e^8g^5|c||d|\right)\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}}{7\left(c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2cde^8fg^5-a^3e^9g^6\right)\left(c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] 2/7*(sqrt(-c*d^2*e + a*e^3)*c^3*d^6*abs(c)*abs(d) - 3*sqrt(-c*d^2*e + a*e^3
)*a*c^2*d^4*e^2*abs(c)*abs(d) + 3*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4*abs(
c)*abs(d) - sqrt(-c*d^2*e + a*e^3)*a^3*e^6*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2
*f - c^2*d^3*e*g)*c*d*e^3*f^4 - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^2*e
^2*f^3*g - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*e^4*f^3*g + 3*sqrt(c^2*d^2*e
^2*f - c^2*d^3*e*g)*c*d^3*e*f^2*g^2 + 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a
*d*e^3*f^2*g^2 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c*d^4*f*g^3 - 3*sqrt(c^2
*d^2*e^2*f - c^2*d^3*e*g)*a*d^2*e^2*f*g^3 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*
g)*a*d^3*e*g^4) + 2/7*(c^8*d^8*e^6*f^2*g^3*abs(c)*abs(d) - 2*a*c^7*d^7*e^7*
f*g^4*abs(c)*abs(d) + a^2*c^6*d^6*e^8*g^5*abs(c)*abs(d))*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(7/2)/((c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3
*a^2*c*d*e^8*f*g^5 - a^3*e^9*g^6)*(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(7/2))
```

Mupad [B] (verification not implemented)

Time = 12.82 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.16

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e\,x}{7\,a\,e\,g^4-7\,c\,d}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(7\,c\,d\,f^4-7\,a\,e\,f^3\,g\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,\left(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,\left(21\,c\,d\,f^3\,g\right.}{7\,a\,e\,g^4-}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 - 7
*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*x)
/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^3))
```

)/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 - 7*a*e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(1/2)*(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) - (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3))

3.758 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(9/2)+4/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{(2cd)\int\frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx}{9(cdf-aeg)}\\ &= \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}(-7aeg+cd(9f+2gx))}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2)*(-7*a*e*g + c*d*(9*f + 2*g*x)))/(63*(c*d
*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+7aeg-9cdf)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63(gx+f)^{\frac{9}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+c^{2}d^{2}f^{2}\right)(ex+d)^{\frac{5}{2}}}$	99
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-2c^{3}d^{3}g\,x^{3}+3a\,c^{2}d^{2}eg\,x^{2}-9c^{3}d^{3}f\,x^{2}+12a^{2}cd\,e^{2}gx-18a\,c^{2}d^{2}efx+7a^{3}e^{3}g-9a^{2}cd\,e^{2}f\right)(cdx+ae)}{63\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^{2}}$	136

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(113) = 226.

Time = 0.60 (sec) , antiderivative size = 639, normalized size of antiderivative = 4.95

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{}{63\left(c^2d^3f^7-2\,acd^2ef^6g+a^2de^2f^5g^2+\left(c^2d^2ef^2g^5-2\,acde^2fg^6+\right.\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c^
3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*c^
2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*d*e^
2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6 + (5*
c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5 - (2*a
*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^2*f*g^6
 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5)*x^4 +
 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^4*g^
3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2*a^2*d*e^2
*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^4*g^
3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*a*c*d*e^2)*f
^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(113) = 226$.

Time = 1.16 (sec) , antiderivative size = 1275, normalized size of antiderivative = 9.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] 2/63*(9*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e*f*abs(c)*abs(d) - 27*sqrt(-c*d^2*e
 + a*e^3)*a*c^3*d^5*e^3*f*abs(c)*abs(d) + 27*sqrt(-c*d^2*e + a*e^3)*a^2*c^2
*d^3*e^5*f*abs(c)*abs(d) - 9*sqrt(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*abs(c)*ab
s(d) - 2*sqrt(-c*d^2*e + a*e^3)*c^4*d^8*g*abs(c)*abs(d) - sqrt(-c*d^2*e + a
*e^3)*a*c^3*d^6*e^2*g*abs(c)*abs(d) + 15*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4
*e^4*g*abs(c)*abs(d) - 19*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6*g*abs(c)*abs
(d) + 7*sqrt(-c*d^2*e + a*e^3)*a^4*e^8*g*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f
 - c^2*d^3*e*g)*c^2*d^2*e^4*f^6 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d
^3*e^3*f^5*g - 2*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d*e^5*f^5*g + 6*sqrt
(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^4*e^2*f^4*g^2 + 8*sqrt(c^2*d^2*e^2*f -
```

```
c^2*d^3*e*g)*a*c*d^2*e^4*f^4*g^2 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*e^
6*f^4*g^2 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^5*e*f^3*g^3 - 12*sqrt
(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^3*e^3*f^3*g^3 - 4*sqrt(c^2*d^2*e^2*f -
c^2*d^3*e*g)*a^2*d*e^5*f^3*g^3 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^2*d^6*
f^2*g^4 + 8*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c*d^4*e^2*f^2*g^4 + 6*sqrt(
c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^2*e^4*f^2*g^4 - 2*sqrt(c^2*d^2*e^2*f - c
^2*d^3*e*g)*a*c*d^5*e*f*g^5 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^3*e
^3*f*g^5 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*d^4*e^2*g^6) + 2/63*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*(2*(c^10*d^10*e^8*f^2*g^5*abs(c)*abs(d)
 - 2*a*c^9*d^9*e^9*f*g^6*abs(c)*abs(d) + a^2*c^8*d^8*e^10*g^7*abs(c)*abs(d)
)*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e^8*f^4*g^4 - 4*a*c^3*d^3*e^
9*f^3*g^5 + 6*a^2*c^2*d^2*e^10*f^2*g^6 - 4*a^3*c*d*e^11*f*g^7 + a^4*e^12*g^
8) + 9*(c^11*d^11*e^10*f^3*g^4*abs(c)*abs(d) - 3*a*c^10*d^10*e^11*f^2*g^5*a
bs(c)*abs(d) + 3*a^2*c^9*d^9*e^12*f*g^6*abs(c)*abs(d) - a^3*c^8*d^8*e^13*g^
7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4 - 4*a*c^3*d^3*e^9*f^3*g^5 + 6*a^2*c^2
*d^2*e^10*f^2*g^6 - 4*a^3*c*d*e^11*f*g^7 + a^4*e^12*g^8))/(c^2*d^2*e^2*f -
a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.44

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,(7\,a\,e\,g-9\,c\,d\,f)}{63\,g^4\,{(a\,e\,g-c\,d\,f)}^2}-\frac{4\,c^4\,d^4\,x^4}{63\,g^3\,{(a\,e\,g-c\,d\,f)}^2}+\frac{2\,c^3\,d^3\,x^3\,(a\,e\,g-9\,c\,d\,f)}{63\,g^4\,{(a\,e\,g-c\,d\,f)}^2}+\frac{2\,a^2\,c\,d\,e^2\,x\,(19\,a\cdots}{63\,g^4\,(a\,e\,g-\cdots}\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\cdots}{}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c*
d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f)^
2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a^2*
c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*d^2*
e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.759 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{7/2}}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(11/2)+8/99*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(7/2)/(g*x+f)^(9/2)+16/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
```

```
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{(4cd)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx}{11(cdf-aeg)}\\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}\\
&\quad+\frac{(8c^2d^2)\int\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx}{99(cdf-aeg)^2}\\
&= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}\\
&\quad+\frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{693(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(63a^2e^2g^2-14acdeg(11f+2gx)}{693(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{11/}}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*d
*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d*f
 - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2c^2d^2-28acde\,g^2x+44c^2d^2fgx+63a^2e^2g^2-154acdefg+99c^2d^2f^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693(gx+f)^{\frac{11}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)(ex+d)^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(8c^4d^4g^2x^4-12a\,c^3d^3e\,g^2x^3+44c^4d^4fg\,x^3+15a^2c^2d^2e^2g^2x^2-66a\,c^3d^3efg\,x^2+99c^4d^4f^2x^2+98a^3cd\,e^3g^2x-26\right.}{693\sqrt{ex+d}\,(gx+f)^{\frac{11}{2}}\,(aeg-cdf)^3}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63*
a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*
g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(174) = 348.

Time = 1.08 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{}{693\left(c^3d^4f^9-3\,ac^2d^3ef^8g+3\,a^2cd^2e^2f^7g^2-a^3de^3f^6g^3+(c^3d^3ef^3\right.}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g + 6
3*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^2
```

```
- 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2 -
330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3*e
^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9 - 3*a*
c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3*d^3*e*
f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g^9)*x^7
+ (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e^2)*f^3*g
^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 - 2*a^3*e^4
)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c^3*d^4 - 15
*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3*g^6 + (6*a^
2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3 - 3*a^3*d*e^
3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^2*d^3*e - 4*a^
2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*x^4 + 5*(3*c^3*
d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^6*g^3
 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*d^2*e^2 - a^3*e^4
)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g^5 + (5*c^3*d^4 -
6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^6*g^3 + (15*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e*f^9 - 6*a^3*d*e^3*f^5*
g^4 + 3*(2*c^3*d^4 - a*c^2*d^2*e^2)*f^8*g - 3*(6*a*c^2*d^3*e - a^2*c*d*e^3)
*f^7*g^2 + (18*a^2*c*d^2*e^2 - a^3*e^4)*f^6*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(13/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(174) = 348$.

Time = 1.77 (sec) , antiderivative size = 2167, normalized size of antiderivative = 10.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{13/2}} dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] 2/693*(99*sqrt(-c*d^2*e + a*e^3)*c^5*d^8*e^2*f^2*abs(c)*abs(d) - 297*sqrt(-
c*d^2*e + a*e^3)*a*c^4*d^6*e^4*f^2*abs(c)*abs(d) + 297*sqrt(-c*d^2*e + a*e^
3)*a^2*c^3*d^4*e^6*f^2*abs(c)*abs(d) - 99*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^
2*e^8*f^2*abs(c)*abs(d) - 44*sqrt(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g*abs(c)*ab
s(d) - 22*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^7*e^3*f*g*abs(c)*abs(d) + 330*sqrt
(-c*d^2*e + a*e^3)*a^2*c^3*d^5*e^5*f*g*abs(c)*abs(d) - 418*sqrt(-c*d^2*e +
a*e^3)*a^3*c^2*d^3*e^7*f*g*abs(c)*abs(d) + 154*sqrt(-c*d^2*e + a*e^3)*a^4*c
*d*e^9*f*g*abs(c)*abs(d) + 8*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^2*abs(c)*abs
(d) + 4*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2*g^2*abs(c)*abs(d) + 3*sqrt(-c*
d^2*e + a*e^3)*a^2*c^3*d^6*e^4*g^2*abs(c)*abs(d) - 113*sqrt(-c*d^2*e + a*e^
3)*a^3*c^2*d^4*e^6*g^2*abs(c)*abs(d) + 161*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2
*e^8*g^2*abs(c)*abs(d) - 63*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^2*abs(c)*abs(
d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^3*e^5*f^8 - 5*sqrt(c^2*d^2*e^2
*f - c^2*d^3*e*g)*c^3*d^4*e^4*f^7*g - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a
*c^2*d^2*e^6*f^7*g + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^3*d^5*e^3*f^6*g
^2 + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^3*e^5*f^6*g^2 + 3*sqrt(c^
2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d*e^7*f^6*g^2 - 10*sqrt(c^2*d^2*e^2*f - c^
2*d^3*e*g)*c^3*d^6*e^2*f^5*g^3 - 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2
*d^4*e^4*f^5*g^3 - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^2*e^6*f^5*g
^3 - sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*e^8*f^5*g^3 + 5*sqrt(c^2*d^2*e^2
*f - c^2*d^3*e*g)*c^3*d^7*e*f^4*g^4 + 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*
a*c^2*d^5*e^3*f^4*g^4 + 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^3*e^5*
f^4*g^4 + 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d*e^7*f^4*g^4 - sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*c^3*d^8*f^3*g^5 - 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g)*a*c^2*d^6*e^2*f^3*g^5 - 30*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^4
*e^4*f^3*g^5 - 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^2*e^6*f^3*g^5 + 3
*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^2*d^7*e*f^2*g^6 + 15*sqrt(c^2*d^2*e^
2*f - c^2*d^3*e*g)*a^2*c*d^5*e^3*f^2*g^6 + 10*sqrt(c^2*d^2*e^2*f - c^2*d^3*
e*g)*a^3*d^3*e^5*f^2*g^6 - 3*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c*d^6*e^
2*f*g^7 - 5*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*d^4*e^4*f*g^7 + sqrt(c^2*
d^2*e^2*f - c^2*d^3*e*g)*a^3*d^5*e^3*g^8) + 2/693*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(7/2)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*(c^12*d^12*e^10*
f^2*g^7*abs(c)*abs(d) - 2*a*c^11*d^11*e^11*f*g^8*abs(c)*abs(d) + a^2*c^10*d
^10*e^12*g^9*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^5*d^5*e^
```

```
10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^12*f^3*g^7 - 10*a^
3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^9 - a^5*e^15*g^10) + 11*(c^13*d
^13*e^12*f^3*g^6*abs(c)*abs(d) - 3*a*c^12*d^12*e^13*f^2*g^7*abs(c)*abs(d) +
 3*a^2*c^11*d^11*e^14*f*g^8*abs(c)*abs(d) - a^3*c^10*d^10*e^15*g^9*abs(c)*a
bs(d))/(c^5*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^
12*f^3*g^7 - 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^9 - a^5*e^15*
g^10)) + 99*(c^14*d^14*e^14*f^4*g^5*abs(c)*abs(d) - 4*a*c^13*d^13*e^15*f^3*
g^6*abs(c)*abs(d) + 6*a^2*c^12*d^12*e^16*f^2*g^7*abs(c)*abs(d) - 4*a^3*c^11
*d^11*e^17*f*g^8*abs(c)*abs(d) + a^4*c^10*d^10*e^18*g^9*abs(c)*abs(d))/(c^5
*d^5*e^10*f^5*g^5 - 5*a*c^4*d^4*e^11*f^4*g^6 + 10*a^2*c^3*d^3*e^12*f^3*g^7
- 10*a^3*c^2*d^2*e^13*f^2*g^8 + 5*a^4*c*d*e^14*f*g^9 - a^5*e^15*g^10))/(c^2
*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(11/2
)
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.35

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{126\,a^5\,e^5\,g^2-308\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,(a\,e\,g-c\,d\,f)^3}+\frac{x^3\,(6\,a^2\,c^3\,d^3\,e^2\,g^2-44\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,\cdots}{693\,g^5\,(a\,e\,g-c\,d\,f)^3}\cdots\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\cdots}{\cdots}}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^3
*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^3*
(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5*(a
*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^4*d^
4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2*x*(1
61*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*
f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*a*c*d*e
*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)^(1/2) +
 (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1/2)*(d +
e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (10*f^2*x^3
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^3)
```

3.760 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} \\ &+ \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{9/2}} \\ &+ \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003(cdf-aeg)^4(d+ex)^{7/2}(f+gx)^{7/2}}\end{aligned}$$

```
[Out] 2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(13/2)+12/143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(11/2)+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(9/2)+32/3003*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{886, 874\}$

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{15/2}}\,dx = \frac{32c^3d^3(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{3003(d + ex)^{7/2}(f + gx)^{7/2}(cdf - aeg)^4} + \frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{429(d + ex)^{7/2}(f + gx)^{9/2}(cdf - aeg)^3} + \frac{12cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{143(d + ex)^{7/2}(f + gx)^{11/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{13(d + ex)^{7/2}(f + gx)^{13/2}(cdf - aeg)}$$

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(15/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\text{integral} = \frac{2(ade + (cd^2 + ae^2)\,x + cdex^2)^{7/2}}{13(cdf - aeg)(d + ex)^{7/2}(f + gx)^{13/2}} + \frac{(6cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx}{13(cdf - aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{(24c^2d^2)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx}{143(cdf-aeg)^2}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{429(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{(16c^3d^3)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx}{429(cdf-aeg)^3}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{429(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{32c^3d^3(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{3003(cdf-aeg)^4(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.61

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-231a^3e^3g^3+63a^2cde^2g^2(13f+}{3003(c}$$

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(15/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^2
*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2*x
^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(3003*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+56a\,c^2d^2e\,g^3x^2-104c^3d^3f\,g^2x^2-126a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-286c^3d^3f^2gx+231a^3e^3g^3-819a^2cd\,e}{3003(gx+f)^{\frac{13}{2}}(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16c^5d^5g^3x^5+24a\,c^4d^4e\,g^3x^4-104c^5d^5f\,g^2x^4-30a^2c^3d^3e^2g^3x^3+156a\,c^4d^4ef\,g^2x^3-286c^5d^5f^2g\,x^3+35a^3c^2}{}$

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^3
*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*g*
x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^4-4*
a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4
)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. $2(235) = 470$.

Time = 1.57 (sec) , antiderivative size = 1648, normalized size of antiderivative = 6.17

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^4
*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 - a*
c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2*c^
4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*c^4*
d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2145*a
^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e^4*g^3
)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 2093*a^4*c
^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^4*e*f^10*
g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e^4*f^7*g^4
 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*e^3*f^2*g^9
 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*g^6 + a^4*d*
e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d^4*e - 21*a^2
*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4)*f^2*g^9 - (4
*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6*g^5 + a^4*d*e^
4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^3*d^4*e - 9*a^2*
c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^8 - (4*a^3
*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7*g^4 + 3*a^4*d*e^4
*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2*a*c^3*d^4*e - 5*a^
2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^4*g^7 - (
12*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^3*g^8)*x^5 + 35*(c^4*d^4*e*f^8*g^3 + a^4*d*
e^4*f^3*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^7*g^4 - 2*(2*a*c^3*d^4*e - 3*a^
2*c^2*d^2*e^3)*f^6*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^5*g^6 - (4
*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^7)*x^4 + 7*(3*c^4*d^4*e*f^9*g^2 + 5*a^4*d*e
^4*f^4*g^7 + (5*c^4*d^5 - 12*a*c^3*d^3*e^2)*f^8*g^3 - 2*(10*a*c^3*d^4*e - 9
*a^2*c^2*d^2*e^3)*f^7*g^4 + 6*(5*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^5 -
```

```
(20*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^5*g^6)*x^3 + 7*(c^4*d^4*e*f^10*g + 3*a^4*
d*e^4*f^5*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^9*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^8*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^7*g^4 -
(12*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^5)*x^2 + (c^4*d^4*e*f^11 + 7*a^4*d*e^4*f
^6*g^5 + (7*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^10*g - 2*(14*a*c^3*d^4*e - 3*a^2*c
^2*d^2*e^3)*f^9*g^2 + 2*(21*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^8*g^3 - (28*
a^3*c*d^2*e^3 - a^4*e^5)*f^7*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(15/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(15/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3241 vs. $2(235) = 470$.

Time = 2.66 (sec) , antiderivative size = 3241, normalized size of antiderivative = 12.14

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="giac")
```

```
[Out] 2/3003*(429*sqrt(-c*d^2*e + a*e^3)*c^6*d^9*e^3*f^3*abs(c)*abs(d) - 1287*sqr
t(-c*d^2*e + a*e^3)*a*c^5*d^7*e^5*f^3*abs(c)*abs(d) + 1287*sqrt(-c*d^2*e +
```

a*e^3)*a^2*c^4*d^5*e^7*f^3*abs(c)*abs(d) - 429*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^3*e^9*f^3*abs(c)*abs(d) - 286*sqrt(-c*d^2*e + a*e^3)*c^6*d^10*e^2*f^2*g*abs(c)*abs(d) - 143*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^8*e^4*f^2*g*abs(c)*abs(d) + 2145*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^6*e^6*f^2*g*abs(c)*abs(d) - 2717*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^4*e^8*f^2*g*abs(c)*abs(d) + 1001*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^2*e^10*f^2*g*abs(c)*abs(d) + 104*sqrt(-c*d^2*e + a*e^3)*c^6*d^11*e*f*g^2*abs(c)*abs(d) + 52*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^9*e^3*f*g^2*abs(c)*abs(d) + 39*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^7*e^5*f*g^2*abs(c)*abs(d) - 1469*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^5*e^7*f*g^2*abs(c)*abs(d) + 2093*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^3*e^9*f*g^2*abs(c)*abs(d) - 819*sqrt(-c*d^2*e + a*e^3)*a^5*c*d*e^11*f*g^2*abs(c)*abs(d) - 16*sqrt(-c*d^2*e + a*e^3)*c^6*d^12*g^3*abs(c)*abs(d) - 8*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2*g^3*abs(c)*abs(d) - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4*g^3*abs(c)*abs(d) - 5*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6*g^3*abs(c)*abs(d) + 371*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8*g^3*abs(c)*abs(d) - 567*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10*g^3*abs(c)*abs(d) + 231*sqrt(-c*d^2*e + a*e^3)*a^6*e^12*g^3*abs(c)*abs(d))/(sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^4*e^6*f^10 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^5*e^5*f^9*g - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^3*e^7*f^9*g + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^6*e^4*f^8*g^2 + 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^4*e^6*f^8*g^2 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^2*e^8*f^8*g^2 - 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^7*e^3*f^7*g^3 - 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^5*e^5*f^7*g^3 - 36*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^3*e^7*f^7*g^3 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d*e^9*f^7*g^3 + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^8*e^2*f^6*g^4 + 80*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^6*e^4*f^6*g^4 + 90*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^4*e^6*f^6*g^4 + 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^2*e^8*f^6*g^4 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*e^10*f^6*g^4 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^9*e*f^5*g^5 - 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^7*e^3*f^5*g^5 - 120*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^5*e^5*f^5*g^5 - 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^3*e^7*f^5*g^5 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d*e^9*f^5*g^5 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*c^4*d^10*f^4*g^6 + 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^8*e^2*f^4*g^6 + 90*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^6*e^4*f^4*g^6 + 80*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^4*e^6*f^4*g^6 + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^2*e^8*f^4*g^6 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a*c^3*d^9*e*f^3*g^7 - 36*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^7*e^3*f^3*g^7 - 60*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^5*e^5*f^3*g^7 - 20*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^3*e^7*f^3*g^7 + 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^2*c^2*d^8*e^2*f^2*g^8 + 24*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^6*e^4*f^2*g^8 + 15*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^4*e^6*f^2*g^8 - 4*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^3*c*d^7*e^3*f*g^9 - 6*sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^5*e^5*f*g^9 + sqrt(c^2*d^2*e^2*f - c^2*d^3*e*g)*a^4*d^6*e^4*g^10) + 2/3003*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*(2*((e*x + d)*c*d*e - c*d

```
^2*e + a*e^3)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*(c^14*d^14*e^12*f^2
*g^9*abs(c)*abs(d) - 2*a*c^13*d^13*e^13*f*g^10*abs(c)*abs(d) + a^2*c^12*d^1
2*e^14*g^11*abs(c)*abs(d))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^6*d^6*e^1
2*f^6*g^6 - 6*a*c^5*d^5*e^13*f^5*g^7 + 15*a^2*c^4*d^4*e^14*f^4*g^8 - 20*a^3
*c^3*d^3*e^15*f^3*g^9 + 15*a^4*c^2*d^2*e^16*f^2*g^10 - 6*a^5*c*d*e^17*f*g^1
1 + a^6*e^18*g^12) + 13*(c^15*d^15*e^14*f^3*g^8*abs(c)*abs(d) - 3*a*c^14*d^
14*e^15*f^2*g^9*abs(c)*abs(d) + 3*a^2*c^13*d^13*e^16*f*g^10*abs(c)*abs(d) -
 a^3*c^12*d^12*e^17*g^11*abs(c)*abs(d))/(c^6*d^6*e^12*f^6*g^6 - 6*a*c^5*d^5
*e^13*f^5*g^7 + 15*a^2*c^4*d^4*e^14*f^4*g^8 - 20*a^3*c^3*d^3*e^15*f^3*g^9 +
 15*a^4*c^2*d^2*e^16*f^2*g^10 - 6*a^5*c*d*e^17*f*g^11 + a^6*e^18*g^12)) + 1
43*(c^16*d^16*e^16*f^4*g^7*abs(c)*abs(d) - 4*a*c^15*d^15*e^17*f^3*g^8*abs(c
)*abs(d) + 6*a^2*c^14*d^14*e^18*f^2*g^9*abs(c)*abs(d) - 4*a^3*c^13*d^13*e^1
9*f*g^10*abs(c)*abs(d) + a^4*c^12*d^12*e^20*g^11*abs(c)*abs(d))/(c^6*d^6*e^
12*f^6*g^6 - 6*a*c^5*d^5*e^13*f^5*g^7 + 15*a^2*c^4*d^4*e^14*f^4*g^8 - 20*a^
3*c^3*d^3*e^15*f^3*g^9 + 15*a^4*c^2*d^2*e^16*f^2*g^10 - 6*a^5*c*d*e^17*f*g^
11 + a^6*e^18*g^12)) + 429*(c^17*d^17*e^18*f^5*g^6*abs(c)*abs(d) - 5*a*c^16
*d^16*e^19*f^4*g^7*abs(c)*abs(d) + 10*a^2*c^15*d^15*e^20*f^3*g^8*abs(c)*abs
(d) - 10*a^3*c^14*d^14*e^21*f^2*g^9*abs(c)*abs(d) + 5*a^4*c^13*d^13*e^22*f*
g^10*abs(c)*abs(d) - a^5*c^12*d^12*e^23*g^11*abs(c)*abs(d))/(c^6*d^6*e^12*f
^6*g^6 - 6*a*c^5*d^5*e^13*f^5*g^7 + 15*a^2*c^4*d^4*e^14*f^4*g^8 - 20*a^3*c^
3*d^3*e^15*f^3*g^9 + 15*a^4*c^2*d^2*e^16*f^2*g^10 - 6*a^5*c*d*e^17*f*g^11 +
 a^6*e^18*g^12))/(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)*c*d*g)^(13/2)
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{462\,a^6\,e^6\,g^3-1638\,a^5\,c\,d\,e^5\,f\,g^2+2002\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,g^6\,\left(a\,e\,g-c\,d\,f\right)^4}-\frac{x^3\,\left(-10\,a^3\,c^3\,d^3\,e^3\,g^3\right.}{}\right.}{x^6}$$

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^3
*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(30
03*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g^3
+ 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g - c*d
*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^4*(3*
a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c*d*f)^
4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4) + (2*
a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*e*f^2*g
```

```
- 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2*d^2*e*x^
2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g - 1469*a^2*c
*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (1
5*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4)
```

3.761 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{(ae+cdx)(d+ex)^{5/2}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(5/2)*(g*x+f)^(1+n)*hypergeom([1, -1/2+n],[2+n],c*d*(g
*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f -
a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{(f+gx)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int\frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd((ae+cdx)(d+ex))^{3/2}}$$

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e +
 c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*d*
(f + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{5}{2}}\,(gx+f)^n}{(ade+(e^2a+c\,d^2)\,x+cde\,x^2)^{\frac{5}{2}}}dx$$

```
[In] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c^2
*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

```
[In] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2), x)
```

3.762 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{(ae+cdx)(d+ex)^{3/2}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(3/2)*(g*x+f)^(1+n)*hypergeom([1, 1/2+n],[2+n],c*d*(g*
x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\
&= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c*
d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(ade+(e^2a+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2*e
 + a^2*e^3)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2), x)
```

3.763 $\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{(ae+cdx)\sqrt{d+ex}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g
))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int\frac{(f+gx)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\
&= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int\frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\
&= \frac{2(ae+cdx)\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{d+ex}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2
, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n)
```

Maple [F]

$$\int \frac{\sqrt{ex+d}\,(gx+f)^n}{\sqrt{ade+(e^2a+c\,d^2)\,x+cde\,x^2}}dx$$

```
[In] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**n/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^n\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

3.764 $\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = -\frac{(ae+cdx)(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{d+ex}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 5/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^n(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right.}{3cd\sqrt{d+ex}}$$

```
[In] Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*Hy
pergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c*d
*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\int\sqrt{ae+cdx}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}} \\
&= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\right)\int\sqrt{ae+cdx}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}} \\
&= \frac{2(ae+cdx)(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd(d+ex)^{3/2}}$$

```
[In] Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n, 5
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(f
 + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]

$$\int \frac{(gx+f)^n\,\sqrt{ade+(e^2a+c\,d^2)\,x+cde\,x^2}}{\sqrt{ex+d}}dx$$

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x +
 d), x)
```

Sympy [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^n}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**n/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

Giac [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{(f+g\,x)^n\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2), x)
```

3.765 $\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}} \, dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}} \, dx = -\frac{(ae+cdx)(f+gx)^{1+n} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} \text{Hypergeometric2F1}\left(1,\frac{7}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{3/2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*hypergeo
m([1, 7/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}} \, dx = \frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\cdots\right.}{5cd\sqrt{d\cdots}}$$

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\int(ae+cdx)^{3/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$

$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\right)\int(ae+cdx)^{3/2}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$

$$= \frac{2(ae+cdx)^2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^n\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hyperg}}{5cd(d+ex)^{5/2}}$$

[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2),x]

[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n, 7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)

Maple [F]

$$\int \frac{(gx+f)^n\,(ade+(e^2a+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}dx$$

[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)

[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)

Fricas [F]

$$\int \frac{(f+gx)^n\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*(g*x + f)^n/sqrt(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(f+gx)^n\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

Giac [F]

$$\int \frac{(f+gx)^n\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(f+g\,x)^n\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2), x)
```

3.766 $\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 104

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{(ae+cdx)(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\text{Hypergeometric2F1}\left(1,\frac{9}{2}+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{5/2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*hypergeo
m([1, 9/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^n(ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{c}{c}\right.}{7cd\sqrt{d}}$$

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[7/2, -n, 9/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(7*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\int(ae+cdx)^{5/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$

$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\right)\int(ae+cdx)^{5/2}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$

$$= \frac{2(ae+cdx)^3(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/2),x]

[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[7/2, -n, 9/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*c*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)

Maple [F]

$$\int \frac{(gx+f)^n \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}dx$$

[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)

[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)

Fricas [F]

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] integral((c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
d)^(5/2), x)
```

Giac [F]

$$\int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(f+g\,x)^n\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2), x)
```

3.767 $\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 44, antiderivative size = 103

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(ae+cdx)(d+ex)^m(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(1,2-m+n,2\right.}{(cdf-aeg)(1+n)}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^m*(g*x+f)^(1+n)*hypergeom([1, 2-m+n],[2+n],c*d*(g*x+f)
/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {905, 72, 71}

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= \frac{(d+ex)^m(f+gx)^{n+1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\text{Hypergeometric2F1}\left(m,n+1,n+\right.}{g(n+1)}$$

```
[In] Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(ae+cdx)^{-m}(f+gx)^n\,dx$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(f+gx)^n\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}dx$$

$$= \frac{\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(m,1+n;2+n;\frac{cd(f+gx}{cdf-aeg}\right.}{g(1+n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(m,1+n,2+n,\frac{cd(f+gx)}{cdf-aeg}\right)}{g(1+n)}$$

```
[In] Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] (((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*((a*e + c*d*x)*(d + e*x))^m)
```

Maple [F]

$$\int \left(ex+d\right)^{m} \left(gx+f\right)^{n} \left(ade+\left(e^{2}a+c\,d^{2}\right)x+cde\,x^{2}\right)^{-m}dx$$

```
[In] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m,
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= \int \frac{(ex+d)^m (gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m} dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

Giac [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$=\int \frac{(f+g\,x)^n\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

```
[In] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
[Out] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

3.768 $\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 44, antiderivative size = 343

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^4d^4e(1-m)(2-m)(3-m)(4-m)} + \frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)} + \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)} + \frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)}$$

```
[Out] -6*(-a*e*g+c*d*f)^2*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^4/d^4/e/(m^2-7*m+12)/(m^2-3*m+2)+6*g*
(-a*e*g+c*d*f)^2*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/
e/(2-m)/(3-m)/(4-m)+3*(-a*e*g+c*d*f)*(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/(3-m)/(4-m)+(e*x+d)^(-1+m)*(g*x+f)^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(4-m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {884, 808, 662}

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}dx =$$
$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}$$
$$+\frac{6g(d+ex)^m(cdf-aeg)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)}$$
$$+\frac{3(f+gx)^2(d+ex)^{m-1}(cdf-aeg)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)}$$
$$+\frac{(f+gx)^3(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

```
[In] Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)} \\
&\quad+\frac{(3(cdf-aeg))\int(d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx}{cd(4-m)} \\
&= \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)} \\
&\quad+\frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)} \\
&\quad+\frac{\left(6(cdf-aeg)^2\right)\int(d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx}{c^2d^2(3-m)(4-m)} \\
&= \frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)} \\
&\quad+\frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)} \\
&\quad+\frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)} \\
&\quad-\frac{\left(6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\right)\int(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}}{c^3d^3e(2-m)(3-m)(4-m)} \\
&= \\
&\quad-\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1}}{c^4d^4e(1-m)(2-m)(3-m)(4-m)} \\
&\quad+\frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)} \\
&\quad+\frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)} \\
&\quad+\frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.39

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(-\frac{(cdf-aeg)^3}{-1+m}-\frac{3g(cdf-aeg)^2(ae+cdx)}{-2+m}+\frac{3g^2(-cdf+aeg)(ae+cdx)^2}{-3+m}-\frac{g^3(ae+cdx)^3}{-4+m}\right)}{c^4d^4}$$

```
[In] Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3/
(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*d*
f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))/(c
^4*d^4)
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.54

method	result
gosper	$-\frac{(ex+d)^m\left(c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3m\right.}{}$
risch	$-\frac{\left(c^4d^4g^3m^3x^4+ac^3d^3eg^3m^3x^3+3c^4d^4fg^2m^3x^3-6c^4d^4g^3m^2x^4+3ac^3d^3efg^2m^3x^2-3ac^3d^3eg^3m^2x^3+3c^4d^4f^2gm^3x^2-\right.}{}$
parallelrisch	Expression too large to display

```
[In] int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RET
URNVERBOSE)
```

```
[Out] -(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*x
^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x^2
+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+c^3
*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^3*x^
3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m*x+6*
a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3*f*g^2
*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*m+24*a*
c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3-24*a^2*
c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c*d*e*x^2+
a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(321) = 642.

Time = 0.33 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.06

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(ac^3d^3ef^3m^3-24\,ac^3d^3ef^3+36\,a^2c^2d^2e^2f^2g-24\,a^3cde^3fg^2+6\,a^4e^4g^3+(c^4d^4g^3m^3-6\,c^4d^4g^3m^2+11}{}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24*
a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2 +
11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f*g^
2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 - 2*(2
1*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2*c^2*
d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3*e*f*g
^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m^2
 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m)*x^2 +
(26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g^2)*m - (
24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c^4*d^4*f^3
 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^4*d^4*f^3 +
 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^3*g^3)*m)*x)
*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m
 + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.97

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= -\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)} - \frac{3\,(c^2d^2(m-1)x^2+acdemx+a^2e^2)f^2g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$
$$- \frac{3\,((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3)fg^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}$$
$$- \frac{((m^3-6\,m^2+11\,m-6)c^4d^4x^4+(m^3-3\,m^2+2\,m)ac^3d^3ex^3+3\,(m^2-m)a^2c^2d^2e^2x^2+6\,a^3cde^3mx+}{(m^4-10\,m^3+35\,m^2-50\,m+24)(cdx+ae)^m c^4d^4}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - 3
*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m
*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3) -
((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*e*x^3
 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g^3/((m
^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. 2(321) = 642.

Time = 0.49 (sec) , antiderivative size = 1926, normalized size of antiderivative = 5.62

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((e*x + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
+ 3*(e*x + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)) + (e*x + d)^m*a*c^3*d^3*e*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) - 6*(e*x + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + 3*(e*x + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e) - m*
log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f*g^2*m^3*x^2*e^(-m*log(c*d*x + a
*e) - m*log(e*x + d)) - 21*(e*x + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*log(c*d*
x + a*e) - m*log(e*x + d)) - 3*(e*x + d)^m*a*c^3*d^3*e*g^3*m^2*x^3*e^(-m*lo
```

```
g(c*d*x + a*e) - m*log(e*x + d)) + 11*(e*x + d)^m*c^4*d^4*g^3*m*x^4*e^(-m*l
og(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*c^4*d^4*f^3*m^3*x*e^(-m*log
(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f^2*g*m^3*x*e^(
-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x + d)^m*c^4*d^4*f^2*g*m^2*x^
2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 15*(e*x + d)^m*a*c^3*d^3*e*f*g
^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x + d)^m*a^2*c^2
*d^2*e^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 42*(e*x + d
)^m*c^4*d^4*f*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 2*(e*x +
 d)^m*a*c^3*d^3*e*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 6*(e
*x + d)^m*c^4*d^4*g^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x +
 d)^m*a*c^3*d^3*e*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 9*(e*x
 + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 21*(e*
x + d)^m*a*c^3*d^3*e*f^2*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) +
 6*(e*x + d)^m*a^2*c^2*d^2*e^2*f*g^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d)) + 57*(e*x + d)^m*c^4*d^4*f^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*lo
g(e*x + d)) + 12*(e*x + d)^m*a*c^3*d^3*e*f*g^2*m*x^2*e^(-m*log(c*d*x + a*e)
 - m*log(e*x + d)) - 3*(e*x + d)^m*a^2*c^2*d^2*e^2*g^3*m*x^2*e^(-m*log(c*d*
x + a*e) - m*log(e*x + d)) - 24*(e*x + d)^m*c^4*d^4*f*g^2*x^3*e^(-m*log(c*d
*x + a*e) - m*log(e*x + d)) - 9*(e*x + d)^m*a*c^3*d^3*e*f^3*m^2*e^(-m*log(c
*d*x + a*e) - m*log(e*x + d)) + 3*(e*x + d)^m*a^2*c^2*d^2*e^2*f^2*g*m^2*e^(
-m*log(c*d*x + a*e) - m*log(e*x + d)) + 26*(e*x + d)^m*c^4*d^4*f^3*m*x*e^(-
m*log(c*d*x + a*e) - m*log(e*x + d)) + 36*(e*x + d)^m*a*c^3*d^3*e*f^2*g*m*x
*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x + d)^m*a^2*c^2*d^2*e^2*
f*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 6*(e*x + d)^m*a^3*c*d*
e^3*g^3*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 36*(e*x + d)^m*c^4*d
^4*f^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 26*(e*x + d)^m*a*c^
3*d^3*e*f^3*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 21*(e*x + d)^m*a^2
*c^2*d^2*e^2*f^2*g*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 6*(e*x + d)
^m*a^3*c*d*e^3*f*g^2*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x +
 d)^m*c^4*d^4*f^3*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x + d)
^m*a*c^3*d^3*e*f^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 36*(e*x + d)^
m*a^2*c^2*d^2*e^2*f^2*g*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x
+ d)^m*a^3*c*d*e^3*f*g^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 6*(e*x
+ d)^m*a^4*e^4*g^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c^4*d^4*m^4 -
 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m + 24*c^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.79

$$\int (d+ex)^m(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\frac{g^3\,x^4\,(d+e\,x)^m\,(m^3-6\,m^2+11\,m-6)}{m^4-10\,m^3+35\,m^2-50\,m+24}+\frac{x\,(d+e\,x)^m\,(6\,a^3\,c\,d\,e^3\,g^3\,m+6\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m^2-24\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m+3\,a\,c^3\,d^3\,e\,f^2\,g\,m^3-21\,a\,c^3\,d}{c^4\,d^4\,(m^4-10\,m^3+35\,m^2}}{}$$

[In] int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)

[Out] -((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 + m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4*f^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^2*m + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*e*f^2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^3*f^3*m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*a^2*c*d*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c^2*d^2*e*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*x^2*(m - 1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m + c^2*d^2*f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3*c*d*f*m)*(m^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m

3.769 $\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 44, antiderivative size = 246

$$\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(1-m)(2-m)(3-m)} + \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)} + \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)}$$

```
[Out] -2*(-a*e*g+c*d*f)*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/e/(1-m)/(2-m)/(3-m)+2*g*(-a*e*g+c
*d*f)*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/e/(2-m)/(3-
m)+(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(3-
m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used

= {884, 808, 662}

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)} + \frac{2g(d+ex)^m(cdf-aeg)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)} + \frac{(f+gx)^2(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)} + \frac{(2(cdf-aeg))\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx}{cd(3-m)}$$

$$= \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)} + \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)} - \frac{\left(2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\right)\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{c^2d^2e(2-m)(3-m)}$$

$$= -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-}}{c^3d^3e(1-m)(2-m)(3-m)} + \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)} + \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.53

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(2a^2e^2g^2+2acdeg(f(-3+m)+g(-1+m)x)+c^2d^2(f^2(6-5m\right.}{c^3d^3(-3+m)(-2+m)(-1+m)}$$

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2*
a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*f*
g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 + m)
*(-1 + m)))
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{(cdx+ae)(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2+2acdefgm-2ac}{c^3d^3(m^3-6m^2+11m-}$
risch	$-\frac{(c^3d^3g^2m^2x^3+a\,c^2d^2e\,g^2m^2x^2+2c^3d^3fg\,m^2x^2-3c^3d^3g^2m\,x^3+2a\,c^2d^2efg\,m^2x-a\,c^2d^2e\,g^2m\,x^2+c^3d^3f^2m^2x-8c^3d^3fgm}{}$
parallelrisch	$\frac{(-x^3(ex+d)^m c^3d^3e\,g^2m^2+3x^3(ex+d)^m c^3d^3e\,g^2m-x(ex+d)^m c^3d^3e\,f^2m^2-6x^2(ex+d)^m c^3d^3efg+5x(ex+d)^m c^3d^3e\,f^2m-(}{}$

```
[In] int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RET
URNVERBOSE)
```

```
[Out] -(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+2
*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*c*
d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-6*a
*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c
^3/d^3/(m^3-6*m^2+11*m-6)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.42

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(ac^2d^2ef^2m^2+6\,ac^2d^2ef^2-6\,a^2cde^2fg+2\,a^3e^3g^2+(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2)x^3+(6\,c^3d^3fg}{}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*g
^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3*f
*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*e*g
^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2 + (
c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*f*g -
 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^
3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\begin{aligned}\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx \\ = -\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)} - \frac{2\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)fg}{(m^2-3m+2)(cdx+ae)^m c^2d^2} \\ -\frac{\left((m^2-3m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2a^2cde^2mx+2a^3e^3\right)g^2}{(m^3-6m^2+11m-6)(cdx+ae)^m c^3d^3}\end{aligned}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - ((m
^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*x +
 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(232) = 464.

Time = 0.54 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.78

$$\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(ex+d)^m c^3d^3g^2m^2x^3e^{(-m\log(cdx+ae)-m\log(ex+d))}+2(ex+d)^m c^3d^3fgm^2x^2e^{(-m\log(cdx+ae)-m\log(ex+d))}+(e}{}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((e*x + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
+ 2*(e*x + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
) + (e*x + d)^m*a*c^2*d^2*e*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) - 3*(e*x + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) + (e*x + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)) + 2*(e*x + d)^m*a*c^2*d^2*e*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) - 8*(e*x + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) - (e*x + d)^m*a*c^2*d^2*e*g^2*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + 2*(e*x + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) + (e*x + d)^m*a*c^2*d^2*e*f^2*m^2*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) - 5*(e*x + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) - 6*(e*x + d)^m*a*c^2*d^2*e*f*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + 6*(e*x + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) - 5*(e*x + d)^m*a*c^2*d^2*e*f^2*m*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*f*g*m*e^(-m*log(c*d*x + a*e) - m*log(e*
x + d)) + 6*(e*x + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)) + 6*(e*x + d)^m*a*c^2*d^2*e*f^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
) - 6*(e*x + d)^m*a^2*c*d*e^2*f*g*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
+ 2*(e*x + d)^m*a^3*e^3*g^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c^3*
d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^3*d^3*m - 6*c^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.33

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=$$

$$-\frac{\frac{g^2\,x^3\,{\left(d+e\,x\right)}^m\,\left(m^2-3\,m+2\right)}{m^3-6\,m^2+11\,m-6}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+}{\left(c\,d\,e\,x^2+\left(c\,d^2\right.\right.}$$

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2*g
^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*m^2
 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*d^2*
f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(11*m
- 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f + 2*c*d
*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^m
```

3.770 $\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} d$

Optimal result

Integrand size = 42, antiderivative size = 150

$$\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(1-m)(2-m)} + \frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}$$

```
[Out] -(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1-m)/c^2/d^2/e/(1-m)/(2-m)+g*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1-m)/c/d/e/(2-m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {808, 662}

$$\int (d+ex)^m(f+gx) \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)} - \frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}$$

```
[In] Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

integral

$$
\begin{aligned}
&=\frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}\\
&\quad-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx}{cde(2-m)}\\
&=-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(1-m)(2-m)}\\
&\quad+\frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.45

$$
\begin{aligned}
&\int(d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx\\
&=-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}(aeg+cd(f(-2+m)+g(-1+m)x))}{c^2d^2(-2+m)(-1+m)}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-2
 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

method	result
gosper	$-\frac{(ex+d)^{m}(cdgmx+cdfm-cdgx+aeg-2cdf)(cdx+ae)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{-m}}{c^{2}d^{2}(m^{2}-3m+2)}$
risch	$-\frac{\left(g\,x^{2}c^{2}d^{2}m+acdegmx+c^{2}d^{2}fmx-g\,x^{2}c^{2}d^{2}+acdefm-2c^{2}d^{2}fx+a^{2}e^{2}g-2acdef\right)(cdx+ae)^{-m}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(i(cdx+ae)(ex+d))m(-}{}}}{c^{2}d^{2}(-2+m)(-1+m)}$
parallelrisch	$-\frac{(x^{2}(ex+d)^{m}c^{2}d^{2}eg\,m^{2}-x^{2}(ex+d)^{m}c^{2}d^{2}egm+x(ex+d)^{m}acd\,e^{2}g\,m^{2}+x(ex+d)^{m}c^{2}d^{2}ef\,m^{2}-2x(ex+d)^{m}c^{2}d^{2}efm+(ex+d)^{n}}{me\,c^{2}d^{2}(-2+m)(-1+m)}$

```
[In] int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETUR
NVERBOSE)
```

```
[Out] -(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(acdefm-2\,acdef+a^2e^2g+(c^2d^2gm-c^2d^2g)x^2-(2\,c^2d^2f-(c^2d^2f+acdeg)m)x)(ex+d)^m}{(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] -(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 - (
2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3*c
^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)}-\frac{(c^2d^2(m-1)x^2+acdemx+a^2e^2)g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] -(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*c
*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(142) = 284.

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.31

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(ex+d)^m c^2d^2gmx^2e^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m c^2d^2fmxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d}{}$$

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] -((e*x + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e
*x + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d
)^m*a*c*d*e*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - (e*x + d)^m*c^
2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a*c*d*e*
f*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*c^2*d^2*f*x*e^
(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*a*c*d*e*f*e^(-m*log(
c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a^2*e^2*g*e^(-m*log(c*d*x + a*
e) - m*log(e*x + d)))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m)}{c\,d\,(m^2-3\,m+2)}+\frac{a\,e\,(d+e\,x)^m\,(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m)}{c^2\,d^2\,(m^2-3\,m+2)}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

```
[In] int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m - 2
*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*d*
f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m
```

3.771 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[Out] (e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {662}

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{m-1}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
```

```
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}}{cd(-1+m)}$$

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
[Out] -((((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
gosper	$-\frac{(cdx+ae)(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{cd(-1+m)}$
parallelrisch	$\frac{\left(-x(ex+d)^m cde-(ex+d)^m a\,e^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{cde(-1+m)}$
norman	$\left(-\frac{x\,\mathrm{e}^{m\ln(ex+d)}}{-1+m}-\frac{ae\,\mathrm{e}^{m\ln(ex+d)}}{cd(-1+m)}\right)\mathrm{e}^{-m\ln\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)}$
risch	$-\frac{(cdx+ae)(cdx+ae)^{-m}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(i(cdx+ae)(ex+d))m(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(ex+}{2}}}{cd(-1+m)}$

```
[In] int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETURNVERBOSE
)
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=-\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cd}$$

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*
a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c*d*m - c*d)
```

Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(a\,e+c\,d\,x)\,(d+e\,x)^m}{c\,d\,(m-1)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

```
[In] int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m)
```

3.772 $\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx} dx$

Optimal result

Integrand size = 44, antiderivative size = 99

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx} dx = \frac{(ae+cdx)(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m} \text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)(1-m)}$$

```
[Out] (c*d*x+a*e)*(e*x+d)^m*hypergeom([1, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f
))/(-a*e*g+c*d*f)/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {905, 70}

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx} dx = \frac{(d+ex)^m(ae+cdx)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{-m} \text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

```
[In] Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e + c
*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{f+gx}\,dx$$

$$=\frac{(ae+cdx)(d+ex)^m\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)(1-m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int\frac{(d+ex)^m\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{f+gx}\,dx$$
$$=-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,\text{Hypergeometric2F1}\left(1,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)(-1+m)}$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[1
, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(-1
 + m)))
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{gx+f}dx$$

```
[In] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
[Out] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
[Out] integral((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.773 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 101

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \frac{cd(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^2(1-m)}$$

```
[Out] c*d*(c*d*x+a*e)*(e*x+d)^m*hypergeom([2, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c
*d*f))/(-a*e*g+c*d*f)^2/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {905, 70}

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

```
[In] Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*e
 + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{(f+gx)^2}\,dx$$

$$=\frac{cd(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^2(1-m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int\frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx=$$
$$-\frac{cd(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,\text{Hypergeometric2F1}\left(2,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^2(-1+m)}$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2
F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)
^2*(-1 + m)))
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+(e^2a+c\,d^2)\,x+cde\,x^2\right)^{-m}}{(gx+f)^2}dx$$

```
[In] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
[Out] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
[Out] integral((e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m\,(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m\,(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{(f+gx)^2}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.774 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 105

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+c}{cdf-ae}\right.}{(cdf-aeg)^3(1-m)}$$

```
[Out] c^2*d^2*(c*d*x+a*e)*(e*x+d)^m*hypergeom([3, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e
*g+c*d*f))/(-a*e*g+c*d*f)^3/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {905, 70}

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+cd}{cdf-ae}\right.}{(1-m)(cdf-aeg)^3}$$

```
[In] Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g*
(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{(f+gx)^3}\,dx$$

$$=\frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^3(1-m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int\frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx =$$
$$-\frac{c^2d^2(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,\text{Hypergeometric2F1}\left(3,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^3(-1+m)}$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeomet
ric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*
e*g)^3*(-1 + m)))
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+(e^2a+c\,d^2)\,x+cde\,x^2\right)^{-m}}{(gx+f)^3}dx$$

```
[In] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
[Out] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
[Out] integral((e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$$
$$=\int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$$
$$=\int \frac{(d+e\,x)^m}{(f+g\,x)^3\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.775 $\int (d+ex)^m(f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m(f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{-m} dx = \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m(f+gx)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \text{Hypergeometric2F}}{5g}$$

```
[Out] 2/5*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(5/2)*hypergeom([5/
2, m],[7/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int (d+ex)^m(f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x +cdex^2\right)^{-m} dx = \frac{2(f+gx)^{5/2}(d+ex)^m \left(x(ae^2+cd^2)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F}}{5g}$$

```
[In] Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(ae+cdx)^{-m}(f+gx)^{3/2}\,dx\\ &= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(f+gx)^{3/2}\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}dx\\ &= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}{}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},m,\frac{7}{2},\right.}{5g}$$

```
[In] Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F]

$$\int (ex+d)^m (gx+f)^{\frac{3}{2}} \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

```
[In] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int\frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^m}\,dx$$

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
[Out] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

3.776 $\int (d+ex)^m \sqrt{f+gx} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m \sqrt{f+gx} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+g}{cdf-a}\right.}{3g}$$

```
[Out] 2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(3/2)*hypergeom([3/
2, m],[5/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int (d+ex)^m \sqrt{f+gx} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+g}{cdf-ae}\right.}{3g}$$

```
[In] Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(ae+cdx)^{-m}\sqrt{f+gx}\,dx \\
&= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x\right.\right. \\
&\qquad \left.\left.+cdex^2\right)^{-m}\right)\int\sqrt{f+gx}\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}dx \\
&= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}{}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

```
[In] Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F]

$$\int (ex+d)^m\sqrt{gx+f}\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

```
[In] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Giac [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\sqrt{f+g\,x}\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

3.777 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx}{cdf-aeg}\right)}{g}$$

```
[Out] 2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([1/2, m],[3/2],c*d*
(g*x+f)/(-a*e*g+c*d*f))*(g*x+f)^(1/2)/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx}{cdf-aeg}\right)}{g}$$

```
[In] Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{\sqrt{f+gx}}\,dx$$

$$=\left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}}{\sqrt{f+gx}}\,dx$$

$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}\sqrt{f+gx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

```
[In] Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m),x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*
x)*(d + e*x))^m)
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{\sqrt{gx+f}}dx$$

```
[In] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$$
$$=\int \frac{(d+e\,x)^m}{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.778 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[Out] -2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-1/2, m],[1/2],c*
d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)/(g*x+f)^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[In] Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{(f+gx)^{3/2}}\,dx$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}}{(f+gx)^{3/2}}\,dx$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*x)
)^m*Sqrt[f + g*x])
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{3}{2}}}dx$$

```
[In] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
[Out] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.779 $\int \frac{(d+ex)^m(ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int \frac{(d+ex)^m(ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(ade+(cd^2+ae^2)x+cdex^2)^{-m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[Out] -2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-3/2, m],[-1/2]
,c*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {905, 72, 71}

$$\int \frac{(d+ex)^m(ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2(d+ex)^m(x(ae^2+cd^2)+ade+cdex^2)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[In] Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{(ae+cdx)^{-m}}{(f+gx)^{5/2}}\,dx$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{\left(-\frac{aeg}{cdf-aeg}-\frac{cdgx}{cdf-aeg}\right)^{-m}}{(f+gx)^{5/2}}\,dx$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d + e
*x))^m*(f + g*x)^(3/2))
```

Maple [F]

$$\int \frac{(ex+d)^m\left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{5}{2}}}dx$$

```
[In] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^
3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m\,(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)^{5/2}\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.780 $\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 47, antiderivative size = 65

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(ae+cdx)^n(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

```
[Out] (c*d*x+a*e)^n*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1
-m+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {872}

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}(ae+cdx)^n}{cd(-m+n+1)}$$

```
[In] Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
```

```
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && EqQ[c*e*f + c*d*g -
 b*e*g, 0] && NeQ[m - n - 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(ae+cdx)^n (d+ex)^{-1+m} \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int (ae+cdx)^n (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(ae+cdx)^{1+n}(d+ex)^m((ae+cdx)(d+ex))^{-m}}{cd(1-m+n)}$$

```
[In] Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
[Out] ((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/(c*d*(1 - m + n)*((a*e + c*d*x)*(d + e*
x))^m)
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result
gosper	$-\frac{(ex+d)^m (cdx+ae)^{1+n} \left(cde\,x^2+a\,e^2 x+c\,d^2 x+ade\right)^{-m}}{cd(-1+m-n)}$
parallelrisch	$-\frac{\left(x(ex+d)^m (cdx+ae)^n cdem+(ex+d)^m (cdx+ae)^n a\,e^2 m\right)\left(cde\,x^2+a\,e^2 x+c\,d^2 x+ade\right)^{-m}}{mcde(-1+m-n)}$
risch	$-\frac{(cdx+ae)^n (cdx+ae)(cdx+ae)^{-m} \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(i(cdx+ae)(ex+d))m(-\mathrm{csgn}(i(cdx+ae)(ex+d))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(cdx+ae)(ex+}{2}}}{cd(-1+m-n)}$

```
[In] int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=
_RETURNVERBOSE)
[Out] -1/c/d/(-1+m-n)*(e*x+d)^m*(c*d*x+a*e)^(1+n)/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^m)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)(cdx+ae)^n(ex+d)^m e^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] -(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d))/(c*d*m - c*d*n - c*d)
```

Sympy [F(-1)]

Timed out.

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

```
[In] integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)e^{(-m\log(cdx+ae)+n\log(cdx+ae))}}{cd(m-n-1)}$$

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] -(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n - 1
))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(cdx+ae)^n(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(cdx+ae)^n(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] -((c*d*x + a*e)^n*(e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
) + (c*d*x + a*e)^n*(e*x + d)^m*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
))/(c*d*m - c*d*n - c*d)
```

Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(a\,e+c\,d\,x)^{n+1}\,(d+e\,x)^m}{c\,d\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m\,(n-m+1)}$$

```
[In] int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] ((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m*(n - m + 1))
```

3.781 $\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} (a$

Optimal result

Integrand size = 73, antiderivative size = 78

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= -\frac{(d+ex)^m \left(-ae^3g-cde^2gx\right)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \log(ae+cdx)}{cde^2g}$$

```
[Out] -(e*x+d)^m*(-c*d*e^2*g*x-a*e^3*g)^m*ln(c*d*x+a*e)/c/d/e^2/g/((a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = {905, 23, 31}

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$= -\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx) \left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

```
[In] Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))/
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
```

```
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int(ae+cdx)^{-m}\left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\,dx\\
&= \left((d+ex)^m\left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int\frac{1}{cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx}\,dx\\
&= -\frac{(d+ex)^m\left(-ae^3g-cde^2gx\right)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\log(ae+cdx)}{cde^2g}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int(d+ex)^m\left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\left(-e^2g(ae+cdx)\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\log(ae+cdx)}{cde^2g}$$

```
[In] Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
+ m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

Maple [F]

$$\int (ex+d)^m \left(c\,d^2eg-e\left(e^2a+c\,d^2\right)g-cd\,e^2gx\right)^{-1+m} \left(ade+\left(e^2a+c\,d^2\right)x+cde\,x^2\right)^{-m}dx$$

```
[In] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\log\left(cdx+ae\right)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

```
[Out] -log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/
((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{e^{2m-2}(-g)^m\log\left(cdx+ae\right)}{cdg}$$

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

```
[Out] -e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

Giac [F]

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\left(-cde^2gx+cd^2eg-(cd^2+ae^2)eg\right)^{m-1}(ex+d)^m}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^m}\,dx$$

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

```
[Out] integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x + d
)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{{\left(d+e\,x\right)}^m\,{\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)}^{m-1}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}\,dx$$

```
[In] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

3.782 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 213

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}}+\frac{(2ae^2g(1+n)+cd(ef-dg(3+2n)))(ae+cdx)\sqrt{d+ex}(f+gx)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,2+n,\right.}{cdg(cdf-aeg)(1+n)(3+2n)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] (2*a*e^2*g*(1+n)+c*d*(e*f-d*g*(3+2*n)))*(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom
([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))*(e*x+d)^(1/2)/c/d/g/(-a*e*g+c
*d*f)/(1+n)/(3+2*n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e*(g*x+f)^(1+
n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(3+2*n)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {894, 905, 72, 71}

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2e(f+gx)^{n+1}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)(2ae^2g(n+1)+cd(ef-dg(2n+3)))\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{c^2d^2g(2n+3)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*
(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*n)
))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{(2ae^2g(1+n)+cd(ef-dg(3+2n)))\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdg(3+2n)}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left((2ae^2g(1+n)+cd(ef-dg(3+2n)))\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}\,dx}{cdg(3+2n)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left((2ae^2g(1+n)+cd(ef-dg(3+2n)))\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\right.}{\sqrt{ae+}}}{cdg(3+2n)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{2(2ae^2g(1+n)+cd(ef-dg(3+2n)))(ae+cdx)\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2}\right.}{c^2d^2g(3+2n)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(cde(f+gx)+(-2ae^2g(1+n)+cd\right.}{c^2d^2g\,(}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g*
(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n))/(
c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}} (gx+f)^n}{\sqrt{ade+(e^2a+c\,d^2)\,x+cde\,x^2}}dx$$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c*d*x + a*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

3.783 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 501

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{128(cdf-aeg)^3(10ae^2g+cd(ef-11dg))(2ae^2g-cd(3ef-dg))\sqrt{}}{3465c^6d^6eg\sqrt{d+ex}}$$
$$-\frac{128(cdf-aeg)^3(10ae^2g+cd(ef-11dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3465c^5d^5e}$$
$$-\frac{32(cdf-aeg)^2(10ae^2g+cd(ef-11dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1155c^4d^4g\sqrt{d+ex}}$$
$$-\frac{16(cdf-aeg)(10ae^2g+cd(ef-11dg))(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{693c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2(10ae^2g+cd(ef-11dg))(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{11cdg\sqrt{d+ex}}$$

```
[Out] 128/3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(2*a*e^2*g-c*d*(-d
*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^6/d^6/e/g/(e*x+d)^(1/2
)-32/1155*(-a*e*g+c*d*f)^2*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/g/(e*x+d)^(1/2)-16/693*(-a*e*g+c*d
*f)*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/99*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+
f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/11*e
*(g*x+f)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)-128/
3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(e*x+d)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 884, 808, 662}

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \frac{128\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11d}{3465c^6d^6eg\sqrt{d+ex}}$$
$$-\frac{128\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11dg))}{3465c^5d^5e}$$
$$-\frac{32(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(10ae^2g+cd(ef-11dg))}{1155c^4d^4g\sqrt{d+ex}}$$
$$-\frac{16(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(10ae^2g+cd(ef-11dg))}{693c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}(10ae^2g+cd(ef-11dg))}{99c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
```

```
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{11cdg\sqrt{d+ex}} \\
&\quad -\frac{1}{11}\left(-11d+\frac{10ae^2}{cd}+\frac{ef}{g}\right)\int\frac{\sqrt{d+ex}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{2(10ae^2g+cd(ef-11dg))\,(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} \\
&\quad +\frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{11cdg\sqrt{d+ex}} \\
&\quad -\frac{(8(cdf-aeg)\,(10ae^2g+cd(ef-11dg)))\int\frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{99c^2d^2g}
\end{aligned}$$

$$= -\frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{693c^3d^3g\sqrt{d+ex}} - \frac{2(10ae^2g+cd(ef-11dg))(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{\left(16(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)\right)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{231c^3d^3g}$$

$$= -\frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1155c^4d^4g\sqrt{d+ex}} - \frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{693c^3d^3g\sqrt{d+ex}} - \frac{2(10ae^2g+cd(ef-11dg))(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{\left(64(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\right)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{1155c^4d^4g}$$

$$= -\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3465c^5d^5e} - \frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{1155c^4d^4g\sqrt{d+ex}} - \frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{693c^3d^3g\sqrt{d+ex}} - \frac{2(10ae^2g+cd(ef-11dg))(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{11cdg\sqrt{d+ex}} + \frac{\left(64(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3465c^5d^5eg}$$

$$= \frac{128(cdf-aeg)^3\,(10ae^2g+cd(ef-11dg))\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3465c^6d^6eg\sqrt{d+ex}}$$
$$- \frac{128(cdf-aeg)^3\,(10ae^2g+cd(ef-11dg))\,\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3465c^5d^5e}$$
$$- \frac{32(cdf-aeg)^2\,(10ae^2g+cd(ef-11dg))\,(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{1155c^4d^4g\sqrt{d+ex}}$$
$$- \frac{16(cdf-aeg)\,(10ae^2g+cd(ef-11dg))\,(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{693c^3d^3g\sqrt{d+ex}}$$
$$- \frac{2(10ae^2g+cd(ef-11dg))\,(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{99c^2d^2g\sqrt{d+ex}}$$
$$+ \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{11cdg\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-1280a^5e^6g^4+128a^4cde^4g^3(44ef+11dg+$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1280*a^5*e^6*g^4 + 128*a^4*c*d*e^4*g^3*(
44*e*f + 11*d*g + 5*e*g*x) - 32*a^3*c^2*d^2*e^3*g^2*(22*d*g*(9*f + g*x) + e
*(297*f^2 + 88*f*g*x + 15*g^2*x^2)) + 16*a^2*c^3*d^3*e^2*g*(33*d*g*(21*f^2
+ 6*f*g*x + g^2*x^2) + e*(462*f^3 + 297*f^2*g*x + 132*f*g^2*x^2 + 25*g^3*x^
3)) - 2*a*c^4*d^4*e*(44*d*g*(105*f^3 + 63*f^2*g*x + 27*f*g^2*x^2 + 5*g^3*x^
3) + e*(1155*f^4 + 1848*f^3*g*x + 1782*f^2*g^2*x^2 + 880*f*g^3*x^3 + 175*g^
4*x^4)) + c^5*d^5*(11*d*(315*f^4 + 420*f^3*g*x + 378*f^2*g^2*x^2 + 180*f*g^
3*x^3 + 35*g^4*x^4) + e*x*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 154
0*f*g^3*x^3 + 315*g^4*x^4))))/(3465*c^6*d^6*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.24

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+17$
gosper	$-\frac{2(cdx+ae)(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+1760a\,c^4d^4e$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3465/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-315*c^5*d^5*e*g^4*x^5+3
50*a*c^4*d^4*e^2*g^4*x^4-385*c^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a
^2*c^3*d^3*e^3*g^4*x^3+440*a*c^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3
-1980*c^5*d^6*f*g^3*x^3-2970*c^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*
x^2-528*a^2*c^3*d^4*e^2*g^4*x^2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d
^5*e*f*g^3*x^2+3564*a*c^4*d^4*e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772
*c^5*d^5*e*f^3*g*x^2-640*a^4*c*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x+2816*a
^3*c^2*d^2*e^4*f*g^3*x-3168*a^2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^3*e^3*f^
2*g^2*x+5544*a*c^4*d^5*e*f^2*g^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620*c^5*d^6*
f^3*g*x-1155*c^5*d^5*e*f^4*x+1280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*g^4-5632*a
^4*c*d*e^5*f*g^3+6336*a^3*c^2*d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4*f^2*g^2-11
088*a^2*c^3*d^4*e^2*f^2*g^2-7392*a^2*c^3*d^3*e^3*f^3*g+9240*a*c^4*d^5*e*f^3
*g+2310*a*c^4*d^4*e^2*f^4-3465*c^5*d^6*f^4)/c^6/d^6
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(315\,c^5d^5eg^4x^5+1155\,(3\,c^5d^6-2\,ac^4d^4e^2)f^4-1848\,(5\,ac^4d^5e-4\,a^2$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 18
48*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 - 6*
a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3 +
128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*c^5*
d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c^5*d^
6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*g^4)*x
^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g^2 - 44
```

```
*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2 - 10*a^3
*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*a*c^4*d^4
*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 352*(9*a^2*
c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3 - 10*a^4*c*
d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c
^6*d^6*e*x + c^6*d^7)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^4}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**4/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)\,f^4}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{8\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)\,f^3g}{15\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{4\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{8\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e^2}{315\,\sqrt{cdx+ae}c^5d^5}$$
$$+\frac{2\,(315\,c^6d^6ex^6+1408\,a^5cd^2e^5-1280\,a^6e^7+35\,(11\,c^6d^7-ac^5d^5e^2)x^5-5\,(11\,ac^5d^6e-10\,a^2c^4d^4e^3)x^4+8\,(}{3465\,\sqrt{cdx+}}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*a^3*
c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
 - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f^2*g^
```

```
2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4
 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2
*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^
3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^5) + 2/346
5*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(11*c^6*d^7 -
 a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x^4 + 8*(11*a
^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*e^3 - 10*a^4*
c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x)*g^4/(sqrt(c*
d*x + a*e)*c^6*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(465) = 930$.

Time = 0.37 (sec) , antiderivative size = 1789, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/3465*e*(3465*(c^5*d^6*f^4 - a*c^4*d^4*e^2*f^4 - 4*a*c^4*d^5*e*f^3*g + 4*a
^2*c^3*d^3*e^3*f^3*g + 6*a^2*c^3*d^4*e^2*f^2*g^2 - 6*a^3*c^2*d^2*e^4*f^2*g^
2 - 4*a^3*c^2*d^3*e^3*f*g^3 + 4*a^4*c*d*e^5*f*g^3 + a^4*c*d^2*e^4*g^4 - a^5
*e^6*g^4)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^6*d^6*e) - 2*(1155*sqr
t(-c*d^2*e + a*e^3)*c^5*d^6*e^4*f^4 - 1155*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^4
*e^6*f^4 - 924*sqrt(-c*d^2*e + a*e^3)*c^5*d^7*e^3*f^3*g - 2772*sqrt(-c*d^2*
e + a*e^3)*a*c^4*d^5*e^5*f^3*g + 3696*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^3*e^
7*f^3*g + 594*sqrt(-c*d^2*e + a*e^3)*c^5*d^8*e^2*f^2*g^2 + 990*sqrt(-c*d^2*
e + a*e^3)*a*c^4*d^6*e^4*f^2*g^2 + 3168*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^4*
e^6*f^2*g^2 - 4752*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^2*e^8*f^2*g^2 - 220*sqr
t(-c*d^2*e + a*e^3)*c^5*d^9*e*f*g^3 - 308*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^7*
e^3*f*g^3 - 528*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^5*e^5*f*g^3 - 1760*sqrt(-c
*d^2*e + a*e^3)*a^3*c^2*d^3*e^7*f*g^3 + 2816*sqrt(-c*d^2*e + a*e^3)*a^4*c*d
*e^9*f*g^3 + 35*sqrt(-c*d^2*e + a*e^3)*c^5*d^10*g^4 + 45*sqrt(-c*d^2*e + a*
e^3)*a*c^4*d^8*e^2*g^4 + 64*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4*g^4 + 11
2*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6*g^4 + 384*sqrt(-c*d^2*e + a*e^3)*a
^4*c*d^2*e^8*g^4 - 640*sqrt(-c*d^2*e + a*e^3)*a^5*e^10*g^4)/(c^6*d^6*e^5) +
 (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^8*f^4 + 4620*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^5*e^7*f^3*g - 9240*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^9*f^3*g - 13860*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^4*e^8*f^2*g^2 + 20790*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a^2*c^2*d^2*e^10*f^2*g^2 + 13860*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a^2*c^2*d^3*e^9*f*g^3 - 18480*((e*x + d)*c*d*e - c*d
```

```
^2*e + a*e^3)^(3/2)*a^3*c*d*e^11*f*g^3 - 4620*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a^3*c*d^2*e^10*g^4 + 5775*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^4*e^12*g^4 + 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^3
*e^6*f^3*g + 4158*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^4*e^5*f^2
*g^2 - 12474*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^2*d^2*e^7*f^2*g^
2 - 8316*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^2*d^3*e^6*f*g^3 + 16
632*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c*d*e^8*f*g^3 + 4158*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c*d^2*e^7*g^4 - 6930*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9*g^4 + 2970*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(7/2)*c^2*d^2*e^4*f^2*g^2 + 1980*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(7/2)*c^2*d^3*e^3*f*g^3 - 7920*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2
)*a*c*d*e^5*f*g^3 - 1980*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c*d^2*
e^4*g^4 + 4950*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6*g^4 + 1540
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*c*d*e^2*f*g^3 + 385*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(9/2)*c*d^2*e*g^4 - 1925*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(9/2)*a*e^3*g^4 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*
g^4)/(c^6*d^6*e^10))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,(2560\,a^5\,e^6\,g^4-2\right.}{}$$

```
[In] int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2))
/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*a*
c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g + 12
672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5*f*g
^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(3465*
c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^3*g -
1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2*f^3*g
- 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3*c^2*d^2
*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^2*(d + e*
x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a^3*c^2*d^2
*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4224*a^2*c^3
*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4*g^2*x^3*(d
+ e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3*f*g - 44*a*c
*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4*(d + e*x)^(1/
2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/(x + d/e)
```

3.784 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 412

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade}}{315c^5d^5eg\sqrt{d+ex}}$$
$$-\frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{315c^4d^4e}$$
$$-\frac{4(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{9cdg\sqrt{d+ex}}$$

```
[Out] 16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3
*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e/g/(e*x+d)^(1/2)-4/
105*(-a*e*g+c*d*f)*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/63*(8*a*e^2*g+c*d*(-9*d*g+
e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(
1/2)+2/9*e*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^
(1/2)-16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(e*x+d)^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {894, 884, 808, 662}

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{16\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2\,(8ae^2g+cd(ef-9dg)}{315c^5d^5eg\sqrt{d+ex}}$$
$$-\frac{16\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2\,(8ae^2g+cd(ef-9dg))}{315c^4d^4e}$$
$$-\frac{4(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)\,(8ae^2g+cd(ef-9dg))}{105c^3d^3g\sqrt{d+ex}}$$
$$-\frac{2(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(8ae^2g+cd(ef-9dg))}{63c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{9cdg\sqrt{d+ex}} \\
&\quad -\frac{1}{9}\left(-9d+\frac{8ae^2}{cd}+\frac{ef}{g}\right)\int\frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{2(8ae^2g+cd(ef-9dg))\,(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} \\
&\quad +\frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{9cdg\sqrt{d+ex}} \\
&\quad -\frac{(2(cdf-aeg)\,(8ae^2g+cd(ef-9dg)))\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{21c^2d^2g} \\
&= -\frac{4(cdf-aeg)\,(8ae^2g+cd(ef-9dg))\,(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{105c^3d^3g\sqrt{d+ex}} \\
&\quad -\frac{2(8ae^2g+cd(ef-9dg))\,(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} \\
&\quad +\frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{9cdg\sqrt{d+ex}} \\
&\quad -\frac{(8(cdf-aeg)^2\,(8ae^2g+cd(ef-9dg)))\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{105c^3d^3g}
\end{aligned}$$

$$= -\frac{16(cdf-aeg)^2 (8ae^2g+cd(ef-9dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{315c^4d^4e} - \frac{4(cdf-aeg)(8ae^2g+cd(ef-9dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3g\sqrt{d+ex}} - \frac{2(8ae^2g+cd(ef-9dg))(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{9cdg\sqrt{d+ex}} + \frac{(8(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg)))\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{315c^4d^4eg}$$

$$= \frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{315c^5d^5eg\sqrt{d+ex}} - \frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{315c^4d^4e} - \frac{4(cdf-aeg)(8ae^2g+cd(ef-9dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3g\sqrt{d+ex}} - \frac{2(8ae^2g+cd(ef-9dg))(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{9cdg\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(128a^4e^5g^3-16a^3cde^3g^2(27ef+9dg+4egx)}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27*
e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*f^
2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*g^2
*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^4*(9
*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 189*f^
2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.99

method	result
default	$2\sqrt{(cdx+ae)(ex+d)}\,(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f$
gosper	$2(cdx+ae)(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f\,g^2x^2+189$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/315/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(35*c^4*d^4*e*g^3*x^4-40*a*
c^3*d^3*e^2*g^3*x^3+45*c^4*d^5*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d
^2*e^3*g^3*x^2-54*a*c^3*d^4*e*g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^4*d
^5*f*g^2*x^2+189*c^4*d^4*e*f^2*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^3*e^
2*g^3*x+216*a^2*c^2*d^2*e^3*f*g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*d^3*e
^2*f^2*g*x+315*c^4*d^5*f^2*g*x+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-144*a^3*
c*d^2*e^3*g^3-432*a^3*c*d*e^4*f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^2*c^2*d
^2*e^3*f^2*g-630*a*c^3*d^4*e*f^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^5*f^3)/c
^5/d^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(35\,c^4d^4eg^3x^4+105\left(3\,c^4d^5-2\,ac^3d^3e^2\right)f^3-126\left(5\,ac^3d^4e-4\,a^2c^2d\right)\right)}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*(
5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*
d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*g^2
 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(7*c^
4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)*g^3)
*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g - 36*(7*
a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e
^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*
d^5*e*x + c^5*d^6)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^3}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)f^2g}{5\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{2\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-\dots}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{2\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e^{\dots}}{315\,\sqrt{cdx+ae}c^5d^5}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^
3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f*g^2/(
sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4 +
128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^
3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c
^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(382) = 764$.

Time = 0.34 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/315*e*(315*(c^4*d^5*f^3 - a*c^3*d^3*e^2*f^3 - 3*a*c^3*d^4*e*f^2*g + 3*a^2
*c^2*d^2*e^3*f^2*g + 3*a^2*c^2*d^3*e^2*f*g^2 - 3*a^3*c*d*e^4*f*g^2 - a^3*c*
d^2*e^3*g^3 + a^4*e^5*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^5*d^5
*e) - 2*(105*sqrt(-c*d^2*e + a*e^3)*c^4*d^5*e^3*f^3 - 105*sqrt(-c*d^2*e + a
*e^3)*a*c^3*d^3*e^5*f^3 - 63*sqrt(-c*d^2*e + a*e^3)*c^4*d^6*e^2*f^2*g - 189
*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^4*e^4*f^2*g + 252*sqrt(-c*d^2*e + a*e^3)*a^
2*c^2*d^2*e^6*f^2*g + 27*sqrt(-c*d^2*e + a*e^3)*c^4*d^7*e*f*g^2 + 45*sqrt(-
c*d^2*e + a*e^3)*a*c^3*d^5*e^3*f*g^2 + 144*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d
^3*e^5*f*g^2 - 216*sqrt(-c*d^2*e + a*e^3)*a^3*c*d*e^7*f*g^2 - 5*sqrt(-c*d^2
*e + a*e^3)*c^4*d^8*g^3 - 7*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2*g^3 - 12*s
qrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4*g^3 - 40*sqrt(-c*d^2*e + a*e^3)*a^3*c
*d^2*e^6*g^3 + 64*sqrt(-c*d^2*e + a*e^3)*a^4*e^8*g^3)/(c^5*d^5*e^4) + (105*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e^6*f^3 + 315*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*e^5*f^2*g - 630*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^7*f^2*g - 630*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a*c^2*d^3*e^6*f*g^2 + 945*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^2*c*d*e^8*f*g^2 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*c*d^2*e^7*g^3 - 420*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9*g^
3 + 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^2*e^4*f^2*g + 189*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^3*e^3*f*g^2 - 567*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c*d*e^5*f*g^2 - 189*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2)*a*c*d^2*e^4*g^3 + 378*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2)*a^2*e^6*g^3 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c*d*e
^2*f*g^2 + 45*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c*d^2*e*g^3 - 180*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3*g^3 + 35*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(9/2)*g^3)/(c^5*d^5*e^8))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.40 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-864\right.}{}\right.}{}$$

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*e
^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3 +
1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4*e*
f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^(1/2
))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g + 14
4*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g + 432
*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (2*g*x^2
*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*g - 18*a
*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d + e*x)^(1
/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d/e)
```

3.785 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 321

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+}}{105c^4d^4eg\sqrt{d+ex}}$$
$$-\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3e}$$
$$-\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cdg\sqrt{d+ex}}$$

```
[Out] 8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3*e*
f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e/g/(e*x+d)^(1/2)-2/35*
(6*a*e^2*g+c*d*(-7*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/7*e*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c/d/g/(e*x+d)^(1/2)-8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*
f))*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 884, 808, 662}

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{8\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)\,(6ae^2g+cd(ef-7dg))\,(}{105c^4d^4eg\sqrt{d+ex}}$$
$$-\frac{8\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)\,(6ae^2g+cd(ef-7dg))}{105c^3d^3e}$$
$$-\frac{2(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(6ae^2g+cd(ef-7dg))}{35c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
```

```
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cdg\sqrt{d+ex}} \\
&\quad -\frac{1}{7}\left(-7d+\frac{6ae^2}{cd}+\frac{ef}{g}\right)\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx \\
&= -\frac{2(6ae^2g+cd(ef-7dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}} \\
&\quad +\frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cdg\sqrt{d+ex}} \\
&\quad -\frac{(4(cdf-aeg)(6ae^2g+cd(ef-7dg)))\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{35c^2d^2g} \\
&= -\frac{8(cdf-aeg)(6ae^2g+cd(ef-7dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^3d^3e} \\
&\quad -\frac{2(6ae^2g+cd(ef-7dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}} \\
&\quad +\frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7cdg\sqrt{d+ex}} \\
&\quad +\frac{(4(cdf-aeg)(6ae^2g+cd(ef-7dg))(2ae^2g-cd(3ef-dg)))\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{105c^3d^3eg}
\end{aligned}$$

$$= \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{105c^4d^4eg\sqrt{d+ex}}$$
$$-\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{105c^3d^3e}$$
$$-\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^2d^2g\sqrt{d+ex}}$$
$$+\frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cdg\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(-48a^3e^4g^2+8a^2cde^2g(14ef+7dg+3egx)\right.}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*f
 + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*f*
g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(35*
f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.74

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(-15g^2e\,x^3c^3d^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2fg\right.}{105\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)\left(-15g^2e\,x^3c^3d^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2fgx-70c^3d^4\right.}{105c^4d^4\sqrt{cde\,x^2+a\,e^2x+c\,d}}$

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-15*c^3*d^3*e*g^2*x^3+18*
a*c^2*d^2*e^2*g^2*x^2-21*c^3*d^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^
3*g^2*x+28*a*c^2*d^3*e*g^2*x+56*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3
*d^3*e*f^2*x+48*a^3*e^4*g^2-56*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*
c^2*d^3*e*f*g+70*a*c^2*d^2*e^2*f^2-105*c^3*d^4*f^2)/c^4/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(15\,c^3d^3eg^2x^3+35\left(3\,c^3d^4-2\,ac^2d^2e^2\right)f^2-28\left(5\,ac^2d^3e-4\,a^2cde^3\right)\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5*
a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 + 3*
(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3*e*
f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c*d*e
^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*
d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}(f+gx)^2}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^2}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{4\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,ac^2d^3e-4\,a^2cde^3\right)x\right)fg}{15\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{2\left(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\left(7\,c^4d^5-ac^3d^3e^2\right)x^3-\left(7\,ac^3d^4e-6\,a^2c^2d^2e^3\right)x^2+4\left(7\,a^2c^2d^3e^2-\right.\right.}{105\,\sqrt{cdx+ae}c^4d^4}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g^2/(sq
rt(c*d*x + a*e)*c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(297) = 594$.

Time = 0.32 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.19

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{105\,(c^3d^4f^2-ac^2d^2e^2f^2-2\,ac^2d^3efg+2\,a^2cde^3fg+a^2cd^2e^2g^2-a^3e^4g^2)\sqrt{(ex+d)cde-}}{c^4d^4e}\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/105*e*(105*(c^3*d^4*f^2 - a*c^2*d^2*e^2*f^2 - 2*a*c^2*d^3*e*f*g + 2*a^2*c
*d*e^3*f*g + a^2*c*d^2*e^2*g^2 - a^3*e^4*g^2)*sqrt((e*x + d)*c*d*e - c*d^2*
e + a*e^3)/(c^4*d^4*e) - 2*(35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*e^2*f^2 - 35*
sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^4*f^2 - 14*sqrt(-c*d^2*e + a*e^3)*c^3*d^
5*e*f*g - 42*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^3*e^3*f*g + 56*sqrt(-c*d^2*e +
a*e^3)*a^2*c*d*e^5*f*g + 3*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g^2 + 5*sqrt(-c*d
^2*e + a*e^3)*a*c^2*d^4*e^2*g^2 + 16*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4*g
^2 - 24*sqrt(-c*d^2*e + a*e^3)*a^3*e^6*g^2)/(c^4*d^4*e^3) + (35*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*f^2 + 70*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*c^2*d^3*e^3*f*g - 140*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a*c*d*e^5*f*g - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d^
2*e^4*g^2 + 105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6*g^2 + 42*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d*e^2*f*g + 21*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(5/2)*c*d^2*e*g^2 - 63*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2)*a*e^3*g^2 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^2)/(c^
4*d^4*e^6))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,(96\,a^3\,e^4\,g^2-112\,a}{}\right.}{}$$

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2))
/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2*d
^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e^3*
f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*d^4*
f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g))/(10
5*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c*d*e*f
))/(35*c^2*d^2*e)))/(x + d/e)
```

3.786 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 209

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{4(cd^2-ae^2)(4ae^2g-cd(5ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} - \frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^2d^2e} + \frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5cde}$$

```
[Out] 2/5*g*(e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-4/15*(-a*
e^2+c*d^2)*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c^3/d^3/e/(e*x+d)^(1/2)-2/15*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(e*x+d)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used

$= \{808, 670, 662\}$

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{4(cd^2-ae^2)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(4ae^2g-cd(5ef-dg))}{15c^3d^3e\sqrt{d+ex}} - \frac{2\sqrt{d+ex}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(4ae^2g-cd(5ef-dg))}{15c^2d^2e} + \frac{2g(d+ex)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5cde}$$

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cde} \\
&\quad+\frac{1}{5}\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right)\int\frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= -\frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e} \\
&\quad+\frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cde} \\
&\quad+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{15d} \\
&= -\frac{4(cd^2-ae^2)\,(4ae^2g-cd(5ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} \\
&\quad-\frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e} \\
&\quad+\frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cde}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.46

$$
\int\frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(8a^2e^3g-2acde(5ef+5dg+2egx)+c^2d^2(5}{15c^3d^3\sqrt{d+ex}}
$$

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sqrt
[d + e*x])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)}{15\sqrt{ex+d}\,c^3d^3}$	113
gosper	$\frac{2(cdx+ae)(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	131

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*e*g*x^2-4*a*c*d*e
^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*e^
2*f+15*c^2*d^3*f)/c^3/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(3\,c^2d^2egx^2+5\,(3\,c^2d^3-2\,acde^2)f-2\,(5\,acd^2e-4\,a^2e^3)g+(5\,c^2d^2}{15\,(c^3d^3ex}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f}{3\,\sqrt{cdx+ae}c^2d^2} + \frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)g}{15\,\sqrt{cdx+ae}c^3d^3}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 + 8
*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3
)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{15\,(c^2d^3f-acde^2f-acd^2eg+a^2e^3g)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e} - \frac{2\left(5\sqrt{-cd^2e+ae^3}c^2d\right.}{}\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2/15*e*(15*(c^2*d^3*f - a*c*d*e^2*f - a*c*d^2*e*g + a^2*e^3*g)*sqrt((e*x +
d)*c*d*e - c*d^2*e + a*e^3)/(c^3*d^3*e) - 2*(5*sqrt(-c*d^2*e + a*e^3)*c^2*d
^3*e*f - 5*sqrt(-c*d^2*e + a*e^3)*a*c*d*e^3*f - sqrt(-c*d^2*e + a*e^3)*c^2*
d^4*g - 3*sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2*g + 4*sqrt(-c*d^2*e + a*e^3)*a
^2*e^4*g)/(c^3*d^3*e^2) + (5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*
e^2*f + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^2*e*g - 10*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g + 3*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(5/2)*g)/(c^3*d^3*e^4))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\,e^2+\right.}{15\,c^3\,d^3\,e}\right.}{x+\frac{d}{e}}$$

[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)

[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (2*g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e^2*g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)

3.787 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 109

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd}$$

```
[Out] 4/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^
(1/2)+2/3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {670, 662}

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{4(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rubi steps

$$\begin{aligned}&\text{integral}\\&=\frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3d}\\&=\frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int\frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(-2ae^2+cd(3d+ex))}{3c^2d^2\sqrt{d+ex}}$$

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-cdex+2e^2a-3c\,d^2)}{3\sqrt{ex+d}\,c^2d^2}$	51
gosper	$-\frac{2(cdx+ae)(-cdex+2e^2a-3c\,d^2)\sqrt{ex+d}}{3c^2d^2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	69

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*e*x+2*a*e^2-3*c*d^2)/c
^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdex+3\,cd^2-2\,ae^2)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^
2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)}{3\,\sqrt{cdx+ae}c^2d^2}$$

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,(cd^2-ae^2)}{c^2d^2e} - \frac{2\,\left(\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2\right)}{c^2d^2e} + \frac{((e\cdots}{}\right.}{3\,|e|}$$

[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")

[Out] 2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2 - a*e^2)/(c^2*d^2*e
) - 2*(sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c^2*d^
2*e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2))/abs(e)

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d} - \frac{(4\,a\,e^2-6\,c\,d^2)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)

[Out] (((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3*
c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)

3.788 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 139

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] -2*(-d*g+e*f)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)+2*e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {894, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cdg\sqrt{d+ex}} \\
&\quad -\frac{\left(2\left(\frac{1}{2}cde^2f-\frac{1}{2}cd^2eg\right)\right)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{cdeg} \\
&= \frac{2e\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cdg\sqrt{d+ex}} \\
&\quad -\frac{(2e^2(ef-dg))\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g} \\
&= \frac{2e\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{d+ex}\Big(e\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd(-ef+dg)\sqrt{ae}}{cdg^{3/2}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d+}}$$

[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e *x^2]),x]

[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f) + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a *e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\,d^{2}g-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdef-e\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,dcg\sqrt{(aeg-cdf)g}}$	153

[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method= _RETURNVERBOSE)

[Out] -2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)* g)^(1/2))*c*d^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d* e*f-e*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^ (1/2)/d/c/g/((a*e*g-c*d*f)*g)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \Bigg[\frac{(cd^2ef-cd^3g+(cde^2f-cd^2eg)x)\sqrt{-cdfg+aeg^2}\log\Big(-}{}$$

[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")

[Out] [((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)

```
*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)
*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2*g^2
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2
 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*f - c*
d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c
*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*f*g - a*e^2*g^2)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2 - a*c
*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(123) = 246.

Time = 0.41 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e}{g|e|} - \frac{(cde^3f-cd^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\cdots}{\sqrt{cdfg-aeg^2}e}\cdots\right.}{\sqrt{cdfg-aeg^2}eg|e|}\cdots\right.}{cd} + \frac{2\left(cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right) - \sqrt{-cd^2e+ae^3}\sqrt{cdfg-aeg^2}e\right)}{\sqrt{cdfg-aeg^2}cdg|e|}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e/(g*abs(e)) - (c*d*e^3*f - c*d^
2*e^2*g)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a
*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g*abs(e)))/(c*d) + 2*(c*d*e^2*f*arct
an(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - c*d^2*e*g*arctan
(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c*d^2*e + a*
e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*c*d*g*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

3.789 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 170

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{(2ae^2g-cd(ef+dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}$$

```
[Out] -(2*a*e^2*g-c*d*(d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-(-d
*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e
*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {892, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(2ae^2g-cd(dg+ef))\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}} - \frac{(ef-dg)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} \\
&\quad+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{3}{2}cd^2eg-e(cd^2+ae^2)\,g\right)\right)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{g\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} \\
&\quad-\frac{\left(e^2(2ae^2g-cd(ef+dg))\right)\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{g(cdf-aeg)}
\end{aligned}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{(2ae^2g-cd(ef+dg))\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(-\frac{\sqrt{g}(-ef+dg)(ae+cdx)}{(-cdf+aeg)(f+gx)} + \frac{(-2ae^2g+cd(ef+dg))\sqrt{ae+cdx}\,\text{arcta}}{(cdf-aeg)^{3/2}}\right.}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/((-(c*d*f) + a*e*g
)*(f + g*x))) + ((-2*a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[(S
qrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(3/2)))/(g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(154) = 308.

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.98

method	result
default	$\frac{\left(-2\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,e^2g^2x+\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\,d^2g^2x+\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdefgx-2\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g}$

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] (-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*g^2*x+arctan
h(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d^2*g^2*x+arctanh(g*(c*d*x
+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*e*f*g*x-2*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c*d^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c*d*e*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*d*g+(c*d*x+a*e)^(1
/2)*((a*e*g-c*d*f)*g)^(1/2)*e*f)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/
(c*d*x+a*e)^(1/2)/g/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(154) = 308$.

Time = 0.32 (sec) , antiderivative size = 896, normalized size of antiderivative = 5.27

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \left[-\frac{(cd^2ef^2+(cd^3-2\,ade^2)fg+(cde^2fg+(cd^2e-2\,ae^3)\right.$$

$$-\frac{(cd^2ef^2+(cd^3-2\,ade^2)fg+(cde^2fg+(cd^2e-2\,ae^3)g^2)x^2+(cde^2f^2+2\,(cd^2e-ae^3)fg+(cd^3-2\,ade}{c^2d^3f^3g^2-2\,acd^2ef^2g^3+a^2de^2fg^4+(c^2d^2ef^2g^3-2\,a}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2
*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*
e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^
2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^
2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 + (c*d^3 - 2
*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2
+ 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*
f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f^2*g^3 + a^2*d*e
^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^2 + (c^2
*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g^3 - (2*a*c*d
^2*e - a^2*e^3)*f*g^4)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(154) = 308.

Time = 0.41 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$

$$-\frac{cde^2f\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+cd^2eg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-2\,ae^3g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-\sqrt{-cd^2e+ae^3}\sqrt{}}{\sqrt{cdfg-aeg^2}cdfg|e|-\sqrt{cdfg-aeg^2}aeg^2|e|}$$

$$+\frac{e\left(\frac{(c^2d^2e^2f+c^2d^3eg-2\,acde^3g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(cdfg|e|-aeg^2|e|)\sqrt{cdfg-aeg^2}e}-\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2e^2f-\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^3eg}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)(cdfg|e|-aeg^2|e|)}\right)}{cd}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] -(c*d*e^2*f*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) +
c*d^2*e*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 2*
a*e^3*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt
(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*e)/(sqrt(c*d*f*g - a*e*g^2)*c*d*
f*g*abs(e) - sqrt(c*d*f*g - a*e*g^2)*a*e*g^2*abs(e)) + e*((c^2*d^2*e^2*f +
c^2*d^3*e*g - 2*a*c*d*e^3*g)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c*d*f*g*abs(e) - a*e*g^2*abs(e))*sqrt(c*d
*f*g - a*e*g^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^2*f
 - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^3*e*g)/((c*d*e^2*f - a*e^3
*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f*g*abs(e) - a*e*g^2*abs(e
))))/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.790 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 261

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} - \frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} - \frac{cd(4ae^2g-cd(ef+3dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}$$

```
[Out] -1/4*c*d*(4*a*e^2*g-c*d*(3*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)
^(5/2)-1/2*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d
*f)/(g*x+f)^2/(e*x+d)^(1/2)-1/4*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {892, 886, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{cd(4ae^2g-cd(3dg+ef))\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}} - \frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(4ae^2g-cd(3dg+ef))}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)^2}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -1/2*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]
*(f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/
(4*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\
&\quad+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{7}{2}cd^2eg-2e(cd^2+ae^2)g\right)\right)\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{2g\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\
&\quad-\frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\
&\quad-\frac{(cd(4ae^2g-cd(ef+3dg)))\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g(cdf-aeg)^2} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\
&\quad-\frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\
&\quad-\frac{(cde^2(4ae^2g-cd(ef+3dg)))\,\text{Subst}\left(\int\frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{4g(cdf-aeg)^2} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\
&\quad-\frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\
&\quad-\frac{cd(4ae^2g-cd(ef+3dg))\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{cd\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)(-2aeg(dg+e(f+2gx))+cd(ef(-f+gx)+dg(5f+}{cd(cdf-aeg)^2(f+gx)^2}\right.}{4g^{3/2}\sqrt{(ae+cdx)}}$$

[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d *e*x^2]),x]

[Out] (c*d*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(-2*a*e*g*(d*g + e*(f + 2*g*x)) + c*d*(e*f*(-f + g*x) + d*g*(5*f + 3*g*x))))/(c*d*(c*d*f - a*e*g)^2*(f + g* x)^2) + ((-4*a*e^2*g + c*d*(e*f + 3*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g] *Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(5/2)))/(4*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(235) = 470.

Time = 0.58 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(4\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acde^2g^3x^2-3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^3g^3x^2-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2efg^2\right.}{}$

[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho d=_RETURNVERBOSE)

[Out] 1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d* f)*g)^(1/2))*a*c*d*e^2*g^3*x^2-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f) *g)^(1/2))*c^2*d^3*g^3*x^2-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1 /2))*c^2*d^2*e*f*g^2*x^2+8*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1 /2))*a*c*d*e^2*f*g^2*x-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^3*f*g^2*x-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c ^2*d^2*e*f^2*g*x+4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c *d*e^2*f^2*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^3 *f^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*e*f^3-4 *a*e^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c*d^2*g^2*x*(c*d*x +a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+c*d*e*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c *d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-2*a*e^ 2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c*d*x+a*e)^(1/ 2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1

```
/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*g-c*d*f)^2/(c*d
*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(235) = 470$.

Time = 0.35 (sec) , antiderivative size = 1704, normalized size of antiderivative = 6.53

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g
^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d
^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2
*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2
)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*d*e^2)*f
^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3*c^2*d^3
- 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f
^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3*e*f^3*g^4 -
 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^3 + (2*c^3*
d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*
c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x
^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^2*c*d^2*e^2 - a^3
*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g
 + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e
^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*
g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*
d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e
*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g
^4 - (5*c^2*d^3 - a*c*d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c
^2*d^2*e*f^2*g^2 + (3*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e
^3)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3
*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*
f^2*g^5 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^
6 - a^3*e^4*g^7)*x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*
```

```
a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c
*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 +
 (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^
3*g^4 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)(ae+cdx)}\,(f+gx)^3}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^3}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(235) = 470.

Time = 0.53 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{e^2\left(\frac{(c^3d^3ef+3\,c^3d^4g-4\,ac^2d^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2f^2g|e|-2\,acdefg^2|e|+a^2e^2g^3|e|)\sqrt{cdfg-aeg^2}e}-\frac{\sqrt{(e}}{}\right.}{} - \frac{c^2d^2e^3f^2\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)+2\,c^2d^3e^2fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-4\,acde^4fg\arctan\left(\frac{\sqrt{-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)-3\,c^2}{4\left(\sqrt{cdfg-aeg^2}c^2d^2ef^3g|e|-\sqrt{cdfg-aeg^2}c^2d^3f^2\right.}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/4*e^2*((c^3*d^3*e*f + 3*c^3*d^4*g - 4*a*c^2*d^2*e^2*g)*arctan(sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g
*abs(e) - 2*a*c*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*sqrt(c*d*f*g - a*e*g
^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^3*f^2 - 5*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^5*e^2*f*g + 3*sqrt((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*a*c^3*d^3*e^4*f*g + 5*sqrt((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*a*c^3*d^4*e^3*g^2 - 4*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^
2*d^2*e^5*g^2 - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e*f*g - 3
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*g^2 + 4*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^2*g^2)/((c^2*d^2*f^2*g*abs(e) - 2*a*c
*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*g)^2))/(c*d) - 1/4*(c^2*d^2*e^3*f^2*arctan(sqrt(-c*
d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 2*c^2*d^3*e^2*f*g*arctan(sq
rt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 4*a*c*d*e^4*f*g*arcta
n(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*c^2*d^4*e*g^2*a
rctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 4*a*c*d^2*e^3
*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - sqrt(-c
*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c*d*e^2*f + 3*sqrt(-c*d^2*e + a*e^3
)*sqrt(c*d*f*g - a*e*g^2)*c*d^2*e*g - 2*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g
 - a*e*g^2)*a*e^3*g)/(sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e*f^3*g*abs(e) - sqrt
(c*d*f*g - a*e*g^2)*c^2*d^3*f^2*g^2*abs(e) - 2*sqrt(c*d*f*g - a*e*g^2)*a*c*
d*e^2*f^2*g^2*abs(e) + 2*sqrt(c*d*f*g - a*e*g^2)*a*c*d^2*e*f*g^3*abs(e) + s
qrt(c*d*f*g - a*e*g^2)*a^2*e^3*f*g^3*abs(e) - sqrt(c*d*f*g - a*e*g^2)*a^2*d
*e^2*g^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.791 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 351

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} - \frac{(6ae^2g-cd(ef+5dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{cd(6ae^2g-cd(ef+5dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{c^2d^2(6ae^2g-cd(ef+5dg))\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}$$

```
[Out] -1/8*c^2*d^2*(6*a*e^2*g-c*d*(5*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*
d*f)^(7/2)-1/3*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g
+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)-1/12*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)-1/
8*c*d*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {892, 886, 888, 211}

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{c^2d^2(6ae^2g-cd(5dg+ef))\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}} - \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}(6ae^2g-cd(5dg+ef))}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(6ae^2g-cd(5dg+ef))}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} - \frac{(ef-dg)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -1/3*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x
]*(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (
c^2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*
(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
&\quad+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{11}{2}cd^2eg-3e(cd^2+ae^2)\,g\right)\right)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3g\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
&\quad-\frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\
&\quad-\frac{(cd(6ae^2g-cd(ef+5dg)))\int\frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{8g(cdf-aeg)^2} \\
&= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
&\quad-\frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\
&\quad-\frac{cd(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} \\
&\quad-\frac{(c^2d^2(6ae^2g-cd(ef+5dg)))\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{16g(cdf-aeg)^3}
\end{aligned}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} - \frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{cd(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{(c^2d^2e^2(6ae^2g-cd(ef+5dg)))\,\mathrm{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(ef+dg)+e^2gx^2}\,dx, x, \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\right)}{8g(cdf-aeg)^3}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} - \frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} - \frac{cd(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{c^2d^2(6ae^2g-cd(ef+5dg))\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{c^2d^2\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)(4a^2e^2g^2(2dg+e(f+3gx))-2acdeg(dg(13f+5gx)\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (c^2*d^2*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(4*a^2*e^2*g^2*(2*d*g + e*(f
 + 3*g*x)) - 2*a*c*d*e*g*(d*g*(13*f + 5*g*x) + e*(8*f^2 + 25*f*g*x + 9*g^2*
x^2)) + c^2*d^2*(e*f*(-3*f^2 + 8*f*g*x + 3*g^2*x^2) + d*g*(33*f^2 + 40*f*g*
x + 15*g^2*x^2))))/(c^2*d^2*(c*d*f - a*e*g)^3*(f + g*x)^3) + (3*(-6*a*e^2*g
 + c*d*(e*f + 5*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(7/2)))/(24*g^(3/2)*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. 2(319) = 638.

Time = 0.56 (sec) , antiderivative size = 1132, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1132

```
[In] int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(40*c^2*d^3*f*g^2*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*a*c^2*d^2*e^2*g^4*x^3-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^3*d^3*e*f*g^3*x^3-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^3*d^3*e*f^2*g^2*x^2-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*c^3*d^3*e*f^3*g*x+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*a*c^2*d^2*e^2*f^3*g+8*a^2*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)+4*a^2*e^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+33*c^2*d^
3*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-3*c^2*d^2*e*f^3*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^4*f*g^3*x^2-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^4*f^2*g^2*x-50*a*c*d*e^2*f*g^2*x*(c*d*x+a*e)^(1/2)*((a*e
*g-c*d*f)*g)^(1/2)-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*
c^3*d^4*g^4*x^3-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3
*d^4*f^3*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e
*f^4+54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*
f*g^3*x^2-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-16*a
*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+54*arctanh(g*(c*d*
x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f^2*g^2*x-18*a*c*d*e^2*
g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c^2*d^2*e*f*g^2*x^2*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-10*a*c*d^2*e*g^3*x*(c*d*x+a*e)^(1/2)
*((a*e*g-c*d*f)*g)^(1/2)+8*c^2*d^2*e*f^2*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)+15*c^2*d^3*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+12
*a^2*e^3*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c
^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(319) = 638$.

Time = 0.96 (sec) , antiderivative size = 2736, normalized size of antiderivative = 7.79

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*e
^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*g^
2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)
*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^2 +
(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3*d^4*
e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(
3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*e^2)*f^3*g
^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*e^2)*f*g^4 -
 (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3*g^2 + (20*c^
3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*c*d*e^3)*f*g^4
 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f^6*g^3 + 6*a^2*c
^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f^3*g^6 + (c^4*d^4
*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*
d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f
^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 3
*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 -
4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 +
2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*c*d^2*e^3 - a^4*e^5)
*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^7 + (3*c^4*d^5 - 4*a
*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(9*
a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3
*g^6)*x), -1/24*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (
c^3*d^3*e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e
^2*f^2*g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2
*d^3*e^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f
^2*g^2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4
*c^3*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
```

```
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^
2 + a*e^2)*g*x)) + (3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*
a*c^2*d^2*e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*
c^2*d^2*e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^
3*e*f^3*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 3
1*a^2*c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4
*e*f^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^
4*f^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^
3*f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
 a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e -
9*a^2*c^2*d^2*e^3)*f^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 -
(4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4
*f*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2
*d^2*e^3)*f^4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*
c*d^2*e^3 - a^4*e^5)*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^
7 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*
e^3)*f^5*g^4 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^
2*e^3 - a^4*e^5)*f^3*g^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1879 vs. $2(319) = 638$.

Time = 0.74 (sec) , antiderivative size = 1879, normalized size of antiderivative = 5.35

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 1/24*e^3*(3*(c^4*d^4*e*f + 5*c^4*d^5*g - 6*a*c^3*d^3*e^2*g)*arctan(sqrt((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*e*
f^3*g*abs(e) - 3*a*c^2*d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(e)
- a^3*e^4*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (3*sqrt((e*x + d)*c*d*e
- c*d^2*e + a*e^3)*c^6*d^6*e^5*f^3 - 33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c^6*d^7*e^4*f^2*g + 24*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^5*d
^5*e^6*f^2*g + 66*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^5*d^6*e^5*f*g
^2 - 57*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^4*e^7*f*g^2 - 33*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^5*e^6*g^3 + 30*sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^3*d^3*e^8*g^3 - 8*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*c^5*d^5*e^3*f^2*g - 40*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*c^5*d^6*e^2*f*g^2 + 56*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*a*c^4*d^4*e^4*f*g^2 + 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^4*d
^5*e^3*g^3 - 48*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^3*d^3*e^5*g
^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^4*e*f*g^2 - 15*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^5*g^3 + 18*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2)*a*c^3*d^3*e^2*g^3)/((c^3*d^3*e*f^3*g*abs(e) - 3*a*c^2*
d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(e) - a^3*e^4*g^4*abs(e))*(
c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3))/(c*d) - 1/
24*(3*c^3*d^3*e^4*f^3*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e)) + 9*c^3*d^4*e^3*f^2*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g
 - a*e*g^2)*e)) - 18*a*c^2*d^2*e^5*f^2*g*arctan(sqrt(-c*d^2*e + a*e^3)*g/(s
qrt(c*d*f*g - a*e*g^2)*e)) - 27*c^3*d^5*e^2*f*g^2*arctan(sqrt(-c*d^2*e + a*
e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 36*a*c^2*d^3*e^4*f*g^2*arctan(sqrt(-c
*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) + 15*c^3*d^6*e*g^3*arctan(sq
rt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 18*a*c^2*d^4*e^3*g^3*
arctan(sqrt(-c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e)) - 3*sqrt(-c*d^
2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^2*e^3*f^2 + 22*sqrt(-c*d^2*e + a
*e^3)*sqrt(c*d*f*g - a*e*g^2)*c^2*d^3*e^2*f*g - 16*sqrt(-c*d^2*e + a*e^3)*s
qrt(c*d*f*g - a*e*g^2)*a*c*d*e^4*f*g - 15*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f
*g - a*e*g^2)*c^2*d^4*e*g^2 + 8*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g
^2)*a*c*d^2*e^3*g^2 + 4*sqrt(-c*d^2*e + a*e^3)*sqrt(c*d*f*g - a*e*g^2)*a^2*
e^5*g^2)/(sqrt(c*d*f*g - a*e*g^2)*c^3*d^3*e^2*f^5*g*abs(e) - 2*sqrt(c*d*f*g
 - a*e*g^2)*c^3*d^4*e*f^4*g^2*abs(e) - 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^2*
```

```
e^3*f^4*g^2*abs(e) + sqrt(c*d*f*g - a*e*g^2)*c^3*d^5*f^3*g^3*abs(e) + 6*sqr
t(c*d*f*g - a*e*g^2)*a*c^2*d^3*e^2*f^3*g^3*abs(e) + 3*sqrt(c*d*f*g - a*e*g^
2)*a^2*c*d*e^4*f^3*g^3*abs(e) - 3*sqrt(c*d*f*g - a*e*g^2)*a*c^2*d^4*e*f^2*g
^4*abs(e) - 6*sqrt(c*d*f*g - a*e*g^2)*a^2*c*d^2*e^3*f^2*g^4*abs(e) - sqrt(c
*d*f*g - a*e*g^2)*a^3*e^5*f^2*g^4*abs(e) + 3*sqrt(c*d*f*g - a*e*g^2)*a^2*c*
d^3*e^2*f*g^5*abs(e) + 2*sqrt(c*d*f*g - a*e*g^2)*a^3*d*e^4*f*g^5*abs(e) - s
qrt(c*d*f*g - a*e*g^2)*a^3*d^2*e^3*g^6*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.792 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} \\ &\quad -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d^2x^2}}{16d^6} \\ &\quad -\frac{b(12c^2+5b^2d^2+30acd^2)\,x^2\sqrt{1-d^2x^2}}{15d^4} \\ &\quad -\frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-d^2x^2}}{24d^4}-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \\ &\quad +\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)\arcsin(dx)}{16d^7}\end{aligned}$$

```
[Out] 1/16*(16*a^3*d^6+24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3)
*arcsin(d*x)/d^7-1/15*b*(45*a^2*d^4+60*a*c*d^2+10*b^2*d^2+24*c^2)*(-d^2*x^2
+1)^(1/2)/d^6-1/16*(24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c
^3)*x*(-d^2*x^2+1)^(1/2)/d^6-1/15*b*(30*a*c*d^2+5*b^2*d^2+12*c^2)*x^2*(-d^2
*x^2+1)^(1/2)/d^4-1/24*c*(18*a*c*d^2+18*b^2*d^2+5*c^2)*x^3*(-d^2*x^2+1)^(1/
2)/d^4-3/5*b*c^2*x^4*(-d^2*x^2+1)^(1/2)/d^2-1/6*c^3*x^5*(-d^2*x^2+1)^(1/2)/
d^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {913, 1829, 655, 222}

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= -\frac{b\sqrt{1-d^2x^2}(45a^2d^4+60acd^2+10b^2d^2+24c^2)}{15d^6} - \frac{x\sqrt{1-d^2x^2}(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3)}{16d^6} + \frac{\arcsin(dx)\,(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3)}{16d^7} - \frac{bx^2\sqrt{1-d^2x^2}(30acd^2+5b^2d^2+12c^2)}{15d^4} - \frac{cx^3\sqrt{1-d^2x^2}(18acd^2+18b^2d^2+5c^2)}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

```
[In] Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/15*(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])
/d^6 - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx+cx^2)^3}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-6a^3d^2-18a^2bd^2x-18a(b^2+ac)d^2x^2-6b(b^2+6ac)d^2x^3-c(5c^2+18b^2d^2+18acd^2)x^4-18bc^2d^2x^5}{\sqrt{1-d^2x^2}}\,dx}{6d^2}$$

$$= -\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} + \frac{\int \frac{30a^3d^4+90a^2bd^4x+90a(b^2+ac)d^4x^2+6bd^2(12c^2+5b^2d^2+30acd^2)x^3+5cd^2(5c^2+18b^2d^2+18acd^2)x^4}{\sqrt{1-d^2x^2}}\,dx}{30d^4}$$

$$= -\frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-120a^3d^6-360a^2bd^6x-15d^2(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)x^2-24bd^4(12c^2+5b^2d^2+30acd^2)x^3}{\sqrt{1-d^2x^2}}\,dx}{120d^6}$$

$$= -\frac{b(12c^2+5b^2d^2+30acd^2)x^2\sqrt{1-d^2x^2}}{15d^4} - \frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} + \frac{\int \frac{360a^3d^8+24bd^4(24c^2+10b^2d^2+60acd^2+45a^2d^4)x+45d^4(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)x^2}{\sqrt{1-d^2x^2}}\,dx}{360d^8}$$

$$= -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)x\sqrt{1-d^2x^2}}{16d^6} - \frac{b(12c^2+5b^2d^2+30acd^2)x^2\sqrt{1-d^2x^2}}{15d^4} - \frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-45d^4(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)-48bd^6(24c^2+10b^2d^2+60acd^2+45a^2d^4)x}{\sqrt{1-d^2x^2}}\,dx}{720d^{10}}$$

$$
\begin{aligned}
&= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} \\
&- \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d^2x^2}}{16d^6} \\
&- \frac{b(12c^2+5b^2d^2+30acd^2)\,x^2\sqrt{1-d^2x^2}}{15d^4} \\
&- \frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \\
&+ \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)\int\frac{1}{\sqrt{1-d^2x^2}}\,dx}{16d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} \\
&- \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d^2x^2}}{16d^6} \\
&- \frac{b(12c^2+5b^2d^2+30acd^2)\,x^2\sqrt{1-d^2x^2}}{15d^4} \\
&- \frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \\
&+ \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)\sin^{-1}(dx)}{16d^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.76

$$
\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx
$$

$$
= \frac{-d\sqrt{1-d^2x^2}(80b^3d^2(2+d^2x^2)+90b^2d^2x(4ad^2+c(3+2d^2x^2))+48b(15a^2d^4+10acd^2(2+d^2x^2)+c^2(8
$$

```
[In] Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(d*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 +
c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8 +
 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2) +
 c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 30*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2
*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcTan[(d*x)/(-1 + Sqrt[1
- d^2*x^2])])/(240*d^7)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.03

method	result
risch	$\frac{\left(40 c^3 x^5 d^4+144 b\,c^2 x^4 d^4+180 a\,c^2 d^4 x^3+180 b^2 c\,d^4 x^3+480 a b c\,d^4 x^2+80 b^3 d^4 x^2+360 a^2 c\,d^4 x+360 a\,b^2 d^4 x+50 c^3 d^2 x^3+720 b\,a^2 d^4+192 b\,c^2\right.}{240 d^6 \sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(40\ \mathrm{csgn}(d) c^3 d^5 x^5 \sqrt{-d^2x^2+1}+144\ \mathrm{csgn}(d) b\,c^2 d^5 x^4 \sqrt{-d^2x^2+1}+180\ \mathrm{csgn}(d) a\,c^2 d^5 x^3 \sqrt{-d^2x^2+1}+180\ \mathrm{csgn}(d) b\right.}{}$

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(40*c^3*d^4*x^5+144*b*c^2*d^4*x^4+180*a*c^2*d^4*x^3+180*b^2*c*d^4*x^3
+480*a*b*c*d^4*x^2+80*b^3*d^4*x^2+360*a^2*c*d^4*x+360*a*b^2*d^4*x+50*c^3*d^
2*x^3+720*a^2*b*d^4+192*b*c^2*d^2*x^2+270*a*c^2*d^2*x+270*b^2*c*d^2*x+960*a
*b*c*d^2+160*b^3*d^2+75*c^3*x+384*b*c^2)*(d*x-1)*(d*x+1)^(1/2)/d^6/(-(d*x-1
)*(d*x+1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16*(16*a^3*d^6+2
4*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3)/d^6/(d^2)^(1/2)*a
rctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(
1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(40\,c^3d^5x^5+144\,bc^2d^5x^4+720\,a^2bd^5+384\,bc^2d+160\,(b^3+6\,abc)d^3+10\,(5\,c^3d^3+18\,(b^2c+ac^2)d^5)x^3}{}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d +
 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 + 16
*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5 +
5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(16*
a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arctan((
sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = &-\frac{\sqrt{-d^2x^2+1}c^3x^5}{6\,d^2} - \frac{3\sqrt{-d^2x^2+1}bc^2x^4}{5\,d^2} \\ &+ \frac{a^3\arcsin(dx)}{d} - \frac{5\sqrt{-d^2x^2+1}c^3x^3}{24\,d^4} \\ &- \frac{3\sqrt{-d^2x^2+1}(b^2c+ac^2)x^3}{4\,d^2} - \frac{3\sqrt{-d^2x^2+1}a^2b}{d^2} \\ &- \frac{4\sqrt{-d^2x^2+1}bc^2x^2}{5\,d^4} - \frac{\sqrt{-d^2x^2+1}(b^3+6\,abc)x^2}{3\,d^2} \\ &- \frac{3\sqrt{-d^2x^2+1}(ab^2+a^2c)x}{2\,d^2} + \frac{3\,(ab^2+a^2c)\arcsin(dx)}{2\,d^3} \\ &- \frac{5\sqrt{-d^2x^2+1}c^3x}{16\,d^6} - \frac{9\sqrt{-d^2x^2+1}(b^2c+ac^2)x}{8\,d^4} \\ &- \frac{8\sqrt{-d^2x^2+1}bc^2}{5\,d^6} - \frac{2\sqrt{-d^2x^2+1}(b^3+6\,abc)}{3\,d^4} \\ &+ \frac{5\,c^3\arcsin(dx)}{16\,d^7} + \frac{9\,(b^2c+ac^2)\arcsin(dx)}{8\,d^5}\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
+ a^3*arcsin(d*x)/d - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d^4 - 3/4*sqrt(-d^2*x
^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*a^2*b/d^2 - 4/5*sqrt
(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)*x^2/d
^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2*(a*b^2 + a^2*c)*arc
sin(d*x)/d^3 - 5/16*sqrt(-d^2*x^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2 + 1)*(
b^2*c + a*c^2)*x/d^4 - 8/5*sqrt(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-d^2*x^2
 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/16*c^3*arcsin(d*x)/d^7 + 9/8*(b^2*c + a*c^2)*
arcsin(d*x)/d^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(720\,a^2bd^5 - 360\,ab^2d^4 - 360\,a^2cd^4 + 240\,b^3d^3 + 1440\,abcd^3 - 450\,b^2cd^2 - 450\,ac^2d^2 + 720\,bc^2d - 165\,c^3}{}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/240*((720*a^2*b*d^5 - 360*a*b^2*d^4 - 360*a^2*c*d^4 + 240*b^3*d^3 + 1440
*a*b*c*d^3 - 450*b^2*c*d^2 - 450*a*c^2*d^2 + 720*b*c^2*d - 165*c^3 + (360*a
*b^2*d^4 + 360*a^2*c*d^4 - 160*b^3*d^3 - 960*a*b*c*d^3 + 810*b^2*c*d^2 + 81
0*a*c^2*d^2 - 960*b*c^2*d + 425*c^3 + 2*(40*b^3*d^3 + 240*a*b*c*d^3 - 270*b
^2*c*d^2 - 270*a*c^2*d^2 + 528*b*c^2*d - 275*c^3 + (90*b^2*c*d^2 + 90*a*c^2
*d^2 - 288*b*c^2*d + 225*c^3 + 4*(5*(d*x + 1)*c^3 + 18*b*c^2*d - 25*c^3)*(d
*x + 1))*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30
*(16*a^3*d^6 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 18*b^2*c*d^2 + 18*a*c^2*d^2 +
5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^7
```

Mupad [B] (verification not implemented)

Time = 39.01 (sec) , antiderivative size = 1768, normalized size of antiderivative = 5.46

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^23*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6
*a^2*c*d^4 + (9*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2)
 - 1)*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (9*b^2*c*d
^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((175*c^3)/12 + 6*
a*b^2*d^4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1
)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((175*c^3)/12 + 6*a*b^2*d^4 + (1
05*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^2
1 + (((1 - d*x)^(1/2) - 1)^5*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)
/2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^19*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)/2 + 126*a^
2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^19 + (((1 - d*x)^(1/2)
- 1)^7*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 +
 (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^17*(
```

(8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 + (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^11*((25295*c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(1/2) - 1)^13*((25295*c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^9*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^15*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^6*((1024*b^3*d^3)/3 + 1080*a^2*b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^18*((1024*b^3*d^3)/3 + 1080*a^2*b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^18 + (((1 - d*x)^(1/2) - 1)^10*(1024*b^3*d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(1024*b^3*d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^12*((3200*b^3*d^3)/3 + 6048*a^2*b*d^5 + (32768*b*c^2*d)/5 + 6400*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^4*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2) - 1)^8*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^16*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^16 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^7 + (12*d^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^7*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^7*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^7*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (792*d^7*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (924*d^7*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^7*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^7*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^7*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (66*d^7*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (12*d^7*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^7*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(5*c^3 + 16*a^3*d^6 + 24*a*b^2*d^4 + 18*a*c^2*d^2 + 24*a^2*c*d^4 + 18*b^2*c*d^2))/(4*d^7)

3.793 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 166

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{\left(4b^2+c\left(8a+\frac{3c}{d^2}\right)\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{(3c^2+4b^2d^2+8acd^2+8a^2d^4)\arcsin(dx)}{8d^5}$$

```
[Out] 1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)*arcsin(d*x)/d^5-2/3*b*(3*a*d^2+2*
c)*(-d^2*x^2+1)^(1/2)/d^4-1/8*(4*b^2+c*(8*a+3*c/d^2))*x*(-d^2*x^2+1)^(1/2)/
d^2-2/3*b*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/4*c^2*x^3*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {913, 1829, 655, 222}

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\arcsin(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5} - \frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2} - \frac{2b\sqrt{1-d^2x^2}(3ad^2+2c)}{3d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

```
[In] Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a+bx+cx^2)^2}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}-\frac{\int \frac{-4a^2d^2-8abd^2x-(3c^2+4b^2d^2+8acd^2)x^2-8bcd^2x^3}{\sqrt{1-d^2x^2}}\,dx}{4d^2}\\
&= -\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}+\frac{\int \frac{12a^2d^4+8bd^2(2c+3ad^2)x+3d^2(3c^2+4b^2d^2+8acd^2)x^2}{\sqrt{1-d^2x^2}}\,dx}{12d^4}\\
&= -\frac{(3c^2+4b^2d^2+8acd^2)\,x\sqrt{1-d^2x^2}}{8d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}\\
&\quad-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}-\frac{\int \frac{-3d^2(3c^2+4b^2d^2+8acd^2+8a^2d^4)-16bd^4(2c+3ad^2)x}{\sqrt{1-d^2x^2}}\,dx}{24d^6}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)\,x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} \\
&\quad - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{(3c^2+4b^2d^2+8acd^2+8a^2d^4)\int\frac{1}{\sqrt{1-d^2x^2}}\,dx}{8d^4} \\
&= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)\,x\sqrt{1-d^2x^2}}{8d^4} \\
&\quad - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{(3c^2+4b^2d^2+8acd^2+8a^2d^4)\sin^{-1}(dx)}{8d^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$
\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx
$$

$$
= \frac{-d\sqrt{1-d^2x^2}(12b^2d^2x+16b(3ad^2+c(2+d^2x^2))+3cx(8ad^2+c(3+2d^2x^2)))+6(3c^2+4b^2d^2+8acd^2+}{24d^5}
$$

```
[In] Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*b^2*d^2*x + 16*b*(3*a*d^2 + c*(2 + d^2*x^2)) + 3
*c*x*(8*a*d^2 + c*(3 + 2*d^2*x^2)))) + 6*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8
*a^2*d^4)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^5)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

method	result
risch	$\frac{(6c^2x^3d^2+16bc\,x^2d^2+24ac\,d^2x+12b^2d^2x+48ba\,d^2+9c^2x+32bc)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{24d^4\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}} + \frac{(8a^2d^4+8c\,d^2a+4b^2d^2+3c^2)\,\mathrm{a}}{8d^4\sqrt{d^2}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6\,\mathrm{csgn}(d)c^2d^3x^3\sqrt{-d^2x^2+1}+16\,\mathrm{csgn}(d)bc\,d^3x^2\sqrt{-d^2x^2+1}+24\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3acx+12\sqrt{-d^2x^2+1}\,\mathrm{csgn}(\right.}{}$

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*c^2*d^2*x^3+16*b*c*d^2*x^2+24*a*c*d^2*x+12*b^2*d^2*x+48*a*b*d^2+9*c
^2*x+32*b*c)*(d*x-1)*(d*x+1)^(1/2)/d^4/(-(d*x-1)*(d*x+1))^(1/2)*((-d*x+1)*(
d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)/d^4/
(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/
2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,c^2d^3x^3+16\,bcd^3x^2+48\,abd^3+32\,bcd+3\,(4\,(b^2+2\,ac)d^3+3\,c^2d)x)\sqrt{dx+1}\sqrt{-dx+1}+6\,(8\,a^2d^4 -}{24\,d^5}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2
+ 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 + 4*
(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = & -\frac{\sqrt{-d^2x^2+1}c^2x^3}{4\,d^2}-\frac{2\,\sqrt{-d^2x^2+1}bcx^2}{3\,d^2}+\frac{a^2\arcsin(dx)}{d} \\ & -\frac{2\,\sqrt{-d^2x^2+1}ab}{d^2}-\frac{\sqrt{-d^2x^2+1}(b^2+2\,ac)x}{2\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}c^2x}{8\,d^4} \\ & +\frac{(b^2+2\,ac)\arcsin(dx)}{2\,d^3}-\frac{4\,\sqrt{-d^2x^2+1}bc}{3\,d^4}+\frac{3\,c^2\arcsin(dx)}{8\,d^5}\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
a^2*arcsin(d*x)/d - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(
b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c)*a
rcsin(d*x)/d^3 - 4/3*sqrt(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d*x)/d^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(48\,abd^3-12\,b^2d^2-24\,acd^2+48\,bcd+(12\,b^2d^2+24\,acd^2-32\,bcd+2\,(3\,(dx+1)c^2+8\,bcd-9\,c^2)(dx}{2}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((48*a*b*d^3 - 12*b^2*d^2 - 24*a*c*d^2 + 48*b*c*d + (12*b^2*d^2 + 24*
a*c*d^2 - 32*b*c*d + 2*(3*(d*x + 1)*c^2 + 8*b*c*d - 9*c^2)*(d*x + 1) + 27*c
^2)*(d*x + 1) - 15*c^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*a^2*d^4 + 4*b^2
*d^2 + 8*a*c*d^2 + 3*c^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 897, normalized size of antiderivative = 5.40

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{(\sqrt{1-d\,x}-1)^{15}\left(2\,b^2\,d^2+\frac{3\,c^2}{2}+4\,a\,c\,d^2\right)}{(\sqrt{d\,x+1}-1)^{15}}+\frac{(\sqrt{1-d\,x}-1)^{3}\left(6\,b^2\,d^2-\frac{23\,c^2}{2}+12\,a\,c\,d^2\right)}{(\sqrt{d\,x+1}-1)^{3}}-\frac{(\sqrt{1-d\,x}-1)^{13}\left(6\,b^2\,d^2-\frac{23\,c^2}{2}+12\,a\,c\,d^2\right)}{(\sqrt{d\,x+1}-1)^{13}}+\frac{(\sqrt{1-}}{}}{}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\,\left(8\,a^2\,d^4+8\,a\,c\,d^2+4\,b^2\,d^2+3\,c^2\right)}{2\,d^5}$$

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^15*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1
)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^3*(6*b^2*d^2 - (23*c^2)/2 + 12*a*c
*d^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*(6*b^2*d^2 - (23
*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*
((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^11*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2)
 - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*(22*b^2*d^2 - (671*c^2)/2 + 44*a*c*d^2)
)/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*(22*b^2*d^2 - (671*c^2
)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(128*
b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(1
28*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8
*((256*b*c*d)/3 + 320*a*b*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2)
```

```
 - 1)^6*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x
)^(1/2) - 1)^10*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^10 - (
((1 - d*x)^(1/2) - 1)*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1)^(1/2)
 - 1) + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (16*
a*b*d^3*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1
- d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) -
1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)
^(1/2) - 1)^16) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(3*c^2
 + 8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2))/(2*d^5)
```

3.794 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1829, 655, 222}

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(2ad^2+c)\arcsin(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
+ Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(\sqrt{-d^{2}x^{2}+1}\,\mathrm{csgn}(d)dcx-2\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}+2\,\mathrm{csgn}(d)d\sqrt{-d^{2}x^{2}+1}\,b-\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^{2}x^{2}+1}}\right)c\right)\mathrm{csgn}(d)}{2d^{3}\sqrt{-d^{2}x^{2}+1}}$
risch	$\frac{(cx+2b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}+\frac{(2a\,d^{2}+c)\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1} - 2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,c\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,c\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.795 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)} dx$

Optimal result

Integrand size = 32, antiderivative size = 282

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)} dx = -\frac{\sqrt{2}c\operatorname{arctanh}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} + \frac{\sqrt{2}c\operatorname{arctanh}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}}$$

```
[Out] -c*arctanh(1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2
)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2
)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+c*arctanh(1/2*
(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*
d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+
2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {913, 999, 739, 212}

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \frac{\sqrt{2}c\,\text{arctanh}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}} - \frac{\sqrt{2}c\,\text{arctanh}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx+cx^2)\sqrt{1-d^2x^2}}\,dx \\
&= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} \\
&= -\frac{(2c)\text{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}} \\
&\quad + \frac{(2c)\text{Subst}\left(\int \frac{1}{4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}} \\
&= -\frac{\sqrt{2}c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} \\
&\quad + \frac{\sqrt{2}c\tanh^{-1}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx$$

$$= -\text{RootSum}\Big[ad^4-2bd^2\#1+4c\#1^2+2ad^2\#1^2-2b\#1^3$$

$$+a\#1^4\&, \frac{d^2\log(x)-d^2\log\left(-1+\sqrt{1-d^2x^2}-x\#1\right)+\log(x)\#1^2-\log\left(-1+\sqrt{1-d^2x^2}-x\#1\right)\#1^2}{bd^2-4c\#1-2ad^2\#1+3b\#1^2-2a\#1^3}$$

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] -RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 &
 , (d^2*Log[x] - d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + Log[x]*#1^2 - Log
[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c*#1 - 2*a*d^2*#1 + 3*b*#1
^2 - 2*a*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 1759, normalized size of antiderivative = 6.24

method	result	size
default	Expression too large to display	1759

```
[In] int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*((-4*a*c+b^2)^(1/2)*d^2
*x+b*d^2*x+(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/
2)+2*a*c-b^2)/a^2/c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2
)^(1/2)))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+
b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x
+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))
*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)
+2*a*c-b^2)/a^2/c^2)^(1/2)+2*ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-(b*(-
4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/
c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*a*c*d^2*
(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b
^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-(b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)
*(-d^2*x^2+1)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a
*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^
2)^(1/2)-2*ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(
-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^
2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*a*c*d^2*(-(b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)
+ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2
)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/
2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a
*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)+ln(2*((-4
*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2
+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*(-d^2*x^2+1)^(1/2)*c+2*c)/(
b+2*c*x+(-4*a*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2
*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b
```

```
^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a
^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a
*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c
+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2))/(-d^2*x^2+1)^(1/2)/(b*d-d*(-4*a*c+b^
2)^(1/2)-2*c)/(d*(-4*a*c+b^2)^(1/2)+b*d-2*c)/(b*d-d*(-4*a*c+b^2)^(1/2)+2*c)
/(d*(-4*a*c+b^2)^(1/2)+b*d+2*c)/(-4*a*c+b^2)^(1/2)/(-(b*(-4*a*c+b^2)^(1/2)-
2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)/(-(b*(
-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^
2/c^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4313 vs. $2(248) = 496$.

Time = 0.43 (sec) , antiderivative size = 4313, normalized size of antiderivative = 15.29

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^
2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*
a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^
3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)
*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*
c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((
a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^
4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^
6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c
- 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2
*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b
^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*
c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 -
 ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^
2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c +
```

8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) + 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*

```
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sq
rt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4
 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqr
t(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d
^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4
*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^
5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 +
(b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d
^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^
2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4
*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5
 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b
^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (
b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx=\int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx=\int \frac{1}{(cx^2+bx+a)\sqrt{dx+1}\sqrt{-dx+1}}\,dx$$

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(248) = 496.

Time = 0.42 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx =$$

$$-\left(\frac{(ad^2-bd+c)\left(\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}-1\right)\sqrt{\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}\operatorname{arctan}}{\left(ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}\right)\sqrt{-(ad^2+bd+c)(ad}}\right.$$

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*d^2 - b*d + c)*((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
 + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt((a*d^2 - c + sqrt(-(a*d^2 +
b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c))*arctan(-1/2
*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(
-d*x + 1)))/sqrt((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (
a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a
*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
 + (a*d^2 - c)^2)) - (a*d^2 - b*d + c)*((a*d^2 - c - sqrt(-(a*d^2 + b*d + c
)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt((a*d^2 -
c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*
d + c))*arctan(-1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x +
1)/(sqrt(2) - sqrt(-d*x + 1)))/sqrt((a*d^2 - c - sqrt(-(a*d^2 + b*d + c)*(a
*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c - sqrt(-(
a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c
)*(a*d^2 - b*d + c) + (a*d^2 - c)^2)))*d
```

Mupad [B] (verification not implemented)

Time = 96.42 (sec) , antiderivative size = 33018, normalized size of antiderivative = 117.09

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
```

2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d
^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16
*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^
(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2
*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32
*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^
2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*
a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(
1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*
c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*
a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2
 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a
^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1
6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1
/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*
d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628388371660
8*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12
 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392
*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12
- 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a
^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*
d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 -
263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473
5460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792
*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1
/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340
029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5
*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 +
 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*
a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c
^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 -
 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992
8776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b
^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^
5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054
7964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4
*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4831838208
0*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14
- 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a

^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 348751344435

2*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d
^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 1047972020
2240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) -
1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*
a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 16
32087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^
2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11
))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*
c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 3006
4771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3
*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 687194
76736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233
382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^
3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x +
 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + ((
(1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 1
0960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^
2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 31525
05995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2)
- 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450
944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2
*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*
a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1
 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 14
6028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^1
0 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3
- 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*
a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4
 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 +
 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^
10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1
/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8
*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^
4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (
-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^
2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a
^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2
*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4
+ b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c
^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-
(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2
*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^
2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*

c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*
a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10
 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473
216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^
14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 569
34086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480
704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1
)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*
c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*
a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(107374
1824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 3628
3883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c
^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267
742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*
c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 858993
4592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^
4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10
 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 41
73634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 1402392721
48992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^
6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1
073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9
 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091
648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*
d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 6871947
67360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*
b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^
11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 37
45211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 3435973836
80*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2
147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^
6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224
*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 1
15964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^
8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 5497558
13888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2
*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 +
5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 824633720
8320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*
b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^1
6) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^
4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889
785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^
6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9

- 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 121977071206 4*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 3006477107 2*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(10737418

24*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 68719476

7360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4* b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2))

)^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b
^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*
d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*
a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^
2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b
^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2
*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^
4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d
^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 -
d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*
d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 472446
40256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*
c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 177
21035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026
624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b
^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 +
b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*
c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a
^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)
*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^1
4 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a
^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 +
29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*
b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 6
6571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*
b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8
- 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263
779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473546
0352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^
5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*
d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2)
- 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029
796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*
c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42
949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2
*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*
d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57
999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992877
6704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*
c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b
^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054796
4928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^
14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a

*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 2418925581107 2*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 377957

1220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) + 283467841536*a*c^4*d^8 + (2*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^8 + 1073741824*a*b^2*c^2*d^10))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*c^2*d^10 + (34359738368*a*b*c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))*(-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*2i - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 -

8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 357

3412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b

^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^

3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928 *a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2

- 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c
*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2
)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c
^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c
d^2)))^(1/2)((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 -
 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*
d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
+ b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)
/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^
3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*
d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648
*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628
3883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6
*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760
209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^
8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 1814194
1858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3
*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^
4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12
+ 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478
196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*
a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1
- d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d
^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 421937587
15904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c
^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 107202
38370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*
a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c
^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13
- 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 1675037
24544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073
741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8
 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 1161359156838
4*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4
8318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^
6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 61847
5290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^
6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^1
0 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492
016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*
a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2
*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 176737904230
4*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^

12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244
096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^
5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^
12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*
x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 +
57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^
6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 12
8849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808
*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c
^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 -
1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 12884901
8880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 123
6950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b
*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 300
64771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^
3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10
+ 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 24051816
8576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(26
80059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b
^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 +
64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^
2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3
*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x +
1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 214748
3648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 +
85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*
a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10
- 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*
a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*
c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12
+ 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*
b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458
496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6
+ 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^
2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 +
1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 10286446
67392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b
^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 348
7513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^
3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 1
0479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)
^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 136064
56393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5
*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892
805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b

^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 8933531975 68*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^ 10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 80745385164 8*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d ^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 1245540515 84*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4* d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^ 6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 254691560 6528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^1 2 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^ 3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 429 49672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1) ^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4 *d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^ 3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) - (-(8* a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^ 2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b ^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2 *d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3* c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2) ^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d ^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^ 2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^ 12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8* a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c ^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a ^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^ 2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16 *a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/ 2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a ^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^ 3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^ 2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(17781164 60544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^ 2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618 094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b* c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564 961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/

2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*
a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4
 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(
1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16
- 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784
*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14
- 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*
a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 -
8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*
a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^
3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^1
0 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 1402
39272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556
224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)
^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c
^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544
872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*
b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 6
87194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 1013612281856
0*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*
c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^1
1 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 3435
97383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) -
1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*
c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357
556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d
^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*
a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 5
49755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a
^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d
^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 824
6337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 94918777241
6*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551
296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13
+ 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*
a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^
4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4
260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712
064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*
c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*
a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 -
 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^
4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 +

910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*

```
b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 343597383
68*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) + 283467841536*a*c^4*d^8 + (2
*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^8 + 1073741824*a*b^2*c^2*d^1
0))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*c^2*d^10 + (34359738368*a*b*
c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))*(-(8*a*c^3 - 2*b^2*c
^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d
^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3
*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4
*c*d^2)))^(1/2)*2i
```

3.796 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 571

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx$$
$$= -\frac{\left(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x\right)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$
$$-\frac{c\left(4c^3+12ac^2d^2-ab\left(b+\sqrt{b^2-4ac}\right)d^4-cd^2\left(5b^2-b\sqrt{b^2-4ac}-8a^2d^2\right)\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\sqrt{b\cdots}\right.}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\cdots\right.}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-(c+ad^2)^2\right)}$$
$$+\frac{c\left(4c^3+12ac^2d^2-2ab^2d^4-b\left(b+\sqrt{b^2-4ac}\right)d^2(c-ad^2)-4cd^2(b^2-2a^2d^2)\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\cdots\right.}{\sqrt{2}\sqrt{2c^2+2acd^2-\cdots}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\left(b^2d^2-(c+ad^2)^2\right)}$$

```
[Out] -(b*(b^2*d^2-c*(3*a*d^2+c))-c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)*(-d^2*x^2+1)^(1/
2)/(-4*a*c+b^2)/(b^2*d^2-(a*d^2+c)^2)/(c*x^2+b*x+a)-1/2*c*arctanh(1/2*(2*c+
d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b
*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3+12*a*c^2*d^2-a*b*d^4*(b+(-4*a*c+
b^2)^(1/2))-c*d^2*(5*b^2-8*a^2*d^2-b*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2
)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^
2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3
+12*a*c^2*d^2-2*a*b^2*d^4-4*c*d^2*(-2*a^2*d^2+b^2)-b*d^2*(-a*d^2+c)*(b+(-4*
a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 3.91 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {913, 989, 1048, 739, 212}

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx =$$

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\operatorname{arctanh}\left(\frac{d^2x\left(b-\right.}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-(ad^2+c)^2\right)}$$

$$+\frac{c\left(-4cd^2(b^2-2a^2d^2)-bd^2\left(\sqrt{b^2-4ac}+b\right)(c-ad^2)-2ab^2d^4+12ac^2d^2+4c^3\right)\operatorname{arctanh}\left(\frac{d^2x(}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-b}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\left(b^2d^2-(ad^2+c)^2\right)}$$

$$-\frac{\sqrt{1-d^2x^2}(b(b^2d^2-c(3ad^2+c))-cx(2acd^2-b^2d^2+2c^2))}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(a+bx+cx^2)}$$

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx+cx^2)^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{\left(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x\right)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}\\
&\quad-\frac{\int \frac{-2c^3-6ac^2d^2+ab^2d^4+2cd^2(b^2-2a^2d^2)-bcd^2(c-ad^2)x}{(a+bx+cx^2)\sqrt{1-d^2x^2}}\,dx}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)}
\end{aligned}$$

$$= -\frac{(b(b^2d^2 - c(c+3ad^2)) - c(2c^2 - b^2d^2 + 2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2 - (c+ad^2)^2\right)(a+bx+cx^2)} + \frac{\left(c\left(4c^3 + 12ac^2d^2 - ab\left(b+\sqrt{b^2-4ac}\right)d^4 - cd^2\left(5b^2 - b\sqrt{b^2-4ac} - 8a^2d^2\right)\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{}}}{(b^2-4ac)^{3/2}\left(b^2d^2 - (c+ad^2)^2\right)} + \frac{\left(bc\left(b+\sqrt{b^2-4ac}\right)d^2(c-ad^2) + 2c(-2c^3 - 6ac^2d^2 + ab^2d^4 + 2cd^2(b^2 - 2a^2d^2))\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2\right.}}{(b^2-4ac)^{3/2}\left(b^2d^2 - (c+ad^2)^2\right)}$$

$$= -\frac{(b(b^2d^2 - c(c+3ad^2)) - c(2c^2 - b^2d^2 + 2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2 - (c+ad^2)^2\right)(a+bx+cx^2)} - \frac{\left(c\left(4c^3 + 12ac^2d^2 - ab\left(b+\sqrt{b^2-4ac}\right)d^4 - cd^2\left(5b^2 - b\sqrt{b^2-4ac} - 8a^2d^2\right)\right)\right)\text{Subst}\left(\int \frac{}{4c^2 - \left(b - \sqrt{\right.}}\right.}{(b^2-4ac)^{3/2}\left(b^2d^2 - (c+ad^2)^2\right)} - \frac{\left(bc\left(b+\sqrt{b^2-4ac}\right)d^2(c-ad^2) + 2c(-2c^3 - 6ac^2d^2 + ab^2d^4 + 2cd^2(b^2 - 2a^2d^2))\right)\text{Subst}\left(\int \frac{}{4c^2 - \left(\right.}\right.}{(b^2-4ac)^{3/2}\left(b^2d^2 - (c+ad^2)^2\right)}$$

$$= -\frac{(b(b^2d^2 - c(c+3ad^2)) - c(2c^2 - b^2d^2 + 2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2 - (c+ad^2)^2\right)(a+bx+cx^2)} - \frac{c\left(4c^3 + 12ac^2d^2 - ab\left(b+\sqrt{b^2-4ac}\right)d^4 - cd^2\left(5b^2 - b\sqrt{b^2-4ac} - 8a^2d^2\right)\right)\tanh^{-1}\left(\frac{2c}{\sqrt{2}\sqrt{2c^2+2ac}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2 + 2acd^2 - b\left(b - \sqrt{b^2-4ac}\right)d^2}\left(b^2d^2 - (c+ad^2)^2\right.} - \frac{c\left(b\left(b+\sqrt{b^2-4ac}\right)d^2(c-ad^2) - 2(2c^3 + 6ac^2d^2 - ab^2d^4 - 2cd^2(b^2 - 2a^2d^2))\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2c^2+}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2 + 2acd^2 - b\left(b + \sqrt{b^2-4ac}\right)d^2}\left(b^2d^2 - (c+ad^2\right.}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.67 (sec) , antiderivative size = 1548, normalized size of antiderivative = 2.71

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx$$
$$= \frac{(-b^3d^2+bc(c+3ad^2)-b^2cd^2x+2c^2(c+ad^2)\,x)\,\sqrt{1-d^2x^2}}{(b^2-4ac)\,(-c+d(b-ad))(c+d(b+ad))(a+x(b+cx))}$$
$$+ \frac{\text{RootSum}\left[ad^4-2bd^2\#1+4c\#1^2+2ad^2\#1^2-2b\#1^3+a\#1^4\&, \frac{-4b^2\log(x)+4ac\log(x)-a^2d^2\log(x)+4b^2\log\left(-1\cdots\right.}{}\right.}{}$$
$$- \frac{\text{RootSum}\left[ad^4-2bd^2\#1+4c\#1^2+2ad^2\#1^2-2b\#1^3+a\#1^4\&, \frac{4b^4c^2\log(x)-20ab^2c^3\log(x)+16a^2c^4\log(x)-4b^6d\cdots}{}\right.}{}$$

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] ((-(b^3*d^2) + b*c*(c + 3*a*d^2) - b^2*c*d^2*x + 2*c^2*(c + a*d^2)*x)*Sqrt[
1 - d^2*x^2])/((b^2 - 4*a*c)*(-c + d*(b - a*d))*(c + d*(b + a*d))*(a + x*(b
 + c*x))) + RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3
 + a*#1^4 & , (-4*b^2*Log[x] + 4*a*c*Log[x] - a^2*d^2*Log[x] + 4*b^2*Log[-1
 + Sqrt[1 - d^2*x^2] - x*#1] - 4*a*c*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + a
^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 2*a*b*Log[x]*#1 + 2*a*b*Log[-1
+ Sqrt[1 - d^2*x^2] - x*#1]*#1 - a^2*Log[x]*#1^2 + a^2*Log[-1 + Sqrt[1 - d^
2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c*#1 - 2*a*d^2*#1 + 3*b*#1^2 - 2*a*#1^3) &
]/a^3 - RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a
*#1^4 & , (4*b^4*c^2*Log[x] - 20*a*b^2*c^3*Log[x] + 16*a^2*c^4*Log[x] - 4*b
^6*d^2*Log[x] + 28*a*b^4*c*d^2*Log[x] - 55*a^2*b^2*c^2*d^2*Log[x] + 30*a^3*
c^3*d^2*Log[x] + 3*a^2*b^4*d^4*Log[x] - 16*a^3*b^2*c*d^4*Log[x] + 14*a^4*c^
2*d^4*Log[x] - 4*b^4*c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 20*a*b^2*c^3*
Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 16*a^2*c^4*Log[-1 + Sqrt[1 - d^2*x^2]
- x*#1] + 4*b^6*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 28*a*b^4*c*d^2*Log
[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 55*a^2*b^2*c^2*d^2*Log[-1 + Sqrt[1 - d^2*
x^2] - x*#1] - 30*a^3*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 3*a^2*b^
4*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 16*a^3*b^2*c*d^4*Log[-1 + Sqrt[1
 - d^2*x^2] - x*#1] - 14*a^4*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 2
*a*b^3*c^2*Log[x]*#1 - 8*a^2*b*c^3*Log[x]*#1 - 2*a*b^5*d^2*Log[x]*#1 + 12*a
^2*b^3*c*d^2*Log[x]*#1 - 18*a^3*b*c^2*d^2*Log[x]*#1 + 2*a^3*b^3*d^4*Log[x]*
#1 - 6*a^4*b*c*d^4*Log[x]*#1 - 2*a*b^3*c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#
1]*#1 + 8*a^2*b*c^3*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 2*a*b^5*d^2*Log
[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 12*a^2*b^3*c*d^2*Log[-1 + Sqrt[1 - d^2
*x^2] - x*#1]*#1 + 18*a^3*b*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 -
 2*a^3*b^3*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 6*a^4*b*c*d^4*Log[-1
```

```
 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + a^2*b^2*c^2*Log[x]*#1^2 - 2*a^3*c^3*Log[x
]*#1^2 - a^2*b^4*d^2*Log[x]*#1^2 + 4*a^3*b^2*c*d^2*Log[x]*#1^2 - 2*a^4*c^2*
d^2*Log[x]*#1^2 - a^2*b^2*c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + 2*a
^3*c^3*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + a^2*b^4*d^2*Log[-1 + Sqrt[
1 - d^2*x^2] - x*#1]*#1^2 - 4*a^3*b^2*c*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*
#1]*#1^2 + 2*a^4*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 -
4*c*#1 - 2*a*d^2*#1 + 3*b*#1^2 - 2*a*#1^3) & ]/(a^3*(-b^2 + 4*a*c)*(c + d*(
-b + a*d))*(c + d*(b + a*d)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 41837, normalized size of antiderivative = 73.27

method	result	size
default	Expression too large to display	41837

```
[In] int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35403 vs. 2(529) = 1058.

Time = 33.26 (sec) , antiderivative size = 35403, normalized size of antiderivative = 62.00

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\,(a+bx+cx^2)^2}\,dx$$

```
[In] integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{(cx^2+bx+a)^2\sqrt{dx+1}\sqrt{-dx+1}}\,dx$$

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \text{Hanged}$$

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] \text{Hanged}
```

3.797 $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 276

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}$$
$$+\frac{b(5c^2+b^2d^2+6acd^2)\sqrt{1-d^2x^2}}{d^6}+\frac{c(7c^2+12b^2d^2+12acd^2)\,x\sqrt{1-d^2x^2}}{8d^6}$$
$$+\frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$
$$-\frac{3(5c^3+12b^2cd^2+12ac^2d^2+8ab^2d^4+8a^2cd^4)\arcsin(dx)}{8d^7}$$

```
[Out] -3/8*(8*a^2*c*d^4+8*a*b^2*d^4+12*a*c^2*d^2+12*b^2*c*d^2+5*c^3)*arcsin(d*x)/
d^7+(b*(3*a^2+3*c^2/d^4+b^2/d^2+6*a*c/d^2)*d^4+(a*d^2+c)*(a^2*d^4+2*a*c*d^2
+3*b^2*d^2+c^2)*x)/d^6/(-d^2*x^2+1)^(1/2)+b*(6*a*c*d^2+b^2*d^2+5*c^2)*(-d^2
*x^2+1)^(1/2)/d^6+1/8*c*(12*a*c*d^2+12*b^2*d^2+7*c^2)*x*(-d^2*x^2+1)^(1/2)/
d^6+b*c^2*x^2*(-d^2*x^2+1)^(1/2)/d^4+1/4*c^3*x^3*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

$= \{913, 1828, 1829, 655, 222\}$

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = -\frac{3\arcsin(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)}{8d^7} + \frac{x(ad^2+c)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}} + \frac{cx\sqrt{1-d^2x^2}(12acd^2+12b^2d^2+7c^2)}{8d^6} + \frac{b\sqrt{1-d^2x^2}(6acd^2+b^2d^2+5c^2)}{d^6} + \frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

```
[In] Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx+cx^2)^3}{(1-d^2x^2)^{3/2}}\,dx$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} - \int \frac{\frac{c^3+3ac^2d^2+3ab^2d^4+3cd^2(b^2+a^2d^2)}{d^6}+\frac{b\left(b^2+3c\left(2a+\frac{c}{d^2}\right)\right)x}{d^2}+\frac{c\left(3b^2+c\left(3a+\frac{c}{d^2}\right)\right)x^2}{d^2}+\frac{3bc^2x^3}{d^2}+\frac{c^3x^4}{d^2}}{\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} + \frac{\int \frac{-\frac{4\left(c^3+3ac^2d^2+3ab^2d^4+3cd^2(b^2+a^2d^2)\right)}{d^4}-4b\left(b^2+3c\left(2a+\frac{c}{d^2}\right)\right)x-c\left(12b^2+c\left(12a+\frac{7c}{d^2}\right)\right)x^2-12bc^2x^3}{\sqrt{1-d^2x^2}}\,dx}{4d^2}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} - \frac{\int \frac{12\left(3b^2(c+ad^2)+c\left(3ac+\frac{c^2}{d^2}+3a^2d^2\right)\right)+12b(5c^2+b^2d^2+6acd^2)x+3c(7c^2+12b^2d^2+12acd^2)x^2}{\sqrt{1-d^2x^2}}\,dx}{12d^4}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{c(7c^2+12b^2d^2+12acd^2)\,x\sqrt{1-d^2x^2}}{8d^6}+\frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} + \frac{\int \frac{-9(5c^3+12b^2cd^2+12ac^2d^2+8ab^2d^4+8a^2cd^4)-24bd^2(5c^2+b^2d^2+6acd^2)x}{\sqrt{1-d^2x^2}}\,dx}{24d^6}$$

$$= \frac{b\left(3a^2 + \frac{3c^2}{d^4} + \frac{b^2}{d^2} + \frac{6ac}{d^2}\right) d^4 + (c + ad^2)\left(c^2 + 3b^2d^2 + 2acd^2 + a^2d^4\right) x}{d^6\sqrt{1 - d^2x^2}}$$
$$+ \frac{b(5c^2 + b^2d^2 + 6acd^2)\sqrt{1 - d^2x^2}}{d^6} + \frac{c(7c^2 + 12b^2d^2 + 12acd^2)\, x\sqrt{1 - d^2x^2}}{8d^6}$$
$$+ \frac{bc^2x^2\sqrt{1 - d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1 - d^2x^2}}{4d^4}$$
$$- \frac{(3(5c^3 + 12b^2cd^2 + 12ac^2d^2 + 8ab^2d^4 + 8a^2cd^4)) \int \frac{1}{\sqrt{1-d^2x^2}}\, dx}{8d^6}$$

$$= \frac{b\left(3a^2 + \frac{3c^2}{d^4} + \frac{b^2}{d^2} + \frac{6ac}{d^2}\right) d^4 + (c + ad^2)\left(c^2 + 3b^2d^2 + 2acd^2 + a^2d^4\right) x}{d^6\sqrt{1 - d^2x^2}}$$
$$+ \frac{b(5c^2 + b^2d^2 + 6acd^2)\sqrt{1 - d^2x^2}}{d^6} + \frac{c(7c^2 + 12b^2d^2 + 12acd^2)\, x\sqrt{1 - d^2x^2}}{8d^6}$$
$$+ \frac{bc^2x^2\sqrt{1 - d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1 - d^2x^2}}{4d^4}$$
$$- \frac{3(5c^3 + 12b^2cd^2 + 12ac^2d^2 + 8ab^2d^4 + 8a^2cd^4)\sin^{-1}(dx)}{8d^7}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.88

$$\int \frac{(a + bx + cx^2)^3}{(1 - dx)^{3/2}(1 + dx)^{3/2}}\, dx =$$
$$-\frac{\frac{d\left(8b^3d^2\left(-2+d^2x^2\right)+12b^2d^2x\left(-2ad^2+c\left(-3+d^2x^2\right)\right)+8b\left(-3a^2d^4+6acd^2\left(-2+d^2x^2\right)+c^2\left(-8+4d^2x^2+d^4x^4\right)\right)+x\left(-24a^2cd^4-8a^3d^6+12ac^2d^2\left(-\right.\right.\right.}{\sqrt{1-d^2x^2}}}{8d^7}$$

```
[In] Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] -1/8*((d*(8*b^3*d^2*(-2 + d^2*x^2) + 12*b^2*d^2*x*(-2*a*d^2 + c*(-3 + d^2*x
^2)) + 8*b*(-3*a^2*d^4 + 6*a*c*d^2*(-2 + d^2*x^2) + c^2*(-8 + 4*d^2*x^2 + d
^4*x^4)) + x*(-24*a^2*c*d^4 - 8*a^3*d^6 + 12*a*c^2*d^2*(-3 + d^2*x^2) + c^3
*(-15 + 5*d^2*x^2 + 2*d^4*x^4))))/Sqrt[1 - d^2*x^2] + 6*(5*c^3 + 12*b^2*c*d
^2 + 12*a*c^2*d^2 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcTan[(d*x)/(-1 + Sqrt[1 -
d^2*x^2])])/d^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(260) = 520.

Time = 0.62 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{(2c^3d^2x^3+8b\,c^2d^2x^2+12a\,c^2d^2x+12b^2c\,d^2x+48abc\,d^2+8b^3d^2+7c^3x+40b\,c^2)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{8d^6\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}-\frac{\left(\frac{15c^3\arctan\left(\ldots\right.}{\sqrt{d\ldots}}\right.}{\ldots}$
default	$-\frac{\sqrt{-dx+1}\left(96\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,abc+8\,\mathrm{csgn}(d)d^7\sqrt{-d^2x^2+1}\,a^3x-12\,\mathrm{csgn}(d)b^2c\,d^5x^3\sqrt{-d^2x^2+1}+24\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a^2c\ldots\right.}{\ldots}$

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(2*c^3*d^2*x^3+8*b*c^2*d^2*x^2+12*a*c^2*d^2*x+12*b^2*c*d^2*x+48*a*b*c*
d^2+8*b^3*d^2+7*c^3*x+40*b*c^2)*(d*x-1)*(d*x+1)^(1/2)/d^6/(-(d*x-1)*(d*x+1)
)^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/8/d^6*(15*c^3/(d^2)^(1/2)
*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))+24*a^2*c*d^4/(d^2)^(1/2)*arctan((
d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))+24*b^2*d^4*a/(d^2)^(1/2)*arctan((d^2)^(1/2
)*x/(-d^2*x^2+1)^(1/2))+36*a*c^2*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2
*x^2+1)^(1/2))+36*b^2*c*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(
1/2))+(4*a^3*d^6+12*a^2*b*d^5+12*a^2*c*d^4+12*a*b^2*d^4+24*a*b*c*d^3+4*b^3*
d^3+12*a*c^2*d^2+12*b^2*c*d^2+12*b*c^2*d+4*c^3)/d^2/(x-1/d)*(-d^2*(x-1/d)^2
-2*d*(x-1/d))^(1/2)-(-4*a^3*d^6+12*a^2*b*d^5-12*a^2*c*d^4-12*a*b^2*d^4+24*a
*b*c*d^3+4*b^3*d^3-12*a*c^2*d^2-12*b^2*c*d^2+12*b*c^2*d-4*c^3)/d^2/(x+1/d)*
(-d^2*(x+1/d)^2+2*d*(x+1/d))^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)
/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$

$$-\frac{24\,a^2bd^5+64\,bc^2d+16\,(b^3+6\,abc)d^3-8\,(3\,a^2bd^7+8\,bc^2d^3+2\,(b^3+6\,abc)d^5)x^2-(2\,c^3d^5x^5+8\,bc^2d^5x^4-\ldots}{\ldots}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7 +
 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*x^
```

```
4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12*(b
^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8*a^3
*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(
d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*c + a*
c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^3}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
[Out] Integral((a + b*x + c*x**2)**3/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =& -\frac{c^3x^5}{4\sqrt{-d^2x^2+1}d^2} - \frac{bc^2x^4}{\sqrt{-d^2x^2+1}d^2} \\ &+ \frac{a^3x}{\sqrt{-d^2x^2+1}} - \frac{5\,c^3x^3}{8\sqrt{-d^2x^2+1}d^4} - \frac{3\,(b^2c+ac^2)x^3}{2\sqrt{-d^2x^2+1}d^2} + \frac{3\,a^2b}{\sqrt{-d^2x^2+1}d^2} \\ &- \frac{4\,bc^2x^2}{\sqrt{-d^2x^2+1}d^4} - \frac{(b^3+6\,abc)x^2}{\sqrt{-d^2x^2+1}d^2} + \frac{3\,(ab^2+a^2c)x}{\sqrt{-d^2x^2+1}d^2} - \frac{3\,(ab^2+a^2c)\arcsin(dx)}{d^3} \\ &+ \frac{15\,c^3x}{8\sqrt{-d^2x^2+1}d^6} + \frac{9\,(b^2c+ac^2)x}{2\sqrt{-d^2x^2+1}d^4} - \frac{15\,c^3\arcsin(dx)}{8\,d^7} \\ &- \frac{9\,(b^2c+ac^2)\arcsin(dx)}{2\,d^5} + \frac{8\,bc^2}{\sqrt{-d^2x^2+1}d^6} + \frac{2\,(b^3+6\,abc)}{\sqrt{-d^2x^2+1}d^4}\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
[Out] -1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
+ a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^
2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2
) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x
^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^
2*c)*arcsin(d*x)/d^3 + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2*c + a
*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d*x)/d^7 - 9/2*(b^2*c +
a*c^2)*arcsin(d*x)/d^5 + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*
c)/(sqrt(-d^2*x^2 + 1)*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(260) = 520.

Time = 0.36 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.67

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\left(\left((dx+1)\left(2\,(dx+1)\left(\frac{(dx+1)c^3}{d^6}+\frac{4\,bc^2d^{31}-5\,c^3d^{30}}{d^{36}}\right)+\frac{12\,b^2cd^{32}+12\,ac^2d^{32}-32\,bc^2d^{31}+25\,c^3d^{30}}{d^{36}}\right)+\frac{8\,b^3d^{33}+4\cdots}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((((d*x + 1)*(2*(d*x + 1)*((d*x + 1)*c^3/d^6 + (4*b*c^2*d^31 - 5*c^3*d^
30)/d^36) + (12*b^2*c*d^32 + 12*a*c^2*d^32 - 32*b*c^2*d^31 + 25*c^3*d^30)/d
^36) + (8*b^3*d^33 + 48*a*b*c*d^33 - 36*b^2*c*d^32 - 36*a*c^2*d^32 + 80*b*c
^2*d^31 - 35*c^3*d^30)/d^36)*(d*x + 1) - 2*(2*a^3*d^36 + 6*a^2*b*d^35 + 6*a
*b^2*d^34 + 6*a^2*c*d^34 + 10*b^3*d^33 + 60*a*b*c*d^33 - 6*b^2*c*d^32 - 6*a
*c^2*d^32 + 54*b*c^2*d^31 - 7*c^3*d^30)/d^36)*sqrt(d*x + 1)*sqrt(-d*x + 1)/
(d*x - 1) - 6*(8*a*b^2*d^4 + 8*a^2*c*d^4 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 5*
c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^6 + 2*(a^3*d^6*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1
) + 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*c*d^4*(sqr
t(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-d*x + 1))/s
qrt(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*b^2
*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*(sqrt(2) - sq
rt(-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
 + 1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^6 - 2*(a^3*d^6 - 3*
a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*c*d^2
 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^6*(sqrt(2) - sqrt(-d*x +
 1))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(c\,x^2+b\,x+a\right)^3}{\left(1-d\,x\right)^{3/2}\,\left(d\,x+1\right)^{3/2}}\,dx$$

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.798 $\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{2b\left(a+\frac{c}{d^2}\right)d^2+(c^2+b^2d^2+2acd^2+a^2d^4)\,x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} - \frac{\left(2b^2+c\left(4a+\frac{3c}{d^2}\right)\right)\arcsin(dx)}{2d^3}$$

```
[Out] -1/2*(2*b^2+c*(4*a+3*c/d^2))*arcsin(d*x)/d^3+(2*b*(a+c/d^2)*d^2+(a^2*d^4+2*
a*c*d^2+b^2*d^2+c^2)*x)/d^4/(-d^2*x^2+1)^(1/2)+2*b*c*(-d^2*x^2+1)^(1/2)/d^4
+1/2*c^2*x*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {913, 1828, 1829, 655, 222}

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}} - \frac{\arcsin(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

```
[In] Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+bx+cx^2\right)^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} - \int \frac{\frac{c^2+b^2d^2+2acd^2}{d^4}+\frac{2bcx}{d^2}+\frac{c^2x^2}{d^2}}{\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \frac{\int \frac{-2b^2-c\left(4a+\frac{3c}{d^2}\right)-4bcx}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}}+\frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4}-\frac{\left(2b^2+c\left(4a+\frac{3c}{d^2}\right)\right)\int\frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}}+\frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4}-\frac{\left(2b^2+c\left(4a+\frac{3c}{d^2}\right)\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\sqrt{1-d^2x^2}(-8bc-4abd^2-3c^2x-2b^2d^2x-4acd^2x-2a^2d^4x+4bcd^2x^2+c}{2d^4\left(-1+d^2x^2\right)} + \frac{\left(-3c^2-2b^2d^2-4acd^2\right)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^5}$$

```
[In] Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(-8*b*c - 4*a*b*d^2 - 3*c^2*x - 2*b^2*d^2*x - 4*a*c*d^2*
x - 2*a^2*d^4*x + 4*b*c*d^2*x^2 + c^2*d^2*x^3))/(2*d^4*(-1 + d^2*x^2)) + ((
-3*c^2 - 2*b^2*d^2 - 4*a*c*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(125) = 250.

Time = 0.62 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{c(cx+4b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^4\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}-\frac{\left(\frac{3c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}+\frac{2b^2d^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}+\frac{4c\,d^2a\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}\right.}{}$
default	$-\frac{\sqrt{-dx+1}\left(2\,\mathrm{csgn}(d)d^5\sqrt{-d^2x^2+1}\,a^2x-\mathrm{csgn}(d)c^2d^3x^3\sqrt{-d^2x^2+1}-4\,\mathrm{csgn}(d)bc\,d^3x^2\sqrt{-d^2x^2+1}+4\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)ac\,d^4x^2\right.}{}$

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c*(c*x+4*b)*(d*x-1)*(d*x+1)^(1/2)/d^4/(-(d*x-1)*(d*x+1))^(1/2)*((-d*x+
1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/2/d^4*(3*c^2/(d^2)^(1/2)*arctan((d^2)^(1
/2)*x/(-d^2*x^2+1)^(1/2))+2*b^2*d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*
x^2+1)^(1/2))+4*c*d^2*a/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2)
```

```
)+(a^2*d^4+2*a*b*d^3+2*a*c*d^2+b^2*d^2+2*b*c*d+c^2)/d^2/(x-1/d)*(-d^2*(x-1/
d)^2-2*d*(x-1/d))^(1/2)-(-a^2*d^4+2*a*b*d^3-2*a*c*d^2-b^2*d^2+2*b*c*d-c^2)/
d^2/(x+1/d)*(-d^2*(x+1/d)^2+2*d*(x+1/d))^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-
d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = -\frac{4\,abd^3+8\,bcd-4\,(abd^5+2\,bcd^3)x^2-(c^2d^3x^3+4\,bcd^3x^2-4\,abd^3-8\,bcd-(2\,a^2d^5+2\,(b^2+2\,ac)d^3+3\,c^2}{2\,(d^7x^2}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
[Out] -1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4*
b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^
2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2 + 2
*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/(d^7*x^2 - d^5)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^2}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
[Out] Integral((a + b*x + c*x**2)**2/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx &= \frac{a^2x}{\sqrt{-d^2x^2+1}} - \frac{c^2x^3}{2\sqrt{-d^2x^2+1}d^2} - \frac{2\,bcx^2}{\sqrt{-d^2x^2+1}d^2} \\ &\quad + \frac{2\,ab}{\sqrt{-d^2x^2+1}d^2} + \frac{(b^2+2\,ac)x}{\sqrt{-d^2x^2+1}d^2} - \frac{(b^2+2\,ac)\arcsin(dx)}{d^3} \\ &\quad + \frac{3\,c^2x}{2\sqrt{-d^2x^2+1}d^4} - \frac{3\,c^2\arcsin(dx)}{2\,d^5} + \frac{4\,bc}{\sqrt{-d^2x^2+1}d^4}\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^2
/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c)*
x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d*x)/d^3 + 3/2*c^2*x/(sqr
t(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d*x)/d^5 + 4*b*c/(sqrt(-d^2*x^2 + 1)*
d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(125) = 250$.

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.90

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\frac{2\sqrt{dx+1}\sqrt{-dx+1}\left((dx+1)\left(\frac{(dx+1)c^2}{d^4}+\frac{4\,bcd^{13}-3\,c^2d^{12}}{d^{16}}\right)-\frac{a^2d^{16}+2\,abd^{15}+b^2d^{14}+2\,acd^{14}+10\,bcd^{13}-c^2d^{12}}{d^{16}}}{dx-1}}{}$$

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*((d*x + 1)*c^2/d^4 + (4*b*c*
d^13 - 3*c^2*d^12)/d^16) - (a^2*d^16 + 2*a*b*d^15 + b^2*d^14 + 2*a*c*d^14 +
 10*b*c*d^13 - c^2*d^12)/d^16)/(d*x - 1) - 4*(2*b^2*d^2 + 4*a*c*d^2 + 3*c^2
)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4 + (a^2*d^4*(sqrt(2) - sqrt(-d*x + 1
))/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2
*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1)
+ c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^4 - (a^2*d^4 - 2*a*b*d^3
+ b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^4*(sqrt(2) - sqrt(-
d*x + 1))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{{\left(c\,x^2+b\,x+a\right)}^2}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.799 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 40

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx=\frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}}-\frac{c\arcsin(dx)}{d^3}$$

```
[Out] -c*arcsin(d*x)/d^3+(b+(a*d^2+c)*x)/d^2/(-d^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1828, 12, 222}

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx=\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\arcsin(dx)}{d^3}$$

```
[In] Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{a+bx+cx^2}{(1-d^2x^2)^{3/2}}\,dx\\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \int \frac{c}{d^2\sqrt{1-d^2x^2}}\,dx\\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \frac{c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{d^2}\\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{\frac{d(b+(c+ad^2)x)}{\sqrt{1-d^2x^2}} - 2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
[Out] ((d*(b + (c + a*d^2)*x))/Sqrt[1 - d^2*x^2] - 2*c*ArcTan[(d*x)/(-1 + Sqrt[1
- d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.78

method	result
default	$\frac{\left(-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^3ax-\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-(dx-1)(dx+1)}}\right)c\,d^2x^2-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)dcx-\operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,b+\arctan\left(\frac{\operatorname{csgn}(d)\cdots}{\sqrt{-(dx-1)(\cdots}}\right.\right.}{(dx-1)\sqrt{-d^2x^2+1}\,d^3\sqrt{dx+1}}$

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*a*x-arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^
(1/2))*c*d^2*x^2-(-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-csgn(d)*d*(-d^2*x^2+1)^(1
/2)*b+arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^(1/2))*c)*(-d*x+1)^(1/2)*csgn(d
)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.52

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{bd^3x^2-(bd+(ad^3+cd)x)\sqrt{dx+1}\sqrt{-dx+1}-bd+2\left(cd^2x^2-c\right)\arctan\left(\cdots\right.}{d^5x^2-d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d + 2
*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x^2
 - d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \frac{ax}{\sqrt{-d^2x^2+1}}+\frac{cx}{\sqrt{-d^2x^2+1}d^2}-\frac{c\arcsin{(dx)}}{d^3}+\frac{b}{\sqrt{-d^2x^2+1}d^2}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d*x)/d^3 +
 b/(sqrt(-d^2*x^2 + 1)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 4.65

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx =$$

$$-\frac{\frac{8\,c\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{d^2}-\frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{d^2}+\frac{(ad^2-bd+c)\sqrt{dx+1}}{d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}+\frac{2\,(ad^4+bd^3+cd^2)\sqrt{dx+1}\sqrt{-d}}{(dx-1)d^4}}{4\,d}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(8*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2 - (a*d^2*(sqrt(2) - sqrt(-d
*x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c*(
sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^2 + (a*d^2 - b*d + c)*sqrt(d*x +
 1)/(d^2*(sqrt(2) - sqrt(-d*x + 1))) + 2*(a*d^4 + b*d^3 + c*d^2)*sqrt(d*x +
 1)*sqrt(-d*x + 1)/((d*x - 1)*d^4))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{c\,x^2+b\,x+a}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.800 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 443

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \frac{d^2(b-(c+ad^2)\,x)}{\left(b^2d^2-(c+ad^2)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\operatorname{arctanh}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\,\left(b^2d^2-(c+ad^2)^2\right)} - \frac{c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)\operatorname{arctanh}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\,\left(b^2d^2-(c+ad^2)^2\right)}$$

```
[Out] d^2*(b-(a*d^2+c)*x)/(b^2*d^2-(a*d^2+c)^2)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(
1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*
a*c+b^2)^(1/2)))/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*c*arctanh(1/2*(2*c+d^2*x*(b
+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))/
(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {913, 990, 1048, 739, 212}

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\operatorname{arctanh}\left(\frac{d^2}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{}}\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\,(}$$

$$-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\operatorname{arctanh}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

$$+\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/((((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
```

```
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 990

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2*
p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx+cx^2)\,(1-d^2x^2)^{3/2}}\,dx \\
&= \frac{d^2(b-(c+ad^2)\,x)}{\left(b^2d^2-(c+ad^2)^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{2d^2(c^2-b^2d^2+acd^2)-2bcd^4x}{(a+bx+cx^2)\sqrt{1-d^2x^2}}\,dx}{2d^2\left(b^2d^2-(c+ad^2)^2\right)} \\
&= \frac{d^2(b-(c+ad^2)\,x)}{\left(b^2d^2-(c+ad^2)^2\right)\sqrt{1-d^2x^2}} \\
&\quad + \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}\left(b^2d^2-(c+ad^2)^2\right)} \\
&\quad - \frac{\left(c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}\left(b^2d^2-(c+ad^2)^2\right)}
\end{aligned}$$

$$= \frac{d^2(b-(c+ad^2)x)}{(b^2d^2-(c+ad^2)^2)\sqrt{1-d^2x^2}}$$
$$-\frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)\right)\text{Subst}\left(\int \frac{1}{4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right.}{\sqrt{b^2-4ac}\,(b^2d^2-(c+ad^2)^2)}$$
$$+\frac{\left(c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\right)\text{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right.}{\sqrt{b^2-4ac}\,(b^2d^2-(c+ad^2)^2)}$$
$$= \frac{d^2(b-(c+ad^2)x)}{(b^2d^2-(c+ad^2)^2)\sqrt{1-d^2x^2}}$$
$$+\frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\,(b^2d^2-(c+ad^2)^2)}$$
$$-\frac{c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)\tanh^{-1}\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\,(b^2d^2-(c+ad^2)^2)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \frac{d^2(b-(c+ad^2)\,x)\,\sqrt{1-d^2x^2}+(1-d^2x^2)\,\text{RootSum}\left[ad^4-\right.}{}$$

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(b - (c + a*d^2)*x)*Sqrt[1 - d^2*x^2] + (1 - d^2*x^2)*RootSum[a*d^4 -
2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 & , (-(c^2*d^2*Log
[x]) + b^2*d^4*Log[x] - a*c*d^4*Log[x] + c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2]
 - x*#1] - b^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + a*c*d^4*Log[-1 + Sq
rt[1 - d^2*x^2] - x*#1] - 2*b*c*d^2*Log[x]*#1 + 2*b*c*d^2*Log[-1 + Sqrt[1 -
 d^2*x^2] - x*#1]*#1 - c^2*Log[x]*#1^2 + b^2*d^2*Log[x]*#1^2 - a*c*d^2*Log[
x]*#1^2 + c^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - b^2*d^2*Log[-1 + Sq
rt[1 - d^2*x^2] - x*#1]*#1^2 + a*c*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#
1^2)/(-(b*d^2) + 4*c*#1 + 2*a*d^2*#1 - 3*b*#1^2 + 2*a*#1^3) & ])/((c + d*(-
b + a*d))*(c + d*(b + a*d))*(-1 + d*x)*(1 + d*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.61 (sec) , antiderivative size = 11142, normalized size of antiderivative = 25.15

method	result	size
default	Expression too large to display	11142

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21628 vs. $2(404) = 808$.

Time = 6.57 (sec) , antiderivative size = 21628, normalized size of antiderivative = 48.82

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas
")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
[Out] Integral(1/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima
")
[Out] integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)),x)
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)), x)
```

3.801 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 939

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx =$$

$$-\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-c(6ab^2d^4+\cdots}{(b^2-4ac)\,(c-bd+ad^2)^2\,(c+bd+ad^2)^2\,\sqrt{1-d^2x^2}}$$

$$-\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$+\frac{c\left(4c^5+24ac^4d^2+3ab^3\left(b+\sqrt{b^2-4ac}\right)d^6-c^3d^2\left(9b^2-b\sqrt{b^2-4ac}-36a^2d^2\right)-2ac^2d^4\left(7b^2+5b\sqrt{b^2-4ac}\cdots\right.\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{2c^2+2acd^2-b\,(\cdots}}$$

$$+\frac{c\left(b\left(b+\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-2(2c^5d^2+12ac^4d^4+3ab^4d^8+2b^2cd^6(\cdots\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\,d^2\sqrt{2c^2+2acd^2-b\,(b+\cdots}}$$

```
[Out] -d^2*(b*(-11*a^2*c*d^4+3*a*b^2*d^4-10*a*c^2*d^2+2*b^2*c*d^2+c^3)-(2*c^4+b^2
*d^4*(a^2*d^2+2*b^2)-c^2*d^2*(6*a^2*d^2+b^2)-c*(4*a^3*d^6+6*a*b^2*d^4))*x)/
(-4*a*c+b^2)/(a*d^2-b*d+c)^2/(a*d^2+b*d+c)^2/(-d^2*x^2+1)^(1/2)+(-b*(b^2*d^
2-c*(3*a*d^2+c))+c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)/(-4*a*c+b^2)/(b^2*d^2-(a*d^
2+c)^2)/(c*x^2+b*x+a)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b-(-
4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4
*a*c+b^2)^(1/2)))^(1/2))*(4*c^5+24*a*c^4*d^2+3*a*b^3*d^6*(b+(-4*a*c+b^2)^(1
/2))-c^3*d^2*(9*b^2-36*a^2*d^2-b*(-4*a*c+b^2)^(1/2))-2*a*c^2*d^4*(7*b^2-8*a
^2*d^2+5*b*(-4*a*c+b^2)^(1/2))+b*c*d^4*(2*b^3-17*a^2*b*d^2+2*b^2*(-4*a*c+b^
```

```
2)^(1/2)-11*a^2*d^2*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(a^2*d^4+2*a*c*
d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2
+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*c^5*d^2
-24*a*c^4*d^4-6*a*b^4*d^8-4*b^2*c*d^6*(-7*a^2*d^2+b^2)+2*c^3*(-18*a^2*d^6+4
*b^2*d^4)+8*c^2*(-2*a^3*d^8+3*a*b^2*d^6)+b*d^4*(-11*a^2*c*d^4+3*a*b^2*d^4-1
0*a*c^2*d^2+2*b^2*c*d^2+c^3)*(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/d^2
/(a^2*d^4+2*a*c*d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 11.13 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {913, 989, 1076, 1048, 739, 212}

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx =$$

$$-\frac{(b(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3)-(2c^4-d^2(b^2+6a^2d^2)\,c^2-(4a^3d^6+6ab^2d^4)\,c+b^2d^4(2b^2}{(b^2-4ac)\,(ad^2-bd+c)^2\,(ad^2+bd+c)^2\,\sqrt{1-d^2x^2}}$$

$$+\frac{c\left(3ab^3\left(b+\sqrt{b^2-4ac}\right)d^6-2ac^2\left(7b^2+5\sqrt{b^2-4ac}b-8a^2d^2\right)d^4+bc\left(2b^3+2\sqrt{b^2-4ac}b^2-17a^2d^2b-11\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\,\sqrt{2c^2+2ad^2c-}}$$

$$-\frac{b(b^2d^2-c(3ad^2+c))-c(2c^2+2ad^2c-b^2d^2)\,x}{(b^2-4ac)\left(b^2d^2-(ad^2+c)^2\right)(cx^2+bx+a)\sqrt{1-d^2x^2}}$$

$$-\frac{c\left(6ab^4d^8+4b^2c(b^2-7a^2d^2)\,d^6+24ac^4d^4-b\left(b+\sqrt{b^2-4ac}\right)(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\,\sqrt{2c^2+2ad^2c-b\left(b+\sqrt{b\right.}}}$$

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]

[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
```

```
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) - (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*
c*d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2
- 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sq
rt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d^2
- b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d
+ e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c
- a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d - c*(2*a*f)) + c*(A*(2*c^2*d - c*(2*a*
f)) - B*(-2*a*c*e) + C*(-2*a*(c*d - a*f)))*x), x] + Dist[1/((-4*a*c)*(a*c*e
^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*S
imp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p + 1) + (2*(A*c*(c*
d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*
c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) -
(2*f*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p
 + q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*((-c)*e*(2*p
+ q + 4)))*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p +
 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e
^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !Int
egerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+bx+cx^2)^2\,(1-d^2x^2)^{3/2}}\,dx$$

$$= -\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}} - \frac{\int \frac{-2c^3-6ac^2d^2+ab^2d^4+2cd^2(b^2-2a^2d^2)+bd^2(c^2-2b^2d^2+7acd^2)x+2cd^2(2c^2-b^2d^2+2acd^2)x^2}{(a+bx+cx^2)(1-d^2x^2)^{3/2}}\,dx}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)^2\sqrt{1-d^2x^2}} - \frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}} - \frac{\int \frac{2\left(2c^5d^2+12ac^4d^4+3ab^4d^8+2b^2cd^6(b^2-7a^2d^2)-\frac{1}{2}c^3(8b^2d^4-36a^2d^6)-4c^2(3ab^2d^6-2a^3d^8)\right)+2bcd^4(c^3+2b^2cd^2-10ac^2d^2+3ab}{(a+bx+cx^2)\sqrt{1-d^2x^2}}}{2(b^2-4ac)\,d^2\left(b^2d^2-(c+ad^2)^2\right)^2}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)^2\sqrt{1-d^2x^2}}$$

$$-\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$-\frac{\left(c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b-\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2\right.\right.}{(b^2-4ac)^{3/2}\,d^2\,(b^2}$$

$$+\frac{\left(c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b+\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2\right.\right.}{(b^2-4ac)^{3/2}\,d^2\,(b^2}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)^2\sqrt{1-d^2x^2}}$$

$$-\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$+\frac{\left(c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b-\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2\right.\right.}{(b^2 -}$$

$$-\frac{\left(c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b+\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2\right.\right.}{(b^2 -}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)^2\sqrt{1-d^2x^2}}$$

$$-\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$+\frac{c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b-\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2-\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\,d^2\sqrt{2c^2+2ac}}$$

$$-\frac{c\left(4c^5d^2+24ac^4d^4+6ab^4d^8+4b^2cd^6(b^2-7a^2d^2)-b\left(b+\sqrt{b^2-4ac}\right)d^4(c^3+2b^2cd^2-10ac^2d^2-\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\,d^2\sqrt{2c^2+2ac}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.47 (sec) , antiderivative size = 3830, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Result too large to show}$$

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
[Out] (-((Sqrt[1 - d^2*x^2]*(b^5*d^4*(-1 + 2*d^2*x^2) + 2*c*(c + a*d^2)^2*x*(-2*a
^2*d^4 - 2*a*c*d^4*x^2 + c^2*(-1 + d^2*x^2)) + b^2*d^2*x*(a^3*d^6 + a*c^2*d
^2*(13 - 6*d^2*x^2) + c^3*(2 - d^2*x^2) + a^2*c*d^4*(6 + d^2*x^2)) + b^4*d^
4*x*(-(a*d^2) + c*(-3 + 2*d^2*x^2)) + b*c*(c + a*d^2)*(-4*a^2*d^4*(-2 + d^2
*x^2) + c^2*(-1 + d^2*x^2) + a*c*d^2*(-5 + 9*d^2*x^2)) + b^3*(a^2*d^6*(-2 +
 d^2*x^2) + c^2*(2*d^2 - 3*d^4*x^2) + a*c*(3*d^4 - 9*d^6*x^2)))))/((b^2 - 4*
a*c)*(-1 + d*x)*(1 + d*x)*(a + x*(b + c*x)))) + RootSum[a*d^4 - 2*b*d^2*#1
+ 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 & , (-4*b^4*c^4*Log[x] + 20*a
*b^2*c^5*Log[x] - 16*a^2*c^6*Log[x] + 8*b^6*c^2*d^2*Log[x] - 56*a*b^4*c^3*d
^2*Log[x] + 107*a^2*b^2*c^4*d^2*Log[x] - 46*a^3*c^5*d^2*Log[x] - 4*b^8*d^4*
Log[x] + 36*a*b^6*c*d^4*Log[x] - 110*a^2*b^4*c^2*d^4*Log[x] + 132*a^3*b^2*c
^3*d^4*Log[x] - 44*a^4*c^4*d^4*Log[x] + 3*a^2*b^6*d^6*Log[x] - 22*a^3*b^4*c
*d^6*Log[x] + 43*a^4*b^2*c^2*d^6*Log[x] - 14*a^5*c^3*d^6*Log[x] + 4*b^4*c^4
*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 20*a*b^2*c^5*Log[-1 + Sqrt[1 - d^2*x^
2] - x*#1] + 16*a^2*c^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 8*b^6*c^2*d^2*
Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 56*a*b^4*c^3*d^2*Log[-1 + Sqrt[1 - d^2
*x^2] - x*#1] - 107*a^2*b^2*c^4*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 46
*a^3*c^5*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 4*b^8*d^4*Log[-1 + Sqrt[1
 - d^2*x^2] - x*#1] - 36*a*b^6*c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 1
10*a^2*b^4*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 132*a^3*b^2*c^3*d^4
*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 44*a^4*c^4*d^4*Log[-1 + Sqrt[1 - d^2*
x^2] - x*#1] - 3*a^2*b^6*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 22*a^3*b^
4*c*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 43*a^4*b^2*c^2*d^6*Log[-1 + Sq
rt[1 - d^2*x^2] - x*#1] + 14*a^5*c^3*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]
 - 2*a*b^3*c^4*Log[x]*#1 + 8*a^2*b*c^5*Log[x]*#1 + 4*a*b^5*c^2*d^2*Log[x]*#
1 - 24*a^2*b^3*c^3*d^2*Log[x]*#1 + 34*a^3*b*c^4*d^2*Log[x]*#1 - 2*a*b^7*d^4
*Log[x]*#1 + 16*a^2*b^5*c*d^4*Log[x]*#1 - 42*a^3*b^3*c^2*d^4*Log[x]*#1 + 36
*a^4*b*c^3*d^4*Log[x]*#1 + 2*a^3*b^5*d^6*Log[x]*#1 - 10*a^4*b^3*c*d^6*Log[x
]*#1 + 10*a^5*b*c^2*d^6*Log[x]*#1 + 2*a*b^3*c^4*Log[-1 + Sqrt[1 - d^2*x^2]
- x*#1]*#1 - 8*a^2*b*c^5*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 4*a*b^5*c^
2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 24*a^2*b^3*c^3*d^2*Log[-1 + S
qrt[1 - d^2*x^2] - x*#1]*#1 - 34*a^3*b*c^4*d^2*Log[-1 + Sqrt[1 - d^2*x^2] -
 x*#1]*#1 + 2*a*b^7*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 16*a^2*b^5*
c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 42*a^3*b^3*c^2*d^4*Log[-1 + S
qrt[1 - d^2*x^2] - x*#1]*#1 - 36*a^4*b*c^3*d^4*Log[-1 + Sqrt[1 - d^2*x^2] -
```

x*#1]*#1 - 2*a^3*b^5*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 10*a^4*b^3*c*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 10*a^5*b*c^2*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - a^2*b^2*c^4*Log[x]*#1^2 + 2*a^3*c^5*Log[x]*#1^2 + 2*a^2*b^4*c^2*d^2*Log[x]*#1^2 - 8*a^3*b^2*c^3*d^2*Log[x]*#1^2 + 4*a^4*c^4*d^2*Log[x]*#1^2 - a^2*b^6*d^4*Log[x]*#1^2 + 6*a^3*b^4*c*d^4*Log[x]*#1^2 - 9*a^4*b^2*c^2*d^4*Log[x]*#1^2 + 2*a^5*c^3*d^4*Log[x]*#1^2 + a^2*b^2*c^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 2*a^3*c^5*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 2*a^2*b^4*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + 8*a^3*b^2*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 4*a^4*c^4*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + a^2*b^6*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 6*a^3*b^4*c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + 9*a^4*b^2*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 2*a^5*c^3*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c*#1 - 2*a*d^2*#1 + 3*b*#1^2 - 2*a*#1^3) &]/(a^3*(-b^2 + 4*a*c)) + RootSum[a*d^4 - 2*b*d^2*#1 + 4*c*#1^2 + 2*a*d^2*#1^2 - 2*b*#1^3 + a*#1^4 & , (-4*b^2*c^4*Log[x] + 4*a*c^5*Log[x] + 8*b^4*c^2*d^2*Log[x] - 24*a*b^2*c^3*d^2*Log[x] + 11*a^2*c^4*d^2*Log[x] - 4*b^6*d^4*Log[x] + 20*a*b^4*c*d^4*Log[x] - 30*a^2*b^2*c^2*d^4*Log[x] + 8*a^3*c^3*d^4*Log[x] + 3*a^2*b^4*d^6*Log[x] - 8*a^3*b^2*c*d^6*Log[x] - a^4*c^2*d^6*Log[x] + 3*a^4*b^2*d^8*Log[x] - 2*a^5*c*d^8*Log[x] + 4*b^2*c^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 4*a*c^5*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 8*b^4*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 24*a*b^2*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 11*a^2*c^4*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 4*b^6*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 20*a*b^4*c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 30*a^2*b^2*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 8*a^3*c^3*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 3*a^2*b^4*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 8*a^3*b^2*c*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + a^4*c^2*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 3*a^4*b^2*d^8*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] + 2*a^5*c*d^8*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1] - 2*a*b*c^4*Log[x]*#1 + 4*a*b^3*c^2*d^2*Log[x]*#1 - 8*a^2*b*c^3*d^2*Log[x]*#1 - 2*a*b^5*d^4*Log[x]*#1 + 8*a^2*b^3*c*d^4*Log[x]*#1 - 14*a^3*b*c^2*d^4*Log[x]*#1 + 2*a^3*b^3*d^6*Log[x]*#1 - 8*a^4*b*c*d^6*Log[x]*#1 + 2*a*b*c^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 4*a*b^3*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 8*a^2*b*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 2*a*b^5*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 8*a^2*b^3*c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 14*a^3*b*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - 2*a^3*b^3*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 + 8*a^4*b*c*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1 - a^2*c^4*Log[x]*#1^2 + 2*a^2*b^2*c^2*d^2*Log[x]*#1^2 - 4*a^3*c^3*d^2*Log[x]*#1^2 - a^2*b^4*d^4*Log[x]*#1^2 + 4*a^3*b^2*c*d^4*Log[x]*#1^2 - 5*a^4*c^2*d^4*Log[x]*#1^2 + 3*a^4*b^2*d^6*Log[x]*#1^2 - 2*a^5*c*d^6*Log[x]*#1^2 + a^2*c^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 2*a^2*b^2*c^2*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + 4*a^3*c^3*d^2*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + a^2*b^4*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 4*a^3*b^2*c*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 + 5*a^4*c^2*d^4*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2 - 3*a^4*b^2*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#

```
1]*#1^2 + 2*a^5*c*d^6*Log[-1 + Sqrt[1 - d^2*x^2] - x*#1]*#1^2)/(b*d^2 - 4*c
*#1 - 2*a*d^2*#1 + 3*b*#1^2 - 2*a*#1^3) & ]/a^3)/((c + d*(-b + a*d))^2*(c +
 d*(b + a*d))^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.58 (sec) , antiderivative size = 108969, normalized size of antiderivative = 116.05

method	result	size
default	Expression too large to display	108969

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{(cx^2+bx+a)^2(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{{(1-d\,x)}^{3/2}\,{(d\,x+1)}^{3/2}\,{(c\,x^2+b\,x+a)}^2}\,dx$$

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2), x)
```

3.802 $\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

```
[Out] x*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)/((1+c*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {531, 441, 440}

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

```
[In] Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c
*x^2)/a)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a+cx^2)^p (1-e^2x^2)^m \, dx \\
&= \left((a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right) \int \left(1+\frac{cx^2}{a}\right)^p (1-e^2x^2)^m \, dx \\
&= x(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 167 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int (1-ex)^m (1+ex)^m (a+cx^2)^p \, dx$$
$$= \frac{3ax(a+cx^2)^p (1-e^2x^2)^m \operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)}{3a \operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)+2x^2\left(cp \operatorname{AppellF1}\left(\frac{3}{2},1-p,-m,\frac{5}{2},-\frac{cx^2}{a},e^2x^2\right)-ae^2m \operatorname{AppellF1}\right.}$$

```
[In] Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a
), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x^2
*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*Appell
F1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

Maple [F]

$$\int \left(-e x +1\right)^{m} \left(e x +1\right)^{m} \left(c \,x^{2}+a \right)^{p}d x$$

```
[In] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
[Out] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\, dx = \int \left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m\, dx$$

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\, dx = \text{Timed out}$$

```
[In] integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p\, dx = \int \left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m\, dx$$

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Giac [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int \left(cx^2+a\right)^p (ex+1)^m (-ex+1)^m \, dx$$

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int \left(c\,x^2+a\right)^p \, (1-e\,x)^m \, (e\,x+1)^m \, dx$$

```
[In] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m,x)
[Out] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m, x)
```

3.803 $\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \, dx = x(d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-c*x^
2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {533, 441, 440}

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \, dx = x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m(d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[In] Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^
2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \left((d-ex)^m(d+ex)^m\left(d^2-e^2x^2\right)^{-m}\right)\int\left(a+cx^2\right)^p\left(d^2-e^2x^2\right)^m\,dx$$

$$=\left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2-e^2x^2\right)^{-m}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(d^2-e^2x^2\right)^m\,dx$$

$$=\left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^m\,dx$$

$$=x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Mathematica [F]

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \,dx = \int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \,dx$$

```
[In] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

Maple [F]

$$\int \left(-ex+d\right)^{m} \left(ex+d\right)^{m} \left(c\,x^{2}+a\right)^{p}dx$$

```
[In] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
[Out] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \,dx = \text{Timed out}$$

```
[In] integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int (cx^2+a)^p(ex+d)^m(-ex+d)^m\,dx$$

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Giac [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int (cx^2+a)^p(ex+d)^m(-ex+d)^m\,dx$$

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^m\,\left(d-e\,x\right)^m\,dx$$

```
[In] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m,x)
[Out] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m, x)
```

3.804 $\int (d+ex)^m (df-efx)^m \left(a+cx^2\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d+ex)^m (df-efx)^m \left(a+cx^2\right)^p \, dx = x(d+ex)^m (df-efx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-
c*x^2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {533, 441, 440}

$$\int (d+ex)^m (df-efx)^m \left(a+cx^2\right)^p \, dx = x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[In] Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((
c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((d+ex)^m(df-efx)^m\left(d^2f-e^2fx^2\right)^{-m}\right)\int\left(a+cx^2\right)^p\left(d^2f-e^2fx^2\right)^m\,dx\\
&= \left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2f-e^2fx^2\right)^{-m}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(d^2f-e^2fx^2\right)^m\,dx\\
&= \left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^m\,dx\\
&= x(d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$$

```
[In] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

Maple [F]

$$\int \left(ex+d\right)^{m}\left(-efx+df\right)^{m}\left(c\,x^{2}+a\right)^{p}dx$$

```
[In] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
[Out] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (-efx+df)^m(cx^2+a)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \int (-efx+df)^m (cx^2+a)^p (ex+d)^m \,dx$$

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \int (-efx+df)^m (cx^2+a)^p (ex+d)^m \,dx$$

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \int (d\,f-e\,f\,x)^m \left(c\,x^2+a\right)^p (d+e\,x)^m \,dx$$

```
[In] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m,x)
[Out] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m, x)
```

3.805 $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 275

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = -\frac{(ef-dg)^3 \left(ag^2+cf(ef-2dg)\right) (f+gx)^{1+n}}{g^6(1+n)} + \frac{(ef-dg)^2 \left(3aeg^2+c(5e^2f^2-10defg+2d^2g^2)\right) (f+gx)^{2+n}}{g^6(2+n)} - \frac{e(ef-dg) \left(3aeg^2+c(10e^2f^2-20defg+7d^2g^2)\right) (f+gx)^{3+n}}{g^6(3+n)} + \frac{e^2\left(aeg^2+c(10e^2f^2-20defg+9d^2g^2)\right) (f+gx)^{4+n}}{g^6(4+n)} - \frac{5ce^3(ef-dg)(f+gx)^{5+n}}{g^6(5+n)} + \frac{ce^4(f+gx)^{6+n}}{g^6(6+n)}$$

```
[Out] -(-d*g+e*f)^3*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^6/(1+n)+(-d*g+e*f)^2
*(3*a*e*g^2+c*(2*d^2*g^2-10*d*e*f*g+5*e^2*f^2))*(g*x+f)^(2+n)/g^6/(2+n)-e*(
-d*g+e*f)*(3*a*e*g^2+c*(7*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3+n)/g^6
/(3+n)+e^2*(a*e*g^2+c*(9*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(4+n)/g^6/
(4+n)-5*c*e^3*(-d*g+e*f)*(g*x+f)^(5+n)/g^6/(5+n)+c*e^4*(g*x+f)^(6+n)/g^6/(6
+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {961}

$$\begin{aligned}&\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx\\&=\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c(2d^2g^2-10defg+5e^2f^2)\right)}{g^6(n+2)}\\&\quad-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c(7d^2g^2-20defg+10e^2f^2)\right)}{g^6(n+3)}\\&\quad+\frac{e^2(f+gx)^{n+4}\left(aeg^2+c(9d^2g^2-20defg+10e^2f^2)\right)}{g^6(n+4)}\\&\quad-\frac{(ef-dg)^3(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^6(n+1)}\\&\quad-\frac{5ce^3(ef-dg)(f+gx)^{n+5}}{g^6(n+5)}+\frac{ce^4(f+gx)^{n+6}}{g^6(n+6)}\end{aligned}$$

```
[In] Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(ef-dg)^3\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^5} + \frac{(ef-dg)^2\left(3aeg^2+c(5e^2f^2-10defg+2d^2g^2)\right)(f+gx)^{1+n}}{g^5} + \frac{e(ef-dg)\left(-3aeg^2-c(10e^2f^2-20defg+7d^2g^2)\right)(f+gx)^{2+n}}{g^5} + \frac{e^2(aeg^2+c(10e^2f^2-20defg+9d^2g^2))(f+gx)^{3+n}}{g^5} - \frac{5ce^3(ef-dg)(f+gx)^{4+n}}{g^5} + \frac{ce^4(f+gx)^{5+n}}{g^5}\right) dx \\ &= -\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)} + \frac{(ef-dg)^2\left(3aeg^2+c(5e^2f^2-10defg+2d^2g^2)\right)(f+gx)^{2+n}}{g^6(2+n)} - \frac{e(ef-dg)\left(3aeg^2+c(10e^2f^2-20defg+7d^2g^2)\right)(f+gx)^{3+n}}{g^6(3+n)} + \frac{e^2(aeg^2+c(10e^2f^2-20defg+9d^2g^2))(f+gx)^{4+n}}{g^6(4+n)} - \frac{5ce^3(ef-dg)(f+gx)^{5+n}}{g^6(5+n)} + \frac{ce^4(f+gx)^{6+n}}{g^6(6+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right) dx = \frac{(f+gx)^{1+n}\left(-\frac{(ef-dg)^3(ag^2+cf(ef-2dg))}{1+n} + \frac{(ef-dg)^2(3aeg^2+c(5e^2f^2-10defg+2d^2g^2))(f+gx)}{2+n} - \frac{e(ef-dg)(3aeg^2+c(10e^2f^2-2}{3+n}\right.}{g^6}$$

```
[In] Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n))
+ ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(f +
g*x))/(2 + n) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g + 7*
d^2*g^2))*(f + g*x)^2)/(3 + n) + (e^2*(a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g
 + 9*d^2*g^2))*(f + g*x)^3)/(4 + n) - (5*c*e^3*(e*f - d*g)*(f + g*x)^4)/(5
+ n) + (c*e^4*(f + g*x)^5)/(6 + n)))/g^6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. 2(275) = 550.

Time = 0.58 (sec) , antiderivative size = 1786, normalized size of antiderivative = 6.49

method	result	size
norman	Expression too large to display	1786
gosper	Expression too large to display	2017
risch	Expression too large to display	2642
parallelrisch	Expression too large to display	3960

```
[In] int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
[Out] c*e^4/(6+n)*x^6*exp(n*ln(g*x+f))+f*(a*d^3*g^5*n^5-2*c*d^4*f*g^4*n^4+20*a*d^
3*g^5*n^4-3*a*d^2*e*f*g^4*n^4-36*c*d^4*f*g^4*n^3+14*c*d^3*e*f^2*g^3*n^3+155
*a*d^3*g^5*n^3-54*a*d^2*e*f*g^4*n^3+6*a*d*e^2*f^2*g^3*n^3-238*c*d^4*f*g^4*n
^2+210*c*d^3*e*f^2*g^3*n^2-54*c*d^2*e^2*f^3*g^2*n^2+580*a*d^3*g^5*n^2-357*a
*d^2*e*f*g^4*n^2+90*a*d*e^2*f^2*g^3*n^2-6*a*e^3*f^3*g^2*n^2-684*c*d^4*f*g^4
*n+1036*c*d^3*e*f^2*g^3*n-594*c*d^2*e^2*f^3*g^2*n+120*c*d*e^3*f^4*g*n+1044*
a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+444*a*d*e^2*f^2*g^3*n-66*a*e^3*f^3*g^2*n-7
20*c*d^4*f*g^4+1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*e^3*f^4*
g-120*c*e^4*f^5+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g^3-180*a*
e^3*f^3*g^2)/g^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(
g*x+f))+(2*c*d^4*g^4*n^4+7*c*d^3*e*f*g^3*n^4+3*a*d^2*e*g^4*n^4+3*a*d*e^2*f*
g^3*n^4+36*c*d^4*g^4*n^3+105*c*d^3*e*f*g^3*n^3-27*c*d^2*e^2*f^2*g^2*n^3+54*
a*d^2*e*g^4*n^3+45*a*d*e^2*f*g^3*n^3-3*a*e^3*f^2*g^2*n^3+238*c*d^4*g^4*n^2+
518*c*d^3*e*f*g^3*n^2-297*c*d^2*e^2*f^2*g^2*n^2+60*c*d*e^3*f^3*g*n^2+357*a*
d^2*e*g^4*n^2+222*a*d*e^2*f*g^3*n^2-33*a*e^3*f^2*g^2*n^2+684*c*d^4*g^4*n+84
0*c*d^3*e*f*g^3*n-810*c*d^2*e^2*f^2*g^2*n+360*c*d*e^3*f^3*g*n-60*c*e^4*f^4*
n+1026*a*d^2*e*g^4*n+360*a*d*e^2*f*g^3*n-90*a*e^3*f^2*g^2*n+720*c*d^4*g^4+1
080*a*d^2*e*g^4)/g^4/(n^5+20*n^4+155*n^3+580*n^2+1044*n+720)*x^2*exp(n*ln(g
*x+f))+(2*c*d^4*f*g^4*n^5+a*d^3*g^5*n^5+3*a*d^2*e*f*g^4*n^5+36*c*d^4*f*g^4*
n^4-14*c*d^3*e*f^2*g^3*n^4+20*a*d^3*g^5*n^4+54*a*d^2*e*f*g^4*n^4-6*a*d*e^2*
f^2*g^3*n^4+238*c*d^4*f*g^4*n^3-210*c*d^3*e*f^2*g^3*n^3+54*c*d^2*e^2*f^3*g^
2*n^3+155*a*d^3*g^5*n^3+357*a*d^2*e*f*g^4*n^3-90*a*d*e^2*f^2*g^3*n^3+6*a*e^
3*f^3*g^2*n^3+684*c*d^4*f*g^4*n^2-1036*c*d^3*e*f^2*g^3*n^2+594*c*d^2*e^2*f^
3*g^2*n^2-120*c*d*e^3*f^4*g*n^2+580*a*d^3*g^5*n^2+1026*a*d^2*e*f*g^4*n^2-44
4*a*d*e^2*f^2*g^3*n^2+66*a*e^3*f^3*g^2*n^2+720*c*d^4*f*g^4*n-1680*c*d^3*e*f
^2*g^3*n+1620*c*d^2*e^2*f^3*g^2*n-720*c*d*e^3*f^4*g*n+120*c*e^4*f^5*n+1044*
a*d^3*g^5*n+1080*a*d^2*e*f*g^4*n-720*a*d*e^2*f^2*g^3*n+180*a*e^3*f^3*g^2*n+
720*a*d^3*g^5)/g^5/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n
*ln(g*x+f))+(9*c*d^2*g^2*n^2+5*c*d*e*f*g*n^2+a*e*g^2*n^2+99*c*d^2*g^2*n+30*
c*d*e*f*g*n-5*c*e^2*f^2*n+11*a*e*g^2*n+270*c*d^2*g^2+30*a*e*g^2)*e^2/g^2/(n
^3+15*n^2+74*n+120)*x^4*exp(n*ln(g*x+f))+(7*c*d^3*g^3*n^3+9*c*d^2*e*f*g^2*n
^3+3*a*d*e*g^3*n^3+a*e^2*f*g^2*n^3+105*c*d^3*g^3*n^2+99*c*d^2*e*f*g^2*n^2-2
```

```
0*c*d*e^2*f^2*g*n^2+45*a*d*e*g^3*n^2+11*a*e^2*f*g^2*n^2+518*c*d^3*g^3*n+270
*c*d^2*e*f*g^2*n-120*c*d*e^2*f^2*g*n+20*c*e^3*f^3*n+222*a*d*e*g^3*n+30*a*e^
2*f*g^2*n+840*c*d^3*g^3+360*a*d*e*g^3)*e/g^3/(n^4+18*n^3+119*n^2+342*n+360)
*x^3*exp(n*ln(g*x+f))+(5*d*g*n+e*f*n+30*d*g)*c*e^3/g/(n^2+11*n+30)*x^5*exp(
n*ln(g*x+f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. 2(275) = 550.

Time = 0.35 (sec) , antiderivative size = 2032, normalized size of antiderivative = 7.39

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
[Out] (a*d^3*f*g^5*n^5 - 120*c*e^4*f^6 + 720*c*d*e^3*f^5*g + 720*a*d^3*f*g^5 - 18
0*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 240*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 360
*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4 + (c*e^4*g^6*n^5 + 15*c*e^4*g^6*n^4 + 85*c*e
^4*g^6*n^3 + 225*c*e^4*g^6*n^2 + 274*c*e^4*g^6*n + 120*c*e^4*g^6)*x^6 + (72
0*c*d*e^3*g^6 + (c*e^4*f*g^5 + 5*c*d*e^3*g^6)*n^5 + 10*(c*e^4*f*g^5 + 8*c*d
*e^3*g^6)*n^4 + 5*(7*c*e^4*f*g^5 + 95*c*d*e^3*g^6)*n^3 + 50*(c*e^4*f*g^5 +
26*c*d*e^3*g^6)*n^2 + 12*(2*c*e^4*f*g^5 + 135*c*d*e^3*g^6)*n)*x^5 + (20*a*d
^3*f*g^5 - (2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n^4 + (180*(9*c*d^2*e^2 + a*e^3)*
g^6 + (5*c*d*e^3*f*g^5 + (9*c*d^2*e^2 + a*e^3)*g^6)*n^5 - (5*c*e^4*f^2*g^4
- 60*c*d*e^3*f*g^5 - 17*(9*c*d^2*e^2 + a*e^3)*g^6)*n^4 - (30*c*e^4*f^2*g^4
- 235*c*d*e^3*f*g^5 - 107*(9*c*d^2*e^2 + a*e^3)*g^6)*n^3 - (55*c*e^4*f^2*g^
4 - 360*c*d*e^3*f*g^5 - 307*(9*c*d^2*e^2 + a*e^3)*g^6)*n^2 - 6*(5*c*e^4*f^2
*g^4 - 30*c*d*e^3*f*g^5 - 66*(9*c*d^2*e^2 + a*e^3)*g^6)*n)*x^4 + (155*a*d^3
*f*g^5 + 2*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 18*(2*c*d^4 + 3*a*d^2*e)*f^2*g
^4)*n^3 + (240*(7*c*d^3*e + 3*a*d*e^2)*g^6 + ((9*c*d^2*e^2 + a*e^3)*f*g^5 +
 (7*c*d^3*e + 3*a*d*e^2)*g^6)*n^5 - 2*(10*c*d*e^3*f^2*g^4 - 7*(9*c*d^2*e^2
+ a*e^3)*f*g^5 - 9*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^4 + (20*c*e^4*f^3*g^3 - 1
80*c*d*e^3*f^2*g^4 + 65*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 121*(7*c*d^3*e + 3*a*
d*e^2)*g^6)*n^3 + 4*(15*c*e^4*f^3*g^3 - 100*c*d*e^3*f^2*g^4 + 28*(9*c*d^2*e
^2 + a*e^3)*f*g^5 + 93*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^2 + 4*(10*c*e^4*f^3*g
^3 - 60*c*d*e^3*f^2*g^4 + 15*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 127*(7*c*d^3*e +
 3*a*d*e^2)*g^6)*n)*x^3 + (580*a*d^3*f*g^5 - 6*(9*c*d^2*e^2 + a*e^3)*f^4*g^
2 + 30*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 119*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)
*n^2 + (360*(2*c*d^4 + 3*a*d^2*e)*g^6 + ((7*c*d^3*e + 3*a*d*e^2)*f*g^5 + (2
*c*d^4 + 3*a*d^2*e)*g^6)*n^5 - (3*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 16*(7*c*d
^3*e + 3*a*d*e^2)*f*g^5 - 19*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^4 + (60*c*d*e^3*f
^3*g^3 - 36*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 + 89*(7*c*d^3*e + 3*a*d*e^2)*f*g^
5 + 137*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^3 - (60*c*e^4*f^4*g^2 - 420*c*d*e^3*f^
3*g^3 + 123*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 194*(7*c*d^3*e + 3*a*d*e^2)*f*g
```

```
^5 - 461*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^2 - 6*(10*c*e^4*f^4*g^2 - 60*c*d*e^3*
f^3*g^3 + 15*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 20*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 117*(2*c*d^4 + 3*a*d^2*e)*g^6)*n)*x^2 + 2*(60*c*d*e^3*f^5*g + 522*a*d^
3*f*g^5 - 33*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^3
*g^3 - 171*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n + (720*a*d^3*g^6 + (a*d^3*g^6 +
 (2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^5 + 2*(10*a*d^3*g^6 - (7*c*d^3*e + 3*a*d*e^
2)*f^2*g^4 + 9*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^4 + (155*a*d^3*g^6 + 6*(9*c*d
^2*e^2 + a*e^3)*f^3*g^3 - 30*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 + 119*(2*c*d^4
 + 3*a*d^2*e)*f*g^5)*n^3 - 2*(60*c*d*e^3*f^4*g^2 - 290*a*d^3*g^6 - 33*(9*c*
d^2*e^2 + a*e^3)*f^3*g^3 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 - 171*(2*c*d^
4 + 3*a*d^2*e)*f*g^5)*n^2 + 12*(10*c*e^4*f^5*g - 60*c*d*e^3*f^4*g^2 + 87*a*
d^3*g^6 + 15*(9*c*d^2*e^2 + a*e^3)*f^3*g^3 - 20*(7*c*d^3*e + 3*a*d*e^2)*f^2
*g^4 + 30*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n)*x)*(g*x + f)^n/(g^6*n^6 + 21*g^6*
n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24206 vs. 2(260) = 520.

Time = 4.36 (sec) , antiderivative size = 24206, normalized size of antiderivative = 88.02

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4
/4 + c*d**4*x**2 + 7*c*d**3*e*x**3/3 + 9*c*d**2*e**2*x**4/4 + c*d*e**3*x**5
 + c*e**4*x**6/6), Eq(g, 0)), (-12*a*d**3*g**5/(60*f**5*g**6 + 300*f**4*g**
7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11
*x**5) - 9*a*d**2*e*f*g**4/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*
x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 45*a*d**2*e
*g**5*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**
9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 6*a*d*e**2*f**2*g**3/(60*f**5*
g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g*
*10*x**4 + 60*g**11*x**5) - 30*a*d*e**2*f*g**4*x/(60*f**5*g**6 + 300*f**4*g
**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**
11*x**5) - 60*a*d*e**2*g**5*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3
*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 3*a*e
**3*f**3*g**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f*
*2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 15*a*e**3*f**2*g**3*x/(6
0*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 3
00*f*g**10*x**4 + 60*g**11*x**5) - 30*a*e**3*f*g**4*x**2/(60*f**5*g**6 + 30
0*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
+ 60*g**11*x**5) - 30*a*e**3*g**5*x**3/(60*f**5*g**6 + 300*f**4*g**7*x + 60
0*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) -
```

6*c*d**4*f*g**4/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600
*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 30*c*d**4*g**5*x/(60*
f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300
*f*g**10*x**4 + 60*g**11*x**5) - 14*c*d**3*e*f**2*g**3/(60*f**5*g**6 + 300*
f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 +
60*g**11*x**5) - 70*c*d**3*e*f*g**4*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600
*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) -
140*c*d**3*e*g**5*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2
 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 27*c*d**2*e**2*
f**3*g**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g
9*x3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 135*c*d**2*e**2*f**2*g**3*x/
(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 +
 300*f*g**10*x**4 + 60*g**11*x**5) - 270*c*d**2*e**2*f*g**4*x**2/(60*f**5*g
6 + 300*f4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**
10*x**4 + 60*g**11*x**5) - 270*c*d**2*e**2*g**5*x**3/(60*f**5*g**6 + 300*f*
*4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60
*g**11*x**5) - 60*c*d*e**3*f**4*g/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**
3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 300*
c*d*e**3*f**3*g**2*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 +
 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 600*c*d*e**3*f**2
*g**3*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*
g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 600*c*d*e**3*f*g**4*x**3/(6
0*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 3
00*f*g**10*x**4 + 60*g**11*x**5) - 300*c*d*e**3*g**5*x**4/(60*f**5*g**6 + 3
00*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
 + 60*g**11*x**5) + 60*c*e**4*f**5*log(f/g + x)/(60*f**5*g**6 + 300*f**4*g*
*7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**1
1*x**5) + 137*c*e**4*f**5/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x
2 + 600*f2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 300*c*e**4*f
4*g*x*log(f/g + x)/(60*f5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 +
 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 625*c*e**4*f**4*g
*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**
3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 600*c*e**4*f**3*g**2*x**2*log(f/g +
 x)/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x*
*3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 1100*c*e**4*f**3*g**2*x**2/(60*f**
5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*
g**10*x**4 + 60*g**11*x**5) + 600*c*e**4*f**2*g**3*x**3*log(f/g + x)/(60*f*
*5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f
*g**10*x**4 + 60*g**11*x**5) + 900*c*e**4*f**2*g**3*x**3/(60*f**5*g**6 + 30
0*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
+ 60*g**11*x**5) + 300*c*e**4*f*g**4*x**4*log(f/g + x)/(60*f**5*g**6 + 300*
f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 +
60*g**11*x**5) + 300*c*e**4*f*g**4*x**4/(60*f**5*g**6 + 300*f**4*g**7*x + 6
00*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5)
+ 60*c*e**4*g**5*x**5*log(f/g + x)/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f*

*3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5), Eq(n
, -6)), (-3*a*d**3*g**5/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2
+ 48*f*g**9*x**3 + 12*g**10*x**4) - 3*a*d**2*e*f*g**4/(12*f**4*g**6 + 48*f*
*3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 12*a*d**2
*e*g**5*x/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x*
*3 + 12*g**10*x**4) - 3*a*d*e**2*f**2*g**3/(12*f**4*g**6 + 48*f**3*g**7*x +
72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 12*a*d*e**2*f*g**4*x
/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g
10*x4) - 18*a*d*e**2*g**5*x**2/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2
*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 3*a*e**3*f**3*g**2/(12*f**4*
g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4)
- 12*a*e**3*f**2*g**3*x/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2
+ 48*f*g**9*x**3 + 12*g**10*x**4) - 18*a*e**3*f*g**4*x**2/(12*f**4*g**6 +
48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 12*a
*e**3*g**5*x**3/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g
9*x3 + 12*g**10*x**4) - 2*c*d**4*f*g**4/(12*f**4*g**6 + 48*f**3*g**7*x
+ 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 8*c*d**4*g**5*x/(12
*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10
*x**4) - 7*c*d**3*e*f**2*g**3/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8
*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 28*c*d**3*e*f*g**4*x/(12*f**4*g**
6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) -
42*c*d**3*e*g**5*x**2/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 +
48*f*g**9*x**3 + 12*g**10*x**4) - 27*c*d**2*e**2*f**3*g**2/(12*f**4*g**6 +
48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 108*
c*d**2*e**2*f**2*g**3*x/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2
+ 48*f*g**9*x**3 + 12*g**10*x**4) - 162*c*d**2*e**2*f*g**4*x**2/(12*f**4*g*
*6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) -
108*c*d**2*e**2*g**5*x**3/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x*
*2 + 48*f*g**9*x**3 + 12*g**10*x**4) + 60*c*d*e**3*f**4*g*log(f/g + x)/(12*
f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*
x**4) + 125*c*d*e**3*f**4*g/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x
2 + 48*f*g9*x**3 + 12*g**10*x**4) + 240*c*d*e**3*f**3*g**2*x*log(f/g +
x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12
*g**10*x**4) + 440*c*d*e**3*f**3*g**2*x/(12*f**4*g**6 + 48*f**3*g**7*x + 72
*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) + 360*c*d*e**3*f**2*g**3*
x**2*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f
*g**9*x**3 + 12*g**10*x**4) + 540*c*d*e**3*f**2*g**3*x**2/(12*f**4*g**6 + 4
8*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) + 240*c
*d*e**3*f*g**4*x**3*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g
8*x2 + 48*f*g**9*x**3 + 12*g**10*x**4) + 240*c*d*e**3*f*g**4*x**3/(12*f
4*g6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x
4) + 60*c*d*e3*g**5*x**4*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x +
72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 60*c*e**4*f**5*log(f/
g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3
+ 12*g**10*x**4) - 125*c*e**4*f**5/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2

*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 240*c*e**4*f**4*g*x*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 440*c*e**4*f**4*g*x/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 360*c*e**4*f**3*g**2*x**2*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 540*c*e**4*f**3*g**2*x**2/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 240*c*e**4*f**2*g**3*x**3*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 240*c*e**4*f**2*g**3*x**3/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) - 60*c*e**4*f*g**4*x**4*log(f/g + x)/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4) + 12*c*e**4*g**5*x**5/(12*f**4*g**6 + 48*f**3*g**7*x + 72*f**2*g**8*x**2 + 48*f*g**9*x**3 + 12*g**10*x**4), Eq(n, -5)), (-2*a*d**3*g**5/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 3*a*d**2*e*f*g**4/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 9*a*d**2*e*g**5*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 6*a*d*e**2*f**2*g**3/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 18*a*d*e**2*f*g**4*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 18*a*d*e**2*g**5*x**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 6*a*e**3*f**3*g**2*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 11*a*e**3*f**3*g**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 18*a*e**3*f**2*g**3*x*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 27*a*e**3*f**2*g**3*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 18*a*e**3*f*g**4*x**2*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 18*a*e**3*f*g**4*x**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 6*a*e**3*g**5*x**3*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 2*c*d**4*f*g**4/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 6*c*d**4*g**5*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 14*c*d**3*e*f**2*g**3/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 42*c*d**3*e*f*g**4*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 42*c*d**3*e*g**5*x**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 54*c*d**2*e**2*f**3*g**2*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 99*c*d**2*e**2*f**3*g**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 162*c*d**2*e**2*f**2*g**3*x*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 243*c*d**2*e**2*f**2*g**3*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 162*c*d**2*e**2*f*g**4*x**2*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 162*c*d**2*e**2*f*g**4*x**2/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 54*c*d**2*e**2*g**5*x**3*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 120*c*d*e**3*f**4*g*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 220*c*d*e**3*f**4*g/(6*f**3*g**6 + 1

8*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 360*c*d*e**3*f**3*g**2*x*lo
g(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) -
540*c*d*e**3*f**3*g**2*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6
*g**9*x**3) - 360*c*d*e**3*f**2*g**3*x**2*log(f/g + x)/(6*f**3*g**6 + 18*f*
*2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 360*c*d*e**3*f**2*g**3*x**2/(6*
f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 120*c*d*e**3*f
*g**4*x**3*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*
g**9*x**3) + 30*c*d*e**3*g**5*x**4/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**
8*x**2 + 6*g**9*x**3) + 60*c*e**4*f**5*log(f/g + x)/(6*f**3*g**6 + 18*f**2*
g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 110*c*e**4*f**5/(6*f**3*g**6 + 18*
f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 180*c*e**4*f**4*g*x*log(f/g +
 x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 270*c*e
4*f4*g*x/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3)
+ 180*c*e**4*f**3*g**2*x**2*log(f/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18
*f*g**8*x**2 + 6*g**9*x**3) + 180*c*e**4*f**3*g**2*x**2/(6*f**3*g**6 + 18*f
2*g7*x + 18*f*g**8*x**2 + 6*g**9*x**3) + 60*c*e**4*f**2*g**3*x**3*log(f
/g + x)/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*x**3) - 15*
c*e**4*f*g**4*x**4/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 + 6*g**9*
x**3) + 3*c*e**4*g**5*x**5/(6*f**3*g**6 + 18*f**2*g**7*x + 18*f*g**8*x**2 +
 6*g**9*x**3), Eq(n, -4)), (-3*a*d**3*g**5/(6*f**2*g**6 + 12*f*g**7*x + 6*g
8*x2) - 9*a*d**2*e*f*g**4/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 1
8*a*d**2*e*g**5*x/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 18*a*d*e**2*f
2*g3*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 27*a*d*e*
*2*f**2*g**3/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 36*a*d*e**2*f*g**4
*x*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 36*a*d*e**2*f*g
4*x/(6*f2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 18*a*d*e**2*g**5*x**2*log
(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 18*a*e**3*f**3*g**2*l
og(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 27*a*e**3*f**3*g**2
/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 36*a*e**3*f**2*g**3*x*log(f/g
+ x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 36*a*e**3*f**2*g**3*x/(6*f
2*g6 + 12*f*g**7*x + 6*g**8*x**2) - 18*a*e**3*f*g**4*x**2*log(f/g + x)/
(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 6*a*e**3*g**5*x**3/(6*f**2*g**6
 + 12*f*g**7*x + 6*g**8*x**2) - 6*c*d**4*f*g**4/(6*f**2*g**6 + 12*f*g**7*x
+ 6*g**8*x**2) - 12*c*d**4*g**5*x/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2)
 + 42*c*d**3*e*f**2*g**3*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x
2) + 63*c*d3*e*f**2*g**3/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 84
*c*d**3*e*f*g**4*x*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) +
 84*c*d**3*e*f*g**4*x/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 42*c*d**3
*e*g**5*x**2*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 162*c
*d**2*e**2*f**3*g**2*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2)
 - 243*c*d**2*e**2*f**3*g**2/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 32
4*c*d**2*e**2*f**2*g**3*x*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*
x**2) - 324*c*d**2*e**2*f**2*g**3*x/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**
2) - 162*c*d**2*e**2*f*g**4*x**2*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x +
6*g**8*x**2) + 54*c*d**2*e**2*g**5*x**3/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8

*x**2) + 180*c*d*e**3*f**4*g*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g*
*8*x**2) + 270*c*d*e**3*f**4*g/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) +
360*c*d*e**3*f**3*g**2*x*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x
2) + 360*c*d*e3*f**3*g**2*x/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) +
180*c*d*e**3*f**2*g**3*x**2*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g*
*8*x**2) - 60*c*d*e**3*f*g**4*x**3/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2
) + 15*c*d*e**3*g**5*x**4/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 60*c*
e**4*f**5*log(f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 90*c*e**
4*f**5/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 120*c*e**4*f**4*g*x*log(
f/g + x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 120*c*e**4*f**4*g*x/(6
*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 60*c*e**4*f**3*g**2*x**2*log(f/g
+ x)/(6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) + 20*c*e**4*f**2*g**3*x**3/(
6*f**2*g**6 + 12*f*g**7*x + 6*g**8*x**2) - 5*c*e**4*f*g**4*x**4/(6*f**2*g**
6 + 12*f*g**7*x + 6*g**8*x**2) + 2*c*e**4*g**5*x**5/(6*f**2*g**6 + 12*f*g**
7*x + 6*g**8*x**2), Eq(n, -3)), (-12*a*d**3*g**5/(12*f*g**6 + 12*g**7*x) +
36*a*d**2*e*f*g**4*log(f/g + x)/(12*f*g**6 + 12*g**7*x) + 36*a*d**2*e*f*g**
4/(12*f*g**6 + 12*g**7*x) + 36*a*d**2*e*g**5*x*log(f/g + x)/(12*f*g**6 + 12
*g**7*x) - 72*a*d*e**2*f**2*g**3*log(f/g + x)/(12*f*g**6 + 12*g**7*x) - 72*
a*d*e**2*f**2*g**3/(12*f*g**6 + 12*g**7*x) - 72*a*d*e**2*f*g**4*x*log(f/g +
x)/(12*f*g**6 + 12*g**7*x) + 36*a*d*e**2*g**5*x**2/(12*f*g**6 + 12*g**7*x)
+ 36*a*e**3*f**3*g**2*log(f/g + x)/(12*f*g**6 + 12*g**7*x) + 36*a*e**3*f**
3*g**2/(12*f*g**6 + 12*g**7*x) + 36*a*e**3*f**2*g**3*x*log(f/g + x)/(12*f*g
6 + 12*g7*x) - 18*a*e**3*f*g**4*x**2/(12*f*g**6 + 12*g**7*x) + 6*a*e**3
*g**5*x**3/(12*f*g**6 + 12*g**7*x) + 24*c*d**4*f*g**4*log(f/g + x)/(12*f*g*
*6 + 12*g**7*x) + 24*c*d**4*f*g**4/(12*f*g**6 + 12*g**7*x) + 24*c*d**4*g**5
*x*log(f/g + x)/(12*f*g**6 + 12*g**7*x) - 168*c*d**3*e*f**2*g**3*log(f/g +
x)/(12*f*g**6 + 12*g**7*x) - 168*c*d**3*e*f**2*g**3/(12*f*g**6 + 12*g**7*x)
- 168*c*d**3*e*f*g**4*x*log(f/g + x)/(12*f*g**6 + 12*g**7*x) + 84*c*d**3*e
*g**5*x**2/(12*f*g**6 + 12*g**7*x) + 324*c*d**2*e**2*f**3*g**2*log(f/g + x)
/(12*f*g**6 + 12*g**7*x) + 324*c*d**2*e**2*f**3*g**2/(12*f*g**6 + 12*g**7*x
) + 324*c*d**2*e**2*f**2*g**3*x*log(f/g + x)/(12*f*g**6 + 12*g**7*x) - 162*
c*d**2*e**2*f*g**4*x**2/(12*f*g**6 + 12*g**7*x) + 54*c*d**2*e**2*g**5*x**3/
(12*f*g**6 + 12*g**7*x) - 240*c*d*e**3*f**4*g*log(f/g + x)/(12*f*g**6 + 12*
g**7*x) - 240*c*d*e**3*f**4*g/(12*f*g**6 + 12*g**7*x) - 240*c*d*e**3*f**3*g
2*x*log(f/g + x)/(12*f*g6 + 12*g**7*x) + 120*c*d*e**3*f**2*g**3*x**2/(1
2*f*g**6 + 12*g**7*x) - 40*c*d*e**3*f*g**4*x**3/(12*f*g**6 + 12*g**7*x) + 2
0*c*d*e**3*g**5*x**4/(12*f*g**6 + 12*g**7*x) + 60*c*e**4*f**5*log(f/g + x)/
(12*f*g**6 + 12*g**7*x) + 60*c*e**4*f**5/(12*f*g**6 + 12*g**7*x) + 60*c*e**
4*f**4*g*x*log(f/g + x)/(12*f*g**6 + 12*g**7*x) - 30*c*e**4*f**3*g**2*x**2/
(12*f*g**6 + 12*g**7*x) + 10*c*e**4*f**2*g**3*x**3/(12*f*g**6 + 12*g**7*x)
- 5*c*e**4*f*g**4*x**4/(12*f*g**6 + 12*g**7*x) + 3*c*e**4*g**5*x**5/(12*f*g
6 + 12*g7*x), Eq(n, -2)), (a*d**3*log(f/g + x)/g - 3*a*d**2*e*f*log(f/g
+ x)/g**2 + 3*a*d**2*e*x/g + 3*a*d*e**2*f**2*log(f/g + x)/g**3 - 3*a*d*e**
2*f*x/g**2 + 3*a*d*e**2*x**2/(2*g) - a*e**3*f**3*log(f/g + x)/g**4 + a*e**3
*f**2*x/g**3 - a*e**3*f*x**2/(2*g**2) + a*e**3*x**3/(3*g) - 2*c*d**4*f*log(

f/g + x)/g**2 + 2*c*d**4*x/g + 7*c*d**3*e*f**2*log(f/g + x)/g**3 - 7*c*d**3
*e*f*x/g**2 + 7*c*d**3*e*x**2/(2*g) - 9*c*d**2*e**2*f**3*log(f/g + x)/g**4
+ 9*c*d**2*e**2*f**2*x/g**3 - 9*c*d**2*e**2*f*x**2/(2*g**2) + 3*c*d**2*e**2
*x**3/g + 5*c*d*e**3*f**4*log(f/g + x)/g**5 - 5*c*d*e**3*f**3*x/g**4 + 5*c*
d*e**3*f**2*x**2/(2*g**3) - 5*c*d*e**3*f*x**3/(3*g**2) + 5*c*d*e**3*x**4/(4
*g) - c*e**4*f**5*log(f/g + x)/g**6 + c*e**4*f**4*x/g**5 - c*e**4*f**3*x**2
/(2*g**4) + c*e**4*f**2*x**3/(3*g**3) - c*e**4*f*x**4/(4*g**2) + c*e**4*x**
5/(5*g), Eq(n, -1)), (a*d**3*f*g**5*n**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*
n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g
6) + 20*a*d3*f*g**5*n**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g
6*n4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 155*a
*d**3*f*g**5*n**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 +
735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 580*a*d**3*f*g**
5*n**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n*
*3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1044*a*d**3*f*g**5*n*(f + g
*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**
6*n**2 + 1764*g**6*n + 720*g**6) + 720*a*d**3*f*g**5*(f + g*x)**n/(g**6*n**
6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g*
*6*n + 720*g**6) + a*d**3*g**6*n**5*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**
5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6
) + 20*a*d**3*g**6*n**4*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6
*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 155*a*d*
*3*g**6*n**3*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735
*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 580*a*d**3*g**6*n**
2*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3
+ 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1044*a*d**3*g**6*n*x*(f + g*x)
n/(g6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n
2 + 1764*g6*n + 720*g**6) + 720*a*d**3*g**6*x*(f + g*x)**n/(g**6*n**6 +
21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*
n + 720*g**6) - 3*a*d**2*e*f**2*g**4*n**4*(f + g*x)**n/(g**6*n**6 + 21*g**6
*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*
g**6) - 54*a*d**2*e*f**2*g**4*n**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 +
175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) -
357*a*d**2*e*f**2*g**4*n**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g
6*n4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1026*
a*d**2*e*f**2*g**4*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1080*a*d**2*e
*f**2*g**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**
6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 3*a*d**2*e*f*g**5*n**5*
x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 54*a*d**2*e*f*g**5*n**4*x*(f + g
*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**
6*n**2 + 1764*g**6*n + 720*g**6) + 357*a*d**2*e*f*g**5*n**3*x*(f + g*x)**n/
(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2
+ 1764*g**6*n + 720*g**6) + 1026*a*d**2*e*f*g**5*n**2*x*(f + g*x)**n/(g**6*

n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764
*g**6*n + 720*g**6) + 1080*a*d**2*e*f*g**5*n*x*(f + g*x)**n/(g**6*n**6 + 21
*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n +
720*g**6) + 3*a*d**2*e*g**6*n**5*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n*
*5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**
6) + 57*a*d**2*e*g**6*n**4*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 17
5*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 41
1*a*d**2*e*g**6*n**3*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6
*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1383*a*d
2*e*g6*n**2*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 2106*a*d**2*e
*g**6*n*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g
6*n3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1080*a*d**2*e*g**6*x*
2(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 6*a*d*e**2*f**3*g**3*n**3*(f +
g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g*
*6*n**2 + 1764*g**6*n + 720*g**6) + 90*a*d*e**2*f**3*g**3*n**2*(f + g*x)**n
/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2
+ 1764*g**6*n + 720*g**6) + 444*a*d*e**2*f**3*g**3*n*(f + g*x)**n/(g**6*n*
*6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g
6*n + 720*g6) + 720*a*d*e**2*f**3*g**3*(f + g*x)**n/(g**6*n**6 + 21*g**
6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720
*g**6) - 6*a*d*e**2*f**2*g**4*n**4*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5
+ 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6)
- 90*a*d*e**2*f**2*g**4*n**3*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 17
5*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 44
4*a*d*e**2*f**2*g**4*n**2*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g*
*6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 720*a*
d*e**2*f**2*g**4*n*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 3*a*d*e**2*f*
g**5*n**5*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735
*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 48*a*d*e**2*f*g**5*
n**4*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6
*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 267*a*d*e**2*f*g**5*n**3
*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**
3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 582*a*d*e**2*f*g**5*n**2*x**
2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 360*a*d*e**2*f*g**5*n*x**2*(f +
g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g*
*6*n**2 + 1764*g**6*n + 720*g**6) + 3*a*d*e**2*g**6*n**5*x**3*(f + g*x)**n/
(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2
+ 1764*g**6*n + 720*g**6) + 54*a*d*e**2*g**6*n**4*x**3*(f + g*x)**n/(g**6*n
6 + 21*g6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*
g**6*n + 720*g**6) + 363*a*d*e**2*g**6*n**3*x**3*(f + g*x)**n/(g**6*n**6 +
21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n

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 + 720*g**6) + 1116*a*d*e**2*g**6*n**2*x**3*(f + g*x)**n/(g**6*n**6 + 21*g*
*6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 72
0*g**6) + 1524*a*d*e**2*g**6*n*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5
+ 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6)
+ 720*a*d*e**2*g**6*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*
n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 6*a*e**3*
f**4*g**2*n**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735
*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 66*a*e**3*f**4*g**2
*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 180*a*e**3*f**4*g**2*(f + g*x)*
*n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n*
*2 + 1764*g**6*n + 720*g**6) + 6*a*e**3*f**3*g**3*n**3*x*(f + g*x)**n/(g**6
*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 176
4*g**6*n + 720*g**6) + 66*a*e**3*f**3*g**3*n**2*x*(f + g*x)**n/(g**6*n**6 +
 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*
n + 720*g**6) + 180*a*e**3*f**3*g**3*n*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*
n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g
**6) - 3*a*e**3*f**2*g**4*n**4*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5
+ 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6)
- 36*a*e**3*f**2*g**4*n**3*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 17
5*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 12
3*a*e**3*f**2*g**4*n**2*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g
**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 90*a*
e**3*f**2*g**4*n*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**
4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + a*e**3*f*g**
5*n**5*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g*
*6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 14*a*e**3*f*g**5*n**4*
x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3
 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 65*a*e**3*f*g**5*n**3*x**3*(f
 + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624
*g**6*n**2 + 1764*g**6*n + 720*g**6) + 112*a*e**3*f*g**5*n**2*x**3*(f + g*x
)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*
n**2 + 1764*g**6*n + 720*g**6) + 60*a*e**3*f*g**5*n*x**3*(f + g*x)**n/(g**6
*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 176
4*g**6*n + 720*g**6) + a*e**3*g**6*n**5*x**4*(f + g*x)**n/(g**6*n**6 + 21*g
**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 7
20*g**6) + 17*a*e**3*g**6*n**4*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5
+ 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6)
+ 107*a*e**3*g**6*n**3*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g*
*6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 307*a*
e**3*g**6*n**2*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 396*a*e**3*g**
6*n*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*
n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 180*a*e**3*g**6*x**4*(f +
 g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g
```

```
**6*n**2 + 1764*g**6*n + 720*g**6) - 2*c*d**4*f**2*g**4*n**4*(f + g*x)**n/(
g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 +
 1764*g**6*n + 720*g**6) - 36*c*d**4*f**2*g**4*n**3*(f + g*x)**n/(g**6*n**6
 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**
6*n + 720*g**6) - 238*c*d**4*f**2*g**4*n**2*(f + g*x)**n/(g**6*n**6 + 21*g*
*6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 72
0*g**6) - 684*c*d**4*f**2*g**4*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 1
75*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 7
20*c*d**4*f**2*g**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 2*c*d**4*f*g**
5*n**5*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*
n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 36*c*d**4*f*g**5*n**4*x*(
f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 162
4*g**6*n**2 + 1764*g**6*n + 720*g**6) + 238*c*d**4*f*g**5*n**3*x*(f + g*x)*
*n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n*
*2 + 1764*g**6*n + 720*g**6) + 684*c*d**4*f*g**5*n**2*x*(f + g*x)**n/(g**6*
n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764
*g**6*n + 720*g**6) + 720*c*d**4*f*g**5*n*x*(f + g*x)**n/(g**6*n**6 + 21*g*
*6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 72
0*g**6) + 2*c*d**4*g**6*n**5*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 +
175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) +
38*c*d**4*g**6*n**4*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*
n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 274*c*d**
4*g**6*n**3*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 7
35*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 922*c*d**4*g**6*n
**2*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*
n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1404*c*d**4*g**6*n*x**2*(
f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 162
4*g**6*n**2 + 1764*g**6*n + 720*g**6) + 720*c*d**4*g**6*x**2*(f + g*x)**n/(
g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 +
 1764*g**6*n + 720*g**6) + 14*c*d**3*e*f**3*g**3*n**3*(f + g*x)**n/(g**6*n*
*6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g
**6*n + 720*g**6) + 210*c*d**3*e*f**3*g**3*n**2*(f + g*x)**n/(g**6*n**6 + 2
1*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n
+ 720*g**6) + 1036*c*d**3*e*f**3*g**3*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n
**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g*
*6) + 1680*c*d**3*e*f**3*g**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*
g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 14*c
*d**3*e*f**2*g**4*n**4*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*
n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 210*c*d**
3*e*f**2*g**4*n**3*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1036*c*d**3*e
*f**2*g**4*n**2*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 +
735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1680*c*d**3*e*f*
*2*g**4*n*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g*
```

*6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 7*c*d**3*e*f*g**5*n**5
*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**
3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 112*c*d**3*e*f*g**5*n**4*x**
2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 623*c*d**3*e*f*g**5*n**3*x**2*(f
 + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624
*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1358*c*d**3*e*f*g**5*n**2*x**2*(f +
g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g*
*6*n**2 + 1764*g**6*n + 720*g**6) + 840*c*d**3*e*f*g**5*n*x**2*(f + g*x)**n
/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2
 + 1764*g**6*n + 720*g**6) + 7*c*d**3*e*g**6*n**5*x**3*(f + g*x)**n/(g**6*n
6 + 21*g6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*
g**6*n + 720*g**6) + 126*c*d**3*e*g**6*n**4*x**3*(f + g*x)**n/(g**6*n**6 +
21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n
 + 720*g**6) + 847*c*d**3*e*g**6*n**3*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**
6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720
*g**6) + 2604*c*d**3*e*g**6*n**2*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**
5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6
) + 3556*c*d**3*e*g**6*n*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*
g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1680
*c*d**3*e*g**6*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 54*c*d**2*e**2
*f**4*g**2*n**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 73
5*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 594*c*d**2*e**2*f*
*4*g**2*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6
*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1620*c*d**2*e**2*f**4*g*
2(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 54*c*d**2*e**2*f**3*g**3*n**3*x
*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1
624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 594*c*d**2*e**2*f**3*g**3*n**2*x*
(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 16
24*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1620*c*d**2*e**2*f**3*g**3*n*x*(f
+ g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*
g**6*n**2 + 1764*g**6*n + 720*g**6) - 27*c*d**2*e**2*f**2*g**4*n**4*x**2*(f
 + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624
*g**6*n**2 + 1764*g**6*n + 720*g**6) - 324*c*d**2*e**2*f**2*g**4*n**3*x**2*
(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 16
24*g**6*n**2 + 1764*g**6*n + 720*g**6) - 1107*c*d**2*e**2*f**2*g**4*n**2*x*
2(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 810*c*d**2*e**2*f**2*g**4*n*x**
2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 9*c*d**2*e**2*f*g**5*n**5*x**3*(
f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 162
4*g**6*n**2 + 1764*g**6*n + 720*g**6) + 126*c*d**2*e**2*f*g**5*n**4*x**3*(f
 + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624

*g**6*n**2 + 1764*g**6*n + 720*g**6) + 585*c*d**2*e**2*f*g**5*n**3*x**3*(f
+ g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*
g**6*n**2 + 1764*g**6*n + 720*g**6) + 1008*c*d**2*e**2*f*g**5*n**2*x**3*(f
+ g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*
g**6*n**2 + 1764*g**6*n + 720*g**6) + 540*c*d**2*e**2*f*g**5*n*x**3*(f + g*
x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6
*n**2 + 1764*g**6*n + 720*g**6) + 9*c*d**2*e**2*g**6*n**5*x**4*(f + g*x)**n
/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2
 + 1764*g**6*n + 720*g**6) + 153*c*d**2*e**2*g**6*n**4*x**4*(f + g*x)**n/(g
6*n6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 +
1764*g**6*n + 720*g**6) + 963*c*d**2*e**2*g**6*n**3*x**4*(f + g*x)**n/(g**6
*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 176
4*g**6*n + 720*g**6) + 2763*c*d**2*e**2*g**6*n**2*x**4*(f + g*x)**n/(g**6*n
6 + 21*g6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*
g**6*n + 720*g**6) + 3564*c*d**2*e**2*g**6*n*x**4*(f + g*x)**n/(g**6*n**6 +
 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*
n + 720*g**6) + 1620*c*d**2*e**2*g**6*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**
6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720
*g**6) + 120*c*d*e**3*f**5*g*n*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175
*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 720
*c*d*e**3*f**5*g*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 7
35*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 120*c*d*e**3*f**4
*g**2*n**2*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g
6*n3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 720*c*d*e**3*f**4*g**
2*n*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**
3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 60*c*d*e**3*f**3*g**3*n**3*x
2*(f + g*x)n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3
+ 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 420*c*d*e**3*f**3*g**3*n**2*x*
2(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 +
 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 360*c*d*e**3*f**3*g**3*n*x**2*(
f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 162
4*g**6*n**2 + 1764*g**6*n + 720*g**6) - 20*c*d*e**3*f**2*g**4*n**4*x**3*(f
+ g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*
g**6*n**2 + 1764*g**6*n + 720*g**6) - 180*c*d*e**3*f**2*g**4*n**3*x**3*(f +
 g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g
6*n2 + 1764*g**6*n + 720*g**6) - 400*c*d*e**3*f**2*g**4*n**2*x**3*(f +
g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g*
*6*n**2 + 1764*g**6*n + 720*g**6) - 240*c*d*e**3*f**2*g**4*n*x**3*(f + g*x)
n/(g6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n
2 + 1764*g6*n + 720*g**6) + 5*c*d*e**3*f*g**5*n**5*x**4*(f + g*x)**n/(g
6*n6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 +
1764*g**6*n + 720*g**6) + 60*c*d*e**3*f*g**5*n**4*x**4*(f + g*x)**n/(g**6*n
6 + 21*g6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*
g**6*n + 720*g**6) + 235*c*d*e**3*f*g**5*n**3*x**4*(f + g*x)**n/(g**6*n**6
+ 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6

*n + 720*g**6) + 360*c*d*e**3*f*g**5*n**2*x**4*(f + g*x)**n/(g**6*n**6 + 21
*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n +
720*g**6) + 180*c*d*e**3*f*g**5*n*x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n
5 + 175*g6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g*
*6) + 5*c*d*e**3*g**6*n**5*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 17
5*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 80
*c*d*e**3*g**6*n**4*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*
n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 475*c*d*e
3*g6*n**3*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 +
735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1300*c*d*e**3*g
6*n2*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*
g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 1620*c*d*e**3*g**6*n
*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**
3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 720*c*d*e**3*g**6*x**5*(f +
g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g*
*6*n**2 + 1764*g**6*n + 720*g**6) - 120*c*e**4*f**6*(f + g*x)**n/(g**6*n**6
+ 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**
6*n + 720*g**6) + 120*c*e**4*f**5*g*n*x*(f + g*x)**n/(g**6*n**6 + 21*g**6*n
5 + 175*g6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g*
*6) - 60*c*e**4*f**4*g**2*n**2*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5
+ 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6)
- 60*c*e**4*f**4*g**2*n*x**2*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g
6*n4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 20*c*
e**4*f**3*g**3*n**3*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*
n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 60*c*e**4
*f**3*g**3*n**2*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4
+ 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 40*c*e**4*f**
3*g**3*n*x**3*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*
g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 5*c*e**4*f**2*g**4*n
4*x4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*
n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 30*c*e**4*f**2*g**4*n**3*
x**4*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3
+ 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 55*c*e**4*f**2*g**4*n**2*x**4
*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1
624*g**6*n**2 + 1764*g**6*n + 720*g**6) - 30*c*e**4*f**2*g**4*n*x**4*(f + g
*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**
6*n**2 + 1764*g**6*n + 720*g**6) + c*e**4*f*g**5*n**5*x**5*(f + g*x)**n/(g*
*6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1
764*g**6*n + 720*g**6) + 10*c*e**4*f*g**5*n**4*x**5*(f + g*x)**n/(g**6*n**6
+ 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**
6*n + 720*g**6) + 35*c*e**4*f*g**5*n**3*x**5*(f + g*x)**n/(g**6*n**6 + 21*g
6*n5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 7
20*g**6) + 50*c*e**4*f*g**5*n**2*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**
5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6
) + 24*c*e**4*f*g**5*n*x**5*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g*

```
*6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + c*e**4
*g**6*n**5*x**6*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 73
5*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 15*c*e**4*g**6*n**
4*x**6*(f + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n*
*3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g**6) + 85*c*e**4*g**6*n**3*x**6*(f
 + g*x)**n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624
*g**6*n**2 + 1764*g**6*n + 720*g**6) + 225*c*e**4*g**6*n**2*x**6*(f + g*x)*
*n/(g**6*n**6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n*
*2 + 1764*g**6*n + 720*g**6) + 274*c*e**4*g**6*n*x**6*(f + g*x)**n/(g**6*n*
*6 + 21*g**6*n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g
**6*n + 720*g**6) + 120*c*e**4*g**6*x**6*(f + g*x)**n/(g**6*n**6 + 21*g**6*
n**5 + 175*g**6*n**4 + 735*g**6*n**3 + 1624*g**6*n**2 + 1764*g**6*n + 720*g
**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(275) = 550$.

Time = 0.27 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.95

$$
\begin{aligned}
\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx &= \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^4}{(n^2+3n+2)g^2} \\
&+ \frac{7\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cd^3e}{(n^3+6n^2+11n+6)g^3} \\
&+ \frac{3\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ad^2e}{(n^2+3n+2)g^2} + \frac{(gx+f)^{n+1}ad^3}{g(n+1)} \\
&+ \frac{9\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^{n}}{(n^4+10n^3+35n^2+50n+24)g^4} \\
&+ \frac{3\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n ade^2}{(n^3+6n^2+11n+6)g^3} \\
&+ \frac{5\left((n^4+10n^3+35n^2+50n+24)g^5x^5+(n^4+6n^3+11n^2+6n)fg^4x^4-4(n^3+3n^2+2n)f^2g^3x^3\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)g^5} \\
&+ \frac{\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n a}{(n^4+10n^3+35n^2+50n+24)g^4} \\
&+ \frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)g^6x^6+(n^5+10n^4+35n^3+50n^2+24n)fg^5x^5-5(n^4\right.}{(n^6+21n^5+175n^4+}
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^4/((n^2 + 3*n + 2)*g^2)
 + 7*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^3*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 3*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d^2*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)
*a*d^3/(g*(n + 1)) + 9*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2
```

```
*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*
c*d^2*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + 3*((n^2 + 3*n + 2)*g^
3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*d*e^2/((n^
3 + 6*n^2 + 11*n + 6)*g^3) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5
 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x
^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*d*e^3/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5) + ((n^3 + 6*n^2 + 11*
n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 +
6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*a*e^3/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)
*g^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^6*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*f*g^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
f^2*g^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*f^3*g^3*x^3 - 60*(n^2 + n)*f^4*g^2*x^2
 + 120*f^5*g*n*x - 120*f^6)*(g*x + f)^n*c*e^4/((n^6 + 21*n^5 + 175*n^4 + 73
5*n^3 + 1624*n^2 + 1764*n + 720)*g^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. $2(275) = 550$.

Time = 0.33 (sec) , antiderivative size = 3830, normalized size of antiderivative = 13.93

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*e^4*g^6*n^5*x^6 + (g*x + f)^n*c*e^4*f*g^5*n^5*x^5 + 5*(g*x +
 f)^n*c*d*e^3*g^6*n^5*x^5 + 15*(g*x + f)^n*c*e^4*g^6*n^4*x^6 + 5*(g*x + f)^
n*c*d*e^3*f*g^5*n^5*x^4 + 9*(g*x + f)^n*c*d^2*e^2*g^6*n^5*x^4 + 10*(g*x + f
)^n*c*e^4*f*g^5*n^4*x^5 + 80*(g*x + f)^n*c*d*e^3*g^6*n^4*x^5 + 85*(g*x + f)
^n*c*e^4*g^6*n^3*x^6 + 9*(g*x + f)^n*c*d^2*e^2*f*g^5*n^5*x^3 + 7*(g*x + f)^
n*c*d^3*e*g^6*n^5*x^3 - 5*(g*x + f)^n*c*e^4*f^2*g^4*n^4*x^4 + 60*(g*x + f)^
n*c*d*e^3*f*g^5*n^4*x^4 + 153*(g*x + f)^n*c*d^2*e^2*g^6*n^4*x^4 + (g*x + f)
^n*a*e^3*g^6*n^5*x^4 + 35*(g*x + f)^n*c*e^4*f*g^5*n^3*x^5 + 475*(g*x + f)^n
*c*d*e^3*g^6*n^3*x^5 + 225*(g*x + f)^n*c*e^4*g^6*n^2*x^6 + 7*(g*x + f)^n*c*
d^3*e*f*g^5*n^5*x^2 + 2*(g*x + f)^n*c*d^4*g^6*n^5*x^2 - 20*(g*x + f)^n*c*d*
e^3*f^2*g^4*n^4*x^3 + 126*(g*x + f)^n*c*d^2*e^2*f*g^5*n^4*x^3 + 126*(g*x +
f)^n*c*d^3*e*g^6*n^4*x^3 + (g*x + f)^n*a*e^3*f*g^5*n^5*x^3 + 3*(g*x + f)^n*
a*d*e^2*g^6*n^5*x^3 - 30*(g*x + f)^n*c*e^4*f^2*g^4*n^3*x^4 + 235*(g*x + f)^
n*c*d*e^3*f*g^5*n^3*x^4 + 963*(g*x + f)^n*c*d^2*e^2*g^6*n^3*x^4 + 17*(g*x +
 f)^n*a*e^3*g^6*n^4*x^4 + 50*(g*x + f)^n*c*e^4*f*g^5*n^2*x^5 + 1300*(g*x +
f)^n*c*d*e^3*g^6*n^2*x^5 + 274*(g*x + f)^n*c*e^4*g^6*n*x^6 + 2*(g*x + f)^n*
c*d^4*f*g^5*n^5*x - 27*(g*x + f)^n*c*d^2*e^2*f^2*g^4*n^4*x^2 + 112*(g*x + f
)^n*c*d^3*e*f*g^5*n^4*x^2 + 38*(g*x + f)^n*c*d^4*g^6*n^4*x^2 + 3*(g*x + f)^
n*a*d*e^2*f*g^5*n^5*x^2 + 3*(g*x + f)^n*a*d^2*e*g^6*n^5*x^2 + 20*(g*x + f)^
n*c*e^4*f^3*g^3*n^3*x^3 - 180*(g*x + f)^n*c*d*e^3*f^2*g^4*n^3*x^3 + 585*(g*
```

x + f)^n*c*d^2*e^2*f*g^5*n^3*x^3 + 847*(g*x + f)^n*c*d^3*e*g^6*n^3*x^3 + 14
*(g*x + f)^n*a*e^3*f*g^5*n^4*x^3 + 54*(g*x + f)^n*a*d*e^2*g^6*n^4*x^3 - 55*
(g*x + f)^n*c*e^4*f^2*g^4*n^2*x^4 + 360*(g*x + f)^n*c*d*e^3*f*g^5*n^2*x^4 +
 2763*(g*x + f)^n*c*d^2*e^2*g^6*n^2*x^4 + 107*(g*x + f)^n*a*e^3*g^6*n^3*x^4
 + 24*(g*x + f)^n*c*e^4*f*g^5*n*x^5 + 1620*(g*x + f)^n*c*d*e^3*g^6*n*x^5 +
120*(g*x + f)^n*c*e^4*g^6*x^6 - 14*(g*x + f)^n*c*d^3*e*f^2*g^4*n^4*x + 36*(
g*x + f)^n*c*d^4*f*g^5*n^4*x + 3*(g*x + f)^n*a*d^2*e*f*g^5*n^5*x + (g*x + f
)^n*a*d^3*g^6*n^5*x + 60*(g*x + f)^n*c*d*e^3*f^3*g^3*n^3*x^2 - 324*(g*x + f
)^n*c*d^2*e^2*f^2*g^4*n^3*x^2 + 623*(g*x + f)^n*c*d^3*e*f*g^5*n^3*x^2 + 274
*(g*x + f)^n*c*d^4*g^6*n^3*x^2 - 3*(g*x + f)^n*a*e^3*f^2*g^4*n^4*x^2 + 48*(
g*x + f)^n*a*d*e^2*f*g^5*n^4*x^2 + 57*(g*x + f)^n*a*d^2*e*g^6*n^4*x^2 + 60*
(g*x + f)^n*c*e^4*f^3*g^3*n^2*x^3 - 400*(g*x + f)^n*c*d*e^3*f^2*g^4*n^2*x^3
 + 1008*(g*x + f)^n*c*d^2*e^2*f*g^5*n^2*x^3 + 2604*(g*x + f)^n*c*d^3*e*g^6*
n^2*x^3 + 65*(g*x + f)^n*a*e^3*f*g^5*n^3*x^3 + 363*(g*x + f)^n*a*d*e^2*g^6*
n^3*x^3 - 30*(g*x + f)^n*c*e^4*f^2*g^4*n*x^4 + 180*(g*x + f)^n*c*d*e^3*f*g^
5*n*x^4 + 3564*(g*x + f)^n*c*d^2*e^2*g^6*n*x^4 + 307*(g*x + f)^n*a*e^3*g^6*
n^2*x^4 + 720*(g*x + f)^n*c*d*e^3*g^6*x^5 - 2*(g*x + f)^n*c*d^4*f^2*g^4*n^4
 + (g*x + f)^n*a*d^3*f*g^5*n^5 + 54*(g*x + f)^n*c*d^2*e^2*f^3*g^3*n^3*x - 2
10*(g*x + f)^n*c*d^3*e*f^2*g^4*n^3*x + 238*(g*x + f)^n*c*d^4*f*g^5*n^3*x -
6*(g*x + f)^n*a*d*e^2*f^2*g^4*n^4*x + 54*(g*x + f)^n*a*d^2*e*f*g^5*n^4*x +
20*(g*x + f)^n*a*d^3*g^6*n^4*x - 60*(g*x + f)^n*c*e^4*f^4*g^2*n^2*x^2 + 420
*(g*x + f)^n*c*d*e^3*f^3*g^3*n^2*x^2 - 1107*(g*x + f)^n*c*d^2*e^2*f^2*g^4*n
^2*x^2 + 1358*(g*x + f)^n*c*d^3*e*f*g^5*n^2*x^2 + 922*(g*x + f)^n*c*d^4*g^6
*n^2*x^2 - 36*(g*x + f)^n*a*e^3*f^2*g^4*n^3*x^2 + 267*(g*x + f)^n*a*d*e^2*f
*g^5*n^3*x^2 + 411*(g*x + f)^n*a*d^2*e*g^6*n^3*x^2 + 40*(g*x + f)^n*c*e^4*f
^3*g^3*n*x^3 - 240*(g*x + f)^n*c*d*e^3*f^2*g^4*n*x^3 + 540*(g*x + f)^n*c*d^
2*e^2*f*g^5*n*x^3 + 3556*(g*x + f)^n*c*d^3*e*g^6*n*x^3 + 112*(g*x + f)^n*a*
e^3*f*g^5*n^2*x^3 + 1116*(g*x + f)^n*a*d*e^2*g^6*n^2*x^3 + 1620*(g*x + f)^n
*c*d^2*e^2*g^6*x^4 + 396*(g*x + f)^n*a*e^3*g^6*n*x^4 + 14*(g*x + f)^n*c*d^3
*e*f^3*g^3*n^3 - 36*(g*x + f)^n*c*d^4*f^2*g^4*n^3 - 3*(g*x + f)^n*a*d^2*e*f
^2*g^4*n^4 + 20*(g*x + f)^n*a*d^3*f*g^5*n^4 - 120*(g*x + f)^n*c*d*e^3*f^4*g
^2*n^2*x + 594*(g*x + f)^n*c*d^2*e^2*f^3*g^3*n^2*x - 1036*(g*x + f)^n*c*d^3
*e*f^2*g^4*n^2*x + 684*(g*x + f)^n*c*d^4*f*g^5*n^2*x + 6*(g*x + f)^n*a*e^3*
f^3*g^3*n^3*x - 90*(g*x + f)^n*a*d*e^2*f^2*g^4*n^3*x + 357*(g*x + f)^n*a*d^
2*e*f*g^5*n^3*x + 155*(g*x + f)^n*a*d^3*g^6*n^3*x - 60*(g*x + f)^n*c*e^4*f^
4*g^2*n*x^2 + 360*(g*x + f)^n*c*d*e^3*f^3*g^3*n*x^2 - 810*(g*x + f)^n*c*d^2
*e^2*f^2*g^4*n*x^2 + 840*(g*x + f)^n*c*d^3*e*f*g^5*n*x^2 + 1404*(g*x + f)^n
*c*d^4*g^6*n*x^2 - 123*(g*x + f)^n*a*e^3*f^2*g^4*n^2*x^2 + 582*(g*x + f)^n*
a*d*e^2*f*g^5*n^2*x^2 + 1383*(g*x + f)^n*a*d^2*e*g^6*n^2*x^2 + 1680*(g*x +
f)^n*c*d^3*e*g^6*x^3 + 60*(g*x + f)^n*a*e^3*f*g^5*n*x^3 + 1524*(g*x + f)^n*
a*d*e^2*g^6*n*x^3 + 180*(g*x + f)^n*a*e^3*g^6*x^4 - 54*(g*x + f)^n*c*d^2*e^
2*f^4*g^2*n^2 + 210*(g*x + f)^n*c*d^3*e*f^3*g^3*n^2 - 238*(g*x + f)^n*c*d^4
*f^2*g^4*n^2 + 6*(g*x + f)^n*a*d*e^2*f^3*g^3*n^3 - 54*(g*x + f)^n*a*d^2*e*f
^2*g^4*n^3 + 155*(g*x + f)^n*a*d^3*f*g^5*n^3 + 120*(g*x + f)^n*c*e^4*f^5*g*
n*x - 720*(g*x + f)^n*c*d*e^3*f^4*g^2*n*x + 1620*(g*x + f)^n*c*d^2*e^2*f^3*

```
g^3*n*x - 1680*(g*x + f)^n*c*d^3*e*f^2*g^4*n*x + 720*(g*x + f)^n*c*d^4*f*g^
5*n*x + 66*(g*x + f)^n*a*e^3*f^3*g^3*n^2*x - 444*(g*x + f)^n*a*d*e^2*f^2*g^
4*n^2*x + 1026*(g*x + f)^n*a*d^2*e*f*g^5*n^2*x + 580*(g*x + f)^n*a*d^3*g^6*
n^2*x + 720*(g*x + f)^n*c*d^4*g^6*x^2 - 90*(g*x + f)^n*a*e^3*f^2*g^4*n*x^2
+ 360*(g*x + f)^n*a*d*e^2*f*g^5*n*x^2 + 2106*(g*x + f)^n*a*d^2*e*g^6*n*x^2
+ 720*(g*x + f)^n*a*d*e^2*g^6*x^3 + 120*(g*x + f)^n*c*d*e^3*f^5*g*n - 594*(
g*x + f)^n*c*d^2*e^2*f^4*g^2*n + 1036*(g*x + f)^n*c*d^3*e*f^3*g^3*n - 684*(
g*x + f)^n*c*d^4*f^2*g^4*n - 6*(g*x + f)^n*a*e^3*f^4*g^2*n^2 + 90*(g*x + f)
^n*a*d*e^2*f^3*g^3*n^2 - 357*(g*x + f)^n*a*d^2*e*f^2*g^4*n^2 + 580*(g*x + f
)^n*a*d^3*f*g^5*n^2 + 180*(g*x + f)^n*a*e^3*f^3*g^3*n*x - 720*(g*x + f)^n*a
*d*e^2*f^2*g^4*n*x + 1080*(g*x + f)^n*a*d^2*e*f*g^5*n*x + 1044*(g*x + f)^n*
a*d^3*g^6*n*x + 1080*(g*x + f)^n*a*d^2*e*g^6*x^2 - 120*(g*x + f)^n*c*e^4*f^
6 + 720*(g*x + f)^n*c*d*e^3*f^5*g - 1620*(g*x + f)^n*c*d^2*e^2*f^4*g^2 + 16
80*(g*x + f)^n*c*d^3*e*f^3*g^3 - 720*(g*x + f)^n*c*d^4*f^2*g^4 - 66*(g*x +
f)^n*a*e^3*f^4*g^2*n + 444*(g*x + f)^n*a*d*e^2*f^3*g^3*n - 1026*(g*x + f)^n
*a*d^2*e*f^2*g^4*n + 1044*(g*x + f)^n*a*d^3*f*g^5*n + 720*(g*x + f)^n*a*d^3
*g^6*x - 180*(g*x + f)^n*a*e^3*f^4*g^2 + 720*(g*x + f)^n*a*d*e^2*f^3*g^3 -
1080*(g*x + f)^n*a*d^2*e*f^2*g^4 + 720*(g*x + f)^n*a*d^3*f*g^5)/(g^6*n^6 +
21*g^6*n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^
6)
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 1943, normalized size of antiderivative = 7.07

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

```
[In] int((f + g*x)^n*(d + e*x)^3*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(720*a*d^3*g^6 + 580*a*d^3*g^6*n^2 + 155*a*d^3*g^6*n^3 + 20*
a*d^3*g^6*n^4 + a*d^3*g^6*n^5 + 1044*a*d^3*g^6*n + 720*c*d^4*f*g^5*n + 120*
c*e^4*f^5*g*n + 180*a*e^3*f^3*g^3*n + 684*c*d^4*f*g^5*n^2 + 238*c*d^4*f*g^5
*n^3 + 36*c*d^4*f*g^5*n^4 + 2*c*d^4*f*g^5*n^5 + 66*a*e^3*f^3*g^3*n^2 + 6*a*
e^3*f^3*g^3*n^3 - 444*a*d*e^2*f^2*g^4*n^2 - 90*a*d*e^2*f^2*g^4*n^3 - 6*a*d*
e^2*f^2*g^4*n^4 + 1620*c*d^2*e^2*f^3*g^3*n - 120*c*d*e^3*f^4*g^2*n^2 - 1036
*c*d^3*e*f^2*g^4*n^2 - 210*c*d^3*e*f^2*g^4*n^3 - 14*c*d^3*e*f^2*g^4*n^4 + 1
080*a*d^2*e*f*g^5*n + 594*c*d^2*e^2*f^3*g^3*n^2 + 54*c*d^2*e^2*f^3*g^3*n^3
- 720*a*d*e^2*f^2*g^4*n + 1026*a*d^2*e*f*g^5*n^2 + 357*a*d^2*e*f*g^5*n^3 +
54*a*d^2*e*f*g^5*n^4 + 3*a*d^2*e*f*g^5*n^5 - 720*c*d*e^3*f^4*g^2*n - 1680*c
*d^3*e*f^2*g^4*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n
^6 + 720)) - ((f + g*x)^n*(120*c*e^4*f^6 + 180*a*e^3*f^4*g^2 + 720*c*d^4*f^
2*g^4 - 720*a*d^3*f*g^5 - 720*c*d*e^3*f^5*g - 1044*a*d^3*f*g^5*n - 720*a*d*
e^2*f^3*g^3 + 1080*a*d^2*e*f^2*g^4 - 1680*c*d^3*e*f^3*g^3 - 580*a*d^3*f*g^5
*n^2 - 155*a*d^3*f*g^5*n^3 - 20*a*d^3*f*g^5*n^4 - a*d^3*f*g^5*n^5 + 66*a*e^
```

3*f^4*g^2*n + 684*c*d^4*f^2*g^4*n + 1620*c*d^2*e^2*f^4*g^2 + 6*a*e^3*f^4*g^
2*n^2 + 238*c*d^4*f^2*g^4*n^2 + 36*c*d^4*f^2*g^4*n^3 + 2*c*d^4*f^2*g^4*n^4
- 90*a*d*e^2*f^3*g^3*n^2 + 357*a*d^2*e*f^2*g^4*n^2 - 6*a*d*e^2*f^3*g^3*n^3
+ 54*a*d^2*e*f^2*g^4*n^3 + 3*a*d^2*e*f^2*g^4*n^4 + 594*c*d^2*e^2*f^4*g^2*n
- 210*c*d^3*e*f^3*g^3*n^2 - 14*c*d^3*e*f^3*g^3*n^3 - 120*c*d*e^3*f^5*g*n +
54*c*d^2*e^2*f^4*g^2*n^2 - 444*a*d*e^2*f^3*g^3*n + 1026*a*d^2*e*f^2*g^4*n -
1036*c*d^3*e*f^3*g^3*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) + (c*e^4*x^6*(f + g*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n
^4 + n^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 72
0) + (x^2*(f + g*x)^n*(n + 1)*(720*c*d^4*g^4 + 238*c*d^4*g^4*n^2 + 36*c*d^4
*g^4*n^3 + 2*c*d^4*g^4*n^4 + 1080*a*d^2*e*g^4 + 684*c*d^4*g^4*n - 60*c*e^4*
f^4*n + 1026*a*d^2*e*g^4*n + 357*a*d^2*e*g^4*n^2 + 54*a*d^2*e*g^4*n^3 + 3*a
*d^2*e*g^4*n^4 - 90*a*e^3*f^2*g^2*n - 33*a*e^3*f^2*g^2*n^2 - 3*a*e^3*f^2*g^
2*n^3 - 810*c*d^2*e^2*f^2*g^2*n + 360*a*d*e^2*f*g^3*n + 360*c*d*e^3*f^3*g*n
+ 840*c*d^3*e*f*g^3*n - 297*c*d^2*e^2*f^2*g^2*n^2 - 27*c*d^2*e^2*f^2*g^2*n
^3 + 222*a*d*e^2*f*g^3*n^2 + 45*a*d*e^2*f*g^3*n^3 + 3*a*d*e^2*f*g^3*n^4 + 6
0*c*d*e^3*f^3*g*n^2 + 518*c*d^3*e*f*g^3*n^2 + 105*c*d^3*e*f*g^3*n^3 + 7*c*d
^3*e*f*g^3*n^4))/(g^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6
+ 720)) + (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(840*c*d^3*g^3 + 105*c*d^3*g^
3*n^2 + 7*c*d^3*g^3*n^3 + 360*a*d*e*g^3 + 518*c*d^3*g^3*n + 20*c*e^3*f^3*n
+ 45*a*d*e*g^3*n^2 + 3*a*d*e*g^3*n^3 + 30*a*e^2*f*g^2*n + 11*a*e^2*f*g^2*n^
2 + a*e^2*f*g^2*n^3 + 222*a*d*e*g^3*n - 120*c*d*e^2*f^2*g*n + 270*c*d^2*e*f
*g^2*n - 20*c*d*e^2*f^2*g*n^2 + 99*c*d^2*e*f*g^2*n^2 + 9*c*d^2*e*f*g^2*n^3)
)/(g^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (e^2
x^4(f + g*x)^n*(11*n + 6*n^2 + n^3 + 6)*(270*c*d^2*g^2 + 30*a*e*g^2 + 9*c
*d^2*g^2*n^2 + 11*a*e*g^2*n + a*e*g^2*n^2 + 99*c*d^2*g^2*n - 5*c*e^2*f^2*n
+ 5*c*d*e*f*g*n^2 + 30*c*d*e*f*g*n))/(g^2*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (c*e^3*x^5*(f + g*x)^n*(30*d*g + 5*d*g*n + e
*f*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(g*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720))

3.806 $\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 208

$$\begin{aligned}\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) \, dx &= \frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)} \\ &\quad - \frac{2(ef-dg)\left(aeg^2+c(2e^2f^2-4defg+d^2g^2)\right)(f+gx)^{2+n}}{g^5(2+n)} \\ &\quad + \frac{e\left(aeg^2+c(6e^2f^2-12defg+5d^2g^2)\right)(f+gx)^{3+n}}{g^5(3+n)} \\ &\quad - \frac{4ce^2(ef-dg)(f+gx)^{4+n}}{g^5(4+n)} + \frac{ce^3(f+gx)^{5+n}}{g^5(5+n)}\end{aligned}$$

```
[Out] (-d*g+e*f)^2*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^5/(1+n)-2*(-d*g+e*f)*
(a*e*g^2+c*(d^2*g^2-4*d*e*f*g+2*e^2*f^2))*(g*x+f)^(2+n)/g^5/(2+n)+e*(a*e*g^
2+c*(5*d^2*g^2-12*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3+n)/g^5/(3+n)-4*c*e^2*(-d*g
+e*f)*(g*x+f)^(4+n)/g^5/(4+n)+c*e^3*(g*x+f)^(5+n)/g^5/(5+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used

$= \{961\}$

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx$$
$$= -\frac{2(ef-dg)(f+gx)^{n+2} \left(aeg^2+c(d^2g^2-4defg+2e^2f^2)\right)}{g^5(n+2)} + \frac{e(f+gx)^{n+3} \left(aeg^2+c(5d^2g^2-12defg+6e^2f^2)\right)}{g^5(n+3)} + \frac{(ef-dg)^2(f+gx)^{n+1} \left(ag^2+cf(ef-2dg)\right)}{g^5(n+1)} - \frac{4ce^2(ef-dg)(f+gx)^{n+4}}{g^5(n+4)} + \frac{ce^3(f+gx)^{n+5}}{g^5(n+5)}$$

```
[In] Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ef-dg)^2 \left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^4} + \frac{2(ef-dg)\left(-aeg^2-c(2e^2f^2-4defg+d^2g^2)\right)(f+gx)^{1+n}}{g^4} + \frac{e(aeg^2+c(6e^2f^2-12defg+5d^2g^2))(f+gx)^{2+n}}{g^4} - \frac{4ce^2(ef-dg)(f+gx)^{3+n}}{g^4} + \frac{ce^3(f+gx)^{4+n}}{g^4}\right) dx$$

$$= \frac{(ef-dg)^2(ag^2+cf(ef-2dg))(f+gx)^{1+n}}{g^5(1+n)} - \frac{2(ef-dg)(aeg^2+c(2e^2f^2-4defg+d^2g^2))(f+gx)^{2+n}}{g^5(2+n)} + \frac{e(aeg^2+c(6e^2f^2-12defg+5d^2g^2))(f+gx)^{3+n}}{g^5(3+n)} - \frac{4ce^2(ef-dg)(f+gx)^{4+n}}{g^5(4+n)} + \frac{ce^3(f+gx)^{5+n}}{g^5(5+n)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$= \frac{(f+gx)^{1+n}\left(\frac{(ef-dg)^2(ag^2+cf(ef-2dg))}{1+n} - \frac{2(ef-dg)(aeg^2+c(2e^2f^2-4defg+d^2g^2))(f+gx)}{2+n} + \frac{e(aeg^2+c(6e^2f^2-12defg+5d^2g^2))(f+}{3+n}\right.}{g^5}$$

```
[In] Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n) - (
2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x))/(2
 + n) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^2)/
(3 + n) - (4*c*e^2*(e*f - d*g)*(f + g*x)^3)/(4 + n) + (c*e^3*(f + g*x)^4)/(
5 + n)))/g^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(208) = 416.

Time = 0.56 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.95

method	result	size
norman	Expression too large to display	1029
gosper	Expression too large to display	1048
risch	Expression too large to display	1438
parallelrisch	Expression too large to display	2213

```
[In] int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*e^3/(5+n)*x^5*exp(n*ln(g*x+f))+f*(a*d^2*g^4*n^4-2*c*d^3*f*g^3*n^3+14*a*d^
2*g^4*n^3-2*a*d*e*f*g^3*n^3-24*c*d^3*f*g^3*n^2+10*c*d^2*e*f^2*g^2*n^2+71*a*
d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+2*a*e^2*f^2*g^2*n^2-94*c*d^3*f*g^3*n+90*c*d^
2*e*f^2*g^2*n-24*c*d*e^2*f^3*g*n+154*a*d^2*g^4*n-94*a*d*e*f*g^3*n+18*a*e^2*
```

```
f^2*g^2*n-120*c*d^3*f*g^3+200*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e^3*f^
4+120*a*d^2*g^4-120*a*d*e*f*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*n^3+22
5*n^2+274*n+120)*exp(n*ln(g*x+f))+(2*c*d^3*g^3*n^3+5*c*d^2*e*f*g^2*n^3+2*a*
d*e*g^3*n^3+a*e^2*f*g^2*n^3+24*c*d^3*g^3*n^2+45*c*d^2*e*f*g^2*n^2-12*c*d*e^
2*f^2*g*n^2+24*a*d*e*g^3*n^2+9*a*e^2*f*g^2*n^2+94*c*d^3*g^3*n+100*c*d^2*e*f
*g^2*n-60*c*d*e^2*f^2*g*n+12*c*e^3*f^3*n+94*a*d*e*g^3*n+20*a*e^2*f*g^2*n+12
0*c*d^3*g^3+120*a*d*e*g^3)/g^3/(n^4+14*n^3+71*n^2+154*n+120)*x^2*exp(n*ln(g
*x+f))+(2*c*d^3*f*g^3*n^4+a*d^2*g^4*n^4+2*a*d*e*f*g^3*n^4+24*c*d^3*f*g^3*n^
3-10*c*d^2*e*f^2*g^2*n^3+14*a*d^2*g^4*n^3+24*a*d*e*f*g^3*n^3-2*a*e^2*f^2*g^
2*n^3+94*c*d^3*f*g^3*n^2-90*c*d^2*e*f^2*g^2*n^2+24*c*d*e^2*f^3*g*n^2+71*a*d
^2*g^4*n^2+94*a*d*e*f*g^3*n^2-18*a*e^2*f^2*g^2*n^2+120*c*d^3*f*g^3*n-200*c*
d^2*e*f^2*g^2*n+120*c*d*e^2*f^3*g*n-24*c*e^3*f^4*n+154*a*d^2*g^4*n+120*a*d*
e*f*g^3*n-40*a*e^2*f^2*g^2*n+120*a*d^2*g^4)/g^4/(n^5+15*n^4+85*n^3+225*n^2+
274*n+120)*x*exp(n*ln(g*x+f))+(5*c*d^2*g^2*n^2+4*c*d*e*f*g*n^2+a*e*g^2*n^2+
45*c*d^2*g^2*n+20*c*d*e*f*g*n-4*c*e^2*f^2*n+9*a*e*g^2*n+100*c*d^2*g^2+20*a*
e*g^2)*e/g^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(g*x+f))+(4*d*g*n+e*f*n+20*d*
g)*c/g*e^2/(n^2+9*n+20)*x^4*exp(n*ln(g*x+f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. 2(208) = 416.

Time = 0.39 (sec) , antiderivative size = 1122, normalized size of antiderivative = 5.39

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f*g^4 + 40*
(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g^3 + (c*e^3*g^5*n^4
+ 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c*e^3*g^5*n + 24*c*e^3*g^5)*x^5
+ (120*c*d*e^2*g^5 + (c*e^3*f*g^4 + 4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g^4 +
 22*c*d*e^2*g^5)*n^3 + (11*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c*e^3*
f*g^4 + 122*c*d*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*f^2*g^
3)*n^3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e + a*e^
2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^2*e + a*e^2)*g^
5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49*(5*c*d^2*e + a*e^2)*g^5)
*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f*g^4 - 39*(5*c*d^2*e + a*e^2)*g^5)*
n)*x^3 + (71*a*d^2*f*g^4 + 2*(5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c*d^3 + a*d*
e)*f^2*g^3)*n^2 + (120*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^2)*f*g^4 + 2
*(c*d^3 + a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*e + a*e^2)*f*
g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*d*e^2*f^2*g^3
+ 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^5)*n^2 + 2*(6*c*e^3*
f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e + a*e^2)*f*g^4 + 107*(c*d^3 +
a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2*f^4*g - 77*a*d^2*f*g^4 - 9*(5*c*d^2*e +
```

```
a*e^2)*f^3*g^2 + 47*(c*d^3 + a*d*e)*f^2*g^3)*n + (120*a*d^2*g^5 + (a*d^2*g^
5 + 2*(c*d^3 + a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c*d^2*e + a*e^2)*f^2
*g^3 + 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f^3*g^2 + 71*a*d^2*g^5 -
 18*(5*c*d^2*e + a*e^2)*f^2*g^3 + 94*(c*d^3 + a*d*e)*f*g^4)*n^2 - 2*(12*c*e
^3*f^4*g - 60*c*d*e^2*f^3*g^2 - 77*a*d^2*g^5 + 20*(5*c*d^2*e + a*e^2)*f^2*g
^3 - 60*(c*d^3 + a*d*e)*f*g^4)*n)*x)*(g*x + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85
*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11946 vs. 2(197) = 394.

Time = 2.21 (sec) , antiderivative size = 11946, normalized size of antiderivative = 57.43

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c*
d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*g**4/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 2*a*d*e*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**
2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48*f**3
*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2*f**2*
g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*
g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a*e**2*g**4*x**2/(12*f**4*g*
*5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
2*c*d**3*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g
**8*x**3 + 12*g**9*x**4) - 8*c*d**3*g**4*x/(12*f**4*g**5 + 48*f**3*g**6*x +
 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 5*c*d**2*e*f**2*g**2/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 20*c*d**2*e*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g*
*7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 30*c*d**2*e*g**4*x**2/(12*f**4*g
**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
 12*c*d*e**2*f**3*g/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*f**2*g**2*x/(12*f**4*g**5 + 48*f
**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 72*c*d*e*
*2*f*g**3*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g*
*8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*g**4*x**3/(12*f**4*g**5 + 48*f**3*g**
6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*f**4*l
og(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*
x**3 + 12*g**9*x**4) + 25*c*e**3*f**4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f
**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f**3*g*x*log(f/g
 + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
```

12*g**9*x**4) + 88*c*e**3*f**3*g*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 72*c*e**3*f**2*g**2*x**2*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 108*c*e**3*f**2*g**2*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*g**4*x**4*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4), Eq(n, -5)), (-a*d**2*g**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*d*e*f*g**3/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*d*e*g**4*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*e**2*f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - c*d**3*f*g**3/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*c*d**3*g**4*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 5*c*d**2*e*f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*c*d**2*e*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*c*d**2*e*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*f**3*g*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 22*c*d*e**2*f**3*g/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f**2*g**2*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 54*c*d*e**2*f**2*g**2*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*g**4*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f**4*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 22*c*e**3*f**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**3*g*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 54*c*e**3*f**3*g*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f*g**3*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 3*c*e**3*g**4*x**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3), Eq(n, -4)), (-a*d**2*g**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 2*a*d*e*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*a*d*e*g**4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 2*a*e**2*f**2*g**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 3*a*e**2*f**2*g**2/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x/(2*f**2*g**5 + 4*f*g**6*

x + 2*g**7*x**2) + 2*a*e**2*g**4*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) - 2*c*d**3*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2)
 - 4*c*d**3*g**4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*f
2*g2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 15*c*d**2*
e*f**2*g**2/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3*x
*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3
*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*g**4*x**2*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f**3*g*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 36*c*d*e**2*f**3*g/(2*f**2
*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x*log(f/g + x)/(2
*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x/(2*f**2*g*
*5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f*g**3*x**2*log(f/g + x)/(2*f*
*2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 8*c*d*e**2*g**4*x**3/(2*f**2*g**5 + 4
*f*g**6*x + 2*g**7*x**2) + 12*c*e**3*f**4*log(f/g + x)/(2*f**2*g**5 + 4*f*g
6*x + 2*g7*x**2) + 18*c*e**3*f**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 12*c*e*
*3*f**2*g**2*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4
*c*e**3*f*g**3*x**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + c*e**3*g**4*
x**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2), Eq(n, -3)), (-3*a*d**2*g**4/
(3*f*g**5 + 3*g**6*x) + 6*a*d*e*f*g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) +
 6*a*d*e*f*g**3/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*g**4*x*log(f/g + x)/(3*f*g*
*5 + 3*g**6*x) - 6*a*e**2*f**2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 6*
a*e**2*f**2*g**2/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f*g**3*x*log(f/g + x)/(3*
f*g**5 + 3*g**6*x) + 3*a*e**2*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*
g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*g**3/(3*f*g**5 + 3*g**
6*x) + 6*c*d**3*g**4*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**
2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**2*g**2/(3*f*g**5
 + 3*g**6*x) - 30*c*d**2*e*f*g**3*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 15
*c*d**2*e*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g*log(f/g + x)
/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g/(3*f*g**5 + 3*g**6*x) + 36*c*d*
e**2*f**2*g**2*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 18*c*d*e**2*f*g**3*x*
*2/(3*f*g**5 + 3*g**6*x) + 6*c*d*e**2*g**4*x**3/(3*f*g**5 + 3*g**6*x) - 12*
c*e**3*f**4*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**4/(3*f*g**5 +
 3*g**6*x) - 12*c*e**3*f**3*g*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*e*
*3*f**2*g**2*x**2/(3*f*g**5 + 3*g**6*x) - 2*c*e**3*f*g**3*x**3/(3*f*g**5 +
3*g**6*x) + c*e**3*g**4*x**4/(3*f*g**5 + 3*g**6*x), Eq(n, -2)), (a*d**2*log
(f/g + x)/g - 2*a*d*e*f*log(f/g + x)/g**2 + 2*a*d*e*x/g + a*e**2*f**2*log(f
/g + x)/g**3 - a*e**2*f*x/g**2 + a*e**2*x**2/(2*g) - 2*c*d**3*f*log(f/g + x
)/g**2 + 2*c*d**3*x/g + 5*c*d**2*e*f**2*log(f/g + x)/g**3 - 5*c*d**2*e*f*x/
g**2 + 5*c*d**2*e*x**2/(2*g) - 4*c*d*e**2*f**3*log(f/g + x)/g**4 + 4*c*d*e*
*2*f**2*x/g**3 - 2*c*d*e**2*f*x**2/g**2 + 4*c*d*e**2*x**3/(3*g) + c*e**3*f*
*4*log(f/g + x)/g**5 - c*e**3*f**3*x/g**4 + c*e**3*f**2*x**2/(2*g**3) - c*e
3*f*x3/(3*g**2) + c*e**3*x**4/(4*g), Eq(n, -1)), (a*d**2*f*g**4*n**4*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g

5*n + 120*g5) + 14*a*d**2*f*g**4*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*f
*g**4*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 154*a*d**2*f*g**4*n*(f + g*x)**n/(g**5*n**
5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
120*a*d**2*f*g**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 2
25*g**5*n**2 + 274*g**5*n + 120*g**5) + a*d**2*g**5*n**4*x*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 14*a*d**2*g**5*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*g**5*n**2*x*(f
+ g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 154*a*d**2*g**5*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d**2*g**
5*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
 274*g**5*n + 120*g**5) - 2*a*d*e*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 +
15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*a
*d*e*f**2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 94*a*d*e*f**2*g**3*n*(f + g*x)**n
/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 12
0*g**5) - 120*a*d*e*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
5*n3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*d*e*f*g**4*n**4*x*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 24*a*d*e*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g*
*5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*a*d*e*
f*g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
5*n2 + 274*g**5*n + 120*g**5) + 120*a*d*e*f*g**4*n*x*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 2*a*d*e*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*a*d*e*g**5*n**3*x**2*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
5*n + 120*g5) + 118*a*d*e*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g
5*n4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*a*d*
e*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
5*n2 + 274*g**5*n + 120*g**5) + 120*a*d*e*g**5*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*a*e**2*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 18*a*e**2*f**3*g**2*n*(f +
 g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
5*n + 120*g**5) + 40*a*e**2*f**3*g**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**
4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*a*e**2*f**2*g
3*n3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) - 18*a*e**2*f**2*g**3*n**2*x*(f + g*x)**n/(g
5*n5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
5) - 40*a*e2*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + a*e**2*f*g**4*n**4*x**

2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 10*a*e**2*f*g**4*n**3*x**2*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 29
*a*e**2*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
3 + 225*g5*n**2 + 274*g**5*n + 120*g**5) + 20*a*e**2*f*g**4*n*x**2*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
5*n + 120*g**5) + a*e**2*g**5*n**4*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n
4 + 85*g5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 12*a*e**2*g**
5*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 49*a*e**2*g**5*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 78*a*e**2*g**5*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 40*a*e**2*g**5*x**3*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) - 2*c*d**3*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d**3*f*
*2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) - 94*c*d**3*f**2*g**3*n*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) - 120*c*d**3*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
3 + 225*g5*n**2 + 274*g**5*n + 120*g**5) + 2*c*d**3*f*g**4*n**4*x*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + 24*c*d**3*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*c*d**3*f*
g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*f*g**4*n*x*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*c*d**3*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*c*d**3*g**5*n**3*x**2*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 118*c*d**3*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15
*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*c*
d**3*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 22
5*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*g**5*x**2*(f + g*x)**n/(g
5*n5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
5) + 10*c*d2*e*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 90*c*d**2*e*f**3*g*
*2*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 200*c*d**2*e*f**3*g**2*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 10*
c*d**2*e*f**2*g**3*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 90*c*d**2*e*f**2*g**3*n**2*
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 200*c*d**2*e*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*

c*d**2*e*f*g**4*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 50*c*d**2*e*f*g**4*n**3*x**
2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 145*c*d**2*e*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n*
*5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
 100*c*d**2*e*f*g**4*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*c*d**2*e*g**5*n**4*x**3
*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 27
4*g**5*n + 120*g**5) + 60*c*d**2*e*g**5*n**3*x**3*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 245
*c*d**2*e*g**5*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
3 + 225*g5*n**2 + 274*g**5*n + 120*g**5) + 390*c*d**2*e*g**5*n*x**3*(f
+ g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 200*c*d**2*e*g**5*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d*e**2
*f**4*g*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*
n**2 + 274*g**5*n + 120*g**5) - 120*c*d*e**2*f**4*g*(f + g*x)**n/(g**5*n**5
 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2
4*c*d*e**2*f**3*g**2*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*f**3*g**2*n*
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 12*c*d*e**2*f**2*g**3*n**3*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 - 72*c*d*e**2*f**2*g**3*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 60*c*d*e**2*f**2*g
3*n*x2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 4*c*d*e**2*f*g**4*n**4*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 32*c*d*e**2*f*g**4*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 68*c*d*e**2*f*g*
*4*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) + 40*c*d*e**2*f*g**4*n*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 4*c*d*e**2*g**5*n**4*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 44*c*d*e**2*g**5*n*
*3*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 164*c*d*e**2*g**5*n**2*x**4*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 244*c*d*e**2*g**5*n*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
5*n3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*g**5*x**4*
(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274
*g**5*n + 120*g**5) + 24*c*e**3*f**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*e**3*f**4*g
*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 12*c*e**3*f**3*g**2*n**2*x**2*(f + g*x)**n/(g**5

```
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 12*c*e**3*f**3*g**2*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 4*c*e**3*f**2*g**3*n**
3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**
2 + 274*g**5*n + 120*g**5) - 12*c*e**3*f**2*g**3*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) - 8*c*e**3*f**2*g**3*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + c*e**3*f*g**4*n**4*x*
*4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n**3*x**4*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 11
*c*e**3*f*g**4*n**2*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n*x**4*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + c*e**3*g**5*n**4*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 10*c*e**3*g**5
*n**3*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 35*c*e**3*g**5*n**2*x**5*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 50*c*e**3*g**5*n*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*c*e**3*g**5*x**5*(f + g
*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*
n + 120*g**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(208) = 416$.

Time = 0.23 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.46

$$
\begin{aligned}
\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx &= \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^3}{(n^2+3n+2)g^2} \\
&+ \frac{5\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cd^2e}{(n^3+6n^2+11n+6)g^3} \\
&+ \frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ade}{(n^2+3n+2)g^2} + \frac{(gx+f)^{n+1}ad^2}{g(n+1)} \\
&+ \frac{4\left((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3(n^2+n)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n c}{(n^4+10n^3+35n^2+50n+24)g^4} \\
&+ \frac{\left((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n ae^2}{(n^3+6n^2+11n+6)g^3} \\
&+ \frac{\left((n^4+10n^3+35n^2+50n+24)g^5x^5+(n^4+6n^3+11n^2+6n)fg^4x^4-4(n^3+3n^2+2n)f^2g^3x^3+1}{(n^5+15n^4+85n^3+225n^2+274n+120)g^5}
\end{aligned}
$$

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^3/((n^2 + 3*n + 2)*g^2)
 + 5*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^2*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 2*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a
*d^2/(g*(n + 1)) + 4*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n
)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*
d*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + ((n^2 + 3*n + 2)*g^3*x^3
+ (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*e^2/((n^3 + 6*n^
2 + 11*n + 6)*g^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x^3 + 12*(n
^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*e^3/((n^5 + 15*n
^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(208) = 416$.

Time = 0.31 (sec) , antiderivative size = 2133, normalized size of antiderivative = 10.25

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*e^3*g^5*n^4*x^5 + (g*x + f)^n*c*e^3*f*g^4*n^4*x^4 + 4*(g*x +
 f)^n*c*d*e^2*g^5*n^4*x^4 + 10*(g*x + f)^n*c*e^3*g^5*n^3*x^5 + 4*(g*x + f)^
n*c*d*e^2*f*g^4*n^4*x^3 + 5*(g*x + f)^n*c*d^2*e*g^5*n^4*x^3 + 6*(g*x + f)^n
*c*e^3*f*g^4*n^3*x^4 + 44*(g*x + f)^n*c*d*e^2*g^5*n^3*x^4 + 35*(g*x + f)^n*
c*e^3*g^5*n^2*x^5 + 5*(g*x + f)^n*c*d^2*e*f*g^4*n^4*x^2 + 2*(g*x + f)^n*c*d
^3*g^5*n^4*x^2 - 4*(g*x + f)^n*c*e^3*f^2*g^3*n^3*x^3 + 32*(g*x + f)^n*c*d*e
^2*f*g^4*n^3*x^3 + 60*(g*x + f)^n*c*d^2*e*g^5*n^3*x^3 + (g*x + f)^n*a*e^2*g
^5*n^4*x^3 + 11*(g*x + f)^n*c*e^3*f*g^4*n^2*x^4 + 164*(g*x + f)^n*c*d*e^2*g
^5*n^2*x^4 + 50*(g*x + f)^n*c*e^3*g^5*n*x^5 + 2*(g*x + f)^n*c*d^3*f*g^4*n^4
*x - 12*(g*x + f)^n*c*d*e^2*f^2*g^3*n^3*x^2 + 50*(g*x + f)^n*c*d^2*e*f*g^4*
n^3*x^2 + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + (g*x + f)^n*a*e^2*f*g^4*n^4*x^
2 + 2*(g*x + f)^n*a*d*e*g^5*n^4*x^2 - 12*(g*x + f)^n*c*e^3*f^2*g^3*n^2*x^3
+ 68*(g*x + f)^n*c*d*e^2*f*g^4*n^2*x^3 + 245*(g*x + f)^n*c*d^2*e*g^5*n^2*x^
3 + 12*(g*x + f)^n*a*e^2*g^5*n^3*x^3 + 6*(g*x + f)^n*c*e^3*f*g^4*n*x^4 + 24
4*(g*x + f)^n*c*d*e^2*g^5*n*x^4 + 24*(g*x + f)^n*c*e^3*g^5*x^5 - 10*(g*x +
f)^n*c*d^2*e*f^2*g^3*n^3*x + 24*(g*x + f)^n*c*d^3*f*g^4*n^3*x + 2*(g*x + f)
^n*a*d*e*f*g^4*n^4*x + (g*x + f)^n*a*d^2*g^5*n^4*x + 12*(g*x + f)^n*c*e^3*f
^3*g^2*n^2*x^2 - 72*(g*x + f)^n*c*d*e^2*f^2*g^3*n^2*x^2 + 145*(g*x + f)^n*c
*d^2*e*f*g^4*n^2*x^2 + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2 + 10*(g*x + f)^n*a
*e^2*f*g^4*n^3*x^2 + 26*(g*x + f)^n*a*d*e*g^5*n^3*x^2 - 8*(g*x + f)^n*c*e^3
*f^2*g^3*n*x^3 + 40*(g*x + f)^n*c*d*e^2*f*g^4*n*x^3 + 390*(g*x + f)^n*c*d^2
*e*g^5*n*x^3 + 49*(g*x + f)^n*a*e^2*g^5*n^2*x^3 + 120*(g*x + f)^n*c*d*e^2*g
```

```
^5*x^4 - 2*(g*x + f)^n*c*d^3*f^2*g^3*n^3 + (g*x + f)^n*a*d^2*f*g^4*n^4 + 24
*(g*x + f)^n*c*d*e^2*f^3*g^2*n^2*x - 90*(g*x + f)^n*c*d^2*e*f^2*g^3*n^2*x +
 94*(g*x + f)^n*c*d^3*f*g^4*n^2*x - 2*(g*x + f)^n*a*e^2*f^2*g^3*n^3*x + 24*
(g*x + f)^n*a*d*e*f*g^4*n^3*x + 14*(g*x + f)^n*a*d^2*g^5*n^3*x + 12*(g*x +
f)^n*c*e^3*f^3*g^2*n*x^2 - 60*(g*x + f)^n*c*d*e^2*f^2*g^3*n*x^2 + 100*(g*x
+ f)^n*c*d^2*e*f*g^4*n*x^2 + 214*(g*x + f)^n*c*d^3*g^5*n*x^2 + 29*(g*x + f)
^n*a*e^2*f*g^4*n^2*x^2 + 118*(g*x + f)^n*a*d*e*g^5*n^2*x^2 + 200*(g*x + f)^
n*c*d^2*e*g^5*x^3 + 78*(g*x + f)^n*a*e^2*g^5*n*x^3 + 10*(g*x + f)^n*c*d^2*e
*f^3*g^2*n^2 - 24*(g*x + f)^n*c*d^3*f^2*g^3*n^2 - 2*(g*x + f)^n*a*d*e*f^2*g
^3*n^3 + 14*(g*x + f)^n*a*d^2*f*g^4*n^3 - 24*(g*x + f)^n*c*e^3*f^4*g*n*x +
120*(g*x + f)^n*c*d*e^2*f^3*g^2*n*x - 200*(g*x + f)^n*c*d^2*e*f^2*g^3*n*x +
 120*(g*x + f)^n*c*d^3*f*g^4*n*x - 18*(g*x + f)^n*a*e^2*f^2*g^3*n^2*x + 94*
(g*x + f)^n*a*d*e*f*g^4*n^2*x + 71*(g*x + f)^n*a*d^2*g^5*n^2*x + 120*(g*x +
 f)^n*c*d^3*g^5*x^2 + 20*(g*x + f)^n*a*e^2*f*g^4*n*x^2 + 214*(g*x + f)^n*a*
d*e*g^5*n*x^2 + 40*(g*x + f)^n*a*e^2*g^5*x^3 - 24*(g*x + f)^n*c*d*e^2*f^4*g
*n + 90*(g*x + f)^n*c*d^2*e*f^3*g^2*n - 94*(g*x + f)^n*c*d^3*f^2*g^3*n + 2*
(g*x + f)^n*a*e^2*f^3*g^2*n^2 - 24*(g*x + f)^n*a*d*e*f^2*g^3*n^2 + 71*(g*x
+ f)^n*a*d^2*f*g^4*n^2 - 40*(g*x + f)^n*a*e^2*f^2*g^3*n*x + 120*(g*x + f)^n
*a*d*e*f*g^4*n*x + 154*(g*x + f)^n*a*d^2*g^5*n*x + 120*(g*x + f)^n*a*d*e*g^
5*x^2 + 24*(g*x + f)^n*c*e^3*f^5 - 120*(g*x + f)^n*c*d*e^2*f^4*g + 200*(g*x
 + f)^n*c*d^2*e*f^3*g^2 - 120*(g*x + f)^n*c*d^3*f^2*g^3 + 18*(g*x + f)^n*a*
e^2*f^3*g^2*n - 94*(g*x + f)^n*a*d*e*f^2*g^3*n + 154*(g*x + f)^n*a*d^2*f*g^
4*n + 120*(g*x + f)^n*a*d^2*g^5*x + 40*(g*x + f)^n*a*e^2*f^3*g^2 - 120*(g*x
 + f)^n*a*d*e*f^2*g^3 + 120*(g*x + f)^n*a*d^2*f*g^4)/(g^5*n^5 + 15*g^5*n^4
+ 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 1133, normalized size of antiderivative = 5.45

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$

$$= \frac{(f+g\,x)^n\,(-2\,c\,d^3\,f^2\,g^3\,n^3-24\,c\,d^3\,f^2\,g^3\,n^2-94\,c\,d^3\,f^2\,g^3\,n-120\,c\,d^3\,f^2\,g^3+10\,c\,d^2\,e\,f^3\,g^2\,n^2+90\,c\,d^2\,e}{}$$

$$+\frac{x\,(f+g\,x)^n\,(2\,c\,d^3\,f\,g^4\,n^4+24\,c\,d^3\,f\,g^4\,n^3+94\,c\,d^3\,f\,g^4\,n^2+120\,c\,d^3\,f\,g^4\,n-10\,c\,d^2\,e\,f^2\,g^3\,n^3-90\,c\,d^2}{}$$

$$+\frac{c\,e^3\,x^5\,(f+g\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}$$

$$+\frac{x^2\,(f+g\,x)^n\,(n+1)\,\left(2\,c\,d^3\,g^3\,n^3+24\,c\,d^3\,g^3\,n^2+94\,c\,d^3\,g^3\,n+120\,c\,d^3\,g^3+5\,c\,d^2\,e\,f\,g^2\,n^3+45\,c\,d^2\,e\,f\right.}{}$$

$$+\frac{e\,x^3\,(f+g\,x)^n\,(n^2+3\,n+2)\,\left(5\,c\,d^2\,g^2\,n^2+45\,c\,d^2\,g^2\,n+100\,c\,d^2\,g^2+4\,c\,d\,e\,f\,g\,n^2+20\,c\,d\,e\,f\,g\,n-4\,c\right.}{g^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}$$

$$+\frac{c\,e^2\,x^4\,(f+g\,x)^n\,(20\,d\,g+4\,d\,g\,n+e\,f\,n)\,(n^3+6\,n^2+11\,n+6)}{g\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}$$

```
[In] int((f + g*x)^n*(d + e*x)^2*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] ((f + g*x)^n*(24*c*e^3*f^5 + 40*a*e^2*f^3*g^2 - 120*c*d^3*f^2*g^3 + 120*a*d
^2*f*g^4 - 120*a*d*e*f^2*g^3 - 120*c*d*e^2*f^4*g + 154*a*d^2*f*g^4*n + 200*
c*d^2*e*f^3*g^2 + 71*a*d^2*f*g^4*n^2 + 14*a*d^2*f*g^4*n^3 + a*d^2*f*g^4*n^4
 + 18*a*e^2*f^3*g^2*n - 94*c*d^3*f^2*g^3*n + 2*a*e^2*f^3*g^2*n^2 - 24*c*d^3
*f^2*g^3*n^2 - 2*c*d^3*f^2*g^3*n^3 + 10*c*d^2*e*f^3*g^2*n^2 - 94*a*d*e*f^2*
g^3*n - 24*c*d*e^2*f^4*g*n - 24*a*d*e*f^2*g^3*n^2 - 2*a*d*e*f^2*g^3*n^3 + 9
0*c*d^2*e*f^3*g^2*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (x*(f + g*x)^n*(120*a*d^2*g^5 + 71*a*d^2*g^5*n^2 + 14*a*d^2*g^5*n^3 + a*
d^2*g^5*n^4 + 154*a*d^2*g^5*n + 120*c*d^3*f*g^4*n - 24*c*e^3*f^4*g*n - 40*a
*e^2*f^2*g^3*n + 94*c*d^3*f*g^4*n^2 + 24*c*d^3*f*g^4*n^3 + 2*c*d^3*f*g^4*n^
4 - 18*a*e^2*f^2*g^3*n^2 - 2*a*e^2*f^2*g^3*n^3 + 120*a*d*e*f*g^4*n + 24*c*d
*e^2*f^3*g^2*n^2 - 90*c*d^2*e*f^2*g^3*n^2 - 10*c*d^2*e*f^2*g^3*n^3 + 94*a*d
*e*f*g^4*n^2 + 24*a*d*e*f*g^4*n^3 + 2*a*d*e*f*g^4*n^4 + 120*c*d*e^2*f^3*g^2
*n - 200*c*d^2*e*f^2*g^3*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5
+ 120)) + (c*e^3*x^5*(f + g*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*
n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^2*(f + g*x)^n*(n + 1)*(120*
c*d^3*g^3 + 24*c*d^3*g^3*n^2 + 2*c*d^3*g^3*n^3 + 120*a*d*e*g^3 + 94*c*d^3*g
^3*n + 12*c*e^3*f^3*n + 24*a*d*e*g^3*n^2 + 2*a*d*e*g^3*n^3 + 20*a*e^2*f*g^2
*n + 9*a*e^2*f*g^2*n^2 + a*e^2*f*g^2*n^3 + 94*a*d*e*g^3*n - 60*c*d*e^2*f^2*
g*n + 100*c*d^2*e*f*g^2*n - 12*c*d*e^2*f^2*g*n^2 + 45*c*d^2*e*f*g^2*n^2 + 5
*c*d^2*e*f*g^2*n^3))/(g^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(100*c*d^2*g^2 + 20*a*e*g^2 + 5*c*d^2*
g^2*n^2 + 9*a*e*g^2*n + a*e*g^2*n^2 + 45*c*d^2*g^2*n - 4*c*e^2*f^2*n + 4*c*
d*e*f*g*n^2 + 20*c*d*e*f*g*n))/(g^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^
5 + 120)) + (c*e^2*x^4*(f + g*x)^n*(20*d*g + 4*d*g*n + e*f*n)*(11*n + 6*n^2
 + n^3 + 6))/(g*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

3.807 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\begin{aligned}\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx &= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)} \\ &\quad +\frac{\left(aeg^2+c(3e^2f^2-6defg+2d^2g^2)\right)(f+gx)^{2+n}}{g^4(2+n)} \\ &\quad -\frac{3ce(ef-dg)(f+gx)^{3+n}}{g^4(3+n)}+\frac{ce^2(f+gx)^{4+n}}{g^4(4+n)}\end{aligned}$$

```
[Out] -(-d*g+e*f)*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^4/(1+n)+(a*e*g^2+c*(2*
d^2*g^2-6*d*e*f*g+3*e^2*f^2))*(g*x+f)^(2+n)/g^4/(2+n)-3*c*e*(-d*g+e*f)*(g*x
+f)^(3+n)/g^4/(3+n)+c*e^2*(g*x+f)^(4+n)/g^4/(4+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {785}

$$\begin{aligned}\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx &= \frac{(f+gx)^{n+2}\left(aeg^2+c(2d^2g^2-6defg+3e^2f^2)\right)}{g^4(n+2)} \\ &\quad -\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)} \\ &\quad -\frac{3ce(ef-dg)(f+gx)^{n+3}}{g^4(n+3)}+\frac{ce^2(f+gx)^{n+4}}{g^4(n+4)}\end{aligned}$$

```
[In] Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(ef-dg)\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^3} + \frac{\left(aeg^2+c(3e^2f^2-6defg+2d^2g^2)\right)(f+gx)^{1+n}}{g^3} - \frac{3ce(ef-dg)(f+gx)^{2+n}}{g^3} + \frac{ce^2(f+gx)^{3+n}}{g^3}\right) dx \\ &= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)} + \frac{\left(aeg^2+c(3e^2f^2-6defg+2d^2g^2)\right)(f+gx)^{2+n}}{g^4(2+n)} - \frac{3ce(ef-dg)(f+gx)^{3+n}}{g^4(3+n)} + \frac{ce^2(f+gx)^{4+n}}{g^4(4+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \frac{(f+gx)^{1+n}\left(-\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)}{1+n}+\frac{\left(aeg^2+c(3e^2f^2-6defg+2d^2g^2)\right)(f+gx)}{2+n}-\frac{3ce(ef-dg)(f+gx)^2}{3+n}+\frac{ce^2(f+gx)^3}{4+n}\right)}{g^4}$$

```
[In] Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(-(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n)) +
((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x))/(2 + n) - (3*
c*e*(e*f - d*g)*(f + g*x)^2)/(3 + n) + (c*e^2*(f + g*x)^3)/(4 + n)))/g^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(146) = 292.

Time = 0.50 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.08

method	result
gosper	$\frac{(gx+f)^{1+n}(c\,e^2g^3n^3x^3+3cde\,g^3n^3x^2+6c\,e^2g^3n^2x^3+2c\,d^2g^3n^3x+21cde\,g^3n^2x^2-3c\,e^2f\,g^2n^2x^2+11c\,e^2g^3n\,x^3+ae\,g^3n^3x+16c}{}$
norman	$\frac{c\,e^2x^4\mathrm{e}^{n\ln(gx+f)}}{4+n}+\frac{f\left(ad\,g^3n^3-2c\,d^2f\,g^2n^2+9ad\,g^3n^2-aef\,g^2n^2-14c\,d^2f\,g^2n+6cde\,f^2gn+26ad\,g^3n-7aef\,g^2n-24c\,d^2f\,g^2+\right.}{g^4(n^4+10n^3+35n^2+50n+24)}$
risch	$\frac{(c\,e^2g^4n^3x^4+3cde\,g^4n^3x^3+c\,e^2f\,g^3n^3x^3+6c\,e^2g^4n^2x^4+2c\,d^2g^4n^3x^2+3cdef\,g^3n^3x^2+21cde\,g^4n^2x^3+3c\,e^2f\,g^3n^2x^3+11c\,e^2g^4n}{}$
parallelrisch	Expression too large to display

```
[In] int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^4*(g*x+f)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(c*e^2*g^3*n^3*x^3+3*c*d*e*
g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*c*
e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*c*d
*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a*d*g
^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g^2*n*
x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-a*e*f*
g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g*n-24*c
*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*x-24*c*d
^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^3+24*a*d*g^3-12*a*e*f*g^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(146) = 292.

Time = 0.33 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.76

$$\int(d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$
$$=\frac{(adfg^3n^3-6\,ce^2f^4+24\,cdef^3g+24\,adfg^3-12\,(2\,cd^2+ae)f^2g^2+(ce^2g^4n^3+6\,ce^2g^4n^2+11\,ce^2g^4n+6\,c}{}$$

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*f*g^3*n^3 - 6*c*e^2*f^4 + 24*c*d*e*f^3*g + 24*a*d*f*g^3 - 12*(2*c*d^2
+ a*e)*f^2*g^2 + (c*e^2*g^4*n^3 + 6*c*e^2*g^4*n^2 + 11*c*e^2*g^4*n + 6*c*e^
2*g^4)*x^4 + (24*c*d*e*g^4 + (c*e^2*f*g^3 + 3*c*d*e*g^4)*n^3 + 3*(c*e^2*f*g
^3 + 7*c*d*e*g^4)*n^2 + 2*(c*e^2*f*g^3 + 21*c*d*e*g^4)*n)*x^3 + (9*a*d*f*g^
3 - (2*c*d^2 + a*e)*f^2*g^2)*n^2 + (12*(2*c*d^2 + a*e)*g^4 + (3*c*d*e*f*g^3
 + (2*c*d^2 + a*e)*g^4)*n^3 - (3*c*e^2*f^2*g^2 - 15*c*d*e*f*g^3 - 8*(2*c*d^
2 + a*e)*g^4)*n^2 - (3*c*e^2*f^2*g^2 - 12*c*d*e*f*g^3 - 19*(2*c*d^2 + a*e)*
```

```
g^4)*n)*x^2 + (6*c*d*e*f^3*g + 26*a*d*f*g^3 - 7*(2*c*d^2 + a*e)*f^2*g^2)*n
+ (24*a*d*g^4 + (a*d*g^4 + (2*c*d^2 + a*e)*f*g^3)*n^3 - (6*c*d*e*f^2*g^2 -
9*a*d*g^4 - 7*(2*c*d^2 + a*e)*f*g^3)*n^2 + 2*(3*c*e^2*f^3*g - 12*c*d*e*f^2*
g^2 + 13*a*d*g^4 + 6*(2*c*d^2 + a*e)*f*g^3)*n)*x)*(g*x + f)^n/(g^4*n^4 + 10
*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4952 vs. $2(134) = 268$.

Time = 1.16 (sec) , antiderivative size = 4952, normalized size of antiderivative = 33.92

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x**
4/4), Eq(g, 0)), (-2*a*d*g**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - a*e*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - 3*a*e*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x
**2 + 6*g**7*x**3) - 2*c*d**2*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g
**6*x**2 + 6*g**7*x**3) - 6*c*d**2*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 1
8*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d*e*f**2*g/(6*f**3*g**4 + 18*f**2*g**5*x
 + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*f*g**2*x/(6*f**3*g**4 + 18*f**2
*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*g**3*x**2/(6*f**3*g**4 +
 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*f**3*log(f/g + x
)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 11*c*e**2
*f**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*
e**2*f**2*g*x*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 +
 6*g**7*x**3) + 27*c*e**2*f**2*g*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**
6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2*log(f/g + x)/(6*f**3*g**4 + 1
8*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2/(6*f*
*3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*g**3*x*
*3*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**
3), Eq(n, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - a*e*f
*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a*e*g**3*x/(2*f**2*g**4
+ 4*f*g**5*x + 2*g**6*x**2) - 2*c*d**2*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2
*g**6*x**2) - 4*c*d**2*g**3*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*
c*d*e*f**2*g*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 9*c*d*
e*f**2*g/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x*log(f
/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x/(2*f**
2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*g**3*x**2*log(f/g + x)/(2*f**2
*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2*f**3*log(f/g + x)/(2*f**2*g**4
 + 4*f*g**5*x + 2*g**6*x**2) - 9*c*e**2*f**3/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*
```

g**6*x**2) - 12*c*e**2*f**2*g*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) -
6*c*e**2*f*g**2*x**2*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2)
+ 2*c*e**2*g**3*x**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2), Eq(n, -3)),
(-2*a*d*g**3/(2*f*g**4 + 2*g**5*x) + 2*a*e*f*g**2*log(f/g + x)/(2*f*g**4 +
2*g**5*x) + 2*a*e*f*g**2/(2*f*g**4 + 2*g**5*x) + 2*a*e*g**3*x*log(f/g + x)/
(2*f*g**4 + 2*g**5*x) + 4*c*d**2*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x)
+ 4*c*d**2*f*g**2/(2*f*g**4 + 2*g**5*x) + 4*c*d**2*g**3*x*log(f/g + x)/(2*f
*g**4 + 2*g**5*x) - 12*c*d*e*f**2*g*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 12
*c*d*e*f**2*g/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f*g**2*x*log(f/g + x)/(2*f*g
4 + 2*g5*x) + 6*c*d*e*g**3*x**2/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3*l
og(f/g + x)/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3/(2*f*g**4 + 2*g**5*x) + 6
*c*e**2*f**2*g*x*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 3*c*e**2*f*g**2*x**2/
(2*f*g**4 + 2*g**5*x) + c*e**2*g**3*x**3/(2*f*g**4 + 2*g**5*x), Eq(n, -2)),
(a*d*log(f/g + x)/g - a*e*f*log(f/g + x)/g**2 + a*e*x/g - 2*c*d**2*f*log(f
/g + x)/g**2 + 2*c*d**2*x/g + 3*c*d*e*f**2*log(f/g + x)/g**3 - 3*c*d*e*f*x/
g**2 + 3*c*d*e*x**2/(2*g) - c*e**2*f**3*log(f/g + x)/g**4 + c*e**2*f**2*x/g
3 - c*e2*f*x**2/(2*g**2) + c*e**2*x**3/(3*g), Eq(n, -1)), (a*d*f*g**3*n
3*(f + g*x)n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 9*a*d*f*g**3*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 26*a*d*f*g**3*n*(f + g*x)**n/(g**4*n**4 + 10*
g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*f*g**3*(f + g*x)**
n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*d*g**
4*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n
+ 24*g**4) + 9*a*d*g**4*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*g**4*n*x*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*g**4*x*(f + g
*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - a*
e*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50
*g**4*n + 24*g**4) - 7*a*e*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 12*a*e*f**2*g**2*(f + g*x)**n/(g
4*n4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*f*g**3*
n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n +
24*g**4) + 7*a*e*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*f*g**3*n*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*g**4*n**3*x**2
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) + 8*a*e*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 19*a*e*g**4*n*x**2*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*g**4*x**2*(f +
g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 2
*c*d**2*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**
2 + 50*g**4*n + 24*g**4) - 14*c*d**2*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d**2*f**2*g**2*(f
+ g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 2*c*d**2*f*g**3*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n

```
**2 + 50*g**4*n + 24*g**4) + 14*c*d**2*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d**2*f*g**3*n
*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g
**4) + 2*c*d**2*g**4*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 16*c*d**2*g**4*n**2*x**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 38*c*d**2*g
**4*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 24*c*d**2*g**4*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*d*e*f**3*g*n*(f + g*x)**n/(g**4*
n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d*e*f**3*g
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) - 6*c*d*e*f**2*g**2*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g
**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d*e*f**2*g**2*n*x*(f + g*x)**n/(g**4
*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c*d*e*f*g**3
*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 15*c*d*e*f*g**3*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*
n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*c*d*e*f*g**3*n*x**2*(f + g*
x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c
*d*e*g**4*n**3*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 +
 50*g**4*n + 24*g**4) + 21*c*d*e*g**4*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 1
0*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 42*c*d*e*g**4*n*x**3*(f
 + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 24*c*d*e*g**4*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 6*c*e**2*f**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e**2*f**3*g*n*x*(f + g*x)**n
/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 3*c*e**2
*f**2*g**2*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 3*c*e**2*f**2*g**2*n*x**2*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*f*g**3*n**3*x
**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 3*c*e**2*f*g**3*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
35*g**4*n**2 + 50*g**4*n + 24*g**4) + 2*c*e**2*f*g**3*n*x**3*(f + g*x)**n/(
g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*g**
4*n**3*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4
*n + 24*g**4) + 6*c*e**2*g**4*n**2*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n
**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 11*c*e**2*g**4*n*x**4*(f + g*x)
**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e
**2*g**4*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g*
*4*n + 24*g**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.98

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd^2}{(n^2+3\,n+2)g^2} + \frac{3\,((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n cde}{(n^3+6\,n^2+11\,n+6)g^3} + \frac{(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n ae}{(n^2+3\,n+2)g^2} + \frac{(gx+f)^{n+1}ad}{g(n+1)} + \frac{((n^3+6\,n^2+11\,n+6)g^4x^4+(n^3+3\,n^2+2\,n)fg^3x^3-3\,(n^2+n)f^2g^2x^2+6\,f^3gnx-6\,f^4)(gx+f)^n ce^2}{(n^4+10\,n^3+35\,n^2+50\,n+24)g^4}$$

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^2/((n^2 + 3*n + 2)*g^2)
 + 3*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g^2*(n + 1)*x^2 + f*g*n
*x - f^2)*(g*x + f)^n*a*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a*d/(g*
(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^
3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*e^2/((n^4
+ 10*n^3 + 35*n^2 + 50*n + 24)*g^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(146) = 292.

Time = 0.28 (sec) , antiderivative size = 1008, normalized size of antiderivative = 6.90

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \frac{(gx+f)^n ce^2g^4n^3x^4+(gx+f)^n ce^2fg^3n^3x^3+3\,(gx+f)^n cdeg^4n^3x^3+6\,(gx+f)^n ce^2g^4n^2x^4+3\,(gx+f)^n \cdots}{\cdots}$$

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*e^2*g^4*n^3*x^4 + (g*x + f)^n*c*e^2*f*g^3*n^3*x^3 + 3*(g*x +
 f)^n*c*d*e*g^4*n^3*x^3 + 6*(g*x + f)^n*c*e^2*g^4*n^2*x^4 + 3*(g*x + f)^n*c
*d*e*f*g^3*n^3*x^2 + 2*(g*x + f)^n*c*d^2*g^4*n^3*x^2 + 3*(g*x + f)^n*c*e^2*
f*g^3*n^2*x^3 + 21*(g*x + f)^n*c*d*e*g^4*n^2*x^3 + 11*(g*x + f)^n*c*e^2*g^4
*n*x^4 + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x - 3*(g*x + f)^n*c*e^2*f^2*g^2*n^2*
x^2 + 15*(g*x + f)^n*c*d*e*f*g^3*n^2*x^2 + 16*(g*x + f)^n*c*d^2*g^4*n^2*x^2
 + (g*x + f)^n*a*e*g^4*n^3*x^2 + 2*(g*x + f)^n*c*e^2*f*g^3*n*x^3 + 42*(g*x
+ f)^n*c*d*e*g^4*n*x^3 + 6*(g*x + f)^n*c*e^2*g^4*x^4 - 6*(g*x + f)^n*c*d*e*
```

```
f^2*g^2*n^2*x + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*a*e*f*g^3*n^
3*x + (g*x + f)^n*a*d*g^4*n^3*x - 3*(g*x + f)^n*c*e^2*f^2*g^2*n*x^2 + 12*(g
*x + f)^n*c*d*e*f*g^3*n*x^2 + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 + 8*(g*x + f)^
n*a*e*g^4*n^2*x^2 + 24*(g*x + f)^n*c*d*e*g^4*x^3 - 2*(g*x + f)^n*c*d^2*f^2*
g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3 + 6*(g*x + f)^n*c*e^2*f^3*g*n*x - 24*(g
*x + f)^n*c*d*e*f^2*g^2*n*x + 24*(g*x + f)^n*c*d^2*f*g^3*n*x + 7*(g*x + f)^
n*a*e*f*g^3*n^2*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x + f)^n*c*d^2*g^4*
x^2 + 19*(g*x + f)^n*a*e*g^4*n*x^2 + 6*(g*x + f)^n*c*d*e*f^3*g*n - 14*(g*x
+ f)^n*c*d^2*f^2*g^2*n - (g*x + f)^n*a*e*f^2*g^2*n^2 + 9*(g*x + f)^n*a*d*f*
g^3*n^2 + 12*(g*x + f)^n*a*e*f*g^3*n*x + 26*(g*x + f)^n*a*d*g^4*n*x + 12*(g
*x + f)^n*a*e*g^4*x^2 - 6*(g*x + f)^n*c*e^2*f^4 + 24*(g*x + f)^n*c*d*e*f^3*
g - 24*(g*x + f)^n*c*d^2*f^2*g^2 - 7*(g*x + f)^n*a*e*f^2*g^2*n + 26*(g*x +
f)^n*a*d*f*g^3*n + 24*(g*x + f)^n*a*d*g^4*x - 12*(g*x + f)^n*a*e*f^2*g^2 +
24*(g*x + f)^n*a*d*f*g^3)/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 2
4*g^4)
```

Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.92

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right)\,dx$$
$$= \frac{x\,(f+g\,x)^n\,(2\,c\,d^2\,f\,g^3\,n^3+14\,c\,d^2\,f\,g^3\,n^2+24\,c\,d^2\,f\,g^3\,n-6\,c\,d\,e\,f^2\,g^2\,n^2-24\,c\,d\,e\,f^2\,g^2\,n+a\,d\,g^4\,n^3+}{g^4\,(n^4+10\,n^3+35\,n^2+}$$
$$-\frac{(f+g\,x)^n\,(2\,c\,d^2\,f^2\,g^2\,n^2+14\,c\,d^2\,f^2\,g^2\,n+24\,c\,d^2\,f^2\,g^2-6\,c\,d\,e\,f^3\,g\,n-24\,c\,d\,e\,f^3\,g-a\,d\,f\,g^3\,n^3-9}{g^4\,(n^4+10\,n^3+35\,n^2}$$
$$+\frac{c\,e^2\,x^4\,(f+g\,x)^n\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24}$$
$$+\frac{x^2\,(f+g\,x)^n\,(n+1)\,(2\,c\,d^2\,g^2\,n^2+14\,c\,d^2\,g^2\,n+24\,c\,d^2\,g^2+3\,c\,d\,e\,f\,g\,n^2+12\,c\,d\,e\,f\,g\,n-3\,c\,e^2\,f^2\,n+}{g^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$
$$+\frac{c\,e\,x^3\,(f+g\,x)^n\,(12\,d\,g+3\,d\,g\,n+e\,f\,n)\,(n^2+3\,n+2)}{g\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$

```
[In] int((f + g*x)^n*(d + e*x)*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(24*a*d*g^4 + 26*a*d*g^4*n + 9*a*d*g^4*n^2 + a*d*g^4*n^3 + 7
*a*e*f*g^3*n^2 + a*e*f*g^3*n^3 + 24*c*d^2*f*g^3*n + 6*c*e^2*f^3*g*n + 14*c*
d^2*f*g^3*n^2 + 2*c*d^2*f*g^3*n^3 + 12*a*e*f*g^3*n - 24*c*d*e*f^2*g^2*n - 6
*c*d*e*f^2*g^2*n^2))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - ((f + g*x)
^n*(6*c*e^2*f^4 + 24*c*d^2*f^2*g^2 - 24*a*d*f*g^3 + 12*a*e*f^2*g^2 - 9*a*d*
f*g^3*n^2 - a*d*f*g^3*n^3 + 7*a*e*f^2*g^2*n + a*e*f^2*g^2*n^2 + 14*c*d^2*f^
2*g^2*n - 24*c*d*e*f^3*g - 26*a*d*f*g^3*n + 2*c*d^2*f^2*g^2*n^2 - 6*c*d*e*f
^3*g*n))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e^2*x^4*(f + g*x)^n
*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) + (x^2*(f +
```

g*x)^n*(n + 1)*(24*c*d^2*g^2 + 12*a*e*g^2 + 2*c*d^2*g^2*n^2 + 7*a*e*g^2*n + a*e*g^2*n^2 + 14*c*d^2*g^2*n - 3*c*e^2*f^2*n + 3*c*d*e*f*g*n^2 + 12*c*d*e*f*g*n))/(g^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e*x^3*(f + g*x)^n*(12*d*g + 3*d*g*n + e*f*n)*(3*n + n^2 + 2))/(g*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))

3.808 $\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{(ag^2+cf(ef-2dg))\,(f+gx)^{1+n}}{g^3(1+n)} - \frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)} + \frac{ce(f+gx)^{3+n}}{g^3(3+n)}$$

```
[Out] (a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^3/(1+n)-2*c*(-d*g+e*f)*(g*x+f)^(2+
n)/g^3/(2+n)+c*e*(g*x+f)^(3+n)/g^3/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {712}

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{(f+gx)^{n+1}\,(ag^2+cf(ef-2dg))}{g^3(n+1)} - \frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)} + \frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

```
[In] Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(ag^2+cf(ef-2dg))(f+gx)^n}{g^2}+\frac{2c(-ef+dg)(f+gx)^{1+n}}{g^2}+\frac{ce(f+gx)^{2+n}}{g^2}\right)dx\\ &= \frac{(ag^2+cf(ef-2dg))(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+gx)^{3+n}}{g^3(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int (f+gx)^n\left(a+2cdx+cex^2\right)dx=\frac{(f+gx)^{1+n}\left(\frac{ag^2+cf(ef-2dg)}{1+n}-\frac{2c(ef-dg)(f+gx)}{2+n}+\frac{ce(f+gx)^2}{3+n}\right)}{g^3}$$

```
[In] Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
[Out] ((f + g*x)^(1 + n)*((a*g^2 + c*f*(e*f - 2*d*g))/(1 + n) - (2*c*(e*f - d*g)*
(f + g*x))/(2 + n) + (c*e*(f + g*x)^2)/(3 + n)))/g^3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

method	result
gosper	$\frac{(gx+f)^{1+n}(ce\,g^2n^2x^2+2cd\,g^2n^2x+3ce\,g^2n\,x^2+8cd\,g^2nx-2cef\,gnx+2e\,x^2c\,g^2+a\,g^2n^2-2cdf\,gn+6cd\,g^2x-2cef\,gx+5a\,g^2n-6cd}{g^3(n^3+6n^2+11n+6)}$
norman	$\frac{ce\,x^3e^{n\ln(gx+f)}}{3+n}+\frac{f(a\,g^2n^2-2cdf\,gn+5a\,g^2n-6cdf\,g+2ce\,f^2+6a\,g^2)e^{n\ln(gx+f)}}{g^3(n^3+6n^2+11n+6)}+\frac{(2cdf\,g\,n^2+a\,g^2n^2+6cdf\,gn-2ce\,f^2n+5a}{g^2(n^3+6n^2+11n+}$
risch	$\frac{(ce\,g^3n^2x^3+2cd\,g^3n^2x^2+cef\,g^2n^2x^2+3ce\,g^3n\,x^3+2cdf\,g^2n^2x+8cd\,g^3n\,x^2+cef\,g^2n\,x^2+2ce\,x^3g^3+a\,g^3n^2x+6cdf\,g^2nx+6cd\,g^3x}{(2+n)(3+n)(1+n)g^3}$
parallelrisch	$\frac{x^3(gx+f)^ncef\,g^3n^2+3x^3(gx+f)^ncef\,g^3n+2x^2(gx+f)^ncdf\,g^3n^2+x^2(gx+f)^nce\,f^2g^2n^2+2x^3(gx+f)^ncef\,g^3+8x^2(gx+f)^ncdf}{}$

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
[Out] 1/g^3*(g*x+f)^(1+n)/(n^3+6*n^2+11*n+6)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c
*e*g^2*n*x^2+8*c*d*g^2*n*x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*
n+6*c*d*g^2*x-2*c*e*f*g*x+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(84) = 168.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.60

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \frac{(afg^2n^2+2\,cef^3-6\,cdf^2g+6\,afg^2+(ceg^3n^2+3\,ceg^3n+2\,ceg^3)x^3+(6\,cdg^3+(cefg^2+2\,cdg^3)n^2+(c}{g^3n^3+6\,g}$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*g
^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f*g
^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*
g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x)*(g*x + f)^n/
(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. 2(75) = 150.

Time = 0.58 (sec) , antiderivative size = 1489, normalized size of antiderivative = 17.73

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2*
g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c*e*
f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2/(2*
f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f**2*g*
*3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**
5*x**2), Eq(n, -3)), (-a*g**2/(f*g**3 + g**4*x) + 2*c*d*f*g*log(f/g + x)/(f
*g**3 + g**4*x) + 2*c*d*f*g/(f*g**3 + g**4*x) + 2*c*d*g**2*x*log(f/g + x)/(
f*g**3 + g**4*x) - 2*c*e*f**2*log(f/g + x)/(f*g**3 + g**4*x) - 2*c*e*f**2/(
f*g**3 + g**4*x) - 2*c*e*f*g*x*log(f/g + x)/(f*g**3 + g**4*x) + c*e*g**2*x*
*2/(f*g**3 + g**4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/
g**2 + 2*c*d*x/g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*
g), Eq(n, -1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**
```

```
3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3
*n + 6*g**3) - 2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) - 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2
+ 11*g**3*n + 6*g**3) + 6*c*d*f*g**2*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n
**2 + 11*g**3*n + 6*g**3) + 2*c*d*g**3*n**2*x**2*(f + g*x)**n/(g**3*n**3 +
6*g**3*n**2 + 11*g**3*n + 6*g**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n*
*3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3
*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*n
**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f + g*x)**n/(g*
*3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n**2*x**2*(f + g*x
)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n*x**2*(f
+ g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x*
*3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3
*n*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e
*g**3*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\int (f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd}{(n^2+3\,n+2)g^2} + \frac{((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n ce}{(n^3+6\,n^2+11\,n+6)g^3} + \frac{(gx+f)^{n+1}a}{g(n+1)}$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d/((n^2 + 3*n + 2)*g^2) +
 ((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x
 + f)^n*c*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g*x + f)^(n + 1)*a/(g*(n + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.36

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{(gx+f)^n ceg^3n^2x^3 + (gx+f)^n cefg^2n^2x^2 + 2\,(gx+f)^n cdg^3n^2x^2 + 3\,(gx+f)^n ceg^3nx^3 + 2\,(gx+f)^n cdf}{}$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*e*g^3*n^2*x^3 + (g*x + f)^n*c*e*f*g^2*n^2*x^2 + 2*(g*x + f)^
n*c*d*g^3*n^2*x^2 + 3*(g*x + f)^n*c*e*g^3*n*x^3 + 2*(g*x + f)^n*c*d*f*g^2*n
^2*x + (g*x + f)^n*c*e*f*g^2*n*x^2 + 8*(g*x + f)^n*c*d*g^3*n*x^2 + 2*(g*x +
 f)^n*c*e*g^3*x^3 - 2*(g*x + f)^n*c*e*f^2*g*n*x + 6*(g*x + f)^n*c*d*f*g^2*n
*x + (g*x + f)^n*a*g^3*n^2*x + 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)^n*c*
d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x + 2*(g*x + f)
^n*c*e*f^3 - 6*(g*x + f)^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(g*x + f
)^n*a*g^3*x + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^
3)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = (f+g\,x)^n \left(\frac{f\,(2\,c\,e\,f^2-2\,c\,d\,f\,g\,n-6\,c\,d\,f\,g+a\,g^2\,n^2+5\,a\,g^2\,n+6\,a\,g^2)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{x\,(-2\,c\,e\,f^2\,g\,n+2\,c\,d\,f\,g^2\,n^2+6\,c\,d\,f\,g^2\,n+a\,g^3\,n^2+5\,a\,g^3\,n+6\,a\,g^3)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{c\,e\,x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6} + \frac{c\,x^2\,(n+1)\,\left(6\,d\,g+2\,d\,g\,n+e\,f\,n\right)}{g\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

```
[In] int((f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (f + g*x)^n*((f*(6*a*g^2 + a*g^2*n^2 + 2*c*e*f^2 + 5*a*g^2*n - 6*c*d*f*g -
2*c*d*f*g*n))/(g^3*(11*n + 6*n^2 + n^3 + 6)) + (x*(6*a*g^3 + a*g^3*n^2 + 5*
a*g^3*n + 2*c*d*f*g^2*n^2 + 6*c*d*f*g^2*n - 2*c*e*f^2*g*n))/(g^3*(11*n + 6*
n^2 + n^3 + 6)) + (c*e*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (c*x
^2*(n + 1)*(6*d*g + 2*d*g*n + e*f*n))/(g*(11*n + 6*n^2 + n^3 + 6)))
```

3.809 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx = -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)}+\frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{(cd^2-ae)\,(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)(1+n)}$$

```
[Out] -c*(-d*g+e*f)*(g*x+f)^(1+n)/e/g^2/(1+n)+c*(g*x+f)^(2+n)/g^2/(2+n)+(c*d^2-a*
e)*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f
)/(1+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {965, 81, 70}

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx = \frac{(cd^2-ae)\,(f+gx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)} - \frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] -((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n)
)/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\int \frac{(f+gx)^n(-eg(cdf-ag)(2+n)-ceg(ef-dg)(2+n)x)}{d+ex}\,dx}{eg^2(2+n)} \\
&= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} - \frac{(cd^2-ae)\int \frac{(f+gx)^n}{d+ex}\,dx}{e} \\
&= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} \\
&\quad + \frac{(cd^2-ae)(f+gx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx = \frac{(f+gx)^{1+n}\left(\frac{c(-ef+dg(2+n)+eg(1+n)x)}{g^2(2+n)}+\frac{(cd^2-ae)\text{ Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}\right)}{e(1+n)}$$

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
[Out] ((f + g*x)^(1 + n)*((c*(-(e*f) + d*g*(2 + n) + e*g*(1 + n)*x))/(g^2*(2 + n)
) + ((c*d^2 - a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f -
d*g)])/(e*f - d*g)))/(e*(1 + n))
```

Maple [F]

$$\int \frac{(gx+f)^n(ce\,x^2+2cdx+a)}{ex+d}dx$$

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="fricas")
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d),x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

Giac [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="giac")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x), x)
```

3.810 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 88

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2} dx = \frac{c(f+gx)^{1+n}}{eg(1+n)} - \frac{(cd^2-ae)\, g(f+gx)^{1+n} \operatorname{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}$$

```
[Out] c*(g*x+f)^(1+n)/e/g/(1+n)-(c*d^2-a*e)*g*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2
+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^2/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {961, 70}

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2} dx = \frac{c(f+gx)^{n+1}}{eg(n+1)} - \frac{g(cd^2-ae)\,(f+gx)^{n+1} \operatorname{Hypergeometric2F1}\left(2,n+1,n+2,\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] (c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*Hy
pergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)
^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c(f+gx)^n}{e} + \frac{(-cd^2+ae)\,(f+gx)^n}{e(d+ex)^2}\right) dx \\
&= \frac{c(f+gx)^{1+n}}{eg(1+n)} + \frac{(-cd^2+ae)\int \frac{(f+gx)^n}{(d+ex)^2}\,dx}{e} \\
&= \frac{c(f+gx)^{1+n}}{eg(1+n)} - \frac{(cd^2-ae)\,g(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \frac{(f+gx)^{1+n}\left(c(ef-dg)^2+(-cd^2+ae)\,g^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)\right)}{eg(ef-dg)^2(1+n)}$$

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] ((f + g*x)^(1 + n)*(c*(e*f - d*g)^2 + (-(c*d^2) + a*e)*g^2*Hypergeometric2F
1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*g*(e*f - d*g)^2*(1 + n))
```

Maple [F]

$$\int \frac{(gx+f)^n (ce\,x^2+2cdx+a)}{(ex+d)^2}dx$$

[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)

[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)

Fricas [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^2*x^2 + 2*d*e*x + d^2), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**2,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="maxima")

[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)

Giac [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^2}\,dx$$

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2, x)
```

3.811 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 193

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{(cd^2-ae)\,g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} + \frac{(aeg^2(1-n)n-c(2e^2f^2-4defg+d^2g^2(2+n-n^2)))\,(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n\right.}{2e(ef-dg)^3(1+n)}$$

```
[Out] -1/2*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^2-1/2*(c*d^2-a*e)*g*(1-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)+1/2*(a*e*g^2*(1-n)*n-c*(2*e^2*f^2-4*d
*e*f*g+d^2*g^2*(-n^2+n+2)))*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f
)/(-d*g+e*f))/e/(-d*g+e*f)^3/(1+n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {963, 79, 70}

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx = \frac{(f+gx)^{n+1}\,(aeg^2(1-n)n-c(d^2g^2(-n^2+n+2)-4defg+2e^2f^2))\,\text{Hypergeometric2F1}\left(1,n+1,n+2\right.}{2e(n+1)(ef-dg)^3} - \frac{g(1-n)\,(cd^2-ae)\,(f+gx)^{n+1}}{2e(d+ex)(ef-dg)^2} - \frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -1/2*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^2) - ((c*d^
2 - a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)^2*(d + e*x)) + ((a*e
*g^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(
2*e*(e*f - d*g)^3*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{\int \frac{(f+gx)^n\left(ag(1-n)-\frac{cd(2ef-dg(1+n))}{e}-2c(ef-dg)x\right)}{(d+ex)^2}\,dx}{2(ef-dg)} \\
&= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{(cd^2-ae)\,g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} \\
&\quad -\frac{(aeg^2(1-n)n-c(2e^2f^2-4defg+d^2g^2(2+n-n^2)))\int \frac{(f+gx)^n}{d+ex}\,dx}{2e(ef-dg)^2}
\end{aligned}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{(cd^2-ae)\,g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} + \frac{(aeg^2(1-n)n-c(2e^2f^2-4defg+d^2g^2(2+n-n^2)))\,(f+gx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{2e(ef-dg)^3(1+n)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx = -\frac{(f+gx)^{1+n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)+(-cd^2+ae)\,g^2\,\text{Hypergeometr}\right.}{e(ef-dg)^3(1+n)}$$

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -(((f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[3, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^3*(1 + n)))
```

Maple [F]

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^3}dx$$

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx = \int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx$$

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**3,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3} dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^3} dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^3} dx$$

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3, x)
```

3.812 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{(cd^2-ae)\,g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2}+\frac{g\left(aeg^2(2-3n+n^2)+c(6e^2f^2-12defg+d^2g^2(4+3n-n^2))\right)(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+\dots\right.}{6e(ef-dg)^4(1+n)}$$

```
[Out] -1/3*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^3-1/6*(c*d^2-a*e)*g*(2-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)^2+1/6*g*(a*e*g^2*(n^2-3*n+2)+c*(6*e^2
*f^2-12*d*e*f*g+d^2*g^2*(-n^2+3*n+4)))*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2+
n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^4/(1+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {963, 79, 70}

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx = \frac{g(f+gx)^{n+1}\left(aeg^2(n^2-3n+2)+c(d^2g^2(-n^2+3n+4)-12defg+6e^2f^2)\right)\text{Hypergeometric2F1}\left(2,n+\dots\right.}{6e(n+1)(ef-dg)^4}-\frac{g(2-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{6e(d+ex)^2(ef-dg)^2}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] -1/3*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^3) - ((c*d^
2 - a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(6*e*(e*f - d*g)^2*(d + e*x)^2) + (g*
(a*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n - n
^2)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e
*f - d*g)])/(6*e*(e*f - d*g)^4*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{\int \frac{(f+gx)^n\left(ag(2-n)-\frac{cd(3ef-dg(1+n))}{e}-3c(ef-dg)x\right)}{(d+ex)^3}\,dx}{3(ef-dg)}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{(cd^2-ae)\,g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{(aeg^2(2-3n+n^2)+c(6e^2f^2-12defg+d^2g^2(4+3n-n^2)))\int \frac{(f+gx)^n}{(d+ex)^2}\,dx}{6e(ef-dg)^2}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{(cd^2-ae)\,g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{g(aeg^2(2-3n+n^2)+c(6e^2f^2-12defg+d^2g^2(4+3n-n^2)))\,(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n\right.}{6e(ef-dg)^4(1+n)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \frac{g(f+gx)^{1+n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{e(f+gx)}{ef-dg}\right)+(-cd^2+ae)\,g^2\,\text{Hypergeometric}\right.}{e(ef-dg)^4(1+n)}$$

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
[Out] (g*(f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[2, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[4, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^4*(1 + n))
```

Maple [F]

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^4}dx$$

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2
*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx$$

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**4,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

Giac [F]

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4}\,dx$$

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx = \int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{(d+e\,x)^4}\,dx$$

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4, x)
```

3.813 $\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 231

$$\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx$$
$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}$$
$$+ \frac{(c(ef-dg)(2+m)(ef(1+m)+dg(1+n)) + g(2+m+n)(aeg(3+m+n) - cd(ef(2+m)+dg(1+}{e^2g^2(1+m)(2+m+n)}$$

```
[Out] -c*(-d*g+e*f)*(2+m)*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e/g^2/(2+m+n)/(3+m+n)+c*(e*
x+d)^(2+m)*(g*x+f)^(1+n)/e/g/(3+m+n)+(c*(-d*g+e*f)*(2+m)*(e*f*(1+m)+d*g*(1+
n))+g*(2+m+n)*(a*e*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n))))*(e*x+d)^(1+m)*(g*x
+f)^n*hypergeom([-n, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^2/g^2/(1+m)/(2+m+n
)/(3+m+n)/((e*(g*x+f)/(-d*g+e*f))^n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {965, 81, 72, 71}

$$\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx$$
$$= \frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} \left(aeg(m+n+3) + \frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)} - cd(dg(n+1)+ef(\right.}{e^2g(m+1)(m+n+3)}$$
$$- \frac{c(m+2)(ef-dg)(d+ex)^{m+1}(f+gx)^{n+1}}{eg^2(m+n+2)(m+n+3)} + \frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -((c*(e*f - d*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(e*g^2*(2 + m
 + n)*(3 + m + n))) + (c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m +
 n)) + ((a*e*g*(3 + m + n) + (c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 +
 n)))/(g*(2 + m + n)) - c*d*(e*f*(2 + m) + d*g*(1 + n)))*(d + e*x)^(1 + m)*
(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g)
)])/(e^2*g*(1 + m)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\int (d+ex)^m(f+gx)^n\,(e(aeg(3+m+n)-cd(ef(2+m)+dg(1+n)))-ce^2(ef-dg)(2+m)x)\,dx}{e^2g(3+m+n)}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\left(aeg(3+m+n)+\frac{c(ef-dg)(2+m)(ef(1+m)+dg(1+n))}{g(2+m+n)}-cd(ef(2+m)+dg(1+n))\right)\int (d+ex)^m(f+}{eg(3+m+n)}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\left(\left(aeg(3+m+n)+\frac{c(ef-dg)(2+m)(ef(1+m)+dg(1+n))}{g(2+m+n)}-cd(ef(2+m)+dg(1+n))\right)(f+gx)^n\left(\frac{e(f}{ef}\right.\right.}{eg(3+m+n)}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\left(aeg(3+m+n)+\frac{c(ef-dg)(2+m)(ef(1+m)+dg(1+n))}{g(2+m+n)}-cd(ef(2+m)+dg(1+n))\right)(d+ex)^{1+m}(f-}{e^2g(1+m)(3+m+n)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.77

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$

$$= \frac{(d+ex)^{1+m}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{g(d+ex)}{-ef+dg}\right)-2c\right.}{}$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] - 2*c*(e*f - d*g)^2*Hypergeometr
ic2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(a*g^2 + c*f*
(e*f - 2*d*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x))/(-(e*f) +
d*g)]))/(e^2*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(ce\,x^2+2cdx+a\right) dx$$

[In] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)

[Out] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)

Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n\,dx$$

[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")

[Out] integral((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n\,dx$$

[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")

[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)

Giac [F]

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \int \left(cex^2+2\,cdx+a\right)(ex+d)^m(gx+f)^n \, dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(a+2cdx+cex^2\right) \, dx = \int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,e\,x^2+2\,c\,d\,x+a\right) \, dx$$

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2),x)
[Out] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2), x)
```

3.814 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)} \, dx = \frac{cx}{eg} + \frac{(cd^2-bde+ae^2)\log(d+ex)}{e^2(ef-dg)} - \frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}$$

```
[Out] c*x/e/g+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^2/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)*ln
(g*x+f)/g^2/(-d*g+e*f)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {907}

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)} \, dx = \frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)} - \frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)} + \frac{cx}{eg}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c}{eg} + \frac{cd^2 - bde + ae^2}{e(ef - dg)(d + ex)} + \frac{cf^2 - bfg + ag^2}{g(-ef + dg)(f + gx)}\right) dx \\ &= \frac{cx}{eg} + \frac{(cd^2 - bde + ae^2)\log(d + ex)}{e^2(ef - dg)} - \frac{(cf^2 - bfg + ag^2)\log(f + gx)}{g^2(ef - dg)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{a + bx + cx^2}{(d + ex)(f + gx)} dx = \frac{cx}{eg} - \frac{(-cd^2 + bde - ae^2)\log(d + ex)}{e^2(ef - dg)} - \frac{(cf^2 - bfg + ag^2)\log(f + gx)}{g^2(ef - dg)}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
[Out] (c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) -
((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

method	result
default	$\frac{cx}{eg}+\frac{\left(-e^{2} a+bde-c\,d^{2}\right)\ln\left(ex+d\right)}{\left(dg-ef\right)e^{2}}+\frac{\left(a\,g^{2}-bfg+c\,f^{2}\right)\ln\left(gx+f\right)}{g^{2}\left(dg-ef\right)}$
norman	$\frac{cx}{eg}+\frac{\left(a\,g^{2}-bfg+c\,f^{2}\right)\ln\left(gx+f\right)}{g^{2}\left(dg-ef\right)}-\frac{\left(e^{2} a-bde+c\,d^{2}\right)\ln\left(ex+d\right)}{\left(dg-ef\right)e^{2}}$
parallelrisch	$-\frac{\ln\left(ex+d\right)a\,e^{2} g^{2}-\ln\left(ex+d\right)bde\,g^{2}+\ln\left(ex+d\right)c\,d^{2} g^{2}-\ln\left(gx+f\right)a\,e^{2} g^{2}+\ln\left(gx+f\right)b\,e^{2} fg-\ln\left(gx+f\right)c\,e^{2} f^{2}-xcde\,g^{2}+xc\,e^{2} fg}{e^{2} g^{2}\left(dg-ef\right)}$
risch	$\frac{cx}{eg}-\frac{\ln\left(ex+d\right)a}{dg-ef}+\frac{\ln\left(ex+d\right)bd}{\left(dg-ef\right)e}-\frac{\ln\left(ex+d\right)c\,d^{2}}{\left(dg-ef\right)e^{2}}+\frac{\ln\left(-gx-f\right)a}{dg-ef}-\frac{\ln\left(-gx-f\right)bf}{g\left(dg-ef\right)}+\frac{\ln\left(-gx-f\right)c\,f^{2}}{g^{2}\left(dg-ef\right)}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
[Out] c*x/e/g+(-a*e^2+b*d*e-c*d^2)/(d*g-e*f)/e^2*ln(e*x+d)+1/g^2*(a*g^2-b*f*g+c*f
^2)/(d*g-e*f)*ln(g*x+f)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)g^2\log{(ex+d)}+(ce^2fg-cdeg^2)x-(ce^2f^2-be^2fg+ae^2g^2)\log{(gx+f)}}{e^3fg^2-de^2g^3}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*g^2*log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c*
e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(70) = 140.

Time = 78.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 5.06

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{cx}{eg} + \frac{(ag^2-bfg+cf^2)\log\left(x+\frac{adeg^2+ae^2fg-2bdefg+cd^2fg+cdef^2-\frac{d^2eg\left(ag^2-bfg+cf^2\right)}{dg-ef}+\frac{2de^2f\left(ag^2-bfg+cf^2\right)}{dg-ef}-\frac{e^3f^2\left(ag^2-bfg+c\right.}{g(dg-ef)}}{2ae^2g^2-bdeg^2-be^2fg+cd^2g^2+ce^2f^2}\right.}{g^2\left(dg-ef\right)} - \frac{(ae^2-bde+cd^2)\log\left(x+\frac{adeg^2+ae^2fg-2bdefg+cd^2fg+cdef^2+\frac{d^2g^3\left(ae^2-bde+cd^2\right)}{e(dg-ef)}-\frac{2dfg^2\left(ae^2-bde+cd^2\right)}{dg-ef}+\frac{ef^2g\left(ae^2-bde+cd^2\right)}{dg-ef}}{2ae^2g^2-bdeg^2-be^2fg+cd^2g^2+ce^2f^2}\right.}{e^2\left(dg-ef\right)}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*
b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/(d
*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a
*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e*
*2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*d*e +
 c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*
e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f*g**2*(
a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c*d**2)/(
d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c*e**2
*f**2))/(e**2*(d*g - e*f))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\log(ex+d)}{e^3f-de^2g} - \frac{(cf^2-bfg+ag^2)\log(gx+f)}{efg^2-dg^3} + \frac{cx}{eg}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
[Out] (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g + a
*g^2)*log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\log(|ex+d|)}{e^3f-de^2g} - \frac{(cf^2-bfg+ag^2)\log(|gx+f|)}{efg^2-dg^3} + \frac{cx}{eg}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
[Out] (c*d^2 - b*d*e + a*e^2)*log(abs(e*x + d))/(e^3*f - d*e^2*g) - (c*f^2 - b*f*
g + a*g^2)*log(abs(g*x + f))/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx = \frac{\ln(d+e\,x)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^3\,f-d\,e^2\,g} + \frac{\ln(f+g\,x)\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^2\,\left(d\,g-e\,f\right)} + \frac{c\,x}{e\,g}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)*(d + e*x)),x)
[Out] (log(d + e*x)*(a*e^2 + c*d^2 - b*d*e))/(e^3*f - d*e^2*g) + (log(f + g*x)*(a
*g^2 + c*f^2 - b*f*g))/(g^2*(d*g - e*f)) + (c*x)/(e*g)
```

3.815 $\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 184

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))\,x}{e^3g^3} - \frac{c(cef+cdg-2beg)x^2}{2e^2g^2} + \frac{c^2x^3}{3eg} + \frac{(cd^2-bde+ae^2)^2\log(d+ex)}{e^4(ef-dg)} - \frac{(cf^2-bfg+ag^2)^2\log(f+gx)}{g^4(ef-dg)}$$

```
[Out] (b^2*e^2*g^2-2*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x/
e^3/g^3-1/2*c*(-2*b*e*g+c*d*g+c*e*f)*x^2/e^2/g^2+1/3*c^2*x^3/e/g+(a*e^2-b*d
*e+c*d^2)^2*ln(e*x+d)/e^4/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^2*ln(g*x+f)/g^4/(-
d*g+e*f)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {907}

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{x(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{e^3g^3} + \frac{\log(d+ex)\,(ae^2-bde+cd^2)^2}{e^4(ef-dg)} - \frac{\log(f+gx)\,(ag^2-bfg+cf^2)^2}{g^4(ef-dg)} - \frac{cx^2(-2beg+cdg+cef)}{2e^2g^2} + \frac{c^2x^3}{3eg}$$

```
[In] Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^2e^2g^2 - 2ceg(bef + bdg - aeg) + c^2(e^2f^2 + defg + d^2g^2)}{e^3g^3} - \frac{c(cef + cdg - 2beg)x}{e^2g^2} + \frac{c^2x^2}{eg} + \frac{(cd^2 - bde + ae^2)^2}{e^3(ef - dg)(d + ex)} + \frac{(cf^2 - bfg + ag^2)^2}{g^3(-ef + dg)(f + gx)} \right) dx$$

$$= \frac{(b^2e^2g^2 - 2ceg(bef + bdg - aeg) + c^2(e^2f^2 + defg + d^2g^2))\,x}{e^3g^3} - \frac{c(cef + cdg - 2beg)x^2}{2e^2g^2} + \frac{c^2x^3}{3eg} + \frac{(cd^2 - bde + ae^2)^2\log(d + ex)}{e^4(ef - dg)} - \frac{(cf^2 - bfg + ag^2)^2\log(f + gx)}{g^4(ef - dg)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int \frac{(a + bx + cx^2)^2}{(d + ex)(f + gx)}\,dx = -\frac{eg(-ef + dg)x(6b^2e^2g^2 + 6ceg(2aeg + b(-2ef - 2dg + egx)) + c^2(6d^2g^2 - 3deg(-2f + gx) + e^2(6f^2 - \ldots}{6e^4g^4(ef - \ldots}$$

```
[In] Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/6*(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f -
2*d*g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*
g*x + 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6*e^
4*(c*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(e^4*g^4*(e*f - d*g))
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.42

method	result
norman	$\frac{(2ac\,e^2g^2+b^2e^2g^2-2bcde\,g^2-2bc\,e^2fg+c^2d^2g^2+c^2defg+c^2e^2f^2)x}{e^3g^3}+\frac{c^2x^3}{3eg}+\frac{c(2beg-cdg-cef)x^2}{2e^2g^2}+\frac{(a^2g^4-2abf\,g^3+2a}{}$
default	$\frac{\frac{1}{3}c^2x^3e^2g^2+bc\,e^2g^2x^2-\frac{1}{2}c^2de\,g^2x^2-\frac{1}{2}c^2e^2fg\,x^2+2ac\,e^2g^2x+b^2e^2g^2x-2bcde\,g^2x-2bc\,e^2fgx+c^2d^2g^2x+c^2defgx+c^2e^2f^2x}{e^3g^3}$
parallelrisch	$-\frac{-2x^3c^2d\,e^3g^4+2x^3c^2e^4f\,g^3+3x^2c^2d^2e^2g^4-3x^2c^2e^4f^2g^2-6x\,b^2d\,e^3g^4+6x\,b^2e^4f\,g^3-6x\,c^2d^3e\,g^4+6x\,c^2e^4f^3g+6\ln(ex+d)b}{}$
risch	$\frac{c^2x^3}{3eg}+\frac{bc\,x^2}{eg}-\frac{c^2d\,x^2}{2e^2g}-\frac{c^2f\,x^2}{2e\,g^2}+\frac{2acx}{eg}+\frac{b^2x}{eg}-\frac{2bcdx}{e^2g}-\frac{2bcfx}{e\,g^2}+\frac{c^2d^2x}{e^3g}+\frac{c^2dfx}{e^2g^2}+\frac{c^2f^2x}{e\,g^3}-\frac{\ln(ex+d)a^2}{dg-ef}$

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] (2*a*c*e^2*g^2+b^2*e^2*g^2-2*b*c*d*e*g^2-2*b*c*e^2*f*g+c^2*d^2*g^2+c^2*d*e*
f*g+c^2*e^2*f^2)/e^3/g^3*x+1/3*c^2*x^3/e/g+1/2*c/e^2/g^2*(2*b*e*g-c*d*g-c*e
*f)*x^2+1/g^4*(a^2*g^4-2*a*b*f*g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c^
2*f^4)/(d*g-e*f)*ln(g*x+f)-(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2
*b*c*d^3*e+c^2*d^4)/(d*g-e*f)/e^4*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{6\,(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+(b^2+2\,ac)d^2e^2)g^4\log(ex+d)+2\,(c^2e^4fg^3-c^2de^3g^4)x^3-3\,(c^2e^4f^2}{}$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e
^2)*g^4*log(e*x + d) + 2*(c^2*e^4*f*g^3 - c^2*d*e^3*g^4)*x^3 - 3*(c^2*e^4*f
^2*g^2 - 2*b*c*e^4*f*g^3 - (c^2*d^2*e^2 - 2*b*c*d*e^3)*g^4)*x^2 + 6*(c^2*e^
4*f^3*g - 2*b*c*e^4*f^2*g^2 + (b^2 + 2*a*c)*e^4*f*g^3 - (c^2*d^3*e - 2*b*c*
d^2*e^2 + (b^2 + 2*a*c)*d*e^3)*g^4)*x - 6*(c^2*e^4*f^4 - 2*b*c*e^4*f^3*g -
2*a*b*e^4*f*g^3 + a^2*e^4*g^4 + (b^2 + 2*a*c)*e^4*f^2*g^2)*log(g*x + f))/(e
^5*f*g^4 - d*e^4*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+(b^2+2\,ac)d^2e^2)\log(ex+d)}{e^5f-de^4g} - \frac{(c^2f^4-2\,bcf^3g-2\,abfg^3+a^2g^4+(b^2+2\,ac)f^2g^2)\log(gx+f)}{efg^4-dg^5} + \frac{2\,c^2e^2g^2x^3-3\,(c^2e^2fg+(c^2de-2\,bce^2)g^2)x^2+6\,(c^2e^2f^2+(c^2de-2\,bce^2)fg+(c^2d^2-2\,bcde+(b^2+2$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log
(e*x + d)/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^
4 + (b^2 + 2*a*c)*f^2*g^2)*log(g*x + f)/(e*f*g^4 - d*g^5) + 1/6*(2*c^2*e^2*
g^2*x^3 - 3*(c^2*e^2*f*g + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*f^2
+ (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2)*g^2
)*x)/(e^3*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = \frac{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\log(|ex+d|)}{e^5f-de^4g} - \frac{(c^2f^4-2\,bcf^3g+b^2f^2g^2+2\,acf^2g^2-2\,abfg^3+a^2g^4)\log(|gx+f|)}{efg^4-dg^5} + \frac{2\,c^2e^2g^2x^3-3\,c^2e^2fgx^2-3\,c^2deg^2x^2+6\,bce^2g^2x^2+6\,c^2e^2f^2x+6\,c^2defgx-12\,bce^2fgx+6\,c^2d^2g^2x-1}{6\,e^3g^3}$$

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*log(abs(e*x + d))/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g + b^2*f^2*g
^2 + 2*a*c*f^2*g^2 - 2*a*b*f*g^3 + a^2*g^4)*log(abs(g*x + f))/(e*f*g^4 - d*
g^5) + 1/6*(2*c^2*e^2*g^2*x^3 - 3*c^2*e^2*f*g*x^2 - 3*c^2*d*e*g^2*x^2 + 6*b
*c*e^2*g^2*x^2 + 6*c^2*e^2*f^2*x + 6*c^2*d*e*f*g*x - 12*b*c*e^2*f*g*x + 6*c
^2*d^2*g^2*x - 12*b*c*d*e*g^2*x + 6*b^2*e^2*g^2*x + 12*a*c*e^2*g^2*x)/(e^3*
g^3)
```

Mupad [B] (verification not implemented)

Time = 12.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx = x\left(\frac{b^2+2\,a\,c}{e\,g}+\frac{\left(\frac{c^2\,(d\,g+e\,f)}{e^2\,g^2}-\frac{2\,b\,c}{e\,g}\right)\,(d\,g+e\,f)}{e\,g}-\frac{c^2\,d\,f}{e^2\,g^2}\right)-x^2\left(\frac{c^2\,(d\,g+e\,f)}{2\,e^2\,g^2}-\frac{b\,c}{e\,g}\right) + \frac{\ln(d+e\,x)\,\left(e^2\,(b^2\,d^2+2\,a\,c\,d^2)+a^2\,e^4+c^2\,d^4-2\,a\,b\,d\,e^3-2\,b\,c\,d^3\,e\right)}{e^5\,f-d\,e^4\,g} + \frac{\ln(f+g\,x)\,\left(g^2\,(b^2\,f^2+2\,a\,c\,f^2)+a^2\,g^4+c^2\,f^4-2\,a\,b\,f\,g^3-2\,b\,c\,f^3\,g\right)}{d\,g^5-e\,f\,g^4}+\frac{c^2\,x^3}{3\,e\,g}$$

```
[In] int((a + b*x + c*x^2)^2/((f + g*x)*(d + e*x)),x)
```

```
[Out] x*((2*a*c + b^2)/(e*g) + (((c^2*(d*g + e*f))/(e^2*g^2) - (2*b*c)/(e*g))*(d*
g + e*f))/(e*g) - (c^2*d*f)/(e^2*g^2)) - x^2*((c^2*(d*g + e*f))/(2*e^2*g^2)
 - (b*c)/(e*g)) + (log(d + e*x)*(e^2*(b^2*d^2 + 2*a*c*d^2) + a^2*e^4 + c^2*
d^4 - 2*a*b*d*e^3 - 2*b*c*d^3*e))/(e^5*f - d*e^4*g) + (log(f + g*x)*(g^2*(b
^2*f^2 + 2*a*c*f^2) + a^2*g^4 + c^2*f^4 - 2*a*b*f*g^3 - 2*b*c*f^3*g))/(d*g^
5 - e*f*g^4) + (c^2*x^3)/(3*e*g)
```

3.816 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 531

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx =$$

$$-\frac{(b^2e^3g^3(bef+bdg-3aeg)-c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)-3ce^2g^2(a^2e^2g^2-2abeg(e}{}$$

$$+\frac{(b^3e^3g^3-3bce^2g^2(bef+bdg-2aeg)-c^3(e^3f^3+de^2f^2g+d^2efg^2+d^3g^3)-3c^2eg(aeg(ef+dg)-b(e^2}{2e^4g^4}$$

$$+\frac{c(3b^2e^2g^2-3ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))\,x^3}{3e^3g^3}$$

$$-\frac{c^2(cef+cdg-3beg)x^4}{4e^2g^2}+\frac{c^3x^5}{5eg}+\frac{(cd^2-bde+ae^2)^3\log(d+ex)}{e^6(ef-dg)}$$

$$-\frac{(cf^2-bfg+ag^2)^3\log(f+gx)}{g^6(ef-dg)}$$

```
[Out] -(b^2*e^3*g^3*(-3*a*e*g+b*d*g+b*e*f)-c^3*(d^4*g^4+d^3*e*f*g^3+d^2*e^2*f^2*g
^2+d*e^3*f^3*g+e^4*f^4)-3*c*e^2*g^2*(a^2*e^2*g^2-2*a*b*e*g*(d*g+e*f)+b^2*(d
^2*g^2+d*e*f*g+e^2*f^2))-3*c^2*e*g*(a*e*g*(d^2*g^2+d*e*f*g+e^2*f^2)-b*(d^3*
g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)))*x/e^5/g^5+1/2*(b^3*e^3*g^3-3*b*c*e^2
*g^2*(-2*a*e*g+b*d*g+b*e*f)-c^3*(d^3*g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)-3
*c^2*e*g*(a*e*g*(d*g+e*f)-b*(d^2*g^2+d*e*f*g+e^2*f^2)))*x^2/e^4/g^4+1/3*c*(
3*b^2*e^2*g^2-3*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x
^3/e^3/g^3-1/4*c^2*(-3*b*e*g+c*d*g+c*e*f)*x^4/e^2/g^2+1/5*c^3*x^5/e/g+(a*e^
2-b*d*e+c*d^2)^3*ln(e*x+d)/e^6/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^3*ln(g*x+f)/g
^6/(-d*g+e*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {907}

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx =$$
$$-\frac{x(-3ce^2g^2(a^2e^2g^2-2abeg(dg+ef)+b^2(d^2g^2+defg+e^2f^2))+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg(\cdots}{\cdots}$$
$$+\frac{x^2(-3c^2eg(aeg(dg+ef)-b(d^2g^2+defg+e^2f^2))-3bce^2g^2(-2aeg+bdg+bef)+b^3e^3g^3-(c^3(d^3g^3\cdots}{2e^4g^4}$$
$$+\frac{cx^3(-3ceg(-aeg+bdg+bef)+3b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{3e^3g^3}$$
$$+\frac{\log(d+ex)\,(ae^2-bde+cd^2)^3}{e^6(ef-dg)}-\frac{\log(f+gx)\,(ag^2-bfg+cf^2)^3}{g^6(ef-dg)}$$
$$-\frac{c^2x^4(-3beg+cdg+cef)}{4e^2g^2}+\frac{c^3x^5}{5eg}$$

```
[In] Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -((((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

integral

$$= \int \left(\frac{-b^2e^3g^3(bef+bdg-3aeg)+c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)+3ce^2g^2(a^2e^2g^2-2abeg\ldots}{\ldots} + \frac{(b^3e^3g^3-3bce^2g^2(bef+bdg-2aeg)-c^3(e^3f^3+de^2f^2g+d^2efg^2+d^3g^3)-3c^2eg(aeg(ef+dg)-b(e^2\ldots}{e^4g^4} + \frac{c(3b^2e^2g^2-3ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))x^2}{e^3g^3} - \frac{c^2(cef+cdg-3beg)x^3}{e^2g^2} + \frac{c^3x^4}{eg} + \frac{(cd^2-bde+ae^2)^3}{e^5(ef-dg)(d+ex)} + \frac{(cf^2-bfg+ag^2)^3}{g^5(-ef+dg)(f+gx)} \right) dx$$

$$= -\frac{(b^2e^3g^3(bef+bdg-3aeg)-c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)-3ce^2g^2(a^2e^2g^2-2\ldots}{\ldots} + \frac{(b^3e^3g^3-3bce^2g^2(bef+bdg-2aeg)-c^3(e^3f^3+de^2f^2g+d^2efg^2+d^3g^3)-3c^2eg(aeg(ef+dg\ldots}{2e^4g^4} + \frac{c(3b^2e^2g^2-3ceg(bef+bdg-aeg)+c^2(e^2f^2+defg+d^2g^2))x^3}{3e^3g^3} - \frac{c^2(cef+cdg-3beg)x^4}{4e^2g^2} + \frac{c^3x^5}{5eg} + \frac{(cd^2-bde+ae^2)^3\log(d+ex)}{e^6(ef-dg)} - \frac{(cf^2-bfg+ag^2)^3\log(f+gx)}{g^6(ef-dg)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = -\frac{egx(-30b^2e^3g^3(ef-dg)(6aeg+b(-2ef-2dg+egx))+c^3(60d^5g^5-30d^4eg^5x+20d^3e^2g^5x^2-15d^2e^3\ldots}{\ldots}$$

```
[In] Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/60*(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*
g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*
g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 +
 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*
a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) +
e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d
^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12
*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*
```

```
f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b*d) + a*e
))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(
e^6*g^6*(e*f - d*g))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.58

method	result
norman	$\frac{(3a^2c e^4g^4+3a b^2e^4g^4-6abcd e^3g^4-6abc e^4f g^3+3a c^2d^2e^2g^4+3a c^2d e^3f g^3+3a c^2e^4f^2g^2-b^3d e^3g^4-b^3e^4f g^3+3b^2c d^2e^2g^4+}{}$
default	$\frac{3a c^2d^2e^2g^4x+3a c^2e^4f^2g^2x-6abc e^4f g^3x+3a c^2d e^3f g^3x+3b^2cd e^3f g^3x+c^3d^4g^4x+c^3e^4f^4x+\frac{1}{2}b^3e^4g^4x^2+\frac{1}{5}c^3x^5e^4g^4+a c^2}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] (3*a^2*c*e^4*g^4+3*a*b^2*e^4*g^4-6*a*b*c*d*e^3*g^4-6*a*b*c*e^4*f*g^3+3*a*c^
2*d^2*e^2*g^4+3*a*c^2*d*e^3*f*g^3+3*a*c^2*e^4*f^2*g^2-b^3*d*e^3*g^4-b^3*e^4
*f*g^3+3*b^2*c*d^2*e^2*g^4+3*b^2*c*d*e^3*f*g^3+3*b^2*c*e^4*f^2*g^2-3*b*c^2*
d^3*e*g^4-3*b*c^2*d^2*e^2*f*g^3-3*b*c^2*d*e^3*f^2*g^2-3*b*c^2*e^4*f^3*g+c^3
*d^4*g^4+c^3*d^3*e*f*g^3+c^3*d^2*e^2*f^2*g^2+c^3*d*e^3*f^3*g+c^3*e^4*f^4)/e
^5/g^5*x+1/5*c^3*x^5/e/g+1/2/e^4/g^4*(6*a*b*c*e^3*g^3-3*a*c^2*d*e^2*g^3-3*a
*c^2*e^3*f*g^2+b^3*e^3*g^3-3*b^2*c*d*e^2*g^3-3*b^2*c*e^3*f*g^2+3*b*c^2*d^2*
e*g^3+3*b*c^2*d*e^2*f*g^2+3*b*c^2*e^3*f^2*g-c^3*d^3*g^3-c^3*d^2*e*f*g^2-c^3
*d*e^2*f^2*g-c^3*e^3*f^3)*x^2+1/3*c/e^3/g^3*(3*a*c*e^2*g^2+3*b^2*e^2*g^2-3*
b*c*d*e*g^2-3*b*c*e^2*f*g+c^2*d^2*g^2+c^2*d*e*f*g+c^2*e^2*f^2)*x^3+1/4*c^2/
e^2/g^2*(3*b*e*g-c*d*g-c*e*f)*x^4+1/g^6*(a^3*g^6-3*a^2*b*f*g^5+3*a^2*c*f^2*
g^4+3*a*b^2*f^2*g^4-6*a*b*c*f^3*g^3+3*a*c^2*f^4*g^2-b^3*f^3*g^3+3*b^2*c*f^4
*g^2-3*b*c^2*f^5*g+c^3*f^6)/(d*g-e*f)*ln(g*x+f)-(a^3*e^6-3*a^2*b*d*e^5+3*a^
2*c*d^2*e^4+3*a*b^2*d^2*e^4-6*a*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^3*d^3*e^3+3*b
^2*c*d^4*e^2-3*b*c^2*d^5*e+c^3*d^6)/(d*g-e*f)/e^6*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 2.80 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \frac{60\,(c^3d^6-3\,bc^2d^5e-3\,a^2bde^5+a^3e^6+3\,(b^2c+ac^2)d^4e^2-(b^3+6\,abc)d^3e^3+3\,(ab^2+a^2c)d^2e^4)g^6\log(ex}{}$$

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/60*(60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*
c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*g^6*log
(e*x + d) + 12*(c^3*e^6*f*g^5 - c^3*d*e^5*g^6)*x^5 - 15*(c^3*e^6*f^2*g^4 -
3*b*c^2*e^6*f*g^5 - (c^3*d^2*e^4 - 3*b*c^2*d*e^5)*g^6)*x^4 + 20*(c^3*e^6*f^
3*g^3 - 3*b*c^2*e^6*f^2*g^4 + 3*(b^2*c + a*c^2)*e^6*f*g^5 - (c^3*d^3*e^3 -
3*b*c^2*d^2*e^4 + 3*(b^2*c + a*c^2)*d*e^5)*g^6)*x^3 - 30*(c^3*e^6*f^4*g^2 -
 3*b*c^2*e^6*f^3*g^3 + 3*(b^2*c + a*c^2)*e^6*f^2*g^4 - (b^3 + 6*a*b*c)*e^6*
f*g^5 - (c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 - (b^3 +
 6*a*b*c)*d*e^5)*g^6)*x^2 + 60*(c^3*e^6*f^5*g - 3*b*c^2*e^6*f^4*g^2 + 3*(b^
2*c + a*c^2)*e^6*f^3*g^3 - (b^3 + 6*a*b*c)*e^6*f^2*g^4 + 3*(a*b^2 + a^2*c)*
e^6*f*g^5 - (c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c + a*c^2)*d^3*e^3 - (b^3
 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*g^6)*x - 60*(c^3*e^6*f^6 - 3
*b*c^2*e^6*f^5*g - 3*a^2*b*e^6*f*g^5 + a^3*e^6*g^6 + 3*(b^2*c + a*c^2)*e^6*
f^4*g^2 - (b^3 + 6*a*b*c)*e^6*f^3*g^3 + 3*(a*b^2 + a^2*c)*e^6*f^2*g^4)*log(
g*x + f))/(e^7*f*g^6 - d*e^6*g^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \frac{(c^3d^6 - 3\,bc^2d^5e - 3\,a^2bde^5 + a^3e^6 + 3\,(b^2c + ac^2)d^4e^2 - (b^3 + 6\,abc)d^3e^3 + 3\,(ab^2 + a^2c)d^2e^4)\log(ex+d)}{e^7f - de^6g} - \frac{(c^3f^6 - 3\,bc^2f^5g - 3\,a^2bfg^5 + a^3g^6 + 3\,(b^2c + ac^2)f^4g^2 - (b^3 + 6\,abc)f^3g^3 + 3\,(ab^2 + a^2c)f^2g^4)\log(gx+}{efg^6 - dg^7} + \frac{12\,c^3e^4g^4x^5 - 15\,(c^3e^4fg^3 + (c^3de^3 - 3\,bc^2e^4)g^4)x^4 + 20\,(c^3e^4f^2g^2 + (c^3de^3 - 3\,bc^2e^4)fg^3 + (c^3d^2e^2 - 3\,b}{}$$

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*
e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x + d)/(e^
```

```
7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*(b^
2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*g^4)
*log(g*x + f)/(e*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*(c^3*e^4*f*
g^3 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*e^4*f^2*g^2 + (c^3*d*e^3
 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^
4)*g^4)*x^3 - 30*(c^3*e^4*f^3*g + (c^3*d*e^3 - 3*b*c^2*e^4)*f^2*g^2 + (c^3*
d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f*g^3 + (c^3*d^3*e - 3*b*c
^2*d^2*e^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*g^4)*x^2 + 60*(
c^3*e^4*f^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (c^3*d^2*e^2 - 3*b*c^2*d*e^
3 + 3*(b^2*c + a*c^2)*e^4)*f^2*g^2 + (c^3*d^3*e - 3*b*c^2*d^2*e^2 + 3*(b^2*
c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*f*g^3 + (c^3*d^4 - 3*b*c^2*d^3*e +
3*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + 3*(a*b^2 + a^2*c)*e^4)*
g^4)*x)/(e^5*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$
$$= \frac{(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 + 3\,ac^2d^4e^2 - b^3d^3e^3 - 6\,abcd^3e^3 + 3\,ab^2d^2e^4 + 3\,a^2cd^2e^4 - 3\,a^2bde^5 + a^3e^6)\,\mathrm{lo}}{e^7f - de^6g}$$
$$- \frac{(c^3f^6 - 3\,bc^2f^5g + 3\,b^2cf^4g^2 + 3\,ac^2f^4g^2 - b^3f^3g^3 - 6\,abcf^3g^3 + 3\,ab^2f^2g^4 + 3\,a^2cf^2g^4 - 3\,a^2bfg^5 + a^3}{efg^6 - dg^7}$$
$$+ \frac{12\,c^3e^4g^4x^5 - 15\,c^3e^4fg^3x^4 - 15\,c^3de^3g^4x^4 + 45\,bc^2e^4g^4x^4 + 20\,c^3e^4f^2g^2x^3 + 20\,c^3de^3fg^3x^3 - 60\,bc^2e^4f}{}$$

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3
- 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3
*e^6)*log(abs(e*x + d))/(e^7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g + 3*b^
2*c*f^4*g^2 + 3*a*c^2*f^4*g^2 - b^3*f^3*g^3 - 6*a*b*c*f^3*g^3 + 3*a*b^2*f^2
*g^4 + 3*a^2*c*f^2*g^4 - 3*a^2*b*f*g^5 + a^3*g^6)*log(abs(g*x + f))/(e*f*g^
6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*c^3*e^4*f*g^3*x^4 - 15*c^3*d*e^3
*g^4*x^4 + 45*b*c^2*e^4*g^4*x^4 + 20*c^3*e^4*f^2*g^2*x^3 + 20*c^3*d*e^3*f*g
^3*x^3 - 60*b*c^2*e^4*f*g^3*x^3 + 20*c^3*d^2*e^2*g^4*x^3 - 60*b*c^2*d*e^3*g
^4*x^3 + 60*b^2*c*e^4*g^4*x^3 + 60*a*c^2*e^4*g^4*x^3 - 30*c^3*e^4*f^3*g*x^2
 - 30*c^3*d*e^3*f^2*g^2*x^2 + 90*b*c^2*e^4*f^2*g^2*x^2 - 30*c^3*d^2*e^2*f*g
^3*x^2 + 90*b*c^2*d*e^3*f*g^3*x^2 - 90*b^2*c*e^4*f*g^3*x^2 - 90*a*c^2*e^4*f
*g^3*x^2 - 30*c^3*d^3*e*g^4*x^2 + 90*b*c^2*d^2*e^2*g^4*x^2 - 90*b^2*c*d*e^3
*g^4*x^2 - 90*a*c^2*d*e^3*g^4*x^2 + 30*b^3*e^4*g^4*x^2 + 180*a*b*c*e^4*g^4*
x^2 + 60*c^3*e^4*f^4*x + 60*c^3*d*e^3*f^3*g*x - 180*b*c^2*e^4*f^3*g*x + 60*
```

```
c^3*d^2*e^2*f^2*g^2*x - 180*b*c^2*d*e^3*f^2*g^2*x + 180*b^2*c*e^4*f^2*g^2*x
 + 180*a*c^2*e^4*f^2*g^2*x + 60*c^3*d^3*e*f*g^3*x - 180*b*c^2*d^2*e^2*f*g^3
*x + 180*b^2*c*d*e^3*f*g^3*x + 180*a*c^2*d*e^3*f*g^3*x - 60*b^3*e^4*f*g^3*x
 - 360*a*b*c*e^4*f*g^3*x + 60*c^3*d^4*g^4*x - 180*b*c^2*d^3*e*g^4*x + 180*b
^2*c*d^2*e^2*g^4*x + 180*a*c^2*d^2*e^2*g^4*x - 60*b^3*d*e^3*g^4*x - 360*a*b
*c*d*e^3*g^4*x + 180*a*b^2*e^4*g^4*x + 180*a^2*c*e^4*g^4*x)/(e^5*g^5)
```

Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$$

$$= x^4\left(\frac{3\,b\,c^2}{4\,e\,g}-\frac{c^3\,(d\,g+e\,f)}{4\,e^2\,g^2}\right)-x^3\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{3\,e\,g}-\frac{c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{3\,e^2\,g^2}\right)$$

$$+x^2\left(\frac{b^3+6\,a\,c\,b}{2\,e\,g}+\frac{(d\,g+e\,f)\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{2\,e\,g}-\frac{d\,f\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{2\,e\,g}\right)$$

$$+x\left(\frac{3\,a\,(b^2+a\,c)}{e\,g}-\frac{(d\,g+e\,f)\,\left(\frac{b^3+6\,a\,c\,b}{e\,g}+\frac{(d\,g+e\,f)\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,\left(b^2+a\,c\right)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{e\,g}-\frac{d\,f\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}\right)}{e\,g}+\frac{d\,f\,\left(\frac{(d\,g+e\,f)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,(d\,g+e\,f)}{e^2\,g^2}\right)}{e\,g}-\frac{3\,c\,(b^2+a\,c)}{e\,g}+\frac{c^3\,d\,f}{e^2\,g^2}\right)}{e\,g}\right)$$

$$+\frac{\ln(d+e\,x)\,\left(e^4\,(3\,c\,a^2\,d^2+3\,a\,b^2\,d^2)+e^2\,(3\,b^2\,c\,d^4+3\,a\,c^2\,d^4)-e^3\,(b^3\,d^3+6\,a\,c\,b\,d^3)+a^3\,e^6+c^3\,d^6-3\cdots\right.}{e^7\,f-d\,e^6\,g}$$

$$+\frac{\ln(f+g\,x)\,\left(g^4\,(3\,c\,a^2\,f^2+3\,a\,b^2\,f^2)+g^2\,(3\,b^2\,c\,f^4+3\,a\,c^2\,f^4)-g^3\,(b^3\,f^3+6\,a\,c\,b\,f^3)+a^3\,g^6+c^3\,f^6-\cdots\right.}{d\,g^7-e\,f\,g^6}$$

$$+\frac{c^3\,x^5}{5\,e\,g}$$

```
[In] int((a + b*x + c*x^2)^3/((f + g*x)*(d + e*x)),x)

[Out] x^4*((3*b*c^2)/(4*e*g) - (c^3*(d*g + e*f))/(4*e^2*g^2)) - x^3*(((d*g + e*f)
*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(3*e*g) - (c*(a*c + b^2))
/(e*g) + (c^3*d*f)/(3*e^2*g^2)) + x^2*((b^3 + 6*a*b*c)/(2*e*g) + ((d*g + e*
f)*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (
3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(2*e*g) - (d*f*((3*b*c^2)/(e
*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(2*e*g)) + x*((3*a*(a*c + b^2))/(e*g) -
 ((d*g + e*f)*((b^3 + 6*a*b*c)/(e*g) + ((d*g + e*f)*(((d*g + e*f)*((3*b*c^2
)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (
c^3*d*f)/(e^2*g^2)))/(e*g) - (d*f*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2
*g^2)))/(e*g)))/(e*g) + (d*f*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e
*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(e
*g)) + (log(d + e*x)*(e^4*(3*a*b^2*d^2 + 3*a^2*c*d^2) + e^2*(3*a*c^2*d^4 +
3*b^2*c*d^4) - e^3*(b^3*d^3 + 6*a*b*c*d^3) + a^3*e^6 + c^3*d^6 - 3*a^2*b*d*
e^5 - 3*b*c^2*d^5*e))/(e^7*f - d*e^6*g) + (log(f + g*x)*(g^4*(3*a*b^2*f^2 +
 3*a^2*c*f^2) + g^2*(3*a*c^2*f^4 + 3*b^2*c*f^4) - g^3*(b^3*f^3 + 6*a*b*c*f^
3) + a^3*g^6 + c^3*f^6 - 3*a^2*b*f*g^5 - 3*b*c^2*f^5*g))/(d*g^7 - e*f*g^6)
+ (c^3*x^5)/(5*e*g)
```

3.817 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx = -\frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))}+\frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)\,(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\,(cf^2-bfg+ag^2)}-\frac{(cef+cdg-beg)\log\left(a+bx+cx^2\right)}{2\,(cd^2-bde+ae^2)\,(cf^2-g(bf-ag))}$$

```
[Out] e^2*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)-g^2*ln(g*x+f)/(-d*g+e*f)/(a*g^
2-b*f*g+c*f^2)-1/2*(-b*e*g+c*d*g+c*e*f)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)
/(c*f^2-g*(-a*g+b*f))-(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*arctanh((
2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(c*f^2-g*(-a*g+b*f))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {907, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)} - \frac{\log\left(a+bx+cx^2\right)\left(-beg+cdg+cef\right)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)} + \frac{e^2\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e^3}{(cd^2-bde+ae^2)(-ef+dg)(d+ex)} - \frac{g^3}{(ef-dg)(cf^2-bfg+ag^2)(f+gx)} + \frac{c^2df+b^2eg-c(bef+bdg+aeg)-c(cef+cdg-beg)x}{(cd^2-bde+ae^2)(cf^2-bfg+ag^2)(a+bx+cx^2)}\right) dx \\
&= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} + \frac{\int \frac{c^2df+b^2eg-c(bef+bdg+aeg)-c(cef+cdg-beg)x}{a+bx+cx^2}\,dx}{(cd^2-bde+ae^2)(cf^2-g(bf-ag))} \\
&= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} + \frac{(-cef-cdg+beg)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2(cd^2-bde+ae^2)(cf^2-g(bf-ag))} + \frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\int \frac{1}{a+bx+cx^2}\,dx}{2(cd^2-bde+ae^2)(cf^2-g(bf-ag))} \\
&= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} - \frac{(cef+cdg-beg)\log(a+bx+cx^2)}{2(cd^2-bde+ae^2)(cf^2-g(bf-ag))} - \frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{(cd^2-bde+ae^2)(cf^2-g(bf-ag))}
\end{aligned}$$

$$= -\frac{(2c^2df + b^2eg - c(bef + bdg + 2aeg))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(cd^2 - bde + ae^2)\,(cf^2 - g(bf - ag))} + \frac{e^2\log(d+ex)}{(cd^2 - bde + ae^2)\,(ef - dg)} - \frac{g^2\log(f+gx)}{(ef - dg)\,(cf^2 - bfg + ag^2)} - \frac{(cef + cdg - beg)\log(a + bx + cx^2)}{2\,(cd^2 - bde + ae^2)\,(cf^2 - g(bf - ag))}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(f+gx)\,(a+bx+cx^2)}\,dx = \frac{(2c^2df + b^2eg - c(bef + bdg + 2aeg))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,(cd^2 + e(-bd + ae))\,(cf^2 + g(-bf + ag))} + \frac{e^2\log(d+ex)}{(cd^2 + e(-bd + ae))\,(ef - dg)} - \frac{g^2\log(f+gx)}{(ef - dg)\,(cf^2 + g(-bf + ag))} - \frac{(cef + cdg - beg)\log(a + x(b + cx))}{2\,(cd^2 + e(-bd + ae))\,(cf^2 + g(-bf + ag))}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 + g
*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f - d
*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((c*e*
f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(c*f
^2 + g*(-(b*f) + a*g)))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.99

method	result
default	$-\frac{e^2\ln(ex+d)}{(dg-ef)(e^2a-bde+c\,d^2)} + \frac{\frac{\left(bceg-c^2dg-c^2ef\right)\ln\left(c\,x^2+bx+a\right)}{2c} + \frac{2\left(-aceg+b^2eg-bcdg-bcef+c^2df-\frac{\left(bceg-c^2dg-c^2ef\right)b}{2c}\right)\arctan\left(\frac{2cx+}{\sqrt{4ac-}}\right.}{\sqrt{4ac-b^2}}}{(e^2a-bde+c\,d^2)(a\,g^2-bfg+c\,f^2)}$
risch	Expression too large to display

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*ln(e*x+d)+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b
*f*g+c*f^2)*(1/2*(b*c*e*g-c^2*d*g-c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*c*e*g+b^
```

```
2*e*g-b*c*d*g-b*c*e*f+c^2*d*f-1/2*(b*c*e*g-c^2*d*g-c^2*e*f)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))+g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f
^2)*ln(g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.58

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \frac{e^3\log\left(|ex+d|\right)}{cd^2e^2f-bde^3f+ae^4f-cd^3eg+bd^2e^2g-ade^3g} - \frac{g^3\log\left(|gx+f|\right)}{cef^3g-cdf^2g^2-bef^2g^2+bdfg^3+aefg^3-adg^4} - \frac{(cef+cdg-beg)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2f^2-bcdef^2+ace^2f^2-bcd^2fg+b^2defg-abe^2fg+acd^2g^2-abdeg^2+a^2e^2g^2\right)} + \frac{\left(2\,c^2df-bcef-bcdg+b^2eg-2\,aceg\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2f^2-bcdef^2+ace^2f^2-bcd^2fg+b^2defg-abe^2fg+acd^2g^2-abdeg^2+a^2e^2g^2\right)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] e^3*log(abs(e*x + d))/(c*d^2*e^2*f - b*d*e^3*f + a*e^4*f - c*d^3*e*g + b*d^
2*e^2*g - a*d*e^3*g) - g^3*log(abs(g*x + f))/(c*e*f^3*g - c*d*f^2*g^2 - b*e
*f^2*g^2 + b*d*f*g^3 + a*e*f*g^3 - a*d*g^4) - 1/2*(c*e*f + c*d*g - b*e*g)*l
og(c*x^2 + b*x + a)/(c^2*d^2*f^2 - b*c*d*e*f^2 + a*c*e^2*f^2 - b*c*d^2*f*g
+ b^2*d*e*f*g - a*b*e^2*f*g + a*c*d^2*g^2 - a*b*d*e*g^2 + a^2*e^2*g^2) + (2
*c^2*d*f - b*c*e*f - b*c*d*g + b^2*e*g - 2*a*c*e*g)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d*e*f^2 + a*c*e^2*f^2 - b*c*d^2*f*g + b
^2*d*e*f*g - a*b*e^2*f*g + a*c*d^2*g^2 - a*b*d*e*g^2 + a^2*e^2*g^2)*sqrt(-b
^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 32.76 (sec) , antiderivative size = 12173, normalized size of antiderivative = 49.48

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 - a^3*b^2*e^5*
g^5*(b^2 - 4*a*c)^(1/2) - c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) - c^5*d^5*f^3
*g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 1
8*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 + 4*a^4*c*e
^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*
a*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^
2*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 +
a^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g
^5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*
```

b^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f^3*g^2*x - 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b*c^3*e^5*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d^5*f*g^4*(b^2 - 4*a*c)^(1/2) + 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c*d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*x + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x - 6*b^5*c*e^5*f^3*g^2*x + 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - a^2*b^3*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21*a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^3 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4*g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3 - b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3*x + 3*a*b*c^4*d*e^4*f^5 + 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^4*d^5*f*g^4 + 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4*f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x + a*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5*f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e^2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c^4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^4*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g^5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d^3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x + 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f^3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x + 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 24*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) + b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3*d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2* g^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*b^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f^2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17*b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*e^3*f^2*g^3*x - 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*c*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b^5*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4 + 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4*c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a*b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x + 4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*x + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*g^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4*f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x + 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^3*e^2*f^4*g*(b^2 - 4*a*c)^(1/2) + b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*c*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*c*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^4*e*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d*e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^2*b*c^3*d^3*e^2*f*g^4 + 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 15*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 25*b^3*c^2*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^2*e^3*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 12*b^2*c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 8*b^4*c*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3*e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*x - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*

c^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f
*g^4*x + 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^3*d^3*e^
2*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/
2) + 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d*e^4*f^2
g^3(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) +
20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*g^5*
x*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 13
*a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 4
1*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x - 20*a*c^4*d^2*e^3*f^3*g
^2*x*(b^2 - 4*a*c)^(1/2) - 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) +
a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + a^2*c^3*d^2*e^3*f*g^4*x*(b^2
- 4*a*c)^(1/2) - 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^
2*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*
a*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x - 14*a*b*c^3*d*e^4*f^4*g*(b^2
- 4*a*c)^(1/2) - 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*d^
4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2)
+ 13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 13*a*c^4*d^4*e*f*g^4*x*(b^2
- 4*a*c)^(1/2) - 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 60*a*b*
c^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b
^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 18*a
*b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 -
4*a*c)^(1/2) - 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2
*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^2*c*d*g - 4*a*c^2*d*g - 4*a*c^2*e*f
- b^3*e*g + b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*
c)^(1/2) + 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a
*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2
*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^
2*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b
^2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*
c*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) - (
log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 + a^3*b^2*e^5*g
^5*(b^2 - 4*a*c)^(1/2) + c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) + c^5*d^5*f^3*
g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 18
*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 - 4*a^4*c*e^
5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*a
*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^2
*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 + a
^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g^
5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*b
^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f^
3*g^2*x + 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b*c^3*e^5*f^5*(b^2 -
4*a*c)^(1/2) - 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) - 7*a*c^4*d^5*f*g^4*(b
^2 - 4*a*c)^(1/2) - 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^5*g^5
x(b^2 - 4*a*c)^(1/2) - 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^
2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c*

d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*x + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x - 6*b^5*c*e^5*f^3*g^2*x - 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - a^2*b^3*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) + a^2*b^3*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c^3*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21*a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^3 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4*g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3 - b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3*x + 3*a*b*c^4*d*e^4*f^5 - 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^4*d^5*f*g^4 - 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4*f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x + a*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5*f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e^2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c^4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^4*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g^5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d^3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x + 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f^3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x - 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) + 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 24*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) - b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 9*a^2*c^3*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3*d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*g^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*b^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f^2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17*b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*e^3*f^2*g^3*x + 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*c*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) + 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c *e^5*g^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b^5*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4 - 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4*c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a*b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x + 4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*x + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*g^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4*f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x - 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^3*e^2*f^4*g*(b^2 - 4*a*c)^(1/2) - b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^4*e*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d*e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^2*b*c^3*d^3*e^2*f*g^4 - 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 15*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 25*b^3*c^2*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) - 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*b^2*c*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^2*e^3*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 8*b^4*c*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3*e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*x - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*c^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f*g^4*x - 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^3*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*g^5*x

*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 13*
a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 41
*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x + 20*a*c^4*d^2*e^3*f^3*g^
2*x*(b^2 - 4*a*c)^(1/2) + 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) -
a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - a^2*c^3*d^2*e^3*f*g^4*x*(b^2
- 4*a*c)^(1/2) + 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2
*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*a
*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x + 14*a*b*c^3*d*e^4*f^4*g*(b^2 -
4*a*c)^(1/2) + 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*d^4
*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) -
13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 13*a*c^4*d^4*e*f*g^4*x*(b^2 -
4*a*c)^(1/2) + 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 60*a*b*c
^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b^
2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 18*a*
b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 -
4*a*c)^(1/2) + 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*b*c^2*
d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^3*e*g + 4*a*c^2*d*g + 4*a*c^2*e*f - b
^2*c*d*g - b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*c
)^(1/2) - 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a*
c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*
g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2
*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^
2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c
*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) + (e
^2*log(d + e*x))/(a*e^3*f - c*d^3*g - a*d*e^2*g - b*d*e^2*f + b*d^2*e*g + c
*d^2*e*f) + (g^2*log(f + g*x))/(a*d*g^3 - c*e*f^3 - a*e*f*g^2 - b*d*f*g^2 +
b*e*f^2*g + c*d*f^2*g)

3.818 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 644

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2} dx =$$
$$-\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df+b^2eg-c(bef+bdg+2aeg))x}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+bx+cx^2\right)}$$
$$+\frac{2c(2c^2df+b^2eg-c(bef+bdg+2aeg))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)}$$
$$+\frac{(b^2e^2g^2(bef+bdg-2aeg)-2c^3df(e^2f^2+defg+d^2g^2)+2ceg(a^2e^2g^2+abeg(ef+dg)-b^2(ef+dg)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2(cf^2-g}$$
$$+\frac{e^4\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2(ef-dg)}-\frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2}$$
$$-\frac{(cef+cdg-beg)\left(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg))\right)\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2\left(cf^2-g(bf-ag)\right)^2}$$

```
[Out] (-b^3*e*g+b^2*c*(d*g+e*f)-2*a*c^2*(d*g+e*f)-b*c*(-3*a*e*g+c*d*f)-c*(2*c^2*d
*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
f^2-g*(-a*g+b*f))/(c*x^2+b*x+a)+2*c*(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e
*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+
c*d^2)/(c*f^2-g*(-a*g+b*f))+e^4*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)-
g^4*ln(g*x+f)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2-1/2*(-b*e*g+c*d*g+c*e*f)*(c*
(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c
*d^2)^2/(c*f^2-g*(-a*g+b*f))^2+(b^2*e^2*g^2*(-2*a*e*g+b*d*g+b*e*f)-2*c^3*d*
f*(d^2*g^2+d*e*f*g+e^2*f^2)+2*c*e*g*(a^2*e^2*g^2+a*b*e*g*(d*g+e*f)-b^2*(d*g
+e*f)^2)-c^2*(4*a*d*e^2*f*g^2-b*(d^3*g^3+5*d^2*e*f*g^2+5*d*e^2*f^2*g+e^3*f^
3)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^2/(c*f^2-g*(
-a*g+b*f))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {907, 652, 632, 212, 648, 642}

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2)+b^2e^2g^2(-2aeg+bdg+bef)-c^2(4ade^2\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(b\right.}$$

$$+\frac{2c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}$$

$$-\frac{cx(-c(2aeg+bdg+bef)+b^2eg+2c^2df)+bc(cdf-3aeg)+2ac^2(dg+ef)+b^3eg-b^2c(dg+ef)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}$$

$$-\frac{\log\left(a+bx+cx^2\right)\left(-beg+cdg+cef\right)\left(eg(2aeg-b(dg+ef))+c(d^2g^2+e^2f^2)\right)}{2\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}$$

$$+\frac{e^4\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)^2}-\frac{g^4\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

integral

$$= \int \left(-\frac{e^5}{(cd^2 - bde + ae^2)^2 (-ef + dg)(d + ex)} - \frac{g^5}{(ef - dg)(cf^2 - bfg + ag^2)^2 (f + gx)} + \frac{c^2df + b^2eg - c(bef + bdg + aeg) - c(cef + cdg - beg)x}{(cd^2 - bde + ae^2)(cf^2 - bfg + ag^2)(a + bx + cx^2)^2} + \frac{-b^2e^2g^2(bef + bdg - 2aeg) + c^3df(e^2f^2 + defg + d^2g^2) + c^2(2ade^2fg^2 - b(ef + dg)^3) - ceg(a^2e^2g^2 + \cdots}{(cd^2 - bde + ae^2)^2 (c \cdots}$$

$$= \frac{e^4 \log(d+ex)}{(cd^2-bde+ae^2)^2 (ef-dg)} - \frac{g^4 \log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)^2}$$

$$+ \frac{\int \frac{-b^2e^2g^2(bef+bdg-2aeg)+c^3df(e^2f^2+defg+d^2g^2)+c^2(2ade^2fg^2-b(ef+dg)^3)-ceg(a^2e^2g^2+2abeg(ef+dg)-b^2(2e^2f^2+3defg+}{a+bx+cx^2}}{(cd^2-bde+ae^2)^2(cf^2-g(bf-ag))^2}$$

$$+ \frac{\int \frac{c^2df+b^2eg-c(bef+bdg+aeg)-c(cef+cdg-beg)x}{(a+bx+cx^2)^2}\,dx}{(cd^2-bde+ae^2)(cf^2-g(bf-ag))}$$

$$=$$

$$-\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df+b^2eg-c(bef+bdg+2aeg))}{(b^2-4ac)(cd^2-bde+ae^2)(cf^2-g(bf-ag))(a+bx+cx^2)}$$

$$+ \frac{e^4 \log(d+ex)}{(cd^2-bde+ae^2)^2 (ef-dg)} - \frac{g^4 \log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)^2}$$

$$- \frac{(c(2c^2df+b^2eg-c(bef+bdg+2aeg)))\int \frac{1}{a+bx+cx^2}\,dx}{(b^2-4ac)(cd^2-bde+ae^2)(cf^2-g(bf-ag))}$$

$$- \frac{((cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg))))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2(cd^2-bde+ae^2)^2(cf^2-g(bf-ag))^2}$$

$$- \frac{(b^2e^2g^2(bef+bdg-2aeg)-2c^3df(e^2f^2+defg+d^2g^2)+2ceg(a^2e^2g^2+abeg(ef+dg)-b^2(ef}{2(cd^2-bde+ae^2)^2(cf^2-g(}$$

$$=$$

$$-\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df+b^2eg-c(bef+bdg+2aeg))}{(b^2-4ac)(cd^2-bde+ae^2)(cf^2-g(bf-ag))(a+bx+cx^2)}$$

$$+ \frac{e^4 \log(d+ex)}{(cd^2-bde+ae^2)^2 (ef-dg)} - \frac{g^4 \log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)^2}$$

$$- \frac{(cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))\log(a+bx+cx^2)}{2(cd^2-bde+ae^2)^2(cf^2-g(bf-ag))^2}$$

$$+ \frac{(2c(2c^2df+b^2eg-c(bef+bdg+2aeg)))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{(b^2-4ac)(cd^2-bde+ae^2)(cf^2-g(bf-ag))}$$

$$+ \frac{(b^2e^2g^2(bef+bdg-2aeg)-2c^3df(e^2f^2+defg+d^2g^2)+2ceg(a^2e^2g^2+abeg(ef+dg)-b^2(ef}{(cd^2-bde+ae^2)^2}$$

$$= -\frac{b^3eg - b^2c(ef+dg) + 2ac^2(ef+dg) + bc(cdf-3aeg) + c(2c^2df + b^2eg - c(bef+bdg+2aeg)}{(b^2-4ac)(cd^2-bde+ae^2)(cf^2-g(bf-ag))(a+bx+cx^2)}$$
$$+ \frac{2c(2c^2df+b^2eg-c(bef+bdg+2aeg))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}(cd^2-bde+ae^2)(cf^2-g(bf-ag))}$$
$$+ \frac{(b^2e^2g^2(bef+bdg-2aeg) - 2c^3df(e^2f^2+defg+d^2g^2) + 2ceg(a^2e^2g^2+abeg(ef+dg)-b^2(e}{\sqrt{b^2-4ac}(cd^2-bde+ae^2)^2(}$$
$$+ \frac{e^4\log(d+ex)}{(cd^2-bde+ae^2)^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)^2}$$
$$- \frac{(cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))\log(a+bx+cx^2)}{2(cd^2-bde+ae^2)^2(cf^2-g(bf-ag))^2}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^2}\,dx$$
$$= \frac{-b^3eg + b^2c(dg+e(f-gx)) - 2c^2(adg+cdfx+ae(f-gx)) + bc(3aeg+c(-df+efx+dgx))}{(b^2-4ac)(-cd^2+e(bd-ae))(-cf^2+g(bf-ag))(a+x(b+cx))}$$
$$+ \frac{(4c^5d^3f^3 + b^4e^2g^2(bef+bdg-2aeg) - 2b^2ceg(-6a^2e^2g^2+2abeg(ef+dg)+b^2(e^2f^2+defg+d^2g^2))}{}$$
$$+ \frac{e^4\log(d+ex)}{(cd^2+e(-bd+ae))^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)(cf^2+g(-bf+ag))^2}$$
$$- \frac{(cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))\log(a+x(b+cx))}{2(cd^2+e(-bd+ae))^2(cf^2+g(-bf+ag))^2}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f -
 g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*c^5
*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*e^2*
g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c^4*d*
f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^2*(-12
*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 + d*e*f*
g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3)) - 2*c^3
*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) + a*b*(3*e
^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*(c*
f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))
```

```
^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g)
)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(
e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(c*f^2 +
 g*(-(b*f) + a*g))^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2227 vs. 2(635) = 1270.

Time = 1.49 (sec) , antiderivative size = 2228, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	2228
risch	Expression too large to display	29824

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)-1/(a*e^2-b*d*e+c*d^2)^2/(a*g
^2-b*f*g+c*f^2)^2*((c*(2*a^3*c*e^3*g^3-a^2*b^2*e^3*g^3-a^2*b*c*d*e^2*g^3-a^
2*b*c*e^3*f*g^2+2*a^2*c^2*d^2*e*g^3-2*a^2*c^2*d*e^2*f*g^2+2*a^2*c^2*e^3*f^2
*g+a*b^3*d*e^2*g^3+a*b^3*e^3*f*g^2-2*a*b^2*c*d^2*e*g^3-2*a*b^2*c*e^3*f^2*g+
a*b*c^2*d^3*g^3+a*b*c^2*d^2*e*f*g^2+a*b*c^2*d*e^2*f^2*g+a*b*c^2*e^3*f^3-2*a
*c^3*d^3*f*g^2+2*a*c^3*d^2*e*f^2*g-2*a*c^3*d*e^2*f^3-b^4*d*e^2*f*g^2+2*b^3*
c*d^2*e*f*g^2+2*b^3*c*d*e^2*f^2*g-b^2*c^2*d^3*f*g^2-5*b^2*c^2*d^2*e*f^2*g-b
^2*c^2*d*e^2*f^3+3*b*c^3*d^3*f^2*g+3*b*c^3*d^2*e*f^3-2*c^4*d^3*f^3)/(4*a*c-
b^2)*x+(3*a^3*b*c*e^3*g^3-2*a^3*c^2*d*e^2*g^3-2*a^3*c^2*e^3*f*g^2-a^2*b^3*e
^3*g^3-2*a^2*b^2*c*d*e^2*g^3-2*a^2*b^2*c*e^3*f*g^2+5*a^2*b*c^2*d^2*e*g^3+3*
a^2*b*c^2*d*e^2*f*g^2+5*a^2*b*c^2*e^3*f^2*g-2*a^2*c^3*d^3*g^3-2*a^2*c^3*d^2
*e*f*g^2-2*a^2*c^3*d*e^2*f^2*g-2*a^2*c^3*e^3*f^3+a*b^4*d*e^2*g^3+a*b^4*e^3*
f*g^2-2*a*b^3*c*d^2*e*g^3+a*b^3*c*d*e^2*f*g^2-2*a*b^3*c*e^3*f^2*g+a*b^2*c^2
*d^3*g^3-3*a*b^2*c^2*d^2*e*f*g^2-3*a*b^2*c^2*d*e^2*f^2*g+a*b^2*c^2*e^3*f^3+
a*b*c^3*d^3*f*g^2+7*a*b*c^3*d^2*e*f^2*g+a*b*c^3*d*e^2*f^3-2*a*c^4*d^3*f^2*g
-2*a*c^4*d^2*e*f^3-b^5*d*e^2*f*g^2+2*b^4*c*d^2*e*f*g^2+2*b^4*c*d*e^2*f^2*g-
b^3*c^2*d^3*f*g^2-4*b^3*c^2*d^2*e*f^2*g-b^3*c^2*d*e^2*f^3+2*b^2*c^3*d^3*f^2
*g+2*b^2*c^3*d^2*e*f^3-b*c^4*d^3*f^3)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b
^2)*(1/2*(-8*a^2*b*c^2*e^3*g^3+8*a^2*c^3*d*e^2*g^3+8*a^2*c^3*e^3*f*g^2+2*a*
b^3*c*e^3*g^3+2*a*b^2*c^2*d*e^2*g^3+2*a*b^2*c^2*e^3*f*g^2-8*a*b*c^3*d^2*e*g
^3-8*a*b*c^3*d*e^2*f*g^2-8*a*b*c^3*e^3*f^2*g+4*a*c^4*d^3*g^3+4*a*c^4*d^2*e*
f*g^2+4*a*c^4*d*e^2*f^2*g+4*a*c^4*e^3*f^3-b^4*c*d*e^2*g^3-b^4*c*e^3*f*g^2+2
*b^3*c^2*d^2*e*g^3+2*b^3*c^2*d*e^2*f*g^2+2*b^3*c^2*e^3*f^2*g-b^2*c^3*d^3*g^
3-b^2*c^3*d^2*e*f*g^2-b^2*c^3*d*e^2*f^2*g-b^2*c^3*e^3*f^3)/c*ln(c*x^2+b*x+a
)+2*(7*a^2*b*c^2*d*e^2*g^3+7*a^2*b*c^2*e^3*f*g^2+2*b^4*c*d^2*e*g^3+2*b^4*c*
e^3*f^2*g-10*a^2*b^2*c*e^3*g^3+2*a^2*c^3*d^2*e*g^3+2*a^2*c^3*e^3*f^2*g+5*a*
b*c^3*d^3*g^3+5*a*b*c^3*e^3*f^3-6*a*c^4*d^3*f*g^2-6*a*c^4*d*e^2*f^3+3*b*c^4
*d^3*f^2*g+3*b*c^4*d^2*e*f^3+2*a*b^4*e^3*g^3-b^5*d*e^2*g^3-b^5*e^3*f*g^2-b^
```

```
3*c^2*d^3*g^3-b^3*c^2*e^3*f^3+6*a^3*c^2*e^3*g^3-b^3*c^2*d^2*e*f*g^2-b^3*c^2
*d*e^2*f^2*g-4*b^2*c^3*d^2*e*f^2*g-10*a^2*c^3*d*e^2*f*g^2+3*a*b^3*c*d*e^2*g
^3+3*a*b^3*c*e^3*f*g^2-10*a*b^2*c^2*d^2*e*g^3-1/2*(-8*a^2*b*c^2*e^3*g^3+8*a
^2*c^3*d*e^2*g^3+8*a^2*c^3*e^3*f*g^2+2*a*b^3*c*e^3*g^3+2*a*b^2*c^2*d*e^2*g^
3+2*a*b^2*c^2*e^3*f*g^2-8*a*b*c^3*d^2*e*g^3-8*a*b*c^3*d*e^2*f*g^2-8*a*b*c^3
*e^3*f^2*g+4*a*c^4*d^3*g^3+4*a*c^4*d^2*e*f*g^2+4*a*c^4*d*e^2*f^2*g+4*a*c^4*
e^3*f^3-b^4*c*d*e^2*g^3-b^4*c*e^3*f*g^2+2*b^3*c^2*d^2*e*g^3+2*b^3*c^2*d*e^2
*f*g^2+2*b^3*c^2*e^3*f^2*g-b^2*c^3*d^3*g^3-b^2*c^3*d^2*e*f*g^2-b^2*c^3*d*e^
2*f^2*g-b^2*c^3*e^3*f^3)*b/c-2*c^5*d^3*f^3-10*a*b^2*c^2*d*e^2*f*g^2+13*a*b*
c^3*d^2*e*f*g^2+13*a*b*c^3*d*e^2*f^2*g-10*a*b^2*c^2*e^3*f^2*g-2*a*c^4*d^2*e
*f^2*g+2*b^4*c*d*e^2*f*g^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))))+g^4/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2*ln(g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3400 vs. $2(634) = 1268$.

Time = 0.30 (sec) , antiderivative size = 3400, normalized size of antiderivative = 5.28

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] e^5*log(abs(e*x + d))/(c^2*d^4*e^2*f - 2*b*c*d^3*e^3*f + b^2*d^2*e^4*f + 2*
a*c*d^2*e^4*f - 2*a*b*d*e^5*f + a^2*e^6*f - c^2*d^5*e*g + 2*b*c*d^4*e^2*g -
 b^2*d^3*e^3*g - 2*a*c*d^3*e^3*g + 2*a*b*d^2*e^4*g - a^2*d*e^5*g) - g^5*log
(abs(g*x + f))/(c^2*e*f^5*g - c^2*d*f^4*g^2 - 2*b*c*e*f^4*g^2 + 2*b*c*d*f^3
*g^3 + b^2*e*f^3*g^3 + 2*a*c*e*f^3*g^3 - b^2*d*f^2*g^4 - 2*a*c*d*f^2*g^4 -
2*a*b*e*f^2*g^4 + 2*a*b*d*f*g^5 + a^2*e*f*g^5 - a^2*d*g^6) - 1/2*(c^2*e^3*f
^3 + c^2*d*e^2*f^2*g - 2*b*c*e^3*f^2*g + c^2*d^2*e*f*g^2 - 2*b*c*d*e^2*f*g^
2 + b^2*e^3*f*g^2 + 2*a*c*e^3*f*g^2 + c^2*d^3*g^3 - 2*b*c*d^2*e*g^3 + b^2*d
*e^2*g^3 + 2*a*c*d*e^2*g^3 - 2*a*b*e^3*g^3)*log(c*x^2 + b*x + a)/(c^4*d^4*f
^4 - 2*b*c^3*d^3*e*f^4 + b^2*c^2*d^2*e^2*f^4 + 2*a*c^3*d^2*e^2*f^4 - 2*a*b*
c^2*d*e^3*f^4 + a^2*c^2*e^4*f^4 - 2*b*c^3*d^4*f^3*g + 4*b^2*c^2*d^3*e*f^3*g
 - 2*b^3*c*d^2*e^2*f^3*g - 4*a*b*c^2*d^2*e^2*f^3*g + 4*a*b^2*c*d*e^3*f^3*g
- 2*a^2*b*c*e^4*f^3*g + b^2*c^2*d^4*f^2*g^2 + 2*a*c^3*d^4*f^2*g^2 - 2*b^3*c
*d^3*e*f^2*g^2 - 4*a*b*c^2*d^3*e*f^2*g^2 + b^4*d^2*e^2*f^2*g^2 + 4*a*b^2*c*
d^2*e^2*f^2*g^2 + 4*a^2*c^2*d^2*e^2*f^2*g^2 - 2*a*b^3*d*e^3*f^2*g^2 - 4*a^2
*b*c*d*e^3*f^2*g^2 + a^2*b^2*e^4*f^2*g^2 + 2*a^3*c*e^4*f^2*g^2 - 2*a*b*c^2*
d^4*f*g^3 + 4*a*b^2*c*d^3*e*f*g^3 - 2*a*b^3*d^2*e^2*f*g^3 - 4*a^2*b*c*d^2*e
^2*f*g^3 + 4*a^2*b^2*d*e^3*f*g^3 - 2*a^3*b*e^4*f*g^3 + a^2*c^2*d^4*g^4 - 2*
a^2*b*c*d^3*e*g^4 + a^2*b^2*d^2*e^2*g^4 + 2*a^3*c*d^2*e^2*g^4 - 2*a^3*b*d*e
^3*g^4 + a^4*e^4*g^4) - (4*c^5*d^3*f^3 - 6*b*c^4*d^2*e*f^3 + 12*a*c^4*d*e^2
*f^3 + b^3*c^2*e^3*f^3 - 6*a*b*c^3*e^3*f^3 - 6*b*c^4*d^3*f^2*g + 8*b^2*c^3*
d^2*e*f^2*g + 4*a*c^4*d^2*e*f^2*g + b^3*c^2*d*e^2*f^2*g - 22*a*b*c^3*d*e^2*
f^2*g - 2*b^4*c*e^3*f^2*g + 12*a*b^2*c^2*e^3*f^2*g - 4*a^2*c^3*e^3*f^2*g +
12*a*c^4*d^3*f*g^2 + b^3*c^2*d^2*e*f*g^2 - 22*a*b*c^3*d^2*e*f*g^2 - 2*b^4*c
*d*e^2*f*g^2 + 12*a*b^2*c^2*d*e^2*f*g^2 + 20*a^2*c^3*d*e^2*f*g^2 + b^5*e^3*
f*g^2 - 4*a*b^3*c*e^3*f*g^2 - 6*a^2*b*c^2*e^3*f*g^2 + b^3*c^2*d^3*g^3 - 6*a
*b*c^3*d^3*g^3 - 2*b^4*c*d^2*e*g^3 + 12*a*b^2*c^2*d^2*e*g^3 - 4*a^2*c^3*d^2
*e*g^3 + b^5*d*e^2*g^3 - 4*a*b^3*c*d*e^2*g^3 - 6*a^2*b*c^2*d*e^2*g^3 - 2*a*
b^4*e^3*g^3 + 12*a^2*b^2*c*e^3*g^3 - 12*a^3*c^2*e^3*g^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^4*f^4 - 4*a*c^5*d^4*f^4 - 2*b^3*c^3*d^3*e*
f^4 + 8*a*b*c^4*d^3*e*f^4 + b^4*c^2*d^2*e^2*f^4 - 2*a*b^2*c^3*d^2*e^2*f^4 -
 8*a^2*c^4*d^2*e^2*f^4 - 2*a*b^3*c^2*d*e^3*f^4 + 8*a^2*b*c^3*d*e^3*f^4 + a^
2*b^2*c^2*e^4*f^4 - 4*a^3*c^3*e^4*f^4 - 2*b^3*c^3*d^4*f^3*g + 8*a*b*c^4*d^4
*f^3*g + 4*b^4*c^2*d^3*e*f^3*g - 16*a*b^2*c^3*d^3*e*f^3*g - 2*b^5*c*d^2*e^2
*f^3*g + 4*a*b^3*c^2*d^2*e^2*f^3*g + 16*a^2*b*c^3*d^2*e^2*f^3*g + 4*a*b^4*c
```

```
*d*e^3*f^3*g - 16*a^2*b^2*c^2*d*e^3*f^3*g - 2*a^2*b^3*c*e^4*f^3*g + 8*a^3*b
*c^2*e^4*f^3*g + b^4*c^2*d^4*f^2*g^2 - 2*a*b^2*c^3*d^4*f^2*g^2 - 8*a^2*c^4*
d^4*f^2*g^2 - 2*b^5*c*d^3*e*f^2*g^2 + 4*a*b^3*c^2*d^3*e*f^2*g^2 + 16*a^2*b*
c^3*d^3*e*f^2*g^2 + b^6*d^2*e^2*f^2*g^2 - 12*a^2*b^2*c^2*d^2*e^2*f^2*g^2 -
16*a^3*c^3*d^2*e^2*f^2*g^2 - 2*a*b^5*d*e^3*f^2*g^2 + 4*a^2*b^3*c*d*e^3*f^2*
g^2 + 16*a^3*b*c^2*d*e^3*f^2*g^2 + a^2*b^4*e^4*f^2*g^2 - 2*a^3*b^2*c*e^4*f^
2*g^2 - 8*a^4*c^2*e^4*f^2*g^2 - 2*a*b^3*c^2*d^4*f*g^3 + 8*a^2*b*c^3*d^4*f*g
^3 + 4*a*b^4*c*d^3*e*f*g^3 - 16*a^2*b^2*c^2*d^3*e*f*g^3 - 2*a*b^5*d^2*e^2*f
*g^3 + 4*a^2*b^3*c*d^2*e^2*f*g^3 + 16*a^3*b*c^2*d^2*e^2*f*g^3 + 4*a^2*b^4*d
*e^3*f*g^3 - 16*a^3*b^2*c*d*e^3*f*g^3 - 2*a^3*b^3*e^4*f*g^3 + 8*a^4*b*c*e^4
*f*g^3 + a^2*b^2*c^2*d^4*g^4 - 4*a^3*c^3*d^4*g^4 - 2*a^2*b^3*c*d^3*e*g^4 +
8*a^3*b*c^2*d^3*e*g^4 + a^2*b^4*d^2*e^2*g^4 - 2*a^3*b^2*c*d^2*e^2*g^4 - 8*a
^4*c^2*d^2*e^2*g^4 - 2*a^3*b^3*d*e^3*g^4 + 8*a^4*b*c*d*e^3*g^4 + a^4*b^2*e^
4*g^4 - 4*a^5*c*e^4*g^4)*sqrt(-b^2 + 4*a*c)) - (b*c^4*d^3*f^3 - 2*b^2*c^3*d
^2*e*f^3 + 2*a*c^4*d^2*e*f^3 + b^3*c^2*d*e^2*f^3 - a*b*c^3*d*e^2*f^3 - a*b^
2*c^2*e^3*f^3 + 2*a^2*c^3*e^3*f^3 - 2*b^2*c^3*d^3*f^2*g + 2*a*c^4*d^3*f^2*g
 + 4*b^3*c^2*d^2*e*f^2*g - 7*a*b*c^3*d^2*e*f^2*g - 2*b^4*c*d*e^2*f^2*g + 3*
a*b^2*c^2*d*e^2*f^2*g + 2*a^2*c^3*d*e^2*f^2*g + 2*a*b^3*c*e^3*f^2*g - 5*a^2
*b*c^2*e^3*f^2*g + b^3*c^2*d^3*f*g^2 - a*b*c^3*d^3*f*g^2 - 2*b^4*c*d^2*e*f*
g^2 + 3*a*b^2*c^2*d^2*e*f*g^2 + 2*a^2*c^3*d^2*e*f*g^2 + b^5*d*e^2*f*g^2 - a
*b^3*c*d*e^2*f*g^2 - 3*a^2*b*c^2*d*e^2*f*g^2 - a*b^4*e^3*f*g^2 + 2*a^2*b^2*
c*e^3*f*g^2 + 2*a^3*c^2*e^3*f*g^2 - a*b^2*c^2*d^3*g^3 + 2*a^2*c^3*d^3*g^3 +
 2*a*b^3*c*d^2*e*g^3 - 5*a^2*b*c^2*d^2*e*g^3 - a*b^4*d*e^2*g^3 + 2*a^2*b^2*
c*d*e^2*g^3 + 2*a^3*c^2*d*e^2*g^3 + a^2*b^3*e^3*g^3 - 3*a^3*b*c*e^3*g^3 + (
2*c^5*d^3*f^3 - 3*b*c^4*d^2*e*f^3 + b^2*c^3*d*e^2*f^3 + 2*a*c^4*d*e^2*f^3 -
 a*b*c^3*e^3*f^3 - 3*b*c^4*d^3*f^2*g + 5*b^2*c^3*d^2*e*f^2*g - 2*a*c^4*d^2*
e*f^2*g - 2*b^3*c^2*d*e^2*f^2*g - a*b*c^3*d*e^2*f^2*g + 2*a*b^2*c^2*e^3*f^2
*g - 2*a^2*c^3*e^3*f^2*g + b^2*c^3*d^3*f*g^2 + 2*a*c^4*d^3*f*g^2 - 2*b^3*c^
2*d^2*e*f*g^2 - a*b*c^3*d^2*e*f*g^2 + b^4*c*d*e^2*f*g^2 + 2*a^2*c^3*d*e^2*f
*g^2 - a*b^3*c*e^3*f*g^2 + a^2*b*c^2*e^3*f*g^2 - a*b*c^3*d^3*g^3 + 2*a*b^2*
c^2*d^2*e*g^3 - 2*a^2*c^3*d^2*e*g^3 - a*b^3*c*d*e^2*g^3 + a^2*b*c^2*d*e^2*g
^3 + a^2*b^2*c*e^3*g^3 - 2*a^3*c^2*e^3*g^3)*x)/((c*d^2 - b*d*e + a*e^2)^2*(
c*f^2 - b*f*g + a*g^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 54.58 (sec) , antiderivative size = 130035, normalized size of antiderivative = 201.92

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((b^3*e*g + 2*a*c^2*d*g + 2*a*c^2*e*f + b*c^2*d*f - b^2*c*d*g - b^2*c*e*f -
 3*a*b*c*e*g)/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*
```

c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*c*d *e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g) - (x*(2*a*c^2*e*g - 2*c^3*d*f + b*c ^2*d*g + b*c^2*e*f - b^2*c*e*g))/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b ^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^ 3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g))/(a + b*x + c*x^ 2) + symsum(log((12*a^2*c^5*e^6*g^6 - 3*b^2*c^5*d^2*e^4*g^6 - 3*b^2*c^5*e^6 *f^2*g^4 + 4*c^7*d^2*e^4*f^2*g^4 - 2*a*b^2*c^4*e^6*g^6 + 16*a*c^6*d^2*e^4*g ^6 + 3*b^3*c^4*d*e^5*g^6 + 16*a*c^6*e^6*f^2*g^4 + 3*b^3*c^4*e^6*f*g^5 - 4*b *c^6*d*e^5*f^2*g^4 - 4*b*c^6*d^2*e^4*f*g^5 - 16*a*b*c^5*d*e^5*g^6 - 16*a*b* c^5*e^6*f*g^5 + 16*a*c^6*d*e^5*f*g^5)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e ^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32 *a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c ^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2 *f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^ 3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b *c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^ 3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c ^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5* c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^ 2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e ^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^ 2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2 *d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3 *c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^ 4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32* a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f ^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a *b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f ^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5 *c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3* d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64* a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6 *b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^ 5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f ^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2* z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 5 12*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10

*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^1
0*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*
e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g
^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4
+ 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a
^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^
5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9
*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*
z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*
a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8
*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f
^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*
z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16
*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*
c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3
*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*
z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*
b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5
*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*
a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 56
32*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 +
 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^
4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7
*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*
g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f
^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^
4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5
*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*
d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c
^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^
2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6
*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*
a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 33
60*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 -
 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^
4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7
*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*
g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f
^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^
2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3
*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*
d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c
^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^
6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5
*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*

a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 17
28*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 -
1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^
4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2
*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*
g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f
^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^
8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7
*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*
d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c
^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^
3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4
*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*
a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 13
60*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 -
1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^
4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8
*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*
g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f
^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7
*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^
2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e
^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*
e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8
*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^
5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d
^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*
d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2
*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^
4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c
^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*
c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7
*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^
7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b
^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*
b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3
*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^
3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a
^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*
a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504
*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 50
4*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 4
16*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 -
352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 -
352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4

- 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4
 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^
4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z
^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*
z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^
7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*
g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5
*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^
2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f
^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4
*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e
^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*
e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*
e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e
^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^
4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3
*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2
*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^
5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4
 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4
+ 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 -
 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 -
2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2
112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 21
12*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 166
4*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664
*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*
a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a
^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^
6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5
*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*
b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b
^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*
c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c
^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*
c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c
^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^
4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5
*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d
^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*
e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3
*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*
g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8
*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^
4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 +

640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 64
0*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a
^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*
b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c
^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^
3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d
^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^
9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5
*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*
g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g
*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^
4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 -
 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 28
8*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a
^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*
b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c
^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*
d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2
*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^
7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f
^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*
g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^
6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*
z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4
 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 -
196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192
*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^
2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b
^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*
c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2
*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*
f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5
*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^
3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z
^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4
+ 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 2
8*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a
^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^
11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c
^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8
*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^
5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f
^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^
3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 -
 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*

c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*
e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*
g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 -
5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c
^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^
6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5
*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*
g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z
^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 -
2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*
a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c
^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^
4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8
*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*
g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z
^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 +
40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b
^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e
^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*
g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4
- 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*
b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*
e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6
*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*
z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 -
240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*
a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^
6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*
e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^
7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^
4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 3
20*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^
4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2
*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*
d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^
2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z
^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6
*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*
b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c
^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^
8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6
*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*
z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 +
48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*
b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8

*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*
e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f
^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z
^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 +
90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*
b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^1
0*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^
8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 -
14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^
7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4
*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11
*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^
9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a
^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*
e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*
z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*
a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3
*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*
g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^
4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^
8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4
*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^
9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4
 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*
c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10
*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z
^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6
*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*
f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4
 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a
^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*
d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*
z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^
9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7
*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^1
0*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*
a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d
^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z
^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*
a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7
*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^
10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*
a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e
*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 1
28*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9

*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 -
256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^
5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^
4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6
*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4
 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^
10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6
*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^
5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^
10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*
c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^1
0*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a
^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g
^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^
6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^1
0*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6
*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b
^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*
z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^
8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^
6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e
^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^
6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 +
 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4
*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^
6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g
^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 -
60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^
8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7
*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*
z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b
^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^
7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e
^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*
e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3
*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^
3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d
^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*
d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d
^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^
2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2
*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*
e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^
6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2
 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d

*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^
2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 +
 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 20
8*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a
^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*
b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c
^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^
5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5
*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5
*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*
g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z
^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 -
54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*
b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5
*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*
d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e
^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*
g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z
^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 -
32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*
b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c
^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^
2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f
^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*
z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 5
0*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7
*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*
f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z
^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6
*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^
4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z
^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*
c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2
*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*
z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 3
76*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*
c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5
*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^
6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z
^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 31
0*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b
^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*
e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*
g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 -
 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^

3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^

8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^
2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*
g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 4
4*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c
^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*
g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*
b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e
^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4
*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^
6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*
c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*
b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4
*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*
c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4
*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*
d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g
^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a
^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g
^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g
*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7
*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5
*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5
*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*
z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*
e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 1
6*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2
*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b
^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^
6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3
*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z
- 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*
d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a
*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4
*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*
a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z
 - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^
7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3
*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g
^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g
^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7
*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z +
3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*
z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c
^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5
*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g

^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*((13*a^2*b^5*c^2*e^7*g^7 - 56*a^3*b^3*c^3*e^7*g^7 + 24*a^2*c^7*d^5*e^2*g^7 - 2*b^4*c^5*d^5*e^2*g^7 + b^5*c^4*d^4*e^3*g^7 + b^6*c^3*d^3*e^4*g^7 - 2*b^7*c^2*d^2*e^5*g^7 + 24*a^2*c^7*e^7*f^5*g^2 - 2*b^4*c^5*e^7*f^5*g^2 + b^5*c^4*e^7*f^4*g^3 + b^6*c^3*e^7*f^3*g^4 - 2*b^7*c^2*e^7*f^2*g^5 - a*b^7*c*e^7*g^7 + b^8*c*d*e^6*g^7 + b^8*c*e^7*f*g^6 + 80*a^4*b*c^4*e^7*g^7 - 28*a^4*c^5*d*e^6*g^7 + b^3*c^6*d^6*e*g^7 - 28*a^4*c^5*e^7*f*g^6 + b^3*c^6*e^7*f^6*g + 4*c^9*d^3*e^4*f^6*g + 4*c^9*d^6*e*f^3*g^4 - 12*a*b^6*c^2*d*e^6*g^7 - 12*a*b^6*c^2*e^7*f*g^6 - 4*b*c^8*d^2*e^5*f^6*g - 4*b*c^8*d^6*e*f^2*g^5 - b^2*c^7*d*e^6*f^6*g - b^2*c^7*d^6*e*f*g^6 - 2*b^7*c^2*d*e^6*f*g^6 + 2*a*b^2*c^6*d^5*e^2*g^7 + 10*a*b^3*c^5*d^4*e^3*g^7 - 20*a*b^4*c^4*d^3*e^4*g^7 + 25*a*b^5*c^3*d^2*e^5*g^7 - 56*a^2*b*c^6*d^4*e^3*g^7 + 44*a^2*b^4*c^3*d*e^6*g^7 + 76*a^3*b*c^5*d^2*e^5*g^7 - 40*a^3*b^2*c^4*d*e^6*g^7 + 2*a*b^2*c^6*e^7*f^5*g^2 + 10*a*b^3*c^5*e^7*f^4*g^3 - 20*a*b^4*c^4*e^7*f^3*g^4 + 25*a*b^5*c^3*e^7*f^2*g^5 - 56*a^2*b*c^6*e^7*f^4*g^3 + 44*a^2*b^4*c^3*e^7*f*g^6 + 76*a^3*b*c^5*e^7*f^2*g^5 - 40*a^3*b^2*c^4*e^7*f*g^6 + 16*a*c^8*d^2*e^5*f^5*g^2 + 24*a*c^8*d^3*e^4*f^4*g^3 + 24*a*c^8*d^4*e^3*f^3*g^4 + 16*a*c^8*d^5*e^2*f^2*g^5 + 28*a^2*c^7*d*e^6*f^4*g^3 + 28*a^2*c^7*d^4*e^3*f*g^6 - 80*a^3*c^6*d*e^6*f^2*g^5 - 80*a^3*c^6*d^2*e^5*f*g^6 - 12*b*c^8*d^3*e^4*f^5*g^2 - 12*b*c^8*d^5*e^2*f^3*g^4 + 6*b^3*c^6*d*e^6*f^5*g^2 + 6*b^3*c^6*d^5*e^2*f*g^6 - 9*b^4*c^5*d*e^6*f^4*g^3 - 9*b^4*c^5*d^4*e^3*f*g^6 + 4*b^5*c^4*d*e^6*f^3*g^4 + 4*b^5*c^4*d^3*e^4*f*g^6 + b^6*c^3*d*e^6*f^2*g^5 + b^6*c^3*d^2*e^5*f*g^6 - 4*a*b*c^7*d^6*e*g^7 - 4*a*b*c^7*e^7*f^6*g + 8*a*c^8*d*e^6*f^6*g + 8*a*c^8*d^6*e*f*g^6 + 65*a^2*b^2*c^5*d^3*e^4*g^7 - 88*a^2*b^3*c^4*d^2*e^5*g^7 + 65*a^2*b^2*c^5*e^7*f^3*g^4 - 88*a^2*b^3*c^4*e^7*f^2*g^5 + 68*a^2*c^7*d^2*e^5*f^3*g^4 + 68*a^2*c^7*d^3*e^4*f^2*g^5 + 8*b^2*c^7*d^2*e^5*f^5*g^2 + 9*b^2*c^7*d^3*e^4*f^4*g^3 + 9*b^2*c^7*d^4*e^3*f^3*g^4 + 8*b^2*c^7*d^5*e^2*f^2*g^5 - b^3*c^6*d^2*e^5*f^4*g^3 + 4*b^3*c^6*d^3*e^4*f^3*g^4 - b^3*c^6*d^4*e^3*f^2*g^5 - 9*b^4*c^5*d^2*e^5*f^3*g^4 - 9*b^4*c^5*d^3*e^4*f^2*g^5 + 7*b^5*c^4*d^2*e^5*f^2*g^5 + 74*a*b^2*c^6*d^2*e^5*f^3*g^4 + 74*a*b^2*c^6*d^3*e^4*f^2*g^5 - 28*a*b^3*c^5*d^2*e^5*f^2*g^5 - 120*a^2*b*c^6*d^2*e^5*f^2*g^5 + 159*a^2*b^2*c^5*d*e^6*f^2*g^5 + 159*a^2*b^2*c^5*d^2*e^5*f*g^6 - 36*a*b*c^7*d*e^6*f^5*g^2 - 36*a*b*c^7*d^5*e^2*f*g^6 + 28*a*b^5*c^3*d*e^6*f*g^6 + 104*a^3*b*c^5*d*e^6*f*g^6 - 56*a*b*c^7*d^2*e^5*f^4*g^3 - 96*a*b*c^7*d^3*e^4*f^3*g^4 - 56*a*b*c^7*d^4*e^3*f^2*g^5 + 44*a*b^2*c^6*d*e^6*f^4*g^3 + 44*a*b^2*c^6*d^4*e^3*f*g^6 - 32*a*b^4*c^4*d*e^6*f^2*g^5 - 32*a*b^4*c^4*d^2*e^5*f*g^6 - 116*a^2*b*c^6*d*e^6*f^3*g^4 - 116*a^2*b*c^6*d^3*e^4*f*g^6 - 112*a^2*b^3*c^4*d*e^6*f*g^6)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a

^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c
^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*
c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^
2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5
*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*
e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3
*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*
f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2
*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^
2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3
*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a
*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*
a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2
*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*
a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3
*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b
^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^
2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*
d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) -
root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 -
792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9
*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*
d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f
^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*
z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 +
256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a
^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7
*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*
e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9
*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4
+ 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^
10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c
*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^
7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4
 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b
^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^
4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f
^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^
4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*
a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*
c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^
5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 -
 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^
4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4
*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*

g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f
^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^
5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3
*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*
d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c
^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^
3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7
*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*
a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 37
76*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 +
3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^
4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2
*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*
g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f
^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^
3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3
*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*
d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c
^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^
4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2
*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*
a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 18
28*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 -
1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^
4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8
*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*
g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f
^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^
8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7
*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*
d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c
^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^
2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7
*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*
a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 14
08*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 +
1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^
4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5
*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*
g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f
^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^
5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2
*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*
d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c
^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*

c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5
*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^
6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b
^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*
b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5
*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^
2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a
^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694
*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 69
4*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 6
92*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 +
692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 +
672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4
+ 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4
- 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^
4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z
^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4
*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^
6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g
^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*
g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4
*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^
7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f
^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*
f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8
*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^
6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e
^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^
3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d
^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*
d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4
*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^
5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10
*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^
7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b
^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^
5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5
*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*
c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c
^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*
c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*
c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3
*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^

3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f
*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*
g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g
^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^
5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9
*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*
z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z
^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^
4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4
 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4
 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 -
 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 -
1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1
280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 12
80*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 115
2*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152
*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*
a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a
^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*
b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5
*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^
4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d
^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^
8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f
^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*
g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g
*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^
4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 -
 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 40
0*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a
^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*
b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6
*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^
3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d
^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*
e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4
*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*
g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3
*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^
4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 -
 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 25
6*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a
^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^
6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7
*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*

d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3
*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*
e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4
*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^
5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g
^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*
z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4
 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 +
156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*
a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8
*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c
*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^
7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e
^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*
f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2
*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^
6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^
4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 +
24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a
^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^1
1*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11
*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*
d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4
*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^
3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 2
0*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c
^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^
6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2
*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 46
08*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^
7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6
*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*
e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f
^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^
7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4
 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 17
92*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^
9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9
*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*
e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f
^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7
*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20
*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^1
1*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e
^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g

^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 -
5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^1
2*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*
e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*
f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g
^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^
4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68
*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b
^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*
e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^
4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7
*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4
- 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192
*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b
^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*
d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f
^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z
^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 +
320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a
^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^
3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5
*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^
2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10
*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 -
6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3
*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*
c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d
^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^
4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10
*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 +
48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2
*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9
*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z
^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*
c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7
*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*
b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f
^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 -
768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9
*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^
3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 -
512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^
10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*
f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 -
14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e

^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4
+ 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^
6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5
*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*
a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*
c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*
g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 1
2*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3
*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g
^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 +
 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c
^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8
*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 76
8*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*
c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^
6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4
- 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9
*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*
f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4
 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*
b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e
^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4
+ 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d
*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4
- 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*
e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^
4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5
*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^
2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c
^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6
*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^
3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g
^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^
10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^
8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^
7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8
*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 -
6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5
*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 1
2*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^1
0*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4
*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a
^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10
*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e
^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a

^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^
4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7
*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^
5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 -
76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*
c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e
^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*
z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*
a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c
*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2
*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 +
2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2
*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^
2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b
^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*
b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3
*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^
2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^
2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2
*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*
b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b
^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^
5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*
c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g
*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^
8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5
*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*
g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7
*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^
2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 +
 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 12
2*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a
^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^
3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5
*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*
e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f
^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^
7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2
 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48
*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*
b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c
^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*
e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f
^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^
4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2

- 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*

z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d
*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 20
4*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*
f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^
2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8
*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^
5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2
 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^
6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^
9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*
b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 -
 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*
z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8
*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^
8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 -
 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2
*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c
^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^
2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^
4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*
a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^
8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^
8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^
4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*
z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2
 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2
*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^
5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5
*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b
*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^
4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 2
8*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d
*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z
+ 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e
^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*
c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b
^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*
f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b
^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*
f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*
c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*
f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z -
12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*
z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z
 + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g

^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d
^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z +
b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^
6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^
3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z +
 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*
f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a
^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*
g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*
c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*
z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*
d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*
b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*
c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*
a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z
+ b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^
7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*
g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^
6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^
7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*
z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e
^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*
a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d
*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*
f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 -
16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*
c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*(root(1
120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^
5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4
*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^
2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8
*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 +
512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^
10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c
^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e
^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^
9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4
 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*
a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2
*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e
^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*
z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*
a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8
*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*
f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3

*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 13

60*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 -
1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^
4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5
*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*
g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f
^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^
3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8
*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^
2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d
^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*
d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4
*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^
7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c
^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*
c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8
*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^
8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b
^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*
b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4
*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^
3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a
^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*
a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544
*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 54
4*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 5
36*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 -
536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 -
504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4
- 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4
+ 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^
4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z
^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*
z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3
*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^
2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g
^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*
g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8
*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*
f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6
*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^
5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e
^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*
e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d
^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*
d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4

*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*
d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d
^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^
3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4
*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^
2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^
9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9
*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*
z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z
^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^
4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4
 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4
+ 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 +
 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 +
1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1
536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 15
36*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 153
6*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536
*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*
a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a
^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^
7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6
*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*
b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b
^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^
5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5
*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^
2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d
^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^
9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7
*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*
g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2
*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^
4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 +
 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 51
2*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a
^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*
b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4
*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^
2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d
^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*
e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f
^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*
g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9
*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^

4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 -
288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 28
0*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a
^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*
b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c
^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*
d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2
*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^
3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f
^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^
6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g
^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*
z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4
- 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 -
192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156
*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^
2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^
8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d
^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e
^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*
f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3
*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^
6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z
^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4
+ 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24
*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2
*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9
*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*
d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4
*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^
5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g
^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4
- 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*
c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5
*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*
g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 -
4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7
*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*
d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e
^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^
5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3
*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4
- 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 179
2*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6
*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*

d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e
^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^
2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8
*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 +
 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b
^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*
e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2
*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4
 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*
b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*
e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f
^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^
3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4
 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 19
2*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b
^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2
*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f
^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*
z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 +
 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*
a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b
^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6
*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f
^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2
*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 4
80*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^
8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4
*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*
d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e
^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10
*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 +
 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*
b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c
^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^
5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4
*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10
*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 +
 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2
*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c
^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6
*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 1
4*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*
e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z
^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^
6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2

*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*

b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*
z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*
b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10
*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f
^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6
*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 -
 b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^1
0*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2
*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*
e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g
^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2
+ 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4
*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*
e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7
*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 +
44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*
c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e
^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z
^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^
7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d
^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*
d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4
*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^
5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c
^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*
c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*
c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c
^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^
6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3
*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*
d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^
7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 -
 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*
e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7
*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^
2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 -
 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 16
4*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a
^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^
2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3
*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*
d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e
^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^
2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4
*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2

- 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*
a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b
^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^
2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e
^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^
6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2
*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2
- 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*
a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5
*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3
*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^
3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6
*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 3
2*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7
*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*
f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z
^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*
c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4
*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^
2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c
^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*
f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5
*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 +
320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3
*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^
6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f
^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*
z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 2
32*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*
b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5
*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*
g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2
+ 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a
^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*
c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6
*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*
z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^
3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3
*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 -
4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5
*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2
- 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c
^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^
7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 54
8*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^

2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z
^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2
*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^
5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 -
68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*
d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z
^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d
*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 -
24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f
*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*
b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*
z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^
7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 +
128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*
e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a
*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*
g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*
f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^
3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*
c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 -
2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z
^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*
g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^
6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z
^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6
*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 18
8*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*
f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^
8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2
+ 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*
g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*
z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 -
48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^1
0*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d
*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*
g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 1
20*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d
^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^
6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^
4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f
^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a
*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g
^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*
e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6
*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^

5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5
*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^
4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^
3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^
5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2
*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*
a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*
a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z +
 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4
*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^
2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z +
52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*
f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3
*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^
5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2
*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z
- 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^
2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32
*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^
5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*
d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^
7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^
6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*
c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z -
96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z -
10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z -
10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z +
128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*
c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g
^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*
e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g
^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5
- 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c
^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^
6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*((64*a^6*c^7*
d^7*e^2*g^9 + 64*a^7*c^6*d^5*e^4*g^9 - 64*a^8*c^5*d^3*e^6*g^9 + 64*a^6*c^7*
e^9*f^7*g^2 + 64*a^7*c^6*e^9*f^5*g^4 - 64*a^8*c^5*e^9*f^3*g^6 - 64*a^9*c^4*
d*e^8*g^9 - 64*a^9*c^4*e^9*f*g^8 + 16*a^5*b*c^7*d^8*e*g^9 + a^6*b^6*c*d*e^8
*g^9 + 16*a^5*b*c^7*e^9*f^8*g + a^6*b^6*c*e^9*f*g^8 - 128*a^5*c^8*d*e^8*f^8
*g - 128*a^5*c^8*d^8*e*f*g^8 + a^3*b^5*c^5*d^8*e*g^9 + a^3*b^9*c*d^4*e^5*g^
9 - 8*a^4*b^3*c^6*d^8*e*g^9 - a^4*b^8*c*d^3*e^6*g^9 - a^5*b^7*c*d^2*e^7*g^9
 - 144*a^6*b*c^6*d^6*e^3*g^9 - 80*a^7*b*c^5*d^4*e^5*g^9 - 12*a^7*b^4*c^2*d*
e^8*g^9 + 80*a^8*b*c^4*d^2*e^7*g^9 + 48*a^8*b^2*c^3*d*e^8*g^9 + a^3*b^5*c^5
*e^9*f^8*g + a^3*b^9*c*e^9*f^4*g^5 - 8*a^4*b^3*c^6*e^9*f^8*g - a^4*b^8*c*e^
9*f^3*g^6 - a^5*b^7*c*e^9*f^2*g^7 - 144*a^6*b*c^6*e^9*f^6*g^3 - 80*a^7*b*c^

5*e^9*f^4*g^5 - 12*a^7*b^4*c^2*e^9*f*g^8 + 80*a^8*b*c^4*e^9*f^2*g^7 + 48*a^
8*b^2*c^3*e^9*f*g^8 - 128*a^3*c^10*d^5*e^4*f^8*g - 128*a^3*c^10*d^8*e*f^5*g
^4 - 256*a^4*c^9*d^3*e^6*f^8*g - 256*a^4*c^9*d^8*e*f^3*g^6 - 448*a^6*c^7*d*
e^8*f^6*g^3 - 448*a^6*c^7*d^6*e^3*f*g^8 - 576*a^7*c^6*d*e^8*f^4*g^5 - 576*a
^7*c^6*d^4*e^5*f*g^8 - 320*a^8*c^5*d*e^8*f^2*g^7 - 320*a^8*c^5*d^2*e^7*f*g^
8 + b^5*c^8*d^6*e^3*f^8*g + b^5*c^8*d^8*e*f^6*g^3 - b^6*c^7*d^5*e^4*f^8*g -
 b^6*c^7*d^8*e*f^5*g^4 - b^7*c^6*d^4*e^5*f^8*g - b^7*c^6*d^8*e*f^4*g^5 + b^
8*c^5*d^3*e^6*f^8*g + b^8*c^5*d^8*e*f^3*g^6 + b^12*c*d^3*e^6*f^4*g^5 + b^12
*c*d^4*e^5*f^3*g^6 - 4*a^3*b^6*c^4*d^7*e^2*g^9 + 6*a^3*b^7*c^3*d^6*e^3*g^9
- 4*a^3*b^8*c^2*d^5*e^4*g^9 + 36*a^4*b^4*c^5*d^7*e^2*g^9 - 57*a^4*b^5*c^4*d
^6*e^3*g^9 + 37*a^4*b^6*c^3*d^5*e^4*g^9 - 7*a^4*b^7*c^2*d^4*e^5*g^9 - 96*a^
5*b^2*c^6*d^7*e^2*g^9 + 168*a^5*b^3*c^5*d^6*e^3*g^9 - 100*a^5*b^4*c^4*d^5*e
^4*g^9 + 3*a^5*b^5*c^3*d^4*e^5*g^9 + 10*a^5*b^6*c^2*d^3*e^6*g^9 + 48*a^6*b^
2*c^5*d^5*e^4*g^9 + 56*a^6*b^3*c^4*d^4*e^5*g^9 - 36*a^6*b^4*c^3*d^3*e^6*g^9
 + 13*a^6*b^5*c^2*d^2*e^7*g^9 + 64*a^7*b^2*c^4*d^3*e^6*g^9 - 56*a^7*b^3*c^3
*d^2*e^7*g^9 - 4*a^3*b^6*c^4*e^9*f^7*g^2 + 6*a^3*b^7*c^3*e^9*f^6*g^3 - 4*a^
3*b^8*c^2*e^9*f^5*g^4 + 36*a^4*b^4*c^5*e^9*f^7*g^2 - 57*a^4*b^5*c^4*e^9*f^6
*g^3 + 37*a^4*b^6*c^3*e^9*f^5*g^4 - 7*a^4*b^7*c^2*e^9*f^4*g^5 - 96*a^5*b^2*
c^6*e^9*f^7*g^2 + 168*a^5*b^3*c^5*e^9*f^6*g^3 - 100*a^5*b^4*c^4*e^9*f^5*g^4
 + 3*a^5*b^5*c^3*e^9*f^4*g^5 + 10*a^5*b^6*c^2*e^9*f^3*g^6 + 48*a^6*b^2*c^5*
e^9*f^5*g^4 + 56*a^6*b^3*c^4*e^9*f^4*g^5 - 36*a^6*b^4*c^3*e^9*f^3*g^6 + 13*
a^6*b^5*c^2*e^9*f^2*g^7 + 64*a^7*b^2*c^4*e^9*f^3*g^6 - 56*a^7*b^3*c^3*e^9*f
^2*g^7 + 64*a^3*c^10*d^6*e^3*f^7*g^2 + 64*a^3*c^10*d^7*e^2*f^6*g^3 + 192*a^
4*c^9*d^4*e^5*f^7*g^2 - 320*a^4*c^9*d^5*e^4*f^6*g^3 - 320*a^4*c^9*d^6*e^3*f
^5*g^4 + 192*a^4*c^9*d^7*e^2*f^4*g^5 + 192*a^5*c^8*d^2*e^7*f^7*g^2 - 832*a^
5*c^8*d^3*e^6*f^6*g^3 - 192*a^5*c^8*d^4*e^5*f^5*g^4 - 192*a^5*c^8*d^5*e^4*f
^4*g^5 - 832*a^5*c^8*d^6*e^3*f^3*g^6 + 192*a^5*c^8*d^7*e^2*f^2*g^7 + 64*a^6
*c^7*d^2*e^7*f^5*g^4 - 960*a^6*c^7*d^3*e^6*f^4*g^5 - 960*a^6*c^7*d^4*e^5*f^
3*g^6 + 64*a^6*c^7*d^5*e^4*f^2*g^7 - 448*a^7*c^6*d^2*e^7*f^3*g^6 - 448*a^7*
c^6*d^3*e^6*f^2*g^7 - 2*b^5*c^8*d^7*e^2*f^7*g^2 + 2*b^6*c^7*d^6*e^3*f^7*g^2
 + 2*b^6*c^7*d^7*e^2*f^6*g^3 - 2*b^7*c^6*d^5*e^4*f^7*g^2 - 6*b^7*c^6*d^6*e^
3*f^6*g^3 - 2*b^7*c^6*d^7*e^2*f^5*g^4 + 6*b^8*c^5*d^4*e^5*f^7*g^2 + 8*b^8*c
^5*d^5*e^4*f^6*g^3 + 8*b^8*c^5*d^6*e^3*f^5*g^4 + 6*b^8*c^5*d^7*e^2*f^4*g^5
- 4*b^9*c^4*d^3*e^6*f^7*g^2 - 11*b^9*c^4*d^4*e^5*f^6*g^3 - 10*b^9*c^4*d^5*e
^4*f^5*g^4 - 11*b^9*c^4*d^6*e^3*f^4*g^5 - 4*b^9*c^4*d^7*e^2*f^3*g^6 + 6*b^1
0*c^3*d^3*e^6*f^6*g^3 + 9*b^10*c^3*d^4*e^5*f^5*g^4 + 9*b^10*c^3*d^5*e^4*f^4
*g^5 + 6*b^10*c^3*d^6*e^3*f^3*g^6 - 4*b^11*c^2*d^3*e^6*f^5*g^4 - 4*b^11*c^2
*d^4*e^5*f^4*g^5 - 4*b^11*c^2*d^5*e^4*f^3*g^6 + 16*a*b^3*c^9*d^7*e^2*f^7*g^
2 - 12*a*b^4*c^8*d^6*e^3*f^7*g^2 - 12*a*b^4*c^8*d^7*e^2*f^6*g^3 + 30*a*b^5*
c^7*d^5*e^4*f^7*g^2 + 30*a*b^5*c^7*d^6*e^3*f^6*g^3 + 30*a*b^5*c^7*d^7*e^2*f
^5*g^4 - 100*a*b^6*c^6*d^4*e^5*f^7*g^2 - 56*a*b^6*c^6*d^5*e^4*f^6*g^3 - 56*
a*b^6*c^6*d^6*e^3*f^5*g^4 - 100*a*b^6*c^6*d^7*e^2*f^4*g^5 + 62*a*b^7*c^5*d^
3*e^6*f^7*g^2 + 128*a*b^7*c^5*d^4*e^5*f^6*g^3 + 42*a*b^7*c^5*d^5*e^4*f^5*g^
4 + 128*a*b^7*c^5*d^6*e^3*f^4*g^5 + 62*a*b^7*c^5*d^7*e^2*f^3*g^6 + 4*a*b^8*
c^4*d^2*e^7*f^7*g^2 - 76*a*b^8*c^4*d^3*e^6*f^6*g^3 - 48*a*b^8*c^4*d^4*e^5*f

^5*g^4 - 48*a*b^8*c^4*d^5*e^4*f^4*g^5 - 76*a*b^8*c^4*d^6*e^3*f^3*g^6 + 4*a*
b^8*c^4*d^7*e^2*f^2*g^7 - 6*a*b^9*c^3*d^2*e^7*f^6*g^3 + 28*a*b^9*c^3*d^3*e^
6*f^5*g^4 - 20*a*b^9*c^3*d^4*e^5*f^4*g^5 + 28*a*b^9*c^3*d^5*e^4*f^3*g^6 - 6
*a*b^9*c^3*d^6*e^3*f^2*g^7 + 4*a*b^10*c^2*d^2*e^7*f^5*g^4 + 14*a*b^10*c^2*d
^3*e^6*f^4*g^5 + 14*a*b^10*c^2*d^4*e^5*f^3*g^6 + 4*a*b^10*c^2*d^5*e^4*f^2*g
^7 - 32*a^2*b*c^10*d^7*e^2*f^7*g^2 + 48*a^2*b^2*c^9*d^5*e^4*f^8*g + 48*a^2*
b^2*c^9*d^8*e*f^5*g^4 - 168*a^2*b^3*c^8*d^4*e^5*f^8*g - 168*a^2*b^3*c^8*d^8
*e*f^4*g^5 + 80*a^2*b^4*c^7*d^3*e^6*f^8*g + 80*a^2*b^4*c^7*d^8*e*f^3*g^6 +
27*a^2*b^5*c^6*d^2*e^7*f^8*g + 27*a^2*b^5*c^6*d^8*e*f^2*g^7 + 4*a^2*b^7*c^4
*d*e^8*f^7*g^2 + 4*a^2*b^7*c^4*d^7*e^2*f*g^8 - 6*a^2*b^8*c^3*d*e^8*f^6*g^3
- 6*a^2*b^8*c^3*d^6*e^3*f*g^8 + 4*a^2*b^9*c^2*d*e^8*f^5*g^4 + 4*a^2*b^9*c^2
*d^5*e^4*f*g^8 + 16*a^2*b^10*c*d^2*e^7*f^3*g^6 + 16*a^2*b^10*c*d^3*e^6*f^2*
g^7 + 224*a^3*b*c^9*d^5*e^4*f^7*g^2 - 288*a^3*b*c^9*d^6*e^3*f^6*g^3 + 224*a
^3*b*c^9*d^7*e^2*f^5*g^4 - 32*a^3*b^2*c^8*d^3*e^6*f^8*g - 32*a^3*b^2*c^8*d^
8*e*f^3*g^6 - 168*a^3*b^3*c^7*d^2*e^7*f^8*g - 168*a^3*b^3*c^7*d^8*e*f^2*g^7
 - 14*a^3*b^5*c^5*d*e^8*f^7*g^2 - 14*a^3*b^5*c^5*d^7*e^2*f*g^8 + 40*a^3*b^6
*c^4*d*e^8*f^6*g^3 + 40*a^3*b^6*c^4*d^6*e^3*f*g^8 - 44*a^3*b^7*c^3*d*e^8*f^
5*g^4 - 44*a^3*b^7*c^3*d^5*e^4*f*g^8 + 24*a^3*b^8*c^2*d*e^8*f^4*g^5 + 24*a^
3*b^8*c^2*d^4*e^5*f*g^8 - 30*a^3*b^9*c*d^2*e^7*f^2*g^7 + 544*a^4*b*c^8*d^3*
e^6*f^7*g^2 + 256*a^4*b*c^8*d^4*e^5*f^6*g^3 + 1632*a^4*b*c^8*d^5*e^4*f^5*g^
4 + 256*a^4*b*c^8*d^6*e^3*f^4*g^5 + 544*a^4*b*c^8*d^7*e^2*f^3*g^6 - 80*a^4*
b^3*c^6*d*e^8*f^7*g^2 - 80*a^4*b^3*c^6*d^7*e^2*f*g^8 - 60*a^4*b^4*c^5*d*e^8
*f^6*g^3 - 60*a^4*b^4*c^5*d^6*e^3*f*g^8 + 234*a^4*b^5*c^4*d*e^8*f^5*g^4 + 2
34*a^4*b^5*c^4*d^5*e^4*f*g^8 - 208*a^4*b^6*c^3*d*e^8*f^4*g^5 - 208*a^4*b^6*
c^3*d^4*e^5*f*g^8 + 50*a^4*b^7*c^2*d*e^8*f^3*g^6 + 50*a^4*b^7*c^2*d^3*e^6*f
*g^8 + 416*a^5*b*c^7*d^2*e^7*f^6*g^3 + 2592*a^5*b*c^7*d^3*e^6*f^5*g^4 + 105
6*a^5*b*c^7*d^4*e^5*f^4*g^5 + 2592*a^5*b*c^7*d^5*e^4*f^3*g^6 + 416*a^5*b*c^
7*d^6*e^3*f^2*g^7 + 96*a^5*b^2*c^6*d*e^8*f^6*g^3 + 96*a^5*b^2*c^6*d^6*e^3*f
*g^8 - 784*a^5*b^3*c^5*d*e^8*f^5*g^4 - 784*a^5*b^3*c^5*d^5*e^4*f*g^8 + 732*
a^5*b^4*c^4*d*e^8*f^4*g^5 + 732*a^5*b^4*c^4*d^4*e^5*f*g^8 - 18*a^5*b^5*c^3*
d*e^8*f^3*g^6 - 18*a^5*b^5*c^3*d^3*e^6*f*g^8 - 184*a^5*b^6*c^2*d*e^8*f^2*g^
7 - 184*a^5*b^6*c^2*d^2*e^7*f*g^8 + 1024*a^6*b*c^6*d^2*e^7*f^4*g^5 + 3552*a
^6*b*c^6*d^3*e^6*f^3*g^6 + 1024*a^6*b*c^6*d^4*e^5*f^2*g^7 - 736*a^6*b^2*c^5
*d*e^8*f^4*g^5 - 736*a^6*b^2*c^5*d^4*e^5*f*g^8 - 720*a^6*b^3*c^4*d*e^8*f^3*
g^6 - 720*a^6*b^3*c^4*d^3*e^6*f*g^8 + 684*a^6*b^4*c^3*d*e^8*f^2*g^7 + 684*a
^6*b^4*c^3*d^2*e^7*f*g^8 + 992*a^7*b*c^5*d^2*e^7*f^2*g^7 - 736*a^7*b^2*c^4*
d*e^8*f^2*g^7 - 736*a^7*b^2*c^4*d^2*e^7*f*g^8 - 10*a^5*b^7*c*d*e^8*f*g^8 +
608*a^8*b*c^4*d*e^8*f*g^8 - 144*a^2*b^3*c^8*d^5*e^4*f^7*g^2 + 48*a^2*b^3*c^
8*d^6*e^3*f^6*g^3 - 144*a^2*b^3*c^8*d^7*e^2*f^5*g^4 + 524*a^2*b^4*c^7*d^4*e
^5*f^7*g^2 + 44*a^2*b^4*c^7*d^5*e^4*f^6*g^3 + 44*a^2*b^4*c^7*d^6*e^3*f^5*g^
4 + 524*a^2*b^4*c^7*d^7*e^2*f^4*g^5 - 270*a^2*b^5*c^6*d^3*e^6*f^7*g^2 - 480
*a^2*b^5*c^6*d^4*e^5*f^6*g^3 + 246*a^2*b^5*c^6*d^5*e^4*f^5*g^4 - 480*a^2*b^
5*c^6*d^6*e^3*f^4*g^5 - 270*a^2*b^5*c^6*d^7*e^2*f^3*g^6 - 90*a^2*b^6*c^5*d^
2*e^7*f^7*g^2 + 286*a^2*b^6*c^5*d^3*e^6*f^6*g^3 - 180*a^2*b^6*c^5*d^4*e^5*f
^5*g^4 - 180*a^2*b^6*c^5*d^5*e^4*f^4*g^5 + 286*a^2*b^6*c^5*d^6*e^3*f^3*g^6

- 90*a^2*b^6*c^5*d^7*e^2*f^2*g^7 + 104*a^2*b^7*c^4*d^2*e^7*f^6*g^3 + 4*a^2*
b^7*c^4*d^3*e^6*f^5*g^4 + 520*a^2*b^7*c^4*d^4*e^5*f^4*g^5 + 4*a^2*b^7*c^4*d
^5*e^4*f^3*g^6 + 104*a^2*b^7*c^4*d^6*e^3*f^2*g^7 - 30*a^2*b^8*c^3*d^2*e^7*f
^5*g^4 - 186*a^2*b^8*c^3*d^3*e^6*f^4*g^5 - 186*a^2*b^8*c^3*d^4*e^5*f^3*g^6
- 30*a^2*b^8*c^3*d^5*e^4*f^2*g^7 - 27*a^2*b^9*c^2*d^2*e^7*f^4*g^5 + 70*a^2*
b^9*c^2*d^3*e^6*f^3*g^6 - 27*a^2*b^9*c^2*d^4*e^5*f^2*g^7 - 928*a^3*b^2*c^8*
d^4*e^5*f^7*g^2 + 288*a^3*b^2*c^8*d^5*e^4*f^6*g^3 + 288*a^3*b^2*c^8*d^6*e^3
*f^5*g^4 - 928*a^3*b^2*c^8*d^7*e^2*f^4*g^5 + 208*a^3*b^3*c^7*d^3*e^6*f^7*g^
2 + 512*a^3*b^3*c^7*d^4*e^5*f^6*g^3 - 1424*a^3*b^3*c^7*d^5*e^4*f^5*g^4 + 51
2*a^3*b^3*c^7*d^6*e^3*f^4*g^5 + 208*a^3*b^3*c^7*d^7*e^2*f^3*g^6 + 540*a^3*b
^4*c^6*d^2*e^7*f^7*g^2 - 228*a^3*b^4*c^6*d^3*e^6*f^6*g^3 + 1428*a^3*b^4*c^6
*d^4*e^5*f^5*g^4 + 1428*a^3*b^4*c^6*d^5*e^4*f^4*g^5 - 228*a^3*b^4*c^6*d^6*e
^3*f^3*g^6 + 540*a^3*b^4*c^6*d^7*e^2*f^2*g^7 - 518*a^3*b^5*c^5*d^2*e^7*f^6*
g^3 - 190*a^3*b^5*c^5*d^3*e^6*f^5*g^4 - 2110*a^3*b^5*c^5*d^4*e^5*f^4*g^5 -
190*a^3*b^5*c^5*d^5*e^4*f^3*g^6 - 518*a^3*b^5*c^5*d^6*e^3*f^2*g^7 - 88*a^3*
b^6*c^4*d^2*e^7*f^5*g^4 + 368*a^3*b^6*c^4*d^3*e^6*f^4*g^5 + 368*a^3*b^6*c^4
*d^4*e^5*f^3*g^6 - 88*a^3*b^6*c^4*d^5*e^4*f^2*g^7 + 404*a^3*b^7*c^3*d^2*e^7
*f^4*g^5 + 12*a^3*b^7*c^3*d^3*e^6*f^3*g^6 + 404*a^3*b^7*c^3*d^4*e^5*f^2*g^7
- 140*a^3*b^8*c^2*d^2*e^7*f^3*g^6 - 140*a^3*b^8*c^2*d^3*e^6*f^2*g^7 - 1024
*a^4*b^2*c^7*d^2*e^7*f^7*g^2 - 128*a^4*b^2*c^7*d^3*e^6*f^6*g^3 - 2016*a^4*b
^2*c^7*d^4*e^5*f^5*g^4 - 2016*a^4*b^2*c^7*d^5*e^4*f^4*g^5 - 128*a^4*b^2*c^7
*d^6*e^3*f^3*g^6 - 1024*a^4*b^2*c^7*d^7*e^2*f^2*g^7 + 688*a^4*b^3*c^6*d^2*e
^7*f^6*g^3 - 720*a^4*b^3*c^6*d^3*e^6*f^5*g^4 + 2160*a^4*b^3*c^6*d^4*e^5*f^4
*g^5 - 720*a^4*b^3*c^6*d^5*e^4*f^3*g^6 + 688*a^4*b^3*c^6*d^6*e^3*f^2*g^7 +
1124*a^4*b^4*c^5*d^2*e^7*f^5*g^4 + 1060*a^4*b^4*c^5*d^3*e^6*f^4*g^5 + 1060*
a^4*b^4*c^5*d^4*e^5*f^3*g^6 + 1124*a^4*b^4*c^5*d^5*e^4*f^2*g^7 - 1616*a^4*b
^5*c^4*d^2*e^7*f^4*g^5 - 674*a^4*b^5*c^4*d^3*e^6*f^3*g^6 - 1616*a^4*b^5*c^4
*d^4*e^5*f^2*g^7 + 186*a^4*b^6*c^3*d^2*e^7*f^3*g^6 + 186*a^4*b^6*c^3*d^3*e^
6*f^2*g^7 + 334*a^4*b^7*c^2*d^2*e^7*f^2*g^7 - 2208*a^5*b^2*c^6*d^2*e^7*f^5*
g^4 - 2592*a^5*b^2*c^6*d^3*e^6*f^4*g^5 - 2592*a^5*b^2*c^6*d^4*e^5*f^3*g^6 -
2208*a^5*b^2*c^6*d^5*e^4*f^2*g^7 + 1728*a^5*b^3*c^5*d^2*e^7*f^4*g^5 - 304*
a^5*b^3*c^5*d^3*e^6*f^3*g^6 + 1728*a^5*b^3*c^5*d^4*e^5*f^2*g^7 + 1108*a^5*b
^4*c^4*d^2*e^7*f^3*g^6 + 1108*a^5*b^4*c^4*d^3*e^6*f^2*g^7 - 1170*a^5*b^5*c^
3*d^2*e^7*f^2*g^7 - 2432*a^6*b^2*c^5*d^2*e^7*f^3*g^6 - 2432*a^6*b^2*c^5*d^3
*e^6*f^2*g^7 + 1008*a^6*b^3*c^4*d^2*e^7*f^2*g^7 - 8*a*b^3*c^9*d^6*e^3*f^8*g
- 8*a*b^3*c^9*d^8*e*f^6*g^3 + 27*a*b^5*c^7*d^4*e^5*f^8*g + 27*a*b^5*c^7*d^
8*e*f^4*g^5 - 18*a*b^6*c^6*d^3*e^6*f^8*g - 18*a*b^6*c^6*d^8*e*f^3*g^6 - a*b
^7*c^5*d^2*e^7*f^8*g - a*b^7*c^5*d^8*e*f^2*g^7 - a*b^11*c*d^2*e^7*f^4*g^5 -
10*a*b^11*c*d^3*e^6*f^3*g^6 - a*b^11*c*d^4*e^5*f^2*g^7 + 16*a^2*b*c^10*d^6
*e^3*f^8*g + 16*a^2*b*c^10*d^8*e*f^6*g^3 - a^2*b^6*c^5*d*e^8*f^8*g - a^2*b^
6*c^5*d^8*e*f*g^8 - a^2*b^10*c*d*e^8*f^4*g^5 - a^2*b^10*c*d^4*e^5*f*g^8 + 3
04*a^3*b*c^9*d^4*e^5*f^8*g + 304*a^3*b*c^9*d^8*e*f^4*g^5 - 6*a^3*b^9*c*d*e^
8*f^3*g^6 - 6*a^3*b^9*c*d^3*e^6*f*g^8 + 304*a^4*b*c^8*d^2*e^7*f^8*g + 304*a
^4*b*c^8*d^8*e*f^2*g^7 + 48*a^4*b^2*c^7*d*e^8*f^8*g + 48*a^4*b^2*c^7*d^8*e*
f*g^8 + 16*a^4*b^8*c*d*e^8*f^2*g^7 + 16*a^4*b^8*c*d^2*e^7*f*g^8 + 288*a^5*b

*c^7*d*e^8*f^7*g^2 + 288*a^5*b*c^7*d^7*e^2*f*g^8 + 1184*a^6*b*c^6*d*e^8*f^5 *g^4 + 1184*a^6*b*c^6*d^5*e^4*f*g^8 + 118*a^6*b^5*c^2*d*e^8*f*g^8 + 1504*a^ 7*b*c^5*d*e^8*f^3*g^6 + 1504*a^7*b*c^5*d^3*e^6*f*g^8 - 464*a^7*b^3*c^3*d*e^ 8*f*g^8)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^ 4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2 *b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^ 2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e ^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^ 4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3 *b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2 *d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c* d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2* d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d ^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^ 4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f ^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^ 3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f ^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2* e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c ^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b ^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^ 3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e ^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a* b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f ^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5 *c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d* e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - (x *(128*a^9*c^4*e^9*g^9 + 24*a^7*b^4*c^2*e^9*g^9 - 96*a^8*b^2*c^3*e^9*g^9 + 2 88*a^6*c^7*d^6*e^3*g^9 + 416*a^7*c^6*d^4*e^5*g^9 + 352*a^8*c^5*d^2*e^7*g^9 + 288*a^6*c^7*e^9*f^6*g^3 + 416*a^7*c^6*e^9*f^4*g^5 + 352*a^8*c^5*e^9*f^2*g ^7 - 2*a^6*b^6*c*e^9*g^9 + 96*a^5*c^8*d^8*e*g^9 + 96*a^5*c^8*e^9*f^8*g + 6* a^5*b^7*c*d*e^8*g^9 - 384*a^8*b*c^4*d*e^8*g^9 + 6*a^5*b^7*c*e^9*f*g^8 - 384 *a^8*b*c^4*e^9*f*g^8 + 64*a^8*c^5*d*e^8*f*g^8 - 2*a^2*b^6*c^5*d^8*e*g^9 - 2 *a^2*b^10*c*d^4*e^5*g^9 + 22*a^3*b^4*c^6*d^8*e*g^9 + 6*a^3*b^9*c*d^3*e^6*g^ 9 - 80*a^4*b^2*c^7*d^8*e*g^9 - 8*a^4*b^8*c*d^2*e^7*g^9 - 416*a^5*b*c^7*d^7* e^2*g^9 - 960*a^6*b*c^6*d^5*e^4*g^9 - 72*a^6*b^5*c^2*d*e^8*g^9 - 928*a^7*b* c^5*d^3*e^6*g^9 + 288*a^7*b^3*c^3*d*e^8*g^9 - 2*a^2*b^6*c^5*e^9*f^8*g - 2*a ^2*b^10*c*e^9*f^4*g^5 + 22*a^3*b^4*c^6*e^9*f^8*g + 6*a^3*b^9*c*e^9*f^3*g^6 - 80*a^4*b^2*c^7*e^9*f^8*g - 8*a^4*b^8*c*e^9*f^2*g^7 - 416*a^5*b*c^7*e^9*f^ 7*g^2 - 960*a^6*b*c^6*e^9*f^5*g^4 - 72*a^6*b^5*c^2*e^9*f*g^8 - 928*a^7*b*c^ 5*e^9*f^3*g^6 + 288*a^7*b^3*c^3*e^9*f*g^8 - 32*a^2*c^11*d^6*e^3*f^8*g - 32* a^2*c^11*d^8*e*f^6*g^3 + 32*a^3*c^10*d^4*e^5*f^8*g + 32*a^3*c^10*d^8*e*f^4* g^5 + 160*a^4*c^9*d^2*e^7*f^8*g + 160*a^4*c^9*d^8*e*f^2*g^7 + 64*a^5*c^8*d* e^8*f^7*g^2 + 64*a^5*c^8*d^7*e^2*f*g^8 + 192*a^6*c^7*d*e^8*f^5*g^4 + 192*a^6*c^7*d^5*e^4*f*g^8 + 192*a^7*c^6*d*e^8*f^3*g^6 + 192*a^7*c^6*d^3*e^6*f*g^8 - 2*b^4*c^9*d^6*e^3*f^8*g - 2*b^4*c^9*d^8*e*f^6*g^3 + 6*b^5*c^8*d^5*e^4*f^8*g + 6*b^5*c^8*d^8*e*f^5*g^4 - 8*b^6*c^7*d^4*e^5*f^8*g - 8*b^6*c^7*d^8*e*f^4*g^5 + 6*b^7*c^6*d^3*e^6*f^8*g + 6*b^7*c^6*d^8*e*f^3*g^6 - 2*b^8*c^5*d^2*e^7*f^8*g - 2*b^8*c^5*d^8*e*f^2*g^7 - 2*b^12*c*d^2*e^7*f^4*g^5 + 2*b^12*c*d^3*e^6*f^3*g^6 - 2*b^12*c*d^4*e^5*f^2*g^7 + 8*a^2*b^7*c^4*d^7*e^2*g^9 - 12*a^2*b^8*c^3*d^6*e^3*g^9 + 8*a^2*b^9*c^2*d^5*e^4*g^9 - 90*a^3*b^5*c^5*d^7*e^2*g^9 + 132*a^3*b^6*c^4*d^6*e^3*g^9 - 76*a^3*b^7*c^3*d^5*e^4*g^9 + 6*a^3*b^8*c^2*d^4*e^5*g^9 + 336*a^4*b^3*c^6*d^7*e^2*g^9 - 462*a^4*b^4*c^5*d^6*e^3*g^9 + 164*a^4*b^5*c^4*d^5*e^4*g^9 + 106*a^4*b^6*c^3*d^4*e^5*g^9 - 56*a^4*b^7*c^2*d^3*e^6*g^9 + 432*a^5*b^2*c^6*d^6*e^3*g^9 + 288*a^5*b^3*c^5*d^5*e^4*g^9 - 598*a^5*b^4*c^4*d^4*e^5*g^9 + 102*a^5*b^5*c^3*d^3*e^6*g^9 + 90*a^5*b^6*c^2*d^2*e^7*g^9 + 720*a^6*b^2*c^5*d^4*e^5*g^9 + 336*a^6*b^3*c^4*d^3*e^6*g^9 - 314*a^6*b^4*c^3*d^2*e^7*g^9 + 240*a^7*b^2*c^4*d^2*e^7*g^9 + 8*a^2*b^7*c^4*e^9*f^7*g^2 - 12*a^2*b^8*c^3*e^9*f^6*g^3 + 8*a^2*b^9*c^2*e^9*f^5*g^4 - 90*a^3*b^5*c^5*e^9*f^7*g^2 + 132*a^3*b^6*c^4*e^9*f^6*g^3 - 76*a^3*b^7*c^3*e^9*f^5*g^4 + 6*a^3*b^8*c^2*e^9*f^4*g^5 + 336*a^4*b^3*c^6*e^9*f^7*g^2 - 462*a^4*b^4*c^5*e^9*f^6*g^3 + 164*a^4*b^5*c^4*e^9*f^5*g^4 + 106*a^4*b^6*c^3*e^9*f^4*g^5 - 56*a^4*b^7*c^2*e^9*f^3*g^6 + 432*a^5*b^2*c^6*e^9*f^6*g^3 + 288*a^5*b^3*c^5*e^9*f^5*g^4 - 598*a^5*b^4*c^4*e^9*f^4*g^5 + 102*a^5*b^5*c^3*e^9*f^3*g^6 + 90*a^5*b^6*c^2*e^9*f^2*g^7 + 720*a^6*b^2*c^5*e^9*f^4*g^5 + 336*a^6*b^3*c^4*e^9*f^3*g^6 - 314*a^6*b^4*c^3*e^9*f^2*g^7 + 240*a^7*b^2*c^4*e^9*f^2*g^7 + 64*a^2*c^11*d^7*e^2*f^7*g^2 + 192*a^3*c^10*d^5*e^4*f^7*g^2 - 320*a^3*c^10*d^6*e^3*f^6*g^3 + 192*a^3*c^10*d^7*e^2*f^5*g^4 + 192*a^4*c^9*d^3*e^6*f^7*g^2 - 256*a^4*c^9*d^4*e^5*f^6*g^3 + 576*a^4*c^9*d^5*e^4*f^5*g^4 - 256*a^4*c^9*d^6*e^3*f^4*g^5 + 192*a^4*c^9*d^7*e^2*f^3*g^6 + 320*a^5*c^8*d^2*e^7*f^6*g^3 + 576*a^5*c^8*d^3*e^6*f^5*g^4 - 192*a^5*c^8*d^4*e^5*f^4*g^5 + 576*a^5*c^8*d^5*e^4*f^3*g^6 + 320*a^5*c^8*d^6*e^3*f^2*g^7 + 512*a^6*c^7*d^2*e^7*f^4*g^5 + 576*a^6*c^7*d^3*e^6*f^3*g^6 + 512*a^6*c^7*d^4*e^5*f^2*g^7 + 704*a^7*c^6*d^2*e^7*f^2*g^7 + 4*b^4*c^9*d^7*e^2*f^7*g^2 - 6*b^5*c^8*d^6*e^3*f^7*g^2 - 6*b^5*c^8*d^7*e^2*f^6*g^3 - 6*b^6*c^7*d^5*e^4*f^7*g^2 + 26*b^6*c^7*d^6*e^3*f^6*g^3 - 6*b^6*c^7*d^7*e^2*f^5*g^4 + 22*b^7*c^6*d^4*e^5*f^7*g^2 - 22*b^7*c^6*d^5*e^4*f^6*g^3 - 22*b^7*c^6*d^6*e^3*f^5*g^4 + 22*b^7*c^6*d^7*e^2*f^4*g^5 - 22*b^8*c^5*d^3*e^6*f^7*g^2 - 12*b^8*c^5*d^4*e^5*f^6*g^3 + 42*b^8*c^5*d^5*e^4*f^5*g^4 - 12*b^8*c^5*d^6*e^3*f^4*g^5 - 22*b^8*c^5*d^7*e^2*f^3*g^6 + 8*b^9*c^4*d^2*e^7*f^7*g^2 + 28*b^9*c^4*d^3*e^6*f^6*g^3 - 16*b^9*c^4*d^4*e^5*f^5*g^4 - 16*b^9*c^4*d^5*e^4*f^4*g^5 + 28*b^9*c^4*d^6*e^3*f^3*g^6 + 8*b^9*c^4*d^7*e^2*f^2*g^7 - 12*b^10*c^3*d^2*e^7*f^6*g^3 - 12*b^10*c^3*d^3*e^6*f^5*g^4 + 18*b^10*c^3*d^4*e^5*f^4*g^5 - 12*b^10*c^3*d^5*e^4*f^3*g^6 - 12*b^10*c^3*d^6*e^3*f^2*g^7 + 8*b^11*c^2*d^2*e^7*f^5*g^4 - 2*b^11*c^2*d^3*e^6*f^4*g^5 - 2*b^11*c^2*d^4*e^5*f^3*g^6 + 8*b^11*c^2*d^5*e^4*f^2*g^7 - 32*a*b^2*c^10*d^7*e^2*f^7*g^2 + 48*a*b^3*c^9*d^6*e^3*f^7*g^2 + 48*a*b^3*c^9*d^7*e^2*f^6*g^3 + 60*a*b^4*c^8*d^5*e^4*f^7*g^2 - 228*a*b^4*c^8*d^6*e^3*f^6*g^3 + 60*a*b^4*c^8*d^7*e^2*f^5*g^4 - 214*a*b^5*c^7*d^4*e^5*f^7*g^2 + 194*a*b^5*c^

7*d^5*e^4*f^6*g^3 + 194*a*b^5*c^7*d^6*e^3*f^5*g^4 - 214*a*b^5*c^7*d^7*e^2*f
^4*g^5 + 216*a*b^6*c^6*d^3*e^6*f^7*g^2 + 148*a*b^6*c^6*d^4*e^5*f^6*g^3 - 40
8*a*b^6*c^6*d^5*e^4*f^5*g^4 + 148*a*b^6*c^6*d^6*e^3*f^4*g^5 + 216*a*b^6*c^6
*d^7*e^2*f^3*g^6 - 62*a*b^7*c^5*d^2*e^7*f^7*g^2 - 302*a*b^7*c^5*d^3*e^6*f^6
*g^3 + 150*a*b^7*c^5*d^4*e^5*f^5*g^4 + 150*a*b^7*c^5*d^5*e^4*f^4*g^5 - 302*
a*b^7*c^5*d^6*e^3*f^3*g^6 - 62*a*b^7*c^5*d^7*e^2*f^2*g^7 + 100*a*b^8*c^4*d^
2*e^7*f^6*g^3 + 136*a*b^8*c^4*d^3*e^6*f^5*g^4 - 200*a*b^8*c^4*d^4*e^5*f^4*g
^5 + 136*a*b^8*c^4*d^5*e^4*f^3*g^6 + 100*a*b^8*c^4*d^6*e^3*f^2*g^7 - 68*a*b
^9*c^3*d^2*e^7*f^5*g^4 + 32*a*b^9*c^3*d^3*e^6*f^4*g^5 + 32*a*b^9*c^3*d^4*e^
5*f^3*g^6 - 68*a*b^9*c^3*d^5*e^4*f^2*g^7 + 14*a*b^10*c^2*d^2*e^7*f^4*g^5 -
32*a*b^10*c^2*d^3*e^6*f^3*g^6 + 14*a*b^10*c^2*d^4*e^5*f^2*g^7 - 96*a^2*b*c^
10*d^6*e^3*f^7*g^2 - 96*a^2*b*c^10*d^7*e^2*f^6*g^3 - 144*a^2*b^2*c^9*d^4*e^
5*f^8*g - 144*a^2*b^2*c^9*d^8*e*f^4*g^5 + 128*a^2*b^3*c^8*d^3*e^6*f^8*g + 1
28*a^2*b^3*c^8*d^8*e*f^3*g^6 - 6*a^2*b^4*c^7*d^2*e^7*f^8*g - 6*a^2*b^4*c^7*
d^8*e*f^2*g^7 + 174*a^2*b^6*c^5*d*e^8*f^7*g^2 + 174*a^2*b^6*c^5*d^7*e^2*f*g
^8 - 260*a^2*b^7*c^4*d*e^8*f^6*g^3 - 260*a^2*b^7*c^4*d^6*e^3*f*g^8 + 156*a^
2*b^8*c^3*d*e^8*f^5*g^4 + 156*a^2*b^8*c^3*d^5*e^4*f*g^8 - 18*a^2*b^9*c^2*d*
e^8*f^4*g^5 - 18*a^2*b^9*c^2*d^4*e^5*f*g^8 - 6*a^2*b^10*c*d^2*e^7*f^2*g^7 -
 608*a^3*b*c^9*d^4*e^5*f^7*g^2 + 288*a^3*b*c^9*d^5*e^4*f^6*g^3 + 288*a^3*b*
c^9*d^6*e^3*f^5*g^4 - 608*a^3*b*c^9*d^7*e^2*f^4*g^5 - 112*a^3*b^2*c^8*d^2*e
^7*f^8*g - 112*a^3*b^2*c^8*d^8*e*f^2*g^7 - 620*a^3*b^4*c^6*d*e^8*f^7*g^2 -
620*a^3*b^4*c^6*d^7*e^2*f*g^8 + 894*a^3*b^5*c^5*d*e^8*f^6*g^3 + 894*a^3*b^5
*c^5*d^6*e^3*f*g^8 - 384*a^3*b^6*c^4*d*e^8*f^5*g^4 - 384*a^3*b^6*c^4*d^5*e^
4*f*g^8 - 140*a^3*b^7*c^3*d*e^8*f^4*g^5 - 140*a^3*b^7*c^3*d^4*e^5*f*g^8 + 9
2*a^3*b^8*c^2*d*e^8*f^3*g^6 + 92*a^3*b^8*c^2*d^3*e^6*f*g^8 - 928*a^4*b*c^8*
d^2*e^7*f^7*g^2 - 160*a^4*b*c^8*d^3*e^6*f^6*g^3 - 672*a^4*b*c^8*d^4*e^5*f^5
*g^4 - 672*a^4*b*c^8*d^5*e^4*f^4*g^5 - 160*a^4*b*c^8*d^6*e^3*f^3*g^6 - 928*
a^4*b*c^8*d^7*e^2*f^2*g^7 + 704*a^4*b^2*c^7*d*e^8*f^7*g^2 + 704*a^4*b^2*c^7
*d^7*e^2*f*g^8 - 816*a^4*b^3*c^6*d*e^8*f^6*g^3 - 816*a^4*b^3*c^6*d^6*e^3*f*
g^8 - 308*a^4*b^4*c^5*d*e^8*f^5*g^4 - 308*a^4*b^4*c^5*d^5*e^4*f*g^8 + 898*a
^4*b^5*c^4*d*e^8*f^4*g^5 + 898*a^4*b^5*c^4*d^4*e^5*f*g^8 - 150*a^4*b^6*c^3*
d*e^8*f^3*g^6 - 150*a^4*b^6*c^3*d^3*e^6*f*g^8 - 154*a^4*b^7*c^2*d*e^8*f^2*g
^7 - 154*a^4*b^7*c^2*d^2*e^7*f*g^8 - 1824*a^5*b*c^7*d^2*e^7*f^5*g^4 - 1056*
a^5*b*c^7*d^3*e^6*f^4*g^5 - 1056*a^5*b*c^7*d^4*e^5*f^3*g^6 - 1824*a^5*b*c^7
*d^5*e^4*f^2*g^7 + 1440*a^5*b^2*c^6*d*e^8*f^5*g^4 + 1440*a^5*b^2*c^6*d^5*e^
4*f*g^8 - 976*a^5*b^3*c^5*d*e^8*f^4*g^5 - 976*a^5*b^3*c^5*d^4*e^5*f*g^8 - 6
44*a^5*b^4*c^4*d*e^8*f^3*g^6 - 644*a^5*b^4*c^4*d^3*e^6*f*g^8 + 498*a^5*b^5*
c^3*d*e^8*f^2*g^7 + 498*a^5*b^5*c^3*d^2*e^7*f*g^8 - 1888*a^6*b*c^6*d^2*e^7*
f^3*g^6 - 1888*a^6*b*c^6*d^3*e^6*f^2*g^7 + 1600*a^6*b^2*c^5*d*e^8*f^3*g^6 +
 1600*a^6*b^2*c^5*d^3*e^6*f*g^8 - 176*a^6*b^3*c^4*d*e^8*f^2*g^7 - 176*a^6*b
^3*c^4*d^2*e^7*f*g^8 + 4*a*b^7*c^5*d*e^8*f^8*g + 4*a*b^7*c^5*d^8*e*f*g^8 +
4*a*b^11*c*d*e^8*f^4*g^5 + 4*a*b^11*c*d^4*e^5*f*g^8 - 160*a^4*b*c^8*d*e^8*f
^8*g - 160*a^4*b*c^8*d^8*e*f*g^8 - 14*a^4*b^8*c*d*e^8*f*g^8 - 192*a^2*b^2*c
^9*d^5*e^4*f^7*g^2 + 576*a^2*b^2*c^9*d^6*e^3*f^6*g^3 - 192*a^2*b^2*c^9*d^7*
e^2*f^5*g^4 + 656*a^2*b^3*c^8*d^4*e^5*f^7*g^2 - 496*a^2*b^3*c^8*d^5*e^4*f^6

*g^3 - 496*a^2*b^3*c^8*d^6*e^3*f^5*g^4 + 656*a^2*b^3*c^8*d^7*e^2*f^4*g^5 -
660*a^2*b^4*c^7*d^3*e^6*f^7*g^2 - 624*a^2*b^4*c^7*d^4*e^5*f^6*g^3 + 1284*a^
2*b^4*c^7*d^5*e^4*f^5*g^4 - 624*a^2*b^4*c^7*d^6*e^3*f^4*g^5 - 660*a^2*b^4*c
^7*d^7*e^2*f^3*g^6 + 54*a^2*b^5*c^6*d^2*e^7*f^7*g^2 + 1062*a^2*b^5*c^6*d^3*
e^6*f^6*g^3 - 474*a^2*b^5*c^6*d^4*e^5*f^5*g^4 - 474*a^2*b^5*c^6*d^5*e^4*f^4
*g^5 + 1062*a^2*b^5*c^6*d^6*e^3*f^3*g^6 + 54*a^2*b^5*c^6*d^7*e^2*f^2*g^7 -
130*a^2*b^6*c^5*d^2*e^7*f^6*g^3 - 482*a^2*b^6*c^5*d^3*e^6*f^5*g^4 + 850*a^2
*b^6*c^5*d^4*e^5*f^4*g^5 - 482*a^2*b^6*c^5*d^5*e^4*f^3*g^6 - 130*a^2*b^6*c^
5*d^6*e^3*f^2*g^7 + 108*a^2*b^7*c^4*d^2*e^7*f^5*g^4 - 228*a^2*b^7*c^4*d^3*e
^6*f^4*g^5 - 228*a^2*b^7*c^4*d^4*e^5*f^3*g^6 + 108*a^2*b^7*c^4*d^5*e^4*f^2*
g^7 - 18*a^2*b^8*c^3*d^2*e^7*f^4*g^5 + 192*a^2*b^8*c^3*d^3*e^6*f^3*g^6 - 18
*a^2*b^8*c^3*d^4*e^5*f^2*g^7 - 2*a^2*b^9*c^2*d^2*e^7*f^3*g^6 - 2*a^2*b^9*c^
2*d^3*e^6*f^2*g^7 + 544*a^3*b^2*c^8*d^3*e^6*f^7*g^2 + 960*a^3*b^2*c^8*d^4*e
^5*f^6*g^3 - 1440*a^3*b^2*c^8*d^5*e^4*f^5*g^4 + 960*a^3*b^2*c^8*d^6*e^3*f^4
*g^5 + 544*a^3*b^2*c^8*d^7*e^2*f^3*g^6 + 496*a^3*b^3*c^7*d^2*e^7*f^7*g^2 -
1168*a^3*b^3*c^7*d^3*e^6*f^6*g^3 + 688*a^3*b^3*c^7*d^4*e^5*f^5*g^4 + 688*a^
3*b^3*c^7*d^5*e^4*f^4*g^5 - 1168*a^3*b^3*c^7*d^6*e^3*f^3*g^6 + 496*a^3*b^3*
c^7*d^7*e^2*f^2*g^7 - 668*a^3*b^4*c^6*d^2*e^7*f^6*g^3 + 436*a^3*b^4*c^6*d^3
*e^6*f^5*g^4 - 1820*a^3*b^4*c^6*d^4*e^5*f^4*g^5 + 436*a^3*b^4*c^6*d^5*e^4*f
^3*g^6 - 668*a^3*b^4*c^6*d^6*e^3*f^2*g^7 + 238*a^3*b^5*c^5*d^2*e^7*f^5*g^4
+ 734*a^3*b^5*c^5*d^3*e^6*f^4*g^5 + 734*a^3*b^5*c^5*d^4*e^5*f^3*g^6 + 238*a
^3*b^5*c^5*d^5*e^4*f^2*g^7 + 144*a^3*b^6*c^4*d^2*e^7*f^4*g^5 - 416*a^3*b^6*
c^4*d^3*e^6*f^3*g^6 + 144*a^3*b^6*c^4*d^4*e^5*f^2*g^7 - 156*a^3*b^7*c^3*d^2
*e^7*f^3*g^6 - 156*a^3*b^7*c^3*d^3*e^6*f^2*g^7 + 44*a^3*b^8*c^2*d^2*e^7*f^2
*g^7 + 1344*a^4*b^2*c^7*d^2*e^7*f^6*g^3 + 192*a^4*b^2*c^7*d^3*e^6*f^5*g^4 +
1920*a^4*b^2*c^7*d^4*e^5*f^4*g^5 + 192*a^4*b^2*c^7*d^5*e^4*f^3*g^6 + 1344*
a^4*b^2*c^7*d^6*e^3*f^2*g^7 + 80*a^4*b^3*c^6*d^2*e^7*f^5*g^4 - 560*a^4*b^3*
c^6*d^3*e^6*f^4*g^5 - 560*a^4*b^3*c^6*d^4*e^5*f^3*g^6 + 80*a^4*b^3*c^6*d^5*
e^4*f^2*g^7 - 1280*a^4*b^4*c^5*d^2*e^7*f^4*g^5 - 220*a^4*b^4*c^5*d^3*e^6*f^
3*g^6 - 1280*a^4*b^4*c^5*d^4*e^5*f^2*g^7 + 714*a^4*b^5*c^4*d^2*e^7*f^3*g^6
+ 714*a^4*b^5*c^4*d^3*e^6*f^2*g^7 + 58*a^4*b^6*c^3*d^2*e^7*f^2*g^7 + 2304*a
^5*b^2*c^6*d^2*e^7*f^4*g^5 + 1248*a^5*b^2*c^6*d^3*e^6*f^3*g^6 + 2304*a^5*b^
2*c^6*d^4*e^5*f^2*g^7 - 272*a^5*b^3*c^5*d^2*e^7*f^3*g^6 - 272*a^5*b^3*c^5*d
^3*e^6*f^2*g^7 - 996*a^5*b^4*c^4*d^2*e^7*f^2*g^7 + 1600*a^6*b^2*c^5*d^2*e^7
*f^2*g^7 + 16*a*b^2*c^10*d^6*e^3*f^8*g + 16*a*b^2*c^10*d^8*e*f^6*g^3 - 48*a
*b^3*c^9*d^5*e^4*f^8*g - 48*a*b^3*c^9*d^8*e*f^5*g^4 + 66*a*b^4*c^8*d^4*e^5*
f^8*g + 66*a*b^4*c^8*d^8*e*f^4*g^5 - 52*a*b^5*c^7*d^3*e^6*f^8*g - 52*a*b^5*
c^7*d^8*e*f^3*g^6 + 14*a*b^6*c^6*d^2*e^7*f^8*g + 14*a*b^6*c^6*d^8*e*f^2*g^7
- 16*a*b^8*c^4*d*e^8*f^7*g^2 - 16*a*b^8*c^4*d^7*e^2*f*g^8 + 24*a*b^9*c^3*d
*e^8*f^6*g^3 + 24*a*b^9*c^3*d^6*e^3*f*g^8 - 16*a*b^10*c^2*d*e^8*f^5*g^4 - 1
6*a*b^10*c^2*d^5*e^4*f*g^8 + 2*a*b^11*c*d^2*e^7*f^3*g^6 + 2*a*b^11*c*d^3*e^
6*f^2*g^7 + 96*a^2*b*c^10*d^5*e^4*f^8*g + 96*a^2*b*c^10*d^8*e*f^5*g^4 - 42*
a^2*b^5*c^6*d*e^8*f^8*g - 42*a^2*b^5*c^6*d^8*e*f*g^8 - 10*a^2*b^10*c*d*e^8*
f^3*g^6 - 10*a^2*b^10*c*d^3*e^6*f*g^8 - 64*a^3*b*c^9*d^3*e^6*f^8*g - 64*a^3
*b*c^9*d^8*e*f^3*g^6 + 144*a^3*b^3*c^7*d*e^8*f^8*g + 144*a^3*b^3*c^7*d^8*e*

f*g^8 + 14*a^3*b^9*c*d*e^8*f^2*g^7 + 14*a^3*b^9*c*d^2*e^7*f*g^8 - 544*a^5*b
*c^7*d*e^8*f^6*g^3 - 544*a^5*b*c^7*d^6*e^3*f*g^8 + 168*a^5*b^6*c^2*d*e^8*f*
g^8 - 992*a^6*b*c^6*d*e^8*f^4*g^5 - 992*a^6*b*c^6*d^4*e^5*f*g^8 - 668*a^6*b
^4*c^3*d*e^8*f*g^8 - 992*a^7*b*c^5*d*e^8*f^2*g^7 - 992*a^7*b*c^5*d^2*e^7*f*
g^8 + 864*a^7*b^2*c^4*d*e^8*f*g^8))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 +
16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4
*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 -
8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a
^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5
*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f
^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 -
2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*
c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c
^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*
c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2
*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*
e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*
e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3
*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*
g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d
^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c
^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*
c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*
b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2
*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b
^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3
*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c
*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g -
64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^
3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^
4*b^2*c^2*d*e^3*f*g^3)) - (48*a^6*b^2*c^3*e^8*g^8 - 12*a^5*b^4*c^2*e^8*g^8
- 64*a^7*c^4*e^8*g^8 + 40*a^4*c^7*d^6*e^2*g^8 + 80*a^5*c^6*d^4*e^4*g^8 - 24
*a^6*c^5*d^2*e^6*g^8 + 40*a^4*c^7*e^8*f^6*g^2 + 80*a^5*c^6*e^8*f^4*g^4 - 24
*a^6*c^5*e^8*f^2*g^6 + a^4*b^6*c*e^8*g^8 + a*b^5*c^5*d^7*e*g^8 + a*b^9*c*d^
3*e^5*g^8 + 20*a^3*b*c^7*d^7*e*g^8 + a*b^5*c^5*e^8*f^7*g + a*b^9*c*e^8*f^3*
g^5 + 20*a^3*b*c^7*e^8*f^7*g + 8*a*c^10*d^5*e^3*f^7*g + 8*a*c^10*d^7*e*f^5*
g^3 + 8*a^3*c^8*d*e^7*f^7*g + 8*a^3*c^8*d^7*e*f*g^7 + 304*a^6*c^5*d*e^7*f*g
^7 - b^6*c^5*d*e^7*f^7*g - b^6*c^5*d^7*e*f*g^7 - b^10*c*d*e^7*f^3*g^5 - b^1
0*c*d^3*e^5*f*g^7 - 4*a*b^6*c^4*d^6*e^2*g^8 + 6*a*b^7*c^3*d^5*e^3*g^8 - 4*a
*b^8*c^2*d^4*e^4*g^8 - 9*a^2*b^3*c^6*d^7*e*g^8 - 2*a^2*b^8*c*d^2*e^6*g^8 -
88*a^4*b*c^6*d^5*e^3*g^8 - 172*a^5*b*c^5*d^3*e^5*g^8 - 4*a*b^6*c^4*e^8*f^6*
g^2 + 6*a*b^7*c^3*e^8*f^5*g^3 - 4*a*b^8*c^2*e^8*f^4*g^4 - 9*a^2*b^3*c^6*e^8
*f^7*g - 2*a^2*b^8*c*e^8*f^2*g^6 - 88*a^4*b*c^6*e^8*f^5*g^3 - 172*a^5*b*c^5
*e^8*f^3*g^5 - 16*a*c^10*d^6*e^2*f^6*g^2 + 16*a^2*c^9*d^3*e^5*f^7*g + 16*a^
2*c^9*d^7*e*f^3*g^5 + 192*a^4*c^7*d*e^7*f^5*g^3 + 192*a^4*c^7*d^5*e^3*f*g^7

+ 488*a^5*c^6*d*e^7*f^3*g^5 + 488*a^5*c^6*d^3*e^5*f*g^7 - 2*b^2*c^9*d^5*e^3*f^7*g - 2*b^2*c^9*d^7*e*f^5*g^3 + 3*b^3*c^8*d^4*e^4*f^7*g + 3*b^3*c^8*d^7*e*f^4*g^4 + 4*b^7*c^4*d*e^7*f^6*g^2 + 4*b^7*c^4*d^6*e^2*f*g^7 - 6*b^8*c^3*d*e^7*f^5*g^3 - 6*b^8*c^3*d^5*e^3*f*g^7 + 4*b^9*c^2*d*e^7*f^4*g^4 + 4*b^9*c^2*d^4*e^4*f*g^7 - b^10*c*d^2*e^6*f^2*g^6 + 38*a^2*b^4*c^5*d^6*e^2*g^8 - 58*a^2*b^5*c^4*d^5*e^3*g^8 + 36*a^2*b^6*c^3*d^4*e^4*g^8 - 5*a^2*b^7*c^2*d^3*e^5*g^8 - 98*a^3*b^2*c^6*d^6*e^2*g^8 + 158*a^3*b^3*c^5*d^5*e^3*g^8 - 80*a^3*b^4*c^4*d^4*e^4*g^8 - 22*a^3*b^5*c^3*d^3*e^5*g^8 + 22*a^3*b^6*c^2*d^2*e^6*g^8 - 20*a^4*b^2*c^5*d^4*e^4*g^8 + 147*a^4*b^3*c^4*d^3*e^5*g^8 - 80*a^4*b^4*c^3*d^2*e^6*g^8 + 102*a^5*b^2*c^4*d^2*e^6*g^8 + 38*a^2*b^4*c^5*e^8*f^6*g^2 - 58*a^2*b^5*c^4*e^8*f^5*g^3 + 36*a^2*b^6*c^3*e^8*f^4*g^4 - 5*a^2*b^7*c^2*e^8*f^3*g^5 - 98*a^3*b^2*c^6*e^8*f^6*g^2 + 158*a^3*b^3*c^5*e^8*f^5*g^3 - 80*a^3*b^4*c^4*e^8*f^4*g^4 - 22*a^3*b^5*c^3*e^8*f^3*g^5 + 22*a^3*b^6*c^2*e^8*f^2*g^6 - 20*a^4*b^2*c^5*e^8*f^4*g^4 + 147*a^4*b^3*c^4*e^8*f^3*g^5 - 80*a^4*b^4*c^3*e^8*f^2*g^6 + 102*a^5*b^2*c^4*e^8*f^2*g^6 - 56*a^2*c^9*d^4*e^4*f^6*g^2 + 80*a^2*c^9*d^5*e^3*f^5*g^3 - 56*a^2*c^9*d^6*e^2*f^4*g^4 + 264*a^3*c^8*d^3*e^5*f^5*g^3 - 96*a^3*c^8*d^4*e^4*f^4*g^4 + 264*a^3*c^8*d^5*e^3*f^3*g^5 + 40*a^4*c^7*d^2*e^6*f^4*g^4 + 736*a^4*c^7*d^3*e^5*f^3*g^5 + 40*a^4*c^7*d^4*e^4*f^2*g^6 + 16*a^5*c^6*d^2*e^6*f^2*g^6 + 4*b^2*c^9*d^6*e^2*f^6*g^2 - 3*b^3*c^8*d^5*e^3*f^6*g^2 - 3*b^3*c^8*d^6*e^2*f^5*g^3 - 4*b^4*c^7*d^4*e^4*f^6*g^2 + 8*b^4*c^7*d^5*e^3*f^5*g^3 - 4*b^4*c^7*d^6*e^2*f^4*g^4 - b^6*c^5*d^2*e^6*f^6*g^2 - b^6*c^5*d^3*e^5*f^5*g^3 - b^6*c^5*d^4*e^4*f^4*g^4 - b^6*c^5*d^5*e^3*f^3*g^5 - b^6*c^5*d^6*e^2*f^2*g^6 + 4*b^7*c^4*d^2*e^6*f^5*g^3 + 4*b^7*c^4*d^3*e^5*f^4*g^4 + 4*b^7*c^4*d^4*e^4*f^3*g^5 + 4*b^7*c^4*d^5*e^3*f^2*g^6 - 6*b^8*c^3*d^2*e^6*f^4*g^4 - 6*b^8*c^3*d^3*e^5*f^3*g^5 - 6*b^8*c^3*d^4*e^4*f^2*g^6 + 4*b^9*c^2*d^2*e^6*f^3*g^5 + 4*b^9*c^2*d^3*e^5*f^2*g^6 + 30*a*b^2*c^8*d^4*e^4*f^6*g^2 - 52*a*b^2*c^8*d^5*e^3*f^5*g^3 + 30*a*b^2*c^8*d^6*e^2*f^4*g^4 - 6*a*b^3*c^7*d^3*e^5*f^6*g^2 + 8*a*b^3*c^7*d^4*e^4*f^5*g^3 + 8*a*b^3*c^7*d^5*e^3*f^4*g^4 - 6*a*b^3*c^7*d^6*e^2*f^3*g^5 + 20*a*b^4*c^6*d^2*e^6*f^6*g^2 + 26*a*b^4*c^6*d^3*e^5*f^5*g^3 - 28*a*b^4*c^6*d^4*e^4*f^4*g^4 + 26*a*b^4*c^6*d^5*e^3*f^3*g^5 + 20*a*b^4*c^6*d^6*e^2*f^2*g^6 - 61*a*b^5*c^5*d^2*e^6*f^5*g^3 - 43*a*b^5*c^5*d^3*e^5*f^4*g^4 - 43*a*b^5*c^5*d^4*e^4*f^3*g^5 - 61*a*b^5*c^5*d^5*e^3*f^2*g^6 + 80*a*b^6*c^4*d^2*e^6*f^4*g^4 + 68*a*b^6*c^4*d^3*e^5*f^3*g^5 + 80*a*b^6*c^4*d^4*e^4*f^2*g^6 - 44*a*b^7*c^3*d^2*e^6*f^3*g^5 - 44*a*b^7*c^3*d^3*e^5*f^2*g^6 + 4*a*b^8*c^2*d^2*e^6*f^2*g^6 + 24*a^2*b*c^8*d^3*e^5*f^6*g^2 - 32*a^2*b*c^8*d^4*e^4*f^5*g^3 - 32*a^2*b*c^8*d^5*e^3*f^4*g^4 + 24*a^2*b*c^8*d^6*e^2*f^3*g^5 + 113*a^2*b^3*c^6*d*e^7*f^6*g^2 + 113*a^2*b^3*c^6*d^6*e^2*f*g^7 - 152*a^2*b^4*c^5*d*e^7*f^5*g^3 - 152*a^2*b^4*c^5*d^5*e^3*f*g^7 + 34*a^2*b^5*c^4*d*e^7*f^4*g^4 + 34*a^2*b^5*c^4*d^4*e^4*f*g^7 + 64*a^2*b^6*c^3*d*e^7*f^3*g^5 + 64*a^2*b^6*c^3*d^3*e^5*f*g^7 - 31*a^2*b^7*c^2*d*e^7*f^2*g^6 - 31*a^2*b^7*c^2*d^2*e^6*f*g^7 - 260*a^3*b*c^7*d^2*e^6*f^5*g^3 - 476*a^3*b*c^7*d^3*e^5*f^4*g^4 - 476*a^3*b*c^7*d^4*e^4*f^3*g^5 - 260*a^3*b*c^7*d^5*e^3*f^2*g^6 - 16*a^3*b^2*c^6*d*e^7*f^5*g^3 - 16*a^3*b^2*c^6*d^5*e^3*f*g^7 + 282*a^3*b^3*c^5*d*e^7*f^4*g^4 + 282*a^3*b^3*c^5*d^4*e^4*f*g^7 - 316*a^3*b^4*c^4*d*e^7*f^3*g^5 - 316*a^3*b^4*c^4*d^3*e^5*f*g^7

+ 70*a^3*b^5*c^3*d*e^7*f^2*g^6 + 70*a^3*b^5*c^3*d^2*e^6*f*g^7 - 928*a^4*b*
c^6*d^2*e^6*f^3*g^5 - 928*a^4*b*c^6*d^3*e^5*f^2*g^6 + 246*a^4*b^2*c^5*d*e^7
*f^3*g^5 + 246*a^4*b^2*c^5*d^3*e^5*f*g^7 + 173*a^4*b^3*c^4*d*e^7*f^2*g^6 +
173*a^4*b^3*c^4*d^2*e^6*f*g^7 - 12*a*b*c^9*d^4*e^4*f^7*g - 12*a*b*c^9*d^7*e
*f^4*g^4 + 10*a*b^4*c^6*d*e^7*f^7*g + 10*a*b^4*c^6*d^7*e*f*g^7 + 3*a*b^9*c*
d*e^7*f^2*g^6 + 3*a*b^9*c*d^2*e^6*f*g^7 - 2*a^2*b^8*c*d*e^7*f*g^7 - 64*a^2*
b^2*c^7*d^2*e^6*f^6*g^2 - 154*a^2*b^2*c^7*d^3*e^5*f^5*g^3 + 152*a^2*b^2*c^7
*d^4*e^4*f^4*g^4 - 154*a^2*b^2*c^7*d^5*e^3*f^3*g^5 - 64*a^2*b^2*c^7*d^6*e^2
*f^2*g^6 + 245*a^2*b^3*c^6*d^2*e^6*f^5*g^3 + 227*a^2*b^3*c^6*d^3*e^5*f^4*g^
4 + 227*a^2*b^3*c^6*d^4*e^4*f^3*g^5 + 245*a^2*b^3*c^6*d^5*e^3*f^2*g^6 - 346
*a^2*b^4*c^5*d^2*e^6*f^4*g^4 - 280*a^2*b^4*c^5*d^3*e^5*f^3*g^5 - 346*a^2*b^
4*c^5*d^4*e^4*f^2*g^6 + 120*a^2*b^5*c^4*d^2*e^6*f^3*g^5 + 120*a^2*b^5*c^4*d
^3*e^5*f^2*g^6 + 70*a^2*b^6*c^3*d^2*e^6*f^2*g^6 + 478*a^3*b^2*c^6*d^2*e^6*f
^4*g^4 + 232*a^3*b^2*c^6*d^3*e^5*f^3*g^5 + 478*a^3*b^2*c^6*d^4*e^4*f^2*g^6
+ 200*a^3*b^3*c^5*d^2*e^6*f^3*g^5 + 200*a^3*b^3*c^5*d^3*e^5*f^2*g^6 - 528*a
^3*b^4*c^4*d^2*e^6*f^2*g^6 + 988*a^4*b^2*c^5*d^2*e^6*f^2*g^6 + 12*a*b*c^9*d
^5*e^3*f^6*g^2 + 12*a*b*c^9*d^6*e^2*f^5*g^3 - 4*a*b^2*c^8*d^3*e^5*f^7*g - 4
*a*b^2*c^8*d^7*e*f^3*g^5 - 2*a*b^3*c^7*d^2*e^6*f^7*g - 2*a*b^3*c^7*d^7*e*f^
2*g^6 - 41*a*b^5*c^5*d*e^7*f^6*g^2 - 41*a*b^5*c^5*d^6*e^2*f*g^7 + 60*a*b^6*
c^4*d*e^7*f^5*g^3 + 60*a*b^6*c^4*d^5*e^3*f*g^7 - 34*a*b^7*c^3*d*e^7*f^4*g^4
 - 34*a*b^7*c^3*d^4*e^4*f*g^7 + 2*a*b^8*c^2*d*e^7*f^3*g^5 + 2*a*b^8*c^2*d^3
*e^5*f*g^7 + 8*a^2*b*c^8*d^2*e^6*f^7*g + 8*a^2*b*c^8*d^7*e*f^2*g^6 - 26*a^2
*b^2*c^7*d*e^7*f^7*g - 26*a^2*b^2*c^7*d^7*e*f*g^7 - 52*a^3*b*c^7*d*e^7*f^6*
g^2 - 52*a^3*b*c^7*d^6*e^2*f*g^7 + 24*a^3*b^6*c^2*d*e^7*f*g^7 - 520*a^4*b*c
^6*d*e^7*f^4*g^4 - 520*a^4*b*c^6*d^4*e^4*f*g^7 - 80*a^4*b^4*c^3*d*e^7*f*g^7
 - 596*a^5*b*c^5*d*e^7*f^2*g^6 - 596*a^5*b*c^5*d^2*e^6*f*g^7 - 12*a^5*b^2*c
^4*d*e^7*f*g^7)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4
+ 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d
^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^
4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 +
 b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^
5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5
*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4
- 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*
b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2
*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5
*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*
b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b
*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*
d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d
^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^
2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4
*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^
3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2
+ 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g

- 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) + (x*(48*a^4*b^5*c^2*e^8*g^8 - 192*a^5*b^3*c^3*e^8*g^8 - 256*a^4*c^7*d^5*e^3*g^8 - 464*a^5*c^6*d^3*e^5*g^8 - 256*a^4*c^7*e^8*f^5*g^3 - 464*a^5*c^6*e^8*f^3*g^5 - 4*a^3*b^7*c*e^8*g^8 + 256*a^6*b*c^4*e^8*g^8 - 48*a^3*c^8*d^7*e*g^8 - 256*a^6*c^5*d*e^7*g^8 - 48*a^3*c^8*e^8*f^7*g - 256*a^6*c^5*e^8*f*g^7 - 2*a*b^4*c^6*d^7*e*g^8 - 2*a*b^9*c*d^2*e^6*g^8 + 6*a^2*b^8*c*d*e^7*g^8 - 2*a*b^4*c^6*e^8*f^7*g - 2*a*b^9*c*e^8*f^2*g^6 + 6*a^2*b^8*c*e^8*f*g^7 - 16*a*c^10*d^4*e^4*f^7*g - 16*a*c^10*d^7*e*f^4*g^4 + 2*b^5*c^6*d*e^7*f^7*g + 2*b^5*c^6*d^7*e*f*g^7 + 2*b^10*c*d*e^7*f^2*g^6 + 2*b^10*c*d^2*e^6*f*g^7 + 6*a*b^5*c^5*d^6*e^2*g^8 - 4*a*b^6*c^4*d^5*e^3*g^8 - 4*a*b^7*c^3*d^4*e^4*g^8 + 6*a*b^8*c^2*d^3*e^5*g^8 + 20*a^2*b^2*c^7*d^7*e*g^8 + 144*a^3*b*c^7*d^6*e^2*g^8 - 68*a^3*b^6*c^2*d*e^7*g^8 + 640*a^4*b*c^6*d^4*e^4*g^8 + 240*a^4*b^4*c^3*d*e^7*g^8 + 848*a^5*b*c^5*d^2*e^6*g^8 - 192*a^5*b^2*c^4*d*e^7*g^8 + 6*a*b^5*c^5*e^8*f^6*g^2 - 4*a*b^6*c^4*e^8*f^5*g^3 - 4*a*b^7*c^3*e^8*f^4*g^4 + 6*a*b^8*c^2*e^8*f^3*g^5 + 20*a^2*b^2*c^7*e^8*f^7*g + 144*a^3*b*c^7*e^8*f^6*g^2 - 68*a^3*b^6*c^2*e^8*f*g^7 + 640*a^4*b*c^6*e^8*f^4*g^4 + 240*a^4*b^4*c^3*e^8*f*g^7 + 848*a^5*b*c^5*e^8*f^2*g^6 - 192*a^5*b^2*c^4*e^8*f*g^7 + 16*a*c^10*d^5*e^3*f^6*g^2 + 16*a*c^10*d^6*e^2*f^5*g^3 - 64*a^2*c^9*d^2*e^6*f^7*g - 64*a^2*c^9*d^7*e*f^2*g^6 + 48*a^3*c^8*d*e^7*f^6*g^2 + 48*a^3*c^8*d^6*e^2*f*g^7 - 304*a^5*c^6*d*e^7*f^2*g^6 - 304*a^5*c^6*d^2*e^6*f*g^7 + 4*b^2*c^9*d^4*e^4*f^7*g + 4*b^2*c^9*d^7*e*f^4*g^4 - 8*b^3*c^8*d^3*e^5*f^7*g - 8*b^3*c^8*d^7*e*f^3*g^5 + 2*b^4*c^7*d^2*e^6*f^7*g + 2*b^4*c^7*d^7*e*f^2*g^6 - 6*b^6*c^5*d*e^7*f^6*g^2 - 6*b^6*c^5*d^6*e^2*f*g^7 + 4*b^7*c^4*d*e^7*f^5*g^3 + 4*b^7*c^4*d^5*e^3*f*g^7 + 4*b^8*c^3*d*e^7*f^4*g^4 + 4*b^8*c^3*d^4*e^4*f*g^7 - 6*b^9*c^2*d*e^7*f^3*g^5 - 6*b^9*c^2*d^3*e^5*f*g^7 - 60*a^2*b^3*c^6*d^6*e^2*g^8 + 30*a^2*b^4*c^5*d^5*e^3*g^8 + 64*a^2*b^5*c^4*d^4*e^4*g^8 - 72*a^2*b^6*c^3*d^3*e^5*g^8 + 12*a^2*b^7*c^2*d^2*e^6*g^8 + 8*a^3*b^2*c^6*d^5*e^3*g^8 - 352*a^3*b^3*c^5*d^4*e^4*g^8 + 268*a^3*b^4*c^4*d^3*e^5*g^8 + 52*a^3*b^5*c^3*d^2*e^6*g^8 - 188*a^4*b^2*c^5*d^3*e^5*g^8 - 484*a^4*b^3*c^4*d^2*e^6*g^8 - 60*a^2*b^3*c^6*e^8*f^6*g^2 + 30*a^2*b^4*c^5*e^8*f^5*g^3 + 64*a^2*b^5*c^4*e^8*f^4*g^4 - 72*a^2*b^6*c^3*e^8*f^3*g^5 + 12*a^2*b^7*c^2*e^8*f^2*g^6 + 8*a^3*b^2*c^6*e^8*f^5*g^3 - 352*a^3*b^3*c^5*e^8*f^4*g^4 + 268*a^3*b^4*c^4*e^8*f^3*g^5 + 52*a^3*b^5*c^3*e^8*f^2*g^6 - 188*a^4*b^2*c^5*e^8*f^3*g^5 - 484*a^4*b^3*c^4*e^8*f^2*g^6 + 64*a^2*c^9*d^3*e^5*f^6*g^2 + 64*a^2*c^9*d^6*e^2*f^3*g^5 - 272*a^3*c^8*d^2*e^6*f^5*g^3 + 16*a^3*c^8*d^3*e^5*f^4*g^4 + 16*a^3*c^8*d^4*e^4*f^3*g^5 - 272*a^3*c^8*d^5*e^3*f^2*g^6 - 512*a^4*c^7*d^2*e^6*f^3*g^5 - 512*a^4*c^7*d^3*e^5*f^2*g^6 - 4*b^2*c^9*d^5*e^3*f^6*g^2 - 4*b^2*c^9*d^6*e^2*f^5*g^3 - 4*b^3*c^8*d^4*e^4*f^6*g^2 + 24*b^3*c^8*d^5*e^3*f^5*g^3 - 4*b^3*c^8*d^6*e^2*f^4*g^4 + 22*b^4*c^7*d^3*e^5*f^6*g^2 - 24*b^4*c^7*d^4*e^4*f^5

*g^3 - 24*b^4*c^7*d^5*e^3*f^4*g^4 + 22*b^4*c^7*d^6*e^2*f^3*g^5 - 8*b^5*c^6*
d^2*e^6*f^6*g^2 - 14*b^5*c^6*d^3*e^5*f^5*g^3 + 40*b^5*c^6*d^4*e^4*f^4*g^4 -
14*b^5*c^6*d^5*e^3*f^3*g^5 - 8*b^5*c^6*d^6*e^2*f^2*g^6 + 14*b^6*c^5*d^2*e^
6*f^5*g^3 - 4*b^6*c^5*d^3*e^5*f^4*g^4 - 4*b^6*c^5*d^4*e^4*f^3*g^5 + 14*b^6*
c^5*d^5*e^3*f^2*g^6 - 16*b^7*c^4*d^2*e^6*f^4*g^4 - 4*b^7*c^4*d^3*e^5*f^3*g^
5 - 16*b^7*c^4*d^4*e^4*f^2*g^6 + 14*b^8*c^3*d^2*e^6*f^3*g^5 + 14*b^8*c^3*d^
3*e^5*f^2*g^6 - 8*b^9*c^2*d^2*e^6*f^2*g^6 - 8*a*b^9*c*d*e^7*f*g^7 - 104*a*b
^2*c^8*d^3*e^5*f^6*g^2 + 96*a*b^2*c^8*d^4*e^4*f^5*g^3 + 96*a*b^2*c^8*d^5*e^
3*f^4*g^4 - 104*a*b^2*c^8*d^6*e^2*f^3*g^5 + 104*a*b^3*c^7*d^3*e^5*f^5*g^3 -
160*a*b^3*c^7*d^4*e^4*f^4*g^4 + 104*a*b^3*c^7*d^5*e^3*f^3*g^5 - 78*a*b^4*c
^6*d^2*e^6*f^5*g^3 - 42*a*b^4*c^6*d^3*e^5*f^4*g^4 - 42*a*b^4*c^6*d^4*e^4*f^
3*g^5 - 78*a*b^4*c^6*d^5*e^3*f^2*g^6 + 166*a*b^5*c^5*d^2*e^6*f^4*g^4 + 88*a
*b^5*c^5*d^3*e^5*f^3*g^5 + 166*a*b^5*c^5*d^4*e^4*f^2*g^6 - 148*a*b^6*c^4*d^
2*e^6*f^3*g^5 - 148*a*b^6*c^4*d^3*e^5*f^2*g^6 + 60*a*b^7*c^3*d^2*e^6*f^2*g^
6 + 128*a^2*b*c^8*d^2*e^6*f^6*g^2 - 192*a^2*b*c^8*d^3*e^5*f^5*g^3 - 192*a^2
*b*c^8*d^5*e^3*f^3*g^5 + 128*a^2*b*c^8*d^6*e^2*f^2*g^6 - 212*a^2*b^2*c^7*d*
e^7*f^6*g^2 - 212*a^2*b^2*c^7*d^6*e^2*f*g^7 + 96*a^2*b^3*c^6*d*e^7*f^5*g^3
+ 96*a^2*b^3*c^6*d^5*e^3*f*g^7 + 266*a^2*b^4*c^5*d*e^7*f^4*g^4 + 266*a^2*b^
4*c^5*d^4*e^4*f*g^7 - 196*a^2*b^5*c^4*d*e^7*f^3*g^5 - 196*a^2*b^5*c^4*d^3*e
^5*f*g^7 - 108*a^2*b^6*c^3*d*e^7*f^2*g^6 - 108*a^2*b^6*c^3*d^2*e^6*f*g^7 +
656*a^3*b*c^7*d^2*e^6*f^4*g^4 - 64*a^3*b*c^7*d^3*e^5*f^3*g^5 + 656*a^3*b*c^
7*d^4*e^4*f^2*g^6 - 488*a^3*b^2*c^6*d*e^7*f^4*g^4 - 488*a^3*b^2*c^6*d^4*e^4
*f*g^7 + 16*a^3*b^3*c^5*d*e^7*f^3*g^5 + 16*a^3*b^3*c^5*d^3*e^5*f*g^7 + 612*
a^3*b^4*c^4*d*e^7*f^2*g^6 + 612*a^3*b^4*c^4*d^2*e^6*f*g^7 + 1536*a^4*b*c^6*
d^2*e^6*f^2*g^6 - 772*a^4*b^2*c^5*d*e^7*f^2*g^6 - 772*a^4*b^2*c^5*d^2*e^6*f
*g^7 + 32*a*b*c^9*d^3*e^5*f^7*g + 32*a*b*c^9*d^7*e*f^3*g^5 - 24*a*b^3*c^7*d
*e^7*f^7*g - 24*a*b^3*c^7*d^7*e*f*g^7 + 64*a^2*b*c^8*d*e^7*f^7*g + 64*a^2*b
*c^8*d^7*e*f*g^7 + 608*a^5*b*c^5*d*e^7*f*g^7 + 156*a^2*b^2*c^7*d^2*e^6*f^5*
g^3 + 228*a^2*b^2*c^7*d^3*e^5*f^4*g^4 + 228*a^2*b^2*c^7*d^4*e^4*f^3*g^5 + 1
56*a^2*b^2*c^7*d^5*e^3*f^2*g^6 - 572*a^2*b^3*c^6*d^2*e^6*f^4*g^4 - 272*a^2*
b^3*c^6*d^3*e^5*f^3*g^5 - 572*a^2*b^3*c^6*d^4*e^4*f^2*g^6 + 424*a^2*b^4*c^5
*d^2*e^6*f^3*g^5 + 424*a^2*b^4*c^5*d^3*e^5*f^2*g^6 + 24*a^2*b^5*c^4*d^2*e^6
*f^2*g^6 - 96*a^3*b^2*c^6*d^2*e^6*f^3*g^5 - 96*a^3*b^2*c^6*d^3*e^5*f^2*g^6
- 928*a^3*b^3*c^5*d^2*e^6*f^2*g^6 + 16*a*b*c^9*d^4*e^4*f^6*g^2 - 96*a*b*c^9
*d^5*e^3*f^5*g^3 + 16*a*b*c^9*d^6*e^2*f^4*g^4 + 8*a*b^2*c^8*d^2*e^6*f^7*g +
8*a*b^2*c^8*d^7*e*f^2*g^6 + 74*a*b^4*c^6*d*e^7*f^6*g^2 + 74*a*b^4*c^6*d^6*
e^2*f*g^7 - 48*a*b^5*c^5*d*e^7*f^5*g^3 - 48*a*b^5*c^5*d^5*e^3*f*g^7 - 52*a*
b^6*c^4*d*e^7*f^4*g^4 - 52*a*b^6*c^4*d^4*e^4*f*g^7 + 64*a*b^7*c^3*d*e^7*f^3
*g^5 + 64*a*b^7*c^3*d^3*e^5*f*g^7 - 6*a*b^8*c^2*d*e^7*f^2*g^6 - 6*a*b^8*c^2
*d^2*e^6*f*g^7 + 84*a^2*b^7*c^2*d*e^7*f*g^7 + 128*a^3*b*c^7*d*e^7*f^5*g^3 +
128*a^3*b*c^7*d^5*e^3*f*g^7 - 248*a^3*b^5*c^3*d*e^7*f*g^7 + 512*a^4*b*c^6*
d*e^7*f^3*g^5 + 512*a^4*b*c^6*d^3*e^5*f*g^7 + 8*a^4*b^3*c^4*d*e^7*f*g^7))/(
16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*
f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*
c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e

^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*
f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2
 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*
b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f
*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4
 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4
- 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4
+ 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g -
 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g +
16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2
*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^
7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^
2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 -
6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3
*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3
*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*
e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2
 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*
e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*
a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2
*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b
^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^
2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(b^8*c*e
^7*g^7 + 104*a^4*c^5*e^7*g^7 + 50*a^2*b^4*c^3*e^7*g^7 - 96*a^3*b^2*c^4*e^7*
g^7 + 36*a^2*c^7*d^4*e^3*g^7 + 72*a^3*c^6*d^2*e^5*g^7 - 2*b^3*c^6*d^5*e^2*g
^7 + b^4*c^5*d^4*e^3*g^7 + b^6*c^3*d^2*e^5*g^7 + 36*a^2*c^7*e^7*f^4*g^3 + 7
2*a^3*c^6*e^7*f^2*g^5 - 2*b^3*c^6*e^7*f^5*g^2 + b^4*c^5*e^7*f^4*g^3 + b^6*c
^3*e^7*f^2*g^5 - 12*a*b^6*c^2*e^7*g^7 + b^2*c^7*d^6*e*g^7 - 2*b^7*c^2*d*e^6
*g^7 + b^2*c^7*e^7*f^6*g - 2*b^7*c^2*e^7*f*g^6 + 4*c^9*d^2*e^5*f^6*g + 4*c^
9*d^6*e*f^2*g^5 - 4*a*b*c^7*d^5*e^2*g^7 + 22*a*b^5*c^3*d*e^6*g^7 - 16*a^3*b
*c^5*d*e^6*g^7 - 4*a*b*c^7*e^7*f^5*g^2 + 22*a*b^5*c^3*e^7*f*g^6 - 16*a^3*b*
c^5*e^7*f*g^6 + 8*a*c^8*d*e^6*f^5*g^2 + 8*a*c^8*d^5*e^2*f*g^6 - 112*a^3*c^6
*d*e^6*f*g^6 + 4*b^6*c^3*d*e^6*f*g^6 + 2*a*b^2*c^6*d^4*e^3*g^7 + 10*a*b^3*c
^5*d^3*e^4*g^7 - 18*a*b^4*c^4*d^2*e^5*g^7 - 80*a^2*b*c^6*d^3*e^4*g^7 - 56*a
^2*b^3*c^4*d*e^6*g^7 + 2*a*b^2*c^6*e^7*f^4*g^3 + 10*a*b^3*c^5*e^7*f^3*g^4 -
 18*a*b^4*c^4*e^7*f^2*g^5 - 80*a^2*b*c^6*e^7*f^3*g^4 - 56*a^2*b^3*c^4*e^7*f
*g^6 + 40*a*c^8*d^2*e^5*f^4*g^3 + 40*a*c^8*d^4*e^3*f^2*g^5 + 16*a^2*c^7*d*e
^6*f^3*g^4 + 16*a^2*c^7*d^3*e^4*f*g^6 - 12*b*c^8*d^2*e^5*f^5*g^2 - 12*b*c^8
*d^5*e^2*f^2*g^5 + 10*b^2*c^7*d*e^6*f^5*g^2 + 10*b^2*c^7*d^5*e^2*f*g^6 - 14
*b^4*c^5*d*e^6*f^3*g^4 - 14*b^4*c^5*d^3*e^4*f*g^6 + 6*b^5*c^4*d*e^6*f^2*g^5
 + 6*b^5*c^4*d^2*e^5*f*g^6 - 4*b*c^8*d*e^6*f^6*g - 4*b*c^8*d^6*e*f*g^6 + 54
*a^2*b^2*c^5*d^2*e^5*g^7 + 54*a^2*b^2*c^5*e^7*f^2*g^5 + 168*a^2*c^7*d^2*e^5
*f^2*g^5 + 5*b^2*c^7*d^2*e^5*f^4*g^3 + 5*b^2*c^7*d^4*e^3*f^2*g^5 + 10*b^3*c
^6*d^2*e^5*f^3*g^4 + 10*b^3*c^6*d^3*e^4*f^2*g^5 - 12*b^4*c^5*d^2*e^5*f^2*g^
5 + 36*a*b^2*c^6*d^2*e^5*f^2*g^5 - 60*a*b*c^7*d*e^6*f^4*g^3 - 60*a*b*c^7*d^

4*e^3*f*g^6 - 72*a*b^4*c^4*d*e^6*f*g^6 - 80*a*b*c^7*d^2*e^5*f^3*g^4 - 80*a*
b*c^7*d^3*e^4*f^2*g^5 + 92*a*b^2*c^6*d*e^6*f^3*g^4 + 92*a*b^2*c^6*d^3*e^4*f
*g^6 + 6*a*b^3*c^5*d*e^6*f^2*g^5 + 6*a*b^3*c^5*d^2*e^5*f*g^6 - 192*a^2*b*c^
6*d*e^6*f^2*g^5 - 192*a^2*b*c^6*d^2*e^5*f*g^6 + 276*a^2*b^2*c^5*d*e^6*f*g^6
))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*
e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*
b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d
^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*
e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2
*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*
a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e
^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3
*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*
g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*
g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3
*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*
g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2
 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g -
2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 1
6*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^
4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4
*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d
*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*
c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2
*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*
d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g -
 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2
*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a
^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^
2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(4*b
^3*c^4*e^6*g^6 - 16*a*b*c^5*e^6*g^6 + 16*a*c^6*d*e^5*g^6 + 16*a*c^6*e^6*f*g
^5 - 4*b^2*c^5*d*e^5*g^6 - 4*b^2*c^5*e^6*f*g^5))/(16*a^2*c^6*d^4*f^4 + a^4*
b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 1
6*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c
^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e
^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2
 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 +
b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5
*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*
g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4
 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4
+ 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g
- 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 -
 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 -
32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*

a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3))*root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b

^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^
5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776
*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3
456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4
+ 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z
^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^
8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6
*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*
f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e
^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^
7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3
*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*
c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b
^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^
5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828
*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1
828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4
+ 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z
^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^
4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8
*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*
f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e
^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^
2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6
*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*
c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b
^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^
7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408
*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1
408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4
+ 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z
^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^
5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4
*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*
f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e
^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^
8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2
*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c
^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*
c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6
*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^
3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b
^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*
b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2
*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^

3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a
^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694
*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 69
4*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 6
92*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 +
692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 +
672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4
+ 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4
+ 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^
4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z
^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7
*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^
4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g
^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*
g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6
*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^
2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f
^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*
f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2
*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^
2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*
e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^
5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d
^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*
d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5
*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c
^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7
*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^
7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^
5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5
*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*
c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c
^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c
^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*
c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e
*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^
7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f
^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*
g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9
*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^
5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g
*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*
z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z

^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^
4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4
 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4
+ 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 -
 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 -
1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1
280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 12
80*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 115
2*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152
*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*
a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a
^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7
*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^
5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c
^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*
d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e
^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*
f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f
*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^
3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z
^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4
+ 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 4
80*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*
a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3
*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^
6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c
^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*
d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3
*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*
f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6
*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*
g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z
^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4
- 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 2
56*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*
a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7
*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^
7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6
*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^
4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*
e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^
3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f
^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*
g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9
*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^

4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 +
 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 15
6*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5
*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^
5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d
^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*
e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7
*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^
7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g
^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*
z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 +
 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24
*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^
11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^1
1*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2
*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*
e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6
*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4
 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^1
1*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^
8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6
*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4
 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*
a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c
^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^
3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4
*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*
g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z
^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 +
1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*
a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c
^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^
6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8
*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*
g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4
 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*
b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^
4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f
^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^
4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2
*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^
4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^1
0*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4
*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*
z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 +

90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^1
0*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c
^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*
f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g
^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^
4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 1
92*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^
2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c
^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10
*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g
^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4
- 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320
*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*
b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c
^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6
*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g
^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4
- 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a
^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^
3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6
*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*
e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f
^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4
+ 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a
^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*
c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^
9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4
*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e
^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^
4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*
d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z
^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^
5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*
e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*
z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a
^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^
7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z
^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*
b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g
^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3
*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f
^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 -
768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*
b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10
*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^

4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b
^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f
^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z
^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^
4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^1
0*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^
4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8
*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5
*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10
*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a
^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^
9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*
z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68
*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*
d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^1
0*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6
*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g
^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8
*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*
g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b
^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*
f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 -
 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f
^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 +
256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^
10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 +
384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*
d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^
4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d
^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*
z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7
*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z
^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b
^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4
- b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g
^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^1
0*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*
b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^
4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b
^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*
e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f
^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^
2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^
3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d
^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*

g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 +
 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5
*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^
7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z
^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^
7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4
*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^
2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b
^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*
b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3
*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^
2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2
*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*
b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b
^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^
5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*
c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e
*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4
*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^
4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3
*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^
3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z
^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2
+ 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 1
22*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*
a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b
^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c
^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^
2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4
*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*
g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z
^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 +
48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^
2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3
*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^
6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2
*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*
g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z
^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 +
20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*
b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c
^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^
7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g
^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 +
24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^

6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2
*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z
^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*
c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5
*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^
2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c
^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^
6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g
^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2
- 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a
^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*
d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4
*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^
4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 +
 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^
4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^
2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5
*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^
2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229
*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^
3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^
4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8
*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2
*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^
2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*
b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f
^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2
- 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^
8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g
^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 18
2*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e
^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^
2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b
*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*
e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 5
8*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d
^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2
+ 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*
f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*
a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7
*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*
c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2
 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^
8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 4
8*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5

*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2
*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^
2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g
^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^
3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d
^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5
*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 -
8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^
4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^
8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 +
36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2
*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a
^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*
z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^
2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 4
5*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 +
4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b
^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*
c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*
b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^
3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2
*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 1
20*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^
3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4
*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2
*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2
*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*
b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z
- 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*
g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d
^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*
z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d
^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z
+ b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2
*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4
*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c
*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6
*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c
*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12
*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f
*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^
6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*
a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2
*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5
*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*

```
z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^
2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4
*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*
e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^
2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g
^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*
z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^
4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5
*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*
d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8
*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c
*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c
*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2
*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^
7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^
7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5
+ 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b
*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5
*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6
*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^
6 - 36*a^2*c^4*e^6*g^6, z, k), k, 1, 4)
```

3.819 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 287

$$\begin{aligned}\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx &= -\frac{2(ef-dg)^3\,(cf^2-bfg+ag^2)\,\sqrt{f+gx}}{g^6} \\ &\quad+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))(f+gx)^{3/2}}{3g^6} \\ &\quad+\frac{2(ef-dg)\,(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2))\,(f+gx)^{5/2}}{5g^6} \\ &\quad-\frac{2e(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2))\,(f+gx)^{7/2}}{7g^6} \\ &\quad-\frac{2e^2(5cef-3cdg-beg)(f+gx)^{9/2}}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}\end{aligned}$$

```
[Out] 2/3*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(3
/2)/g^6+2/5*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+1
0*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*
g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*e^2*(-b*e*g-3*c*d*g+5*c*e
*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e*f)^3*(a*g^2-b
*f*g+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {911, 1167}

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$= -\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}$$
$$+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}$$
$$-\frac{2\sqrt{f+gx}(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6}$$
$$+\frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{3g^6}$$
$$-\frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3 \left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right) dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^3(cf^2-bfg+ag^2)}{g^5}+\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))x^2}{g^5}+\frac{(ef-dg)(3eg(2bef-bdg-aeg)-c}{g^5}\right.\right.}{}$$

$$= -\frac{2(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^6} + \frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))(f+gx)^{3/2}}{3g^6} + \frac{2(ef-dg)(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2))(f+gx)^{5/2}}{5g^6} - \frac{2e(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{7/2}}{7g^6} - \frac{2e^2(5cef-3cdg-beg)(f+gx)^{9/2}}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(c(231d^3g^3(8f^2-4fgx+3g^2x^2)+297d^2eg^2(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+33de^2g(128f}{}$$

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e*
g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4 -
 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f^5
- 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5
)) + 11*g*(9*a*g*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^2*g*(8*f^2
 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^
3)) + b*(105*d^3*g^3*(-2*f + g*x) + 63*d^2*e*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x
^2) + 27*d*e^2*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^3*(128
*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)))))/(3465*g
^6)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c\,e^{3}(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^{2}c+e^{3}(bg-2cf)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^{2}ec+3(dg-ef)e^{2}(bg-2cf)+e^{3}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{7}{2}}}{7}$
default	$\frac{2c\,e^{3}(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^{2}c+e^{3}(bg-2cf)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^{2}ec+3(dg-ef)e^{2}(bg-2cf)+e^{3}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{7}{2}}}{7}$
pseudoelliptic	$2\sqrt{gx+f}\left(\left(\frac{x^{3}\left(\frac{7}{11}c\,x^{2}+\frac{7}{9}bx+a\right)e^{3}}{7}+\frac{3d\left(\frac{5}{9}c\,x^{2}+\frac{5}{7}bx+a\right)x^{2}e^{2}}{5}+d^{2}x\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)e+d^{3}\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)\right)g^{5}-2\left(\left(\frac{5}{99}c\,x\right.\right.\right.$
gosper	$2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148$
trager	$2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148$
risch	$2\sqrt{gx+f}\,(315c\,e^{3}x^{5}g^{5}+385b\,e^{3}g^{5}x^{4}+1155cd\,e^{2}g^{5}x^{4}-350c\,e^{3}f\,g^{4}x^{4}+495a\,e^{3}g^{5}x^{3}+1485bd\,e^{2}g^{5}x^{3}-440b\,e^{3}f\,g^{4}x^{3}+148$

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c+e^3*(b*g-2*c*f))*(g
*x+f)^(9/2)+1/7*(3*(d*g-e*f)^2*e*c+3*(d*g-e*f)*e^2*(b*g-2*c*f)+e^3*(a*g^2-b
*f*g+c*f^2))*(g*x+f)^(7/2)+1/5*((d*g-e*f)^3*c+3*(d*g-e*f)^2*e*(b*g-2*c*f)+3
*(d*g-e*f)*e^2*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(5/2)+1/3*((d*g-e*f)^3*(b*g-2*c
*f)+3*(d*g-e*f)^2*e*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)^3*(a*g^2-b
*f*g+c*f^2)*(g*x+f)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\,(315\,ce^3g^5x^5-1280\,ce^3f^5+3465\,ad^3g^5+1408\,(3\,cde^2+be^3)f^4g-1584\,(3\,cd^2e+3\,bde^2+ae^3)f^3g^2+\cdots}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 1408*(3*c*d*e
^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 + 1848*(c*
d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(b*d^3 + 3*a*d^2*e)*f*g^4 - 35*
(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 8
```

```
8*(3*c*d*e^2 + b*e^3)*f*g^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 -
 3*(160*c*e^3*f^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^2*e +
3*b*d*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (
640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 1155*(b*
d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(291) = 582$.

Time = 1.23 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{11}{2}}}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\left(be^3g+3cde^2g-5ce^3f\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(ae^3g^2+3bde^2g^2-4be^3fg+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+3\right.}{}\right.}{} \\ \frac{ad^3x+\frac{ce^3x^6}{6}+\frac{x^5\left(be^3+3cde^2\right)}{5}+\frac{x^4\left(ae^3+3bde^2+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3\right)}{3}+\frac{x^2\cdot\left(3ad^2e+bd^3\right)}{2}}{\sqrt{f}} \end{cases}$$

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*e**3*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(b*e**3
*g + 3*c*d*e**2*g - 5*c*e**3*f)/(9*g**5) + (f + g*x)**(7/2)*(a*e**3*g**2 +
3*b*d*e**2*g**2 - 4*b*e**3*f*g + 3*c*d**2*e*g**2 - 12*c*d*e**2*f*g + 10*c*e
**3*f**2)/(7*g**5) + (f + g*x)**(5/2)*(3*a*d*e**2*g**3 - 3*a*e**3*f*g**2 +
3*b*d**2*e*g**3 - 9*b*d*e**2*f*g**2 + 6*b*e**3*f**2*g + c*d**3*g**3 - 9*c*d
**2*e*f*g**2 + 18*c*d*e**2*f**2*g - 10*c*e**3*f**3)/(5*g**5) + (f + g*x)**(
3/2)*(3*a*d**2*e*g**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 + b*d**3*g**
4 - 6*b*d**2*e*f*g**3 + 9*b*d*e**2*f**2*g**2 - 4*b*e**3*f**3*g - 2*c*d**3*f
*g**3 + 9*c*d**2*e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(3*g**5)
 + sqrt(f + g*x)*(a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 -
a*e**3*f**3*g**2 - b*d**3*f*g**4 + 3*b*d**2*e*f**2*g**3 - 3*b*d*e**2*f**3*g
**2 + b*e**3*f**4*g + c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*
f**4*g - c*e**3*f**5)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5
*(b*e**3 + 3*c*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3
*(3*a*d*e**2 + 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/sqrt(
f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3 - 385\,(5\,ce^3f - (3\,cde^2+be^3)g)(gx+f)^{\frac{9}{2}} + 495\,(10\,ce^3f^2 - 4\,(3\,cde^2+be^3)fg + (3\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g
)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 6*(3*c*
d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2) - 3
465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 +
3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(265) = 530$.

Time = 0.28 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(3465\,\sqrt{gx+f}ad^3 + \frac{1155\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)bd^3}{g} + \frac{3465\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ad^2e}{g} + \frac{231\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 1\right.}{g^2}\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(3*
(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2 +
693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d^2
*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)^(7/2)
```

```
- 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*d
*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*
f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f
^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x +
 f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*s
qrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{9/2}\,(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g)}{9\,g^6} + \frac{(f+g\,x)^{7/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2)}{7\,g^6} + \frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2)}{5\,g^6} + \frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^3\,(c\,f^2-b\,f\,g+a\,g^2)}{g^6} + \frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)^2\,(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g)}{3\,g^6} + \frac{2\,c\,e^3\,(f+g\,x)^{11/2}}{11\,g^6}$$

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(9/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(9*g^6) + ((f + g*
x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g^2 + 6*c*d^
2*e*g^2 - 24*c*d*e^2*f*g))/(7*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*e^
2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8*c*d*e*f*g)
)/(5*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^3*(a*g^2 + c*f^2 - b*f*g))/g^6 +
 (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^2 + 5*c*e*f^2 - 4*b*e*
f*g - 2*c*d*f*g))/(3*g^6) + (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

3.820 $\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 212

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2(ef-dg)^2\,(cf^2-bfg+ag^2)\,\sqrt{f+gx}}{g^5} - \frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))(f+gx)^{3/2}}{3g^5} - \frac{2(eg(3bef-2bdg-aeg)-c(6e^2f^2-6defg+d^2g^2))\,(f+gx)^{5/2}}{5g^5} - \frac{2e(4cef-2cdg-beg)(f+gx)^{7/2}}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

```
[Out] -2/3*(-d*g+e*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(3/
2)/g^5-2/5*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(
g*x+f)^(5/2)/g^5-2/7*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(7/2)/g^5+2/9*c*e^2
*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used

$= \{911, 1167\}$

$$\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= -\frac{2(f+gx)^{5/2}(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{5g^5}$$
$$+\frac{2\sqrt{f+gx}(ef-dg)^2(ag^2-bfg+cf^2)}{g^5}$$
$$-\frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5}$$
$$-\frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^2(cf^2-bfg+ag^2)}{g^4}+\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))x^2}{g^4}+\frac{(-eg(3bef-2bdg-aeg)+c(6e^2f^2\cdots}{g^4}\right.\right.}{g}$$

$$
\begin{aligned}
&= \frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5} \\
&- \frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))(f+gx)^{3/2}}{3g^5} \\
&- \frac{2(eg(3bef-2bdg-aeg)-c(6e^2f^2-6defg+d^2g^2))\,(f+gx)^{5/2}}{5g^5} \\
&- \frac{2e(4cef-2cdg-beg)(f+gx)^{7/2}}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21

$$
\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx
= \frac{2\sqrt{f+gx}(c(21d^2g^2(8f^2-4fgx+3g^2x^2)+18deg(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+e^2(128f^4-64f}{}
$$

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18*d*e*g*(-
16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 - 64*f^3*g*x +
 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)) + 3*g*(7*a*g*(15*d^2*g^2 + 10
*d*e*g*(-2*f + g*x) + e^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + b*(35*d^2*g^2*(-
2*f + g*x) + 14*d*e*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) - 3*e^2*(16*f^3 - 8*f^2
*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))))/(315*g^5)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{\left(\frac{5}{9}c\,x^{2}+\frac{5}{7}bx+a\right)x^{2}e^{2}}{5}+\frac{2d\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)xe}{3}+d^{2}\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)\right)g^{4}-\frac{4\left(\frac{\left(\frac{10}{21}c\,x^{2}+\frac{9}{14}bx+a\right)x\,e^{2}}{5}+d\left(\frac{9}{35}c\,x^{2}+\frac{2}{5}\right.\right.}{3}\right.}{g^{5}}$
derivativedivides	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^{2}(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^{2}c+2(dg-ef)e(bg-2cf)+e^{2}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-\right.}{}}{g^{5}}$
default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^{2}(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^{2}c+2(dg-ef)e(bg-2cf)+e^{2}\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-\right.}{}}{g^{5}}$
gosper	$2\sqrt{gx+f}\,(35c\,e^{2}x^{4}g^{4}+45b\,e^{2}g^{4}x^{3}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+126bde\,g^{4}x^{2}-54b\,e^{2}f\,g^{3}x^{2}+63c\,d^{2}g^{4}x^{2}-108c$
trager	$2\sqrt{gx+f}\,(35c\,e^{2}x^{4}g^{4}+45b\,e^{2}g^{4}x^{3}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+126bde\,g^{4}x^{2}-54b\,e^{2}f\,g^{3}x^{2}+63c\,d^{2}g^{4}x^{2}-108c$
risch	$2\sqrt{gx+f}\,(35c\,e^{2}x^{4}g^{4}+45b\,e^{2}g^{4}x^{3}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+126bde\,g^{4}x^{2}-54b\,e^{2}f\,g^{3}x^{2}+63c\,d^{2}g^{4}x^{2}-108c$

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*((1/5*(5/9*c*x^2+5/7*b*x+a)*x^2*e^2+2/3*d*(3/7*c*x^2+3/5*b*
x+a)*x*e+d^2*(1/5*c*x^2+1/3*b*x+a))*g^4-4/3*(1/5*(10/21*c*x^2+9/14*b*x+a)*x
*e^2+d*(9/35*c*x^2+2/5*b*x+a)*e+1/2*d^2*(2/5*c*x+b))*f*g^3+8/15*((2/7*c*x^2
+3/7*b*x+a)*e^2+2*d*(3/7*c*x+b)*e+c*d^2)*f^2*g^2-16/35*e*((4/9*c*x+b)*e+2*c
*d)*f^3*g+128/315*c*e^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,ce^2g^4x^4+128\,ce^2f^4+315\,ad^2g^4-144\,(2\,cde+be^2)f^3g+168\,(cd^2+2\,bde+ae^2)f^2g^2-210\,(bd^2+2\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 + 315*a*d^2*g^4 - 144*(2*c*d*e + b*
e^2)*f^3*g + 168*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 - 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 5*(8*c*e^2*f*g^3 - 9*(2*c*d*e + b*e^2)*g^4)*x^3 + 3*(16*c*e^2*f^2*g
^2 - 18*(2*c*d*e + b*e^2)*f*g^3 + 21*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (
64*c*e^2*f^3*g - 72*(2*c*d*e + b*e^2)*f^2*g^2 + 84*(c*d^2 + 2*b*d*e + a*e^2
)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^2 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{9}{2}}}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\left(be^2g+2cdeg-4ce^2f\right)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2f^2\right)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(2adeg^3-2ae^2fg^2+bd^2g^3-4b\right.}{}\right.}{g} \\ \frac{ad^2x+\frac{ce^2x^5}{5}+\frac{x^4\left(be^2+2cde\right)}{4}+\frac{x^3\left(ae^2+2bde+cd^2\right)}{3}+\frac{x^2\cdot\left(2ade+bd^2\right)}{2}}{\sqrt{f}} \end{cases}$$

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*e**2*(f + g*x)**(9/2)/(9*g**4) + (f + g*x)**(7/2)*(b*e**2*g
 + 2*c*d*e*g - 4*c*e**2*f)/(7*g**4) + (f + g*x)**(5/2)*(a*e**2*g**2 + 2*b*d
*e*g**2 - 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**4
) + (f + g*x)**(3/2)*(2*a*d*e*g**3 - 2*a*e**2*f*g**2 + b*d**2*g**3 - 4*b*d*
e*f*g**2 + 3*b*e**2*f**2*g - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f*
*3)/(3*g**4) + sqrt(f + g*x)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g*
*2 - b*d**2*f*g**3 + 2*b*d*e*f**2*g**2 - b*e**2*f**3*g + c*d**2*f**2*g**2 -
 2*c*d*e*f**3*g + c*e**2*f**4)/g**4)/g, Ne(g, 0)), ((a*d**2*x + c*e**2*x**5
/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*
(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2-45\,(4\,ce^2f-(2\,cde+be^2)g)(gx+f)^{\frac{7}{2}}+63\,(6\,ce^2f^2-3\,(2\,cde+be^2)fg+(cd^2+2\,b\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*x
 + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
+ a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2*
g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f)^(3
/2) + 315*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2*b*d
*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\left(315\sqrt{gx+f}ad^2+\frac{105\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)bd^2}{g}+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ade}{g}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+\ldots}\right.}{g^2}\right.}{\ldots}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x +
f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(3*(
g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2 + 2
1*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e^2/g
^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f
)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx &= \frac{(f+g\,x)^{7/2}\,\left(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g\right)}{7\,g^5}\\ &+\frac{(f+g\,x)^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2\right)}{5\,g^5}\\ &+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,\left(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{3\,g^5}\\ &+\frac{2\,\sqrt{f+g\,x}\,{(d\,g-e\,f)}^2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^5}+\frac{2\,c\,e^2\,{(f+g\,x)}^{9/2}}{9\,g^5}\end{aligned}$$

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(7*g^5) + ((f + g*x)^
(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d*e*g^2 - 6*b*e^2*f*g
 - 12*c*d*e*f*g))/(5*g^5) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(2*a*e*g^2 + b*d
*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*
g - e*f)^2*(a*g^2 + c*f^2 - b*f*g))/g^5 + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

3.821 $\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = -\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4} + \frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))(f+gx)^{3/2}}{3g^4} - \frac{2(3cef-cdg-beg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[Out] 2/3*(c*f*(-2*d*g+3*e*f)-g*(-a*e*g-b*d*g+2*b*e*f))*(g*x+f)^(3/2)/g^4-2/5*(-b
*e*g-c*d*g+3*c*e*f)*(g*x+f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f
)*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {785}

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4} + \frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4} - \frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}} + \frac{(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^3} + \frac{(-3cef+cdg+beg)(f+gx)^{3/2}}{g^3} + \frac{ce(f+gx)^{5/2}}{g^3}\right) dx \\ &= -\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4} + \frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))(f+gx)^{3/2}}{3g^4} - \frac{2(3cef-cdg-beg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\left(7g\left(5bdg(-2f+gx)+5ag(-2ef+3dg+egx)+be\left(8f^2-4fgx+3g^2x^2\right)\right)+c\left(7dg\left(8f^2-4fgx\right.\right.\right.}{105g^4}$$

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*x
) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^2*
x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)xe}{3}+d\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)\right)g^{3}-\frac{2f\left(\left(\frac{9}{35}c\,x^{2}+\frac{2}{5}bx+a\right)e+d\left(\frac{2cx}{5}+b\right)\right)g^{2}}{3}+\frac{8\left(\left(\frac{3cx}{7}+b\right)e+cd\right)f^{2}g}{15}-\frac{1}{}\right.\right.}{g^{4}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}-bfg+c\right.}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^{2}-bfg+c\,f^{2}\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^{2}-bfg+c\right.}{g^{4}}$
gosper	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\right.}{105g^{4}}$
trager	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\right.}{105g^{4}}$
risch	$\frac{2\sqrt{gx+f}\,\left(15ce\,x^{3}g^{3}+21be\,g^{3}x^{2}+21cd\,g^{3}x^{2}-18cef\,g^{2}x^{2}+35ae\,g^{3}x+35bd\,g^{3}x-28bef\,g^{2}x-28cdf\,g^{2}x+24ce\,f^{2}gx+105a\right.}{105g^{4}}$

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*((1/3*(3/7*c*x^2+3/5*b*x+a)*x*e+d*(1/5*c*x^2+1/3*b*x+a))*g^
3-2/3*f*((9/35*c*x^2+2/5*b*x+a)*e+d*(2/5*c*x+b))*g^2+8/15*((3/7*c*x+b)*e+c*
d)*f^2*g-16/35*c*e*f^3)/g^4
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(15\,ceg^3x^3-48\,cef^3+105\,adg^3+56\,(cd+be)f^2g-70\,(bd+ae)fg^2-3\,(6\,cefg^2-7\,(cd+be)g^3)x^2+\right.}{105\,g^4}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 105*a*d*g^3 + 56*(c*d + b*e)*f^2*g - 7
0*(b*d + a*e)*f*g^2 - 3*(6*c*e*f*g^2 - 7*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2
*g - 28*(c*d + b*e)*f*g^2 + 35*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{7}{2}}}{7g^3}+\frac{(f+gx)^{\frac{5}{2}}(beg+cdg-3cef)}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}\left(aeg^2+bdg^2-2befg-2cdfg+3cef^2\right)}{3g^3}+\frac{\sqrt{f+gx}\left(adg^3-aefg^2-bdfg^2+bef^2g+cdf^2g-cef^3\right)}{g^3}\right)}{g} & \text{for } g \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{\sqrt{f}} & \text{othe} \end{cases}$$

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*e*(f + g*x)**(7/2)/(7*g**3) + (f + g*x)**(5/2)*(b*e*g + c*d
*g - 3*c*e*f)/(5*g**3) + (f + g*x)**(3/2)*(a*e*g**2 + b*d*g**2 - 2*b*e*f*g
- 2*c*d*f*g + 3*c*e*f**2)/(3*g**3) + sqrt(f + g*x)*(a*d*g**3 - a*e*f*g**2 -
 b*d*f*g**2 + b*e*f**2*g + c*d*f**2*g - c*e*f**3)/g**3)/g, Ne(g, 0)), ((a*d
*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d)/2)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce-21\,(3\,cef-(cd+be)g)(gx+f)^{\frac{5}{2}}+35\,(3\,cef^2-2\,(cd+be)fg+(bd+ae)g^2)(gx+f)\right.}{105\,g^4}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(5/2
) + 35*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2) -
105*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)*sqrt(g*x +
f))/g^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\left(105\sqrt{gx+f}ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)bd}{g}+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ae}{g}+\frac{7\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right.}{105\,g}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*d
/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)
- 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^(5/
2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^
3)*c*e/g^3)/g
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx &= \frac{(f+g\,x)^{5/2}\,(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f)}{5\,g^4} \\ &\quad+\frac{(f+g\,x)^{3/2}\,(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g)}{3\,g^4} \\ &\quad+\frac{2\sqrt{f+g\,x}\,(d\,g-e\,f)\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^4}+\frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4}\end{aligned}$$

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(5*g^4) + ((f + g*x)^(3/2)*
(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g - 4*c*d*f*g))/(3*g^4) + (2*(
f + g*x)^(1/2)*(d*g - e*f)*(a*g^2 + c*f^2 - b*f*g))/g^4 + (2*c*e*(f + g*x)^
(7/2))/(7*g^4)
```

3.822 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx=\frac{2(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] -2/3*(-b*g+2*c*f)*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2-b*f*g+
c*f^2)*(g*x+f)^(1/2)/g^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {712}

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(ag^2-bfg+cf^2)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[In] Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{cf^2 - bfg + ag^2}{g^2\sqrt{f+gx}} + \frac{(-2cf+bg)\sqrt{f+gx}}{g^2} + \frac{c(f+gx)^{3/2}}{g^2} \right) dx$$
$$= \frac{2(cf^2 - bfg + ag^2)\sqrt{f+gx}}{g^3} - \frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(5g(-2bf+3ag+bgx)+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
[Out] (2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2
*x^2)))/(15*g^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{2\left(\left(\frac{1}{5}c\,x^2+\frac{1}{3}bx+a\right)g^2-\frac{2f\left(\frac{2cx}{5}+b\right)g}{3}+\frac{8c\,f^2}{15}\right)\sqrt{gx+f}}{g^3}$	46
gosper	$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2+5b\,g^2x-4cfxg+15a\,g^2-10bfg+8c\,f^2\right)}{15g^3}$	53
trager	$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2+5b\,g^2x-4cfxg+15a\,g^2-10bfg+8c\,f^2\right)}{15g^3}$	53
risch	$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2+5b\,g^2x-4cfxg+15a\,g^2-10bfg+8c\,f^2\right)}{15g^3}$	53
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^2\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^2\sqrt{gx+f}}{g^3}$	75
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^2\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^2\sqrt{gx+f}}{g^3}$	75

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*((1/5*c*x^2+1/3*b*x+a)*g^2-2/3*f*(2/5*c*x+b)*g+8/15*c*f^2)*(g*x+f)^(1/2)/
g^3
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(3\,cg^2x^2+8\,cf^2-10\,bfg+15\,ag^2-(4\,cfg-5\,bg^2)x\right)\sqrt{gx+f}}{15\,g^3}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*
sqrt(g*x + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2a\sqrt{f+gx}+\frac{2b\left(-f\sqrt{f+gx}+\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}+\frac{2c\left(f^2\sqrt{f+gx}-\frac{2f(f+gx)^{\frac{3}{2}}}{3}+\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((2*a*sqrt(f + g*x) + 2*b*(-f*sqrt(f + g*x) + (f + g*x)**(3/2)/3)
/g + 2*c*(f**2*sqrt(f + g*x) - 2*f*(f + g*x)**(3/2)/3 + (f + g*x)**(5/2)/5)
/g**2)/g, Ne(g, 0)), ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)b}{g}+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15\,g}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a + \frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)b}{g} + \frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+g\,x}\,\left(3\,c\,{(f+g\,x)}^2+15\,a\,g^2+15\,c\,f^2+5\,b\,g\,(f+g\,x)-10\,c\,f\,(f+g\,x)-15\,b\,f\,g\right)}{15\,g^3}$$

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(1/2),x)
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 + 5*b*g*(f + g*x)
 - 10*c*f*(f + g*x) - 15*b*f*g))/(15*g^3)
```

3.823 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}} dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}} dx = \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2} - \frac{2(cd^2-bde+ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/
2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)+2*(b*e*g-c*(d*g+e*f))*(g*x+f)
^(1/2)/e^2/g^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {911, 1167, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}} dx = -\frac{2(ae^2-bde+cd^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}} + \frac{2\sqrt{f+gx}(beg-c(dg+ef))}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{beg-c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2-bde+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}\\
&= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}\\
&\quad+\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{e^2g}\\
&= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(3beg+c(-2ef-3dg+egx))}{3e^2g^2} + \frac{2(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(3*b*e*g + c*(-2*e*f - 3*d*g + e*g*x)))/(3*e^2*g^2) + (2*(
c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]
])/(e^(5/2)*Sqrt[-(e*f) + d*g])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{2(cegx+3beg-3cdg-2cef)\sqrt{gx+f}}{3g^2e^2}+\frac{2(e^2a-bde+c\,d^2)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}$	93
pseudoelliptic	$\frac{\frac{2\sqrt{gx+f}\,(cegx+3beg-3cdg-2cef)}{3}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{e^2g^2}$	94
derivativedivides	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	115
default	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	115

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(c*e*g*x+3*b*e*g-3*c*d*g-2*c*e*f)*(g*x+f)^(1/2)/g^2/e^2+2*(a*e^2-b*d*e+
c*d^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.94

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \left[\frac{3\,(cd^2-bde+ae^2)\sqrt{e^2f-deg}g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\,(2\,ce^3f^2+(cde^2-3\,be^3)fg-3\,(}{3\,(e^4fg^2-de^3g^3)}\right.$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 - b*d*e + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f
- d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 +
(c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)*g^2 - (c*e^3*f*g - c*d*e^2*
g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 - b*d*e + a*e
^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g
*x + e*f)) - (2*c*e^3*f^2 + (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)
*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3)]
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3eg}+\frac{\sqrt{f+gx}(beg-cdg-cef)}{e^2g}+\frac{g\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^3\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g\neq 0 \\ \frac{\frac{cx^2}{2e}+\frac{x(be-cd)}{e^2}+\frac{\left(ae^2-bde+cd^2\right)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((2*(c*(f + g*x)**(3/2)/(3*e*g) + sqrt(f + g*x)*(b*e*g - c*d*g - c
*e*f)/(e**2*g) + g*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g -
 e*f)/e))/(e**3*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), ((c*x**2/(2*e) + x*(b*e
 - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d
+ e*x)/e, True))/e**2)/sqrt(f), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^2} + \frac{2\left((gx+f)^{\frac{3}{2}}ce^2g^4-3\sqrt{gx+f}ce^2fg^4-3\sqrt{gx+f}cdeg^5+3\sqrt{gx+f}be^2g^5\right)}{3\,e^3g^6}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqr
t(-e^2*f + d*e*g)*e^2) + 2/3*((g*x + f)^(3/2)*c*e^2*g^4 - 3*sqrt(g*x + f)*c
*e^2*f*g^4 - 3*sqrt(g*x + f)*c*d*e*g^5 + 3*sqrt(g*x + f)*b*e^2*g^5)/(e^3*g^
6)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \sqrt{f+g\,x}\left(\frac{2\,b\,g-4\,c\,f}{e\,g^2}-\frac{2\,c\,(d\,g^3-e\,f\,g^2)}{e^2\,g^4}\right) + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,(c\,d^2-b\,d\,e+a\,e^2)}{e^{5/2}\,\sqrt{d\,g-e\,f}} + \frac{2\,c\,{(f+g\,x)}^{3/2}}{3\,e\,g^2}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (f + g*x)^(1/2)*((2*b*g - 4*c*f)/(e*g^2) - (2*c*(d*g^3 - e*f*g^2))/(e^2*g^4
)) + (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2 -
b*d*e))/(e^(5/2)*(d*g - e*f)^(1/2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.824 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

```
[Out] (c*d*(-3*d*g+4*e*f)-e*(-a*e*g-b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+d*(-
b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {911, 1171, 396, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}-\frac{\sqrt{f+gx}(ae^2-bde+cd^2)}{e^2(d+ex)(ef-dg)}+\frac{2c\sqrt{f+gx}}{e^2g}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x])/(e^2*
(e*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g
```

```
))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(
3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$=-\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)}+\frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{2cf^2}{g^2}+\frac{2bf}{g}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} - \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{e^2g(ef-dg)}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}(e(bd-ae)g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)} - \frac{(cd(-4ef+3dg)+e(2bef-bdg-aeg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x)
)))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b
*d*g - a*e*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)
*(-(e*f) + d*g)^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

method	result
risch	$\frac{2c\sqrt{gx+f}}{e^2g}+\frac{\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2g+bdeg-2b\,e^2f-3c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}}{e^2}$
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2g+bdeg-2b\,e^2f-3c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2g+bdeg-2b\,e^2f-3c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
pseudoelliptic	$\frac{(ex+d)g((ag-2bf)e^2+d(bg+4cf)e-3c\,d^2g)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,((-2cfx+ag)e^2-d((-2cx+b)g}{\sqrt{(dg-ef)e}\,g\,e^2(dg-ef)(ex+d)}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g+1/e^2*(g*(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2
)/(e*(g*x+f)+d*g-e*f)+(a*e^2*g+b*d*e*g-2*b*e^2*f-3*c*d^2*g+4*c*d*e*f)/(d*g-
e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(126) = 252.

Time = 0.49 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.55

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$=\left[-\frac{\sqrt{e^2f-deg}(2\,(2\,cd^2e-bde^2)fg-(3\,cd^3-bd^2e-ade^2)g^2+(2\,(2\,cde^2-be^3)fg-(3\,cd^2e-bde^2-a}{2\,(de}\right.$$
$$-\frac{\sqrt{-e^2f+deg}(2\,(2\,cd^2e-bde^2)fg-(3\,cd^3-bd^2e-ade^2)g^2+(2\,(2\,cde^2-be^3)fg-(3\,cd^2e-bde^2-}{de^5f^2g}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(e^2*f - d*e*g)*(2*(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*
e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^
3)*g^2)*x)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/
(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 + a*e^4)*f*g + (3*c*
```

```
d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2
*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6
*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -(sqrt(-e^2*f + d*e*g)*(2*(2*c*d^
2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*
e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^3)*g^2)*x)*arctan(sqrt(-e^2*f + d*e*g
)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 +
a*e^4)*f*g + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e
^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 +
d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{(4\,cdef-2\,be^2f-3\,cd^2g+bdeg+ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f-de^2g)\sqrt{-e^2f+deg}} - \frac{\sqrt{gx+f}cd^2g-\sqrt{gx+f}bdeg+\sqrt{gx+f}ae^2g}{(e^3f-de^2g)((gx+f)e-ef+dg)} + \frac{2\sqrt{gx+f}c}{e^2g}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -(4*c*d*e*f - 2*b*e^2*f - 3*c*d^2*g + b*d*e*g + a*e^2*g)*arctan(sqrt(g*x +
f)*e/sqrt(-e^2*f + d*e*g))/((e^3*f - d*e^2*g)*sqrt(-e^2*f + d*e*g)) - (sqrt
(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*e*g + sqrt(g*x + f)*a*e^2*g)/((e^3*f
- d*e^2*g)*((g*x + f)*e - e*f + d*g)) + 2*sqrt(g*x + f)*c/(e^2*g)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(a\,e^2\,g-2\,b\,e^2\,f-3\,c\,d^2\,g+b\,d\,e\,g+4\,c\,d\,e\,f\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2-b\,g\,d\,e+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)} + \frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 2*b*e^2*f - 3
*c*d^2*g + b*d*e*g + 4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(
1/2)*(a*e^2*g + c*d^2*g - b*d*e*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d
*e^2*g)) + (2*c*(f + g*x)^(1/2))/(e^2*g)
```

3.825 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{(eg(4bef-bdg-3aeg)-c(8e^2f^2-8defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

```
[Out] 1/4*(e*g*(-3*a*e*g-b*d*g+4*b*e*f)-c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctan
h(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+d
*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(-5*d*g+8*e*f)
-e*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {911, 1171, 393, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(eg(-3aeg-bdg+4bef)-c(3d^2g^2-8defg+8e^2f^2))}{4e^{5/2}(ef-dg)^{5/2}} - \frac{\sqrt{f+gx}(ae^2-bde+cd^2)}{2e^2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}(cd(8ef-5dg)-e(-3aeg-bdg+4bef))}{4e^2(d+ex)(ef-dg)^2}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d*g)*(d + e*x)^2)
+ ((c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*
e^2*(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2
*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d
*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2 - bde + ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{4cf^2}{g^2}+\frac{4bf}{g}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{(cd^2 - bde + ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg) - e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{(eg(4bef-bdg-3aeg) - c(8e^2f^2-8defg+3d^2g^2))\,\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{4e^2g(ef-dg)^2}$$

$$= -\frac{(cd^2 - bde + ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg) - e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{(eg(4bef-bdg-3aeg) - c(8e^2f^2-8defg+3d^2g^2))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \frac{\frac{\sqrt{e}\sqrt{f+gx}\left(cd\left(-3d^2g+8e^2fx+de(6f-5gx)\right)+e\left(ae(-2ef+5dg+3egx)-b\left(2def+d^2g+4e^2fx-degx\right)\right)\right)}{(ef-dg)^2(d+ex)^2} + \frac{(eg(-4bef+bdg+3aeg)+c(8e^2f^2-8de}{(-ef+dg)}}{4e^{5/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((Sqrt[e]*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 8*e^2*f*x + d*e*(6*f - 5*g*x)) + e
*(a*e*(-2*e*f + 5*d*g + 3*e*g*x) - b*(2*d*e*f + d^2*g + 4*e^2*f*x - d*e*g*x
))))/((e*f - d*g)^2*(d + e*x)^2) + ((e*g*(-4*b*e*f + b*d*g + 3*a*e*g) + c*(
8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e
*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{5\sqrt{gx+f}\left(\frac{(3agx-2f(2bx+a))e^3}{5}+d\left(\left(\frac{bx}{5}+a\right)g-\frac{2f(-4cx+b)}{5}\right)e^2-\frac{d^2((5cx+b)g-6cf)e}{5}-\frac{3c\,d^3g}{5}\right)\sqrt{(dg-ef)e}+3(ex+d)^2\left(\left(\right.\right.}{4\sqrt{(dg-ef)e}\,(dg-ef)^2e^2(ex+d)^2}$
derivativedivides	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+bde\,g^2-4b\,e^2f}{4(d^2g}$
default	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+bde\,g^2-4b\,e^2f}{4(d^2g}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(5*(g*x+f)^(1/2)*(1/5*(3*a*g*x-2*f*(2*b*x+a))*e^3+d*((1/5*b*x+a)*g-2/5*
f*(-4*c*x+b))*e^2-1/5*d^2*((5*c*x+b)*g-6*c*f)*e-3/5*c*d^3*g)*((d*g-e*f)*e)^
(1/2)+3*(e*x+d)^2*((a*g^2-4/3*b*f*g+8/3*c*f^2)*e^2+1/3*d*g*(b*g-8*c*f)*e+c*
d^2*g^2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/2)/(
d*g-e*f)^2/e^2/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(186) = 372$.

Time = 0.49 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.32

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(8\,cd^2e^2f^2-4\,(2\,cd^3e+bd^2e^2)fg+(3\,cd^4+bd^3e+3\,ad^2e^2)g^2+(8\,ce^4f^2-4\,(2\,cde^3+be^4)fg+(3\,cd^2e}{}\right.$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d^4 + b*d^3*e
 + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g + (3*c*d^2*e
^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^2*e^2 + b*d*
e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*
log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d))
+ 2*(2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a
*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b
*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2
*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d
^4 + b*d^3*e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g
```

```
+ (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^
2*e^2 + b*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(-e^
2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a*d*e^4)
*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b*e^5)*f
^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2*e^3 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(186) = 372$.

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.86

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{(8\,ce^2f^2 - 8\,cdefg - 4\,be^2fg + 3\,cd^2g^2 + bdeg^2 + 3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^2 - 2\,de^3fg + d^2e^2g^2)\sqrt{-e^2f+deg}} + \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg - 4\,(gx+f)^{\frac{3}{2}}be^3fg - 8\,\sqrt{gx+f}cde^2f^2g + 4\,\sqrt{gx+f}be^3f^2g - 5\,(gx+f)^{\frac{3}{2}}cd^2eg^2 +}{}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c*e^2*f^2 - 8*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 + b*d*e*g^2 + 3*
a*e^2*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^2 - 2*d*e^3
*f*g + d^2*e^2*g^2)*sqrt(-e^2*f + d*e*g)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*
f*g - 4*(g*x + f)^(3/2)*b*e^3*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g + 4*sqrt(
g*x + f)*b*e^3*f^2*g - 5*(g*x + f)^(3/2)*c*d^2*e*g^2 + (g*x + f)^(3/2)*b*d*
e^2*g^2 + 3*(g*x + f)^(3/2)*a*e^3*g^2 + 11*sqrt(g*x + f)*c*d^2*e*f*g^2 - 3*
sqrt(g*x + f)*b*d*e^2*f*g^2 - 5*sqrt(g*x + f)*a*e^3*f*g^2 - 3*sqrt(g*x + f)
*c*d^3*g^3 - sqrt(g*x + f)*b*d^2*e*g^3 + 5*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4
*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)*((g*x + f)*e - e*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.31

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+b\,d\,e\,g^2+8\,c\,e^2\,f^2-4\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}} - \frac{\frac{\sqrt{f+g\,x}\,\left(3\,c\,d^2\,g^2+b\,d\,e\,g^2-8\,c\,f\,d\,e\,g-5\,a\,e^2\,g^2+4\,b\,f\,e^2\,g\right)}{4\,e^2\,\left(d\,g-e\,f\right)}-\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+b\,d\,e\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2-4\,b\,f\,e^2\,g\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2 + 3*c*d^2*g
^2 + 8*c*e^2*f^2 + b*d*e*g^2 - 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g
- e*f)^(5/2)) - ((((f + g*x)^(1/2)*(3*c*d^2*g^2 - 5*a*e^2*g^2 + b*d*e*g^2 +
4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^2*(d*g - e*f)) - ((f + g*x)^(3/2)*(3*a*e^2
*g^2 - 5*c*d^2*g^2 + b*d*e*g^2 - 4*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*
f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 + e^2*f^2
 - 2*d*e*f*g)
```

3.826 $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 285

$$\begin{aligned}\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3\,(cf^2-bfg+ag^2)}{g^6\sqrt{f+gx}}\\ &+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))\sqrt{f+gx}}{g^6}\\ &+\frac{2(ef-dg)\,(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2))\,(f+gx)^{3/2}}{3g^6}\\ &-\frac{2e(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2))\,(f+gx)^{5/2}}{5g^6}\\ &-\frac{2e^2(5cef-3cdg-beg)(f+gx)^{7/2}}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}\end{aligned}$$

```
[Out] 2/3*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+10*e^2*f^
2))*(g*x+f)^(3/2)/g^6-2/5*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*g^2-12*d
*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e^2*(-b*e*g-3*c*d*g+5*c*e*f)*(g*x
+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2*(-d*g+e*f)^3*(a*g^2-b*f*g+c*f^2
)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*
b*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {911, 1275}

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2e(f+gx)^{5/2}\,(eg(-aeg-3bdg+4bef)-c(3d^2g^2-12defg+10e^2f^2))}{5g^6} + \frac{2(f+gx)^{3/2}(ef-dg)\,(3eg(-aeg-bdg+2bef)-c(d^2g^2-8defg+10e^2f^2))}{3g^6} + \frac{2(ef-dg)^3\,(ag^2-bfg+cf^2)}{g^6\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6} - \frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int\left(\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^5}+\frac{(-ef+dg)^3(cf^2-bfg+ag^2)}{g^5x^2}+\frac{(ef-dg)(3eg(2bef-bdg-aeg)-c(10\ldots}{g^5}\right.\right.}{}$$

$$= \frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}} + \frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))\sqrt{f+gx}}{g^6} + \frac{2(ef-dg)\left(3eg(2bef-bdg-aeg)-c(10e^2f^2-8defg+d^2g^2)\right)(f+gx)^{3/2}}{3g^6} - \frac{2e\left(eg(4bef-3bdg-aeg)-c(10e^2f^2-12defg+3d^2g^2)\right)(f+gx)^{5/2}}{5g^6} - \frac{2e^2(5cef-3cdg-beg)(f+gx)^{7/2}}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2(c(105d^3g^3(-8f^2-4fgx+g^2x^2)+189d^2eg^2(16f^3+8f^2gx-2fg^2x^2+g\ldots}{}$$

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(c*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8
*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*
f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*
f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)) + 9*g*(7*a*g*(-5*
d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4*f*g*x + g^2*x^2)
 + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + b*(35*d^3*g^3*(2*f +
 g*x) + 35*d^2*e*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 21*d*e^2*g*(16*f^3 + 8*
f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + e^3*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*
x^2 - 8*f*g^3*x^3 + 5*g^4*x^4)))))/(315*g^6*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{\left(\frac{5}{9}c\,x^{2}+\frac{5}{7}bx+a\right)x^{3}e^{3}}{5}-d\left(\frac{3}{7}c\,x^{2}+\frac{3}{5}bx+a\right)x^{2}e^{2}-3d^{2}\left(\frac{1}{5}c\,x^{2}+\frac{1}{3}bx+a\right)xe+d^{3}\left(a-\frac{1}{3}c\,x^{2}-bx\right)\right)g^{5}-6\left(-\frac{\left(\frac{25}{63}c\,x^{2}+\right.}{}\right.\right.}{}$
risch	$2(35c\,e^{3}x^{4}g^{4}+45b\,e^{3}g^{4}x^{3}+135cd\,e^{2}g^{4}x^{3}-85c\,e^{3}f\,g^{3}x^{3}+63a\,e^{3}g^{4}x^{2}+189bd\,e^{2}g^{4}x^{2}-117b\,e^{3}f\,g^{3}x^{2}+189c\,d^{2}e\,g^{4}x^{2}-35$
gosper	$-\frac{2(-35c\,e^{3}x^{5}g^{5}-45b\,e^{3}g^{5}x^{4}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189bd\,e^{2}g^{5}x^{3}+72b\,e^{3}f\,g^{4}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+}{}$
trager	$-\frac{2(-35c\,e^{3}x^{5}g^{5}-45b\,e^{3}g^{5}x^{4}-135cd\,e^{2}g^{5}x^{4}+50c\,e^{3}f\,g^{4}x^{4}-63a\,e^{3}g^{5}x^{3}-189bd\,e^{2}g^{5}x^{3}+72b\,e^{3}f\,g^{4}x^{3}-189c\,d^{2}e\,g^{5}x^{3}+}{}$
derivativedivides	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}-12ad\,e^{2}f\,g^{3}\sqrt{gx+f}-12b\,d^{2}ef\,g^{3}\sqrt{gx+f}-6bd\,e^{2}f\,g^{2}(gx+f)^{\frac{3}{2}}-6c\,d^{2}ef\,g^{2}(gx+f)^{\frac{3}{2}}+}{}$
default	$\frac{\frac{2c\,e^{3}(gx+f)^{\frac{9}{2}}}{9}-\frac{24cd\,e^{2}fg(gx+f)^{\frac{5}{2}}}{5}-12ad\,e^{2}f\,g^{3}\sqrt{gx+f}-12b\,d^{2}ef\,g^{3}\sqrt{gx+f}-6bd\,e^{2}f\,g^{2}(gx+f)^{\frac{3}{2}}-6c\,d^{2}ef\,g^{2}(gx+f)^{\frac{3}{2}}+}{}$

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(g*x+f)^(1/2)*((-1/5*(5/9*c*x^2+5/7*b*x+a)*x^3*e^3-d*(3/7*c*x^2+3/5*b*x+
a)*x^2*e^2-3*d^2*(1/5*c*x^2+1/3*b*x+a)*x*e+d^3*(a-1/3*c*x^2-b*x))*g^5-6*(-1
/15*(25/63*c*x^2+4/7*b*x+a)*x^2*e^3-2/3*d*(6/35*c*x^2+3/10*b*x+a)*x*e^2+d^2
*(-1/5*c*x^2-2/3*b*x+a)*e+1/3*d^3*(-2/3*c*x+b))*f*g^4+8*(-1/5*x*(10/63*c*x^
2+2/7*b*x+a)*e^3+d*(-6/35*c*x^2-3/5*b*x+a)*e^2+d^2*(-3/5*c*x+b)*e+1/3*c*d^3
)*f^2*g^3-16/5*e*f^3*((-10/63*c*x^2-4/7*b*x+a)*e^2+3*d*(-4/7*c*x+b)*e+3*c*d
^2)*g^2+128/35*((-5/9*c*x+b)*e+3*c*d)*e^2*f^4*g-256/63*c*e^3*f^5)/g^6
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(35\,ce^3g^5x^5+1280\,ce^3f^5-315\,ad^3g^5-1152\,(3\,cde^2+be^3)f^4g+1008\,(3}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 315*a*d^3*g^5 - 1152*(3*c*d*e^2
+ b*e^3)*f^4*g + 1008*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - 840*(c*d^3
+ 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + 630*(b*d^3 + 3*a*d^2*e)*f*g^4 - 5*(10*c*
e^3*f*g^4 - 9*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + (80*c*e^3*f^2*g^3 - 72*(3*c*d*
e^2 + b*e^3)*f*g^4 + 63*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - (160*c*e
^3*f^3*g^2 - 144*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 126*(3*c*d^2*e + 3*b*d*e^2 +
 a*e^3)*f*g^4 - 105*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f
```

```
^4*g - 576*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 504*(3*c*d^2*e + 3*b*d*e^2 + a*e^3
)*f^2*g^3 - 420*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 315*(b*d^3 + 3*a*d^
2*e)*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] (verification not implemented)

Time = 32.42 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{9}{2}}}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(be^3g+3cde^2g-5ce^3f\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^3g^2+3bde^2g^2-4be^3fg+3cd^2eg^2-12cde^2fg+\cdots\right.}{5g^5}\cdots}{\cdots} \\ \frac{ad^3x+\frac{ce^3x^6}{6}+\frac{x^5\left(be^3+3cde^2\right)}{5}+\frac{x^4\left(ae^3+3bde^2+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3\right)}{3}+\frac{x^2\cdot\left(3ad^2e+bd^3\right)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e**3*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(b*e**3*g
 + 3*c*d*e**2*g - 5*c*e**3*f)/(7*g**5) + (f + g*x)**(5/2)*(a*e**3*g**2 + 3*
b*d*e**2*g**2 - 4*b*e**3*f*g + 3*c*d**2*e*g**2 - 12*c*d*e**2*f*g + 10*c*e**
3*f**2)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d*e**2*g**3 - 3*a*e**3*f*g**2 + 3*
b*d**2*e*g**3 - 9*b*d*e**2*f*g**2 + 6*b*e**3*f**2*g + c*d**3*g**3 - 9*c*d**
2*e*f*g**2 + 18*c*d*e**2*f**2*g - 10*c*e**3*f**3)/(3*g**5) + sqrt(f + g*x)*
(3*a*d**2*e*g**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 + b*d**3*g**4 - 6
*b*d**2*e*f*g**3 + 9*b*d*e**2*f**2*g**2 - 4*b*e**3*f**3*g - 2*c*d**3*f*g**3
 + 9*c*d**2*e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/g**5 - (d*g -
 e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*
d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c*d*e**2)/5 + x**4*(a*e**3 + 3*b*
d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3)/3 + x**2*(
3*a*d**2*e + b*d**3)/2)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\left(5\,ce^3f-\left(3\,cde^2+be^3\right)g\right)(gx+f)^{\frac{7}{2}}+63\left(10\,ce^3f^2-4\left(3\,cde^2+be^3\right)fg+\left(3\,cd^2e+3\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g)*(
g*x + f)^(7/2) + 63*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*e^3*f^3 - 6*(3*c*d*e^
```

```
2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 4*(3*c*d*e^2 +
b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^5 + 315
*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d*e^
2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 + 3*a
*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(265) = 530$.

Time = 0.31 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(ce^3f^5 - 3\,cde^2f^4g - be^3f^4g + 3\,cd^2ef^3g^2 + 3\,bde^2f^3g^2 + ae^3f^3g^2 - cd^3}{\sqrt{gx+f}g^6}$$
$$+ \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^3g^{48} - 225\,(gx+f)^{\frac{7}{2}}ce^3fg^{48} + 630\,(gx+f)^{\frac{5}{2}}ce^3f^2g^{48} - 1050\,(gx+f)^{\frac{3}{2}}ce^3f^3g^{48} + 1575\right.}{}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*e^3*f^5 - 3*c*d*e^2*f^4*g - b*e^3*f^4*g + 3*c*d^2*e*f^3*g^2 + 3*b*d*e^
2*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3*f^2*g^3 - 3*b*d^2*e*f^2*g^3 - 3*a*d*e^2*f
^2*g^3 + b*d^3*f*g^4 + 3*a*d^2*e*f*g^4 - a*d^3*g^5)/(sqrt(g*x + f)*g^6) + 2
/315*(35*(g*x + f)^(9/2)*c*e^3*g^48 - 225*(g*x + f)^(7/2)*c*e^3*f*g^48 + 63
0*(g*x + f)^(5/2)*c*e^3*f^2*g^48 - 1050*(g*x + f)^(3/2)*c*e^3*f^3*g^48 + 15
75*sqrt(g*x + f)*c*e^3*f^4*g^48 + 135*(g*x + f)^(7/2)*c*d*e^2*g^49 + 45*(g*
x + f)^(7/2)*b*e^3*g^49 - 756*(g*x + f)^(5/2)*c*d*e^2*f*g^49 - 252*(g*x + f
)^(5/2)*b*e^3*f*g^49 + 1890*(g*x + f)^(3/2)*c*d*e^2*f^2*g^49 + 630*(g*x + f
)^(3/2)*b*e^3*f^2*g^49 - 3780*sqrt(g*x + f)*c*d*e^2*f^3*g^49 - 1260*sqrt(g*
x + f)*b*e^3*f^3*g^49 + 189*(g*x + f)^(5/2)*c*d^2*e*g^50 + 189*(g*x + f)^(5
/2)*b*d*e^2*g^50 + 63*(g*x + f)^(5/2)*a*e^3*g^50 - 945*(g*x + f)^(3/2)*c*d^
2*e*f*g^50 - 945*(g*x + f)^(3/2)*b*d*e^2*f*g^50 - 315*(g*x + f)^(3/2)*a*e^3
*f*g^50 + 2835*sqrt(g*x + f)*c*d^2*e*f^2*g^50 + 2835*sqrt(g*x + f)*b*d*e^2*
f^2*g^50 + 945*sqrt(g*x + f)*a*e^3*f^2*g^50 + 105*(g*x + f)^(3/2)*c*d^3*g^5
1 + 315*(g*x + f)^(3/2)*b*d^2*e*g^51 + 315*(g*x + f)^(3/2)*a*d*e^2*g^51 - 6
30*sqrt(g*x + f)*c*d^3*f*g^51 - 1890*sqrt(g*x + f)*b*d^2*e*f*g^51 - 1890*sq
rt(g*x + f)*a*d*e^2*f*g^51 + 315*sqrt(g*x + f)*b*d^3*g^52 + 945*sqrt(g*x +
f)*a*d^2*e*g^52)/g^54
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{7/2}\,(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g)}{7\,g^6}$$
$$-\frac{2\,c\,d^3\,f^2\,g^3-2\,b\,d^3\,f\,g^4+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2+6\,b\,d^2\,e\,f^2\,g^3-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g-6\,b\,d\,e^2\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}$$
$$+\frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2)}{5\,g^6}$$
$$+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{3\,g^6}$$
$$+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g)}{g^6}+\frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6}$$

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(7*g^6) - (2*a*d^3
*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g^3 - 2*b*d^3*f*g^4 + 2*
b*e^3*f^4*g - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*b*d
*e^2*f^3*g^2 + 6*b*d^2*e*f^2*g^3 - 6*c*d^2*e*f^3*g^2)/(g^6*(f + g*x)^(1/2))
 + ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g
^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f*g))/(5*g^6) + (2*(f + g*x)^(3/2)*(d*g - e
*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8
*c*d*e*f*g))/(3*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^
2 + 5*c*e*f^2 - 4*b*e*f*g - 2*c*d*f*g))/g^6 + (2*c*e^3*(f + g*x)^(9/2))/(9*
g^6)
```

3.827 $\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\begin{aligned}\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx &= -\frac{2(ef-dg)^2(cf^2-bfg+ag^2)}{g^5\sqrt{f+gx}}\\ &-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))\sqrt{f+gx}}{g^5}\\ &-\frac{2(eg(3bef-2bdg-aeg)-c(6e^2f^2-6defg+d^2g^2))(f+gx)^{3/2}}{3g^5}\\ &-\frac{2e(4cef-2cdg-beg)(f+gx)^{5/2}}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}\end{aligned}$$

```
[Out] -2/3*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)
^(3/2)/g^5-2/5*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+
f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)/g^5/(g*x+f)^(1/2)-2*(-d*g+e
*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/g^5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used

$= \{911, 1275\}$

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2(f+gx)^{3/2}\,(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(ef-dg)^2\,(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}} - \frac{2\sqrt{f+gx}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5} - \frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))}{g^4}+\frac{(-ef+dg)^2(cf^2-bfg+ag^2)}{g^4x^2}+\frac{(-eg(3bef-2bdg-aeg)+c(6e^2f^2-\cdots}{g^4}\right.\right.}{g}$$

$$
\begin{aligned}
&= -\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}} \\
&\quad -\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))\sqrt{f+gx}}{g^5} \\
&\quad -\frac{2\left(eg(3bef-2bdg-aeg)-c\left(6e^2f^2-6defg+d^2g^2\right)\right)(f+gx)^{3/2}}{3g^5} \\
&\quad -\frac{2e(4cef-2cdg-beg)(f+gx)^{5/2}}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.20

$$
\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2(c(35d^2g^2(-8f^2-4fgx+g^2x^2)+42deg(16f^3+8f^2gx-2fg^2x^2+g^3x^3
$$

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f^2*g
*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*x^2
+ 8*f*g^3*x^3 - 5*g^4*x^4)) + 7*g*(5*a*g*(-3*d^2*g^2 + 6*d*e*g*(2*f + g*x)
+ e^2*(-8*f^2 - 4*f*g*x + g^2*x^2)) + b*(15*d^2*g^2*(2*f + g*x) + 10*d*e*g*
(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^
3*x^3)))))/(105*g^5*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\left(\left(30c\,x^4+42b\,x^3+70a\,x^2\right)e^2+420d\left(\frac{1}{5}c\,x^2+\frac{1}{3}bx+a\right)xe-210d^2\left(a-\frac{1}{3}c\,x^2-bx\right)\right)g^4+840f\left(-\frac{\left(\frac{6}{35}c\,x^2+\frac{3}{10}bx+a\right)x\,e^2}{3}+d\left(-\right.\right.}{1}$
risch	$\frac{2\left(15c\,e^2x^3g^3+21b\,e^2g^3x^2+42cde\,g^3x^2-39c\,e^2f\,g^2x^2+35a\,e^2g^3x+70bde\,g^3x-63b\,e^2f\,g^2x+35c\,d^2g^3x-126cdef\,g^2x+87\right.}{105}$
gosper	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\right.}{}$
trager	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\right.}{}$
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^2(gx+f)^{\frac{3}{2}}}{3}-2b\,e^2fg(gx+f)^{\frac{3}{2}}+2}{}$
default	$\frac{\frac{2c\,e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^2(gx+f)^{\frac{3}{2}}}{3}-2b\,e^2fg(gx+f)^{\frac{3}{2}}+2}{}$

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(((30*c*x^4+42*b*x^3+70*a*x^2)*e^2+420*d*(1/5*c*x^2+1/3*b*x+a)*x*e-21
0*d^2*(a-1/3*c*x^2-b*x))*g^4+840*f*(-1/3*(6/35*c*x^2+3/10*b*x+a)*x*e^2+d*(-
1/5*c*x^2-2/3*b*x+a)*e+1/2*d^2*(-2/3*c*x+b))*g^3-560*((-6/35*c*x^2-3/5*b*x+
a)*e^2+2*d*(-3/5*c*x+b)*e+c*d^2)*f^2*g^2+672*e*((-4/7*c*x+b)*e+2*c*d)*f^3*g
-768*c*e^2*f^4)/(g*x+f)^(1/2)/g^5
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\left(15\,ce^2g^4x^4-384\,ce^2f^4-105\,ad^2g^4+336\,(2\,cde+be^2)f^3g-280\,(cd^2+2\cdots\right.}{\cdots}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 - 105*a*d^2*g^4 + 336*(2*c*d*e + b*
e^2)*f^3*g - 280*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 + 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 3*(8*c*e^2*f*g^3 - 7*(2*c*d*e + b*e^2)*g^4)*x^3 + (48*c*e^2*f^2*g^2
 - 42*(2*c*d*e + b*e^2)*f*g^3 + 35*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (19
2*c*e^2*f^3*g - 168*(2*c*d*e + b*e^2)*f^2*g^2 + 140*(c*d^2 + 2*b*d*e + a*e^
2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{7}{2}}}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\left(be^2g+2cdeg-4ce^2f\right)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\left(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2f^2\cdots\right)}{3g^4}\cdots\right.}{\cdots} \\ \frac{ad^2x+\frac{ce^2x^5}{5}+\frac{x^4\left(be^2+2cde\right)}{4}+\frac{x^3\left(ae^2+2bde+cd^2\right)}{3}+\frac{x^2\cdot\left(2ade+bd^2\right)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e**2*(f + g*x)**(7/2)/(7*g**4) + (f + g*x)**(5/2)*(b*e**2*g
 + 2*c*d*e*g - 4*c*e**2*f)/(5*g**4) + (f + g*x)**(3/2)*(a*e**2*g**2 + 2*b*d
*e*g**2 - 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g**4
) + sqrt(f + g*x)*(2*a*d*e*g**3 - 2*a*e**2*f*g**2 + b*d**2*g**3 - 4*b*d*e*f
*g**2 + 3*b*e**2*f**2*g - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)
/g**4 - (d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**4*sqrt(f + g*x)))/g, N
e(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(a*
e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-21\,(4\,ce^2f-(2\,cde+be^2)g)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-3\,(2\,cde+be^2)fg+(cd^2+2\,bde}{}\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
 + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2
*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*x +
f))/g^4 - 105*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2
*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(194) = 388$.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx =$$
$$-\frac{2\,(ce^2f^4-2\,cdef^3g-be^2f^3g+cd^2f^2g^2+2\,bdef^2g^2+ae^2f^2g^2-bd^2fg^3-2\,adefg^3+ad^2g^4)}{\sqrt{gx+f}g^5}$$
$$+\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce^2g^{30}-84\,(gx+f)^{\frac{5}{2}}ce^2fg^{30}+210\,(gx+f)^{\frac{3}{2}}ce^2f^2g^{30}-420\,\sqrt{gx+f}ce^2f^3g^{30}+42\,(gx+\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*e^2*f^4 - 2*c*d*e*f^3*g - b*e^2*f^3*g + c*d^2*f^2*g^2 + 2*b*d*e*f^2*g
^2 + a*e^2*f^2*g^2 - b*d^2*f*g^3 - 2*a*d*e*f*g^3 + a*d^2*g^4)/(sqrt(g*x + f
)*g^5) + 2/105*(15*(g*x + f)^(7/2)*c*e^2*g^30 - 84*(g*x + f)^(5/2)*c*e^2*f*
g^30 + 210*(g*x + f)^(3/2)*c*e^2*f^2*g^30 - 420*sqrt(g*x + f)*c*e^2*f^3*g^3
0 + 42*(g*x + f)^(5/2)*c*d*e*g^31 + 21*(g*x + f)^(5/2)*b*e^2*g^31 - 210*(g*
x + f)^(3/2)*c*d*e*f*g^31 - 105*(g*x + f)^(3/2)*b*e^2*f*g^31 + 630*sqrt(g*x
 + f)*c*d*e*f^2*g^31 + 315*sqrt(g*x + f)*b*e^2*f^2*g^31 + 35*(g*x + f)^(3/2
)*c*d^2*g^32 + 70*(g*x + f)^(3/2)*b*d*e*g^32 + 35*(g*x + f)^(3/2)*a*e^2*g^3
2 - 210*sqrt(g*x + f)*c*d^2*f*g^32 - 420*sqrt(g*x + f)*b*d*e*f*g^32 - 210*s
qrt(g*x + f)*a*e^2*f*g^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 210*sqrt(g*x + f
)*a*d*e*g^33)/g^35
```

Mupad [B] (verification not implemented)

Time = 11.86 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^2\,(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g)}{5\,g^5}$$
$$-\frac{2\,c\,d^2\,f^2\,g^2-2\,b\,d^2\,f\,g^3+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g+4\,b\,d\,e\,f^2\,g^2-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4-2\,b\,e^2\,f^3\,g+2\,a\,e^2\,f}{g^5\,\sqrt{f+g\,x}}$$
$$+\frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2)}{3\,g^5}$$
$$+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,\left(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{g^5}+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5}$$

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(5*g^5) - (2*a*d^2*g^
4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 - 2*b*d^2*f*g^3 - 2*b*e
^2*f^3*g + 4*b*d*e*f^2*g^2 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^
(1/2)) + ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d
*e*g^2 - 6*b*e^2*f*g - 12*c*d*e*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*g - e
*f)*(2*a*e*g^2 + b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/g^5 + (2*c*e
^2*(f + g*x)^(7/2))/(7*g^5)
```

3.828 $\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(ef-dg)(cf^2-bfg+ag^2)}{g^4\sqrt{f+gx}} + \frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4} - \frac{2(3cef-cdg-beg)(f+gx)^{3/2}}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[Out] -2/3*(-b*e*g-c*d*g+3*c*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(
-d*g+e*f)*(a*g^2-b*f*g+c*f^2)/g^4/(g*x+f)^(1/2)+2*(c*f*(-2*d*g+3*e*f)-g*(-a
*e*g-b*d*g+2*b*e*f))*(g*x+f)^(1/2)/g^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {785}

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(ef-dg)(ag^2-bfg+cf^2)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4} - \frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(2bef-bdg-aeg)}{g^3\sqrt{f+gx}}\right.\\ &\qquad\left.+\frac{(-3cef+cdg+beg)\sqrt{f+gx}}{g^3}+\frac{ce(f+gx)^{3/2}}{g^3}\right)dx\\ &= \frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4}\\ &\quad-\frac{2(3cef-cdg-beg)(f+gx)^{3/2}}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx=\frac{2(5g(3bdg(2f+gx)+3ag(2ef-dg+egx)+be(-8f^2-4fgx+g^2x^2))+c}{15g^4\sqrt{f+g}}$$

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 +
 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{((6c x^3+10b x^2+30ax)e-30d(a-\frac{1}{3}c x^2-bx))g^3+60((-\frac{1}{5}c x^2-\frac{2}{3}bx+a)e+d(-\frac{2cx}{3}+b))f g^2-80((-\frac{3cx}{5}+b)e+cd)f^2g+}{15\sqrt{gx+f}\, g^4}$
risch	$\frac{2(3ce x^2g^2+5bex g^2+5cd g^2x-9cefgx+15ae g^2+15bd g^2-25befg-25cdfg+33ce f^2)\sqrt{gx+f}}{15g^4}-\frac{2(ad g^3-aef g^2-bdf g}{g^4\sqrt{}}$
gosper	$-\frac{2(-3ce x^3g^3-5be g^3x^2-5cd g^3x^2+6cef g^2x^2-15ae g^3x-15bd g^3x+20bef g^2x+20cdf g^2x-24ce f^2gx+15ad g^3-30a}{15\sqrt{gx+f}\, g^4}$
trager	$-\frac{2(-3ce x^3g^3-5be g^3x^2-5cd g^3x^2+6cef g^2x^2-15ae g^3x-15bd g^3x+20bef g^2x+20cdf g^2x-24ce f^2gx+15ad g^3-30a}{15\sqrt{gx+f}\, g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae g^2\sqrt{gx+f}+2bd g^2\sqrt{gx+f}-4befg\sqrt{gx+f}-4cdfg\sqrt{gx+}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae g^2\sqrt{gx+f}+2bd g^2\sqrt{gx+f}-4befg\sqrt{gx+f}-4cdfg\sqrt{gx+}}{g^4}$

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(((6*c*x^3+10*b*x^2+30*a*x)*e-30*d*(a-1/3*c*x^2-b*x))*g^3+60*((-1/5*c*
x^2-2/3*b*x+a)*e+d*(-2/3*c*x+b))*f*g^2-80*((-3/5*c*x+b)*e+c*d)*f^2*g+96*c*e
*f^3)/(g*x+f)^(1/2)/g^4
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(3\,ceg^3x^3+48\,cef^3-15\,adg^3-40\,(cd+be)f^2g+30\,(bd+ae)fg^2-(6\,c\right.}{15\,(}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*(
b*d + a*e)*f*g^2 - (6*c*e*f*g^2 - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2*g -
20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{5}{2}}}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}(beg+cdg-3cef)}{3g^3}+\frac{\sqrt{f+gx}\left(aeg^2+bdg^2-2befg-2cdfg+3cef^2\right)}{g^3}-\frac{(dg-ef)\left(ag^2-bfg\right.}{g^3\sqrt{f+gx}}\right.}{g} \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{f^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*e*(f + g*x)**(5/2)/(5*g**3) + (f + g*x)**(3/2)*(b*e*g + c*d
*g - 3*c*e*f)/(3*g**3) + sqrt(f + g*x)*(a*e*g**2 + b*d*g**2 - 2*b*e*f*g - 2
*c*d*f*g + 3*c*e*f**2)/g**3 - (d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**3*s
qrt(f + g*x)))/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x*
*2*(a*e + b*d)/2)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{3(gx+f)^{\frac{5}{2}}ce-5(3cef-(cd+be)g)(gx+f)^{\frac{3}{2}}+15(3cef^2-2(cd+be)fg+(bd+ae)g^2)\sqrt{gx+f}}{g^3}+\frac{15(c\dots}{\dots}\right)}{15g}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(3/2)
+ 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3 +
 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(sqrt(g*x +
 f)*g^3))/g
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(cef^3-cdf^2g-bef^2g+bdfg^2+aefg^2-adg^3)}{\sqrt{gx+f}g^4} + \frac{2\left(3(gx+f)^{\frac{5}{2}}ceg^{16}-15(gx+f)^{\frac{3}{2}}cefg^{16}+45\sqrt{gx+f}cef^2g^{16}+5(gx+f)^{\frac{3}{2}}cdg^{17}+5(gx+f)^{\frac{3}{2}}beg^{17}-3\dots\right)}{15g^{20}}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*e*f^3 - c*d*f^2*g - b*e*f^2*g + b*d*f*g^2 + a*e*f*g^2 - a*d*g^3)/(sqrt
(g*x + f)*g^4) + 2/15*(3*(g*x + f)^(5/2)*c*e*g^16 - 15*(g*x + f)^(3/2)*c*e*
f*g^16 + 45*sqrt(g*x + f)*c*e*f^2*g^16 + 5*(g*x + f)^(3/2)*c*d*g^17 + 5*(g*
x + f)^(3/2)*b*e*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 - 30*sqrt(g*x + f)*b*e*
f*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 15*sqrt(g*x + f)*a*e*g^18)/g^20
```

Mupad [B] (verification not implemented)

Time = 11.75 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{3/2}\,(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f)}{3\,g^4} - \frac{2\,a\,d\,g^3-2\,c\,e\,f^3-2\,a\,e\,f\,g^2-2\,b\,d\,f\,g^2+2\,b\,e\,f^2\,g+2\,c\,d\,f^2\,g}{g^4\,\sqrt{f+g\,x}} + \frac{\sqrt{f+g\,x}\,(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g)}{g^4} + \frac{2\,c\,e\,(f+g\,x)^{5/2}}{5\,g^4}$$

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(3*g^4) - (2*a*d*g^3 - 2*c*
e*f^3 - 2*a*e*f*g^2 - 2*b*d*f*g^2 + 2*b*e*f^2*g + 2*c*d*f^2*g)/(g^4*(f + g*
x)^(1/2)) + ((f + g*x)^(1/2)*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g
 - 4*c*d*f*g))/g^4 + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

3.829 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(cf^2-bfg+ag^2)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2-b*f*g+c*f^2)/g^3/(g*x+f)^(1/2)-2*(-b*g+2*c
*f)*(g*x+f)^(1/2)/g^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {712}

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx=-\frac{2(ag^2-bfg+cf^2)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[In] Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{cf^2 - bfg + ag^2}{g^2(f+gx)^{3/2}} + \frac{-2cf+bg}{g^2\sqrt{f+gx}} + \frac{c\sqrt{f+gx}}{g^2} \right) dx$$
$$= -\frac{2(cf^2 - bfg + ag^2)}{g^3\sqrt{f+gx}} - \frac{2(2cf-bg)\sqrt{f+gx}}{g^3} + \frac{2c(f+gx)^{3/2}}{3g^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{6g(2bf-ag+bgx)+2c(-8f^2-4fgx+g^2x^2)}{3g^3\sqrt{f+gx}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
[Out] (6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt[
f + g*x])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^{2}+3bx-3a\right)g^{2}}{3}+4f\left(-\frac{2cx}{3}+b\right)g-\frac{16c\,f^{2}}{3}}{\sqrt{gx+f}\,g^{3}}$	47
gosper	$-\frac{2\left(-c\,x^{2}g^{2}-3b\,g^{2}x+4cfxg+3a\,g^{2}-6bfg+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	53
trager	$-\frac{2\left(-c\,x^{2}g^{2}-3b\,g^{2}x+4cfxg+3a\,g^{2}-6bfg+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	53
risch	$\frac{2(cxg+3bg-5cf)\sqrt{gx+f}}{3g^{3}}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{g^{3}\sqrt{gx+f}}$	55
derivativedivides	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}+2bg\sqrt{gx+f}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	63
default	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}+2bg\sqrt{gx+f}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	63

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*((c*x^2+3*b*x-3*a)*g^2+6*f*(-2/3*c*x+b)*g-8*c*f^2)/(g*x+f)^(1/2)/g^3
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\,(cg^2x^2-8\,cf^2+6\,bfg-3\,ag^2-(4\,cfg-3\,bg^2)x)\sqrt{gx+f}}{3\,(g^4x+fg^3)}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt(
g*x + f)/(g^4*x + f*g^3)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3g^2}+\frac{\sqrt{f+gx}(bg-2cf)}{g^2}-\frac{ag^2-bfg+cf^2}{g^2\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{bx^2}{2}+\frac{cx^3}{3}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*(f + g*x)**(3/2)/(3*g**2) + sqrt(f + g*x)*(b*g - 2*c*f)/g**
2 - (a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*x + b
*x**2/2 + c*x**3/3)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-3\,(2\,cf-bg)\sqrt{gx+f}}{g^2}-\frac{3\,(cf^2-bfg+ag^2)}{\sqrt{gx+f}g^2}\right)}{3\,g}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 - b
*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{2\,(cf^2-bfg+ag^2)}{\sqrt{gx+f}g^3} + \frac{2\left((gx+f)^{\frac{3}{2}}cg^6-6\,\sqrt{gx+f}cfg^6+3\,\sqrt{gx+f}bg^7\right)}{3\,g^9}$$

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6
 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2\,c\,{(f+g\,x)}^2-6\,a\,g^2-6\,c\,f^2+6\,b\,g\,(f+g\,x)-12\,c\,f\,(f+g\,x)+6\,b\,f\,g}{3\,g^3\,\sqrt{f+g\,x}}$$

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] (2*c*(f + g*x)^2 - 6*a*g^2 - 6*c*f^2 + 6*b*g*(f + g*x) - 12*c*f*(f + g*x) +
 6*b*f*g)/(3*g^3*(f + g*x)^(1/2))
```

3.830 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(cd^2-bde+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3
/2)/(-d*g+e*f)^(3/2)+2*(a*g^2-b*f*g+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c
*(g*x+f)^(1/2)/e/g^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {911, 1275, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2(ae^2-bde+cd^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2-bfg+cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx,x,\sqrt{f+gx}\right)}{g}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2-bde+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)\,dx,x,\sqrt{f+gx}\right)}{g}\\
&= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}\\
&\quad -\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx,x,\sqrt{f+gx}\right)}{e(ef-dg)}\\
&= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(eg(bf-ag)+cdg(f+gx)-cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}} - \frac{2(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f)
+ d*g)*Sqrt[f + g*x]) - (2*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqrt[
f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^2a-bde+c\,d^2\right)g^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	122
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^2a-bde+c\,d^2\right)g^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	122
pseudoelliptic	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(e^2a-bde+c\,d^2\right)g^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	122
risch	$\frac{2c\sqrt{gx+f}}{e\,g^2}+\frac{-\frac{2g^2\left(e^2a-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}-\frac{2e\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2e}$	128

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(c/e*(g*x+f)^(1/2)-(a*g^2-b*f*g+c*f^2)/(d*g-e*f)/(g*x+f)^(1/2)-(a*e^2
-b*d*e+c*d^2)/e*g^2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((
d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(108) = 216$.

Time = 0.43 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.43

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \left[-\frac{((cd^2-bde+ae^2)g^3x+(cd^2-bde+ae^2)fg^2)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+\cdots}{\cdots}\right)\cdots}{e^4f^3\cdots}\right.$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*d^2 - b*d*e + a*e^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(e^2
*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))
/(e*x + d)) - 2*(2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c*d*e^2 + b*e^3)*f^2*g + (c
*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*
g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^
4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x), 2*(((c*d^2 - b*d*e + a*e^2)*g^
3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*
f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c
*d*e^2 + b*e^3)*f^2*g + (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g -
2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2
*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.41

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{f+gx}}{eg}-\frac{ag^2-bfg+cf^2}{g\sqrt{f+gx}(dg-ef)}-\frac{g(ae^2-bde+cd^2)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}(dg-ef)}\right)}{g} & \text{for}\ g\neq 0 \\ \frac{\frac{cx^2}{2e}+\frac{x(be-cd)}{e^2}+\frac{(ae^2-bde+cd^2)\left(\begin{cases}\frac{x}{d} & \text{for}\ e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{f^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] Piecewise((2*(c*sqrt(f + g*x)/(e*g) - (a*g**2 - b*f*g + c*f**2)/(g*sqrt(f +
 g*x)*(d*g - e*f)) - g*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d
*g - e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne(g, 0)), ((c*x**
2/(2*e) + x*(b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*Piecewise((x/d, Eq
(e, 0)), (log(d + e*x)/e, True))/e**2)/f**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^2f-deg)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2-bfg+ag^2)}{(efg^2-dg^3)\sqrt{gx+f}} + \frac{2\sqrt{gx+f}c}{eg^2}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^
2*f - d*e*g)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 - b*f*g + a*g^2)/((e*f*g^2 -
d*g^3)*sqrt(g*x + f)) + 2*sqrt(g*x + f)*c/(e*g^2)
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(e^2\,f-d\,e\,g\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{3/2}\,\left(2\,c\,d^2-2\,b\,d\,e+2\,a\,e^2\right)}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}} - \frac{2\,\left(c\,e\,f^2-b\,e\,f\,g+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*c*(f + g*x)^(1/2))/(e*g^2) + (2*atan((2*(f + g*x)^(1/2)*(e^2*f - d*e*g)*
(a*e^2 + c*d^2 - b*d*e))/(e^(1/2)*(d*g - e*f)^(3/2)*(2*a*e^2 + 2*c*d^2 - 2*
b*d*e)))*(a*e^2 + c*d^2 - b*d*e))/(e^(3/2)*(d*g - e*f)^(3/2)) - (2*(a*e*g^2
 + c*e*f^2 - b*e*f*g))/(e*g^2*(f + g*x)^(1/2)*(d*g - e*f))
```

3.831 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 165

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e(2bef+bdg-3aeg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

```
[Out] (c*d*(-d*g+4*e*f)-e*(-3*a*e*g+b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2-b*f*g+c*f^2)/g/(-d*g+e
*f)^2/(g*x+f)^(1/2)-(a*e^2-b*d*e+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {911, 1273, 464, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}} - \frac{\sqrt{f+gx}(ae^2-bde+cd^2)}{e(d+ex)(ef-dg)^2} - \frac{2(ag^2-bfg+cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
```

```
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(e(bd-ae)g^2+c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}dx, x, \sqrt{f+gx}\right)}{e^2(ef-dg)^2}$$

$$= -\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(cd(4ef-dg)-e(2bef+bdg-3aeg))\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}dx, x, \sqrt{f+gx}\right)}{eg(ef-dg)^2}$$

$$= -\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e(2bef+bdg-3aeg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))+eg(b(3df+2efx+dgx)-a(ef+2dg+}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}} + \frac{(cd(-4ef+dg)+e(2bef+bdg-3aeg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) + e*g*(b*(3*d*f + 2*e*f*x
 + d*g*x) - a*(e*f + 2*d*g + 3*e*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f
 + g*x]) + ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTan[(Sq
rt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(5/2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-bdeg-2b\,e^2f-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
default	$-\frac{\frac{2g\left(\frac{g\left(e^2a-bde+c\,d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a\,e^2g-bdeg-2b\,e^2f-c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^2\sqrt{gx+f}}}{g}$
pseudoelliptic	$-\frac{2\left(\frac{3\sqrt{gx+f}\,(ex+d)g\left(\left(ag-\frac{2bf}{3}\right)e^2-\frac{d(bg-4cf)e}{3}-\frac{c\,d^2g}{3}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2}+\left(\left(\frac{3a\,g^2x}{2}+\frac{f(-2bx+a)g}{2}+c\,f^2x\right)e^2+d\left(\left(-\right.\right.\right.\right.}{\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,g(ex+d)(dg-ef)^2e}$

[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2-b*d*e+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)+1/2*(3*a*e^2*g-b*d*e*g-2*b*e^2*f-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2-b*f*g+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(151) = 302.

Time = 0.50 (sec) , antiderivative size = 1088, normalized size of antiderivative = 6.59

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \left[\frac{(2\,(2\,cd^2e-bde^2)f^2g-(cd^3+bd^2e-3\,ade^2)fg^2+(2\,(2\,cde^2-be^3)fg^2-(cd\right.$$

$$-\frac{(2\,(2\,cd^2e-bde^2)f^2g-(cd^3+bd^2e-3\,ade^2)fg^2+(2\,(2\,cde^2-be^3)fg^2-(cd^2e+bde^2-3\,ae^3)g^3)x^2+(2\,(}{}$$

[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")

[Out] [1/2*((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2

```
 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x), -((2*(
2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2 + (2*(2*c*
d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (2*(2*c*d*e
^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + b*d^2*e
- 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(
g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3
*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 -
 (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*
d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^
5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g
^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{(4\,cdef-2\,be^2f-cd^2g-bdeg+3\,ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f^2-2\,de^2fg+d^2eg^2)\sqrt{-e^2f+deg}} - \frac{2\,(gx+f)ce^2f^2-2\,ce^2f^3-2\,(gx+f)be^2fg+2\,cdef^2g+2\,be^2f^2g+(gx+f)cd^2g^2-(gx+f)bdeg^2+3\,(}{(e^3f^2g-2\,de^2fg^2+d^2eg^3)\left((gx+f)^{\frac{3}{2}}e-\sqrt{gx+f}ef+\sqrt{gx+}\right.}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*c*d*e*f - 2*b*e^2*f - c*d^2*g - b*d*e*g + 3*a*e^2*g)*arctan(sqrt(g*x +
f)*e/sqrt(-e^2*f + d*e*g))/((e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*sqrt(-e^2*f
 + d*e*g)) - (2*(g*x + f)*c*e^2*f^2 - 2*c*e^2*f^3 - 2*(g*x + f)*b*e^2*f*g +
 2*c*d*e*f^2*g + 2*b*e^2*f^2*g + (g*x + f)*c*d^2*g^2 - (g*x + f)*b*d*e*g^2
+ 3*(g*x + f)*a*e^2*g^2 - 2*b*d*e*f*g^2 - 2*a*e^2*f*g^2 + 2*a*d*e*g^3)/((e^
3*f^2*g - 2*d*e^2*f*g^2 + d^2*e*g^3)*((g*x + f)^(3/2)*e - sqrt(g*x + f)*e*f
 + sqrt(g*x + f)*d*g))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.32

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{5/2}}\right)\,\left(2\,b\,e^2\,f-3\,a\,e^2\,g+c\,d^2\,g+b\,d\,e\,g-4\,c\,d\,e\,f\right.}{e^{3/2}\,\left(d\,g-e\,f\right)^{5/2}} - \frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2-b\,d\,e\,g^2+2\,c\,e^2\,f^2-2\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{e\,\left(d\,g-e\,f\right)^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,\left(f+g\,x\right)^{3/2}}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))/(e^(1/2)*(d*g -
 e*f)^(5/2)))*(2*b*e^2*f - 3*a*e^2*g + c*d^2*g + b*d*e*g - 4*c*d*e*f))/(e^(
3/2)*(d*g - e*f)^(5/2)) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f +
g*x)*(3*a*e^2*g^2 + c*d^2*g^2 + 2*c*e^2*f^2 - b*d*e*g^2 - 2*b*e^2*f*g))/(e*
(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f + g*x)^(3/2))
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{(c(8e^2f^2+8defg-d^2g^2)+3eg(5aeg-b(4ef+dg)))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

```
[Out] -1/4*(c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2)+3*e*g*(5*a*e*g-b*(d*g+4*e*f)))*arcta
nh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^
2-b*f*g+c*f^2)/(-d*g+e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2-b*d*e+c*d^2)*(g*x+f)^(
1/2)/e/(-d*g+e*f)^2/(e*x+d)^2+1/4*(c*d*(-d*g+8*e*f)-e*(-7*a*e*g+3*b*d*g+4*b
*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {911, 1273, 467, 464, 214}

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c(-d^2g^2+8defg+8e^2f^2)\right)}{4e^{3/2}(ef-dg)^{7/2}} - \frac{\sqrt{f+gx}(ae^2-bde+cd^2)}{2e(d+ex)^2(ef-dg)^2} + \frac{2(ag^2-bfg+cf^2)}{\sqrt{f+gx}(ef-dg)^3} + \frac{\sqrt{f+gx}(cd(8ef-dg)-e(-7aeg+3bdg+4bef))}{4e(d+ex)(ef-dg)^3}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(3e(bd-ae)g^2+c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2e^2(ef-dg)^2}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\text{Subst}\left(\int \frac{\frac{8e^2\left(cf^2-bfg+ag^2\right)}{g^4}+\frac{e(cd(8ef-dg)-e(4bef+3bdg-7aeg))x^2}{g^3(ef-dg)}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{4e^2(ef-dg)^2}$$

$$= \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2}$$
$$+ \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}$$
$$+ \frac{(c(8e^2f^2+8defg-d^2g^2)+3eg(5aeg-b(4ef+dg)))\,\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{4eg(ef-dg)^3}$$
$$= \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2}$$
$$+ \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}$$
$$- \frac{(c(8e^2f^2+8defg-d^2g^2)+3eg(5aeg-b(4ef+dg)))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.20

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\frac{\sqrt{e}\left(c\left(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e\left(14f^2+5fgx-g^2x^2\right)\right)-e\left(a\left(-8d^2g^2-deg(9f+25gx)+e^2(2\right.\right.\right.}{(ef-dg)^3(d+ex)^2\sqrt{f+g}}}{}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) + d
^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) - e*(a*(-8*d^2*g^2 - d*e*g*(9*f + 25*g*x
) + e^2*(2*f^2 - 5*f*g*x - 15*g^2*x^2)) + b*(4*e^2*f*x*(f + 3*g*x) + d^2*g*
(13*f + 5*g*x) + d*e*(2*f^2 + 21*f*g*x + 3*g^2*x^2)))))/((e*f - d*g)^3*(d +
 e*x)^2*Sqrt[f + g*x]) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a
*e*g - b*(4*e*f + d*g)))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]
)/(-(e*f) + d*g)^(7/2))/(4*e^(3/2))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{15\left(\sqrt{gx+f}\,(ex+d)^2\left(\left(a\,g^2-\frac{4}{5}bfg+\frac{8}{15}c\,f^2\right)e^2-\frac{dg\left(bg-\frac{8cf}{3}\right)e}{5}-\frac{c\,d^2g^2}{15}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\frac{8\left(\left(\frac{15a\,g^2x^2}{8}+\frac{5\left(-\frac{12}{}\right.}{}\right.\right.}{}\right.}{}$
derivativedivides	$-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg\right.}{}}{(e(gx+f)+dg-ef)^2}\right.}{}$
default	$-\frac{2\left(a\,g^2-bfg+c\,f^2\right)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg\right.}{}}{(e(gx+f)+dg-ef)^2}\right.}{}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/4/(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*((g*x+f)^(1/2)*(e*x+d)^2*((a*g^2-4/
5*b*f*g+8/15*c*f^2)*e^2-1/5*d*g*(b*g-8/3*c*f)*e-1/15*c*d^2*g^2)*arctan(e*(g
*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+8/15*((15/8*a*g^2*x^2+5/8*(-12/5*b*x+a)*x*
f*g-1/4*f^2*(-4*c*x^2+2*b*x+a))*e^3+9/8*d*((-1/3*b*x^2+25/9*a*x)*g^2+f*(8/9
*c*x^2-7/3*b*x+a)*g-2/9*f^2*(-12*c*x+b))*e^2+d^2*((a-1/8*c*x^2-5/8*b*x)*g^2
-13/8*(-5/13*c*x+b)*f*g+7/4*c*f^2)*e+1/8*c*d^3*g*(g*x+f))*((d*g-e*f)*e)^(1/
2))/(d*g-e*f)^3/(e*x+d)^2/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(226) = 452$.

Time = 0.59 (sec) , antiderivative size = 1883, normalized size of antiderivative = 7.59

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g - (c*d^4 + 3*b*
d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 - 3*b*e^4)*f*g^
2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 12*(2*c*d*
e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g^2 - 2*(c*d^3
*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*(c*d^2*e^2 -
 b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2 - (c*d^4 + 3*
b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*
g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a*d^3*e^2*g^3 -
2*(7*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a
```

```
*d*e^4)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
 12*b*e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2
+ 3*b*d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d
^2*e^3 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e
^4)*f*g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(
d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6
*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f
*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*
d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d
^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^
6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g -
(c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 -
3*b*e^4)*f*g^2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3
 + 12*(2*c*d*e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g
^2 - 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24
*(c*d^2*e^2 - b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2
- (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sq
rt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c
*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a*d*e^4
)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 12*b*
e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 + 3*b*
d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d^2*e^3
 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e^4)*f*
g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^
6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f
*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 +
 d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^
5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6
*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*
g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + bx + cx^2}{(d + ex)^3 (f + gx)^{3/2}} \, dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(226) = 452.

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.90

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(8\,ce^2f^2+8\,cdefg-12\,be^2fg-cd^2g^2-3\,bdeg^2+15\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+de}}\right.}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2-bfg+ag^2)}{(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\sqrt{gx+f}} + \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-4\,(gx+f)^{\frac{3}{2}}be^3fg-8\,\sqrt{gx+f}cde^2f^2g+4\,\sqrt{gx+f}be^3f^2g-(gx+f)^{\frac{3}{2}}cd^2eg^2-3\,(g}{4\,(e^4f^3}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c*e^2*f^2 + 8*c*d*e*f*g - 12*b*e^2*f*g - c*d^2*g^2 - 3*b*d*e*g^2 + 1
5*a*e^2*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^3 - 3*d*e
^3*f^2*g + 3*d^2*e^2*f*g^2 - d^3*e*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 -
b*f*g + a*g^2)/((e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*sqrt(g*
x + f)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 4*(g*x + f)^(3/2)*b*e^3*f*g
- 8*sqrt(g*x + f)*c*d*e^2*f^2*g + 4*sqrt(g*x + f)*b*e^3*f^2*g - (g*x + f)^(
3/2)*c*d^2*e*g^2 - 3*(g*x + f)^(3/2)*b*d*e^2*g^2 + 7*(g*x + f)^(3/2)*a*e^3*
g^2 + 7*sqrt(g*x + f)*c*d^2*e*f*g^2 + sqrt(g*x + f)*b*d*e^2*f*g^2 - 9*sqrt(
g*x + f)*a*e^3*f*g^2 + sqrt(g*x + f)*c*d^3*g^3 - 5*sqrt(g*x + f)*b*d^2*e*g^
3 + 9*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^
2 - d^3*e*g^3)*((g*x + f)*e - e*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2 -\right.}{4\,e^{3/2}\,\left(d\,g-e\,f\right)^{7/2}}$$

$$-\frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,\left(d\,g-e\,f\right)^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g-5\,b\,d\,e\,g^2+16\,c\,e\right.}{4\,e\,\left(d\,g-e\,f\right)^2}}{e^2\,\left(f+g\,x\right)^{5/2}-\left(f+g\,x\right)^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 -
 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) -
 ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*
d^2*g^2 + 8*c*e^2*f^2 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*(d*g
- e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 - 5*b*d*e*g
^2 - 20*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2)
 - (f + g*x)^(3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2
 - 2*d*e*f*g))
```

3.833 $\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\operatorname{arccosh}(x)+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right)$$

```
[Out] -arccosh(x)+1/5*arctan((1+x)^(1/2)/(-1+x)^(1/2)/(-2+5^(1/2))^(1/2))*(-10+10
*5^(1/2))^(1/2)+1/5*arctanh((1+x)^(1/2)/(-1+x)^(1/2)/(2+5^(1/2))^(1/2))*(10
+10*5^(1/2))^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(91) = 182.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.10, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{915, 1005, 223, 212, 1048, 739, 210\}$

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \frac{\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\sqrt{x-1}\sqrt{x+1}\arctan\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}} - \frac{\sqrt{x-1}\sqrt{x+1}\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}} - \frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\sqrt{x-1}\sqrt{x+1}\operatorname{arctanh}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}$$

```
[In] Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[5])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[(2 - (1 - Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x
]*Sqrt[1 + x]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5
])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1005

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{\sqrt{-1+x^2}}{1+x-x^2}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}+\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{(1+x-x^2)\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}\\
&\quad+\frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\left(1-\sqrt{5}-2x\right)\sqrt{-1+x^2}}\,dx}{5\sqrt{-1+x^2}}\\
&\quad+\frac{\left(\left(5+\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\left(1+\sqrt{5}-2x\right)\sqrt{-1+x^2}}\,dx}{5\sqrt{-1+x^2}}
\end{aligned}$$

$$= -\frac{\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{-4+\left(1-\sqrt{5}\right)^2-x^2}\,dx, x, \frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)}{5\sqrt{-1+x^2}} - \frac{\left(\left(5+\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{-4+\left(1+\sqrt{5}\right)^2-x^2}\,dx, x, \frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{-1+x^2}}\right)}{5\sqrt{-1+x^2}}$$

$$= \frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\sqrt{-1+x}\sqrt{1+x}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\sqrt{-2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right) - 2\text{arctanh}\left(\sqrt{\frac{-1+x}{1+x}}\right) + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right)$$

```
[In] Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -(Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x
)]]) - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTan
h[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(65) = 130$.

Time = 0.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\sqrt{1+x}\,\sqrt{-1+x}\,\sqrt{5}\left(\sqrt{5}\,\sqrt{2\sqrt{5}-2}\,\sqrt{2\sqrt{5}+2}\,\ln\left(x+\sqrt{x^{2}-1}\right)-\sqrt{5}\,\sqrt{2\sqrt{5}-2}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^{2}-1}}\right)-\sqrt{5}\,\sqrt{2\sqrt{5}+2}\,\arctan\left(\frac{}{\sqrt{}}\right.\right.}{5\sqrt{2\sqrt{5}-2}\,\sqrt{2\sqrt{5}+2}\,\sqrt{x^{2}-1}}$

```
[In] int((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(1+x)^(1/2)*(-1+x)^(1/2)*5^(1/2)*(5^(1/2)*(2*5^(1/2)-2)^(1/2)*(2*5^(1/
2)+2)^(1/2)*ln(x+(x^2-1)^(1/2))-5^(1/2)*(2*5^(1/2)-2)^(1/2)*arctanh((5^(1/2
)*x+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))-5^(1/2)*(2*5^(1/2)+2)^(1/2)*arc
tan((5^(1/2)*x-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))-(2*5^(1/2)-2)^(1/2)*
arctanh((5^(1/2)*x+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))+(2*5^(1/2)+2)^(1
/2)*arctan((5^(1/2)*x-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2)))/(2*5^(1/2)-2
)^(1/2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(65) = 130$.

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx &= \frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}+\sqrt{2\sqrt{5}+2}\right.\\&\left.+1\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}\right.\\&\left.-\sqrt{2\sqrt{5}+2}+1\right)-\frac{1}{10}\sqrt{5}\sqrt{-2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}\right.\\&\left.-2x-\sqrt{5}+\sqrt{-2\sqrt{5}+2}+1\right)\\&+\frac{1}{10}\sqrt{5}\sqrt{-2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x-\sqrt{5}\right.\\&\left.-\sqrt{-2\sqrt{5}+2}+1\right)+\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)\end{aligned}$$

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqrt
(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqr
t(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt
(5)*sqrt(-2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x - sqrt(5) + sq
```

```
rt(-2*sqrt(5) + 2) + 1) + 1/10*sqrt(5)*sqrt(-2*sqrt(5) + 2)*log(2*sqrt(x +
1)*sqrt(x - 1) - 2*x - sqrt(5) - sqrt(-2*sqrt(5) + 2) + 1) + log(sqrt(x + 1
)*sqrt(x - 1) - x)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -\int \frac{\sqrt{x-1}\sqrt{x+1}}{x^2-x-1}\,dx$$

```
[In] integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)
[Out] -Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \int -\frac{\sqrt{x+1}\sqrt{x-1}}{x^2-x-1}\,dx$$

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
[Out] -integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
[Out] log((sqrt(x + 1) - sqrt(x - 1))^2)
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 916, normalized size of antiderivative = 10.07

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = -4\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)$$

$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{3408370\sqrt{10}\sqrt{\sqrt{5}+1}-\sqrt{10}\sqrt{\sqrt{5}+1}\sqrt{x-1}\,300730\mathrm{i}-3408370\sqrt{10}\sqrt{\sqrt{5}+1}\sqrt{x+1}-1771398\sqrt{5}\sqrt{10}\sqrt{\sqrt{5}+1}+7836865\sqrt{10}\,x}{}\right.}{}$$

$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{3408370\sqrt{10}\sqrt{1-\sqrt{5}}+3066340\sqrt{10}\,x^2\sqrt{1-\sqrt{5}}-\sqrt{10}\sqrt{1-\sqrt{5}}\sqrt{x-1}\,300730\mathrm{i}-3408370\sqrt{10}\sqrt{1-\sqrt{5}}\sqrt{x+1}+1771398\sqrt{5}\sqrt{10}}{}\right.}{}$$

```
[In] int(((x - 1)^(1/2)*(x + 1)^(1/2))/(x - x^2 + 1),x)
```

```
[Out] - 4*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) - (10^(1/2)*atan((34083
70*10^(1/2)*(5^(1/2) + 1)^(1/2) - 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2
)*300730i - 3408370*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 1771398*5^
(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2) + 7836865*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
 + 3066340*10^(1/2)*x^2*(5^(1/2) + 1)^(1/2) - 1294942*5^(1/2)*10^(1/2)*x^2*
(5^(1/2) + 1)^(1/2) + 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1
/2)*300730i - 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i +
1771398*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(5^
(1/2) + 1)^(1/2)*(x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(5^(1/2) + 1)^(
1/2)*(x + 1)^(1/2) - 3475583*5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2) + 5^(1/
2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i + 10^(1
/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i - 5^(1/2)*10^
(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i + 2589884*5^(1/2)*10^(1/2
)*x*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) + 5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1
/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x + 1)^(1
/2) - 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) - 7067880*5^(1/2) - 39924
30*5^(1/2)*x^2 + 12033450*x^2 + 7067880*5^(1/2)*(x + 1)^(1/2) + 7984860*5^(
1/2)*x*(x + 1)^(1/2) + 10104760))*(5^(1/2) + 1)^(1/2)*1i)/5 - (10^(1/2)*ata
n((3408370*10^(1/2)*(1 - 5^(1/2))^(1/2) + 3066340*10^(1/2)*x^2*(1 - 5^(1/2)
)^(1/2) - 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*300730i - 3408370*10^(
1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) + 1771398*5^(1/2)*10^(1/2)*(1 - 5^(1
/2))^(1/2) + 7836865*10^(1/2)*x*(1 - 5^(1/2))^(1/2) + 3475583*5^(1/2)*10^(1
/2)*x*(1 - 5^(1/2))^(1/2) + 1294942*5^(1/2)*10^(1/2)*x^2*(1 - 5^(1/2))^(1/2
) + 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*300730i + 5^(1
/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 1771398*5^(1/2)*10
^(1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(
x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2)
 - 5^(1/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i
 + 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i + 5^(
1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 2589884*5^(1/2)
*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 5^(1/2)*10^(1/2)*x*(1 - 5^(
```

1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x + 1)^(1/2) + 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) + 7067880*5^(1/2) + 3992430*5^(1/2)*x^2 + 12033450*x^2 - 7067880*5^(1/2)*(x + 1)^(1/2) - 7984860*5^(1/2)*x*(x + 1)^(1/2) + 10104760))*(1 - 5^(1/2))^(1/2)*1i)/5

3.834 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

```
[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh
(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)
^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*
x+f)^(1/2)/e^2/g^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {965, 81, 65, 223, 212}

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2)
+ (c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e
*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}(4ae^2g-cd(3ef+dg))-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{8e^2g^2}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{4e^3g^2}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{4e^3g^2}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3ef-3dg+2egx))}{4e^2g^2} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*e
^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f
 + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5
/2)*g^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(138) = 276.

Time = 0.48 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\frac{\left(4\sqrt{eg}\,\sqrt{(gx+f)(ex+d)}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cdefg+3\ln\left(\right.\right.}{}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((g*
x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2
*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*
g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*
g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+
d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f+8*(e*g)^
(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\right.}{}\right.$$
$$-\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-2}{8\,e^3g^3}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
```

```
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$=\frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2-4\,be^6g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg-4\,be^2fg+3\,cd^2g^2-4\,bdeg}{}\right.}{4\,|e|}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3*
g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2 - 4*b*e^6*g^2)/(e^8*g^3)) - (3*c*e^2*f^2
+ 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8*a*e^2*g^2)*log(
abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e*x + d)*e*g - d*e*g)))/(sqrt(
e*g)*e^2*g^2))*e/abs(e)
```

Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$= \frac{\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}$$

$$- \frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\left(\frac{3\,c\,d^2\,g^2}{2}+c\,d\right.}{e^2\,g^3\left(\sqrt{f+g\,x}-\sqrt{\right.}}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\left(\sqrt{\right.}}{g^3\left(\sqrt{f\right.}}}$$

$$- \frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}} - \frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)(d\,g+e\,f)}{e^{3/2}\,g^{3/2}}$$

$$+ \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2)}{2\,e^{5/2}\,g^{5/2}}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/((((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
```

```
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

3.835 $\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 333

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4} + \frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3} - \frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{64e^{5/2}g^{9/2}}$$

```
[Out] 1/64*(-d*g+e*f)^2*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*
g+5*e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(
9/2)+1/96*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e*f)
))*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^3-1/24*(-8*b*e*g+9*c*d*g+7*c*e*f)*(e*x
+d)^(5/2)*(g*x+f)^(1/2)/e^2/g^2+1/4*c*(e*x+d)^(7/2)*(g*x+f)^(1/2)/e^2/g-1/6
4*(-d*g+e*f)*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e
*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {965, 81, 52, 65, 223, 212}

$$\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(ef-dg)^2\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10d}{64e^{5/2}g^{9/2}}$$
$$-\frac{\sqrt{d+ex}\sqrt{f+gx}(ef-dg)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^2g^4}$$
$$+\frac{(d+ex)^{3/2}\sqrt{f+gx}(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{96e^2g^3}$$
$$-\frac{(d+ex)^{5/2}\sqrt{f+gx}(-8beg+9cdg+7cef)}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

```
[In] Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -1/64*((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*
g - b*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^4) + ((c*(35*e^2*
f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x
)^(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e
*x)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(
4*e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*
(6*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[
f + g*x])])/(64*e^(5/2)*g^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}+\frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8ae^2g-cd(7ef+dg)\right)-\frac{1}{2}e(7cef+9cdg-8beg)x\right)}{\sqrt{f+gx}}\,dx}{4e^2g}\\
&= -\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}\\
&\quad+\frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}}\,dx}{48e^2g^2}\\
&= \frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}\\
&\quad-\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}\\
&\quad-\frac{\left((ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\,dx}{64e^2g^3}
\end{aligned}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$
$$+\frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}$$
$$-\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$
$$+\frac{\left((ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{128e^2g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$
$$+\frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}$$
$$-\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$
$$+\frac{\left((ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, \ldots}{64e^3g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$
$$+\frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}$$
$$-\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$
$$+\frac{\left((ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\right)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{e}}\,dx, x, \ldots}{64e^3g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$
$$+\frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}$$
$$-\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$
$$+\frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{64e^{5/2}g^{9/2}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(c(-9d^3g^3+3d^2eg^2(-5f+2gx)+de^2g(145f^2-92fgx}{}$$

$$+\frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{64e^{5/2}g^{9/2}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(c*(-9*d^3*g^3 + 3*d^2*e*g^2*(-5*f + 2*g*x) +
d*e^2*g*(145*f^2 - 92*f*g*x + 72*g^2*x^2) + e^3*(-105*f^3 + 70*f^2*g*x - 56
*f*g^2*x^2 + 48*g^3*x^3)) + 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x) + b*(
3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))
))/(192*e^2*g^4) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2)
+ 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[
g]*Sqrt[d + e*x])])/(64*e^(5/2)*g^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. 2(295) = 590.

Time = 0.48 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1207

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(-184*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
*c*d*e^2*f*g^2*x+144*c*d*e^2*g^3*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-11
2*c*e^3*f*g^2*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+192*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)*a*e^3*g^3*x+12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*e*f*g^3+48*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)*b*d^2*e*g^3+240*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*e^3*f^2*
g-288*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*a*e^3*f*g^2+480*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)*a*d*e^2*g^3-180*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2
)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^3*g+54*ln(1/2*(2*e*g*x+2*((g*
x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f^2*g^2+216
*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2)
)*b*d*e^3*f^2*g^2-288*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*a*d*e^3*f*g^3-72*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e^2*f*g^3+9*ln(1/2*(2*e*g*x+
2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^4+105*l
```

```
n(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*
c*e^4*f^4+128*b*e^3*g^3*x^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+96*c*e^3*g^
3*x^3*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*
x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^3*e*g^4+224*((g*x+f)*(e*x
+d))^(1/2)*(e*g)^(1/2)*b*d*e^2*g^3*x-160*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2
)*b*e^3*f*g^2*x-30*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d^2*e*f*g^2-18*((g
*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d^3*g^3-352*((g*x+f)*(e*x+d))^(1/2)*(e*g
)^(1/2)*b*d*e^2*f*g^2+290*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d*e^2*f^2*g
-210*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*e^3*f^3+144*ln(1/2*(2*e*g*x+2*((
g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*d^2*e^2*g^4+144*l
n(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*
a*e^4*f^2*g^2-120*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g
+e*f)/(e*g)^(1/2))*b*e^4*f^3*g+12*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d^2
*e*g^3*x+140*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*e^3*f^2*g*x)/g^4/e^2/((g
*x+f)*(e*x+d))^(1/2)/(e*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \left[\frac{3\,(35\,ce^4f^4-20\,(3\,cde^3+2\,be^4)f^3g+6\,(3\,cd^2e^2+12\,bde^3+8\,ae^4)f^2g^2\right.$$

$$-\frac{3\,(35\,ce^4f^4-20\,(3\,cde^3+2\,be^4)f^3g+6\,(3\,cd^2e^2+12\,bde^3+8\,ae^4)f^2g^2+4\,(cd^3e-6\,bd^2e^2-24\,ade^3)fg^3}{}$$

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c*e^4*f^4 - 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 +
 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g
^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*sqrt(e*g)*log(8*e^2*g^2*x^2
+ e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*
x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(48*c*e^4*g^4*x^3 - 105
*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (15*c*d^2*e^2 + 176*b*d*
e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*g^4 - 8*(
7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^2 - 2*(2
3*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*g^4)*x)
*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5), -1/384*(3*(35*c*e^4*f^4 - 20*(3*c*
d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4
*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d
^2*e^2)*g^4)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*
x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(
```

```
48*c*e^4*g^4*x^3 - 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (1
5*c*d^2*e^2 + 176*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 -
 80*a*d*e^3)*g^4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(3
5*c*e^4*f^2*g^2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e
^3 + 48*a*e^4)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\left(2\,(ex+d)\left(4\,(ex+d)\left(\frac{6\,(ex+d)c}{e^3g}-\frac{7\,ce^7fg^5+9\,cde}{e^9g}\right.\right.\right.\right.}{}$$

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e*x + d)*(4*(e*x + d)*(6*(e*
x + d)*c/(e^3*g) - (7*c*e^7*f*g^5 + 9*c*d*e^6*g^6 - 8*b*e^7*g^6)/(e^9*g^7))
 + (35*c*e^8*f^2*g^4 + 10*c*d*e^7*f*g^5 - 40*b*e^8*f*g^5 + 3*c*d^2*e^6*g^6
```

```
- 8*b*d*e^7*g^6 + 48*a*e^8*g^6)/(e^9*g^7)) - 3*(35*c*e^9*f^3*g^3 - 25*c*d*e
^8*f^2*g^4 - 40*b*e^9*f^2*g^4 - 7*c*d^2*e^7*f*g^5 + 32*b*d*e^8*f*g^5 + 48*a
*e^9*f*g^5 - 3*c*d^3*e^6*g^6 + 8*b*d^2*e^7*g^6 - 48*a*d*e^8*g^6)/(e^9*g^7))
*sqrt(e*x + d) - 3*(35*c*e^4*f^4 - 60*c*d*e^3*f^3*g - 40*b*e^4*f^3*g + 18*c
*d^2*e^2*f^2*g^2 + 72*b*d*e^3*f^2*g^2 + 48*a*e^4*f^2*g^2 + 4*c*d^3*e*f*g^3
- 24*b*d^2*e^2*f*g^3 - 96*a*d*e^3*f*g^3 + 3*c*d^4*g^4 - 8*b*d^3*e*g^4 + 48*
a*d^2*e^2*g^4)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)))/(sqrt(e*g)*e^2*g^4))*e/abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^{3/2}\,(c\,x^2+b\,x+a)}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2), x)
```

3.836 $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 246

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} - \frac{(ef-dg)\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{8e^{5/2}g^{7/2}}$$

```
[Out] -1/8*(-d*g+e*f)*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*(4*a*e*g-b*(d*g+3*e*
f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(7/2)-1
/12*(-6*b*e*g+7*c*d*g+5*c*e*f)*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^2+1/3*c*(e
*x+d)^(5/2)*(g*x+f)^(1/2)/e^2/g+1/8*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*
(4*a*e*g-b*(d*g+3*e*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used

$= \{965, 81, 52, 65, 223, 212\}$

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= -\frac{(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^{5/2}g^{7/2}}$$
$$+\frac{\sqrt{d+ex}\sqrt{f+gx}(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^2g^3}$$
$$-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{12e^2g^2}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(6ae^2g-cd(5ef+dg)\right)-\frac{1}{2}e(5cef+7cdg-6beg)x\right)}{\sqrt{f+gx}}\,dx}{3e^2g}$$

$$= -\frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\,dx}{8e^2g^2}$$

$$= \frac{\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} - \frac{\left((ef-dg)\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{16e^2g^3}$$

$$= \frac{\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} - \frac{\left((ef-dg)\left(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d\ldots}\right.}{8e^3g^3}$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3}$$
$$- \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$
$$- \frac{((ef-dg)\,(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg))))\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{8e^3g^3}$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3}$$
$$- \frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$
$$- \frac{(ef-dg)\,(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{8e^{5/2}g^{7/2}}$$

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{\sqrt{d+ex}\sqrt{f+gx}(6eg(4aeg+b(-3ef+dg+2egx))+c(-3d^2g^2+2deg(-2f+gx)+e^2(15f^2-10fgx}{24e^2g^3}$$
$$+ \frac{(ef-dg)\,(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\left(\sqrt{d-\frac{ef}{g}}-\sqrt{d+ex}\right)}\right)}{4e^{5/2}g^{7/2}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2*e*g*x))
+ c*(-3*d^2*g^2 + 2*d*e*g*(-2*f + g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2
))))/(24*e^2*g^3) + ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e
*g*(4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*(S
qrt[d - (e*f)/g] - Sqrt[d + e*x]))])/(4*e^(5/2)*g^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(214) = 428$.

Time = 0.49 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.10

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\,\left(16c\,e^2g^2x^2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+24\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)ad\,e^2g^3-24\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{}}{2\sqrt{eg}}\right.\right.}{}$

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*c*e^2*g^2*x^2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(2*e*g
*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g^
2+18*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^2*f^2*g-15*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^3*
f^3+24*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*b*e^2*g^2*x+4*((g*x+f)*(e*x+d))^
(1/2)*(e*g)^(1/2)*c*d*e*g^2*x-20*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*e^2*
f*g*x+48*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*a*e^2*g^2+12*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e^2*f
*g-6*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)*c*d^2*g^2-8*((g*x+f)*(e*x+d))^(1/2
)*(e*g)^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e^2*f^2)/g
^3/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*g)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\left[-\frac{3\left(5\,ce^3f^3-3\left(cde^2+2\,be^3\right)f^2g-\left(cd^2e-4\,bde^2-8\,ae^3\right)fg^2-\left(cd^3-2\,bd^2e+8\,ade^2\right)g^3\right)\sqrt{eg}\log\left(8\,e^2g\right.}{}\right.$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^2
 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*g^3)*sqrt(e*g)*log(8*e^
2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*
g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) - 4*(8*c*e^3*g^3*
x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*e - 2*b*d*e
^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e^3*g^4), 1/48*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3
)*f^2*g - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*
d*e^2)*g^3)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x
 + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) + 2*(8
*c*e^3*g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*
e - 2*b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*
x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^4)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(2\,(ex+d)\left(\frac{4\,(ex+d)c}{e^3g}-\frac{5\,ce^7fg^3+7\,cde^6g^4-6\,be^7g^4}{e^9g^5}\right)+\frac{3\,(5\,ce^8f^2g^2+2\,cde^7fg^3-6\,be}{}\right.\right.}{}$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*(4*(e*
x + d)*c/(e^3*g) - (5*c*e^7*f*g^3 + 7*c*d*e^6*g^4 - 6*b*e^7*g^4)/(e^9*g^5))
 + 3*(5*c*e^8*f^2*g^2 + 2*c*d*e^7*f*g^3 - 6*b*e^8*f*g^3 + c*d^2*e^6*g^4 - 2
*b*d*e^7*g^4 + 8*a*e^8*g^4)/(e^9*g^5)) + 3*(5*c*e^3*f^3 - 3*c*d*e^2*f^2*g -
 6*b*e^3*f^2*g - c*d^2*e*f*g^2 + 4*b*d*e^2*f*g^2 + 8*a*e^3*f*g^2 - c*d^3*g^
3 + 2*b*d^2*e*g^3 - 8*a*d*e^2*g^3)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(
e^2*f + (e*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^3))*e/abs(e)
```

Mupad [B] (verification not implemented)

Time = 109.82 (sec) , antiderivative size = 1832, normalized size of antiderivative = 7.45

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] (((2*a*d*g + 2*a*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(g^2*((f + g*x)^(1/2)
- f^(1/2))^3) + ((2*a*e^2*f + 2*a*d*e*g)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*
((f + g*x)^(1/2) - f^(1/2))) - (8*a*d^(1/2)*e*f^(1/2)*((d + e*x)^(1/2) - d^
(1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2
))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((
c*d^3*e^3*g^3)/4 - (5*c*e^6*f^3)/4 + (3*c*d*e^5*f^2*g)/4 + (c*d^2*e^4*f*g^2
)/4))/(g^9*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^5*((
33*c*e^4*f^3)/2 + (19*c*d^3*e*g^3)/2 + (313*c*d*e^3*f^2*g)/2 + (275*c*d^2*e
^2*f*g^2)/2))/(g^7*((f + g*x)^(1/2) - f^(1/2))^5) - (((d + e*x)^(1/2) - d^(
1/2))^7*((19*c*d^3*g^3)/2 + (33*c*e^3*f^3)/2 + (313*c*d*e^2*f^2*g)/2 + (275
*c*d^2*e*f*g^2)/2))/(g^6*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2)
 - d^(1/2))^3*((17*c*d^3*e^2*g^3)/12 - (85*c*e^5*f^3)/12 + (17*c*d*e^4*f^2*
g)/4 + (91*c*d^2*e^3*f*g^2)/4))/(g^8*((f + g*x)^(1/2) - f^(1/2))^3) + (((d
```

+ e*x)^(1/2) - d^(1/2))^11*((c*d^3*g^3)/4 - (5*c*e^3*f^3)/4 + (3*c*d*e^2*f^
2*g)/4 + (c*d^2*e*f*g^2)/4))/(e^2*g^4*((f + g*x)^(1/2) - f^(1/2))^11) - (((
d + e*x)^(1/2) - d^(1/2))^9*((17*c*d^3*g^3)/12 - (85*c*e^3*f^3)/12 + (17*c*
d*e^2*f^2*g)/4 + (91*c*d^2*e*f*g^2)/4))/(e*g^5*((f + g*x)^(1/2) - f^(1/2))^
9) + (d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6*(128*c*e^3*f^2 + 64*c*d
^2*e*g^2 + (704*c*d*e^2*f*g)/3))/(g^6*((f + g*x)^(1/2) - f^(1/2))^6) + (d^(
1/2)*f^(1/2)*(32*c*d^2*g + 96*c*d*e*f)*((d + e*x)^(1/2) - d^(1/2))^8)/(g^4*
((f + g*x)^(1/2) - f^(1/2))^8) + (d^(1/2)*f^(1/2)*(96*c*d*e^3*f + 32*c*d^2*
e^2*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^6*((f + g*x)^(1/2) - f^(1/2))^4))/
(((d + e*x)^(1/2) - d^(1/2))^12/((f + g*x)^(1/2) - f^(1/2))^12 + e^6/g^6 -
(6*e*((d + e*x)^(1/2) - d^(1/2))^10)/(g*((f + g*x)^(1/2) - f^(1/2))^10) - (
6*e^5*((d + e*x)^(1/2) - d^(1/2))^2)/(g^5*((f + g*x)^(1/2) - f^(1/2))^2) +
(15*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4)
- (20*e^3*((d + e*x)^(1/2) - d^(1/2))^6)/(g^3*((f + g*x)^(1/2) - f^(1/2))^6
) + (15*e^2*((d + e*x)^(1/2) - d^(1/2))^8)/(g^2*((f + g*x)^(1/2) - f^(1/2))
^8)) + ((((d + e*x)^(1/2) - d^(1/2))*((b*d^2*e^2*g^2)/2 - (3*b*e^4*f^2)/2 +
b*d*e^3*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) + (((d + e*x)^(1/2) - d^(1
/2))^3*((11*b*e^3*f^2)/2 + (7*b*d^2*e*g^2)/2 + 23*b*d*e^2*f*g))/(g^5*((f +
g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^5*((7*b*d^2*g^2)/2
+ (11*b*e^2*f^2)/2 + 23*b*d*e*f*g))/(g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (
((d + e*x)^(1/2) - d^(1/2))^7*((b*d^2*g^2)/2 - (3*b*e^2*f^2)/2 + b*d*e*f*g)
)/(e*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (d^(1/2)*f^(1/2)*(32*b*e^2*f + 16
*b*d*e*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4
) - (8*b*d^(3/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6)/(g^2*((f + g*x)^(1/
2) - f^(1/2))^6) - (8*b*d^(3/2)*e^2*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^2)/
(g^4*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g
*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*(
(f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*
((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2
*((f + g*x)^(1/2) - f^(1/2))^4)) + (2*a*atanh((g^(1/2)*((d + e*x)^(1/2) - d
^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d*g - e*f))/(e^(1/2)*g^(3/
2)) - (b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1
/2) - f^(1/2))))*(d*g - e*f)*(d*g + 3*e*f))/(2*e^(3/2)*g^(5/2)) + (c*atanh(
(g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2)))
)*(d*g - e*f)*(d^2*g^2 + 5*e^2*f^2 + 2*d*e*f*g))/(4*e^(5/2)*g^(7/2))

3.837 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

```
[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh
(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)
^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*
x+f)^(1/2)/e^2/g^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {965, 81, 65, 223, 212}

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2)
+ (c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e
*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}(4ae^2g-cd(3ef+dg))-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$
$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{8e^2g^2}$$
$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{4e^3g^2}$$
$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{4e^3g^2}$$
$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3ef-3dg+2egx))}{4e^2g^2} + \frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*e
^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f
 + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5
/2)*g^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.

Time = 0.48 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\frac{\left(4\sqrt{eg}\,\sqrt{(gx+f)(ex+d)}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cdefg+3\ln\right.}{}$

[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f+8*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\right.}{}\right.$$
$$-\frac{(3\,ce^2f^2+2\,(cde-2\,be^2)fg+(3\,cd^2-4\,bde+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-}{8\,e^3g^3}$$

[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*

```
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2-4\,be^6g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg-4\,be^2fg+3\,cd^2g^2-4\,bdeg^2+}{}\right.}{4\,|e|}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3*
g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2 - 4*b*e^6*g^2)/(e^8*g^3)) - (3*c*e^2*f^2
+ 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8*a*e^2*g^2)*log(
abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2*f + (e*x + d)*e*g - d*e*g)))/(sqrt(
e*g)*e^2*g^2))*e/abs(e)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \frac{\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{(2\,b\,d\,g+2\,b\,e\,f)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}$$
$$-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\left(\frac{3\,c\,d^2\,g^2}{2}+\cdots\right)}{e^2\,g^3\left(\sqrt{f+g\,x}-\cdots\right)}\cdots}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\left(\cdots\right.}{g^3\left(\cdots\right.}\cdots}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)(d\,g+e\,f)}{e^{3/2}\,g^{3/2}}$$
$$+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] ((((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/((((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
```

```
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

3.838 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 129

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}$$

```
[Out] -(-2*b*e*g+3*c*d*g+c*e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/
2))/e^(5/2)/g^(3/2)-2*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
^(1/2)+c*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {963, 81, 65, 223, 212}

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(-2beg+3cdg+cef)}{e^{5/2}g^{3/2}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}-\frac{2\int\frac{\frac{(cd-be)(ef-dg)}{2e^2}-\frac{c(ef-dg)x}{2e}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{ef-dg}$$

$$=-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\text{Subst}\left(\int\frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\text{Subst}\left(\int\frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int\frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}(2e(bd-ae)g+c(-3d^2g+e^2fx+de(f-gx)))}{e^2g(ef-dg)\sqrt{d+ex}} + \frac{(2beg-c(ef+3dg))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}g^{3/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
[Out] (Sqrt[f + g*x]*(2*e*(b*d - a*e)*g + c*(-3*d^2*g + e^2*f*x + d*e*(f - g*x)))
)/(e^2*g*(e*f - d*g)*Sqrt[d + e*x]) + ((2*b*e*g - c*(e*f + 3*d*g))*ArcTanh[
(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(e^(5/2)*g^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. 2(109) = 218.

Time = 0.48 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.40

method	result
default	$\frac{\sqrt{gx+f}\left(2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)bd\,e^2g^2x-2\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)b\,e^3fgx-3\ln\left(\frac{2egx+2\sqrt{(gx+f)}}{2}\right.\right.}{}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*b*d*e^2*g^2*x-2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^3*f*g*x-3*ln(1/2*(2*e*g*x+2*((g*
x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*g^2*x+2*ln(1/
2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*
e^2*f*g*x+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*c*e^3*f^2*x+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1
/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e*g*x+2*((
g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*
e*f*g+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^
(1/2))*c*d*e^2*f^2+2*c*d*e*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-2*c*e^2*
f*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((g*x+f)*(e*x+d))^(1/2)*(
e*g)^(1/2)-4*b*d*e*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*c*d^2*g*((g*x+f)
*(e*x+d))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2))/
(e*g)^(1/2)/g/(d*g-e*f)/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(109) = 218$.

Time = 1.51 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.56

$$\int \frac{a + bx + cx^2}{(d + ex)^{3/2}\sqrt{f + gx}}\,dx = \left[-\frac{(cde^2f^2 + 2\,(cd^2e - bde^2)fg - (3\,cd^3 - 2\,bd^2e)g^2 + (ce^3f^2 + 2\,(cde^2 - be^3)fg\right.$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2
 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*s
qrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e
*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x)
- 4*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g - c*d
*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (e^5
*f*g^2 - d*e^4*g^3)*x), 1/2*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*
c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e
- 2*b*d*e^2)*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x
)) + 2*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g -
c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (
e^5*f*g^2 - d*e^4*g^3)*x)]
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{4\left(cd^2g-bdeg+ae^2g\right)}{\left(e^2f-deg-\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)\sqrt{eg}|e|} + \frac{(cef+3\,cdg-2\,beg)\log\left(\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)}{2\,\sqrt{eg}eg|e|} + \frac{\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}c|e|}{e^4g}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] -4*(c*d^2*g - b*d*e*g + a*e^2*g)/((e^2*f - d*e*g - (sqrt(e*g)*sqrt(e*x + d)
 - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)*sqrt(e*g)*abs(e)) + 1/2*(c*e*f +
 3*c*d*g - 2*b*e*g)*log((sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e
*g - d*e*g))^2)/(sqrt(e*g)*e*g*abs(e)) + sqrt(e^2*f + (e*x + d)*e*g - d*e*g
)*sqrt(e*x + d)*c*abs(e)/(e^4*g)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)),x)
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.839 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}}+\frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}+\frac{2c\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

```
[Out] 2*c*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(1/2)-2/
3*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(3/2)+2/3*(c*(-4*d^
2*g+6*d*e*f)-e*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*
x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {963, 79, 65, 223, 212}

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\frac{2\sqrt{f+gx}(c(6def-4d^2g)-e(-2aeg-bdg+3bef))}{3e^2\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
```

```
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}}-\frac{2\int\frac{\frac{cd(3ef-dg)-e(3bef-bdg-2aeg)}{2e^2}-\frac{3}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{e^2}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}dx, x, \sqrt{d+ex}\right)}{e^3}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{e^3}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(cd(-3d^2g+6e^2fx+de(5f-4gx))+e^2(b(-2df-3efx+dgx)+}{3e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2c\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}\sqrt{g}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]

[Out] (2*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 6*e^2*f*x + d*e*(5*f - 4*g*x)) + e^2*(b*(
-2*d*f - 3*e*f*x + d*g*x) + a*(-(e*f) + 3*d*g + 2*e*g*x))))/(3*e^2*(e*f - d
*g)^2*(d + e*x)^(3/2)) + (2*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt
[d + e*x])])/(e^(5/2)*Sqrt[g])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(136) = 272.

Time = 0.48 (sec) , antiderivative size = 773, normalized size of antiderivative = 4.83

method	result
default	$\frac{\sqrt{gx+f}\left(3\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^{2}e^{2}g^{2}x^{2}-6\ln\left(\frac{2egx+2\sqrt{(gx+f)(ex+d)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cd\,e^{3}fg\,x^{2}+3\ln\left(\frac{2egx+2\sqrt{(gx+}}{}\right.\right.}{}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*g^2*x^2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d
))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f*g*x^2+3*ln(1/2*(2*e*g*
x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^4*f^2*x^2
+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/
2))*c*d^3*e*g^2*x-12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f*g*x+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^2*x+3*ln(1/2*(2*e*g*x+2*
((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^2-6*ln(1/
2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^
3*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*c*d^2*e^2*f^2+4*a*e^3*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+2*
b*d*e^2*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*b*e^3*f*x*((g*x+f)*(e*x+d
))^(1/2)*(e*g)^(1/2)-8*c*d^2*e*g*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+12*c
*d*e^2*f*x*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*((g*x+f)*(e*x+d)
)^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-4*b*d*e^2
*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+10*c*d^2*e*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2)/(d*
g-e*f)^2/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(137) = 274.

Time = 3.31 (sec) , antiderivative size = 792, normalized size of antiderivative = 4.95

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \left[\frac{3\,(cd^2e^2f^2-2\,cd^3efg+cd^4g^2+(ce^4f^2-2\,cde^3fg+cd^2e^2g^2)x^2+2\,(cde^3f^2-\right.$$

$$-\frac{3\,(cd^2e^2f^2-2\,cd^3efg+cd^4g^2+(ce^4f^2-2\,cde^3fg+cd^2e^2g^2)x^2+2\,(cde^3f^2-2\,cd^2e^2fg+cd^3eg^2)x)\sqrt{-eg}}{3\,(d^2e^5f^2g-2\,d^3e^4fg^2-}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^3
*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^2)
*x)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*
x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2
)*x) + 4*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*e^3)*g^2
 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*e^3*g^3
+ (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g - 2*d^2*e^
5*f*g^2 + d^3*e^4*g^3)*x), -1/3*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g
^2 + (c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c
*d^2*e^2*f*g + c*d^3*e*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*
sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g +
d*e*g^2)*x)) - 2*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*
e^3)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*
e^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g -
2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x)]
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(5/2)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(137) = 274.

Time = 0.38 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.07

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{c\log\left(\left(\sqrt{eg}\sqrt{ex+d}-\sqrt{e^2f+(ex+d)eg-deg}\right)^2\right)}{\sqrt{eg}e|e|} + \frac{4\left(6\,cde^4f^2g-3\,be^5f^2g-10\,cd^2e^3fg^2+4\,bde^4fg^2+2\,ae^5fg^2+4\,cd^3e^2g^3-bd^2e^3g^3-2\,ade^4g^3-12\left(\sqrt{eg}\right.\right.}{}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -c*log((sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)/(
sqrt(e*g)*e*abs(e)) + 4/3*(6*c*d*e^4*f^2*g - 3*b*e^5*f^2*g - 10*c*d^2*e^3*f
*g^2 + 4*b*d*e^4*f*g^2 + 2*a*e^5*f*g^2 + 4*c*d^3*e^2*g^3 - b*d^2*e^3*g^3 -
2*a*d*e^4*g^3 - 12*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g -
d*e*g))^2*c*d*e^2*f*g + 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)
*e*g - d*e*g))^2*b*e^3*f*g + 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g))^2*c*d^2*e*g^2 - 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g))^2*a*e^3*g^2 + 6*(sqrt(e*g)*sqrt(e*x + d) - sqrt(
e^2*f + (e*x + d)*e*g - d*e*g))^4*c*d*g - 3*(sqrt(e*g)*sqrt(e*x + d) - sqrt
(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*e*g)/((e^2*f - d*e*g - (sqrt(e*g)*sqrt
(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)^3*sqrt(e*g)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.840 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 198

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}}\\ &+\frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}}\\ &+\frac{2(2eg(5bef-bdg-4aeg)-c(15e^2f^2-10defg+3d^2g^2))\sqrt{f+gx}}{15e^2(ef-dg)^3\sqrt{d+ex}}\end{aligned}$$

```
[Out] -2/5*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(5/2)+2/15*(2*c*
d*(-3*d*g+5*e*f)-e*(-4*a*e*g-b*d*g+5*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(3/2)+2/15*(2*e*g*(-4*a*e*g-b*d*g+5*b*e*f)-c*(3*d^2*g^2-10*d*e*f*g
+15*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {963, 79, 37}

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx &= \frac{2\sqrt{f+gx}(2eg(-4aeg-bdg+5bef)-c(3d^2g^2-10defg+15e^2f^2))}{15e^2\sqrt{d+ex}(ef-dg)^3}\\ &-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{15e^2(d+ex)^{3/2}(ef-dg)^2}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}}-\frac{2\int\frac{\frac{cd(5ef-dg)-e(5bef-bdg-4aeg)}{2e^2}-\frac{5}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{5(ef-dg)}\\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}}+\frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}}\\
&\quad-\frac{(2eg(5bef-bdg-4aeg)-c(15e^2f^2-10defg+3d^2g^2))\int\frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{15e^2(ef-dg)^2}
\end{aligned}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2(2eg(5bef-bdg-4aeg)-c(15e^2f^2-10defg+3d^2g^2))\sqrt{f+gx}}{15e^2(ef-dg)^3\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\left(15cf^2-15bfg+15ag^2-\frac{10cdf(f+gx)}{d+ex}+\frac{5bef(f+gx)}{d+ex}+\frac{5bdg(f+gx)}{d+ex}-\frac{10aeg(f+gx)}{d+ex}+\frac{3cd^2(f+gx)^2}{(d+ex)^2}-\frac{3bde(}{(d}\right.}{15(ef-dg)^3\sqrt{d+ex}}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(15*c*f^2 - 15*b*f*g + 15*a*g^2 - (10*c*d*f*(f + g*x))/(d
 + e*x) + (5*b*e*f*(f + g*x))/(d + e*x) + (5*b*d*g*(f + g*x))/(d + e*x) - (
10*a*e*g*(f + g*x))/(d + e*x) + (3*c*d^2*(f + g*x)^2)/(d + e*x)^2 - (3*b*d*
e*(f + g*x)^2)/(d + e*x)^2 + (3*a*e^2*(f + g*x)^2)/(d + e*x)^2))/(15*(e*f -
 d*g)^3*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.06

method	result
default	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefgx+5b\,e}{15(ex+d)^{\frac{5}{2}}(dg-ef)^3}$
gosper	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefgx+5b\,e}{15(ex+d)^{\frac{5}{2}}(d^3g^3-3d^2ef\,g^2+3d\,e^2f^2g-e^3f^3}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d^
2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+5*
b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15*a*
d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/(e*x
+d)^(5/2)/(d*g-e*f)^3
```

Fricas [A] (verification not implemented)

none

Time = 7.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.78

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx =$$
$$-\frac{2\,(15\,ad^2g^2+(8\,cd^2+2\,bde+3\,ae^2)f^2-10\,(bd^2+ade)fg+(15\,ce^2f^2-10\,(cde+be^2)fg+(3\,cd^2+2\,bde}{15\,(d^3e^3f^3-3\,d^4e^2f^2g+3\,d^5efg^2-d^6g^3+(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3)x^3+3\,(d}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*a*d^2*g^2 + (8*c*d^2 + 2*b*d*e + 3*a*e^2)*f^2 - 10*(b*d^2 + a*d*e
)*f*g + (15*c*e^2*f^2 - 10*(c*d*e + b*e^2)*f*g + (3*c*d^2 + 2*b*d*e + 8*a*e
^2)*g^2)*x^2 + (5*(4*c*d*e + b*e^2)*f^2 - 2*(2*c*d^2 + 13*b*d*e + 2*a*e^2)*
f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^3*e^3*f^3
- 3*d^4*e^2*f^2*g + 3*d^5*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*
d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e
^3*f*g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*
f*g^2 - d^5*e*g^3)*x)
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(7/2)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(180) = 360$.

Time = 0.42 (sec) , antiderivative size = 1175, normalized size of antiderivative = 5.93

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx =$$

$$-\frac{4\left(15\sqrt{eg}ce^8f^4 - 40\sqrt{eg}cde^7f^3g - 10\sqrt{eg}be^8f^3g + 38\sqrt{eg}cd^2e^6f^2g^2 + 22\sqrt{eg}bde^7f^2g^2 + 8\sqrt{eg}ae^8f^2g^2\right.}{}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -4/15*(15*sqrt(e*g)*c*e^8*f^4 - 40*sqrt(e*g)*c*d*e^7*f^3*g - 10*sqrt(e*g)*b
*e^8*f^3*g + 38*sqrt(e*g)*c*d^2*e^6*f^2*g^2 + 22*sqrt(e*g)*b*d*e^7*f^2*g^2
+ 8*sqrt(e*g)*a*e^8*f^2*g^2 - 16*sqrt(e*g)*c*d^3*e^5*f*g^3 - 14*sqrt(e*g)*b
*d^2*e^6*f*g^3 - 16*sqrt(e*g)*a*d*e^7*f*g^3 + 3*sqrt(e*g)*c*d^4*e^4*g^4 + 2
*sqrt(e*g)*b*d^3*e^5*g^4 + 8*sqrt(e*g)*a*d^2*e^6*g^4 - 60*sqrt(e*g)*(sqrt(e
*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*e^6*f^3 + 80*s
qrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*
c*d*e^5*f^2*g + 50*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x +
 d)*e*g - d*e*g))^2*b*e^6*f^2*g - 20*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - s
qrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*d^2*e^4*f*g^2 - 60*sqrt(e*g)*(sqrt(
e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d*e^5*f*g^2 -
 40*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g
))^2*a*e^6*f*g^2 + 10*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*
x + d)*e*g - d*e*g))^2*b*d^2*e^4*g^3 + 40*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d
) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*d*e^5*g^3 + 90*sqrt(e*g)*(sqrt
(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*e^4*f^2 - 40
*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^
4*c*d*e^3*f*g - 70*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x +
 d)*e*g - d*e*g))^4*b*e^4*f*g + 30*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqr
t(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*d^2*e^2*g^2 - 10*sqrt(e*g)*(sqrt(e*g)
*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*d*e^3*g^2 + 80*sq
rt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*a
*e^4*g^2 - 60*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e
*g - d*e*g))^6*c*e^2*f + 30*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g))^6*b*e^2*g + 15*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d
) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^8*c)/((e^2*f - d*e*g - (sqrt(e*g)*
sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)^5*e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.31

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+g\,x}\,\left(\frac{16\,c\,d^2\,f^2-20\,b\,d^2\,f\,g+30\,a\,d^2\,g^2+4\,b\,d\,e\,f^2-20\,a\,d\,e\,f\,g+6\,a\,e^2\,f^2}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x\,\left(-8\,c\,d^2\,f\,g+10\,b\,d^2\,g^2\right.}{}\right.}{x^2\,\sqrt{d+}}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((30*a*d^2*g^2 + 6*a*e^2*f^2 + 16*c*d^2*f^2 + 4*b*d*e*f^2
- 20*b*d^2*f*g - 20*a*d*e*f*g)/(15*e^2*(d*g - e*f)^3) + (x*(10*b*d^2*g^2 +
10*b*e^2*f^2 + 40*a*d*e*g^2 + 40*c*d*e*f^2 - 8*a*e^2*f*g - 8*c*d^2*f*g - 52
*b*d*e*f*g))/(15*e^2*(d*g - e*f)^3) + (x^2*(16*a*e^2*g^2 + 6*c*d^2*g^2 + 30
*c*e^2*f^2 + 4*b*d*e*g^2 - 20*b*e^2*f*g - 20*c*d*e*f*g))/(15*e^2*(d*g - e*f
)^3)))/(x^2*(d + e*x)^(1/2) + (d^2*(d + e*x)^(1/2))/e^2 + (2*d*x*(d + e*x)^
(1/2))/e)
```

3.841 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 281

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = &-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}\\
&+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}}\\
&+\frac{2(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^3(d+ex)^{3/2}}\\
&-\frac{4g(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^4\sqrt{d+ex}}\end{aligned}$$

```
[Out] -2/7*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(7/2)+2/35*(2*c*
d*(-4*d*g+7*e*f)-e*(-6*a*e*g-b*d*g+7*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(5/2)+2/105*(4*e*g*(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*
g+35*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(3/2)-4/105*g*(4*e*g*
(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*g+35*e^2*f^2))*(g*x+f)^(1/2)
/e^2/(-d*g+e*f)^4/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

= {963, 79, 47, 37}

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = -\frac{4g\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2\sqrt{d+ex}(ef-dg)^4} + \frac{2\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2(d+ex)^{3/2}(ef-dg)^3} - \frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{35e^2(d+ex)^{5/2}(ef-dg)^2}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2)
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}-\frac{2\int\frac{\frac{cd(7ef-dg)-e(7bef-bdg-6aeg)}{2e^2}-\frac{7}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx}{7(ef-dg)} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} \\
&\quad -\frac{(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\int\frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{35e^2(ef-dg)^2} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} \\
&\quad +\frac{2(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^3(d+ex)^{3/2}} \\
&\quad +\frac{(2g(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2)))\int\frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{105e^2(ef-dg)^3} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} \\
&\quad +\frac{2(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^3(d+ex)^{3/2}} \\
&\quad -\frac{4g(4eg(7bef-bdg-6aeg)-c(35e^2f^2-14defg+3d^2g^2))\sqrt{f+gx}}{105e^2(ef-dg)^4\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(-105cf^2g(d+ex)^3+105bfg^2(d+ex)^3-105ag^3(d+ex)^3+35cef^2(d+ex)^2(f+gx)+70cdfg}{}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(-105*c*f^2*g*(d + e*x)^3 + 105*b*f*g^2*(d + e*x)^3 - 105
*a*g^3*(d + e*x)^3 + 35*c*e*f^2*(d + e*x)^2*(f + g*x) + 70*c*d*f*g*(d + e*x
)^2*(f + g*x) - 70*b*e*f*g*(d + e*x)^2*(f + g*x) - 35*b*d*g^2*(d + e*x)^2*(
f + g*x) + 105*a*e*g^2*(d + e*x)^2*(f + g*x) - 42*c*d*e*f*(d + e*x)*(f + g*
x)^2 + 21*b*e^2*f*(d + e*x)*(f + g*x)^2 - 21*c*d^2*g*(d + e*x)*(f + g*x)^2
+ 42*b*d*e*g*(d + e*x)*(f + g*x)^2 - 63*a*e^2*g*(d + e*x)*(f + g*x)^2 + 15*
c*d^2*e*(f + g*x)^3 - 15*b*d*e^2*(f + g*x)^3 + 15*a*e^3*(f + g*x)^3))/(105*
(e*f - d*g)^4*(d + e*x)^(7/2))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.52

method	result
default	$\frac{2\sqrt{gx+f}\,(48a\,e^3g^3x^3+8bd\,e^2g^3x^3-56b\,e^3f\,g^2x^3+6c\,d^2e\,g^3x^3-28cd\,e^2f\,g^2x^3+70c\,e^3f^2g\,x^3+168ad\,e^2g^3x^2-24a\,e^3f\,g^2x^2+28b\,d^2e\,g^3}{}$
gosper	$\frac{2\sqrt{gx+f}\,(48a\,e^3g^3x^3+8bd\,e^2g^3x^3-56b\,e^3f\,g^2x^3+6c\,d^2e\,g^3x^3-28cd\,e^2f\,g^2x^3+70c\,e^3f^2g\,x^3+168ad\,e^2g^3x^2-24a\,e^3f\,g^2x^2+28b\,d^2e\,g^3}{}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+
6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x
^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2
*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^
3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^
3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x
+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*
d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c
*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d*g-e*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(257) = 514$.

Time = 22.89 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.28

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \frac{2\left(105\,ad^3g^3-(8\,cd^2e+6\,bde^2+15\,ae^3)f^3+7\,(8\,cd^3+4\,bd^2e+9\,ade^2)f^2g-\right.}{105\,(d^4e^4f^4-4\,d^5e}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(105*a*d^3*g^3 - (8*c*d^2*e + 6*b*d*e^2 + 15*a*e^3)*f^3 + 7*(8*c*d^3
+ 4*b*d^2*e + 9*a*d*e^2)*f^2*g - 35*(2*b*d^3 + 3*a*d^2*e)*f*g^2 + 2*(35*c*e
^3*f^2*g - 14*(c*d*e^2 + 2*b*e^3)*f*g^2 + (3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3
)*g^3)*x^3 - (35*c*e^3*f^3 - 7*(37*c*d*e^2 + 4*b*e^3)*f^2*g + (101*c*d^2*e
+ 200*b*d*e^2 + 24*a*e^3)*f*g^2 - 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)*g^3)
*x^2 - (7*(4*c*d*e^2 + 3*b*e^3)*f^3 - (200*c*d^2*e + 101*b*d*e^2 + 18*a*e^3
)*f^2*g + 7*(4*c*d^3 + 37*b*d^2*e + 12*a*d*e^2)*f*g^2 - 35*(b*d^3 + 6*a*d^2
*e)*g^3)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^4*e^4*f^4 - 4*d^5*e^3*f^3*g + 6*
d^6*e^2*f^2*g^2 - 4*d^7*e*f*g^3 + d^8*g^4 + (e^8*f^4 - 4*d*e^7*f^3*g + 6*d^
2*e^6*f^2*g^2 - 4*d^3*e^5*f*g^3 + d^4*e^4*g^4)*x^4 + 4*(d*e^7*f^4 - 4*d^2*e
^6*f^3*g + 6*d^3*e^5*f^2*g^2 - 4*d^4*e^4*f*g^3 + d^5*e^3*g^4)*x^3 + 6*(d^2*
e^6*f^4 - 4*d^3*e^5*f^3*g + 6*d^4*e^4*f^2*g^2 - 4*d^5*e^3*f*g^3 + d^6*e^2*g
^4)*x^2 + 4*(d^3*e^5*f^4 - 4*d^4*e^4*f^3*g + 6*d^5*e^3*f^2*g^2 - 4*d^6*e^2*
f*g^3 + d^7*e*g^4)*x)
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \int \frac{a+bx+cx^2}{(d+ex)^{\frac{9}{2}}\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(9/2)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(257) = 514$.

Time = 0.52 (sec) , antiderivative size = 2035, normalized size of antiderivative = 7.24

$$\int \frac{a + bx + cx^2}{(d + ex)^{9/2}\sqrt{f + gx}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(35*sqrt(e*g)*c*e^10*f^5*g - 119*sqrt(e*g)*c*d*e^9*f^4*g^2 - 28*sqrt(
e*g)*b*e^10*f^4*g^2 + 150*sqrt(e*g)*c*d^2*e^8*f^3*g^3 + 88*sqrt(e*g)*b*d*e^
9*f^3*g^3 + 24*sqrt(e*g)*a*e^10*f^3*g^3 - 86*sqrt(e*g)*c*d^3*e^7*f^2*g^4 -
96*sqrt(e*g)*b*d^2*e^8*f^2*g^4 - 72*sqrt(e*g)*a*d*e^9*f^2*g^4 + 23*sqrt(e*g
)*c*d^4*e^6*f*g^5 + 40*sqrt(e*g)*b*d^3*e^7*f*g^5 + 72*sqrt(e*g)*a*d^2*e^8*f
*g^5 - 3*sqrt(e*g)*c*d^5*e^5*g^6 - 4*sqrt(e*g)*b*d^4*e^6*g^6 - 24*sqrt(e*g)
*a*d^3*e^7*g^6 - 245*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x
 + d)*e*g - d*e*g))^2*c*e^8*f^4*g + 588*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d)
- sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*d*e^7*f^3*g^2 + 196*sqrt(e*g)*(s
qrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*e^8*f^3*g
^2 - 462*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g -
d*e*g))^2*c*d^2*e^6*f^2*g^3 - 420*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt
(e^2*f + (e*x + d)*e*g - d*e*g))^2*b*d*e^7*f^2*g^3 - 168*sqrt(e*g)*(sqrt(e*
g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*e^8*f^2*g^3 + 1
40*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
)^2*c*d^3*e^5*f*g^4 + 252*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g))^2*b*d^2*e^6*f*g^4 + 336*sqrt(e*g)*(sqrt(e*g)*sqrt(
e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*d*e^7*f*g^4 - 21*sqrt(e
*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2*c*d^4
*e^4*g^5 - 28*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e
*g - d*e*g))^2*b*d^3*e^5*g^5 - 168*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqr
t(e^2*f + (e*x + d)*e*g - d*e*g))^2*a*d^2*e^6*g^5 + 630*sqrt(e*g)*(sqrt(e*g
)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*e^6*f^3*g - 714*
sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4
*c*d*e^5*f^2*g^2 - 588*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e
*x + d)*e*g - d*e*g))^4*b*e^6*f^2*g^2 + 42*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x +
d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*c*d^2*e^4*f*g^3 + 672*sqrt(e*g)
*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*d*e^5*
f*g^3 + 504*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g
 - d*e*g))^4*a*e^6*f*g^3 + 42*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2
*f + (e*x + d)*e*g - d*e*g))^4*c*d^3*e^3*g^4 - 84*sqrt(e*g)*(sqrt(e*g)*sqrt
(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*b*d^2*e^4*g^4 - 504*sqrt
(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^4*a*d
*e^5*g^4 - 770*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*
e*g - d*e*g))^6*c*e^4*f^2*g + 140*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt
```

```
(e^2*f + (e*x + d)*e*g - d*e*g))^6*c*d*e^3*f*g^2 + 700*sqrt(e*g)*(sqrt(e*g)
*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^6*b*e^4*f*g^2 - 210*s
qrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^6*
c*d^2*e^2*g^3 + 140*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g))^6*b*d*e^3*g^3 - 840*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) -
 sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^6*a*e^4*g^3 + 455*sqrt(e*g)*(sqrt(e*g
)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^8*c*e^2*f*g + 105*sq
rt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^8*c
*d*e*g^2 - 280*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e^2*f + (e*x + d)*
e*g - d*e*g))^8*b*e^2*g^2 - 105*sqrt(e*g)*(sqrt(e*g)*sqrt(e*x + d) - sqrt(e
^2*f + (e*x + d)*e*g - d*e*g))^10*c*g)/((e^2*f - d*e*g - (sqrt(e*g)*sqrt(e*
x + d) - sqrt(e^2*f + (e*x + d)*e*g - d*e*g))^2)^7*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.61

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+g\,x}\left(\frac{x^3\,\left(12\,c\,d^2\,e\,g^3-56\,c\,d\,e^2\,f\,g^2+16\,b\,d\,e^2\,g^3+140\,c\,e^3\,f^2\,g-112\,b\,e^3\,f\,g^2+96\,a\,e^3\,g^3\right)}{105\,e^3\,(d\,g-e\,f)^4} - \frac{-112}{}\right.}{}$$

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((x^3*(96*a*e^3*g^3 + 16*b*d*e^2*g^3 + 12*c*d^2*e*g^3 - 11
2*b*e^3*f*g^2 + 140*c*e^3*f^2*g - 56*c*d*e^2*f*g^2))/(105*e^3*(d*g - e*f)^4
) - (30*a*e^3*f^3 - 210*a*d^3*g^3 + 12*b*d*e^2*f^3 + 16*c*d^2*e*f^3 + 140*b
*d^3*f*g^2 - 112*c*d^3*f^2*g - 126*a*d*e^2*f^2*g + 210*a*d^2*e*f*g^2 - 56*b
*d^2*e*f^2*g)/(105*e^3*(d*g - e*f)^4) + (x*(70*b*d^3*g^3 - 42*b*e^3*f^3 + 4
20*a*d^2*e*g^3 - 56*c*d*e^2*f^3 + 36*a*e^3*f^2*g - 56*c*d^3*f*g^2 - 168*a*d
*e^2*f*g^2 + 202*b*d*e^2*f^2*g - 518*b*d^2*e*f*g^2 + 400*c*d^2*e*f^2*g))/(1
05*e^3*(d*g - e*f)^4) + (2*x^2*(7*d*g - e*f)*(24*a*e^2*g^2 + 3*c*d^2*g^2 +
35*c*e^2*f^2 + 4*b*d*e*g^2 - 28*b*e^2*f*g - 14*c*d*e*f*g))/(105*e^3*(d*g -
e*f)^4)))/(x^3*(d + e*x)^(1/2) + (d^3*(d + e*x)^(1/2))/e^3 + (3*d*x^2*(d +
e*x)^(1/2))/e + (3*d^2*x*(d + e*x)^(1/2))/e^2)
```

3.842 $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 249

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} - \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}$$

```
[Out] -1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*e^4))*arctanh
(f^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(f*x+e)^(1/2))/e^(3/2)/f^(7/2)+2*(a+e*(-b*f+
c*e)/f^2)*(e*x+d)^(3/2)/(-d*f+e^2)/(f*x+e)^(1/2)+1/2*c*(e*x+d)^(3/2)*(f*x+e
)^(1/2)/e/f^2+1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*
e^4))*(e*x+d)^(1/2)*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used

$= \{963, 81, 52, 65, 223, 212\}$

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4e^{3/2}f^{7/2}} + \frac{\sqrt{d+ex}\sqrt{e+fx}(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4ef^3(e^2-df)} + \frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2}$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{f\left(3be^2-bdf-2aef\right)-c\left(3e^3-def\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\int \frac{\sqrt{d+ex}}{\sqrt{e+fx}}\,dx}{4ef^2(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} - \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\int \frac{1}{\sqrt{d+ex}\sqrt{e+fx}}\,dx}{8ef^3}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} - \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{df}{e}+\frac{fx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{4e^2f^3}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} - \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\,\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{e+fx}}\right)}{4e^2f^3}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\sqrt{e+fx}}{4ef^3(e^2-df)} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} - \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{\sqrt{d+ex}(4ef(3be-2af+bfx)+c(-15e^3-5e^2fx+df^2x+ef(d+2fx^2)}{4ef^3\sqrt{e+fx}} + \frac{(4ef(-3be^2+bdf+2aef)+c(15e^4-6de^2f-d^2f^2))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*f*x + d*
f^2*x + e*f*(d + 2*f*x^2))))/(4*e*f^3*Sqrt[e + f*x]) + ((4*e*f*(-3*b*e^2 +
b*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[
d + e*x])/(Sqrt[e]*Sqrt[e + f*x])])/(4*e^(3/2)*f^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(219) = 438$.

Time = 0.47 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.35

method	result
default	$\frac{\sqrt{ex+d}\,\left(8\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}+df+e^2}{2\sqrt{ef}}\right)a\,e^2 f^3 x+4\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}+df+e^2}{2\sqrt{ef}}\right)bde\,f^3 x-12\ln\left(\frac{2efx+2\sqrt{(ex+d)(f\cdots}{2\sqrt{e\cdots}}\right.\right.}{}$

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+
d*f+e^2)/(e*f)^(1/2))*a*e^2*f^3*x+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/
2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*d*e*f^3*x-12*ln(1/2*(2*e*f*x+2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^3*f^2*x-ln(1/2*(2*
e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*f^3
*x-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(
1/2))*c*d*e^2*f^2*x+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2
)+d*f+e^2)/(e*f)^(1/2))*c*e^4*f*x+4*c*e*f^2*x^2*((e*x+d)*(f*x+e))^(1/2)*(e*
f)^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/
2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))
^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*x+2*((e*x+d)
*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-6*ln(1/2*(2*e
*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d*e^3*f+
15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*e^5+8*b*e*f^2*x*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+2*c*d*f^2*x*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-10*c*e^2*f*x*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(
1/2)-16*a*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+24*b*e^2*f*((e*x+d)*(f*
x+e))^(1/2)*(e*f)^(1/2)+2*c*d*e*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-30*c*
e^3*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2))/(e*f)^(1/2)/e/((e*x+d)*(f*x+e))^(1
/2)/f^3/(f*x+e)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \left[\frac{(15\,ce^5-(cd^2e-4\,bde^2-8\,ae^3)f^2-6\,(cde^3+2\,be^4)f+(15\,ce^4f-(cd^2\right.$$

$$-\frac{(15\,ce^5-(cd^2e-4\,bde^2-8\,ae^3)f^2-6\,(cde^3+2\,be^4)f+(15\,ce^4f-(cd^2-4\,bde-8\,ae^2)f^3-6\,(cde^2+2}{}$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
[Out] [1/16*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*e
^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*e^
3)*f^2)*x)*sqrt(e*f)*log(8*e^2*f^2*x^2 + e^4 + 6*d*e^2*f + d^2*f^2 + 4*(2*e
*f*x + e^2 + d*f)*sqrt(e*f)*sqrt(e*x + d)*sqrt(f*x + e) + 8*(e^3*f + d*e*f^
2)*x) + 4*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2 + 12*b*e^3
)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sqrt(f*x + e
))/(e^2*f^5*x + e^3*f^4), -1/8*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)
*f^2 - 6*(c*d*e^3 + 2*b*e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*
f^3 - 6*(c*d*e^2 + 2*b*e^3)*f^2)*x)*sqrt(-e*f)*arctan(1/2*(2*e*f*x + e^2 +
d*f)*sqrt(-e*f)*sqrt(e*x + d)*sqrt(f*x + e)/(e^2*f^2*x^2 + d*e^2*f + (e^3*f
 + d*e*f^2)*x)) - 2*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2
+ 12*b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sq
rt(f*x + e))/(e^2*f^5*x + e^3*f^4)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{(e+fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{\sqrt{ex+d}\left((ex+d)\left(\frac{2\,(ex+d)c}{f|e|} - \frac{5\,ce^4f^3+3\,cde^2f^4-4\,be^3f^4}{e^2f^5|e|}\right) - \frac{15\,ce^6f^2-6\,cde^4f^3-12\,be^5f^3\cdots}{e^2f\cdots}\right.}{4\sqrt{e^3+(ex+d)ef-def}} - \frac{(15\,ce^4-6\,cde^2f-12\,be^3f-cd^2f^2+4\,bdef^2+8\,ae^2f^2)\log\left(\left|-\sqrt{ef}\sqrt{ex+d}+\sqrt{e^3+(ex+d)ef-def}\right|\cdots\right.}{4\sqrt{ef}f^3|e|}$$

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(e*x + d)*((e*x + d)*(2*(e*x + d)*c/(f*abs(e)) - (5*c*e^4*f^3 + 3*c
*d*e^2*f^4 - 4*b*e^3*f^4)/(e^2*f^5*abs(e))) - (15*c*e^6*f^2 - 6*c*d*e^4*f^3
 - 12*b*e^5*f^3 - c*d^2*e^2*f^4 + 4*b*d*e^3*f^4 + 8*a*e^4*f^4)/(e^2*f^5*abs
(e)))/sqrt(e^3 + (e*x + d)*e*f - d*e*f) - 1/4*(15*c*e^4 - 6*c*d*e^2*f - 12*
b*e^3*f - c*d^2*f^2 + 4*b*d*e*f^2 + 8*a*e^2*f^2)*log(abs(-sqrt(e*f)*sqrt(e*
x + d) + sqrt(e^3 + (e*x + d)*e*f - d*e*f)))/(sqrt(e*f)*f^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}\,\left(c\,x^2+b\,x+a\right)}{{\left(e+f\,x\right)}^{3/2}}\,dx$$

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.843 $\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 240

$$\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{(bd-ae)(73b^2d^2-90abde+35a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{(73b^2d^2-90abde+35a^2e^2)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{(bd-ae)^2(73b^2d^2-90abde+35a^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

```
[Out] 2*e*(b*x+a)^(3/2)*(e*x+d)^(5/2)/b^2+1/8*(-a*e+b*d)^2*(35*a^2*e^2-90*a*b*d*e
+73*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(9/2)/e
^(1/2)+1/12*(35*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/
b^3+1/3*(-13*a*e+17*b*d)*(e*x+d)^(5/2)*(b*x+a)^(1/2)/b^2+1/8*(-a*e+b*d)*(35
*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {965, 81, 52, 65, 223, 212}

$$\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{(bd-ae)^2(35a^2e^2-90abde+73b^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}} + \frac{\sqrt{a+bx}\sqrt{d+ex}(bd-ae)(35a^2e^2-90abde+73b^2d^2)}{8b^4} + \frac{\sqrt{a+bx}(d+ex)^{3/2}(35a^2e^2-90abde+73b^2d^2)}{12b^3} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\sqrt{a+bx}(d+ex)^{5/2}(17bd-13ae)}{3b^2}$$

```
[In] Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{\int \frac{(d+ex)^{3/2}\left(4e\left(15b^2d^2-3abde-5a^2e^2\right)+4be^2(17bd-13ae)x\right)}{\sqrt{a+bx}}\,dx}{4b^2e}\\
&= \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}\\
&\quad+\frac{\left(73b^2d^2-90abde+35a^2e^2\right)\int \frac{(d+ex)^{3/2}}{\sqrt{a+bx}}\,dx}{6b^2}\\
&= \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3}+\frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2}\\
&\quad+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{\left((bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx}}\,dx}{8b^3}
\end{aligned}
$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\left((bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{16b^4}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\left((bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{8b^5}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\left((bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\right)\text{Subst}\left(\int\frac{1}{1-\frac{ex^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{8b^5}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{(bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\sqrt{d+ex}(-105a^3e^3+5a^2be^2(89d+14ex)-ab^2e(725d$$

$$+\frac{(bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

```
[In] Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x
]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[d + e*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x + 2
32*d*e^2*x^2 + 48*e^3*x^3)))/(24*b^4) + ((b*d - a*e)^2*(73*b^2*d^2 - 90*a*b
*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])]
)/(8*b^(9/2)*Sqrt[e])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(204) = 408.

Time = 0.44 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.38

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{bx+a}\left(96b^3e^3x^3\sqrt{(bx+a)(ex+d)}\,\sqrt{be}-112a\,b^2e^3x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+464b^3d\,e^2x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+105\ln\left(\frac{2bex}{}\right.\right.}{}$

```
[In] int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*b^3*e^3*x^3*((b*x+a)*(e*x+d))^(1/2)*(b
*e)^(1/2)-112*a*b^2*e^3*x^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+464*b^3*d*e
^2*x^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+105*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^4*e^4-480*ln(1/2*(2*b*e*x
+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^3*b*d*e^3+86
4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2
))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2
)+a*e+b*d)/(b*e)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))
^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^4*d^4+140*((b*x+a)*(e*x+d))^(1/2
)*(b*e)^(1/2)*a^2*b*e^3*x-584*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a*b^2*d*e
^2*x+932*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^2*e*x-210*((b*x+a)*(e*x+
d))^(1/2)*(b*e)^(1/2)*a^3*e^3+890*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a^2*b
*d*e^2-1450*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a*b^2*d^2*e+1002*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^{3/2}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \left[\frac{3\,(73\,b^4d^4-236\,ab^3d^3e+288\,a^2b^2d^2e^2-160\,a^3bde^3+35\,a^4e^4)\sqrt{\ }}{}\right.$$

$$-\frac{3\,(73\,b^4d^4-236\,ab^3d^3e+288\,a^2b^2d^2e^2-160\,a^3bde^3+35\,a^4e^4)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-}{}$$

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d*
e^3 + 35*a^4*e^4)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e
^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2
*d*e + a*b*e^2)*x) + 4*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^2
+ 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2 +
2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*sqr
t(e*x + d))/(b^5*e), -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d
^2*e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d
 + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^
2*d*e + a*b*e^2)*x)) - 2*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^
2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2
+ 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*s
qrt(e*x + d))/(b^5*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\cdot(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(15*d**2 + 20*d*e*x + 8*e**2*x**2)/sqrt(a + b*x),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(204) = 408$.

Time = 0.37 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.08

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = -\frac{\frac{360\left(\frac{(b^2d-abe)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}}-\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\right)d^3|b|}{b^2} - \frac{28\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(2(bx+a)\left(\right.\right.\right.}{}}{}$$

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/24*(360*((b^2*d - a*b*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d +
 (b*x + a)*b*e - a*b*e)))/sqrt(b*e) - sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*s
qrt(b*x + a))*d^3*abs(b)/b^2 - 28*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt
(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4)/(b^7*e
^4)) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)/(b^7*e^4)) - 3*(b^3
*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*log(abs(-sqrt(b*e)*sqrt(b*x
 + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b*e^2))*d*e^2*abs(
b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*
(b*x + a)/b^3 + (b^12*d*e^5 - 25*a*b^11*e^6)/(b^14*e^6)) - (5*b^13*d^2*e^4
+ 14*a*b^12*d*e^5 - 163*a^2*b^11*e^6)/(b^14*e^6)) + 3*(5*b^14*d^3*e^3 + 9*a
*b^13*d^2*e^4 + 15*a^2*b^12*d*e^5 - 93*a^3*b^11*e^6)/(b^14*e^6))*sqrt(b*x +
 a) + 3*(5*b^4*d^4 + 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 - 3
5*a^4*e^4)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e -
a*b*e)))/(sqrt(b*e)*b^2*e^3))*e^3*abs(b)/b^2 - 210*(sqrt(b^2*d + (b*x + a)*
b*e - a*b*e)*(2*b*x + 2*a + (b*d*e - 5*a*e^2)/e^2)*sqrt(b*x + a) + (b^3*d^2
 + 2*a*b^2*d*e - 3*a^2*b*e^2)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d
 + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*e))*d^2*e*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}\,\left(15\,d^2+20\,d\,e\,x+8\,e^2\,x^2\right)}{\sqrt{a+b\,x}}\,dx$$

```
[In] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2), x)
```

3.844 $\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 176

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3}+\frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}+\frac{(bd-ae)\left(11b^2d^2-13abde+5a^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}$$

```
[Out] 8/3*e*(b*x+a)^(3/2)*(e*x+d)^(3/2)/b^2+(-a*e+b*d)*(5*a^2*e^2-13*a*b*d*e+11*b
^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(7/2)/e^(1/2
)+2*(-3*a*e+4*b*d)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/b^2+(5*a^2*e^2-13*a*b*d*e+11
*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {965, 81, 52, 65, 223, 212}

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}(5a^2e^2-13abde+11b^2d^2)}{b^3}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}+\frac{2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}$$

```
[In] Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
```

```
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{d+ex}(3e(3bd-2ae)(5bd+2ae)+12be^2(4bd-3ae)x)}{\sqrt{a+bx}}\,dx}{3b^2e}$$

$$= \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{(11b^2d^2-13abde+5a^2e^2)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{((bd-ae)(11b^2d^2-13abde+5a^2e^2))\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^3}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{((bd-ae)(11b^2d^2-13abde+5a^2e^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^4}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{((bd-ae)(11b^2d^2-13abde+5a^2e^2))\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b^4}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{(bd-ae)(11b^2d^2-13abde+5a^2e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\sqrt{d+ex}\left(\sqrt{a+bx}(15a^2e^2-abe(49d+10ex)+b^2(57d^2+32dex+8e^2x^2))+\frac{3\sqrt{bd-ae}(11b^2d^2-13abde+5a^2e^2)\text{arc}}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}\right.}{3b^3}$$

```
[In] Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(57
*d^2 + 32*d*e*x + 8*e^2*x^2)) + (3*Sqrt[b*d - a*e]*(11*b^2*d^2 - 13*a*b*d*e
 + 5*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*Sq
rt[(b*(d + e*x))/(b*d - a*e)])))/(3*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(150) = 300.

Time = 0.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{ex+d}\,\sqrt{bx+a}\left(-16b^2e^2x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+15\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^3e^3-54\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right.\right.}{}$

```
[In] int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*b^2*e^2*x^2*((b*x+a)*(e*x+d))^(1/2)*(
b*e)^(1/2)+15*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d
)/(b*e)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(
1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d
))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^3*d^3+20*(b*e)
^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b*e^2*x-64*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(
1/2)*b^2*d*e*x-30*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(b*e)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*b
^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$
$$= \left[-\frac{3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{be}\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2-4\,(2\,bex+bd+\right.}{}\right.$$
$$-\frac{3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-2\,(8\,b^3e^3x^2+}{6\,b^4e}$$

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(b
*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(2*b*e*x + b*d +
a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) - 4*(
8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*
e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e), -1/6*(3*(11*b^3
*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan(1/2*(
2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 +
a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*
b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*
sqrt(e*x + d))/(b^4*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{d+ex}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(15*d**2 + 20*d*e*x + 8*e**2*x**2)/sqrt(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(150) = 300$.

Time = 0.34 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = -\frac{\frac{45\left(\frac{(b^2d-abe)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{be}}-\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\right)d^2|b|}{b^2} - \frac{\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4}{\cdots}\right.\right.\right.}{\cdots}}{\cdots}$$

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/3*(45*((b^2*d - a*b*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (
b*x + a)*b*e - a*b*e)))/sqrt(b*e) - sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqr
t(b*x + a))*d^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x
+ a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4)/(b^7*e^4))
- 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)/(b^7*e^4)) - 3*(b^3*d^3
+ a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*log(abs(-sqrt(b*e)*sqrt(b*x + a)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b*e^2))*e^2*abs(b)/b^2
- 15*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + 2*a + (b*d*e - 5*a*e^2)/
e^2)*sqrt(b*x + a) + (b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*log(abs(-sqrt(b*
e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*e))*d*e
*abs(b)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 107.00 (sec) , antiderivative size = 1797, normalized size of antiderivative = 10.21

$$\int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

```
[In] int(((d + e*x)^(1/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^3*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e
))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))*(10*b
^3*d^3 + 20*a*b^2*d^2*e - 30*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/2))
) - (160*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2)
 - d^(1/2))^6) + (((a + b*x)^(1/2) - a^(1/2))^7*(10*b^2*d^3 - 30*a^2*d*e^2
+ 20*a*b*d^2*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) + (((a + b*x)^(1/2)
- a^(1/2))^5*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e))/(b*e^2*((d + e*x
)^(1/2) - d^(1/2))^5) - (a^(1/2)*d^(1/2)*(320*b*d^2 + 640*a*d*e)*((a + b*x)
^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (160*a^(1/2)*b^2
*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2)
)/(((a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 -
(4*b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) +
 (6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4)
- (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) -
((((a + b*x)^(1/2) - a^(1/2))*(2*b^5*d^3 - 10*a^3*b^2*e^3 + 6*a^2*b^3*d*e^2
 + 2*a*b^4*d^2*e))/(e^6*((d + e*x)^(1/2) - d^(1/2))) - (((a + b*x)^(1/2) -
a^(1/2))^5*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*b^2*d^2*e + 1252*a^2*b*d*e^2)
)/(e^4*((d + e*x)^(1/2) - d^(1/2))^5) - (((a + b*x)^(1/2) - a^(1/2))^3*((34
*b^4*d^3)/3 - (170*a^3*b*e^3)/3 + 34*a^2*b^2*d*e^2 + 182*a*b^3*d^2*e))/(e^5
*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^11*(2*b^3*d^
3 - 10*a^3*e^3 + 2*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(b^3*e*((d + e*x)^(1/2) -
d^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^9*((34*b^3*d^3)/3 - (170*a^3*e^
3)/3 + 182*a*b^2*d^2*e + 34*a^2*b*d*e^2))/(b^2*e^2*((d + e*x)^(1/2) - d^(1/
2))^9) - (((a + b*x)^(1/2) - a^(1/2))^7*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*
b^2*d^2*e + 1252*a^2*b*d*e^2))/(b*e^3*((d + e*x)^(1/2) - d^(1/2))^7) + (a^(
1/2)*d^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*(1024*a^2*e^2 + 512*b^2*d^2 + (5
632*a*b*d*e)/3))/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (a^(1/2)*d^(1/2)*(25
6*b*d^2 + 768*a*d*e)*((a + b*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) -
 d^(1/2))^8) + (a^(1/2)*d^(1/2)*(256*b^3*d^2 + 768*a*b^2*d*e)*((a + b*x)^(1
/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/2) -
a^(1/2))^12/((d + e*x)^(1/2) - d^(1/2))^12 + b^6/e^6 - (6*b^5*((a + b*x)^(1
/2) - a^(1/2))^2)/(e^5*((d + e*x)^(1/2) - d^(1/2))^2) + (15*b^4*((a + b*x)^
(1/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4) - (20*b^3*((a + b*x
)^(1/2) - a^(1/2))^6)/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (15*b^2*((a + b
*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) - d^(1/2))^8) - (6*b*((a + b*
x)^(1/2) - a^(1/2))^10)/(e*((d + e*x)^(1/2) - d^(1/2))^10)) + (((30*b*d^3 +
```

```
 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2) - d^(1/2)))
 - (120*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d + e*x)^(1/2)
- d^(1/2))^2) + ((30*b*d^3 + 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*
e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/((d + e*x)
^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d
+ e*x)^(1/2) - d^(1/2))^2)) - (2*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2))
)/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d)*(5*a^2*e^2 + b^2*d^2 +
 2*a*b*d*e))/(b^(7/2)*e^(1/2)) - (30*d^2*atanh((e^(1/2)*((a + b*x)^(1/2) -
a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d))/(b^(3/2)*e^(1
/2)) + (10*d*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x
)^(1/2) - d^(1/2))))*(a*e - b*d)*(3*a*e + b*d))/(b^(5/2)*e^(1/2))
```

3.845 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 122

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx=\frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2(8b^2d^2-8abde+3a^2e^2)\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}$$

```
[Out] 2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*
x+d)^(1/2))/b^(5/2)/e^(1/2)+4*e*(b*x+a)^(3/2)*(e*x+d)^(1/2)/b^2+2*(-5*a*e+7
*b*d)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {965, 81, 65, 223, 212}

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx=\frac{2(3a^2e^2-8abde+8b^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}$$

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\int \frac{2e\left(15b^2d^2-6abde-2a^2e^2\right)+4be^2(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^2e}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{(8b^2d^2-8abde+3a^2e^2)\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{b^2}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{(2(8b^2d^2-8abde+3a^2e^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^3}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{(2(8b^2d^2-8abde+3a^2e^2))\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b^3}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2(8b^2d^2-8abde+3a^2e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-3ae+2bex)}{b^2} + \frac{2(8b^2d^2-8abde+3a^2e^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{e}}$$

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*Sqrt[d + e*x]*(7*b*d - 3*a*e + 2*b*e*x))/b^2 + (2*(8*b^2*d
^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a
 + b*x])])/(b^(5/2)*Sqrt[e])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.48 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.02

method	result
default	$\frac{\left(4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,bex+3\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^2-8\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abde+8\ln\left(\frac{2bex}{}\right.\right.}{\sqrt{be}\,b^2\sqrt{(bx+a)(ex+d)}}$

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] (4*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*b*e*x+3*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*e^2-8*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e+8*ln(1/2
*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d
^2-6*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(b*e)^(1/2)*((b*x+a)*(e*x+d
))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(b*e)^(1/2)/b^2/((b*x+a)*(e*x+d))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.52

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \left[\frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)\sqrt{be}\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2+4\,(2\,bex+bd+ae)\sqrt{be}\sqrt{bx+a}\sqrt{ex}\right.}{2\,b^3e}\right.$$
$$-\frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-2\,(2\,b^2e^2x+7\,b^2de-3\,abe^2)\sqrt{bx}}{b^3e}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2
*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a
)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + 4*(2*b^2*e^2*x + 7*b^2*d*e - 3
*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e), -((8*b^2*d^2 - 8*a*b*d*e +
3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x
+ a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(2*
b^2*e^2*x + 7*b^2*d*e - 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e)]
```

Sympy [F]

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$$

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\,dx = \frac{2\left(\sqrt{b^2d + (bx + a)be - abe}\sqrt{bx + a}\left(\frac{2\,(bx+a)e}{b^3} + \frac{7\,b^6de^2 - 5\,ab^5e^3}{b^8e^2}\right) - \frac{(8\,b^2d^2 - 8\,abde + 3\,a^2e^2)\log\left(\left|-\sqrt{be}\sqrt{bx+a} + \sqrt{b^2d + (bx+a)be - abe}\right|\right)}{\sqrt{be}b^2}\right)b}{|b|}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 + (
7*b^6*d*e^2 - 5*a*b^5*e^3)/(b^8*e^2)) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)
*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/(
sqrt(b*e)*b^2))*b/abs(b)
```

Mupad [B] (verification not implemented)

Time = 32.86 (sec) , antiderivative size = 893, normalized size of antiderivative = 7.32

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$$

$$= \frac{\frac{(40\,b\,d^2+40\,a\,e\,d)\,(\sqrt{a+b\,x}-\sqrt{a})}{e^2\,(\sqrt{d+e\,x}-\sqrt{d})} - \frac{160\,\sqrt{a}\,d^{3/2}\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e\,(\sqrt{d+e\,x}-\sqrt{d})^2} + \frac{(40\,b\,d^2+40\,a\,e\,d)\,(\sqrt{a+b\,x}-\sqrt{a})^3}{b\,e\,(\sqrt{d+e\,x}-\sqrt{d})^3}}{\frac{(\sqrt{a+b\,x}-\sqrt{a})^4}{(\sqrt{d+e\,x}-\sqrt{d})^4} + \frac{b^2}{e^2} - \frac{2\,b\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e\,(\sqrt{d+e\,x}-\sqrt{d})^2}}$$

$$- \frac{\frac{(\sqrt{a+b\,x}-\sqrt{a})\,(12\,a^2\,b\,e^2+8\,a\,b^2\,d\,e+12\,b^3\,d^2)}{e^4\,(\sqrt{d+e\,x}-\sqrt{d})} - \frac{(\sqrt{a+b\,x}-\sqrt{a})^3\,(44\,a^2\,e^2+200\,a\,b\,d\,e+44\,b^2\,d^2)}{e^3\,(\sqrt{d+e\,x}-\sqrt{d})^3} + \frac{(\sqrt{a+b\,x}-\sqrt{a})^7\,(12\,a^2\,e^2+8\,a\,b\,d\,e+12\,b}{b^2\,e\,(\sqrt{d+e\,x}-\sqrt{d})^7}}{\frac{(\sqrt{a+b\,x}-\sqrt{a})^8}{(\sqrt{d+e\,x}-\sqrt{d})^8} + \frac{b^4}{e^4} - \frac{4\,b^3\,(\sqrt{a+b\,x}-\sqrt{a})^2}{e^3\,(\sqrt{d+e\,x}-\sqrt{d})^2} + \frac{6\,b^2\,(\sqrt{a+b\,x}-\sqrt{}}{e^2\,(\sqrt{d+e\,x}-\sqrt{}}}$$

$$- \frac{60\,d^2\,\mathrm{atan}\left(\frac{b\,(\sqrt{d+e\,x}-\sqrt{d})}{\sqrt{-b\,e}\,(\sqrt{a+b\,x}-\sqrt{a})}\right)}{\sqrt{-b\,e}} - \frac{2\,\ln\left(\frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{d+e\,x}-\sqrt{d}} - \sqrt{b}\right)\,(3\,a^2\,e^2+2\,a\,b\,d\,e+3\,b^2\,d^2)}{b^{5/2}\,\sqrt{e}}$$

$$+ \frac{\ln\left(\sqrt{b} + \frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{d+e\,x}-\sqrt{d}}\right)\,(6\,a^2\,e^2+4\,a\,b\,d\,e+6\,b^2\,d^2)}{b^{5/2}\,\sqrt{e}}$$

$$- \frac{40\,d\,\mathrm{atanh}\left(\frac{\sqrt{e}\,(\sqrt{a+b\,x}-\sqrt{a})}{\sqrt{b}\,(\sqrt{d+e\,x}-\sqrt{d})}\right)\,(a\,e+b\,d)}{b^{3/2}\,\sqrt{e}}$$

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2)
- d^(1/2))) - (160*a^(1/2)*d^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d +
e*x)^(1/2) - d^(1/2))^2) + ((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2
))^3)/(b*e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/(
(d + e*x)^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2
)/(e*((d + e*x)^(1/2) - d^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(1/2))*(12*b^
3*d^2 + 12*a^2*b*e^2 + 8*a*b^2*d*e))/(e^4*((d + e*x)^(1/2) - d^(1/2))) - ((
(a + b*x)^(1/2) - a^(1/2))^3*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(e^3*
((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^7*(12*a^2*e^2
 + 12*b^2*d^2 + 8*a*b*d*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) - (((a +
b*x)^(1/2) - a^(1/2))^5*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(b*e^2*((d
 + e*x)^(1/2) - d^(1/2))^5) + (a^(1/2)*d^(1/2)*(256*a*e + 256*b*d)*((a + b*
x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/
2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 - (4*b^3*((a + b*x)
^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) + (6*b^2*((a + b*x
)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (4*b*((a + b*x)
^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) - (60*d^2*atan((b*(
(d + e*x)^(1/2) - d^(1/2)))/((-b*e)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(-
```

```
b*e)^(1/2) - (2*log((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2)
- d^(1/2)) - b^(1/2))*(3*a^2*e^2 + 3*b^2*d^2 + 2*a*b*d*e))/(b^(5/2)*e^(1/2)
) + (log(b^(1/2) + (e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2) -
 d^(1/2)))*(6*a^2*e^2 + 6*b^2*d^2 + 4*a*b*d*e))/(b^(5/2)*e^(1/2)) - (40*d*a
tanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1
/2))))*(a*e + b*d))/(b^(3/2)*e^(1/2))
```

3.846 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 108

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{8(2bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}$$

```
[Out] 8*(-a*e+2*b*d)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(3/2)
/e^(1/2)+6*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(1/2)+8*(b*x+a)^(1/2)*(e*x+
d)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {963, 81, 65, 223, 212}

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{8(2bd-ae)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}} + \frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{2\int \frac{6d(bd-ae)+4e(bd-ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{bd-ae}\\ &= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{(4(2bd-ae))\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{b}\end{aligned}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{b}\sqrt{a+bx}(-4ae(d+ex)+bd(7d+4ex))}{\sqrt{d+ex}} + \frac{4(2b^2d^2-3abde+a^2e^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{e}}\right)}{b^{3/2}(bd-ae)}$$

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x
]
```

```
[Out] (2*((Sqrt[b]*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/Sqrt[d +
 e*x] + (4*(2*b^2*d^2 - 3*a*b*d*e + a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x]
)/(Sqrt[b]*Sqrt[d + e*x])])/Sqrt[e]))/(b^(3/2)*(b*d - a*e))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(88) = 176$.

Time = 0.48 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.06

method	result
default	$-\frac{2\sqrt{bx+a}\left(2\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^3x-6\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abde^2x+4\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex}{2\sqrt{be}}\right.\right.}{}$

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a
*e+b*d)/(b*e)^(1/2))*a^2*e^3*x-6*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*
(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e^2*x+4*ln(1/2*(2*b*e*x+2*((b*x+a)*
(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^2*e*x+2*ln(1/2*(2*b*
e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d*e^2-6
```

```
*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2)
)*a*b*d^2*e+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d
)/(b*e)^(1/2))*b^2*d^3-4*a*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+4*b*d*
e*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^(1/2)*(b*
e)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/b/(b*e)^(1/2)/(a*e-b*
d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.29

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \left[-\frac{2\left((2\,b^2d^3-3\,abd^2e+a^2de^2+(2\,b^2d^2e-3\,abde^2+a^2e^3)x\right)\sqrt{be}\log\left(8\,b^2e\right.}{}\right.$$

$$-\frac{2\left(2\,(2\,b^2d^3-3\,abd^2e+a^2de^2+(2\,b^2d^2e-3\,abde^2+a^2e^3)x\right)\sqrt{-be}\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\,(b^2e^2x^2+abde+(b^2de+abe^2)x)}\right)-}{b^3d^2e-ab^2de^2+(b^3de^2-ab^2e^3)x}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-2*((2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2 + a^
2*e^3)*x)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(
2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a
*b*e^2)*x) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b
*x + a)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x
), -2*(2*(2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2
+ a^2*e^3)*x)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b
*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7
*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b*x + a)*sqrt(e*
x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x)]
```

Sympy [F]

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\left(\frac{4(b^3de^3 - ab^2e^4)(bx+a)}{b^3de^2|b| - ab^2e^3|b|} + \frac{7b^4d^2e^2 - 8ab^3de^3 + 4a^2b^2e^4}{b^3de^2|b| - ab^2e^3|b|}\right)}{\sqrt{b^2d + (bx+a)be - abe}} - \frac{8(2bd - ae)\log\left(\left|-\sqrt{be}\sqrt{bx+a} + \sqrt{b^2d + (bx+a)be - abe}\right|\right)}{\sqrt{be}|b|}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*sqrt(b*x + a)*(4*(b^3*d*e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*e^2*abs(b) - a*
b^2*e^3*abs(b)) + (7*b^4*d^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*e^
2*abs(b) - a*b^2*e^3*abs(b)))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e) - 8*(2*b*
d - a*e)*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*
b*e)))/(sqrt(b*e)*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx = \int \frac{15\,d^2 + 20\,d\,e\,x + 8\,e^2\,x^2}{\sqrt{a+b\,x}\,{(d+e\,x)}^{3/2}}\,dx$$

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.847 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 116

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[Out] 16*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(1/2)/e^(1/2)+2*d
^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(3/2)+4*d*(-2*a*e+3*b*d)*(b*x+a)^(1/2)/
(-a*e+b*d)^2/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {963, 79, 65, 223, 212}

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \frac{16\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}} + \frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)} + \frac{4d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)^2}$$

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```

```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{2\int \frac{3d(7bd-6ae)+12e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx}{3(bd-ae)} \\ &= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + 8\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx\end{aligned}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \frac{2d\sqrt{a+bx}\left(7bd-4ae-\frac{de(a+bx)}{d+ex}\right)}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{arctanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x
]
```

```
[Out] (2*d*Sqrt[a + b*x]*(7*b*d - 4*a*e - (d*e*(a + b*x))/(d + e*x)))/((b*d - a*e
)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d +
e*x])])/(Sqrt[b]*Sqrt[e])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(96) = 192$.

Time = 0.48 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.18

method	result
default	$\frac{2\sqrt{bx+a}\left(4\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)a^2e^4x^2-8\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}+ae+bd}{2\sqrt{be}}\right)abde^3x^2+4\ln\left(\frac{2bex+2\sqrt{(bx+a)(e}{2\sqrt{}}\right.\right.}{}$

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*
e+b*d)/(b*e)^(1/2))*a^2*e^4*x^2-8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)
*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d*e^3*x^2+4*ln(1/2*(2*b*e*x+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^2*e^2*x^2+8*ln(1/
2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*
d*e^3*x-16*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(
```

```
b*e)^(1/2))*a*b*d^2*e^2*x+8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)
^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^3*e*x+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+
d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d^3*e+4*ln(1
/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2
*d^4-4*a*d*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+6*b*d^2*e*x*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+7*b
*d^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)/(a*e-b*d)^2/((b*x+a)*
(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(96) = 192$.

Time = 0.64 (sec) , antiderivative size = 665, normalized size of antiderivative = 5.73

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \left[\frac{2\left(2\left(b^2d^4 - 2\,abd^3e + a^2d^2e^2 + (b^2d^2e^2 - 2\,abde^3 + a^2e^4)x^2 + 2\,(b^2d^3e - 2\,ab\right.\right.}{}\right.$$

$$-\frac{2\left(4\left(b^2d^4 - 2\,abd^3e + a^2d^2e^2 + (b^2d^2e^2 - 2\,abde^3 + a^2e^4)x^2 + 2\,(b^2d^3e - 2\,abd^2e^2 + a^2de^3)x\right)\sqrt{-be}\arctan\left(\right.\right.}{b^3d^4e - 2\,ab^2d^3e^2 + a^2bd^2e^3 + (b^3d^2e^3 - 2\,ab^2de^4 + a^2}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [2*(2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a
^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(b*e)*log(8*
b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(
b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + (7*b^2*d^3*e
- 5*a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x
 + d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^
2*d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*
x), -2*(4*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3
 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(-b*e)*a
rctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2
*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7*b^2*d^3*e - 5*a*b*d^2*e^2
 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*d^4
*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^2*d*e^4 + a^2*b
*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*x)]
```

Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(5/2)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(96) = 192$.

Time = 0.35 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.90

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = -\frac{16\,b\log\left(\left|-\sqrt{be}\sqrt{bx+a} + \sqrt{b^2d + (bx+a)be - abe}\right|\right)}{\sqrt{be}|b|} + \frac{2\sqrt{bx+a}\left(\frac{2\,(3\,b^6d^2e^2 - 2\,ab^5de^3)(bx+a)}{b^4d^2e|b| - 2\,ab^3de^2|b| + a^2b^2e^3|b|} + \frac{7\,b^7d^3e - 11\,ab^6d^2e^2 + 4\,a^2b^5de^3}{b^4d^2e|b| - 2\,ab^3de^2|b| + a^2b^2e^3|b|}\right)}{(b^2d + (bx+a)be - abe)^{\frac{3}{2}}}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="giac")
[Out] -16*b*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e
)))/(sqrt(b*e)*abs(b)) + 2*sqrt(b*x + a)*(2*(3*b^6*d^2*e^2 - 2*a*b^5*d*e^3)
*(b*x + a)/(b^4*d^2*e*abs(b) - 2*a*b^3*d*e^2*abs(b) + a^2*b^2*e^3*abs(b)) +
 (7*b^7*d^3*e - 11*a*b^6*d^2*e^2 + 4*a^2*b^5*d*e^3)/(b^4*d^2*e*abs(b) - 2*a
*b^3*d*e^2*abs(b) + a^2*b^2*e^3*abs(b)))/(b^2*d + (b*x + a)*b*e - a*b*e)^(3
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx = \int \frac{15\,d^2 + 20\,d\,e\,x + 8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)),x)
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.848 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{16(23b^2d^2-35abde+15a^2e^2)\sqrt{a+bx}}{15(bd-ae)^3\sqrt{d+ex}}$$

```
[Out] 6/5*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(5/2)+8/15*d*(-5*a*e+8*b*d)*(b*x+a
)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(3/2)+16/15*(15*a^2*e^2-35*a*b*d*e+23*b^2*d^2)
*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {963, 79, 37}

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{16\sqrt{a+bx}(15a^2e^2-35abde+23b^2d^2)}{15\sqrt{d+ex}(bd-ae)^3} + \frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)} + \frac{8d\sqrt{a+bx}(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}}+\frac{2\int\frac{6d(6bd-5ae)+20e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx}{5(bd-ae)}\\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}}+\frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}}\\
&\quad+\frac{(8(23b^2d^2-35abde+15a^2e^2))\int\frac{1}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx}{15(bd-ae)^2}\\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}}+\frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}}+\frac{16(23b^2d^2-35abde+15a^2e^2)\sqrt{a+bx}}{15(bd-ae)^3\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{2\sqrt{a+bx}\Big(225b^2d^2-300abde+120a^2e^2+\frac{9d^2e^2(a+bx)^2}{(d+ex)^2}-\frac{50bd^2e(a+bx)}{d+ex}+\frac{20ade^2(}{d+}$$
$$15(bd-ae)^3\sqrt{d+ex}$$

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(225*b^2*d^2 - 300*a*b*d*e + 120*a^2*e^2 + (9*d^2*e^2*(a +
 b*x)^2)/(d + e*x)^2 - (50*b*d^2*e*(a + b*x))/(d + e*x) + (20*a*d*e^2*(a +
b*x))/(d + e*x)))/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result	si
default	$-\frac{2\sqrt{bx+a}\,(120a^2e^4x^2-280abd\,e^3x^2+184b^2d^2e^2x^2+260a^2d\,e^3x-612ab\,d^2e^2x+400b^2d^3ex+149a^2d^2e^2-350ab\,d^3e+225b^2d^4)}{15(ex+d)^{\frac{5}{2}}\,(ae-bd)^3}$	12
gosper	$-\frac{2\sqrt{bx+a}\,(120a^2e^4x^2-280abd\,e^3x^2+184b^2d^2e^2x^2+260a^2d\,e^3x-612ab\,d^2e^2x+400b^2d^3ex+149a^2d^2e^2-350ab\,d^3e+225b^2d^4)}{15(ex+d)^{\frac{5}{2}}\,(a^3e^3-3a^2bd\,e^2+3a\,b^2d^2e-b^3d^3)}$	15

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2+
260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b*d
^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a*e-b*d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(115) = 230$.

Time = 0.90 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.20

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{2\,(225\,b^2d^4-350\,abd^3e+149\,a^2d^2e^2+8\,(23\,b^2d^2e^2-3}{15\,(b^3d^6-3\,ab^2d^5e+3\,a^2bd^4e^2-a^3d^3e^3+(b^3d^3e^3-3\,ab^2d^2e^4+3\,a^2bde^5-}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(225*b^2*d^4 - 350*a*b*d^3*e + 149*a^2*d^2*e^2 + 8*(23*b^2*d^2*e^2 - 3
5*a*b*d*e^3 + 15*a^2*e^4)*x^2 + 4*(100*b^2*d^3*e - 153*a*b*d^2*e^2 + 65*a^2
```

```
*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d
^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3
*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d*e^5)
*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x)
```

Sympy [F]

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx=\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(7/2)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e-b*d)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(115) = 230$.

Time = 0.38 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.70

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx=\frac{2\left(4\,(bx+a)\left(\frac{2\,(23\,b^8d^2e^4-35\,ab^7de^5+15\,a^2b^6e^6)(bx+a)}{b^5d^3e^2|b|-3\,ab^4d^2e^3|b|+3\,a^2b^3de^4|b|-a^3b^2e^5|b|}+\frac{5\,(20\,b^9d^3e^3-49\,ab^8d^2e^4+41\,a^2b^7d}{b^5d^3e^2|b|-3\,ab^4d^2e^3|b|+3\,a^2b^3de^4}\right.\right.}{15\,(b^2d+(bx+a}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="giac")
[Out] 2/15*(4*(b*x + a)*(2*(23*b^8*d^2*e^4 - 35*a*b^7*d*e^5 + 15*a^2*b^6*e^6)*(b*
x + a)/(b^5*d^3*e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) + 3*a^2*b^3*d*e^4*abs(b
```

```
) - a^3*b^2*e^5*abs(b)) + 5*(20*b^9*d^3*e^3 - 49*a*b^8*d^2*e^4 + 41*a^2*b^7
*d*e^5 - 12*a^3*b^6*e^6)/(b^5*d^3*e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) + 3*a
^2*b^3*d*e^4*abs(b) - a^3*b^2*e^5*abs(b))) + 15*(15*b^10*d^4*e^2 - 50*a*b^9
*d^3*e^3 + 63*a^2*b^8*d^2*e^4 - 36*a^3*b^7*d*e^5 + 8*a^4*b^6*e^6)/(b^5*d^3*
e^2*abs(b) - 3*a*b^4*d^2*e^3*abs(b) + 3*a^2*b^3*d*e^4*abs(b) - a^3*b^2*e^5*
abs(b)))*sqrt(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.02

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = -\frac{\sqrt{d+e\,x}\left(\frac{x^2\,\left(240\,a^3\,e^4-40\,a^2\,b\,d\,e^3-856\,a\,b^2\,d^2\,e^2+800\,b^3\,d^3\,e\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{x\,\left(520\,a^3\,d\,e^3-926\,a^2\,b\,d^2\,e^2+100\,a\,b^2\,d^3\,e+450\,b^3\,d^4\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{2\,a\,d^2\,\left(149\,a^2 \cdots\right.}{1 \cdots}\right.}{x^3\,\sqrt{a+b\,x}+\frac{d^3\,\sqrt{a+b\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{a+b\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{a+b\,x}}{e^2}}$$

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] -((d + e*x)^(1/2)*((x^2*(240*a^3*e^4 + 800*b^3*d^3*e - 856*a*b^2*d^2*e^2 -
40*a^2*b*d*e^3))/(15*e^3*(a*e - b*d)^3) + (x*(450*b^3*d^4 + 520*a^3*d*e^3 -
 926*a^2*b*d^2*e^2 + 100*a*b^2*d^3*e))/(15*e^3*(a*e - b*d)^3) + (2*a*d^2*(1
49*a^2*e^2 + 225*b^2*d^2 - 350*a*b*d*e))/(15*e^3*(a*e - b*d)^3) + (16*b*x^3
*(15*a^2*e^2 + 23*b^2*d^2 - 35*a*b*d*e))/(15*e*(a*e - b*d)^3)))/(x^3*(a + b
*x)^(1/2) + (d^3*(a + b*x)^(1/2))/e^3 + (3*d*x^2*(a + b*x)^(1/2))/e + (3*d^
2*x*(a + b*x)^(1/2))/e^2)
```

3.849 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 189

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx=\frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}}+\frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}}+\frac{16(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^3(d+ex)^{3/2}}+\frac{32b(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^4\sqrt{d+ex}}$$

```
[Out] 6/7*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(7/2)+4/35*d*(-14*a*e+23*b*d)*(b*x
+a)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(5/2)+16/105*(35*a^2*e^2-84*a*b*d*e+58*b^2*d
^2)*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(3/2)+32/105*b*(35*a^2*e^2-84*a*b*d*
e+58*b^2*d^2)*(b*x+a)^(1/2)/(-a*e+b*d)^4/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {963, 79, 47, 37}

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx=\frac{32b\sqrt{a+bx}(35a^2e^2-84abde+58b^2d^2)}{105\sqrt{d+ex}(bd-ae)^4}+\frac{16\sqrt{a+bx}(35a^2e^2-84abde+58b^2d^2)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a+bx}(23bd-14ae)}{35(d+ex)^{5/2}(bd-ae)^2}$$

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx}{7(bd-ae)} \\
&= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} \\
&\quad + \frac{(8(58b^2d^2-84abde+35a^2e^2))\int \frac{1}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx}{35(bd-ae)^2} \\
&= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} \\
&\quad + \frac{16(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^3(d+ex)^{3/2}} \\
&\quad + \frac{(16b(58b^2d^2-84abde+35a^2e^2))\int \frac{1}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx}{105(bd-ae)^3} \\
&= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} \\
&\quad + \frac{16(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^3(d+ex)^{3/2}} \\
&\quad + \frac{32b(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^4\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{2\sqrt{a+bx}\Big(1575b^3d^2-2100ab^2de+840a^2be^2-\frac{45d^2e^3(a+bx)^3}{(d+ex)^3}+\frac{273bd^2e^2(a+bx)^2}{(d+ex)^2}-}{105(bd-ae)^4\sqrt{d+e}}$$

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(1575*b^3*d^2 - 2100*a*b^2*d*e + 840*a^2*b*e^2 - (45*d^2*e
^3*(a + b*x)^3)/(d + e*x)^3 + (273*b*d^2*e^2*(a + b*x)^2)/(d + e*x)^2 - (84
*a*d*e^3*(a + b*x)^2)/(d + e*x)^2 - (875*b^2*d^2*e*(a + b*x))/(d + e*x) + (
840*a*b*d*e^2*(a + b*x))/(d + e*x) - (280*a^2*e^3*(a + b*x))/(d + e*x)))/(1
05*(b*d - a*e)^4*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

method	result
default	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3d\,e^4}{105(ex+d)^{\frac{7}{2}}(ae-bd)^4}$
gosper	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3d\,e^4}{105(ex+d)^{\frac{7}{2}}(e^4a^4-4bd\,e^3a^3+6b^2d^2e^2a^2-4}$

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*e
^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b^3
*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3850
*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b^3*d
^5)/(e*x+d)^(7/2)/(a*e-b*d)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(165) = 330$.

Time = 1.66 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.58

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{2\,(1575\,b^3d^5-2975\,ab^2d^4e+1953\,a^2bd^3e^2-40}{105\,(b^4d^8-4\,ab^3d^7e+6\,a^2b^2d^6e^2-4\,a^3bd^5e^3+a^4d^4e^4+(b^4d^4e^4-4\,ab^3d^3e^5}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/105*(1575*b^3*d^5 - 2975*a*b^2*d^4*e + 1953*a^2*b*d^3*e^2 - 409*a^3*d^2*e
^3 + 16*(58*b^3*d^2*e^3 - 84*a*b^2*d*e^4 + 35*a^2*b*e^5)*x^3 + 8*(406*b^3*d
^3*e^2 - 646*a*b^2*d^2*e^3 + 329*a^2*b*d*e^4 - 35*a^3*e^5)*x^2 + 2*(1925*b^
3*d^4*e - 3332*a*b^2*d^3*e^2 + 1945*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*sqrt(
b*x + a)*sqrt(e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 - 4*a^3
*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e
^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4 + 6*a^
2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 - 4*a*b^3
*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2 + 4*(b^4*
d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a^4*d^3*e^5
)*x)
```

Sympy [F]

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{9/2}}\,dx = \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(9/2)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(a*e-b*d)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(165) = 330$.

Time = 0.44 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.96

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{9/2}}\,dx = \frac{2\left(2\left(4\,(bx + a)\left(\frac{2\,(58\,b^{10}d^2e^6 - 84\,ab^9de^7 + 35\,a^2b^8e^8)\,(bx+a)}{b^6d^4e^3|b| - 4\,ab^5d^3e^4|b| + 6\,a^2b^4d^2e^5|b| - 4\,a^3b^3de^6|b| + a^4b^2e^7|b|} + \frac{7\,(58\,b^{11}d^3e}{b^6d^4e^3|b| - 4\,ab^5}\right.\right.\right.}{}$$

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="giac")
[Out] 2/105*(2*(4*(b*x + a)*(2*(58*b^10*d^2*e^6 - 84*a*b^9*d*e^7 + 35*a^2*b^8*e^8
)*(b*x + a)/(b^6*d^4*e^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*e^
5*abs(b) - 4*a^3*b^3*d*e^6*abs(b) + a^4*b^2*e^7*abs(b)) + 7*(58*b^11*d^3*e^
5 - 142*a*b^10*d^2*e^6 + 119*a^2*b^9*d*e^7 - 35*a^3*b^8*e^8)/(b^6*d^4*e^3*a
bs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*e^5*abs(b) - 4*a^3*b^3*d*e^6
*abs(b) + a^4*b^2*e^7*abs(b))) + 35*(55*b^12*d^4*e^4 - 188*a*b^11*d^3*e^5 +
 243*a^2*b^10*d^2*e^6 - 142*a^3*b^9*d*e^7 + 32*a^4*b^8*e^8)/(b^6*d^4*e^3*ab
```

```
s(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*e^5*abs(b) - 4*a^3*b^3*d*e^6*
abs(b) + a^4*b^2*e^7*abs(b)))*(b*x + a) + 105*(15*b^13*d^5*e^3 - 65*a*b^12*
d^4*e^4 + 113*a^2*b^11*d^3*e^5 - 99*a^3*b^10*d^2*e^6 + 44*a^4*b^9*d*e^7 - 8
*a^5*b^8*e^8)/(b^6*d^4*e^3*abs(b) - 4*a*b^5*d^3*e^4*abs(b) + 6*a^2*b^4*d^2*
e^5*abs(b) - 4*a^3*b^3*d*e^6*abs(b) + a^4*b^2*e^7*abs(b)))*sqrt(b*x + a)/(b
^2*d + (b*x + a)*b*e - a*b*e)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.06

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{\sqrt{d+e\,x}\,\left(\frac{-818\,a^4\,d^2\,e^3+3906\,a^3\,b\,d^3\,e^2-5950\,a^2\,b^2\,d^4\,e+3150\,a\,b^3\,d^5}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{x\,\left(-1288\,a^4\,d\,e^4+6962\,a^3\right.}{}\right.}{}$$

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((d + e*x)^(1/2)*((3150*a*b^3*d^5 - 818*a^4*d^2*e^3 - 5950*a^2*b^2*d^4*e +
3906*a^3*b*d^3*e^2)/(105*e^4*(a*e - b*d)^4) + (x*(3150*b^4*d^5 - 1288*a^4*d
*e^4 + 6962*a^3*b*d^2*e^3 - 9422*a^2*b^2*d^3*e^2 + 1750*a*b^3*d^4*e))/(105*
e^4*(a*e - b*d)^4) - (x^2*(560*a^4*e^5 - 7700*b^4*d^4*e + 6832*a*b^3*d^3*e^
2 + 2556*a^2*b^2*d^2*e^3 - 3976*a^3*b*d*e^4))/(105*e^4*(a*e - b*d)^4) + (32
*b^2*x^4*(35*a^2*e^2 + 58*b^2*d^2 - 84*a*b*d*e))/(105*e*(a*e - b*d)^4) + (1
6*b*x^3*(35*a^3*e^3 + 406*b^3*d^3 - 530*a*b^2*d^2*e + 161*a^2*b*d*e^2))/(10
5*e^2*(a*e - b*d)^4)))/(x^4*(a + b*x)^(1/2) + (d^4*(a + b*x)^(1/2))/e^4 + (
6*d^2*x^2*(a + b*x)^(1/2))/e^2 + (4*d*x^3*(a + b*x)^(1/2))/e + (4*d^3*x*(a
+ b*x)^(1/2))/e^3)
```

3.850 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 417

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} - \frac{2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)-2*
arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/
(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(e*(-b*e+2*c*d)+(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-2*arctanh((e*x+d)^(1/2)*(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^
(1/2))/c/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {923, 65, 223, 212, 6860, 95, 214}

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}} - \frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}} + \frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 923

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}+\frac{cd^2-ae^2+e(2cd-be)x}{c\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\right)dx\\
&= \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\sqrt{f+gx}(a+bx+cx^2)}\,dx}{c}+\frac{e^2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}\\
&= \frac{\int \left(\frac{e(2cd-be)+\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{e(2cd-be)-\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{c}
\end{aligned}$$

$$= \frac{(2e)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{c} + \frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)x^2}dx,x\right.}{c} + \frac{\left(2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}dx,x\right.}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} - \frac{2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{c\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}dx = \frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\sqrt{cd^2+e(-bd+ae)}\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2ae}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}}}{}$$

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e
)]*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*
```

```
f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g
]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*
a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c*d + (
b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan[(Sqrt[2]*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2
- 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(S
qrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sq
rt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x]
)/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 11685 vs. $2(361) = 722$.

Time = 0.65 (sec) , antiderivative size = 11686, normalized size of antiderivative = 28.02

method	result	size
default	Expression too large to display	11686

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**(3/2)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)), x)
```

3.851 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 285

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/
2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+2*arctanh(
(e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {923, 95, 214}

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$
$$= \frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 923

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e+\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e-\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right) dx$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d}}{\sqrt{f}}\right.$$
$$+ \left(2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d}}{\sqrt{f}}\right.$$

$$= -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

Mathematica [A] (verified)

Time = 10.70 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2\left(-\frac{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(-((Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b -
 Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c]
)*e]*Sqrt[f + g*x])])/Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) + (Sqrt[-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a
*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g
*x])])/Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]))/Sqrt[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5481 vs. 2(241) = 482.

Time = 0.67 (sec) , antiderivative size = 5482, normalized size of antiderivative = 19.24

method	result	size
default	Expression too large to display	5482

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. 2(241) = 482.

Time = 27.87 (sec) , antiderivative size = 4471, normalized size of antiderivative = 15.69

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

[Out] 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 + sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g

```
^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) + 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b
*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*
b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
- (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2
*g^2 - 2*(b*d*e - a*e^2)*f^2 + sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)
*d*f*g - ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3
*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d
*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*
b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (
b*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*
a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a
*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g
 - (b*d^2 - 4*a*d*e)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c
)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2
 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g +
d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*
a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a
^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((b
^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^
2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^
2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*
g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f
*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a
*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g - ((b^3*c -
4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c
)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
(b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c -
8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((
b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e
^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c
^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f
*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*
f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e
)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^
2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*
b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2
- 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2
```

```
)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Hanged}$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
[Out] \text{Hanged}
```

3.852 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 287

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{4c\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{4c\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

```
[Out] -4*c*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+4*c*arct
anh((e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {925, 95, 214}

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{4c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}} - \frac{4c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{2c}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx\\
&= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{(4c)\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}}\\
&\quad - \frac{(4c)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}}\\
&= -\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\\
&\quad + \frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{\sqrt{2}\sqrt{cd^2+e(-bd+ae)}\left(\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}\sqrt{d+ex}}\right)}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2aeg}} + \frac{\left(2cd+\left(-b+\right.\right.}{}\right.}{\sqrt{b^2-4ac}\left(-cd^2+e(bd-ae)\right)}$$

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])
*e)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d
*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*
g]*Sqrt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g -
 Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g] + ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*A
rcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f +
b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sq
rt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt
[b^2 - 4*a*c]*d*g - 2*a*e*g]))/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e)
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5506 vs. $2(243) = 486$.

Time = 0.66 (sec) , antiderivative size = 5507, normalized size of antiderivative = 19.19

method	result	size
default	Expression too large to display	5507

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19727 vs. $2(243) = 486$.

Time = 118.80 (sec) , antiderivative size = 19727, normalized size of antiderivative = 68.74

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Hanged}$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),x)
[Out] \text{Hanged}
```

3.853 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+bx+cx^2)} dx$

Optimal result

Integrand size = 31, antiderivative size = 429

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{8c^2\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{8c^2\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

```
[Out] 4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2
)^(1/2)/(e*x+d)^(1/2)-4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(-4*a*c+b^2)^(1/2)/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)^(1/2)-8*c^2*arctanh((e*x+d)^(1/2)*(2*
c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(2
*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+8*c^2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {925, 98, 95, 214}

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = -\frac{8c^2\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}} + \frac{8c^2\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}} + \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2c}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\right) dx \\
&= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} \\
&\quad - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} \\
&\quad + \frac{(4c^2)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)} - \frac{(4c^2)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)}
\end{aligned}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} + \frac{(8c^2)\,\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)} - \frac{(8c^2)\,\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{8c^2\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{8c^2\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}$$

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \frac{2e^2\sqrt{f+gx}}{(cd^2+e(-bd+ae))\,(-ef+dg)\sqrt{d+ex}} + \frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4acd}}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}ef+bdg-\sqrt{b^2-4ac}dg-2ae}} + \frac{\sqrt{2}\left(-2c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2+2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f}}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2}}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef-\sqrt{b^2-4ac}ef+bdg+\sqrt{b^2-4ac}dg-2c}}$$

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*Sqrt[d + e
*x]) + (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
```

```
qrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[f + g*x])/(Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^
2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-
(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g -
 Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^
2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^
2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/
(Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*f + b*e*f -
 Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 47350 vs. 2(369) = 738.

Time = 0.68 (sec) , antiderivative size = 47351, normalized size of antiderivative = 110.38

method	result	size
default	Expression too large to display	47351

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{(cx^2+bx+a)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,{(d+e\,x)}^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)), x)
```

3.854 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 532

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2f^2-3defg+d^2g^2)+2ceg(5b^2e^2g^2 \cdots}{64c^3e^4} + \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} - \frac{\left(4ce(2cd-be)\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right)-2\left(4c^2d^2-\frac{b^2e^2}{2}-2ce(bd-ae)\right)g(5 \cdots}{128c^{7/2}e^5} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

```
[Out] 1/24*g^2*(-5*b*e*g-14*c*d*g+24*c*e*f)*(c*x^2+b*x+a)^(3/2)/c^2/e^2+1/4*g^3*(
e*x+d)*(c*x^2+b*x+a)^(3/2)/c/e^2-1/128*(4*c*e*(-b*e+2*c*d)*(16*c^2*e^2*f^3+
5*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-2*b*d*g+6*b*e*f))-2*(4*c^2*d^2-1/2*b^2*e^2-2
*c*e*(-a*e+b*d))*g*(5*b^2*e^2*g^2-4*c*e*g*(a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d
^2*g^2-3*d*e*f*g+3*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(7/2)/e^5+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^5+1/6
4*(5*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-2*b*d*g+6*b*e*f)+
16*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(5*b^2*e^2*g^2-4*c*e*g*(
a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+b*x+
a)^(1/2)/c^3/e^4
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(2cd-be)\left(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3\right)-2g\left(-2c\right.\right.}{128c^{7/2}}$$
$$+\frac{(ef-dg)^3\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}$$
$$+\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2(d^2g^2-3defg+3e^2f^2)\right)-4bce^2g^2(\right.}{64c^3e^4}$$
$$+\frac{g^2(a+bx+cx^2)^{3/2}(-5beg-14cdg+24cef)}{24c^2e^2}+\frac{g^3(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}$$

```
[In] Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}e\left(8ce^2f^3-d(3bd+2ae)g^3\right)-eg\left(e(4bd+ae)g^2-3c\left(4e^2f^2-d^2g^2\right)\right)x+\frac{1}{2}e^2g^2(24cef-14cdg-5beg)x^2\right)}{d+ex}\,dx}{4ce^3}$$

$$= \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\left(\frac{3}{4}e^3\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right)+\frac{3}{4}e^3g\left(5b^2e^2g^2-4ceg(6bef-2bdg+aeg)+16c^2\left(3e^2f^2-3defg+d^2g^2\right)\right)x\right)\sqrt{a}}{d+ex}}{12c^2e^5}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+d^2g^2\right)+2ce\right.}{64c^3e^4} + \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} - \int \frac{\frac{3}{8}e^3\left(4ce(bd-2ae)\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right)-d\left(4bcd-b^2e-4ace\right)g\left(5b^2e^2g^2-4ceg(6bef-2bdg+aeg)+16c^2\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+d^2g^2\right)+2ce\right.}{64c^3e^4} + \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)^3\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^5} - \frac{\left(4ce(2cd-be)\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right)-2\left(4c^2d^2-\frac{b^2e^2}{2}-2ce(bd-\right.\right.}{128c^3}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+d^2g^2\right)+2ce\right.}{64c^3e^4} + \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)(ef-dg)^3\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^5} - \frac{\left(4ce(2cd-be)\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right)-2\left(4c^2d^2-\frac{b^2e^2}{2}-2ce(bd-\right.\right.}{}$$

$$= \frac{(5b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(6bef-2bdg+aeg) + 16bc^2eg(3e^2f^2-3defg+d^2g^2) + 2ceg}{64c^3e^4}$$

$$+ \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

$$- \frac{\left(4ce(2cd-be)\left(16c^2e^2f^3+5b^2deg^3-4cdg^2(6bef-2bdg+aeg)\right) - 2\left(4c^2d^2 - \frac{b^2e^2}{2} - 2ce(bd-a\right.\right.}{128}$$

$$+ \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$= \frac{\frac{2e\sqrt{a+x(b+cx)}(15b^3e^3g^3-2bce^2g^2(26aeg+b(36ef-12dg+5egx))+16c^3(-12d^3g^3+6d^2eg^2(6f+gx)-2de^2g(18f^2+9fgx+2g^2x^2)+3e^3(4f^3+6f^2g}{c^3}}{}$$

```
[In] Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 - 2*b*c*e^2*g^2*(26*a*e*g + b*(
36*e*f - 12*d*g + 5*e*g*x)) + 16*c^3*(-12*d^3*g^3 + 6*d^2*e*g^2*(6*f + g*x)
 - 2*d*e^2*g*(18*f^2 + 9*f*g*x + 2*g^2*x^2) + 3*e^3*(4*f^3 + 6*f^2*g*x + 4*
f*g^2*x^2 + g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-8*d*g + 3*e*(8*f + g*x)) + b*(6*
d^2*g^2 - 2*d*e*g*(9*f + g*x) + e^2*(18*f^2 + 6*f*g*x + g^2*x^2)))))/c^3 -
768*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sqrt[c]*(d + e*
x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(-5*b^4*
e^4*g^3 + 128*c^4*d*(-(e*f) + d*g)^3 + 8*b^2*c*e^3*g^2*(3*b*e*f - b*d*g + 3
*a*e*g) - 16*c^2*e^2*g*(a^2*e^2*g^2 + 2*a*b*e*g*(3*e*f - d*g) + b^2*(3*e^2*
f^2 - 3*d*e*f*g + d^2*g^2)) + 64*c^3*e*(b*(e*f - d*g)^3 + a*e*g*(3*e^2*f^2
- 3*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/
c^(7/2))/(384*e^5)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.65

method	result
default	$\frac{(-d^3g^3+3d^2efg^2-3de^2f^2g+e^3f^3)\left(\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+cd^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-\cdots}{}}\right.}{2e\sqrt{c}}\right.}{}$
risch	$-\frac{(-48g^3c^3e^3x^3-8bc^2e^3g^3x^2+64c^3de^2g^3x^2-192c^3e^3fg^2x^2-24ac^2e^3g^3x+10b^2ce^3g^3x+16bc^2de^2g^3x-48bc^2e^3fg^2x-96c^3d^2eg\cdots}{}$

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(((x+d/e)^2*c+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2
*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e)))+g/e^3*(d^2*g^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e^2*g^2*
(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/
4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1
/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+3*e^2*
f^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-3*d*e*f*g*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
+(-d*e*g^2+3*e^2*f*g)*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{{\left(f+g\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x} \,d x$$

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.855 $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2(a+bx+cx^2)^{3/2}}{3ce} + \frac{\left((8c^2d^2-b^2e^2-4ce(bd-ae))\,g(4cef-2cdg-beg)-4ce(2cd-be)\,(2cef^2-bdg^2)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}e^4} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/16*((8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*
g*(-b*e*g-2*c*d*g+4*c*e*f)-4*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*c*e*f^2))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^4+(-d*g+e*f)^2*arctan
h(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4-1/8*(b^2*e^2*g^2-8*c^2*(-d*g+e*f)^2-2*b*
c*e*g*(-d*g+2*e*f)-2*c*e*g*(-b*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(1/2)/
c^2/e^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g(-4ce(bd-ae)-b^2e^2+8c^2d^2)\left(-beg-2cdg+4cef\right)-4ce(2cd-be)\left(2cef^2-\right.\right.}{16c^{5/2}e^4}$$
$$+\frac{(ef-dg)^2\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4}$$
$$-\frac{\sqrt{a+bx+cx^2}\left(b^2e^2g^2-2cegx(-beg-2cdg+4cef)-2bceg(2ef-dg)-8c^2(ef-dg)^2\right)}{8c^2e^3}$$
$$+\frac{g^2(a+bx+cx^2)^{3/2}}{3ce}$$

```
[In] Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*
g*(4*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(c^2*e^3) + (g^2*(a
 + b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e)
)*g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2)
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4)
+ (Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^2(a+bx+cx^2)^{3/2}}{3ce} + \frac{\int \frac{\left(\frac{3}{2}e(2cef^2-bdg^2)+\frac{3}{2}eg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{3ce^2}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} - \frac{\int \frac{-\frac{3}{4}e\left(d\left(4bcd-b^2e-4ace\right)g(4cef-2cdg-beg)-4ce(bd-2ae)\left(2cef^2-bdg^2\right)\right)-\frac{3}{4}e\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-\right.}{(d+ex)\sqrt{a+bx+cx^2}}}{12c^2e^4}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^4} + \frac{\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)\int \frac{1}{\sqrt{a+bx+c}}}{16c^2e^4}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} - \frac{\left(2\left(cd^2-bde+ae^2\right)(ef-dg)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^4} + \frac{\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)\text{Subst}\left(\int\right.}{8c^2e^4}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} + \frac{\left(\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)g(4cef-2cdg-beg)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)\tanh^{-1}\left(\right.}{16c^{5/2}e^4} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$=\frac{\frac{2e\sqrt{a+x(b+cx)}(-3b^2e^2g^2+2ceg(4aeg+b(6ef-3dg+egx))+4c^2(6d^2g^2-3deg(4f+gx)+2e^2(3f^2+3fgx+g^2x^2)))}{c^2}+96\sqrt{-cd^2+bde-ae}}{}$$

[In] Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]

[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(-3*b^2*e^2*g^2 + 2*c*e*g*(4*a*e*g + b*(6*e*f - 3*d*g + e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(4*f + g*x) + 2*e^2*(3*f^2 + 3*f*g*x + g^2*x^2))))/c^2 + 96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2 *ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b* d - a*e)]] + (3*(-(b^3*e^3*g^2) + 16*c^3*d*(e*f - d*g)^2 + 2*b*c*e^2*g*(2*b *e*f - b*d*g + 2*a*e*g) - 8*c^2*e*(b*(e*f - d*g)^2 + a*e*g*(2*e*f - d*g))* Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(48*e^4)

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(8c^2e^2g^2x^2+2bc\,e^2g^2x-12c^2de\,g^2x+24c^2e^2fgx+8ac\,e^2g^2-3b^2e^2g^2-6bcde\,g^2+12bc\,e^2fg+24c^2d^2g^2-48c^2defg+24c^2e^2f^2)\sqrt{c\,x^2+bx}}{24c^2e^3}$
default	$-\frac{g\left(dg\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)-2ef\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)\right.}{e^2}$

[In] int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/24*(8*c^2*e^2*g^2*x^2+2*b*c*e^2*g^2*x-12*c^2*d*e*g^2*x+24*c^2*e^2*f*g*x+8 *a*c*e^2*g^2-3*b^2*e^2*g^2-6*b*c*d*e*g^2+12*b*c*e^2*f*g+24*c^2*d^2*g^2-48*c ^2*d*e*f*g+24*c^2*e^2*f^2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3-1/16/c^2/e^3*(16*(a* d^2*e^2*g^2-2*a*d*e^3*f*g+a*e^4*f^2-b*d^3*e*g^2+2*b*d^2*e^2*f*g-b*d*e^3*f^2 +c*d^4*g^2-2*c*d^3*e*f*g+c*d^2*e^2*f^2)*c^2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c* d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)

```
^(1/2))/(x+d/e))+(4*a*b*c*e^3*g^2+8*a*c^2*d*e^2*g^2-16*a*c^2*e^3*f*g-b^3*e^
3*g^2-2*b^2*c*d*e^2*g^2+4*b^2*c*e^3*f*g-8*b*c^2*d^2*e*g^2+16*b*c^2*d*e^2*f*
g-8*b*c^2*e^3*f^2+16*c^3*d^3*g^2-32*c^3*d^2*e*f*g+16*c^3*d*e^2*f^2)/e*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+g\,x)^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}e^3} + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3}$$

```
[Out] -1/8*(b^2*e^2*g+8*c^2*d*(-d*g+e*f)-4*c*e*(a*e*g-b*d*g+b*e*f))*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^3+(-d*g+e*f)*arctanh(1/2*(b
*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*
e^2-b*d*e+c*d^2)^(1/2)/e^3+1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2+b*x
+a)^(1/2)/c/e^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{828, 857, 635, 212, 738\}$

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}$$
$$+ \frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3}$$
$$+ \frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

```
[In] Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} \\
&\quad - \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)f+4acdeg-bd(4cd-be)g)+\frac{1}{2}\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{4ce^2} \\
&= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} \\
&\quad + \frac{\left((cd^2-bde+ae^2)(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^3} \\
&\quad - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8ce^3} \\
&= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} \\
&\quad - \frac{\left(2(cd^2-bde+ae^2)(ef-dg)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^3} \\
&\quad - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4ce^3} \\
&= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} \\
&\quad - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}e^3} \\
&\quad + \frac{\sqrt{cd^2-bde+ae^2}(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}(beg+2c(2ef-2dg+egx))-8c\sqrt{-cd^2+bde-ae^2}(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e}{\sqrt{-cd^2+}}\right.\right.}{8c^{3/2}e^3}$$

```
[In] Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) -
 8*c*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e*x
) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) + (-(b^2*e^2*
g) + 8*c^2*d*(-(e*f) + d*g) + 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*e^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(2cegx+beg-4cdg+4cef)\sqrt{c\,x^2+bx+a}}{4c\,e^2}+\frac{\frac{\left(4ac\,e^2g-b^2e^2g-4bcdeg+4bc\,e^2f+8c^2d^2g-8c^2def\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{8\left(a\,e^2gd-a\,e^3f-\right.}{}}{}$
default	$\frac{g\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{e}+\frac{(-dg+ef)\left(\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}+\frac{(be-2cd)}{}\right.}{}$

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/8/c/e^2*(
(4*a*c*e^2*g-b^2*e^2*g-4*b*c*d*e*g+4*b*c*e^2*f+8*c^2*d^2*g-8*c^2*d*e*f)/e*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+8*(a*d*e^2*g-a*e^3*f-b*d
^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{(f+gx)\,\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.857 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex} dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/c^
(1/2)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+(c*x^2+b*x+a)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {748, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{a+bx+cx^2}}{e}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
```

```
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$= \frac{e\sqrt{a+x(b+cx)} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+x(b+cx)}}\right) + 2\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{e^2}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
[Out] (e*Sqrt[a + x*(b + c*x)] + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[-(
c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])] +
 2*Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] + (b*e*
ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c])/e^2
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.57

method	result
risch	$\frac{\sqrt{cx^2+bx+a}}{e}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{e\sqrt{c}}-\frac{\left(2e^2a-2bde+2cd^2\right)\ln\left(\frac{\frac{2e^2a-2bde+2cd^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+cd^2}{e^2}}\sqrt{\left(x+\cdots\right.}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{e^2a-bde+cd^2}{e^2}}}}{2e}$
default	$\frac{\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+cd^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+cd^2}{e^2}}\right)}{2e\sqrt{c}}-\frac{\left(e^2a-bde\cdots\right.}{\cdots}}{e}$

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/e+1/2/e*((b*e-2*c*d)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))/c^(1/2)-(2*a*e^2-2*b*d*e+2*c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \left[\frac{4\sqrt{cx^2+bx+a}ce-(2cd-be)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+2\cdots}{\cdots}\right.$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
- (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2),
 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2
 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2), 1/4*(4*
```

```
sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b
*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c*e^2)
, 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arcta
n(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)
*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c*e
^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

3.858 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 228

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} + \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} - \frac{\sqrt{cf^2-bfg+ag^2}\,\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/e/g+arctanh(1/2*
(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(
a*e^2-b*d*e+c*d^2)^(1/2)/e/(-d*g+e*f)-arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x
)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/
g/(-d*g+e*f)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

$= \{909, 738, 212, 857, 635\}$

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \frac{\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)} - \frac{\sqrt{ag^2-bfg+cf^2}\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}+\frac{(cd^2-bde+ae^2)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}\\
&= \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{eg}\\
&\quad -\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)}\\
&\quad -\frac{(cf^2-bfg+ag^2)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)}\\
&= \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)}+\frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{eg}\\
&\quad +\frac{(2(cf^2-bfg+ag^2))\,\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}\\
&= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}+\frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)}\\
&\quad -\frac{\sqrt{cf^2-bfg+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \frac{2\sqrt{-cd^2+e(bd-ae)}g\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)-2e\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+}}{\sqrt{-cf^2+bfg-ag^2}}\right.}{eg(ef-dg)}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

[Out] (2*Sqrt[-(c*d^2) + e*(b*d - a*e)]*g*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - 2*e*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] + Sqrt[c]*(-(e*f) + d*g)*Log[e*g*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(204) = 408.

Time = 0.76 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.95

method	result
default	$\frac{\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^2-bfg+c\,f^2}{g^2}}+\frac{(bg-2cf)\ln\left(\frac{\frac{bg-2cf}{2g}+c\left(x+\frac{f}{g}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^2-bfg+c\,f^2}{g^2}}\right)}{2g\sqrt{c}}-\frac{\left(a\,g^2-\cdots\right.}{\cdots}}{dg-ef}$

[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)

[Out] 1/(d*g-e*f)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx=\text{Timed out}$$

[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{(f+g\,x)\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)), x)
```

3.859 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 490

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2} - \frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)} + \frac{\sqrt{cd^2-bde+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} + \frac{(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)\sqrt{cf^2-bfg+ag^2}} - \frac{e\sqrt{cf^2-bfg+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+
e*f)^2/c^(1/2)+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/g/(-d*g+e*f)^2/c^(1/2)-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))*c^(1/2)/g/(-d*g+e*f)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e*
f)^2+1/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1/2)-e*a
rctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^2+(c*x^2+b*x+a)^(1/2)/(-d*
g+e*f)/(g*x+f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {974, 748, 857, 635, 212, 738, 746}

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \frac{\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2} - \frac{e\sqrt{ag^2-bfg+cf^2}\,\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2} + \frac{(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)\sqrt{ag^2-bfg+cf^2}} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2} + \frac{\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^2} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} \right) dx$$

$$= \frac{e^2 \int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{ef-dg}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{e\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} - \frac{\int \frac{b+2cx}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{(cd^2-bde+ae^2)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} + \frac{(e(2cf-bg))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)^2} - \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)} + \frac{(2cf-bg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)} - \frac{(e(cf^2-bfg+ag^2))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} + \frac{(e(2cf-bg))\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} - \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)} - \frac{(2cf-bg)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)} + \frac{(2e(cf^2-bfg+ag^2))\,\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} + \frac{(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)\sqrt{cf^2-bfg+ag^2}} - \frac{e\sqrt{cf^2-bfg+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$$

$$= \frac{\frac{(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right) - \frac{\sqrt{-cf^2+bfg-ag^2}(2cdf+2aeg-b(ef+dg))}{cf^2+g(-bf+}}{(ef-dg)^2}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (((e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[-(c*d^2) + b*d*e -
a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) +
 e*(b*d - a*e)]] - (Sqrt[-(c*f^2) + b*f*g - a*g^2]*(2*c*d*f + 2*a*e*g - b*(
e*f + d*g))*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f
^2) + g*(b*f - a*g)]])/(c*f^2 + g*(-(b*f) + a*g)))/(e*f - d*g)^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. 2(434) = 868.

Time = 0.90 (sec) , antiderivative size = 1359, normalized size of antiderivative = 2.77

method	result	size
default	Expression too large to display	1359

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e/(d*g-e*f)^2*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2
*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d
*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g/(d*g-e*f)*(-
1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2
-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*
c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln
((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f
/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln(
(1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))))-e/(d*g-e*f)^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(
x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)
/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^
2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2))/(x+f/g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)^2}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)), x)
```

3.860 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 673

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx &= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}-\frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2}\\
&\quad-\frac{e(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3}\\
&\quad+\frac{e^2(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3}\\
&\quad-\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}\\
&\quad+\frac{e\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3}\\
&\quad+\frac{\left(b^2-4ac\right)g\,\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}}\\
&\quad+\frac{e(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^2\sqrt{cf^2-bfg+ag^2}}\\
&\quad-\frac{e^2\sqrt{cf^2-bfg+ag^2}\,\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}\end{aligned}$$

```
[Out] 1/8*(-4*a*c+b^2)*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*e*(-
b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/
c^(1/2)+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
```

```
/2))/g/(-d*g+e*f)^3/c^(1/2)-e*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*c^(1/2)/g/(-d*g+e*f)^2+e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e
*f)^3+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*
g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(1/2
)-e^2*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^3+e*(c*x^2+b*x+a)^(
1/2)/(-d*g+e*f)^2/(g*x+f)-1/4*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {974, 748, 857, 635, 212, 738, 734, 746}

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \frac{g(b^2-4ac)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}} + \frac{e\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3} - \frac{e^2\sqrt{ag^2-bfg+cf^2}\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3} + \frac{e^2(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3} + \frac{e(2cf-bg)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} - \frac{\sqrt{c}e\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} - \frac{e(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3} - \frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)} + \frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
```

```
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
+ ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(ef-dg)^3(d+ex)}-\frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^3}-\frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^2}\right.\\ &\qquad\left.-\frac{e^2g\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)}\right)dx\\ &= \frac{e^3\int\frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^3}-\frac{(e^2g)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^3}-\frac{(eg)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^2}-\frac{g\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{ef-dg}\end{aligned}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e^2\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^3} + \frac{e^2\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^3} - \frac{e\int \frac{b+2cx}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{\left(\left(b^2-4ac\right)g\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{(e(2cd-be))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^3} + \frac{\left(e\left(cd^2-bde+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} + \frac{\left(e^2(2cf-bg)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)^3} - \frac{(ce)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)^2} + \frac{(e(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)^2} - \frac{\left(\left(b^2-4ac\right)g\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{4(ef-dg)\left(cf^2-bfg+ag^2\right)} - \frac{\left(e^2\left(cf^2-bfg+ag^2\right)\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{\left(b^2-4ac\right)g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{(e(2cd-be))\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3} - \frac{\left(2e\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3} + \frac{\left(e^2(2cf-bg)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3} - \frac{(2ce)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} - \frac{(e(2cf-bg))\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} + \frac{\left(2e^2\left(cf^2-bfg+ag^2\right)\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}$$

$$
\begin{aligned}
&= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} \\
&\quad - \frac{e(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3} \\
&\quad + \frac{e^2(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} \\
&\quad + \frac{e\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3} \\
&\quad + \frac{(b^2-4ac)\,g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}} \\
&\quad + \frac{e(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^2\sqrt{cf^2-bfg+ag^2}} \\
&\quad - \frac{e^2\sqrt{cf^2-bfg+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.90 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$$

$$= \frac{\frac{8e(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + \frac{2g(ef-dg)^2(-bf+2ag-2cfx+bgx)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2} + \frac{4e(-2cd+be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + 8e\sqrt{cd^2+}}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-(
b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f)
+ a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])
/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f - b*g)*ArcTa
```

```
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(e*f - d*g)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(603) = 1206.

Time = 0.99 (sec) , antiderivative size = 2512, normalized size of antiderivative = 3.73

method	result	size
default	Expression too large to display	2512

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b*f
*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*
(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g
-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1
/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)
/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/
g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-
2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/
g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*
(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))-e^2/(d*g-e*f)^3*(((x+d/e)^2*c+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((
1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2))/(x+d/e)))+e^2/(d*g-e*f)^3*(((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f
)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
```

```
(1/2))/(x+f/g)))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*
f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)
+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)
-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g
+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g
)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(
a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^
2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)^3}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^3}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1824 vs. $2(603) = 1206$.

Time = 1.07 (sec) , antiderivative size = 1824, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e - b*d*e^2 + a*e^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((e^3*f^3 - 3*d*e^2*f^2*g + 3*
d^2*e*f*g^2 - d^3*g^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(8*c^2*d*e*f^3 -
 4*b*c*e^2*f^3 - 12*b*c*d*e*f^2*g + 3*b^2*e^2*f^2*g + 12*a*c*e^2*f^2*g + 6*
b^2*d*e*f*g^2 - 12*a*b*e^2*f*g^2 - b^2*d^2*g^3 + 4*a*c*d^2*g^3 - 4*a*b*d*e*
g^3 + 8*a^2*e^2*g^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(
c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*e^3*f^5 - 3*c*d*e^2*f^4*g - b*e^3*f
^4*g + 3*c*d^2*e*f^3*g^2 + 3*b*d*e^2*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3*f^2*g^
3 - 3*b*d^2*e*f^2*g^3 - 3*a*d*e^2*f^2*g^3 + b*d^3*f*g^4 + 3*a*d^2*e*f*g^4 -
 a*d^3*g^5)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*c^2*d*f^2*g^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e
*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^3 + 3*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*f*g^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*c*e*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^4 + 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a*b*e*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*
e*f^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g - 20*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*f^3*g + 9*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^2*sqrt(c)*e*f^2*g^2 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a*c^(3/2)*e*f^2*g^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sq
rt(c)*d*f*g^3 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^3 -
 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e*f*g^3 + 8*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*g^4 - 8*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^2*sqrt(c)*e*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
2*e*f^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g - 16*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^2*c*e*f^3*g - 16*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a*c^2*e*f^3*g - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g
^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^2 + 5*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*b^3*e*f^2*g^2 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a*b*c*e*f^2*g^2 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^3 + 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^3 - 9*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^2*e*f*g^3 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*
e*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^4 + 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^2*c*d*g^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^2*b*e*g^4 + 2*b^2*c^(3/2)*e*f^4 + 2*b^2*c^(3/2)*d*f^3*g - 3*b^3*sqrt(c)*e
```

```
*f^3*g - 8*a*b*c^(3/2)*e*f^3*g - b^3*sqrt(c)*d*f^2*g^2 - 4*a*b*c^(3/2)*d*f^
2*g^2 + 15*a*b^2*sqrt(c)*e*f^2*g^2 + 4*a^2*c^(3/2)*e*f^2*g^2 + a*b^2*sqrt(c
)*d*f*g^3 + 4*a^2*c^(3/2)*d*f*g^3 - 20*a^2*b*sqrt(c)*e*f*g^3 + 8*a^3*sqrt(c
)*e*g^4)/((c*e^2*f^4*g - 2*c*d*e*f^3*g^2 - b*e^2*f^3*g^2 + c*d^2*f^2*g^3 +
2*b*d*e*f^2*g^3 + a*e^2*f^2*g^3 - b*d^2*f*g^4 - 2*a*d*e*f*g^4 + a*d^2*g^5)*
((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*sqrt(c)*f + b*f - a*g)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^3\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)), x)
```

3.861 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 933

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^3} - \frac{e^2(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^4} + \frac{e^3(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^4} - \frac{\sqrt{c}e^2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3} + \frac{e^2\sqrt{cd^2-bde+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^4} + \frac{\left(b^2-4ac\right)g(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{16(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}} + \frac{\left(b^2-4ac\right)eg\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}} + \frac{e^2(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} - \frac{e^3\sqrt{cf^2-bfg+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^4}$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^3+1/16*(
-4*a*c+b^2)*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*
f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)+
1/8*(-4*a*c+b^2)*e*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*
f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*
e^2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e
*f)^4/c^(1/2)+1/2*e^3*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/g/(-d*g+e*f)^4/c^(1/2)-e^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
```

```
b*x+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)^3+e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d
)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/
2)/(-d*g+e*f)^4+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)
/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^3/(a*g^2-b*f*g
+c*f^2)^(1/2)-e^3*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^4+e^2*(
c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^3/(g*x+f)-1/8*g*(-b*g+2*c*f)*(b*f-2*a*g+(-b*g
+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2-1
/4*e*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b
*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used

$= \{974, 748, 857, 635, 212, 738, 744, 734, 746\}$

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \frac{(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}g(ef-dg)^4} - \frac{\sqrt{cf^2-bgf+ag^2}\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4} - \frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{g(ef-dg)^3} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{2\sqrt{c}(ef-dg)^4} + \frac{\sqrt{cd^2-bed+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^2}{(ef-dg)^4} + \frac{(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{2g(ef-dg)^3\sqrt{cf^2-bgf+ag^2}} + \frac{\sqrt{cx^2+bx+a}e^2}{(ef-dg)^3(f+gx)} + \frac{(b^2-4ac)\,g\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{8(ef-dg)^2\left(cf^2-bgf+ag^2\right)^{3/2}} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{cx^2+bx+a}e}{4(ef-dg)^2\left(cf^2-bgf+ag^2\right)(f+gx)^2} + \frac{g^2\left(cx^2+bx+a\right)^{3/2}}{3(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)^3} + \frac{(b^2-4ac)\,g(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{16(ef-dg)\left(cf^2-bgf+ag^2\right)^{5/2}} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{cx^2+bx+a}}{8(ef-dg)\left(cf^2-bgf+ag^2\right)^2(f+gx)^2}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
```

```
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
```

```
Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(ef-dg)^4(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^4} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^3} - \frac{e^2g\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)^2} - \frac{e^3g\sqrt{a+bx+cx^2}}{(ef-dg)^4(f+gx)} \right) dx$$

$$= \frac{e^4\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^4} - \frac{(e^3g)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^4} - \frac{(e^2g)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^4}\,dx}{ef-dg}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)^2} + \frac{g^2(a+bx+cx^2)^{3/2}}{3(ef-dg)(cf^2-bfg+ag^2)(f+gx)^3} - \frac{e^3\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^4} + \frac{e^3\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^4} - \frac{e^2\int \frac{b+2cx}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^3} + \frac{((b^2-4ac)eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8(ef-dg)^2(cf^2-bfg+ag^2)} - \frac{(g(2cf-bg))\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{2(ef-dg)(cf^2-bfg+ag^2)}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)^2} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)^2} + \frac{g^2(a+bx+cx^2)^{3/2}}{3(ef-dg)(cf^2-bfg+ag^2)(f+gx)^3} - \frac{(e^2(2cd-be))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^4} + \frac{(e^2(cd^2-bde+ae^2))\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^4} + \frac{(e^3(2cf-bg))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)^4} - \frac{(ce^2)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)^3} + \frac{(e^2(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2g(ef-dg)^3} + \frac{((b^2-4ac)g(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{16(ef-dg)(cf^2-bfg+ag^2)^2} - \frac{((b^2-4ac)eg)\,\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{4(ef-dg)^2(cf^2-bfg+ag^2)} - \frac{(e^3(cf^2-bfg+ag^2))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g(ef-dg)^4}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2}$$
$$- \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2}$$
$$+ \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^3}$$
$$+ \frac{\left(b^2-4ac\right)eg\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}$$
$$- \frac{\left(e^2(2cd-be)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^4}$$
$$- \frac{\left(2e^2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^4}$$
$$+ \frac{\left(e^3(2cf-bg)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^4}$$
$$- \frac{\left(2ce^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}$$
$$- \frac{\left(e^2(2cf-bg)\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}$$
$$- \frac{\left(\left(b^2-4ac\right)g(2cf-bg)\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2}$$
$$+ \frac{\left(2e^3\left(cf^2-bfg+ag^2\right)\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^4}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2}$$
$$- \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2}$$
$$+ \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^3} - \frac{e^2(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^4}$$
$$+ \frac{e^3(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^4} - \frac{\sqrt{c}e^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^3}$$
$$+ \frac{e^2\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^4}$$
$$+ \frac{\left(b^2-4ac\right)g(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{16(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}}$$
$$+ \frac{\left(b^2-4ac\right)eg\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}$$
$$+ \frac{e^2(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g(ef-dg)^3\sqrt{cf^2-bfg+ag^2}}$$
$$- \frac{e^3\sqrt{cf^2-bfg+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^4}$$

Mathematica [A] (verified)

Time = 12.45 (sec) , antiderivative size = 858, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$$

$$= \frac{\frac{48e^2(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + \frac{12eg(ef-dg)^2(-bf+2ag-2cfx+bgx)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2} - \frac{16g^2(-ef+dg)^3(a+x(b+cx))^{3/2}}{(cf^2+g(-bf+ag))(f+gx)^3} + 24e^2\left(\frac{(-2cd+be)a}{\right.}}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] ((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g)
^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-
(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^(3/
2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*d^2
```

```
+ e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*(e*f -
d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
 a*g)]*Sqrt[a + x*(b + c*x)])]))/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (3*g*(2*
c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*f*x + b*(f
 - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*a*c)*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 + g*(-(b*f) +
a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^2 + g
*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. 2(845) = 1690.

Time = 1.16 (sec) , antiderivative size = 3777, normalized size of antiderivative = 4.05

method	result	size
default	Expression too large to display	3777

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] e^3/(d*g-e*f)^4*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)
^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b
*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*
e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g^3/(d*g-e*f
)*(-1/3/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/
2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2
-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b
*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)
/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g
-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1
```

/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*
g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f
*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*
g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f
*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g)))))+1/g*e^2/(d*g-e*f)^3*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(
x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/
2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/
g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/
2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(
a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f
/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2
-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g
))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))))-1/g^2*e/(d*g-e*f)^2*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(
a*g^2-b*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*
g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*
f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g
-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^
2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*
f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*g+c*f
^2)*g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+
1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f
^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))-e^3/(d*g-e*f)^4*(((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*
f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f
/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a

```
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx=\int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^4}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8076 vs. $2(845) = 1690$.

Time = 5.56 (sec) , antiderivative size = 8076, normalized size of antiderivative = 8.66

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
[Out] 2*(c*d^2*e^2 - b*d*e^3 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((e^4*f^4 - 4*d*e^3*f^3*g +
```

6*d^2*e^2*f^2*g^2 - 4*d^3*e*f*g^3 + d^4*g^4)*sqrt(-c*d^2 + b*d*e - a*e^2))
- 1/8*(16*c^3*d*e^2*f^5 - 8*b*c^2*e^3*f^5 - 40*b*c^2*d*e^2*f^4*g + 12*b^2*c
*e^3*f^4*g + 32*a*c^2*e^3*f^4*g + 42*b^2*c*d*e^2*f^3*g^2 - 8*a*c^2*d*e^2*f^
3*g^2 - 5*b^3*e^3*f^3*g^2 - 60*a*b*c*e^3*f^3*g^2 - 8*b^2*c*d^2*e*f^2*g^3 +
32*a*c^2*d^2*e*f^2*g^3 - 15*b^3*d*e^2*f^2*g^3 - 20*a*b*c*d*e^2*f^2*g^3 + 30
*a*b^2*e^3*f^2*g^3 + 40*a^2*c*e^3*f^2*g^3 + 2*b^2*c*d^3*f*g^4 - 8*a*c^2*d^3
*f*g^4 + 5*b^3*d^2*e*f*g^4 - 20*a*b*c*d^2*e*f*g^4 + 20*a*b^2*d*e^2*f*g^4 -
40*a^2*b*e^3*f*g^4 - b^3*d^3*g^5 + 4*a*b*c*d^3*g^5 - 2*a*b^2*d^2*e*g^5 + 8*
a^2*c*d^2*e*g^5 - 8*a^2*b*d*e^2*g^5 + 16*a^3*e^3*g^5)*arctan(-((sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*
e^4*f^8 - 4*c^2*d*e^3*f^7*g - 2*b*c*e^4*f^7*g + 6*c^2*d^2*e^2*f^6*g^2 + 8*b
*c*d*e^3*f^6*g^2 + b^2*e^4*f^6*g^2 + 2*a*c*e^4*f^6*g^2 - 4*c^2*d^3*e*f^5*g^
3 - 12*b*c*d^2*e^2*f^5*g^3 - 4*b^2*d*e^3*f^5*g^3 - 8*a*c*d*e^3*f^5*g^3 - 2*
a*b*e^4*f^5*g^3 + c^2*d^4*f^4*g^4 + 8*b*c*d^3*e*f^4*g^4 + 6*b^2*d^2*e^2*f^4
*g^4 + 12*a*c*d^2*e^2*f^4*g^4 + 8*a*b*d*e^3*f^4*g^4 + a^2*e^4*f^4*g^4 - 2*b
*c*d^4*f^3*g^5 - 4*b^2*d^3*e*f^3*g^5 - 8*a*c*d^3*e*f^3*g^5 - 12*a*b*d^2*e^2
*f^3*g^5 - 4*a^2*d*e^3*f^3*g^5 + b^2*d^4*f^2*g^6 + 2*a*c*d^4*f^2*g^6 + 8*a*
b*d^3*e*f^2*g^6 + 6*a^2*d^2*e^2*f^2*g^6 - 2*a*b*d^4*f*g^7 - 4*a^2*d^3*e*f*g
^7 + a^2*d^4*g^8)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 1/24*(48*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*c^3*d*e*f^4*g^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*b*c^2*e^2*f^4*g^3 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d*
e*f^3*g^4 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*e^2*f^3*g^4 + 48
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*e^2*f^3*g^4 + 66*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^2*c*d*e*f^2*g^5 + 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a*c^2*d*e*f^2*g^5 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3
*e^2*f^2*g^5 - 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e^2*f^2*g^5 -
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d^2*f*g^6 + 24*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a*c^2*d^2*f*g^6 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*b^3*d*e*f*g^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d*e
*f*g^6 + 42*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e^2*f*g^6 + 24*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*e^2*f*g^6 + 3*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*b^3*d^2*g^7 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b
*c*d^2*g^7 + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*d*e*g^7 + 24*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*d*e*g^7 - 24*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^2*b*e^2*g^7 + 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
c^(7/2)*d*e*f^5*g^2 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*e
^2*f^5*g^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d^2*f^4*g^3 -
432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d*e*f^4*g^3 + 180*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*e^2*f^4*g^3 + 192*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*e^2*f^4*g^3 + 96*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*b*c^(5/2)*d^2*f^3*g^4 + 234*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*b^2*c^(3/2)*d*e*f^3*g^4 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a*c^(5/2)*d*e*f^3*g^4 - 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(
c)*e^2*f^3*g^4 - 324*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*e^2*
f^3*g^4 - 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^2*f^2*g^5

+ 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d^2*f^2*g^5 - 12*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*d*e*f^2*g^5 - 144*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d*e*f^2*g^5 + 162*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a*b^2*sqrt(c)*e^2*f^2*g^5 + 24*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a^2*c^(3/2)*e^2*f^2*g^5 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*b^3*sqrt(c)*d^2*f*g^6 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c
^(3/2)*d^2*f*g^6 - 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*d
*e*f*g^6 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*d*e*f*g^6
- 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*e^2*f*g^6 - 48*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*d^2*g^7 + 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*d*e*g^7 - 48*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a^3*sqrt(c)*e^2*g^7 + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*c^4*e^2*f^7 + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d*e*f^6*g - 30
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*e^2*f^6*g - 32*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*c^4*d^2*f^5*g^2 + 32*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b*c^3*d*e*f^5*g^2 + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c
^2*e^2*f^5*g^2 + 256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*e^2*f^5*g^
2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^2*f^4*g^3 - 396*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*d*e*f^4*g^3 - 272*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a*c^3*d*e*f^4*g^3 - 14*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*b^3*c*e^2*f^4*g^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^
2*e^2*f^4*g^3 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*d^2*f^3*g^
4 + 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*d^2*f^3*g^4 + 280*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d*e*f^3*g^4 + 672*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*b*c^2*d*e*f^3*g^4 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b^4*e^2*f^3*g^4 - 204*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c
*e^2*f^3*g^4 - 336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2*e^2*f^3*g^
4 - 74*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d^2*f^2*g^5 - 120*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d^2*f^2*g^5 - 16*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*b^4*d*e*f^2*g^5 - 612*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a*b^2*c*d*e*f^2*g^5 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2
*d*e*f^2*g^5 + 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*e^2*f^2*g^5
+ 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e^2*f^2*g^5 + 8*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d^2*f*g^6 + 144*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b^2*c*d^2*f*g^6 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^2*c^2*d^2*f*g^6 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d*e*f*g^
6 + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d*e*f*g^6 - 144*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e^2*f*g^6 - 288*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a^3*c*e^2*f*g^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a*b^3*d^2*g^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d^2*g^7
+ 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*e^2*g^7 + 96*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*e^2*f^7 + 240*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*b*c^(7/2)*d*e*f^6*g - 408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*b^2*c^(5/2)*e^2*f^6*g - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/
2)*e^2*f^6*g - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*d^2*f^5*g

^2 - 252*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*d*e*f^5*g^2 - 24
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*d*e*f^5*g^2 + 402*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*e^2*f^5*g^2 + 1080*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*e^2*f^5*g^2 + 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^2*c^(5/2)*d^2*f^4*g^3 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a*c^(7/2)*d^2*f^4*g^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^3*c^(3/2)*d*e*f^4*g^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5
/2)*d*e*f^4*g^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*e^2
*f^4*g^3 - 1068*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*e^2*f^4
g^3 - 816(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*e^2*f^4*g^3 +
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*d^2*f^3*g^4 + 72*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*d^2*f^3*g^4 + 96*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*d*e*f^3*g^4 + 156*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*a*b^2*c^(3/2)*d*e*f^3*g^4 + 336*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^2*c^(5/2)*d*e*f^3*g^4 + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a*b^3*sqrt(c)*e^2*f^3*g^4 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^2*b*c^(3/2)*e^2*f^3*g^4 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sq
rt(c)*d^2*f^2*g^5 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*
d^2*f^2*g^5 - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*d^2*f^2
g^5 - 288(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d*e*f^2*g^5
- 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(3/2)*d*e*f^2*g^5 - 48*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*e^2*f^2*g^5 - 288*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*e^2*f^2*g^5 + 72*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d^2*f*g^6 + 48*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^2*b*c^(3/2)*d^2*f*g^6 + 240*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^2*b^2*sqrt(c)*d*e*f*g^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^3*c^(3/2)*d*e*f*g^6 - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3
*b*sqrt(c)*e^2*f*g^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqr
t(c)*d^2*g^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*d*e*g
^7 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*sqrt(c)*e^2*g^7 + 48*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*e^2*f^7 + 120*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b^2*c^3*d*e*f^6*g - 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*c^2*e^2*f^6*g - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*e^2*f^6
g - 24(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*d^2*f^5*g^2 - 156*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d*e*f^5*g^2 - 240*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*b*c^3*d*e*f^5*g^2 + 150*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*b^4*c*e^2*f^5*g^2 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*
e^2*f^5*g^2 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*e^2*f^5*g^2 +
24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^2*f^4*g^3 + 48*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b*c^3*d^2*f^4*g^3 + 54*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^4*c*d*e*f^4*g^3 + 252*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
2*c^2*d*e*f^4*g^3 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d*e*f^4
g^3 - 33(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*e^2*f^4*g^3 - 714*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c*e^2*f^4*g^3 - 1152*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*b*c^2*e^2*f^4*g^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x +

a))*b^4*c*d^2*f^3*g^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*d
^2*f^3*g^4 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d^2*f^3*g^4 + 1
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*d*e*f^3*g^4 - 120*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^3*c*d*e*f^3*g^4 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a*b^4*e^2*f^3*g^4 + 1272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
2*c*e^2*f^3*g^4 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^2*e^2*f^3*g
^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*d^2*f^2*g^5 + 18*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b^3*c*d^2*f^2*g^5 - 120*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b*c^2*d^2*f^2*g^5 - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a*b^4*d*e*f^2*g^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*d*e*f^2
g^5 - 72(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^2*d*e*f^2*g^5 - 165*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*e^2*f^2*g^5 - 972*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^3*b*c*e^2*f^2*g^5 + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a*b^4*d^2*f*g^6 + 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*d^2
*f*g^6 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^2*d^2*f*g^6 + 24*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d*e*f*g^6 + 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^3*b*c*d*e*f*g^6 + 102*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^3*b^2*e^2*f*g^6 + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e^2*f*g^
6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d^2*g^7 - 36*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^3*b*c*d^2*g^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^3*b^2*d*e*g^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*d*e*g^7
- 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*e^2*g^7 + 8*b^3*c^(5/2)*e^2
*f^7 + 20*b^3*c^(5/2)*d*e*f^6*g - 26*b^4*c^(3/2)*e^2*f^6*g - 48*a*b^2*c^(5/
2)*e^2*f^6*g - 4*b^3*c^(5/2)*d^2*f^5*g^2 - 26*b^4*c^(3/2)*d*e*f^5*g^2 - 60*
a*b^2*c^(5/2)*d*e*f^5*g^2 + 15*b^5*sqrt(c)*e^2*f^5*g^2 + 182*a*b^3*c^(3/2)*
e^2*f^5*g^2 + 48*a^2*b*c^(5/2)*e^2*f^5*g^2 + 4*b^4*c^(3/2)*d^2*f^4*g^3 + 12
*a*b^2*c^(5/2)*d^2*f^4*g^3 + 12*b^5*sqrt(c)*d*e*f^4*g^3 + 56*a*b^3*c^(3/2)*
d*e*f^4*g^3 + 96*a^2*b*c^(5/2)*d*e*f^4*g^3 - 120*a*b^4*sqrt(c)*e^2*f^4*g^3
- 360*a^2*b^2*c^(3/2)*e^2*f^4*g^3 - 16*a^3*c^(5/2)*e^2*f^4*g^3 - 3*b^5*sqrt
(c)*d^2*f^3*g^4 + 2*a*b^3*c^(3/2)*d^2*f^3*g^4 - 24*a^2*b*c^(5/2)*d^2*f^3*g^
4 - 30*a*b^4*sqrt(c)*d*e*f^3*g^4 - 54*a^2*b^2*c^(3/2)*d*e*f^3*g^4 - 40*a^3*
c^(5/2)*d*e*f^3*g^4 + 315*a^2*b^3*sqrt(c)*e^2*f^3*g^4 + 292*a^3*b*c^(3/2)*e
^2*f^3*g^4 + 6*a*b^4*sqrt(c)*d^2*f^2*g^5 - 18*a^2*b^2*c^(3/2)*d^2*f^2*g^5 +
8*a^3*c^(5/2)*d^2*f^2*g^5 + 24*a^2*b^3*sqrt(c)*d*e*f^2*g^5 + 16*a^3*b*c^(3
/2)*d*e*f^2*g^5 - 378*a^3*b^2*sqrt(c)*e^2*f^2*g^5 - 88*a^4*c^(3/2)*e^2*f^2*
g^5 - 3*a^2*b^3*sqrt(c)*d^2*f*g^6 + 28*a^3*b*c^(3/2)*d^2*f*g^6 - 6*a^3*b^2*
sqrt(c)*d*e*f*g^6 + 8*a^4*c^(3/2)*d*e*f*g^6 + 216*a^4*b*sqrt(c)*e^2*f*g^6 -
16*a^4*c^(3/2)*d^2*g^7 - 48*a^5*sqrt(c)*e^2*g^7)/((c^2*e^3*f^7*g - 3*c^2*d
*e^2*f^6*g^2 - 2*b*c*e^3*f^6*g^2 + 3*c^2*d^2*e*f^5*g^3 + 6*b*c*d*e^2*f^5*g^
3 + b^2*e^3*f^5*g^3 + 2*a*c*e^3*f^5*g^3 - c^2*d^3*f^4*g^4 - 6*b*c*d^2*e*f^4
*g^4 - 3*b^2*d*e^2*f^4*g^4 - 6*a*c*d*e^2*f^4*g^4 - 2*a*b*e^3*f^4*g^4 + 2*b*
c*d^3*f^3*g^5 + 3*b^2*d^2*e*f^3*g^5 + 6*a*c*d^2*e*f^3*g^5 + 6*a*b*d*e^2*f^3
*g^5 + a^2*e^3*f^3*g^5 - b^2*d^3*f^2*g^6 - 2*a*c*d^3*f^2*g^6 - 6*a*b*d^2*e*
f^2*g^6 - 3*a^2*d*e^2*f^2*g^6 + 2*a*b*d^3*f*g^7 + 3*a^2*d^2*e*f*g^7 - a^2*d
^3*g^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^

```
2 + b*x + a))*sqrt(c)*f + b*f - a*g)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{(f+g\,x)^4\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)), x)
```

3.862 $\int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1098

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx =$$

$$-\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef-3bdg+8aeg)+8bc^2e\right.}{}$$

$$+\frac{(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg(3e^2f^2-3defg+d^2g^2)+2ceg(7b^2e^2g}{192c^3e^4}$$

$$+\frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$+\frac{\left(4ce(2cd-be)\left(8ce(bd-2ae)\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)-d(8bcd-3b^2e-4ace)\right.}{}$$

$$+\frac{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^7}$$

```
[Out] 1/192*(7*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-3*b*d*g+9*b*e
*f)+24*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(7*b^2*e^2*g^2-4*c*e
*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+
b*x+a)^(3/2)/c^3/e^4+1/60*g^2*(-7*b*e*g-22*c*d*g+36*c*e*f)*(c*x^2+b*x+a)^(5
/2)/c^2/e^2+1/6*g^3*(e*x+d)*(c*x^2+b*x+a)^(5/2)/c/e^2+1/3072*(4*c*e*(-b*e+2
*c*d)*(8*c*e*(-2*a*e+b*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*
b*d*g+9*b*e*f))-d*(-4*a*c*e-3*b^2*e+8*b*c*d)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*
g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))-2*(4*c^2*d^2-1/2*
b^2*e^2-2*c*e*(-a*e+b*d))*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3
-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*
c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d
```

```
*e*f*g+3*e^2*f^2))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
9/2)/e^7+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*
e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^7-1/1536*(21*b
^5*e^5*g^3-1536*c^5*d^2*(-d*g+e*f)^3+384*c^4*e*(-4*a*e+5*b*d)*(-d*g+e*f)^3-
12*b^3*c*e^4*g^2*(8*a*e*g-3*b*d*g+9*b*e*f)+24*b*c^2*e^3*g*(2*a^2*e^2*g^2+6*
a*b*e*g*(-d*g+3*e*f)+3*b^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))-96*b*c^3*e^2*(2*b
*(-d*g+e*f)^3+3*a*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))+2*c*e*(8*c*e*(-b*e+2*c
*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c
^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b
*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))*x)*(c*x^2+b*x+a)^(1/2)
/c^4/e^6
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(d+ex)\left(cx^2+bx+a\right)^{5/2} g^3}{6ce^2}$$
$$+\frac{(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2} g^2}{60c^2e^2}$$
$$+\frac{(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2+24bc^2e(3e^2f^2-3degf+d^2g^2)\,g+2ce(24(3e^2f^2-3degf+d^2g^2)}{192c^3e^4}$$
$$+\frac{\Big(4ce(2cd-be)\,(8ce(bd-2ae)\,(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg))-d(-3eb^2+8cdb-4ace)}{}$$
$$+\frac{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{e^7}$$
$$-\frac{\Big(3(-512d^2(ef-dg)^3c^5+128e(5bd-4ae)(ef-dg)^3c^4-32be^2(2b(ef-dg)^3+3aeg(3e^2f^2-3degf+d^2g}{}$$

```
[In] Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -1/1536*((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d -
 4*a*e)*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b
*c^2*e^3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*
d*e*f*g + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2
- 3*d*e*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b
^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*
d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*
d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x +
 c*x^2])/(c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2
*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^
```

```
2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*
(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4
) + (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e
^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d -
b*e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*
e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2
- 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2
*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b
*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)
) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2
- 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2
*g^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9
/2)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])]
)/e^7
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
```

```
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(\frac{1}{2}e\left(12ce^2f^3-d(5bd+2ae)g^3\right)-eg\left(e(6bd+ae)g^2-c\left(18e^2f^2-5d^2g^2\right)\right)x+\frac{1}{2}e^2g^2(36cef-22cdg-7beg)x^2\right)}{d+ex}\,dx}{6ce^3}$$

$$= \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \frac{\left(\frac{5}{4}e^3\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)+\frac{5}{4}e^3g\left(7b^2e^2g^2-4ceg(9bef-3bdg+aeg)+24c^2\left(3e^2f^2-3defg+d^2g^2\right)\right)x\right)(a+b}{d+ex}}{30c^2e^5}$$

$$= \frac{(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg(3e^2f^2-3defg+d^2g^2)+2ceg}{192c^3e^4} + \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} - \frac{\int \frac{\left(\frac{5}{8}e^3\left(8ce(bd-2ae)\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)+2\left(2acde-bd\left(4cd-\frac{3be}{2}\right)\right)g\left(7b^2e^2g^2-4ceg(9bef-3bdg+aeg)+\right.\right.\right.}{}}{}$$

$$= -\frac{\left(3(7b^5e^5g^3 - 512c^5d^2(ef-dg)^3 + 128c^4e(5bd-4ae)(ef-dg)^3 - 4b^3ce^4g^2(9bef-3bdg+8ae\right.}{}$$

$$+\frac{(7b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(9bef-3bdg+aeg) + 24bc^2eg(3e^2f^2-3defg+d^2g^2) + 2}{192c^3}$$

$$+\frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$+\frac{\int \frac{\frac{5}{16}e^3\left(4ce(bd-2ae)\left(8ce(bd-2ae)\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)-d\left(8bcd-3b^2e-4ace\right)g\left(7b^2e^2g^2-4ceg(9bef-3\right.\right.\right.}{}}{}$$

$$= -\frac{\left(3(7b^5e^5g^3 - 512c^5d^2(ef-dg)^3 + 128c^4e(5bd-4ae)(ef-dg)^3 - 4b^3ce^4g^2(9bef-3bdg+8ae\right.}{}$$

$$+\frac{(7b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(9bef-3bdg+aeg) + 24bc^2eg(3e^2f^2-3defg+d^2g^2) + 2}{192c^3}$$

$$+\frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$+\frac{\left(\left(cd^2-bde+ae^2\right)^2(ef-dg)^3\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^7}$$

$$+\frac{\left(4ce(2cd-be)\left(8ce(bd-2ae)\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)-d(8bcd-3b\right.\right.}{}$$

$$= -\frac{\left(3(7b^5e^5g^3 - 512c^5d^2(ef-dg)^3 + 128c^4e(5bd-4ae)(ef-dg)^3 - 4b^3ce^4g^2(9bef-3bdg+8ae\right.}{}$$

$$+\frac{(7b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(9bef-3bdg+aeg) + 24bc^2eg(3e^2f^2-3defg+d^2g^2) + 2}{192c^3}$$

$$+\frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$-\frac{\left(2\left(cd^2-bde+ae^2\right)^2(ef-dg)^3\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^7}$$

$$+\frac{\left(4ce(2cd-be)\left(8ce(bd-2ae)\left(24c^2e^2f^3+7b^2deg^3-4cdg^2(9bef-3bdg+aeg)\right)-d(8bcd-3b\right.\right.}{}$$

$$= -\frac{\left(3(7b^5e^5g^3 - 512c^5d^2(ef-dg)^3 + 128c^4e(5bd-4ae)(ef-dg)^3 - 4b^3ce^4g^2(9bef-3bdg+8aeg\right.}{}$$

$$+\frac{(7b^3e^3g^3 + 64c^3(ef-dg)^3 - 4bce^2g^2(9bef-3bdg+aeg) + 24bc^2eg(3e^2f^2-3defg+d^2g^2) + 2c}{192c^3e}$$

$$+\frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$+\frac{\left(4ce(2cd-be)\left(8ce(bd-2ae)\left(24c^2e^2f^3 + 7b^2deg^3 - 4cdg^2(9bef-3bdg+aeg)\right) - d(8bcd-3b^2\right.}{}$$

$$+\frac{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^7}$$

Mathematica [A] (verified)

Time = 11.51 (sec) , antiderivative size = 743, normalized size of antiderivative = 0.68

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{5120(ef-dg)^3(a+x(b+cx))^{3/2} + \frac{1920eg(ef-dg)^2(b+2cx)(a+x(b+cx))^{3/2}}{c} + \frac{307}{}}{}$$

```
[In] Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b + c
*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (360*(
b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[
a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)
^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2) + (e^3*
g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 + 7*b^2*g^2
 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b
^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c^2 + (9
60*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*
a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(
e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d
+ 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
c*x)])])))/(c^(3/2)*e^3))/(15360*e^4)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 1409, normalized size of antiderivative = 1.28

method	result	size
default	Expression too large to display	1409
risch	Expression too large to display	2365

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(1/3*((x+d/e)^2*c+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*
(x+d/e)+(b*e-2*c*d)/e)/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)
*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*(((x+d/e)^2*c+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/
2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e))))+g/e^3*(d^2*g^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)
^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b
^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+e^2*g^2*(1/6*x*(c
*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x
+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*
c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2)))))+3*e^2*f^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3*d*e*f*g*(1/8*(2*c*x+b)/c*
(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(-d*
e*g^2+3*e^2*f*g)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2
+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \int \frac{(f+gx)^3 (a+bx+cx^2)^{\frac{3}{2}}}{d+ex} \, dx$$

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
[Out] Integral((f + g*x)**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx = \int \frac{(f+g\,x)^3 \,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x} \, dx$$

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.863 $\int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 662

$$\int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2\cdots}{\cdots}$$
$$-\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x)(a+bx+cx^2)^{3/2}}{48c^2e^3}$$
$$+\frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$
$$-\frac{(3b^5e^5g^2+256c^5d^3(ef-dg)^2-384c^4de(bd-ae)(ef-dg)^2-6b^3ce^4g(2bef-bdg+4aeg)+16bc^2e^3(3a^2e\cdots}{\cdots}$$
$$+\frac{(cd^2-bde+ae^2)^{3/2}(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^6}$$

```
[Out] -1/48*(3*b^2*e^2*g^2-16*c^2*(-d*g+e*f)^2-6*b*c*e*g*(-d*g+2*e*f)-6*c*e*g*(-b
*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(3/2)/c^2/e^3+1/5*g^2*(c*x^2+b*x+a)^
(5/2)/c/e-1/256*(3*b^5*e^5*g^2+256*c^5*d^3*(-d*g+e*f)^2-384*c^4*d*e*(-a*e+b
*d)*(-d*g+e*f)^2-6*b^3*c*e^4*g*(4*a*e*g-b*d*g+2*b*e*f)+16*b*c^2*e^3*(3*a^2*
e^2*g^2+b^2*(-d*g+e*f)^2+3*a*b*e*g*(-d*g+2*e*f))+96*c^3*e^2*(b^2*d*(-d*g+e*
f)^2-2*a*b*e*(-d*g+e*f)^2-a^2*e^2*g*(-d*g+2*e*f)))*arctanh(1/2*(2*c*x+b)/c^
(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/e^6+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)
^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/e^6+1/128*(3*b^4*e^4*g^2+128*c^4*d^2*(-d*g+e*f)^2-32*c^3*e*(-
4*a*e+5*b*d)*(-d*g+e*f)^2-6*b^2*c*e^3*g*(2*a*e*g-b*d*g+2*b*e*f)+8*b*c^2*e^2
*(2*b*(-d*g+e*f)^2+3*a*e*g*(-d*g+2*e*f))+2*c*e*((16*c^2*d^2-3*b^2*e^2-4*c*e
*(-3*a*e+2*b*d))*g*(-b*e*g-2*c*d*g+4*c*e*f)-8*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*
c*e*f^2))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^5
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx =$$
$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3e^2(-a^2e^2g(2ef-dg)-2abe(ef-dg)^2+b^2d(ef-dg)^2)+16bc^2e^3(3a^2e^2g^2\right.}{}$$
$$+\frac{(ef-dg)^2\,(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^6}$$
$$-\frac{(a+bx+cx^2)^{3/2}\,(3b^2e^2g^2-6cegx(-beg-2cdg+4cef)-6bceg(2ef-dg)-16c^2(ef-dg)^2)}{48c^2e^3}$$
$$+\frac{\sqrt{a+bx+cx^2}(2cex(g(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2)\,(-beg-2cdg+4cef)-8ce(2cd-be)\,(2cef^2\right.}{}$$
$$+\frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$

```
[In] Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f
 - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e
*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c
*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*
c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*e^5) - ((3*b^2*e^2*g
^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*
c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c
*x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^
4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g)
 + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g
)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2
*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^2(a+bx+cx^2)^{5/2}}{5ce} + \frac{\int \frac{\left(\frac{5}{2}e(2cef^2-bdg^2)+\frac{5}{2}eg(4cef-2cdg-beg)x\right)(a+bx+cx^2)^{3/2}}{d+ex}\,dx}{5ce^2}$$

$$= -\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x)\,(a+bx+cx^2)^{3/2}}{48c^2e^3} + \frac{g^2(a+bx+cx^2)^{5/2}}{5ce} - \frac{\int \frac{\left(-\frac{5}{4}e\left(d\left(8bcd-3b^2e-4ace\right)g(4cef-2cdg-beg)-8ce(bd-2ae)\left(2cef^2-bdg^2\right)\right)-\frac{5}{4}e\left(\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)g(4cef-2cdg\right.}{d+ex}}{40c^2e^4}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+2aeg)+8bc}{}$$

$$-\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x)\,(a+bx+cx^2)^{3/2}}{48c^2e^3} + \frac{g^2(a+bx+cx^2)^{5/2}}{5ce} + \frac{\int \frac{-\frac{5}{8}e\left(4ce(bd-2ae)\left(d\left(8bcd-3b^2e-4ace\right)g(4cef-2cdg-beg)-8ce(bd-2ae)\left(2cef^2-bdg^2\right)\right)-d\left(4bcd-b^2e-4ace\right)\left(\left(16c^2d^2-3b^2e^2-\right.\right.\right.}{}}{}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+2aeg)+8bc}{}$$

$$-\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x)\,(a+bx+cx^2)^{3/2}}{48c^2e^3} + \frac{g^2(a+bx+cx^2)^{5/2}}{5ce} + \frac{\left(\left(cd^2-bde+ae^2\right)^2(ef-dg)^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^6} - \frac{(3b^5e^5g^2+256c^5d^3(ef-dg)^2-384c^4de(bd-ae)(ef-dg)^2-6b^3ce^4g(2bef-bdg+4aeg)+1}{}$$

$$= \frac{(3b^4e^4g^2 + 128c^4d^2(ef-dg)^2 - 32c^3e(5bd-4ae)(ef-dg)^2 - 6b^2ce^3g(2bef-bdg+2aeg) + 8bc^2e}{}$$

$$- \frac{(3b^2e^2g^2 - 16c^2(ef-dg)^2 - 6bceg(2ef-dg) - 6ceg(4cef-2cdg-beg)x)(a+bx+cx^2)^{3/2}}{48c^2e^3}$$

$$+ \frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$

$$- \frac{\left(2(cd^2-bde+ae^2)^2(ef-dg)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^6}$$

$$- \frac{(3b^5e^5g^2 + 256c^5d^3(ef-dg)^2 - 384c^4de(bd-ae)(ef-dg)^2 - 6b^3ce^4g(2bef-bdg+4aeg) + 16}{}$$

$$= \frac{(3b^4e^4g^2 + 128c^4d^2(ef-dg)^2 - 32c^3e(5bd-4ae)(ef-dg)^2 - 6b^2ce^3g(2bef-bdg+2aeg) + 8bc^2e}{}$$

$$- \frac{(3b^2e^2g^2 - 16c^2(ef-dg)^2 - 6bceg(2ef-dg) - 6ceg(4cef-2cdg-beg)x)(a+bx+cx^2)^{3/2}}{48c^2e^3}$$

$$+ \frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$

$$- \frac{(3b^5e^5g^2 + 256c^5d^3(ef-dg)^2 - 384c^4de(bd-ae)(ef-dg)^2 - 6b^3ce^4g(2bef-bdg+4aeg) + 16}{}$$

$$+ \frac{(cd^2-bde+ae^2)^{3/2}(ef-dg)^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^6}$$

Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{1280(ef-dg)^2(a+x(b+cx))^{3/2} + \frac{480eg(ef-dg)(b+2cx)(a+x(b+cx))^{3/2}}{c} + \frac{768e^2}{}}{}$$

```
[In] Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
[Out] (1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c +
(90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/
2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2
)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-
2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2
```

```
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*
e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[
(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])])))/(c^(3/2)*e^3))/(3840*e^3)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 998, normalized size of antiderivative = 1.51

method	result
default	$-\frac{g\left(dg\left(\frac{(2cx+b)\left(cx^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)-2ef\left(\frac{(2cx+b)\left(cx^2+bx+a\right.}{8c}\right.\right.}{}$
risch	Expression too large to display

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -g/e^2*(d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))))-2*e*f*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4
*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-e*g*(1/5*(c*x^2+b*x+a)^(5/2)/c-
1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x
+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/3*((x+d/e)^2*c+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(
2*c*(x+d/e)+(b*e-2*c*d)/e)/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(
3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*(((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln
((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+gx)^2 (a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
[Out] Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 (a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.864 $\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 441

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3aeg)+2ce(3b^2e^2g+16c^2}{64c^2e^4} + \frac{(8cef-8cdg+3beg+6cegx)(a+bx+cx^2)^{3/2}}{24ce^2} + \frac{(3b^4e^4g-128c^4d^3(ef-dg)+192c^3de(bd-ae)(ef-dg)-8b^2ce^3(bef-bdg+3aeg)+48c^2e^2(a^2e^2g-b^2c}{128c^{5/2}e^5} + \frac{(cd^2-bde+ae^2)^{3/2}(ef-dg)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

```
[Out] 1/24*(6*c*e*g*x+3*b*e*g-8*c*d*g+8*c*e*f)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(3
*b^4*e^4*g-128*c^4*d^3*(-d*g+e*f)+192*c^3*d*e*(-a*e+b*d)*(-d*g+e*f)-8*b^2*c
*e^3*(3*a*e*g-b*d*g+b*e*f)+48*c^2*e^2*(a^2*e^2*g-b^2*d*(-d*g+e*f)+2*a*b*e*(
-d*g+e*f)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^5+
(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5-1/64*(3*b^3*e^3*g-64*c^
3*d^2*(-d*g+e*f)+16*c^2*e*(-4*a*e+5*b*d)*(-d*g+e*f)-4*b*c*e^2*(3*a*e*g-2*b*
d*g+2*b*e*f)+2*c*e*(3*b^2*e^2*g+16*c^2*d*(-d*g+e*f)-4*c*e*(3*a*e*g-2*b*d*g+
2*b*e*f))*x)*(c*x^2+b*x+a)^(1/2)/c^2/e^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {828, 857, 635, 212, 738}

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-d\right.\right.}{}$$
$$+\frac{(ef-dg)\left(ae^2-bde+cd^2\right)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}$$
$$-\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg\right.}{64c^2e^4}$$
$$+\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

```
[In] Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -1/64*((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*
f - d*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 1
6*c^2*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x
+ c*x^2])/(c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x +
 c*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c
^3*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48
*c^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 -
b*d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} - \frac{\int \frac{\left(\frac{1}{2}\left(8ce(bd-2ae)f+4acdeg-2bd\left(4cd-\frac{3be}{2}\right)g\right)+\frac{1}{2}\left(3b^2e^2g+16c^2d(ef-dg)-4ce(2bef-2bdg+3aeg)\right)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{8ce^2}$$

$$= -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3aeg)+2ce(3b^2\cdots}{64c^2e^4} + \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} + \frac{\int \frac{\frac{1}{4}\left(4ce(bd-2ae)(8ce(bd-2ae)f+4acdeg-bd(8cd-3be)g)-d\left(4bcd-b^2e-4ace\right)\left(3b^2e^2g+16c^2d(ef-dg)-4ce(2bef-2bdg+3aeg)\right)\right)+\cdots}{(d+ex)\sqrt{a-\cdots}}}{32c\cdots}$$

$$= -\frac{(3b^3e^3g - 64c^3d^2(ef-dg) + 16c^2e(5bd-4ae)(ef-dg) - 4bce^2(2bef-2bdg+3aeg) + 2ce(3a}{64c^2e^4}$$
$$+ \frac{(8cef - 8cdg + 3beg + 6cegx)(a+bx+cx^2)^{3/2}}{24ce^2}$$
$$+ \frac{\left((cd^2-bde+ae^2)^2(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^5}$$
$$+ \frac{(3b^4e^4g - 128c^4d^3(ef-dg) + 192c^3de(bd-ae)(ef-dg) - 8b^2ce^3(bef-bdg+3aeg) + 48c^2e^2}{128c^2e^5}$$
$$= -\frac{(3b^3e^3g - 64c^3d^2(ef-dg) + 16c^2e(5bd-4ae)(ef-dg) - 4bce^2(2bef-2bdg+3aeg) + 2ce(3a}{64c^2e^4}$$
$$+ \frac{(8cef - 8cdg + 3beg + 6cegx)(a+bx+cx^2)^{3/2}}{24ce^2}$$
$$- \frac{\left(2(cd^2-bde+ae^2)^2(ef-dg)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^5}$$
$$+ \frac{(3b^4e^4g - 128c^4d^3(ef-dg) + 192c^3de(bd-ae)(ef-dg) - 8b^2ce^3(bef-bdg+3aeg) + 48c^2e^2}{64c^2e^5}$$
$$= -\frac{(3b^3e^3g - 64c^3d^2(ef-dg) + 16c^2e(5bd-4ae)(ef-dg) - 4bce^2(2bef-2bdg+3aeg) + 2ce(3a}{64c^2e^4}$$
$$+ \frac{(8cef - 8cdg + 3beg + 6cegx)(a+bx+cx^2)^{3/2}}{24ce^2}$$
$$+ \frac{(3b^4e^4g - 128c^4d^3(ef-dg) + 192c^3de(bd-ae)(ef-dg) - 8b^2ce^3(bef-bdg+3aeg) + 48c^2e^2}{128c^{5/2}e^5}$$
$$+ \frac{(cd^2-bde+ae^2)^{3/2}(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{2e\sqrt{a+x(b+cx)}\left(-9b^3e^3g - 16c^3\left(12d^3g - 6d^2e(2f+gx) + 2de^2x(3f+2gx) - e^3x^2(4f+3gx)\right) + 6bce^2(1}{}$$

```
[In] Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(-9*b^3*e^3*g - 16*c^3*(12*d^3*g - 6*d^2*e*(2*f
 + g*x) + 2*d*e^2*x*(3*f + 2*g*x) - e^3*x^2*(4*f + 3*g*x)) + 6*b*c*e^2*(10*
a*e*g + b*(4*e*f - 4*d*g + e*g*x)) + 8*c^2*e*(a*e*(32*e*f - 32*d*g + 15*e*g
```

```
*x) + b*(30*d^2*g - 2*d*e*(15*f + 7*g*x) + e^2*x*(14*f + 9*g*x)))))/c^2 - 7
68*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)
*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*
d - a*e)]] - (3*(3*b^4*e^4*g + 128*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*
d - a*e)*(-(e*f) + d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^
2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g) + b^2*d*(-(e*f) + d*g)))*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(384*e^5)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.67

method	result
risch	$\frac{(48g\,e^3x^3c^3+72b\,c^2e^3g\,x^2-64c^3d\,e^2g\,x^2+64c^3e^3f\,x^2+120a\,c^2e^3gx+6b^2c\,e^3gx-112b\,c^2d\,e^2gx+112b\,c^2e^3fx+96c^3d^2egx-96c^3d\,e^2fx+}{192}$
default	$\frac{g\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{e}+\frac{(-dg+ef)\left(\left(\left(x+\frac{d}{e}\right)^2c+\frac{(be-2c}{}\right.\right.}{}$

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/c^2*(48*c^3*e^3*g*x^3+72*b*c^2*e^3*g*x^2-64*c^3*d*e^2*g*x^2+64*c^3*e^
3*f*x^2+120*a*c^2*e^3*g*x+6*b^2*c*e^3*g*x-112*b*c^2*d*e^2*g*x+112*b*c^2*e^3
*f*x+96*c^3*d^2*e*g*x-96*c^3*d*e^2*f*x+60*a*b*c*e^3*g-256*a*c^2*d*e^2*g+256
*a*c^2*e^3*f-9*b^3*e^3*g-24*b^2*c*d*e^2*g+24*b^2*c*e^3*f+240*b*c^2*d^2*e*g-
240*b*c^2*d*e^2*f-192*c^3*d^3*g+192*c^3*d^2*e*f)*(c*x^2+b*x+a)^(1/2)/e^4+1/
128/e^4/c^2*(128*(a^2*d*e^4*g-a^2*e^5*f-2*a*b*d^2*e^3*g+2*a*b*d*e^4*f+2*a*c
*d^3*e^2*g-2*a*c*d^2*e^3*f+b^2*d^3*e^2*g-b^2*d^2*e^3*f-2*b*c*d^4*e*g+2*b*c*
d^3*e^2*f+c^2*d^5*g-c^2*d^4*e*f)*c^2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))+(48*a^2*c^2*e^4*g-24*a*b^2*c*e^4*g-96*a*b*c^2*d*e^3*g+96*a*b*c^2*
e^4*f+192*a*c^3*d^2*e^2*g-192*a*c^3*d*e^3*f+3*b^4*e^4*g+8*b^3*c*d*e^3*g-8*b
^3*c*e^4*f+48*b^2*c^2*d^2*e^2*g-48*b^2*c^2*d*e^3*f-192*b*c^3*d^3*e*g+192*b*
```

```
c^3*d^2*e^2*f+128*c^4*d^4*g-128*c^4*d^3*e*f)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+gx)(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
[Out] Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.865 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/e-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*
e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^4+(a
*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e
+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e
+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used

$= \{748, 828, 857, 635, 212, 738\}$

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{16c^{3/2}e^4} + \frac{(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e} \\
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} \\
&\quad + \frac{(a+bx+cx^2)^{3/2}}{3e} + \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)^2-d(2cd-be)(4bcd-b^2e-4ace))-\frac{1}{2}(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8ce^3} \\
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} \\
&\quad + \frac{(a+bx+cx^2)^{3/2}}{3e} + \frac{(cd^2-bde+ae^2)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^4} \\
&\quad - \frac{((2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16ce^4}
\end{aligned}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{\left(2(cd^2-bde+ae^2)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^4} - \frac{((2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8ce^4}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{\frac{e\sqrt{a+x(b+cx)}(3b^2e^2+2ce(-15bd+16ae+7bex)+4c^2(6d^2-3dex+2e^2x^2))}{c} + 48\sqrt{-cd^2+bde-ae^2}(cd^2 \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x) +
 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c + 48*Sqrt[-(c*d^2) + b*d*e - a*e^2
]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(Sqr
t[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])] - (3*(2*c*d - b*e)*(8*c^2*d^2 -
b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a +
x*(b + c*x)])])/c^(3/2))/(24*e^4)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.52

method	result
risch	$\frac{\left(8c^{2}x^{2}e^{2}+14bc\,e^{2}x-12c^{2}dex+32ac\,e^{2}+3b^{2}e^{2}-30bcde+24c^{2}d^{2}\right)\sqrt{c\,x^{2}+bx+a}}{24c\,e^{3}}+\frac{\frac{\left(12c\,e^{3}ba-24a\,c^{2}d\,e^{2}-b^{3}e^{3}-6b^{2}cd\,e^{2}+24b\,c^{2}d^{2}e-16c^{3}\cdots\right.}{e\sqrt{c}}}{\cdots}$
default	$\frac{\frac{\left(\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a-bde+c\,d^{2}}{e^{2}}\right)^{\frac{3}{2}}}{3}+\frac{(be-2cd)\left(\frac{\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)\sqrt{\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^{2}a-bde+c\,d^{2}}{e^{2}}}}{4c}+\frac{\left(4c\left(e^{2}a\cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/c*(8*c^2*e^2*x^2+14*b*c*e^2*x-12*c^2*d*e*x+32*a*c*e^2+3*b^2*e^2-30*b*c
*d*e+24*c^2*d^2)*(c*x^2+b*x+a)^(1/2)/e^3+1/16/e^3/c*((12*a*b*c*e^3-24*a*c^2
*d*e^2-b^3*e^3-6*b^2*c*d*e^2+24*b*c^2*d^2*e-16*c^3*d^3)/e*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^
2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e)))
```

Fricas [A] (verification not implemented)

none

Time = 150.15 (sec) , antiderivative size = 1523, normalized size of antiderivative = 6.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 -
 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*s
qrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 -
 (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*(8*c^3
*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*
c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/48*(3*(16*c
^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a*b*c)*e^3)
```

```
*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 +
b*c*x + a*c)) + 24*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(c*d^2 - b*d*e + a
*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d
*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*
x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2
+ 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^3*e^3*x^2 + 24*c^3*d^2
*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e
^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/96*(96*(c^3*d^2 - b*c^2*d*e + a*
c^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e
^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*
e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*
e^2)*x)) - 3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^
3 - 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 3
0*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)
*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/48*(48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^
2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^
2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x
)) + 3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12
*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) + 2*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^
2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2
+ b*x + a))/(c^2*e^4)]
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

3.866 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 491

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} - \frac{(2cd-be)(cd^2-bde+ae^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^3(ef-dg)} + \frac{(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg(3bef^2-ag(3ef-dg)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}eg^3(ef-dg)} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3(ef-dg)} - \frac{(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d
*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(3/
2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/g^3/(-d*g+e*f)-1/2*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)/c^(1/2)+1/8*(8*c^2
*e*f^3+b*g^2*(-4*a*e*g+b*d*g+3*b*e*f)-4*c*g*(3*b*e*f^2-a*g*(-d*g+3*e*f)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/g^3/(-d*g+e*f)/c^(1/2)+
(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(1/2)/e^2/(-d*g+e*f)-1/4*(4*c*e*f^2-g*(-4
*a*e*g-b*d*g+5*b*e*f)-2*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(1/2)/e/g^2/(-d*g+e
*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {909, 748, 857, 635, 212, 738, 828}

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg(3bef^2-ag(3ef-dg))+bg^2(-4aeg+bdg+3bef\right.}{8\sqrt{c}eg^3(ef-dg)}$$
$$+\frac{(ae^2-bde+cd^2)^{3/2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3(ef-dg)}$$
$$-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(ae^2-bde+cd^2)}{2\sqrt{c}e^3(ef-dg)}$$
$$-\frac{(ag^2-bfg+cf^2)^{3/2}\,\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}{e^2(ef-dg)}$$
$$-\frac{\sqrt{a+bx+cx^2}(-g(-4aeg-bdg+5bef)-2cgx(ef-dg)+4cef^2)}{4eg^2(ef-dg)}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
```

```
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{e(ef-dg)} + \frac{(cd^2-bde+ae^2)\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} - \frac{(cd^2-bde+ae^2)\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e^2(ef-dg)} + \frac{\int \frac{\frac{1}{2}c\left(f\left(4bcf-b^2g-4acg\right)(ef-dg)+4g(bf-2ag)(cdf-bef+aeg)\right)+\frac{1}{2}c\left(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg\left(3bef^2-ag(3ef-dg)\right)\right)x}{(f+gx)\sqrt{a+bx+cx^2}}}{4ceg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} - \frac{((2cd-be)(cd^2-bde+ae^2))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^3(ef-dg)} + \frac{(cd^2-bde+ae^2)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^3(ef-dg)} - \frac{(cf^2-bfg+ag^2)^2\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g^3(ef-dg)} + \frac{(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg(3bef^2-ag(3ef-dg)))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8eg^3(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} - \frac{((2cd-be)(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^3(ef-dg)} - \frac{\left(2(cd^2-bde+ae^2)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^3(ef-dg)} + \frac{\left(2(cf^2-bfg+ag^2)^2\right)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)} + \frac{(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg(3bef^2-ag(3ef-dg)))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4eg^3(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)} - \frac{(2cd-be)(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^3(ef-dg)} + \frac{(8c^2ef^3+bg^2(3bef+bdg-4aeg)-4cg(3bef^2-ag(3ef-dg)))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}eg^3(ef-dg)} + \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3(ef-dg)} - \frac{(cf^2-bfg+ag^2)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)}$$

Mathematica [A] (verified)

Time = 10.72 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \frac{\frac{(3b^2e^2g^2-12ceg(bef+bdg-aeg)+8c^2(e^2f^2+defg+d^2g^2))\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + \frac{2\left(eg(ef-dg)\sqrt{a+}\right.}{}}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e*
f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqr
```

```
t[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f - 4*d
*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g - 2
*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))
/(e*f - d*g))/(8*e^3*g^3)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(2cegx+5beg-4cdg-4cef)\sqrt{c\,x^2+bx+a}}{4e^2g^2}+\frac{\frac{\left(12ac\,e^2g^2+3b^2e^2g^2-12bcde\,g^2-12bc\,e^2fg+8c^2d^2g^2+8c^2defg+8c^2e^2f^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b\dots}\right.}{eg\sqrt{c}}}{\dots}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*e*g*x+5*b*e*g-4*c*d*g-4*c*e*f)*(c*x^2+b*x+a)^(1/2)/e^2/g^2+1/8/e^2
/g^2*((12*a*c*e^2*g^2+3*b^2*e^2*g^2-12*b*c*d*e*g^2-12*b*c*e^2*f*g+8*c^2*d^2
*g^2+8*c^2*d*e*f*g+8*c^2*e^2*f^2)/e/g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))/c^(1/2)+8/e^2*g^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b
*c*d^3*e+c^2*d^4)/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-8*e
^2/g^2*(a^2*g^4-2*a*b*f*g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c^2*f^4)/
(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*
g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*
f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)\,(f+gx)}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)), x)
```

3.867 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 787

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}$$
$$+\frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)}$$
$$-\frac{e(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2}$$
$$+\frac{(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)}-\frac{(2cd-be)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^2(ef-dg)^2}$$
$$+\frac{e(2cf-bg)\,(8c^2f^2-b^2g^2-4cg(2bf-3ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^2}$$
$$-\frac{3(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)}$$
$$+\frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2(ef-dg)^2}$$
$$+\frac{3(2cf-bg)\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)}$$
$$-\frac{e(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^2}$$

```
[Out] (c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2
-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
```

```
^(3/2)/e^2/(-d*g+e*f)^2+1/16*e*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*
g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-
d*g+e*f)^2+(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(-d*g+e*f)^2-e*(a*g^2-b
*f*g+c*f^2)^(3/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*
(-a*g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+
e*f)/c^(1/2)+3/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2
-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g^3/(-d*
g+e*f)+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c
*x^2+b*x+a)^(1/2)/c/e/(-d*g+e*f)^2+3/4*(-2*c*g*x-3*b*g+4*c*f)*(c*x^2+b*x+a)
^(1/2)/g^2/(-d*g+e*f)-1/8*e*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+5*b*f)-2*c*g*(
-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^2
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used

$= \{974, 748, 828, 857, 635, 212, 738, 746\}$

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = -\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4cg(2bf-ag)+b^2g^2+8c^2f^2)}{8\sqrt{c}g^3(ef-dg)} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{16c^{3/2}e^2(ef-dg)^2} + \frac{e(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4cg(2bf-3ag)-b^2g^2+8c^2f^2)}{16c^{3/2}g^3(ef-dg)^2} + \frac{(ae^2-bde+cd^2)^{3/2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ef-dg)^2} - \frac{e(ag^2-bfg+cf^2)^{3/2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)^2} + \frac{3(2cf-bg)\sqrt{ag^2-bfg+cf^2}\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g^3(ef-dg)} + \frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce(ef-dg)^2} - \frac{e\sqrt{a+bx+cx^2}(-2cg(5bf-4ag)+b^2g^2-2cgx(2cf-bg)+8c^2f^2)}{8cg^2(ef-dg)^2} + \frac{3\sqrt{a+bx+cx^2}(-3bg+4cf-2cgx)}{4g^2(ef-dg)} + \frac{(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2]])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
```

```
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
```

```
- g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2(a+bx+cx^2)^{3/2}}{(ef-dg)^2(d+ex)} - \frac{g(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)^2} - \frac{eg(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)}\right) dx \\
&= \frac{e^2 \int \frac{(a+bx+cx^2)^{3/2}}{d+ex} \, dx}{(ef-dg)^2} - \frac{(eg) \int \frac{(a+bx+cx^2)^{3/2}}{f+gx} \, dx}{(ef-dg)^2} - \frac{g \int \frac{(a+bx+cx^2)^{3/2}}{(f+gx)^2} \, dx}{ef-dg} \\
&= \frac{(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)} - \frac{e \int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex} \, dx}{2(ef-dg)^2} \\
&\quad + \frac{e \int \frac{(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{f+gx} \, dx}{2(ef-dg)^2} - \frac{3 \int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{f+gx} \, dx}{2(ef-dg)}
\end{aligned}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)} - \frac{e\left(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x\right)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2} + \frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)} + \frac{\int \frac{\frac{1}{2}\left(4ce(bd-2ae)^2-d(2cd-be)\left(4bcd-b^2e-4ace\right)\right)-\frac{1}{2}(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8ce(ef-dg)^2} - \frac{e\int \frac{\frac{1}{2}\left(4cg(bf-2ag)^2-f(2cf-bg)\left(4bcf-b^2g-4acg\right)\right)-\frac{1}{2}(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8cg^2(ef-dg)^2} + \frac{3\int \frac{c\left(3b^2fg+4acfg-4b\left(cf^2+ag^2\right)\right)-c\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8cg^2(ef-dg)}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)} - \frac{e\left(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x\right)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2} + \frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)} + \frac{\left(cd^2-bde+ae^2\right)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2(ef-dg)^2} - \frac{\left((2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16ce^2(ef-dg)^2} + \frac{\left(3(2cf-bg)\left(cf^2-bfg+ag^2\right)\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2g^3(ef-dg)} - \frac{\left(e\left(cf^2-bfg+ag^2\right)^2\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g^3(ef-dg)^2} + \frac{\left(e(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cg^3(ef-dg)^2} - \frac{\left(3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8g^3(ef-dg)}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)} - \frac{e\left(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x\right)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2} + \frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)} - \frac{\left(2\left(cd^2-bde+ae^2\right)^2\right)\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^2(ef-dg)^2} - \frac{\left((2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8ce^2(ef-dg)^2} - \frac{\left(3(2cf-bg)\left(cf^2-bfg+ag^2\right)\right)\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)} + \frac{\left(2e\left(cf^2-bfg+ag^2\right)^2\right)\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^2} + \frac{\left(e(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cg^3(ef-dg)^2} - \frac{\left(3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4g^3(ef-dg)}$$

$$
\begin{aligned}
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} \\
&+ \frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)} \\
&- \frac{e(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^2} \\
&+ \frac{(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)} \\
&- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^2(ef-dg)^2} \\
&+ \frac{e(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^2} \\
&- \frac{3(8c^2f^2+b^2g^2-4cg(2bf-ag))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)} \\
&+ \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2(ef-dg)^2} \\
&+ \frac{3(2cf-bg)\sqrt{cf^2-bfg+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)} \\
&- \frac{e(cf^2-bfg+ag^2)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.93 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.45

$$
\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \frac{-\sqrt{c}(ef-dg)^2(4cef+2cdg-3beg)(f+gx)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2(cd^2+
$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*e)
)^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)*Sqr
t[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x))
- e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*f) + 3
*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3*(e*f -
d*g)^2*(f + g*x))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.14

method	result
risch	$\frac{\sqrt{c x^2+bx+a}\,c}{g^2 e}+\frac{\frac{\sqrt{c}\,(3beg-2cdg-4cef)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{eg}+\frac{2e\left(a^2g^4-2abf\,g^3+2ac\,f^2g^2+b^2f^2g^2-2bc\,f^3g+c^2f^4\right)\left(-\frac{g^2\sqrt{\left(x+\frac{f}{g}\right)^2c+\cdots}}{(a\,g\cdots}\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^2/e*(c*x^2+b*x+a)^(1/2)*c+1/2/g^2/e*(c^(1/2)*(3*b*e*g-2*c*d*g-4*c*e*f)/
e/g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*e/g^3*(a^2*g^4-2*a*b*f*g^
3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c^2*f^4)/(d*g-e*f)*(-1/(a*g^2-b*f*g
+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))/(x+f/g)))-2/e^2*g^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2
-2*b*c*d^3*e+c^2*d^4)/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
))+2*e/g^2*(a^2*e*g^4-2*a*b*d*g^4+4*a*c*d*f*g^3-2*a*c*e*f^2*g^2+2*b^2*d*f*g
^3-b^2*e*f^2*g^2-6*b*c*d*f^2*g^2+4*b*c*e*f^3*g+4*c^2*d*f^3*g-3*c^2*e*f^4)/(
d*g-e*f)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b
*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c
*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)^2\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)), x)
```

3.868 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1066

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3}$$
$$+\frac{3e(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)^2}-\frac{3(4cf-bg+2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)(f+gx)}$$
$$-\frac{e^2(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^3}$$
$$+\frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2}+\frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)}$$
$$-\frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e(ef-dg)^3}$$
$$+\frac{3\sqrt{c}(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)}$$
$$+\frac{e^2(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^3}$$
$$-\frac{3e(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)^2}$$
$$+\frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)^3}$$
$$+\frac{3e(2cf-bg)\sqrt{cf^2-bfg+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef-dg)^2}$$
$$-\frac{e^2(cf^2-bfg+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^3}$$
$$-\frac{3(8c^2f^2+b^2g^2-4cg(2bf-ag))\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8g^3(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

```
[Out] 1/2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)^2+e*(c*x^2+b*x+a)^(3/2)/(-d*g+e*
f)^2/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arc
tanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/(-d*g+e*f)^3+1/16
*e^2*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*g+2*b*f))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-d*g+e*f)^3+(a*e^2-b*d*e+c*
d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^2+b*x+a)^(1/2))/e/(-d*g+e*f)^3-e^2*(a*g^2-b*f*g+c*f^2)^(3/2)*arctanh(
1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2
```

```
))/g^3/(-d*g+e*f)^3-3/8*e*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2/c^(1/2)+3/2*(-b*g
+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/g^3/(-d*
g+e*f)-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-
b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)
/(a*g^2-b*f*g+c*f^2)^(1/2)+3/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+
2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2
)^(1/2)/g^3/(-d*g+e*f)^2+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*
(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/(-d*g+e*f)^3+3/4*e*(-2*c*g*x-3*b*g+4*
c*f)*(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)^2-3/4*(2*c*g*x-b*g+4*c*f)*(c*x^2+b*
x+a)^(1/2)/g^2/(-d*g+e*f)/(g*x+f)-1/8*e^2*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+
5*b*f)-2*c*g*(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^3
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 1066, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used

$= \{974, 748, 828, 857, 635, 212, 738, 746, 826\}$

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}$$
$$-\frac{(cf^2-bgf+ag^2)^{3/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{g^3(ef-dg)^3}$$
$$-\frac{\left(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x\right)\sqrt{cx^2+bx+a}\,e^2}{8cg^2(ef-dg)^3}$$
$$+\frac{(cx^2+bx+a)^{3/2}e}{(ef-dg)^2(f+gx)}-\frac{3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e}{8\sqrt{c}g^3(ef-dg)^2}$$
$$+\frac{3(2cf-bg)\sqrt{cf^2-bgf+ag^2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{2g^3(ef-dg)^2}$$
$$+\frac{3(4cf-3bg-2cgx)\sqrt{cx^2+bx+a}\,e}{4g^2(ef-dg)^2}+\frac{(cx^2+bx+a)^{3/2}}{2(ef-dg)(f+gx)^2}$$
$$+\frac{3\sqrt{c}(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{2g^3(ef-dg)}$$
$$-\frac{3\left(8c^2f^2+b^2g^2-4cg(2bf-ag)\right)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8g^3(ef-dg)\sqrt{cf^2-bgf+ag^2}}$$
$$+\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{cx^2+bx+a}}{8c(ef-dg)^3}$$
$$-\frac{3(4cf-bg+2cgx)\sqrt{cx^2+bx+a}}{4g^2(ef-dg)(f+gx)}$$
$$-\frac{(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{16c^{3/2}(ef-dg)^3e}$$
$$+\frac{(cd^2-bed+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{(ef-dg)^3e}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
```

```
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)^3) + (
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^3(a+bx+cx^2)^{3/2}}{(ef-dg)^3(d+ex)} - \frac{g(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)^3} - \frac{eg(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g(a+bx+cx^2)^{3/2}}{(ef-dg)^3(f+gx)}\right) dx$$

$$= \frac{e^3\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{(e^2g)\int \frac{(a+bx+cx^2)^{3/2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{(a+bx+cx^2)^{3/2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int \frac{(a+bx+cx^2)^{3/2}}{(f+gx)^3}\,dx}{ef-dg}$$

$$= \frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{e^2\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2(ef-dg)^3} + \frac{e^2\int \frac{(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{2(ef-dg)^3} - \frac{(3e)\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{2(ef-dg)^2} - \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{4(ef-dg)}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)^2} - \frac{3(4cf-bg+2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)(f+gx)} - \frac{e^2(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^3} + \frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)} + \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)^2-d(2cd-be)(4bcd-b^2e-4ace))-\frac{1}{2}(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8c(ef-dg)^3} - \frac{e^2\int \frac{\frac{1}{2}(4cg(bf-2ag)^2-f(2cf-bg)(4bcf-b^2g-4acg))-\frac{1}{2}(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag))x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8cg^2(ef-dg)^3} + \frac{(3e)\int \frac{c(3b^2fg+4acfg-4b(cf^2+ag^2))-c(8c^2f^2+b^2g^2-4cg(2bf-ag))x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8cg^2(ef-dg)^2} + \frac{3\int \frac{4bcf-b^2g-4acg+4c(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8g^2(ef-dg)}$$

$$= \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x)\sqrt{a + bx + cx^2}}{8c(ef - dg)^3}$$

$$+ \frac{3e(4cf - 3bg - 2cgx)\sqrt{a + bx + cx^2}}{4g^2(ef - dg)^2} - \frac{3(4cf - bg + 2cgx)\sqrt{a + bx + cx^2}}{4g^2(ef - dg)(f + gx)}$$

$$- \frac{e^2(8c^2f^2 + b^2g^2 - 2cg(5bf - 4ag) - 2cg(2cf - bg)x)\sqrt{a + bx + cx^2}}{8cg^2(ef - dg)^3}$$

$$+ \frac{(a + bx + cx^2)^{3/2}}{2(ef - dg)(f + gx)^2} + \frac{e(a + bx + cx^2)^{3/2}}{(ef - dg)^2(f + gx)}$$

$$+ \frac{(cd^2 - bde + ae^2)^2 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef - dg)^3}$$

$$- \frac{((2cd - be)(8c^2d^2 - b^2e^2 - 4ce(2bd - 3ae))) \int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16ce(ef - dg)^3}$$

$$+ \frac{(3c(2cf - bg)) \int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2g^3(ef - dg)}$$

$$+ \frac{(3e(2cf - bg)(cf^2 - bfg + ag^2)) \int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2g^3(ef - dg)^2}$$

$$- \frac{\left(e^2(cf^2 - bfg + ag^2)^2\right) \int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g^3(ef - dg)^3}$$

$$+ \frac{(e^2(2cf - bg)(8c^2f^2 - b^2g^2 - 4cg(2bf - 3ag))) \int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cg^3(ef - dg)^3}$$

$$- \frac{(3e(8c^2f^2 + b^2g^2 - 4cg(2bf - ag))) \int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8g^3(ef - dg)^2}$$

$$- \frac{(3(8c^2f^2 + b^2g^2 - 4cg(2bf - ag))) \int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8g^3(ef - dg)}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)^2} - \frac{3(4cf-bg+2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)(f+gx)} - \frac{e^2(8c^2f^2+b^2g^2-2cg(5bf-4ag)-2cg(2cf-bg)x)\sqrt{a+bx+cx^2}}{8cg^2(ef-dg)^3} + \frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{\left(2(cd^2-bde+ae^2)^2\right)\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)^3} - \frac{((2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8ce(ef-dg)^3} + \frac{(3c(2cf-bg))\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)} - \frac{(3e(2cf-bg)(cf^2-bfg+ag^2))\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^2} + \frac{\left(2e^2(cf^2-bfg+ag^2)^2\right)\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^3(ef-dg)^3} + \frac{(e^2(2cf-bg)(8c^2f^2-b^2g^2-4cg(2bf-3ag)))\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cg^3(ef-dg)^3} - \frac{(3e(8c^2f^2+b^2g^2-4cg(2bf-ag)))\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4g^3(ef-dg)^2} + \frac{(3(8c^2f^2+b^2g^2-4cg(2bf-ag)))\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{4g^3(ef-dg)}$$

$$
\begin{aligned}
&= \frac{\left(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x\right)\sqrt{a + bx + cx^2}}{8c(ef - dg)^3} \\
&+ \frac{3e(4cf - 3bg - 2cgx)\sqrt{a + bx + cx^2}}{4g^2(ef - dg)^2} - \frac{3(4cf - bg + 2cgx)\sqrt{a + bx + cx^2}}{4g^2(ef - dg)(f + gx)} \\
&- \frac{e^2\left(8c^2f^2 + b^2g^2 - 2cg(5bf - 4ag) - 2cg(2cf - bg)x\right)\sqrt{a + bx + cx^2}}{8cg^2(ef - dg)^3} \\
&+ \frac{\left(a + bx + cx^2\right)^{3/2}}{2(ef - dg)(f + gx)^2} + \frac{e\left(a + bx + cx^2\right)^{3/2}}{(ef - dg)^2(f + gx)} \\
&- \frac{(2cd - be)\left(8c^2d^2 - b^2e^2 - 4ce(2bd - 3ae)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e(ef - dg)^3} \\
&+ \frac{3\sqrt{c}(2cf - bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef - dg)} \\
&+ \frac{e^2(2cf - bg)\left(8c^2f^2 - b^2g^2 - 4cg(2bf - 3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef - dg)^3} \\
&- \frac{3e\left(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef - dg)^2} \\
&+ \frac{\left(cd^2 - bde + ae^2\right)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef - dg)^3} \\
&+ \frac{3e(2cf - bg)\sqrt{cf^2 - bfg + ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2g^3(ef - dg)^2} \\
&- \frac{e^2\left(cf^2 - bfg + ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^3(ef - dg)^3} \\
&- \frac{3\left(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)\right)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8g^3(ef - dg)\sqrt{cf^2 - bfg + ag^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 12.29 (sec) , antiderivative size = 1036, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \frac{1}{4}\left(\frac{2(a+x(b+cx))^{3/2}}{(ef-dg)(f+gx)^2}+\frac{4e(a+x(b+cx))^{3/2}}{(ef-dg)^2(f+gx)}\right.$$

$$+\frac{-\left((2cd-be)\left(8c^2d^2-b^2e^2+4ce(-2bd+3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}(-b^2e^2\right.}{4c^{3/2}e(}$$

$$-\frac{3e\left(\left(8c^2f^2+b^2g^2+4cg(-2bf+ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(g(-4cf+3bg+2cgx)\sqrt{a+x(b+c\right.\right.}{2\sqrt{c}g^3(ef-dg)^2}$$

$$+\frac{3\left(\frac{(-2cf+bg)(a+x(b+cx))^{3/2}}{f+gx}-\frac{\sqrt{a+x(b+cx)}\left(b^2g^2+2c^2f(2f-gx)+cg(-5bf+2ag+bgx)\right)}{g^2}+\frac{4\sqrt{c}(2cf-bg)\left(cf^2+g(-bf+ag)\right)\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.}{}\right.}{(ef-dg)\left(cf^2+g(-bf+a\right.}$$

$$-\frac{e^2\left(\left(2cf-bg\right)\left(8c^2f^2-b^2g^2+4cg(-2bf+3ag)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(g\sqrt{a+x(b+cx)}(-b^2g^2\right.\right.}{4c^{3/2}g}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b + c
*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-(((2*c*d - b*e)*(8*c^2*d^2 - b^2*e
^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d
+ e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*((8*c^2
*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[a + x*(b
+ c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) +
2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b +
c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*(a + x*(b +
c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c^2*f*(2*f - g
*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f - b*g)*(c*f^2
```

```
+ g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 + g*(-(b*f) + a*g
)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a
*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a
*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g*(-2*b*f + 3*a*g))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*Sqrt[
a + x*(b + c*x)]*(-(b^2*g^2) + 4*c^2*f*(-2*f + g*x) - 2*c*g*(-5*b*f + 4*a*g
 + b*g*x)) + 8*c*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g -
 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)))/(4*c^(3/2)*g^3*(-(e*f) + d*g)^3))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4241 vs. $2(968) = 1936$.

Time = 0.96 (sec) , antiderivative size = 4242, normalized size of antiderivative = 3.98

method	result	size
default	Expression too large to display	4242

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+1/4*(b*g-2*c*f)*g/(a*g^2-b*f
*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*
(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*
(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b
*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f
^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+
1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f
^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+4*c/(a*g^2-b*f*g+c*f^
2)*g^2*(1/8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c
*f)^2/g^2)/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*
g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))+3/2*c/(a*g^2-b*
f*g+c*f^2)*g^2*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/
g^2)^(3/2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*
c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*
```

g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))))-e^2/(d*g-e*f)^3*(1/3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*(((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))+e^2/(d*g-e*f)^3*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+4*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g

```
^2)/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f
)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)(gx+f)^3}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)^3\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)), x)
```

3.869 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 886

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \frac{(cd^2-bde+ae^2)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx}}{8ce^4(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(13ef-d}{64ceg^4(ef-}$$
$$+\frac{(cd^2-bde+ae^2)\left(a+bx+cx^2\right)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(a+bx+cx^2\right)^{3/2}}{24eg^2(ef-dg)}$$
$$-\frac{(2cd-be)\left(cd^2-bde+ae^2\right)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^5(ef-dg)}$$
$$+\frac{(128c^4ef^5-320c^3ef^3g(bf-ag)-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abef^2g+a^2g^2(5ef-}{128c^{3/2}eg^5(ef-dg)}$$
$$+\frac{\left(cd^2-bde+ae^2\right)^{5/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5(ef-dg)}$$
$$-\frac{\left(cf^2-bfg+ag^2\right)^{5/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^5(ef-dg)}$$

```
[Out] 1/3*(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(3/2)/e^2/(-d*g+e*f)-1/24*(8*c*e*f^2-
g*(-8*a*e*g-3*b*d*g+11*b*e*f)-6*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(3/2)/e/g^2
/(-d*g+e*f)-1/16*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2-b^2*e^2-4*c*e*
(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/
e^5/(-d*g+e*f)+1/128*(128*c^4*e*f^5-320*c^3*e*f^3*g*(-a*g+b*f)-b^3*g^4*(-8*
```

```
a*e*g+3*b*d*g+5*b*e*f)+48*c^2*g^2*(5*b^2*e*f^3-10*a*b*e*f^2*g+a^2*g^2*(-d*g
+5*e*f))-8*b*c*g^3*(5*b^2*e*f^2+12*a^2*e*g^2-3*a*b*g*(d*g+5*e*f)))*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/g^5/(-d*g+e*f)+(a*e^2-
b*d*e+c*d^2)^(5/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(5/2)*arct
anh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/g^5/(-d*g+e*f)+1/8*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*
a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^4/(-d*g+e*f)-1/64*
(64*c^3*e*f^4-16*c^2*e*f^2*g*(-8*a*g+9*b*f)-b^2*g^3*(-8*a*e*g+3*b*d*g+5*b*e
*f)+4*c*g^2*(22*b^2*e*f^2+16*a^2*e*g^2-3*a*b*g*(-d*g+13*e*f))-2*c*g*(16*c^2
*e*f^3+b*g^2*(-8*a*e*g+3*b*d*g+5*b*e*f)-4*c*g*(6*b*e*f^2-a*g*(-3*d*g+7*e*f)
))*x)*(c*x^2+b*x+a)^(1/2)/c/e/g^4/(-d*g+e*f)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {909, 748, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)^{5/2}}{e^5(ef-dg)}$$
$$+\frac{(cx^2+bx+a)^{3/2}(cd^2-bed+ae^2)}{3e^2(ef-dg)}$$
$$-\frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)}{16c^{3/2}e^5(ef-dg)}$$
$$+\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{cx^2+bx+a}(cd^2-bed+ae^2)}{8ce^4(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)(cx^2+bx+a)^{3/2}}{24eg^2(ef-dg)}$$
$$+\frac{(128c^4ef^5-320c^3eg(bf-ag)f^3-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abegf^2+a^2g^2(5ef-dg$$
$$128c^{3/2}eg^5(ef-dg)$$
$$-\frac{(cf^2-bgf+ag^2)^{5/2}\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{g^5(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(13ef-dg}{64ceg^4(ef-d}$$

```
[In] Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
```

```
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\,(a+bx+cx^2)^{3/2}}{24eg^2(ef-dg)}$$
$$-\frac{(cd^2-bde+ae^2)\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e^2(ef-dg)}$$
$$+\frac{\int \frac{\left(\frac{1}{2}c\left(f\left(8bcf-3b^2g-4acg\right)(ef-dg)+8g(bf-2ag)(cdf-bef+aeg)\right)+\frac{1}{2}c\left(16c^2ef^3+bg^2(5bef+3bdg-8aeg)-4cg\left(6bef^2-ag(7ef-3\right.\right.\right.}{f+gx}}{8ceg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\,(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3ab}{64}$$
$$+\frac{(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\,(a+bx+cx^2)^{3/2}}{24eg^2(ef-dg)}$$
$$+\frac{(cd^2-bde+ae^2)\int \frac{\frac{1}{2}\left(4ce(bd-2ae)^2-d(2cd-be)\left(4bcd-b^2e-4ace\right)\right)-\frac{1}{2}(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{8ce^4(ef-dg)}$$
$$-\frac{\int \frac{1}{4}c\left(4cg(bf-2ag)\left(f\left(8bcf-3b^2g-4acg\right)(ef-dg)+8g(bf-2ag)(cdf-bef+aeg)\right)-f\left(4bcf-b^2g-4acg\right)\left(16c^2ef^3+bg^2(5bef+3bdg-\right.\right.}{}$$

$$= \frac{(cd^2-bde+ae^2)\,(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3ab}{64}$$
$$+\frac{(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\,(a+bx+cx^2)^{3/2}}{24eg^2(ef-dg)}$$
$$+\frac{(cd^2-bde+ae^2)^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^5(ef-dg)}$$
$$-\frac{((2cd-be)\,(cd^2-bde+ae^2)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16ce^5(ef-dg)}$$
$$-\frac{(cf^2-bfg+ag^2)^3\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{g^5(ef-dg)}$$
$$+\frac{(128c^4ef^5-320c^3ef^3g(bf-ag)-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abef^2g+a}{128ceg^5(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\,(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(}{64c}$$
$$+\frac{(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\,(a+bx+cx^2)^{3/2}}{24eg^2(ef-dg)}$$
$$-\frac{\left(2(cd^2-bde+ae^2)^3\right)\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^5(ef-dg)}$$
$$-\frac{((2cd-be)\,(cd^2-bde+ae^2)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8ce^5(ef-dg)}$$
$$+\frac{\left(2(cf^2-bfg+ag^2)^3\right)\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{g^5(ef-dg)}$$
$$+\frac{(128c^4ef^5-320c^3ef^3g(bf-ag)-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abef^2g+a^2g}{64ceg^5(ef}$$

$$= \frac{(cd^2-bde+ae^2)\,(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}$$
$$-\frac{(64c^3ef^4-16c^2ef^2g(9bf-8ag)-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(}{64c}$$
$$+\frac{(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)}$$
$$-\frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)\,(a+bx+cx^2)^{3/2}}{24eg^2(ef-dg)}$$
$$-\frac{(2cd-be)\,(cd^2-bde+ae^2)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^5(ef-dg)}$$
$$+\frac{(128c^4ef^5-320c^3ef^3g(bf-ag)-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abef^2g+a^2g}{128c^{3/2}eg^5(ef-}$$
$$+\frac{(cd^2-bde+ae^2)^{5/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5(ef-dg)}$$
$$-\frac{(cf^2-bfg+ag^2)^{5/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g^5(ef-dg)}$$

Mathematica [A] (verified)

Time = 11.70 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \frac{3(5b^4e^4g^4(-ef+dg)-40b^2ce^3g^3(ef-dg)(bef+bdg-3aeg)+320c^3eg(-be^4f^4}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b*
d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*d^
3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^2*e
^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-(e*f)
 + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*e*g +
b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^2*(-2*f
 + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6*f^2*g*x
 + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g + 27*e*g*
x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f*g*x + 17*
g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x
)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.41

method	result	size
risch	Expression too large to display	1248
default	Expression too large to display	2107

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/c*(48*c^3*e^3*g^3*x^3+136*b*c^2*e^3*g^3*x^2-64*c^3*d*e^2*g^3*x^2-64*c
^3*e^3*f*g^2*x^2+216*a*c^2*e^3*g^3*x+118*b^2*c*e^3*g^3*x-208*b*c^2*d*e^2*g^
3*x-208*b*c^2*e^3*f*g^2*x+96*c^3*d^2*e*g^3*x+96*c^3*d*e^2*f*g^2*x+96*c^3*e^
3*f^2*g*x+556*a*b*c*e^3*g^3-448*a*c^2*d*e^2*g^3-448*a*c^2*e^3*f*g^2+15*b^3*
e^3*g^3-264*b^2*c*d*e^2*g^3-264*b^2*c*e^3*f*g^2+432*b*c^2*d^2*e*g^3+432*b*c
^2*d*e^2*f*g^2+432*b*c^2*e^3*f^2*g-192*c^3*d^3*g^3-192*c^3*d^2*e*f*g^2-192*
c^3*d*e^2*f^2*g-192*c^3*e^3*f^3)*(c*x^2+b*x+a)^(1/2)/e^4/g^4+1/128/g^4/e^4/
c*(128/e^2*g^4*c*(a^3*e^6-3*a^2*b*d*e^5+3*a^2*c*d^2*e^4+3*a*b^2*d^2*e^4-6*a
*b*c*d^3*e^3+3*a*c^2*d^4*e^2-b^3*d^3*e^3+3*b^2*c*d^4*e^2-3*b*c^2*d^5*e+c^3*
```

```
d^6)/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-128*e^4/g^2*c*(a
^3*g^6-3*a^2*b*f*g^5+3*a^2*c*f^2*g^4+3*a*b^2*f^2*g^4-6*a*b*c*f^3*g^3+3*a*c^
2*f^4*g^2-b^3*f^3*g^3+3*b^2*c*f^4*g^2-3*b*c^2*f^5*g+c^3*f^6)/(d*g-e*f)/((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+
f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+(240*a^2*c^2*e^4*g^4+120*a*b^2*c*e^
4*g^4-480*a*b*c^2*d*e^3*g^4-480*a*b*c^2*e^4*f*g^3+320*a*c^3*d^2*e^2*g^4+320
*a*c^3*d*e^3*f*g^3+320*a*c^3*e^4*f^2*g^2-5*b^4*e^4*g^4-40*b^3*c*d*e^3*g^4-4
0*b^3*c*e^4*f*g^3+240*b^2*c^2*d^2*e^2*g^4+240*b^2*c^2*d*e^3*f*g^3+240*b^2*c
^2*e^4*f^2*g^2-320*b*c^3*d^3*e*g^4-320*b*c^3*d^2*e^2*f*g^3-320*b*c^3*d*e^3*
f^2*g^2-320*b*c^3*e^4*f^3*g+128*c^4*d^4*g^4+128*c^4*d^3*e*f*g^3+128*c^4*d^2
*e^2*f^2*g^2+128*c^4*d*e^3*f^3*g+128*c^4*e^4*f^4)/e/g*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{5/2}}{(f+g\,x)\,(d+e\,x)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)), x)
```

3.870 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 431

$$\begin{aligned}\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3} \\ &+ \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3}+\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} \\ &- \frac{g(5b^3e^3g^3-6bce^2g^2(4bef-bdg+2aeg)-16c^3(4e^3f^3-6de^2f^2g+4d^2efg^2-d^3g^3)+8c^2eg(aeg(4ef-}{16c^{7/2}e^4} \\ &+ \frac{(ef-dg)^4\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

```
[Out] -1/16*g*(5*b^3*e^3*g^3-6*b*c*e^2*g^2*(2*a*e*g-b*d*g+4*b*e*f)-16*c^3*(-d^3*g
^3+4*d^2*e*f*g^2-6*d*e^2*f^2*g+4*e^3*f^3)+8*c^2*e*g*(a*e*g*(-d*g+4*e*f)+b*(
d^2*g^2-4*d*e*f*g+6*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(7/2)/e^4+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)+1/
24*g^2*(15*b^2*e^2*g^2-4*c*e*g*(4*a*e*g-7*b*d*g+18*b*e*f)+4*c^2*(11*d^2*g^2
-36*d*e*f*g+36*e^2*f^2))*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/12*g^3*(-5*b*e*g-14*
c*d*g+24*c*e*f)*(e*x+d)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/3*g^4*(e*x+d)^2*(c*x^
2+b*x+a)^(1/2)/c/e^3
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1667, 857, 635, 212, 738}

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{g\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg(aeg(4ef-dg)+b(d^2g^2-4defg+6e^2f^2))-6bce^2g^2(2aeg-bdg+4b\right.}{16c^{7/2}e^4}$$
$$+\frac{(ef-dg)^4\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}}$$
$$+\frac{g^2\sqrt{a+bx+cx^2}(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2(11d^2g^2-36defg+36e^2f^2))}{24c^3e^3}$$
$$+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg-14cdg+24cef)}{12c^2e^3}+\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

```
[In] Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{2}e\left(6ce^3f^4-d^2(bd+4ae)g^4\right)-\frac{1}{2}eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right)x-\frac{1}{2}e^2g^2\left(e(11bd+4ae)g^2-c\left(36e^2f^2-10d^2g^2\right)\right)x^2+\frac{1}{2}e^3g^3(24cef-14}{(d+ex)\sqrt{a+bx+cx^2}}}{3ce^4}$$

$$= \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{4}e^4\left(24c^2e^3f^4+5bde(bd+2ae)g^4-2cdg^3(bd(12ef-5dg)+6ae(4ef-dg))\right)+\frac{1}{2}e^4g\left(5be^2(2bd+ae)g^3+2c^2\left(24e^3f^3-12d^2efg^2+5d^3g^3\right)}{(d+ex)\sqrt{a+b}}}{6c^2e}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{a+bx+cx^2}}{24c^3e^3} + \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{3}{8}e^6\left(16c^3e^3f^4-5b^3de^2g^4+6bcdeg^3(4bef-bdg+2aeg)-8c^2dg^2\left(aeg(4ef-dg)+b\left(6e^2f^2-4defg+d^2g^2\right)\right)\right)-\frac{3}{8}e^6g\left(5b^3e^3g^3-6bce^2g^2\right.}{(d+ex)\sqrt{a+bx+cx^2}}}{6c^3e^9}$$

$$= \frac{g^2(15b^2e^2g^2 - 4ceg(18bef - 7bdg + 4aeg) + 4c^2(36e^2f^2 - 36defg + 11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3} + \frac{g^3(24cef - 14cdg - 5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{(ef-dg)^4\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^4} - \frac{(g(5b^3e^3g^3 - 6bce^2g^2(4bef - bdg + 2aeg) - 16c^3(4e^3f^3 - 6de^2f^2g + 4d^2efg^2 - d^3g^3) + 8c^2eg(}{16c^3e^4}$$

$$= \frac{g^2(15b^2e^2g^2 - 4ceg(18bef - 7bdg + 4aeg) + 4c^2(36e^2f^2 - 36defg + 11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3} + \frac{g^3(24cef - 14cdg - 5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} - \frac{(2(ef-dg)^4)\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^4} - \frac{(g(5b^3e^3g^3 - 6bce^2g^2(4bef - bdg + 2aeg) - 16c^3(4e^3f^3 - 6de^2f^2g + 4d^2efg^2 - d^3g^3) + 8c^2eg(}{8c^3e^4}$$

$$= \frac{g^2(15b^2e^2g^2 - 4ceg(18bef - 7bdg + 4aeg) + 4c^2(36e^2f^2 - 36defg + 11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3} + \frac{g^3(24cef - 14cdg - 5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} - \frac{g(5b^3e^3g^3 - 6bce^2g^2(4bef - bdg + 2aeg) - 16c^3(4e^3f^3 - 6de^2f^2g + 4d^2efg^2 - d^3g^3) + 8c^2eg(a}{16c^{7/2}e^4} + \frac{(ef-dg)^4\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{2eg^2\sqrt{a+x(b+cx)}(15b^2e^2g^2 - 2ceg(8aeg + b(36ef - 9dg + 5egx)) + 4c^2(6d^2g^2 - 3deg(8f+gx) + 2e^2(18f^2 + 6fgx + g^2x^2)))}{c^3} + \frac{96\sqrt{-cd^2+bde-ae^2}(}{}}{}$$

```
[In] Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)]*(15*b^2*e^2*g^2 - 2*c*e*g*(8*a*e*g + b*(36*
e*f - 9*d*g + 5*e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(8*f + g*x) + 2*e^2*(1
```

8*f^2 + 6*f*g*x + g^2*x^2))))/c^3 + (96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (3*g*(5*b^3*e^3*g^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(7/2))/(48*e^4)

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{g^2(-8c^2e^2g^2x^2+10bce^2g^2x+12c^2deg^2x-48c^2e^2fgx+16ace^2g^2-15b^2e^2g^2-18bcdeg^2+72bce^2fg-24c^2d^2g^2+96c^2defg-144c^2e^2f}{24c^3e^3}$
default	$-\frac{(g^4d^4-4d^3efg^3+6d^2e^2f^2g^2-4de^3f^3g+e^4f^4)\ln\left(\frac{\frac{2e^2a-2bde+2cd^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}}}{x+\frac{d}{e}}\right.}{e^5\sqrt{\frac{e^2a-bde+cd^2}{e^2}}}$

[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/24*g^2*(-8*c^2*e^2*g^2*x^2+10*b*c*e^2*g^2*x+12*c^2*d*e*g^2*x-48*c^2*e^2*f*g*x+16*a*c*e^2*g^2-15*b^2*e^2*g^2-18*b*c*d*e*g^2+72*b*c*e^2*f*g-24*c^2*d^2*g^2+96*c^2*d*e*f*g-144*c^2*e^2*f^2)*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/16/c^3/e^3*(-16*(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)*c^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+g*(12*a*b*c*e^3*g^3+8*a*c^2*d*e^2*g^3-32*a*c^2*e^3*f*g^2-5*b^3*e^3*g^3-6*b^2*c*d*e^2*g^3+24*b^2*c*e^3*f*g^2-8*b*c^2*d^2*e*g^3+32*b*c^2*d*e^2*f*g^2-48*b*c^2*e^3*f^2*g-16*c^3*d^3*g^3+64*c^3*d^2*e*f*g^2-96*c^3*d*e^2*f^2*g+64*c^3*e^3*f^3)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{{\left(f+g\,x\right)}^4}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.871 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 270

$$\begin{aligned}\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}\\ &\quad+\frac{g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^3}\\ &\quad+\frac{(ef-dg)^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

```
[Out] 1/8*g*(3*b^2*e^2*g^2-4*c*e*g*(a*e*g-b*d*g+3*b*e*f)+8*c^2*(d^2*g^2-3*d*e*f*g
+3*e^2*f^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3
+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1
/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)+3/4*g^2*(-b*e*g-2*c*
d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*g^3*(e*x+d)*(c*x^2+b*x+a)^(1/2
)/c/e^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1667, 857, 635, 212, 738}

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2(d^2g^2-3defg+3e^2f^2))}{8c^{5/2}e^3}$$
$$+\frac{(ef-dg)^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}}$$
$$+\frac{3g^2\sqrt{a+bx+cx^2}(-beg-2cdg+4cef)}{4c^2e^2}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

```
[In] Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{2}e\left(4ce^2f^3-d(bd+2ae)g^3\right)-eg\left(e(2bd+ae)g^2-c\left(6e^2f^2-d^2g^2\right)\right)x+\frac{3}{2}e^2g^2(4cef-2cdg-beg)x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2ce^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{4}e^3\left(8c^2e^2f^3+3b^2deg^3-4cdg^2(3bef-bdg+aeg)\right)+\frac{1}{4}e^3g\left(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2\left(3e^2f^2-3defg+d^2g^2\right)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2c^2e^5}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^3} + \frac{\left(g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^2e^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} - \frac{\left(2(ef-dg)^3\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^3} + \frac{\left(g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+}{\sqrt{a+b}}\right.}{4c^2e^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^3} + \frac{(ef-dg)^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{-\frac{2eg^2\sqrt{a+x(b+cx)}(-3beg+2c(6ef-2dg+egx))}{c^2} + \frac{16\sqrt{-cd^2+bde-ae^2}(-ef+dg)^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{g(3b^2e^2g^2-4ceg(3}{}}{8e^3}$$

```
[In] Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/8*((-2*e*g^2*Sqrt[a + x*(b + c*x)]*(-3*b*e*g + 2*c*(6*e*f - 2*d*g + e*g*
x)))/c^2 + (16*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sqrt
[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(
c*d^2 + e*(-(b*d) + a*e)) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*g +
a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Log[b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + x*(b + c*x)]])/c^(5/2))/e^3
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{g^2(-2cegx+3beg+4cdg-12cef)\sqrt{cx^2+bx+a}}{4c^2e^2} - \frac{8\left(d^3g^3-3d^2efg^2+3de^2f^2g-e^3f^3\right)c^2\ln\left(\frac{\frac{2e^2a-2bde+2cd^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{e^2}}{}\right)}{e^2\sqrt{\frac{e^2a-bde+cd^2}{e^2}}}$
default	$-\frac{\left(-d^3g^3+3d^2efg^2-3de^2f^2g+e^3f^3\right)\ln\left(\frac{\frac{2e^2a-2bde+2cd^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde-}{e^2}}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{e^2a-bde+cd^2}{e^2}}}$

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*g^2*(-2*c*e*g*x+3*b*e*g+4*c*d*g-12*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2-
1/8/c^2/e^2*(-8*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)*c^2/e^2/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d
/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+g*(4*a*c*e^2*g^2-3*b^2*e^2*g^2-4*b*c
*d*e*g^2+12*b*c*e^2*f*g-8*c^2*d^2*g^2+24*c^2*d*e*f*g-24*c^2*e^2*f^2)/e*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^3}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.872 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}} \, dx = \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{g(4cef-2cdg-beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{(ef-dg)^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/2*g*(-b*e*g-2*c*d*g+4*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(3/2)/e^2+(-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+g^
2*(c*x^2+b*x+a)^(1/2)/c/e
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1667, 857, 635, 212, 738}

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}} \, dx = \frac{g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2} + \frac{(ef-dg)^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}} + \frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

```
[In] Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{\int \frac{\frac{1}{2}e(2cef^2-bdg^2)+\frac{1}{2}eg(4cef-2cdg-beg)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ce^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(g(4cef-2cdg-beg))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ce^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} - \frac{(2(ef-dg)^2)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^2} + \frac{(g(4cef-2cdg-beg))\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ce^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{(ef-dg)^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{2eg^2\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cd^2+bde-ae^2}(ef-dg)^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} - \frac{g(-4cef+2cdg+beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}}{2e^2}$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f
 - d*g)^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2
) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) - (g*(-4*c*e*f + 2*c*d*g +
b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2))/(2*
e^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.47

method	result
risch	$\frac{g^{2}\sqrt{c\,x^{2}+b x+a}}{c e}-\frac{\frac{g\left(b e g+2 c d g-4 c e f\right)\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{e\sqrt{c}}+\frac{2\left(d^{2} g^{2}-2 d e f g+e^{2} f^{2}\right) c\ln\left(\frac{2 e^{2} a-2 b d e+2 c\,d^{2}}{e^{2}}+\frac{\left(b e-2 c d\right)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{e^{2} a-b}\right.}{e^{2}\sqrt{\frac{e^{2} a-b d e+c\,d^{2}}{e^{2}}}}}{2 e c}$
default	$-\frac{g\left(\frac{d g\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-\frac{2 e f\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-e g\left(\frac{\sqrt{c\,x^{2}+b x+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{2 c^{\frac{3}{2}}}\right)\right)}{e^{2}}-\frac{\left(d^{2} g^{2}-2 d e f g+\right.}{}$

[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] g^2*(c*x^2+b*x+a)^(1/2)/c/e-1/2/e/c*(g*(b*e*g+2*c*d*g-4*c*e*f)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*(d^2*g^2-2*d*e*f*g+e^2*f^2)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^2}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^2}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.873 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e} + \frac{(ef-dg)\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

```
[Out] g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/c^(1/2)+(-d*g+e*f)*a
rctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {857, 635, 212, 738}

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{(ef-dg)\mathrm{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}} + \frac{g\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}$$

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e}+\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e}\\
&= \frac{(2g)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e}\\
&\quad -\frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e}\\
&= \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}+\frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{\frac{2\sqrt{-cd^2+bde-ae^2}(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{g\log\left(e\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}}{e}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e*
x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(
-(b*d) + a*e)) + (g*Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/S
qrt[c])/e)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

method	result
default	$\frac{g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}-\frac{(-dg+ef)\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-}{}}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}$

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(-d*g+e*f)/e^2/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+
d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(115) = 230$.

Time = 21.85 (sec) , antiderivative size = 1071, normalized size of antiderivative = 8.18

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{(cd^2-bde+ae^2)\sqrt{c}g\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-\sqrt{cd^2-bde+}}{} \right.$$

$$-\frac{2(cd^2-bde+ae^2)\sqrt{-c}g\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)+\sqrt{cd^2-bde+ae^2}(cef-cdg)\log\left(\frac{8abde-8a^2e}{}\right.}{2(c^2d^2e-b}$$

$$-\frac{(cd^2-bde+ae^2)\sqrt{-c}g\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)-\sqrt{-cd^2+bde-ae^2}(cef-cdg)\arctan\left(-\frac{}{2(ac}\right.}{c^2d^2e-bcde^2+ace^3}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e + a
*e^2)*(c*e*f - c*d*g)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c
^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)
*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a
*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b
*c*d*e^2 + a*c*e^3), 1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2
- 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f - c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*
e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 -
 a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(c*d^2 - b*d*e
+ a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*(c*e*f - c*d*g)*log((
8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
 - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -((c*d
^2 - b*d*e + a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f -
c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
- 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*
e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^
2 + a*c*e^3)]
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.874 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}$$

```
[Out] arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {738, 212}

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x^2}\,dx, x, \frac{-bd + 2ae - (2cd - be)x}{\sqrt{a + bx + cx^2}}\right)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2 - bde + ae^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}}\,dx = \frac{2\sqrt{-cd^2 + bde - ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2 + e(-bd + ae)}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*
(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(71) = 142.

Time = 0.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}}$	157

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(71) = 142.

Time = 0.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.34

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-(b^2+4\,ac)d^2-(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^2-4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+a}(bd-2\,ae+(2\,cd-be)x)-2\,(4\,bcd^2+4\,abe^2}{e^2x^2+2\,dex+d^2}\right.}{2\,\sqrt{cd^2-bde+ae^2}}\right.$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sqr
t(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x
^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2
+ (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x))/(c*
d^2 - b*d*e + a*e^2)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.875 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 182

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)} - \frac{g\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

```
[Out] e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)/(a*e^2-b*d*e+c*d^2)^(1/2)-g*arctanh(1/2*(b*f-2*a*g+
(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(
a*g^2-b*f*g+c*f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {974, 738, 212}

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}} - \frac{g\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

```
[In] Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
```

```
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e}{(ef-dg)(d+ex)\sqrt{a+bx+cx^2}}-\frac{g}{(ef-dg)(f+gx)\sqrt{a+bx+cx^2}}\right)dx\\
&= \frac{e\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ef-dg}-\frac{g\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{ef-dg}\\
&= -\frac{(2e)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg}\\
&\quad+\frac{(2g)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg}\\
&= \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)}-\frac{g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{2e\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))\,(-ef+dg)}$$
$$-\frac{2g\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\,(cf^2+g(-bf+ag))}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d) + a*e))
*(-(e*f) + d*g)) - (2*g*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*(f +
 g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/((e*f - d
*g)*(c*f^2 + g*(-(b*f) + a*g)))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\ln\left(\frac{\frac{2a\,g^2-2bfg+2c\,f^2}{g^2}+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{a\,g^2-bfg+c\,f^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^2-bfg+c\,f^2}{g^2}}}{x+\frac{f}{g}}\right)}{(dg-ef)\sqrt{\frac{a\,g^2-bfg+c\,f^2}{g^2}}}+\frac{\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}}{}\right.}{}$

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+
(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/(d*g-e*f)/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(166) = 332.

Time = 85.21 (sec) , antiderivative size = 1952, normalized size of antiderivative = 10.73

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((8*a*b*f*g
 - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)*g
^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g
+ (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)/(g^
2*x^2 + 2*f*g*x + f^2)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^2 - b*d*e
+ a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c*d*e^2 + a
*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^
2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*g^3),
-1/2*(2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*g*arctan(-1/2*
sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g + (2*c*f -
b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a*c*g^2)*x^2 +
(b*c*f^2 - b^2*f*g + a*b*g^2)*x)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*
e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c
*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*
c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e
^2)*g^3), -1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((
8*a*b*f*g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 +
 4*a*c)*g^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f
 - 2*a*g + (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*
g)*x)/(g^2*x^2 + 2*f*g*x + f^2)) - 2*(c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(-c*
d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 +
b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^
2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/((c^2*d
^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f
^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e
 + a^2*d*e^2)*g^3), -((c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*
g*arctan(-1/2*sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a
*g + (2*c*f - b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a
*c*g^2)*x^2 + (b*c*f^2 - b^2*f*g + a*b*g^2)*x)) - (c*e*f^2 - b*e*f*g + a*e*
```

```
g^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*
sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e +
a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)
*x)))/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 -
a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^
3 - a*b*d^2*e + a^2*d*e^2)*g^3)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.876 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 340

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} - \frac{g(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{eg\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\sqrt{cf^2-bfg+ag^2}}$$

```
[Out] -1/2*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)+e^2*arc
tanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/(-d*g+e*f)^2/(a*e^2-b*d*e+c*d^2)^(1/2)-e*g*arctanh(1/2*(b*f-2*a*g+(
-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/
(a*g^2-b*f*g+c*f^2)^(1/2)+g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c
*f^2)/(g*x+f)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {974, 738, 212, 744}

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{e^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}} - \frac{eg\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} - \frac{g(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}} + \frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
```

```
Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^2\sqrt{a+bx+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)\sqrt{a+bx+cx^2}}\right) dx \\
&= \frac{e^2 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{ef-dg} \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{(2e^2)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} \\
&\quad + \frac{(2eg)\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{(g(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)\left(cf^2-bfg+ag^2\right)} \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} \\
&\quad - \frac{eg\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\sqrt{cf^2-bfg+ag^2}} + \frac{(g(2cf-bg))\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)}
\end{aligned}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\,(cf^2-bfg+ag^2)\,(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} - \frac{g(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)\,(cf^2-bfg+ag^2)^{3/2}} - \frac{eg\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\sqrt{cf^2-bfg+ag^2}}$$

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{2g^2(-ef+dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} - \frac{2e^2\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}} + \frac{g(2cf(2ef-dg)+g(-3bef+bdg+2aeg))\text{arctanh}\left(\frac{\cdots}{2\sqrt{cf^2\cdots}}\right)}{(cf^2+g(-bf+ag))^{3/2}}}{2(ef-dg)^2}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*
g))*(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d^2 + e*(-(b*d) + a
*e)] + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(-
2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x
*(b + c*x)])]])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d*g)^2
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.79

method	result
default	$-\frac{e\ln\left(\frac{\frac{2e^2a-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(dg-ef)^2\sqrt{\frac{e^2a-bde+c\,d^2}{e^2}}} + -\frac{g^2\sqrt{\left(x+\frac{f}{g}\right)^2c+\frac{(bg-2c\cdots}{}}}{\left(a\,g^2-bfg\cdots\right)}$

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+1/g/(d*g-e*f)*(-
1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2
```

```
-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+e/(d*g-e*f)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2))/(x+f/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.877 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 587

$$\begin{aligned}
&\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} \\
&\quad + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^3} \\
&\quad - \frac{eg(2cf-bg)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{e^2g\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} \\
&\quad - \frac{g\left(8c^2f^2+3b^2g^2-4cg(2bf+ag)\right)\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}}
\end{aligned}$$

```
[Out] -1/2*e*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/8
*g*(8*c^2*f^2+3*b^2*g^2-4*c*g*(a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c
*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)^(5/2)+e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/(a*e^2-b*d*e+c*d^2)^(1/2)-e^2*g
*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(1/2)+1/2*g^2*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2+3/4*g^2*(-b*g+2*c*f)*(c*x^2+b*
x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)+e*g^2*(c*x^2+b*x+a)^(1/
2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {974, 738, 212, 758, 820, 744}

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{g(-4cg(ag+2bf)+3b^2g^2+8c^2f^2)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}$$
$$+\frac{e^3\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}}$$
$$-\frac{eg(2cf-bg)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}$$
$$+\frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$
$$+\frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^3\sqrt{a+bx+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^2\sqrt{a+bx+cx^2}} - \frac{e^2g}{(ef-dg)^3(f+gx)\sqrt{a+bx+cx^2}} \right) dx$$

$$= \frac{e^3 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{(e^2g)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{1}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^3\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3} + \frac{(2e^2g)\,\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3} - \frac{(eg(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)} + \frac{g\int \frac{\frac{1}{2}(-4cf+3bg)+cgx}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^3} - \frac{e^2g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} + \frac{(eg(2cf-bg))\text{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)} - \frac{\left(g\left(8c^2f^2+3b^2g^2-4cg(2bf+ag)\right)\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^3} - \frac{eg(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{e^2g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} + \frac{(g(8c^2f^2+3b^2g^2-4cg(2bf+ag)))\,\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^3} - \frac{eg(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}} - \frac{e^2g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\sqrt{cf^2-bfg+ag^2}} - \frac{g(8c^2f^2+3b^2g^2-4cg(2bf+ag))\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 11.31 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{4g^2(ef-dg)^2\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2} + \frac{8eg^2(ef-dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} + \frac{8e^3\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}} + \frac{4eg(-2cf+bg)(ef-dg)\text{arct}}{(cf^2\text{-}}}{}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f
 + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*
d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f*x +
```

```
b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*
f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x + b*(f -
 g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^
2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a + x*(b +
 c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 + 3*b^2*g^2
- 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f
^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^
(5/2)))/(8*(e*f - d*g)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(541) = 1082.

Time = 0.91 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.02

method	result	size
default	Expression too large to display	1185

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/4*(b*g-2*c*f)*g/(a*g^2-b*f
*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+e^2/(d*g-e*f)^3/((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2))/(x+d/e))-e^2/(d*g-e*f)^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^
2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2))/(x+f/g))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)
*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c
*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^3\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^3}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2239 vs. $2(541) = 1082$.

Time = 0.77 (sec) , antiderivative size = 2239, normalized size of antiderivative = 3.81

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] -2*e^3*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 + b*d*e - a*e^2))/((e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*s
qrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(24*c^2*e^2*f^4*g - 24*c^2*d*e*f^3*g^2 -
 36*b*c*e^2*f^3*g^2 + 8*c^2*d^2*f^2*g^3 + 28*b*c*d*e*f^2*g^3 + 15*b^2*e^2*f
```

^2*g^3 + 20*a*c*e^2*f^2*g^3 - 8*b*c*d^2*f*g^4 - 10*b^2*d*e*f*g^4 - 20*a*b*e ^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 + 4*a*b*d*e*g^5 + 8*a^2*e^2*g^5)*a rctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b* f*g - a*g^2))/((c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g - 2*b*c*e^3*f^6*g + 3*c^2*d ^2*e*f^5*g^2 + 6*b*c*d*e^2*f^5*g^2 + b^2*e^3*f^5*g^2 + 2*a*c*e^3*f^5*g^2 - c^2*d^3*f^4*g^3 - 6*b*c*d^2*e*f^4*g^3 - 3*b^2*d*e^2*f^4*g^3 - 6*a*c*d*e^2*f ^4*g^3 - 2*a*b*e^3*f^4*g^3 + 2*b*c*d^3*f^3*g^4 + 3*b^2*d^2*e*f^3*g^4 + 6*a* c*d^2*e*f^3*g^4 + 6*a*b*d*e^2*f^3*g^4 + a^2*e^3*f^3*g^4 - b^2*d^3*f^2*g^5 - 2*a*c*d^3*f^2*g^5 - 6*a*b*d^2*e*f^2*g^5 - 3*a^2*d*e^2*f^2*g^5 + 2*a*b*d^3* f*g^6 + 3*a^2*d^2*e*f*g^6 - a^2*d^3*g^7)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 1/ 4*(16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*f^3*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*f^2*g^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^4 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*f*g^4 + 4*(sqrt(c)*x - sqrt (c*x^2 + b*x + a))^3*a*c*e*f*g^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3* b^2*d*g^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^5 - 4*(sqrt(c)* x - sqrt(c*x^2 + b*x + a))^3*a*b*e*g^5 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*e*f^4*g - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d *f^3*g^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*f^3*g^2 + 2 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*f^2*g^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*e*f^2*g^3 - 4*(sqrt(c)*x - sqrt(c*x ^2 + b*x + a))^2*a*c^(3/2)*e*f^2*g^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e*f*g^4 - 8 *(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*e*g^5 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*e*f^4*g - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^2 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*e*f^3*g^2 - 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*e*f^3*g^2 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*e*f^2*g^3 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*e*f^2*g^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^4 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)) *a*b*c*d*f*g^4 - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e*f*g^4 - 28* (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e*f*g^4 + 5*(sqrt(c)*x - sqrt(c*x ^2 + b*x + a))*a*b^2*d*g^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d* g^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*e*g^5 + 10*b^2*c^(3/2)*e* f^4*g - 6*b^2*c^(3/2)*d*f^3*g^2 - 7*b^3*sqrt(c)*e*f^3*g^2 - 32*a*b*c^(3/2)* e*f^3*g^2 + 3*b^3*sqrt(c)*d*f^2*g^3 + 20*a*b*c^(3/2)*d*f^2*g^3 + 27*a*b^2*s qrt(c)*e*f^2*g^3 + 20*a^2*c^(3/2)*e*f^2*g^3 - 11*a*b^2*sqrt(c)*d*f*g^4 - 12 *a^2*c^(3/2)*d*f*g^4 - 28*a^2*b*sqrt(c)*e*f*g^4 + 8*a^2*b*sqrt(c)*d*g^5 + 8 *a^3*sqrt(c)*e*g^5)/((c^2*e^2*f^6 - 2*c^2*d*e*f^5*g - 2*b*c*e^2*f^5*g + c^2 *d^2*f^4*g^2 + 4*b*c*d*e*f^4*g^2 + b^2*e^2*f^4*g^2 + 2*a*c*e^2*f^4*g^2 - 2* b*c*d^2*f^3*g^3 - 2*b^2*d*e*f^3*g^3 - 4*a*c*d*e*f^3*g^3 - 2*a*b*e^2*f^3*g^3 + b^2*d^2*f^2*g^4 + 2*a*c*d^2*f^2*g^4 + 4*a*b*d*e*f^2*g^4 + a^2*e^2*f^2*g^ 4 - 2*a*b*d^2*f*g^5 - 2*a^2*d*e*f*g^5 + a^2*d^2*g^6)*((sqrt(c)*x - sqrt(c*x

```
^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f
- a*g)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^3\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.878 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 496

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$
$$-\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))+bc(c^2df^4+}{}$$
$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{g^3(8cef-2cdg-3beg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}e^2}$$
$$+\frac{(ef-dg)^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\left(cd^2-bde+ae^2\right)^{3/2}}$$

```
[Out] 1/2*g^3*(-3*b*e*g-2*c*d*g+8*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(5/2)/e^2+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(3/2
)-2*(a*b^3*d*g^4-b^2*(a^2*e*g^4+4*a*c*d*f*g^3+c^2*e*f^4)+2*a*c*(a^2*e*g^4+c
^2*f^3*(-4*d*g+e*f)-2*a*c*f*g^2*(-2*d*g+3*e*f))+b*c*(c^2*d*f^4+a^2*g^3*(-3*
d*g+4*e*f)+2*a*c*f^2*g*(3*d*g+2*e*f))+(2*c^4*d*f^4+b^3*(-a*e+b*d)*g^4-b*c*g
^3*(4*b^2*d*f-3*a^2*e*g-4*a*b*(-d*g+e*f))+2*c^2*g^2*(3*b^2*d*f^2-3*a*b*f*(-
2*d*g+e*f)-a^2*g*(-d*g+4*e*f))+c^3*f^2*(4*a*g*(-3*d*g+2*e*f)-b*f*(4*d*g+e*f
)))*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)+g^4*(c*x^2+
b*x+a)^(1/2)/c^2/e
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1660, 1667, 857, 635, 212, 738}

$$\int \frac{(f+gx)^4}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2(-b^2(a^2eg^4+4acdfg^3+c^2ef^4)+x(2c^2g^2(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2)-bcg^3(-3a^2e}{}$$

$$+\frac{g^3\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{2c^{5/2}e^2}$$

$$+\frac{(ef-dg)^4\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ae^2-bde+cd^2)^{3/2}}+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}$$

```
[In] Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral =

$$-\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))+bc(c^2df}{}$$

$$-\frac{2\int\frac{-\frac{(b^2-4ac)\left(bd(bd-ae)g^4-cdg^3(4bdf-4aef+adg)+c^2f^2\left(e^2f^2-4defg+6d^2g^2\right)\right)}{2c^2\left(cd^2-bde+ae^2\right)}-\frac{(b^2-4ac)g^3(4cf-bg)x}{2c^2}-\frac{(b^2-4ac)g^4x^2}{2c}}{(d+ex)\sqrt{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))}{}$$

$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}$$

$$-\frac{2\int\frac{-\frac{(b^2-4ac)e(3bde(bd-ae)g^4+2c^2ef^2(e^2f^2-4defg+6d^2g^2)+cdg^3(2ae(4ef-dg)-bd(8ef+dg)))}{4c(cd^2-bde+ae^2)}-\frac{(b^2-4ac)eg^3(8cef-2cdg-3beg)x}{4c}}{(d+ex)\sqrt{a+bx+cx^2}}dx}{c(b^2-4ac)e^2}$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))}{}$$

$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{(ef-dg)^4\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx}{e^2(cd^2-bde+ae^2)}$$

$$+\frac{(g^3(8cef-2cdg-3beg))\int\frac{1}{\sqrt{a+bx+cx^2}}dx}{2c^2e^2}$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))}{}$$

$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}$$

$$-\frac{(2(ef-dg)^4)\,\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^2(cd^2-bde+ae^2)}$$

$$+\frac{(g^3(8cef-2cdg-3beg))\,\text{Subst}\left(\int\frac{1}{4c-x^2}dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c^2e^2}$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2(3ef-2dg))}{}$$

$$+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{g^3(8cef-2cdg-3beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}e^2}$$

$$+\frac{(ef-dg)^4\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 12.46 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2e\left(-3b^4deg^4x+b^3g^3(3aeg(-d+ex)+cdx(8ef+dg-egx))+b^2\left(3a^2e^2g^4+c^2\left(2e^2f^4-12def^2g^2x+d^2g\right.\right.\right.}{}$$

```
[In] Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*g
 - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2*g
^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) - 2*b
*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*
x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x -
 g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d
*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d*e*g*
(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Log[d + e*x
])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Lo
g[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) - (2*(e*f - d*g)^4*
Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqr
t[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(474) = 948.

Time = 1.04 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	1026
risch	Expression too large to display	4958

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)/e^5*(1/(a*e
^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4
*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-g/e^4*(2*d^3*g^3*(2*c*x+b)/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)-8*e^3*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-e^3*g^
3*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/
```

```
c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^
(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-8*d^2*e*f*g^2*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+12*d*e^2*f^2*g*(2*c*x+b)/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)+(d*e^2*g^3-4*e^3*f*g^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*
(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/
c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-d^2*e*g^3+4*d*e^2*f*
g^2-6*e^3*f^2*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((f + g*x)**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{(f+g\,x)^4}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.879 $\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 357

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2\cdots}{\cdots} + \frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e} + \frac{(ef-dg)^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\,(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] g^3*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e+(-d*g+e*f)
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*(a*d*g^3+c*e*f^3)-2*a*c*(c
*f^2*(-3*d*g+e*f)-a*g^2*(-d*g+3*e*f))-b*(c^2*d*f^3+a^2*e*g^3+3*a*c*f*g*(d*g
+e*f))-(2*c^3*d*f^3-b^2*(-a*e+b*d)*g^3+c*g^2*(-2*a^2*e*g+3*a*b*d*g-3*a*b*e*
f+3*b^2*d*f)+c^2*f*(6*a*g*(-d*g+e*f)-b*f*(3*d*g+e*f)))*x)/c/(-4*a*c+b^2)/(a
*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1660, 857, 635, 212, 738}

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(-x(cg^2(-2a^2eg+3abdg-3abef+3b^2df)-b^2g^3(bd-ae)+c^2f(6ag\cdots}{\cdots} + \frac{g^3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e} + \frac{(ef-dg)^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\,(ae^2-bde+cd^2)^{3/2}}$$

```
[In] Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
```

```
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2eg^3+3acfg(ef+dg))-(2c^3df}{c(b^2-4ac)(cd^2-bde+}$$

$$-\frac{2\int\frac{\frac{(b^2-4ac)(d(bd-ae)g^3-cf(e^2f^2-3defg+3d^2g^2))}{2c(cd^2-bde+ae^2)}-\frac{(b^2-4ac)g^3x}{2c}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2eg^3+3acfg(ef+dg))-}{c(b^2-4ac)(cd^2-}$$

$$+\frac{g^3\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{ce}+\frac{(ef-dg)^3\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(cd^2-bde+ae^2)}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2eg^3+3acfg(ef+dg))-}{c(b^2-4ac)(cd^2-}$$

$$+\frac{(2g^3)\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ce}$$

$$-\frac{(2(ef-dg)^3)\,\mathrm{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e(cd^2-bde+ae^2)}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2eg^3+3acfg(ef+dg))-}{c(b^2-4ac)(cd^2-}$$

$$+\frac{g^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}+\frac{(ef-dg)^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.99

$$\int\frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(-b^3dg^3x+b^2(ag^3(-d+ex)+c(-ef^3+3dfg^2x))+b(a^2eg^3+c^2f^2(-}{}$$

$$+\frac{2(-ef+dg)^3\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\sqrt{-cd^2+e(bd-ae)}\,(cd^2+e(-bd+ae))}-\frac{g^3\log\left(ce\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}e}$$

```
[In] Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

[Out] (2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) + b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x) + d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]) + (2*(-(e*f) + d*g)^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))) - (g^3*Log[c*e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(c^(3/2)*e)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(339) = 678.

Time = 0.84 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.07

method	result
default	$\frac{(-d^3g^3+3d^2efg^2-3de^2f^2g+e^3f^3)\left(\frac{e^2}{(e^2a-bde+cd^2)\sqrt{(x+\frac{d}{e})^2c+\frac{(be-2cd)(x+\frac{d}{e})}{e}+\frac{e^2a-bde+cd^2}{e^2}}}-\frac{(be-\cdots}{(e^2a-bde+cd^2)\left(\frac{4c(e^2a-bde+cd^2)}{e^2}\cdots\right.}\right.}{\cdots}$

[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] (-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+g/e^3*(2*d^2*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+e^2*g^2*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+6*e^2*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-6*d*e*f*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(-d*e*g^2+3*e^2*f*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((f + g*x)**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^3}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.880 $\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 240

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{}} + \frac{(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

```
[Out] (-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*e*f^2+2*a*(a*e*g^2
-c*f*(-2*d*g+e*f))-b*(c*d*f^2+a*g*(d*g+2*e*f))-(2*c^2*d*f^2+b*(-a*e+b*d)*g^
2+c*(2*a*g*(-d*g+2*e*f)-b*f*(2*d*g+e*f)))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1660, 12, 738, 212}

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{(ef-dg)^2\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}} + \frac{2(-x(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2)-b(ag(dg+2ef)+cdf^2)+2a(aeg^2-}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2+b(bd-ae)g^2+c(2ag(2ef-dg)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$-\frac{2\int -\frac{(b^2-4ac)(ef-dg)^2}{2(cd^2-bde+ae^2)(d+ex)\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= \frac{2(b^2ef^2 + 2a(aeg^2 - cf(ef - 2dg)) - b(cdf^2 + ag(2ef + dg)) - (2c^2df^2 + b(bd - ae)g^2 + c(2ag(2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}}$$
$$+ \frac{(ef - dg)^2 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\, dx}{cd^2 - bde + ae^2}$$

$$= \frac{2(b^2ef^2 + 2a(aeg^2 - cf(ef - 2dg)) - b(cdf^2 + ag(2ef + dg)) - (2c^2df^2 + b(bd - ae)g^2 + c(2ag(2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}}$$
$$- \frac{(2(ef - dg)^2)\, \text{Subst}\left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x^2}\, dx, x, \frac{-bd + 2ae - (2cd - be)x}{\sqrt{a + bx + cx^2}}\right)}{cd^2 - bde + ae^2}$$

$$= \frac{2(b^2ef^2 + 2a(aeg^2 - cf(ef - 2dg)) - b(cdf^2 + ag(2ef + dg)) - (2c^2df^2 + b(bd - ae)g^2 + c(2ag(2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}}$$
$$+ \frac{(ef - dg)^2 \tanh^{-1}\left(\frac{bd - 2ae + (2cd - be)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx + cx^2}}\right)}{(cd^2 - bde + ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{(f + gx)^2}{(d + ex)(a + bx + cx^2)^{3/2}}\, dx = 2\left(\frac{-2a^2eg^2 + 2c^2df^2x - 2acdg(2f + gx) + 2acef(f + 2gx) + abg(2ef +}{(b^2 - 4ac)(-cd^2 + e(bd - ae}\right.$$
$$\left. + \frac{\sqrt{-cd^2 + bde - ae^2}(ef - dg)^2 \arctan\left(\frac{\sqrt{c}(d+ex) - e\sqrt{a + x(b + cx)}}{\sqrt{-cd^2 + e(bd - ae)}}\right)}{(cd^2 + e(-bd + ae))^2}\right)$$

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] 2*((-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f + 2
*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-(e
*f*x) + d*(f - 2*g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a +
x*(b + c*x)]) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c
*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(230) = 460.

Time = 0.77 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.31

method	result
default	$-\frac{g\left(\frac{2dg(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{4ef(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-eg\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)\right)}{e^2}+\frac{\left(d^2g^2-2defg+e^2f^2\right)\left(\frac{}{\left(e^2a-\right.}\right.}{}$

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-4*e*f*(2*c*x+b)/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)-e*g*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/(a*e^2-b
*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(
a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(230) = 460.

Time = 3.33 (sec) , antiderivative size = 2023, normalized size of antiderivative = 8.43

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4
*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g
+ (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*
b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log(
(8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*
d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d
*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
```

```
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x), (((a
*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d
^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c
- 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*
f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

Sympy [F]

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((f + g*x)**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(230) = 460.

Time = 0.29 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.22

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{(2c^3d^3f^2-3bc^2d^2ef^2+b^2cde^2f^2+2ac^2de^2f^2-abce^3f^2-2bc^2d^3fg+2b^2cd^2efg+4ac^2d^2efg-6abcde^2fg+4a^2ce^3fg+b^2cd^3g^2-2ac^2d^3g^2-b^3\cdots}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e\cdots}\right.}{\cdots}$$

$$+\frac{2\left(e^2f^2-2defg+d^2g^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f^2 - 3*b*c^2*d^2*e*f^2 + b^2*c*d*e^2*f^2 + 2*a*c^2*d*e^2*f^
2 - a*b*c*e^3*f^2 - 2*b*c^2*d^3*f*g + 2*b^2*c*d^2*e*f*g + 4*a*c^2*d^2*e*f*g
 - 6*a*b*c*d*e^2*f*g + 4*a^2*c*e^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2 -
b^3*d^2*e*g^2 + a*b*c*d^2*e*g^2 + 2*a*b^2*d*e^2*g^2 - 2*a^2*c*d*e^2*g^2 - a
^2*b*e^3*g^2)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*
e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8
*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 2*b^2*c*d^2*
e*f^2 + 2*a*c^2*d^2*e*f^2 + b^3*d*e^2*f^2 - a*b*c*d*e^2*f^2 - a*b^2*e^3*f^2
 + 2*a^2*c*e^3*f^2 - 4*a*c^2*d^3*f*g + 6*a*b*c*d^2*e*f*g - 2*a*b^2*d*e^2*f*
g - 4*a^2*c*d*e^2*f*g + 2*a^2*b*e^3*f*g + a*b*c*d^3*g^2 - a*b^2*d^2*e*g^2 -
 2*a^2*c*d^2*e*g^2 + 3*a^2*b*d*e^2*g^2 - 2*a^3*e^3*g^2)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e -
a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^2}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,dx$$

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.881 $\int \frac{f+gx}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}+\frac{e(ef-dg)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

```
[Out] e*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d*f-b^2*e*f+2*a*c*
e*f-2*a*c*d*g+a*b*e*g+c*(2*c*d*f+2*a*e*g-b*(d*g+e*f))*x)/(-4*a*c+b^2)/(a*e^
2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {836, 12, 738, 212}

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{e(ef-dg)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx+cx^2}} \\ &\quad -\frac{2\int -\frac{(b^2-4ac)e(ef-dg)}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)\,(cd^2-bde+ae^2)}\end{aligned}$$

$$= -\frac{2(bcdf - b^2ef + 2acef - 2acdg + abeg + c(2cdf + 2aeg - b(ef + dg))x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}} + \frac{(e(ef - dg))\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2 - bde + ae^2}$$

$$= -\frac{2(bcdf - b^2ef + 2acef - 2acdg + abeg + c(2cdf + 2aeg - b(ef + dg))x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}} - \frac{(2e(ef - dg))\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{cd^2 - bde + ae^2}$$

$$= -\frac{2(bcdf - b^2ef + 2acef - 2acdg + abeg + c(2cdf + 2aeg - b(ef + dg))x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}} + \frac{e(ef - dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2 - bde + ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06

$$\int \frac{f + gx}{(d + ex)(a + bx + cx^2)^{3/2}}\,dx = \frac{-2b^2ef + 2b(aeg - cefx + cd(f - gx)) + 4c(-adg + cdfx + ae(f + gx))}{(b^2 - 4ac)(-cd^2 + e(bd - ae))\sqrt{a + x(b + cx)}} - \frac{2e\sqrt{-cd^2 + bde - ae^2}(-ef + dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2 + e(-bd + ae))^2}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f
*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(
b + c*x)]) - (2*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqr
t[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/
(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(177) = 354.

Time = 0.70 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.38

method	result
default	$\frac{2g(2cx+b)}{e(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{(-dg+ef)\left(\frac{e^2}{(e^2a-bde+c\,d^2)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{e^2a-bde+c\,d^2}{e^2}}}-\frac{}{(e^2a-bde+c\,d^2)\left(\frac{4c\left(e^2a-bde+c\right.}{e^2}\right.}\right.}{}$

[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*g/e*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(-d*g+e*f)/e^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(177) = 354$.

Time = 2.95 (sec) , antiderivative size = 1663, normalized size of antiderivative = 8.89

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")

[Out] [-1/2*(((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 4*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f - (2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)

```
*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c
)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/
2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
+ (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 -
 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f -
 (2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((
2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2
*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^
2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^
4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*
c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a
^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*
a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a
^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

Sympy [F]

$$\int \frac{f + gx}{(d + ex)\left(a + bx + cx^2\right)^{3/2}}\,dx = \int \frac{f + gx}{(d + ex)\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((f + g*x)/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f + gx}{(d + ex)\left(a + bx + cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.12

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(\frac{(2c^3d^3f-3bc^2d^2ef+b^2cde^2f+2ac^2de^2f-abce^3f-bc^2d^3g+b^2cd^2eg+2ac^2d^2eg-3abcde^2g+2a^2ce^3g)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f-2b^2cd^2ef+2ac^2 \cdots}{b^2c^2d^4-4ac^3d^4-2b^3 \cdots}\cdots}{\sqrt{cx^2+bx+a}} + \frac{2\left(e^2f-deg\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f - 3*b*c^2*d^2*e*f + b^2*c*d*e^2*f + 2*a*c^2*d*e^2*f - a*b*
c*e^3*f - b*c^2*d^3*g + b^2*c*d^2*e*g + 2*a*c^2*d^2*e*g - 3*a*b*c*d*e^2*g +
 2*a^2*c*e^3*g)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^
3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 +
 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*b^2*c*d^2*
e*f + 2*a*c^2*d^2*e*f + b^3*d*e^2*f - a*b*c*d*e^2*f - a*b^2*e^3*f + 2*a^2*c
*e^3*f - 2*a*c^2*d^3*g + 3*a*b*c*d^2*e*g - a*b^2*d*e^2*g - 2*a^2*c*d*e^2*g
+ a^2*b*e^3*g)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e
 + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*
a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(e^2*
f - d*e*g)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.882 $\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

```
[Out] e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c
*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {754, 12, 738, 212}

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{e^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}} - \frac{2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
```

```
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)(cd^2-bde+ae^2)}\\
&= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}+\frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}\\
&= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}\\
&\quad-\frac{(2e^2)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{cd^2-bde+ae^2}
\end{aligned}$$

$$= -\frac{2(bcd - b^2e + 2ace + c(2cd - be)x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx + cx^2}} + \frac{e^2 \tanh^{-1}\left(\frac{bd - 2ae + (2cd - be)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx + cx^2}}\right)}{(cd^2 - bde + ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d + ex)(a + bx + cx^2)^{3/2}}\,dx =$$

$$-\frac{2\Big((cd^2 + e(-bd + ae))(-b^2e + 2c(ae + cdx) + bc(d - ex)) + (-b^2 + 4ac)e^2\sqrt{-cd^2 + bde - ae^2}\sqrt{a + x(b}{(b^2 - 4ac)(cd^2 + e(-bd + ae))^2\sqrt{a + x(b + cx)}}$$

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*((c*d^2 + e*(-(b*d) + a*e))*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*
x)) + (-b^2 + 4*a*c)*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b + c*x
)]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a
+ x*(b + c*x)])]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(
b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(145) = 290.

Time = 0.69 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.58

method	result
default	$\frac{\frac{e^2}{(e^2a - bde + c\,d^2)\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{(be - 2cd)\left(x + \frac{d}{e}\right)}{e} + \frac{e^2a - bde + c\,d^2}{e^2}}} - \frac{(be - 2cd)e\left(2c\left(x + \frac{d}{e}\right) + \frac{be - 2cd}{e}\right)}{(e^2a - bde + c\,d^2)\left(\frac{4c(e^2a - bde + c\,d^2)}{e^2} - \frac{(be - 2cd)^2}{e^2}\right)\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{(be - 2cd)\left(x + \right.}{e}}}}{e}$

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2
*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(145) = 290$.

Time = 0.60 (sec) , antiderivative size = 1349, normalized size of antiderivative = 8.70

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) -
4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^
2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*
(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a
^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*
d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 -
 (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c +
2*a*c^2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 -
 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*
c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
- 4*a^3*b*c)*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(145) = 290.

Time = 0.28 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.97

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\,e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}} - \frac{2\left(\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{bc\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+\cdots}\cdots}{\sqrt{cx^2+bx+a}}$$

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] 2*e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
 - 2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)
*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e
^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^
3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e
 + b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^
3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 -
 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*
c*e^4))/sqrt(c*x^2 + b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.883 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 352

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^{3/2}}\,dx = -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}} + \frac{e^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}(ef-dg)} - \frac{g^3\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)(cf^2-bfg+ag^2)^{3/2}}$$

```
[Out] e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)-g^3*arctanh(1/2*(b*f-2*
a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*
f)/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2
*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(c
*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {974, 754, 12, 738, 212}

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^{3/2}}\,dx = \frac{e^3\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\,(ae^2-bde+cd^2)^{3/2}} - \frac{g^3\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\,(ag^2-bfg+cf^2)^{3/2}} - \frac{2e(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)\,(ae^2-bde+cd^2)} + \frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)\,(ag^2-bfg+cf^2)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&= \int \left(\frac{e}{(ef-dg)(d+ex)\left(a+bx+cx^2\right)^{3/2}}-\frac{g}{(ef-dg)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\right)dx\\
&= \frac{e\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx}{ef-dg}-\frac{g\int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}}\,dx}{ef-dg}\\
&= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}}\\
&\quad+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}}\\
&\quad-\frac{(2e)\int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)}+\frac{(2g)\int -\frac{(b^2-4ac)g^2}{2(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)}\\
&= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}}\\
&\quad+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}}\\
&\quad+\frac{e^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(cd^2-bde+ae^2\right)(ef-dg)}-\frac{g^3\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\right)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a+bx+cx^2}} \\
&\quad+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{a+bx+cx^2}} \\
&\quad-\frac{(2e^3)\,\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)\,(ef-dg)} \\
&\quad+\frac{(2g^3)\,\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\,(cf^2-bfg+ag^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a+bx+cx^2}} \\
&\quad+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{a+bx+cx^2}} \\
&\quad+\frac{e^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}\,(ef-dg)}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\,(cf^2-bfg+ag^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int\frac{1}{(d+ex)(f+gx)\,(a+bx+cx^2)^{3/2}}\,dx &= \frac{2(-b^3eg+b^2c(dg+e(f-gx))-2c^2(adg+cdfx+ae(f-gx)}{(b^2-4ac)\,(-cd^2+e(bd-ae))\,(-cf^2+g(bf-} \\
&\quad-\frac{2e^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))^2\,(-ef+dg)} \\
&\quad-\frac{2g^3\sqrt{-cf^2+bfg-ag^2}\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\,(cf^2+g(-bf+ag))^2}
\end{aligned}
$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(
f - g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x))))/((b^2 - 4*a*c)*(-(
c*d^2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) -
 (2*e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
 + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d) + a*e
))^2*(-(e*f) + d*g)) - (2*g^3*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c
]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/((e
*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(332) = 664.

Time = 0.77 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.32

method	result
default	$\frac{\frac{g^{2}}{\left(a\,g^{2}-b f g+c\,f^{2}\right)\sqrt{\left(x+\frac{f}{g}\right)^{2} c+\frac{\left(b g-2 c f\right)\left(x+\frac{f}{g}\right)}{g}+\frac{a\,g^{2}-b f g+c\,f^{2}}{g^{2}}}}-\frac{\left(b g-2 c f\right) g\left(2 c\left(x+\frac{f}{g}\right)+\frac{b g-2 c f}{g}\right)}{\left(a\,g^{2}-b f g+c\,f^{2}\right)\left(\frac{4 c\left(a\,g^{2}-b f g+c\,f^{2}\right)}{g^{2}}-\frac{\left(b g-2 c f\right)^{2}}{g^{2}}\right)\sqrt{\left(x+\frac{f}{g}\right)^{2} c+\frac{\left(b g-2 c f\right)}{g}}}}{d g-}$

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*g-e*f)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g
)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^
2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2
*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/
g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*(1/(a*e^2-b
*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(
a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^{2}\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(d+ex)\,(f+gx)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(f+g\,x\right)\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.884 $\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 642

$$\begin{aligned}
&\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \\
&-\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} \\
&+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}} \\
&+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)\sqrt{a+bx+cx^2}} \\
&+\frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)} \\
&+\frac{e^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}(ef-dg)^2} \\
&-\frac{3g^3(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)\left(cf^2-bfg+ag^2\right)^{5/2}} \\
&-\frac{eg^3\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}
\end{aligned}$$

```
[Out] e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^2-3/2*g^3*(-b*g+2*c*f)*
arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)-e*g^3*arctanh(1/2*(b*f-2*a*
g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)
```

```
^2/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c
*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c
*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*
c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+g^2*(4*c^
2*f^2+3*b^2*g^2-4*c*g*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e
*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {974, 754, 12, 738, 212, 820}

$$\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}}\,dx = \frac{e^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2(ae^2-bde+cd^2)^{3/2}} - \frac{eg^3\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}} - \frac{3g^3(2cf-bg)\text{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{5/2}} + \frac{g^2\sqrt{a+bx+cx^2}(-4cg(2ag+bf)+3b^2g^2+4c^2f^2)}{(b^2-4ac)(f+gx)(ef-dg)(ag^2-bfg+cf^2)^2} - \frac{2e^2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ae^2-bde+cd^2)} + \frac{2eg(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ag^2-bfg+cf^2)} + \frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)(f+gx)\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
```

```
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}-\frac{g}{(ef-dg)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}-\frac{eg}{(ef-dg)^2(f+gx)\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$=\frac{e^2\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx}{(ef-dg)^2}-\frac{(eg)\int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}}\,dx}{(ef-dg)^2}-\frac{g\int \frac{1}{(f+gx)^2(a+bx+cx^2)^{3/2}}\,dx}{ef-dg}$$

$$=-\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}}+\frac{2eg\left(bcf-b^2g+2acg+c(2cf-bg)x\right)}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}}+\frac{2g\left(bcf-b^2g+2acg+c(2cf-bg)x\right)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)\sqrt{a+bx+cx^2}}-\frac{\left(2e^2\right)\int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^2}+\frac{(2eg)\int -\frac{(b^2-4ac)g^2}{2(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)}+\frac{(2g)\int \frac{\frac{1}{2}g(2bcf-3b^2g+8acg)+cg(2cf-bg)x}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}} + \frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}} + \frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)} + \frac{e^4\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(cd^2-bde+ae^2)(ef-dg)^2} - \frac{(3g^3(2cf-bg))\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)(cf^2-bfg+ag^2)^2} - \frac{(eg^3)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2(cf^2-bfg+ag^2)}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}} + \frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}} + \frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)} - \frac{(2e^4)\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)(ef-dg)^2} + \frac{(3g^3(2cf-bg))\,\mathrm{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)(cf^2-bfg+ag^2)^2} + \frac{(2eg^3)\,\mathrm{Subst}\left(\int \frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx, x, \frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2(cf^2-bfg+ag^2)}$$

$$= -\frac{2e^2(bcd - b^2e + 2ace + c(2cd - be)x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)(ef - dg)^2\sqrt{a + bx + cx^2}} + \frac{2eg(bcf - b^2g + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)^2(cf^2 - bfg + ag^2)\sqrt{a + bx + cx^2}} + \frac{2g(bcf - b^2g + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)(cf^2 - bfg + ag^2)(f + gx)\sqrt{a + bx + cx^2}} + \frac{g^2(4c^2f^2 + 3b^2g^2 - 4cg(bf + 2ag))\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(ef - dg)(cf^2 - bfg + ag^2)^2(f + gx)} + \frac{e^4\tanh^{-1}\left(\frac{bd - 2ae + (2cd - be)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx + cx^2}}\right)}{(cd^2 - bde + ae^2)^{3/2}(ef - dg)^2} - \frac{3g^3(2cf - bg)\tanh^{-1}\left(\frac{bf - 2ag + (2cf - bg)x}{2\sqrt{cf^2 - bfg + ag^2}\sqrt{a + bx + cx^2}}\right)}{2(ef - dg)(cf^2 - bfg + ag^2)^{5/2}} - \frac{eg^3\tanh^{-1}\left(\frac{bf - 2ag + (2cf - bg)x}{2\sqrt{cf^2 - bfg + ag^2}\sqrt{a + bx + cx^2}}\right)}{(ef - dg)^2(cf^2 - bfg + ag^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 12.75 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d + ex)(f + gx)^2(a + bx + cx^2)^{3/2}}\,dx = -\frac{2e^2(b^2e - 2c(ae + cdx) + bc(-d + ex))}{(b^2 - 4ac)(-cd^2 + e(bd - ae))(ef - dg)^2\sqrt{a + x(b + cx)}} + \frac{2eg(b^2g - 2c(ag + cfx) + bc(-f + gx))}{(b^2 - 4ac)(ef - dg)^2(-cf^2 + g(bf - ag))\sqrt{a + x(b + cx)}} - \frac{2g(b^2g - 2c(ag + cfx) + bc(-f + gx))}{(b^2 - 4ac)(-ef + dg)(-cf^2 + g(bf - ag))(f + gx)\sqrt{a + x(b + cx)}} + \frac{g^2\left(-\frac{2(4c^2f^2 + 3b^2g^2 - 4cg(bf + 2ag))\sqrt{a + x(b + cx)}}{(b^2 - 4ac)(cf^2 + g(-bf + ag))^2(f + gx)} + \frac{3g(-2cf + bg)\operatorname{arctanh}\left(\frac{-bf + 2ag - 2cfx + bgx}{2\sqrt{cf^2 + g(-bf + ag)}\sqrt{a + x(b + cx)}}\right)}{(cf^2 + g(-bf + ag))^{5/2}}\right)}{2(-ef + dg)} + \frac{e^4\operatorname{arctanh}\left(\frac{-2ae + 2cdx + b(d - ex)}{2\sqrt{cd^2 + e(-bd + ae)}\sqrt{a + x(b + cx)}}\right)}{(cd^2 + e(-bd + ae))^{3/2}(ef - dg)^2} - \frac{eg^3\operatorname{arctanh}\left(\frac{-2ag + 2cfx + b(f - gx)}{2\sqrt{cf^2 + g(-bf + ag)}\sqrt{a + x(b + cx)}}\right)}{(ef - dg)^2(cf^2 + g(-bf + ag))^{3/2}}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g -
```

```
 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2
) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c*f*x)
 + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f - a*g)
)*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (g^2*((-2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c
*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f)
+ a*g))^2*(f + g*x)) + (3*g*(-2*c*f + b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*
x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^
2 + g*(-(b*f) + a*g))^(5/2)))/(2*(-(e*f) + d*g)) + (e^4*ArcTanh[(-2*a*e + 2
*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x
)])])/((c*d^2 + e*(-(b*d) + a*e))^(3/2)*(e*f - d*g)^2) - (e*g^3*ArcTanh[(-2
*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*
(b + c*x)])])/((e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. $2(608) = 1216$.

Time = 0.95 (sec) , antiderivative size = 1481, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1481

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/(d*g-e*f)^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d
/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)
^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*
d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g/(d*g-e*f)*(-1/(a*
g^2-b*f*g+c*f^2)*g^2/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^2-b*f*g+c
*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-
b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f
*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2))/(x+f/g)))-4*c/(a*g^2-b*f*g+c*f^2)*g^2*(2*c*(x+f/g)+(b*g-2*c*f)
/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))-e/(d*g-e*f)^2*(1/(a*g^2-b*f*g+c
*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-
b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g
```

```
^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f
*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2))/(x+f/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.885 $\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 1064

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)^3\,(cf^2-bfg+ag^2)\,\sqrt{a+bx+cx^2}} + \frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)\,(f+gx)^2\sqrt{a+bx+cx^2}} + \frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)^2\,(cf^2-bfg+ag^2)\,(f+gx)\sqrt{a+bx+cx^2}} + \frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\,\sqrt{a+bx+cx^2}}{2\,(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)^2\,(f+gx)^2} + \frac{eg^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\,\sqrt{a+bx+cx^2}}{(b^2-4ac)\,(ef-dg)^2\,(cf^2-bfg+ag^2)^2\,(f+gx)} + \frac{g^2(2cf-bg)\,(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\,\sqrt{a+bx+cx^2}}{4\,(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)^3\,(f+gx)} + \frac{e^5\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}\,(ef-dg)^3} - \frac{3eg^3(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2\,(cf^2-bfg+ag^2)^{5/2}} - \frac{e^2g^3\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3\,(cf^2-bfg+ag^2)^{3/2}} - \frac{3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag))\,\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)\,(cf^2-bfg+ag^2)^{7/2}}$$

```
[Out] e^5*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^3-3/2*e*g^3*(-b*g+2*c*f
)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(5/2)-e^2*g^3*arctanh(1/2*(b*
f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*
g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(3/2)-3/8*g^3*(16*c^2*f^2+5*b^2*g^2-4*c*g*(a*g
+4*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(7/2)-2*e^3*(b*c*d-b^2*e
+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^3/(c
*x^2+b*x+a)^(1/2)+2*e^2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^
```

```
2)/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*
a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)
^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c
+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+1/2*g^2*
(8*c^2*f^2+5*b^2*g^2-4*c*g*(3*a*g+2*b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/
(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2+e*g^2*(4*c^2*f^2+3*b^2*g^2-4*c*g
*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*
f^2)^2/(g*x+f)+1/4*g^2*(-b*g+2*c*f)*(8*c^2*f^2+15*b^2*g^2-4*c*g*(13*a*g+2*b
*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^3/(g*x
+f)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {974, 754, 12, 738, 212, 848, 820}

$$
\begin{aligned}
\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}dx = & \frac{\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3} \\
& -\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}} \\
& -\frac{g^3\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^2}{(ef-dg)^3\left(cf^2-bgf+ag^2\right)^{3/2}} \\
& +\frac{2g\left(-gb^2+cfb+2acg+c(2cf-bg)x\right)e^2}{\left(b^2-4ac\right)(ef-dg)^3\left(cf^2-bgf+ag^2\right)\sqrt{cx^2+bx+a}} \\
& -\frac{3g^3(2cf-bg)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e}{2(ef-dg)^2\left(cf^2-bgf+ag^2\right)^{5/2}} \\
& +\frac{g^2\left(4c^2f^2+3b^2g^2-4cg(bf+2ag)\right)\sqrt{cx^2+bx+a}e}{\left(b^2-4ac\right)(ef-dg)^2\left(cf^2-bgf+ag^2\right)^2(f+gx)} \\
& +\frac{2g\left(-gb^2+cfb+2acg+c(2cf-bg)x\right)e}{\left(b^2-4ac\right)(ef-dg)^2\left(cf^2-bgf+ag^2\right)(f+gx)\sqrt{cx^2+bx+a}} \\
& -\frac{3g^3\left(16c^2f^2+5b^2g^2-4cg(4bf+ag)\right)\text{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8(ef-dg)\left(cf^2-bgf+ag^2\right)^{7/2}} \\
& +\frac{g^2(2cf-bg)\left(8c^2f^2+15b^2g^2-4cg(2bf+13ag)\right)\sqrt{cx^2+bx+a}}{4\left(b^2-4ac\right)(ef-dg)\left(cf^2-bgf+ag^2\right)^3(f+gx)} \\
& +\frac{g^2\left(8c^2f^2+5b^2g^2-4cg(2bf+3ag)\right)\sqrt{cx^2+bx+a}}{2\left(b^2-4ac\right)(ef-dg)\left(cf^2-bgf+ag^2\right)^2(f+gx)^2} \\
& +\frac{2g\left(-gb^2+cfb+2acg+c(2cf-bg)x\right)}{\left(b^2-4ac\right)(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)^2\sqrt{cx^2+bx+a}}
\end{aligned}
$$

```
[In] Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
```

```
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}} - \frac{eg}{(ef-dg)^2(f+gx)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{e^2g}{(ef-dg)^3(f+gx)\left(a+bx+cx^2\right)^{3/2}} \right) dx$$

$$= \frac{e^3 \int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}} \, dx}{(ef-dg)^3} - \frac{(e^2 g) \int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}} \, dx}{(ef-dg)^3} - \frac{(eg) \int \frac{1}{(f+gx)^2(a+bx+cx^2)^{3/2}} \, dx}{(ef-dg)^2} - \frac{g \int \frac{1}{(f+gx)^3(a+bx+cx^2)^{3/2}} \, dx}{ef-dg}$$

$$= -\frac{2e^3(bcd - b^2e + 2ace + c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g(bcf - b^2g + 2acg + c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}} + \frac{2g(bcf - b^2g + 2acg + c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2\sqrt{a+bx+cx^2}} + \frac{2eg(bcf - b^2g + 2acg + c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}} - \frac{(2e^3) \int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}} \, dx}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3} + \frac{(2e^2 g) \int -\frac{(b^2-4ac)g^2}{2(f+gx)\sqrt{a+bx+cx^2}} \, dx}{(b^2-4ac)(ef-dg)^3(cf^2-bfg+ag^2)} + \frac{(2eg) \int \frac{\frac{1}{2}g(2bcf-3b^2g+8acg)+cg(2cf-bg)x}{(f+gx)^2\sqrt{a+bx+cx^2}} \, dx}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)} + \frac{(2g) \int \frac{\frac{1}{2}g(4bcf-5b^2g+12acg)+2cg(2cf-bg)x}{(f+gx)^3\sqrt{a+bx+cx^2}} \, dx}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)}$$

$$
\begin{aligned}
&= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} \\
&\quad + \frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^3\left(cf^2-bfg+ag^2\right)\sqrt{a+bx+cx^2}} \\
&\quad + \frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2\sqrt{a+bx+cx^2}} \\
&\quad + \frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)\sqrt{a+bx+cx^2}} \\
&\quad + \frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{a+bx+cx^2}}{2\left(b^2-4ac\right)(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} \\
&\quad + \frac{eg^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)^2\left(cf^2-bfg+ag^2\right)^2(f+gx)} \\
&\quad + \frac{e^5\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(cd^2-bde+ae^2\right)(ef-dg)^3} - \frac{(3eg^3(2cf-bg))\int\frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^2} \\
&\quad - \frac{g\int\frac{\frac{1}{4}g\left(28b^2cfg-80ac^2fg-15b^3g^2-4bc\left(2cf^2-13ag^2\right)\right)-\frac{1}{2}cg\left(8c^2f^2+5b^2g^2-4cg(2bf+3ag)\right)x}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)(ef-dg)\left(cf^2-bfg+ag^2\right)^2} \\
&\quad - \frac{(e^2g^3)\int\frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3\left(cf^2-bfg+ag^2\right)}
\end{aligned}
$$

$$= -\frac{2e^3(bcd - b^2e + 2ace + c(2cd - be)x)}{(b^2 - 4ac)(cd^2 - bde + ae^2)(ef - dg)^3\sqrt{a + bx + cx^2}}$$
$$+ \frac{2e^2g(bcf - b^2g + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)^3(cf^2 - bfg + ag^2)\sqrt{a + bx + cx^2}}$$
$$+ \frac{2g(bcf - b^2g + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)(cf^2 - bfg + ag^2)(f + gx)^2\sqrt{a + bx + cx^2}}$$
$$+ \frac{2eg(bcf - b^2g + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)^2(cf^2 - bfg + ag^2)(f + gx)\sqrt{a + bx + cx^2}}$$
$$+ \frac{g^2(8c^2f^2 + 5b^2g^2 - 4cg(2bf + 3ag))\sqrt{a + bx + cx^2}}{2(b^2 - 4ac)(ef - dg)(cf^2 - bfg + ag^2)^2(f + gx)^2}$$
$$+ \frac{eg^2(4c^2f^2 + 3b^2g^2 - 4cg(bf + 2ag))\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(ef - dg)^2(cf^2 - bfg + ag^2)^2(f + gx)}$$
$$+ \frac{g^2(2cf - bg)(8c^2f^2 + 15b^2g^2 - 4cg(2bf + 13ag))\sqrt{a + bx + cx^2}}{4(b^2 - 4ac)(ef - dg)(cf^2 - bfg + ag^2)^3(f + gx)}$$
$$- \frac{(2e^5)\,\text{Subst}\left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x^2}\,dx, x, \frac{-bd + 2ae - (2cd - be)x}{\sqrt{a + bx + cx^2}}\right)}{(cd^2 - bde + ae^2)(ef - dg)^3}$$
$$+ \frac{(3eg^3(2cf - bg))\,\text{Subst}\left(\int \frac{1}{4cf^2 - 4bfg + 4ag^2 - x^2}\,dx, x, \frac{-bf + 2ag - (2cf - bg)x}{\sqrt{a + bx + cx^2}}\right)}{(ef - dg)^2(cf^2 - bfg + ag^2)^2}$$
$$+ \frac{(2e^2g^3)\,\text{Subst}\left(\int \frac{1}{4cf^2 - 4bfg + 4ag^2 - x^2}\,dx, x, \frac{-bf + 2ag - (2cf - bg)x}{\sqrt{a + bx + cx^2}}\right)}{(ef - dg)^3(cf^2 - bfg + ag^2)}$$
$$- \frac{(3g^3(16c^2f^2 + 5b^2g^2 - 4cg(4bf + ag)))\int \frac{1}{(f + gx)\sqrt{a + bx + cx^2}}\,dx}{8(ef - dg)(cf^2 - bfg + ag^2)^3}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}}$$
$$+\frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}}$$
$$+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}}$$
$$+\frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{a+bx+cx^2}}{2(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)^2}$$
$$+\frac{eg^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)^2(f+gx)}$$
$$+\frac{g^2(2cf-bg)(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\sqrt{a+bx+cx^2}}{4(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^3(f+gx)}$$
$$+\frac{e^5\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}(ef-dg)^3}$$
$$-\frac{3eg^3(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2(cf^2-bfg+ag^2)^{5/2}}$$
$$-\frac{e^2g^3\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3(cf^2-bfg+ag^2)^{3/2}}$$
$$+\frac{(3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag)))\,\text{Subst}\left(\int\frac{1}{4cf^2-4bfg+4ag^2-x^2}\,dx,x,\frac{-bf+2ag-(2cf-bg)x}{\sqrt{a+bx+cx^2}}\right)}{4(ef-dg)(cf^2-bfg+ag^2)^3}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}}$$
$$+\frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^3(cf^2-bfg+ag^2)\sqrt{a+bx+cx^2}}$$
$$+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)(f+gx)\sqrt{a+bx+cx^2}}$$
$$+\frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{a+bx+cx^2}}{2(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^2(f+gx)^2}$$
$$+\frac{eg^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+bx+cx^2}}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)^2(f+gx)}$$
$$+\frac{g^2(2cf-bg)(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\sqrt{a+bx+cx^2}}{4(b^2-4ac)(ef-dg)(cf^2-bfg+ag^2)^3(f+gx)}$$
$$+\frac{e^5\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}(ef-dg)^3}$$
$$-\frac{3eg^3(2cf-bg)\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{2(ef-dg)^2(cf^2-bfg+ag^2)^{5/2}}$$
$$-\frac{e^2g^3\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^3(cf^2-bfg+ag^2)^{3/2}}$$
$$-\frac{3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag))\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{8(ef-dg)(cf^2-bfg+ag^2)^{7/2}}$$

Mathematica [A] (verified)

Time = 15.23 (sec) , antiderivative size = 1013, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2e^3(b^2e-2c(ae+cdx)+bc(-d+ex))}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(ef-dg)^3\sqrt{a+x(b+cx)}} - \frac{2e^2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(-ef+dg)^3\left(-cf^2+g(bf-ag)\right)\sqrt{a+x(b+cx)}} - \frac{2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(-ef+dg)\left(-cf^2+g(bf-ag)\right)(f+gx)^2\sqrt{a+x(b+cx)}} + \frac{2eg(b^2g-2c(ag+cfx)+bc(-f+gx))}{\left(b^2-4ac\right)(ef-dg)^2\left(-cf^2+g(bf-ag)\right)(f+gx)\sqrt{a+x(b+cx)}} + \frac{eg^2\left(\frac{2\left(4c^2f^2+3b^2g^2-4cg(bf+2ag)\right)\sqrt{a+x(b+cx)}}{(b^2-4ac)(cf^2+g(-bf+ag))^2(f+gx)} + \frac{3g(2cf-bg)\text{arctanh}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{\left(cf^2+g(-bf+ag)\right)^{5/2}}\right)}{2(ef-dg)^2} - \frac{g^2\left(\frac{4\left(8c^2f^2+5b^2g^2-4cg(2bf+3ag)\right)\sqrt{a+x(b+cx)}}{(f+gx)^2} + \frac{2(2cf-bg)\left(8c^2f^2+15b^2g^2-4cg(2bf+13ag)\right)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} + \frac{3\left(b^2-4ac\right)g\left(16c^2f^2+5b^2 \cdots\right.}{\cdots}\right)}{8\left(b^2-4ac\right)(-ef+dg)\left(cf^2+g(-bf+ag)\right)^2} - \frac{e^5\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\left(cd^2+e(-bd+ae)\right)^{3/2}(-ef+dg)^3} - \frac{e^2g^3\text{arctanh}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(ef-dg)^3\left(cf^2+g(-bf+ag)\right)^{3/2}}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2*g
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*(-(
c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c
*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g + c*f*
x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*f - a*g
))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f
) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f
*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f
^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*c^2*f^2 + 5*
b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g*x)^2 + (2*(2
*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))*Sqrt[a + x*(b
```

```
 + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 - 4*a*c)*g*(16*c
^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x
 + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2
 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 + g*(-(
b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))
^(3/2)*(-(e*f) + d*g)^3) - (e^2*g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^3
*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2684 vs. $2(1010) = 2020$.

Time = 1.14 (sec) , antiderivative size = 2685, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	2685

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2/((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-5/4*(b*g-2*c*f)*g/(a*g^2-b*f
*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*
(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+
c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f
)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*
f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f
/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-4*c/(a*g^2-b*f*g+c*f^2)*g^2*(2*c*(x
+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))-3/2*c/(a*g^2-b
*f*g+c*f^2)*g^2*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x
+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+
c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b
*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c
*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))-e^2/(d*g-e*f)^3*(1
/(a*e^2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/
e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e
```

```
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+e^2/(d*g-e*f)^3*(1/(a*g^2-b*f*g+c*f^2
)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*
g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*
g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))/(x+f/g)))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)/((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2*
c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c
*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f
^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/
g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a
*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*
f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-4*c/(a*g
^2-b*f*g+c*f^2)*g^2*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^
2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)(gx+f)^3}\,dx$$

[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14979 vs. 2(1010) = 2020.

Time = 4.14 (sec) , antiderivative size = 14979, normalized size of antiderivative = 14.08

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")

[Out] 2*e^5*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c*d^2*e^3*f^3 - b*d*e^4*f^3 + a*e^5*f^3 - 3*c*d^3*e^2*f^2*g + 3*b*d^2*e^3*f^2*g - 3*a*d*e^4*f^2*g + 3*c*d^4*e*f*g^2 - 3*b*d^3*e^2*f*g^2 + 3*a*d^2*e^3*f*g^2 - c*d^5*g^3 + b*d^4*e*g^3 - a*d^3*e^2*g^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 2*((2*c^9*d^3*f^9 - 3*b*c^8*d^2*e*f^9 + b^2*c^7*d*e^2*f^9 + 2*a*c^8*d*e^2*f^9 - a*b*c^7*e^3*f^9 - 9*b*c^8*d^3*f^8*g + 15*b^2*c^7*d^2*e*f^8*g - 6*a*c^8*d^2*e*f^8*g - 6*b^3*c^6*d*e^2*f^8*g - 3*a*b*c^7*d*e^2*f^8*g + 6*a*b^2*c^6*e^3*f^8*g - 6*a^2*c^7*e^3*f^8*g + 18*b^2*c^7*d^3*f^7*g^2 - 33*b^3*c^6*d^2*e*f^7*g^2 + 24*a*b*c^7*d^2*e*f^7*g^2 + 15*b^4*c^5*d*e^2*f^7*g^2 - 6*a*b^2*c^6*d*e^2*f^7*g^2 - 15*a*b^3*c^5*e^3*f^7*g^2 + 24*a^2*b*c^6*e^3*f^7*g^2 - 21*b^3*c^6*d^3*f^6*g^3 + 41*b^4*c^5*d^2*e*f^6*g^3 - 34*a*b^2*c^6*d^2*e*f^6*g^3 - 16*a^2*c^7*d^2*e*f^6*g^3 - 20*b^5*c^4*d*e^2*f^6*g^3 + 13*a*b^3*c^5*d*e^2*f^6*g^3 + 16*a^2*b*c^6*d*e^2*f^6*g^3 + 20*a*b^4*c^4*e^3*f^6*g^3 - 34*a^2*b^2*c^5*e^3*f^6*g^3 - 16*a^3*c^6*e^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c^7*d^3*f^5*g^4 - 30*b^5*c^4*d^2*e*f^5*g^4 + 9*a*b^3*c^5*d^2*e*f^5*g^4 + 66*a^2*b*c^6*d^2*e*f^5*g^4 + 15*b^6*c^3*d*e^2*f^5*g^4 - 48*a^2*b^2*c^5*d*e^2*f^5*g^4 - 12*a^3*c^6*d*e^2*f^5*g^4 - 15*a*b^5*c^3*e^3*f^5*g^4 + 15*a^2*b^3*c^4*e^3*f^5*g^4 + 54*a^3*b*c^5*e^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a*b^3*c^5*d^3*f^4*g^5 + 30*a^2*b*c^6*d^3*f^4*g^5 + 12*b^6*c^3*d^2*e*f^4*g^5 + 24*a*b^4*c^4*d^2*e*f^4*g^5 - 96*a^2*b^2*c^5*d^2*e*f^4*g^5 - 12*a^3*c^6*d^2*e*f^4*g^5 - 6*b^7*c^2*d*e^2*f^4*g^5 - 15*a*b^5*c^3*d*e^2*f^4*g^5 + 51*a^2*b^3*c^4*d*e^2*f^4*g^5 + 42*a^3*b*c^5*d*e^2*f^4*g^5 + 6*a*b^6*c^2*e^3*f^4*g^5 + 9*a^2*b^4*c^3*e^3*f^4*g^5 - 66*a^3*b^2*c^4*e^3*f^4*g^5 - 12*a^4*c^5*e^3*f^4*g^5 + b^6*c^3 *d^3*f^3*g^6 + 12*a*b^4*c^4*d^3*f^3*g^6 - 18*a^2*b^2*c^5*d^3*f^3*g^6 - 16*a^3*c^6*d^3*f^3*g^6 - 2*b^7*c^2*d^2*e*f^3*g^6 - 23*a*b^5*c^3*d^2*e*f^3*g^6 + 49*a^2*b^3*c^4*d^2*e*f^3*g^6 + 48*a^3*b*c^5*d^2*e*f^3*g^6 + b^8*c*d*e^2*f^3*g^6 + 12*a*b^6*c^2*d*e^2*f^3*g^6 - 19*a^2*b^4*c^3*d*e^2*f^3*g^6 - 50*a^3*b^2*c^4*d*e^2*f^3*g^6 - 16*a^4*c^5*d*e^2*f^3*g^6 - a*b^7*c*e^3*f^3*g^6 - 11*a^2*b^5*c^2*e^3*f^3*g^6 + 31*a^3*b^3*c^3*e^3*f^3*g^6 + 32*a^4*b*c^4*e^3*f^3*g^6 - 3*a*b^5*c^3*d^3*f^2*g^7 - 3*a^2*b^3*c^4*d^3*f^2*g^7 + 24*a^3*b*c^5*d^3*f^2*g^7 + 6*a*b^6*c^2*d^2*e*f^2*g^7 + 3*a^2*b^4*c^3*d^2*e*f^2*g^7 - 54*a^3*b^2*c^4*d^2*e*f^2*g^7 - 3*a*b^7*c*d*e^2*f^2*g^7 - 3*a^2*b^5*c^2*d*e^2*f^2*g^7 + 27*a^3*b^3*c^3*d*e^2*f^2*g^7 + 24*a^4*b*c^4*d*e^2*f^2*g^7 + 3*a^2*b^6*c*e^3*f^2*g^7 - 30*a^4*b^2*c^3*e^3*f^2*g^7 + 3*a^2*b^4*c^3*d^3*f*g^8 - 6*a^3*b^2*c^4*d^3*f*g^8 - 6*a^4*c^5*d^3*f*g^8 - 6*a^2*b^5*c^2*d^2*e*f*g^8 + 15*a^3*b^3*c^3*d^2*e*f*g^8 + 9*a^4*b*c^4*d^2*e*f*g^8 + 3*a^2*b^6*c*d*e^2*f*g^8 - 6*a^3*b^4*c^2*d*e^2*f*g^8 - 9*a^4*b^2*c^3*d*e^2*f*g^8 - 6*a^5*c^4*d*e^2*f*g^8 - 3*a^3*b^5*c*e^3*f*g^8 + 9*a^4*b^3*c^2*e^3*f*g^8 + 3*a^5*b*c^3*e^3*f*g^8 - a^3*b^3*c^3*d^3*g^9 + 3*a^4*b*c^4*d^3*g^9 + 2*a^3*b^4*c^2*d^2*e*g^9 - 7*a^4*b^2*c^3*d^2*e*g^9 + 2*a^5*c^4*d^2*e*g^9 - a^3*b^5*c*d*e^2*g^9 + 3*a^4*b^3*c^2*d*e^2*g^9 + a^5*b*c^3*d*e^2*g^9 + a^4*b^4*c*e^3*g^9 - 4*a^5*b^2*c^2*e^3*g^9 + 2*a^6*c^3*e^3*g^9)*x/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 2*b^3*c^7*d^3*e*f^12 + 8*a*b*c^8*d^3*e*f^12 + b^4*c^6*d^2*e^2*f^12 - 2*a*b^2*c^7*d^2*e^2*f^12 - 8*a^2*c^8*d^2*e^2*f^12 - 2*a*b^3*c^6*d*e^3*f^12 + 8*a^2*b*c^7*d*e^3*f^12 + a^2*b^2*c^6*e^4*f^12 - 4*a^3*c^7*e^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*f^11*g + 12*b^4*c^6*d^3*e*f^11*g - 48*a*b^2*c^7*d^3*e*f^11*g - 6*b^5*c^5*d^2*e^2*f^11*g + 12*a*b^3*c^6*d^2*e^2*f^11*g + 48*a^2*b*c^7*d^2*e^2*f^11*g + 12*a*b^4*c^5*d*e^3*f^11*g - 48*a^2*b^2*c^6*d*e^3*f^11*g - 6*a^2*b^3*c^5*e^4*f^11*g + 24*a^3*b*c^6*e^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d^4*f^10*g^2 - 30*b^5*c^5*d^3*e*f^10*g^2 + 108*a*b^3*c^6*d^3*e*f^10*g^2 + 48*a^2*b*c^7*d^3*e*f^10*g^2 + 15*b^6*c^4*d^2*e^2*f^10*g^2 - 24*a*b^4*c^5*d^2*e^2*f^10*g^2 - 132*a^2*b^2*c^6*d^2*e^2*f^10*g^2 - 48*a^3*c^7*d^2*e^2*f^10*g^2 - 30*a*b^5*c^4*d*e^3*f^10*g^2 + 108*a^2*b^3*c^5*d*e^3*f^10*g^2 + 48*a^3*b*c^6*d*e^3*f^10*g^2 + 15*a^2*b^4*c^4*e^4*f^10*g^2 - 54*a^3*b^2*c^5*e^4*f^10*g^2 - 24*a^4*c^6*e^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 40*b^6*c^4*d^3*e*f^9*g^3 - 100*a*b^4*c^5*d^3*e*f^9*g^3 - 240*a^2*b^2*c^6*d^3*e*f^9*g^3 - 20*b^7*c^3*d^2*e^2*f^9*g^3 + 10*a*b^5*c^4*d^2*e^2*f^9*g^3 + 220*a^2*b^3*c^5*d^2*e^2*f^9*g^3 + 240*a^3*b*c^6*d^2*e^2*f^9*g^3 + 40*a*b^6*c^3*d*e^3*f^9*g^3 - 100*a^2*b^4*c^4*d*e^3*f^9*g^3 - 240*a^3*b^2*c^5*d*e^3*f^9*g^3 - 20*a^2*b^5*c^3*e^4*f^9*g^3 + 50*a^3*b^3*c^4*e^4*f^9*g^3 + 120*a^4*b*c^5*e^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 30*b^7*c^3*d^3*e*f^8*g^4 + 450*a^2*b^3*c^5*d^3*e*f^8*g^4 + 120*a^3*b*c^6*d^3*e*f^8*g^4 + 15*b^8*c^2*d^2*e^2*f^8*g^4 + 30*a*b^6*c^3*d^2*e^2*f^8*g^4 - 225*a^2*b^4*c^4*d^2*e^2*f^8*g^4 - 510*a^3*b^2*c^5*d^2*e^2*f^8*g^4 - 120*a^4*c^6*d^2*e^2*f^8*g^4 - 30*a*b^7*c^2*d*e^3*f^8*g^4 + 450*a^3*b^3*c^4*d*e^3*f^8*g^4 + 120*a^4*b*c^5*d*e^3*f^8*g^4 + 15*a^2*b^6*c^2*e^4*f^8*g^4 - 225*a^4*b^2*c^4*e^4*f^8*g^4 - 60*a^5* c^5*e^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + 12*b^8*c^2*d^3*e*f^7*g^5 + 72*a*b^6*c^3*d^3*e*f^7*g^5 - 360*a^2*b^4*c^4*d^3*e*f^7*g^5 - 480*a^3*b^2*c^5*d^3*e*f^7*g^5 - 6*b^9*c*d^2*e^2*f^7*g^5 - 48*a*b^7*c^2*d^2*e^2*f^7*g^5 + 108*a^2*b^5*c^3*d^2*e^2*f^7*g^5 + 600*a^3*b^3*c^4*d^2*e^2*f^7*g^5 + 480*a^4*b*c^5*d^2*e^2*f^7*g^5 + 12*a*b^8*c*d*e^3*f^7*g^5 + 72*a^2*b^6*c^2*d*e^3*f^7*g^5 - 360*a^3*b^4*c^3*d*e^3*f^7*g^5 - 480*a^4*b^2*c^4*d*e^3*f^7*g^5 - 6*a^2*b^7*c*e^4*f^7*g^5 - 36*a^3*b^5*c^2*e^4*f^7*g^5 + 180*a^4*b^3*c^3*e^4*f^7*g^5 + 240*a^5*b*c^4*e^4*f^7*g^5 + b^8*c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 2*b^9*c*d^3*e*f^6*g^6 - 52*a*b^7*c^2*d^3*e*f^6*g^6 + 60*a^2*b^5*c^3*d^3*e*f^6*g^6 + 680*a^3*b^3*c^4*d^3*e*f^6*g^6 + 160*a^4*b*c^5*d^3*e*f^6*g^6 + b^10*d^2*e^2*f^6*g^6 + 28*a*b^8*c*d^2*e^2*f^6*g^6 + 22*a^2*b^6*c^2*d^2*e^2*f^6*g^6 - 400*a^3*b^4*c^3*d^2*e^2*f^6*g^6 - 760*a^4*b^2*c^4*d^2*e^2*f^6*g^6 - 160*a^5*c^5*d^2*e^2*f^6*g^6 - 2*a*b^9*d*e^3*f^6*g^6 - 52*a^2*b^7*c*d*e^3*f^6*g^6 + 60*a^3*b^5*c^2*d*e^3*f^6*g^6 + 680*a^4*b^3*c^3*d*e^3*f^6*g^6 + 160*a^5*b*c^4*d*e^3*f^6*g^6 + a^2*b^8*e^4*f^6*g^6 + 26*a^3*b^6*c*e^4*f^6*g^6 - 30*a^4*b^4*c^2*e^4*f^6*g^6 - 340*a^5*b^2*c^3*e^4*f^6*g^6 - 80*a^6*c^4*e^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^7 + 12*a*b^8*c*d^3*e*f^5*g^7 + 72*a^2*b^6*c^2*d^3*e*f^5*g^7 - 360*a^3*b^4*c^3*d^3*e*f^5*g^7 - 480*a^4*b^2*c^4*d^3*e*f^5*g^7 - 6*a*b^9*d^2*e^2*f^5*g^7 - 48*a^2*b^7*c*d^2*e^2*f^5*g^7 + 108*a^3*b^5*c^2*d^2*e^2*f^5*g^7 + 600*a^4*b^3*c^3*d^2*e^2*f^5*g^7 + 480*a^5*b*c^4*d^2*e^2*f^5*g^7 + 12*a^2*b^8*d*e^3*f^5*g^7 + 72*a^3*b^6*c*d*e^3*f^5*g^7 - 360*a^4*b^4*c^2*d*e^3*f^5*g^7 - 480*a^5*b^2*c^3*d*e^3*f^5*g^7 - 6*a^3*b^7*e^4*f^5*g^7 - 36*a^4*b^5*c*e^4*f^5*g^7 + 180*a^5*b^3*c^2*e^4*f^5*g^7 + 240*a^6*b*c^3*e^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d^4*f^4*g^8 - 30*a^2*b^7*c*d^3*e*f^4*g^8 + 450*a^4*b^3*c^3*d^3*e*f^4*g^8 + 120*a^5*b*c^4*d^3*e*f^4*g^8 + 15*a^2*b^8*d^2*e^2*f^4*g^8 + 30*a^3*b^6*c*d^2*e^2*f^4*g^8 - 225*a^4*b^4*c^2*d^2*e^2*f^4*g^8 - 510*a^5*b^2*c^3*d^2*e^2*f^4*g^8 - 120*a^6*c^4*d^2*e^2*f^4*g^8 - 30*a^3*b^7*d*e^3*f^4*g^8 + 450*a^5*b^3*c^2*d*e^3*f^4*g^8 + 120*a^6*b*c^3*d*e^3*f^4*g^8 + 15*a^4*b^6*e^4*f^4*g^8 - 225*a^6*b^2*c^2*e^4*f^4*g^8 - 60*a^7*c^3*e^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 40*a^3*b^6*c*d^3*e*f^3*g^9 - 100*a^4*b^4*c^2*d^3*e*f^3*g^9 - 240*a^5*b^2*c^3*d^3*e*f^3*g^9 - 20*a^3*b^7*d^2*e^2*f^3*g^9 + 10*a^4*b^5*c*d^2*e^2*f^3*g^9 + 220*a^5*b^3*c^2*d^2*e^2*f^3*g^9 + 240*a^6*b*c^3*d^2*e^2*f^3*g^9 + 40*a^4*b^6*d*e^3*f^3*g^9 - 100*a^5*b^4*c*d*e^3*f^3*g^9 - 240*a^6*b^2*c^2*d*e^3*f^3*g^9 - 20*a^5*b^5*e^4*f^3*g^9 + 50*a^6*b^3*c*e^4*f^3*g^9 + 120*a^7*b*c^2*e^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 30*a^4*b^5*c*d^3*e*f^2*g^10 + 108*a^5*b^3*c^2*d^3*e*f^2*g^10 + 48*a^6*b*c^3*d^3*e*f^2*g^10 + 15*a^4*b^6*d^2*e^2*f^2*g^10 - 24*a^5*b^4*c*d^2*e^2*f^2*g^10 - 132*a^6*b^2*c^2*d^2*e^2*f^2*g^10 - 48*a^7*c^3*d^2*e^2*f^2*g^10 - 30*a^5*b^5*d*e^3*f^2*g^10 + 108*a^6*b^3*c*d*e^3*f^2*g^10 + 48*a^7*b*c^2*d*e^3*f^2*g^10 + 15*

a^6*b^4*e^4*f^2*g^10 - 54*a^7*b^2*c*e^4*f^2*g^10 - 24*a^8*c^2*e^4*f^2*g^10
- 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*d^4*f*g^11 + 12*a^5*b^4*c*d^3*e*f
*g^11 - 48*a^6*b^2*c^2*d^3*e*f*g^11 - 6*a^5*b^5*d^2*e^2*f*g^11 + 12*a^6*b^3
*c*d^2*e^2*f*g^11 + 48*a^7*b*c^2*d^2*e^2*f*g^11 + 12*a^6*b^4*d*e^3*f*g^11 -
48*a^7*b^2*c*d*e^3*f*g^11 - 6*a^7*b^3*e^4*f*g^11 + 24*a^8*b*c*e^4*f*g^11 +
a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*a^6*b^3*c*d^3*e*g^12 + 8*a^7
*b*c^2*d^3*e*g^12 + a^6*b^4*d^2*e^2*g^12 - 2*a^7*b^2*c*d^2*e^2*g^12 - 8*a^8
*c^2*d^2*e^2*g^12 - 2*a^7*b^3*d*e^3*g^12 + 8*a^8*b*c*d*e^3*g^12 + a^8*b^2*e
^4*g^12 - 4*a^9*c*e^4*g^12) + (b*c^8*d^3*f^9 - 2*b^2*c^7*d^2*e*f^9 + 2*a*c^
8*d^2*e*f^9 + b^3*c^6*d*e^2*f^9 - a*b*c^7*d*e^2*f^9 - a*b^2*c^6*e^3*f^9 + 2
*a^2*c^7*e^3*f^9 - 6*b^2*c^7*d^3*f^8*g + 6*a*c^8*d^3*f^8*g + 12*b^3*c^6*d^2
*e*f^8*g - 21*a*b*c^7*d^2*e*f^8*g - 6*b^4*c^5*d*e^2*f^8*g + 9*a*b^2*c^6*d*e
^2*f^8*g + 6*a^2*c^7*d*e^2*f^8*g + 6*a*b^3*c^5*e^3*f^8*g - 15*a^2*b*c^6*e^3
*f^8*g + 15*b^3*c^6*d^3*f^7*g^2 - 24*a*b*c^7*d^3*f^7*g^2 - 30*b^4*c^5*d^2*e
*f^7*g^2 + 66*a*b^2*c^6*d^2*e*f^7*g^2 + 15*b^5*c^4*d*e^2*f^7*g^2 - 27*a*b^3
*c^5*d*e^2*f^7*g^2 - 24*a^2*b*c^6*d*e^2*f^7*g^2 - 15*a*b^4*c^4*e^3*f^7*g^2
+ 42*a^2*b^2*c^5*e^3*f^7*g^2 - 20*b^4*c^5*d^3*f^6*g^3 + 34*a*b^2*c^6*d^3*f^
6*g^3 + 16*a^2*c^7*d^3*f^6*g^3 + 40*b^5*c^4*d^2*e*f^6*g^3 - 89*a*b^3*c^5*d^
2*e*f^6*g^3 - 32*a^2*b*c^6*d^2*e*f^6*g^3 - 20*b^6*c^3*d*e^2*f^6*g^3 + 35*a*
b^4*c^4*d*e^2*f^6*g^3 + 50*a^2*b^2*c^5*d*e^2*f^6*g^3 + 16*a^3*c^6*d*e^2*f^6
*g^3 + 20*a*b^5*c^3*e^3*f^6*g^3 - 55*a^2*b^3*c^4*e^3*f^6*g^3 - 16*a^3*b*c^5
*e^3*f^6*g^3 + 15*b^5*c^4*d^3*f^5*g^4 - 15*a*b^3*c^5*d^3*f^5*g^4 - 54*a^2*b
*c^6*d^3*f^5*g^4 - 30*b^6*c^3*d^2*e*f^5*g^4 + 45*a*b^4*c^4*d^2*e*f^5*g^4 +
114*a^2*b^2*c^5*d^2*e*f^5*g^4 - 12*a^3*c^6*d^2*e*f^5*g^4 + 15*b^7*c^2*d*e^2
*f^5*g^4 - 15*a*b^5*c^3*d*e^2*f^5*g^4 - 75*a^2*b^3*c^4*d*e^2*f^5*g^4 - 42*a
^3*b*c^5*d*e^2*f^5*g^4 - 15*a*b^6*c^2*e^3*f^5*g^4 + 30*a^2*b^4*c^3*e^3*f^5*
g^4 + 60*a^3*b^2*c^4*e^3*f^5*g^4 - 12*a^4*c^5*e^3*f^5*g^4 - 6*b^6*c^3*d^3*f
^4*g^5 - 9*a*b^4*c^4*d^3*f^4*g^5 + 66*a^2*b^2*c^5*d^3*f^4*g^5 + 12*a^3*c^6*
d^3*f^4*g^5 + 12*b^7*c^2*d^2*e*f^4*g^5 + 12*a*b^5*c^3*d^2*e*f^4*g^5 - 147*a
^2*b^3*c^4*d^2*e*f^4*g^5 + 6*a^3*b*c^5*d^2*e*f^4*g^5 - 6*b^8*c*d*e^2*f^4*g^
5 - 9*a*b^6*c^2*d*e^2*f^4*g^5 + 72*a^2*b^4*c^3*d*e^2*f^4*g^5 + 48*a^3*b^2*c
^4*d*e^2*f^4*g^5 + 12*a^4*c^5*d*e^2*f^4*g^5 + 6*a*b^7*c*e^3*f^4*g^5 + 3*a^2
*b^5*c^2*e^3*f^4*g^5 - 81*a^3*b^3*c^3*e^3*f^4*g^5 + 18*a^4*b*c^4*e^3*f^4*g^
5 + b^7*c^2*d^3*f^3*g^6 + 11*a*b^5*c^3*d^3*f^3*g^6 - 31*a^2*b^3*c^4*d^3*f^3
*g^6 - 32*a^3*b*c^5*d^3*f^3*g^6 - 2*b^8*c*d^2*e*f^3*g^6 - 21*a*b^6*c^2*d^2*
e*f^3*g^6 + 74*a^2*b^4*c^3*d^2*e*f^3*g^6 + 46*a^3*b^2*c^4*d^2*e*f^3*g^6 - 1
6*a^4*c^5*d^2*e*f^3*g^6 + b^9*d*e^2*f^3*g^6 + 11*a*b^7*c*d*e^2*f^3*g^6 - 32
*a^2*b^5*c^2*d*e^2*f^3*g^6 - 45*a^3*b^3*c^3*d*e^2*f^3*g^6 - 16*a^4*b*c^4*d*
e^2*f^3*g^6 - a*b^8*e^3*f^3*g^6 - 10*a^2*b^6*c*e^3*f^3*g^6 + 43*a^3*b^4*c^2
*e^3*f^3*g^6 + 14*a^4*b^2*c^3*e^3*f^3*g^6 - 16*a^5*c^4*e^3*f^3*g^6 - 3*a*b^
6*c^2*d^3*f^2*g^7 + 30*a^3*b^2*c^4*d^3*f^2*g^7 + 6*a*b^7*c*d^2*e*f^2*g^7 -
3*a^2*b^5*c^2*d^2*e*f^2*g^7 - 63*a^3*b^3*c^3*d^2*e*f^2*g^7 + 24*a^4*b*c^4*d
^2*e*f^2*g^7 - 3*a*b^8*d*e^2*f^2*g^7 + 33*a^3*b^4*c^2*d*e^2*f^2*g^7 + 6*a^4
*b^2*c^3*d*e^2*f^2*g^7 + 3*a^2*b^7*e^3*f^2*g^7 - 3*a^3*b^5*c*e^3*f^2*g^7 -
33*a^4*b^3*c^2*e^3*f^2*g^7 + 24*a^5*b*c^3*e^3*f^2*g^7 + 3*a^2*b^5*c^2*d^3*f

*g^8 - 9*a^3*b^3*c^3*d^3*f*g^8 - 3*a^4*b*c^4*d^3*f*g^8 - 6*a^2*b^6*c*d^2*e*
f*g^8 + 21*a^3*b^4*c^2*d^2*e*f*g^8 - 6*a^5*c^4*d^2*e*f*g^8 + 3*a^2*b^7*d*e^
2*f*g^8 - 9*a^3*b^5*c*d*e^2*f*g^8 - 6*a^4*b^3*c^2*d*e^2*f*g^8 + 3*a^5*b*c^3
*d*e^2*f*g^8 - 3*a^3*b^6*e^3*f*g^8 + 12*a^4*b^4*c*e^3*f*g^8 - 3*a^5*b^2*c^2
*e^3*f*g^8 - 6*a^6*c^3*e^3*f*g^8 - a^3*b^4*c^2*d^3*g^9 + 4*a^4*b^2*c^3*d^3*
g^9 - 2*a^5*c^4*d^3*g^9 + 2*a^3*b^5*c*d^2*e*g^9 - 9*a^4*b^3*c^2*d^2*e*g^9 +
 7*a^5*b*c^3*d^2*e*g^9 - a^3*b^6*d*e^2*g^9 + 4*a^4*b^4*c*d*e^2*g^9 - a^5*b^
2*c^2*d*e^2*g^9 - 2*a^6*c^3*d*e^2*g^9 + a^4*b^5*e^3*g^9 - 5*a^5*b^3*c*e^3*g
^9 + 5*a^6*b*c^2*e^3*g^9)/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 2*b^3*c^7*
d^3*e*f^12 + 8*a*b*c^8*d^3*e*f^12 + b^4*c^6*d^2*e^2*f^12 - 2*a*b^2*c^7*d^2*
e^2*f^12 - 8*a^2*c^8*d^2*e^2*f^12 - 2*a*b^3*c^6*d*e^3*f^12 + 8*a^2*b*c^7*d*
e^3*f^12 + a^2*b^2*c^6*e^4*f^12 - 4*a^3*c^7*e^4*f^12 - 6*b^3*c^7*d^4*f^11*g
 + 24*a*b*c^8*d^4*f^11*g + 12*b^4*c^6*d^3*e*f^11*g - 48*a*b^2*c^7*d^3*e*f^1
1*g - 6*b^5*c^5*d^2*e^2*f^11*g + 12*a*b^3*c^6*d^2*e^2*f^11*g + 48*a^2*b*c^7
*d^2*e^2*f^11*g + 12*a*b^4*c^5*d*e^3*f^11*g - 48*a^2*b^2*c^6*d*e^3*f^11*g -
 6*a^2*b^3*c^5*e^4*f^11*g + 24*a^3*b*c^6*e^4*f^11*g + 15*b^4*c^6*d^4*f^10*g
^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d^4*f^10*g^2 - 30*b^5*c^5*d^3*e
*f^10*g^2 + 108*a*b^3*c^6*d^3*e*f^10*g^2 + 48*a^2*b*c^7*d^3*e*f^10*g^2 + 15
*b^6*c^4*d^2*e^2*f^10*g^2 - 24*a*b^4*c^5*d^2*e^2*f^10*g^2 - 132*a^2*b^2*c^6
*d^2*e^2*f^10*g^2 - 48*a^3*c^7*d^2*e^2*f^10*g^2 - 30*a*b^5*c^4*d*e^3*f^10*g
^2 + 108*a^2*b^3*c^5*d*e^3*f^10*g^2 + 48*a^3*b*c^6*d*e^3*f^10*g^2 + 15*a^2*
b^4*c^4*e^4*f^10*g^2 - 54*a^3*b^2*c^5*e^4*f^10*g^2 - 24*a^4*c^6*e^4*f^10*g^
2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b*c^7*d^4*f
^9*g^3 + 40*b^6*c^4*d^3*e*f^9*g^3 - 100*a*b^4*c^5*d^3*e*f^9*g^3 - 240*a^2*b
^2*c^6*d^3*e*f^9*g^3 - 20*b^7*c^3*d^2*e^2*f^9*g^3 + 10*a*b^5*c^4*d^2*e^2*f^
9*g^3 + 220*a^2*b^3*c^5*d^2*e^2*f^9*g^3 + 240*a^3*b*c^6*d^2*e^2*f^9*g^3 + 4
0*a*b^6*c^3*d*e^3*f^9*g^3 - 100*a^2*b^4*c^4*d*e^3*f^9*g^3 - 240*a^3*b^2*c^5
*d*e^3*f^9*g^3 - 20*a^2*b^5*c^3*e^4*f^9*g^3 + 50*a^3*b^3*c^4*e^4*f^9*g^3 +
120*a^4*b*c^5*e^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^
8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 30*b^7*c^3*d^3*e*f^8*g^4 + 450*a^2*b^3*c^5
*d^3*e*f^8*g^4 + 120*a^3*b*c^6*d^3*e*f^8*g^4 + 15*b^8*c^2*d^2*e^2*f^8*g^4 +
 30*a*b^6*c^3*d^2*e^2*f^8*g^4 - 225*a^2*b^4*c^4*d^2*e^2*f^8*g^4 - 510*a^3*b
^2*c^5*d^2*e^2*f^8*g^4 - 120*a^4*c^6*d^2*e^2*f^8*g^4 - 30*a*b^7*c^2*d*e^3*f
^8*g^4 + 450*a^3*b^3*c^4*d*e^3*f^8*g^4 + 120*a^4*b*c^5*d*e^3*f^8*g^4 + 15*a
^2*b^6*c^2*e^4*f^8*g^4 - 225*a^4*b^2*c^4*e^4*f^8*g^4 - 60*a^5*c^5*e^4*f^8*g
^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4
*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + 12*b^8*c^2*d^3*e*f^7*g^5 + 72*a*b^6*
c^3*d^3*e*f^7*g^5 - 360*a^2*b^4*c^4*d^3*e*f^7*g^5 - 480*a^3*b^2*c^5*d^3*e*f
^7*g^5 - 6*b^9*c*d^2*e^2*f^7*g^5 - 48*a*b^7*c^2*d^2*e^2*f^7*g^5 + 108*a^2*b
^5*c^3*d^2*e^2*f^7*g^5 + 600*a^3*b^3*c^4*d^2*e^2*f^7*g^5 + 480*a^4*b*c^5*d^
2*e^2*f^7*g^5 + 12*a*b^8*c*d*e^3*f^7*g^5 + 72*a^2*b^6*c^2*d*e^3*f^7*g^5 - 3
60*a^3*b^4*c^3*d*e^3*f^7*g^5 - 480*a^4*b^2*c^4*d*e^3*f^7*g^5 - 6*a^2*b^7*c*
e^4*f^7*g^5 - 36*a^3*b^5*c^2*e^4*f^7*g^5 + 180*a^4*b^3*c^3*e^4*f^7*g^5 + 24
0*a^5*b*c^4*e^4*f^7*g^5 + b^8*c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 -
30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f

^6*g^6 - 2*b^9*c*d^3*e*f^6*g^6 - 52*a*b^7*c^2*d^3*e*f^6*g^6 + 60*a^2*b^5*c^3*d^3*e*f^6*g^6 + 680*a^3*b^3*c^4*d^3*e*f^6*g^6 + 160*a^4*b*c^5*d^3*e*f^6*g^6 + b^10*d^2*e^2*f^6*g^6 + 28*a*b^8*c*d^2*e^2*f^6*g^6 + 22*a^2*b^6*c^2*d^2*e^2*f^6*g^6 - 400*a^3*b^4*c^3*d^2*e^2*f^6*g^6 - 760*a^4*b^2*c^4*d^2*e^2*f^6*g^6 - 160*a^5*c^5*d^2*e^2*f^6*g^6 - 2*a*b^9*d*e^3*f^6*g^6 - 52*a^2*b^7*c*d*e^3*f^6*g^6 + 60*a^3*b^5*c^2*d*e^3*f^6*g^6 + 680*a^4*b^3*c^3*d*e^3*f^6*g^6 + 160*a^5*b*c^4*d*e^3*f^6*g^6 + a^2*b^8*e^4*f^6*g^6 + 26*a^3*b^6*c*e^4*f^6*g^6 - 30*a^4*b^4*c^2*e^4*f^6*g^6 - 340*a^5*b^2*c^3*e^4*f^6*g^6 - 80*a^6*c^4*e^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^7 + 12*a*b^8*c*d^3*e*f^5*g^7 + 72*a^2*b^6*c^2*d^3*e*f^5*g^7 - 360*a^3*b^4*c^3*d^3*e*f^5*g^7 - 480*a^4*b^2*c^4*d^3*e*f^5*g^7 - 6*a*b^9*d^2*e^2*f^5*g^7 - 48*a^2*b^7*c*d^2*e^2*f^5*g^7 + 108*a^3*b^5*c^2*d^2*e^2*f^5*g^7 + 600*a^4*b^3*c^3*d^2*e^2*f^5*g^7 + 480*a^5*b*c^4*d^2*e^2*f^5*g^7 + 12*a^2*b^8*d*e^3*f^5*g^7 + 72*a^3*b^6*c*d*e^3*f^5*g^7 - 360*a^4*b^4*c^2*d*e^3*f^5*g^7 - 480*a^5*b^2*c^3*d*e^3*f^5*g^7 - 6*a^3*b^7*e^4*f^5*g^7 - 36*a^4*b^5*c*e^4*f^5*g^7 + 180*a^5*b^3*c^2*e^4*f^5*g^7 + 240*a^6*b*c^3*e^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d^4*f^4*g^8 - 30*a^2*b^7*c*d^3*e*f^4*g^8 + 450*a^4*b^3*c^3*d^3*e*f^4*g^8 + 120*a^5*b*c^4*d^3*e*f^4*g^8 + 15*a^2*b^8*d^2*e^2*f^4*g^8 + 30*a^3*b^6*c*d^2*e^2*f^4*g^8 - 225*a^4*b^4*c^2*d^2*e^2*f^4*g^8 - 510*a^5*b^2*c^3*d^2*e^2*f^4*g^8 - 120*a^6*c^4*d^2*e^2*f^4*g^8 - 30*a^3*b^7*d*e^3*f^4*g^8 + 450*a^5*b^3*c^2*d*e^3*f^4*g^8 + 120*a^6*b*c^3*d*e^3*f^4*g^8 + 15*a^4*b^6*e^4*f^4*g^8 - 225*a^6*b^2*c^2*e^4*f^4*g^8 - 60*a^7*c^3*e^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 40*a^3*b^6*c*d^3*e*f^3*g^9 - 100*a^4*b^4*c^2*d^3*e*f^3*g^9 - 240*a^5*b^2*c^3*d^3*e*f^3*g^9 - 20*a^3*b^7*d^2*e^2*f^3*g^9 + 10*a^4*b^5*c*d^2*e^2*f^3*g^9 + 220*a^5*b^3*c^2*d^2*e^2*f^3*g^9 + 240*a^6*b*c^3*d^2*e^2*f^3*g^9 + 40*a^4*b^6*d*e^3*f^3*g^9 - 100*a^5*b^4*c*d*e^3*f^3*g^9 - 240*a^6*b^2*c^2*d*e^3*f^3*g^9 - 20*a^5*b^5*e^4*f^3*g^9 + 50*a^6*b^3*c*e^4*f^3*g^9 + 120*a^7*b*c^2*e^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 30*a^4*b^5*c*d^3*e*f^2*g^10 + 108*a^5*b^3*c^2*d^3*e*f^2*g^10 + 48*a^6*b*c^3*d^3*e*f^2*g^10 + 15*a^4*b^6*d^2*e^2*f^2*g^10 - 24*a^5*b^4*c*d^2*e^2*f^2*g^10 - 132*a^6*b^2*c^2*d^2*e^2*f^2*g^10 - 48*a^7*c^3*d^2*e^2*f^2*g^10 - 30*a^5*b^5*d*e^3*f^2*g^10 + 108*a^6*b^3*c*d*e^3*f^2*g^10 + 48*a^7*b*c^2*d*e^3*f^2*g^10 + 15*a^6*b^4*e^4*f^2*g^10 - 54*a^7*b^2*c*e^4*f^2*g^10 - 24*a^8*c^2*e^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*d^4*f*g^11 + 12*a^5*b^4*c*d^3*e*f*g^11 - 48*a^6*b^2*c^2*d^3*e*f*g^11 - 6*a^5*b^5*d^2*e^2*f*g^11 + 12*a^6*b^3*c*d^2*e^2*f*g^11 + 48*a^7*b*c^2*d^2*e^2*f*g^11 + 12*a^6*b^4*d*e^3*f*g^11 - 48*a^7*b^2*c*d*e^3*f*g^11 - 6*a^7*b^3*e^4*f*g^11 + 24*a^8*b*c*e^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*a^6*b^3*c*d^3*e*g^12 + 8*a^7*b*c^2*d^3*e*g^12 + a^6*b^4*d^2*e^2*g^12 - 2*a^7*b^2*c*d^2*e^2*g^12 - 8*a^8*c^2*d^2*e^2*g^12 - 2*a^7*b^3*d*e^3*g^12 + 8*a^8*b*c*d*e^3*g^12 + a^8*b^2*e^4*g^12 - 4*a^9*c*e^4*g^12))/sqrt(c*x^2 + b*x + a) - 1/4*(80*c^2*e^2*f^4*g^3 - 120*c^2*d*e*f^3*g^4 - 100*b*c*e^2*f^3*g^4 + 48*c^2*d^2*f^2*g^5 + 132*b*c*d*e*f^2*g^5

+ 35*b^2*e^2*f^2*g^5 + 28*a*c*e^2*f^2*g^5 - 48*b*c*d^2*f*g^6 - 42*b^2*d*e*
f*g^6 - 28*a*b*e^2*f*g^6 + 15*b^2*d^2*g^7 - 12*a*c*d^2*g^7 + 12*a*b*d*e*g^7
 + 8*a^2*e^2*g^7)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*
f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^3*e^3*f^9 - 3*c^3*d*e^2*f^8*g - 3*b*c^
2*e^3*f^8*g + 3*c^3*d^2*e*f^7*g^2 + 9*b*c^2*d*e^2*f^7*g^2 + 3*b^2*c*e^3*f^7
*g^2 + 3*a*c^2*e^3*f^7*g^2 - c^3*d^3*f^6*g^3 - 9*b*c^2*d^2*e*f^6*g^3 - 9*b^
2*c*d*e^2*f^6*g^3 - 9*a*c^2*d*e^2*f^6*g^3 - b^3*e^3*f^6*g^3 - 6*a*b*c*e^3*f
^6*g^3 + 3*b*c^2*d^3*f^5*g^4 + 9*b^2*c*d^2*e*f^5*g^4 + 9*a*c^2*d^2*e*f^5*g^
4 + 3*b^3*d*e^2*f^5*g^4 + 18*a*b*c*d*e^2*f^5*g^4 + 3*a*b^2*e^3*f^5*g^4 + 3*
a^2*c*e^3*f^5*g^4 - 3*b^2*c*d^3*f^4*g^5 - 3*a*c^2*d^3*f^4*g^5 - 3*b^3*d^2*e
*f^4*g^5 - 18*a*b*c*d^2*e*f^4*g^5 - 9*a*b^2*d*e^2*f^4*g^5 - 9*a^2*c*d*e^2*f
^4*g^5 - 3*a^2*b*e^3*f^4*g^5 + b^3*d^3*f^3*g^6 + 6*a*b*c*d^3*f^3*g^6 + 9*a*
b^2*d^2*e*f^3*g^6 + 9*a^2*c*d^2*e*f^3*g^6 + 9*a^2*b*d*e^2*f^3*g^6 + a^3*e^3
*f^3*g^6 - 3*a*b^2*d^3*f^2*g^7 - 3*a^2*c*d^3*f^2*g^7 - 9*a^2*b*d^2*e*f^2*g^
7 - 3*a^3*d*e^2*f^2*g^7 + 3*a^2*b*d^3*f*g^8 + 3*a^3*d^2*e*f*g^8 - a^3*d^3*g
^9)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 1/4*(32*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*c^2*e*f^3*g^4 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g
^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*f^2*g^5 + 24*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^6 + 11*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b^2*e*f*g^6 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*f*g^6 -
 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^7 + 4*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a*c*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e
g^7 + 72(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*e*f^4*g^3 - 56*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g^4 - 68*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*b*c^(3/2)*e*f^3*g^4 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*b*c^(3/2)*d*f^2*g^5 + 17*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*
sqrt(c)*e*f^2*g^5 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*e*f^
2*g^5 - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^6 + 28*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^6 + 12*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e*f*g^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a*b*sqrt(c)*d*g^7 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*s
qrt(c)*e*g^7 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*e*f^4*g^3 - 56*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^4 - 64*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b^2*c*e*f^3*g^4 - 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*c^2*e*f^3*g^4 + 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^5 +
 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^5 + 13*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b^3*e*f^2*g^5 + 104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a*b*c*e*f^2*g^5 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^6 - 60
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^6 - 17*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b^2*e*f*g^6 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2
*c*e*f*g^6 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^7 + 4*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^7 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*e*g^7 + 18*b^2*c^(3/2)*e*f^4*g^3 - 14*b^2*c^(3/2)*d*f^3*g^4 - 11
*b^3*sqrt(c)*e*f^3*g^4 - 56*a*b*c^(3/2)*e*f^3*g^4 + 7*b^3*sqrt(c)*d*f^2*g^5
 + 44*a*b*c^(3/2)*d*f^2*g^5 + 39*a*b^2*sqrt(c)*e*f^2*g^5 + 36*a^2*c^(3/2)*e

```
*f^2*g^5 - 23*a*b^2*sqrt(c)*d*f*g^6 - 28*a^2*c^(3/2)*d*f*g^6 - 36*a^2*b*sqr
t(c)*e*f*g^6 + 16*a^2*b*sqrt(c)*d*g^7 + 8*a^3*sqrt(c)*e*g^7)/((c^3*e^2*f^8
- 2*c^3*d*e*f^7*g - 3*b*c^2*e^2*f^7*g + c^3*d^2*f^6*g^2 + 6*b*c^2*d*e*f^6*g
^2 + 3*b^2*c*e^2*f^6*g^2 + 3*a*c^2*e^2*f^6*g^2 - 3*b*c^2*d^2*f^5*g^3 - 6*b^
2*c*d*e*f^5*g^3 - 6*a*c^2*d*e*f^5*g^3 - b^3*e^2*f^5*g^3 - 6*a*b*c*e^2*f^5*g
^3 + 3*b^2*c*d^2*f^4*g^4 + 3*a*c^2*d^2*f^4*g^4 + 2*b^3*d*e*f^4*g^4 + 12*a*b
*c*d*e*f^4*g^4 + 3*a*b^2*e^2*f^4*g^4 + 3*a^2*c*e^2*f^4*g^4 - b^3*d^2*f^3*g^
5 - 6*a*b*c*d^2*f^3*g^5 - 6*a*b^2*d*e*f^3*g^5 - 6*a^2*c*d*e*f^3*g^5 - 3*a^2
*b*e^2*f^3*g^5 + 3*a*b^2*d^2*f^2*g^6 + 3*a^2*c*d^2*f^2*g^6 + 6*a^2*b*d*e*f^
2*g^6 + a^3*e^2*f^2*g^6 - 3*a^2*b*d^2*f*g^7 - 2*a^3*d*e*f*g^7 + a^3*d^2*g^8
)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*sqrt(c)*f + b*f - a*g)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.886 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1551

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-798d^3efg^3}{}$$

$$+\frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$

$$+\frac{2(48b^3e^3g^3+bce^2g^2(67bef-198bdg-157aeg)+c^3(233e^3f^3-843de^2f^2g+1107d^2efg^2-567d^3g^3)-}{3465c^3g^4}$$

$$-\frac{2e(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+c^2(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{693c^2g^4}$$

$$+\frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(128b^5e^3g^5-8b^3ce^2g^4(7bef+66bdg+87aeg)+2c^5f^2(64e^3f^3-264de^2f^2g+396d^2efg^2-}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(64b^4e^3g^4+4b^2ce^2g^3(7bef-66bdg-69aeg)-2c^4f(64e^3f^3-264de^2f}{}$$

```
[Out] 2/3465*(48*b^3*e^3*g^3+b*c*e^2*g^2*(-157*a*e*g-198*b*d*g+67*b*e*f)+c^3*(-56
7*d^3*g^3+1107*d^2*e*f*g^2-843*d*e^2*f^2*g+233*e^3*f^3)-c^2*e*g*(2*a*e*g*(-
231*d*g+74*e*f)-3*b*(99*d^2*g^2-88*d*e*f*g+24*e^2*f^2)))*(g*x+f)^(3/2)*(c*x
^2+b*x+a)^(1/2)/c^3/g^4-2/693*e*(8*b^2*e^2*g^2+c*e*g*(-18*a*e*g-33*b*d*g+19
```

```
*b*e*f)+c^2*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+b*x+a)
^(1/2)/c^2/g^4+2/99*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(7/2)*(c*x^2+b*x+a)^(
1/2)/c/g^4-2/3465*(64*b^4*e^4*g^4+4*b^2*c*e^3*g^3*(-69*a*e*g-66*b*d*g+7*b*e
*f)+c^4*(315*d^4*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+1
87*e^4*f^4)+3*c^2*e^2*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(
44*d^2*g^2-11*d*e*f*g+e^2*f^2))-c^3*e*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*
e^2*f^2)+b*(231*d^3*g^3-99*d^2*e*f*g^2+8*e^3*f^3)))*(g*x+f)^(1/2)*(c*x^2+b*
x+a)^(1/2)/c^4/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3
465*(128*b^5*e^3*g^5-8*b^3*c*e^2*g^4*(87*a*e*g+66*b*d*g+7*b*e*f)+2*c^5*f^2*
(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+b*c^2*e*g^3*(771*
a^2*e^2*g^2+6*a*b*e*g*(396*d*g+43*e*f)-b^2*(-792*d^2*g^2-264*d*e*f*g+37*e^2
*f^2))-c^4*g*(b*f*(-462*d^3*g^3+495*d^2*e*f*g^2-264*d*e^2*f^2*g+56*e^3*f^3)
-18*a*g*(77*d^3*g^3+88*d^2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^3))-c^3*g^2*(6*a^
2*e^2*g^2*(231*d*g+26*e*f)-9*a*b*e*g*(-319*d^2*g^2-110*d*e*f*g+15*e^2*f^2)+
b^2*(462*d^3*g^3+495*d^2*e*f*g^2-198*d*e^2*f^2*g+37*e^3*f^3)))*EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^5/g^5/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3465*(a*
g^2-b*f*g+c*f^2)*(64*b^4*e^3*g^4+4*b^2*c*e^2*g^3*(-69*a*e*g-66*b*d*g+7*b*e*
f)-2*c^4*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*
e*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(44*d^2*g^2-11*d*e*f*
g+e^2*f^2))-c^3*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+b*(231*d^3*g^
3-99*d^2*e*f*g^2+8*e^3*f^3)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^5/g^5/(g
*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 5.02 (sec) , antiderivative size = 1551, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

$= \{932, 1667, 857, 732, 435, 430\}$

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^4}{11e}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2f^2(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3)\,c^5-g(bf(56e^3f^3-264de^2gf^2+495d^2e}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)\,(-2f(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3)\,c^4-g(6aeg(2e^2f^2}{}$$

$$+\frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{cx^2+bx+a}}{99cg^4}$$

$$-\frac{2e((29e^2f^2-96degf+81d^2g^2)\,c^2+eg(19bef-33bdg-18aeg)c+8b^2e^2g^2)\,(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{693c^2g^4}$$

$$+\frac{2((233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3)\,c^3-eg(2aeg(74ef-231dg)-3b(24e^2f^2-88degf+}{3465c^3g^4}$$

$$-\frac{2((187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d^3eg^3f+315d^4g^4)\,c^4-eg(6aeg(2e^2f^2-33degf+165d^2}{}$$

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) +
 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 -
 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2*g^2
*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g +
1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b
*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*
x^2])/(3465*c^3*g^4) - (2*e*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 1
8*a*e*g) + c^2*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt
[a + b*x + c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(12
8*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e*g) + 2*c^5*f^2
*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + b*c^2*e*g
^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*
d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^
2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*
```

```
g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b*e*g*
(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 - 198*d*e^2*f^2*
g + 495*d^2*e*f*g^2 + 462*d^3*g^3)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(3465*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*d*g - 69*a
*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g
^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2
*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g +
 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{\int \frac{(d+ex)^3\left(bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+beg)x^2\right)}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4} - \frac{2\int \frac{\frac{1}{2}g\left(b^2e^4f^4g+7abe^4f^3g^2+acg\left(7e^4f^4-21de^3f^3g-27d^3efg^3+9d^4g^4\right)+bc\left(e^4f^5-3de^3f^4g+9d^4fg^4\right)\right)+\frac{1}{2}g\left(be^4f^2g^2(11bf+21ag)-\right.}{}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$
$$- \frac{2e(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+c^2(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+b}}{693c^2g^4}$$
$$+ \frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$
$$- \frac{4\int \frac{-\frac{1}{4}g^5(8b^3e^4f^3g^2+b^2e^3f^2g(40aeg^2+3cf(4ef-11dg))+bcf(ae^3fg^2(28ef-165dg)+c(22e^4f^4-75de^3f^3g+81d^2e^2f^2g^2-63d^4g}{}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$
$$+ \frac{2(48b^3e^3g^3+bce^2g^2(67bef-198bdg-157aeg)+c^3(233e^3f^3-843de^2f^2g+1107d^2efg^2-567}{346}$$
$$- \frac{2e(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+c^2(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+b}}{693c^2g^4}$$
$$+ \frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$
$$- \frac{8\int \frac{\frac{3}{8}g^8(16b^4e^4f^2g^3+3b^3e^3fg^2(16aeg^2+cf(3ef-22dg))-b^2ce^2fg(2aeg^2(26ef+99dg)-cf(4e^2f^2-33defg+99d^2g^2))+ac^2g(2ae}{}}{}$$

$$=$$
$$- \frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-79}{}$$
$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$
$$+ \frac{2(48b^3e^3g^3+bce^2g^2(67bef-198bdg-157aeg)+c^3(233e^3f^3-843de^2f^2g+1107d^2efg^2-567}{346}$$
$$- \frac{2e(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+c^2(29e^2f^2-96defg+81d^2g^2))(f+gx)^{5/2}\sqrt{a+b}}{693c^2g^4}$$
$$+ \frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$
$$- \frac{16\int \frac{-\frac{3}{16}eg^{10}(64b^5e^3fg^4+4b^4e^2g^3(16aeg^2-cf(5ef+66dg))-b^3ceg^2(8aeg^2(49ef+33dg)+9cf(2e^2f^2-11defg-44d^2g^2))+2ac^2g}{}}{}$$

$$= -\frac{2(64b^4e^4g^4 + 4b^2ce^3g^3(7bef - 66bdg - 69aeg) + c^4(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2 - 798}{}$$

$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$

$$+ \frac{2(48b^3e^3g^3 + bce^2g^2(67bef - 198bdg - 157aeg) + c^3(233e^3f^3 - 843de^2f^2g + 1107d^2efg^2 - 567}{3465}$$

$$- \frac{2e(8b^2e^2g^2 + ceg(19bef - 33bdg - 18aeg) + c^2(29e^2f^2 - 96defg + 81d^2g^2))(f+gx)^{5/2}\sqrt{a+bx}}{693c^2g^4}$$

$$+ \frac{2e^2(cef - 3cdg + beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$

$$+ \frac{((cf^2 - bfg + ag^2)(64b^4e^3g^4 + 4b^2ce^2g^3(7bef - 66bdg - 69aeg) - 2c^4f(64e^3f^3 - 264de^2f^2g + 3}{}$$

$$+ \frac{(128b^5e^3g^5 - 8b^3ce^2g^4(7bef + 66bdg + 87aeg) + 2c^5f^2(64e^3f^3 - 264de^2f^2g + 396d^2efg^2 - 231d}{}$$

$$= -\frac{2(64b^4e^4g^4 + 4b^2ce^3g^3(7bef - 66bdg - 69aeg) + c^4(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2 - 798}{}$$

$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$

$$+ \frac{2(48b^3e^3g^3 + bce^2g^2(67bef - 198bdg - 157aeg) + c^3(233e^3f^3 - 843de^2f^2g + 1107d^2efg^2 - 567}{3465}$$

$$- \frac{2e(8b^2e^2g^2 + ceg(19bef - 33bdg - 18aeg) + c^2(29e^2f^2 - 96defg + 81d^2g^2))(f+gx)^{5/2}\sqrt{a+bx}}{693c^2g^4}$$

$$+ \frac{2e^2(cef - 3cdg + beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$

$$+ \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(128b^5e^3g^5 - 8b^3ce^2g^4(7bef + 66bdg + 87aeg) + 2c^5f^2(64e^3f^3 - 264de^2f^2g + 39\right.}{}$$

$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(cf^2 - bfg + ag^2)(64b^4e^3g^4 + 4b^2ce^2g^3(7bef - 66bdg - 69aeg) - 2c^4f(64e^3f^3 -\right.}{}$$

$$= -\frac{2(64b^4e^4g^4 + 4b^2ce^3g^3(7bef - 66bdg - 69aeg) + c^4(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2 - 79}{}$$

$$+ \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e}$$

$$+ \frac{2(48b^3e^3g^3 + bce^2g^2(67bef - 198bdg - 157aeg) + c^3(233e^3f^3 - 843de^2f^2g + 1107d^2efg^2 - 567}{346}$$

$$- \frac{2e(8b^2e^2g^2 + ceg(19bef - 33bdg - 18aeg) + c^2(29e^2f^2 - 96defg + 81d^2g^2))(f+gx)^{5/2}\sqrt{a+b}}{693c^2g^4}$$

$$+ \frac{2e^2(cef - 3cdg + beg)(f+gx)^{7/2}\sqrt{a+bx+cx^2}}{99cg^4}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(128b^5e^3g^5 - 8b^3ce^2g^4(7bef + 66bdg + 87aeg) + 2c^5f^2(64e^3f^3 - 264de^2f^2g + 396}{}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2 - bfg + ag^2)(64b^4e^3g^4 + 4b^2ce^2g^3(7bef - 66bdg - 69aeg) - 2c^4f(64e^3f^3 -}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.07 (sec) , antiderivative size = 26600, normalized size of antiderivative = 17.15

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3253 vs. 2(1469) = 2938.

Time = 3.12 (sec) , antiderivative size = 3254, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	3254
risch	Expression too large to display	11966
default	Expression too large to display	32647

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/11*e^3*x
^4*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/9*(b*e^3*g+3*c*d*e^2*g
+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*x^3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f
*x+a*f)^(1/2)+2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11
*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c
*f))/c/g*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(
1/2)+2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e^2*f*d+c*d^3*g+3*c*d^2*
e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(7/2*a*g+7
/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(
9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c
/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b
*f*x+a*f)^(1/2)+2/3*(3*a*d^2*e*g+3*a*d*e^2*f+b*d^3*g+3*b*d^2*e*f+c*d^3*f-2/
3*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*f*a-2/7*(a*e^3*g
+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9
*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/
g*(5/2*a*g+5/2*b*f)-2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e^2*f*d+c
*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))
/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e
^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(
5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*
(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^3*f-2/5*(3*a*e^2*g*d
+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e^2*f*d+c*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*
d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+
3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*
(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g
*(3*b*g+3*c*f))/c/g*f*a-2/3*(3*a*d^2*e*g+3*a*d*e^2*f+b*d^3*g+3*b*d^2*e*f+c*
d^3*f-2/3*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*f*a-2/7*
(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*
b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*
c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e
^2*f*d+c*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g
+5*c*f))/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g
+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/
11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*
f))/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
```

```
/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+
b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2))+2*(a*d^3*g+3*a*d^2*e*f+b*d^3*f-4/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3
*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+
f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*f*a-2/5*(3*a*e^2*g*d
+3/11*a*e^3*f+3*b*d^2*e*g+3*b*e^2*f*d+c*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*
d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+
3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*
(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g
*(3*b*g+3*c*f))/c/g*(3/2*a*g+3/2*b*f)-2/3*(3*a*d^2*e*g+3*a*d*e^2*f+b*d^3*g+
3*b*d^2*e*f+c*d^3*f-2/3*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f)
)/c/g*f*a-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3
*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))
/c/g*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(3*a*e^2*g*d+3/11*a*e^3*f+3*b
*d^2*e*g+3*b*e^2*f*d+c*d^3*g+3*c*d^2*e*f-2/9*(b*e^3*g+3*c*d*e^2*g+f*c*e^3-2
/11*e^3*(5*b*g+5*c*f))/c/g*(7/2*a*g+7/2*b*f)-2/7*(a*e^3*g+3*b*d*e^2*g+b*e^3
*f+3*c*d^2*e*g+3*c*d*e^2*f-2/11*e^3*(9/2*a*g+9/2*b*f)-2/9*(b*e^3*g+3*c*d*e^
2*g+f*c*e^3-2/11*e^3*(5*b*g+5*c*f))/c/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c
/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g
)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2
)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1741, normalized size of antiderivative = 1.12

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)

[Out] -2/10395*((128*c^6*e^3*f^6 - 24*(22*c^6*d*e^2 + 5*b*c^5*e^3)*f^5*g + 3*(264
*c^6*d^2*e + 176*b*c^5*d*e^2 - (11*b^2*c^4 - 68*a*c^5)*e^3)*f^4*g^2 - (462*
c^6*d^3 + 891*b*c^5*d^2*e - 165*(b^2*c^4 - 6*a*c^5)*d*e^2 + (20*b^3*c^3 - 8
7*a*b*c^4)*e^3)*f^3*g^3 + 3*(231*b*c^5*d^3 - 66*(2*b^2*c^4 - 11*a*c^5)*d^2*
```

```
e + 11*(5*b^3*c^3 - 21*a*b*c^4)*d*e^2 - (11*b^4*c^2 - 53*a*b^2*c^3 + 34*a^2
*c^4)*e^3)*f^2*g^4 + 3*(231*(b^2*c^4 - 6*a*c^5)*d^3 - 33*(9*b^3*c^3 - 41*a*
b*c^4)*d^2*e + 22*(8*b^4*c^2 - 42*a*b^2*c^3 + 33*a^2*c^4)*d*e^2 - (40*b^5*c
 - 246*a*b^3*c^2 + 329*a^2*b*c^3)*e^3)*f*g^5 - (231*(2*b^3*c^3 - 9*a*b*c^4)
*d^3 - 99*(8*b^4*c^2 - 41*a*b^2*c^3 + 30*a^2*c^4)*d^2*e + 33*(16*b^5*c - 96
*a*b^3*c^2 + 123*a^2*b*c^3)*d*e^2 - (128*b^6 - 888*a*b^4*c + 1599*a^2*b^2*c
^2 - 450*a^3*c^3)*e^3)*g^6)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*
c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*
(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x +
c*f + b*g)/(c*g)) + 3*(128*c^6*e^3*f^5*g - 8*(66*c^6*d*e^2 + 7*b*c^5*e^3)*f
^4*g^2 + (792*c^6*d^2*e + 264*b*c^5*d*e^2 - (37*b^2*c^4 - 108*a*c^5)*e^3)*f
^3*g^3 - (462*c^6*d^3 + 495*b*c^5*d^2*e - 198*(b^2*c^4 - 3*a*c^5)*d*e^2 + (
37*b^3*c^3 - 135*a*b*c^4)*e^3)*f^2*g^4 + (462*b*c^5*d^3 - 99*(5*b^2*c^4 - 1
6*a*c^5)*d^2*e + 66*(4*b^3*c^3 - 15*a*b*c^4)*d*e^2 - 2*(28*b^4*c^2 - 129*a*
b^2*c^3 + 78*a^2*c^4)*e^3)*f*g^5 - (462*(b^2*c^4 - 3*a*c^5)*d^3 - 99*(8*b^3
*c^3 - 29*a*b*c^4)*d^2*e + 66*(8*b^4*c^2 - 36*a*b^2*c^3 + 21*a^2*c^4)*d*e^2
 - (128*b^5*c - 696*a*b^3*c^2 + 771*a^2*b*c^3)*e^3)*g^6)*sqrt(c*g)*weierstr
assZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3
*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(
c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(
c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2
*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) - 3*(315*
c^6*e^3*g^6*x^4 - 64*c^6*e^3*f^4*g^2 + 4*(66*c^6*d*e^2 + 5*b*c^5*e^3)*f^3*g
^3 - (396*c^6*d^2*e + 99*b*c^5*d*e^2 - 2*(9*b^2*c^4 - 23*a*c^5)*e^3)*f^2*g^
4 + (231*c^6*d^3 + 198*b*c^5*d^2*e - 33*(3*b^2*c^4 - 8*a*c^5)*d*e^2 + 10*(2
*b^3*c^3 - 7*a*b*c^4)*e^3)*f*g^5 + (231*b*c^5*d^3 - 198*(2*b^2*c^4 - 5*a*c^
5)*d^2*e + 33*(8*b^3*c^3 - 27*a*b*c^4)*d*e^2 - 2*(32*b^4*c^2 - 138*a*b^2*c^
3 + 75*a^2*c^4)*e^3)*g^6 + 35*(c^6*e^3*f*g^5 + (33*c^6*d*e^2 + b*c^5*e^3)*g
^6)*x^3 - 5*(8*c^6*e^3*f^2*g^4 - (33*c^6*d*e^2 + 2*b*c^5*e^3)*f*g^5 - (297*
c^6*d^2*e + 33*b*c^5*d*e^2 - 2*(4*b^2*c^4 - 9*a*c^5)*e^3)*g^6)*x^2 + (48*c^
6*e^3*f^3*g^3 - (198*c^6*d*e^2 + 13*b*c^5*e^3)*f^2*g^4 + (297*c^6*d^2*e + 6
6*b*c^5*d*e^2 - (13*b^2*c^4 - 32*a*c^5)*e^3)*f*g^5 + (693*c^6*d^3 + 297*b*c
^5*d^2*e - 66*(3*b^2*c^4 - 7*a*c^5)*d*e^2 + (48*b^3*c^3 - 157*a*b*c^4)*e^3)
*g^6)*x)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^6*g^6)
```

Sympy [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^3\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2), x)
```

3.887 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1015

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2(2ae(ef-}{315c^3eg^3}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}$$
$$-\frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{315c^2g^3}$$
$$+\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8b^4e^2g^4-4b^2ceg^3(bef+6bdg+9aeg)+c^4f^2(8e^2f^2-24defg+21d^2g^2)+3c^2g^2(7a^2e^2g}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(8b^3e^2g^3+3bceg^2(bef-8bdg-9aeg)-2c^3f(8e^2f^2-24defg+21d^2g}{}$$

```
[Out] -4/315*(3*b^2*e^2*g^2+c*e*g*(-7*a*e*g-9*b*d*g+4*b*e*f)+c^2*(21*d^2*g^2-24*d
*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/63*e*(b*e*g-
3*c*d*g+c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^3+2/315*(8*b^3*e^3*g^3
+3*b*c*e^2*g^2*(-9*a*e*g-8*b*d*g+b*e*f)+c^3*(-35*d^3*g^3+63*d^2*e*f*g^2-57*
d*e^2*f^2*g+19*e^3*f^3)-3*c^2*e*g^2*(2*a*e*(-10*d*g+e*f)+b*d*(-7*d*g+2*e*f)
))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/2)*
```

```
(c*x^2+b*x+a)^(1/2)/e-2/315*(8*b^4*e^2*g^4-4*b^2*c*e*g^3*(9*a*e*g+6*b*d*g+b
*e*f)+c^4*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)+3*c^2*g^2*(7*a^2*e^2*g^2+a*
b*e*g*(29*d*g+5*e*f)-b^2*(-7*d^2*g^2-5*d*e*f*g+e^2*f^2))+c^3*g*(3*a*g*(-21*
d^2*g^2-16*d*e*f*g+3*e^2*f^2)-b*f*(21*d^2*g^2-15*d*e*f*g+4*e^2*f^2)))*Ellip
ticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/g^4
/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
15*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^2*g^3+3*b*c*e*g^2*(-9*a*e*g-8*b*d*g+b*e*f)-
2*c^3*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)-3*c^2*g^2*(2*a*e*(-10*d*g+e*f)+b*
d*(-7*d*g+2*e*f)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^4/(g*x+f)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 1015, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {932, 1667, 857, 732, 435, 430}

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9e}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(f^2(8e^2f^2-24degf+21d^2g^2)\,c^4+g(3ag(3e^2f^2-16degf-21d^2g^2)-bf(4e^2f^2-15de}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)\,(-2f(8e^2f^2-24degf+21d^2g^2)\,c^3-3g^2(2ae(ef-10dg)+bd(2ef-}{}$$

$$+\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{63cg^3}$$

$$-\frac{4((8e^2f^2-24degf+21d^2g^2)\,c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2)\,(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{315c^2g^3}$$

$$+\frac{2((19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3)\,c^3-3eg^2(2ae(ef-10dg)+bd(2ef-7dg))c^2+3be^2g^2(b}{315c^3eg^3}$$

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^3*(19*e^3
*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^2*(2*a*e*(
```

```
e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/
(315*c^3*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e)
 - (4*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2
 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^
2*g^3) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2*c*e*g^
3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^
2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5
*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2)
 - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a
*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e
*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{\int \frac{(d+ex)^2(bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+beg)x^2)}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3} - \frac{2\int \frac{\frac{1}{2}g(b^2e^3f^3g+acg(5e^3f^3-15de^2f^2g-21d^2efg^2+7d^3g^3)+bf(5ae^3fg^2+c(e^3f^3-3de^2f^2g+7d^3g^3)))+g(be^3fg^2(4bf+5ag)+c^2}{}}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{315c^2g^3} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3} - \frac{4\int \frac{-\frac{1}{4}g^4(6b^3e^3f^2g^2+3b^2e^2fg(6aeg^2+cf(ef-6dg))-bcf(3ae^2g^2(5ef+18dg)-c(11e^3f^3-33de^2f^2g+42d^2efg^2-35d^3g^3))-acg(4}{}}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2(2}{315c^3eg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{315c^2g^3} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3} - \frac{8\int \frac{\frac{3}{8}eg^6(8b^4e^2fg^3+b^3eg^2(8aeg^2-3cf(ef+8dg))-3b^2cg(2aeg^2(7ef+4dg)+cf(e^2f^2-4defg-7d^2g^2))-bc(27a^2e^2g^4-3acg^2(3e^2f^2}{}}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2(2}{315c^3eg^3} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{315c^2g^3} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3} - \frac{((cf^2-bfg+ag^2)(8b^3e^2g^3+3bceg^2(bef-8bdg-9aeg)-2c^3f(8e^2f^2-24defg+21d^2g^2)-3}{315c^3g^4} - \frac{(2(8b^4e^2g^4-4b^2ceg^3(bef+6bdg+9aeg)+c^4f^2(8e^2f^2-24defg+21d^2g^2)+3c^2g^2(7a^2e^2g^2+a}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2}{315c^3eg^3}$$

$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}$$

$$-\frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+c}}{315c^2g^3}$$

$$+\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3}$$

$$-\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(8b^4e^2g^4-4b^2ceg^3(bef+6bdg+9aeg)+c^4f^2(8e^2f^2-24defg+21d^2g^2)+3c\right.}{}$$

$$-\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\left(8b^3e^2g^3+3bceg^2(bef-8bdg-9aeg)-2c^3f(8e^2f^2-24de\right.\right.}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2}{315c^3eg^3}$$

$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e}$$

$$-\frac{4(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2(8e^2f^2-24defg+21d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+c}}{315c^2g^3}$$

$$+\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^3}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8b^4e^2g^4-4b^2ceg^3(bef+6bdg+9aeg)+c^4f^2(8e^2f^2-24defg+21d^2g^2)+3c^2}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(8b^3e^2g^3+3bceg^2(bef-8bdg-9aeg)-2c^3f(8e^2f^2-24def}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.97 (sec) , antiderivative size = 15781, normalized size of antiderivative = 15.55

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. 2(939) = 1878.

Time = 3.52 (sec) , antiderivative size = 1936, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1936
risch	Expression too large to display	7219
default	Expression too large to display	20224

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*e^2*x^
3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/7*(b*e^2*g+2*c*d*e*g+c*
e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a
*f)^(1/2)+2/5*(a*e^2*g+2*b*d*e*g+b*e^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g
+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+
3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(2*a*d*e*
g+1/3*a*e^2*f+b*d^2*g+2*b*d*e*f+c*d^2*f-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*
e^2*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a*e^2*g+2*b*d*e*g+b*e^2*f+c*d
^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9
*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*
x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^2*f-2/5*(a*e^2*g+2*b*d*e*g+b*e^2*
f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*
f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*f*a-2/3*(2*a*d*e*g+1/3*a*e^
2*f+b*d^2*g+2*b*d*e*f+c*d^2*f-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g
+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a*e^2*g+2*b*d*e*g+b*e^2*f+c*d^2*g+2*c*d
*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*
g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))*(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
```

```
))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*d^2*g+2*a*d*e*f+
b*d^2*f-4/7*(b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*f*a-2/5*(
a*e^2*g+2*b*d*e*g+b*e^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(
b*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(3/
2*a*g+3/2*b*f)-2/3*(2*a*d*e*g+1/3*a*e^2*f+b*d^2*g+2*b*d*e*f+c*d^2*f-2/7*(b*
e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a
*e^2*g+2*b*d*e*g+b*e^2*f+c*d^2*g+2*c*d*e*f-2/9*e^2*(7/2*a*g+7/2*b*f)-2/7*(b
*e^2*g+2*c*d*e*g+c*e^2*f-2/9*e^2*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b
*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b
*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.12

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/945*((16*c^5*e^2*f^5 - 16*(3*c^5*d*e + b*c^4*e^2)*f^4*g + (42*c^5*d^2 + 5
4*b*c^4*d*e - 5*(b^2*c^3 - 6*a*c^4)*e^2)*f^3*g^2 - (63*b*c^4*d^2 - 12*(2*b^
2*c^3 - 11*a*c^4)*d*e + (5*b^3*c^2 - 21*a*b*c^3)*e^2)*f^2*g^3 - (63*(b^2*c^
3 - 6*a*c^4)*d^2 - 6*(9*b^3*c^2 - 41*a*b*c^3)*d*e + 2*(8*b^4*c - 42*a*b^2*c
^2 + 33*a^2*c^3)*e^2)*f*g^4 + (21*(2*b^3*c^2 - 9*a*b*c^3)*d^2 - 6*(8*b^4*c
- 41*a*b^2*c^2 + 30*a^2*c^3)*d*e + (16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*e^
2)*g^5)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c
)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*
g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) +
6*(8*c^5*e^2*f^4*g - 4*(6*c^5*d*e + b*c^4*e^2)*f^3*g^2 + 3*(7*c^5*d^2 + 5*b
*c^4*d*e - (b^2*c^3 - 3*a*c^4)*e^2)*f^2*g^3 - (21*b*c^4*d^2 - 3*(5*b^2*c^3
- 16*a*c^4)*d*e + (4*b^3*c^2 - 15*a*b*c^3)*e^2)*f*g^4 + (21*(b^2*c^3 - 3*a*
c^4)*d^2 - 3*(8*b^3*c^2 - 29*a*b*c^3)*d*e + (8*b^4*c - 36*a*b^2*c^2 + 21*a^
2*c^3)*e^2)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 -
```

```
3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^
2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f
^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2
*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c
*g*x + c*f + b*g)/(c*g))) + 3*(35*c^5*e^2*g^5*x^3 + 8*c^5*e^2*f^3*g^2 - 3*(
8*c^5*d*e + b*c^4*e^2)*f^2*g^3 + (21*c^5*d^2 + 12*b*c^4*d*e - (3*b^2*c^3 -
8*a*c^4)*e^2)*f*g^4 + (21*b*c^4*d^2 - 12*(2*b^2*c^3 - 5*a*c^4)*d*e + (8*b^3
*c^2 - 27*a*b*c^3)*e^2)*g^5 + 5*(c^5*e^2*f*g^4 + (18*c^5*d*e + b*c^4*e^2)*g
^5)*x^2 - (6*c^5*e^2*f^2*g^3 - 2*(9*c^5*d*e + b*c^4*e^2)*f*g^4 - (63*c^5*d^
2 + 18*b*c^4*d*e - 2*(3*b^2*c^3 - 7*a*c^4)*e^2)*g^5)*x)*sqrt(c*x^2 + b*x +
a)*sqrt(g*x + f))/(c^5*g^5)
```

Sympy [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^2\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2), x)
```

3.888 $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 652

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\sqrt{f+gx}(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x)\sqrt{a+bx+c}}{105c^2g^2}$$

$$+\frac{2e\sqrt{f+gx}(a+bx+cx^2)^{3/2}}{7c}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}((cef+7cdg-4beg)(8c^2f^2-2b^2g^2-3cg(bf-2ag))-5cg(2cf-bg)(7cdf-e(3bf+ag}{105c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(4b^2eg^2-2c^2f(4ef-7dg)+cg(bef-7bdg-10aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4a}\right.}}}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx}}$$

```
[Out] 2/7*e*(c*x^2+b*x+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(4*b^2*e*g^2+c^2*f*(-7*d*g+
4*e*f)-c*g*(-5*a*e*g+7*b*d*g+2*b*e*f)-3*c*g*(-4*b*e*g+7*c*d*g+c*e*f)*x)*(g*
x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+1/105*((-4*b*e*g+7*c*d*g+c*e*f)*(8*c
^2*f^2-2*b^2*g^2-3*c*g*(-2*a*g+b*f))-5*c*g*(-b*g+2*c*f)*(7*c*d*f-e*(a*g+3*b
*f)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)/c^3/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2)+2/105*(a*g^2-b*f*g+c*f^2)*(4*b^2*e*g^2-2*c^2*f*(-7*d*g+4*e*f)+c*g*(
-10*a*e*g-7*b*d*g+b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
```

```
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^
2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f
)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {846, 828, 857, 732, 435, 430}

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(cg(-10aeg-7bdg+bef)+4b^2eg^2-\ldots}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx\ldots}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}((-3cg(bf-2ag)-2b^2g^2+8c^2f^2)(-4beg+7cdg+cef)-5cg(\ldots}{105c^3g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(\ldots}{2cf-g(\ldots}}}$$

$$-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-cg(-5aeg+7bdg+2bef)+4b^2eg^2-3cgx(-4beg+7cdg+cef)+c^2f(4e\ldots}{105c^2g^2}$$

$$+\frac{2e\sqrt{f+gx}(a+bx+cx^2)^{3/2}}{7c}$$

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b
*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2
])/(105*c^2*g^2) + (2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2
- 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^3*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(
4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(1
05*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-3bef-aeg)+\frac{1}{2}(cef+7cdg-4beg)x\right)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x\right)\sqrt{a}}{105c^2g^2} + \frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c} - \frac{4\int \frac{\frac{1}{4}\left(5cg(bf-2ag)(7cdf-e(3bf+ag))-2(cef+7cdg-4beg)\left(\frac{1}{2}bf(4cf-bg)-ag\left(cf+\frac{bg}{2}\right)\right)\right)-\frac{1}{4}\left((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-\right.\right.}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x\right)\sqrt{a}}{105c^2g^2} + \frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c} + \frac{\left(\left(cf^2-bfg+ag^2\right)\left(4b^2eg^2-2c^2f(4ef-7dg)+cg(bef-7bdg-10aeg)\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{105c^2g^3} + \frac{\left((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-e(3bf+ag))\right)\int}{105c^2g^3}$$

$$= -\frac{2\sqrt{f+gx}(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x)\sqrt{a\cdots}}{105c^2g^2}$$
$$+\frac{2e\sqrt{f+gx}(a+bx+cx^2)^{3/2}}{7c}$$
$$+\frac{\left(\sqrt{2}\sqrt{b^2-4ac}((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-e\cdots\right.}{105c^3g^3\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\cdots}}\cdots}$$
$$+\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\left(4b^2eg^2-2c^2f(4ef-7dg)+cg(bef-7bdg-10aeg)\right)\sqrt{\frac{\cdots}{2cf-\cdots}}\right.}{105c^3g^3\sqrt{f+gx}\sqrt{a+\cdots}}$$
$$= -\frac{2\sqrt{f+gx}(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x)\sqrt{a\cdots}}{105c^2g^2}$$
$$+\frac{2e\sqrt{f+gx}(a+bx+cx^2)^{3/2}}{7c}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}((cef+7cdg-4beg)\left(8c^2f^2-2b^2g^2-3cg(bf-2ag)\right)-5cg(2cf-bg)(7cdf-e(3\cdots}{105c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\cdots}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\left(4b^2eg^2-2c^2f(4ef-7dg)+cg(bef-7bdg-10aeg)\right)\sqrt{\frac{c\cdots}{2cf-(\cdots}}}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+\cdots}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.21 (sec) , antiderivative size = 8432, normalized size of antiderivative = 12.93

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. 2(588) = 1176.

Time = 1.95 (sec) , antiderivative size = 1229, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1229
risch	Expression too large to display	3893
default	Expression too large to display	10711

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e*x^2*
(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(b*e*g+c*d*g+c*e*f-2/7*
e*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*
(a*e*g+b*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*g+c*e*f-2/7
*e*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f
*x+a*f)^(1/2)+2*(a*d*f-2/5*(b*e*g+c*d*g+c*e*f-2/7*e*(3*b*g+3*c*f))/c/g*f*a-
2/3*(a*e*g+b*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*g+c*e*f
-2/7*e*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*d*g+3/7*a*e*f+b*d*f-2/5
*(b*e*g+c*d*g+c*e*f-2/7*e*(3*b*g+3*c*f))/c/g*(3/2*a*g+3/2*b*f)-2/3*(a*e*g+b
*d*g+b*e*f+c*d*f-2/7*e*(5/2*a*g+5/2*b*f)-2/5*(b*e*g+c*d*g+c*e*f-2/7*e*(3*b*
g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))
/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*
c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x
^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Ell
ipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+
(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.11

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\left((8\,c^4ef^4-(14\,c^4d+9\,bc^3e)f^3g+(21\,bc^3d-2\,(2\,b^2c^2-11\,ac^3)e)f^2g^2+(21\,(b^2c^2-6\,ac^3)d-(9\,b^3c-\right.}{}$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*((8*c^4*e*f^4 - (14*c^4*d + 9*b*c^3*e)*f^3*g + (21*b*c^3*d - 2*(2*b^
2*c^2 - 11*a*c^3)*e)*f^2*g^2 + (21*(b^2*c^2 - 6*a*c^3)*d - (9*b^3*c - 41*a*
b*c^2)*e)*f*g^3 - (7*(2*b^3*c - 9*a*b*c^2)*d - (8*b^4 - 41*a*b^2*c + 30*a^2
*c^2)*e)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 -
 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c
^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*
g)) + 3*(8*c^4*e*f^3*g - (14*c^4*d + 5*b*c^3*e)*f^2*g^2 + (14*b*c^3*d - (5*
b^2*c^2 - 16*a*c^3)*e)*f*g^3 - (14*(b^2*c^2 - 3*a*c^3)*d - (8*b^3*c - 29*a*
b*c^2)*e)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*
a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)
*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g
 - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g
*x + c*f + b*g)/(c*g))) - 3*(15*c^4*e*g^4*x^2 - 4*c^4*e*f^2*g^2 + (7*c^4*d
+ 2*b*c^3*e)*f*g^3 + (7*b*c^3*d - 2*(2*b^2*c^2 - 5*a*c^3)*e)*g^4 + 3*(c^4*e
*f*g^3 + (7*c^4*d + b*c^3*e)*g^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(
c^4*g^4)
```

Sympy [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+g\,x}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2), x)
```

3.889 $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 513

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(c^2f^2+b^2g^2-cg(bf+3ag))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2cf-(}\right.}{15c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2}}{\sqrt{}}}}{}\right.\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/g-2/15*(-b*g+2*c*f)*(g*x+f)^(1/2)*(c*
x^2+b*x+a)^(1/2)/c/g-2/15*(c^2*f^2+b^2*g^2-c*g*(3*a*g+b*f))*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^
(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g^2/(c*x^2+b*
x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-b*g+2*
c*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {748, 846, 857, 732, 435, 430}

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\ldots\right)}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\ldots\right)}{15c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{15cg}$$

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c*g) + (2*(f + g
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(15*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\int \frac{\sqrt{f+gx}(bf-2ag+(2cf-bg)x)}{\sqrt{a+bx+cx^2}}\,dx}{5g}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\int \frac{\frac{1}{2}(bcf^2+b^2fg-8acfg+abg^2)+(c^2f^2+b^2g^2-cg(bf+3ag))x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{15cg}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} + \frac{((2cf-bg)(cf^2-bfg+ag^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{15cg^2} - \frac{(2(c^2f^2+b^2g^2-cg(bf+3ag)))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{15cg^2}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(c^2f^2+b^2g^2-cg(bf+3ag))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}}{2cf-bg-\sqrt{b^2}}}}{\sqrt{1-x^2}}\right.}{15c^2g^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x}}\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(c^2f^2+b^2g^2-cg(bf+3ag))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{15c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b}}{\sqrt{}}}}{}\right.\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.55 (sec) , antiderivative size = 1052, normalized size of antiderivative = 2.05

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+x(b+cx))(bg+c(f+3gx))}{cg}+\frac{(f+gx)\left(-\frac{4g^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c^2f^2+b^2g^2-cg(bf+3ag)\right)(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{2}\left(2cf-bg+\sqrt{(b\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*g + c*(f + 3*g*x)))/(c*g) + ((f + g
*x)*((-4*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*
a*c)*g^2])]*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*(a + x*(b + c*x)))/(f +
 g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(c^2*f^2 + b^2
*g^2 - c*g*(b*f + 3*a*g))*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*
f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] -
2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g +
Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f
^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x
]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*(b^3*g^3 - b^2*g^2*(2*c*f + Sqrt[
(b^2 - 4*a*c)*g^2]) + b*c*g*(-4*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-(c
*f^2*Sqrt[(b^2 - 4*a*c)*g^2]) + a*g^2*(8*c*f + 3*Sqrt[(b^2 - 4*a*c)*g^2])))
*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a
*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g
*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 -
 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*
(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c
*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqr
```

```
t[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g
*x]))/(c^2*g^3*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])])))/(15*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 892, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5}+\frac{2\left(\frac{bg}{5}+\frac{cf}{5}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(\frac{3fa}{5}-\frac{2\left(\frac{bg}{5}+\frac{cf}{5}\right)\left(\frac{ag}{2}\right.}{3cg}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*x*(c*g
*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(1/5*b*g+1/5*c*f)/c/g*(c*g*
x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(3/5*f*a-2/3*(1/5*b*g+1/5*c*f)
/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-
f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f
*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2))+2*(2/5*a*g+2/5*b*f-2/3*(1/5*b*g+1/5*c*f)/c/g*(b*g+c*f))*(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*(
(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.94

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2\left((2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2}{3c^2g^2}\right.\right.}{}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a
*b*c)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*
a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)
*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))
 + 6*(c^3*f^2*g - b*c^2*f*g^2 + (b^2*c - 3*a*c^2)*g^3)*sqrt(c*g)*weierstras
sZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f
^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^
3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^
2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b
^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(3*c^3*
g^3*x + c^3*f*g^2 + b*c^2*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^3*g^
3)
```

Sympy [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}\sqrt{gx+f}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Giac [F]

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}\sqrt{gx+f}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2), x)
```

3.890 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 764

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(eg(bef-3bdg+2aeg)+c(-e^2f^2+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\right.}{3ce^3g\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$-\frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3*(b*e*g-3*c*d*g+c*e*f)*EllipticE
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*
(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/g/(c*
x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3*(e*
g*(2*a*e*g-3*b*d*g+b*e*f)+c*(3*d^2*g^2-e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)
^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c/e^3/g/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-(a*e^2-b*d*e+c*d^2)*
```

```
EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(
1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*
c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)
/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 969, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {932, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2g\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\text{ar}\right.}{3ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$-\frac{\sqrt{2}(cd^2-bed+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{EllipticP}}{\sqrt{c}e^3\sqrt{cx^2+bx+a}}$$
$$+\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3e}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(3*c*e^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*(c*e*f - 3*c*d*g + b*
```

```
e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(3*c*d*(e*f - d*g) - e*(2*b*e*f - 3*b*d*g +
2*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f -
b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt
[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b
*x + c*x^2])
```

<u>Rule 175</u>

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

<u>Rule 430</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

<u>Rule 435</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

<u>Rule 551</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \frac{bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+beg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \left(\frac{3cd(ef-dg)-e(2bef-3bdg+2aeg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(cef-3cdg+beg)x}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{3(cd^2-bde+ae^2)(ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{((cd^2-bde+ae^2)(ef-dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^3} + \frac{(cef-3cdg+beg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e^2} - \frac{(3cd(ef-dg)-e(2bef-3bdg+2aeg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e^3}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{(cef-3cdg+beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2g} - \frac{(f(cef-3cdg+beg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e^2g} + \frac{\left((cd^2-bde+ae^2)(ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b}}}{e^3\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subs}}{3ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin\right.}{3ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(cd^2-bde+ae^2)(ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{}}{2cf-b}}}\right.}{3ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2}}{2cf-\left(b+\sqrt{}\right.}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin\right.}{3ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(cd^2-bde+ae^2)(ef-dg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-dg}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-}}{2cf-\left(b+\sqrt{b}\right.}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-}\right.}{3ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(cd^2-bde+ae^2)(ef-dg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-e}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-}}{2cf-\left(b+\sqrt{b}\right.}\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-}\right.}{3ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\right.}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.44 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+x(b+cx)}}{3e}$$

$$+\frac{(f+gx)^{3/2}\sqrt{a+x(b+cx)}\left(4e(cef-3cdg+beg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\cdots\right)}{f+gx}\right.\right.}{\cdots}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/(3*e) + ((f + g*x)^(3/2)*Sqrt[a + x
*(b + c*x)]*(4*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b -
 (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) - (I*Sqrt[2]*e*(c*e*f - 3*c
*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a
*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)
/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a
*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/
(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*e*(-6*c^2*d*f*g
+ b*e*g*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 + Sqrt[(b^2 - 4*
a*c)*g^2]*(e*f - 3*d*g) + 3*b*g*(e*f + d*g)))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2]
 - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*
x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2]
 + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x
)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*
Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]])/Sqr
t[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt
[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - ((6*I)*Sqrt[2]*c*(-(c*d^2) + e*(b*d
 - a*e))*g^2*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-
1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-
1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4
```

```
*a*c)*g^2])]*EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)*g^2]
))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))])/Sqrt[f + g*x]))/(6*c*e^3*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + b*x + c*x^2]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.63

method	result	size
elliptic	Expression too large to display	1245
risch	Expression too large to display	2338
default	Expression too large to display	6812

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/e*(c*g
*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*((a*e^2*g-b*d*e*g+b*e^2*f+c*d
^2*g-c*d*e*f)/e^3-2/3/e*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c
+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^
2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2))+2*(1/e^2*(b*e*g-c*d*g+c*e*f)-2/3/e*(b*g+c*f))*(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-2*(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*
d*e^2*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.891 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 743

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2beg-c(ef+3dg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^3\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-(b+}}}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{a+}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)+3/2*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2
)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(
g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+(2*b*e*g-c*(3*d*g+e*f))*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*
f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
```

```
1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)
*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(
-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))
^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^3/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 934, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {930, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$$

$$= \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)\cdots}\right.}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.\cdots}{ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f\cdots}{2cf-\left(b+\cdots\right.}}}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{cx^2+\cdots}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{e(d+ex)}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*
c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^2*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2])
```

```
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*e*f - 3*c*d*g + 2*b
*e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticP
i[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2
]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqr
t[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*
Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{2cef-3cdg+2beg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(bef-2bdg+aeg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2} + \frac{(2cef-3cdg+2beg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^3} - \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^3}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{\cdots}{\sqrt{b-\cdots}}}{2e^3\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{\cdots}}{2cf-b\cdots}}}\cdots\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx,x,\frac{\sqrt{b+\sqrt{}}}{}\right.}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}}{e^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-}}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{a}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.83 (sec) , antiderivative size = 1473, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+x(b+cx)}}{e(d+ex)} - \frac{(f+gx)^{3/2}\sqrt{a+x(b+cx)}\left(-12e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)\right.}{}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/(e*(d + e*x))) - ((f + g*x)^(3/2)*S
qrt[a + x*(b + c*x)]*(-12*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b -
 (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((3*I)*Sqrt[2]*e*(-(e*f)
+ d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2
] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g
*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2
] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*
x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]])/Sq
rt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqr
t[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*e*(2*a*e*g^2 - 3*e*f*Sq
rt[(b^2 - 4*a*c)*g^2] + 3*d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*(e*f - 3*d*g) +
 c*(-4*e*f^2 + 6*d*f*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x
) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x
) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g +
 a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))
])/Sqrt[f + g*x] - ((2*I)*Sqrt[2]*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d
*g + a*e*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-
1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-
1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4
```

```
*a*c)*g^2])]*EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)*g^2]
))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))])/Sqrt[f + g*x]))/(4*e^3*g*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g)
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + b*x + c*x^2]*Sqrt[((f
+ g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x
)))/(f + g*x)))/g^2])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 1190, normalized size of antiderivative = 1.60

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	16696

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/e*(c*g*
x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+2*((b*e*g-2*c*d*g+c*e*f)
/e^3+1/2*c*d/e^3*g)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a
*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
))+3*c/e^2*g*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF((
(x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/e^4*(a*e^2*g-2*
b*d*e*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x
+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^
(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*
x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-
f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2,x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.892 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 1034

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3} dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$+ \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\,(cd^2-bde+ae^2)\,(ef-dg)(d+ex)}$$

$$- \frac{\sqrt{b^2-4ac}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{4\sqrt{2}e^2\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{b^2-4ac}(-cd(2ef+3dg)+e(bef+4bdg-5aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{2\sqrt{2}e^3\,(cd^2+e(-bd+ae))\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$+ \frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}(b^2e^4f^2+a^2e^4g^2+c^2d^3g(4ef-3dg)-2ace^2(2e^2f^2-6defg+3d^2g^2)-2beg(}{4\sqrt{2}\sqrt{c}e^3}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)^2+1/4*(c*d*(-3*d*g+2*e*f)-
e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)/(e*x+d)-1/8*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+
```

```
(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(-c*d*(3*d*g+2*e*f)+e*(-5*a*e*g+4*b*d*g+b*e
*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))*2^(1/2)/(g*x+f)
^(1/2)/(a+x*(c*x+b))^(1/2)+1/8*(b^2*e^4*f^2+a^2*e^4*g^2+c^2*d^3*g*(-3*d*g+4
*e*f)-2*a*c*e^2*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2)-2*b*e*g*(a*e^3*f+c*d^2*(-2*
d*g+3*e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f)
,((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2
*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)^2*
2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A] (verified)

Time = 4.90 (sec) , antiderivative size = 1705, normalized size of antiderivative = 1.65, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {930, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$$

$$= \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)},\text{ar}\right.}{4\sqrt{2}\sqrt{c}e^3\left(cd^2-bed+ae^2\right)(ef-dg)}$$

$$-\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(cd(2ef-3dg)-e(}{4\sqrt{2}e^2\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(}{2\sqrt{2}e^2\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{2\sqrt{2}e^3\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}(cd(2ef-3dg)-e(bef-2bdg+aeg))}{4e\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\right.}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x)^2) + ((c*d*(2*e*f -
 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
/(4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(
c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
```

```
- (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f
- d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x
 + c*x^2]) + (3*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^3*Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(c*d*(2*e*f - 3*d*g
) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt
[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(2*Sqrt[2]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c
*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*El
lipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[
(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (
b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(S
qrt[2]*Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b -
Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))^2
*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*
c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*
g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f
 + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] -
 (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^3*(c
*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
```

```
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}+\frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e}\\ &= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}\\ &\quad+\frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{-cd(2ef-3dg)+e(bef-2bdg+aeg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{2(cef-3cdg+beg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{4e}\end{aligned}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^3} + \frac{(cef-3cdg+beg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^3} - \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^3}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\,(cd^2-bde+ae^2)\,(ef-dg)(d+ex)} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^3\,(cd^2-bde+ae^2)\,(ef-dg)} + \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}}}}{2e^3\sqrt{a+bx+cx^2}} + \frac{\left(3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{2}}\right.}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\,(cd^2-bde+ae^2)\,(ef-dg)(d+ex)} + \frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\int \left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(b}{(d+ex)\sqrt{f+gx}\sqrt{}}\right.}{8e^3\,(cd^2-bde+ae^2)\,(ef-dg)} - \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-}}}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$
$$+\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$
$$+\frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{(cdg(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^3\left(cd^2-bde+ae^2\right)(ef-dg)}$$
$$-\frac{(cg(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)}$$
$$-\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))^2\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^3\left(cd^2-bde+ae^2\right)(ef-dg)}$$
$$-\frac{\left((cef-3cdg+beg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{}}}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$
$$+\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$
$$+\frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{(c(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)}$$
$$+\frac{(cf(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)}$$
$$-\frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int\frac{}{\sqrt{b-}}}{8e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}}$$
$$-\frac{\left(\sqrt{b^2-4ac}dg(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\Bigg(}{2\sqrt{2}e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx-}}$$
$$-\frac{\left((cef-3cdg+beg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\Bigg(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{}{(b-}}}}{e^3\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$+\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$+\frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{b^2-4ac}dg(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\right.}{2\sqrt{2}e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\right.}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{a+bx+cx}}$$

$$+\frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subs}}{4e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a}}$$

$$-\frac{\left(\sqrt{b^2-4ac}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2}{2cf-}}}{\sqrt{}}\right.}{4\sqrt{2}e^2\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+c}}$$

$$+\frac{\left(\sqrt{b^2-4ac}f(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\right.}{2\sqrt{2}e^2\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2}$$

$$+ \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$- \frac{\sqrt{b^2-4ac}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{4\sqrt{2}e^2\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{3\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{b^2-4ac}f(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\right.\right.}{2\sqrt{2}e^2\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{b^2-4ac}dg(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\right.\right.}{2\sqrt{2}e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\right.}{\sqrt{2}\sqrt{c}e^3(ef-dg)\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))^2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}}{4e^3\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a-}}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.67 (sec) , antiderivative size = 33765, normalized size of antiderivative = 32.65

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 1593, normalized size of antiderivative = 1.54

method	result	size
elliptic	Expression too large to display	1593
default	Expression too large to display	55368

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2/e*(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+1/4*(a*e^2*g-2*b*d*e
*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)/e/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*
d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+2*
(c*g/e^3-1/8*c*g*(3*a*d*e^2*g-2*a*e^3*f-4*b*d^2*e*g+3*b*d*e^2*f+5*c*d^3*g-4
*c*d^2*e*f)/e^3/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f))*
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(
((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-1/4*c*g*(a*e^2*g-
2*b*d*e*g+b*e^2*f+3*c*d^2*g-2*c*d*e*f)/e^2/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d
*e^2*f+c*d^3*g-c*d^2*e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b
*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/4*
(a^2*e^4*g^2-2*a*b*e^4*f*g-6*a*c*d^2*e^2*g^2+12*a*c*d*e^3*f*g-4*a*c*e^4*f^2
+b^2*e^4*f^2+4*b*c*d^3*e*g^2-6*b*c*d^2*e^2*f*g-3*c^2*d^4*g^2+4*c^2*d^3*e*f*
g)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticP
i(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$$

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^3} \,dx$$

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.893 $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1098

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+336d^2efg^2-70d^3g^3)-3c^2eg(6ae}{315c^3g^4}$$
$$+\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}$$
$$-\frac{2e(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2(64e^2f^2-111defg+42d^2g^2))(f+gx)^{3/2}\sqrt{a+bx+cx}}{315c^2g^4}$$
$$-\frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(16b^4e^3g^4+8b^2ce^2g^3(2bef-9bdg-9aeg)-2c^4f(64e^3f^3-216de^2f^2g+252d^2efg^2-105}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)+2c^3(64e^3f^3-216de^2f^2g+}{}$$

```
[Out] -2/315*e*(6*b^2*e^2*g^2+c*e*g*(-14*a*e*g-27*b*d*g+17*b*e*f)-2*c^2*(42*d^2*g
^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^4-2/63*
e^2*(-b*e*g-6*c*d*g+8*c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^4+2/315*
(8*b^3*e^3*g^3+3*b*c*e^2*g^2*(-9*a*e*g-12*b*d*g+5*b*e*f)-c^3*(-70*d^3*g^3+3
36*d^2*e*f*g^2-408*d*e^2*f^2*g+152*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*
```

```
f)-b*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c^3/g^4+2/9*(e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/315*(16*b^4*e^3
*g^4+8*b^2*c*e^2*g^3*(-9*a*e*g-9*b*d*g+2*b*e*f)-2*c^4*f*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*e*g^2*(14*a^2*e^2*g^2-a*b*e*g*
(-87*d*g+19*e*f)+b^2*(42*d^2*g^2-27*d*e*f*g+7*e^2*f^2))-c^3*g*(6*a*e*g*(63*
d^2*g^2-39*d*e*f*g+10*e^2*f^2)-b*(-105*d^3*g^3+189*d^2*e*f*g^2-144*d*e^2*f^
2*g+40*e^3*f^3)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)/c^4/g^5/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)-2/315*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^3*g^3+3*b*c*e^2*g^2*
(-9*a*e*g-12*b*d*g+5*b*e*f)+2*c^3*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f
^2*g+64*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*f)-b*(21*d^2*g^2-24*d*e*f*g
+8*e^2*f^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c
*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^5/(g*x+f)^(1/2)/(c*x
^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {934, 1667, 857, 732, 435, 430}

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9g}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(-2f(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3)\,c^4-g(6aeg(10e^2f^2-39degf+63d^2}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)\,(2(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3)\,c^3-3eg(6aeg(2ef-}{}$$

$$-\frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{63cg^4}$$

$$-\frac{2e(-2(64e^2f^2-111degf+42d^2g^2)\,c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2)\,(f+gx)^{3/2}\sqrt{cx^2+bx}}{315c^2g^4}$$

$$+\frac{2(-((152e^3f^3-408de^2gf^2+336d^2eg^2f-70d^3g^3)\,c^3)-3eg(6aeg(2ef-5dg)-b(8e^2f^2-24degf+2}{315c^3g^4}$$

```
[In] Int[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152
*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a
*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*
g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3
/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*
g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt[2]*Sqrt[b^2 -
4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*
c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^
2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d
*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^
2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2
*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g +
 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8
*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
```

```
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m - d
*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2*
c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2(bdf+6aef-8adg+(2cdf+7bef-7bdg-2aeg)x+(8cef-6cdg-beg)x^2)}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4} - \frac{2\int \frac{\frac{1}{2}g\left(b^2e^3f^3g-2acg\left(20e^3f^3-15de^2f^2g-21d^2efg^2+28d^3g^3\right)+bf\left(5ae^3fg^2-c\left(8e^3f^3-6de^2f^2g-7d^3g^3\right)\right)\right)+\frac{1}{2}g\left(2be^3fg^2(4bf+5ag\right.}{}}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e\left(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+}}{315c^2g^4} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4} - \frac{4\int \frac{-\frac{1}{4}g^4\left(6b^3e^3f^2g^2+3b^2e^2fg(6aeg^2+cf(4ef-9dg))-2acg\left(21ae^3fg^2+c\left(92e^3f^3-258de^2f^2g+231d^2efg^2-140d^3g^3\right)\right)+bcf\left(3ae^2\right.}{}}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-70d^3g^3\right)-3c\right.}{315c^3g^4} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e\left(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+}}{315c^2g^4} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4} - \frac{8\int \frac{\frac{3}{8}g^6\left(8b^4e^3fg^3+b^3e^2g^2(8aeg^2+9cf(ef-4dg))-3b^2ceg\left(2aeg^2(5ef+6dg)-cf\left(4e^2f^2-15defg+21d^2g^2\right)\right)+2ac^2g\left(3ae^2g^2(ef+15\right.\right.}{}}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-70d^3g^3\right)-3c\right.}{315c^3g^4} + \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e\left(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{a+}}{315c^2g^4} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4} - \frac{\left(\left(cf^2-bfg+ag^2\right)\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)+2c^3\left(64e^3f^3-216de^2f^2g+252d^2\right.\right.\right.}{315} - \frac{\left(16b^4e^3g^4+8b^2ce^2g^3(2bef-9bdg-9aeg)-2c^4f\left(64e^3f^3-216de^2f^2g+252d^2efg^2-105d^3g^3\right)\right.}{}$$

$$= \frac{2(8b^3e^3g^3 + 3bce^2g^2(5bef - 12bdg - 9aeg) - c^3(152e^3f^3 - 408de^2f^2g + 336d^2efg^2 - 70d^3g^3) - 3}{315c^3g^4}$$

$$+ \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}$$

$$- \frac{2e(6b^2e^2g^2 + ceg(17bef - 27bdg - 14aeg) - 2c^2(64e^2f^2 - 111defg + 42d^2g^2))(f+gx)^{3/2}\sqrt{a}}{315c^2g^4}$$

$$- \frac{2e^2(8cef - 6cdg - beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4}$$

$$- \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(16b^4e^3g^4 + 8b^2ce^2g^3(2bef - 9bdg - 9aeg) - 2c^4f(64e^3f^3 - 216de^2f^2g + 252d^2\right.}{}$$

$$- \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(cf^2 - bfg + ag^2)(8b^3e^3g^3 + 3bce^2g^2(5bef - 12bdg - 9aeg) + 2c^3(64e^3f^3 - 21\right.}{}$$

$$= \frac{2(8b^3e^3g^3 + 3bce^2g^2(5bef - 12bdg - 9aeg) - c^3(152e^3f^3 - 408de^2f^2g + 336d^2efg^2 - 70d^3g^3) - 3}{315c^3g^4}$$

$$+ \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g}$$

$$- \frac{2e(6b^2e^2g^2 + ceg(17bef - 27bdg - 14aeg) - 2c^2(64e^2f^2 - 111defg + 42d^2g^2))(f+gx)^{3/2}\sqrt{a}}{315c^2g^4}$$

$$- \frac{2e^2(8cef - 6cdg - beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(16b^4e^3g^4 + 8b^2ce^2g^3(2bef - 9bdg - 9aeg) - 2c^4f(64e^3f^3 - 216de^2f^2g + 252d^2e}{}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2 - bfg + ag^2)(8b^3e^3g^3 + 3bce^2g^2(5bef - 12bdg - 9aeg) + 2c^3(64e^3f^3 - 216}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.00 (sec) , antiderivative size = 17771, normalized size of antiderivative = 16.18

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 1845, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1845
risch	Expression too large to display	7892
default	Expression too large to display	22215

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*e^3/g*
x^3*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/7*(b*e^3+3*c*d*e^2-2/
9*e^3/g*(4*b*g+4*c*f))/c/g*x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1
/2)+2/5*(a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3
*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2
+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(3*a*d*e^2+3*b*d^2*e+c*d^3-2/3*e^3/g*f*
a-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(
a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-
2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3
+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d^3-2/5*(a*e^3+3*b*d*e^2+3*c*d
^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*
f))/c/g*(3*b*g+3*c*f))/c/g*f*a-2/3*(3*a*d*e^2+3*b*d^2*e+c*d^3-2/3*e^3/g*f*a
-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a
*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-2
/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+
1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*
EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(3*a*
```

```
d^2*e+b*d^3-4/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*f*a-2/5*(a*e^
3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-2/9*
e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c/g*(3/2*a*g+3/2*b*f)-2/3*(3*a*d*e^
2+3*b*d^2*e+c*d^3-2/3*e^3/g*f*a-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f
))/c/g*(5/2*a*g+5/2*b*f)-2/5*(a*e^3+3*b*d*e^2+3*c*d^2*e-2/9*e^3/g*(7/2*a*g+
7/2*b*f)-2/7*(b*e^3+3*c*d*e^2-2/9*e^3/g*(4*b*g+4*c*f))/c/g*(3*b*g+3*c*f))/c
/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g
)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2
)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1241, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/945*((128*c^5*e^3*f^5 - 8*(54*c^5*d*e^2 + 13*b*c^4*e^3)*f^4*g + (504*c^5
*d^2*e + 360*b*c^4*d*e^2 - (25*b^2*c^3 - 156*a*c^4)*e^3)*f^3*g^2 - (210*c^5
*d^3 + 441*b*c^4*d^2*e - 18*(5*b^2*c^3 - 31*a*c^4)*d*e^2 + 5*(2*b^3*c^2 - 9
*a*b*c^3)*e^3)*f^2*g^3 + (210*b*c^4*d^3 - 126*(b^2*c^3 - 6*a*c^4)*d^2*e + 9
*(5*b^3*c^2 - 22*a*b*c^3)*d*e^2 - (8*b^4*c - 39*a*b^2*c^2 + 24*a^2*c^3)*e^3
)*f*g^4 + (105*(b^2*c^3 - 6*a*c^4)*d^3 - 63*(2*b^3*c^2 - 9*a*b*c^3)*d^2*e +
 9*(8*b^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*d*e^2 - (16*b^5 - 96*a*b^3*c + 123
*a^2*b*c^2)*e^3)*g^5)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g
+ (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c
 - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f +
b*g)/(c*g)) + 3*(128*c^5*e^3*f^4*g - 8*(54*c^5*d*e^2 + 5*b*c^4*e^3)*f^3*g^2
 + 3*(168*c^5*d^2*e + 48*b*c^4*d*e^2 - (7*b^2*c^3 - 20*a*c^4)*e^3)*f^2*g^3
- (210*c^5*d^3 + 189*b*c^4*d^2*e - 9*(9*b^2*c^3 - 26*a*c^4)*d*e^2 + (16*b^3
*c^2 - 57*a*b*c^3)*e^3)*f*g^4 + (105*b*c^4*d^3 - 126*(b^2*c^3 - 3*a*c^4)*d^
2*e + 9*(8*b^3*c^2 - 29*a*b*c^3)*d*e^2 - 2*(8*b^4*c - 36*a*b^2*c^2 + 21*a^2
*c^3)*e^3)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3
```

```
*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2
)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^
2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*
g*x + c*f + b*g)/(c*g))) - 3*(35*c^5*e^3*g^5*x^3 - 64*c^5*e^3*f^3*g^2 + 12*
(18*c^5*d*e^2 + b*c^4*e^3)*f^2*g^3 - (252*c^5*d^2*e + 45*b*c^4*d*e^2 - (9*b
^2*c^3 - 22*a*c^4)*e^3)*f*g^4 + (105*c^5*d^3 + 63*b*c^4*d^2*e - 18*(2*b^2*c
^3 - 5*a*c^4)*d*e^2 + (8*b^3*c^2 - 27*a*b*c^3)*e^3)*g^5 - 5*(8*c^5*e^3*f*g^
4 - (27*c^5*d*e^2 + b*c^4*e^3)*g^5)*x^2 + (48*c^5*e^3*f^2*g^3 - (162*c^5*d*
e^2 + 7*b*c^4*e^3)*f*g^4 + (189*c^5*d^2*e + 27*b*c^4*d*e^2 - 2*(3*b^2*c^3 -
 7*a*c^4)*e^3)*g^5)*x)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^5*g^6)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.894 $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 755

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^2g^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3} + \frac{\sqrt{2}\sqrt{b^2-4ac}(8b^3e^2g^3+bceg^2(9bef-28bdg-29aeg)-2c^3f(24e^2f^2-56defg+35d^2g^2)-c^2g(2aeg(1\ldots}{105c^3g^4\sqrt{\ldots}} + \frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)+c^2(24e^2f^2-56defg+35d^2g^2))\ldots}{105c^3g^4\sqrt{f+gx}\sqrt{a+\ldots}}$$

```
[Out] -2/35*e*(-b*e*g-4*c*d*g+6*c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/g^3-4/
105*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)-c^2*(10*d^2*g^2-34*d*e*
f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/7*(e*x+d)^2*(g
*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g+1/105*(8*b^3*e^2*g^3+b*c*e*g^2*(-29*a*e*g
-28*b*d*g+9*b*e*f)-2*c^3*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2)-c^2*g*(2*a*e*
g*(-42*d*g+13*e*f)-b*(35*d^2*g^2-42*d*e*f*g+16*e^2*f^2)))*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^4/(c*x^2+b*x+
```

```
a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/105*(a*g^2-b*
f*g+c*f^2)*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)+c^2*(35*d^2*g^2-
56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2
)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^4/(g*x+f)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {934, 1667, 857, 732, 435, 430}

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2}{105c^3g^4\sqrt{f+gx}\sqrt{a+}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-c^2g(2aeg(13ef-42dg)-b(35d^2g^2-42defg+16e^2f^2))+bce}{105c^3g^4\sqrt{}}$$

$$-\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2}(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2-(c^2(10d^2g^2-34defg+21e^2f^2)))}{105c^2g^3}$$
$$-\frac{2e(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-beg-4cdg+6cef)}{35cg^3}+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$

```
[In] Int[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2
- 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^2*g
^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - (2*e*(6*c
*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) +
 (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g
- 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e
*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt[f +
g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^4*Sqrt[(c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f
```

```
- 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(105*c^3*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m - d
*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2*
c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
```

```
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(bdf+4aef-6adg+(2cdf+5bef-5bdg-2aeg)x+(6cef-4cdg-beg)x^2\right)}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3} - \frac{2\int \frac{\frac{1}{2}g\left(b^2e^2f^2g-2acg\left(9e^2f^2-16defg+15d^2g^2\right)+bf\left(3ae^2g^2-c\left(6e^2f^2-4defg-5d^2g^2\right)\right)\right)+\frac{1}{2}g\left(be^2g^2(5bf+3ag)-2c^2f\left(6e^2f^2-4d\right.\right.}{\sqrt{f}}}{}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx+cx}}{105c^2g^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3} - \frac{4\int \frac{-\frac{1}{4}g^3\left(4b^3e^2fg^2+b^2eg\left(4aeg^2+cf(5ef-14dg)\right)-bc\left(aeg^2(11ef+14dg)+cf\left(24e^2f^2-56defg+35d^2g^2\right)\right)-2acg\left(5ae^2g^2-c\left(6e^2f\right.\right.\right.}{}}{}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^2g^3}$$

$$+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$

$$-\frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3}$$

$$+\frac{(2(cf^2-bfg+ag^2)(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)+c^2(24e^2f^2-56defg+35d^2g^2)))\int \frac{}{\sqrt{}}}{105c^2g^4}$$

$$+\frac{(8b^3e^2g^3+bceg^2(9bef-28bdg-29aeg)-2c^3f(24e^2f^2-56defg+35d^2g^2)-c^2g(2aeg(13ef-}{105c^2g^4}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^2g^3}$$

$$+\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$

$$-\frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3}$$

$$+\frac{\left(\sqrt{2}\sqrt{b^2-4ac}(8b^3e^2g^3+bceg^2(9bef-28bdg-29aeg)-2c^3f(24e^2f^2-56defg+35d^2g^2)-c^2\right.}{10}$$

$$+\frac{\left(4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)+c^2(24e^2f^2-56defg+\right.}{105c^3g^4\sqrt{f+}}$$

$$= -\frac{4(2b^2e^2g^2 + ceg(4bef - 7bdg - 5aeg) - c^2(21e^2f^2 - 34defg + 10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx}}{105c^2g^3}$$
$$+ \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$
$$- \frac{2e(6cef - 4cdg - beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3}$$
$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(8b^3e^2g^3 + bceg^2(9bef - 28bdg - 29aeg) - 2c^3f(24e^2f^2 - 56defg + 35d^2g^2) - c^2}{105}$$
$$+ \frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2 - bfg + ag^2)(2b^2e^2g^2 + ceg(4bef - 7bdg - 5aeg) + c^2(24e^2f^2 - 56defg +}{105c^3g^4\sqrt{f+gx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.65 (sec) , antiderivative size = 10030, normalized size of antiderivative = 13.28

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 1272, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1272
risch	Expression too large to display	4566
default	Expression too large to display	12922

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e^2/g*
x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(b*e^2+2*c*d*e-2/7*
e^2/g*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+
```

```
2/3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e^2+2*c*d*e-2/7
*e^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x
+b*f*x+a*f)^(1/2)+2*(a*d^2-2/5*(b*e^2+2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f))/c/g*
f*a-2/3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e^2+2*c*d*e
-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g
)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2*a*d*e+b*d^2-4/7*e^2
/g*f*a-2/5*(b*e^2+2*c*d*e-2/7*e^2/g*(3*b*g+3*c*f))/c/g*(3/2*a*g+3/2*b*f)-2/
3*(e^2*a+2*b*d*e+c*d^2-2/7*e^2/g*(5/2*a*g+5/2*b*f)-2/5*(b*e^2+2*c*d*e-2/7*e
^2/g*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g
*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+
1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left((48\,c^4e^2f^4-8\,(14\,c^4de+5\,bc^3e^2)f^3g+2\,(35\,c^4d^2+49\,bc^3de-(5\,b^2c^2-31\,ac^3)e^2)f^2g^2-(70\,bc^3d^2-28\right.}{}$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/315*((48*c^4*e^2*f^4 - 8*(14*c^4*d*e + 5*b*c^3*e^2)*f^3*g + 2*(35*c^4*d^2
 + 49*b*c^3*d*e - (5*b^2*c^2 - 31*a*c^3)*e^2)*f^2*g^2 - (70*b*c^3*d^2 - 28*
(b^2*c^2 - 6*a*c^3)*d*e + (5*b^3*c - 22*a*b*c^2)*e^2)*f*g^3 - (35*(b^2*c^2
- 6*a*c^3)*d^2 - 14*(2*b^3*c - 9*a*b*c^2)*d*e + (8*b^4 - 41*a*b^2*c + 30*a^
2*c^2)*e^2)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^
2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*
a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/
```

```
(c*g)) + 3*(48*c^4*e^2*f^3*g - 16*(7*c^4*d*e + b*c^3*e^2)*f^2*g^2 + (70*c^4
*d^2 + 42*b*c^3*d*e - (9*b^2*c^2 - 26*a*c^3)*e^2)*f*g^3 - (35*b*c^3*d^2 - 2
8*(b^2*c^2 - 3*a*c^3)*d*e + (8*b^3*c - 29*a*b*c^2)*e^2)*g^4)*sqrt(c*g)*weie
rstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^
3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^
2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2
+ (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(
15*c^4*e^2*g^4*x^2 + 24*c^4*e^2*f^2*g^2 - (56*c^4*d*e + 5*b*c^3*e^2)*f*g^3
+ (35*c^4*d^2 + 14*b*c^3*d*e - 2*(2*b^2*c^2 - 5*a*c^3)*e^2)*g^4 - 3*(6*c^4*
e^2*f*g^3 - (14*c^4*d*e + b*c^3*e^2)*g^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(g*x
 + f))/(c^4*g^5)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}} \,d x$$

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.895 $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 519

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\Bigg(\arcsin}{15c^2g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8cef-10cdg+beg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\Bigg(\text{ar}}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/15*(-3*c*e*g*x-b*e*g-5*c*d*g+4*c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c/g^2-1/15*(2*b^2*e*g^2-2*c^2*f*(-5*d*g+4*e*f)+c*g*(-6*a*e*g-5*b*d*g+3*b*e*
f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)/c^2/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)-2/15*(b*e*g-10*c*d*g+8*c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^
2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)
^(1/2))))^(1/2)/c^2/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {828, 857, 732, 435, 430}

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(beg-10cdg+8cef)\,\text{EllipticF}\left(\text{arcs}\right.}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))\,E\left(\text{arcsin}\right.}{15c^2g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

```
[In] Int[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c*
x^2])/(15*c*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f
 - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
15*c^2*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{2\int \frac{\frac{1}{2}\left(5cdg(bf-2ag)-bef(4cf-bg)+2aeg\left(cf+\frac{bg}{2}\right)\right)+\frac{1}{2}\left(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg)\right)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{15cg^2}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{((8cef-10cdg+beg)\left(cf^2-bfg+ag^2\right))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{15cg^3} - \frac{(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{15cg^3}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\Big(}{15c^2g^3\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(8cef-10cdg+beg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\Big(\int \frac{}{\sqrt{1-}}}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2eg^2-2c^2f(4ef-5dg)+cg(3bef-5bdg-6aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\Big(\sin^{-1}\Big(}{15c^2g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(8cef-10cdg+beg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\Big(\sin^{-1}\Big(\frac{\sqrt{b+}}{}}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.60 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+x(b+cx))(beg+c(-4ef+5dg+3egx))}{cg^2}+\frac{2(f+gx)\left(\frac{g^2\left(-2b^2eg^2+2c^2f(4ef-5dg)+cg(-3bef+5bdg+6aeg)\right)(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{1-\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*e*g + c*(-4*e*f + 5*d*g + 3*e*g*x))
)/(c*g^2) + (2*(f + g*x)*((g^2*(-2*b^2*e*g^2 + 2*c^2*f*(4*e*f - 5*d*g) + c*
g*(-3*b*e*f + 5*b*d*g + 6*a*e*g))*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*S
qrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g
^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(2*b^2*e*g^2 + 2*c^2*f*(-4*e*f + 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*
g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (2*b^3*e*g^3 - b^
2*g^2*(-(c*e*f) + 5*c*d*g + 2*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g*(-8*a*e*g^
2 + Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e*f + 5*d*g)) + 2*c*(c*f*Sqrt[(b^2 - 4*a*c)
*g^2]*(4*e*f - 5*d*g) + a*g^2*(-2*c*e*f + 10*c*d*g + 3*e*Sqrt[(b^2 - 4*a*c)
*g^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqr
t[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt
[f + g*x])))/(c^2*g^4)))/(15*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(461) = 922$.

Time = 2.70 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2ex\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5g}+\frac{2\left(be+cd-\frac{2(2bg+2cf)e}{5g}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(ad-\frac{2fae}{5g}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e/g*x*
(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(b*e+c*d-2/5/g*(2*b*g+2
*c*f)*e)/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a*d-2/5*f*a
/g*e-2/3*(b*e+c*d-2/5/g*(2*b*g+2*c*f)*e)/c/g*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*e+b*d-2/5*e/g*(3/2*a*g+
3/2*b*f)-2/3*(b*e+c*d-2/5/g*(2*b*g+2*c*f)*e)/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(
(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx =$$
$$-\frac{2\left((8c^3ef^3-(10c^3d+7bc^2e)f^2g+2(5bc^2d-(b^2c-6ac^2)e)fg^2+(5(b^2c-6ac^2)d-(2b^3-9abc)e\right.}{}$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*((8*c^3*e*f^3 - (10*c^3*d + 7*b*c^2*e)*f^2*g + 2*(5*b*c^2*d - (b^2*c
- 6*a*c^2)*e)*f*g^2 + (5*(b^2*c - 6*a*c^2)*d - (2*b^3 - 9*a*b*c)*e)*g^3)*sq
rt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^
2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b
^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8*c^3*e
*f^2*g - (10*c^3*d + 3*b*c^2*e)*f*g^2 + (5*b*c^2*d - 2*(b^2*c - 3*a*c^2)*e)
*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)
/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
(2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*
g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2
*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f
+ b*g)/(c*g))) - 3*(3*c^3*e*g^3*x - 4*c^3*e*f*g^2 + (5*c^3*d + b*c^2*e)*g^3
)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^3*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

```
[In] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.896 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 444

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/3*(-b*g+2*c*f)*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b
^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/
2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/g^2/(c*x^2+b*x+a)^
(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/3*(a*g^2-b*f*g+c
*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)
/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {748, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(2cf-bg)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,cg}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*g
^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*c*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\int \frac{bf-2ag+(2cf-bg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3g} \\
&= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{(2cf-bg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3g^2} \\
&\quad + \frac{(2(cf^2-bfg+ag^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3g^2}
\end{aligned}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}\cdots}{\sqrt{b^2-4a\cdots}}}}{\sqrt{2}}\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} + \frac{\left(4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4\cdots}}{2cf-bg-\sqrt{\cdots}}}}\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{4\sqrt{2}\sqrt{b^2-4ac}(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\cdots\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.42 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(4g^2(a+x(b+cx)) + \frac{(f+gx)\left(\frac{4g^2(-2cf+bg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{2}(2cf-bg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*(4*g^2*(a + x*(b + c*x)) + ((f + g*x)*((4*g^2*(-2*c*f + b*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g)*(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g
*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(b^2*g^2 - 4*a*c*g^2 + 2*c*f*Sqrt[(b
^2 - 4*a*c)*g^2] - b*g*Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^
2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/
((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt
[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g
*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f
 - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*Sqrt[(c*f^2 + g*(-(
b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])))/(6*g^3*Sqrt[a + x
*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(386) = 772.

Time = 1.28 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3g}+\frac{2\left(a-\frac{2\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3g}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-}}{2}}}}{\sqrt{cg\,x^3+bg\,x^2+}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/g*(c*g
*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(a-2/3/g*(1/2*a*g+1/2*b*f))*(
f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF((
(x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(b-2/3/g*(b*g+c*
f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(
f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2g^2+(2c^2f^2-2bcfg-(b^2-6ac)g^2)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+}{3c^2}\right.\right.}{}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*g^2 + (2*c^2*f^2 - 2*b*c*f*g
 - (b^2 - 6*a*c)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g
+ (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c
 - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f +
b*g)/(c*g)) + 3*(2*c^2*f*g - b*c*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^
2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstra
ssPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*
c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3
)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^2*g^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.897 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 700

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticP}}{\sqrt{c}e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*
(-b*e*g+c*d*g+c*e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/g/(g*x+f)^(1/
2)/(c*x^2+b*x+a)^(1/2)-(a*e^2-b*d*e+c*d^2)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
```

```
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))
))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(-d*g+e*f)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {936, 948, 175, 552, 551, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx =$$

$$-\frac{\sqrt{2}(ae^2-bde+cd^2)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}}{\sqrt{c}e^2\sqrt{a+bx+cx^2}(ef-dg)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(e*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
+ b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2
*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/
```

```
(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x
 + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
```

```
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 936

```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[1/((d
+ e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d
- b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d
^2 - b*d*e + a*e^2, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{cd-be-cex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2}+\frac{(cd^2-bde+ae^2)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2}\\
&= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{eg}-\frac{(cef+cdg-beg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2g}\\
&\quad+\frac{\left((cd^2-bde+ae^2)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2}}}{e^2\sqrt{a+bx+cx^2}}
\end{aligned}
$$

$$= -\frac{\left(2(cd^2-bde+ae^2)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-}}{2cf-bg-}}}\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$=\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2(cd^2-bde+ae^2)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$=\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2(cd^2-bde+ae^2)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{}{(b}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(cef+cdg-beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}(cd^2-bde+ae^2)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\right.}{\sqrt{c}e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.83 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

$$= \frac{(f+gx)^{3/2}\left(\frac{4eg^2(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2} - \frac{i\sqrt{2}e(-ef+dg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx)+}{\left(2cf-bg+\sqrt{(b^2-4\right.}}}}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] ((f + g*x)^(3/2)*((4*e*g^2*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 - (
I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2
*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x
 + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[
(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))*Sqrt[(-
```

```
2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*
x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]]
, -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2]))])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*(-(c*d^2) + e*(b*d - a*e))*g^2*
Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2
*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticPi[((e*f - d*g)*(2*c*f -
b*g - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh
[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g
^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f -
b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(2*e^2*g^2*(-(e*f) + d*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.59

method	result	size
elliptic	Expression too large to display	1114
default	Expression too large to display	3126

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*(b*e-c*d
)/e^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c
+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Elli
pticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*c/e*(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(a*e^2-b*d*e+c*d^2)/e^3*(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
```

```
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*Elliptic
Pi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+bx+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.898 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 736

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)}$$
$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}g}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{E}}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{a+bx+c}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)+1/2*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x
+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/(-d*g+e*f)*2^(1/2)/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/2*(e^2
```

```
*(-a*g+b*f)-c*d*(-d*g+2*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c
*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c
/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))
^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-
(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/e^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 957, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {938, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\cdots}{2cf-(\cdots}\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\cdots\right.}\right.}{e(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\left(e^2(bf-ag)-cd(2ef-dg)\right)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\cdots}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{cx^2+bx\cdots}}$$
$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(ef-dg)(d+ex)}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + (Sqrt[b^
2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f
```

```
- (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(
e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(2*e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g
) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Ell
ipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(
Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
 - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sq
rt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 938

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[
b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{bf-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{e^2(bf-ag)-cd(2ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2(ef-dg)} + \frac{\left(bf-ag-\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} + \frac{\left(\left(bf-ag-\frac{cd(2ef-dg)}{e^2}\right)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2}}}}{2(ef-dg)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2cf-\left(b-\right.}\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(\left(bf-ag-\frac{cd(2ef-dg)}{e^2}\right)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx,x,\frac{\sqrt{b+}}{}\right.}{e(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{e(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2cf-\left(b-\right.}\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(\left(bf-ag-\frac{cd(2ef-dg)}{e^2}\right)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\left(\left(bf-ag-\frac{cd(2ef-dg)}{e^2}\right)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1\cdots}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\left(e^2(bf-ag)-cd(2ef-dg)\right)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+g\cdots}{2cf-\left(b+\sqrt{b^2\cdots}\right.}}}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{a+bx+\cdots}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.66 (sec) , antiderivative size = 1471, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{a+x(b+cx)}}{(-ef+dg)(d+ex)}$$

$$+\frac{(f+gx)^{3/2}\sqrt{a+x(b+cx)}\left(-4e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/((-(e*f) + d*g)*(d + e*x)) + ((f + g*
x)^(3/2)*Sqrt[a + x*(b + c*x)]*(-4*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f)
 + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2
+ (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + (I*Sqrt[2]*e*(-(
e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c
)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(
f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c
)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f
 + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sq
rt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])
])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*e*(-2*a*e*g^2 - e*
f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*(3*e*f - d*g)
 + 2*c*f*(-2*e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x
) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x
) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g +
 a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))
])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*g*(e^2*(b*f - a*g) + c*d*(-2*e*f + d*g))*
Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*
x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*
x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
```

```
EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^
2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f +
 g*x]))/(4*e^2*g*(-(e*f) + d*g)^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + b*x + c*x^2]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.64

method	result	size
elliptic	Expression too large to display	1208
default	Expression too large to display	13874

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/(d*g-e*f
)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+2*(c/e^2-1/2*c*d/
e^2*g/(d*g-e*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-
c*g/(d*g-e*f)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^
(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Elliptic
F(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+(a*e^2*g-b*e^2
*f-c*d^2*g+2*c*d*e*f)/e^3/(d*g-e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x
^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.899 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1049

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}$$
$$+\frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$
$$-\frac{\sqrt{b^2-4ac}(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}e\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{b^2-4ac}(e^2(bf-ag)+cd(-2ef+dg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2-}}}}{\sqrt{2}}\right.\right.}{2\sqrt{2}e^2\left(cd^2+e(-bd+ae)\right)(ef-dg)\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$
$$-\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}(3a^2e^4g^2+c^2d^3g(4ef-dg)+b^2e^3f(-ef+4dg)+2ace^2(2e^2f^2-2defg+3}{4}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(d*g+2
*e*f)-e*(-3*a*e*g+2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)^2/(e*x+d)-1/8*(c*d*(d*g+2*e*f)-e*(-3*a*e*g+2*b*d*g+
b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(e^2*(-a*g+b*f)+c*d*(d*g-2*e*f))*E
```

```
llipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(
-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)*2^(1/2)/(g
*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/8*(3*a^2*e^4*g^2+c^2*d^3*g*(-d*g+4*e*f)+b
^2*e^3*f*(4*d*g-e*f)+2*a*c*e^2*(3*d^2*g^2-2*d*e*f*g+2*e^2*f^2)-2*b*e^2*g*(3
*c*d^2*f+a*e*(2*d*g+e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*
c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))
/(-d*g+e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A] (verified)

Time = 4.94 (sec) , antiderivative size = 1747, normalized size of antiderivative = 1.67, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {938, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx =$$

$$-\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(cd(2ef+dg)-e}{4\sqrt{2}e\,(cd^2-bed+ae^2)\,(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{2\sqrt{2}e\,(cd^2-bed+ae^2)\,(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{2\sqrt{2}e^2\,(cd^2-bed+ae^2)\,(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-(b+}}{4\sqrt{2}\sqrt{c}e^2\,(cd^2}$$

$$+\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}(cd(2ef+dg)-e(bef+2bdg-3aeg))}{4\,(cd^2-bed+ae^2)\,(ef-dg)^2(d+ex)}$$

$$-\frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef+cdg-beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\right.}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x)^2) + ((c*
d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2
- 4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
```

```
g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^
2)*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f
*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/
(2*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b
*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d
*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)
*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g]*(c*e*f + c*d*g - b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(
e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[
b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*
c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
 + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f + d*g) - e*(b*e*f +
 2*b*d*g - 3*a*e*g))*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sq
rt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
 Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 938

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
```

```
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[
b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}+\frac{\int \frac{bf-3ag+2(cf-bg)x-cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4(ef-dg)}\\
&= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}\\
&\quad+\frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{-cd(2ef+dg)+e(bef+2bdg-3aeg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{2(cef+cdg-beg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{4(ef-dg)}
\end{aligned}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(cef+cdg-beg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2(ef-dg)} - \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)^2} + \frac{\left((cef+cdg-beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}}}}{2e^2(ef-dg)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{2}}\right.}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\int \left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e}{(d+ex)\sqrt{f+g}}\right.}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)^2} - \frac{\left((cef+cdg-beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-}}}\right.}{e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(cdg(cd(2ef+dg)-e(bef+2bdg-3aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)^2} - \frac{(cg(cd(2ef+dg)-e(bef+2bdg-3aeg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e\left(cd^2-bde+ae^2\right)(ef-dg)^2} - \frac{((cd(2ef+dg)-e(bef+2bdg-3aeg))(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a}}}{8e^2\left(cd^2-bde+ae^2\right)(ef-dg)^2} - \frac{\left((cef+cdg-beg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-}}}\right.}{e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(c(cd(2ef+dg)-e(bef+2bdg-3aeg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{8e(cd^2-bde+ae^2)(ef-dg)^2} + \frac{(cf(cd(2ef+dg)-e(bef+2bdg-3aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e(cd^2-bde+ae^2)(ef-dg)^2} - \frac{\left((cd(2ef+dg)-e(bef+2bdg-3aeg))(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4}}\right.}{8e^2(cd^2-bde+ae^2)(ef-dg)} - \frac{\left(\sqrt{b^2-4ac}dg(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}}{2\sqrt{2}e^2(cd^2-bde+ae^2)(ef-dg)^2\sqrt{f+gx}\sqrt{a+bx}} - \frac{\left((cef+cdg-beg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{}{(b-}}}\right.}{e^2(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}$$

$$+ \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$- \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{b^2-4ac}dg(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\right.\right.}{2\sqrt{2}e^2\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef+cdg-beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e(}{}\right.}{\sqrt{2}\sqrt{c}e^2(ef-dg)^2\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left((cd(2ef+dg)-e(bef+2bdg-3aeg))(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4a}}\right.}{4e^2\left(cd^2-bde+\right.}$$

$$- \frac{\left(\sqrt{b^2-4ac}(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b}}{2cf-bg}}}{\sqrt{1-}}\right.}{4\sqrt{2}e\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4acg}}}\sqrt{a+bx+cx}}$$

$$+ \frac{\left(\sqrt{b^2-4ac}f(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4acg}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\right.}{2\sqrt{2}e\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{f+gx}\sqrt{a+bx+}}$$

$$= \text{Too large to display}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.62 (sec) , antiderivative size = 36617, normalized size of antiderivative = 34.91

$$\int\frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 1634, normalized size of antiderivative = 1.56

method	result	size
elliptic	Expression too large to display	1634
default	Expression too large to display	57841

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/2/(d*g-e
*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+1/4*(3*a*e^2*
g-2*b*d*e*g-b*e^2*f+c*d^2*g+2*c*d*e*f)/(d*g-e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e
*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(
1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f-3*b*d*e^2*f-c*d^3*g+4*c*d^2*e*f)/
(d*g-e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e^2*(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x
+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-1/4*c*g*(3*a*e^2*g-2
*b*d*e*g-b*e^2*f+c*d^2*g+2*c*d*e*f)/(d*g-e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+
b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/
g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
+1/4*(3*a^2*e^4*g^2-4*a*b*d*e^3*g^2-2*a*b*e^4*f*g+6*a*c*d^2*e^2*g^2-4*a*c*d
*e^3*f*g+4*a*c*e^4*f^2+4*b^2*d*e^3*f*g-b^2*e^4*f^2-6*b*c*d^2*e^2*f*g-c^2*d^
4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2
*e*f)/(d*g-e*f)/e^3*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a
*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\,\sqrt{f+gx}}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.900 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 774

$$\begin{aligned}\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx &= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2}\\ &+\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}\\ &+\frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2}\\ &-\frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3e^3g^3-8bce^2g^2(2bef+21bdg+13aeg)-c^3(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g\cdots}{105\cdots}\\ &-\frac{2\sqrt{2}\sqrt{b^2-4ace}(cf^2-bfg+ag^2)(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)+c^2(8e^2f^2-42defg+105\cdots}{105c^4g^3\sqrt{f+gx}\sqrt{\cdots}}\end{aligned}$$

```
[Out] 2/35*e^2*(-6*b*e*g+11*c*d*g+c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^
2+2/105*e*(24*b^2*e^2*g^2+c*e*g*(-25*a*e*g-84*b*d*g+13*b*e*f)-c^2*(-90*d^2*
g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/g^2+2/7*e*
(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c-1/105*(48*b^3*e^3*g^3-8*b*c*e
^2*g^2*(13*a*e*g+21*b*d*g+2*b*e*f)-c^3*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^
2*f^2*g+8*e^3*f^3)+c^2*e*g*(a*e*g*(189*d*g+19*e*f)-b*(-210*d^2*g^2-63*d*e*f
*g+9*e^2*f^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
```

```
/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))
))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)/c^4/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/105*e*(a*g^2-b*f*g+c*f^2)*(24*b^2*e^2*g^2+c*e*g*(-25*a*
e*g-84*b*d*g+13*b*e*f)+c^2*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/c^4/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {955, 1667, 857, 732, 435, 430}

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(ceg(-25aeg-84bdg+13bef)+2}{105c^4g^3\sqrt{f+gx}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(c^2eg(aeg(189dg+19ef)-b(-210d^2g^2-63defg+9e^2f^2))-8b}{1}$$

$$+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2-(c^2(-90d^2g^2+12defg+7e^2f^2}{105c^3g^2}$$

$$+\frac{2e^2(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-6beg+11cdg+cef)}{35c^2g^2}$$

$$+\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*
f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c
^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*
e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35
*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b
*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63
```

```
*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*
f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^4*g^3*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 955

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
```

```
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(d
*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*
m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f
 + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+e(bdf+4aef+adg)-(cd(12ef+7dg)-e(5bef+2bdg+5aeg))x-e(cef+11cdg-6beg)x^2\right)}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2} - \frac{2\int \frac{-\frac{1}{2}g\left(6b^2e^3f^2g+bef\left(18ae^2g^2-c\left(e^2f^2+11defg+5d^2g^2\right)\right)+cg\left(35cd^3fg-ae\left(3e^2f^2+53defg+5d^2g^2\right)\right)\right)-\frac{1}{2}g\left(6be^3g^2(5bf+3ag)-c^2\left(2e^3f^3\right.\right.}{}}{}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2} - \frac{4\int \frac{\frac{1}{4}g^3\left(24b^3e^3fg^2+b^2e^2g\left(24aeg^2-cf(5ef+84dg)\right)-bce\left(6aeg^2(11ef+14dg)+cf\left(4e^2f^2-21defg-105d^2g^2\right)\right)-cg\left(105c^2d^3fg+25a^2e^3g^2-a\right.\right.}{}}{}$$

$$= \frac{2e(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) - c^2(7e^2f^2 + 12defg - 90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2}$$
$$+ \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef + 11cdg - 6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2}$$
$$- \frac{(e(cf^2 - bfg + ag^2)(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) + c^2(8e^2f^2 - 42defg + 105d^2g^2)))\int \frac{}{\sqrt{f+g}}}{105c^3g^3}$$
$$- \frac{(48b^3e^3g^3 - 8bce^2g^2(2bef + 21bdg + 13aeg) - c^3(8e^3f^3 - 42de^2f^2g + 105d^2efg^2 + 105d^3g^3) + c^2eg(aeg}{105c^3g^3}$$

$$= \frac{2e(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) - c^2(7e^2f^2 + 12defg - 90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2}$$
$$+ \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef + 11cdg - 6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2}$$
$$- \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(48b^3e^3g^3 - 8bce^2g^2(2bef + 21bdg + 13aeg) - c^3(8e^3f^3 - 42de^2f^2g + 105d^2efg^2 + 105d^3\right.}{1}$$
$$- \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}e(cf^2 - bfg + ag^2)(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) + c^2(8e^2f^2 - 42defg + 10\right.}{105c^4g^3\sqrt{f+gx}\sqrt{}}$$

$$= \frac{2e(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) - c^2(7e^2f^2 + 12defg - 90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2}$$
$$+ \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef + 11cdg - 6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2}$$
$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3e^3g^3 - 8bce^2g^2(2bef + 21bdg + 13aeg) - c^3(8e^3f^3 - 42de^2f^2g + 105d^2efg^2 + 105d^3g}{105c}$$
$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cf^2 - bfg + ag^2)(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) + c^2(8e^2f^2 - 42defg + 105c}{105c^4g^3\sqrt{f+gx}\sqrt{a+b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.65 (sec) , antiderivative size = 1402, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{\sqrt{f+gx}(a+bx+cx^2)\left(-\frac{2e(4c^2e^2f^2-21c^2defg+5bce^2fg-105c^2d^2g^2+84bcdeg^2-24b^2e^2g^2+25ace^2g^2)}{105c^3g^2}-\frac{2e^2(-cef-21cdg+6beg}{35c^2g}\right.}{\sqrt{a+x(b+cx)}}$$
$$-\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(-\left((-48b^3e^3g^3+8bce^2g^2(2bef+21bdg+13aeg)+c^3(8e^3f^3-42de^2f^2g\right.\right.}{}$$

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*(a + b*x + c*x^2)*((-2*e*(4*c^2*e^2*f^2 - 21*c^2*d*e*f*g + 5
*b*c*e^2*f*g - 105*c^2*d^2*g^2 + 84*b*c*d*e*g^2 - 24*b^2*e^2*g^2 + 25*a*c*e
^2*g^2))/(105*c^3*g^2) - (2*e^2*(-(c*e*f) - 21*c*d*g + 6*b*e*g)*x)/(35*c^2*
g) + (2*e^3*x^2)/(7*c)))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqrt[a
+ b*x + c*x^2]*(-((-48*b^3*e^3*g^3 + 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d*g + 13
*a*e*g) + c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3)
- c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 + 63*d*e*f*g + 210*d^2*
g^2)))*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x))
)/(f + g*x))) - ((I/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a
*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d
*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^
3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 + 63*d*e*f*g + 2
10*d^2*g^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (48*b^4
*e^3*g^4 - 8*b^3*e^2*g^3*(8*c*e*f + 21*c*d*g + 6*e*Sqrt[(b^2 - 4*a*c)*g^2])
 + b^2*c*e*g^2*(-152*a*e^2*g^2 + 8*e*Sqrt[(b^2 - 4*a*c)*g^2]*(2*e*f + 21*d*
g) + c*(e^2*f^2 + 231*d*e*f*g + 210*d^2*g^2)) - b*(-104*a*c*e^3*g^3*Sqrt[(b
^2 - 4*a*c)*g^2] + 105*c^3*d^2*g^3*(3*e*f + d*g) + c^2*e*g*(-(a*e*g^2*(151*
e*f + 357*d*g)) + 3*Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e^2*f^2 + 21*d*e*f*g + 70*d
```

```
^2*g^2))) + c^2*(50*a^2*e^3*g^4 - a*e*g^2*(e*Sqrt[(b^2 - 4*a*c)*g^2]*(19*e*
f + 189*d*g) + c*(4*e^2*f^2 + 294*d*e*f*g + 210*d^2*g^2)) + c*(210*c*d^3*f*
g^3 + Sqrt[(b^2 - 4*a*c)*g^2]*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2
 + 105*d^3*g^3))))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(S
qrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g
^2])]*Sqrt[f + g*x])))/(105*c^4*g^4*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2
*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x))))/(f +
 g*x)))/g^2])
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 1283, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1283
risch	Expression too large to display	4891
default	Expression too large to display	14978

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*e^3/c*
x^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/5*(3*d*e^2*g+e^3*f-2/
7*e^3/c*(3*b*g+3*c*f))/c/g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2
)+2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b*f)-2/5*(3*d*e^2*g+e^3*f
-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a
*g*x+b*f*x+a*f)^(1/2)+2*(d^3*f-2/5*(3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f)
)/c/g*f*a-2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b*f)-2/5*(3*d*e^2
*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(1/2*a*g+1/2*b*f))
*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF
(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(d^3*g+3*d^2*e
*f-4/7*a/c*f*e^3-2/5*(3*d*e^2*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(3/2*a*g
+3/2*b*f)-2/3*(3*d^2*e*g+3*d*e^2*f-2/7*e^3/c*(5/2*a*g+5/2*b*f)-2/5*(3*d*e^2
*g+e^3*f-2/7*e^3/c*(3*b*g+3*c*f))/c/g*(2*b*g+2*c*f))/c/g*(b*g+c*f))*(f/g-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1
```

```
/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left((8\,c^4e^3f^4-(42\,c^4de^2-5\,bc^3e^3)f^3g+(105\,c^4d^2e-42\,bc^3de^2+(10\,b^2c^2-13\,ac^3)e^3)f^2g^2-(210\,c^4d^3\right.}{}$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*((8*c^4*e^3*f^4 - (42*c^4*d*e^2 - 5*b*c^3*e^3)*f^3*g + (105*c^4*d^2*
e - 42*b*c^3*d*e^2 + (10*b^2*c^2 - 13*a*c^3)*e^3)*f^2*g^2 - (210*c^4*d^3 -
210*b*c^3*d^2*e + 21*(7*b^2*c^2 - 12*a*c^3)*d*e^2 - (40*b^3*c - 113*a*b*c^2
)*e^3)*f*g^3 + (105*b*c^3*d^3 - 105*(2*b^2*c^2 - 3*a*c^3)*d^2*e + 21*(8*b^3
*c - 21*a*b*c^2)*d*e^2 - (48*b^4 - 176*a*b^2*c + 75*a^2*c^2)*e^3)*g^4)*sqrt
(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*
g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3
 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8*c^4*e^3
*f^3*g - 3*(14*c^4*d*e^2 - 3*b*c^3*e^3)*f^2*g^2 + (105*c^4*d^2*e - 63*b*c^3
*d*e^2 + (16*b^2*c^2 - 19*a*c^3)*e^3)*f*g^3 + (105*c^4*d^3 - 210*b*c^3*d^2*
e + 21*(8*b^2*c^2 - 9*a*c^3)*d*e^2 - 8*(6*b^3*c - 13*a*b*c^2)*e^3)*g^4)*sqr
t(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2
), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 -
9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2
- 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*
c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c
*g))) - 3*(15*c^4*e^3*g^4*x^2 - 4*c^4*e^3*f^2*g^2 + (21*c^4*d*e^2 - 5*b*c^3
*e^3)*f*g^3 + (105*c^4*d^2*e - 84*b*c^3*d*e^2 + (24*b^2*c^2 - 25*a*c^3)*e^3
)*g^4 + 3*(c^4*e^3*f*g^3 + 3*(7*c^4*d*e^2 - 2*b*c^3*e^3)*g^4)*x)*sqrt(c*x^2
 + b*x + a)*sqrt(g*x + f))/(c^5*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.901 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 567

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}\left(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2\left(2e^2f^2-10defg-15d^2g^2\right)\right)\sqrt{f+gx}\sqrt{-\frac{c(a+bx}{b^2-}}}{15c^3g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{4\sqrt{2}\sqrt{b^2-4ac}\,e(cef-5cdg+2beg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\text{ar}\right.}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/15*e*(-4*b*e*g+7*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g+2/5
*e*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/15*(8*b^2*e^2*g^2-c*e*g*(9
*a*e*g+20*b*d*g+3*b*e*f)-c^2*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f^2))*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^2/(c*x
^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/15*e*(
2*b*e*g-5*c*d*g+c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+
b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)/c^3/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {955, 1667, 857, 732, 435, 430}

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{4\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(2beg-5cdg+cef)\,\text{EllipticF}\left(\arcsin\right.}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-(c^2(-15d^2g^2-10defg+}{15c^3g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(-4beg+7cdg+cef)}{15c^2g}+\frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(c*e*f + 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c
^2*g) + (2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) -
 c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15
*c^3*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 955

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(d
*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*
m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f
 + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+e(bdf+2aef+adg)-(cd(8ef+5dg)-e(3bef+2bdg+3aeg))x-e(cef+7cdg-4beg)x^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}g(4b^2e^2fg+be(4aeg^2-cf(ef+10dg))+cg(15cd^2f-ae(7ef+10dg)))-\frac{1}{2}g(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2(2e^2f^2-10d}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{15c^2g^2}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{(2e(cef-5cdg+2beg)(cf^2-bfg+ag^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{15c^2g^2} + \frac{(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2(2e^2f^2-10defg-15d^2g^2))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{15c^2g^2}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2(2e^2f^2-10defg-15d^2g^2))\sqrt{f+gx}\sqrt{\right.}{15c^3g^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4acg}}}\sqrt{a+bx+}} + \frac{\left(4\sqrt{2}\sqrt{b^2-4ac}e(cef-5cdg+2beg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4acg}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2e^2g^2-ceg(3bef+20bdg+9aeg)-c^2(2e^2f^2-10defg-15d^2g^2))\sqrt{f+gx}\sqrt{}}{15c^3g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+c}} + \frac{4\sqrt{2}\sqrt{b^2-4ac}e(cef-5cdg+2beg)(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-}\right.}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.19 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(-\frac{2e(-cef-10cdg+4beg)}{15c^2g}+\frac{2e^2x}{5c}\right)\sqrt{f+gx}(a+bx+cx^2)}{\sqrt{a+x(b+cx)}} - \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left((-8b^2e^2g^2+ceg(3bef+20bdg+9aeg)+c^2(2e^2f^2-10defg-15d^2g^2))\right.}{}$$

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((-2*e*(-(c*e*f) - 10*c*d*g + 4*b*e*g))/(15*c^2*g) + (2*e^2*x)/(5*c))*Sqrt
[f + g*x]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqr
t[a + b*x + c*x^2]*((-8*b^2*e^2*g^2 + c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g)
+ c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((I/2)*Sqrt[1 - (2*(c*
f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x)
)]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g
^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) + c^2*(-2*e^2*f^2 + 10*d*e*f*g +
15*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (-30*c^3
*d^2*f*g^2 + 8*b^2*e^2*g^2*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*e*g*(-17*a*b
*e*g^2 + 9*a*e*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*Sqrt[(b^2 - 4*a*c)*g^2]*(3*e*f
 + 20*d*g) - b^2*g*(11*e*f + 20*d*g)) - c^2*(-15*b*d*g^2*(2*e*f + d*g) - 2*
a*e*g^2*(7*e*f + 10*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(-2*e^2*f^2 + 10*d*e*f*g
 + 15*d^2*g^2)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqr
t[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[f + g*x])))/(15*c^3*g^3*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5c}+\frac{2\left(2deg+e^2f-\frac{2e^2(2bg+2cf)}{5c}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d^2f\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e^2/c*
x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(2*d*e*g+e^2*f-2/5*e^
2/c*(2*b*g+2*c*f))/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d
^2*f-2/5*e^2/c*f*a-2/3*(2*d*e*g+e^2*f-2/5*e^2/c*(2*b*g+2*c*f))/c/g*(1/2*a*g
+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)
*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(d^2
*g+2*d*e*f-2/5*e^2/c*(3/2*a*g+3/2*b*f)-2/3*(2*d*e*g+e^2*f-2/5*e^2/c*(2*b*g+
2*c*f))/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f
/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*
x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*E
llipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\left(2c^3e^2f^3-2\left(5c^3de-bc^2e^2\right)f^2g+\left(30c^3d^2-20bc^2de+\left(7b^2c-12ac^2\right)e^2\right)fg^2-\left(15bc^2d^2-10\left(2b^2c\right.\right.\right.\right.}{}$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/45*((2*c^3*e^2*f^3 - 2*(5*c^3*d*e - b*c^2*e^2)*f^2*g + (30*c^3*d^2 - 20*b
*c^2*d*e + (7*b^2*c - 12*a*c^2)*e^2)*f*g^2 - (15*b*c^2*d^2 - 10*(2*b^2*c -
3*a*c^2)*d*e + (8*b^3 - 21*a*b*c)*e^2)*g^3)*sqrt(c*g)*weierstrassPInverse(4
/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*
b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3),
 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(2*c^3*e^2*f^2*g - (10*c^3*d*e - 3*b*
c^2*e^2)*f*g^2 - (15*c^3*d^2 - 20*b*c^2*d*e + (8*b^2*c - 9*a*c^2)*e^2)*g^3)
*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2
*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^
3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (
b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c -
6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g
)/(c*g))) + 3*(3*c^3*e^2*g^3*x + c^3*e^2*f*g^2 + 2*(5*c^3*d*e - 2*b*c^2*e^2
)*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^4*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.902 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 452

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}e(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \ldots\right)}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/3*(-2*b*e*g+3*c*d*g+c*e*f)*Elli
pticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(
-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g/
(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*
e*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g/(g*x+f)^(1/2
)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {846, 857, 732, 435, 430}

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3*c^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}+\frac{2\int \frac{\frac{1}{2}(3cdf-e(bf+ag))+\frac{1}{2}(cef+3cdg-2beg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3c}\\ &= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}+\frac{(cef+3cdg-2beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg}\\ &\quad -\frac{(e(cf^2-bfg+ag^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3cg}\end{aligned}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}e(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-}}{2cf-bg-}}}\right.}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-}}{2cf-\left(b+\sqrt{}\right.}\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\right.}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.34 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(ce(a+x(b+cx)) + \frac{(f+gx)\left(\frac{g^2(cef+3cdg-2beg)(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{1-\frac{2\left(cf^2+g(-bf+ag)\right)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2\left(cf^2+g(\cdots\right)}{\left(-2cf+bg+\sqrt{(b^2\cdots}\right)\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*(a + x*(b + c*x)) + ((f + g*x)*((g^2*(c*e*f + 3*c*d*g
 - 2*b*e*g)*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*
(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[
1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(2*b*e*g - c*(e*f +
3*d*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b
^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (6*c^2*d*f*g
+ 2*b*e*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*(e*f + d*
g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)))*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt
[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/g^2))/(3*c^2*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(394) = 788.

Time = 1.54 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3c}+\frac{2\left(df-\frac{2e\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3c}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{}}{}}{-\frac{f}{g}-\frac{-b+}{}}}}{\sqrt{cg\,x^3+bg\,x^2}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/c*e*(c
*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d*f-2/3/c*e*(1/2*a*g+1/2*b
*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(d*g+e*f-2
/3/c*e*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f
)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellipt
icF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2eg^2-(c^2ef^2-2(3c^2d-bce)fg+(3bcd-(2b^2-3ac)e)g^2)\sqrt{cg}\text{weierstra}\right.}{}$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*e*g^2 - (c^2*e*f^2 - 2*(3*c^
2*d - b*c*e)*f*g + (3*b*c*d - (2*b^2 - 3*a*c)*e)*g^2)*sqrt(c*g)*weierstrass
PInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^
3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/
(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(c^2*e*f*g + (3*c^2*d - 2*b
*c*e)*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)
*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g
^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b
*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3
*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x +
 c*f + b*g)/(c*g))))/(c^3*g^2)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(d+e\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2), x)
```

3.903 $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/
(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {732, 435}

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a +
b*x + c*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

integral

$$= \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.60 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{\right.}\right.}}\right.\right.\right.}{\sqrt{2}cg\sqrt{\frac{}{-2cf+(}}}$$

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (I*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 + (-b + Sqrt[b^2 - 4*a*c])*g)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f
 + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]))/(Sqrt[2]*c*g
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(164) = 328.

Time = 0.64 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.97

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2f\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b}}{2c}}}\right.}{\sqrt{cgx^3+bgx^2+cfx^2+agx+bfx+fa}}\right.}{}$
default	$\frac{\sqrt{gx+f}\sqrt{cx^2+bx+a}\left(g\sqrt{-4ac+b^2}+bg-2cf\right)\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}}\sqrt{\frac{\left(-b-2cx+\sqrt{-4ac+b^2}\right)g}{2cf-bg+g\sqrt{-4ac+b^2}}}\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}\left(F\left(\sqrt{\right.\right.}}{}$

```
[In] int((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*f*(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/
g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*g*(f/g-1/2*(b+(-4*a*c
+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c
*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{cg}cg\text{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+(b^2-3\,ac)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3\right)}{27\,c^3g^3},\text{weierstra}\right.\right.}{}$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*c*g*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c
)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*
g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 -
b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g -
3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x
+ c*f + b*g)/(c*g))) - (2*c*f - b*g)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2
*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f
^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3
*c*g*x + c*f + b*g)/(c*g)))/(c^2*g)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

3.904 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 467

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right.}{2c(ef-dg)}\right.}{\sqrt{c}e\sqrt{a+bx+cx^2}}$$

```
[Out] 2*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2
)-EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*
a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+
f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/c^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {957, 732, 430, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4\right.}}{2c(ef-dg)}\right.}{\sqrt{c}e\sqrt{a+bx+cx^2}}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e*Sqrt[a + b*x + c*x^
2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+ex)\sqrt{}}}{e\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{b-}}{}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}}\right.}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(ef-dg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\frac{2c}{g}}}\right.}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(ef-dg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4ac}-\right.}}}\right.}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right.}{2c(ef-dg)}\right.}{\sqrt{c}e\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.14 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{i\sqrt{2}\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\right)\right.\right.}{e\sqrt{\frac{}{-2cf+}}}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c]
)*g)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)] - EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2
*c*(e*f - d*g)), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)]))/(e*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.44

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2g\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b}}{2c}}}\right.}{e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}\right.}{}$
default	$\frac{\left(-F\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)g\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,\Pi\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\frac{(g\sqrt{-4ac+b^2}}{2c(dg}\right.\right.}{}$

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*g/e*(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2*(d*g-e*f)/e^2*(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*Elli
pticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.905 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 994

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)\,(d+ex)}$$
$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{e\,(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}{\sqrt{2}\sqrt{c}e\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a}}$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)+1/2*Ellipt
icE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
```

```
)^(1/2)-f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*
x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(g*x+f)^(1
/2)/(c*x^2+b*x+a)^(1/2)+d*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c
+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d
*e+c*d^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^2*(-a*g+b*f)-c*d*(-d*g+2
*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1
/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g
-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/
(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {959, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e}{(cd^2-bed+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bed+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}} - \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}e} + \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{E}}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)\sqrt{cx^2}}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x
))) + (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sq
rt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c
*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(c*d^2 - b
*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f +
 g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 959

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*(
```

```
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e
*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} - \frac{\int \frac{-2cdf+bef-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cgx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{e^2(bf-ag)-cd(2ef-dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{2(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(cd^2-bde+ae^2)} - \frac{(e^2(bf-ag)-cd(2ef-dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)} - \frac{\left((e^2(bf-ag)-cd(2ef-dg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+2c}}}{2e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{b-}}{}\right.}{e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\left((e^2(bf-ag)-cd(2ef-dg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int\frac{}{(ef-dg-}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{\sqrt{2}(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b}}{\sqrt{}}}}{}\right.}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\left((e^2(bf-ag)-cd(2ef-dg))\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\left((e^2(bf-ag)-cd(2ef-dg))\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-d}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+}{2cf-\left(b+\sqrt{\right.}}}}{\sqrt{2}\sqrt{c}e(cd^2-bde+ae^2)(ef-dg)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.19 (sec) , antiderivative size = 1502, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}(a+bx+cx^2)}{(cd^2-bde+ae^2)\,(d+ex)\sqrt{a+x(b+cx)}}$$

$$-\frac{(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(-4e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)+\cdots\right.}{\cdots}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(d + e*x)*Sq
rt[a + x*(b + c*x)])) - ((f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*e*(-(e*f
) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)
*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)
))/(f + g*x)) + (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 +
f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c
)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 +
f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]
 - (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] -
(2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x))
)/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] +
(2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))
/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f
 + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b
^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - ((2*I)*Sqrt[2]*g*(e^2*(b*f - a*g) + c*
d*(-2*e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c
*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^
2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c
*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^
```

```
2 - 4*a*c)*g^2])]*EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)
*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 -
b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -
((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)
*g^2]))])/Sqrt[f + g*x]))/(4*e*(c*d^2 - b*d*e + a*e^2)*g*(-(e*f) + d*g)*Sqr
t[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt
[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)
/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2])
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 1229, normalized size of antiderivative = 1.24

method	result	size
elliptic	Expression too large to display	1229
default	Expression too large to display	13017

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-e/(a*e^2-
b*d*e+c*d^2)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)+c*d*g/
(a*e^2-b*d*e+c*d^2)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x
+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2))+c*g/(a*e^2-b*d*e+c*d^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/
(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*
x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)
/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1
/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+(
a*e^2*g-b*e^2*f-c*d^2*g+2*c*d*e*f)/e^2/(a*e^2-b*d*e+c*d^2)*(f/g-1/2*(b+(-4*
a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1
/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+
f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)^2\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.906 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1786

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2}$$

$$-\frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}\left(cd^2-bde+ae^2\right)^2(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e\left(cd^2-bde+ae^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{b^2-4ac}f(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\text{arcsi}\right.}{2\sqrt{2}\left(cd^2-bde+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{b^2-4ac}dg(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\text{arcs}\right.}{2\sqrt{2}e\left(cd^2-bde+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{EllipticPi}}{\sqrt{2}\sqrt{c}e\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(6ef-5dg)-e(3bef-2bdg-aeg))(cd(2ef-3dg)-e(bef-2bdg+}{4\sqrt{2}\sqrt{c}e\left(cd^2-\right.}$$

```
[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-1/4*
e*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+
a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)/(e*x+d)+1/8*(c*d*(-5*d*g+6*e*f)-e
*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/
2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/2*g*EllipticF(1/2*(
```

```
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
-1/4*f*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2
)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)
^(1/2)+1/4*d*g*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)+1/2*(b*e*g-3*c*d*g+c*e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)
*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2
+b*x+a)^(1/2)-1/8*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(c*d*(-3*
d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f
-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] (verified)

Time = 4.99 (sec) , antiderivative size = 1786, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {959, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{cx^2+bx+ae}}{4\left(cd^2-bed+ae^2\right)^2(ef-dg)(d+ex)}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+ae}}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}$$

$$+\frac{\sqrt{b^2-4ac}(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{b^2-4ac}f(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\text{arcsi}\right.}{2\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+ae}}$$

$$+\frac{\sqrt{b^2-4ac}dg(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\text{arcs}\right.}{2\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+ae}}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}}{\sqrt{2}\sqrt{c}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{cx^2+b}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(6ef-5dg)-e(3bef-2bdg-aeg))(cd(2ef-3dg)-e(bef-2bdg+a}{4\sqrt{2}\sqrt{c}\left(cd^2-\right.}$$

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d +
e*x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d +
e*x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*
e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
```

```
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*(c*
d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*
x^2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g -
a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
 (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d
*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e
*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[
2]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x
^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g
+ Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
 g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (
2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 -
 b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*(c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/
(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
- 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f
 - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 959

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e
*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\,(cd^2-bde+ae^2)\,(d+ex)^2}-\frac{\int\frac{-4cdf+3bef-aeg+2(cef-2cdg+beg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4\,(cd^2-bde+ae^2)}\\ &= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\,(cd^2-bde+ae^2)\,(d+ex)^2}\\ &\quad-\frac{\int\left(\frac{cg}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{-cd(6ef-5dg)+e(3bef-2bdg-aeg)}{e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{2(cef-3cdg+beg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{4\,(cd^2-bde+ae^2)}\end{aligned}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e(cd^2-bde+ae^2)} - \frac{(cef-3cdg+beg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(cd^2-bde+ae^2)} + \frac{(cd(6ef-5dg)-e(3bef-2bdg-aeg))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)(d+ex)} - \frac{(cd(6ef-5dg)-e(3bef-2bdg-aeg))\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e(cd^2-bde+ae^2)^2(ef-dg)} - \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4a}}}}{2e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{2}}\right.}{\sqrt{2}e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)(d+ex)} - \frac{\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(cd(6ef-5dg)-e(3bef-2bdg-aeg))\int \left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+}{(d+ex)\sqrt{f+}}\right.}{8e(cd^2-bde+ae^2)^2(ef-dg)} + \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2}}}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(d+ex)^2}$$
$$-\frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)(d+ex)}$$
$$-\frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$+\frac{(cg(cd(6ef-5dg)-e(3bef-2bdg-aeg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8(cd^2-bde+ae^2)^2(ef-dg)}$$
$$+\frac{(cdg(cd(6ef-5dg)-e(3bef-2bdg-aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{8e(cd^2-bde+ae^2)^2(ef-dg)}$$
$$+\frac{((cd(6ef-5dg)-e(3bef-2bdg-aeg))(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{}}}{8e(cd^2-bde+ae^2)^2(ef-dg)}$$
$$+\frac{\left((cef-3cdg+beg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2}}}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)(d+ex)}$$

$$- \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(cd^2-bde+ae^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+ \frac{(c(cd(6ef-5dg)-e(3bef-2bdg-aeg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{8(cd^2-bde+ae^2)^2(ef-dg)}$$

$$- \frac{(cf(cd(6ef-5dg)-e(3bef-2bdg-aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8(cd^2-bde+ae^2)^2(ef-dg)}$$

$$+ \frac{\left((cd(6ef-5dg)-e(3bef-2bdg-aeg))(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-}\right.}{8e(cd^2-bde+ae^2)^2(ef-dg}$$

$$+ \frac{\left(\sqrt{b^2-4ac}dg(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}}{2\sqrt{2}e(cd^2-bde+ae^2)^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx}}$$

$$+ \frac{\left((cef-3cdg+beg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{}{(b}}}\right.}{e(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.79 (sec) , antiderivative size = 36634, normalized size of antiderivative = 20.51

$$\int\frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 1698, normalized size of antiderivative = 0.95

method	result	size
elliptic	Expression too large to display	1698
default	Expression too large to display	59522

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2*e/(a*
e^2-b*d*e+c*d^2)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+
1/4*e*(a*e^2*g+2*b*d*e*g-3*b*e^2*f-5*c*d^2*g+6*c*d*e*f)/(a*d*e^2*g-a*e^3*f-
b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/(a*e^2-b*d*e+c*d^2)*(c*g*x^3+b*g*x^2
+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(3*a*d*e^2*g-2*a*e^3*f-b*d*
e^2*f-3*c*d^3*g+4*c*d^2*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g
-c*d^2*e*f)/(a*e^2-b*d*e+c*d^2)/e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/
g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2
+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))-1/4*c*g*(a*e^2*g+2*b*d*e*g-3*b*e^2*f-5*c*d^2*g+6*c*d*e*f)/
(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/(a*e^2-b*d*e+c*d^
2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/4*(a^2*e^4*g^2-4*a*b*d*e^
3*g^2+2*a*b*e^4*f*g+10*a*c*d^2*e^2*g^2-12*a*c*d*e^3*f*g+4*a*c*e^4*f^2+4*b^2
*d*e^3*f*g-3*b^2*e^4*f^2-10*b*c*d^2*e^2*f*g+8*b*c*d*e^3*f^2-3*c^2*d^4*g^2+1
2*c^2*d^3*e*f*g-8*c^2*d^2*e^2*f^2)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c
*d^3*g-c*d^2*e*f)/(a*e^2-b*d*e+c*d^2)/e^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c
)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2
+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e
),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{(d+ex)^3\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/((d + e*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.907 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 675

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{}}{2cf-(}\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b}}{2cf-(b+}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(ef-dg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg\right.}{2c(}\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$

```
[Out] g*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/e/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+
2*g*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(
c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(g*x+f)^(1/2)/(c*x^
2+b*x+a)^(1/2)-(-d*g+e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g
*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d
*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g
```

```
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2)/e^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {971, 732, 430, 948, 175, 552, 551, 435}

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}(ef-dg)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\right.}{2c(e}\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+bx+cx^2}} \right) dx$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+ex)\sqrt{f}}}{e^2\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,d\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2(ef-dg)^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2c\cdots}{g}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2(ef-dg)^2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\cdots}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\left(2(ef-dg)^2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2\cdots}{\left(b-\sqrt{b^2-4ac}\cdots\right.}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2cf-(b+}\right.}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$- \frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(ef-dg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e(2cf-b}{2c(}\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. $2(675) = 1350$.

Time = 13.60 (sec) , antiderivative size = 1385, normalized size of antiderivative = 2.05

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(\frac{2fg\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{-b+\sqrt{}}{}}}{}\right.\right.}{ce\sqrt{\frac{-b+\sqrt{b^2-4ac}-2cx}{\sqrt{b^2-4ac}}}}\right.}{}$$

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*((2*f*g*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g)])/(c*e*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]) - (d
*g^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2
- 4*a*c]*g)])/(c*e^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c
]]) + (g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*((-2*c*f + (b + Sqrt[b^2 - 4*
a*c])*g)*EllipticE[ArcSin[Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)] - (b + Sqrt[b^2 - 4*a*c])*g*EllipticF[ArcSin[Sqrt[2]*Sqrt[(
c*(f + g*x))/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2
```

```
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]))/(2*c^2*e*Sqrt[(g*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]) - (4*Sq
rt[b^2 - 4*a*c]*f^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e) + (8*Sqrt[b^2 - 4*a*c]*d*f*g*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
, ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(e*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)) - (4*Sqrt[b^2 - 4*a*c]*d^2*g^2*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e
 + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g)])/(e^2*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))))/(Sqrt[f + g*x]*Sqrt
[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1122
default	Expression too large to display	1879

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*g*(d*g-
2*e*f)/e^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)
*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*g^2/
e*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(d^2*g^2-2*d*e*f*g+e^2*f^2
)/e^3*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c
```

```
+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/
g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**(3/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^{3/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.908 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1138

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2\cdots}}{2cf-(b+\cdots}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\cdots\right.}{3c^2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg\cdots\right.}{2c(\cdots}\right.}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/3*g*(-b*g+2*c*f)*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/e/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(-d*g+e*
f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/c/e^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)+2*g*(-d*g+e*f)^2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/
(c*x^2+b*x+a)^(1/2)-2/3*g*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/c^2/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-(-d*g+e*f)^2*EllipticPi(2^(1/
2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b
-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x
+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules

used = {971, 732, 430, 948, 175, 552, 551, 435, 756, 857}

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}g^2}{3ce}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)g}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)g}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-}}{2cf-\left(b+\sqrt{\right.}}\right.}{ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{3c^2e\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\text{EllipticPi}\left(\frac{e\left(2cf-bg\right.}{2c(e}\right.}{\sqrt{c}e^3\sqrt{cx^2+bx+a}}$$

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
 + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a +
```

```
b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*g + a*g^2)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f + g*
x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
(b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[
b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{g(ef-dg)\sqrt{f+gx}}{e^2\sqrt{a+bx+cx^2}} + \frac{g(f+gx)^{3/2}}{e\sqrt{a+bx+cx^2}}\right) dx$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(g(ef-dg)^2)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^3} + \frac{(ef-dg)^3\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^3}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}(3cf^2-g(bf+ag))+g(2cf-bg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+e}}{e^3\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4}}{\sqrt{b^2-}}}}{\sqrt{2}}\right.}{ce^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}}}}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{\ }}{2cf-(b}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(2g(2cf-bg))\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3ce} - \frac{(g(cf^2-bfg+ag^2))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3ce} - \frac{\left(2(ef-dg)^3\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int\frac{1}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2cf}{g}\ldots}}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{\ }}{2cf-(b}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2-}}}}{\sqrt{2}}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}g(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-}}{2cf-bg-}}}\right.}{3c^2e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\left(2(ef-dg)^3\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{1}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\frac{2c}{g}\ldots}}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2cf-}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}g(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3c^2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2(ef-dg)^3\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4ac}\right.}}}\right.}{e^3\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{}}{2cf-(b}\right.}{ce^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}g(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\right.}{3c^2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e(2cf-}{2}\right.}{\sqrt{c}e^3\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.53 (sec) , antiderivative size = 37137, normalized size of antiderivative = 32.63

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.09

method	result	size
elliptic	Expression too large to display	1245
risch	Expression too large to display	2354
default	Expression too large to display	7464

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/e*g^2/
c*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(g*(d^2*g^2-3*d*e*f*g+3
*e^2*f^2)/e^3-2/3/e*g^2/c*(1/2*a*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*
x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2))+2*(-g^2/e^2*(d*g-3*e*f)-2/3/e*g^2/c*(b*g+c*f))*
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-2*(d^3*g^3-3*d^2*e*f*g^2+3*d*e
^2*f^2*g-e^3*f^3)/e^4*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x
+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^{5/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.909 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 631

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}e(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{-\frac{c(a+bx}{b^2-}}}{15c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(4be^3g^2(bf-ag)+c^2(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3)-ce^2g(ag(7ef-15dg)-}{15c^3g^3\sqrt{f+}}$$

```
[Out] -8/15*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+2
/5*e^2*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g+1/15*e*(8*b^2*e^2*g^2+
c*e*g*(-9*a*e*g-30*b*d*g+7*b*e*f)+c^2*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/
g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/15*(4*b*e^3*g^2*(-a*g+b*f)+c^2*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*e^2*f^2*g
+8*e^3*f^3)-c*e^2*g*(a*g*(-15*d*g+7*e*f)-3*b*f*(-5*d*g+e*f)))*EllipticF(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2
```

```
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {944, 1667, 857, 732, 435, 430}

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2(45d^2g^2-30defg+8e}{15c^3g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf}{15c^3g^3\sqrt{f}}$$

$$-\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{15c^2g^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}$$

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-8*e^2*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*g^2) + (2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) +
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9
*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*
f*g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 944

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*f
 + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f
 + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*
d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{\int \frac{bde^2f-5cd^3g+ae^2(2ef+dg)+e(cd(2ef-15dg)+e(3bef+2bdg+3aeg))x+4e^2(cef-3cdg+beg)x^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{5cg}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{2\int \frac{-\frac{1}{2}g(4b^2e^3fg+be^2(4aeg^2+cf(4ef-15dg))+cg(15cd^3g-ae^2(2ef+15dg)))-\frac{1}{2}eg(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{15c^2g^3}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} + \frac{(e(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2)))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{15c^2g^3} - \frac{(4be^3g^2(bf-ag)+c^2(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3)-ce^2g(ag(7ef-15dg)-3bf(}{15c^2g^3}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}e(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2))\sqrt{f+g}\right.}{15c^3g^3\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(4be^3g^2(bf-ag)+c^2(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3)-ce^2g(ag(7ef\right.}{15c}$$

$$=-\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}e(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2))\sqrt{f+gx}\sqrt{}}{15c^3g^3\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+c}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(4be^3g^2(bf-ag)+c^2(8e^3f^3-30de^2f^2g+45d^2efg^2-15d^3g^3)-ce^2g(ag(7ef-}{15c^3g^3\sqrt{f}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.49 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$=\frac{\frac{4eg^2(8b^2e^2g^2+ceg(7bef-30bdg-9aeg)+c^2(8e^2f^2-30defg+45d^2g^2))(a+x(b+cx))}{\sqrt{f+gx}}+4ce^2g^2\sqrt{f+gx}(a+x(b+cx))(-4beg+c}{}$$

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*e*g^2*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*a*e*g) + c^2*(8*e^
2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + x*(b + c*x)))/Sqrt[f + g*x] + 4*c*e^
2*g^2*Sqrt[f + g*x]*(a + x*(b + c*x))*(-4*b*e*g + c*(-4*e*f + 15*d*g + 3*e*
g*x)) - (I*(f + g*x)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(e*(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*a*
e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))] - (-8*b^3*e^3*g^3 + b^2*e^2*g^2*(c*e*f + 30*c*d*
g + 8*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*e*g*(-45*c*d^2*g^2 + e*(17*a*e*g^2 +
 Sqrt[(b^2 - 4*a*c)*g^2]*(7*e*f - 30*d*g))) + c*(-(a*e^2*g^2*(4*c*e*f + 30*
c*d*g + 9*e*Sqrt[(b^2 - 4*a*c)*g^2])) + c*(30*c*d^3*g^3 + e*Sqrt[(b^2 - 4*a
*c)*g^2]*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))))*EllipticF[I*ArcSinh[(Sqrt
```

```
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))]]))/Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])])/(30*c^3*g^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 985, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5cg}+\frac{2\left(3d\,e^2-\frac{2(2bg+2cf)e^3}{5cg}\right)\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d^3-\frac{2}{\cdots}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*e^3/c/
g*x*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2/3*(3*d*e^2-2/5/c/g*(2
*b*g+2*c*f)*e^3)/c/g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d^3
-2/5*f*a/c/g*e^3-2/3*(3*d*e^2-2/5/c/g*(2*b*g+2*c*f)*e^3)/c/g*(1/2*a*g+1/2*b
*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(3*d^2*e-2
/5*e^3/c/g*(3/2*a*g+3/2*b*f)-2/3*(3*d*e^2-2/5/c/g*(2*b*g+2*c*f)*e^3)/c/g*(b
*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*(
(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f
/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left((8\,c^3e^3f^3 - 3\,(10\,c^3de^2 - bc^2e^3)f^2g + 3\,(15\,c^3d^2e - 5\,bc^2de^2 + (b^2c - ac^2)e^3)fg^2 - (45\,c^3d^3 - 45\,bc^2d^2\right.}{}$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/45*((8*c^3*e^3*f^3 - 3*(10*c^3*d*e^2 - b*c^2*e^3)*f^2*g + 3*(15*c^3*d^2*
e - 5*b*c^2*d*e^2 + (b^2*c - a*c^2)*e^3)*f*g^2 - (45*c^3*d^3 - 45*b*c^2*d^2
*e + 15*(2*b^2*c - 3*a*c^2)*d*e^2 - (8*b^3 - 21*a*b*c)*e^3)*g^3)*sqrt(c*g)*
weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2),
-4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a
*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(8*c^3*e^3*f^2*g
 - (30*c^3*d*e^2 - 7*b*c^2*e^3)*f*g^2 + (45*c^3*d^2*e - 30*b*c^2*d*e^2 + (8
*b^2*c - 9*a*c^2)*e^3)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*
g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2
*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse
(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 -
3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3
), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) - 3*(3*c^3*e^3*g^3*x - 4*c^3*e^3*f*g^2
 + (15*c^3*d*e^2 - 4*b*c^2*e^3)*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(
c^4*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.910 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 479

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(e^2g(bf-ag)+c(2e^2f^2-6defg+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\ldots\right)}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g-2/3*e*(b*e*g-3*c*d*g+c*e*f)*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)
*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2
/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
+2/3*(e^2*g*(-a*g+b*f)+c*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2))*EllipticF(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {944, 24, 857, 732, 435, 430}

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c(3d^2g^2-6defg+2e^2f^2)\right)\text{EllipticF}\left(\text{a}\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(beg-3cdg+cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4\right.}}\right.}{3c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}$$

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(3*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f -
 a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2
*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 944

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*f
 + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f
 + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*
d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\text{integral} = \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{d(be^2f-3cd^2g+ae^2g)+e(cd(2ef-9dg)+e(bef+2bdg+aeg))x+2e^2(cef-3cdg+beg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{e^2(be^2f-3cd^2g+ae^2g)+2e^3(cef-3cdg+beg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{(2e(cef-3cdg+beg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg^2} + \frac{(e^2g(bf-ag)+c(2e^2f^2-6defg+3d^2g^2))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3cg^2}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x\right.}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(e^2g(bf-ag)+c(2e^2f^2-6defg+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{}}{2cf-(b}\right.}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(e^2g(bf-ag)+c(2e^2f^2-6defg+3d^2g^2))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.40 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(2ce^2g^2(a+x(b+cx)) + \frac{(f+gx)\left(-\frac{4eg^2(cef-3cdg+beg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2} + \frac{i\sqrt{2}e(cef-3cdg+beg)\left(2cf-b\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(2*c*e^2*g^2*(a + x*(b + c*x)) + ((f + g*x)*((-4*e*g^2*(c*e*
f - 3*c*d*g + b*e*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*e*(c*e*f -
3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f
*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f
 - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2
 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*
g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Ellipti
cE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*(3
*c^2*d^2*g^2 + b*e^2*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) - c*e*(3*b*d*g^2 + a
*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g)))*Sqrt[(-2*a*g^2 + f*Sqrt[(b
^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))
/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqr
t[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f +
g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*Arc
Sinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*
c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*
```

```
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/Sqrt[(c*f^2 + g*(-(b*
f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]))/(3*c^2*g^3*Sqrt[a +
x*(b + c*x)])
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3cg}+\frac{2\left(d^2-\frac{2e^2\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3cg}\right)\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-}{-\frac{f}{g}-}}}{\sqrt{cg\,x^3+}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*e^2/c/
g*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)+2*(d^2-2/3*e^2/c/g*(1/2*a
*g+1/2*b*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/
2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2
*d*e-2/3*e^2/c/g*(b*g+c*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(
-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x
+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/
(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/
2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{gx+f}c^2e^2g^2+(2c^2e^2f^2-(6c^2de-bce^2)fg+(9c^2d^2-6bcde+(2b^2-3ac)e^2)g^2\right)\ldots}{\ldots}$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*e^2*g^2 + (2*c^2*e^2*f^2 - (
6*c^2*d*e - b*c*e^2)*f*g + (9*c^2*d^2 - 6*b*c*d*e + (2*b^2 - 3*a*c)*e^2)*g^
2)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2
)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
 (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 6*(c^
2*e^2*f*g - (3*c^2*d*e - b*c*e^2)*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f
^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2
*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstr
assPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^
3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^3*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.911 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 393

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*
(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g/(g*x+f)^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {857, 732, 435, 430}

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\cdots}{2cf-(\cdots}\right.}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
```

```
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{g}$$

$$= \frac{\left(\sqrt{2}\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(-ef+dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,d\right.}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2}}{2cf-(b+\sqrt{}}\right.}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.69 (sec) , antiderivative size = 814, normalized size of antiderivative = 2.07

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{(f+gx)^{3/2}\left(-\frac{4eg^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{2}e\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx)+\sqrt{(b^2-4ac)g^2}\cdots}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx\cdots}}\cdots}{\cdots}\cdots\right.}{\cdots}$$

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]
*e*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g
*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt
[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f
 + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
 + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[
(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x
] - (I*Sqrt[2]*(2*c*d*g + e*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]))*Sqrt[(-2*a*
g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x +
b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b
*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -(
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*
g^2]))])/Sqrt[f + g*x]))/(c*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(345) = 690.

Time = 1.81 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2d\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\right.}{\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}\right.}{}$
default	Expression too large to display

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*d*(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/
g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*e*(f/g-1/2*(b+(-4*a*c
+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c
*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{cg}ceg\text{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+(b^2-3ac)g^2\right)}{3c^2g^2},-\frac{4\left(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3\right)}{27c^3g^3},\text{weiers}\right.\right.}{}$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*c*e*g*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a
*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*
f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
- b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g
- 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*
x + c*f + b*g)/(c*g))) + (c*e*f - (3*c*d - b*e)*g)*sqrt(c*g)*weierstrassPIn
verse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f
^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^
3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)))/(c^2*g^2)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.912 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {732, 430}

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

integral

$$=\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$=\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.45 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{i(f+gx)\sqrt{2-\frac{4(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{c}{-2cf+}}}{\sqrt{}}\right.\right.}{g\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))])/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)F\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}\sqrt{\frac{\left(-b-2cx+\sqrt{-4ac+b^2}\right.}{2cf-bg+g\sqrt{-4ac+b^2}}}}{cg\left(cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa\right)}$
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,F\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+}}{2c}}}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}$

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)/c*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)/g*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)}{cg}$$

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)
/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
(2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))/(c*g)
```

Sympy [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.913 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 280

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right.}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)\sqrt{a+bx+cx^2}}$$

```
[Out] -EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a
*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/c^(
1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {948, 175, 552, 551}

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right.}{2c(ef-dg)}\right.}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)}$$

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))
/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+ex)\sqrt{f+gx}}\,dx}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{b+\sqrt{b}}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{1+}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right.}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.04 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.78

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{i(f+gx)\sqrt{2-\frac{4(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{c}{-2cf+}}}{\sqrt{}}\right.\right.\right.}{(-ef+}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(EllipticF[I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))] - EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(
b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sq
```

```
rt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2]))])]))/((-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.18

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)\Pi\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\frac{\left(g\sqrt{-4ac+b^2}+bg-2cf\right)e}{2c(dg-ef)},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)\sqrt{\frac{\left(b+2cx+\sqrt{-4ac+b^2}\right)g}{g\sqrt{-4ac+b^2}+bg-2cf}}}{c(dg-ef)(cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa)}$
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\Pi\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}\left(-\frac{f}{g}+\frac{d}{e}\right)}$

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g
-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c*(c*x^2+b*x+
a)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.914 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1037

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,ef\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-(b+}}}{\sqrt{2}\sqrt{c}\,(cd^2-bde+ae^2)(ef-dg}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(e*x+
d)+1/2*e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(
a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f
```

```
-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a
*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+d*g*Elliptic
F(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g
*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*
c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)-1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticP
i(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*
e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1
/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*
a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+
f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2
*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 1037, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {953, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx = -\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{(cd^2-bed+ae^2)(ef-dg)(d+ex)}$$
$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)e}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)\right.}}{\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$
$$-\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f\cdot}{2cf-(b+\sqrt{}}}}{\sqrt{2}\sqrt{c}\left(cd^2-bed+ae^2\right)(ef-dg)^{\cdot}}$$

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f -
 d*g)*(d + e*x))) + (Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*e*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*
c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^
2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
```

```
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

integral

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int \left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(bef-2bdg+aeg)}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{2(cd^2-bde+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} + \frac{(ceg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} - \frac{(cef)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} + \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+}}}{2(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}ef\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}ef\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}ef\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\left((cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}ef\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(}{2cf-\left(b+\right.}}}{\sqrt{2}\sqrt{c}(cd^2-bde+ae^2)(ef-dg)^2\sqrt{a}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.53 (sec) , antiderivative size = 1513, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{e^2\sqrt{f+gx}(a+bx+cx^2)}{(cd^2-bde+ae^2)\,(ef-dg)(d+ex)\sqrt{a+x(b+cx)}}$$
$$+ \frac{(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(-4e(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(-1+\frac{f}{f+gx}\right)^2+\frac{g\left(b-\frac{bf}{f+gx}+\frac{ag}{f+gx}\right)}{f+gx}\right)+\right.}{}$$

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g
)*(d + e*x)*Sqrt[a + x*(b + c*x)])) + ((f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2
]*(-4*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt
[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (
a*g)/(f + g*x)))/(f + g*x)) + (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x)
- 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqr
t[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x)
+ 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a
*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])
/Sqrt[f + g*x] - (I*Sqrt[2]*(2*c*d*g*(e*f - 2*d*g) - e*(2*a*e*g^2 + b*g*(e*
f - 3*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - d*g)))*Sqrt[(Sqrt[(b^2 - 4*a*c)
*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f
 + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)
*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f
+ g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*g*(c*d*(2*e*f -
 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a
```

```
*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a
*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticPi[((e*f - d*g)*(2*c*f - b*g
 - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(S
qrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g
 + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(4*(c*d^2 - b*d*e + a*e^2)*g
*(-(e*f) + d*g)^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2
 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*
x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 1347, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	1347
default	Expression too large to display	14048

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(e^2/(a*d*e
^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*f*x^
2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-c*d*g/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2
*f+c*d^3*g-c*d^2*e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x
+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2))-c*e*g/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(f/g-1
/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+(a*e^2*g-2*b*d*e*g+b*e^2*f+3*c*d^2*g
-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/
```

```
2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*El
lipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.915 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1114

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2}$$
$$-\frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)}$$
$$+\frac{3\sqrt{b^2-4ac}\,e(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\cdots\right.}{4\sqrt{2}\left(cd^2-bde+ae^2\right)^2(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{b^2-4ac}(cd(-6ef+7dg)+e(3bef-4bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{2\sqrt{2}\left(cd^2+e(-bd+ae)\right)^2(ef-dg)\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$
$$+\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}\,g}\,(c^2d^2(8e^2f^2-20defg+15d^2g^2)+2ce(bd(-4e^2f^2+11defg-10d^2g^2)+ae(-\cdots}{\cdots}$$

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(
e*x+d)^2-3/4*e^2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)
*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/8*e*(c*d*
(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/4*(c
```

```
*d*(7*d*g-6*e*f)+e*(a*e*g-4*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2
)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x
+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/
(c*d^2+e*(a*e-b*d))^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
1/8*(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+8*e^2*f^2)+2*c*e*(b*d*(-10*d^2*g^2+11*d
*e*f*g-4*e^2*f^2)+a*e*(3*d^2*g^2+2*d*e*f*g-2*e^2*f^2))+e^2*(3*a^2*e^2*g^2+2
*a*b*e*g*(-4*d*g+e*f)+b^2*(8*d^2*g^2-8*d*e*f*g+3*e^2*f^2)))*EllipticPi(2^(1
/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b
*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^
(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^
(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(c*d^2+e*(a*e-b*d))^2/(d*g-e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(
1/2)
```

Rubi [A] (verified)

Time = 4.76 (sec) , antiderivative size = 1762, normalized size of antiderivative = 1.58, number of steps used = 25, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules

used = {953, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{3(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{4\left(cd^2-bed+ae^2\right)^2(ef-dg)^2(d+ex)}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}$$

$$+\frac{3\sqrt{b^2-4ac}(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{4\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$-\frac{3\sqrt{b^2-4ac}f(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\right.}{2\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{3\sqrt{b^2-4ac}dg(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\right.}{2\sqrt{2}\left(cd^2-bed+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{3\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cd(2ef-3dg)-e(bef-2bdg+aeg))^2\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c}{2cf-(b}}}{4\sqrt{2}\sqrt{c}\left(cd^2-bed+ae^2\right)^2(ef-d}$$

$$+\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(cef-3cdg+beg)\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\right.}{\sqrt{2}\sqrt{c}\left(cd^2-bed+ae^2\right)(ef-dg)^2\sqrt{cx^2+b}}$$

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*
f - d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g +
a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(
e*f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2 - 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(
b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
```

```
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[(c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 -
4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (3*Sqrt[b^2 - 4*a*c]*e*f*(c*d*(2*e*
f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]
) + (3*Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*
e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g
)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - S
qrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g
)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a
*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*
x + c*x^2]) - (3*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*
g) - e*(b*e*f - 2*b*d*g + a*e*g))^2*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*
g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*
c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^3*Sqrt[a
 + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(ef-dg)(d+ex)^2} \\
&\quad -\frac{\int \frac{3e^2(bf+ag)-4d(cef-cdg+beg)+2e(cef-2cdg+beg)x+ce^2gx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4(cd^2-bde+ae^2)(ef-dg)} \\
&= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(ef-dg)(d+ex)^2} \\
&\quad -\frac{\int \left(\frac{cg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{3(-cd(2ef-3dg)+e(bef-2bdg+aeg))}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{2(cef-3cdg+beg)}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right)dx}{4(cd^2-bde+ae^2)(ef-dg)} \\
&= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(ef-dg)(d+ex)^2}-\frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4(cd^2-bde+ae^2)(ef-dg)} \\
&\quad -\frac{(cef-3cdg+beg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} \\
&\quad +\frac{(3(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4(cd^2-bde+ae^2)(ef-dg)}
\end{aligned}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)} - \frac{(3(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8\left(cd^2-bde+ae^2\right)^2(ef-dg)^2} - \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}}}}{2\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4}}{\sqrt{b^2-4}}}}{\sqrt{2}}\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(3(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)-}{(d+ex)\sqrt{f+}}\right.}{8\left(cd^2-bde+ae^2\right)^2(ef-dg)^2} + \frac{\left((cef-3cdg+beg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-}}}\right.}{\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(cd^2-bde+ae^2)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4(cd^2-bde+ae^2)^2(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(3cdg(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8(cd^2-bde+ae^2)^2(ef-dg)^2} + \frac{(3ceg(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8(cd^2-bde+ae^2)^2(ef-dg)^2} + \frac{(3(cd(2ef-3dg)-e(bef-2bdg+aeg))^2)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8(cd^2-bde+ae^2)^2(ef-dg)^2} + \frac{\left((cef-3cdg+beg)\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{b+\sqrt{}}}\right.}{(cd^2-bde+ae^2)(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)} - \frac{\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(3ce(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{8\left(cd^2-bde+ae^2\right)^2(ef-dg)^2} - \frac{(3cef(cd(2ef-3dg)-e(bef-2bdg+aeg)))\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{8\left(cd^2-bde+ae^2\right)^2(ef-dg)^2} + \frac{\left(3(cd(2ef-3dg)-e(bef-2bdg+aeg))^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int\frac{}{\sqrt{b-}}}{8\left(cd^2-bde+ae^2\right)^2(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{\left(3\sqrt{b^2-4ac}dg(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\right.}{2\sqrt{2}\left(cd^2-bde+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{a+bx+}} + \frac{\left((cef-3cdg+beg)\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{}{(ef-dg-ex^2)\sqrt{1+\frac{}{(b-}}}\right.}{\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= \text{Too large to display}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.30 (sec) , antiderivative size = 40396, normalized size of antiderivative = 36.26

$$\int\frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 1686, normalized size of antiderivative = 1.51

method	result	size
elliptic	Expression too large to display	1686
default	Expression too large to display	64947

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(1/2*e^2/(a
*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+b*g*x^2+c*
f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)^2+3/4*e^2*(a*e^2*g-2*b*d*e*g+b*e^2*f+3
*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*
f)^2*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e
^2*g+2*a*e^3*f-4*b*d^2*e*g+b*d*e^2*f+7*c*d^3*g-4*c*d^2*e*f)/(a*d*e^2*g-a*e^
3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*
x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2))-3/4*e*c*g*(a*e^2*g-2*b*d*e*g+b*e^2*f+3*c*d^2*g-
2*c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)^2*(f/g
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(
-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/4*(3*a^2*e^4*g^2-8*a*b*d*e^3*g^2
+2*a*b*e^4*f*g+6*a*c*d^2*e^2*g^2+4*a*c*d*e^3*f*g-4*a*c*e^4*f^2+8*b^2*d^2*e^
2*g^2-8*b^2*d*e^3*f*g+3*b^2*e^4*f^2-20*b*c*d^3*e*g^2+22*b*c*d^2*e^2*f*g-8*b
*c*d*e^3*f^2+15*c^2*d^4*g^2-20*c^2*d^3*e*f*g+8*c^2*d^2*e^2*f^2)/(a*d*e^2*g-
a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)^2/e*(f/g-1/2*(b+(-4*a*c+b^2)
^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^
3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/
g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.916 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 553

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}e\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^2\sqrt{a+bx+cx^2}}$$

```
[Out] 2*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(1/2)-g*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*
g+e*f)/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)-e*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*
g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2
))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(-d*g+e*f)^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {971, 758, 21, 732, 435, 948, 175, 552, 551}

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{\sqrt{2}e\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right.}{2c(ef-dg)}\right.}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)^2} + \frac{2g^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
 g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e*Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d
*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
```

```
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right) dx$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} + \frac{(2g)\int \frac{-\frac{cf}{2}-\frac{cgx}{2}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{\left(e\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+ex)\sqrt{f+gx}}\,dx}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\right)} - \frac{\left(2e\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{b+\cdots}}\cdots\right)}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$

$$- \frac{\left(\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2e\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{1}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{1+\cdots}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2e\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{1}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\cdots}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}e\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\cdots\right.}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^2\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.22 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\frac{g^2(a+x(b+cx))}{ef-dg} + \frac{(f+gx)^2\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}-\frac{i\sqrt{1 \cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*((g^2*(a + x*(b + c*x)))/(e*f - d*g) + ((f + g*x)^2*(c + (c*f^2)/(f + g*
x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b*g
)/(f + g*x) - ((I/4)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((e*f - d*g)*(2*c
*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f
^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x
]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2]))] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]) - 2*e*
(c*f^2 + g*(-(b*f) + a*g))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))] + 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticPi[((e*f - d*g)*(2*c*f - b*g -
 Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))]))/((e*f - d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/(-(e*f) + d*g)))
/((c*f^2 + g*(-(b*f) + a*g))*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(490) = 980$.

Time = 3.23 (sec) , antiderivative size = 1264, normalized size of antiderivative = 2.29

method	result	size
elliptic	Expression too large to display	1264
default	Expression too large to display	4757

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*g*x^
2+b*g*x+a*g)/(a*g^2-b*f*g+c*f^2)*g/(d*g-e*f)/((x+f/g)*(c*g*x^2+b*g*x+a*g))^
(1/2)+2*(-g*(b*g-c*f)/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)+b*g^2/(a*g^2-b*f*g+c*f^
2)/(d*g-e*f))*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1
/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*g
^2*c/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f
/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^
2+a*g*x+b*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((
x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b
^2)^(1/2))*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-
f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)))-2/(d*g-e*f)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2),(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^{\frac{3}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.917 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1125

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}}$$
$$+\frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,eg\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}e^2\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4a\right.}}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^3\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(3/2)+4/
3*g^2*(-b*g+2*c*f)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*
```

```
x+f)^(1/2)+2*e*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*
x+f)^(1/2)-2/3*g*(-b*g+2*c*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2
+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(c*x^2+b*x+a)^
(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*g*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a
*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)^2/(a*
g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/3*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-e^2*EllipticPi(2^(1/2)*c^(1/2)*
(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-
4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(
1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules

used = {971, 758, 848, 857, 732, 435, 430, 21, 948, 175, 552, 551}

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\text{EllipticPi}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^3\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)e}{(ef-dg)^2\left(cf^2-bgf+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{2g^2\sqrt{cx^2+bx+a}e}{(ef-dg)^2\left(cf^2-bgf+ag^2\right)\sqrt{f+gx}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3(ef-dg)\left(cf^2-bgf+ag^2\right)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right.}{3(ef-dg)\left(cf^2-bgf+ag^2\right)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{4g^2(2cf-bg)\sqrt{cx^2+bx+a}}{3(ef-dg)\left(cf^2-bgf+ag^2\right)^2\sqrt{f+gx}}+\frac{2g^2\sqrt{cx^2+bx+a}}{3(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)^{3/2}}$$

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*
f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/((e
*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((e*f - d*g)^2
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
```

```
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^3*Sqrt[a + b*x
+ c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a+bx+cx^2}} + \frac{e^2}{(ef-dg)^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right) dx$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} + \frac{(2eg)\int \frac{-\frac{cf}{2}-\frac{cgx}{2}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)} + \frac{(2g)\int \frac{\frac{1}{2}(-3cf+2bg)+\frac{cgx}{2}}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{3(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{\left(e^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+ex)\sqrt{f+gx}}\,dx}{(ef-dg)^2\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{(4g)\int \frac{\frac{1}{4}c\left(3cf^2-g(bf+ag)\right)+\frac{1}{2}cg(2cf-bg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2} - \frac{(ceg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)} - \frac{\left(2e^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{b-\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{b}}\right.}{(ef-dg)^2\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{(2cg(2cf-bg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2} + \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3(ef-dg)\left(cf^2-bfg+ag^2\right)} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}eg\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}} - \frac{\left(2e^2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{1}{(ef-dg-ex^2)\sqrt{b+\sqrt{b^2-4ac}-\frac{2cf}{g}+\frac{2cx^2}{g}}\sqrt{}}\right.}{(ef-dg)^2\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}}$$

$$+ \frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}\,eg\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}\,gx^2}{2cf-bg-\sqrt{b^2-4ac}\,g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-\cdots}}{\sqrt{b^2\cdots}}}}{\sqrt{2}}\cdots\right.}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}\,gx^2}{2cf-bg-\sqrt{b^2-4ac}\,g}}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{\cdots}}{\cdots}}}{\cdots}\right.}{3(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(2e^2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int\frac{1}{(ef-dg-ex^2)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\cdots}}\cdots\right.}{(ef-dg)^2\sqrt{a+bx+cx^2}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)(cf^2-bfg+ag^2)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)(cf^2-bfg+ag^2)^2\sqrt{f+gx}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(cf^2-bfg+ag^2)\sqrt{f+gx}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3(ef-dg)(cf^2-bfg+ag^2)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}eg\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{(ef-dg)^2(cf^2-bfg+ag^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)}\right.}{3(ef-dg)(cf^2-bfg+ag^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}e^2\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-\right.}}{2c(ef-dg)}\right.}{\sqrt{c}(ef-dg)^3\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.11 (sec) , antiderivative size = 14762, normalized size of antiderivative = 13.12

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 1505, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	1505
default	Expression too large to display	27601

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((g*x+f)*(c*x^2+b*x+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/(a*g^
2-b*f*g+c*f^2)/(d*g-e*f)*(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2)/(x
+f/g)^2+2/3*(c*g*x^2+b*g*x+a*g)/(a*g^2-b*f*g+c*f^2)^2*g*(3*a*e*g^2+2*b*d*g^
2-5*b*e*f*g-4*c*d*f*g+7*c*e*f^2)/(d*g-e*f)^2/((x+f/g)*(c*g*x^2+b*g*x+a*g))^
(1/2)+2*(-1/3*c*g/(a*g^2-b*f*g+c*f^2)/(d*g-e*f)+1/3*g*(b*g-c*f)*(3*a*e*g^2+
2*b*d*g^2-5*b*e*f*g-4*c*d*f*g+7*c*e*f^2)/(a*g^2-b*f*g+c*f^2)^2/(d*g-e*f)^2-
1/3*b*g^2/(a*g^2-b*f*g+c*f^2)^2*(3*a*e*g^2+2*b*d*g^2-5*b*e*f*g-4*c*d*f*g+7*
c*e*f^2)/(d*g-e*f)^2)*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/
g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x
+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((
-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2))-2/3*g^2*c*(3*a*e*g^2+2*b*d*g^2-5*b*e*f*g-4*c*d*f*g+7*c*e*f^2)/(a*g^2-b
*f*g+c*f^2)^2/(d*g-e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f
/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b
*f*x+a*f)^(1/2)*((-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+f/g)/(f
/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)*EllipticF(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-f/g+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*e/
(d*g-e*f)^2*(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+f/g)/(f/g-1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-f/g-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)^(1/2
)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-f/g+d/e),((-f/g+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\,(f+gx)^{\frac{5}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{5/2}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.918 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 475

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} \, dx = \frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}$$

```
[Out] (e*x+d)*EllipticPi((g*x+f)^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x+d)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2),e*(2*c*f-g*(b+(-4*a*c+b^2
)^(1/2)))/g/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))),((b*d-2*a*e+d*(-4*a*c+b^2)^(1/
2))*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2))/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(b+2*c*x-(-4*a*c+b^2)^(1/2))^(1
/2)*((-d*g+e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*x+d)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-d*g+e*f)*(2*a+x
*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2)))^(1/2)/g/
(c*x^2+b*x+a)^(1/2)/(c*x+2*a*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {940}

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+\cdots}{(d+ex)\left(f\sqrt{b^2-4ac}\cdots\right.}}}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\sqrt{a+b\cdots}}$$

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c]
 + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^2 -
 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)*Ell
ipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]
)/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d + Sqrt[b^
2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c]) + c*x]*Sqr
t[a + b*x + c*x^2])
```

<u>Rule 940</u>

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

integral

$$=\frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)eg}\sqrt{}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1118 vs. $2(475) = 950$.

Time = 28.87 (sec) , antiderivative size = 1118, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{-\frac{g(cf^2+g(-bf+ag))(d+ex)}{\left(-2cdfg-2aeg^2+ef\sqrt{(b^2-4ac)g^2}-dg\sqrt{(b^2-4ac)g^2}+bg(ef+dg)\right)(f+gx)}}(f+gx)^{3/2}\left(\frac{2ef\sqrt{(b^2-4ac)g^2}\sqrt{-\frac{(cf^2+g(-bf+a}{(b^2-4ac}}}{}\right.}{}$$

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-((g*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x))/((-2*c*d*f*g - 2
*a*e*g^2 + e*f*Sqrt[(b^2 - 4*a*c)*g^2] - d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*
(e*f + d*g))*(f + g*x)))]*(f + g*x)^(3/2)*((2*e*f*Sqrt[(b^2 - 4*a*c)*g^2]*S
qrt[-(((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*
x)^2))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[
(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]]
, (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*S
qrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(
c*f^2 + g*(-(b*f) + a*g)) + (d*g*(2*a*g^2 - f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x - g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))*Sqrt[(2*a*g^2 - 2*c*
f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*
a*c)*g^2]*(f + g*x))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f
- g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a
*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e
```

```
*f + d*g))])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)*Sqrt[(-2*a*g^2 + 2*c*f*g
*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)
*g^2]*(f + g*x))]) - (4*e*Sqrt[(b^2 - 4*a*c)*g^2]*Sqrt[-(((c*f^2 + g*(-(b*f
) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*x)^2))]*EllipticPi[(2*Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]), ArcSin[Sqrt
[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))
/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*
(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2])))/(g^2*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(420) = 840$.

Time = 5.12 (sec) , antiderivative size = 1463, normalized size of antiderivative = 3.08

method	result	size
elliptic	Expression too large to display	1463
default	Expression too large to display	10161

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e*
x+d)^(1/2)*(2*d*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-
d/e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(-d/e+f/g)/(x-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-
b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-d/e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+
d/e)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+d/e
))^(1/2))+2*e*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-d/
e+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(f/g-1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(-d/e+f/g)/(x-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+
(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(
x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*(1/2/c*(-b+(-4*a*c+b^2)^(1/2
))*EllipticF(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-d/e+f/g)/(x-1/
```

```
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*(-f/g+d/e)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(1/2/
c*(-b+(-4*a*c+b^2)^(1/2))+d/e))^(1/2))+(-f/g-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
*EllipticPi(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(-d/e+f/g)/(x-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2),(-d/e+f/g)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g
+d/e)/(-f/g+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+d/
e))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.919 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 588

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(a+bx+cx^2)}{(cf^2-bfg+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}\right)\sqrt{\frac{1-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bfg+ag^2\right)(d+ex)}+\cdots}{\left(1+\frac{\sqrt{cd^2-bde+ae^2}(\cdots}{\sqrt{cf^2-g(bf-ag)}(\cdots}\right.\cdots}}\cdots}{\sqrt[4]{cd^2-bde+ae^2}(ef-dg)\sqrt{a+bx+cx^2}\sqrt{1-\cdots}}$$

```
[Out] -(c*f^2-g*(-a*g+b*f))^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)
*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arc
tan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+
d)^(1/2)))*EllipticF(sin(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(
a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/2*(2+(2*c*d*f+2*a*e*g-b*(d*g+e*f
))/(c*d^2-e*(-a*e+b*d))^(1/2)/(c*f^2-g*(-a*g+b*f))^(1/2))^(1/2))*(1+(g*x+f)
*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))*((-d*g+e*f)^
2*(c*x^2+b*x+a)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^2)^(1/2)*((1-(2*c*d*f+2*a*e*g-b
*(d*g+e*f))*(g*x+f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)
^2/(c*f^2-g*(-a*g+b*f))/(e*x+d)^2)/(1+(g*x+f)*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*
x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/4)/(-d*g+e
*f)/(c*x^2+b*x+a)^(1/2)/(1-(2*c*d*f+2*a*e*g-b*(d*g+e*f))*(g*x+f)/(a*g^2-b*f
*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)^2/(c*f^2-g*(-a*g+b*f))/(e*x+d
)^2)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {949, 1117}

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2-bde+cd^2\right)}{(d+ex)^2\left(cf^2-g(bf-ag)\right)}-\frac{(f+gx)(2ae\cdots}{(d+ex)(\cdots}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag\cdots}}\cdots\right.}}}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+g\cdots}{(d+e\cdots}}}$$

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c
*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e + a*
e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c*d*
f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e*x))
 + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d + e*x)
^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*
g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)*Sqrt[f
 + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*d*f + 2*
a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g*(b*f - a
*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]
*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g +
 a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f
 - a*g))*(d + e*x)^2)]))
```

Rule 949

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2])), Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*e
*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 - b
*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

integral =

$$-\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2(a+bx+cx^2)}{(cf^2-bfg+ag^2)(d+ex)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{(2cdf-bef-bdg+2aeg)x^2}{cf^2-bfg+ag^2}+\frac{\left(cd^2-bde+ae^2\right)x^4}{cf^2-bfg+ag^2}}}dx,x,\frac{\sqrt{f+gx}}{\sqrt{d+ex}}\right)}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$=-\frac{\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(a+bx+cx^2)}{(cf^2-bfg+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}\right)\sqrt{\frac{1-\frac{(2cdf+2aeg-b(ef+dg)}{(cf^2-bfg+ag^2)(d\cdots}}{\left(1+\frac{\sqrt{cd^2\cdots}}{\sqrt{cf^2\cdots}}\cdots\right.}}\cdots}{\sqrt[4]{cd^2-bde+ae^2}(ef-dg)\sqrt{a+bx+cx^2}\sqrt{\cdots}}$$

Mathematica [A] (verified)

Time = 26.55 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.64

$$\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$=\frac{2\sqrt{2}e\sqrt{-\frac{e(cd^2+e(-bd+ae))(f+gx)}{\left(-2cdef+e\sqrt{(b^2-4ac)e^2}f-2ae^2g-d\sqrt{(b^2-4ac)e^2}g+be(ef+dg)\right)(d+ex)}}\sqrt{a+x(b+cx)}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2ae\cdots}{\cdots}}}{\cdots}\right.\right.}{\sqrt{(b^2-4ac)e^2}\sqrt{d+ex}\sqrt{f+gx}\sqrt{-\frac{(cd^2+\cdots}{\cdots}}}$$

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*e*Sqrt[-((e*(c*d^2 + e*(-(b*d) + a*e))*(f + g*x))/((-2*c*d*e*f +
 e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g + b*
e*(e*f + d*g))*(d + e*x)))]*Sqrt[a + x*(b + c*x)]*EllipticF[ArcSin[Sqrt[(2*
a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/(Sq
rt[(b^2 - 4*a*c)*e^2]*(d + e*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f
 - d*g))/(-2*c*d*e*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^
2 - 4*a*c)*e^2]*g + b*e*(e*f + d*g))])/(Sqrt[(b^2 - 4*a*c)*e^2]*Sqrt[d + e*
x]*Sqrt[f + g*x]*Sqrt[-(((c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x)))/((b^
2 - 4*a*c)*(d + e*x)^2))]])
```

Maple [A] (warning: unable to verify)

Time = 6.06 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.02

method	result
default	$\frac{8\left(-2c^2dg\,x^2+2c^2ef\,x^2+2\sqrt{-4ac+b^2}\,cdgx-2\sqrt{-4ac+b^2}\,cefx-2bcdgx+2bcefx+\sqrt{-4ac+b^2}\,bdg-\sqrt{-4ac+b^2}\,bef+2acdg-2acef-b^2d}{\sqrt{-\frac{(gx+f)\left(-b-2}{}}}$
elliptic	$\frac{2\sqrt{(gx+f)(cx^2+bx+a)(ex+d)}\left(-\frac{f}{g}+\frac{d}{e}\right)\sqrt{\frac{\left(-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)\left(x+\frac{f}{g}\right)}{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}\left(x-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2\sqrt{\frac{\left(\frac{-b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\left(x+\frac{b+\sqrt{-4a}}{2c}\right)}{\left(\frac{f}{g}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\left(x-\frac{-b+\sqrt{-4a}}{2c}\right)}}}{\sqrt{gx+f}\,\sqrt{cx^2+bx+a}\,\sqrt{ex+d}\left(-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)\left(\frac{-b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right.}$

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 8*(-2*c^2*d*g*x^2+2*c^2*e*f*x^2+2*(-4*a*c+b^2)^(1/2)*c*d*g*x-2*(-4*a*c+b^2)
^(1/2)*c*e*f*x-2*b*c*d*g*x+2*b*c*e*f*x+(-4*a*c+b^2)^(1/2)*b*d*g-(-4*a*c+b^2
)^(1/2)*b*e*f+2*a*c*d*g-2*a*c*e*f-b^2*d*g+b^2*e*f)*EllipticF((-(e*(-4*a*c+b
^2)^(1/2)-b*e+2*c*d)*(g*x+f)/(d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)
,2*((-4*a*c+b^2)^(1/2)*(d*g-e*f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*(-4*
a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*((2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))*(e*x+d)
/(d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*f-b*g+g*(-4*a*c+b^2)^
(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(-b-2*
c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*(-(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)*(g*x+f)/(
d*g-e*f)/(-b-2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(
1/2)*(e*x+d)^(1/2)/(-1/c*(g*x+f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*(b+2*c*x+(-4
*a*c+b^2)^(1/2))*(e*x+d))^(1/2)/(b*g-2*c*f-g*(-4*a*c+b^2)^(1/2))/(-e*(-4*a*
c+b^2)^(1/2)+b*e-2*c*d)/((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*
e*f + (c*d + b*e)*g)*x^3 + a*d*f + ((c*d + b*e)*f + (b*d + a*e)*g)*x^2 + (a
*d*g + (b*d + a*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="m
axima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="g
iac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.920 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx &= \frac{(cd^2-bde+ae^2)\,(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}\\ &\quad-\frac{(ef-dg)(2cd(ef-2dg)-e(bef-3bdg+2aeg))(d+ex)^{2+m}}{e^5(2+m)}\\ &\quad+\frac{(eg(2bef-3bdg+aeg)+c(e^2f^2-6defg+6d^2g^2))\,(d+ex)^{3+m}}{e^5(3+m)}\\ &\quad+\frac{g(2cef-4cdg+beg)(d+ex)^{4+m}}{e^5(4+m)}+\frac{cg^2(d+ex)^{5+m}}{e^5(5+m)}\end{aligned}$$

```
[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^5/(1+m)-(-d*g+e*f)*(2*c*d*
(-2*d*g+e*f)-e*(2*a*e*g-3*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^5/(2+m)+(e*g*(a*e*g
-3*b*d*g+2*b*e*f)+c*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))*(e*x+d)^(3+m)/e^5/(3+m)+
g*(b*e*g-4*c*d*g+2*c*e*f)*(e*x+d)^(4+m)/e^5/(4+m)+c*g^2*(e*x+d)^(5+m)/e^5/(
5+m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used

$= \{961\}$

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) \, dx$$
$$= \frac{(d+ex)^{m+3} \left(eg(aeg-3bdg+2bef)+c(6d^2g^2-6defg+e^2f^2)\right)}{e^5(m+3)} + \frac{(ef-dg)^2(d+ex)^{m+1} \left(ae^2-bde+cd^2\right)}{e^5(m+1)} - \frac{(ef-dg)(d+ex)^{m+2}(2cd(ef-2dg)-e(2aeg-3bdg+bef))}{e^5(m+2)} + \frac{g(d+ex)^{m+4}(beg-4cdg+2cef)}{e^5(m+4)} + \frac{cg^2(d+ex)^{m+5}}{e^5(m+5)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\text{integral} = \int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^m}{e^4} + \frac{(ef-dg)(-2cd(ef-2dg)+e(bef-3bdg+2aeg))(d+ex)^{1+m}}{e^4} + \frac{\left(eg(2bef-3bdg+aeg)+c(e^2f^2-6defg+6d^2g^2)\right)(d+ex)^{2+m}}{e^4} + \frac{g(2cef-4cdg+beg)(d+ex)^{3+m}}{e^4} + \frac{cg^2(d+ex)^{4+m}}{e^4} \right) dx$$

$$= \frac{(cd^2 - bde + ae^2)(ef - dg)^2(d + ex)^{1+m}}{e^5(1 + m)} - \frac{(ef - dg)(2cd(ef - 2dg) - e(bef - 3bdg + 2aeg))(d + ex)^{2+m}}{e^5(2 + m)} + \frac{(eg(2bef - 3bdg + aeg) + c(e^2f^2 - 6defg + 6d^2g^2))(d + ex)^{3+m}}{e^5(3 + m)} + \frac{g(2cef - 4cdg + beg)(d + ex)^{4+m}}{e^5(4 + m)} + \frac{cg^2(d + ex)^{5+m}}{e^5(5 + m)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.90

$$\int (d + ex)^m (f + gx)^2 \left(a + bx + cx^2\right) dx = \frac{(d + ex)^{1+m} \left(\frac{(cd^2+e(-bd+ae))(ef-dg)^2}{1+m} + \frac{(ef-dg)(2cd(-ef+2dg)+e(bef-3bdg+2aeg))(d+ex)}{2+m} + \frac{(eg(2bef-3bdg+aeg)+c(e^2f^2-6de}{3+m}\right.}{e^5}$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2)/(1 + m) + ((
e*f - d*g)*(2*c*d*(-(e*f) + 2*d*g) + e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*
x))/(2 + m) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g +
6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)
^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(220) = 440.

Time = 0.56 (sec) , antiderivative size = 1249, normalized size of antiderivative = 5.68

method	result	size
norman	Expression too large to display	1249
gosper	Expression too large to display	1347
risch	Expression too large to display	1823
parallelrisch	Expression too large to display	2719

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*g^2/(5+m)*x^5*exp(m*ln(e*x+d))+d*(a*e^4*f^2*m^4-2*a*d*e^3*f*g*m^3+14*a*e^
4*f^2*m^3-b*d*e^3*f^2*m^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2+71*a*e^4*f
^2*m^2+4*b*d^2*e^2*f*g*m^2-12*b*d*e^3*f^2*m^2+2*c*d^2*e^2*f^2*m^2+18*a*d^2*
e^2*g^2*m-94*a*d*e^3*f*g*m+154*a*e^4*f^2*m-6*b*d^3*e*g^2*m+36*b*d^2*e^2*f*g
```

```
*m-47*b*d*e^3*f^2*m-12*c*d^3*e*f*g*m+18*c*d^2*e^2*f^2*m+40*a*d^2*e^2*g^2-12
0*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+80*b*d^2*e^2*f*g-60*b*d*e^3*f^2+
24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^2)/e^5/(m^5+15*m^4+85*m^3+225*m^
2+274*m+120)*exp(m*ln(e*x+d))+(a*e^2*g^2*m^2+b*d*e*g^2*m^2+2*b*e^2*f*g*m^2+
2*c*d*e*f*g*m^2+c*e^2*f^2*m^2+9*a*e^2*g^2*m+5*b*d*e*g^2*m+18*b*e^2*f*g*m-4*
c*d^2*g^2*m+10*c*d*e*f*g*m+9*c*e^2*f^2*m+20*a*e^2*g^2+40*b*e^2*f*g+20*c*e^2
*f^2)/e^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+(a*d*e^2*g^2*m^3+2*a*e^
3*f*g*m^3+2*b*d*e^2*f*g*m^3+b*e^3*f^2*m^3+c*d*e^2*f^2*m^3+9*a*d*e^2*g^2*m^2
+24*a*e^3*f*g*m^2-3*b*d^2*e*g^2*m^2+18*b*d*e^2*f*g*m^2+12*b*e^3*f^2*m^2-6*c
*d^2*e*f*g*m^2+9*c*d*e^2*f^2*m^2+20*a*d*e^2*g^2*m+94*a*e^3*f*g*m-15*b*d^2*e
*g^2*m+40*b*d*e^2*f*g*m+47*b*e^3*f^2*m+12*c*d^3*g^2*m-30*c*d^2*e*f*g*m+20*c
*d*e^2*f^2*m+120*a*e^3*f*g+60*b*e^3*f^2)/e^3/(m^4+14*m^3+71*m^2+154*m+120)*
x^2*exp(m*ln(e*x+d))+(b*e*g*m+c*d*g*m+2*c*e*f*m+5*b*e*g+10*c*e*f)/e*g/(m^2+
9*m+20)*x^4*exp(m*ln(e*x+d))-(-2*a*d*e^3*f*g*m^4-a*e^4*f^2*m^4-b*d*e^3*f^2*
m^4+2*a*d^2*e^2*g^2*m^3-24*a*d*e^3*f*g*m^3-14*a*e^4*f^2*m^3+4*b*d^2*e^2*f*g
*m^3-12*b*d*e^3*f^2*m^3+2*c*d^2*e^2*f^2*m^3+18*a*d^2*e^2*g^2*m^2-94*a*d*e^3
*f*g*m^2-71*a*e^4*f^2*m^2-6*b*d^3*e*g^2*m^2+36*b*d^2*e^2*f*g*m^2-47*b*d*e^3
*f^2*m^2-12*c*d^3*e*f*g*m^2+18*c*d^2*e^2*f^2*m^2+40*a*d^2*e^2*g^2*m-120*a*d
*e^3*f*g*m-154*a*e^4*f^2*m-30*b*d^3*e*g^2*m+80*b*d^2*e^2*f*g*m-60*b*d*e^3*f
^2*m+24*c*d^4*g^2*m-60*c*d^3*e*f*g*m+40*c*d^2*e^2*f^2*m-120*a*e^4*f^2)/e^4/
(m^5+15*m^4+85*m^3+225*m^2+274*m+120)*x*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(220) = 440$.

Time = 0.33 (sec) , antiderivative size = 1381, normalized size of antiderivative = 6.28

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 + 5
0*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*b*e^5*g^2 + (2*c*e^5
*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5*f*g + (6*c*d*e^4 + 11*b*e^5)*
g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^5*f*
g + (6*c*d*e^4 + 61*b*e^5)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3 - 14
*a*d*e^4)*f^2)*m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c*e^5*f
^2 + 2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e^5*f^2
+ 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*g^2)*m^3
+ (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g - (12*c*d^2*e^3 - 17*b*d*e^
4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 2*(5*c*d*e^4 + 39*b*e^5)*f*g - (
4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^2 - 3*b*d^2
*e^3 + 6*a*d*e^4)*f^2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e^3)*f*g + 2*
(12*c*d^5 - 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2 + (2*c*d^3*e^
```

```
2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e^3)*f*g)*m^2 +
 (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^4 + b*e^5)*f^2 + 2*(
b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13*b*e^5)*f^2 - 2*(3*c*d^2*e^3 -
 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*d^2*e^3 - 10*a*d*e^4)*g^2)*m^3 + ((29*c*
d*e^4 + 59*b*e^5)*f^2 - 2*(18*c*d^2*e^3 - 29*b*d*e^4 - 59*a*e^5)*f*g + (12*
c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c*d*e^4 + 107*b*e^5)
*f^2 - 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g + (12*c*d^3*e^2 - 15*b
*d^2*e^3 + 20*a*d*e^4)*g^2)*m)*x^2 + ((18*c*d^3*e^2 - 47*b*d^2*e^3 + 154*a*
d*e^4)*f^2 - 2*(6*c*d^4*e - 18*b*d^3*e^2 + 47*a*d^2*e^3)*f*g - 6*(b*d^4*e -
 3*a*d^3*e^2)*g^2)*m + (120*a*e^5*f^2 + (2*a*d*e^4*f*g + (b*d*e^4 + a*e^5)*
f^2)*m^4 - 2*(a*d^2*e^3*g^2 + (c*d^2*e^3 - 6*b*d*e^4 - 7*a*e^5)*f^2 + 2*(b*
d^2*e^3 - 6*a*d*e^4)*f*g)*m^3 - ((18*c*d^2*e^3 - 47*b*d*e^4 - 71*a*e^5)*f^2
 - 2*(6*c*d^3*e^2 - 18*b*d^2*e^3 + 47*a*d*e^4)*f*g - 6*(b*d^3*e^2 - 3*a*d^2
*e^3)*g^2)*m^2 - 2*((20*c*d^2*e^3 - 30*b*d*e^4 - 77*a*e^5)*f^2 - 10*(3*c*d^
3*e^2 - 4*b*d^2*e^3 + 6*a*d*e^4)*f*g + (12*c*d^4*e - 15*b*d^3*e^2 + 20*a*d^
2*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*
m^2 + 274*e^5*m + 120*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15757 vs. 2(212) = 424.

Time = 2.87 (sec) , antiderivative size = 15757, normalized size of antiderivative = 71.62

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2*
b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5/5
), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*a*d*e**3*f*g/(12*d**4*e**5 +
48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*a*d
*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d
**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*a*e**4*f*g*x/(12*d**4*e*
*5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) -
6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*
d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e*g**2/(12*d**4*e**5 + 48*d**3*e**6*
x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*b*d**2*e**2*f*g/
(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e*
*9*x**4) - 12*b*d**2*e**2*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - b*d*e**3*f**2/(12*d**4*e**5 + 4
8*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*b*d*
e**3*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
```

```
**3 + 12*e**9*x**4) - 18*b*d*e**3*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*b*e**4*f**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - 12*b*e**4*f*g*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**
2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*b*e**4*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*d*
*4*g**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4*g**2/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*d**3*e*f*g/(1
2*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 48*c*d**3*e*g**2*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*g**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - c*d**2*e**2*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) - 24*c*d**2*e**2*f*g*x/(12*d**4*e**5 + 48*
d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 72*c*d**
2*e**2*g**2*x**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7
*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 108*c*d**2*e**2*g**2*x**2/(12*d**4
*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
 - 4*c*d*e**3*f**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) - 36*c*d*e**3*f*g*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3
*g**2*x**3*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*e**
4*f**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*
x**3 + 12*e**9*x**4) - 24*c*e**4*f*g*x**3/(12*d**4*e**5 + 48*d**3*e**6*x +
72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*e**4*g**2*x**4*lo
g(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
**3 + 12*e**9*x**4), Eq(m, -5)), (-2*a*d**2*e**2*g**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*d*e**3*f*g/(6*d**3*e**5 + 18
*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*d*e**3*g**2*x/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*e**4*f**2/(6*d**3
*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*f*g*x/(6*
d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*g**2*
x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*d*
*3*e*g**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e
**8*x**3) + 11*b*d**3*e*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2
 + 6*e**8*x**3) - 4*b*d**2*e**2*f*g/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d**3*e**5 +
18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 27*b*d**2*e**2*g**2*x/(6*d
**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - b*d*e**3*f**2/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*d*e**3*
f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 18*b*
d*e**3*g**2*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
```

2 + 6*e**8*x**3) + 18*b*d*e**3*g**2*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18
*d*e**7*x**2 + 6*e**8*x**3) - 3*b*e**4*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*e**4*f*g*x**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*e**4*g**2*x**3*log(d/e + x)/
(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d**4*g
2*log(d/e + x)/(6*d3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x*
*3) - 44*c*d**4*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**
8*x**3) + 12*c*d**3*e*f*g*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d
*e**7*x**2 + 6*e**8*x**3) + 22*c*d**3*e*f*g/(6*d**3*e**5 + 18*d**2*e**6*x +
 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**3*e*g**2*x*log(d/e + x)/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 108*c*d**3*e*g**2*x/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*c*d**2*e**
2*f**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 36*c
*d**2*e**2*f*g*x*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 54*c*d**2*e**2*f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*
d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2*x**2*log(d/e + x)/(6*d**3*
e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2
*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*d
*e**3*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3)
 + 36*c*d*e**3*f*g*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 36*c*d*e**3*f*g*x**2/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d*e**3*g**2*x**3*log(d/e + x)/(6*d*
*3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*e**4*f**2*x*
*2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 12*c*e**
4*f*g*x**3*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*
e**8*x**3) + 6*c*e**4*g**2*x**4/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x
2 + 6*e8*x**3), Eq(m, -4)), (2*a*d**2*e**2*g**2*log(d/e + x)/(2*d**2*e*
*5 + 4*d*e**6*x + 2*e**7*x**2) + 3*a*d**2*e**2*g**2/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) - 2*a*d*e**3*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*a*d*e**3*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*a*d*e**3*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - a*e**4*f**2/
(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*a*e**4*f*g*x/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) + 2*a*e**4*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) - 6*b*d**3*e*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e
6*x + 2*e7*x**2) - 9*b*d**3*e*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
2) + 4*b*d2*e**2*f*g*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) + 6*b*d**2*e**2*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*d**
2*e**2*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*
d**2*e**2*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*d*e**3*f**2/(
2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x*log(d/e + x)/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) - 6*b*d*e**3*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2) - 2*b*e**4*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e*
*7*x**2) + 4*b*e**4*f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**
7*x**2) + 2*b*e**4*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*

c*d**4*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 18*c*d*
*4*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d**3*e*f*g*log(d/e
+ x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 18*c*d**3*e*f*g/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x*log(d/e + x)/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 2*c*d**2*e**2*f**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 3*c*d**2*e**2*f**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) - 24*c*d**2*e**2*f*g*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*
x**2) - 24*c*d**2*e**2*f*g*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*
c*d**2*e**2*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*c*d*e**3*f**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 4*c*d*e**3*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d*e**3*
f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c*d*e**3
*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*c*e**4*f**2*x**2*lo
g(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*c*e**4*f*g*x**3/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c*e**4*g**2*x**4/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-12*a*d**2*e**2*g**2*log(d/e + x)/(6*d
*e**5 + 6*e**6*x) - 12*a*d**2*e**2*g**2/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3
*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*g/(6*d*e**5 + 6*e**
6*x) - 12*a*d*e**3*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 6*a*e**4*f**
2/(6*d*e**5 + 6*e**6*x) + 12*a*e**4*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x
) + 6*a*e**4*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2/(6*d*e**5 + 6*e**6*x) - 24*b*d**
2*e**2*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2*f*g/(6*d*e**
5 + 6*e**6*x) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) +
6*b*d*e**3*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*b*d*e**3*f**2/(6*d*e
5 + 6*e6*x) - 24*b*d*e**3*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 9*
b*d*e**3*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*b*e**4*f**2*x*log(d/e + x)/(6*
d*e**5 + 6*e**6*x) + 12*b*e**4*f*g*x**2/(6*d*e**5 + 6*e**6*x) + 3*b*e**4*g*
*2*x**3/(6*d*e**5 + 6*e**6*x) - 24*c*d**4*g**2*log(d/e + x)/(6*d*e**5 + 6*e
6*x) - 24*c*d4*g**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g/(6*d*e**5 + 6*e**6*x) - 24*c*d**3
*e*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2*log(d/e
+ x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2/(6*d*e**5 + 6*e**6*x) + 36
*c*d**2*e**2*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*c*d**2*e**2*g**2
*x**2/(6*d*e**5 + 6*e**6*x) - 12*c*d*e**3*f**2*x*log(d/e + x)/(6*d*e**5 + 6
*e**6*x) - 18*c*d*e**3*f*g*x**2/(6*d*e**5 + 6*e**6*x) - 4*c*d*e**3*g**2*x**
3/(6*d*e**5 + 6*e**6*x) + 6*c*e**4*f**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*c*e*
*4*f*g*x**3/(6*d*e**5 + 6*e**6*x) + 2*c*e**4*g**2*x**4/(6*d*e**5 + 6*e**6*x
), Eq(m, -2)), (a*d**2*g**2*log(d/e + x)/e**3 - 2*a*d*f*g*log(d/e + x)/e**2
 - a*d*g**2*x/e**2 + a*f**2*log(d/e + x)/e + 2*a*f*g*x/e + a*g**2*x**2/(2*e
) - b*d**3*g**2*log(d/e + x)/e**4 + 2*b*d**2*f*g*log(d/e + x)/e**3 + b*d**2
*g**2*x/e**3 - b*d*f**2*log(d/e + x)/e**2 - 2*b*d*f*g*x/e**2 - b*d*g**2*x**
2/(2*e**2) + b*f**2*x/e + b*f*g*x**2/e + b*g**2*x**3/(3*e) + c*d**4*g**2*lo
g(d/e + x)/e**5 - 2*c*d**3*f*g*log(d/e + x)/e**4 - c*d**3*g**2*x/e**4 + c*d

2*f2*log(d/e + x)/e**3 + 2*c*d**2*f*g*x/e**3 + c*d**2*g**2*x**2/(2*e**3
) - c*d*f**2*x/e**2 - c*d*f*g*x**2/e**2 - c*d*g**2*x**3/(3*e**2) + c*f**2*x
2/(2*e) + 2*c*f*g*x3/(3*e) + c*g**2*x**4/(4*e), Eq(m, -1)), (2*a*d**3*e
2*g2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
5*m2 + 274*e**5*m + 120*e**5) + 18*a*d**3*e**2*g**2*m*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 40*a*d**3*e**2*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
3 + 225*e5*m**2 + 274*e**5*m + 120*e**5) - 2*a*d**2*e**3*f*g*m**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) - 24*a*d**2*e**3*f*g*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*d**2*e*
*3*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
2 + 274*e5*m + 120*e**5) - 120*a*d**2*e**3*f*g*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*
a*d**2*e**3*g**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 18*a*d**2*e**3*g**2*m**2*x*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
5*m + 120*e5) - 40*a*d**2*e**3*g**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*d*e**4*
f**2*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 14*a*d*e**4*f**2*m**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 71*a*d*e**4*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a*d*e**4*f**2*m*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 120*a*d*e**4*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
5*m3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*d*e**4*f*g*m**4*x*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*a*d*e**4*f*g*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 94*a*d*e*
*4*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
5*m2 + 274*e**5*m + 120*e**5) + 120*a*d*e**4*f*g*m*x*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + a*d*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*a*d*e**4*g**2*m**3*x**
2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) + 29*a*d*e**4*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20
*a*d*e**4*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*e**5*f**2*m**4*x*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) + 14*a*e**5*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a*e**5*f**2*m**2*x
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 154*a*e**5*f**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**

5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5
*f**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 2*a*e**5*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
26*a*e**5*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 118*a*e**5*f*g*m**2*x**2*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 214*a*e**5*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
4 + 85*e5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5*f*
g*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) + a*e**5*g**2*m**4*x**3*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12
*a*e**5*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*a*e**5*g**2*m**2*x**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 78*a*e**5*g**2*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*e**5*g**2
*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
 + 274*e**5*m + 120*e**5) - 6*b*d**4*e*g**2*m*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*b*d*
*4*e*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 4*b*d**3*e**2*f*g*m**2*(d + e*x)**m/(e**5*m
5 + 15*e5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 36*b*d**3*e**2*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*b*d**3*e**2*f*g*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + 6*b*d**3*e**2*g**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 30*b*d**3*e**2*g
2*m*x*(d + e*x)m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) - b*d**2*e**3*f**2*m**3*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 1
2*b*d**2*e**3*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 47*b*d**2*e**3*f**2*m*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
 + 120*e**5) - 60*b*d**2*e**3*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*b*d**2*e**3*f*g*
m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) - 36*b*d**2*e**3*f*g*m**2*x*(d + e*x)**m/(e**5*m
5 + 15*e5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 80*b*d**2*e**3*f*g*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
3 + 225*e5*m**2 + 274*e**5*m + 120*e**5) - 3*b*d**2*e**3*g**2*m**3*x**2
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) - 18*b*d**2*e**3*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) -
15*b*d**2*e**3*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5

*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*f**2*m**4*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*b*d*e**4*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 47*b*d*e**4*f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*b*d*e**4*f**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*b*d*e**4*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*b*d*e**4*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 58*b*d*e**4*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b*d*e**4*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*g**2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 8*b*d*e**4*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 17*b*d*e**4*g**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*b*d*e**4*g**2*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*e**5*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 13*b*e**5*f**2*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 59*b*e**5*f**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 107*b*e**5*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*b*e**5*f**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*b*e**5*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*b*e**5*f*g*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 98*b*e**5*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 156*b*e**5*f*g*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*b*e**5*f*g*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*e**5*g**2*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*b*e**5*g**2*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 41*b*e**5*g**2*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 61*b*e**5*g**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 30*b*e**5*g**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*c*d**5*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*

e**5*m**2 + 274*e**5*m + 120*e**5) - 12*c*d**4*e*f*g*m*(d + e*x)**m/(e**5*m
5 + 15*e5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 60*c*d**4*e*f*g*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
25*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*c*d**4*e*g**2*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 2*c*d**3*e**2*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 18*c*d**3*e**2*f**2*m
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 40*c*d**3*e**2*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*
e**2*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*c*d**3*e**2*f*g*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 12*c*d**3*e**2*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*e**2*
g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) - 2*c*d**2*e**3*f**2*m**3*x*(d + e*x)**m/(e
5*m5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
5) - 18*c*d2*e**3*f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 40*c*d**2*e**3*f**2
*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) - 6*c*d**2*e**3*f*g*m**3*x**2*(d + e*x)**m/(e**5*m
5 + 15*e5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 36*c*d**2*e**3*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*c*d**2*e**3*f*g*m*x
2*(d + e*x)m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) - 4*c*d**2*e**3*g**2*m**3*x**3*(d + e*x)**m/(e**5*m
5 + 15*e5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 12*c*d**2*e**3*g**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*c*d**2*e**3*g**2*m*
x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + c*d*e**4*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10
*c*d*e**4*f**2*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
3 + 225*e5*m**2 + 274*e**5*m + 120*e**5) + 29*c*d*e**4*f**2*m**2*x**2*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 20*c*d*e**4*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
5*m4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*d*e*
*4*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
5*e**5*m**2 + 274*e**5*m + 120*e**5) + 16*c*d*e**4*f*g*m**3*x**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 34*c*d*e**4*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*c*d*e**4*f*g
*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + c*d*e**4*g**2*m**4*x**4*(d + e*x)**m/(e**5*m*

```
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 6*c*d*e**4*g**2*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c*d*e**4*g**2*m**2*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 6*c*d*e**4*g**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*
f**2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*e**5*f**2*m**3*x**3*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 49*c*e**5*f**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 78*c*e**5*f**2*m*x*
*3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 40*c*e**5*f**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*e**5
*f*g*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 22*c*e**5*f*g*m**3*x**4*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 82*c*e**5*f*g*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 122*c*e**5*f*g*m*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 60*c*e**5*f*g*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*g**2
*m**4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 10*c*e**5*g**2*m**3*x**5*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 35*c*e**5*g**2*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c*e**5*g**2*m*x**5*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*c*e**5*g**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(220) = 440$.

Time = 0.24 (sec) , antiderivative size = 684, normalized size of antiderivative = 3.11

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx = \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m bf^2}{(m^2+3m+2)e^2} + \frac{2\,(e^2(m+1)x^2+demx-d^2)(ex+d)^m afg}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}af^2}{e(m+1)} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m cf^2}{(m^3+6m^2+11m+6)e^3} + \frac{2\,((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m bfg}{(m^3+6m^2+11m+6)e^3} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m ag^2}{(m^3+6m^2+11m+6)e^3} + \frac{2\,((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex\cdots}{(m^4+10m^3+35m^2+50m+24)e^4} + \frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex+\cdots}{(m^4+10m^3+35m^2+50m+24)e^4} + \frac{((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4\,(m^3+3m^2+2m)d^2e\cdots}{(m^5+15m^4+85m^3+225m^2+274m+120\cdots}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f^2/((m^2 + 3*m + 2)*e^2) +
 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*f*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f^2/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m +
2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*g^2/(
(m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3
+ 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(
e*x + d)^m*c*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^2
+ 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x
^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 +
11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*
d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c*g^2/((m^5 + 15*m^4 + 85*
m^3 + 225*m^2 + 274*m + 120)*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2735 vs. 2(220) = 440.

Time = 0.29 (sec) , antiderivative size = 2735, normalized size of antiderivative = 12.43

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((e*x + d)^m*c*e^5*g^2*m^4*x^5 + 2*(e*x + d)^m*c*e^5*f*g*m^4*x^4 + (e*x + d
)^m*c*d*e^4*g^2*m^4*x^4 + (e*x + d)^m*b*e^5*g^2*m^4*x^4 + 10*(e*x + d)^m*c*
e^5*g^2*m^3*x^5 + (e*x + d)^m*c*e^5*f^2*m^4*x^3 + 2*(e*x + d)^m*c*d*e^4*f*g
*m^4*x^3 + 2*(e*x + d)^m*b*e^5*f*g*m^4*x^3 + (e*x + d)^m*b*d*e^4*g^2*m^4*x^
3 + (e*x + d)^m*a*e^5*g^2*m^4*x^3 + 22*(e*x + d)^m*c*e^5*f*g*m^3*x^4 + 6*(e
*x + d)^m*c*d*e^4*g^2*m^3*x^4 + 11*(e*x + d)^m*b*e^5*g^2*m^3*x^4 + 35*(e*x
+ d)^m*c*e^5*g^2*m^2*x^5 + (e*x + d)^m*c*d*e^4*f^2*m^4*x^2 + (e*x + d)^m*b*
e^5*f^2*m^4*x^2 + 2*(e*x + d)^m*b*d*e^4*f*g*m^4*x^2 + 2*(e*x + d)^m*a*e^5*f
*g*m^4*x^2 + (e*x + d)^m*a*d*e^4*g^2*m^4*x^2 + 12*(e*x + d)^m*c*e^5*f^2*m^3
*x^3 + 16*(e*x + d)^m*c*d*e^4*f*g*m^3*x^3 + 24*(e*x + d)^m*b*e^5*f*g*m^3*x^
3 - 4*(e*x + d)^m*c*d^2*e^3*g^2*m^3*x^3 + 8*(e*x + d)^m*b*d*e^4*g^2*m^3*x^3
 + 12*(e*x + d)^m*a*e^5*g^2*m^3*x^3 + 82*(e*x + d)^m*c*e^5*f*g*m^2*x^4 + 11
*(e*x + d)^m*c*d*e^4*g^2*m^2*x^4 + 41*(e*x + d)^m*b*e^5*g^2*m^2*x^4 + 50*(e
*x + d)^m*c*e^5*g^2*m*x^5 + (e*x + d)^m*b*d*e^4*f^2*m^4*x + (e*x + d)^m*a*e
^5*f^2*m^4*x + 2*(e*x + d)^m*a*d*e^4*f*g*m^4*x + 10*(e*x + d)^m*c*d*e^4*f^2
*m^3*x^2 + 13*(e*x + d)^m*b*e^5*f^2*m^3*x^2 - 6*(e*x + d)^m*c*d^2*e^3*f*g*m
^3*x^2 + 20*(e*x + d)^m*b*d*e^4*f*g*m^3*x^2 + 26*(e*x + d)^m*a*e^5*f*g*m^3*
x^2 - 3*(e*x + d)^m*b*d^2*e^3*g^2*m^3*x^2 + 10*(e*x + d)^m*a*d*e^4*g^2*m^3*
x^2 + 49*(e*x + d)^m*c*e^5*f^2*m^2*x^3 + 34*(e*x + d)^m*c*d*e^4*f*g*m^2*x^3
 + 98*(e*x + d)^m*b*e^5*f*g*m^2*x^3 - 12*(e*x + d)^m*c*d^2*e^3*g^2*m^2*x^3
+ 17*(e*x + d)^m*b*d*e^4*g^2*m^2*x^3 + 49*(e*x + d)^m*a*e^5*g^2*m^2*x^3 + 1
22*(e*x + d)^m*c*e^5*f*g*m*x^4 + 6*(e*x + d)^m*c*d*e^4*g^2*m*x^4 + 61*(e*x
+ d)^m*b*e^5*g^2*m*x^4 + 24*(e*x + d)^m*c*e^5*g^2*x^5 + (e*x + d)^m*a*d*e^4
*f^2*m^4 - 2*(e*x + d)^m*c*d^2*e^3*f^2*m^3*x + 12*(e*x + d)^m*b*d*e^4*f^2*m
^3*x + 14*(e*x + d)^m*a*e^5*f^2*m^3*x - 4*(e*x + d)^m*b*d^2*e^3*f*g*m^3*x +
 24*(e*x + d)^m*a*d*e^4*f*g*m^3*x - 2*(e*x + d)^m*a*d^2*e^3*g^2*m^3*x + 29*
(e*x + d)^m*c*d*e^4*f^2*m^2*x^2 + 59*(e*x + d)^m*b*e^5*f^2*m^2*x^2 - 36*(e*
x + d)^m*c*d^2*e^3*f*g*m^2*x^2 + 58*(e*x + d)^m*b*d*e^4*f*g*m^2*x^2 + 118*(
e*x + d)^m*a*e^5*f*g*m^2*x^2 + 12*(e*x + d)^m*c*d^3*e^2*g^2*m^2*x^2 - 18*(e
*x + d)^m*b*d^2*e^3*g^2*m^2*x^2 + 29*(e*x + d)^m*a*d*e^4*g^2*m^2*x^2 + 78*(
e*x + d)^m*c*e^5*f^2*m*x^3 + 20*(e*x + d)^m*c*d*e^4*f*g*m*x^3 + 156*(e*x +
d)^m*b*e^5*f*g*m*x^3 - 8*(e*x + d)^m*c*d^2*e^3*g^2*m*x^3 + 10*(e*x + d)^m*b
*d*e^4*g^2*m*x^3 + 78*(e*x + d)^m*a*e^5*g^2*m*x^3 + 60*(e*x + d)^m*c*e^5*f*
g*x^4 + 30*(e*x + d)^m*b*e^5*g^2*x^4 - (e*x + d)^m*b*d^2*e^3*f^2*m^3 + 14*(
e*x + d)^m*a*d*e^4*f^2*m^3 - 2*(e*x + d)^m*a*d^2*e^3*f*g*m^3 - 18*(e*x + d)
```

```
^m*c*d^2*e^3*f^2*m^2*x + 47*(e*x + d)^m*b*d*e^4*f^2*m^2*x + 71*(e*x + d)^m*
a*e^5*f^2*m^2*x + 12*(e*x + d)^m*c*d^3*e^2*f*g*m^2*x - 36*(e*x + d)^m*b*d^2
*e^3*f*g*m^2*x + 94*(e*x + d)^m*a*d*e^4*f*g*m^2*x + 6*(e*x + d)^m*b*d^3*e^2
*g^2*m^2*x - 18*(e*x + d)^m*a*d^2*e^3*g^2*m^2*x + 20*(e*x + d)^m*c*d*e^4*f^
2*m*x^2 + 107*(e*x + d)^m*b*e^5*f^2*m*x^2 - 30*(e*x + d)^m*c*d^2*e^3*f*g*m*
x^2 + 40*(e*x + d)^m*b*d*e^4*f*g*m*x^2 + 214*(e*x + d)^m*a*e^5*f*g*m*x^2 +
12*(e*x + d)^m*c*d^3*e^2*g^2*m*x^2 - 15*(e*x + d)^m*b*d^2*e^3*g^2*m*x^2 + 2
0*(e*x + d)^m*a*d*e^4*g^2*m*x^2 + 40*(e*x + d)^m*c*e^5*f^2*x^3 + 80*(e*x +
d)^m*b*e^5*f*g*x^3 + 40*(e*x + d)^m*a*e^5*g^2*x^3 + 2*(e*x + d)^m*c*d^3*e^2
*f^2*m^2 - 12*(e*x + d)^m*b*d^2*e^3*f^2*m^2 + 71*(e*x + d)^m*a*d*e^4*f^2*m^
2 + 4*(e*x + d)^m*b*d^3*e^2*f*g*m^2 - 24*(e*x + d)^m*a*d^2*e^3*f*g*m^2 + 2*
(e*x + d)^m*a*d^3*e^2*g^2*m^2 - 40*(e*x + d)^m*c*d^2*e^3*f^2*m*x + 60*(e*x
+ d)^m*b*d*e^4*f^2*m*x + 154*(e*x + d)^m*a*e^5*f^2*m*x + 60*(e*x + d)^m*c*d
^3*e^2*f*g*m*x - 80*(e*x + d)^m*b*d^2*e^3*f*g*m*x + 120*(e*x + d)^m*a*d*e^4
*f*g*m*x - 24*(e*x + d)^m*c*d^4*e*g^2*m*x + 30*(e*x + d)^m*b*d^3*e^2*g^2*m*
x - 40*(e*x + d)^m*a*d^2*e^3*g^2*m*x + 60*(e*x + d)^m*b*e^5*f^2*x^2 + 120*(
e*x + d)^m*a*e^5*f*g*x^2 + 18*(e*x + d)^m*c*d^3*e^2*f^2*m - 47*(e*x + d)^m*
b*d^2*e^3*f^2*m + 154*(e*x + d)^m*a*d*e^4*f^2*m - 12*(e*x + d)^m*c*d^4*e*f*
g*m + 36*(e*x + d)^m*b*d^3*e^2*f*g*m - 94*(e*x + d)^m*a*d^2*e^3*f*g*m - 6*(
e*x + d)^m*b*d^4*e*g^2*m + 18*(e*x + d)^m*a*d^3*e^2*g^2*m + 120*(e*x + d)^m
*a*e^5*f^2*x + 40*(e*x + d)^m*c*d^3*e^2*f^2 - 60*(e*x + d)^m*b*d^2*e^3*f^2
+ 120*(e*x + d)^m*a*d*e^4*f^2 - 60*(e*x + d)^m*c*d^4*e*f*g + 80*(e*x + d)^m
*b*d^3*e^2*f*g - 120*(e*x + d)^m*a*d^2*e^3*f*g + 24*(e*x + d)^m*c*d^5*g^2 -
 30*(e*x + d)^m*b*d^4*e*g^2 + 40*(e*x + d)^m*a*d^3*e^2*g^2)/(e^5*m^5 + 15*e
^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 1354, normalized size of antiderivative = 6.15

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)\,dx$$

$$= \frac{(d+e\,x)^m\,(24\,c\,d^5\,g^2-12\,c\,d^4\,e\,f\,g\,m-60\,c\,d^4\,e\,f\,g-6\,b\,d^4\,e\,g^2\,m-30\,b\,d^4\,e\,g^2+2\,c\,d^3\,e^2\,f^2\,m^2+18\,c\,d^3}{}$$

$$+\frac{x\,(d+e\,x)^m\,(-24\,c\,d^4\,e\,g^2\,m+12\,c\,d^3\,e^2\,f\,g\,m^2+60\,c\,d^3\,e^2\,f\,g\,m+6\,b\,d^3\,e^2\,g^2\,m^2+30\,b\,d^3\,e^2\,g^2\,m-2\,c}{}$$

$$+\frac{c\,g^2\,x^5\,(d+e\,x)^m\,(m^4+10\,m^3+35\,m^2+50\,m+24)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}$$

$$+\frac{x^2\,(m+1)\,(d+e\,x)^m\,(12\,c\,d^3\,g^2\,m-6\,c\,d^2\,e\,f\,g\,m^2-30\,c\,d^2\,e\,f\,g\,m-3\,b\,d^2\,e\,g^2\,m^2-15\,b\,d^2\,e\,g^2\,m+c}{}$$

$$+\frac{x^3\,(d+e\,x)^m\,(m^2+3\,m+2)\,(-4\,c\,d^2\,g^2\,m+2\,c\,d\,e\,f\,g\,m^2+10\,c\,d\,e\,f\,g\,m+b\,d\,e\,g^2\,m^2+5\,b\,d\,e\,g^2\,m+}{e^2\,(m^5+15\,m^4+}$$

$$+\frac{g\,x^4\,(d+e\,x)^m\,(m^3+6\,m^2+11\,m+6)\,(5\,b\,e\,g+10\,c\,e\,f+b\,e\,g\,m+c\,d\,g\,m+2\,c\,e\,f\,m)}{e\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}$$

[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2),x)

[Out] ((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 - 60*b*d^2*e^3*f^2 + 40*c*d^3*e^2*f^2 + 120*a*d*e^4*f^2 - 30*b*d^4*e*g^2 - 120*a*d^2*e^3*f*g + 80*b*d^3*e^2*f*g + 154*a*d*e^4*f^2*m - 6*b*d^4*e*g^2*m + 71*a*d*e^4*f^2*m^2 + 14*a*d*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m - 47*b*d^2*e^3*f^2*m + 18*c*d^3*e^2*f^2*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 - 12*b*d^2*e^3*f^2*m^2 - b*d^2*e^3*f^2*m^3 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4*e*f*g*m - 94*a*d^2*e^3*f*g*m + 36*b*d^3*e^2*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e^3*f*g*m^3 + 4*b*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (x*(d + e*x)^m*(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e^5*f^2*m^3 + a*e^5*f^2*m^4 + 154*a*e^5*f^2*m + 60*b*d*e^4*f^2*m - 24*c*d^4*e*g^2*m - 40*a*d^2*e^3*g^2*m + 47*b*d*e^4*f^2*m^2 + 12*b*d*e^4*f^2*m^3 + b*d*e^4*f^2*m^4 + 30*b*d^3*e^2*g^2*m - 40*c*d^2*e^3*f^2*m - 18*a*d^2*e^3*g^2*m^2 - 2*a*d^2*e^3*g^2*m^3 + 6*b*d^3*e^2*g^2*m^2 - 18*c*d^2*e^3*f^2*m^2 - 2*c*d^2*e^3*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^4*f*g*m^3 + 2*a*d*e^4*f*g*m^4 - 80*b*d^2*e^3*f*g*m + 60*c*d^3*e^2*f*g*m - 36*b*d^2*e^3*f*g*m^2 - 4*b*d^2*e^3*f*g*m^3 + 12*c*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (x^2*(m + 1)*(d + e*x)^m*(60*b*e^3*f^2 + 12*b*e^3*f^2*m^2 + b*e^3*f^2*m^3 + 120*a*e^3*f*g + 47*b*e^3*f^2*m + 12*c*d^3*g^2*m + 20*a*d*e^2*g^2*m - 15*b*d^2*e*g^2*m + 20*c*d*e^2*f^2*m + 24*a*e^3*f*g*m^2 + 2*a*e^3*f*g*m^3 + 9*a*d*e^2*g^2*m^2 + a*d*e^2*g^2*m^3 - 3*b*d^2*e*g^2*m^2 + 9*c*d*e^2*f^2*m^2 + c*d*e^2*f^2*m^3 + 94*a*e^3*f*g*m + 40*b*d*e^2*f*g*m - 30*c*d^2*e*f*g*m + 18*b*d*e^2*f*g*m^2 + 2*b*d*e^2*f*g*m^3 - 6*c*d^2*e*f*g*m^2))/(e^3*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(20*a*e^2*g^2 + 20*c*e^2*f^2 + a*e^2*g^2*m^2 + c*e^2*f^2*m^2 + 40*b*e^2*f*g + 9*a*e^2*g^2*m - 4*c*d^2*g^2*m + 9*c*e^2*f^2*m + b*d*e*g^2*m^2 + 2*b*e^2*f*g*m^2 + 5*b*d*e*g^2*m + 18*b*e^2*f*g*m + 2*c*d*e*f*g*m^2 + 10*c*d*e*f*g*m))/(e^2*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(5*b*e*g + 10*c*e*f + b*e*g*m + c*d*g*m + 2*c*e*f*m))/(e*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))

3.921 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\begin{aligned}\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx &= \frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)}\\ &\quad-\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^4(2+m)}\\ &\quad+\frac{(cef-3cdg+beg)(d+ex)^{3+m}}{e^4(3+m)}+\frac{cg(d+ex)^{4+m}}{e^4(4+m)}\end{aligned}$$

```
[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)*(e*x+d)^(1+m)/e^4/(1+m)-(c*d*(-3*d*g+2*e*f)-
e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^4/(2+m)+(b*e*g-3*c*d*g+c*e*f)*(e*x
+d)^(3+m)/e^4/(3+m)+c*g*(e*x+d)^(4+m)/e^4/(4+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {785}

$$\begin{aligned}\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx &= \frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}\\ &\quad-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}\\ &\quad+\frac{(d+ex)^{m+3}(beg-3cdg+cef)}{e^4(m+3)}+\frac{cg(d+ex)^{m+4}}{e^4(m+4)}\end{aligned}$$

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(
2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*
g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(cd^2-bde+ae^2)(ef-dg)(d+ex)^m}{e^3} + \frac{(-cd(2ef-3dg)+e(bef-2bdg+aeg))(d+ex)^{1+m}}{e^3} + \frac{(cef-3cdg+beg)(d+ex)^{2+m}}{e^3} + \frac{cg(d+ex)^{3+m}}{e^3}\right) dx \\
&= \frac{(cd^2-bde+ae^2)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)} - \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^4(2+m)} + \frac{(cef-3cdg+beg)(d+ex)^{3+m}}{e^4(3+m)} + \frac{cg(d+ex)^{4+m}}{e^4(4+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$
\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right) dx = \frac{(d+ex)^{1+m}\left(-\frac{(cd^2+e(-bd+ae))(6cdg+beg(1+m)-2cef(4+m))}{e^2(1+m)} + \frac{(-b^2e^2g(2+m)+2c^2d(3dg-ef(4+m))+ce(bdg(-2+m)+2aeg(3+m)+\cdots}{e^2(2+m)}\right.}{ce^2(3+m)(4+m)}
$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m)
- 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*d*
g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)))*
(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(4 +
m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(144) = 288$.

Time = 0.49 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.49

method	result
gosper	$-\frac{(ex+d)^{1+m}(-c\,e^3 g\,m^3 x^3 - b\,e^3 g\,m^3 x^2 - c\,e^3 f\,m^3 x^2 - 6c\,e^3 g\,m^2 x^3 - a\,e^3 g\,m^3 x - b\,e^3 f\,m^3 x - 7b\,e^3 g\,m^2 x^2 + 3cd\,e^2 g\,m^2 x^2 - 7c}{}$
norman	$\frac{cg\,x^4 \mathrm{e}^{m\ln(ex+d)}}{4+m} + \frac{(begm+cdgm+cefm+4beg+4cef)x^3 \mathrm{e}^{m\ln(ex+d)}}{e(m^2+7m+12)} + \frac{(a\,e^2 g\,m^2 + bdeg\,m^2 + b\,e^2 f\,m^2 + cdef\,m^2 + 7a\,e^2 gm}{e}$
risch	$-\frac{(-c\,e^4 g\,m^3 x^4 - b\,e^4 g\,m^3 x^3 - cd\,e^3 g\,m^3 x^3 - c\,e^4 f\,m^3 x^3 - 6c\,e^4 g\,m^2 x^4 - a\,e^4 g\,m^3 x^2 - bd\,e^3 g\,m^3 x^2 - b\,e^4 f\,m^3 x^2 - 7b\,e^4 g\,m^2 x^3 -}{}$
parallelrisch	Expression too large to display

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^4*(e*x+d)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-c*e^3*g*m^3*x^3-b*e^3*g*
m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b*e
^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^3*f
*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2+2*c
*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-14*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m
^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e^3*f*m
*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+10*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f
*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^2*f*m+8*
b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x+12*a*d*e
^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d*e^2*f+6*c*d^3*g-8*c*d^2*e*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(144) = 288$.

Time = 0.30 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.26

$$\int (d+ex)^m (f+gx)\left(a+bx+cx^2\right)\,dx = \frac{(ade^3fm^3 + (ce^4gm^3 + 6\,ce^4gm^2 + 11\,ce^4gm + 6\,ce^4g)x^4 + (8\,ce^4f + 8\,be^4g + (ce^4f + (cde^3 + be^4)g)m^3}{}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*x
^4 + (8*c*e^4*f + 8*b*e^4*g + (c*e^4*f + (c*d*e^3 + b*e^4)*g)*m^3 + (7*c*e^
4*f + (3*c*d*e^3 + 7*b*e^4)*g)*m^2 + 2*(7*c*e^4*f + (c*d*e^3 + 7*b*e^4)*g)*
m)*x^3 - (a*d^2*e^2*g + (b*d^2*e^2 - 9*a*d*e^3)*f)*m^2 + (12*b*e^4*f + 12*a
*e^4*g + ((c*d*e^3 + b*e^4)*f + (b*d*e^3 + a*e^4)*g)*m^3 + ((5*c*d*e^3 + 8*
b*e^4)*f - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*g)*m^2 + ((4*c*d*e^3 + 19*b*
```

```
e^4)*f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3*e - 3*
b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g + ((2*c*
d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)*m + (24*
a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 - 7*b*d*e^3
 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e^2 - 6*b*d*
e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)*x)*(e*x + d
)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(134) = 268$.

Time = 1.27 (sec) , antiderivative size = 5930, normalized size of antiderivative = 41.18

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
+ c*g*x**4/4), Eq(e, 0)), (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e*g/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - b*d*e**2*f/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2*g*x/(6*d**3*e**4 + 18*d**2*e**5*
x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*b*e**3*f*x/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**3*g*x**2/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*d**3*g*log(d/e + x)/(6*d**3*
e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c*d**3*g/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*c*d**2*e*f/(6*d*
*3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d**2*e*g*x*
log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
+ 27*c*d**2*e*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x
**3) - 6*c*d*e**2*f*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e*
*7*x**3) + 18*c*d*e**2*g*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2/(6*d**3*e**4 + 18*d**2*e
**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*c*e**3*f*x**2/(6*d**3*e**4 + 18*d
**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**3*g*x**3*log(d/e + x)/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (
-a*d*e**2*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*
e**6*x**2) + 2*b*d**2*e*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x
**2) + 3*b*d**2*e*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - b*d*e**2*f/(
2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x*log(d/e + x)/(2*d*
*2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x/(2*d**2*e**4 + 4*d*e**
```

5*x + 2*e**6*x**2) - 2*b*e**3*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*b*e**3*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d**3*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 3*c*d**2*e*f/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 12*c*d**2*e*g*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 12*c*d**2*e*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c*d*e**2*f*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c*d*e**2*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*e**2*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*f*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*g*x**3/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2), Eq(m, -3)), (2*a*d*e**2*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*a*d*e**2*g/(2*d*e**4 + 2*e**5*x) - 2*a*e**3*f/(2*d*e**4 + 2*e**5*x) + 2*a*e**3*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f/(2*d*e**4 + 2*e**5*x) - 4*b*d*e**2*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3*g*x**2/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g/(2*d*e**4 + 2*e**5*x) - 4*c*d**2*e*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d**2*e*f/(2*d*e**4 + 2*e**5*x) + 6*c*d**2*e*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d*e**2*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 3*c*d*e**2*g*x**2/(2*d*e**4 + 2*e**5*x) + 2*c*e**3*f*x**2/(2*d*e**4 + 2*e**5*x) + c*e**3*g*x**3/(2*d*e**4 + 2*e**5*x), Eq(m, -2)), (-a*d*g*log(d/e + x)/e**2 + a*f*log(d/e + x)/e + a*g*x/e + b*d**2*g*log(d/e + x)/e**3 - b*d*f*log(d/e + x)/e**2 - b*d*g*x/e**2 + b*f*x/e + b*g*x**2/(2*e) - c*d**3*g*log(d/e + x)/e**4 + c*d**2*f*log(d/e + x)/e**3 + c*d**2*g*x/e**3 - c*d*f*x/e**2 - c*d*g*x**2/(2*e**2) + c*f*x**2/(2*e) + c*g*x**3/(3*e), Eq(m, -1)), (-a*d**2*e**2*g*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*a*d**2*e**2*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12*a*d**2*e**2*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*f*m**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*d*e**3*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 26*a*d*e**3*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*d*e**3*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*g*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7*a*d*e**3*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*d*e**3*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*f*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*e**4*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 26*a*e**4*f*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*e**4*f*x*(d + e*x)**m/(e**4*m**4 + 10*

e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*g*m**3*x**2*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*
a*e**4*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 19*a*e**4*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*e**4*g*x**2*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*b*d**3
*e*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4) + 8*b*d**3*e*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m*
*2 + 50*e**4*m + 24*e**4) - b*d**2*e**2*f*m**2*(d + e*x)**m/(e**4*m**4 + 10
*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*b*d**2*e**2*f*m*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12
*b*d**2*e**2*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
4*m + 24*e4) - 2*b*d**2*e**2*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*b*d**2*e**2*g*m*x*(d + e*x)
m/(e4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e
3*f*m3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**
4*m + 24*e**4) + 7*b*d*e**3*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*d*e**3*f*m*x*(d + e*x)**m/(e*
*4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e**3*g*m
3*x2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
+ 24*e**4) + 5*b*d*e**3*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
+ 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 4*b*d*e**3*g*m*x**2*(d + e*x)**m/(e
4*m4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*e**4*f*m*
*3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
 24*e**4) + 8*b*e**4*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 19*b*e**4*f*m*x**2*(d + e*x)**m/(e**4*
m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*e**4*f*x**
2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e*
*4) + b*e**4*g*m**3*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
2 + 50*e4*m + 24*e**4) + 7*b*e**4*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 14*b*e**4*g*m*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 8*b*e**4*g*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 5
0*e**4*m + 24*e**4) - 6*c*d**4*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*c*d**3*e*f*m*(d + e*x)**m/(e**4*m**4
 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*d**3*e*f*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*
c*d**3*e*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
4*m + 24*e4) - 2*c*d**2*e**2*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*c*d**2*e**2*f*m*x*(d + e*x)
m/(e4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 3*c*d
2*e2*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
+ 50*e**4*m + 24*e**4) - 3*c*d**2*e**2*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*f*m**3*x**2*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)

```
+ 5*c*d*e**3*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
**2 + 50*e**4*m + 24*e**4) + 4*c*d*e**3*f*m*x**2*(d + e*x)**m/(e**4*m**4 +
10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*g*m**3*x**3*(
d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
 + 3*c*d*e**3*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*
m**2 + 50*e**4*m + 24*e**4) + 2*c*d*e**3*g*m*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*f*m**3*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 7*c*e**4*f*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**
2 + 50*e**4*m + 24*e**4) + 14*c*e**4*f*m*x**3*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*e**4*f*x**3*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**
4*g*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e*
*4*m + 24*e**4) + 6*c*e**4*g*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m*
*3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 11*c*e**4*g*m*x**4*(d + e*x)**m/
(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*c*e**4*
g*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(144) = 288$.

Time = 0.23 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.44

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m bf}{(m^2+3m+2)e^2} + \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m ag}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}af}{e(m+1)} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m cf}{(m^3+6m^2+11m+6)e^3} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m bg}{(m^3+6m^2+11m+6)e^3} + \frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex+}{(m^4+10m^3+35m^2+50m+24)e^4}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f/((m^2 + 3*m + 2)*e^2) + (
e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*g/((m^2 + 3*m + 2)*e^2) + (e
*x + d)^(m + 1)*a*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e
*x + d)^m*b*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^
4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m
*x - 6*d^4)*(e*x + d)^m*c*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(144) = 288$.

Time = 0.28 (sec) , antiderivative size = 1156, normalized size of antiderivative = 8.03

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((e*x + d)^m*c*e^4*g*m^3*x^4 + (e*x + d)^m*c*e^4*f*m^3*x^3 + (e*x + d)^m*c*
d*e^3*g*m^3*x^3 + (e*x + d)^m*b*e^4*g*m^3*x^3 + 6*(e*x + d)^m*c*e^4*g*m^2*x
^4 + (e*x + d)^m*c*d*e^3*f*m^3*x^2 + (e*x + d)^m*b*e^4*f*m^3*x^2 + (e*x + d
)^m*b*d*e^3*g*m^3*x^2 + (e*x + d)^m*a*e^4*g*m^3*x^2 + 7*(e*x + d)^m*c*e^4*f
*m^2*x^3 + 3*(e*x + d)^m*c*d*e^3*g*m^2*x^3 + 7*(e*x + d)^m*b*e^4*g*m^2*x^3
+ 11*(e*x + d)^m*c*e^4*g*m*x^4 + (e*x + d)^m*b*d*e^3*f*m^3*x + (e*x + d)^m*
a*e^4*f*m^3*x + (e*x + d)^m*a*d*e^3*g*m^3*x + 5*(e*x + d)^m*c*d*e^3*f*m^2*x
^2 + 8*(e*x + d)^m*b*e^4*f*m^2*x^2 - 3*(e*x + d)^m*c*d^2*e^2*g*m^2*x^2 + 5*
(e*x + d)^m*b*d*e^3*g*m^2*x^2 + 8*(e*x + d)^m*a*e^4*g*m^2*x^2 + 14*(e*x + d
)^m*c*e^4*f*m*x^3 + 2*(e*x + d)^m*c*d*e^3*g*m*x^3 + 14*(e*x + d)^m*b*e^4*g*
m*x^3 + 6*(e*x + d)^m*c*e^4*g*x^4 + (e*x + d)^m*a*d*e^3*f*m^3 - 2*(e*x + d)
^m*c*d^2*e^2*f*m^2*x + 7*(e*x + d)^m*b*d*e^3*f*m^2*x + 9*(e*x + d)^m*a*e^4*
f*m^2*x - 2*(e*x + d)^m*b*d^2*e^2*g*m^2*x + 7*(e*x + d)^m*a*d*e^3*g*m^2*x +
 4*(e*x + d)^m*c*d*e^3*f*m*x^2 + 19*(e*x + d)^m*b*e^4*f*m*x^2 - 3*(e*x + d)
^m*c*d^2*e^2*g*m*x^2 + 4*(e*x + d)^m*b*d*e^3*g*m*x^2 + 19*(e*x + d)^m*a*e^4
*g*m*x^2 + 8*(e*x + d)^m*c*e^4*f*x^3 + 8*(e*x + d)^m*b*e^4*g*x^3 - (e*x + d
)^m*b*d^2*e^2*f*m^2 + 9*(e*x + d)^m*a*d*e^3*f*m^2 - (e*x + d)^m*a*d^2*e^2*g
*m^2 - 8*(e*x + d)^m*c*d^2*e^2*f*m*x + 12*(e*x + d)^m*b*d*e^3*f*m*x + 26*(e
*x + d)^m*a*e^4*f*m*x + 6*(e*x + d)^m*c*d^3*e*g*m*x - 8*(e*x + d)^m*b*d^2*e
^2*g*m*x + 12*(e*x + d)^m*a*d*e^3*g*m*x + 12*(e*x + d)^m*b*e^4*f*x^2 + 12*(
e*x + d)^m*a*e^4*g*x^2 + 2*(e*x + d)^m*c*d^3*e*f*m - 7*(e*x + d)^m*b*d^2*e^
2*f*m + 26*(e*x + d)^m*a*d*e^3*f*m + 2*(e*x + d)^m*b*d^3*e*g*m - 7*(e*x + d
)^m*a*d^2*e^2*g*m + 24*(e*x + d)^m*a*e^4*f*x + 8*(e*x + d)^m*c*d^3*e*f - 12
*(e*x + d)^m*b*d^2*e^2*f + 24*(e*x + d)^m*a*d*e^3*f - 6*(e*x + d)^m*c*d^4*g
 + 8*(e*x + d)^m*b*d^3*e*g - 12*(e*x + d)^m*a*d^2*e^2*g)/(e^4*m^4 + 10*e^4*
m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.18

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(d+e\,x)^m\,(24\,a\,d\,e^3\,f-6\,c\,d^4\,g+8\,b\,d^3\,e\,g+8\,c\,d^3\,e\,f-12\,a\,d^2\,e^2\,g-12\,b\,d^2\,e^2\,f+9\,a\,d\,e^3\,f\,m^2+a\,d\,e^3}{e^4\,(m^4+10\,m^3+}$$
$$+\frac{x\,(d+e\,x)^m\,(24\,a\,e^4\,f+26\,a\,e^4\,f\,m+9\,a\,e^4\,f\,m^2+a\,e^4\,f\,m^3+7\,a\,d\,e^3\,g\,m^2+7\,b\,d\,e^3\,f\,m^2+a\,d\,e^3\,g\,m}{e^4\,(m^4+1}$$
$$+\frac{x^2\,(m+1)\,(d+e\,x)^m\,(12\,a\,e^2\,g+12\,b\,e^2\,f+7\,a\,e^2\,g\,m+7\,b\,e^2\,f\,m-3\,c\,d^2\,g\,m+a\,e^2\,g\,m^2+b\,e^2\,f\,m^2}{e^2\,(m^4+10\,m^3+35\,m^2+50\,m+24)}$$
$$+\frac{c\,g\,x^4\,(d+e\,x)^m\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$
$$+\frac{x^3\,(d+e\,x)^m\,(m^2+3\,m+2)\,(4\,b\,e\,g+4\,c\,e\,f+b\,e\,g\,m+c\,d\,g\,m+c\,e\,f\,m)}{e\,(m^4+10\,m^3+35\,m^2+50\,m+24)}$$

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*b*d^3*e*g + 8*c*d^3*e*f - 12*a*d
^2*e^2*g - 12*b*d^2*e^2*f + 9*a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2*g
*m - 7*b*d^2*e^2*f*m - a*d^2*e^2*g*m^2 - b*d^2*e^2*f*m^2 + 26*a*d*e^3*f*m +
 2*b*d^3*e*g*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
+ (x*(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*m^3 +
 7*a*d*e^3*g*m^2 + 7*b*d*e^3*f*m^2 + a*d*e^3*g*m^3 + b*d*e^3*f*m^3 - 8*b*d^
2*e^2*g*m - 8*c*d^2*e^2*f*m - 2*b*d^2*e^2*g*m^2 - 2*c*d^2*e^2*f*m^2 + 12*a*
d*e^3*g*m + 12*b*d*e^3*f*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24)) + (x^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 12*b*e^2*f + 7*a*e^2*g
*m + 7*b*e^2*f*m - 3*c*d^2*g*m + a*e^2*g*m^2 + b*e^2*f*m^2 + 4*b*d*e*g*m +
4*c*d*e*f*m + b*d*e*g*m^2 + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^
4 + 24)) + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 +
10*m^3 + m^4 + 24) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(4*b*e*g + 4*c*e*f +
b*e*g*m + c*d*g*m + c*e*f*m))/(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

3.922 $\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx = -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{(cf^2-bfg+ag^2)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)(1+m)}$$

```
[Out] -(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1+m)/e^2/g^2/(1+m)+c*(e*x+d)^(2+m)/e^2/g/(2+
m)+(a*g^2-b*f*g+c*f^2)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-
d*g+e*f))/g^2/(-d*g+e*f)/(1+m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {965, 81, 70}

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx = \frac{(d+ex)^{m+1}(ag^2-bfg+cf^2)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)} - \frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] -(((c*e*f + c*d*g - b*e*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d +
e*x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\int \frac{(d+ex)^m(-e(cdf-aeg)(2+m)-e(cef+cdg-beg)(2+m)x)}{f+gx}\,dx}{e^2g(2+m)} \\
&= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{(cf^2-bfg+ag^2)\int \frac{(d+ex)^m}{f+gx}\,dx}{g^2} \\
&= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} \\
&\quad + \frac{(cf^2-bfg+ag^2)(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \frac{(d+ex)^{1+m}\left(\frac{beg-c(ef+dg)}{e^2(1+m)}+\frac{cg(d+ex)}{e^2(2+m)}+\frac{(cf^2+g(-bf+ag))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{(ef-dg)(1+m)}\right)}{g^2}$$

[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]

[Out] ((d + e*x)^(1 + m)*((b*e*g - c*(e*f + d*g))/(e^2*(1 + m)) + (c*g*(d + e*x))/(e^2*(2 + m)) + ((c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m))))/g^2

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{gx+f}\,dx$$

[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)

[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)

Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f} \, dx$$

[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)

Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx$$

[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f),x)

[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x), x)

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{f+g\,x} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x), x)
```

3.923 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2} dx = \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} + \frac{(cf(2dg-ef(2+m))-g(aegm+b(dg-ef(1+m))))(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+\right.}{g^2(ef-dg)^2(1+m)}$$

```
[Out] c*(e*x+d)^(1+m)/e/g^2/(1+m)+(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(
g*x+f)+(c*f*(2*d*g-e*f*(2+m))-g*(a*e*g*m+b*(d*g-e*f*(1+m))))*(e*x+d)^(1+m)*
hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^2/(1+m)
```

Rubi [A] (verified)

Time = 0.12 (sec), antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {963, 81, 70}

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2} dx = -\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+bdg-bef(m+1))-cf(2dg-}{g^2(m+1)(ef-dg)^2} + \frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)} + \frac{c(d+ex)^{m+1}}{eg^2(m+1)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] (c*(d + e*x)^(1 + m))/(e*g^2*(1 + m)) + ((a + (f*(c*f - b*g))/g^2)*(d + e*x
)^(1 + m))/((e*f - d*g)*(f + g*x)) - ((g*(b*d*g + a*e*g*m - b*e*f*(1 + m))
- c*f*(2*d*g - e*f*(2 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m\left(\frac{cdfg-aeg^2m-cef^2(1+m)-bg(dg-ef(1+m))}{g^2}-c\left(d-\frac{ef}{g}\right)x\right)}{f+gx}\,dx}{ef-dg}$$

$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+m))-cf(2dg-ef(2+m)))\int \frac{(d+ex)^m}{f+gx}\,dx}{g^2(ef-dg)}$$

$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+m))-cf(2dg-ef(2+m)))(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)^2(1+m)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx = \frac{(d+ex)^{1+m}\left(c(ef-dg)^2-e(2cf-bg)(ef-dg)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)+e^2(cf^2\right.}{eg^2(ef-dg)^2(1+m)}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(e*f - d*g)^2 - e*(2*c*f - b*g)*(e*f - d*g)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e^2*(c*f^2 + g*(-
(b*f) + a*g))*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*
g)]))/(e*g^2*(e*f - d*g)^2*(1 + m))
```

Maple [F]

$$\int \frac{(ex+d)^m(c\,x^2+bx+a)}{(gx+f)^2}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(f+g\,x)^2} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2, x)
```

3.924 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3}\,dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{(cf(4dg-ef(3+m))+g(aeg(1-m)-b(2dg-ef(1+m))))(d+ex)^{1+m}}{2g^2(ef-dg)^2(f+gx)} + \frac{(c(2d^2g^2-4defg(1+m)+e^2f^2(2+3m+m^2))-egm(aeg(1-m)-b(2dg-ef(1+m))))\,(d+ex)^{1+}}{2g^2(ef-dg)^3(1+m)}$$

```
[Out] 1/2*(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(g*x+f)^2+1/2*(c*f*(4*d*g
-e*f*(3+m))+g*(a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)/g^2/(-d*g+e*
f)^2/(g*x+f)+1/2*(c*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-e*g*m*(
a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g
*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^3/(1+m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{963, 79, 70\}$

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3}\,dx$$

$$= \frac{(d+ex)^{m+1}\,\mathrm{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)(egm(-aeg(1-m)+2bdg-bef(m+1))}{2g^2(m+1)(ef-dg)^3}$$

$$-\frac{(d+ex)^{m+1}(g(-aeg(1-m)+2bdg-bef(m+1))-cf(4dg-ef(m+3)))}{2g^2(f+gx)(ef-dg)^2}$$

$$+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] ((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) -
 ((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 + m)))
*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((e*g*m*(2*b*d*g - a*
e*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(
2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*
(d + e*x))/(e*f - d*g))])/(2*g^2*(e*f - d*g)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
```

```
b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{cf(2dg-ef(1+m))-g(2bdg-aeg(1-m)-bef(1+m))}{g^2}-2c\left(d-\frac{ef}{g}\right)x\right)}{(f+gx)^2}\,dx}{2(ef-dg)}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-ef(3+m)))(d+ex)^{1+m}}{2g^2(ef-dg)^2(f+gx)} + \frac{(egm(2bdg-aeg(1-m)-bef(1+m))+c(2d^2g^2-4defg(1+m)+e^2f^2(2+3m+m^2)))\int \frac{(d}{}}{2g^2(ef-dg)^2}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-ef(3+m)))(d+ex)^{1+m}}{2g^2(ef-dg)^2(f+gx)} + \frac{(egm(2bdg-aeg(1-m)-bef(1+m))+c(2d^2g^2-4defg(1+m)+e^2f^2(2+3m+m^2)))(d}{2g^2(ef-dg)^3(1+m)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(f+gx)^3}\,dx = -\frac{(d+ex)^{1+m}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)+e\left(-\left((2cf-bg)(ef-dg)\,\text{H}\right.\right.\right.}{g^2}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] -(((d + e*x)^(1 + m)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)*Hypergeometric
2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e*(c*f^2 + g*(-(b*f)
+ a*g))*Hypergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])))
/(g^2*(-(e*f) + d*g)^3*(1 + m)))
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{(gx+f)^3}dx$$

[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)

[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)

Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3} dx$$

[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="fricas")

[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)

Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} dx = \int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} dx$$

[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**3,x)

[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**3, x)

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3} dx$$

[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^3} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(f+g\,x)^3} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3, x)
```

3.925 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 525

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^{1+m}}{e^7(1+m)}$$
$$-\frac{2\left(cd^2-bde+ae^2\right)(ef-dg)(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^7(2+m)}$$
$$+\frac{\left(c^2d^2\left(6e^2f^2-20defg+15d^2g^2\right)+e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(e^2f^2-6defg+6d^2g^2\right)\right)+2ce(\cdots\right.}{e^7(3+m)}$$
$$+\frac{2\left(be^2g(bef-2bdg+aeg)-2c^2d\left(e^2f^2-5defg+5d^2g^2\right)+ce\left(2aeg(ef-2dg)+b\left(e^2f^2-8defg+10\cdots\right.\right.\right.}{e^7(4+m)}$$
$$+\frac{\left(b^2e^2g^2+2ceg(2bef-5bdg+aeg)+c^2\left(e^2f^2-10defg+15d^2g^2\right)\right)(d+ex)^{5+m}}{e^7(5+m)}$$
$$+\frac{2cg(cef-3cdg+beg)(d+ex)^{6+m}}{e^7(6+m)}+\frac{c^2g^2(d+ex)^{7+m}}{e^7(7+m)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^7/(1+m)-2*(a*e^2-b*d*e+c
*d^2)*(-d*g+e*f)*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)
/e^7/(2+m)+(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+6*e^2*f^2)+e^2*(a^2*e^2*g^2+2*a*
b*e*g*(-3*d*g+2*e*f)+b^2*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))+2*c*e*(a*e*(6*d^2*g
^2-6*d*e*f*g+e^2*f^2)-b*d*(10*d^2*g^2-12*d*e*f*g+3*e^2*f^2)))*(e*x+d)^(3+m)
/e^7/(3+m)+2*(b*e^2*g*(a*e*g-2*b*d*g+b*e*f)-2*c^2*d*(5*d^2*g^2-5*d*e*f*g+e^
2*f^2)+c*e*(2*a*e*g*(-2*d*g+e*f)+b*(10*d^2*g^2-8*d*e*f*g+e^2*f^2)))*(e*x+d)
^(4+m)/e^7/(4+m)+(b^2*e^2*g^2+2*c*e*g*(a*e*g-5*b*d*g+2*b*e*f)+c^2*(15*d^2*g
^2-10*d*e*f*g+e^2*f^2))*(e*x+d)^(5+m)/e^7/(5+m)+2*c*g*(b*e*g-3*c*d*g+c*e*f)
*(e*x+d)^(6+m)/e^7/(6+m)+c^2*g^2*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {961}

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx$$
$$= \frac{(d+ex)^{m+3}\left(e^2(a^2e^2g^2+2abeg(2ef-3dg)+b^2(6d^2g^2-6defg+e^2f^2))+2ce(ae(6d^2g^2-6defg+e^2f^2)\right.}{e^7(m+3)}$$
$$+\frac{(d+ex)^{m+5}\left(2ceg(aeg-5bdg+2bef)+b^2e^2g^2+c^2(15d^2g^2-10defg+e^2f^2)\right)}{e^7(m+5)}$$
$$+\frac{2(d+ex)^{m+4}\left(ce(2aeg(ef-2dg)+b(10d^2g^2-8defg+e^2f^2))+be^2g(aeg-2bdg+bef)-2c^2d(5d^2g^2\right.}{e^7(m+4)}$$
$$+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^7(m+1)}$$
$$-\frac{2(ef-dg)(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^7(m+2)}$$
$$+\frac{2cg(d+ex)^{m+6}(beg-3cdg+cef)}{e^7(m+6)}+\frac{c^2g^2(d+ex)^{m+7}}{e^7(m+7)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\text{integral} = \int \left(\frac{(cd^2-bde+ae^2)^2 (ef-dg)^2 (d+ex)^m}{e^6} + \frac{2(cd^2-bde+ae^2)(ef-dg)(-cd(2ef-3dg)+e(bef-2bdg+aeg))(d+ex)^{1+m}}{e^6} + \frac{(c^2d^2(6e^2f^2-20defg+15d^2g^2)+e^2(a^2e^2g^2+2abeg(2ef-3dg)+b^2(e^2f^2-6defg+6d^2g^2))+2ce(}{e^6} + \frac{2(be^2g(bef-2bdg+aeg)-2c^2d(e^2f^2-5defg+5d^2g^2)+ce(2aeg(ef-2dg)+b(e^2f^2-8defg+10}{e^6} + \frac{(b^2e^2g^2+2ceg(2bef-5bdg+aeg)+c^2(e^2f^2-10defg+15d^2g^2))(d+ex)^{4+m}}{e^6} + \frac{2cg(cef-3cdg+beg)(d+ex)^{5+m}}{e^6} + \frac{c^2g^2(d+ex)^{6+m}}{e^6} \right) dx$$

$$= \frac{(cd^2-bde+ae^2)^2 (ef-dg)^2 (d+ex)^{1+m}}{e^7(1+m)} - \frac{2(cd^2-bde+ae^2)(ef-dg)(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^7(2+m)} + \frac{(c^2d^2(6e^2f^2-20defg+15d^2g^2)+e^2(a^2e^2g^2+2abeg(2ef-3dg)+b^2(e^2f^2-6defg+6d^2g^2))}{e^7(3+m)} + \frac{2(be^2g(bef-2bdg+aeg)-2c^2d(e^2f^2-5defg+5d^2g^2)+ce(2aeg(ef-2dg)+b(e^2f^2-8de}{e^7(4+m)} + \frac{(b^2e^2g^2+2ceg(2bef-5bdg+aeg)+c^2(e^2f^2-10defg+15d^2g^2))(d+ex)^{5+m}}{e^7(5+m)} + \frac{2cg(cef-3cdg+beg)(d+ex)^{6+m}}{e^7(6+m)} + \frac{c^2g^2(d+ex)^{7+m}}{e^7(7+m)}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.94

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 dx = \frac{(d+ex)^{1+m} \left(\frac{(cd^2+e(-bd+ae))^2(ef-dg)^2}{1+m} - \frac{2(cd^2+e(-bd+ae))(-ef+dg)(cd(-2ef+3dg)+e(bef-2bdg+aeg))(d+ex)}{2+m} + \frac{(c^2d^2(6e^2f^2}{} \right.}{}$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)^2)/(1 + m) -
(2*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*(c*d*(-2*e*f + 3*d*g) + e*(b*e
*f - 2*b*d*g + a*e*g))*(d + e*x))/(2 + m) + ((c^2*d^2*(6*e^2*f^2 - 20*d*e*f
```

```
*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b^2*(e^2*
f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(b*d*(-3*e^2*f^2 + 12*d*e*f*g - 10*d^
2*g^2) + a*e*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)))*(d + e*x)^2)/(3 + m) + (2*
(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g
^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*g^2)))*(
d + e*x)^3)/(4 + m) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) +
 c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4)/(5 + m) + (2*c*g*(c*
e*f - 3*c*d*g + b*e*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*x)^6)/(7 + m)
))/e^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4652 vs. $2(525) = 1050$.

Time = 0.78 (sec) , antiderivative size = 4653, normalized size of antiderivative = 8.86

method	result	size
norman	Expression too large to display	4653
gosper	Expression too large to display	5890
risch	Expression too large to display	7342
parallelrisch	Expression too large to display	10811

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*e^6*f^2*m^6-2*a^2*d*e^5*f*g*m^5+27*a^2*e^6*f^2*m^5-2*a*b*d*e^5*f^2*m
^5+2*a^2*d^2*e^4*g^2*m^4-50*a^2*d*e^5*f*g*m^4+295*a^2*e^6*f^2*m^4+8*a*b*d^2
*e^4*f*g*m^4-50*a*b*d*e^5*f^2*m^4+4*a*c*d^2*e^4*f^2*m^4+2*b^2*d^2*e^4*f^2*m
^4+44*a^2*d^2*e^4*g^2*m^3-490*a^2*d*e^5*f*g*m^3+1665*a^2*e^6*f^2*m^3-12*a*b
*d^3*e^3*g^2*m^3+176*a*b*d^2*e^4*f*g*m^3-490*a*b*d*e^5*f^2*m^3-24*a*c*d^3*e
^3*f*g*m^3+88*a*c*d^2*e^4*f^2*m^3-12*b^2*d^3*e^3*f*g*m^3+44*b^2*d^2*e^4*f^2
*m^3-12*b*c*d^3*e^3*f^2*m^3+358*a^2*d^2*e^4*g^2*m^2-2350*a^2*d*e^5*f*g*m^2+
5104*a^2*e^6*f^2*m^2-216*a*b*d^3*e^3*g^2*m^2+1432*a*b*d^2*e^4*f*g*m^2-2350*
a*b*d*e^5*f^2*m^2+48*a*c*d^4*e^2*g^2*m^2-432*a*c*d^3*e^3*f*g*m^2+716*a*c*d^
2*e^4*f^2*m^2+24*b^2*d^4*e^2*g^2*m^2-216*b^2*d^3*e^3*f*g*m^2+358*b^2*d^2*e^
4*f^2*m^2+96*b*c*d^4*e^2*f*g*m^2-216*b*c*d^3*e^3*f^2*m^2+24*c^2*d^4*e^2*f^2
*m^2+1276*a^2*d^2*e^4*g^2*m-5508*a^2*d*e^5*f*g*m+8028*a^2*e^6*f^2*m-1284*a*
b*d^3*e^3*g^2*m+5104*a*b*d^2*e^4*f*g*m-5508*a*b*d*e^5*f^2*m+624*a*c*d^4*e^2
*g^2*m-2568*a*c*d^3*e^3*f*g*m+2552*a*c*d^2*e^4*f^2*m+312*b^2*d^4*e^2*g^2*m-
1284*b^2*d^3*e^3*f*g*m+1276*b^2*d^2*e^4*f^2*m-240*b*c*d^5*e*g^2*m+1248*b*c*
d^4*e^2*f*g*m-1284*b*c*d^3*e^3*f^2*m-240*c^2*d^5*e*f*g*m+312*c^2*d^4*e^2*f^
2*m+1680*a^2*d^2*e^4*g^2-5040*a^2*d*e^5*f*g+5040*a^2*e^6*f^2-2520*a*b*d^3*e
^3*g^2+6720*a*b*d^2*e^4*f*g-5040*a*b*d*e^5*f^2+2016*a*c*d^4*e^2*g^2-5040*a*
c*d^3*e^3*f*g+3360*a*c*d^2*e^4*f^2+1008*b^2*d^4*e^2*g^2-2520*b^2*d^3*e^3*f*
g+1680*b^2*d^2*e^4*f^2-1680*b*c*d^5*e*g^2+4032*b*c*d^4*e^2*f*g-2520*b*c*d^3
*e^3*f^2+720*c^2*d^6*g^2-1680*c^2*d^5*e*f*g+1008*c^2*d^4*e^2*f^2)/e^7/(m^7+
```

28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*exp(m*ln(e*x+d))+g
^2*c^2/(7+m)*x^7*exp(m*ln(e*x+d))+(2*a*c*e^2*g^2*m^2+b^2*e^2*g^2*m^2+2*b*c*
d*e*g^2*m^2+4*b*c*e^2*f*g*m^2+2*c^2*d*e*f*g*m^2+c^2*e^2*f^2*m^2+26*a*c*e^2*
g^2*m+13*b^2*e^2*g^2*m+14*b*c*d*e*g^2*m+52*b*c*e^2*f*g*m-6*c^2*d^2*g^2*m+14
*c^2*d*e*f*g*m+13*c^2*e^2*f^2*m+84*a*c*e^2*g^2+42*b^2*e^2*g^2+168*b*c*e^2*f
*g+42*c^2*e^2*f^2)/e^2/(m^3+18*m^2+107*m+210)*x^5*exp(m*ln(e*x+d))+(2*a*b*e
^3*g^2*m^3+2*a*c*d*e^2*g^2*m^3+4*a*c*e^3*f*g*m^3+b^2*d*e^2*g^2*m^3+2*b^2*e^
3*f*g*m^3+4*b*c*d*e^2*f*g*m^3+2*b*c*e^3*f^2*m^3+c^2*d*e^2*f^2*m^3+36*a*b*e^
3*g^2*m^2+26*a*c*d*e^2*g^2*m^2+72*a*c*e^3*f*g*m^2+13*b^2*d*e^2*g^2*m^2+36*b
^2*e^3*f*g*m^2-10*b*c*d^2*e*g^2*m^2+52*b*c*d*e^2*f*g*m^2+36*b*c*e^3*f^2*m^2
-10*c^2*d^2*e*f*g*m^2+13*c^2*d*e^2*f^2*m^2+214*a*b*e^3*g^2*m+84*a*c*d*e^2*g
^2*m+428*a*c*e^3*f*g*m+42*b^2*d*e^2*g^2*m+214*b^2*e^3*f*g*m-70*b*c*d^2*e*g^
2*m+168*b*c*d*e^2*f*g*m+214*b*c*e^3*f^2*m+30*c^2*d^3*g^2*m-70*c^2*d^2*e*f*g
*m+42*c^2*d*e^2*f^2*m+420*a*b*e^3*g^2+840*a*c*e^3*f*g+420*b^2*e^3*f*g+420*b
*c*e^3*f^2)/e^3/(m^4+22*m^3+179*m^2+638*m+840)*x^4*exp(m*ln(e*x+d))+(a^2*e^
4*g^2*m^4+2*a*b*d*e^3*g^2*m^4+4*a*b*e^4*f*g*m^4+4*a*c*d*e^3*f*g*m^4+2*a*c*e
^4*f^2*m^4+2*b^2*d*e^3*f*g*m^4+b^2*e^4*f^2*m^4+2*b*c*d*e^3*f^2*m^4+22*a^2*e
^4*g^2*m^3+36*a*b*d*e^3*g^2*m^3+88*a*b*e^4*f*g*m^3-8*a*c*d^2*e^2*g^2*m^3+72
*a*c*d*e^3*f*g*m^3+44*a*c*e^4*f^2*m^3-4*b^2*d^2*e^2*g^2*m^3+36*b^2*d*e^3*f*
g*m^3+22*b^2*e^4*f^2*m^3-16*b*c*d^2*e^2*f*g*m^3+36*b*c*d*e^3*f^2*m^3-4*c^2*
d^2*e^2*f^2*m^3+179*a^2*e^4*g^2*m^2+214*a*b*d*e^3*g^2*m^2+716*a*b*e^4*f*g*m
^2-104*a*c*d^2*e^2*g^2*m^2+428*a*c*d*e^3*f*g*m^2+358*a*c*e^4*f^2*m^2-52*b^2
*d^2*e^2*g^2*m^2+214*b^2*d*e^3*f*g*m^2+179*b^2*e^4*f^2*m^2+40*b*c*d^3*e*g^2
*m^2-208*b*c*d^2*e^2*f*g*m^2+214*b*c*d*e^3*f^2*m^2+40*c^2*d^3*e*f*g*m^2-52*
c^2*d^2*e^2*f^2*m^2+638*a^2*e^4*g^2*m+420*a*b*d*e^3*g^2*m+2552*a*b*e^4*f*g*
m-336*a*c*d^2*e^2*g^2*m+840*a*c*d*e^3*f*g*m+1276*a*c*e^4*f^2*m-168*b^2*d^2*
e^2*g^2*m+420*b^2*d*e^3*f*g*m+638*b^2*e^4*f^2*m+280*b*c*d^3*e*g^2*m-672*b*c
*d^2*e^2*f*g*m+420*b*c*d*e^3*f^2*m-120*c^2*d^4*g^2*m+280*c^2*d^3*e*f*g*m-16
8*c^2*d^2*e^2*f^2*m+840*a^2*e^4*g^2+3360*a*b*e^4*f*g+1680*a*c*e^4*f^2+840*b
^2*e^4*f^2)/e^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+
d))+(a^2*d*e^4*g^2*m^5+2*a^2*e^5*f*g*m^5+4*a*b*d*e^4*f*g*m^5+2*a*b*e^5*f^2*
m^5+2*a*c*d*e^4*f^2*m^5+b^2*d*e^4*f^2*m^5+22*a^2*d*e^4*g^2*m^4+50*a^2*e^5*f
*g*m^4-6*a*b*d^2*e^3*g^2*m^4+88*a*b*d*e^4*f*g*m^4+50*a*b*e^5*f^2*m^4-12*a*c
*d^2*e^3*f*g*m^4+44*a*c*d*e^4*f^2*m^4-6*b^2*d^2*e^3*f*g*m^4+22*b^2*d*e^4*f^
2*m^4-6*b*c*d^2*e^3*f^2*m^4+179*a^2*d*e^4*g^2*m^3+490*a^2*e^5*f*g*m^3-108*a
*b*d^2*e^3*g^2*m^3+716*a*b*d*e^4*f*g*m^3+490*a*b*e^5*f^2*m^3+24*a*c*d^3*e^2
*g^2*m^3-216*a*c*d^2*e^3*f*g*m^3+358*a*c*d*e^4*f^2*m^3+12*b^2*d^3*e^2*g^2*m
^3-108*b^2*d^2*e^3*f*g*m^3+179*b^2*d*e^4*f^2*m^3+48*b*c*d^3*e^2*f*g*m^3-108
*b*c*d^2*e^3*f^2*m^3+12*c^2*d^3*e^2*f^2*m^3+638*a^2*d*e^4*g^2*m^2+2350*a^2*
e^5*f*g*m^2-642*a*b*d^2*e^3*g^2*m^2+2552*a*b*d*e^4*f*g*m^2+2350*a*b*e^5*f^2
*m^2+312*a*c*d^3*e^2*g^2*m^2-1284*a*c*d^2*e^3*f*g*m^2+1276*a*c*d*e^4*f^2*m^
2+156*b^2*d^3*e^2*g^2*m^2-642*b^2*d^2*e^3*f*g*m^2+638*b^2*d*e^4*f^2*m^2-120
*b*c*d^4*e*g^2*m^2+624*b*c*d^3*e^2*f*g*m^2-642*b*c*d^2*e^3*f^2*m^2-120*c^2*
d^4*e*f*g*m^2+156*c^2*d^3*e^2*f^2*m^2+840*a^2*d*e^4*g^2*m+5508*a^2*e^5*f*g*
m-1260*a*b*d^2*e^3*g^2*m+3360*a*b*d*e^4*f*g*m+5508*a*b*e^5*f^2*m+1008*a*c*d

```
^3*e^2*g^2*m-2520*a*c*d^2*e^3*f*g*m+1680*a*c*d*e^4*f^2*m+504*b^2*d^3*e^2*g^
2*m-1260*b^2*d^2*e^3*f*g*m+840*b^2*d*e^4*f^2*m-840*b*c*d^4*e*g^2*m+2016*b*c
*d^3*e^2*f*g*m-1260*b*c*d^2*e^3*f^2*m+360*c^2*d^5*g^2*m-840*c^2*d^4*e*f*g*m
+504*c^2*d^3*e^2*f^2*m+5040*a^2*e^5*f*g+5040*a*b*e^5*f^2)/e^5/(m^6+27*m^5+2
95*m^4+1665*m^3+5104*m^2+8028*m+5040)*x^2*exp(m*ln(e*x+d))+c*g*(2*b*e*g*m+c
*d*g*m+2*c*e*f*m+14*b*e*g+14*c*e*f)/e/(m^2+13*m+42)*x^6*exp(m*ln(e*x+d))-(-
2*a^2*d*e^5*f*g*m^6-a^2*e^6*f^2*m^6-2*a*b*d*e^5*f^2*m^6+2*a^2*d^2*e^4*g^2*m
^5-50*a^2*d*e^5*f*g*m^5-27*a^2*e^6*f^2*m^5+8*a*b*d^2*e^4*f*g*m^5-50*a*b*d*e
^5*f^2*m^5+4*a*c*d^2*e^4*f^2*m^5+2*b^2*d^2*e^4*f^2*m^5+44*a^2*d^2*e^4*g^2*m
^4-490*a^2*d*e^5*f*g*m^4-295*a^2*e^6*f^2*m^4-12*a*b*d^3*e^3*g^2*m^4+176*a*b
*d^2*e^4*f*g*m^4-490*a*b*d*e^5*f^2*m^4-24*a*c*d^3*e^3*f*g*m^4+88*a*c*d^2*e^
4*f^2*m^4-12*b^2*d^3*e^3*f*g*m^4+44*b^2*d^2*e^4*f^2*m^4-12*b*c*d^3*e^3*f^2*
m^4+358*a^2*d^2*e^4*g^2*m^3-2350*a^2*d*e^5*f*g*m^3-1665*a^2*e^6*f^2*m^3-216
*a*b*d^3*e^3*g^2*m^3+1432*a*b*d^2*e^4*f*g*m^3-2350*a*b*d*e^5*f^2*m^3+48*a*c
*d^4*e^2*g^2*m^3-432*a*c*d^3*e^3*f*g*m^3+716*a*c*d^2*e^4*f^2*m^3+24*b^2*d^4
*e^2*g^2*m^3-216*b^2*d^3*e^3*f*g*m^3+358*b^2*d^2*e^4*f^2*m^3+96*b*c*d^4*e^2
*f*g*m^3-216*b*c*d^3*e^3*f^2*m^3+24*c^2*d^4*e^2*f^2*m^3+1276*a^2*d^2*e^4*g^
2*m^2-5508*a^2*d*e^5*f*g*m^2-5104*a^2*e^6*f^2*m^2-1284*a*b*d^3*e^3*g^2*m^2+
5104*a*b*d^2*e^4*f*g*m^2-5508*a*b*d*e^5*f^2*m^2+624*a*c*d^4*e^2*g^2*m^2-256
8*a*c*d^3*e^3*f*g*m^2+2552*a*c*d^2*e^4*f^2*m^2+312*b^2*d^4*e^2*g^2*m^2-1284
*b^2*d^3*e^3*f*g*m^2+1276*b^2*d^2*e^4*f^2*m^2-240*b*c*d^5*e*g^2*m^2+1248*b*
c*d^4*e^2*f*g*m^2-1284*b*c*d^3*e^3*f^2*m^2-240*c^2*d^5*e*f*g*m^2+312*c^2*d^
4*e^2*f^2*m^2+1680*a^2*d^2*e^4*g^2*m-5040*a^2*d*e^5*f*g*m-8028*a^2*e^6*f^2*
m-2520*a*b*d^3*e^3*g^2*m+6720*a*b*d^2*e^4*f*g*m-5040*a*b*d*e^5*f^2*m+2016*a
*c*d^4*e^2*g^2*m-5040*a*c*d^3*e^3*f*g*m+3360*a*c*d^2*e^4*f^2*m+1008*b^2*d^4
*e^2*g^2*m-2520*b^2*d^3*e^3*f*g*m+1680*b^2*d^2*e^4*f^2*m-1680*b*c*d^5*e*g^2
*m+4032*b*c*d^4*e^2*f*g*m-2520*b*c*d^3*e^3*f^2*m+720*c^2*d^6*g^2*m-1680*c^2
*d^5*e*f*g*m+1008*c^2*d^4*e^2*f^2*m-5040*a^2*e^6*f^2)/e^6/(m^7+28*m^6+322*m
^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*x*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4747 vs. $2(525) = 1050$.

Time = 0.38 (sec) , antiderivative size = 4747, normalized size of antiderivative = 9.04

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g^
2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m + 7
20*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*f*g
 + (c^2*d*e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d*e^6 + 44*
b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d*e^6 + 76*b*c*e^7)*g^2)*m^
```

4 + 5*(328*c^2*e^7*f*g + (45*c^2*d*e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1849*c^
2*e^7*f*g + (137*c^2*d*e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^7*f*g +
(30*c^2*d*e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (2*a*b*d^
2*e^5 - 27*a^2*d*e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*f*g + 100
8*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d*e^6 + 2*b*c*e^7)*f*g + (2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(17*c^2*d*e^
6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d*e^6 - 23*(b^2 + 2*a*c)*e^7)
*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d*e^6 + 138*b*c*e^7)*f*g - (20*c^
2*d^2*e^5 - 70*b*c*d*e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5*(185*c^2*e^7*
f^2 + 2*(59*c^2*d*e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 - 118*b*c*d*e^6
- 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (187*c^2*d*e^6 + 2
144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d*e^6 - 536*(b^2 + 2*a*c)*e^7)
*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d*e^6 + 201*b*c*e^7)*f*g - (12*c
^2*d^2*e^5 - 28*b*c*d*e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*m)*x^5 + (2*a^2*d^3
*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d*e^6 - 2*(b^2 + 2*a*c)*d^3*e^4)*f^2 +
2*(4*a*b*d^3*e^4 - 25*a^2*d^2*e^5)*f*g)*m^4 + (2520*b*c*e^7*f^2 + 2520*a*b
*e^7*g^2 + 2520*(b^2 + 2*a*c)*e^7*f*g + ((c^2*d*e^6 + 2*b*c*e^7)*f^2 + 2*(2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f*g + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*g^2
)*m^6 + ((19*c^2*d*e^6 + 48*b*c*e^7)*f^2 - 2*(5*c^2*d^2*e^5 - 38*b*c*d*e^6
- 24*(b^2 + 2*a*c)*e^7)*f*g - (10*b*c*d^2*e^5 - 48*a*b*e^7 - 19*(b^2 + 2*a*
c)*d*e^6)*g^2)*m^5 + ((131*c^2*d*e^6 + 452*b*c*e^7)*f^2 - 2*(65*c^2*d^2*e^5
- 262*b*c*d*e^6 - 226*(b^2 + 2*a*c)*e^7)*f*g + (30*c^2*d^3*e^4 - 130*b*c*d
^2*e^5 + 452*a*b*e^7 + 131*(b^2 + 2*a*c)*d*e^6)*g^2)*m^4 + ((401*c^2*d*e^6
+ 2112*b*c*e^7)*f^2 - 2*(265*c^2*d^2*e^5 - 802*b*c*d*e^6 - 1056*(b^2 + 2*a*
c)*e^7)*f*g + (180*c^2*d^3*e^4 - 530*b*c*d^2*e^5 + 2112*a*b*e^7 + 401*(b^2
+ 2*a*c)*d*e^6)*g^2)*m^3 + 10*((54*c^2*d*e^6 + 509*b*c*e^7)*f^2 - (83*c^2*d
^2*e^5 - 216*b*c*d*e^6 - 509*(b^2 + 2*a*c)*e^7)*f*g + (33*c^2*d^3*e^4 - 83*
b*c*d^2*e^5 + 509*a*b*e^7 + 54*(b^2 + 2*a*c)*d*e^6)*g^2)*m^2 + 12*(3*(7*c^2
*d*e^6 + 164*b*c*e^7)*f^2 - (35*c^2*d^2*e^5 - 84*b*c*d*e^6 - 492*(b^2 + 2*a
*c)*e^7)*f*g + (15*c^2*d^3*e^4 - 35*b*c*d^2*e^5 + 492*a*b*e^7 + 21*(b^2 + 2
*a*c)*d*e^6)*g^2)*m)*x^4 - ((12*b*c*d^4*e^3 + 490*a*b*d^2*e^5 - 1665*a^2*d*
e^6 - 44*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 2*(88*a*b*d^3*e^4 - 245*a^2*d^2*e^5 -
6*(b^2 + 2*a*c)*d^4*e^3)*f*g + 4*(3*a*b*d^4*e^3 - 11*a^2*d^3*e^4)*g^2)*m^3
+ (6720*a*b*e^7*f*g + 1680*a^2*e^7*g^2 + 1680*(b^2 + 2*a*c)*e^7*f^2 + ((2*
b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f^2 + 2*(2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f*
g + (2*a*b*d*e^6 + a^2*e^7)*g^2)*m^6 - ((4*c^2*d^2*e^5 - 42*b*c*d*e^6 - 25*
(b^2 + 2*a*c)*e^7)*f^2 + 2*(8*b*c*d^2*e^5 - 50*a*b*e^7 - 21*(b^2 + 2*a*c)*d
*e^6)*f*g - (42*a*b*d*e^6 + 25*a^2*e^7 - 4*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^5
- ((64*c^2*d^2*e^5 - 326*b*c*d*e^6 - 247*(b^2 + 2*a*c)*e^7)*f^2 - 2*(20*c^2
*d^3*e^4 - 128*b*c*d^2*e^5 + 494*a*b*e^7 + 163*(b^2 + 2*a*c)*d*e^6)*f*g - (
40*b*c*d^3*e^4 + 326*a*b*d*e^6 + 247*a^2*e^7 - 64*(b^2 + 2*a*c)*d^2*e^5)*g^
2)*m^4 - ((332*c^2*d^2*e^5 - 1134*b*c*d*e^6 - 1219*(b^2 + 2*a*c)*e^7)*f^2 -
2*(200*c^2*d^3*e^4 - 664*b*c*d^2*e^5 + 2438*a*b*e^7 + 567*(b^2 + 2*a*c)*d*
e^6)*f*g + (120*c^2*d^4*e^3 - 400*b*c*d^3*e^4 - 1134*a*b*d*e^6 - 1219*a^2*e
^7 + 332*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^3 - 8*((76*c^2*d^2*e^5 - 211*b*c*d*e

^6 - 389*(b^2 + 2*a*c)*e^7)*f^2 - (115*c^2*d^3*e^4 - 304*b*c*d^2*e^5 + 1556*a*b*e^7 + 211*(b^2 + 2*a*c)*d*e^6)*f*g + (45*c^2*d^4*e^3 - 115*b*c*d^3*e^4 - 211*a*b*d*e^6 - 389*a^2*e^7 + 76*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^2 - 4*((84*c^2*d^2*e^5 - 210*b*c*d*e^6 - 949*(b^2 + 2*a*c)*e^7)*f^2 - 2*(70*c^2*d^3*e^4 - 168*b*c*d^2*e^5 + 1898*a*b*e^7 + 105*(b^2 + 2*a*c)*d*e^6)*f*g + (60*c^2*d^4*e^3 - 140*b*c*d^3*e^4 - 210*a*b*d*e^6 - 949*a^2*e^7 + 84*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m)*x^3 + 168*(6*c^2*d^5*e^2 - 15*b*c*d^4*e^3 - 30*a*b*d^2*e^5 + 30*a^2*d*e^6 + 10*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 168*(10*c^2*d^6*e - 24*b*c*d^5*e^2 - 40*a*b*d^3*e^4 + 30*a^2*d^2*e^5 + 15*(b^2 + 2*a*c)*d^4*e^3)*f*g + 24*(30*c^2*d^7 - 70*b*c*d^6*e - 105*a*b*d^4*e^3 + 70*a^2*d^3*e^4 + 42*(b^2 + 2*a*c)*d^5*e^2)*g^2 + 2*((12*c^2*d^5*e^2 - 108*b*c*d^4*e^3 - 1175*a*b*d^2*e^5 + 2552*a^2*d*e^6 + 179*(b^2 + 2*a*c)*d^3*e^4)*f^2 + (48*b*c*d^5*e^2 + 716*a*b*d^3*e^4 - 1175*a^2*d^2*e^5 - 108*(b^2 + 2*a*c)*d^4*e^3)*f*g - (108*a*b*d^4*e^3 - 179*a^2*d^3*e^4 - 12*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m^2 + (5040*a*b*e^7*f^2 + 5040*a^2*e^7*f*g + (a^2*d*e^6*g^2 + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f^2 + 2*(2*a*b*d*e^6 + a^2*e^7)*f*g)*m^6 - ((6*b*c*d^2*e^5 - 52*a*b*e^7 - 23*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(46*a*b*d*e^6 + 26*a^2*e^7 - 3*(b^2 + 2*a*c)*d^2*e^5)*f*g + (6*a*b*d^2*e^5 - 23*a^2*d*e^6)*g^2)*m^5 + 3*((4*c^2*d^3*e^4 - 38*b*c*d^2*e^5 + 180*a*b*e^7 + 67*(b^2 + 2*a*c)*d*e^6)*f^2 + 2*(8*b*c*d^3*e^4 + 134*a*b*d*e^6 + 90*a^2*e^7 - 19*(b^2 + 2*a*c)*d^2*e^5)*f*g - (38*a*b*d^2*e^5 - 67*a^2*d*e^6 - 4*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^4 + ((168*c^2*d^3*e^4 - 750*b*c*d^2*e^5 + 2840*a*b*e^7 + 817*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(60*c^2*d^4*e^3 - 336*b*c*d^3*e^4 - 1634*a*b*d*e^6 - 1420*a^2*e^7 + 375*(b^2 + 2*a*c)*d^2*e^5)*f*g - (120*b*c*d^4*e^3 + 750*a*b*d^2*e^5 - 817*a^2*d*e^6 - 168*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^3 + 2*((330*c^2*d^3*e^4 - 951*b*c*d^2*e^5 + 3929*a*b*e^7 + 739*(b^2 + 2*a*c)*d*e^6)*f^2 - (480*c^2*d^4*e^3 - 1320*b*c*d^3*e^4 - 2956*a*b*d*e^6 - 3929*a^2*e^7 + 951*(b^2 + 2*a*c)*d^2*e^5)*f*g + (180*c^2*d^5*e^2 - 480*b*c*d^4*e^3 - 951*a*b*d^2*e^5 + 739*a^2*d*e^6 + 330*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^2 + 12*((42*c^2*d^3*e^4 - 105*b*c*d^2*e^5 + 879*a*b*e^7 + 70*(b^2 + 2*a*c)*d*e^6)*f^2 - (70*c^2*d^4*e^3 - 168*b*c*d^3*e^4 - 280*a*b*d*e^6 - 879*a^2*e^7 + 105*(b^2 + 2*a*c)*d^2*e^5)*f*g + (30*c^2*d^5*e^2 - 70*b*c*d^4*e^3 - 105*a*b*d^2*e^5 + 70*a^2*d*e^6 + 42*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m)*x^2 + 4*((78*c^2*d^5*e^2 - 321*b*c*d^4*e^3 - 1377*a*b*d^2*e^5 + 2007*a^2*d*e^6 + 319*(b^2 + 2*a*c)*d^3*e^4)*f^2 - (60*c^2*d^6*e - 312*b*c*d^5*e^2 - 1276*a*b*d^3*e^4 + 1377*a^2*d^2*e^5 + 321*(b^2 + 2*a*c)*d^4*e^3)*f*g - (60*b*c*d^6*e + 321*a*b*d^4*e^3 - 319*a^2*d^3*e^4 - 78*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m + (5040*a^2*e^7*f^2 + (2*a^2*d*e^6*f*g + (2*a*b*d*e^6 + a^2*e^7)*f^2)*m^6 - (2*a^2*d^2*e^5*g^2 - (50*a*b*d*e^6 + 27*a^2*e^7 - 2*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(4*a*b*d^2*e^5 - 25*a^2*d*e^6)*f*g)*m^5 + ((12*b*c*d^3*e^4 + 490*a*b*d*e^6 + 295*a^2*e^7 - 44*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 2*(88*a*b*d^2*e^5 - 245*a^2*d*e^6 - 6*(b^2 + 2*a*c)*d^3*e^4)*f*g + 4*(3*a*b*d^3*e^4 - 11*a^2*d^2*e^5)*g^2)*m^4 - ((24*c^2*d^4*e^3 - 216*b*c*d^3*e^4 - 2350*a*b*d*e^6 - 1665*a^2*e^7 + 358*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(48*b*c*d^4*e^3 + 716*a*b*d^2*e^5 - 1175*a^2*d*e^6 - 108*(b^2 + 2*a*c)*d^3*e^4)*f*g - 2*(108*a*b*d^3*e^4 - 179*a^2*d^2*e^5 - 12*(b^2

```
+ 2*a*c)*d^4*e^3)*g^2)*m^3 - 4*((78*c^2*d^4*e^3 - 321*b*c*d^3*e^4 - 1377*a*
b*d*e^6 - 1276*a^2*e^7 + 319*(b^2 + 2*a*c)*d^2*e^5)*f^2 - (60*c^2*d^5*e^2 -
 312*b*c*d^4*e^3 - 1276*a*b*d^2*e^5 + 1377*a^2*d*e^6 + 321*(b^2 + 2*a*c)*d^
3*e^4)*f*g - (60*b*c*d^5*e^2 + 321*a*b*d^3*e^4 - 319*a^2*d^2*e^5 - 78*(b^2
+ 2*a*c)*d^4*e^3)*g^2)*m^2 - 12*((84*c^2*d^4*e^3 - 210*b*c*d^3*e^4 - 420*a*
b*d*e^6 - 669*a^2*e^7 + 140*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 14*(10*c^2*d^5*e^2
 - 24*b*c*d^4*e^3 - 40*a*b*d^2*e^5 + 30*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^
4)*f*g + 2*(30*c^2*d^6*e - 70*b*c*d^5*e^2 - 105*a*b*d^3*e^4 + 70*a^2*d^2*e^
5 + 42*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6
+ 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m +
 5040*e^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74400 vs. 2(537) = 1074.

Time = 11.37 (sec) , antiderivative size = 74400, normalized size of antiderivative = 141.71

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + a*b*f**2*
x**2 + 4*a*b*f*g*x**3/3 + a*b*g**2*x**4/2 + 2*a*c*f**2*x**3/3 + a*c*f*g*x**
4 + 2*a*c*g**2*x**5/5 + b**2*f**2*x**3/3 + b**2*f*g*x**4/2 + b**2*g**2*x**5
/5 + b*c*f**2*x**4/2 + 4*b*c*f*g*x**5/5 + b*c*g**2*x**6/3 + c**2*f**2*x**5/
5 + c**2*f*g*x**6/3 + c**2*g**2*x**7/7), Eq(e, 0)), (-a**2*d**2*e**4*g**2/(
60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
+ 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a**2*d*e**5*f
*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 6*a**2*d*e*
*5*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*
e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 10*a
**2*e**6*f**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d
**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) -
24*a**2*e**6*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 15*a**2*e**6*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*
e**13*x**6) - 2*a*b*d**3*e**3*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d*
*4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**
5 + 60*e**13*x**6) - 4*a*b*d**2*e**4*f*g/(60*d**6*e**7 + 360*d**5*e**8*x +
900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**
12*x**5 + 60*e**13*x**6) - 12*a*b*d**2*e**4*g**2*x/(60*d**6*e**7 + 360*d**5
*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
```

360*d*e**12*x**5 + 60*e**13*x**6) - 4*a*b*d*e**5*f**2/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 24*a*b*d*e**5*f*g*x/(60*d**6*e**7
+ 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e*
*11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 30*a*b*d*e**5*g**2*x**2/(60*
d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 9
00*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 24*a*b*e**6*f**2*x
/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**
3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*b*e**6*f
*g*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e*
*10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 40*a*b
*e**6*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200
*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6)
- 4*a*c*d**4*e**2*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
+ 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) - 4*a*c*d**3*e**3*f*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**
9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60
*e**13*x**6) - 24*a*c*d**3*e**3*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 90
0*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12
*x**5 + 60*e**13*x**6) - 2*a*c*d**2*e**4*f**2/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) - 24*a*c*d**2*e**4*f*g*x/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*c*d**2*e**4*g**2*x**2/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*
d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*a*c*d*e**5*f**2*x/
(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
+ 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*c*d*e**5*
f*g*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e
10*x3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 80*a*
c*d*e**5*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 30*a*c*e**6*f**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
13*x6) - 80*a*c*e**6*f*g*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
+ 60*e**13*x**6) - 60*a*c*e**6*g**2*x**4/(60*d**6*e**7 + 360*d**5*e**8*x +
900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) - 2*b**2*d**4*e**2*g**2/(60*d**6*e**7 + 360*d**5*
e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
360*d*e**12*x**5 + 60*e**13*x**6) - 2*b**2*d**3*e**3*f*g/(60*d**6*e**7 + 36
0*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*
x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*b**2*d**3*e**3*g**2*x/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*
d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - b**2*d**2*e**4*f**2/(

60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
+ 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*b**2*d**2*e*
*4*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e
10*x3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 30*b*
*2*d**2*e**4*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) - 6*b**2*d*e**5*f**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e*
*9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 6
0*e**13*x**6) - 30*b**2*d*e**5*f*g*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) - 40*b**2*d*e**5*g**2*x**3/(60*d**6*e**7 + 360*d**5
*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
 360*d*e**12*x**5 + 60*e**13*x**6) - 15*b**2*e**6*f**2*x**2/(60*d**6*e**7 +
 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**
11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 40*b**2*e**6*f*g*x**3/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*
d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 30*b**2*e**6*g**2*x**
4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x*
*3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 20*b*c*d**5*
e*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**
10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 8*b*c*d
4*e2*f*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d*
*3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 1
20*b*c*d**4*e**2*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**
2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**1
3*x**6) - 2*b*c*d**3*e**3*f**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e
9*x2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 +
60*e**13*x**6) - 48*b*c*d**3*e**3*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) - 300*b*c*d**3*e**3*g**2*x**2/(60*d**6*e**7 + 360*d
5*e8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**
4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*b*c*d**2*e**4*f**2*x/(60*d**6*e*
*7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2
*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 120*b*c*d**2*e**4*f*g*x**
2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x*
*3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 400*b*c*d**2
*e**4*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200
*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6)
- 30*b*c*d*e**5*f**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x
2 + 1200*d3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e*
*13*x**6) - 160*b*c*d*e**5*f*g*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d
4*e9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x*
*5 + 60*e**13*x**6) - 300*b*c*d*e**5*g**2*x**4/(60*d**6*e**7 + 360*d**5*e**
8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360
*d*e**12*x**5 + 60*e**13*x**6) - 40*b*c*e**6*f**2*x**3/(60*d**6*e**7 + 360*

d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 120*b*c*e**6*f*g*x**4/(60*d**6*e**
7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*
e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 120*b*c*e**6*g**2*x**5/(60
*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 60*c**2*d**6*g**2
*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d
3*e10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) +
147*c**2*d**6*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 20*c**2*d**5*e*f*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**
2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**1
3*x**6) + 360*c**2*d**5*e*g**2*x*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) + 822*c**2*d**5*e*g**2*x/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 2*c**2*d**4*e**2*f**2/(60*d**6*e**
7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*
e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 120*c**2*d**4*e**2*f*g*x/(
60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
+ 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 900*c**2*d**4*e
2*g2*x**2*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
13*x6) + 1875*c**2*d**4*e**2*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x
+ 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e
12*x5 + 60*e**13*x**6) - 12*c**2*d**3*e**3*f**2*x/(60*d**6*e**7 + 360*d
5*e8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**
4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 300*c**2*d**3*e**3*f*g*x**2/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*
d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 1200*c**2*d**3*e**3*g
2*x3*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
+ 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*
x**6) + 2200*c**2*d**3*e**3*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900
*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) - 30*c**2*d**2*e**4*f**2*x**2/(60*d**6*e**7 + 360*d**
5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4
+ 360*d*e**12*x**5 + 60*e**13*x**6) - 400*c**2*d**2*e**4*f*g*x**3/(60*d**6*
e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d*
*2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 900*c**2*d**2*e**4*g**2
*x**4*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) + 1350*c**2*d**2*e**4*g**2*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d*
*4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**
5 + 60*e**13*x**6) - 40*c**2*d*e**5*f**2*x**3/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*

d*e**12*x**5 + 60*e**13*x**6) - 300*c**2*d*e**5*f*g*x**4/(60*d**6*e**7 + 36
0*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*
x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 360*c**2*d*e**5*g**2*x**5*log(d/
e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**
10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 360*c**
2*d*e**5*g**2*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 30*c**2*e**6*f**2*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*
e**13*x**6) - 120*c**2*e**6*f*g*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
5 + 60*e13*x**6) + 60*c**2*e**6*g**2*x**6*log(d/e + x)/(60*d**6*e**7 +
360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**1
1*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6), Eq(m, -7)), (-a**2*d**2*e**4*g*
*2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x*
*3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 3*a**2*d*e**5*f*g/(30*d**5*e**7 +
150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x*
*4 + 30*e**12*x**5) - 5*a**2*d*e**5*g**2*x/(30*d**5*e**7 + 150*d**4*e**8*x
+ 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x*
*5) - 6*a**2*e**6*f**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2
 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 15*a**2*e**6*f
*g*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*
x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 10*a**2*e**6*g**2*x**2/(30*d**5*
e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e
11*x4 + 30*e**12*x**5) - 3*a*b*d**3*e**3*g**2/(30*d**5*e**7 + 150*d**4*
e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e
12*x5) - 4*a*b*d**2*e**4*f*g/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3
*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 15*a
*b*d**2*e**4*g**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 +
300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 3*a*b*d*e**5*f**2
/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3
 + 150*d*e**11*x**4 + 30*e**12*x**5) - 20*a*b*d*e**5*f*g*x/(30*d**5*e**7 +
150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x*
*4 + 30*e**12*x**5) - 30*a*b*d*e**5*g**2*x**2/(30*d**5*e**7 + 150*d**4*e**8
*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12
*x**5) - 15*a*b*e**6*f**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9
*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 40*a*b*e*
*6*f*g*x**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2
*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 30*a*b*e**6*g**2*x**3/(30
*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 1
50*d*e**11*x**4 + 30*e**12*x**5) - 12*a*c*d**4*e**2*g**2/(30*d**5*e**7 + 15
0*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4
 + 30*e**12*x**5) - 6*a*c*d**3*e**3*f*g/(30*d**5*e**7 + 150*d**4*e**8*x + 3
00*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5)
 - 60*a*c*d**3*e**3*g**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*

x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 2*a*c*d**2
*e**4*f**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*
e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 30*a*c*d**2*e**4*f*g*x/(30
*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 1
50*d*e**11*x**4 + 30*e**12*x**5) - 120*a*c*d**2*e**4*g**2*x**2/(30*d**5*e**
7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**1
1*x**4 + 30*e**12*x**5) - 10*a*c*d*e**5*f**2*x/(30*d**5*e**7 + 150*d**4*e**
8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**1
2*x**5) - 60*a*c*d*e**5*f*g*x**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3
*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 120*
a*c*d*e**5*g**2*x**3/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 +
 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 20*a*c*e**6*f**2
*x**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10
*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 60*a*c*e**6*f*g*x**3/(30*d**5*e
7 + 150*d4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e*
*11*x**4 + 30*e**12*x**5) - 60*a*c*e**6*g**2*x**4/(30*d**5*e**7 + 150*d**4*
e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e
12*x5) - 6*b**2*d**4*e**2*g**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d*
*3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 3*
b**2*d**3*e**3*f*g/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 3
00*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 30*b**2*d**3*e**3*
g**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**1
0*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - b**2*d**2*e**4*f**2/(30*d**5*e
7 + 150*d4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e*
*11*x**4 + 30*e**12*x**5) - 15*b**2*d**2*e**4*f*g*x/(30*d**5*e**7 + 150*d**
4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30
*e**12*x**5) - 60*b**2*d**2*e**4*g**2*x**2/(30*d**5*e**7 + 150*d**4*e**8*x
+ 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x*
*5) - 5*b**2*d*e**5*f**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*
x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 30*b**2*d*
e**5*f*g*x**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d*
*2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 60*b**2*d*e**5*g**2*x**
3/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**
3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 10*b**2*e**6*f**2*x**2/(30*d**5*e**
7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**1
1*x**4 + 30*e**12*x**5) - 30*b**2*e**6*f*g*x**3/(30*d**5*e**7 + 150*d**4*e*
*8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**
12*x**5) - 30*b**2*e**6*g**2*x**4/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**
3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 60*
b*c*d**5*e*g**2*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**
9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 137*b*c*
d**5*e*g**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2
*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 24*b*c*d**4*e**2*f*g/(30*
d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 15
0*d*e**11*x**4 + 30*e**12*x**5) + 300*b*c*d**4*e**2*g**2*x*log(d/e + x)/(30

*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 1
50*d*e**11*x**4 + 30*e**12*x**5) + 625*b*c*d**4*e**2*g**2*x/(30*d**5*e**7 +
 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x
4 + 30*e12*x**5) - 3*b*c*d**3*e**3*f**2/(30*d**5*e**7 + 150*d**4*e**8*x
 + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x
5) - 120*b*c*d3*e**3*f*g*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e
9*x2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 600*b*
c*d**3*e**3*g**2*x**2*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d*
*3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 11
00*b*c*d**3*e**3*g**2*x**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*
x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 15*b*c*d**
2*e**4*f**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d*
*2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 240*b*c*d**2*e**4*f*g*x
2/(30*d5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x
3 + 150*d*e11*x**4 + 30*e**12*x**5) + 600*b*c*d**2*e**4*g**2*x**3*log(d
/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**
10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 900*b*c*d**2*e**4*g**2*x**3/(
30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 +
 150*d*e**11*x**4 + 30*e**12*x**5) - 30*b*c*d*e**5*f**2*x**2/(30*d**5*e**7
+ 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*
x**4 + 30*e**12*x**5) - 240*b*c*d*e**5*f*g*x**3/(30*d**5*e**7 + 150*d**4*e*
*8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**
12*x**5) + 300*b*c*d*e**5*g**2*x**4*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e
8*x + 300*d3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e*
*12*x**5) + 300*b*c*d*e**5*g**2*x**4/(30*d**5*e**7 + 150*d**4*e**8*x + 300*
d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) -
30*b*c*e**6*f**2*x**3/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2
+ 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 120*b*c*e**6*f*
g*x**4/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**1
0*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 60*b*c*e**6*g**2*x**5*log(d/e
+ x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*
x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 180*c**2*d**6*g**2*log(d/e + x)/
(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3
+ 150*d*e**11*x**4 + 30*e**12*x**5) - 411*c**2*d**6*g**2/(30*d**5*e**7 + 15
0*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4
 + 30*e**12*x**5) + 60*c**2*d**5*e*f*g*log(d/e + x)/(30*d**5*e**7 + 150*d**
4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30
*e**12*x**5) + 137*c**2*d**5*e*f*g/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d*
*3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 90
0*c**2*d**5*e*g**2*x*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**
3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 187
5*c**2*d**5*e*g**2*x/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 +
 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 6*c**2*d**4*e**2
*f**2/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10
*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 300*c**2*d**4*e**2*f*g*x*log(d/

e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**1
0*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 625*c**2*d**4*e**2*f*g*x/(30*d
5*e7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150
*d*e**11*x**4 + 30*e**12*x**5) - 1800*c**2*d**4*e**2*g**2*x**2*log(d/e + x)
/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3
 + 150*d*e**11*x**4 + 30*e**12*x**5) - 3300*c**2*d**4*e**2*g**2*x**2/(30*d*
*5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*
d*e**11*x**4 + 30*e**12*x**5) - 30*c**2*d**3*e**3*f**2*x/(30*d**5*e**7 + 15
0*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4
 + 30*e**12*x**5) + 600*c**2*d**3*e**3*f*g*x**2*log(d/e + x)/(30*d**5*e**7
+ 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*
x**4 + 30*e**12*x**5) + 1100*c**2*d**3*e**3*f*g*x**2/(30*d**5*e**7 + 150*d*
*4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 3
0*e**12*x**5) - 1800*c**2*d**3*e**3*g**2*x**3*log(d/e + x)/(30*d**5*e**7 +
150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x*
*4 + 30*e**12*x**5) - 2700*c**2*d**3*e**3*g**2*x**3/(30*d**5*e**7 + 150*d**
4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30
*e**12*x**5) - 60*c**2*d**2*e**4*f**2*x**2/(30*d**5*e**7 + 150*d**4*e**8*x
+ 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x*
*5) + 600*c**2*d**2*e**4*f*g*x**3*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**
8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**1
2*x**5) + 900*c**2*d**2*e**4*f*g*x**3/(30*d**5*e**7 + 150*d**4*e**8*x + 300
*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) -
 900*c**2*d**2*e**4*g**2*x**4*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x
+ 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x*
*5) - 900*c**2*d**2*e**4*g**2*x**4/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d*
*3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 60
*c**2*d*e**5*f**2*x**3/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2
 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 300*c**2*d*e**
5*f*g*x**4*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**
2 + 300*d**2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) + 300*c**2*d*e*
*5*f*g*x**4/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2
*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 180*c**2*d*e**5*g**2*x**5
*log(d/e + x)/(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d*
*2*e**10*x**3 + 150*d*e**11*x**4 + 30*e**12*x**5) - 30*c**2*e**6*f**2*x**4/
(30*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3
+ 150*d*e**11*x**4 + 30*e**12*x**5) + 60*c**2*e**6*f*g*x**5*log(d/e + x)/(3
0*d**5*e**7 + 150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 +
150*d*e**11*x**4 + 30*e**12*x**5) + 30*c**2*e**6*g**2*x**6/(30*d**5*e**7 +
150*d**4*e**8*x + 300*d**3*e**9*x**2 + 300*d**2*e**10*x**3 + 150*d*e**11*x*
*4 + 30*e**12*x**5), Eq(m, -6)), (-a**2*d**2*e**4*g**2/(12*d**4*e**7 + 48*d
3*e8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 2*a**2*
d*e**5*f*g/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*
x**3 + 12*e**11*x**4) - 4*a**2*d*e**5*g**2*x/(12*d**4*e**7 + 48*d**3*e**8*x
 + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 3*a**2*e**6*f**2/

(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e
11*x4) - 8*a**2*e**6*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**
9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 6*a**2*e**6*g**2*x**2/(12*d**4*
e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4
) - 6*a*b*d**3*e**3*g**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2
 + 48*d*e**10*x**3 + 12*e**11*x**4) - 4*a*b*d**2*e**4*f*g/(12*d**4*e**7 + 4
8*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 24*a
*b*d**2*e**4*g**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48
*d*e**10*x**3 + 12*e**11*x**4) - 2*a*b*d*e**5*f**2/(12*d**4*e**7 + 48*d**3*
e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 16*a*b*d*e*
*5*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x*
*3 + 12*e**11*x**4) - 36*a*b*d*e**5*g**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*
x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 8*a*b*e**6*f**2*
x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12
*e**11*x**4) - 24*a*b*e**6*f*g*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**
2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 24*a*b*e**6*g**2*x**3/(12*
d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11
*x**4) + 24*a*c*d**4*e**2*g**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x
 + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 50*a*c*d**4*e**2*g
2/(12*d4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 +
12*e**11*x**4) - 12*a*c*d**3*e**3*f*g/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d
2*e9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 96*a*c*d**3*e**3*g**2*x*
log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**1
0*x**3 + 12*e**11*x**4) + 176*a*c*d**3*e**3*g**2*x/(12*d**4*e**7 + 48*d**3*
e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 2*a*c*d**2*
e**4*f**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x
3 + 12*e11*x**4) - 48*a*c*d**2*e**4*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*
x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 144*a*c*d**2*e**
4*g**2*x**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2
 + 48*d*e**10*x**3 + 12*e**11*x**4) + 216*a*c*d**2*e**4*g**2*x**2/(12*d**4*
e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4
) - 8*a*c*d*e**5*f**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2
+ 48*d*e**10*x**3 + 12*e**11*x**4) - 72*a*c*d*e**5*f*g*x**2/(12*d**4*e**7 +
 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 96
*a*c*d*e**5*g**2*x**3*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2
*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 96*a*c*d*e**5*g**2*x**3/(12
*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**1
1*x**4) - 12*a*c*e**6*f**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e*
*9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 48*a*c*e**6*f*g*x**3/(12*d**4*
e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4
) + 24*a*c*e**6*g**2*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*
d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 12*b**2*d**4*e**2*g**2*
log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**1
0*x**3 + 12*e**11*x**4) + 25*b**2*d**4*e**2*g**2/(12*d**4*e**7 + 48*d**3*e*
*8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 6*b**2*d**3*e

3*f*g/(12*d4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**
3 + 12*e**11*x**4) + 48*b**2*d**3*e**3*g**2*x*log(d/e + x)/(12*d**4*e**7 +
48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 88*
b**2*d**3*e**3*g**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 +
48*d*e**10*x**3 + 12*e**11*x**4) - b**2*d**2*e**4*f**2/(12*d**4*e**7 + 48*d
3*e8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 24*b**2
*d**2*e**4*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*
e**10*x**3 + 12*e**11*x**4) + 72*b**2*d**2*e**4*g**2*x**2*log(d/e + x)/(12*
d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11
*x**4) + 108*b**2*d**2*e**4*g**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d
2*e9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 4*b**2*d*e**5*f**2*x/(12
*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**1
1*x**4) - 36*b**2*d*e**5*f*g*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*
e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 48*b**2*d*e**5*g**2*x**3*log
(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x
3 + 12*e11*x**4) + 48*b**2*d*e**5*g**2*x**3/(12*d**4*e**7 + 48*d**3*e**
8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 6*b**2*e**6*f*
*2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**
3 + 12*e**11*x**4) - 24*b**2*e**6*f*g*x**3/(12*d**4*e**7 + 48*d**3*e**8*x +
72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 12*b**2*e**6*g**2*x
4*log(d/e + x)/(12*d4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*
e**10*x**3 + 12*e**11*x**4) - 120*b*c*d**5*e*g**2*log(d/e + x)/(12*d**4*e**
7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) -
250*b*c*d**5*e*g**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 4
8*d*e**10*x**3 + 12*e**11*x**4) + 48*b*c*d**4*e**2*f*g*log(d/e + x)/(12*d**
4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x*
*4) + 100*b*c*d**4*e**2*f*g/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x
2 + 48*d*e10*x**3 + 12*e**11*x**4) - 480*b*c*d**4*e**2*g**2*x*log(d/e +
x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 +
12*e**11*x**4) - 880*b*c*d**4*e**2*g**2*x/(12*d**4*e**7 + 48*d**3*e**8*x +
72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 6*b*c*d**3*e**3*f**2
/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*
e**11*x**4) + 192*b*c*d**3*e**3*f*g*x*log(d/e + x)/(12*d**4*e**7 + 48*d**3*
e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 352*b*c*d**
3*e**3*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**1
0*x**3 + 12*e**11*x**4) - 720*b*c*d**3*e**3*g**2*x**2*log(d/e + x)/(12*d**4
*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**
4) - 1080*b*c*d**3*e**3*g**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*
e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 24*b*c*d**2*e**4*f**2*x/(12*
d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11
*x**4) + 288*b*c*d**2*e**4*f*g*x**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e*
*8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 432*b*c*d**2*
e**4*f*g*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**
10*x**3 + 12*e**11*x**4) - 480*b*c*d**2*e**4*g**2*x**3*log(d/e + x)/(12*d**
4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x*

*4) - 480*b*c*d**2*e**4*g**2*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 36*b*c*d*e**5*f**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 192*b*c*d*e**5*f*g*x**3*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 192*b*c*d*e**5*f*g*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 120*b*c*d*e**5*g**2*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 24*b*c*e**6*f**2*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 48*b*c*e**6*f*g*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 24*b*c*e**6*g**2*x**5/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 180*c**2*d**6*g**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 375*c**2*d**6*g**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 120*c**2*d**5*e*f*g*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 250*c**2*d**5*e*f*g/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 720*c**2*d**5*e*g**2*x*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1320*c**2*d**5*e*g**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 12*c**2*d**4*e**2*f**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 25*c**2*d**4*e**2*f**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 480*c**2*d**4*e**2*f*g*x*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 880*c**2*d**4*e**2*f*g*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1080*c**2*d**4*e**2*g**2*x**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1620*c**2*d**4*e**2*g**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 48*c**2*d**3*e**3*f**2*x*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 88*c**2*d**3*e**3*f**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 720*c**2*d**3*e**3*f*g*x**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 1080*c**2*d**3*e**3*f*g*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 720*c**2*d**3*e**3*g**2*x**3*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 720*c**2*d**3*e**3*g**2*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 72*c**2*d**2*e**4*f**2*x**2*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 108*c**2*d**2*e**4*f**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 480*c**2*d**2*e**4*f*g*x**3*log(d/e + x)/(1

2*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**
11*x**4) - 480*c**2*d**2*e**4*f*g*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*
d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 180*c**2*d**2*e**4*g**2
*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*
d*e**10*x**3 + 12*e**11*x**4) + 48*c**2*d*e**5*f**2*x**3*log(d/e + x)/(12*d
4*e7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*
x**4) + 48*c**2*d*e**5*f**2*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e
9*x2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 120*c**2*d*e**5*f*g*x**4*log(
d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x*
*3 + 12*e**11*x**4) - 36*c**2*d*e**5*g**2*x**5/(12*d**4*e**7 + 48*d**3*e**8
*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 12*c**2*e**6*f*
*2*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 4
8*d*e**10*x**3 + 12*e**11*x**4) + 24*c**2*e**6*f*g*x**5/(12*d**4*e**7 + 48*
d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 6*c**2
*e**6*g**2*x**6/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e
10*x3 + 12*e**11*x**4), Eq(m, -5)), (-a**2*d**2*e**4*g**2/(3*d**3*e**7
+ 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - a**2*d*e**5*f*g/(3*d**3*e
7 + 9*d2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*a**2*d*e**5*g**2*x/
(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - a**2*e**6*f*
*2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*a**2*e*
*6*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*a
2*e6*g**2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x
3) + 6*a*b*d3*e**3*g**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d
*e**9*x**2 + 3*e**10*x**3) + 11*a*b*d**3*e**3*g**2/(3*d**3*e**7 + 9*d**2*e*
*8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 4*a*b*d**2*e**4*f*g/(3*d**3*e**7 + 9
*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*a*b*d**2*e**4*g**2*x*log(
d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 27*
a*b*d**2*e**4*g**2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10
*x**3) - a*b*d*e**5*f**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e
10*x3) - 12*a*b*d*e**5*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x*
*2 + 3*e**10*x**3) + 18*a*b*d*e**5*g**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*
d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*a*b*d*e**5*g**2*x**2/(3*d*
*3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*a*b*e**6*f**2*x
/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 12*a*b*e**6
*f*g*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 6*
a*b*e**6*g**2*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**
2 + 3*e**10*x**3) - 24*a*c*d**4*e**2*g**2*log(d/e + x)/(3*d**3*e**7 + 9*d**
2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 44*a*c*d**4*e**2*g**2/(3*d**3*e*
*7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 12*a*c*d**3*e**3*f*g*l
og(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) +
22*a*c*d**3*e**3*f*g/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10
*x**3) - 72*a*c*d**3*e**3*g**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x
+ 9*d*e**9*x**2 + 3*e**10*x**3) - 108*a*c*d**3*e**3*g**2*x/(3*d**3*e**7 + 9
*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 2*a*c*d**2*e**4*f**2/(3*d**3
*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 36*a*c*d**2*e**4*f*

g*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**
3) + 54*a*c*d**2*e**4*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 +
3*e**10*x**3) - 72*a*c*d**2*e**4*g**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d*
*2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 72*a*c*d**2*e**4*g**2*x**2/(3*d
3*e7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 6*a*c*d*e**5*f**
2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 36*a*c*d
*e**5*f*g*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 +
3*e**10*x**3) + 36*a*c*d*e**5*f*g*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e
9*x2 + 3*e**10*x**3) - 24*a*c*d*e**5*g**2*x**3*log(d/e + x)/(3*d**3*e**
7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 6*a*c*e**6*f**2*x**2/(3
*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 12*a*c*e**6*f*
g*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*
x**3) + 6*a*c*e**6*g**2*x**4/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 +
 3*e**10*x**3) - 12*b**2*d**4*e**2*g**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*
e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 22*b**2*d**4*e**2*g**2/(3*d**3*e**
7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 6*b**2*d**3*e**3*f*g*lo
g(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 1
1*b**2*d**3*e**3*f*g/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10
*x**3) - 36*b**2*d**3*e**3*g**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x
 + 9*d*e**9*x**2 + 3*e**10*x**3) - 54*b**2*d**3*e**3*g**2*x/(3*d**3*e**7 +
9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - b**2*d**2*e**4*f**2/(3*d**3
*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*b**2*d**2*e**4*f
*g*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x*
*3) + 27*b**2*d**2*e**4*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2
+ 3*e**10*x**3) - 36*b**2*d**2*e**4*g**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9
*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 36*b**2*d**2*e**4*g**2*x**2/
(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*b**2*d*e**
5*f**2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*
b**2*d*e**5*f*g*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x
2 + 3*e10*x**3) + 18*b**2*d*e**5*f*g*x**2/(3*d**3*e**7 + 9*d**2*e**8*x
+ 9*d*e**9*x**2 + 3*e**10*x**3) - 12*b**2*d*e**5*g**2*x**3*log(d/e + x)/(3*
d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 3*b**2*e**6*f**
2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 6*b**
2*e**6*f*g*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 +
 3*e**10*x**3) + 3*b**2*e**6*g**2*x**4/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e
9*x2 + 3*e**10*x**3) + 60*b*c*d**5*e*g**2*log(d/e + x)/(3*d**3*e**7 + 9
*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 110*b*c*d**5*e*g**2/(3*d**3*
e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 48*b*c*d**4*e**2*f*g
*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3)
- 88*b*c*d**4*e**2*f*g/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**
10*x**3) + 180*b*c*d**4*e**2*g**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8
*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 270*b*c*d**4*e**2*g**2*x/(3*d**3*e**7
+ 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 6*b*c*d**3*e**3*f**2*log(
d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 11*
b*c*d**3*e**3*f**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x

3) - 144*b*c*d3*e**3*f*g*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 216*b*c*d**3*e**3*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 180*b*c*d**3*e**3*g**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 180*b*c*d**3*e**3*g**2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*b*c*d**2*e**4*f**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 27*b*c*d**2*e**4*f**2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 144*b*c*d**2*e**4*f*g*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 144*b*c*d**2*e**4*f*g*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 60*b*c*d**2*e**4*g**2*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*b*c*d*e**5*f**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 18*b*c*d*e**5*f**2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 48*b*c*d*e**5*f*g*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 15*b*c*d*e**5*g**2*x**4/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 6*b*c*e**6*f**2*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 12*b*c*e**6*f*g*x**4/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 3*b*c*e**6*g**2*x**5/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 60*c**2*d**6*g**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 110*c**2*d**6*g**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 60*c**2*d**5*e*f*g*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 110*c**2*d**5*e*f*g/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 180*c**2*d**5*e*g**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 270*c**2*d**5*e*g**2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 12*c**2*d**4*e**2*f**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 22*c**2*d**4*e**2*f**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 180*c**2*d**4*e**2*f*g*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 270*c**2*d**4*e**2*f*g*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 180*c**2*d**4*e**2*g**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 180*c**2*d**4*e**2*g**2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 36*c**2*d**3*e**3*f**2*x*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 54*c**2*d**3*e**3*f**2*x/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 180*c**2*d**3*e**3*f*g*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 180*c**2*d**3*e**3*f*g*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 60*c**2*d**3*e**3*g**2*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 36*c**2*d**2*e**4*f**2*x**2*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 36*c**2*d**2*e**4*f**2*x**2/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + 60*c**2*d**2*e**4*f*g*x**3*log(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9

*d*e**9*x**2 + 3*e**10*x**3) + 15*c**2*d**2*e**4*g**2*x**4/(3*d**3*e**7 + 9
*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 12*c**2*d*e**5*f**2*x**3*log
(d/e + x)/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) - 15
*c**2*d*e**5*f*g*x**4/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**1
0*x**3) - 3*c**2*d*e**5*g**2*x**5/(3*d**3*e**7 + 9*d**2*e**8*x + 9*d*e**9*x
2 + 3*e10*x**3) + 3*c**2*e**6*f**2*x**4/(3*d**3*e**7 + 9*d**2*e**8*x +
9*d*e**9*x**2 + 3*e**10*x**3) + 3*c**2*e**6*f*g*x**5/(3*d**3*e**7 + 9*d**2*
e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3) + c**2*e**6*g**2*x**6/(3*d**3*e**7 +
 9*d**2*e**8*x + 9*d*e**9*x**2 + 3*e**10*x**3), Eq(m, -4)), (12*a**2*d**2*e
4*g2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 18*a**2
*d**2*e**4*g**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 12*a**2*d*e**
5*f*g/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*a**2*d*e**5*g**2*x*l
og(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*a**2*d*e**5*g*
*2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 6*a**2*e**6*f**2/(12*d**
2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 24*a**2*e**6*f*g*x/(12*d**2*e**7 + 2
4*d*e**8*x + 12*e**9*x**2) + 12*a**2*e**6*g**2*x**2*log(d/e + x)/(12*d**2*e
7 + 24*d*e8*x + 12*e**9*x**2) - 72*a*b*d**3*e**3*g**2*log(d/e + x)/(12*
d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 108*a*b*d**3*e**3*g**2/(12*d**2*e
7 + 24*d*e8*x + 12*e**9*x**2) + 48*a*b*d**2*e**4*f*g*log(d/e + x)/(12*d
2*e7 + 24*d*e**8*x + 12*e**9*x**2) + 72*a*b*d**2*e**4*f*g/(12*d**2*e**7
 + 24*d*e**8*x + 12*e**9*x**2) - 144*a*b*d**2*e**4*g**2*x*log(d/e + x)/(12*
d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 144*a*b*d**2*e**4*g**2*x/(12*d**2
*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 12*a*b*d*e**5*f**2/(12*d**2*e**7 + 24
*d*e**8*x + 12*e**9*x**2) + 96*a*b*d*e**5*f*g*x*log(d/e + x)/(12*d**2*e**7
+ 24*d*e**8*x + 12*e**9*x**2) + 96*a*b*d*e**5*f*g*x/(12*d**2*e**7 + 24*d*e*
*8*x + 12*e**9*x**2) - 72*a*b*d*e**5*g**2*x**2*log(d/e + x)/(12*d**2*e**7 +
 24*d*e**8*x + 12*e**9*x**2) - 24*a*b*e**6*f**2*x/(12*d**2*e**7 + 24*d*e**8
*x + 12*e**9*x**2) + 48*a*b*e**6*f*g*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d
*e**8*x + 12*e**9*x**2) + 24*a*b*e**6*g**2*x**3/(12*d**2*e**7 + 24*d*e**8*x
 + 12*e**9*x**2) + 144*a*c*d**4*e**2*g**2*log(d/e + x)/(12*d**2*e**7 + 24*d
*e**8*x + 12*e**9*x**2) + 216*a*c*d**4*e**2*g**2/(12*d**2*e**7 + 24*d*e**8*
x + 12*e**9*x**2) - 144*a*c*d**3*e**3*f*g*log(d/e + x)/(12*d**2*e**7 + 24*d
*e**8*x + 12*e**9*x**2) - 216*a*c*d**3*e**3*f*g/(12*d**2*e**7 + 24*d*e**8*x
 + 12*e**9*x**2) + 288*a*c*d**3*e**3*g**2*x*log(d/e + x)/(12*d**2*e**7 + 24
*d*e**8*x + 12*e**9*x**2) + 288*a*c*d**3*e**3*g**2*x/(12*d**2*e**7 + 24*d*e
8*x + 12*e9*x**2) + 24*a*c*d**2*e**4*f**2*log(d/e + x)/(12*d**2*e**7 +
24*d*e**8*x + 12*e**9*x**2) + 36*a*c*d**2*e**4*f**2/(12*d**2*e**7 + 24*d*e*
*8*x + 12*e**9*x**2) - 288*a*c*d**2*e**4*f*g*x*log(d/e + x)/(12*d**2*e**7 +
 24*d*e**8*x + 12*e**9*x**2) - 288*a*c*d**2*e**4*f*g*x/(12*d**2*e**7 + 24*d
*e**8*x + 12*e**9*x**2) + 144*a*c*d**2*e**4*g**2*x**2*log(d/e + x)/(12*d**2
*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 48*a*c*d*e**5*f**2*x*log(d/e + x)/(12
*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 48*a*c*d*e**5*f**2*x/(12*d**2*e*
*7 + 24*d*e**8*x + 12*e**9*x**2) - 144*a*c*d*e**5*f*g*x**2*log(d/e + x)/(12
*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 48*a*c*d*e**5*g**2*x**3/(12*d**2
*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*a*c*e**6*f**2*x**2*log(d/e + x)/(1

2*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 48*a*c*e**6*f*g*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 12*a*c*e**6*g**2*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 72*b**2*d**4*e**2*g**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 108*b**2*d**4*e**2*g**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 72*b**2*d**3*e**3*f*g*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 108*b**2*d**3*e**3*f*g/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 144*b**2*d**3*e**3*g**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 144*b**2*d**3*e**3*g**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 12*b**2*d**2*e**4*f**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 18*b**2*d**2*e**4*f**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 144*b**2*d**2*e**4*f*g*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 144*b**2*d**2*e**4*f*g*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 72*b**2*d**2*e**4*g**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*b**2*d*e**5*f**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*b**2*d*e**5*f**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 72*b**2*d*e**5*f*g*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 24*b**2*d*e**5*g**2*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 12*b**2*e**6*f**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*b**2*e**6*f*g*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 6*b**2*e**6*g**2*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 240*b*c*d**5*e*g**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 360*b*c*d**5*e*g**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 288*b*c*d**4*e**2*f*g*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 432*b*c*d**4*e**2*f*g/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 480*b*c*d**4*e**2*g**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 480*b*c*d**4*e**2*g**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 72*b*c*d**3*e**3*f**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 108*b*c*d**3*e**3*f**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 576*b*c*d**3*e**3*f*g*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 576*b*c*d**3*e**3*f*g*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 240*b*c*d**3*e**3*g**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 144*b*c*d**2*e**4*f**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 144*b*c*d**2*e**4*f**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 288*b*c*d**2*e**4*f*g*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 80*b*c*d**2*e**4*g**2*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 72*b*c*d*e**5*f**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 96*b*c*d*e**5*f*g*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 20*b*c*d*e**5*g**2*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*b*c*e**6*f**2*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 24*b*c*e**6*f*g*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 8*b*c*e**6*g**2*x**5/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 180*c**2*d**6*g**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 270*c**2*d**6*g**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 240*c**2*d**5*e*f*g*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 360*c**2*d**5*e*f*g/(12* d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 360*c**2*d**5*e*g**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 360*c**2*d**5*e*g**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 72*c**2*d**4*e**2*f**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 108*c**2*d**4*e**2*f**2/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 480*c**2*d**4*e**2*f*g*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 480*c**2*d**4*e**2*f*g*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 180*c**2*d**4*e**2*g**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 144*c**2*d**3*e**3*f**2*x*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 144*c**2*d**3*e**3*f**2*x/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 240*c**2*d**3*e**3*f*g*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 60*c**2*d**3*e**3*g**2*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 72*c**2*d**2*e**4*f**2*x**2*log(d/e + x)/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 80*c**2*d**2*e**4*f*g*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 15*c**2*d**2*e**4*g**2*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 24*c**2*d*e**5*f**2*x**3/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 20*c**2*d*e**5*f*g*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) - 6*c**2*d*e**5*g**2*x**5/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 6*c**2*e**6*f**2*x**4/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 8*c**2*e**6*f*g*x**5/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2) + 3*c**2*e**6*g**2*x**6/(12*d**2*e**7 + 24*d*e**8*x + 12*e**9*x**2), Eq(m, -3)), (-60*a**2*d**2*e**4*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 60*a**2*d**2*e**4*g**2/(30*d*e**7 + 30*e**8*x) + 60*a**2*d*e**5*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 60*a**2*d*e**5*f*g/(30*d*e**7 + 30*e**8*x) - 60*a**2*d*e**5*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 30*a**2*e**6*f**2/(30*d*e**7 + 30*e**8*x) + 60*a**2*e**6*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 30*a**2*e**6*g**2*x**2/(30*d*e**7 + 30*e**8*x) + 180*a*b*d**3*e**3*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 180*a*b*d**3*e**3*g**2/(30*d*e**7 + 30*e**8*x) - 240*a*b*d**2*e**4*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 240*a*b*d**2*e**4*f*g/(30*d*e**7 + 30*e**8*x) + 180*a*b*d**2*e**4*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 60*a*b*d*e**5*f**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 60*a*b*d*e**5*f**2/(30*d*e**7 + 30*e**8*x) - 240*a*b*d*e**5*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 90*a*b*d*e**5*g**2*x**2/(30*d*e**7 + 30*e**8*x) + 60*a*b*e**6*f**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 120*a*b*e**6*f*g*x**2/(30*d*e**7 + 30*e**8*x) + 30*a*b*e**6*g**2*x**3/(30*d*e**7 + 30*e**8*x) - 240*a*c*d**4*e**2*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 240*a*c*d**4*e**2*g**2/(30*d*e**7 + 30*e**8*x) + 360*a*c*d**3*e**3*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 360*a*c*d**3*e**3*f*g/(30*d*e**7 + 30*e**8*x) - 240*a*c*d**3*e**3*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 120*a*c*d**2*e**4*f**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 120*a*c*d**2*e**4*f**2/(30*d*e**7 + 30*e**8*x) + 360*a*c*d**2*e**4*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 120*a*c*d**2*e**4*g**2*x**2/(30*d*e**7 + 30*e**8*x) - 120*a*c*d*e**5*f**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 180*a*c*d*e**5*f*g*x**2/(30*d*e**7 + 30*e**8*x) - 40*a*c*d*e**5*g**2*x**3/(30*d*e**7 + 30*e**8*x) + 60*a*c*e**6*f**2*x**2/(30*d*e**7 + 30*e**8*x) + 60*a*c*e**

6*f*g*x**3/(30*d*e**7 + 30*e**8*x) + 20*a*c*e**6*g**2*x**4/(30*d*e**7 + 30*e**8*x) - 120*b**2*d**4*e**2*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 120*b**2*d**4*e**2*g**2/(30*d*e**7 + 30*e**8*x) + 180*b**2*d**3*e**3*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 180*b**2*d**3*e**3*f*g/(30*d*e**7 + 30*e**8*x) - 120*b**2*d**3*e**3*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 60*b**2*d**2*e**4*f**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 60*b**2*d**2*e**4*f**2/(30*d*e**7 + 30*e**8*x) + 180*b**2*d**2*e**4*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 60*b**2*d**2*e**4*g**2*x**2/(30*d*e**7 + 30*e**8*x) - 60*b**2*d*e**5*f**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 90*b**2*d*e**5*f*g*x**2/(30*d*e**7 + 30*e**8*x) - 20*b**2*d*e**5*g**2*x**3/(30*d*e**7 + 30*e**8*x) + 30*b**2*e**6*f**2*x**2/(30*d*e**7 + 30*e**8*x) + 30*b**2*e**6*f*g*x**3/(30*d*e**7 + 30*e**8*x) + 10*b**2*e**6*g**2*x**4/(30*d*e**7 + 30*e**8*x) + 300*b*c*d**5*e*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 300*b*c*d**5*e*g**2/(30*d*e**7 + 30*e**8*x) - 480*b*c*d**4*e**2*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 480*b*c*d**4*e**2*f*g/(30*d*e**7 + 30*e**8*x) + 300*b*c*d**4*e**2*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 180*b*c*d**3*e**3*f**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 180*b*c*d**3*e**3*f**2/(30*d*e**7 + 30*e**8*x) - 480*b*c*d**3*e**3*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 150*b*c*d**3*e**3*g**2*x**2/(30*d*e**7 + 30*e**8*x) + 180*b*c*d**2*e**4*f**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 240*b*c*d**2*e**4*f*g*x**2/(30*d*e**7 + 30*e**8*x) + 50*b*c*d**2*e**4*g**2*x**3/(30*d*e**7 + 30*e**8*x) - 90*b*c*d*e**5*f**2*x**2/(30*d*e**7 + 30*e**8*x) - 80*b*c*d*e**5*f*g*x**3/(30*d*e**7 + 30*e**8*x) - 25*b*c*d*e**5*g**2*x**4/(30*d*e**7 + 30*e**8*x) + 30*b*c*e**6*f**2*x**3/(30*d*e**7 + 30*e**8*x) + 40*b*c*e**6*f*g*x**4/(30*d*e**7 + 30*e**8*x) + 15*b*c*e**6*g**2*x**5/(30*d*e**7 + 30*e**8*x) - 180*c**2*d**6*g**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 180*c**2*d**6*g**2/(30*d*e**7 + 30*e**8*x) + 300*c**2*d**5*e*f*g*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 300*c**2*d**5*e*f*g/(30*d*e**7 + 30*e**8*x) - 180*c**2*d**5*e*g**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 120*c**2*d**4*e**2*f**2*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 120*c**2*d**4*e**2*f**2/(30*d*e**7 + 30*e**8*x) + 300*c**2*d**4*e**2*f*g*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) + 90*c**2*d**4*e**2*g**2*x**2/(30*d*e**7 + 30*e**8*x) - 120*c**2*d**3*e**3*f**2*x*log(d/e + x)/(30*d*e**7 + 30*e**8*x) - 150*c**2*d**3*e**3*f*g*x**2/(30*d*e**7 + 30*e**8*x) - 30*c**2*d**3*e**3*g**2*x**3/(30*d*e**7 + 30*e**8*x) + 60*c**2*d**2*e**4*f**2*x**2/(30*d*e**7 + 30*e**8*x) + 50*c**2*d**2*e**4*f*g*x**3/(30*d*e**7 + 30*e**8*x) + 15*c**2*d**2*e**4*g**2*x**4/(30*d*e**7 + 30*e**8*x) - 20*c**2*d*e**5*f**2*x**3/(30*d*e**7 + 30*e**8*x) - 25*c**2*d*e**5*f*g*x**4/(30*d*e**7 + 30*e**8*x) - 9*c**2*d*e**5*g**2*x**5/(30*d*e**7 + 30*e**8*x) + 10*c**2*e**6*f**2*x**4/(30*d*e**7 + 30*e**8*x) + 15*c**2*e**6*f*g*x**5/(30*d*e**7 + 30*e**8*x) + 6*c**2*e**6*g**2*x**6/(30*d*e**7 + 30*e**8*x), Eq(m, -2)), (a**2*d**2*g**2*log(d/e + x)/e**3 - 2*a**2*d*f*g*log(d/e + x)/e**2 - a**2*d*g**2*x/e**2 + a**2*f**2*log(d/e + x)/e + 2*a**2*f*g*x/e + a**2*g**2*x**2/(2*e) - 2*a*b*d**3*g**2*log(d/e + x)/e**4 + 4*a*b*d**2*f*g*log(d/e + x)/e**3 + 2*a*b*d**2*g**2*x/e**3 - 2*a*b*d*f**2*log(d/e + x)/e**2 - 4*a*b*d*f*g*x/e**2 - a*b*d*g**2*x**2/e**2 + 2*a*b*f**2*x/e + 2*a*b*f *g*x**2/e + 2*a*b*g**2*x**3/(3*e) + 2*a*c*d**4*g**2*log(d/e + x)/e**5 - 4*a*c*d**3*f*g*log(d/e + x)/e**4 - 2*a*c*d**3*g**2*x/e**4 + 2*a*c*d**2*f**2*log(d/e + x)/e**3 + 4*a*c*d**2*f*g*x/e**3 + a*c*d**2*g**2*x**2/e**3 - 2*a*c*d*f**2*x/e**2 - 2*a*c*d*f*g*x**2/e**2 - 2*a*c*d*g**2*x**3/(3*e**2) + a*c*f**2*x**2/e + 4*a*c*f*g*x**3/(3*e) + a*c*g**2*x**4/(2*e) + b**2*d**4*g**2*log(d/e + x)/e**5 - 2*b**2*d**3*f*g*log(d/e + x)/e**4 - b**2*d**3*g**2*x/e**4 + b**2*d**2*f**2*log(d/e + x)/e**3 + 2*b**2*d**2*f*g*x/e**3 + b**2*d**2*g**2*x**2/(2*e**3) - b**2*d*f**2*x/e**2 - b**2*d*f*g*x**2/e**2 - b**2*d*g**2*x**3/(3*e**2) + b**2*f**2*x**2/(2*e) + 2*b**2*f*g*x**3/(3*e) + b**2*g**2*x**4/(4*e) - 2*b*c*d**5*g**2*log(d/e + x)/e**6 + 4*b*c*d**4*f*g*log(d/e + x)/e**5 + 2*b*c*d**4*g**2*x/e**5 - 2*b*c*d**3*f**2*log(d/e + x)/e**4 - 4*b*c*d**3*f*g*x/e**4 - b*c*d**3*g**2*x**2/e**4 + 2*b*c*d**2*f**2*x/e**3 + 2*b*c*d**2*f*g*x**2/e**3 + 2*b*c*d**2*g**2*x**3/(3*e**3) - b*c*d*f**2*x**2/e**2 - 4*b*c*d*f*g*x**3/(3*e**2) - b*c*d*g**2*x**4/(2*e**2) + 2*b*c*f**2*x**3/(3*e) + b*c*f*g*x**4/e + 2*b*c*g**2*x**5/(5*e) + c**2*d**6*g**2*log(d/e + x)/e**7 - 2*c**2*d**5*f*g*log(d/e + x)/e**6 - c**2*d**5*g**2*x/e**6 + c**2*d**4*f**2*log(d/e + x)/e**5 + 2*c**2*d**4*f*g*x/e**5 + c**2*d**4*g**2*x**2/(2*e**5) - c**2*d**3*f**2*x/e**4 - c**2*d**3*f*g*x**2/e**4 - c**2*d**3*g**2*x**3/(3*e**4) + c**2*d**2*f**2*x**2/(2*e**3) + 2*c**2*d**2*f*g*x**3/(3*e**3) + c**2*d**2*g**2*x**4/(4*e**3) - c**2*d*f**2*x**3/(3*e**2) - c**2*d*f*g*x**4/(2*e**2) - c**2*d*g**2*x**5/(5*e**2) + c**2*f**2*x**4/(4*e) + 2*c**2*f*g*x**5/(5*e) + c**2*g**2*x**6/(6*e), Eq(m, -1)), (2*a**2*d**3*e**4*g**2*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 44*a**2*d**3*e**4*g**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 358*a**2*d**3*e**4*g**2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1276*a**2*d**3*e**4*g**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1680*a**2*d**3*e**4*g**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2*a**2*d**2*e**5*f*g*m**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 50*a**2*d**2*e**5*f*g*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 490*a**2*d**2*e**5*f*g*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2350*a**2*d**2*e**5*f*g*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5508*a**2*d**2*e**5*f*g*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5040*a**2*d**2*e**5*f*g*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +

13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2*a**2*d**2*e**5*g**2*m**5*x
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 44*a**2*d**2
*e**5*g**2*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) - 358*a**2*d**2*e**5*g**2*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) - 1276*a**2*d**2*e**5*g**2*m**2*x*(d + e*x)**m/(e**7*m
7 + 28*e7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1680*a**2*d**2*e**5*g**2*m*x*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + a**2*d*e**6*f**2*
m**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 27*a**2*
d*e**6*f**2*m**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) + 295*a**2*d*e**6*f**2*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 1665*a**2*d*e**6*f**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 5104*a**2*d*e**6*f**2*m**2*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 8028*a**2*d*e**6*f**2*m*(d + e
*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e*
*7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a**2*d*e**6*f*
2(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a**2*d*e
6*f*g*m6*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
+ 50*a**2*d*e**6*f*g*m**5*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 490*a**2*d*e**6*f*g*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 2350*a**2*d*e**6*f*g*m**3*x*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5508*a**2*d*e**6*f*g*m**2*x*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a**2*d*e**6
*f*g*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + a**2
*d*e**6*g**2*m**6*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 23*a**2*d*e**6*g**2*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
6 + 322*e7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) + 201*a**2*d*e**6*g**2*m**4*x**2*(d + e*x)**m/(e**

7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 817*a**2*d*e**6*g**2*m**3*x**2
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1478*a**2*d*
e**6*g**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 840*a2*d*e**6*g**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + a**2*e**7*f**2*m**6*x*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 27*a**2*e**7*f**2*m**5*x*(d + e*x)**m/(e**7*
m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 131
32*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 295*a**2*e**7*f**2*m**4*x*(d + e
*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e*
*7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1665*a**2*e**7*f**2
*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5104*
a**2*e**7*f**2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 8028*a**2*e**7*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 5040*a**2*e**7*f**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 2*a**2*e**7*f*g*m**6*x**2*(d + e*x)**m/(e**7*m
7 + 28*e7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 52*a**2*e**7*f*g*m**5*x**2*(d + e
*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e*
*7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 540*a**2*e**7*f*g*m
4*x2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2840
*a**2*e**7*f*g*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*
m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) + 7858*a**2*e**7*f*g*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 10548*a**2*e**7*f*g*m*x**2*(d + e*x)**m/(e**7*m
7 + 28*e7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a**2*e**7*f*g*x**2*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + a**2*e**7*g**2*m**6*x*
3(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 25*a**2*e*
*7*g**2*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) + 247*a**2*e**7*g**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e

7*m + 5040*e7) + 1219*a**2*e**7*g**2*m**3*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 3112*a**2*e**7*g**2*m**2*x**3*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3796*a**2*e**7*g**2*
m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1680*a
2*e7*g**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 12*a*b*d**4*e**3*g**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) - 216*a*b*d**4*e**3*g**2*m**2*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) - 1284*a*b*d**4*e**3*g**2*m*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2520*a*b*d**4*e**3*g**2*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 8*a*b*d**3*e**4*
f*g*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 176*
a*b*d**3*e**4*f*g*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 1432*a*b*d**3*e**4*f*g*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 5104*a*b*d**3*e**4*f*g*m*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 6720*a*b*d**3*e**4*f*g*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 12*a*b*d**3*e**4*g**2*m**4*x
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 216*a*b*d**3
*e**4*g**2*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) + 1284*a*b*d**3*e**4*g**2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 2520*a*b*d**3*e**4*g**2*m*x*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) - 2*a*b*d**2*e**5*f**2*m**5*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 50*a*b*d**2*e**5*f**2*m*
4(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 490*a*b*d*
*2*e**5*f**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) - 2350*a*b*d**2*e**5*f**2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +

322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e
7*m + 5040*e7) - 5508*a*b*d**2*e**5*f**2*m*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) - 5040*a*b*d**2*e**5*f**2*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 8*a*b*d**2*e**5*f*g*m**5*x*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 176*a*b*d**2*e**
5*f*g*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
1432*a*b*d**2*e**5*f*g*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) - 5104*a*b*d**2*e**5*f*g*m**2*x*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) - 6720*a*b*d**2*e**5*f*g*m*x*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 6*a*b*d**2*e**5*g**2*m**5*x
2*(d + e*x)m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 114*a*b*d
2*e5*g**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) - 750*a*b*d**2*e**5*g**2*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
 + 13068*e**7*m + 5040*e**7) - 1902*a*b*d**2*e**5*g**2*m**2*x**2*(d + e*x)*
*m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m
3 + 13132*e7*m**2 + 13068*e**7*m + 5040*e**7) - 1260*a*b*d**2*e**5*g**2
*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a*b
*d*e**6*f**2*m**6*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 50*a*b*d*e6*f**2*m**5*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 490*a*b*d*e**6*f**2*m**4*x*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 2350*a*b*d*e**6*f**2*m**3*x*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5508*a*b*d*e**6*f**2*m**2*
x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a*b*d*
e**6*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
 4*a*b*d*e**6*f*g*m**6*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 92*a*b*d*e**6*f*g*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2

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+ 13068*e**7*m + 5040*e**7) + 804*a*b*d*e**6*f*g*m**4*x**2*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3268*a*b*d*e**6*f*g*m**3*x**2
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5912*a*b*d*e
**6*f*g*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) + 3360*a*b*d*e**6*f*g*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 32
2*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**
7*m + 5040*e**7) + 2*a*b*d*e**6*g**2*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) + 42*a*b*d*e**6*g**2*m**5*x**3*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 326*a*b*d*e**6*g**2*m**4*
x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1134*a*b
*d*e**6*g**2*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 1688*a*b*d*e**6*g**2*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 840*a*b*d*e**6*g**2*m*x**3*(d + e*x)**m/(e**7*m
**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a*b*e**7*f**2*m**6*x**2*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 52*a*b*e**7*f**2*m**
5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 540*a*
b*e**7*f**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 2840*a*b*e**7*f**2*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 7858*a*b*e**7*f**2*m**2*x**2*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 10548*a*b*e**7*f**2*m*x**2*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a*b*e**7*f**2*x
**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4*a*b*e**
7*f*g*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 100*a*b*e**7*f*g*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 988*a*b*e**7*f*g*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
 + 13068*e**7*m + 5040*e**7) + 4876*a*b*e**7*f*g*m**3*x**3*(d + e*x)**m/(e*
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*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 12448*a*b*e**7*f*g*m**2*x**3*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 15184*a*b*e**
7*f*g*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
6720*a*b*e**7*f*g*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 2*a*b*e**7*g**2*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 48*a*b*e**7*g**2*m**5*x**4*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 452*a*b*e**7*g**2*m**4*x**4*(d + e*x)*
*m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m
**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2112*a*b*e**7*g**2*m**3
*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5090*a*
b*e**7*g**2*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 5904*a*b*e**7*g**2*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 2520*a*b*e**7*g**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*
e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m*
*2 + 13068*e**7*m + 5040*e**7) + 48*a*c*d**5*e**2*g**2*m**2*(d + e*x)**m/(e
**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 624*a*c*d**5*e**2*g**2*m*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2016*a*c*d**5*e*
*2*g**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 24*a*
c*d**4*e**3*f*g*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) - 432*a*c*d**4*e**3*f*g*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) - 2568*a*c*d**4*e**3*f*g*m*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) - 5040*a*c*d**4*e**3*f*g*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 48*a*c*d**4*e**3*g**2*m**3*x*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 624*a*c*d**4*e*
*3*g**2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 2016*a*c*d**4*e**3*g**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
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m + 5040*e**7) + 4*a*c*d**3*e**4*f**2*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 88*a*c*d**3*e**4*f**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 716*a*c*d**3*e**4*f**2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2552*a*c*d**3*e**4*f**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3360*a*c*d**3*e**4*f**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 24*a*c*d**3*e**4*f*g*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 432*a*c*d**3*e**4*f*g*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2568*a*c*d**3*e**4*f*g*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a*c*d**3*e**4*f*g*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 24*a*c*d**3*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 336*a*c*d**3*e**4*g**2*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1320*a*c*d**3*e**4*g**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1008*a*c*d**3*e**4*g**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 4*a*c*d**2*e**5*f**2*m**5*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 88*a*c*d**2*e**5*f**2*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 716*a*c*d**2*e**5*f**2*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2552*a*c*d**2*e**5*f**2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 3360*a*c*d**2*e**5*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 12*a*c*d**2*e**5*f*g*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 228*a*c*d**2*e**5*f*g*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1500*a*c*d**2*e**5*f*g*m**3*x**2*(

d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 3804*a*c*d**2*
e**5*f*g*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 2520*a*c*d**2*e**5*f*g*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) - 8*a*c*d**2*e**5*g**2*m**5*x**3*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 128*a*c*d**2*e**5*g**2*m**4*x**3*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 664*a*c*d**2*e
5*g2*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 1216*a*c*d**2*e**5*g**2*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
6 + 322*e7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 672*a*c*d**2*e**5*g**2*m*x**3*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a*c*d*e**6*f**2*m**6*x**2*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 46*a*c*d*e**6*f*
*2*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
402*a*c*d*e**6*f**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 1634*a*c*d*e**6*f**2*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 2956*a*c*d*e**6*f**2*m**2*x**2*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1680*a*c*d*e**6*f**2*m
*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4*a*c*d
*e**6*f*g*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 84*a*c*d*e6*f*g*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 652*a*c*d*e**6*f*g*m**4*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 2268*a*c*d*e**6*f*g*m**3*x**3*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3376*a*c*d*e**6*f*g*
m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 168
0*a*c*d*e**6*f*g*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) + 2*a*c*d*e**6*g**2*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*

*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 38*a*c*d*e**6*g**2*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 262*a*c*d*e**6*g**2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 802*a*c*d*e**6*g**2*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1080*a*c*d*e**6*g**2*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 504*a*c*d*e**6*g**2*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a*c*e**7*f**2*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 50*a*c*e**7*f**2*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 494*a*c*e**7*f**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2438*a*c*e**7*f**2*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6224*a*c*e**7*f**2*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 7592*a*c*e**7*f**2*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3360*a*c*e**7*f**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4*a*c*e**7*f*g*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 96*a*c*e**7*f*g*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 904*a*c*e**7*f*g*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4224*a*c*e**7*f*g*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 10180*a*c*e**7*f*g*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 11808*a*c*e**7*f*g*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5040*a*c*e**7*f*g*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*a*c*e**7*g**2*m**6*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5

040*e**7) + 46*a*c*e**7*g**2*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) + 414*a*c*e**7*g**2*m**4*x**5*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1850*a*c*e**7*g**2*m**3*x**5*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4288*a*c*e**7*g**2
*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 48
24*a*c*e**7*g**2*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) + 2016*a*c*e**7*g**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 24*b**2*d**5*e**2*g**2*m**2*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 312*b**2*d**5*e**2*g**2*m*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1008*b**2*d**5*e**2*g**2*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 12*b**2*d**4*
e**3*f*g*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
216*b**2*d**4*e**3*f*g*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) - 1284*b**2*d**4*e**3*f*g*m*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) - 2520*b**2*d**4*e**3*f*g*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) - 24*b**2*d**4*e**3*g**2*m**3*x*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 312*b**2*d**4*e**3
*g**2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
1008*b**2*d**4*e**3*g**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 2*b**2*d**3*e**4*f**2*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 44*b**2*d**3*e**4*f**2*m**3*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 358*b**2*d**3*e**4*f**2*m**2*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1276*b**2*d**3
*e**4*f**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
1680*b**2*d**3*e**4*f**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*

m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) + 12*b**2*d**3*e**4*f*g*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 216*b**2*d**3*e**4*f*g*m**3*x*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1284*b**2*d**3*e**4*f*g*m**2*x
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2520*b**2*d*
*3*e**4*f*g*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 12*b**2*d**3*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 168*b**2*d**3*e**4*g**2*m**3*x**2*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 660*b**2*d**3*e**4*g**2*m**2*x*
2(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 504*b**2*d
3*e4*g**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) - 2*b**2*d**2*e**5*f**2*m**5*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) - 44*b**2*d**2*e**5*f**2*m**4*x*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 358*b**2*d**2*e**5*f**2*m**3*x*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1276*b**2*d**2*e
5*f2*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 - 1680*b**2*d**2*e**5*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 32
2*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**
7*m + 5040*e**7) - 6*b**2*d**2*e**5*f*g*m**5*x**2*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) - 114*b**2*d**2*e**5*f*g*m**4*x**2*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 750*b**2*d**2*e**5
*f*g*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 1902*b**2*d**2*e**5*f*g*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) - 1260*b**2*d**2*e**5*f*g*m*x**2*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 4*b**2*d**2*e**5*g**2*m**5*x**3*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 64*b**2*d**2*e*
*5*g**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +

1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) - 332*b**2*d**2*e**5*g**2*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) - 608*b**2*d**2*e**5*g**2*m**2*x**3*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 336*b**2*d**2*e**5*g**2*m*x*
3(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + b**2*d*e**
6*f**2*m**6*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) + 23*b**2*d*e**6*f**2*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e
7*m + 5040*e7) + 201*b**2*d*e**6*f**2*m**4*x**2*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e
7*m2 + 13068*e**7*m + 5040*e**7) + 817*b**2*d*e**6*f**2*m**3*x**2*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1478*b**2*d*e**6*f
2*m2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
840*b**2*d*e**6*f**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 2*b**2*d*e**6*f*g*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 42*b**2*d*e**6*f*g*m**5*x**3*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 326*b**2*d*e**6*f*g*m**4*x**
3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1134*b**2*d
*e**6*f*g*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 1688*b2*d*e**6*f*g*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 840*b**2*d*e**6*f*g*m*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e
7*m2 + 13068*e**7*m + 5040*e**7) + b**2*d*e**6*g**2*m**6*x**4*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 19*b**2*d*e**6*g**2*m*
*5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 131*b
2*d*e6*g**2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) + 401*b**2*d*e**6*g**2*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 540*b**2*d*e**6*g**2*m**2*x**4*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3

+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 252*b**2*d*e**6*g**2*m*x**
4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + b**2*e**7*f
2*m6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
 25*b**2*e**7*f**2*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 247*b**2*e**7*f**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e
7*m6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**
2 + 13068*e**7*m + 5040*e**7) + 1219*b**2*e**7*f**2*m**3*x**3*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3112*b**2*e**7*f**2*m**2*x
3*(d + e*x)m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3796*b**2
*e**7*f**2*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) + 1680*b**2*e**7*f**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 2*b**2*e**7*f*g*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 48*b**2*e**7*f*g*m**5*x**4*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 452*b**2*e**7*f*g*m**4*x**4*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2112*b**2*e**7*f
*g*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
5090*b**2*e**7*f*g*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 5904*b**2*e**7*f*g*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
 13068*e**7*m + 5040*e**7) + 2520*b**2*e**7*f*g*x**4*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) + b**2*e**7*g**2*m**6*x**5*(d + e*x)*
*m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m
3 + 13132*e7*m**2 + 13068*e**7*m + 5040*e**7) + 23*b**2*e**7*g**2*m**5*
x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 207*b**2
*e**7*g**2*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 925*b**2*e**7*g**2*m**3*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) + 2144*b**2*e**7*g**2*m**2*x**5*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2412*b**2*e**7*g**2*m*x**5*(d + e*

x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1008*b**2*e**7*g**2*
x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 240*b*c*
d**6*e*g**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
1680*b*c*d**6*e*g**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 96*b*c*d**5*e**2*f*g*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 1248*b*c*d**5*e**2*f*g*m*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 4032*b*c*d**5*e**2*f*g*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 240*b*c*d**5*e**2*g**2*m**2*x*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1680*b*c*d**5*
e**2*g**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
12*b*c*d**4*e**3*f**2*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) - 216*b*c*d**4*e**3*f**2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) - 1284*b*c*d**4*e**3*f**2*m*(d + e*x)**m/(e**7*
m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 131
32*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2520*b*c*d**4*e**3*f**2*(d + e*x
)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7
*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 96*b*c*d**4*e**3*f*g*
m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1248*b
*c*d**4*e**3*f*g*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) - 4032*b*c*d**4*e**3*f*g*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) - 120*b*c*d**4*e**3*g**2*m**3*x**2*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 960*b*c*d**4*e**3*g**2*m**2*x*
2(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 840*b*c*d*
*4*e**3*g**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 12*b*c*d3*e**4*f**2*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 216*b*c*d**3*e**4*f**2*m**3*x*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*

e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1284*b*c*d**3*e**4*f**2*m**2*x*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2520*b*c*d**3*e**
4*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 48
*b*c*d**3*e**4*f*g*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 672*b*c*d**3*e**4*f*g*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 2640*b*c*d**3*e**4*f*g*m**2*x**2*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2016*b*c*d**3*e**4*f
*g*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 40*
b*c*d**3*e**4*g**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 400*b*c*d**3*e**4*g**2*m**3*x**3*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) + 920*b*c*d**3*e**4*g**2*m**2*x**3*(d + e
*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e*
*7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 560*b*c*d**3*e**4*g
2*m*x3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 6*
b*c*d**2*e**5*f**2*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) - 114*b*c*d**2*e**5*f**2*m**4*x**2*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) - 750*b*c*d**2*e**5*f**2*m**3*x**2*(d + e
*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e*
*7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1902*b*c*d**2*e**5*
f**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 1260*b*c*d**2*e**5*f**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) - 16*b*c*d**2*e**5*f*g*m**5*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) - 256*b*c*d**2*e**5*f*g*m**4*x**3*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1328*b*c*d**2*e**5
*f*g*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 2432*b*c*d**2*e**5*f*g*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) - 1344*b*c*d**2*e**5*f*g*m*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e

7*m2 + 13068*e**7*m + 5040*e**7) - 10*b*c*d**2*e**5*g**2*m**5*x**4*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 130*b*c*d**2*e**5
*g**2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
- 530*b*c*d**2*e**5*g**2*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) - 830*b*c*d**2*e**5*g**2*m**2*x**4*(d + e*x)**m/(e**7*
m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 131
32*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 420*b*c*d**2*e**5*g**2*m*x**4*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*b*c*d*e**6*f*
*2*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
42*b*c*d*e**6*f**2*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 326*b*c*d*e**6*f**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*
e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m*
*2 + 13068*e**7*m + 5040*e**7) + 1134*b*c*d*e**6*f**2*m**3*x**3*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1688*b*c*d*e**6*f**2*m**
2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 840*b*
c*d*e**6*f**2*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 4*b*c*d*e**6*f*g*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 76*b*c*d*e**6*f*g*m**5*x**4*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 524*b*c*d*e**6*f*g*m**4*x**4*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1604*b*c*d*e**6*f*g*m*
*3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2160*
b*c*d*e**6*f*g*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*
m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) + 1008*b*c*d*e**6*f*g*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) + 2*b*c*d*e**6*g**2*m**6*x**5*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 34*b*c*d*e**6*g**2*m**5*x**5*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 210*b*c*d*e**6*g**
2*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5

90*b*c*d*e**6*g**2*m**3*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 748*b*c*d*e**6*g**2*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*
e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m*
*2 + 13068*e**7*m + 5040*e**7) + 336*b*c*d*e**6*g**2*m*x**5*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2*b*c*e**7*f**2*m**6*x**4*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 48*b*c*e**7*f**
2*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4
52*b*c*e**7*f**2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) + 2112*b*c*e**7*f**2*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 5090*b*c*e**7*f**2*m**2*x**4*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5904*b*c*e**7*f**2*m*x**4*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2520*b*c*e**7*f*
*2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4*b*c
*e**7*f*g*m**6*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
7) + 92*b*c*e7*f*g*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 828*b*c*e**7*f*g*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 3700*b*c*e**7*f*g*m**3*x**5*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 8576*b*c*e**7*f*g*m**2*x**
5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 9648*b*c*e*
*7*f*g*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
 4032*b*c*e**7*f*g*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
5 + 1960*e7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) + 2*b*c*e**7*g**2*m**6*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) + 44*b*c*e**7*g**2*m**5*x**6*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 380*b*c*e**7*g**2*m**4*x**6*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1640*b*c*e**7*g**2*m**
3*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m

4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3698*b
*c*e**7*g**2*m**2*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 4076*b*c*e**7*g**2*m*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 1680*b*c*e**7*g**2*x**6*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 720*c**2*d**7*g**2*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) - 240*c**2*d**6*e*f*g*m*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1680*c**2*d**6*e*f*g*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
7*m3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 720*c**2*d**6*e*g*
*2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
4 + 6769*e7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 24*c**
2*d**5*e**2*f**2*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 312*c**2*d**5*e**2*f**2*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 1008*c**2*d**5*e**2*f**2*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
2 + 13068*e7*m + 5040*e**7) + 240*c**2*d**5*e**2*f*g*m**2*x*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1680*c**2*d**5*e**2*f*g*
m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 360*c**2*
d**5*e**2*g**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*
m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) + 360*c**2*d**5*e**2*g**2*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) - 24*c**2*d**4*e**3*f**2*m**3*x*(d + e*x)**m/(e
7*m7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 312*c**2*d**4*e**3*f**2*m**2
x(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1008*c**2*
d**4*e**3*f**2*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 120*c**2*d**4*e**3*f*g*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) - 960*c**2*d**4*e**3*f*g*m**2*x**2*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 840*c**2*d**4*e**3*f*g*m*x**2
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 120*c**2*d**

4*e**3*g**2*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) - 360*c**2*d**4*e**3*g**2*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) - 240*c**2*d**4*e**3*g**2*m*x**3*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 12*c**2*d**3*e**4*f**2*m**4
*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 168*c**
2*d**3*e**4*f**2*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) + 660*c**2*d**3*e**4*f**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 504*c**2*d**3*e**4*f**2*m*x**2*(d + e*x)
m/(e7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 40*c**2*d**3*e**4*f*g*
m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 400
*c**2*d**3*e**4*f*g*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 920*c**2*d**3*e**4*f*g*m**2*x**3*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 560*c**2*d**3*e**4*f*g*m*x**3*(d + e*x
)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7
*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 30*c**2*d**3*e**4*g**
2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1
80*c**2*d**3*e**4*g**2*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 330*c**2*d**3*e**4*g**2*m**2*x**4*(d + e*x)**m/(e**7*m*
*7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132
*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 180*c**2*d**3*e**4*g**2*m*x**4*(d
+ e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769
*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 4*c**2*d**2*e**5
*f**2*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 - 64*c**2*d**2*e**5*f**2*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) - 332*c**2*d**2*e**5*f**2*m**3*x**3*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 608*c**2*d**2*e**5*f**2*m**2*x*
3(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 336*c**2*d
2*e5*f**2*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*

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e**7) - 10*c**2*d**2*e**5*f*g*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 130*c**2*d**2*e**5*f*g*m**4*x**4*(d + e*x)**m/(e
**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 530*c**2*d**2*e**5*f*g*m**3*
x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 830*c**2
*d**2*e**5*f*g*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*
m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) - 420*c**2*d**2*e**5*f*g*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
 13068*e**7*m + 5040*e**7) - 6*c**2*d**2*e**5*g**2*m**5*x**5*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 60*c**2*d**2*e**5*g**2*m**4
*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 210*c**
2*d**2*e**5*g**2*m**3*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) - 300*c**2*d**2*e**5*g**2*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) - 144*c**2*d**2*e**5*g**2*m*x**5*(d + e*x)
**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + c**2*d*e**6*f**2*m**6*
x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 19*c**2*
d*e**6*f**2*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 131*c**2*d*e**6*f**2*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) + 401*c**2*d*e**6*f**2*m**3*x**4*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 540*c**2*d*e**6*f**2*m**2*x**4
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 252*c**2*d*e
**6*f**2*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 2*c**2*d*e**6*f*g*m**6*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322
*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7
*m + 5040*e**7) + 34*c**2*d*e**6*f*g*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) + 210*c**2*d*e**6*f*g*m**4*x**5*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 590*c**2*d*e**6*f*g*m**3
*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m*
*4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 748*c**
```

2*d*e**6*f*g*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) + 336*c**2*d*e**6*f*g*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) + c**2*d*e**6*g**2*m**6*x**6*(d + e*x)**m/(e**7*m**7
 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 15*c**2*d*e**6*g**2*m**5*x**6*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 85*c**2*d*e**6*g**2*
m**4*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 225
*c**2*d*e**6*g**2*m**3*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 274*c**2*d*e**6*g**2*m**2*x**6*(d + e*x)**m/(e**7*m**7 + 28*
e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m*
*2 + 13068*e**7*m + 5040*e**7) + 120*c**2*d*e**6*g**2*m*x**6*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + c**2*e**7*f**2*m**6*x**5*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 23*c**2*e**7*f*
*2*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
207*c**2*e**7*f**2*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
7*m5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 925*c**2*e**7*f**2*m**3*x**5*(d + e*x)**m/(e**7*m**7 + 28*e
7*m6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**
2 + 13068*e**7*m + 5040*e**7) + 2144*c**2*e**7*f**2*m**2*x**5*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2412*c**2*e**7*f**2*m*x**5
*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1008*c**2*e*
*7*f**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
2*c**2*e**7*f*g*m**6*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) + 44*c**2*e**7*f*g*m**5*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
6 + 322*e7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) + 380*c**2*e**7*f*g*m**4*x**6*(d + e*x)**m/(e**7*m
7 + 28*e7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1640*c**2*e**7*f*g*m**3*x**6*(d +
 e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3698*c**2*e**7*f*
g*m**2*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
7*m4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4
076*c**2*e**7*f*g*m*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*

```
m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5
040*e**7) + 1680*c**2*e**7*f*g*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + c**2*e**7*g**2*m**6*x**7*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 21*c**2*e**7*g**2*m**5*x**7*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 175*c**2*e**7*g**2*m**4*x
**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 735*c**2*
e**7*g**2*m**3*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
**7) + 1624*c**2*e**7*g**2*m**2*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) + 1764*c**2*e**7*g**2*m*x**7*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 720*c**2*e**7*g**2*x**7*(d + e*x)**m/
(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. $2(525) = 1050$.

Time = 0.28 (sec) , antiderivative size = 2034, normalized size of antiderivative = 3.87

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f^2/((m^2 + 3*m + 2)*e^
2) + 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*f*g/((m^2 + 3*m +
2)*e^2) + (e*x + d)^(m + 1)*a^2*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3
+ (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f^2/((m^3 + 6*
m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2
*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 4*
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
+ d)^m*a*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^3*x^3 +
(m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*g^2/((m^3 + 6*m^
2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2
*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*
b*c*f^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m
+ 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*
d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*e^4) + 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3
```

```
 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*f*g/((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4
+ (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6
*d^4)*(e*x + d)^m*a*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^
4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d
^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2*f^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 +
274*m + 120)*e^5) + 4*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 +
 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(
m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*f*g/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((m^4 + 10*m^3 + 35*m^2 + 50
*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2
+ 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x
 + d)^m*b^2*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*(
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*
d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 -
24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*a*c*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^
2 + 274*m + 120)*e^5) + 2*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*
e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^
3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^
2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*f*g/((m^6 + 2
1*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 2*((m^5 + 15*m^
4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m
^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3
 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 12
0*d^6)*(e*x + d)^m*b*c*g^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 +
1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
 + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6
*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*
m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360
*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^2*g^2/((m^7
 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10486 vs. 2(525) = 1050.

Time = 0.36 (sec) , antiderivative size = 10486, normalized size of antiderivative = 19.97

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

[Out] ((e*x + d)^m*c^2*e^7*g^2*m^6*x^7 + 2*(e*x + d)^m*c^2*e^7*f*g*m^6*x^6 + (e*x + d)^m*c^2*d*e^6*g^2*m^6*x^6 + 2*(e*x + d)^m*b*c*e^7*g^2*m^6*x^6 + 21*(e*x + d)^m*c^2*e^7*g^2*m^5*x^7 + (e*x + d)^m*c^2*e^7*f^2*m^6*x^5 + 2*(e*x + d)^m*c^2*d*e^6*f*g*m^6*x^5 + 4*(e*x + d)^m*b*c*e^7*f*g*m^6*x^5 + 2*(e*x + d)^m*b*c*d*e^6*g^2*m^6*x^5 + (e*x + d)^m*b^2*e^7*g^2*m^6*x^5 + 2*(e*x + d)^m*a*c*e^7*g^2*m^6*x^5 + 44*(e*x + d)^m*c^2*e^7*f*g*m^5*x^6 + 15*(e*x + d)^m*c^2*d*e^6*g^2*m^5*x^6 + 44*(e*x + d)^m*b*c*e^7*g^2*m^5*x^6 + 175*(e*x + d)^m*c^2*e^7*g^2*m^4*x^7 + (e*x + d)^m*c^2*d*e^6*f^2*m^6*x^4 + 2*(e*x + d)^m*b*c*e^7*f^2*m^6*x^4 + 4*(e*x + d)^m*b*c*d*e^6*f*g*m^6*x^4 + 2*(e*x + d)^m*b^2*e^7*f*g*m^6*x^4 + 4*(e*x + d)^m*a*c*e^7*f*g*m^6*x^4 + (e*x + d)^m*b^2*d*e^6*g^2*m^6*x^4 + 2*(e*x + d)^m*a*c*d*e^6*g^2*m^6*x^4 + 2*(e*x + d)^m*a*b*e^7*g^2*m^6*x^4 + 23*(e*x + d)^m*c^2*e^7*f^2*m^5*x^5 + 34*(e*x + d)^m*c^2*d*e^6*f*g*m^5*x^5 + 92*(e*x + d)^m*b*c*e^7*f*g*m^5*x^5 - 6*(e*x + d)^m*c^2*d^2*e^5*g^2*m^5*x^5 + 34*(e*x + d)^m*b*c*d*e^6*g^2*m^5*x^5 + 23*(e*x + d)^m*b^2*e^7*g^2*m^5*x^5 + 46*(e*x + d)^m*a*c*e^7*g^2*m^5*x^5 + 380*(e*x + d)^m*c^2*e^7*f*g*m^4*x^6 + 85*(e*x + d)^m*c^2*d*e^6*g^2*m^4*x^6 + 380*(e*x + d)^m*b*c*e^7*g^2*m^4*x^6 + 735*(e*x + d)^m*c^2*e^7*g^2*m^3*x^7 + 2*(e*x + d)^m*b*c*d*e^6*f^2*m^6*x^3 + (e*x + d)^m*b^2*e^7*f^2*m^6*x^3 + 2*(e*x + d)^m*a*c*e^7*f^2*m^6*x^3 + 2*(e*x + d)^m*b^2*d*e^6*f*g*m^6*x^3 + 4*(e*x + d)^m*a*c*d*e^6*f*g*m^6*x^3 + 4*(e*x + d)^m*a*b*e^7*f*g*m^6*x^3 + 2*(e*x + d)^m*a*b*d*e^6*g^2*m^6*x^3 + (e*x + d)^m*a^2*e^7*g^2*m^6*x^3 + 19*(e*x + d)^m*c^2*d*e^6*f^2*m^5*x^4 + 48*(e*x + d)^m*b*c*e^7*f^2*m^5*x^4 - 10*(e*x + d)^m*c^2*d^2*e^5*f*g*m^5*x^4 + 76*(e*x + d)^m*b*c*d*e^6*f*g*m^5*x^4 + 48*(e*x + d)^m*b^2*e^7*f*g*m^5*x^4 + 96*(e*x + d)^m*a*c*e^7*f*g*m^5*x^4 - 10*(e*x + d)^m*b*c*d^2*e^5*g^2*m^5*x^4 + 19*(e*x + d)^m*b^2*d*e^6*g^2*m^5*x^4 + 38*(e*x + d)^m*a*c*d*e^6*g^2*m^5*x^4 + 48*(e*x + d)^m*a*b*e^7*g^2*m^5*x^4 + 207*(e*x + d)^m*c^2*e^7*f^2*m^4*x^5 + 210*(e*x + d)^m*c^2*d*e^6*f*g*m^4*x^5 + 828*(e*x + d)^m*b*c*e^7*f*g*m^4*x^5 - 60*(e*x + d)^m*c^2*d^2*e^5*g^2*m^4*x^5 + 210*(e*x + d)^m*b*c*d*e^6*g^2*m^4*x^5 + 207*(e*x + d)^m*b^2*e^7*g^2*m^4*x^5 + 414*(e*x + d)^m*a*c*e^7*g^2*m^4*x^5 + 1640*(e*x + d)^m*c^2*e^7*f*g*m^3*x^6 + 225*(e*x + d)^m*c^2*d*e^6*g^2*m^3*x^6 + 1640*(e*x + d)^m*b*c*e^7*g^2*m^3*x^6 + 1624*(e*x + d)^m*c^2*e^7*g^2*m^2*x^7 + (e*x + d)^m*b^2*d*e^6*f^2*m^6*x^2 + 2*(e*x + d)^m*a*c*d*e^6*f^2*m^6*x^2 + 2*(e*x + d)^m*a*b*e^7*f^2*m^6*x^2 + 4*(e*x + d)^m*a*b*d*e^6*f*g*m^6*x^2 + 2*(e*x + d)^m*a^2*e^7*f*g*m^6*x^2 + (e*x + d)^m*a^2*d*e^6*g^2*m^6*x^2 - 4*(e*x + d)^m*c^2*d^2*e^5*f^2*m^5*x^3 + 42*(e*x + d)^m*b*c*d*e^6*f^2*m^5*x^3 + 25*(e*x + d)^m*b^2*e^7*f^2*m^5*x^3 + 50*(e*x + d)^m*a*c*e^7*f^2*m^5*x^3 - 16*(e*x + d)^m*b*c*d^2*e^5*f*g*m^5*x^3 + 42*(e*x + d)^m*b^2*d*e^6*f*g*m^5*x^3 + 84*(e*x + d)^m*a*c*d*e^6*f*g*m^5*x^3 + 100*(e*x + d)^m*a*b*e^7*f*g*m^5*x^3 - 4*(e*x + d)^m*b^2*d^2*e^5*g^2*m^5*x^3 - 8*(e*x + d)^m*a*c*d^2*e^5*g^2*m^5*x^3 + 42*(e*x + d)^m*a*b*d*e^6*g^2*m^5*x^3 + 25*(e*x + d)^m*a^2*e^7*g^2*m^5*x^3 + 131*(e*x + d)^m*c^2*d*e^6*f^2*m^4*x^4 + 452*(e*x + d)^m*b*c*e^7*f^2*m^4*x^4 - 130*(e*x + d)^m*c^2*d^2*e^5*f*g*m^4*x^4 + 524*(e*x + d)^m*b*c*d*e^6*f*g*m^4*x^4 + 452*(e*x + d)^m*b^2*e^7*f*g*m^4*x^4 + 904*(e*x + d)^m*a*c*e^7*f*g*m^4*x^4 + 30*(e*x + d)^m*c^2*d^3*e^4*g^2*m^4*x^4 - 130*(e*x + d)^m*b*c*d^2*e^5*g^2*m^4*x^4 + 131*

(e*x + d)^m*b^2*d*e^6*g^2*m^4*x^4 + 262*(e*x + d)^m*a*c*d*e^6*g^2*m^4*x^4 +
 452*(e*x + d)^m*a*b*e^7*g^2*m^4*x^4 + 925*(e*x + d)^m*c^2*e^7*f^2*m^3*x^5
+ 590*(e*x + d)^m*c^2*d*e^6*f*g*m^3*x^5 + 3700*(e*x + d)^m*b*c*e^7*f*g*m^3*
x^5 - 210*(e*x + d)^m*c^2*d^2*e^5*g^2*m^3*x^5 + 590*(e*x + d)^m*b*c*d*e^6*g
^2*m^3*x^5 + 925*(e*x + d)^m*b^2*e^7*g^2*m^3*x^5 + 1850*(e*x + d)^m*a*c*e^7
*g^2*m^3*x^5 + 3698*(e*x + d)^m*c^2*e^7*f*g*m^2*x^6 + 274*(e*x + d)^m*c^2*d
*e^6*g^2*m^2*x^6 + 3698*(e*x + d)^m*b*c*e^7*g^2*m^2*x^6 + 1764*(e*x + d)^m*
c^2*e^7*g^2*m*x^7 + 2*(e*x + d)^m*a*b*d*e^6*f^2*m^6*x + (e*x + d)^m*a^2*e^7
*f^2*m^6*x + 2*(e*x + d)^m*a^2*d*e^6*f*g*m^6*x - 6*(e*x + d)^m*b*c*d^2*e^5*
f^2*m^5*x^2 + 23*(e*x + d)^m*b^2*d*e^6*f^2*m^5*x^2 + 46*(e*x + d)^m*a*c*d*e
^6*f^2*m^5*x^2 + 52*(e*x + d)^m*a*b*e^7*f^2*m^5*x^2 - 6*(e*x + d)^m*b^2*d^2
*e^5*f*g*m^5*x^2 - 12*(e*x + d)^m*a*c*d^2*e^5*f*g*m^5*x^2 + 92*(e*x + d)^m*
a*b*d*e^6*f*g*m^5*x^2 + 52*(e*x + d)^m*a^2*e^7*f*g*m^5*x^2 - 6*(e*x + d)^m*
a*b*d^2*e^5*g^2*m^5*x^2 + 23*(e*x + d)^m*a^2*d*e^6*g^2*m^5*x^2 - 64*(e*x +
d)^m*c^2*d^2*e^5*f^2*m^4*x^3 + 326*(e*x + d)^m*b*c*d*e^6*f^2*m^4*x^3 + 247*
(e*x + d)^m*b^2*e^7*f^2*m^4*x^3 + 494*(e*x + d)^m*a*c*e^7*f^2*m^4*x^3 + 40*
(e*x + d)^m*c^2*d^3*e^4*f*g*m^4*x^3 - 256*(e*x + d)^m*b*c*d^2*e^5*f*g*m^4*x
^3 + 326*(e*x + d)^m*b^2*d*e^6*f*g*m^4*x^3 + 652*(e*x + d)^m*a*c*d*e^6*f*g*
m^4*x^3 + 988*(e*x + d)^m*a*b*e^7*f*g*m^4*x^3 + 40*(e*x + d)^m*b*c*d^3*e^4*
g^2*m^4*x^3 - 64*(e*x + d)^m*b^2*d^2*e^5*g^2*m^4*x^3 - 128*(e*x + d)^m*a*c*
d^2*e^5*g^2*m^4*x^3 + 326*(e*x + d)^m*a*b*d*e^6*g^2*m^4*x^3 + 247*(e*x + d)
^m*a^2*e^7*g^2*m^4*x^3 + 401*(e*x + d)^m*c^2*d*e^6*f^2*m^3*x^4 + 2112*(e*x
+ d)^m*b*c*e^7*f^2*m^3*x^4 - 530*(e*x + d)^m*c^2*d^2*e^5*f*g*m^3*x^4 + 1604
*(e*x + d)^m*b*c*d*e^6*f*g*m^3*x^4 + 2112*(e*x + d)^m*b^2*e^7*f*g*m^3*x^4 +
 4224*(e*x + d)^m*a*c*e^7*f*g*m^3*x^4 + 180*(e*x + d)^m*c^2*d^3*e^4*g^2*m^3
x^4 - 530(e*x + d)^m*b*c*d^2*e^5*g^2*m^3*x^4 + 401*(e*x + d)^m*b^2*d*e^6*
g^2*m^3*x^4 + 802*(e*x + d)^m*a*c*d*e^6*g^2*m^3*x^4 + 2112*(e*x + d)^m*a*b*
e^7*g^2*m^3*x^4 + 2144*(e*x + d)^m*c^2*e^7*f^2*m^2*x^5 + 748*(e*x + d)^m*c^
2*d*e^6*f*g*m^2*x^5 + 8576*(e*x + d)^m*b*c*e^7*f*g*m^2*x^5 - 300*(e*x + d)^
m*c^2*d^2*e^5*g^2*m^2*x^5 + 748*(e*x + d)^m*b*c*d*e^6*g^2*m^2*x^5 + 2144*(e
*x + d)^m*b^2*e^7*g^2*m^2*x^5 + 4288*(e*x + d)^m*a*c*e^7*g^2*m^2*x^5 + 4076
*(e*x + d)^m*c^2*e^7*f*g*m*x^6 + 120*(e*x + d)^m*c^2*d*e^6*g^2*m*x^6 + 4076
*(e*x + d)^m*b*c*e^7*g^2*m*x^6 + 720*(e*x + d)^m*c^2*e^7*g^2*x^7 + (e*x + d
)^m*a^2*d*e^6*f^2*m^6 - 2*(e*x + d)^m*b^2*d^2*e^5*f^2*m^5*x - 4*(e*x + d)^m
*a*c*d^2*e^5*f^2*m^5*x + 50*(e*x + d)^m*a*b*d*e^6*f^2*m^5*x + 27*(e*x + d)^
m*a^2*e^7*f^2*m^5*x - 8*(e*x + d)^m*a*b*d^2*e^5*f*g*m^5*x + 50*(e*x + d)^m*
a^2*d*e^6*f*g*m^5*x - 2*(e*x + d)^m*a^2*d^2*e^5*g^2*m^5*x + 12*(e*x + d)^m*
c^2*d^3*e^4*f^2*m^4*x^2 - 114*(e*x + d)^m*b*c*d^2*e^5*f^2*m^4*x^2 + 201*(e*
x + d)^m*b^2*d*e^6*f^2*m^4*x^2 + 402*(e*x + d)^m*a*c*d*e^6*f^2*m^4*x^2 + 54
0*(e*x + d)^m*a*b*e^7*f^2*m^4*x^2 + 48*(e*x + d)^m*b*c*d^3*e^4*f*g*m^4*x^2
- 114*(e*x + d)^m*b^2*d^2*e^5*f*g*m^4*x^2 - 228*(e*x + d)^m*a*c*d^2*e^5*f*g
*m^4*x^2 + 804*(e*x + d)^m*a*b*d*e^6*f*g*m^4*x^2 + 540*(e*x + d)^m*a^2*e^7*
f*g*m^4*x^2 + 12*(e*x + d)^m*b^2*d^3*e^4*g^2*m^4*x^2 + 24*(e*x + d)^m*a*c*d
^3*e^4*g^2*m^4*x^2 - 114*(e*x + d)^m*a*b*d^2*e^5*g^2*m^4*x^2 + 201*(e*x + d
)^m*a^2*d*e^6*g^2*m^4*x^2 - 332*(e*x + d)^m*c^2*d^2*e^5*f^2*m^3*x^3 + 1134*

(e*x + d)^m*b*c*d*e^6*f^2*m^3*x^3 + 1219*(e*x + d)^m*b^2*e^7*f^2*m^3*x^3 + 2438*(e*x + d)^m*a*c*e^7*f^2*m^3*x^3 + 400*(e*x + d)^m*c^2*d^3*e^4*f*g*m^3*x^3 - 1328*(e*x + d)^m*b*c*d^2*e^5*f*g*m^3*x^3 + 1134*(e*x + d)^m*b^2*d*e^6*f*g*m^3*x^3 + 2268*(e*x + d)^m*a*c*d*e^6*f*g*m^3*x^3 + 4876*(e*x + d)^m*a*b*e^7*f*g*m^3*x^3 - 120*(e*x + d)^m*c^2*d^4*e^3*g^2*m^3*x^3 + 400*(e*x + d)^m*b*c*d^3*e^4*g^2*m^3*x^3 - 332*(e*x + d)^m*b^2*d^2*e^5*g^2*m^3*x^3 - 664*(e*x + d)^m*a*c*d^2*e^5*g^2*m^3*x^3 + 1134*(e*x + d)^m*a*b*d*e^6*g^2*m^3*x^3 + 1219*(e*x + d)^m*a^2*e^7*g^2*m^3*x^3 + 540*(e*x + d)^m*c^2*d*e^6*f^2*m^2*x^4 + 5090*(e*x + d)^m*b*c*e^7*f^2*m^2*x^4 - 830*(e*x + d)^m*c^2*d^2*e^5*f*g*m^2*x^4 + 2160*(e*x + d)^m*b*c*d*e^6*f*g*m^2*x^4 + 5090*(e*x + d)^m*b^2*e^7*f*g*m^2*x^4 + 10180*(e*x + d)^m*a*c*e^7*f*g*m^2*x^4 + 330*(e*x + d)^m*c^2*d^3*e^4*g^2*m^2*x^4 - 830*(e*x + d)^m*b*c*d^2*e^5*g^2*m^2*x^4 + 540*(e*x + d)^m*b^2*d*e^6*g^2*m^2*x^4 + 1080*(e*x + d)^m*a*c*d*e^6*g^2*m^2*x^4 + 5090*(e*x + d)^m*a*b*e^7*g^2*m^2*x^4 + 2412*(e*x + d)^m*c^2*e^7*f^2*m*x^5 + 336*(e*x + d)^m*c^2*d*e^6*f*g*m*x^5 + 9648*(e*x + d)^m*b*c*e^7*f*g*m*x^5 - 144*(e*x + d)^m*c^2*d^2*e^5*g^2*m*x^5 + 336*(e*x + d)^m*b*c*d*e^6*g^2*m*x^5 + 2412*(e*x + d)^m*b^2*e^7*g^2*m*x^5 + 4824*(e*x + d)^m*a*c*e^7*g^2*m*x^5 + 1680*(e*x + d)^m*c^2*e^7*f*g*x^6 + 1680*(e*x + d)^m*b*c*e^7*g^2*x^6 - 2*(e*x + d)^m*a*b*d^2*e^5*f^2*m^5 + 27*(e*x + d)^m*a^2*d*e^6*f^2*m^5 - 2*(e*x + d)^m*a^2*d^2*e^5*f*g*m^5 + 12*(e*x + d)^m*b*c*d^3*e^4*f^2*m^4*x - 44*(e*x + d)^m*b^2*d^2*e^5*f^2*m^4*x - 88*(e*x + d)^m*a*c*d^2*e^5*f^2*m^4*x + 490*(e*x + d)^m*a*b*d*e^6*f^2*m^4*x + 295*(e*x + d)^m*a^2*e^7*f^2*m^4*x + 12*(e*x + d)^m*b^2*d^3*e^4*f*g*m^4*x + 24*(e*x + d)^m*a*c*d^3*e^4*f*g*m^4*x - 176*(e*x + d)^m*a*b*d^2*e^5*f*g*m^4*x + 490*(e*x + d)^m*a^2*d*e^6*f*g*m^4*x + 12*(e*x + d)^m*a*b*d^3*e^4*g^2*m^4*x - 44*(e*x + d)^m*a^2*d^2*e^5*g^2*m^4*x + 168*(e*x + d)^m*c^2*d^3*e^4*f^2*m^3*x^2 - 750*(e*x + d)^m*b*c*d^2*e^5*f^2*m^3*x^2 + 817*(e*x + d)^m*b^2*d*e^6*f^2*m^3*x^2 + 1634*(e*x + d)^m*a*c*d*e^6*f^2*m^3*x^2 + 2840*(e*x + d)^m*a*b*e^7*f^2*m^3*x^2 - 120*(e*x + d)^m*c^2*d^4*e^3*f*g*m^3*x^2 + 672*(e*x + d)^m*b*c*d^3*e^4*f*g*m^3*x^2 - 750*(e*x + d)^m*b^2*d^2*e^5*f*g*m^3*x^2 - 1500*(e*x + d)^m*a*c*d^2*e^5*f*g*m^3*x^2 + 3268*(e*x + d)^m*a*b*d*e^6*f*g*m^3*x^2 + 2840*(e*x + d)^m*a^2*e^7*f*g*m^3*x^2 - 120*(e*x + d)^m*b*c*d^4*e^3*g^2*m^3*x^2 + 168*(e*x + d)^m*b^2*d^3*e^4*g^2*m^3*x^2 + 336*(e*x + d)^m*a*c*d^3*e^4*g^2*m^3*x^2 - 750*(e*x + d)^m*a*b*d^2*e^5*g^2*m^3*x^2 + 817*(e*x + d)^m*a^2*d*e^6*g^2*m^3*x^2 - 608*(e*x + d)^m*c^2*d^2*e^5*f^2*m^2*x^3 + 1688*(e*x + d)^m*b*c*d*e^6*f^2*m^2*x^3 + 3112*(e*x + d)^m*b^2*e^7*f^2*m^2*x^3 + 6224*(e*x + d)^m*a*c*e^7*f^2*m^2*x^3 + 920*(e*x + d)^m*c^2*d^3*e^4*f*g*m^2*x^3 - 2432*(e*x + d)^m*b*c*d^2*e^5*f*g*m^2*x^3 + 1688*(e*x + d)^m*b^2*d*e^6*f*g*m^2*x^3 + 3376*(e*x + d)^m*a*c*d*e^6*f*g*m^2*x^3 + 12448*(e*x + d)^m*a*b*e^7*f*g*m^2*x^3 - 360*(e*x + d)^m*c^2*d^4*e^3*g^2*m^2*x^3 + 920*(e*x + d)^m*b*c*d^3*e^4*g^2*m^2*x^3 - 608*(e*x + d)^m*b^2*d^2*e^5*g^2*m^2*x^3 - 1216*(e*x + d)^m*a*c*d^2*e^5*g^2*m^2*x^3 + 1688*(e*x + d)^m*a*b*d*e^6*g^2*m^2*x^3 + 3112*(e*x + d)^m*a^2*e^7*g^2*m^2*x^3 + 252*(e*x + d)^m*c^2*d*e^6*f^2*m*x^4 + 5904*(e*x + d)^m*b*c*e^7*f^2*m*x^4 - 420*(e*x + d)^m*c^2*d^2*e^5*f*g*m*x^4 + 1008*(e*x + d)^m*b*c*d*e^6*f*g*m*x^4 + 5904*(e*x + d)^m*b^2*e^7*f*g*m*x^4 + 11808*(e*x + d)^m*a*c*e^7*f

*g*m*x^4 + 180*(e*x + d)^m*c^2*d^3*e^4*g^2*m*x^4 - 420*(e*x + d)^m*b*c*d^2*
e^5*g^2*m*x^4 + 252*(e*x + d)^m*b^2*d*e^6*g^2*m*x^4 + 504*(e*x + d)^m*a*c*d
*e^6*g^2*m*x^4 + 5904*(e*x + d)^m*a*b*e^7*g^2*m*x^4 + 1008*(e*x + d)^m*c^2*
e^7*f^2*x^5 + 4032*(e*x + d)^m*b*c*e^7*f*g*x^5 + 1008*(e*x + d)^m*b^2*e^7*g
^2*x^5 + 2016*(e*x + d)^m*a*c*e^7*g^2*x^5 + 2*(e*x + d)^m*b^2*d^3*e^4*f^2*m
^4 + 4*(e*x + d)^m*a*c*d^3*e^4*f^2*m^4 - 50*(e*x + d)^m*a*b*d^2*e^5*f^2*m^4
 + 295*(e*x + d)^m*a^2*d*e^6*f^2*m^4 + 8*(e*x + d)^m*a*b*d^3*e^4*f*g*m^4 -
50*(e*x + d)^m*a^2*d^2*e^5*f*g*m^4 + 2*(e*x + d)^m*a^2*d^3*e^4*g^2*m^4 - 24
*(e*x + d)^m*c^2*d^4*e^3*f^2*m^3*x + 216*(e*x + d)^m*b*c*d^3*e^4*f^2*m^3*x
- 358*(e*x + d)^m*b^2*d^2*e^5*f^2*m^3*x - 716*(e*x + d)^m*a*c*d^2*e^5*f^2*m
^3*x + 2350*(e*x + d)^m*a*b*d*e^6*f^2*m^3*x + 1665*(e*x + d)^m*a^2*e^7*f^2*
m^3*x - 96*(e*x + d)^m*b*c*d^4*e^3*f*g*m^3*x + 216*(e*x + d)^m*b^2*d^3*e^4*
f*g*m^3*x + 432*(e*x + d)^m*a*c*d^3*e^4*f*g*m^3*x - 1432*(e*x + d)^m*a*b*d^
2*e^5*f*g*m^3*x + 2350*(e*x + d)^m*a^2*d*e^6*f*g*m^3*x - 24*(e*x + d)^m*b^2
*d^4*e^3*g^2*m^3*x - 48*(e*x + d)^m*a*c*d^4*e^3*g^2*m^3*x + 216*(e*x + d)^m
*a*b*d^3*e^4*g^2*m^3*x - 358*(e*x + d)^m*a^2*d^2*e^5*g^2*m^3*x + 660*(e*x +
 d)^m*c^2*d^3*e^4*f^2*m^2*x^2 - 1902*(e*x + d)^m*b*c*d^2*e^5*f^2*m^2*x^2 +
1478*(e*x + d)^m*b^2*d*e^6*f^2*m^2*x^2 + 2956*(e*x + d)^m*a*c*d*e^6*f^2*m^2
x^2 + 7858(e*x + d)^m*a*b*e^7*f^2*m^2*x^2 - 960*(e*x + d)^m*c^2*d^4*e^3*f
*g*m^2*x^2 + 2640*(e*x + d)^m*b*c*d^3*e^4*f*g*m^2*x^2 - 1902*(e*x + d)^m*b^
2*d^2*e^5*f*g*m^2*x^2 - 3804*(e*x + d)^m*a*c*d^2*e^5*f*g*m^2*x^2 + 5912*(e*
x + d)^m*a*b*d*e^6*f*g*m^2*x^2 + 7858*(e*x + d)^m*a^2*e^7*f*g*m^2*x^2 + 360
*(e*x + d)^m*c^2*d^5*e^2*g^2*m^2*x^2 - 960*(e*x + d)^m*b*c*d^4*e^3*g^2*m^2*
x^2 + 660*(e*x + d)^m*b^2*d^3*e^4*g^2*m^2*x^2 + 1320*(e*x + d)^m*a*c*d^3*e^
4*g^2*m^2*x^2 - 1902*(e*x + d)^m*a*b*d^2*e^5*g^2*m^2*x^2 + 1478*(e*x + d)^m
*a^2*d*e^6*g^2*m^2*x^2 - 336*(e*x + d)^m*c^2*d^2*e^5*f^2*m*x^3 + 840*(e*x +
 d)^m*b*c*d*e^6*f^2*m*x^3 + 3796*(e*x + d)^m*b^2*e^7*f^2*m*x^3 + 7592*(e*x
+ d)^m*a*c*e^7*f^2*m*x^3 + 560*(e*x + d)^m*c^2*d^3*e^4*f*g*m*x^3 - 1344*(e*
x + d)^m*b*c*d^2*e^5*f*g*m*x^3 + 840*(e*x + d)^m*b^2*d*e^6*f*g*m*x^3 + 1680
*(e*x + d)^m*a*c*d*e^6*f*g*m*x^3 + 15184*(e*x + d)^m*a*b*e^7*f*g*m*x^3 - 24
0*(e*x + d)^m*c^2*d^4*e^3*g^2*m*x^3 + 560*(e*x + d)^m*b*c*d^3*e^4*g^2*m*x^3
 - 336*(e*x + d)^m*b^2*d^2*e^5*g^2*m*x^3 - 672*(e*x + d)^m*a*c*d^2*e^5*g^2*
m*x^3 + 840*(e*x + d)^m*a*b*d*e^6*g^2*m*x^3 + 3796*(e*x + d)^m*a^2*e^7*g^2*
m*x^3 + 2520*(e*x + d)^m*b*c*e^7*f^2*x^4 + 2520*(e*x + d)^m*b^2*e^7*f*g*x^4
 + 5040*(e*x + d)^m*a*c*e^7*f*g*x^4 + 2520*(e*x + d)^m*a*b*e^7*g^2*x^4 - 12
*(e*x + d)^m*b*c*d^4*e^3*f^2*m^3 + 44*(e*x + d)^m*b^2*d^3*e^4*f^2*m^3 + 88*
(e*x + d)^m*a*c*d^3*e^4*f^2*m^3 - 490*(e*x + d)^m*a*b*d^2*e^5*f^2*m^3 + 166
5*(e*x + d)^m*a^2*d*e^6*f^2*m^3 - 12*(e*x + d)^m*b^2*d^4*e^3*f*g*m^3 - 24*(
e*x + d)^m*a*c*d^4*e^3*f*g*m^3 + 176*(e*x + d)^m*a*b*d^3*e^4*f*g*m^3 - 490*
(e*x + d)^m*a^2*d^2*e^5*f*g*m^3 - 12*(e*x + d)^m*a*b*d^4*e^3*g^2*m^3 + 44*(
e*x + d)^m*a^2*d^3*e^4*g^2*m^3 - 312*(e*x + d)^m*c^2*d^4*e^3*f^2*m^2*x + 12
84*(e*x + d)^m*b*c*d^3*e^4*f^2*m^2*x - 1276*(e*x + d)^m*b^2*d^2*e^5*f^2*m^2
x - 2552(e*x + d)^m*a*c*d^2*e^5*f^2*m^2*x + 5508*(e*x + d)^m*a*b*d*e^6*f^
2*m^2*x + 5104*(e*x + d)^m*a^2*e^7*f^2*m^2*x + 240*(e*x + d)^m*c^2*d^5*e^2*
f*g*m^2*x - 1248*(e*x + d)^m*b*c*d^4*e^3*f*g*m^2*x + 1284*(e*x + d)^m*b^2*d

^3*e^4*f*g*m^2*x + 2568*(e*x + d)^m*a*c*d^3*e^4*f*g*m^2*x - 5104*(e*x + d)^
m*a*b*d^2*e^5*f*g*m^2*x + 5508*(e*x + d)^m*a^2*d*e^6*f*g*m^2*x + 240*(e*x +
d)^m*b*c*d^5*e^2*g^2*m^2*x - 312*(e*x + d)^m*b^2*d^4*e^3*g^2*m^2*x - 624*(
e*x + d)^m*a*c*d^4*e^3*g^2*m^2*x + 1284*(e*x + d)^m*a*b*d^3*e^4*g^2*m^2*x -
1276*(e*x + d)^m*a^2*d^2*e^5*g^2*m^2*x + 504*(e*x + d)^m*c^2*d^3*e^4*f^2*m
x^2 - 1260(e*x + d)^m*b*c*d^2*e^5*f^2*m*x^2 + 840*(e*x + d)^m*b^2*d*e^6*f
^2*m*x^2 + 1680*(e*x + d)^m*a*c*d*e^6*f^2*m*x^2 + 10548*(e*x + d)^m*a*b*e^7
*f^2*m*x^2 - 840*(e*x + d)^m*c^2*d^4*e^3*f*g*m*x^2 + 2016*(e*x + d)^m*b*c*d
^3*e^4*f*g*m*x^2 - 1260*(e*x + d)^m*b^2*d^2*e^5*f*g*m*x^2 - 2520*(e*x + d)^
m*a*c*d^2*e^5*f*g*m*x^2 + 3360*(e*x + d)^m*a*b*d*e^6*f*g*m*x^2 + 10548*(e*x
+ d)^m*a^2*e^7*f*g*m*x^2 + 360*(e*x + d)^m*c^2*d^5*e^2*g^2*m*x^2 - 840*(e*
x + d)^m*b*c*d^4*e^3*g^2*m*x^2 + 504*(e*x + d)^m*b^2*d^3*e^4*g^2*m*x^2 + 10
08*(e*x + d)^m*a*c*d^3*e^4*g^2*m*x^2 - 1260*(e*x + d)^m*a*b*d^2*e^5*g^2*m*x
^2 + 840*(e*x + d)^m*a^2*d*e^6*g^2*m*x^2 + 1680*(e*x + d)^m*b^2*e^7*f^2*x^3
+ 3360*(e*x + d)^m*a*c*e^7*f^2*x^3 + 6720*(e*x + d)^m*a*b*e^7*f*g*x^3 + 16
80*(e*x + d)^m*a^2*e^7*g^2*x^3 + 24*(e*x + d)^m*c^2*d^5*e^2*f^2*m^2 - 216*(
e*x + d)^m*b*c*d^4*e^3*f^2*m^2 + 358*(e*x + d)^m*b^2*d^3*e^4*f^2*m^2 + 716*
(e*x + d)^m*a*c*d^3*e^4*f^2*m^2 - 2350*(e*x + d)^m*a*b*d^2*e^5*f^2*m^2 + 51
04*(e*x + d)^m*a^2*d*e^6*f^2*m^2 + 96*(e*x + d)^m*b*c*d^5*e^2*f*g*m^2 - 216
*(e*x + d)^m*b^2*d^4*e^3*f*g*m^2 - 432*(e*x + d)^m*a*c*d^4*e^3*f*g*m^2 + 14
32*(e*x + d)^m*a*b*d^3*e^4*f*g*m^2 - 2350*(e*x + d)^m*a^2*d^2*e^5*f*g*m^2 +
24*(e*x + d)^m*b^2*d^5*e^2*g^2*m^2 + 48*(e*x + d)^m*a*c*d^5*e^2*g^2*m^2 -
216*(e*x + d)^m*a*b*d^4*e^3*g^2*m^2 + 358*(e*x + d)^m*a^2*d^3*e^4*g^2*m^2 -
1008*(e*x + d)^m*c^2*d^4*e^3*f^2*m*x + 2520*(e*x + d)^m*b*c*d^3*e^4*f^2*m*
x - 1680*(e*x + d)^m*b^2*d^2*e^5*f^2*m*x - 3360*(e*x + d)^m*a*c*d^2*e^5*f^2
*m*x + 5040*(e*x + d)^m*a*b*d*e^6*f^2*m*x + 8028*(e*x + d)^m*a^2*e^7*f^2*m*
x + 1680*(e*x + d)^m*c^2*d^5*e^2*f*g*m*x - 4032*(e*x + d)^m*b*c*d^4*e^3*f*g
*m*x + 2520*(e*x + d)^m*b^2*d^3*e^4*f*g*m*x + 5040*(e*x + d)^m*a*c*d^3*e^4*
f*g*m*x - 6720*(e*x + d)^m*a*b*d^2*e^5*f*g*m*x + 5040*(e*x + d)^m*a^2*d*e^6
*f*g*m*x - 720*(e*x + d)^m*c^2*d^6*e*g^2*m*x + 1680*(e*x + d)^m*b*c*d^5*e^2
*g^2*m*x - 1008*(e*x + d)^m*b^2*d^4*e^3*g^2*m*x - 2016*(e*x + d)^m*a*c*d^4*
e^3*g^2*m*x + 2520*(e*x + d)^m*a*b*d^3*e^4*g^2*m*x - 1680*(e*x + d)^m*a^2*d
^2*e^5*g^2*m*x + 5040*(e*x + d)^m*a*b*e^7*f^2*x^2 + 5040*(e*x + d)^m*a^2*e^
7*f*g*x^2 + 312*(e*x + d)^m*c^2*d^5*e^2*f^2*m - 1284*(e*x + d)^m*b*c*d^4*e^
3*f^2*m + 1276*(e*x + d)^m*b^2*d^3*e^4*f^2*m + 2552*(e*x + d)^m*a*c*d^3*e^4
*f^2*m - 5508*(e*x + d)^m*a*b*d^2*e^5*f^2*m + 8028*(e*x + d)^m*a^2*d*e^6*f^
2*m - 240*(e*x + d)^m*c^2*d^6*e*f*g*m + 1248*(e*x + d)^m*b*c*d^5*e^2*f*g*m
- 1284*(e*x + d)^m*b^2*d^4*e^3*f*g*m - 2568*(e*x + d)^m*a*c*d^4*e^3*f*g*m +
5104*(e*x + d)^m*a*b*d^3*e^4*f*g*m - 5508*(e*x + d)^m*a^2*d^2*e^5*f*g*m -
240*(e*x + d)^m*b*c*d^6*e*g^2*m + 312*(e*x + d)^m*b^2*d^5*e^2*g^2*m + 624*(
e*x + d)^m*a*c*d^5*e^2*g^2*m - 1284*(e*x + d)^m*a*b*d^4*e^3*g^2*m + 1276*(e
*x + d)^m*a^2*d^3*e^4*g^2*m + 5040*(e*x + d)^m*a^2*e^7*f^2*x + 1008*(e*x +
d)^m*c^2*d^5*e^2*f^2 - 2520*(e*x + d)^m*b*c*d^4*e^3*f^2 + 1680*(e*x + d)^m*
b^2*d^3*e^4*f^2 + 3360*(e*x + d)^m*a*c*d^3*e^4*f^2 - 5040*(e*x + d)^m*a*b*d
^2*e^5*f^2 + 5040*(e*x + d)^m*a^2*d*e^6*f^2 - 1680*(e*x + d)^m*c^2*d^6*e*f*

```
g + 4032*(e*x + d)^m*b*c*d^5*e^2*f*g - 2520*(e*x + d)^m*b^2*d^4*e^3*f*g - 5
040*(e*x + d)^m*a*c*d^4*e^3*f*g + 6720*(e*x + d)^m*a*b*d^3*e^4*f*g - 5040*(
e*x + d)^m*a^2*d^2*e^5*f*g + 720*(e*x + d)^m*c^2*d^7*g^2 - 1680*(e*x + d)^m
*b*c*d^6*e*g^2 + 1008*(e*x + d)^m*b^2*d^5*e^2*g^2 + 2016*(e*x + d)^m*a*c*d^
5*e^2*g^2 - 2520*(e*x + d)^m*a*b*d^4*e^3*g^2 + 1680*(e*x + d)^m*a^2*d^3*e^4
*g^2)/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 1
3132*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 4871, normalized size of antiderivative = 9.28

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2 +
 1680*b^2*d^3*e^4*f^2 + 1008*b^2*d^5*e^2*g^2 + 1008*c^2*d^5*e^2*f^2 - 1680*
b*c*d^6*e*g^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 358*b^2*d^3*
e^4*f^2*m^2 + 44*a^2*d^3*e^4*g^2*m^3 + 44*b^2*d^3*e^4*f^2*m^3 + 2*a^2*d^3*e
^4*g^2*m^4 + 2*b^2*d^3*e^4*f^2*m^4 + 24*b^2*d^5*e^2*g^2*m^2 + 24*c^2*d^5*e^
2*f^2*m^2 - 5040*a*b*d^2*e^5*f^2 - 2520*a*b*d^4*e^3*g^2 + 3360*a*c*d^3*e^4*
f^2 + 2016*a*c*d^5*e^2*g^2 - 2520*b*c*d^4*e^3*f^2 - 5040*a^2*d^2*e^5*f*g -
2520*b^2*d^4*e^3*f*g + 8028*a^2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^2 + 1665
*a^2*d*e^6*f^2*m^3 + 295*a^2*d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5 + a^2*d*e
^6*f^2*m^6 + 1276*a^2*d^3*e^4*g^2*m + 1276*b^2*d^3*e^4*f^2*m + 312*b^2*d^5*
e^2*g^2*m + 312*c^2*d^5*e^2*f^2*m - 2350*a*b*d^2*e^5*f^2*m^2 - 490*a*b*d^2*
e^5*f^2*m^3 - 50*a*b*d^2*e^5*f^2*m^4 - 2*a*b*d^2*e^5*f^2*m^5 - 216*a*b*d^4*
e^3*g^2*m^2 + 716*a*c*d^3*e^4*f^2*m^2 - 12*a*b*d^4*e^3*g^2*m^3 + 88*a*c*d^3
*e^4*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2 - 216*b*c*d^4
*e^3*f^2*m^2 - 12*b*c*d^4*e^3*f^2*m^3 - 2350*a^2*d^2*e^5*f*g*m^2 - 490*a^2*
d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*e^5*f*g*m^5 - 216*b^2*
d^4*e^3*f*g*m^2 - 12*b^2*d^4*e^3*f*g*m^3 + 6720*a*b*d^3*e^4*f*g - 5040*a*c*
d^4*e^3*f*g + 4032*b*c*d^5*e^2*f*g - 240*b*c*d^6*e*g^2*m - 240*c^2*d^6*e*f*
g*m - 5508*a*b*d^2*e^5*f^2*m - 1284*a*b*d^4*e^3*g^2*m + 2552*a*c*d^3*e^4*f^
2*m + 624*a*c*d^5*e^2*g^2*m - 1284*b*c*d^4*e^3*f^2*m - 5508*a^2*d^2*e^5*f*g
*m - 1284*b^2*d^4*e^3*f*g*m + 1432*a*b*d^3*e^4*f*g*m^2 + 176*a*b*d^3*e^4*f*
g*m^3 + 8*a*b*d^3*e^4*f*g*m^4 - 432*a*c*d^4*e^3*f*g*m^2 - 24*a*c*d^4*e^3*f*
g*m^3 + 96*b*c*d^5*e^2*f*g*m^2 + 5104*a*b*d^3*e^4*f*g*m - 2568*a*c*d^4*e^3*
f*g*m + 1248*b*c*d^5*e^2*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 196
0*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*(d + e*x)^m*(5040*a^2*e^7*f^2
+ 8028*a^2*e^7*f^2*m + 5104*a^2*e^7*f^2*m^2 + 1665*a^2*e^7*f^2*m^3 + 295*a^
2*e^7*f^2*m^4 + 27*a^2*e^7*f^2*m^5 + a^2*e^7*f^2*m^6 - 1276*a^2*d^2*e^5*g^2
*m^2 - 1276*b^2*d^2*e^5*f^2*m^2 - 358*a^2*d^2*e^5*g^2*m^3 - 358*b^2*d^2*e^5
```

*f^2*m^3 - 44*a^2*d^2*e^5*g^2*m^4 - 44*b^2*d^2*e^5*f^2*m^4 - 2*a^2*d^2*e^5*g^2*m^5 - 2*b^2*d^2*e^5*f^2*m^5 - 312*b^2*d^4*e^3*g^2*m^2 - 312*c^2*d^4*e^3*f^2*m^2 - 24*b^2*d^4*e^3*g^2*m^3 - 24*c^2*d^4*e^3*f^2*m^3 - 720*c^2*d^6*e*g^2*m - 1680*a^2*d^2*e^5*g^2*m - 1680*b^2*d^2*e^5*f^2*m - 1008*b^2*d^4*e^3*g^2*m - 1008*c^2*d^4*e^3*f^2*m + 1284*a*b*d^3*e^4*g^2*m^2 - 2552*a*c*d^2*e^5*f^2*m^2 + 216*a*b*d^3*e^4*g^2*m^3 - 716*a*c*d^2*e^5*f^2*m^3 + 12*a*b*d^3*e^4*g^2*m^4 - 88*a*c*d^2*e^5*f^2*m^4 - 4*a*c*d^2*e^5*f^2*m^5 - 624*a*c*d^4*e^3*g^2*m^2 + 1284*b*c*d^3*e^4*f^2*m^2 - 48*a*c*d^4*e^3*g^2*m^3 + 216*b*c*d^3*e^4*f^2*m^3 + 12*b*c*d^3*e^4*f^2*m^4 + 240*b*c*d^5*e^2*g^2*m^2 + 1284*b^2*d^3*e^4*f*g*m^2 + 216*b^2*d^3*e^4*f*g*m^3 + 12*b^2*d^3*e^4*f*g*m^4 + 240*c^2*d^5*e^2*f*g*m^2 + 5040*a*b*d*e^6*f^2*m + 5040*a^2*d*e^6*f*g*m + 5508*a*b*d*e^6*f^2*m^2 + 2350*a*b*d*e^6*f^2*m^3 + 490*a*b*d*e^6*f^2*m^4 + 50*a*b*d*e^6*f^2*m^5 + 2*a*b*d*e^6*f^2*m^6 + 2520*a*b*d^3*e^4*g^2*m - 3360*a*c*d^2*e^5*f^2*m - 2016*a*c*d^4*e^3*g^2*m + 2520*b*c*d^3*e^4*f^2*m + 1680*b*c*d^5*e^2*g^2*m + 5508*a^2*d*e^6*f*g*m^2 + 2350*a^2*d*e^6*f*g*m^3 + 490*a^2*d*e^6*f*g*m^4 + 50*a^2*d*e^6*f*g*m^5 + 2*a^2*d*e^6*f*g*m^6 + 2520*b^2*d^3*e^4*f*g*m + 1680*c^2*d^5*e^2*f*g*m - 5104*a*b*d^2*e^5*f*g*m^2 - 1432*a*b*d^2*e^5*f*g*m^3 - 176*a*b*d^2*e^5*f*g*m^4 - 8*a*b*d^2*e^5*f*g*m^5 + 2568*a*c*d^3*e^4*f*g*m^2 + 432*a*c*d^3*e^4*f*g*m^3 + 24*a*c*d^3*e^4*f*g*m^4 - 1248*b*c*d^4*e^3*f*g*m^2 - 96*b*c*d^4*e^3*f*g*m^3 - 6720*a*b*d^2*e^5*f*g*m + 5040*a*c*d^3*e^4*f*g*m - 4032*b*c*d^4*e^3*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(840*a^2*e^4*g^2 + 840*b^2*e^4*f^2 + 638*a^2*e^4*g^2*m + 638*b^2*e^4*f^2*m - 120*c^2*d^4*g^2*m + 179*a^2*e^4*g^2*m^2 + 179*b^2*e^4*f^2*m^2 + 22*a^2*e^4*g^2*m^3 + 22*b^2*e^4*f^2*m^3 + a^2*e^4*g^2*m^4 + b^2*e^4*f^2*m^4 + 1680*a*c*e^4*f^2 + 1276*a*c*e^4*f^2*m - 52*b^2*d^2*e^2*g^2*m^2 - 52*c^2*d^2*e^2*f^2*m^2 - 4*b^2*d^2*e^2*g^2*m^3 - 4*c^2*d^2*e^2*f^2*m^3 + 358*a*c*e^4*f^2*m^2 + 44*a*c*e^4*f^2*m^3 + 2*a*c*e^4*f^2*m^4 + 3360*a*b*e^4*f*g - 168*b^2*d^2*e^2*g^2*m - 168*c^2*d^2*e^2*f^2*m + 2552*a*b*e^4*f*g*m - 104*a*c*d^2*e^2*g^2*m^2 - 8*a*c*d^2*e^2*g^2*m^3 + 420*a*b*d*e^3*g^2*m + 420*b*c*d*e^3*f^2*m + 280*b*c*d^3*e*g^2*m + 716*a*b*e^4*f*g*m^2 + 88*a*b*e^4*f*g*m^3 + 4*a*b*e^4*f*g*m^4 + 420*b^2*d*e^3*f*g*m + 280*c^2*d^3*e*f*g*m + 214*a*b*d*e^3*g^2*m^2 + 36*a*b*d*e^3*g^2*m^3 + 2*a*b*d*e^3*g^2*m^4 - 336*a*c*d^2*e^2*g^2*m + 214*b*c*d*e^3*f^2*m^2 + 36*b*c*d*e^3*f^2*m^3 + 2*b*c*d*e^3*f^2*m^4 + 40*b*c*d^3*e*g^2*m^2 + 214*b^2*d*e^3*f*g*m^2 + 36*b^2*d*e^3*f*g*m^3 + 2*b^2*d*e^3*f*g*m^4 + 40*c^2*d^3*e*f*g*m^2 - 208*b*c*d^2*e^2*f*g*m^2 - 16*b*c*d^2*e^2*f*g*m^3 + 840*a*c*d*e^3*f*g*m + 428*a*c*d*e^3*f*g*m^2 + 72*a*c*d*e^3*f*g*m^3 + 4*a*c*d*e^3*f*g*m^4 - 672*b*c*d^2*e^2*f*g*m))/(e^4*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(42*b^2*e^2*g^2 + 42*c^2*e^2*f^2 + 13*b^2*e^2*g^2*m - 6*c^2*d^2*g^2*m + 13*c^2*e^2*f^2*m + b^2*e^2*g^2*m^2 + c^2*e^2*f^2*m^2 + 84*a*c*e^2*g^2 + 26*a*c*e^2*g^2*m + 2*a*c*e^2*g^2*m^2 + 168*b*c*e^2*f*g + 14*b*c*d*e*g^2*m + 52*b*c*e^2*f*g*m + 14*c^2*d*e*f*g*m + 2*b*c*d*e*g^2*m^2 + 4*b*c*e^2*f*g*m^2 + 2*c^2*d*e*f*g*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^2* (m + 1)*(d + e*x)^m*(360*c^2*d^5*g^2*m + 5040*a*b*e^5*f^2 + 5040*a^2*e^5*f*g + 5508*a*b*e^5*f^2*m + 5508*a^2*e^5*f*g*m + 156*b^2*d^3*e^2*g^2*m^2 + 156*c^2*d^3*e^2*f^2*m^2 + 12*b^2*d^3*e^2*g^2*m^3 + 12*c^2*d^3*e^2*f^2*m^3 + 2350*a*b*e^5*f^2*m^2 + 490*a*b*e^5*f^2*m^3 + 50*a*b*e^5*f^2*m^4 + 2*a*b*e^5*f^2*m^5 + 840*a^2*d*e^4*g^2*m + 840*b^2*d*e^4*f^2*m + 2350*a^2*e^5*f*g*m^2 + 490*a^2*e^5*f*g*m^3 + 50*a^2*e^5*f*g*m^4 + 2*a^2*e^5*f*g*m^5 + 638*a^2*d*e^4*g^2*m^2 + 638*b^2*d*e^4*f^2*m^2 + 179*a^2*d*e^4*g^2*m^3 + 179*b^2*d*e^4*f^2*m^3 + 22*a^2*d*e^4*g^2*m^4 + 22*b^2*d*e^4*f^2*m^4 + a^2*d*e^4*g^2*m^5 + b^2*d*e^4*f^2*m^5 + 504*b^2*d^3*e^2*g^2*m + 504*c^2*d^3*e^2*f^2*m - 642*a*b*d^2*e^3*g^2*m^2 - 108*a*b*d^2*e^3*g^2*m^3 - 6*a*b*d^2*e^3*g^2*m^4 + 312*a*c*d^3*e^2*g^2*m^2 - 642*b*c*d^2*e^3*f^2*m^2 + 24*a*c*d^3*e^2*g^2*m^3 - 108*b*c*d^2*e^3*f^2*m^3 - 6*b*c*d^2*e^3*f^2*m^4 - 642*b^2*d^2*e^3*f*g*m^2 - 108*b^2*d^2*e^3*f*g*m^3 - 6*b^2*d^2*e^3*f*g*m^4 + 1680*a*c*d*e^4*f^2*m - 840*b*c*d^4*e*g^2*m - 840*c^2*d^4*e*f*g*m - 1260*a*b*d^2*e^3*g^2*m + 1276*a*c*d*e^4*f^2*m^2 + 358*a*c*d*e^4*f^2*m^3 + 44*a*c*d*e^4*f^2*m^4 + 2*a*c*d*e^4*f^2*m^5 + 1008*a*c*d^3*e^2*g^2*m - 1260*b*c*d^2*e^3*f^2*m - 120*b*c*d^4*e*g^2*m^2 - 1260*b^2*d^2*e^3*f*g*m - 120*c^2*d^4*e*f*g*m^2 - 1284*a*c*d^2*e^3*f*g*m^2 - 216*a*c*d^2*e^3*f*g*m^3 - 12*a*c*d^2*e^3*f*g*m^4 + 624*b*c*d^3*e^2*f*g*m^2 + 48*b*c*d^3*e^2*f*g*m^3 + 3360*a*b*d*e^4*f*g*m + 2552*a*b*d*e^4*f*g*m^2 + 716*a*b*d*e^4*f*g*m^3 + 88*a*b*d*e^4*f*g*m^4 + 4*a*b*d*e^4*f*g*m^5 - 2520*a*c*d^2*e^3*f*g*m + 2016*b*c*d^3*e^2*f*g*m))/(e^5*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c^2*g^2*x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*c^2*d^3*g^2*m + 420*a*b*e^3*g^2 + 420*b*c*e^3*f^2 + 420*b^2*e^3*f*g + 214*a*b*e^3*g^2*m + 214*b*c*e^3*f^2*m + 214*b^2*e^3*f*g*m + 36*a*b*e^3*g^2*m^2 + 2*a*b*e^3*g^2*m^3 + 36*b*c*e^3*f^2*m^2 + 2*b*c*e^3*f^2*m^3 + 42*b^2*d*e^2*g^2*m + 42*c^2*d*e^2*f^2*m + 36*b^2*e^3*f*g*m^2 + 2*b^2*e^3*f*g*m^3 + 840*a*c*e^3*f*g + 13*b^2*d*e^2*g^2*m^2 + 13*c^2*d*e^2*f^2*m^2 + b^2*d*e^2*g^2*m^3 + c^2*d*e^2*f^2*m^3 + 428*a*c*e^3*f*g*m + 84*a*c*d*e^2*g^2*m - 70*b*c*d^2*e*g^2*m + 72*a*c*e^3*f*g*m^2 + 4*a*c*e^3*f*g*m^3 - 70*c^2*d^2*e*f*g*m + 26*a*c*d*e^2*g^2*m^2 + 2*a*c*d*e^2*g^2*m^3 - 10*b*c*d^2*e*g^2*m^2 - 10*c^2*d^2*e*f*g*m^2 + 168*b*c*d*e^2*f*g*m + 52*b*c*d*e^2*f*g*m^2 + 4*b*c*d*e^2*f*g*m^3))/(e^3*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c*g*x^6*(d + e*x)^m*(14*b*e*g + 14*c*e*f + 2*b*e*g*m + c*d*g*m + 2*c*e*f*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))

3.926 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)} - \frac{\left(cd^2-bde+ae^2\right)\left(cd(4ef-5dg)-e(2bef-3bdg+aeg)\right)(d+ex)^{2+m}}{e^6(2+m)} + \frac{\left(2c^2d^2(3ef-5dg)+be^2(bef-3bdg+2aeg)+2ce(ae(ef-3dg)-3bd(ef-2dg))\right)(d+ex)^{3+m}}{e^6(3+m)} + \frac{\left(b^2e^2g-2c^2d(2ef-5dg)+2ce(bef-4bdg+aeg)\right)(d+ex)^{4+m}}{e^6(4+m)} + \frac{c(cef-5cdg+2beg)(d+ex)^{5+m}}{e^6(5+m)} + \frac{c^2g(d+ex)^{6+m}}{e^6(6+m)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)*(e*x+d)^(1+m)/e^6/(1+m)-(a*e^2-b*d*e+c*d^2
)*(c*d*(-5*d*g+4*e*f)-e*(a*e*g-3*b*d*g+2*b*e*f))*(e*x+d)^(2+m)/e^6/(2+m)+(2
*c^2*d^2*(-5*d*g+3*e*f)+b*e^2*(2*a*e*g-3*b*d*g+b*e*f)+2*c*e*(a*e*(-3*d*g+e*
f)-3*b*d*(-2*d*g+e*f)))*(e*x+d)^(3+m)/e^6/(3+m)+(b^2*e^2*g-2*c^2*d*(-5*d*g+
2*e*f)+2*c*e*(a*e*g-4*b*d*g+b*e*f))*(e*x+d)^(4+m)/e^6/(4+m)+c*(2*b*e*g-5*c*
d*g+c*e*f)*(e*x+d)^(5+m)/e^6/(5+m)+c^2*g*(e*x+d)^(6+m)/e^6/(6+m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {785}

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2 dx$$
$$= \frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}$$
$$+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(2aeg-3bdg+bef)+2c^2d^2(3ef-5dg)\right)}{e^6(m+3)}$$
$$+\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^6(m+1)}$$
$$-\frac{(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)\left(cd(4ef-5dg)-e(aeg-3bdg+2bef)\right)}{e^6(m+2)}$$
$$+\frac{c(d+ex)^{m+5}(2beg-5cdg+cef)}{e^6(m+5)}+\frac{c^2g(d+ex)^{m+6}}{e^6(m+6)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{(cd^2 - bde + ae^2)^2 (ef - dg)(d + ex)^m}{e^5} + \frac{(cd^2 - bde + ae^2)(-cd(4ef - 5dg) + e(2bef - 3bdg + aeg))(d + ex)^{1+m}}{e^5} + \frac{(2c^2d^2(3ef - 5dg) + be^2(bef - 3bdg + 2aeg) + 2ce(ae(ef - 3dg) - 3bd(ef - 2dg)))(d + ex)^{2+m}}{e^5} + \frac{(b^2e^2g - 2c^2d(2ef - 5dg) + 2ce(bef - 4bdg + aeg))(d + ex)^{3+m}}{e^5} + \frac{c(cef - 5cdg + 2beg)(d + ex)^{4+m}}{e^5} + \frac{c^2g(d + ex)^{5+m}}{e^5} \right) dx$$

$$= \frac{(cd^2 - bde + ae^2)^2 (ef - dg)(d + ex)^{1+m}}{e^6(1 + m)} - \frac{(cd^2 - bde + ae^2)(cd(4ef - 5dg) - e(2bef - 3bdg + aeg))(d + ex)^{2+m}}{e^6(2 + m)} + \frac{(2c^2d^2(3ef - 5dg) + be^2(bef - 3bdg + 2aeg) + 2ce(ae(ef - 3dg) - 3bd(ef - 2dg)))(d + ex)^{3+}}{e^6(3 + m)} + \frac{(b^2e^2g - 2c^2d(2ef - 5dg) + 2ce(bef - 4bdg + aeg))(d + ex)^{4+m}}{e^6(4 + m)} + \frac{c(cef - 5cdg + 2beg)(d + ex)^{5+m}}{e^6(5 + m)} + \frac{c^2g(d + ex)^{6+m}}{e^6(6 + m)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 655 vs. $2(311) = 622$.

Time = 1.11 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.11

$$\int (d + ex)^m (f + gx) (a + bx + cx^2)^2 \, dx$$

$$= \frac{(d + ex)^{1+m} \left((a + x(b + cx))^2 (2beg + c(-5dg + ef(6 + m) + eg(5 + m)x)) + \frac{2\left(\left(cd^2 + e(-bd + ae)\right)\left(b^3e^3g\left(3 + 4m + \right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
+ e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m + m
^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m) +
 b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2*e*f
```

```
*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^2*(1
+ m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)) - b^
2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)) + 2*c
^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4*m + m^2
) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m^2) + b^2
*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f*(42 + 19*
m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)) - (c*e*(4
 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 + m) + c*e*(b*
d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g
+ e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m))) +
c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*
g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b + c*x))))/(c*e^
2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. 2(311) = 622.

Time = 0.64 (sec) , antiderivative size = 2149, normalized size of antiderivative = 6.91

method	result	size
norman	Expression too large to display	2149
gosper	Expression too large to display	2563
risch	Expression too large to display	3326
parallelrisch	Expression too large to display	5114

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] g*c^2/(6+m)*x^6*exp(m*ln(e*x+d))+(2*a*c*e^2*g*m^2+b^2*e^2*g*m^2+2*b*c*d*e*g
*m^2+2*b*c*e^2*f*m^2+c^2*d*e*f*m^2+22*a*c*e^2*g*m+11*b^2*e^2*g*m+12*b*c*d*e
*g*m+22*b*c*e^2*f*m-5*c^2*d^2*g*m+6*c^2*d*e*f*m+60*a*c*e^2*g+30*b^2*e^2*g+6
0*b*c*e^2*f)/e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+(2*a*b*e^3*g*m^
3+2*a*c*d*e^2*g*m^3+2*a*c*e^3*f*m^3+b^2*d*e^2*g*m^3+b^2*e^3*f*m^3+2*b*c*d*e
^2*f*m^3+30*a*b*e^3*g*m^2+22*a*c*d*e^2*g*m^2+30*a*c*e^3*f*m^2+11*b^2*d*e^2*
g*m^2+15*b^2*e^3*f*m^2-8*b*c*d^2*e*g*m^2+22*b*c*d*e^2*f*m^2-4*c^2*d^2*e*f*m
^2+148*a*b*e^3*g*m+60*a*c*d*e^2*g*m+148*a*c*e^3*f*m+30*b^2*d*e^2*g*m+74*b^2
*e^3*f*m-48*b*c*d^2*e*g*m+60*b*c*d*e^2*f*m+20*c^2*d^3*g*m-24*c^2*d^2*e*f*m+
240*a*b*e^3*g+240*a*c*e^3*f+120*b^2*e^3*f)/e^3/(m^4+18*m^3+119*m^2+342*m+36
0)*x^3*exp(m*ln(e*x+d))+(a^2*e^4*g*m^4+2*a*b*d*e^3*g*m^4+2*a*b*e^4*f*m^4+2*
a*c*d*e^3*f*m^4+b^2*d*e^3*f*m^4+18*a^2*e^4*g*m^3+30*a*b*d*e^3*g*m^3+36*a*b*
e^4*f*m^3-6*a*c*d^2*e^2*g*m^3+30*a*c*d*e^3*f*m^3-3*b^2*d^2*e^2*g*m^3+15*b^2
*d*e^3*f*m^3-6*b*c*d^2*e^2*f*m^3+119*a^2*e^4*g*m^2+148*a*b*d*e^3*g*m^2+238*
a*b*e^4*f*m^2-66*a*c*d^2*e^2*g*m^2+148*a*c*d*e^3*f*m^2-33*b^2*d^2*e^2*g*m^2
+74*b^2*d*e^3*f*m^2+24*b*c*d^3*e*g*m^2-66*b*c*d^2*e^2*f*m^2+12*c^2*d^3*e*f*
m^2+342*a^2*e^4*g*m+240*a*b*d*e^3*g*m+684*a*b*e^4*f*m-180*a*c*d^2*e^2*g*m+2
```

```
40*a*c*d*e^3*f*m-90*b^2*d^2*e^2*g*m+120*b^2*d*e^3*f*m+144*b*c*d^3*e*g*m-180
*b*c*d^2*e^2*f*m-60*c^2*d^4*g*m+72*c^2*d^3*e*f*m+360*a^2*e^4*g+720*a*b*e^4*
f)/e^4/(m^5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*exp(m*ln(e*x+d))+(a^2*d*
e^4*g*m^5+a^2*e^5*f*m^5+2*a*b*d*e^4*f*m^5+18*a^2*d*e^4*g*m^4+20*a^2*e^5*f*m
^4-4*a*b*d^2*e^3*g*m^4+36*a*b*d*e^4*f*m^4-4*a*c*d^2*e^3*f*m^4-2*b^2*d^2*e^3
*f*m^4+119*a^2*d*e^4*g*m^3+155*a^2*e^5*f*m^3-60*a*b*d^2*e^3*g*m^3+238*a*b*d
*e^4*f*m^3+12*a*c*d^3*e^2*g*m^3-60*a*c*d^2*e^3*f*m^3+6*b^2*d^3*e^2*g*m^3-30
*b^2*d^2*e^3*f*m^3+12*b*c*d^3*e^2*f*m^3+342*a^2*d*e^4*g*m^2+580*a^2*e^5*f*m
^2-296*a*b*d^2*e^3*g*m^2+684*a*b*d*e^4*f*m^2+132*a*c*d^3*e^2*g*m^2-296*a*c*
d^2*e^3*f*m^2+66*b^2*d^3*e^2*g*m^2-148*b^2*d^2*e^3*f*m^2-48*b*c*d^4*e*g*m^2
+132*b*c*d^3*e^2*f*m^2-24*c^2*d^4*e*f*m^2+360*a^2*d*e^4*g*m+1044*a^2*e^5*f*
m-480*a*b*d^2*e^3*g*m+720*a*b*d*e^4*f*m+360*a*c*d^3*e^2*g*m-480*a*c*d^2*e^3
*f*m+180*b^2*d^3*e^2*g*m-240*b^2*d^2*e^3*f*m-288*b*c*d^4*e*g*m+360*b*c*d^3*
e^2*f*m+120*c^2*d^5*g*m-144*c^2*d^4*e*f*m+720*a^2*e^5*f)/e^5/(m^6+21*m^5+17
5*m^4+735*m^3+1624*m^2+1764*m+720)*x*exp(m*ln(e*x+d))+(2*b*e*g*m+c*d*g*m+c*
e*f*m+12*b*e*g+6*c*e*f)*c/e/(m^2+11*m+30)*x^5*exp(m*ln(e*x+d))-d*(-a^2*e^5*
f*m^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m^4+2*a*b*d*e^4*f*m^4+18*a^2*d*e^4*g*m^3
-155*a^2*e^5*f*m^3-4*a*b*d^2*e^3*g*m^3+36*a*b*d*e^4*f*m^3-4*a*c*d^2*e^3*f*m
^3-2*b^2*d^2*e^3*f*m^3+119*a^2*d*e^4*g*m^2-580*a^2*e^5*f*m^2-60*a*b*d^2*e^3
*g*m^2+238*a*b*d*e^4*f*m^2+12*a*c*d^3*e^2*g*m^2-60*a*c*d^2*e^3*f*m^2+6*b^2*
d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2+12*b*c*d^3*e^2*f*m^2+342*a^2*d*e^4*g*m-1
044*a^2*e^5*f*m-296*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+132*a*c*d^3*e^2*g*m-2
96*a*c*d^2*e^3*f*m+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-48*b*c*d^4*e*g*m+
132*b*c*d^3*e^2*f*m-24*c^2*d^4*e*f*m+360*a^2*d*e^4*g-720*a^2*e^5*f-480*a*b*
d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2*g-480*a*c*d^2*e^3*f+180*b^2*d^3*e
^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+360*b*c*d^3*e^2*f+120*c^2*d^5*g-144*
c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*exp(m*ln(
e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. $2(312) = 624$.

Time = 0.34 (sec) , antiderivative size = 2368, normalized size of antiderivative = 7.61

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 + 2
25*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f +
288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e^6*
f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 + 5*(19*c^2*e^6*f + (7*c^2*d*e^5 + 38
*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^2 + 1
2*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4*g + 2
```

```
*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 + (360*b*c*e^6*f + 180*(b^2 + 2*a*c)*e
^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*g)*m^
5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^5 - 17*(b
^2 + 2*a*c)*e^6)*g)*m^4 + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c^2*d^2*e^4
 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e^5 + 307*b*
c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*e^6)*g)*m^2
+ 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*e^5 - 66*(b^2
 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 - 2*(b^2 + 2*a*
c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 + (480*a*b*e^6*g +
 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*f + (2*a*b*e^
6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b*c*d*e^5 - 9*(b^2
 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 + 2*a*c)*d*e^5)*g)*
m^4 - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*c)*e^6)*f - (20*c^2
*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + 2*a*c)*d*e^5)*g)*m^3 -
4*((20*c^2*d^2*e^4 - 56*b*c*d*e^5 - 93*(b^2 + 2*a*c)*e^6)*f - (15*c^2*d^3*e
^3 - 40*b*c*d^2*e^4 + 186*a*b*e^6 + 28*(b^2 + 2*a*c)*d*e^5)*g)*m^2 - 4*((12
*c^2*d^2*e^4 - 30*b*c*d*e^5 - 127*(b^2 + 2*a*c)*e^6)*f - (10*c^2*d^3*e^3 -
24*b*c*d^2*e^4 + 254*a*b*e^6 + 15*(b^2 + 2*a*c)*d*e^5)*g)*m)*x^3 - (2*(6*b*
c*d^4*e^2 + 119*a*b*d^2*e^4 - 290*a^2*d*e^5 - 15*(b^2 + 2*a*c)*d^3*e^3)*f -
 (60*a*b*d^3*e^3 - 119*a^2*d^2*e^4 - 6*(b^2 + 2*a*c)*d^4*e^2)*g)*m^2 + (720
*a*b*e^6*f + 360*a^2*e^6*g + ((2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*f + (2*a*b*
d*e^5 + a^2*e^6)*g)*m^5 - (2*(3*b*c*d^2*e^4 - 19*a*b*e^6 - 8*(b^2 + 2*a*c)*
d*e^5)*f - (32*a*b*d*e^5 + 19*a^2*e^6 - 3*(b^2 + 2*a*c)*d^2*e^4)*g)*m^4 + (
(12*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 274*a*b*e^6 + 89*(b^2 + 2*a*c)*d*e^5)*f
+ (24*b*c*d^3*e^3 + 178*a*b*d*e^5 + 137*a^2*e^6 - 36*(b^2 + 2*a*c)*d^2*e^4)
*g)*m^3 + (2*(42*c^2*d^3*e^3 - 123*b*c*d^2*e^4 + 461*a*b*e^6 + 97*(b^2 + 2*
a*c)*d*e^5)*f - (60*c^2*d^4*e^2 - 168*b*c*d^3*e^3 - 388*a*b*d*e^5 - 461*a^2
*e^6 + 123*(b^2 + 2*a*c)*d^2*e^4)*g)*m^2 + 6*(2*(6*c^2*d^3*e^3 - 15*b*c*d^2
*e^4 + 117*a*b*e^6 + 10*(b^2 + 2*a*c)*d*e^5)*f - (10*c^2*d^4*e^2 - 24*b*c*d
^3*e^3 - 40*a*b*d*e^5 - 117*a^2*e^6 + 15*(b^2 + 2*a*c)*d^2*e^4)*g)*m)*x^2 +
 24*(6*c^2*d^5*e - 15*b*c*d^4*e^2 - 30*a*b*d^2*e^4 + 30*a^2*d*e^5 + 10*(b^2
 + 2*a*c)*d^3*e^3)*f - 12*(10*c^2*d^6 - 24*b*c*d^5*e - 40*a*b*d^3*e^3 + 30*
a^2*d^2*e^4 + 15*(b^2 + 2*a*c)*d^4*e^2)*g + 2*(2*(6*c^2*d^5*e - 33*b*c*d^4*
e^2 - 171*a*b*d^2*e^4 + 261*a^2*d*e^5 + 37*(b^2 + 2*a*c)*d^3*e^3)*f + (24*b
*c*d^5*e + 148*a*b*d^3*e^3 - 171*a^2*d^2*e^4 - 33*(b^2 + 2*a*c)*d^4*e^2)*g)
*m + (720*a^2*e^6*f + (a^2*d*e^5*g + (2*a*b*d*e^5 + a^2*e^6)*f)*m^5 + 2*((1
8*a*b*d*e^5 + 10*a^2*e^6 - (b^2 + 2*a*c)*d^2*e^4)*f - (2*a*b*d^2*e^4 - 9*a^
2*d*e^5)*g)*m^4 + ((12*b*c*d^3*e^3 + 238*a*b*d*e^5 + 155*a^2*e^6 - 30*(b^2
+ 2*a*c)*d^2*e^4)*f - (60*a*b*d^2*e^4 - 119*a^2*d*e^5 - 6*(b^2 + 2*a*c)*d^3
*e^3)*g)*m^3 - 2*(2*(6*c^2*d^4*e^2 - 33*b*c*d^3*e^3 - 171*a*b*d*e^5 - 145*a
^2*e^6 + 37*(b^2 + 2*a*c)*d^2*e^4)*f + (24*b*c*d^4*e^2 + 148*a*b*d^2*e^4 -
171*a^2*d*e^5 - 33*(b^2 + 2*a*c)*d^3*e^3)*g)*m^2 - 12*((12*c^2*d^4*e^2 - 30
*b*c*d^3*e^3 - 60*a*b*d*e^5 - 87*a^2*e^6 + 20*(b^2 + 2*a*c)*d^2*e^4)*f - (1
0*c^2*d^5*e - 24*b*c*d^4*e^2 - 40*a*b*d^2*e^4 + 30*a^2*d*e^5 + 15*(b^2 + 2*
a*c)*d^3*e^3)*g)*m)*x)*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 73
```

```
5*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32864 vs. 2(309) = 618.

Time = 5.65 (sec) , antiderivative size = 32864, normalized size of antiderivative = 105.67

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*f*x + a**2*g*x**2/2 + a*b*f*x**2 + 2*a*b*g*x**3/3 + 2
*a*c*f*x**3/3 + a*c*g*x**4/2 + b**2*f*x**3/3 + b**2*g*x**4/4 + b*c*f*x**4/2
 + 2*b*c*g*x**5/5 + c**2*f*x**5/5 + c**2*g*x**6/6), Eq(e, 0)), (-3*a**2*d*e
**4*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*a**2*e**5*f/(60*d**5*e**6 + 3
00*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4
 + 60*e**11*x**5) - 15*a**2*e**5*g*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*
d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 4
*a*b*d**2*e**3*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600
*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*b*d*e**4*f/(60*d*
*5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d
*e**10*x**4 + 60*e**11*x**5) - 20*a*b*d*e**4*g*x/(60*d**5*e**6 + 300*d**4*e
**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**
11*x**5) - 30*a*b*e**5*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*
x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 40*a*b*e**5
*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**
9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*c*d**3*e**2*g/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10
*x**4 + 60*e**11*x**5) - 4*a*c*d**2*e**3*f/(60*d**5*e**6 + 300*d**4*e**7*x
 + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**
5) - 30*a*c*d**2*e**3*g*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 20*a*c*d*e**
4*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*a*c*d*e**4*g*x**2/(60*d**5*e*
*6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**1
0*x**4 + 60*e**11*x**5) - 40*a*c*e**5*f*x**2/(60*d**5*e**6 + 300*d**4*e**7*
x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x
**5) - 60*a*c*e**5*g*x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 3*b**2*d**3*
e**2*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9
*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 2*b**2*d**2*e**3*f/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10
*x**4 + 60*e**11*x**5) - 15*b**2*d**2*e**3*g*x/(60*d**5*e**6 + 300*d**4*e**
```

```
7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11
*x**5) - 10*b**2*d*e**4*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8
*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 30*b**2*d*
e**4*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2
*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 20*b**2*e**5*f*x**2/(60*d*
*5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d
*e**10*x**4 + 60*e**11*x**5) - 30*b**2*e**5*g*x**3/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) - 24*b*c*d**4*e*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**
8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*b*c*d**
3*e**2*f/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e*
*9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 120*b*c*d**3*e**2*g*x/(60*d**
5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*
e**10*x**4 + 60*e**11*x**5) - 30*b*c*d**2*e**3*f*x/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) - 240*b*c*d**2*e**3*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600
*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) -
60*b*c*d*e**4*f*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 +
 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 240*b*c*d*e**4*g*
x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x
**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*b*c*e**5*f*x**3/(60*d**5*e**6
+ 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x
**4 + 60*e**11*x**5) - 120*b*c*e**5*g*x**4/(60*d**5*e**6 + 300*d**4*e**7*x
+ 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**
5) + 60*c**2*d**5*g*log(d/e + x)/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3
*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 137*c
**2*d**5*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*
e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*c**2*d**4*e*f/(60*d**5*e
**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**
10*x**4 + 60*e**11*x**5) + 300*c**2*d**4*e*g*x*log(d/e + x)/(60*d**5*e**6 +
 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x*
*4 + 60*e**11*x**5) + 625*c**2*d**4*e*g*x/(60*d**5*e**6 + 300*d**4*e**7*x +
 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5
) - 60*c**2*d**3*e**2*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 600*c**2*d**
3*e**2*g*x**2*log(d/e + x)/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*
x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 1100*c**2*d
**3*e**2*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*
d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 120*c**2*d**2*e**3*f*x
**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x*
*3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 600*c**2*d**2*e**3*g*x**3*log(d/e
+ x)/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x
**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 900*c**2*d**2*e**3*g*x**3/(60*d**
5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*
e**10*x**4 + 60*e**11*x**5) - 120*c**2*d*e**4*f*x**3/(60*d**5*e**6 + 300*d*
```

```
*4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60
*e**11*x**5) + 300*c**2*d*e**4*g*x**4*log(d/e + x)/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) + 300*c**2*d*e**4*g*x**4/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d
**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60
*c**2*e**5*f*x**4/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 60
0*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 60*c**2*e**5*g*x**5*
log(d/e + x)/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**
2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5), Eq(m, -6)), (-a**2*d*e**4*
g/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*
e**10*x**4) - 3*a**2*e**5*f/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x
**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 4*a**2*e**5*g*x/(12*d**4*e**6 + 48*
d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 2*a*b*d
**2*e**3*g/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x
**3 + 12*e**10*x**4) - 2*a*b*d*e**4*f/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d
**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 8*a*b*d*e**4*g*x/(12*d**4
*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4
) - 8*a*b*e**5*f*x/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*
d*e**9*x**3 + 12*e**10*x**4) - 12*a*b*e**5*g*x**2/(12*d**4*e**6 + 48*d**3*e
**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 6*a*c*d**3*e*
*2*g/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 +
12*e**10*x**4) - 2*a*c*d**2*e**3*f/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2
*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 24*a*c*d**2*e**3*g*x/(12*d**
4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**
4) - 8*a*c*d*e**4*f*x/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 +
48*d*e**9*x**3 + 12*e**10*x**4) - 36*a*c*d*e**4*g*x**2/(12*d**4*e**6 + 48*d
**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 12*a*c*e
**5*f*x**2/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x
**3 + 12*e**10*x**4) - 24*a*c*e**5*g*x**3/(12*d**4*e**6 + 48*d**3*e**7*x +
72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 3*b**2*d**3*e**2*g/(1
2*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**1
0*x**4) - b**2*d**2*e**3*f/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x*
*2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 12*b**2*d**2*e**3*g*x/(12*d**4*e**6
+ 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 4*
b**2*d*e**4*f*x/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e
**9*x**3 + 12*e**10*x**4) - 18*b**2*d*e**4*g*x**2/(12*d**4*e**6 + 48*d**3*e
**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 6*b**2*e**5*f
*x**2/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 +
 12*e**10*x**4) - 12*b**2*e**5*g*x**3/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d
**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 24*b*c*d**4*e*g*log(d/e +
 x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 1
2*e**10*x**4) + 50*b*c*d**4*e*g/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e*
*8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 6*b*c*d**3*e**2*f/(12*d**4*e**6
 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 9
6*b*c*d**3*e**2*g*x*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e
```

8*x2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 176*b*c*d**3*e**2*g*x/(12*d**4
*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4
) - 24*b*c*d**2*e**3*f*x/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2
 + 48*d*e**9*x**3 + 12*e**10*x**4) + 144*b*c*d**2*e**3*g*x**2*log(d/e + x)/
(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e*
*10*x**4) + 216*b*c*d**2*e**3*g*x**2/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d*
*2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 36*b*c*d*e**4*f*x**2/(12*d
4*e6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x
4) + 96*b*c*d*e4*g*x**3*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 7
2*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 96*b*c*d*e**4*g*x**3/(
12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**
10*x**4) - 24*b*c*e**5*f*x**3/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8
*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 24*b*c*e**5*g*x**4*log(d/e + x)/(
12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**
10*x**4) - 60*c**2*d**5*g*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*
d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 125*c**2*d**5*g/(12*d**4
*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4
) + 12*c**2*d**4*e*f*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*
e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 25*c**2*d**4*e*f/(12*d**4*e**
6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) -
240*c**2*d**4*e*g*x*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e
8*x2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 440*c**2*d**4*e*g*x/(12*d**4*e
6 + 48*d3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4)
+ 48*c**2*d**3*e**2*f*x*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d*
*2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 88*c**2*d**3*e**2*f*x/(12*
d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*
x**4) - 360*c**2*d**3*e**2*g*x**2*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7
*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 540*c**2*d**3*e*
*2*g*x**2/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x*
*3 + 12*e**10*x**4) + 72*c**2*d**2*e**3*f*x**2*log(d/e + x)/(12*d**4*e**6 +
 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 108
*c**2*d**2*e**3*f*x**2/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 +
 48*d*e**9*x**3 + 12*e**10*x**4) - 240*c**2*d**2*e**3*g*x**3*log(d/e + x)/(
12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**
10*x**4) - 240*c**2*d**2*e**3*g*x**3/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d*
*2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 48*c**2*d*e**4*f*x**3*log(
d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**
3 + 12*e**10*x**4) + 48*c**2*d*e**4*f*x**3/(12*d**4*e**6 + 48*d**3*e**7*x +
 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) - 60*c**2*d*e**4*g*x**
4*log(d/e + x)/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e*
*9*x**3 + 12*e**10*x**4) + 12*c**2*e**5*f*x**4*log(d/e + x)/(12*d**4*e**6 +
 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4) + 12*
c**2*e**5*g*x**5/(12*d**4*e**6 + 48*d**3*e**7*x + 72*d**2*e**8*x**2 + 48*d*
e**9*x**3 + 12*e**10*x**4), Eq(m, -5)), (-a**2*d*e**4*g/(6*d**3*e**6 + 18*d
2*e7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 2*a**2*e**5*f/(6*d**3*e**6 + 1

8*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 3*a**2*e**5*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 4*a*b*d**2*e**3*g/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 2*a*b*d*e**4*f/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 12*a*b*d*e**4*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 6*a*b*e**5*f*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 12*a*b*e**5*g*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 12*a*c*d**3*e**2*g*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 22*a*c*d**3*e**2*g/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 4*a*c*d**2*e**3*f/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*a*c*d**2*e**3*g*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 54*a*c*d**2*e**3*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 12*a*c*d*e**4*f*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*a*c*d*e**4*g*x**2*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*a*c*d*e**4*g*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 12*a*c*e**5*f*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 12*a*c*e**5*g*x**3*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 6*b**2*d**3*e**2*g*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 11*b**2*d**3*e**2*g/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 2*b**2*d**2*e**3*f/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 18*b**2*d**2*e**3*g*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 27*b**2*d**2*e**3*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 6*b**2*d*e**4*f*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 18*b**2*d*e**4*g*x**2*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 18*b**2*d*e**4*g*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 6*b**2*e**5*f*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 6*b**2*e**5*g*x**3*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 48*b*c*d**4*e*g*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 88*b*c*d**4*e*g/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 12*b*c*d**3*e**2*f*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 22*b*c*d**3*e**2*f/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 144*b*c*d**3*e**2*g*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 216*b*c*d**3*e**2*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*b*c*d**2*e**3*f*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 54*b*c*d**2*e**3*f*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 144*b*c*d**2*e**3*g*x**2*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 144*b*c*d**2*e**3*g*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*b*c*d*e**4*f*x**2*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 36*b*c*d*e**4*f*x**2/(6*d**3*e**6 + 18*d**2*e**7*x

+ 18*d*e**8*x**2 + 6*e**9*x**3) - 48*b*c*d*e**4*g*x**3*log(d/e + x)/(6*d**3
*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 12*b*c*e**5*f*x**3
*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3)
 + 12*b*c*e**5*g*x**4/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e*
*9*x**3) + 60*c**2*d**5*g*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d
*e**8*x**2 + 6*e**9*x**3) + 110*c**2*d**5*g/(6*d**3*e**6 + 18*d**2*e**7*x +
 18*d*e**8*x**2 + 6*e**9*x**3) - 24*c**2*d**4*e*f*log(d/e + x)/(6*d**3*e**6
 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 44*c**2*d**4*e*f/(6*d**
3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 180*c**2*d**4*e*g
*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**
3) + 270*c**2*d**4*e*g*x/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6
*e**9*x**3) - 72*c**2*d**3*e**2*f*x*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**
7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 108*c**2*d**3*e**2*f*x/(6*d**3*e**6 +
 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 180*c**2*d**3*e**2*g*x**2
*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3)
 + 180*c**2*d**3*e**2*g*x**2/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2
 + 6*e**9*x**3) - 72*c**2*d**2*e**3*f*x**2*log(d/e + x)/(6*d**3*e**6 + 18*d
2*e7*x + 18*d*e**8*x**2 + 6*e**9*x**3) - 72*c**2*d**2*e**3*f*x**2/(6*d*
*3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 60*c**2*d**2*e**
3*g*x**3*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e*
*9*x**3) - 24*c**2*d*e**4*f*x**3*log(d/e + x)/(6*d**3*e**6 + 18*d**2*e**7*x
 + 18*d*e**8*x**2 + 6*e**9*x**3) - 15*c**2*d*e**4*g*x**4/(6*d**3*e**6 + 18*
d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 6*c**2*e**5*f*x**4/(6*d**3*e*
*6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3) + 3*c**2*e**5*g*x**5/(6
*d**3*e**6 + 18*d**2*e**7*x + 18*d*e**8*x**2 + 6*e**9*x**3), Eq(m, -4)), (-
3*a**2*d*e**4*g/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 3*a**2*e**5*f/(
6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 6*a**2*e**5*g*x/(6*d**2*e**6 + 1
2*d*e**7*x + 6*e**8*x**2) + 12*a*b*d**2*e**3*g*log(d/e + x)/(6*d**2*e**6 +
12*d*e**7*x + 6*e**8*x**2) + 18*a*b*d**2*e**3*g/(6*d**2*e**6 + 12*d*e**7*x
+ 6*e**8*x**2) - 6*a*b*d*e**4*f/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) +
 24*a*b*d*e**4*g*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) +
 24*a*b*d*e**4*g*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 12*a*b*e**5*
f*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 12*a*b*e**5*g*x**2*log(d/e
+ x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 36*a*c*d**3*e**2*g*log(d/e
 + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 54*a*c*d**3*e**2*g/(6*d**
2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 12*a*c*d**2*e**3*f*log(d/e + x)/(6*d*
*2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 18*a*c*d**2*e**3*f/(6*d**2*e**6 + 12
*d*e**7*x + 6*e**8*x**2) - 72*a*c*d**2*e**3*g*x*log(d/e + x)/(6*d**2*e**6 +
 12*d*e**7*x + 6*e**8*x**2) - 72*a*c*d**2*e**3*g*x/(6*d**2*e**6 + 12*d*e**7
*x + 6*e**8*x**2) + 24*a*c*d*e**4*f*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7
*x + 6*e**8*x**2) + 24*a*c*d*e**4*f*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x
2) - 36*a*c*d*e4*g*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**
8*x**2) + 12*a*c*e**5*f*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e*
*8*x**2) + 12*a*c*e**5*g*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 1
8*b**2*d**3*e**2*g*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) -

27*b**2*d**3*e**2*g/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 6*b**2*d**2*e**3*f*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 9*b**2*d**2*e**3*f/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 36*b**2*d**2*e**3*g*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 36*b**2*d**2*e**3*g*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 12*b**2*d*e**4*f*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 12*b**2*d*e**4*f*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 18*b**2*d*e**4*g*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 6*b**2*e**5*f*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 6*b**2*e**5*g*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 72*b*c*d**4*e*g*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 108*b*c*d**4*e*g/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 36*b*c*d**3*e**2*f*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 54*b*c*d**3*e**2*f/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 144*b*c*d**3*e**2*g*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 144*b*c*d**3*e**2*g*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 72*b*c*d**2*e**3*f*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 72*b*c*d**2*e**3*f*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 72*b*c*d**2*e**3*g*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 36*b*c*d*e**4*f*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 24*b*c*d*e**4*g*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 12*b*c*e**5*f*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 6*b*c*e**5*g*x**4/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 60*c**2*d**5*g*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 90*c**2*d**5*g/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 36*c**2*d**4*e*f*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 54*c**2*d**4*e*f/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 120*c**2*d**4*e*g*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 120*c**2*d**4*e*g*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 72*c**2*d**3*e**2*f*x*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 72*c**2*d**3*e**2*f*x/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 60*c**2*d**3*e**2*g*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 36*c**2*d**2*e**3*f*x**2*log(d/e + x)/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 20*c**2*d**2*e**3*g*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 12*c**2*d*e**4*f*x**3/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) - 5*c**2*d*e**4*g*x**4/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 3*c**2*e**5*f*x**4/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2) + 2*c**2*e**5*g*x**5/(6*d**2*e**6 + 12*d*e**7*x + 6*e**8*x**2), Eq(m, -3)), (12*a**2*d*e**4*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 12*a**2*d*e**4*g/(12*d*e**6 + 12*e**7*x) - 12*a**2*e**5*f/(12*d*e**6 + 12*e**7*x) + 12*a**2*e**5*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 48*a*b*d**2*e**3*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 48*a*b*d**2*e**3*g/(12*d*e**6 + 12*e**7*x) + 24*a*b*d*e**4*f*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 24*a*b*d*e**4*f/(12*d*e**6 + 12*e**7*x) - 48*a*b*d*e**4*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 24*a*b*e**5*f*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 24*a*b*e**5*g*x**2/(12*d*e**6 + 12*e**7*x) + 72*a*c*d**3*e**2*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 72*a*c*d**3*e**2*g/(12*d*e**6 + 12*e**7*x) - 48*a*c*d**2*e**3*f*log(d/e + x)/(12*d*e**6 + 12*e

7*x) - 48*a*c*d2*e**3*f/(12*d*e**6 + 12*e**7*x) + 72*a*c*d**2*e**3*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 48*a*c*d*e**4*f*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 36*a*c*d*e**4*g*x**2/(12*d*e**6 + 12*e**7*x) + 24*a*c*e**5*f*x**2/(12*d*e**6 + 12*e**7*x) + 12*a*c*e**5*g*x**3/(12*d*e**6 + 12*e**7*x) + 36*b**2*d**3*e**2*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 36*b**2*d**3*e**2*g/(12*d*e**6 + 12*e**7*x) - 24*b**2*d**2*e**3*f*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 24*b**2*d**2*e**3*f/(12*d*e**6 + 12*e**7*x) + 36*b**2*d**2*e**3*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 24*b**2*d*e**4*f*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 18*b**2*d*e**4*g*x**2/(12*d*e**6 + 12*e**7*x) + 12*b**2*e**5*f*x**2/(12*d*e**6 + 12*e**7*x) + 6*b**2*e**5*g*x**3/(12*d*e**6 + 12*e**7*x) - 96*b*c*d**4*e*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 96*b*c*d**4*e*g/(12*d*e**6 + 12*e**7*x) + 72*b*c*d**3*e**2*f*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 72*b*c*d**3*e**2*f/(12*d*e**6 + 12*e**7*x) - 96*b*c*d**3*e**2*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 72*b*c*d**2*e**3*f*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 48*b*c*d**2*e**3*g*x**2/(12*d*e**6 + 12*e**7*x) - 36*b*c*d*e**4*f*x**2/(12*d*e**6 + 12*e**7*x) - 16*b*c*d*e**4*g*x**3/(12*d*e**6 + 12*e**7*x) + 12*b*c*e**5*f*x**3/(12*d*e**6 + 12*e**7*x) + 8*b*c*e**5*g*x**4/(12*d*e**6 + 12*e**7*x) + 60*c**2*d**5*g*log(d/e + x)/(12*d*e**6 + 12*e**7*x) + 60*c**2*d**5*g/(12*d*e**6 + 12*e**7*x) - 48*c**2*d**4*e*f*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 48*c**2*d**4*e*f/(12*d*e**6 + 12*e**7*x) + 60*c**2*d**4*e*g*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 48*c**2*d**3*e**2*f*x*log(d/e + x)/(12*d*e**6 + 12*e**7*x) - 30*c**2*d**3*e**2*g*x**2/(12*d*e**6 + 12*e**7*x) + 24*c**2*d**2*e**3*f*x**2/(12*d*e**6 + 12*e**7*x) + 10*c**2*d**2*e**3*g*x**3/(12*d*e**6 + 12*e**7*x) - 8*c**2*d*e**4*f*x**3/(12*d*e**6 + 12*e**7*x) - 5*c**2*d*e**4*g*x**4/(12*d*e**6 + 12*e**7*x) + 4*c**2*e**5*f*x**4/(12*d*e**6 + 12*e**7*x) + 3*c**2*e**5*g*x**5/(12*d*e**6 + 12*e**7*x), Eq(m, -2)), (-a**2*d*g*log(d/e + x)/e**2 + a**2*f*log(d/e + x)/e + a**2*g*x/e + 2*a*b*d**2*g*log(d/e + x)/e**3 - 2*a*b*d*f*log(d/e + x)/e**2 - 2*a*b*d*g*x/e**2 + 2*a*b*f*x/e + a*b*g*x**2/e - 2*a*c*d**3*g*log(d/e + x)/e**4 + 2*a*c*d**2*f*log(d/e + x)/e**3 + 2*a*c*d**2*g*x/e**3 - 2*a*c*d*f*x/e**2 - a*c*d*g*x**2/e**2 + a*c*f*x**2/e + 2*a*c*g*x**3/(3*e) - b**2*d**3*g*log(d/e + x)/e**4 + b**2*d**2*f*log(d/e + x)/e**3 + b**2*d**2*g*x/e**3 - b**2*d*f*x/e**2 - b**2*d*g*x**2/(2*e**2) + b**2*f*x**2/(2*e) + b**2*g*x**3/(3*e) + 2*b*c*d**4*g*log(d/e + x)/e**5 - 2*b*c*d**3*f*log(d/e + x)/e**4 - 2*b*c*d**3*g*x/e**4 + 2*b*c*d**2*f*x/e**3 + b*c*d**2*g*x**2/e**3 - b*c*d*f*x**2/e**2 - 2*b*c*d*g*x**3/(3*e**2) + 2*b*c*f*x**3/(3*e) + b*c*g*x**4/(2*e) - c**2*d**5*g*log(d/e + x)/e**6 + c**2*d**4*f*log(d/e + x)/e**5 + c**2*d**4*g*x/e**5 - c**2*d**3*f*x/e**4 - c**2*d**3*g*x**2/(2*e**4) + c**2*d**2*f*x**2/(2*e**3) + c**2*d**2*g*x**3/(3*e**3) - c**2*d*f*x**3/(3*e**2) - c**2*d*g*x**4/(4*e**2) + c**2*f*x**4/(4*e) + c**2*g*x**5/(5*e), Eq(m, -1)), (-a**2*d**2*e**4*g*m**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 18*a**2*d**2*e**4*g*m**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 119*a**2*d**2*e**4*g*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 7

35*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 342*a**2*d**2*e**4*g*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 360*a**2*d**2*e**4*g*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + a**2*d*e**5*f*m**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 20*a**2*d*e**5*f*m**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 155*a**2*d*e**5*f*m**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 580*a**2*d*e**5*f*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 1044*a**2*d*e**5*f*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 720*a**2*d*e**5*f*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + a**2*d*e**5*g*m**5*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 18*a**2*d*e**5*g*m**4*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 119*a**2*d*e**5*g*m**3*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 342*a**2*d*e**5*g*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 360*a**2*d*e**5*g*m*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + a**2*e**6*f*m**5*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 20*a**2*e**6*f*m**4*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 155*a**2*e**6*f*m**3*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 580*a**2*e**6*f*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 1044*a**2*e**6*f*m*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 720*a**2*e**6*f*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + a**2*e**6*g*m**5*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 19*a**2*e**6*g*m**4*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 137*a**2*e**6*g*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 461*a**2*e**6*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 702*a**2*e**6*g*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)

+ 360*a**2*e**6*g*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m*
*4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 4*a*b*d**3*
e**3*g*m**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e*
*6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 60*a*b*d**3*e**3*g*m**
2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 296*a*b*d**3*e**3*g*m*(d + e*x)*
*m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m*
*2 + 1764*e**6*m + 720*e**6) + 480*a*b*d**3*e**3*g*(d + e*x)**m/(e**6*m**6
+ 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6
*m + 720*e**6) - 2*a*b*d**2*e**4*f*m**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m
5 + 175*e6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e*
*6) - 36*a*b*d**2*e**4*f*m**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*
e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 238*
a*b*d**2*e**4*f*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4
 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 684*a*b*d**2*
e**4*f*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*
m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 720*a*b*d**2*e**4*f*(d +
e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e*
*6*m**2 + 1764*e**6*m + 720*e**6) - 4*a*b*d**2*e**4*g*m**4*x*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) - 60*a*b*d**2*e**4*g*m**3*x*(d + e*x)**m/(e**6*m**
6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e*
*6*m + 720*e**6) - 296*a*b*d**2*e**4*g*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*
e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
720*e**6) - 480*a*b*d**2*e**4*g*m*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5
+ 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)
+ 2*a*b*d*e**5*f*m**5*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m
4 + 735*e6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 36*a*b*d*e
5*f*m4*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e
6*m3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 238*a*b*d*e**5*f*m**3
x(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 684*a*b*d*e**5*f*m**2*x*(d + e*
x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6
*m**2 + 1764*e**6*m + 720*e**6) + 720*a*b*d*e**5*f*m*x*(d + e*x)**m/(e**6*m
6 + 21*e6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*
e**6*m + 720*e**6) + 2*a*b*d*e**5*g*m**5*x**2*(d + e*x)**m/(e**6*m**6 + 21*
e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
720*e**6) + 32*a*b*d*e**5*g*m**4*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**
5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6
) + 178*a*b*d*e**5*g*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175
*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 388
*a*b*d*e**5*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m
4 + 735*e6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 240*a*b*d*
e**5*g*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*
e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*a*b*e**6*f*m**5*x*

```
*2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 38*a*b*e**6*f*m**4*x**2*(d + e*
x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6
*m**2 + 1764*e**6*m + 720*e**6) + 274*a*b*e**6*f*m**3*x**2*(d + e*x)**m/(e*
*6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1
764*e**6*m + 720*e**6) + 922*a*b*e**6*f*m**2*x**2*(d + e*x)**m/(e**6*m**6 +
 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*
m + 720*e**6) + 1404*a*b*e**6*f*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m*
*5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**
6) + 720*a*b*e**6*f*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*
m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*a*b*e**
6*g*m**5*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*
e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 36*a*b*e**6*g*m**4*x
**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
+ 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 242*a*b*e**6*g*m**3*x**3*(d +
e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e*
*6*m**2 + 1764*e**6*m + 720*e**6) + 744*a*b*e**6*g*m**2*x**3*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) + 1016*a*b*e**6*g*m*x**3*(d + e*x)**m/(e**6*m**6 +
 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*
m + 720*e**6) + 480*a*b*e**6*g*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5
+ 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)
- 12*a*c*d**4*e**2*g*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 132*a*c*
d**4*e**2*g*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*
e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 360*a*c*d**4*e**2*g*
(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 16
24*e**6*m**2 + 1764*e**6*m + 720*e**6) + 4*a*c*d**3*e**3*f*m**3*(d + e*x)**
m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**
2 + 1764*e**6*m + 720*e**6) + 60*a*c*d**3*e**3*f*m**2*(d + e*x)**m/(e**6*m*
*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
**6*m + 720*e**6) + 296*a*c*d**3*e**3*f*m*(d + e*x)**m/(e**6*m**6 + 21*e**6
*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*
e**6) + 480*a*c*d**3*e**3*f*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e*
*6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 12*a*c
*d**3*e**3*g*m**3*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4
+ 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 132*a*c*d**3*e
**3*g*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e
**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 360*a*c*d**3*e**3*g*m
*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 4*a*c*d**2*e**4*f*m**4*x*(d + e
*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**
6*m**2 + 1764*e**6*m + 720*e**6) - 60*a*c*d**2*e**4*f*m**3*x*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) - 296*a*c*d**2*e**4*f*m**2*x*(d + e*x)**m/(e**6*m*
```

*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
6*m + 720*e6) - 480*a*c*d**2*e**4*f*m*x*(d + e*x)**m/(e**6*m**6 + 21*e*
*6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 72
0*e**6) - 6*a*c*d**2*e**4*g*m**4*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**
5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6
) - 72*a*c*d**2*e**4*g*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 1
75*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 2
46*a*c*d**2*e**4*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e
6*m4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 180*a
*c*d**2*e**4*g*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**
4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*a*c*d*e**5
*f*m**5*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e
6*m3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 32*a*c*d*e**5*f*m**4*
x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 178*a*c*d*e**5*f*m**3*x**2*(d
 + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624
*e**6*m**2 + 1764*e**6*m + 720*e**6) + 388*a*c*d*e**5*f*m**2*x**2*(d + e*x)
m/(e6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m
2 + 1764*e6*m + 720*e**6) + 240*a*c*d*e**5*f*m*x**2*(d + e*x)**m/(e**6*
m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764
*e**6*m + 720*e**6) + 2*a*c*d*e**5*g*m**5*x**3*(d + e*x)**m/(e**6*m**6 + 21
*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
 720*e**6) + 28*a*c*d*e**5*g*m**4*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m*
*5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**
6) + 130*a*c*d*e**5*g*m**3*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 17
5*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 22
4*a*c*d*e**5*g*m**2*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*
m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 120*a*c*d
*e**5*g*m*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735
*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*a*c*e**6*f*m**5*x
3*(d + e*x)m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
+ 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 36*a*c*e**6*f*m**4*x**3*(d + e
*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**
6*m**2 + 1764*e**6*m + 720*e**6) + 242*a*c*e**6*f*m**3*x**3*(d + e*x)**m/(e
6*m6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
1764*e**6*m + 720*e**6) + 744*a*c*e**6*f*m**2*x**3*(d + e*x)**m/(e**6*m**6
+ 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6
*m + 720*e**6) + 1016*a*c*e**6*f*m*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m
5 + 175*e6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e*
*6) + 480*a*c*e**6*f*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*a*c*e*
*6*g*m**5*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735
*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 34*a*c*e**6*g*m**4*
x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 214*a*c*e**6*g*m**3*x**4*(d +

e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e
6*m2 + 1764*e**6*m + 720*e**6) + 614*a*c*e**6*g*m**2*x**4*(d + e*x)**m/
(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2
+ 1764*e**6*m + 720*e**6) + 792*a*c*e**6*g*m*x**4*(d + e*x)**m/(e**6*m**6 +
21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*
m + 720*e**6) + 360*a*c*e**6*g*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5
+ 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)
- 6*b**2*d**4*e**2*g*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 66*b**2*
d**4*e**2*g*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*
e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 180*b**2*d**4*e**2*g
*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1
624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*b**2*d**3*e**3*f*m**3*(d + e*x)
m/(e6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m
2 + 1764*e6*m + 720*e**6) + 30*b**2*d**3*e**3*f*m**2*(d + e*x)**m/(e**6
*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 176
4*e**6*m + 720*e**6) + 148*b**2*d**3*e**3*f*m*(d + e*x)**m/(e**6*m**6 + 21*
e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
720*e**6) + 240*b**2*d**3*e**3*f*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 1
75*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 6
*b**2*d**3*e**3*g*m**3*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*
m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 66*b**2*d
3*e3*g*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 +
735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 180*b**2*d**3*e*
*3*g*m*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*
m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 2*b**2*d**2*e**4*f*m**4*x
*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1
624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 30*b**2*d**2*e**4*f*m**3*x*(d + e
*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**
6*m**2 + 1764*e**6*m + 720*e**6) - 148*b**2*d**2*e**4*f*m**2*x*(d + e*x)**m
/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2
+ 1764*e**6*m + 720*e**6) - 240*b**2*d**2*e**4*f*m*x*(d + e*x)**m/(e**6*m*
*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
6*m + 720*e6) - 3*b**2*d**2*e**4*g*m**4*x**2*(d + e*x)**m/(e**6*m**6 +
21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m
+ 720*e**6) - 36*b**2*d**2*e**4*g*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e
6*m5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 7
20*e**6) - 123*b**2*d**2*e**4*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6
*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*
e**6) - 90*b**2*d**2*e**4*g*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 +
175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) +
b**2*d*e**5*f*m**5*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*
m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 16*b**2*d
*e**5*f*m**4*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 +
735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 89*b**2*d*e**5*f

*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**
6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 194*b**2*d*e**5*f*m**2*
x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 120*b**2*d*e**5*f*m*x**2*(d +
 e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e
6*m2 + 1764*e**6*m + 720*e**6) + b**2*d*e**5*g*m**5*x**3*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) + 14*b**2*d*e**5*g*m**4*x**3*(d + e*x)**m/(e**6*m*
*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
6*m + 720*e6) + 65*b**2*d*e**5*g*m**3*x**3*(d + e*x)**m/(e**6*m**6 + 21
*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
 720*e**6) + 112*b**2*d*e**5*g*m**2*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*
m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e
6) + 60*b2*d*e**5*g*m*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175
*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + b**
2*e**6*f*m**5*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 +
 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 18*b**2*e**6*f*
m**4*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6
*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 121*b**2*e**6*f*m**3*x**
3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 372*b**2*e**6*f*m**2*x**3*(d + e
*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**
6*m**2 + 1764*e**6*m + 720*e**6) + 508*b**2*e**6*f*m*x**3*(d + e*x)**m/(e**
6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 17
64*e**6*m + 720*e**6) + 240*b**2*e**6*f*x**3*(d + e*x)**m/(e**6*m**6 + 21*e
6*m5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 7
20*e**6) + b**2*e**6*g*m**5*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 1
75*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 1
7*b**2*e**6*g*m**4*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m
4 + 735*e6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 107*b**2*e
6*g*m3*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 73
5*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 307*b**2*e**6*g*m*
*2*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m
3 + 1624*e6*m**2 + 1764*e**6*m + 720*e**6) + 396*b**2*e**6*g*m*x**4*(d
+ e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*
e**6*m**2 + 1764*e**6*m + 720*e**6) + 180*b**2*e**6*g*x**4*(d + e*x)**m/(e*
*6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1
764*e**6*m + 720*e**6) + 48*b*c*d**5*e*g*m*(d + e*x)**m/(e**6*m**6 + 21*e**
6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720
*e**6) + 288*b*c*d**5*e*g*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 12*b*c*d
4*e2*f*m**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 73
5*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 132*b*c*d**4*e**2*
f*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
+ 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 360*b*c*d**4*e**2*f*(d + e*x)*

*m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m*
*2 + 1764*e**6*m + 720*e**6) - 48*b*c*d**4*e**2*g*m**2*x*(d + e*x)**m/(e**6
*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 176
4*e**6*m + 720*e**6) - 288*b*c*d**4*e**2*g*m*x*(d + e*x)**m/(e**6*m**6 + 21
*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
 720*e**6) + 12*b*c*d**3*e**3*f*m**3*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m*
*5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**
6) + 132*b*c*d**3*e**3*f*m**2*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 17
5*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 36
0*b*c*d**3*e**3*f*m*x*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**
4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 24*b*c*d**3*
e**3*g*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 7
35*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 168*b*c*d**3*e**3
*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e
6*m3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 144*b*c*d**3*e**3*g*m
*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**
3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 6*b*c*d**2*e**4*f*m**4*x**2*
(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 16
24*e**6*m**2 + 1764*e**6*m + 720*e**6) - 72*b*c*d**2*e**4*f*m**3*x**2*(d +
e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e*
*6*m**2 + 1764*e**6*m + 720*e**6) - 246*b*c*d**2*e**4*f*m**2*x**2*(d + e*x)
m/(e6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m
2 + 1764*e6*m + 720*e**6) - 180*b*c*d**2*e**4*f*m*x**2*(d + e*x)**m/(e*
*6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1
764*e**6*m + 720*e**6) - 8*b*c*d**2*e**4*g*m**4*x**3*(d + e*x)**m/(e**6*m**
6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e*
*6*m + 720*e**6) - 72*b*c*d**2*e**4*g*m**3*x**3*(d + e*x)**m/(e**6*m**6 + 2
1*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m
+ 720*e**6) - 160*b*c*d**2*e**4*g*m**2*x**3*(d + e*x)**m/(e**6*m**6 + 21*e*
*6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 72
0*e**6) - 96*b*c*d**2*e**4*g*m*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5
+ 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)
+ 2*b*c*d*e**5*f*m**5*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**
6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 28*b*c*
d*e**5*f*m**4*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 +
 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 130*b*c*d*e**5*
f*m**3*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e*
*6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 224*b*c*d*e**5*f*m**2*
x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 120*b*c*d*e**5*f*m*x**3*(d +
e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e*
*6*m**2 + 1764*e**6*m + 720*e**6) + 2*b*c*d*e**5*g*m**5*x**4*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) + 24*b*c*d*e**5*g*m**4*x**4*(d + e*x)**m/(e**6*m**
6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e*

*6*m + 720*e**6) + 94*b*c*d*e**5*g*m**3*x**4*(d + e*x)**m/(e**6*m**6 + 21*e
6*m5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 7
20*e**6) + 144*b*c*d*e**5*g*m**2*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**
5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6
) + 72*b*c*d*e**5*g*m*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**
6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 2*b*c*e
6*f*m5*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 73
5*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 34*b*c*e**6*f*m**4
*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**
3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 214*b*c*e**6*f*m**3*x**4*(d
+ e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*
e**6*m**2 + 1764*e**6*m + 720*e**6) + 614*b*c*e**6*f*m**2*x**4*(d + e*x)**m
/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2
 + 1764*e**6*m + 720*e**6) + 792*b*c*e**6*f*m*x**4*(d + e*x)**m/(e**6*m**6
+ 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6
*m + 720*e**6) + 360*b*c*e**6*f*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5
 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6)
 + 2*b*c*e**6*g*m**5*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 32*b*c*e
6*g*m4*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 73
5*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 190*b*c*e**6*g*m**
3*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m*
*3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 520*b*c*e**6*g*m**2*x**5*(d
 + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624
*e**6*m**2 + 1764*e**6*m + 720*e**6) + 648*b*c*e**6*g*m*x**5*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) + 288*b*c*e**6*g*x**5*(d + e*x)**m/(e**6*m**6 + 21
*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
 720*e**6) - 120*c**2*d**6*g*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e
6*m4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 24*c*
*2*d**5*e*f*m*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*
e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 144*c**2*d**5*e*f*(d
 + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624
*e**6*m**2 + 1764*e**6*m + 720*e**6) + 120*c**2*d**5*e*g*m*x*(d + e*x)**m/(
e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 +
 1764*e**6*m + 720*e**6) - 24*c**2*d**4*e**2*f*m**2*x*(d + e*x)**m/(e**6*m*
*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
6*m + 720*e6) - 144*c**2*d**4*e**2*f*m*x*(d + e*x)**m/(e**6*m**6 + 21*e
6*m5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 7
20*e**6) - 60*c**2*d**4*e**2*g*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*
m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e
6) - 60*c2*d**4*e**2*g*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 +
175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) +
12*c**2*d**3*e**3*f*m**3*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*
e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 84*c

2*d3*e**3*f*m**2*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6
*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 72*c**2*
d**3*e**3*f*m*x**2*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 +
 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 20*c**2*d**3*e*
*3*g*m**3*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735
*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 60*c**2*d**3*e**3*g
*m**2*x**3*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**
6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 40*c**2*d**3*e**3*g*m*x
3*(d + e*x)m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3
+ 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) - 4*c**2*d**2*e**4*f*m**4*x**3*(
d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 162
4*e**6*m**2 + 1764*e**6*m + 720*e**6) - 36*c**2*d**2*e**4*f*m**3*x**3*(d +
e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e*
*6*m**2 + 1764*e**6*m + 720*e**6) - 80*c**2*d**2*e**4*f*m**2*x**3*(d + e*x)
m/(e6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m
2 + 1764*e6*m + 720*e**6) - 48*c**2*d**2*e**4*f*m*x**3*(d + e*x)**m/(e*
*6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1
764*e**6*m + 720*e**6) - 5*c**2*d**2*e**4*g*m**4*x**4*(d + e*x)**m/(e**6*m*
*6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e
6*m + 720*e6) - 30*c**2*d**2*e**4*g*m**3*x**4*(d + e*x)**m/(e**6*m**6 +
 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*
m + 720*e**6) - 55*c**2*d**2*e**4*g*m**2*x**4*(d + e*x)**m/(e**6*m**6 + 21*
e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m +
720*e**6) - 30*c**2*d**2*e**4*g*m*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m*
*5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**
6) + c**2*d*e**5*f*m**5*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e
6*m4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 12*c*
*2*d*e**5*f*m**4*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**
4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 47*c**2*d*e*
*5*f*m**3*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735
*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 72*c**2*d*e**5*f*m*
*2*x**4*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m
3 + 1624*e6*m**2 + 1764*e**6*m + 720*e**6) + 36*c**2*d*e**5*f*m*x**4*(d
 + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624
*e**6*m**2 + 1764*e**6*m + 720*e**6) + c**2*d*e**5*g*m**5*x**5*(d + e*x)**m
/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2
 + 1764*e**6*m + 720*e**6) + 10*c**2*d*e**5*g*m**4*x**5*(d + e*x)**m/(e**6*
m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764
*e**6*m + 720*e**6) + 35*c**2*d*e**5*g*m**3*x**5*(d + e*x)**m/(e**6*m**6 +
21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m
 + 720*e**6) + 50*c**2*d*e**5*g*m**2*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6
*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*
e**6) + 24*c**2*d*e**5*g*m*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 17
5*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + c*
*2*e**6*f*m**5*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4

```
+ 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 16*c**2*e**6*f
*m**4*x**5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**
6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 95*c**2*e**6*f*m**3*x**
5*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 +
1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 260*c**2*e**6*f*m**2*x**5*(d + e
*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**
6*m**2 + 1764*e**6*m + 720*e**6) + 324*c**2*e**6*f*m*x**5*(d + e*x)**m/(e**
6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 17
64*e**6*m + 720*e**6) + 144*c**2*e**6*f*x**5*(d + e*x)**m/(e**6*m**6 + 21*e
**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 7
20*e**6) + c**2*e**6*g*m**5*x**6*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 1
75*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 1
5*c**2*e**6*g*m**4*x**6*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m
**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 85*c**2*e*
*6*g*m**3*x**6*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735
*e**6*m**3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 225*c**2*e**6*g*m**
2*x**6*(d + e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m*
*3 + 1624*e**6*m**2 + 1764*e**6*m + 720*e**6) + 274*c**2*e**6*g*m*x**6*(d +
 e*x)**m/(e**6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e
**6*m**2 + 1764*e**6*m + 720*e**6) + 120*c**2*e**6*g*x**6*(d + e*x)**m/(e**
6*m**6 + 21*e**6*m**5 + 175*e**6*m**4 + 735*e**6*m**3 + 1624*e**6*m**2 + 17
64*e**6*m + 720*e**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(312) = 624$.

Time = 0.24 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.59

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f/((m^2 + 3*m + 2)*e^2)
 + (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a^2*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*a*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m
+ 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*b*g
/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^
3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)
*(e*x + d)^m*b*c*f/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^
2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2
*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*g/((m^4 + 10*m^3 + 35*m^2 + 50*
```

```
m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*
e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*g/
((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2
*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x +
d)^m*c^2*f/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*((m^4
+ 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4
*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^
4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*g/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274
*m + 120)*e^5) + ((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 +
 (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^
4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*g/((m^6 + 21*m^5 + 175
*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4936 vs. $2(312) = 624$.

Time = 0.32 (sec) , antiderivative size = 4936, normalized size of antiderivative = 15.87

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((e*x + d)^m*c^2*e^6*g*m^5*x^6 + (e*x + d)^m*c^2*e^6*f*m^5*x^5 + (e*x + d)^
m*c^2*d*e^5*g*m^5*x^5 + 2*(e*x + d)^m*b*c*e^6*g*m^5*x^5 + 15*(e*x + d)^m*c^
2*e^6*g*m^4*x^6 + (e*x + d)^m*c^2*d*e^5*f*m^5*x^4 + 2*(e*x + d)^m*b*c*e^6*f
*m^5*x^4 + 2*(e*x + d)^m*b*c*d*e^5*g*m^5*x^4 + (e*x + d)^m*b^2*e^6*g*m^5*x^
4 + 2*(e*x + d)^m*a*c*e^6*g*m^5*x^4 + 16*(e*x + d)^m*c^2*e^6*f*m^4*x^5 + 10
*(e*x + d)^m*c^2*d*e^5*g*m^4*x^5 + 32*(e*x + d)^m*b*c*e^6*g*m^4*x^5 + 85*(e
*x + d)^m*c^2*e^6*g*m^3*x^6 + 2*(e*x + d)^m*b*c*d*e^5*f*m^5*x^3 + (e*x + d)
^m*b^2*e^6*f*m^5*x^3 + 2*(e*x + d)^m*a*c*e^6*f*m^5*x^3 + (e*x + d)^m*b^2*d*
e^5*g*m^5*x^3 + 2*(e*x + d)^m*a*c*d*e^5*g*m^5*x^3 + 2*(e*x + d)^m*a*b*e^6*g
*m^5*x^3 + 12*(e*x + d)^m*c^2*d*e^5*f*m^4*x^4 + 34*(e*x + d)^m*b*c*e^6*f*m^
4*x^4 - 5*(e*x + d)^m*c^2*d^2*e^4*g*m^4*x^4 + 24*(e*x + d)^m*b*c*d*e^5*g*m^
4*x^4 + 17*(e*x + d)^m*b^2*e^6*g*m^4*x^4 + 34*(e*x + d)^m*a*c*e^6*g*m^4*x^4
 + 95*(e*x + d)^m*c^2*e^6*f*m^3*x^5 + 35*(e*x + d)^m*c^2*d*e^5*g*m^3*x^5 +
190*(e*x + d)^m*b*c*e^6*g*m^3*x^5 + 225*(e*x + d)^m*c^2*e^6*g*m^2*x^6 + (e*
x + d)^m*b^2*d*e^5*f*m^5*x^2 + 2*(e*x + d)^m*a*c*d*e^5*f*m^5*x^2 + 2*(e*x +
 d)^m*a*b*e^6*f*m^5*x^2 + 2*(e*x + d)^m*a*b*d*e^5*g*m^5*x^2 + (e*x + d)^m*a
^2*e^6*g*m^5*x^2 - 4*(e*x + d)^m*c^2*d^2*e^4*f*m^4*x^3 + 28*(e*x + d)^m*b*c
*d*e^5*f*m^4*x^3 + 18*(e*x + d)^m*b^2*e^6*f*m^4*x^3 + 36*(e*x + d)^m*a*c*e^
6*f*m^4*x^3 - 8*(e*x + d)^m*b*c*d^2*e^4*g*m^4*x^3 + 14*(e*x + d)^m*b^2*d*e^
5*g*m^4*x^3 + 28*(e*x + d)^m*a*c*d*e^5*g*m^4*x^3 + 36*(e*x + d)^m*a*b*e^6*g
```

*m^4*x^3 + 47*(e*x + d)^m*c^2*d*e^5*f*m^3*x^4 + 214*(e*x + d)^m*b*c*e^6*f*m^3*x^4 - 30*(e*x + d)^m*c^2*d^2*e^4*g*m^3*x^4 + 94*(e*x + d)^m*b*c*d*e^5*g*m^3*x^4 + 107*(e*x + d)^m*b^2*e^6*g*m^3*x^4 + 214*(e*x + d)^m*a*c*e^6*g*m^3*x^4 + 260*(e*x + d)^m*c^2*e^6*f*m^2*x^5 + 50*(e*x + d)^m*c^2*d*e^5*g*m^2*x^5 + 520*(e*x + d)^m*b*c*e^6*g*m^2*x^5 + 274*(e*x + d)^m*c^2*e^6*g*m*x^6 + 2*(e*x + d)^m*a*b*d*e^5*f*m^5*x + (e*x + d)^m*a^2*e^6*f*m^5*x + (e*x + d)^m*a^2*d*e^5*g*m^5*x - 6*(e*x + d)^m*b*c*d^2*e^4*f*m^4*x^2 + 16*(e*x + d)^m*b^2*d*e^5*f*m^4*x^2 + 32*(e*x + d)^m*a*c*d*e^5*f*m^4*x^2 + 38*(e*x + d)^m*a*b*e^6*f*m^4*x^2 - 3*(e*x + d)^m*b^2*d^2*e^4*g*m^4*x^2 - 6*(e*x + d)^m*a*c*d^2*e^4*g*m^4*x^2 + 32*(e*x + d)^m*a*b*d*e^5*g*m^4*x^2 + 19*(e*x + d)^m*a^2*e^6*g*m^4*x^2 - 36*(e*x + d)^m*c^2*d^2*e^4*f*m^3*x^3 + 130*(e*x + d)^m*b*c*d*e^5*f*m^3*x^3 + 121*(e*x + d)^m*b^2*e^6*f*m^3*x^3 + 242*(e*x + d)^m*a*c*e^6*f*m^3*x^3 + 20*(e*x + d)^m*c^2*d^3*e^3*g*m^3*x^3 - 72*(e*x + d)^m*b*c*d^2*e^4*g*m^3*x^3 + 65*(e*x + d)^m*b^2*d*e^5*g*m^3*x^3 + 130*(e*x + d)^m*a*c*d*e^5*g*m^3*x^3 + 242*(e*x + d)^m*a*b*e^6*g*m^3*x^3 + 72*(e*x + d)^m*c^2*d*e^5*f*m^2*x^4 + 614*(e*x + d)^m*b*c*e^6*f*m^2*x^4 - 55*(e*x + d)^m*c^2*d^2*e^4*g*m^2*x^4 + 144*(e*x + d)^m*b*c*d*e^5*g*m^2*x^4 + 307*(e*x + d)^m*b^2*e^6*g*m^2*x^4 + 614*(e*x + d)^m*a*c*e^6*g*m^2*x^4 + 324*(e*x + d)^m*c^2*e^6*f*m*x^5 + 24*(e*x + d)^m*c^2*d*e^5*g*m*x^5 + 648*(e*x + d)^m*b*c*e^6*g*m*x^5 + 120*(e*x + d)^m*c^2*e^6*g*x^6 + (e*x + d)^m*a^2*d*e^5*f*m^5 - 2*(e*x + d)^m*b^2*d^2*e^4*f*m^4*x - 4*(e*x + d)^m*a*c*d^2*e^4*f*m^4*x + 36*(e*x + d)^m*a*b*d*e^5*f*m^4*x + 20*(e*x + d)^m*a^2*e^6*f*m^4*x - 4*(e*x + d)^m*a*b*d^2*e^4*g*m^4*x + 18*(e*x + d)^m*a^2*d*e^5*g*m^4*x + 12*(e*x + d)^m*c^2*d^3*e^3*f*m^3*x^2 - 72*(e*x + d)^m*b*c*d^2*e^4*f*m^3*x^2 + 89*(e*x + d)^m*b^2*d*e^5*f*m^3*x^2 + 178*(e*x + d)^m*a*c*d*e^5*f*m^3*x^2 + 274*(e*x + d)^m*a*b*e^6*f*m^3*x^2 + 24*(e*x + d)^m*b*c*d^3*e^3*g*m^3*x^2 - 36*(e*x + d)^m*b^2*d^2*e^4*g*m^3*x^2 - 72*(e*x + d)^m*a*c*d^2*e^4*g*m^3*x^2 + 178*(e*x + d)^m*a*b*d*e^5*g*m^3*x^2 + 137*(e*x + d)^m*a^2*e^6*g*m^3*x^2 - 80*(e*x + d)^m*c^2*d^2*e^4*f*m^2*x^3 + 224*(e*x + d)^m*b*c*d*e^5*f*m^2*x^3 + 372*(e*x + d)^m*b^2*e^6*f*m^2*x^3 + 744*(e*x + d)^m*a*c*e^6*f*m^2*x^3 + 60*(e*x + d)^m*c^2*d^3*e^3*g*m^2*x^3 - 160*(e*x + d)^m*b*c*d^2*e^4*g*m^2*x^3 + 112*(e*x + d)^m*b^2*d*e^5*g*m^2*x^3 + 224*(e*x + d)^m*a*c*d*e^5*g*m^2*x^3 + 744*(e*x + d)^m*a*b*e^6*g*m^2*x^3 + 36*(e*x + d)^m*c^2*d*e^5*f*m*x^4 + 792*(e*x + d)^m*b*c*e^6*f*m*x^4 - 30*(e*x + d)^m*c^2*d^2*e^4*g*m*x^4 + 72*(e*x + d)^m*b*c*d*e^5*g*m*x^4 + 396*(e*x + d)^m*b^2*e^6*g*m*x^4 + 792*(e*x + d)^m*a*c*e^6*g*m*x^4 + 144*(e*x + d)^m*c^2*e^6*f*x^5 + 288*(e*x + d)^m*b*c*e^6*g*x^5 - 2*(e*x + d)^m*a*b*d^2*e^4*f*m^4 + 20*(e*x + d)^m*a^2*d*e^5*f*m^4 - (e*x + d)^m*a^2*d^2*e^4*g*m^4 + 12*(e*x + d)^m*b*c*d^3*e^3*f*m^3*x - 30*(e*x + d)^m*b^2*d^2*e^4*f*m^3*x - 60*(e*x + d)^m*a*c*d^2*e^4*f*m^3*x + 238*(e*x + d)^m*a*b*d*e^5*f*m^3*x + 155*(e*x + d)^m*a^2*e^6*f*m^3*x + 6*(e*x + d)^m*b^2*d^3*e^3*g*m^3*x + 12*(e*x + d)^m*a*c*d^3*e^3*g*m^3*x - 60*(e*x + d)^m*a*b*d^2*e^4*g*m^3*x + 119*(e*x + d)^m*a^2*d*e^5*g*m^3*x + 84*(e*x + d)^m*c^2*d^3*e^3*f*m^2*x^2 - 246*(e*x + d)^m*b*c*d^2*e^4*f*m^2*x^2 + 194*(e*x + d)^m*b^2*d*e^5*f*m^2*x^2 + 388*(e*x + d)^m*a*c*d*e^5*f*m^2*x^2 + 922*(e*x + d)^m*a*b*e^6*f*m^2*x^2 - 60*(e*x + d)^m*c^2*d^4*e^2*g*m^2*x^2 + 168*(e*x + d)^m*b*c*d^3* e^3*g*m^2*x^2 - 123*(e*x + d)^m*b^2*d^2*e^4*g*m^2*x^2 - 246*(e*x + d)^m*a*c*d^2*e^4*g*m^2*x^2 + 388*(e*x + d)^m*a*b*d*e^5*g*m^2*x^2 + 461*(e*x + d)^m*a^2*e^6*g*m^2*x^2 - 48*(e*x + d)^m*c^2*d^2*e^4*f*m*x^3 + 120*(e*x + d)^m*b*c*d*e^5*f*m*x^3 + 508*(e*x + d)^m*b^2*e^6*f*m*x^3 + 1016*(e*x + d)^m*a*c*e^6*f*m*x^3 + 40*(e*x + d)^m*c^2*d^3*e^3*g*m*x^3 - 96*(e*x + d)^m*b*c*d^2*e^4*g*m*x^3 + 60*(e*x + d)^m*b^2*d*e^5*g*m*x^3 + 120*(e*x + d)^m*a*c*d*e^5*g*m*x^3 + 1016*(e*x + d)^m*a*b*e^6*g*m*x^3 + 360*(e*x + d)^m*b*c*e^6*f*x^4 + 180*(e*x + d)^m*b^2*e^6*g*x^4 + 360*(e*x + d)^m*a*c*e^6*g*x^4 + 2*(e*x + d)^m*b^2*d^3*e^3*f*m^3 + 4*(e*x + d)^m*a*c*d^3*e^3*f*m^3 - 36*(e*x + d)^m*a*b*d^2*e^4*f*m^3 + 155*(e*x + d)^m*a^2*d*e^5*f*m^3 + 4*(e*x + d)^m*a*b*d^3*e^3*g*m^3 - 18*(e*x + d)^m*a^2*d^2*e^4*g*m^3 - 24*(e*x + d)^m*c^2*d^4*e^2*f*m^2*x + 132*(e*x + d)^m*b*c*d^3*e^3*f*m^2*x - 148*(e*x + d)^m*b^2*d^2*e^4*f*m^2*x - 296*(e*x + d)^m*a*c*d^2*e^4*f*m^2*x + 684*(e*x + d)^m*a*b*d*e^5*f*m^2*x + 580*(e*x + d)^m*a^2*e^6*f*m^2*x - 48*(e*x + d)^m*b*c*d^4*e^2*g*m^2*x + 66*(e*x + d)^m*b^2*d^3*e^3*g*m^2*x + 132*(e*x + d)^m*a*c*d^3*e^3*g*m^2*x - 296*(e*x + d)^m*a*b*d^2*e^4*g*m^2*x + 342*(e*x + d)^m*a^2*d*e^5*g*m^2*x + 72*(e*x + d)^m*c^2*d^3*e^3*f*m*x^2 - 180*(e*x + d)^m*b*c*d^2*e^4*f*m*x^2 + 120*(e*x + d)^m*b^2*d*e^5*f*m*x^2 + 240*(e*x + d)^m*a*c*d*e^5*f*m*x^2 + 1404*(e*x + d)^m*a*b*e^6*f*m*x^2 - 60*(e*x + d)^m*c^2*d^4*e^2*g*m*x^2 + 144*(e*x + d)^m*b*c*d^3*e^3*g*m*x^2 - 90*(e*x + d)^m*b^2*d^2*e^4*g*m*x^2 - 180*(e*x + d)^m*a*c*d^2*e^4*g*m*x^2 + 240*(e*x + d)^m*a*b*d*e^5*g*m*x^2 + 702*(e*x + d)^m*a^2*e^6*g*m*x^2 + 240*(e*x + d)^m*b^2*e^6*f*x^3 + 480*(e*x + d)^m*a*c*e^6*f*x^3 + 480*(e*x + d)^m*a*b*e^6*g*x^3 - 12*(e*x + d)^m*b*c*d^4*e^2*f*m^2 + 30*(e*x + d)^m*b^2*d^3*e^3*f*m^2 + 60*(e*x + d)^m*a*c*d^3*e^3*f*m^2 - 238*(e*x + d)^m*a*b*d^2*e^4*f*m^2 + 580*(e*x + d)^m*a^2*d*e^5*f*m^2 - 6*(e*x + d)^m*b^2*d^4*e^2*g*m^2 - 12*(e*x + d)^m*a*c*d^4*e^2*g*m^2 + 60*(e*x + d)^m*a*b*d^3*e^3*g*m^2 - 119*(e*x + d)^m*a^2*d^2*e^4*g*m^2 - 144*(e*x + d)^m*c^2*d^4*e^2*f*m*x + 360*(e*x + d)^m*b*c*d^3*e^3*f*m*x - 240*(e*x + d)^m*b^2*d^2*e^4*f*m*x - 480*(e*x + d)^m*a*c*d^2*e^4*f*m*x + 720*(e*x + d)^m*a*b*d*e^5*f*m*x + 1044*(e*x + d)^m*a^2*e^6*f*m*x + 120*(e*x + d)^m*c^2*d^5*e*g*m*x - 288*(e*x + d)^m*b*c*d^4*e^2*g*m*x + 180*(e*x + d)^m*b^2*d^3*e^3*g*m*x + 360*(e*x + d)^m*a*c*d^3*e^3*g*m*x - 480*(e*x + d)^m*a*b*d^2*e^4*g*m*x + 360*(e*x + d)^m*a^2*d*e^5*g*m*x + 720*(e*x + d)^m*a*b*e^6*f*x^2 + 360*(e*x + d)^m*a^2*e^6*g*x^2 + 24*(e*x + d)^m*c^2*d^5*e*f*m - 132*(e*x + d)^m*b*c*d^4*e^2*f*m + 148*(e*x + d)^m*b^2*d^3*e^3*f*m + 296*(e*x + d)^m*a*c*d^3*e^3*f*m - 684*(e*x + d)^m*a*b*d^2*e^4*f*m + 1044*(e*x + d)^m*a^2*d*e^5*f*m + 48*(e*x + d)^m*b*c*d^5*e*g*m - 66*(e*x + d)^m*b^2*d^4*e^2*g*m - 132*(e*x + d)^m*a*c*d^4*e^2*g*m + 296*(e*x + d)^m*a*b*d^3*e^3*g*m - 342*(e*x + d)^m*a^2*d^2*e^4*g*m + 720*(e*x + d)^m*a^2*e^6*f*x + 144*(e*x + d)^m*c^2*d^5*e*f - 360*(e*x + d)^m*b*c*d^4*e^2*f + 240*(e*x + d)^m*b^2*d^3*e^3*f + 480*(e*x + d)^m*a*c*d^3*e^3*f - 720*(e*x + d)^m*a*b*d^2*e^4*f + 720*(e*x + d)^m*a^2*d*e^5*f - 120*(e*x + d)^m*c^2*d^6*g + 288*(e*x + d)^m*b*c*d^5*e*g - 180*(e*x + d)^m*b^2*d^4*e^2*g - 360*(e*x + d)^m*a*c*d^4*e^2*g + 480*(e*x + d)^m*a*b*d^3*e^3*g - 360*(e*x + d)^m*a^2*d^2*e^4*g)/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 735*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 2307, normalized size of antiderivative = 7.42

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \text{Too large to display}$$

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
[Out] ((d + e*x)^m*(240*b^2*d^3*e^3*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g - 180*b
^2*d^4*e^2*g + 720*a^2*d*e^5*f + 144*c^2*d^5*e*f - 720*a*b*d^2*e^4*f + 480*
a*b*d^3*e^3*g + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g - 360*b*c*d^4*e^2*f +
 1044*a^2*d*e^5*f*m + 24*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*d*e^
5*f*m^3 + 20*a^2*d*e^5*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m + 148*
b^2*d^3*e^3*f*m - 66*b^2*d^4*e^2*g*m + 288*b*c*d^5*e*g - 119*a^2*d^2*e^4*g*
m^2 + 30*b^2*d^3*e^3*f*m^2 - 18*a^2*d^2*e^4*g*m^3 + 2*b^2*d^3*e^3*f*m^3 - a
^2*d^2*e^4*g*m^4 - 6*b^2*d^4*e^2*g*m^2 + 48*b*c*d^5*e*g*m - 684*a*b*d^2*e^4
*f*m + 296*a*b*d^3*e^3*g*m + 296*a*c*d^3*e^3*f*m - 132*a*c*d^4*e^2*g*m - 13
2*b*c*d^4*e^2*f*m - 238*a*b*d^2*e^4*f*m^2 - 36*a*b*d^2*e^4*f*m^3 - 2*a*b*d^
2*e^4*f*m^4 + 60*a*b*d^3*e^3*g*m^2 + 60*a*c*d^3*e^3*f*m^2 + 4*a*b*d^3*e^3*g
*m^3 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*m^2 - 12*b*c*d^4*e^2*f*m^2))/
(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x*(d
+ e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m^3 + 20*a^2*e^
6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g*m + 120*c^2*d^
5*e*g*m - 240*b^2*d^2*e^4*f*m + 342*a^2*d*e^5*g*m^2 + 119*a^2*d*e^5*g*m^3 +
 18*a^2*d*e^5*g*m^4 + a^2*d*e^5*g*m^5 + 180*b^2*d^3*e^3*g*m - 144*c^2*d^4*e
^2*f*m - 148*b^2*d^2*e^4*f*m^2 - 30*b^2*d^2*e^4*f*m^3 - 2*b^2*d^2*e^4*f*m^4
 + 66*b^2*d^3*e^3*g*m^2 - 24*c^2*d^4*e^2*f*m^2 + 6*b^2*d^3*e^3*g*m^3 + 720*
a*b*d*e^5*f*m + 684*a*b*d*e^5*f*m^2 + 238*a*b*d*e^5*f*m^3 + 36*a*b*d*e^5*f*
m^4 + 2*a*b*d*e^5*f*m^5 - 480*a*b*d^2*e^4*g*m - 480*a*c*d^2*e^4*f*m + 360*a
*c*d^3*e^3*g*m + 360*b*c*d^3*e^3*f*m - 288*b*c*d^4*e^2*g*m - 296*a*b*d^2*e^
4*g*m^2 - 296*a*c*d^2*e^4*f*m^2 - 60*a*b*d^2*e^4*g*m^3 - 60*a*c*d^2*e^4*f*m
^3 - 4*a*b*d^2*e^4*g*m^4 - 4*a*c*d^2*e^4*f*m^4 + 132*a*c*d^3*e^3*g*m^2 + 13
2*b*c*d^3*e^3*f*m^2 + 12*a*c*d^3*e^3*g*m^3 + 12*b*c*d^3*e^3*f*m^3 - 48*b*c*
d^4*e^2*g*m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(120*b^2*e^3*f + 15*b^2*e^3*f*m^
2 + b^2*e^3*f*m^3 + 240*a*b*e^3*g + 240*a*c*e^3*f + 74*b^2*e^3*f*m + 20*c^2
*d^3*g*m + 30*a*b*e^3*g*m^2 + 30*a*c*e^3*f*m^2 + 2*a*b*e^3*g*m^3 + 2*a*c*e^
3*f*m^3 + 30*b^2*d*e^2*g*m - 24*c^2*d^2*e*f*m + 11*b^2*d*e^2*g*m^2 - 4*c^2*
d^2*e*f*m^2 + b^2*d*e^2*g*m^3 + 148*a*b*e^3*g*m + 148*a*c*e^3*f*m + 60*a*c*
d*e^2*g*m + 60*b*c*d*e^2*f*m - 48*b*c*d^2*e*g*m + 22*a*c*d*e^2*g*m^2 + 22*b
*c*d*e^2*f*m^2 + 2*a*c*d*e^2*g*m^3 + 2*b*c*d*e^2*f*m^3 - 8*b*c*d^2*e*g*m^2)
)/(e^3*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x^4
*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*b^2*e^2*g + b^2*e^2*g*m^2 + 60*a*
c*e^2*g + 60*b*c*e^2*f + 11*b^2*e^2*g*m - 5*c^2*d^2*g*m + 2*a*c*e^2*g*m^2 +
```

2*b*c*e^2*f*m^2 + c^2*d*e*f*m^2 + 22*a*c*e^2*g*m + 22*b*c*e^2*f*m + 6*c^2*d*e*f*m + 2*b*c*d*e*g*m^2 + 12*b*c*d*e*g*m))/(e^2*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (c^2*g*x^6*(d + e*x)^m*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (x^2*(m + 1)*(d + e*x)^m*(360*a^2*e^4*g + 119*a^2*e^4*g*m^2 + 18*a^2*e^4*g*m^3 + a^2*e^4*g*m^4 + 720*a*b*e^4*f + 342*a^2*e^4*g*m - 60*c^2*d^4*g*m + 238*a*b*e^4*f*m^2 + 36*a*b*e^4*f*m^3 + 2*a*b*e^4*f*m^4 + 120*b^2*d*e^3*f*m + 72*c^2*d^3*e*f*m + 74*b^2*d*e^3*f*m^2 + 15*b^2*d*e^3*f*m^3 + b^2*d*e^3*f*m^4 - 90*b^2*d^2*e^2*g*m + 12*c^2*d^3*e*f*m^2 + 684*a*b*e^4*f*m - 33*b^2*d^2*e^2*g*m^2 - 3*b^2*d^2*e^2*g*m^3 + 240*a*b*d*e^3*g*m + 240*a*c*d*e^3*f*m + 144*b*c*d^3*e*g*m + 148*a*b*d*e^3*g*m^2 + 148*a*c*d*e^3*f*m^2 + 30*a*b*d*e^3*g*m^3 + 30*a*c*d*e^3*f*m^3 + 2*a*b*d*e^3*g*m^4 + 2*a*c*d*e^3*f*m^4 - 180*a*c*d^2*e^2*g*m - 180*b*c*d^2*e^2*f*m + 24*b*c*d^3*e*g*m^2 - 66*a*c*d^2*e^2*g*m^2 - 66*b*c*d^2*e^2*f*m^2 - 6*a*c*d^2*e^2*g*m^3 - 6*b*c*d^2*e^2*f*m^3))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (c*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(12*b*e*g + 6*c*e*f + 2*b*e*g*m + c*d*g*m + c*e*f*m))/(e*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))

3.927 $\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} dx$

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} dx = \frac{(beg-c(ef+dg))(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))(d+ex)^{1+m}}{e^4g^4(1+m)} + \frac{(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(ef+2dg)))(d+ex)^{2+m}}{e^4g^3(2+m)} - \frac{c(cef+3cdg-2beg)(d+ex)^{3+m}}{e^4g^2(3+m)} + \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(ef-dg)(1+m)}$$

```
[Out] (b*e*g-c*(d*g+e*f))*(c*(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*(e*x+d)
^(1+m)/e^4/g^4/(1+m)+(b^2*e^2*g^2+c^2*(3*d^2*g^2+2*d*e*f*g+e^2*f^2)+2*c*e*g
*(a*e*g-b*(2*d*g+e*f)))*(e*x+d)^(2+m)/e^4/g^3/(2+m)-c*(-2*b*e*g+3*c*d*g+c*e
*f)*(e*x+d)^(3+m)/e^4/g^2/(3+m)+c^2*(e*x+d)^(4+m)/e^4/g/(4+m)+(a*g^2-b*f*g+
c*f^2)^2*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^4/
(-d*g+e*f)/(1+m)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {965, 1634, 70}

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx$$
$$= \frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2(3d^2g^2+2defg+e^2f^2)\right)}{e^4g^3(m+2)} + \frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(2aeg-b(dg+ef))+c(d^2g^2+e^2f^2)\right)}{e^4g^4(m+1)} + \frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2 \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)} - \frac{c(d+ex)^{m+3}(-2beg+3cdg+cef)}{e^4g^2(m+3)} + \frac{c^2(d+ex)^{m+4}}{e^4g(m+4)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d
*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2 +
 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(2 +
 m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 + m))/
(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2 - b*f
*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d
+ e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \frac{(d+ex)^m(-e(c^2d^3f-a^2e^3g)(4+m)+e(2abe^3g-c^2d^2(3ef+dg))(4+m)x+e^2(b^2e^2g+2ace^2g-3c^2d(ef+dg))(4+m)x^2-ce^3(cef+3cdg-2beg)}{f+gx}}{e^4g(4+m)}$$

$$= \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \left(\frac{e(beg-c(ef+dg))(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))(4+m)(d+ex)^m}{g^3} + \frac{e(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(e}{g^2}\right.}{e^4g(4+m}$$

$$= \frac{(beg-c(ef+dg))(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))(d+ex)^{1+m}}{e^4g^4(1+m)} + \frac{(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(ef+2dg)))(d+ex)^{2+m}}{e^4g^3(2+m)} - \frac{c(cef+3cdg-2beg)(d+ex)^{3+m}}{e^4g^2(3+m)} + \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{(cf^2-bfg+ag^2)^2\int\frac{(d+ex)^m}{f+gx}\,dx}{g^4}$$

$$= \frac{(beg-c(ef+dg))(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))(d+ex)^{1+m}}{e^4g^4(1+m)} + \frac{(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(ef+2dg)))(d+ex)^{2+m}}{e^4g^3(2+m)} - \frac{c(cef+3cdg-2beg)(d+ex)^{3+m}}{e^4g^2(3+m)} + \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{g(d+ex)}{ef-dg}\right)}{g^4(ef-dg)(1+m)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} dx = \frac{(d+ex)^{1+m}\left(-\frac{(cef+cdg-beg)(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg)))}{e^4(1+m)} + \frac{g(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2g^2)+2ceg(aeg-b(ef+2dg)))(d}{e^4(2+m)}\right.}{g^4}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g
*(2*a*e*g - b*(e*f + d*g))))/(e^4*(1 + m))) + (g*(b^2*e^2*g^2 + c^2*(e^2*f^
2 + 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x))/
(e^4*(2 + m)) - (c*g^2*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^2)/(e^4*(3 + m
)) + (c^2*g^3*(d + e*x)^3)/(e^4*(4 + m)) + ((c*f^2 + g*(-(b*f) + a*g))^2*Hy
pergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g
)*(1 + m))))/g^4
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)^2}{gx+f} dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} dx = \int \frac{(cx^2+bx+a)^2(ex+d)^m}{gx+f} dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g*x + f), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^2}{f+g\,x} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x), x)
```

3.928 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \frac{\left(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg))\right)(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(cef+cdg-beg)(d+ex)^{2+m}}{e^3g^3(2+m)} + \frac{c^2(d+ex)^{3+m}}{e^3g^2(3+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\left(cf^2-bfg+ag^2\right)\left(cf(4dg-ef(4+m))-g(aegm+b(2dg-ef(2+m)))\right)(d+ex)^{1+m}\,\text{Hypergeometr}}{g^4(ef-dg)^2(1+m)}$$

```
[Out] (b^2*e^2*g^2+c^2*(d^2*g^2+2*d*e*f*g+3*e^2*f^2)+2*c*e*g*(a*e*g-b*(d*g+2*e*f)
))*(e*x+d)^(1+m)/e^3/g^4/(1+m)-2*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(2+m)/e^3/g
^3/(2+m)+c^2*(e*x+d)^(3+m)/e^3/g^2/(3+m)+(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m
)/g^4/(-d*g+e*f)/(g*x+f)+(a*g^2-b*f*g+c*f^2)*(c*f*(4*d*g-e*f*(4+m))-g*(a*e*
g*m+b*(2*d*g-e*f*(2+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)
/(-d*g+e*f))/g^4/(-d*g+e*f)^2/(1+m)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {963, 1634, 70}

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} \, dx$$
$$= \frac{(d+ex)^{m+1} (2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2(d^2g^2+2defg+3e^2f^2))}{e^3g^4(m+1)}$$
$$-\frac{(d+ex)^{m+1} (ag^2-bfg+cf^2)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+2bdg-bef(}{g^4(m+1)(ef-dg)^2}$$
$$+\frac{(d+ex)^{m+1} (ag^2-bfg+cf^2)^2}{g^4(f+gx)(ef-dg)}-\frac{2c(d+ex)^{m+2}(-beg+cdg+cef)}{e^3g^3(m+2)}+\frac{c^2(d+ex)^{m+3}}{e^3g^2(m+3)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c*e*g*(a*e*g - b*
(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*c*(c*e*f + c*d*g
- b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m)) + (c^2*(d + e*x)^(3 + m))/(e^
3*g^2*(3 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f -
d*g)*(f + g*x)) - ((c*f^2 - b*f*g + a*g^2)*(g*(2*b*d*g + a*e*g*m - b*e*f*(2
 + m)) - c*f*(4*d*g - e*f*(4 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m \left(\frac{c^2f^3(dg-ef(1+m))-2cfg(bf-ag)(dg-ef(1+m))-g^2\left(a^2eg^2m-b^2f(dg-ef(1+m))+2abg(dg-ef(1+m))\right)}{g^4}+\frac{(ef-dg)\left(c^2f^2+b^2g^2-2cg(bf-a \cdots\right.}{g^3} \cdots\right.}{f+gx} \cdots}{ef-dg}$$

$$= \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int \left(\frac{(ef-dg)(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))(d+ex)^m}{e^2g^4} - \frac{2c(ef-dg)(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^3} + \cdots\right.}{ef-dg}$$

$$= \frac{(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(cef+cdg-beg)(d+ex)^{2+m}}{e^3g^3(2+m)} + \frac{c^2(d+ex)^{3+m}}{e^3g^2(3+m)} + \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} - \frac{((cf^2-bfg+ag^2)(g(2bdg+aegm-bef(2+m))-cf(4dg-ef(4+m)))) \int \frac{(d+ex)^m}{f+gx}\,dx}{g^4(ef-dg)}$$

$$= \frac{(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(cef+cdg-beg)(d+ex)^{2+m}}{e^3g^3(2+m)} + \frac{c^2(d+ex)^{3+m}}{e^3g^2(3+m)} + \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} - \frac{(cf^2-bfg+ag^2)(g(2bdg+aegm-bef(2+m))-cf(4dg-ef(4+m)))(d+ex)^{1+m}\, {}_2F_1\left(1, \cdots\right.}{g^4(ef-dg)^2(1+m)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx = \frac{(d+ex)^{1+m}\left(\frac{b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg))}{e^3(1+m)} - \frac{2cg(cef+cdg-beg)(d+ex)}{e^3(2+m)} + \frac{c^2g^2(d+ex)^2}{e^3(3+m)} - \frac{2(2cf-bg)(cf \cdots}{\cdots} \cdots\right.}{g^4}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*c*e*g*(a*e*g - b*(2*e*f + d*g)))/(e^3*(1 + m)) - (2*c*g*(c*e*f + c*d*g -
b*e*g)*(d + e*x))/(e^3*(2 + m)) + (c^2*g^2*(d + e*x)^2)/(e^3*(3 + m)) - (2*
(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2 + m,
 (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) + (e*(c*f^2 + g*(-(b*
f) + a*g))^2*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g
)])/((e*f - d*g)^2*(1 + m))))/g^4
```

Maple [F]

$$\int \frac{\left(ex+d\right)^{m} \left(c\,x^{2}+bx+a\right)^{2}}{\left(gx+f\right)^{2}}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx = \int \frac{{\left(cx^{2} + bx + a\right)}^{2}{\left(ex + d\right)}^{m}}{{\left(gx + f\right)}^{2}}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(cx^2+bx+a\right)^2 (ex+d)^m}{\left(gx+f\right)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \int \frac{\left(cx^2+bx+a\right)^2 (ex+d)^m}{\left(gx+f\right)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \int \frac{(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^2}{\left(f+g\,x\right)^2}\,dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2, x)
```

3.929 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 461

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$$
$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)}+\frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)}+\frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2}$$
$$+\frac{\left(cf^2-bfg+ag^2\right)\left(cf(8dg-ef(7+m))+g(aeg(1-m)-b(4dg-ef(3+m)))\right)(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)}$$
$$+\frac{(c^2f^2(12d^2g^2-8defg(3+m)+e^2f^2(12+7m+m^2))-g^2(a^2e^2g^2(1-m)m-2abegm(2dg-ef(1+m}{}$$

```
[Out] -c*(-2*b*e*g+c*d*g+3*c*e*f)*(e*x+d)^(1+m)/e^2/g^4/(1+m)+c^2*(e*x+d)^(2+m)/e
^2/g^3/(2+m)+1/2*(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m)/g^4/(-d*g+e*f)/(g*x+f)
^2+1/2*(a*g^2-b*f*g+c*f^2)*(c*f*(8*d*g-e*f*(7+m))+g*(a*e*g*(1-m)-b*(4*d*g-e
*f*(3+m))))*(e*x+d)^(1+m)/g^4/(-d*g+e*f)^2/(g*x+f)+1/2*(c^2*f^2*(12*d^2*g^2
-8*d*e*f*g*(3+m)+e^2*f^2*(m^2+7*m+12))-g^2*(a^2*e^2*g^2*(1-m)*m-2*a*b*e*g*m
*(2*d*g-e*f*(1+m))-b^2*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2)))+2*c
*g*(a*g*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-b*f*(6*d^2*g^2-6*d*
e*f*g*(2+m)+e^2*f^2*(m^2+5*m+6))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-
g*(e*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^3/(1+m)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {963, 1635, 965, 81, 70}

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} \, dx$$
$$= \frac{(d+ex)^{m+1} \text{Hypergeometric2F1}\left(1, m+1, m+2, -\frac{g(d+ex)}{ef-dg}\right) (-g^2(a^2e^2g^2(1-m)m - 2abegm(2dg-ef}{}$$
$$- \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2) (g(-aeg(1-m)+4bdg-bef(m+3)) - cf(8dg-ef(m+7)))}{2g^4(f+gx)(ef-dg)^2}$$
$$+ \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2)^2}{2g^4(f+gx)^2(ef-dg)} - \frac{c(d+ex)^{m+1}(-2beg+cdg+3cef)}{e^2g^4(m+1)} + \frac{c^2(d+ex)^{m+2}}{e^2g^3(m+2)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] -((c*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 + m))/(e^2*g^4*(1 + m))) + (c
^2*(d + e*x)^(2 + m))/(e^2*g^3*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e
*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) - ((c*f^2 - b*f*g + a*g^2)*(g*
(4*b*d*g - a*e*g*(1 - m) - b*e*f*(3 + m)) - c*f*(8*d*g - e*f*(7 + m)))*(d +
 e*x)^(1 + m))/(2*g^4*(e*f - d*g)^2*(f + g*x)) + ((c^2*f^2*(12*d^2*g^2 - 8*
d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m -
2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^
2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f
^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m
 + m^2))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e
*x))/(e*f - d*g))])/(2*g^4*(e*f - d*g)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rubi steps

$$\text{integral} = \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{c^2f^3(2dg-ef(1+m))-2cfg(bf-ag)(2dg-ef(1+m))+g^2\left(a^2eg^2(1-m)+b^2f(2dg-ef(1+m))-2abg(2dg-ef(1+m))\right)}{g^4}+\frac{2(ef-dg)\left(c^2f^2+b^2g^2-2\right.}{g^3}\right.}{(f+gx)^2}}{2(ef-dg)}$$

$$= \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{(cf^2-bfg+ag^2)(g(4bdg-aeg(1-m)-bef(3+m))-cf(8dg-ef(7+m)))(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)} + \frac{\int \frac{(d+ex)^m\left(\frac{c^2f^2\left(6d^2g^2-4defg(3+2m)+e^2f^2\left(6+7m+m^2\right)\right)-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(1+m))-b^2\left(2d^2g^2-4defg(1+m)+e^2f^2\right.\right.}{g^4}}{}}{}$$

$$= \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{(cf^2-bfg+ag^2)(g(4bdg-aeg(1-m)-bef(3+m))-cf(8dg-ef(7+m)))(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)} + \frac{\int \frac{(d+ex)^m\left(\frac{e(2+m)\left(c^2f\left(10d^2efg^2-2d^3g^3-2de^2f^2g(7+4m)+e^3f^3\left(6+7m+m^2\right)\right)-eg^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(1+m))-b^2\left(2d^2g\right.\right.\right.}{}}{}}{}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{(cf^2-bfg+ag^2)(g(4bdg-aeg(1-m)-bef(3+m))-cf(8dg-ef(7+m)))(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)} + \frac{(c^2f^2(12d^2g^2-8defg(3+m)+e^2f^2(12+7m+m^2))-g^2(a^2e^2g^2(1-m)m-2abegm(2dg-}{}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{(cf^2-bfg+ag^2)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{(cf^2-bfg+ag^2)(g(4bdg-aeg(1-m)-bef(3+m))-cf(8dg-ef(7+m)))(d+ex)^{1+m}}{2g^4(ef-dg)^2(f+gx)} + \frac{(c^2f^2(12d^2g^2-8defg(3+m)+e^2f^2(12+7m+m^2))-g^2(a^2e^2g^2(1-m)m-2abegm(2dg-}{}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^m(a+bx+cx^2)^2}{(f+gx)^3}\,dx = \frac{(d+ex)^{1+m}\left(-\frac{c(3cef+cdg-2beg)}{e^2(1+m)} + \frac{c^2g(d+ex)}{e^2(2+m)} + \frac{(6c^2f^2+b^2g^2+2cg(-3bf+ag))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{(ef-dg)(1+m)} - \frac{2e}{}\right.}{}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((c*(3*c*e*f + c*d*g - 2*b*e*g))/(e^2*(1 + m))) + (c^2
*g*(d + e*x))/(e^2*(2 + m)) + ((6*c^2*f^2 + b^2*g^2 + 2*c*g*(-3*b*f + a*g))
*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f -
d*g)*(1 + m)) - (2*e*(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometri
c2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^2*(1 + m)
) + (e^2*(c*f^2 + g*(-(b*f) + a*g))^2*Hypergeometric2F1[3, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^3*(1 + m))))/g^4
```

Maple [F]

$$\int \frac{\left(ex+d\right)^{m} \left(c\,x^{2}+bx+a\right)^{2}}{\left(gx+f\right)^{3}}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(d+ex\right)^{m}\left(a+bx+cx^{2}\right)^{2}}{\left(f+gx\right)^{3}}\,dx$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} \, dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^3} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} \, dx = \int \frac{(cx^2+bx+a)^2 (ex+d)^m}{(gx+f)^3} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} \, dx = \int \frac{(d+e\,x)^m \,(c\,x^2+b\,x+a)^2}{(f+g\,x)^3} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3, x)
```

3.930 $\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \frac{3687(1+4x)^{1+m}}{64(1+m)}+\frac{207(1+4x)^{2+m}}{32(2+m)}+\frac{27(1+4x)^{3+m}}{64(3+m)} - \frac{3\left(5499-1631\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)} - \frac{3\left(5499+1631\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 3687/64*(1+4*x)^(1+m)/(1+m)+207/32*(1+4*x)^(2+m)/(2+m)+27/64*(1+4*x)^(3+m)/
(3+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2))
)*(5499-1631*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([
1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(5499+1631*13^(1/2))/(1+m)/(13+2*1
3^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used

$= \{1642, 70\}$

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx$$
$$= -\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}$$
$$-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{3687(4x+1)^{m+1}}{64(m+1)}+\frac{207(4x+1)^{m+2}}{32(m+2)}+\frac{27(4x+1)^{m+3}}{64(m+3)}$$

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m)
) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13
])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*x)^(
1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3687}{16}(1+4x)^m+\frac{207}{8}(1+4x)^{1+m}+\frac{27}{16}(1+4x)^{2+m} + \frac{\left(1269+\frac{4893}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(1269-\frac{4893}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} + \frac{1}{13}\left(3\left(5499-1631\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{13}\left(3\left(5499+1631\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} - \frac{3\left(5499-1631\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)} - \frac{3\left(5499+1631\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \frac{3}{832}(1+4x)^{1+m}\left(\frac{15977}{1+m} + \frac{1794(1+4x)}{2+m} + \frac{117(1+4x)^2}{3+m} - \frac{32\left(-5499+1631\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{32\left(5499+1631\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*(15977/(1 + m) + (1794*(1 + 4*x))/(2 + m) + (117*(1 +
4*x)^2)/(3 + m) - (32*(-5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (32*(54
99 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*S
qrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/832
```

Maple [F]

$$\int \frac{(2+3x)^4 (1+4x)^m}{3x^2-5x+1} dx$$

```
[In] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4x+1)^m(3x+2)^4}{3x^2-5x+1} dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(3*x^2 - 5*x
+ 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(3x+2)^4 (4x+1)^m}{3x^2-5x+1} dx$$

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Maxima [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4x+1)^m(3x+2)^4}{3x^2-5x+1} dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

Giac [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^4\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.931 $\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 165

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx = \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} - \frac{3(416-135\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13(13-2\sqrt{13})(1+m)} - \frac{3(416+135\sqrt{13})(1+4x)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13(13+2\sqrt{13})(1+m)}$$

```
[Out] 123/16*(1+4*x)^(1+m)/(1+m)+9/16*(1+4*x)^(2+m)/(2+m)-3/13*(1+4*x)^(1+m)*hype
rgeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(416-135*13^(1/2))/(1+m)/(1
3-2*13^(1/2))-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*1
3^(1/2)))*(416+135*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used

= {1642, 70}

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx$$
$$= -\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}$$
$$-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{123(4x+1)^{m+1}}{16(m+1)}+\frac{9(4x+1)^{m+2}}{16(m+2)}$$

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m)) -
 (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(4
16 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{123}{4}(1+4x)^m+\frac{9}{4}(1+4x)^{1+m}+\frac{\left(192+\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(192-\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$
$$=\frac{123(1+4x)^{1+m}}{16(1+m)}+\frac{9(1+4x)^{2+m}}{16(2+m)}+\frac{1}{13}\left(6\left(416-135\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx$$
$$+\frac{1}{13}\left(6\left(416+135\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx$$

$$= \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} - \frac{3(416-135\sqrt{13})\,(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\,(13-2\sqrt{13})\,(1+m)} - \frac{3(416+135\sqrt{13})\,(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\,(13+2\sqrt{13})\,(1+m)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \frac{(1+4x)^{1+m}\left(117(85+12x+4m(11+3x))+16(-146+71\sqrt{13})\,(2+m)\,\text{Hypergeometric2F1}\left(1,1+m\right.\right.}{624\,(2+3m+\ldots}$$

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117*(85 + 12*x + 4*m*(11 + 3*x)) + 16*(-146 + 71*Sqrt[1
3])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
] - 16*(146 + 71*Sqrt[13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 +
12*x)/(13 + 2*Sqrt[13])]))/(624*(2 + 3*m + m^2))
```

Maple [F]

$$\int \frac{(2+3x)^3\,(1+4x)^m}{3x^2-5x+1}dx$$

```
[In] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)^3\,(4x+1)^m}{3x^2-5x+1}\,dx$$

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Maxima [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

Giac [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^3\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.932 $\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx = \frac{3(1+4x)^{1+m}}{4(1+m)} - \frac{3(117-47\sqrt{13})(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26(13-2\sqrt{13})(1+m)} - \frac{3(117+47\sqrt{13})(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26(13+2\sqrt{13})(1+m)}$$

```
[Out] 3/4*(1+4*x)^(1+m)/(1+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(117-47*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1
+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117+47*13^(1/2))/(
1+m)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used

$= \{1642, 70\}$

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx$$
$$= -\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}$$
$$-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{3(4x+1)^{m+1}}{4(m+1)}$$

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*
(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2
*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(3(1+4x)^m + \frac{\left(27+\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(27-\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$
$$= \frac{3(1+4x)^{1+m}}{4(1+m)} + \frac{1}{13}\left(3\left(117-47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx$$
$$+\frac{1}{13}\left(3\left(117+47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx$$

$$= \frac{3(1+4x)^{1+m}}{4(1+m)} - \frac{3(117-47\sqrt{13})\,(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\,(13-2\sqrt{13})\,(1+m)} - \frac{3(117+47\sqrt{13})\,(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\,(13+2\sqrt{13})\,(1+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \frac{(1+4x)^{1+m}\left(117+(-46+58\sqrt{13})\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)-2(23+29\sqrt{13})\,\text{H}\right.}{156(1+m)}$$

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
[Out] ((1 + 4*x)^(1 + m)*(117 + (-46 + 58*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] - 2*(23 + 29*Sqrt[13])*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(156*(1 + m))
```

Maple [F]

$$\int \frac{(2+3x)^2\,(1+4x)^m}{3x^2-5x+1}dx$$

```
[In] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
[Out] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3x+2)^2\,(4x+1)^m}{3x^2-5x+1}\,dx$$

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Maxima [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

Giac [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)^2\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.933 $\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} \, dx = -\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)} - \frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] -3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13
-9*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {844, 70}

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} \, dx = -\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)} - \frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13 +
 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4
*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\left(3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx\\
&= \frac{1}{13}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx+\frac{1}{13}\left(3\left(13+9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx\\
&= -\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}\\
&\quad-\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx$$
$$=\frac{(1+4x)^{1+m}\left(\left(5+7\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)+\left(5-7\sqrt{13}\right)\text{Hypergeometric2}\right.}{78(1+m)}$$

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

[Out] ((1 + 4*x)^(1 + m)*((5 + 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (5 - 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(78*(1 + m))

Maple [F]

$$\int \frac{(2+3x)(1+4x)^m}{3x^2-5x+1}dx$$

[In] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)

[Out] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)

Fricas [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1} dx$$

[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")

[Out] integral((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)

Sympy [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1} dx$$

[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)

[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)

Maxima [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx = \int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1} dx$$

[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")

[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)

Giac [F]

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(3\,x+2)\,\left(4\,x+1\right)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.934 $\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \frac{3(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} - \frac{3(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))/(1+m
)/(13-2*13^(1/2))*13^(1/2)-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1
+4*x)/(13+2*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {725, 70}

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \frac{3(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} - \frac{3(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 -
 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m)*
```

```
Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[
13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{6(1+4x)^m}{\sqrt{13}\,(5+\sqrt{13}-6x)} - \frac{6(1+4x)^m}{\sqrt{13}\,(-5+\sqrt{13}+6x)}\right) dx$$

$$= -\frac{6\int \frac{(1+4x)^m}{5+\sqrt{13}-6x}\,dx}{\sqrt{13}} - \frac{6\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{\sqrt{13}}$$

$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\,(13-2\sqrt{13})\,(1+m)} - \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\,(13+2\sqrt{13})\,(1+m)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx$$
$$= \frac{(1+4x)^{1+m}\left((13+2\sqrt{13})\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right) + (-13+2\sqrt{13})\,\text{Hypergeomet}\right.}{39\sqrt{13}(1+m)}$$

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])] + (-13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m
, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(39*Sqrt[13]*(1 + m))
```

Maple [F]

$$\int \frac{(1+4x)^m}{3x^2-5x+1}dx$$

```
[In] int((1+4*x)^m/(3*x^2-5*x+1),x)
[Out] int((1+4*x)^m/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1),x)
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.935 $\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)} dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\begin{aligned}\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx &= \frac{3(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{85(1+m)} \\ &+ \frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(1+m)} \\ &+ \frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

```
[Out] 3/85*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/442*(1+4*x
)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))
/(1+m)/(13+2*13^(1/2))+3/442*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

= {974, 70, 844}

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx$$
$$= \frac{3(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{85(m+1)}$$
$$+ \frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}$$
$$+ \frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(m+1)}$$

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/
(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(1 +
 m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3(1+4x)^m}{17(2+3x)}+\frac{(7-3x)(1+4x)^m}{17\,(1-5x+3x^2)}\right)\,dx\\
&= \frac{1}{17}\int \frac{(7-3x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{3}{17}\int \frac{(1+4x)^m}{2+3x}\,dx\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}\\
&\quad+\frac{1}{17}\int \left(\frac{\left(-3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(-3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)\,dx\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}\\
&\quad-\frac{1}{221}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx\\
&\quad-\frac{1}{221}\left(3\left(13+9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}\\
&\quad+\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(1+m)}\\
&\quad+\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \frac{(1+4x)^{1+m}\left(234\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)+5\left(31+11\sqrt{13}\right)\text{Hypergeometric2}\right.}{6630(1+}$$

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
[Out] ((1 + 4*x)^(1 + m)*(234*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5
] + 5*(31 + 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
- 2*Sqrt[13])] + 5*(31 - 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])]))/(6630*(1 + m))
```

Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(3x^2-5x+1)}dx$$

```
[In] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
[Out] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4x+1)^m}{(3x+2)\,(3x^2-5x+1)}\,dx$$

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)\,\left(3\,x^2-5\,x+1\right)}\,dx$$

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)),x)
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)), x)
```

3.936 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 199

$$\begin{aligned}\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx &= \frac{27(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1445(1+m)} \\ &+ \frac{3\left(117+47\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} \\ &+ \frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} \\ &+ \frac{12(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{425(1+m)}\end{aligned}$$

```
[Out] 27/1445*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+12/425*(1
+4*x)^(1+m)*hypergeom([2, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/7514*(1+4*x)^(1+m
)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-47*13^(1/2))/(1+
m)/(13+2*13^(1/2))+3/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(117+47*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {974, 70, 844}

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$$
$$= \frac{27(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{1445(m+1)}$$
$$+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}$$
$$+\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{12(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{425(m+1)}$$

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1
+ m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5])/(425*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3(1+4x)^m}{17(2+3x)^2}+\frac{27(1+4x)^m}{289(2+3x)}+\frac{(46-27x)(1+4x)^m}{289\,(1-5x+3x^2)}\right)dx\\
&= \frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{27}{289}\int \frac{(1+4x)^m}{2+3x}\,dx+\frac{3}{17}\int \frac{(1+4x)^m}{(2+3x)^2}\,dx\\
&= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}\\
&\quad+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{425(1+m)}\\
&\quad+\frac{1}{289}\int \left(\frac{\left(-27+\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(-27-\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx\\
&= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}\\
&\quad+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{425(1+m)}\\
&\quad-\frac{\left(3\left(117-47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{3757}-\frac{\left(3\left(117+47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{3757}\\
&= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}\\
&\quad+\frac{3\left(117+47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)}\\
&\quad+\frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)}\\
&\quad+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{425(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$$
$$= \frac{(1+4x)^{1+m}\left(10530\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)+25\left(211+65\sqrt{13}\right)\text{Hypergeome}\right.}{}$$

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(10530*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))
/5] + 25*(211 + 65*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/
(13 - 2*Sqrt[13])] + 5275*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
 + 2*Sqrt[13])] - 1625*Sqrt[13]*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*
x)/(13 + 2*Sqrt[13])] + 15912*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5]))/(563550*(1 + m))
```

Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2(3x^2-5x+1)}dx$$

```
[In] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

```
[Out] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(27*x^4 - 9*x^3 - 39*x^2 - 8*x + 4), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(1-5x+3x^2)}\,dx = \int \frac{(4x+1)^m}{(3x+2)^2\cdot(3x^2-5x+1)}\,dx$$

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)(3\,x+2)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(1-5x+3x^2)}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)^2\,(3\,x^2-5\,x+1)}\,dx$$

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)),x)
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)), x)
```

3.937 $\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{9(1+4x)^{1+m}}{4(1+m)}+\frac{(844-2355x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)}$$
$$-\frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x}{13-2\sqrt{1}}\right.}{169\left(13-2\sqrt{13}\right)(1+m)}$$
$$-\frac{\left(13689+\sqrt{13}\left(297+4474m+1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x}{13+2\sqrt{1}}\right.}{169\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 9/4*(1+4*x)^(1+m)/(1+m)+1/39*(844-2355*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-1/169
*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13689-1
3^(1/2)*(297+4474*m-1570*m*13^(1/2)))/(1+m)/(13-2*13^(1/2))-1/169*(1+4*x)^(
1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13689+13^(1/2)*(2
97+4474*m+1570*m*13^(1/2)))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{1662, 1642, 70\}$

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx =$$
$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}m+4474m+297\right)\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}$$
$$-\frac{\left(\sqrt{13}\left(1570\sqrt{13}m+4474m+297\right)+13689\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{169\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{(844-2355x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}+\frac{9(4x+1)^{m+1}}{4(m+1)}$$

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] (9*(1 + 4*x)^(1 + m))/(4*(1 + m)) + ((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*
(1 - 5*x + 3*x^2)) - ((13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13]*m))*(
1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*S
qrt[13])])/(169*(13 - 2*Sqrt[13])*(1 + m)) - ((13689 + Sqrt[13]*(297 + 4474
*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(169*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
```

```
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m\left(13(4617+3376m)-39(1521+3140m)x-13689x^2\right)}{1-5x+3x^2}\,dx \\
&= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(-4563(1+4x)^m + \frac{\left(-82134-122460m-6\sqrt{13}(297+4474m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-82134-122460m+6\sqrt{13}(297+4474m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx \\
&= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\left(-82134-122460m+6\sqrt{13}(297+4474m)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{507}\left(82134+122460m+6\sqrt{13}(297+4474m)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx \\
&= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(1+m)} - \frac{\left(13689+\sqrt{13}\left(297+4474m+1570\sqrt{13}m\right)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{169\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.24

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= \frac{(1+4x)^{1+m}\left(\frac{13689}{4+4m}+\frac{39(844-2355x)}{1-5x+3x^2}-\frac{1053\left(-117+128\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{1053\left(117+128\sqrt{13}\right)\text{H}\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(13689/(4 + 4*m) + (39*(844 - 2355*x))/(1 - 5*x + 3*x^2)
 - (1053*(-117 + 128*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x
)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (1053*(117 + 128*Sqrt[
13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13
 + 2*Sqrt[13])*(1 + m)) - (-((-14679*(2 + Sqrt[13]) + 2*(-5731 + 667*Sqrt[1
3])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])]) +
(-14679*(-2 + Sqrt[13]) + 2*(5731 + 667*Sqrt[13])*m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(1 + m)))/1521
```

Maple [F]

$$\int \frac{(2+3x)^4\,(1+4x)^m}{(3x^2-5x+1)^2}dx$$

```
[In] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*x
^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^4\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

Giac [F]

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^4}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^4\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.938 $\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(209-426x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{\left(1521+\sqrt{13}(1701-1168m+568\sqrt{13}m)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(1+m)} + \frac{\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 1/39*(209-426*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/338*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(-1521-7384*m+(1701-1168*m)*13^(1
/2))/(1+m)/(13+2*13^(1/2))-1/338*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(
1+4*x)/(13-2*13^(1/2)))*(1521+13^(1/2)*(1701-1168*m+568*m*13^(1/2)))/(1+m)/
(13-2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{1662, 844, 70\}$

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = -\frac{\left(\sqrt{13}(568\sqrt{13}m-1168m+1701)+1521\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)} + \frac{\left(\sqrt{13}(1701-1168m)-13(568m+117)\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(m+1)} + \frac{(209-426x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((1521 + Sqrt[13
]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(338*(13 - 2*Sqrt[13])*(1 +
 m)) + ((Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(338*(13
+ 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
```

```
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(1143+836m)-39(117+568m)x)}{1-5x+3x^2}\,dx \\
&= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-39(117+568m)-3\sqrt{13}(-1701+1168m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-39(117+568m)+3\sqrt{13}(-1701+1168m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx \\
&= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{169}\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx \\
&\quad + \frac{1}{169}\left(\sqrt{13}(1701-1168m)+13(117+568m)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx \\
&= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(\sqrt{13}(1701-1168m)+13(117+568m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.39

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(1+4x)^{1+m}\left(\frac{5434-11076x}{1-5x+3x^2} - \frac{351\left(-13+27\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{12\left(\sqrt{13}(1215-292m)+1846m\right)\text{Hyp}}{\left(13-2\right.}\right.}{}$$

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5434 - 11076*x)/(1 - 5*x + 3*x^2) - (351*(-13 + 27*Sqr
t[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((
-13 + 2*Sqrt[13])*(1 + m)) - (12*(Sqrt[13]*(1215 - 292*m) + 1846*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 - 2*Sqrt[13
])*(1 + m)) - (351*(13 + 27*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (12*(Sqrt[13]*(1
215 - 292*m) - 1846*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 +
2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/1014
```

Maple [F]

$$\int \frac{(2+3x)^3\,(1+4x)^m}{(3x^2-5x+1)^2}dx$$

```
[In] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2
 - 10*x + 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^3\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

Giac [F]

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^3}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^3\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.939 $\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 179

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2} dx = \frac{(61-87x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{2(153-(23-29\sqrt{13})m)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}(13-2\sqrt{13})(1+m)} + \frac{2(153-(23+29\sqrt{13})m)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}(13+2\sqrt{13})(1+m)}$$

```
[Out] 1/39*(61-87*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-2/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(153-m*(23-29*13^(1/2)))/(1+m)/(13-
2*13^(1/2))*13^(1/2)+2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)
/(13+2*13^(1/2)))*(153-m*(23+29*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1662, 844, 70}

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= -\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} + \frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} + \frac{(61-87x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(153 - (23 - 29
*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(153
 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(26(153+122m)-4524mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-4524m-12\sqrt{13}(-153+23m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-4524m+12\sqrt{13}(-153+23m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} + \frac{\left(4\left(153-\left(23-29\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{13\sqrt{13}} - \frac{\left(4\left(153-\left(23+29\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{13\sqrt{13}}$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{1}{507}(1+4x)^{1+m}\left(\frac{793-1131x}{1-5x+3x^2} - \frac{6\left(-153\sqrt{13}+\left(-377+23\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{6\left(-153\sqrt{13}+\left(377+23\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
[Out] ((1 + 4*x)^(1 + m)*((793 - 1131*x)/(1 - 5*x + 3*x^2) - (6*(-153*Sqrt[13] +
(-377 + 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 -
 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (6*(-153*Sqrt[13] + (377 + 23
*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13
])])/((13 + 2*Sqrt[13])*(1 + m))))/507
```

Maple [F]

$$\int \frac{(2+3x)^2\,(1+4x)^m}{(3x^2-5x+1)^2}dx$$

```
[In] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
[Out] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1
), x)
```

Sympy [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)^2\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

Giac [F]

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)^2}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)^2\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.940 $\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2} \, dx = \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{\left(81+2\left(5+7\sqrt{13}\right)m\right)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{\left(81+2\left(5-7\sqrt{13}\right)m\right)(1+4x)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 1/39*(20-21*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+2*m*(5-7*13^(1/2)))/(1+m)*13^(1
/2)/(13+2*13^(1/2))-1/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(81+2*m*(5+7*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {836, 844, 70}

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= -\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$
$$+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{(20-21x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((81 + 2*(5 + 7*Sq
rt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x
))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + ((81 + 2*(
5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\text{integral} = \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{1}{507}\int \frac{(1+4x)^m(13(81+80m)-1092mx)}{1-5x+3x^2}\,dx$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{1}{507}\int \left(\frac{\left(-1092m+6\sqrt{13}(81+10m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-1092m-6\sqrt{13}(81+10m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} + \frac{1}{169}\left(2\left(182m+\sqrt{13}(81+10m)\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx - \frac{1}{507}\left(-1092m+6\sqrt{13}(81+10m)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)} - \frac{\left(182m+\sqrt{13}(81+10m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(1+m)} + \frac{\left(81+2\left(5-7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{1}{507}(1+4x)^{1+m}\left(\frac{260-273x}{1-5x+3x^2} + \frac{3\left(182m+\sqrt{13}(81+10m)\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} - \frac{3\left(182m-\sqrt{13}(81+10m)\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((260 - 273*x)/(1 - 5*x + 3*x^2) + (3*(182*m + Sqrt[13]*
(81 + 10*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13]
)])/((-13 + 2*Sqrt[13])*(1 + m)) - (3*(182*m - Sqrt[13]*(81 + 10*m))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[1
3])*(1 + m))))/507
```

Maple [F]

$$\int \frac{(2+3x)\,(1+4x)^m}{(3x^2-5x+1)^2}dx$$

```
[In] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
[Out] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m*(3*x + 2)/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3x+2)\,(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

Giac [F]

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m(3\,x+2)}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(3\,x+2)\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.941 $\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{2\left(9+2\left(2-\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 1/39*(7-6*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+2/169*(1+4*x)^(1+m)*hypergeom([1,
1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(9+2*m*(2-13^(1/2)))/(1+m)*13^(1/2)/(
13+2*13^(1/2))-2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2
*13^(1/2)))*(9+2*m*(2+13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

= {754, 844, 70}

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx$$
$$= -\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$
$$+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{(7-6x)(4x+1)^{m+1}}{39\,(3x^2-5x+1)}$$

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(9 + 2*(2 + Sqrt[
13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/
(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(9 + 2*(2
- Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \frac{(1+4x)^m(26(9+14m)-312mx)}{1-5x+3x^2}\,dx \\
&= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \left(\frac{\left(-312m+12\sqrt{13}(9+4m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-312m-12\sqrt{13}(9+4m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx \\
&= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{\left(4\left(9+2\left(2-\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{13\sqrt{13}} + \frac{\left(4\left(9+2\left(2+\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{13\sqrt{13}} \\
&= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{2\left(9+2\left(2-\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx \\
&= \frac{1}{507}(1+4x)^{1+m}\left(\frac{91-78x}{1-5x+3x^2} + \frac{6\left(26m+\sqrt{13}(9+4m)\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)} + \frac{6\sqrt{13}\left(9-2\left(-2+\sqrt{13}\right)m\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)
\end{aligned}
$$

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
[Out] ((1 + 4*x)^(1 + m)*((91 - 78*x)/(1 - 5*x + 3*x^2) + (6*(26*m + Sqrt[13]*(9
+ 4*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/(
(-13 + 2*Sqrt[13])*(1 + m)) + (6*Sqrt[13]*(9 - 2*(-2 + Sqrt[13])*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13
])*(1 + m))))/507
```

Maple [F]

$$\int \frac{(1+4x)^m}{(3x^2-5x+1)^2}dx$$

```
[In] int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
[Out] int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2,x)
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.942 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 340

$$\begin{aligned}\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx &= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} \\ &+ \frac{9(1+4x)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1445(1+m)} \\ &+ \frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} \\ &- \frac{\left(81+\left(62+22\sqrt{13}\right)m\right)(1+4x)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} \\ &+ \frac{9\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} \\ &+ \frac{\left(81+\left(62-22\sqrt{13}\right)m\right)(1+4x)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

```
[Out] 1/663*(43-33*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+9/1445*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[
2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/287
3*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+m*(
62-22*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/7514*(1+4*x)^(1+m)*hyperg
eom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*1
3^(1/2))-1/2873*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(
1/2)))*(81+m*(62+22*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 70, 836, 844}

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$$
$$= \frac{9(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{1445(m+1)} - \frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} + \frac{9\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)} + \frac{\left(\left(62-22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} + \frac{9\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)} + \frac{(43-33x)(4x+1)^{m+1}}{663\left(3x^2-5x+1\right)}$$

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m)) +
(9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (
3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((81 +
(62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m))
+ (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m)) + ((81
+ (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{9(1+4x)^m}{289(2+3x)}+\frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)^2}-\frac{3(-7+3x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right)\,dx\\
&= -\left(\frac{3}{289}\int\frac{(-7+3x)(1+4x)^m}{1-5x+3x^2}\,dx\right)+\frac{9}{289}\int\frac{(1+4x)^m}{2+3x}\,dx+\frac{1}{17}\int\frac{(7-3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx\\
&= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)}+\frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}\\
&\quad-\frac{\int\frac{(1+4x)^m(13(81+172m)-1716mx)}{1-5x+3x^2}\,dx}{8619}\\
&\quad-\frac{3}{289}\int\left(\frac{\left(3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)\,dx
\end{aligned}
$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} - \frac{\int \left(\frac{\left(-1716m+6\sqrt{13}(81+62m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-1716m-6\sqrt{13}(81+62m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right) dx}{8619} - \frac{\left(9\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{3757} - \frac{\left(9\left(13+9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{3757}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} + \frac{9\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} - \frac{\left(2\left(81+\left(62-22\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{221\sqrt{13}} + \frac{\left(2\left(81+\left(62+22\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{221\sqrt{13}}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(1+m)} - \frac{\left(81+\left(62+22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{9\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(1+m)} + \frac{\left(81+\left(62-22\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.81

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \frac{(1+4x)^{1+m}\left(\frac{2210(43-33x)}{1-5x+3x^2}+\frac{9126\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1+m}+\frac{1755\left(13+9\sqrt{13}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\ldots\right)}{\left(13-2\sqrt{13}\right)(1+m)}\ldots\right)}{\ldots}$$

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((2210*(43 - 33*x))/(1 - 5*x + 3*x^2) + (9126*Hypergeome
tric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (1755*(13 + 9*Sqrt[13
])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 -
 2*Sqrt[13])*(1 + m)) + (1755*(13 - 9*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (510*S
qrt[13]*(((81 + (62 + 22*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])])/(-13 + 2*Sqrt[13]) + ((81 + (62 - 22*Sqrt[13])
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(13 +
 2*Sqrt[13])))/(1 + m)))/1465230
```

Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)\left(3x^2-5x+1\right)^2}dx$$

```
[In] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)^2(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(27*x^5 - 72*x^4 + 33*x^3 + 32*x^2 - 17*x + 2), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)^2}\,dx = \int \frac{(4x+1)^m}{(3x+2)\,(3x^2-5x+1)^2}\,dx$$

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)\,(1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)\,(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2),x)
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2), x)
```

3.943 $\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 376

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2}\,dx = \frac{(268-195x)(1+4x)^{1+m}}{11271(1-5x+3x^2)} + \frac{162(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{24565(1+m)} + \frac{9\left(117+64\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(1+m)} - \frac{\left(423+2\left(211+65\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{9\left(117-64\sqrt{13}\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(1+m)} + \frac{\left(423+\left(422-130\sqrt{13}\right)m\right)(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)} + \frac{36(1+4x)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{7225(1+m)}$$

```
[Out] 1/11271*(268-195*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+162/24565*(1+4*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+36/7225*(1+4*x)^(1+m)*hypergeom([2
, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/63869*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-64*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/48
841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(423+
m*(422-130*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/63869*(1+4*x)^(1+m)*
hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(117+64*13^(1/2))/(1+m)
```

```
/(13-2*13^(1/2))-1/48841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(
13-2*13^(1/2)))*(423+2*m*(211+65*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 70, 836, 844}

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2}\,dx$$
$$= \frac{162(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{24565(m+1)}$$
$$-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$
$$+\frac{9\left(117+64\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(m+1)}$$
$$+\frac{\left(\left(422-130\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{9\left(117-64\sqrt{13}\right)(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(m+1)}$$
$$+\frac{36(4x+1)^{m+1}\,\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{3}{5}(4x+1)\right)}{7225(m+1)}$$
$$+\frac{(268-195x)(4x+1)^{m+1}}{11271\left(3x^2-5x+1\right)}$$

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1 +
 m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 + m))
 - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 - 2*Sqrt
[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*Sqrt[13]
)*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sqrt[13]*(13
 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m,
 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{9(1+4x)^m}{289(2+3x)^2} + \frac{162(1+4x)^m}{4913(2+3x)} + \frac{(46-27x)(1+4x)^m}{289\,(1-5x+3x^2)^2} - \frac{3(1+4x)^m(-109+54x)}{4913\,(1-5x+3x^2)}\right) dx$$

$$
\begin{aligned}
&= -\frac{3\int \frac{(1+4x)^m(-109+54x)}{1-5x+3x^2}\,dx}{4913} + \frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx \\
&\quad + \frac{9}{289}\int \frac{(1+4x)^m}{(2+3x)^2}\,dx + \frac{162\int \frac{(1+4x)^m}{2+3x}\,dx}{4913} \\
&= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)} \\
&\quad + \frac{36(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{7225(1+m)} \\
&\quad - \frac{\int \frac{(1+4x)^m(13(423+1072m)-10140mx)}{1-5x+3x^2}\,dx}{146523} \\
&\quad - \frac{3\int \left(\frac{\left(54-\frac{384}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(54+\frac{384}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx}{4913} \\
&= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)} \\
&\quad + \frac{36(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{7225(1+m)} \\
&\quad - \frac{\int \left(\frac{\left(-10140m+6\sqrt{13}(423+422m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-10140m-6\sqrt{13}(423+422m)\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx}{146523} \\
&\quad - \frac{\left(18\left(117-64\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{63869} - \frac{\left(18\left(117+64\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{63869} \\
&= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)} \\
&\quad + \frac{9\left(117+64\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{9\left(117-64\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(1+m)} \\
&\quad + \frac{36(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{7225(1+m)} \\
&\quad - \frac{\left(2\left(423+\left(422-130\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{3757\sqrt{13}} \\
&\quad + \frac{\left(2\left(1690m+\sqrt{13}(423+422m)\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{48841}
\end{aligned}
$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271(1-5x+3x^2)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}$$
$$+ \frac{9(117+64\sqrt{13})(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{63869(13-2\sqrt{13})(1+m)}$$
$$- \frac{(1690m+\sqrt{13}(423+422m))(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{48841(13-2\sqrt{13})(1+m)}$$
$$+ \frac{9(117-64\sqrt{13})(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{63869(13+2\sqrt{13})(1+m)}$$
$$+ \frac{(423+(422-130\sqrt{13})m)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}(13+2\sqrt{13})(1+m)}$$
$$+ \frac{36(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{7225(1+m)}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2}\,dx$$
$$= \frac{(1+4x)^{1+m}\left(\frac{16575(268-195x)}{1-5x+3x^2} + \frac{1232010\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{3}{5}(1+4x)\right)}{1+m} + \frac{26325(117+64\sqrt{13})\,\text{Hypergeometric2F1}\left(1\right.}{(13-2\sqrt{13})(1+m)}\right.}{}$$

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((16575*(268 - 195*x))/(1 - 5*x + 3*x^2) + (1232010*Hype
rgeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (26325*(117 + 6
4*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
])/((13 - 2*Sqrt[13])*(1 + m)) + (26325*(117 - 64*Sqrt[13])*Hypergeometric2
F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 +
m)) - (425*((423*(2 + Sqrt[13]) + (2534 + 682*Sqrt[13])*m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (-423*(-2 + Sqrt[13]) +
(2534 - 682*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
+ 2*Sqrt[13])]))/(1 + m) + (930852*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(
1 + 4*x))/5])/(1 + m)))/186816825
```

Maple [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (3x^2-5x+1)^2} dx$$

```
[In] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} dx = \int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)^2} dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(81*x^6 - 162*x^5 - 45*x^4 + 162*x^3 + 13*x^2 - 28*x +
 4), x)
```

Sympy [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} dx = \int \frac{(4x+1)^m}{(3x+2)^2 (3x^2-5x+1)^2} dx$$

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)**2), x)
```

Maxima [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2} dx = \int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)^2} dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

Giac [F]

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x^2-5\,x+1)^2(3\,x+2)^2}\,dx$$

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x)^m}{(2+3x)^2 (1-5x+3x^2)^2}\,dx = \int \frac{(4\,x+1)^m}{(3\,x+2)^2\,(3\,x^2-5\,x+1)^2}\,dx$$

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2),x)
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2), x)
```

3.944 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 237

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} + \frac{2(c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))-ef(3+2m)(aef(1+2m)+b(df-2e^2(1+m))))(d+}{ef^3(e^2-df)(3+2m)}$$

```
[Out] 2*(a+e*(-b*f+c*e)/f^2)*(e*x+d)^(1+m)/(-d*f+e^2)/(f*x+e)^(1/2)+2*c*(e*x+d)^(
1+m)*(f*x+e)^(1/2)/e/f^2/(3+2*m)+2*(c*(d^2*f^2+4*d*e^2*f*(1+m)-4*e^4*(m^2+3
*m+2))-e*f*(3+2*m)*(a*e*f*(1+2*m)+b*(d*f-2*e^2*(1+m))))*(e*x+d)^m*hypergeom
([1/2, -m],[3/2],e*(f*x+e)/(-d*f+e^2))*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)/(3+2*
m)/((-f*(e*x+d)/(-d*f+e^2))^m)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {963, 81, 72, 71}

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = -\frac{2\sqrt{e+fx}(d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{e(e+fx)}{e^2-df}\right)\left(f(aef(2m+1)+bdf-2be^2\right.}{f^3(e^2-df)} + \frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{2c\sqrt{e+fx}(d+ex)^{m+1}}{ef^2(2m+3)}$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x])
 + (2*c*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*f^2*(3 + 2*m)) - (2*(f*(b*d*f -
 2*b*e^2*(1 + m) + a*e*f*(1 + 2*m)) - (c*(d^2*f^2 + 4*d*e^2*f*(1 + m) - 4*e
^4*(2 + 3*m + m^2)))/(e*(3 + 2*m)))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometri
c2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(f^3*(e^2 - d*f)*(-((f*(d +
e*x))/(e^2 - d*f)))^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2\int \frac{(d+ex)^m\left(\frac{c\left(def-2e^3(1+m)\right)-f\left(bdf-2be^2(1+m)+aef(1+2m)\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(f(bdf-2be^2(1+m)+aef(1+2m)) - \frac{c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))}{e(3+2m)}\right)\int \frac{(d+ex)^m}{\sqrt{e+fx}}\,dx}{f^2(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(\left(f(bdf-2be^2(1+m)+aef(1+2m)) - \frac{c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))}{e(3+2m)}\right)(d+ex)^m\left(\frac{f(d+ex)}{-e^2+df}\right.\right.}{f^2(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{2\left(f(bdf-2be^2(1+m)+aef(1+2m)) - \frac{c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))}{e(3+2m)}\right)(d+ex)^m\left(-\frac{f(d+ex}{e^2-df}\right.}{f^3(e^2-df)}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2(d+ex)^m\left(\frac{f(d+ex)}{-e^2+df}\right)^{-m}\left(-3(ce^2+f(-be+af))\,\text{Hypergeometric2F1}\left(-\right.\right.}{}$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-1/2, -m, 1
/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*b*f)*Hypergeometric2
F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] - c*(e + f*x)*Hypergeometric2F1
[3/2, -m, 5/2, (e*(e + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 +
d*f))^m*Sqrt[e + f*x])
```

Maple [F]

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{(fx+e)^{\frac{3}{2}}}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*sqrt(f*x + e)*(e*x + d)^m/(f^2*x^2 + 2*e*f*x + e
^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(e+f\,x)^{3/2}} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.945 $\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 509

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)}$$

$$+\frac{\left(e(bd-ae)g^2(1+m)+c(3d^2g^2+e^2f^2(4+m)-2defg(4+m))\right)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}}{ce^3(1+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c}{2cd-\left(b+\right.}}}$$

$$-\frac{g(beg(5+2m)+2c(3dg-2ef(4+m)))(d+ex)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\right.}{2ce^3(2+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/e/(4+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
3*d^2*g^2+e^2*f^2*(4+m)-2*d*e*f*g*(4+m)))*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-
1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3/(1+m)/(4+m)/(1-2*c*(e*x+d
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-1/2*g*(b*e*g*(5+2*m)+2*c*(3*d*g-2*e*f*(4+m)))*(e*x+d)
^(2+m)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3
/(2+m)/(4+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1667, 857, 773, 138}

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd\cdots\right.}{ce^2(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$-\frac{g\sqrt{a+bx+cx^2}(d+ex)^{m+2}(beg(2m+5)+6cdg-4cef(m+4))\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\cdots\right.}{2ce^3(m+2)(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{g^2(a+bx+cx^2)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + (((b*d - a*
e)*g^2 + (c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m))))/(e*(1 + m)))
*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(4 + m)*Sqrt[1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]) - (g*(6*c*d*g - 4*c*e*f*(4 + m) + b*e*g*(5 + 2*m))*(
d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (
2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(d +
e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
```

```
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\int (d+ex)^m \left(\frac{1}{2}e(2cef^2(4+m)-g^2(3bd+2ae(1+m))) - \frac{1}{2}eg(6cdg-4cef(4+m)+beg(5+2m))x\right) \sqrt{}}{ce^2(4+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{(g(6cdg-4cef(4+m)+beg(5+2m)))\int (d+ex)^{1+m}\sqrt{a+bx+cx^2}\,dx}{2ce^2(4+m)} + \frac{(e(bd-ae)g^2(1+m)+c(3d^2g^2+e^2f^2(4+m)-2defg(4+m)))\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx}{ce^2(4+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)}$$

$$- \frac{\left(g(6cdg-4cef(4+m)+beg(5+2m))\sqrt{a+bx+cx^2}\right)\operatorname{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{\right.}{2ce^3(4+m)\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$+ \frac{\left((e(bd-ae)g^2(1+m)+c(3d^2g^2+e^2f^2(4+m)-2defg(4+m)))\sqrt{a+bx+cx^2}\right)\operatorname{Subst}\left(\int x\right.}{ce^3(4+m)\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{}{d-\frac{(b-}{}}}}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)}$$

$$+ \frac{(e(bd-ae)g^2(1+m)+c(3d^2g^2+e^2f^2(4+m)-2defg(4+m)))(d+ex)^{1+m}\sqrt{a+bx+cx^2}F}{ce^3(1+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2}}}$$

$$- \frac{g(6cdg-4cef(4+m)+beg(5+2m))(d+ex)^{2+m}\sqrt{a+bx+cx^2}F_1\left(2+m;-\frac{1}{2},-\frac{1}{2};3+m;\frac{}{2c}\right.}{2ce^3(2+m)(4+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)}}}$$

Mathematica [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2], x]
```

Maple [F]

$$\int (ex+d)^m\,(gx+f)^2\,\sqrt{c\,x^2+bx+a}dx$$

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m\,(f+gx)^2\,\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)^2(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \int (f+g\,x)^2\,(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.946 $\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 388

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}} + \frac{g(d+ex)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(2+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+
b*x+a)^(1/2)/e^2/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/
2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(e*x+d)^(2+m)*A
ppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/e^2/(2+m)/(1-
2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {857, 773, 138}

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$
$$=\frac{\sqrt{a+bx+cx^2}(ef-dg)(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{g\sqrt{a+bx+cx^2}(d+ex)^{m+2}\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+2)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[In] Int[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
-1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^
2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{g\int (d+ex)^{1+m}\sqrt{a+bx+cx^2}\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx}{e}$$

$$= \frac{\left(g\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,dx,x,d+ex\right)}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}} + \frac{\left((ef-dg)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,dx,x,d+e\right.}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}} + \frac{g(d+ex)^{2+m}\sqrt{a+bx+cx^2}F_1\left(2+m;-\frac{1}{2},-\frac{1}{2};3+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(2+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]
```

Maple [F]

$$\int (ex+d)^m (gx+f)\sqrt{c\,x^2+bx+a}dx$$

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m\,(f+gx)\,\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \int (f+g\,x)\,(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.947 $\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx = \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2
)/e/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*
x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {773, 138}

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}\,\text{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[In] Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

integral

$$= \frac{\sqrt{a+bx+cx^2}\,\mathrm{Subst}\left(\int x^m\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,dx,x,d+ex\right)}{e\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+x(b+cx)}\,\mathrm{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[In] Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

Maple [F]

$$\int (ex+d)^m \sqrt{c\,x^2+bx+a}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \int (d+ex)^m\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\int (d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.948 $\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} \, dx = \operatorname{Int}\left(\frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx}, x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} \, dx = \int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} \, dx$$

```
[In] Int[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Defer[Int][((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

```
[In] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(ex+d)^m\,\sqrt{c\,x^2+bx+a}}{gx+f}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{(d+ex)^m\,\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{cx^2+bx+a}(ex+d)^m}{gx+f}\,dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 12.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{{\left(d+e\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}}{f+g\,x}\,dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x), x)
```

3.949 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 502

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)}$$

$$+\frac{(e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m)-2defg(2+m)))\,(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1}}{ce^3(1+m)(2+m)\sqrt{a+bx}}$$

$$-\frac{g(beg(3+2m)+c(2dg-4ef(2+m)))(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\mathrm{Appell}}{2ce^3(2+m)^2\sqrt{a+bx+cx^2}}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1/2)/c/e/(2+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
d^2*g^2+e^2*f^2*(2+m)-2*d*e*f*g*(2+m)))*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,
2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(1+m)/(2+m)/(c
*x^2+b*x+a)^(1/2)-1/2*g*(b*e*g*(3+2*m)+c*(2*d*g-4*e*f*(2+m)))*(e*x+d)^(2+m)
*AppellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)/c/e^3/(2+m)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1667, 857, 773, 138}

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{ce^2(m+2)\sqrt{a+bx+cx^2}}$$

$$-\frac{g(d+ex)^{m+2}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(beg(2m+3)+2cdg-4cef(m+2))\,\text{Appell}}{2ce^3(m+2)^2\sqrt{a+bx+cx^2}}$$

$$+\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + (((b*d - a*e)
*g^2 + (c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))/(e*(1 + m)))*(d
+ e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[1
+ m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(2 + m)*Sqrt[a
 + b*x + c*x^2]) - (g*(2*c*d*g - 4*c*e*f*(2 + m) + b*e*g*(3 + 2*m))*(d + e*
x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m,
 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)^2*Sqrt[a
 + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
```

```
e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} \\
&\quad + \frac{\int \frac{(d+ex)^m\left(\frac{1}{2}e\left(2cef^2(2+m)-g^2(bd+2ae(1+m))\right)-\frac{1}{2}eg(2cdg-4cef(2+m)+beg(3+2m))x\right)}{\sqrt{a+bx+cx^2}}\,dx}{ce^2(2+m)} \\
&= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{(g(2cdg-4cef(2+m)+beg(3+2m)))\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{2ce^2(2+m)} \\
&\quad + \frac{(e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m)-2defg(2+m)))\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx}{ce^2(2+m)}
\end{aligned}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)}$$

$$- \frac{\left(g(2cdg-4cef(2+m)+beg(3+2m))\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{2ce^3(2+m)\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left((e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m)-2defg(2+m)))\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\right.}{ce^3(2+m)\sqrt{a+bx+}}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)}$$

$$+ \frac{(e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m)-2defg(2+m)))(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex}{2cd-\left(b-\sqrt{b^2-\right.}}}}{ce^3(1+m)(2+m)\sqrt{}}$$

$$- \frac{g(2cdg-4cef(2+m)+beg(3+2m))(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)}}}{2ce^3(2+m)^2\sqrt{a+bx+cx^2}}$$

Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]
```

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)^2}{\sqrt{c\,x^2+bx+a}}dx$$

```
[In] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m\,(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.950 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 388

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(1+m)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(2+m)\sqrt{a+bx+cx^2}}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e
*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^2/(1+m)/(c*x^2+b*x+a)^(1/2)+g*(e*x+d)^(2+m)*App
ellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x
+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^
2/(2+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {857, 773, 138}

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(}{2cd-\left(b-\right.}\right.}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

$$+\frac{g(d+ex)^{m+2}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4a}\right.}\right.}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

```
[In] Int[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{g\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left(g\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^{1+m}}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}\,dx, x,\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$+ \frac{\left((ef-dg)\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^{m}}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-\right.}}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d\cdots}{2cd-\left(b-\sqrt{\cdots}\right.}\right.}{e^2(1+m)\sqrt{a+bx+cx^2}}$$

$$+ \frac{g(d+ex)^{2+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(2+m;\frac{1}{2},\frac{1}{2};3+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)\cdots}\right.}{e^2(2+m)\sqrt{a+bx+cx^2}}$$

Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

[In] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]

[Out] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2], x]

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)}{\sqrt{c\,x^2+bx+a}}dx$$

[In] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)

[Out] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m\,(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

[In] integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((d + e*x)**m*(f + g*x)/sqrt(a + b*x + c*x**2), x)

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)\,(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.951 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(1+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {773, 138}

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e(m+1)\sqrt{a+bx+cx^2}}$$

```
[In] Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
```

```
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

integral

$$= \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int\frac{x^m}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}\,dx,x,d+ex\right)}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10

$$\int\frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}(d+ex)^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F]

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+bx+a}}dx$$

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2), x)
```

3.952 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] Int[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+bx+a}}dx$$

```
[In] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c*g*x^3 + (c*f + b*g)*x^2 + a*f
 + (b*f + a*g)*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

Mupad [N/A]

Not integrable

Time = 73.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.953 $\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 265

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) \, dx = \frac{(beg(3+m+n)-c(ef(2+m)+dg(4+m+2n)))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{(g(2+m+n)\left(ae^2g(3+m+n)-cd(ef(2+m)+dg(1+n))\right)-(ef(1+m)+dg(1+n))(beg(3+m}{e^3g^2(}$$

```
[Out] (b*e*g*(3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n)))*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e^2
/g^2/(2+m+n)/(3+m+n)+c*(e*x+d)^(2+m)*(g*x+f)^(1+n)/e^2/g/(3+m+n)+(g*(2+m+n)
*(a*e^2*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n)))-(e*f*(1+m)+d*g*(1+n))*(b*e*g*(
3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n))))*(e*x+d)^(1+m)*(g*x+f)^n*hypergeom([-n,
 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^3/g^2/(1+m)/(2+m+n)/(3+m+n)/((e*(g*x+f
)/(-d*g+e*f))^n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{965, 81, 72, 71\}$

$$\int (d+ex)^m(f+gx)^n\left(a+bx+cx^2\right)\,dx$$

$$= \frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g\right.\right.}{e^3g^2(m+}$$

$$-\frac{(d+ex)^{m+1}(f+gx)^{n+1}(-beg(m+n+3)+cdg(m+2n+4)+cef(m+2))}{e^2g^2(m+n+2)(m+n+3)}$$

$$+\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n))*(d + e*x)^(1 +
 m)*(f + g*x)^(1 + n))/(e^2*g^2*(2 + m + n)*(3 + m + n))) + (c*(d + e*x)^(2
 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (((e*f*(1 + m) + d*g*(1 + n)
)*(c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n)) + g*(2 + m + n)
*(a*e^2*g*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))))*(d + e*x)^(1 + m)
*(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g
))])/(e^3*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\int (d+ex)^m(f+gx)^n\,(ae^2g(3+m+n) - cd(ef(2+m)+dg(1+n)) - e(cef(2+m) - beg(3+m+}{e^2g(3+m+n)}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\left(ae^2g(3+m+n) - cd(ef(2+m)+dg(1+n)) + \frac{(ef(1+m)+dg(1+n))(cef(2+m)-beg(3+m+n)+cdg(4+m}{g(2+m+n)}\right.}{e^2g(3+m+n)}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\left(\left(ae^2g(3+m+n) - cd(ef(2+m)+dg(1+n)) + \frac{(ef(1+m)+dg(1+n))(cef(2+m)-beg(3+m+n)+cdg(4+}{g(2+m+n)}\right.\right.}{e^2g(3+m+n)}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\left(ae^2g(3+m+n) - cd(ef(2+m)+dg(1+n)) + \frac{(ef(1+m)+dg(1+n))(cef(2+m)-beg(3+m+n)+cdg(4+m}{g(2+m+n)}\right.}{e^3g(1+m)(3+m+}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx$$
$$= \frac{(d+ex)^{1+m}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} \left(c(ef-dg)^2 \text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{g(d+ex)}{-ef+dg}\right)+e\left(\right.\right.}{}$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)
*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e
*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x
))/(-(e*f) + d*g)])))/(e^3*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(c\,x^2+bx+a\right) dx$$

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx = \int \left(cx^2+bx+a\right)(ex+d)^m (gx+f)^n \, dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) \, dx = \int \left(cx^2+bx+a\right)(ex+d)^m(gx+f)^n \, dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) \, dx = \int \left(cx^2+bx+a\right)(ex+d)^m(gx+f)^n \, dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(a+bx+cx^2\right) \, dx = \int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right) \, dx$$

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x)
[Out] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2), x)
```

3.954 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 525

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx=\frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}$$

$$+\frac{(e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+m+2p)))\,(d+ex)^{1+m}\left(a+bx\right.}{}$$

$$-\frac{g(beg(2+m+p)+2c(dg(1+p)-ef(3+m+2p)))(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right.}{ce^3(2+m)(3}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/e/(3+m+2*p)+(e*(-a*e+b*d)*g^2*(1+m)
+c*(2*d^2*g^2*(p+1)+e^2*f^2*(3+m+2*p)-2*d*e*f*g*(3+m+2*p)))*(e*x+d)^(1+m)*(
c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c/e^3/(1+m)/(3+m+2*p)
/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^p)-g*(b*e*g*(2+m+p)+2*c*(d*g*(p+1)-e*f*(3+m+2*
p)))*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c
/e^3/(2+m)/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/(
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1667, 857, 773, 138}

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+1,-p,\cdots\right.}{ce^2(m+2\cdots}$$
$$-\frac{g(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(beg(m+p+2)+2\cdots}{ce^3(m+2)(\cdots}$$
$$+\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + (((
b*d - a*e)*g^2 + (c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p) - 2*d*e*f*g*
(3 + m + 2*p)))/(e*(1 + m)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1
[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(3 + m + 2*p)
*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(2*c*d*g*(1 + p) + b*e*
g*(2 + m + p) - 2*c*e*f*(3 + m + 2*p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^
p*AppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(2
+ m)*(3 + m + 2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
```

```
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\int (d+ex)^m\left(e(cef^2(3+m+2p)-g^2(ae(1+m)+bd(1+p)))-eg(2cdg(1+p)+beg(2+m+p)-2c\right.}{ce^2(3+m+2p)}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{(g(2cdg(1+p)+beg(2+m+p)-2cef(3+m+2p)))\int (d+ex)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{ce^2(3+m+2p)} + \frac{(e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+m+2p)))\int (d+ex)^m}{ce^2(3+m+2p)}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}$$

$$- \frac{\left(g(2cdg(1+p)+beg(2+m+p)-2cef(3+m+2p))\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-}\right.}{ce}$$

$$+ \frac{\left((e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+m+2p)))\left(a+bx\right.\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}$$

$$+ \frac{(e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+m+2p)))(d+ex)^{1+}}{}$$

$$- \frac{g(2cdg(1+p)+beg(2+m+p)-2cef(3+m+2p))(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c}{2cd-\left(b-\right.}\right.}{ce^3(2+}$$

Mathematica [F]

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]
```

Maple [F]

$$\int (ex+d)^m\,(gx+f)^2\left(c\,x^2+bx+a\right)^p dx$$

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^2\,(a+bx+cx^2)^p\;dx = \int (gx+f)^2(cx^2+bx+a)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\,(a+bx+cx^2)^p\;dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^2\,(a+bx+cx^2)^p\;dx = \int (gx+f)^2(cx^2+bx+a)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^2\,(a+bx+cx^2)^p\;dx = \int (gx+f)^2(cx^2+bx+a)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx=\int (f+g\,x)^2\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.955 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 384

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\text{AppellF1}\left(1+\cdots\right.}{e^2(1+m)} + \frac{g(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\text{AppellF1}\left(2+m,-p\cdots\right.}{e^2(2+m)}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))/e^2/(1+m)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c
*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)+g*(e*x+d)^(2+m)*(c*x^2+b*x+a)
^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e^2/(2+m)/((1-2*c*(e*x+d)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {857, 773, 138}

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\operatorname{AppellF1}\left(m\right.}{e^2(m+1)}$$
$$+\frac{g(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\operatorname{AppellF1}\left(m+2,-\right.}{e^2(m+2)}$$

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[
2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{g\int (d+ex)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\left(a+bx+cx^2\right)^p\,dx}{e}$$

$$= \frac{\left(g(a+bx+cx^2)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int x^{1+m}\left(1-\frac{2c}{2cd-\left(b-\sqrt{\right.}\right.}\right.}{e^2}$$

$$+\frac{\left((ef-dg)\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int x^m\left(1-\right.\right.}{e^2}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(1+\right.}{e^2(1+m)}$$

$$+\frac{g(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(2+m;-p\right.}{e^2(2+m)}$$

Mathematica [F]

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$$

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]
```

Maple [F]

$$\int \left(ex+d\right)^{m} \left(gx+f\right) \left(c\,x^{2}+bx+a\right)^{p}dx$$

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx=\int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] integral((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx=\int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \int (gx+f)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \int (f+g\,x)\,\left(d+e\,x\right)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.956 $\int (d+ex)^m (a+bx+cx^2)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 187

$$\int (d+ex)^m (a+bx+cx^2)^p \, dx = \frac{(d+ex)^{1+m} (a+bx+cx^2)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \text{AppellF1}\left(1+m,-p,-\right.}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e/(1+m
)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {773, 138}

$$\int (d+ex)^m (a+bx+cx^2)^p \, dx = \frac{(d+ex)^{m+1} (a+bx+cx^2)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \text{AppellF1}\left(m+1,-p,-\right.}{e(m+1)}$$

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
```

```
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

integral

$$= \frac{\left((a+bx+cx^2)^p \left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right) \operatorname{Subst}\left(\int x^m \left(1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p \left(\right.\right.}{e}$$

$$= \frac{(d+ex)^{1+m} (a+bx+cx^2)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} F_1\left(1+m;-p,-\right.}{e(1+m)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$
$$= \frac{\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} (d+ex)^{1+m}(a+x(b+cx))^p \operatorname{AppellF1}\left(1+m,-p,-p,2+\right.}{e(1+m)}$$

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)]])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F]

$$\int (ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (d+e\,x)^m \left(c\,x^2+b\,x+a\right)^p \, dx$$

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^p,x)
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.957 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}\,dx = \text{Int}\left(\frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx},x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}\,dx = \int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}\,dx$$

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
[Out] Defer[Int][((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx$$

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)^p}{gx+f}dx$$

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{gx+f} \, dx$$

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 12.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^p}{f+g\,x} \, dx$$

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x)
```

3.958 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=-\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e
*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1588, 947, 174, 552, 551}

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=-\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[In] Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] (-2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{1}{c^2}+x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\left(2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx =$$

$$-\frac{2i\sqrt{\frac{e(-1+cx)}{c(d+ex)}}(d+ex)\sqrt{\frac{e+cex}{cd+cex}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right), \frac{cd-e}{cd+e}\right) - \text{EllipticPi}\left(\frac{cd}{cd+e}, i\text{arcsinh}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right), \frac{cd-e}{cd+e}\right)\right)}{d\sqrt{-\frac{cd+e}{c}}\sqrt{1-\frac{1}{c^2x^2}}x}$$

```
[In] Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] ((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
+ c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d -
e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c)]/
Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 - 1/(
c^2*x^2)]*x)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{2(cd-e)\Pi\left(\sqrt{\frac{c(ex+d)}{cd-e}}, \frac{cd-e}{cd}, \sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{-\frac{(cx+1)e}{cd-e}}\sqrt{-\frac{(cx-1)e}{cd+e}}\sqrt{\frac{c(ex+d)}{cd-e}}}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x\sqrt{ex+d}\,cd}$	148

```
[In] int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi((c*(e*x+d)/(c*d-e))^(1/
2),(c*d-e)/c/d,((c*d-e)/(c*d+e))^(1/2))*(-(c*x+1)*e/(c*d-e))^(1/2)*(-(c*x-1
)*e/(c*d+e))^(1/2)*(c*(e*x+d)/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

Fricas [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*c^2*sqrt((c^2*x^2 - 1)/(c^2*x^2))/(c^2*e*x^3 + c^2*d
*x^2 - e*x - d), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\sqrt{d+ex}}\,dx$$

```
[In] integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=\int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx=\int \frac{1}{x^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{d+e\,x}}\,dx$$

```
[In] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)),x)
[Out] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
    return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
  try:
    if expn.parent() is SR:
      return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
      return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
      return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

  except AttributeError as error:
    print("Exception,AttributeError in  is_atom")
    print ("cought exception" , type(error).__name__ )
    return False


def expnType(expn):

  if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
    return 1
  elif type(expn)==list:   #isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
      return 1
    else:
      return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
    if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
      return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
      if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```